An Anomalous Measurement of $\Delta m_{31}{}^2$ from Neutrino Oscillations at the Daya Bay Reactor Neutrino Experiment

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Abstract:

In 2012, the collaboration overseeing the Daya Bay Reactor Neutrino Experiment announced results which determined the magnitude of the mixing angle θ_{13} with unprecedented precision. However, no attempt was made in the collaboration's publications to predict the value of the most relevant mass-squared difference to the observed oscillation, $\Delta m_{31}{}^2$. This paper presents the results of an analysis which suggests that the Daya Bay data prefers a value of $\Delta m_{31}{}^2$ which is far greater than its presently recognized value. Specifically, it is found that Daya Bay predicts $\Delta m_{31}{}^2 = 3.53^{+.74}_{-1.07} \times 10^{-3} eV^2$, where the cited uncertainties correspond to the 99% confidence bounds. This measurement excludes the most precise current measurement of $\Delta m_{31}{}^2$, the MINOS result, at a 99% confidence level and is in turn excluded by the MINOS data at a 10σ level. The possibility that sterile neutrino effects are the cause of this anomalous result is considered and used to suggest further work.

I. Introduction

A. Motivation and Context

Within the past two decades, the theory of neutrino oscillations has achieved wide acceptance as experimental evidence has rapidly accumulated. Neutrino oscillations predict that a neutrino which originates in a definite flavor state can be detected in a different flavor state after propagation from its origin [1,2]. The central tenets of oscillation theory are that neutrinos are massive particles and that the flavor eigenstates (ν_e, ν_μ, ν_τ) of neutrinos are not identical to mass eigenstates (ν_1, ν_2, ν_3). In this framework, a neutrino of definite flavor can be expressed as a linear combination of the three mass eigenstates. All neutrinos are, of course, created with a definite flavor which conserves lepton number for the reaction in which they were produced (e.g. electron antineutrinos created in beta decay). However, the fact that a flavor state has no definite mass requires that the propagation of the particle be conducted through the mass eigenstates of which it is composed. It is the latter states which represent the physical particles which traverse space, and the relative difference in the phase of these states--induced by the mass differences and changed by propagation--allows flavor oscillations to occur in traveling neutrinos.

A thorough comprehension of the oscillation process requires an understanding of two criteria: the combination of mass states that expresses each flavor state and the differences between the squared mass eigenvalues ($\Delta m_{ij}{}^2 = m_i{}^2 - m_j{}^2$). In the case of the former, knowledge is enshrined in the Maki-Nakagawa-Sakata (MNS) mixing matrix, a unitary matrix which provides a transformation between the mass and flavor bases [2]. The MNS matrix is parameterized using three mixing angles (θ_{12},θ_{13},θ_{23}) and a charge-parity (CP) violation phase (δ) [3]. For the mass differences, there are only two independent parameters ($\Delta m_{21}{}^2$,$\Delta m_{31}{}^2$) from which the remaining difference ($\Delta m_{32}{}^2$) can be calculated. The absolute hierarchy of the neutrino eigenstates remains an open question [4]. The values of these parameters are deduced from experiments of several different types, with significant work remaining to obtain precision measurements.

The results of the Daya Bay Reactor Neutrino Experiment represent a major advancement of the cause of precision. In March 2012, the collaboration overseeing the experiment reported a measurement of $sin^2 2\theta_{13} = 0.092^{+.017}_{-.017}$ [5]; a subsequent update [6] of the data has since revised the measurement to $sin^2 2\theta_{13} = 0.089^{+.011}_{-.011}$. These results represent a sizable improvement over previous determinations of θ_{13}, which were frequently unable to exclude the no oscillation case because of large errors [7]. The primary innovation of the Daya Bay experiment is its scale. Where oscillation experiments have traditionally been limited by small sample sizes, the Daya Bay data is quickly converging to small statistical errors due to its rapid accumulation of neutrino events, a fact attributable to the sizable antineutrino flux employed by the experiment. Combined with low systematic uncertainties in the experiment's detectors, this diminishing statistical error distinguishes the Daya Bay measurement of $sin^2 2\theta_{13}$ as the most precise to date.

Despite--or rather because of--this success, the current use of Daya Bay purely as a measure of θ_{13} neglects the full potential of the apparatus. In presenting their results, the collaboration adopted a fixed value for all parameters except θ_{13}, a step which ignores the remaining uncertainty in those values. In particular, the oscillation event observed at Daya Bay is sensitive to the value of $\Delta m_{31}{}^2$, which has not been examined in the context of the experiment's data. This paper attempts to remedy the absence of such a prediction by presenting a statistical fit to the Daya Bay results with two free parameters, θ_{13} and $\Delta m_{31}{}^2$. Its contents shall demonstrate that not only does the Daya Bay data suffice to predict the value of $\Delta m_{31}{}^2$ but also that the predicted value is not statistically compatible with the fixed mass difference used by the collaboration at the 99% confidence level.

B. Outline of Presented Work

The work presented in this paper is partitioned into three sections, which recapitulate the process of generating and reviewing the Daya Bay analysis. Section II shall provide a description of the method used to fit parameters to the data published by the collaboration, pointing out and justifying assumptions that underpin the procedure. Next, Section III shall present the results obtained from the analysis and illustrate their anomalous nature in comparison with previously established values. Finally, in Section IV, some attempt will be made to reconcile the aberration with the accepted values of parameters through the mediating effect of sterile neutrinos.

Section II. Exposition of Analysis Techniques

The Daya Bay experiment is sited on the southern Chinese coast, near a complex of six identical nuclear reactors. Four of these reactors are located in a set of neighboring power plants known collectively as Ling-Ao, while the remaining pair resides to the south in the Daya Bay power plant. These reactors act as the experiment's antineutrino source, emitting electron antineutrinos through the beta decay of fission products. Detection of the antineutrinos is accomplished using a set of six identically constructed detectors designed to discern inverse beta decay events. Three of the detectors are placed near the reactor complexes, while the remaining three are located at a further distance expected to be near the oscillation minimum. In the context of the experiment, the relevant distances are the flux weighted baselines of the detectors, which measure the average distance an emitted antineutrino travels to reach each detector. For the near detectors, these distances are approximately 500 m, while the far detectors share a weighted baseline of 1628 m.

For the purpose of the experiment, the detectors in close proximity to the reactors act as a measure of the activity rate to be expected at the "far" detection site. After accounting for a decrease in flux due to an increased baseline, any deficit in the antineutrino detection rate at the far site is due purely to oscillation effects, as flavors other than electron are not registered by the detector. After the flux correction, it is expected that the detection rates at the three sites shall be equivalent if no change in flavor composition has occurred during the antineutrinos' propagation. However, the Daya Bay collaboration has reported a deficit at the far neutrino detectors; the ratio of the rate observed at the far hall to that observed in the near halls is $R = .944 \pm .008$ [6]. From this difference in the detection rates, we may infer both the presence of neutrino oscillations and the values of the associated oscillation parameters.

Oscillation probabilities for the vacuum case (neglecting electroweak interactions) may be obtained directly from the MNS matrix and propagation of the mass eigenstates[8]. However, over the relatively short baseline of the Daya Bay experiment, approximations were used by the collaboration to simplify the probability formula. Their expression for the probability of an electron antineutrino of energy E being detected in the electron flavor state at a distance L from its origin is

$$P_{\bar{\nu}_e \to \bar{\nu}_e} = 1 - \sin^2 2\theta_{13} \sin^2 \frac{1.267\, \Delta {m_{31}}^2}{E} L + \cos^2 \theta_{13} \sin^2 2\theta_{12} \sin^2 \frac{1.267\, \Delta {m_{21}}^2}{E} L$$

This approximate formula is quite reasonable given the range of the experiment and is observed to depart at most by $.0007\%$ from the exact probability at the best fit parameters.

In considering the above formula, we note that previous experimental evidence has placed $\Delta {m_{31}}^2$ as two orders of magnitude greater than $\Delta {m_{21}}^2$ [7]. The average energy of the antineutrinos emitted by the reactors based on near detector events is 4.3 MeV. Using the accepted values of the mass differences, the characteristic lengths for the two sine terms in the above equations can be calculated as 1.4 km and 44 km, respectively. Oscillations observed by Daya Bay are therefore almost exclusively dependent upon the first sine term in the formula and are most sensitive to the values of θ_{13} and $\Delta {m_{31}}^2$.

The approach utilized here for fitting these parameters is a straightforward χ^2 fit to the data given in figure 24 of reference [6], which reports the ratio of far hall events to near hall events after background subtraction and a correction for flux difference between the sites. The form of the chi-squared used is

$$\chi^2 = \sum_i \frac{\left(R_{exp}^i - R_{th}^i\right)^2}{{\sigma_i}^2}$$

where R_{exp}^i is the experimentally observed ratio over a given energy bin, R_{th}^i is the theoretically predicted ratio over that bin, and σ_i is the error on the observed ratio. Although ratios served as the data points considered in the χ^2 fit, it is important to note that the total number of events is represented in the errors on each ratio, which were calculated by using the absolute number of counts shown in the upper portion of the figure.

While the form of the χ^2 is fairly simple, the true subtlety in the analysis emerges in the prediction of the event ratio for various parameter values. A direct calculation of the expected reaction rate at the far detector requires knowledge of four factors: the antineutrino flux (Φ), the inverse beta decay cross section (σ), the detector efficiency (ε), and the oscillation probability ($P_{\overline{\nu}_e \to \overline{\nu}_e}$). It is expected that each factor shall vary depending on the antineutrino energy, so a calculation of the expected rate in one energy bin necessitates an integral,

$$\int_{\mathrm{E}}^{\mathrm{E}+\Delta\mathrm{E}} \Phi \varepsilon \sigma P_{\overline{\nu}_e \to \overline{\nu}_e} dE$$

This integral is inconvenient to evaluate, primarily due to the difficulty in determining the proper flux and detector efficiencies from publications. Ambiguity surrounding these experimental constraints is, in fact, a problem manifest in many reports of oscillation experiments and can constitute a formidable obstacle to parameter fits. The case of Daya Bay instantiates this problem, as the absolute magnitude of any reactor flux is not mentioned by the collaboration; instead, their publication indicates that a best-fit normalization was used to properly determine the flux. This lack of information makes a direct calculation of presented integral impractical.

To circumvent these issues, this analysis used an alternative approach to calculating the expected rates which is independent of the unknown factors. As motivation for this method, we note that the data provided by the collaboration is separated into bins across the experiment's energy range, and for good reason: the antineutrino flux, detector efficiency, cross-section, and oscillation probability are all expected to be functions of energy. Variations in $P_{\bar{\nu}_e \to \bar{\nu}_e}$ over the range provide critical information for fitting parameters; a previous χ^2 fit which averaged $P_{\bar{\nu}_e \to \bar{\nu}_e}$ over the energy range was observed to be capable of only providing a one-parameter fit. To preserve this information, the collaboration presumably chose energy bins which divided events into sets which are comparable in their characteristics. Because Daya Bay has recorded a large number of events, there is no great need to 'stretch' the energy bins to include counts for the purpose of making bins significant.

With this framework in mind, we now adopt the following assumption: because each energy bin is considered to occupy a relatively small segment of the overall spectrum, the reactor flux, event cross-section, and detector efficiencies do not vary appreciably over each bin. Essentially, we are asserting that over the range of an energy bin, each of the listed factors can be well approximated by a constant value. The advantage of this stipulation is that it immediately simplifies the integral over each bin. Because Φ, σ, and ε, can be treated as constants, our previous integral reduces to

$$\Phi\varepsilon\sigma \int_{\mathrm{E}}^{\mathrm{E}+\,\Delta\mathrm{E}} P_{\bar{\nu}_e \to \bar{\nu}_e} dE$$

The remaining integration is trivial, as the expression for $P_{\bar{\nu}_e \to \bar{\nu}_e}$ is analytic and known. Notice, also, that the integral is effectively averaging $P_{\bar{\nu}_e \to \bar{\nu}_e}$ over the bin. Using $\mathrm{P_{avg}} = \frac{1}{\Delta\mathrm{E}} \int_{\mathrm{E}}^{\mathrm{E}+\,\Delta\mathrm{E}} P_{\bar{\nu}_e \to \bar{\nu}_e} dE$, our expression for the predicted rate is simply $\Phi\varepsilon\sigma P_{avg}\Delta\mathrm{E}$.

At this point, we may observe that Φ, σ, ΔE, and ε are not functions of the oscillation parameters but--according to our assumption--fixed constants for each bin. Hence, we can treat their product as a simple constant factor, $\rho = \Phi\varepsilon\sigma\Delta\mathrm{E}$. Obviously, if ρ can be determined, then the rate is easily obtained through the product ρP_{avg}. Typically, some information in an experimental publication will allow ρ to be found. Reports of oscillation experiments often include an illustration of the best fit values for each bin, which are calculated using the collaboration's best knowledge of Φ, σ, and ε; dividing these fits by P_{avg} for the authors' best fit parameters then yields ρ. In this case, however, we are particularly fortunate in the collaboration's choice to present the data in ratio form. Consider, for instance, the near and far hall event rates over a single energy bin. Using our previous work, we may write these rates as $\rho_{near} P_{near}$ and $\rho_{far} P_{far}$, respectively. The ratio of the rates for the detectors is simply $\frac{\rho_{far} P_{far}}{\rho_{near} P_{near}}$, which is very nearly the ratio of the oscillation probabilities at the two sites.

The ratio of the rates would, in fact, be just the ratio of the oscillation probabilities if $\rho_{far} = \rho_{near}$. This is definitely not true for Daya Bay; although the cross-section and detector efficiency should be identical for the sites, the flux magnitude is much greater for the near site than the far site given the isotropic nature of the antineutrino emission. Recall, however, that the ratio used in the χ^2 formula has been corrected to make the sites comparable. The data reported by the experimentalists was actually constructed so that the flux magnitude difference was already accounted for, so the χ^2 fit should ignore it. We are then justified in adopting the ratio $\frac{P_{far}}{P_{near}}$ as the event ratio for the two detector sites, as the data set requires that $\rho_{far} = \rho_{near}$.

A slight ambiguity arises in attempting to apply the previous expression, for it treats the probability factors as if a single near site exists. In fact, the three near detectors all have different effective baselines, which means that there are three distinct 'near' oscillation probabilities. To resolve this vagueness in the definition of the oscillation term, this analysis treated P_{near} as the average of the probabilities at the three detector sites. Such a difficulty does not arise in the case of the far detector site, as the baselines for these detectors are identical. With these conventions established, it is trivial to calculate the desired ratios for each energy bin and the χ^2 values associated with each parameter set.

Having adopted this approach for evaluating the fit for a set of parameters, the analysis for this paper focused on determining the two most influential parameters for this oscillation, θ_{13} and $\Delta m_{31}{}^2$. To that end, the remaining parameters were fixed at the values given in reference [6] for consistency with the Daya Bay collaboration. θ_{13} and $\Delta m_{31}{}^2$ were then varied in an attempt to determine the constraints the Daya Bay result imposes on their values. Note that θ_{13} was parameterized with its associated function in the oscillation probability formula, $\sin^2 2\theta_{13}$; in addition, all examined values for $\Delta m_{31}{}^2$ were positive. These choices reflect the insensitivity of the oscillation to the signs of θ_{13} and $\Delta m_{31}{}^2$ and should not be construed as commentary on the true parity of these parameters.

Section III. Presentation of Analysis Results

The results of the chi-squared parameter fit are shown in figure 1, which shows the regions allowed in the parameter space $\sin^2 2\theta_{13}$ and $\Delta m_{31}{}^2$ at the 90% and 99% confidence levels along with the best fit value. These regions are immediately surprising in their location in the space, for they exclude the parameter values reported by the collaboration at a confidence of 99%. This exclusion is due primarily to the χ^2 preference for a larger value of $\Delta m_{31}{}^2$. Using the 99% confidence interval to define the uncertainty in the fit, the figure shows that the Daya Bay data prefers the values $\sin^2 2\theta_{13} = .084^{+.033}_{-.028}$ and $\Delta m_{31}{}^2 = 3.53^{+.74}_{-1.07} \times 10^{-3} eV^2$. While this value of $\sin^2 2\theta_{13}$ falls within the bounds established by the Daya Bay measurement, the best fit value for the $\Delta m_{31}{}^2$ is not compatible with the value assumed in the collaboration's analysis. The deviation of this measurement from the accepted value is even more impressive given the precision to which $\Delta m_{31}{}^2$ is thought to be known. Experimental determination of $\Delta m_{31}{}^2$ is most directly obtained from the MINOS experiment [9], whose most recent work cites a value of $2.32^{+.12}_{-.08} \times 10^{-3} eV^2$. With these reported errors, the mass difference given in this analysis is excluded at a 10σ level by the MINOS data.

Having established that this preferred value of $\Delta m_{31}{}^2$ is a significant departure from the accepted value, it is natural to have some reservations about its validity. Closer examination might allay some of our skepticism, however, as the reported Daya Bay result is compatible with the χ^2 fit. Figure 2 illustrates how this analysis subsumes the published result; the graph shows level curves of the χ^2 surface across the parameter space, increasing in steps of five. In addition, a line across the space denotes the accepted value of $\Delta m_{31}{}^2$. From the chart, it is evident that the Daya Bay result of $\sin^2 2\theta_{13} = .089$ occurs on the line for $\Delta m_{31}{}^2 = 2.32 \times 10^{-3} eV^2$ in the region where χ^2 is minimized. In fact, if the fit is conducted with $\Delta m_{31}{}^2$ set to the accepted value, the best fit value of $\sin^2 2\theta_{13}$ is found to be exactly what is reported by the collaboration.

The characteristic shape of the χ^2 surface can be invoked as an explanation for why this preference for a larger ${\Delta m_{31}}^2$ would not be obvious in the context of the published analysis. As figure 2 shows, for values of ${\Delta m_{31}}^2$ greater than $2.32 \times 10^{-3} eV^2$, the allowed values of $\sin^2 2\theta_{13}$ are restricted to a fairly narrow range. A slight change from the accepted value of ${\Delta m_{31}}^2$, then, should produce little change in the measured value of $\sin^2 2\theta_{13}$. Statements in the collaboration's report indicate that some small variations in ${\Delta m_{31}}^2$ may have been considered; in particular, reference [6] declares that the uncertainty in ${\Delta m_{31}}^2$ induces no significant change in the measure of $\sin^2 2\theta_{13}$. The χ^2 analysis used here also observes no great change in $\sin^2 2\theta_{13}$ over the error range of ${\Delta m_{31}}^2$ indicated by MINOS; at most, $\sin^2 2\theta_{13}$ is changed by .002. This is less than the uncertainty from other factors affecting the measurement and may have been considered expendable by the collaborators. If the examination of the parameter space was restricted to providing an assessment of how the uncertainty in the mass could affect the value of $\sin^2 2\theta_{13}$, there may have been little incentive to consider greater values of ${\Delta m_{31}}^2$.

The fit provided by the parameters given in this analysis can be visually compared with the collaboration's fit through a graph of resulting oscillation curves in figures 3 and 4. These graphs show the oscillation probabilities for the two parameter sets alongside the individual detector data, given in a ratio of observed to expected events. As shown in the curves, the best fit parameters improve on the fit suggested by the published analysis, which can be seen to overestimate the signal at the far detector.

As an aside, a subtlety exists in figures 3 and 4 which should be acknowledged. Recall from section II that the collaboration predicted the absolute magnitude of the flux using their χ^2 fit to the parameters. This is necessary because the experiment cannot directly measure the flux;

as all sites are significantly distant from the detectors, all detectors will perceive oscillation deficits. Thus, the flux magnitude must be inferred from the best fit of the oscillation parameters. This is relevant to figures 3 and 4 because the expected number of events is proportional to the total flux, which implies that changes in the measured flux can effectively scale the data points shown in the figures. Because the parameters are fit with a flux magnitude that minimizes χ^2, to properly represent the fit offered by any parameter values requires scaling the data points to reflect the flux inferred from those values. Figure 3 uses data taken directly from figure 23 in reference [6], which should represent the points scaled according to the collaboration's best fit flux. The fact that the best fit as found through our χ^2 fits the data at this flux implies that the best flux for this analysis is consistent with that of the experimentalists. However, it was necessary to scale the points by 1.009 to achieve the fit of figure 4; this corresponds to a 0.9% decrease from the best fit flux. Curiously, figure 23 in [6] purports to illustrate the same fit as figure 4 using the same data as figure 3 in this analysis. This is demonstrably inaccurate; a graph of the oscillation probability against those data points can be shown to consistently overestimate the data.

Section IV: Examination of Possible Sterile Neutrino Effects

Objectively, the best fit result shown in figure 1 yields the preferred parameter values for observations at Daya Bay, and the new, higher value of $\Delta m_{31}{}^2$ deserves serious consideration. However, the enormous divergence of this result from the MINOS measurement suggests that some effort should be made to reconcile the experiments. Unfortunately, our previous efforts with the χ^2 fit considerably reduce the options available to bridge the divide. As our work replicates the measure of $\sin^2 2\theta_{13}$ reported by the collaboration for $\Delta m_{31}{}^2 = 2.32 \times 10^{-3} eV^2$, it is certain that the curve shown in figure 1 represents the best fit to the data for the accepted mass; hence, variation of $\sin^2 2\theta_{13}$ will not improve the fit. As mentioned in Section II, replacing the oscillation formula used by the collaboration with an exact form causes no substantial change over the experimental baseline. Weak interactions through the Mikheyev-Smirnoff-Wolfenstein (MSW) effect [10] are similarly negligible over the short distance of the experiment. The irrelevance of these changes suggests that the data is incapable of accommodating the MINOS value of $\Delta m_{31}{}^2$ under the oscillation theory as defined, and the aberrant measurement of $\Delta m_{31}{}^2$ can be interpreted as evidence that an expansion of the current theory is required.

Such an extension is available by appealing to the concept of sterile neutrinos. Sterile neutrinos are theoretical leptons which, like the known neutrinos, have no electrical charge [11]. However, sterile neutrinos have no flavor connection to a charged lepton and do not participate in the weak interaction. In essence, sterile neutrinos are not directly observable and are only manifest in the disappearance of an initial neutrino flux through oscillation into a sterile state. For the oscillation model, the addition of a sterile state to the flavor basis would require the complementary addition of a mass eigenstate to preserve the unitary property of the mixing matrix and dimensionality of the neutrino basis. Subsequent reconstruction of the mixing matrix would include parameters to account for mixing with the sterile state along with the three-neutrino parameters, and these extra parameters could potentially provide enough flexibility to fit the Daya Bay data with the reported values of $\Delta m_{31}{}^2$ and $\sin^2 2\theta_{13}$.

To determine whether sterile neutrino effects could be used to account for the anomalous measure of $\Delta m_{31}{}^2$, a χ^2 fit at the fixed values of $(\sin^2 2\theta_{13}, \Delta m_{31}{}^2) = (.089, 2.32 \times 10^{-3} eV^2)$ was performed using additional parameters created by assuming the existence of sterile neutrinos. For the purposes of this trial, it was assumed that only one sterile neutrino state exists. To account for the sterile state, four additional parameters were added to the analysis: three mixing angles $(\theta_{14}, \theta_{24}, \theta_{34})$ and a mass difference $(\Delta m_{34}{}^2)$ [3]. The oscillation probability obtained using these additional parameters was interchanged with the three-neutrino probability in the previous χ^2 analysis to produce a fit across the four parameters.

The conclusions that can be drawn from the fit are mixed. At the assumed values for $\sin^2 2\theta_{13}$ and $\Delta m_{31}{}^2$, the fit indicated non-zero values of θ_{14} and $\Delta m_{34}{}^2$ improved the fit from the three neutrino case. Unfortunately, the χ^2 value associated with this fit indicates that it is still inferior to the best fit in the three-neutrino framework, so any fourth neutrino effects observed at Daya Bay alone are insufficient to reconcile the preferred mass with the MINOS measurement. However, the fact that the fit for the given parameter values improves when place in the four neutrino framework suggests that sterile neutrino phenomena could be used to produce a fit that is superior to that given by the three-neutrino case. Indeed, the χ^2 value of the parameters $(\sin^2 2\theta_{13}, \Delta m_{31}{}^2) = (.089, 2.32 \times 10^{-3} eV^2)$ decreased by 4.8 in the move to the fourth neutrino case, and similar improvements for other parameter sets could be possible. Furthermore, the best fit under sterile neutrino effects need not coincide with the best fit values of $(\sin^2 2\theta_{13}$

,$\Delta m_{31}{}^2) = (.084, 3.53 \times 10^{-3} eV^2)$ in the three neutrino case. If the fourth neutrino case provides a best fit at a lower value of $\Delta m_{31}{}^2$, then some movement toward the accepted value may be achieved. Finally, the potential effect of sterile neutrinos on the MINOS measurement cannot be discarded. If we consider fourth neutrino effects a possible cause of the abnormally large $\Delta m_{31}{}^2$ indicated by Daya Bay, then such phenomena might have also affected the analysis of the MINOS data. If sterile neutrinos are a relevant factor in the analysis of both experiments, it is conceivable that the two measures could converge in the context of a fourth neutrino.

This possible reconciliation of the two measures of $\Delta m_{31}{}^2$ provides a direction for future work. A fit over the four-neutrino parameter space for both MINOS and Daya Bay could confirm or disprove the idea that sterile neutrino effects mediate the observations at both experiments. A combined four-neutrino analysis which indicates agreement between the experiments on the value of $\Delta m_{31}{}^2$ and an improved fit for both experiments over the three-neutrino theory would be considerable evidence for the existence of sterile neutrinos. Inquiries into this possibility are currently being conducted, with results pending.

Conclusion:

Within the three-neutrino oscillation framework, the reported data for Daya Bay is best fit by the parameters $(\sin^2 2\theta_{13}\ ,\Delta {m_{31}}^2) = (.084, 3.53 \times 10^{-3} eV^2)$. While the former is consistent with the result reported by the collaboration, the latter diverges significantly from the most precise current measurement of $\Delta {m_{31}}^2$, excluding the MINOS result at the 99% confidence level. This paper has suggested that the additional oscillation parameters granted by the existence of a sterile neutrino state may be sufficient to reconcile the observations of both experiments. The possibility that a fourth neutrino could be used to resolve the disagreement between the MINOS and Daya Bay values of $\Delta {m_{31}}^2$ provides a direction for further work, as a fit over the additional parameters should be able to discern convergence of the mass measurements under sterile neutrino effects.

Acknowledgements:

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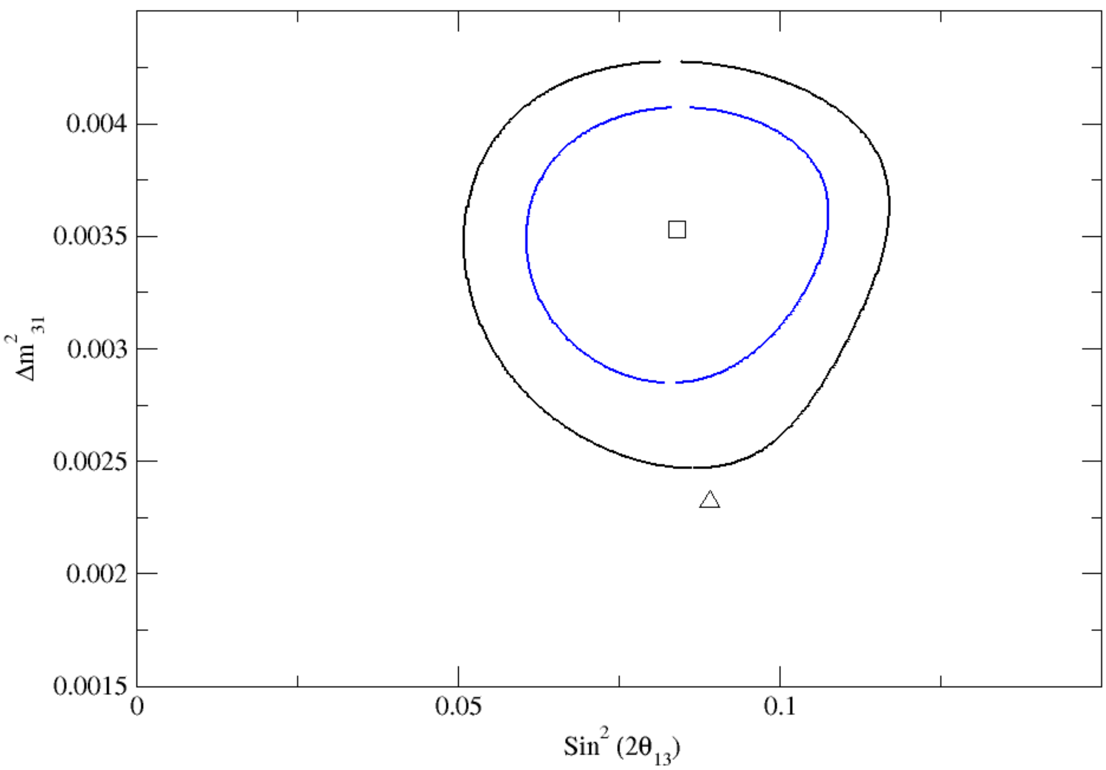

Figure 1: Confidence regions in the parameter space ($sin^2 2\theta_{13}$,$\Delta m_{31}{}^2$). The blue and black curves define the 90% and 99% confidence regions, respectively. The best fit point for the analysis is marked by a square, while the collaboration's reported fit is marked with a triangle.

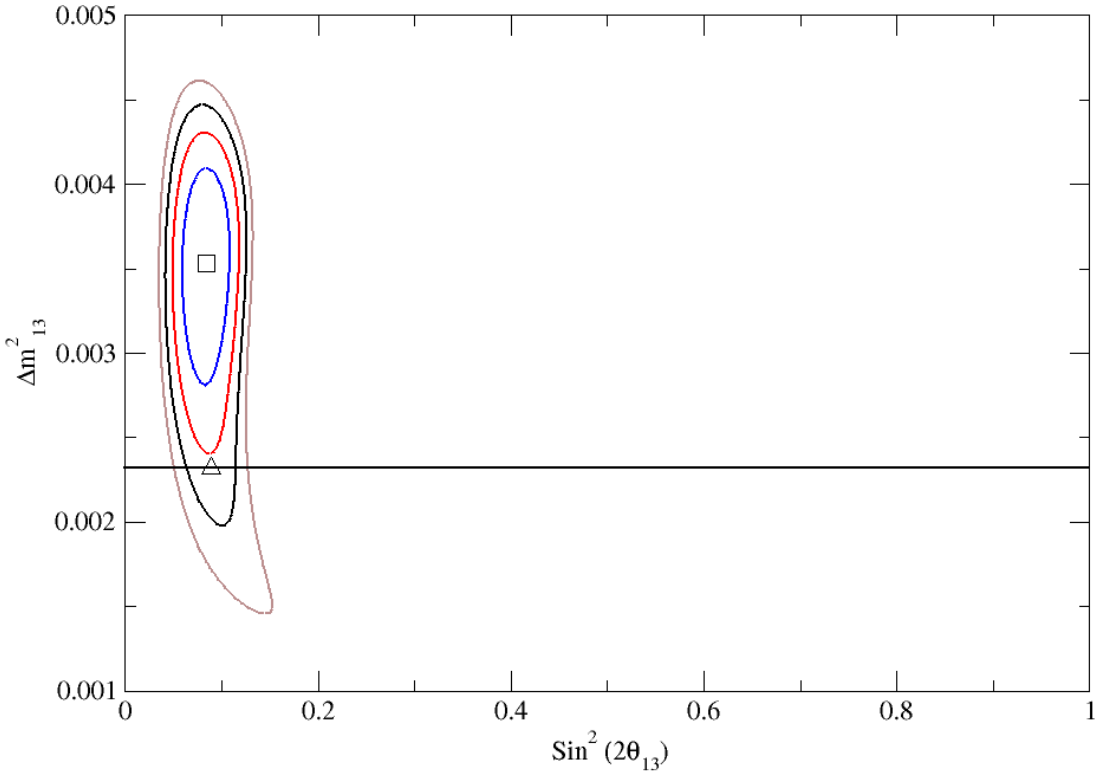

Figure 2: Level curves for the χ^2 surface. The illustrated curves increase in χ^2 by steps of 5 outward from the best fit point. The horizontal line across the space corresponds to $\Delta m_{31}{}^2 = 2.32 \times 10^{-3} eV^2$.

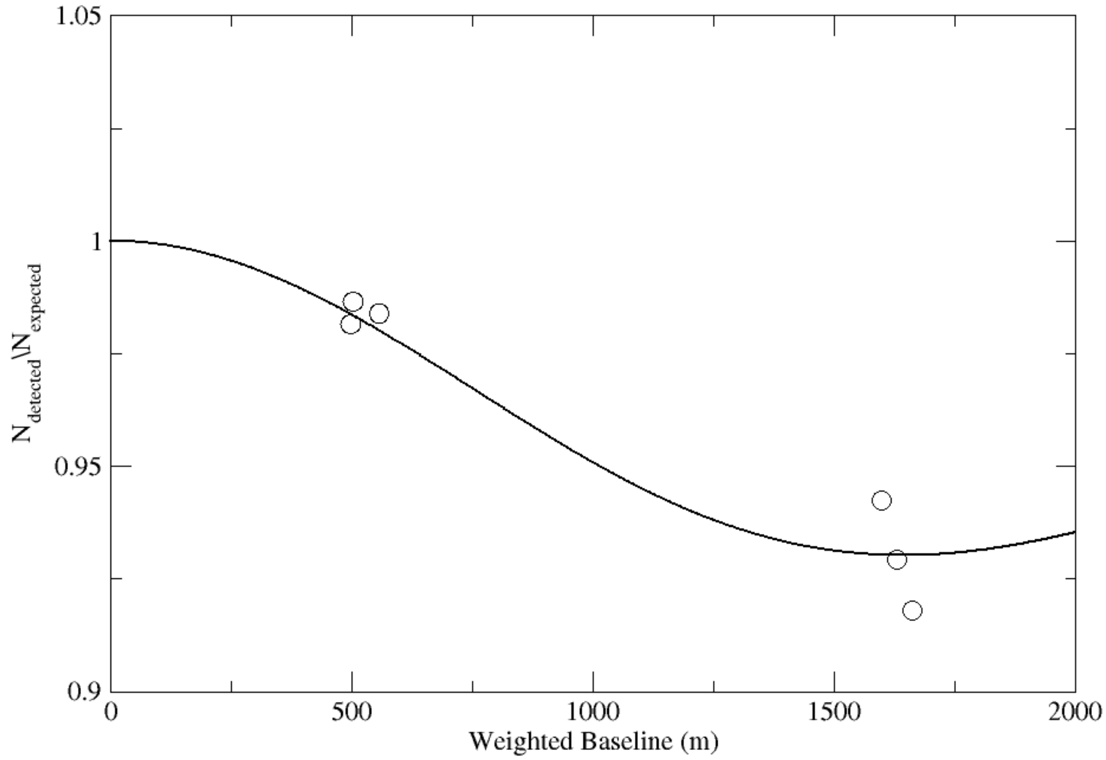

Figure 3: Oscillation curve for the best fit parameters $(\sin^2 2\theta_{13}, \Delta m_{31}{}^2) = (.084, 3.53 \times 10^{-3} eV^2)$. The circles in the graph represent the ratio of observed to expected data for each detector given the experimental best fit to the total flux. Two of the far data points have been displaced by +30 m and -30 m for clarity.

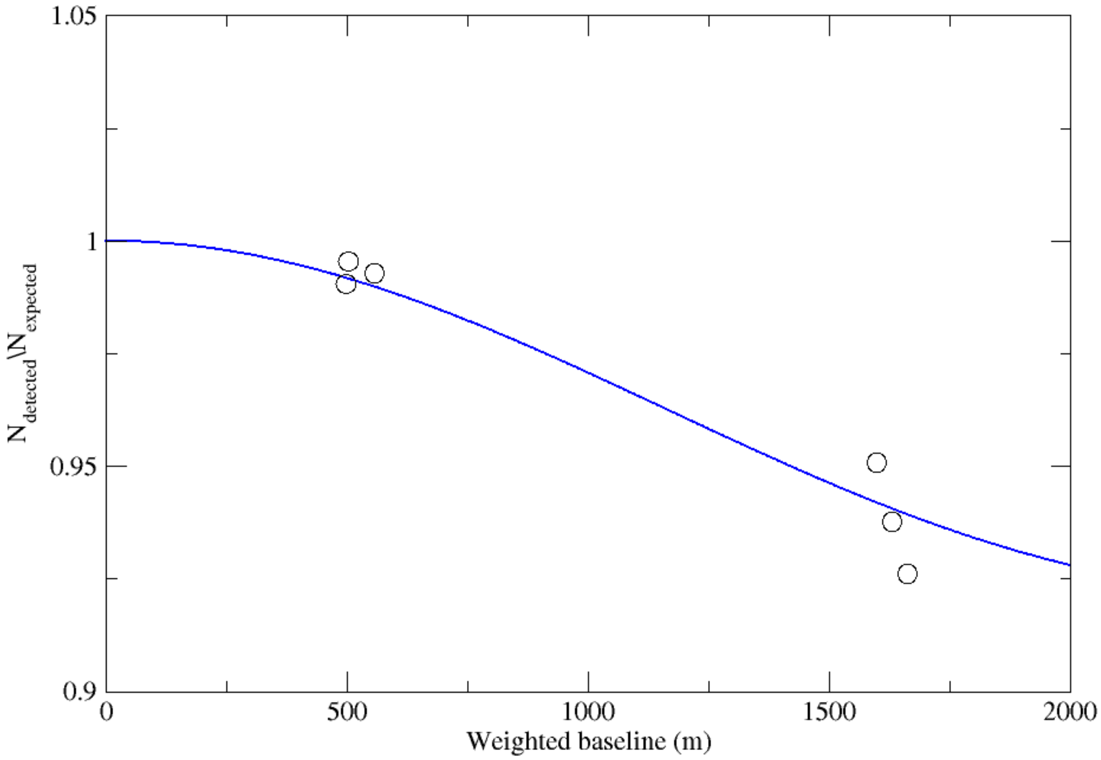

Figure 4: Oscillation curve for the collaboration's fit, $(\sin^2 2\theta_{13}, \Delta m_{31}{}^2) = (.089, 2.32 \times 10^{-3} eV^2)$. The circles in the graph represent the ratio of observed to expected data for each detector given an adjustment to best fit flux magnitude.