Designing for Their Joy:
An ethnographic investigation of middle school students' joyful mathematical learning

By

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Dissertation

Submitted to the Faculty of the
Graduate School of Vanderbilt University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY
in

Learning, Teaching, and Diversity

May 10th, 2024

Nashville, Tennessee

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This dissertation is dedicated to my husband and best friend

Matthew Knowe

and to my children

JB, Kingsley, and Adalynn

Acknowledgements

This work was motivated and sustained by my interactions and entanglements with a community of people who have been to me the personification of love, care, wisdom, sacrifice, passion, empathy, wonder, diligence, and joy. For that community, I am endlessly grateful.

My utmost thanks to Mrs. Fry for your friendship and partnership in teaching and learning. I love you, and I have come to love your students as well. Thank you for pursing this research alongside me and for pursuing your students' joy day-in and day-out as you dedicate yourself to this beautiful career in education. Thank you also to Star, Isaac, Thomas, and Ruqaiya and all the students in the 3rd block 6th grade class of Porter Academic Magnet Middle School. Learning with you and from you gave *me* immense joy. I will forever treasure the shouts and waves across the parking lots, shared snacks, play and stories of your days, and the opportunity to learn about your unique mathematical joy.

I am so thankful for the expertise and passion of the teaching and learning community at Vanderbilt University. Melissa Gresalfi, thank you for your thoughtfulness, foresight, care, and expertise. You've taken time and effort to see me as fully human – as a mom, wife, friend, student, researcher – and your empathy for all those facets of me has sustained my work. You've been such a clear guide and encouraging force in my education. Thank you also to my committee members for inspiring and providing invaluable insights for this work. Thank you, Dr. Joseph, for your work in humanizing mathematics for Black girls and for your encouragement to lean into the inspiration of Black Feminist Theory. To Tesha, for your remarkably insightful, constructive feedback and for your excitement about interesting methodologies and the thoughts and actions of children. Thank you to Doc for providing me with such early feedback and a foundation of critical, scholarly writing. To Marcy, thank you for sharing your wisdom and passion for research practice partnerships and for your inspiring work in schools in Nashville. A special thanks also to Amy Parks, for your care about children's joy in schools, for your expertise in translating theory to practice, your guidance in ethnography and case studies, for your quick provision of the perfect resources, and for your persistent reminders to listen to children. Anita Wager, thank you for always looking for joy in education and for your expertise in designing playful, joyful learning experiences for students. Candice Love and Jamie Vescio, thank you for your friendship, your feedback, and your constant encouragement. Our shared experiences in the Playful Learning Project and the Gresalfi Guild over pastries and Diet Coke have been such a treasure to me. A special thanks also to Barb Stengel who over the past 12 years has been a consistent, loving, scholarly, wise guide for me in the field of education.

While this work developed through the care and provisions of academic resources at Vanderbilt University, I personally was grounded in and sustained by the constant sacrificial love of our friends. To the Maples, Prophaters, Emily Stem, Thompsons, Maples, Melanders, Neus, Leggs, thank you for teaching me what the love of Christ looks like practically - for the meals you've provided and shared, for your generosity, your care for JB, Kingsley, and Ada, for our shared sorrows and celebrations, for your prayers and your action, and for your friendship. A special thanks to Kelsey Maples for caring for Kingsley, teaching her with such kindness, and loving us like your own family. To our Immanuel Nashville Church, you have made the real Jesus nonignorable to me by providing me with the gospel, safety, and time to be loved by God himself through you. You are an immaculate picture of Roman 12:9-18.

And mostly importantly, thank you to my family. Thank you, Matt, for being the most constant example of the joy of self-forgetfulness. Your wonder and curiosity for the world is the most stunning thing about you. Thank you for loving people (especially me) so practically and so well. Thank you for poking your head over the self-checkout station at Harris Teeter to ask me how I was doing on that fateful day, for asking me out for chili cheese fries, for reminding me of Christ, and for traveling across the world to ask me to marry you. Thank you for carrying me through cancer treatment by sitting by my hospital bed to read me Harry Potter, taking me out for snow cones, helping me adjust to a new life without a thyroid, and seeking to understand all of me. Thank you for taking on parenthood (foster, adoption, and biological) with me and through it enduring hardship and celebrating great joys with me. You are the most amazing father to our children. One of my greatest pleasures is to watch you so deeply enjoy reading them books, teaching them to fix their car, talking to them about space, taking them on bird watching walks, cheering on Liverpool with them, teaching them about the Bible, or singing them the doxology as they rest their head on your shoulder (among so many other things). For me, you are the perfect picture of love and you're super-hot. To Kingsley, although you can't read this yet, thank you for being the personification of play, wonder, excitement, and vulnerability. You teach me daily how to be present, how to ask amazing questions, and how to be thrilled by just about anything. The world is magic through your eyes and the closer I get to you the more I see that magic. Adalynn, from the moment you were born you have been overflowing with joy. Your grins and snuggles at even one week old endeared me to you immediately and your contagious, bubbling over laughter is a treasure. I daily look forward to your hugs, grins, precious new words, dancing, and singing. JB, thank you for your humor and smiles even through great suffering. You never cease to amaze me. Thank you for joining our family, for exploring all the complexities of life with us, and for teaching us how to love. Thank you also to my mom and dad, Dadu and Jeffro, for characterizing our family with the laughter and love of Christ. Your encouragement to "never play second fiddle" and to "go big or go home" has led me here. To Lauren, thank you for always protecting me and for being my friend. Your loyalty to those that you love is unbreakable and your commitment to that love is to be admired and emulated. Thank you for joining us in Nashville, for caring for all of us so intentionally, and for being such a crucial part of our lives.

Thank you to each of you for caring for and loving me in your own unique way. I am undeserving of such incredible people and love in my life.

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Introduction

When we commit ourselves to education as the practice of freedom, we participate in the making of an academic community where we can be and become intellectuals in the fullest and deepest sense of the word. We participate in a way of learning and being that makes the world more rather than less real, one that enables us to live life fully and freely.
This is the joy in our quest.
bell hooks (1989, p. 85)

Purpose and Motivation

This empirical study seeks to investigate how we might design formal mathematics learning experiences that support students' joy. Joy, in all its complexity, has been theorized about, questioned, researched, explored, and experienced extensively across cultures, disciplines, and genres (Jaworski, 2012; Lama et al., 2016; Muhammad, 2023; Packnett, 2017; Parks, 2020; Parks, 2021; Volf & Crisp, 2015; Walker, 1992). The diversity of potential definitions and descriptions of joy reflects the diversity of cultures and lived experiences, making joy both endlessly complex and fascinating to research. We might recognize felt and expressed joy in the intimate moments of the birth of a child, or while tinkering with a broken radio, watching in awe as the residents of a coral reef dance below our feet or the Milky Way illuminates over our head, or in singing and dancing together both in celebration of a holiday or in protest of an unjust death. The themes that emerge across varied bodies of literature defining joy informs this research with respect to mathematics learning. Integrating these fields of study has led me to define mathematical joy as *a positionality toward mathematics that animates engagement and leads to feelings of satisfaction and purpose born from the coordination of mental, physical, relational, societal, and intellectual well-being*. Here I ask, what might mathematical joy look like and what conditions might support it for students learning mathematics, a discipline as old as humanity itself, developed over millennia, and explored constantly around the world by both toddlers and astrophysicists?

Exploring, playing with, and mimicking mathematical objects like number, shape, and pattern is deeply rooted in our humanity (Orlin, 2022). Therefore, one would think that engaging in learning mathematics should be fun, pleasing, aesthetic, and overflowing with satisfaction and joy (Su, 2020). But joy and mathematics are two topics not often associated and even commonly perceived as being in opposition. Formal mathematics learning is often experienced and described in ways that are performative, unpleasant, or even traumatic (Boaler, 2021; Martin, 2009b). It is also well documented that the academic discipline of mathematics and those who engage in it have enacted mathematical oppressions, exclusionary practices embedded within systems that have caused harm to children, families, communities, and even democratic society as a whole (Gutiérrez, 2002; Martin, 2009c; Parks, 2020; Singh, 2021; Skovsmose, 1990). As a mathematics learner, teacher, and researcher, I have witnessed this first-hand. I have also experienced great joy in math's mystery, intrigue in the depths of its puzzles, and the transformational power in its application. I have had the distinct bliss of seeing students experience the same. Therefore, in the face of persistent oppressive forms of mathematics education, in this study I ask how we can design formal mathematics learning experiences that support students' joyful math learning.

Our human experiences uniquely reveal for each of us that joy is not merely circumstantial. In fact, joy often emerges within suffering and in resistance to oppression as it defiantly insists that we transcend even the most painful of circumstances. I use Patricia Hill Collins' definition of oppression as "any unjust situation where, systematically and over a long period of time, one group denies another group access to the resources of society" (Collins, 2000, p. 4). For example, it has been well-demonstrated that cultural norms characterize success in mathematics as individualistic, performative, competitive, and exclusive to select few (Boaler, 2021; Christensen et al., 2008; Gutiérrez, 2013; Jaworski, 2012; Joseph, 2021; Louie, 2017; Parks, 2020; Ramirez et al., 2016; Volmink, 1994). This reinforces the dominant, restrictive and hierarchical culture of mathematics education which sorts students based on perceived intelligence, causes unique emotional, physiological, physical, and epistemological harm specifically to minoritized students, and limits mathematical participation from students who could potentially find joy in collaborative, puzzling, aesthetically pleasing mathematics (Gutiérrez, 2002; Louie, 2017; Martin, 2009c). Although the presence of these oppressions does not make joy an impossibility in formal learning spaces, it is evident that making mathematics joyful requires more than simply introducing games or songs into mathematics classrooms. Spaces like mathematics classrooms have the potential to be spaces of great joy for both students and teachers only if we intentionally resist mathematical oppressions.

Alice Walker, a preeminent author, speaker, social activist, Black Feminist Theorist, and Pulitzer Prize winner, provides insights into the interconnectedness of joy and resistance in her novel *Possessing the Secret of Joy*. In the climax of this book, her brave protagonist, Tashi-Evelyn, is facing a moment of crisis of her protest as she endures and counters oppressive systems of intersectional racial and gendered exploitation and suffering. In this moment she is affirmed and cared for by her friends and supporters as they unfurl a banner bearing huge block letters that read "RESISTANCE IS THE SECRET OF JOY!" (Walker, 1992, p. 279). Walker makes clear that the act of resistance to dehumanizing oppression spurs on and inspires our joy. Walker's words and life's work as a Black Feminist theorist has inspired and framed this work about joy in acts of resistance. We see this theme emerge from the profound work of Black Feminist Theory, which through a diversity of multifaceted stances and personal standpoint perspectives (Collins, 2000), can uniquely speak to joy as resistance to oppression by attending to one another's full, complex, affective, embodied humanity. Thus, I take up here a conceptual framework humanizing mathematical to support joy through responsive resistance to dehumanizing mathematics. In humanizing mathematics, we orient ourselves to designing mathematical learning spaces and opportunities by centering students multifaced well-being: their mental, physical, relational, societal, and intellectual well-being. It is with hope and expectation, through resistant, humanizing mathematics, that I look to designing formal mathematical learning experiences that are filled not with trauma but overflowing with joy.

Overview

This research about mathematical joy is situated in the context of a single 6th grade mathematics classroom over the course of one semester. The study developed through a Research Practice Partnership with a close friend and 6th grade teacher. Together, we collaborated in observing, designing, implementing, and reflecting about mathematics lessons that supported (or hindered) students' joy. To explore how to design for students' mathematics joy, we focused on the following research questions:

1) What are some characteristics of joyful mathematical engagement?
2) What conditions support students' joyful mathematical engagement in this formal math classroom?

These research questions are answered through three phases of data collection that focus on baseline understandings of the class, case studies of four focal students, and designing mathematical lessons with the intent of supporting these students' mathematical joy. Analysis involved progressively examining different pieces of the corpus of data to answer the research questions in increasing depth over time through three levels of data analysis.

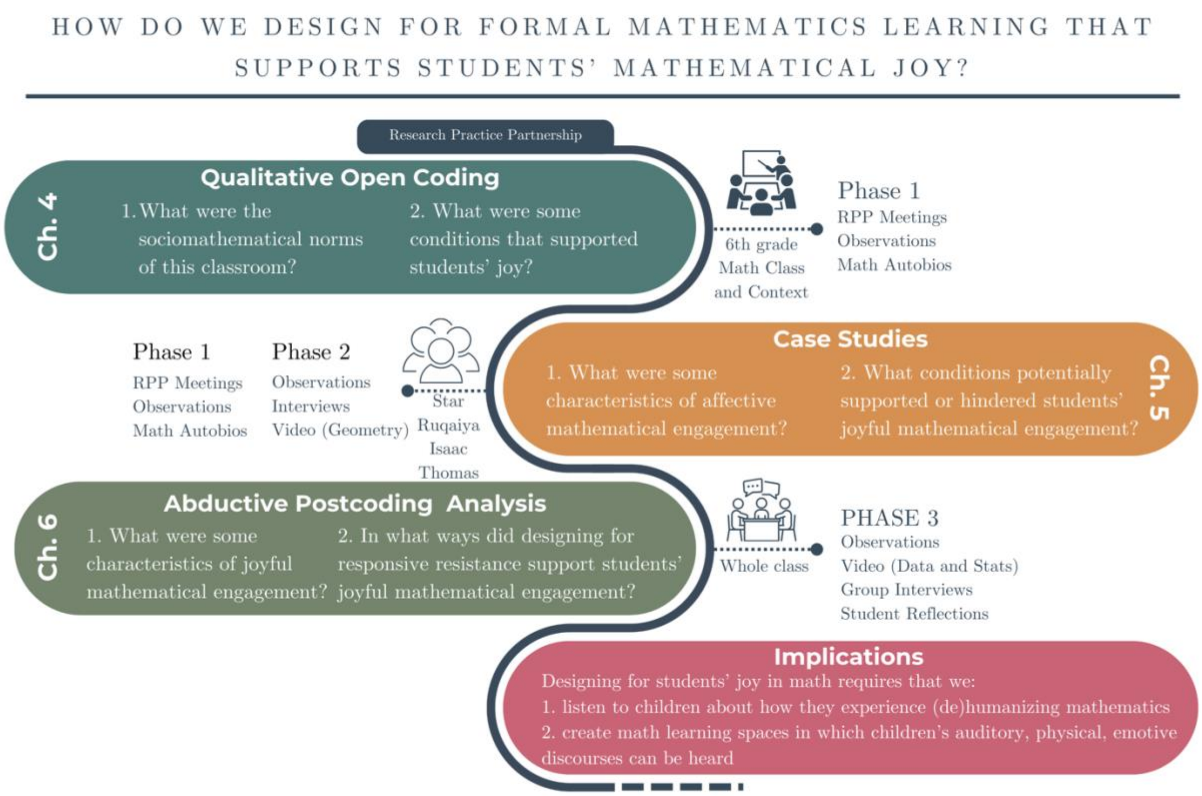


Figure 1: Study overview of procedural and analytic methods with respect to incrementally building research questions

Grounded theory approach of qualitative coding (Charmaz, 2001) revealed that sociomathematical norms of this classroom characterized knowing and doing mathematics as product-oriented, competitive, compliant, and a complex negotiation of fun and stressful. During math lessons, Mrs. Fry and I saw very high participation and engagement but very little positive affect and few moments that we could characterize as students' mathematical joy. This led to a case study analysis (Dyson & Genishi, 2005; Emerson & Mayer, 2002) of four students, Star, Ruqaiya, Isaac, and Thomas, who each uniquely provided a window into the local particulars of how we could support mathematical joy in this math classroom. Ethnographic storytelling and multimodal interaction analysis focused (Jordan & Henderson, 1995; Norris, 2004; Parks & Schmeichel, 2014) on these four students' affect, demonstrated students' desire for mathematical reciprocity, a mutual, benevolent sharing of mathematical ideas and resources, in this math classroom. Therefore, in the third phase of the study we designed a data and statistics math unit that necessitated students' mathematical reciprocity in an attempt to design for these students' unique joyful math engagement. Open-ended abductive postcoding analysis leveraged "joy as methodology" (Parks, 2021) and demonstrated that when these students engaged in tasks that were intentionally designed to support mathematical reciprocity, students across the class demonstrated increased joy in multifaceted ways (for example through humor, movement, debate, and encouragement) that represented their multifaceted well-being.

This is not to generalize to say that we can design for all students' mathematical joy by incorporating reciprocity or in fact any dimensions of design. Indeed, the findings of this study complicate notions of design frameworks that support students' mathematical joy. The joy that emerged for these students was a direct result of listening to students with their varied, complex, and beautiful ways of communicating how they were experiencing both humanizing mathematical experiences and dehumanizing mathematical oppressions in this cultural environment. It was by attending to them, treating them as human, listening to their stories, watching their grins and winces that we could see more clearly, albeit imperfectly, learn how to design more humanizing formal mathematical learning experiences that supported their unique joy.

Positionality

Growing up in a globally Western, upper-middle-class, white family, the ideology that my life would be more joyful if I was more comfortable has driven much of my behavior but conflicted with my lived experience. The moments of exceedingly great joy in my life have coincided with great pain: in the birth of my beloved daughters, in taking up the trauma endured by our son when we had the honor of becoming foster and adoptive parents, in relying on the care of community during cancer treatment, in the confusion of betrayal and comfort of friends who can look you in the eye and say "no way! You too?!", as well as in chanting and singing in solidarity with community members protesting the death of Black Americans at the hands of police officers. While comfort and compliance left me dissatisfied, I have seen joy emerge in the moments of action, interaction, love, and communion despite difference and dissonance, in expression of nonconformity in a society that seeks assimilation, and in the complex dance of pressing forward with resistance. Therefore, in research pursuing mathematical joy I'm leaning into my own experiences of the coupling of resistance and joy.

I approach this work boldly because I care deeply about it but also with caution because my experience with systemic oppression has been limited. As a white woman in the United States, I have largely been encouraged to pursue my passions, education, and lifestyle by family, friends, and exemplary role models. This encouragement overrode potentially oppressive societal messaging aimed at my femininity, youth, or perceived role in society, such as a white male math professor telling me that "women shouldn't be allowed to do math". After graduating with degrees in Pure Mathematics and Chemistry, I pursued a career in education. As a math teacher, I found that my students frequently shared a common experience of trauma associated with formal mathematics education. In my pedagogical practice, I discovered that playful, meaningful mathematics learning in the context of loving relationships helped to heal some of this trauma. But as someone who has been successful in mathematics and in my career as a mathematics educator, I come to the area of inquiry about mathematical trauma and joy as an insider. I acknowledge that this may cause me to overlook the complexity and nuanced hardships of mathematics education for those who have been marginalized by mathematics education or communities whose playful inquiry has been oppressed.

Because of my privileged position as a white woman and as an insider in the area of potentially oppressive mathematics education, I seek to carefully listen to the authors and academics who have been speaking about the intersection of joy and oppression for decades. Black Feminist theorists have been at the center of conceptualizing, theorizing about, and acting on the inseparable concept of resistance and joy in the face of intersectional and generational oppression. And research about humanizing mathematics speaks to the potential for resisting dehumanizing mathematics by centering students' full humanity including attending to students' affect. But both of these fields that are foundational to this work have been written about and designed for the lives of Black and brown people who have experienced unique oppressions at the hands of society and specifically of formal mathematics education. I investigate this work with awareness to heed Alice Walker's warning to white women to resist characterizing joy and especially joy in suffering as uniquely innate to Black people. In the novel Possessing the Secret of Joy (Walker, 1992), the protagonist Tashi-Evelyn quotes a "white colonist author" saying "'Black people are natural' she writes, 'they possess the secret of joy, which is why they can survive the suffering and humiliation inflicted upon them'". Tashi-Evelyn responds to this white author's writing saying "Oh, I say. These settler cannibals. Why don't they just steal our land, mine our gold, chop down our forest, pollute our rivers, enslave us to work on their farms, fuck us, devour our flesh and leave us alone? Why must they also write about how much joy we possess?" (pp. 270). The goal of this work is not to reduce the intellectual legacy of Black Feminist Theory or of Humanizing pedagogical models or to extractively exploit this work. As Collins (2000) addresses, I am aware that my goal in this work is not to explain Black women's theorizing to Black women or Humanizing mathematics to educators or children of color more generally, but rather I write about joy as a form of resistance through the lens of Black Feminists Theory and Humanizing pedagogies to bear witness to this wisdom and learn from their multifaceted perspective. My hope is that this work will not further exploit or demean, but instead provide an avenue for the work of resistance to systemically oppressive systems of formal mathematics education to be realized more completely with joy, especially for Black and brown historically marginalized students.

Chapter 1: Literature Review

Education research has long attended to the multifaceted entanglements of students' personal humanity with their learning including but certainty not limited to their participation, social interactions (Lave & Wenger, 1991; Vygotsky & Cole, 1978), positionality, identity (Cobb et al., 2009; Gresalfi & Hand, 2019; Gresalfi et al., 2009), cultural relations to disciplinary knowledge (Bang, 2015; Nasir et al., 2008) embodiment (Abrahamson & Bakker, 2016; Lakoff & Núñez, 2000), interest, and affect (Hannula, 2012, 2020; Sengupta-Irving & Enyedy, 2015) as they learn mathematics. Despite the focus of the learning sciences on the multiplicity of students' humanity in their learning, little attention has been given explicitly to students' joy in learning. This is particularly true of mathematics education research because mathematics is historically situated as a neutral, objective field and thus necessarily devoid of emotion. The non-neutrality of mathematics education is demonstrated by the corpus of mathematics education research that has highlighted the oppressive norms and practices that formal mathematics education imposes on students (Joseph, 2021; Kumashiro, 2001; Leonard & Martin, 2013; Martin, 2009b, 2009c; Nasir et al., 2009b). Requiring mathematical engagement within these oppressive structures of formal mathematics education suppresses students' joy in math learning. Therefore, the following literature review synthesizes research about mathematics learning with attention on oppressive systemic norms and practices that inhibit students' joy. I then outline the ways that we can resist such forms of formal mathematics learning by supporting students' potential for joy.

Formal Mathematics Learning

In this research, I conceptualize mathematics learning as a social process defined by students' participation in the norms and practices of mathematical communities (Cobb, 2006; Lave & Wenger, 1991), grounded in sociocultural and historical context (Vygotsky & Cole, 1978), and inextricably coordinated with students' identity (Boaler & Greeno, 2000; Gresalfi & Hand, 2019; Nasir, 2002). Mathematics learning involves the co-construction of mathematical learning communities and mathematical identities through social interactions and routines around mathematical tools, ideas, curricular materials, and physical spaces. Therefore, in this research about designing for mathematical joy, the epicenter of mathematical learning will be in interactions between people, mathematical resources, their context, and histories.

Researching how we can design for joy in formal mathematics learning indicates that there is something in how we currently conceptualize and design formal mathematics education that doesn't support students' joy. Paul Lockhark, a mathematics educator and author, even goes so far as to say:

> *If I had to design a mechanism for the express purpose of destroying a child's natural curiosity and love for pattern making, I couldn't possibly do as good a job as is currently being done - I simply wouldn't have the imagination to come up with the kind of senseless, soul crushing ideas that constitute contemporary mathematics education (Lockhart, 2009, p. 2)*

The characterization of formal mathematics education as "soul crushing" (Lockhart, 2009, p. 2) nods to the history of dehumanizing educational practices and norms in mathematics. In the following pages, I hope to motivate the need for radical, liberatory, and joyful change in how we as mathematics educators design systems of formal mathematics learning. Seeking to shift the practices of any system requires that we interrogate the goals and cultural norms of that system to consider our alignment or malalignment with its goals and consequential practices.

Goals of Mathematics Education

The US Department of education claims that their mission is to "promote student achievement and preparation for global competitiveness by fostering educational excellence and ensuring equal access" (*U.S. Department of Education*), but this mission leaves unclear what we are asking students to achieve, in what activities they are competing, how excellence is defined, and to what systems or institutions students are receiving access. Historically, the goals of education change based on societal norms and pressures. Some of the original advocates of public education like Horace Mann were fixated on establishing public education to develop democratic self-governing citizens. He labeled public education as our core civic institution because it brings children of all backgrounds together and because it prepares them for democratic citizenship (Goldstein, 2014; Packer, 2022). The relatively undemocratic nature of schooling reveals the misalignment as to the "aim and method which this magnificent institution serves" in that our democracy is unaware of the magnitude of ethical responsibility of the freedom of mind for students and citizens (Dewey, 1903, p. 193). Public education's current fixation on achievement, competition, and excellence makes evident that a collective, democratic goal of public education is in tension with contrasting individualistic goals of education for financial and personal gain. This goal leads us to systemically conceptualize mathematics education in terms of increased test scores, course enrollments, and workforce preparation, especially for minoritized students who are perceived as falling behind and below standards with respect to this "achievement" (Martin, 2009b). Gholson argues that the ethos of U.S. public schooling has historically been that of "individualism, competition, acquisition, obedience, production of work, and evaluations/rewards based on quantification" (2013, p. 63).

In his recent book, *Chasing Rabbits: A curious guide to a lifetime of mathematical wellness*, Singh proposes a sobering measure of our goals for education by asking, "what if we achieved it?" (Singh, 2021, p. 140). If our goal is to promote students' mathematical achievement, and we measure that achievement by individual standardized mathematics test scores, what would be gained if every student obtained a perfect test score? If our goal is to prepare every student for global competitiveness, and we measure that through college admission status or job market statistics, what would be achieved if every student was accepted into a job or college after their graduation from high school? Singh asks, "Would our society benefit? Would there be more gratitude, kindness, and compassion in the world?" (Singh, 2021, p. 140). The purpose of education as individual achievement and competitiveness reveals the underlying current of mathematics education specifically as a pressurized exclusive performative disciplinary learning experience.

Dehumanizing Norms and Practices of Formal Mathematics Education

In this system students uniquely experience multifaceted forms of mathematical oppression. Indeed, all "teaching, learning, and mathematics curricula exist within a sociopolitical context, which 'allows us to see the historical legacy of mathematics as a tool of oppression'" (Joseph, 2021, p. 76). Among others, these oppressions could be defined by cultural norms of mathematics that define an individual's success in mathematics as performative and competitive, ideologies that define productive positionality towards the discipline as objective and rigorous, and systemic practices that cause emotional, physiological, physical, and epistemological harm specifically to minoritized students in mathematics.

Mathematics as performative and competitive

Given that the goal of public education is competitiveness, excellence, and academic achievement, it is unsurprising that students demonstrate success in mathematics education through excellent performance on competitive standardized assessments. The goal of mathematics and performative and competitive is revealed by the over testing of mathematics in formal learning environments and the consequential perception of success in mathematics by most students as the quick, errorless achievement of an extensive but narrow set of mathematical standards (Boaler, 2021). This often diminishes mathematics from an experiential participation in the discipline of mathematics to a set of procedures by which students gauge their competence and self-worth based on the speed and ease of effort that they can complete these procedures. This has many effects on the outcome of student learning but two are pervasive in students' life-long learning of mathematics. First, students assign their competence in mathematics as something fixed as is evident in the pervasive concept of being a "math person" or not typically based on performance in school-based testing and rote memorization (Boaler, 2022). Secondly, students generally perceive the learning of mathematics as a boring and anxiety inducing activity characterized by rule following that is irreverent to their lives in its symbolic abstraction and in which they have little appreciation or enjoyment (Jaworski, 2012). The prevalence of math anxiety in classrooms and society is undeniably a result of this performative expectation, and there's a growing body of research that demonstrates the negative consequences on students' mathematical learning and wellbeing (Parks, 2020; Ramirez et al., 2016; Singh, 2021). Jaworski, in her writing about mathematics joy and rigor, claims that this is directly related to the school-based mathematics culture and social norms, noting that "we see pupils who are alienated due to the intrinsic nature of mathematics and pupils who underachieve in mathematics because of the practices and social norms within schools where they are taught" (Jaworski, 2012, p. 14). This culture of alienated mathematics is harmful to the engagement of mathematics both in schooling but also across the lifespan.

Mathematics as objective and rigorous

One prevalent ideological oppression and misrepresentation of mathematics is that mathematics is objective and thus necessarily devoid of emotion. Because mathematics is viewed as objective it is often seen as unrelated to emotions or morals. It is also portrayed as the conveyor and authority over "truth" which allows it to maintain an elite status as superior to other fields of intellectual thought and also exclusive to an elite few (Christensen et al., 2008; Gutiérrez, 2013; Volmink, 1994). This elevates mathematics as a field which values objective truth and mental acuity above all else and as a result suspects emotional connection (hooks, 2003). In a chapter of her book *Teaching Community: a pedagogy of hope* titled "Heart to Heart," bell hooks notes that this adherence to the ideology that mathematics is the arbiter of objective truth leads in application to educators perpetuating oppressive conformity and the negation of the need for community based on the prevalence of competition (hooks, 2003). But even in her critique of objectivism and the resistance of emotion in education, hooks echoes the belief that this is suitable for disciplines like mathematics saying, "While objectivism can work well in hard sciences and more fact-oriented subjects, it cannot serve as a useful basis for teaching and learning in humanities classrooms. In these classrooms much of what students seek to know requires engagement not just with the material but with the individual creators whose work we study" (hooks, 2003, p. 129). Following bell hooks' lead, I argue that it is beneficial to learn mathematics by inviting the creators of mathematics, the tools and concepts they built and discovered through their creativity and humanity.

This opens the discipline of mathematics to diversity of histories, thought, and co-creation as we recognize the multifaceted nature of the discipline that has historically leaned into cultures and systems that privilege only some ways of thinking (Kumashiro, 2001). Inquiry that invites humanity and history also inherently evokes emotion that not only resists objectivism but also provides space for community and nurturing toward lifelong interest and enjoyment of mathematics. The learning sciences have increasingly invited research into the importance of positive affect, disposition, and identities in mathematics learning (Esmonde, 2009; Gresalfi, 2009; Gresalfi & Cobb, 2006; Langer-Osuna, 2011; Sengupta-Irving & Enyedy, 2015). These studies demonstrate that we invite these affective qualities of mathematics learning not just to develop proficiency for testing and achievement (although they work as a means to that end as well) but also so that educators and students continue engaging in mathematics over their lifespan. This is necessary if we wish to develop students who become proficient, critical, and thoughtful democratic citizens.

The misrepresentation of mathematics as objective and devoid of emotions also leads to an oppressive ideology that mathematics is rigorous and thus too serious to allow space for joy. Anecdotally, in my own teaching experience, teachers and student would occasionally drop into my classroom which was a space that allowed for passionate discussion and (on good days) joy in mathematics learning. I would get critiqued by students for not having enough “control” of their peers and criticized by my department head for not engaging students in rigorous mathematics as was evident to him by our lack of seriousness. But, as a learner and teacher of mathematics, I knew this dichotomy to be not only false but also harmful to students’ engagement in mathematics.

Barbara Jaworski, emeritus professor in Mathematics Education at Loughborough University, writes extensively about this false dichotomy in her article titled *Mathematics - joy and rigor*. In this piece, Jaworski characterizes rigor as relating to mathematics being powerful, concise, and unambiguous due to its logical consistency, expression of generality, and use of abstraction to capture complex ideas, patterns, and relationships (Jaworski, 2012). Similarly, Dr. Joseph, a mathematics educator and researcher of Black girls engagement in mathematics, defines rigor as Black girls’ development of new knowledge through investigation, meaningful discourse around the mathematical concepts, use of multiple strategies to solve challenging problems, and engagement in mathematical argumentation through justification and reflection (Joseph, 2021, p. 93). Notably, neither of these definitions of rigor relate to mathematics being a set of indelible, undebatable, objective truths or a set of stripped-down rules and procedures to follow – quite the opposite in fact. Jaworski claims that stripping down mathematics to procedures and rules to follow leaves it empty of its meaning, passion, and joy. Therefore, one of our central challenges as math educators is to invite students into this rigor in ways that evoke curiosity and challenge, both of which she argues capture components of rigor and joy. Jaworski proposes that joy could come from pleasure in the collaboration with others about a problem which you are collectively curious, the challenge of developing an argument with others and the puzzling that results from engaging with and exploring mathematical concepts and seeing inside mathematical relationships (Jaworski, 2012). Essentially, joy emerges when we are drawn into the rigor of mathematical discussion, collaboration, and argument in ways that inspire curiosity and challenge for the teacher and the student.

We see here that formal mathematics and the ideological fallacies that have been perpetuated by the culture of mathematics education have act to suppress joy in mathematics and thus exclude people from enjoying productive engagement in the discipline. But the exclusion of students from the joy of mathematics and the oppressions that educators must resist are not merely ideological and not equally distributed to every student or group of students. Oppressions are also systemically built into the practices of mathematics classroom learning, curriculum design, assessment and evaluation standards, and culture of learning and engagement especially for minoritized of students. Research has extensively demonstrated how these forms of resistance have resulted in the exclusion of particular groups of students, primarily students of color, from joyfully engaging in classroom-based mathematics learning and thus mathematics as a discipline.

Mathematics as exclusive

Mathematics has been historically exclusive to those who wield power in our society which means that there has been significant harm done to minoritized students who enter against dehumanizing forms of power such as racism and sexism. Louie calls the influence of these practices in the dominant, restrictive and hierarchical culture of mathematics education the "culture of exclusion" (Louie, 2017). This hierarchical violence against students in math education deepens our understanding of the dehumanizing nature of formal mathematics learning and further necessitates urgent change in the field. In response to the recognition of the dehumanizing nature of formal mathematics education, educators and researchers have called for humanizing mathematics "often in service of addressing long-standing inequities, by shifting away from contextless portrayals of mathematics to reveal its social and cultural dimensions" (Su, 2020, p. 11). In acknowledgement that this dehumanization specifically targets groups of students based on race, Goffney, Gutiérrez, and Boston have focused their work on rehumanizing mathematics specifically for children of color in their book, *Rehumanizing Mathematics for Black, Indigenous, and Latinx Students* (Goffney et al., 2018). Martin, Gholson, Joseph and other prominent scholars focus much of their research on the ways in which mathematics as a discipline and as a formal structure of education have negatively influenced the lives of Black children as well as the often under researched and underappreciated mathematical brilliance of Black children (Gholson, 2013; Gholson & Martin, 2014; Joseph, 2021; Joseph et al., 2019; Joseph & Jordan-Taylor, 2016; Leonard & Martin, 2013; Martin, 2009a, 2009b, 2009c; Martin et al., 2019). The anti-Blackness evident in education and specifically in mathematics education is systemic violence inflicted by an institutional practices and procedures that "adversely impacts on disadvantaged individuals or groups by burdening them psychologically, mentally, culturally, spiritually, economically, or physically" (Martin et al., 2019, p. 36). This violence is enacted through formal school policies and practices such as curriculum design, assessments, instruction, pedagogical practices, and disciplinary norms to serve antiBlackness and maintain the script of White supremacy through their oppressive and exclusionary effect on the lives, stories, and participation of Black children (Martin et al., 2019). This includes performance markers and expectations of productive output that invite students into mathematics almost exclusively with the role of listening compliantly, following precise directions, and computing answers to procedural problems with speed and precision (Boaler, 1998; Louie, 2017) in ways that have actively acted against students' prolonged interest and engagement in mathematics over the lifespan (Singh, 2021). Joseph and Hailu explain this systemic exclusion specifically of Black girls from mathematics by saying that "mathematics is often constructed as a White, male, and exclusionary institutional space" as evidenced by "Black girls' and women's low participation and achievement in mathematics" (Joseph et al., 2019, p. 135). This race-based exclusion is also evidenced more broadly in formal education by Black students' disproportionately high referral to the office, for suspension or expulsion, to special education and Black students' disproportionately low referral to gifted education (Milner et al., 2021). Black students therefore face uniquely dehumanizing personal experiences in their formal mathematics education. This arguments can be broadened from the individual and school/community level to the societal level by saying that mathematics has historically been a tool of colonialism and imperialism by its control of power based on defining who is capable to engage in thinking mathematically about the world (Bishop, 1990; Kumashiro, 2001).

Thus far we have placed the responsibility for Black students' oppression in mathematics on educators, but defining who is capable of mathematics and thus who has access to mathematics as a discipline has largely been the responsibility of researchers and educational policy makers who have perpetuated narratives like the achievement gap between white students and students of color (Gholson, 2013; Martin, 2009a). Research and academic discourse about Black children in mathematics based on comparative data associates race with a hierarchy of mathematical ability with Black students consistently at the bottom (Nasir et al., 2009a) falsely affirming long-standing racist stereotypes of Black students as lazy, anti-social, unintelligent, and not driven, and in doing so serve to devalue the lives and experiences of Black children (Nasir et al., 2009a). This deficit framing of Black students as incapable mathematics learners contributes to the narrative that Black mathematics students need to be saved from their Blackness or the narrative that Black students are expendable to mathematics as a discipline (Joseph & Jordan-Taylor, 2016). This positions math education and math education research as a "cite of degradation for Black students" (Joseph & Jordan-Taylor, 2016, p. 444) thus excluding them not only from mathematics education but also from the joy of engaging in mathematics more generally.

This is critical to investigate with respect to students' mathematics engagement because of students' exclusion from mathematics education generally and because of the dehumanizing exclusion from joyful opportunities to engage in mathematics. Wager and Parks highlight this in their research about the exclusion of students from playful mathematical engagement looking not only at race but also at gender, class, and ethnicity (Wager & Parks, 2014b). In this they note that research about playful mathematics has largely focused on gender differences in math play, but again often with comparative methods that ignore context and sociohistorical influences on playful learning and engagement. Thus, Wager and Parks note that there is often a deficit lens of these differences but also no consensus on these differences if there are any saying that "some of this work has documented significant differences along these lines of difference, while other work has found more similarities than differences" (Wager & Parks, 2014b, p. 222) across class, race and gender lines. They claim that the framing of research that does emphasize difference "when contradictory evidence is widely available, is problematic because it works to reinforce stereotypes of girls as unfit for certain kinds of mathematics" (Wager & Parks, 2014b, p. 222). We see, as with comparative research conducted about race, that this kind of research that finds differences in assessment scores, kinds of play, "school readiness" and attributes those differences to gender, race, class, etc without considering the context or social and historical influences merely perpetuates deficit narratives of belonging with respect to both mathematics generally and especially joyful mathematical learning.

In reflecting on these forms of exclusions, I must consider for whom I'm writing about joy as a form of resistance to oppressive systems of formal mathematics education. Because Black children and specifically Black girls have spent generations under the unique oppression of and isolation from formal disciplinary mathematics, this work is for them and for their mathematical joy. Indeed, "mathematics education cannot truly improve until it adequately addresses the very students who the system has most failed" (Goffney et al., 2018). Because math education is often a generally harmful place for the majority of students, writing about how to design for joy could in turn benefit children from all backgrounds engage in mathematics joyfully. But let's not make this an example of "*interest convergence,* which suggests that members of the dominant group will promote interests for African Americans only when they promote dominant group interests" (Martin, 2009a, p. 18). Because Black children and children of color have suffered uniquely, the findings of this study are ultimately for them, recognizing their unique embodied oppression of joy and seeking potential ways to design for their liberatory engagement in mathematics.

"Resistance is the Secret of Joy" in Mathematics

This maladaptation of goals in formal mathematics education and the resulting oppression on children subjected to it demonstrate the need for and challenge of designing to support students' joy.

Working within a system that oppressions children as they learn mathematics necessitates that we resist those oppressions – especially when designing for students' joy. The bridge between resistance to oppression and joy has already been expertly crafted by generations of Black Feminist Theorists (Collins, 2000; Cooper, 2017; Walker, 1992). Because the work of resisting dehumanizing education is largely crafted for students of color who have been historically marginalized in this space, I have been inspired by the work of Black Feminist Theorists to further explore the claim that joy will emerge when we attend to humanity and resist dehumanizing social constructs and practices in mathematics.

Alice Walker and her novel Possessing the Secret of Joy proclaims "RESISTANCE IS THE SECRET OF JOY!" (Walker, 1992, p. 279). The concept of joy as a form of resistance is deeply rooted in Black history in the United States and has been experienced and explicated throughout generations of Black culture in opposition to oppression through storytelling and song (Lu & Steele, 2019), Black musical traditions such as spirituals, jazz and blues (Morris, 2019; Resistance Revival Chorus - This Joy (Album), 2020), education as a tool for liberation (Brown, 2013; Collins, 2000; Cooper, 2017; Gholson, 2013; Joseph, 2021; Love & Muhammad, 2020), community "uplift" through political and social activism and the empowerment (Collins, 2000, p. 210; Conaway & Waters, 2007), and through the constant advocacy for the recognition of Black people's inherent dignity (Cooper, 2017). Brittany Packnett, a prolific modern-day activist, educator, and writer, writes that her personal mantra is "Truth, love, and yes, joy, are resistance" (Packnett, 2017). She goes on to say that "joy is a moral victory against extremism and a political win, fueling us to persist and resist. Joy is resistance to the hate that fills the front page" (Packnett, 2017). The insistence of Black joy in the face of dehumanizing oppression is crucial to understanding the human experience as we collectively seek to bring justice and lift the burden of oppression in the world today.

The primary political belief of Black Feminist Theory is that "Black women are inherently valuable" (Combahee River Collective, 2019, p. 31) and that Black women's liberation is a necessity because of their personal need as humans for freedom and autonomy, not as conditional to someone else's freedom. Thus, Black Feminist Theory is the production of intellectual thought for Black women in response to oppression through empowerment to stimulate resistance (Collins, 2000) with four ideological tools: inherent dignity, embodiment, intersectionality, and *affect*.

Bringing one's whole humanity into political activism and intellectual thought requires invoking affect. Affective politics as a humanizing theme in Black feminist theory insists that emotion is inseparable from participation in a community (including a political one). Black Feminist Theorists, beginning with Fannie Barrier Williams (1855-1944) place Black women's emotions about racism and white supremacy at the center of their political theorizing because it is necessary to understand how intersecting oppressions influence how Black women feel, think, and take action in the world (Cooper, 2017). Black Feminist Theory consistently demonstrates that affect and rationality are not a dichotomy and that the empowerment that emerges from allowing for and supporting joy and emotion in the production of knowledge that "flows from 'a big love' flies in the face of Western epistemologies that often see emotions and rationality as different and competing concerns (Collins, 2000, p. 167). In this regard, rather than relying on the objective and individualistic nature of knowledge found in Western epistemologies, Collins outlines Black Feminist Epistemology which defines Ethics of Caring as one of its four dimensions of Black Feminist Epistemology. Collins states that "the ethic of caring suggests that personal expressiveness, emotions, and empathy are central to the knowledge validation process" (Collins, 2000, p. 263), making emotionality and intellect inseparable.

Therefore, we draw from the power and conceptual development of the work of Black feminist theorists to better understand the coupling of resistance and joy. This is with the hope that we can design for and support joy in mathematics learning and knowledge construction as a resistance to oppressive formal mathematics education outlined above and in hope for the uplift of a more empowered and affective community of mathematical learners. As mentioned earlier, this work is motivated by the wisdom and insights of Alice Walker who builds her philosophical understanding of resistance being the secret of joy in her work as a Black Feminist Theorist. The legacy of Black Feminist Theory is uniquely positioned to theorize about joy as a form of resistance to systemic oppressions because "U.S. Black women have historically produced social thought designed to oppose oppression" (Collins, 2000, p. 9). It is important to see this lens as a "partial standpoint perspective" (Collins, 2000) that does not offer a more perfect or truthful understanding of joyful resistance, but rather it provides a unique standpoint from which to view the world and tell a story that I, as a white woman, do not personally have access to. Therefore, I have the opportunity to learn from Black feminism to humbly investigate how race, gender, class, sexuality, and other social identities come together and influence how we experience injustices in society as well as the powers that inflict those injustices and the modes of empowerment that can be taken up to resist those oppressions. Thus, this work will be built on the long legacy of Black Feminist Theorists who can shed light on a deeper and more complete understanding of the intersection of joy and oppression through the lens of Alice Walker's poetic insight into resistance being the secret of joy (Walker, 1992).

Mathematical Joy

The desire for joy resonates with each of us on a fundamentally human level. Regardless of its universality, our varied human cultures, experiences, languages, expressiveness, and life circumstances make descriptions of joy multifarious. In literature, joy is described in relation to the affective evidence of joy in positive emotional expression (happiness, delight, pleasure, satisfaction, laughter, amusement, excitement, relief, celebration), foundational pillars of joy (humility, humor, acceptance, forgiveness, gratitude, compassion, and generosity (Lama et al., 2016)), synergistic dispositions for joy (perseverance, freedom, resistance, creative initiative, power to love (Volf & Crisp, 2015)), or emergent descriptors of joy (generative, courage, play, persistence, empowerment, wonder, resilience, defiance, connection, self-expression, hope, and presence (Parks, 2021)). While these words are helpful in the "mapping of the kingdom of joy" they also "convey its complexity and its subtlety" (Lama et al., 2016, p. 49).

Thus, I define mathematical joy here both for clarity of argument and collective understanding between myself and you, the reader. Amy Parks in her work about designing for joy in PK-grade 2 mathematics classrooms evokes a definition of joy from His Holiness the Dali Lama and Archbishop Desmond Tutu in their book *The Book of Joy* saying that "Being joyful is not just about having more fun. We're talking about a more empathetic, more empowered, even more spiritual state of mind that is totally engaged with the world" (Lama et al., 2016, p. 63; Parks, 2020). Together the Dali Lama and Archbishop Desmond Tutu define joy as a state of the heart and the mind that animates our lives and leads to satisfaction and purpose born from the coupling of "deep well-being and benevolence" (Lama et al., 2016, p. 49). Muhammad, in her book *Unearthing Joy* (2023) about finding joy through culturally and historically responsive curriculum and instruction, echoes the insufficiency of emotion as an indicator of joy, and foregrounds the importance of personal well-being, empathy, benevolence, and entanglement with the world in supporting joy. She defines joy as "not a fleeting feeling of happiness, but a sustained sense of fulfillment and self-determination, self-liberation, and self-empowerment" (2023, p. 100) characterized by loving self and humanity, caring for others and the earth, truth, beauty, art, wonder, wellness, personal fulfillment, working for social change. According to these insights, joy is characterized as: 1) a sustained *state of being* of the heart and mind and 2) leading to *affective, animated demonstration of satisfaction* and fulfillment through self-determination, meaning, and purpose, which is 3) supported by and grows out of the coupling of *inward-facing deep, personal well-being* and *outward-facing empathetic, curious, altruistic, invested care for and wonder about the world*. In the following chapters, I refer to the inward-facing deep personal well-being as a coupling of mental and physical well-being. Whereas this outward-facing empathetic, invested care for and wonder about the world, and self-forgetful attention to good, beauty, play, exploration, beauty, liberation in the world all around (Ortlund, 2023; Su, 2020), I will refer to as the combination of relational, societal, and intellectual well-being. Mathematical joy born from these multifaceted forms of well-being may look like learning mathematics through embodied movement, using mathematics in liberatory efforts for the good of others, but it could also look like losing yourself in the beauty of number and shape, pattern and symmetry, chaos and asymmetry, abstraction and structure. Therefore, *mathematical joy here is defined as a positionality toward mathematics that animates engagement and leads to feelings of satisfaction and purpose born from the coordination of mental, physical, relational, societal, and intellectual well-being.*

In research about joy in the literature about young children, we see joy emerge through play, agency, exploration, creativity, empathy, and engagement. There is a heavy emphasis on play and agency in the literature about younger students' affective engagement in mathematics. Wager and Parks in their writing about learning mathematics through play, define play according to Burghardt's five characteristics of play, focusing on it being functional in context, spontaneous, pleasurable, rewarding, voluntary, unserious with respect to its form and timing, repeated, and initiated in the absence of stress (Burghardt, 2011, p. 17; Wager & Parks, 2014b). Interestingly, this paper doesn't define play with the word joy specifically, but it does talk about pleasure, non-seriousness, and absent of stress, aligning with positive affective results of play that would indicate joy. In their work about the joy found in creativity, autonomy, and play in STEAM education Hunter-Doniger defines play as "the serious work of children in which they make meaning of the world through engaging activities that are fun and bring them joy (Hunter-Doniger, 2021, p. 24), again relating the definition to play to that of joy.

In the literature about older students, affect and joy are often associated instead with disciplinary interest and positive identity development. Sengupta-Irving and Enyedy, in their study about designing for student-driven and highly-guided inquiry in a contrasting case study of two 5th grade classrooms, focus on developing students' disciplinary interest which they characterize by increased attention, concentration, and affect. In this study, with respect to affect, they collapse difference in psychological constructs of interest that also align with concepts of joy such as "emotion, affect, enjoyment" (Sengupta-Irving & Enyedy, 2015, p. 554). In this study, Sengupta-Irving and Enyedy demonstrate that students who were assigned to the student-led inquiry class in which students engaged more richly in the Common Core Mathematical Practices (Common Core State Standards Initiative, 2022) also showed more positive affective response and interest toward the mathematics and equivalent improvement on mathematics assessments as compared to the group of students who engaged in highly-guided inquiry (Sengupta-Irving & Enyedy, 2015). Their findings counter the idea that restructuring the goal of mathematics education toward fostering students' interest and enjoyment might "soften" the hard sciences and lead to less rigorous instruction and demonstrate that the growth of ability vs the growth of interest is a false dichotomy (Singh, 2021, p. 9). In fact research has shown that developing student interest and engagement, especially when working in collaboration, leads to increased positive mathematical identity development and that affective and motivational factors of mathematical engagement are central to learning mathematics and continued long-term engagement in mathematics (Esmonde, 2009; Gresalfi, 2009; Gresalfi & Cobb, 2006).

While much of the research about joy in mathematics focuses on play in younger children and disciplinary interest in older children, literature focusing on joyful mathematics learning is increasingly focusing on joyful engagement across the lifespan. This is evident in Su's book, Mathematics for Human Flourishing, where he quotes G.K. Chesterton as saying, "It might reasonably be maintained that the true object of all human life is play" (Su, 2020, p. 49). Su emphasizes the importance of engaging in playful mathematical inquiry, defining play as an "activity engaged in for enjoyment or recreation" but also attuning to specific characteristics of play such as it being voluntary and meaningful and including concepts of structure, freedom, exploration, surprise, imagination (Su, 2020, pp. 49-50). Therefore, as we investigate mathematical joy, we are not merely looking for positive affect, but also engagement in humanizing acts of freedom, ingenuity, imagination and meaning-making and the beneficial virtues that emerge from engaging in those acts. Su claims that play in mathematics "builds hopefulness, curiosity, concentration, confidence in struggle, patience, perseverance, the ability to change perspectives, an openness of spirit" (Su, 2020, p. 61) all of which are not only useful in our everyday lives but also specifically in our disciplinary mathematical endeavors throughout life.

Conceptualizing joy in the math education research as disciplinary interest, exploration, creativity, autonomy, curiosity, meaning-making, and perseverance makes the choice between traditional oppressive forms of mathematics education and joyful modes of learning mathematics seem obvious. Park even notes, "The amazing thing about choosing joy in each of these cases is that the choice also leads to richer and more productive mathematics. So why not choose joy?" (Parks, 2020, p. 63). Hooks notes traditional pedagogies have convinced us that "the classroom ought to be a place where things are said seriously—not without pleasure, not without joy—but seriously, and for serious consideration" (hooks, 1994, p. 150) but from her experience and based on her principles of liberatory pedagogy, "it's important to remind students that joy can be present along with hard work" (hooks, 1994, p. 154). Indeed, joy and the messy work of learning go hand in hand as Singh beautifully states, "our best math moments often occur when joy intentionally selects confusion as a dance partner" (Singh, 2021, p. 4). Singh refers to this as having joy in the muck of learning and makes clear that delight in mathematics often comes when we are in the uncertain, mucky, beautiful, and life affirming space between question and answer (Singh, 2021).

Conclusion

I have made the case here for the need to resist the performative goals, policies, practices, and social norms that exclude many students, especially students of color, from joyful mathematical engagement. This study proceeds with the goal of reorienting formal mathematics education away from dehumanizing forms of mathematics education and toward students' mathematical joy. This mathematical joy here is defined as a positionality toward mathematics that animates engagement and lead to feelings of satisfaction and purpose, born from the coordination of mental, physical, relational, societal, and intellectual well-being. To design for students' joy and reorient away from dehumanizing mathematics, we must draw from the wealth of research conducted about humanizing mathematics. Indeed, for mathematics to be joyful it must be "filled with moments of being human - warmth, compassion, empathy, and kindness. Humanization is all that is left. Mathematics must reclaim it to reimagine it" (Singh, 2021, p. 212). Therefore, in the conceptual framework outlined in the next chapter, I take up pedagogical models and research-based educational practices that center humanizing education through the lens of this definition of mathematical joy in order to outline potential ways that the field of education has provided potential ways to design for students' mathematical joy.

Chapter 2: Conceptual Framework

The goal of this study is to better understand how we can support students' joyful mathematical engagement in formal mathematics learning spaces. The previous chapter made clear that formal mathematics classrooms have generally, historically not been spaces of joy for students because of the systemic oppressive ideologies, norms, and practices that dehumanize children. With inspiration from Black Feminist Theorists, this design work proposes that resistance to students' dehumanization is the secret of joy. Therefore, this work takes up a conceptual framework of humanizing mathematics. Mathematical joy here is defined as a positionality toward mathematics that animates engagement and leads to feelings of satisfaction and purpose *born from mental, physical, relational, societal, and intellectual well-being*. I propose here that by attending to students' multifaceted well-being we provide space for students' full and complex humanity as they engage with the world. Reformed, critical, and humanizing pedagogies are characterized by attending to students' full humanity in a multiplicity of ways. In *Rehumanizing Mathematics,* Gutiérrez (2018a) claims that "a critical aspect of evidence that mathematics is more rehumanized is that it conjures up feelings of joy" (p. 5). This empirical study requires a practice-oriented bridge between the work of humanizing mathematics and the mathematical joy that Gutiérrez claims will be evidence of (re)humanizing mathematics. In this chapter, I take up pedagogical models and research-based educational practices that center humanizing education through the lens of our definition of mathematical joy. Because this definition proposes that joy is born from the coordination of mental, physical, relational, societal, and intellectual well-being, I build here a conceptual understanding of multifarious ways that the field of humanizing education has proposed supporting these diverse aspects of students' well-being. These insights provide metaphorical map of the landscape of the field of humanizing education with which we can explore some of the well-paved boulevards and less traveled dirt trails in our peregrination seeking to design for students' mathematical joy.

Humanizing Mathematics

In recent years, humanizing mathematics education has received significant attention (Goffney et al., 2018; Gutiérrez, 2018b; Mukhopadhyay & Greer, 2015; Singh, 2021; Su, 2020; Yeh & Otis, 2019) and is characterized by emphasizing the importance of attending to students' full humanity. In Chasing Rabbits: A Curious Guide to a Lifetime of Mathematical Wellness, Singh proposes that redesign mathematics education "that explicitly ties into the recognizable pillars of general human well-being: physical, mental, emotional, spiritual, social, and environmental" (Singh, 2021, pp. 11-12). Joseph, designer of Black Feminist Mathematics Pedagogies, proposes structuring mathematics learning spaces that center Black girl's full humanity as "a composite of their personal experiences, backgrounds, histories, languages, intellect, personalities, bodies, and physical and emotional well-being" (Joseph et al., 2019, p. 133). In *Mathematics for Human Flourishing*, Su proposes that in order to support students' human flourishing we must also attend to fulfilling students' human desires for exploration, meaning, play, beauty, permanence, truth, struggle, power, justice, freedom, community, and love (2020). Gutiérrez's framework rehumanizing mathematics education proposing eight dimensions of rehumanizing mathematics: participation/positioning, cultures/histories, windows/mirrors, living practice, creation, broadening mathematics, body/emotions, and ownership (Gutiérrez, 2018a). Each of these cornerstone pieces of literature about humanizing mathematics denotes the complex, compositional nature of our humanity and the entanglements of our personal well-being with the world around us. Indeed, to attend to the full humanity we must attend not only to our own mental and physical health but also to our outward-facing self-forgetful, curious, altruistic engagement with people, literature, music, nature, art, mathematics, communities, issues of social justice, cultures – both the minute and universal facets of the world. Therefore, I propose here that by attending to students' full and complex well-being, we provide space for students' multidimensional humanity as they engage with the world, and thus support their joyful mathematical learning.

Despite Gutiérrez's claim that rehumanized mathematics will lead to joy, few studies about humanizing pedagogy have explicitly attended to affect. Black Feminist scholars most notably highlight the critical relationship between humanizing and liberating pedagogies and joyful affect (hooks, 1994, 2003; Morris, 2022; Muhammad, 2023). These scholars posit that if we wish to make schools more joyful places for learning - especially for students who have been most harmed and denigrated by educational systems - schools must be places of restoration and freedom (Morris, 2019). Black Feminist education scholar, bell hooks, claims that the call for students' freedom and well-being is "potentially disruptive of the atmosphere of seriousness assumed to be essential to the learning process" and "could co-exist with and even stimulate serious intellectual and/or academic engagement" (hooks, 1994, p. 7). In practice and in principle, stimulating affect (what hooks calls excitement) depends on humanizing pedagogy which critically makes space for the collective effort of community, personal satisfaction and wellness, interest in relationships with one another, recognition of one another's voice, and the asset-based, radical pedagogy that insists on the acknowledgement of one another's value, and human dignity (hooks, 1994, p. 8). Therefore, to attend to joy by resisting dehumanizing practices in mathematics, we turn to the ways that researchers and practitioners have designed formal mathematics education opportunities and environments to be more humanizing.

This study draws from any research-based practice-oriented literature about education that centers students' humanity. This includes asset-based and critical pedagogies across disciplines are characterized by centering students' full humanity. This includes, for example, research about funds of knowledge (Moll et al., 1992), antiracist pedagogy (Dunn & Love, 2020), Culturally Relevant Pedagogy (Ladson-Billings, 1995a, 1995b, 2014), Culturally Responsive Teaching (Gay, 2002, 2010), Culturally Sustaining Pedagogy (Paris, 2012; Paris & Alim, 2017), Opportunity Centered Teaching (Milner, 2021; Milner et al., 2021), Black Feminist Pedagogy (Omolade, 1987), and liberatory pedagogies (Morris, 2019). In their own ways, liberatory and other critical pedagogies seeking to resist dehumanization by recognizing leaner's full humanity and fundamentally seeking justice thus fundamentally attending to students' joy. I also draw from mathematics education research which has developed critical, equity-oriented pedagogical models specific to the discipline such as Complex Instruction (Cohen et al., 1999; Rubel, 2017), Mathematics for Social Justice (Gutstein, 2012), healing-informed social justice mathematics (Kokka, 2019), Black Feminist Mathematics Pedagogies (Joseph, 2021), play-based mathematics learning (Parks, 2015; Wager & Parks, 2014a), Rehumanizing Mathematics (Goffney et al., 2018; Gutiérrez, 2018b), as well as literature explicitly about joyful math learning (Hunter - Doniger, 2021; Jaworski, 2012; Parks, 2020). All of these humanizing models, as they center the lives and well-being of students, reasonably contribute to making space for students' joy.

Designing for Mathematical Joy through Well-Being: Connecting Theory to Practice

A conceptual framework built on humanizing mathematics is constructed here as a foundation to design for and better understand students' joyful mathematical experiences. Ladson-Billings (2014) emphasizes the importance of the relationship between theory and practice by saying that if we want more robust and effective pedagogical models then "our pedagogical practice has to be buttressed with significant theoretical grounding." Because of our theoretical definition of mathematical joy as being born from multifaceted well-being, this work will conceptualize the literature of humanizing pedagogical frameworks with respect to well-being in mathematics education. Indeed, in order to humanize mathematical engagement, we must attend to students' well-being (Gutiérrez, 2018a).

Student's well-being is not historically emphasized in mathematics education. Evidences of a shift toward wellness have been the publication of Su's book *Mathematics for Human Flourishing* (2020) and Singh's book *Chasing Rabbits: A Curious Guide to a Lifetime of Mathematical Wellness (Singh, 2021)*. These books support the notion that attending to students' well-being supports students mathematic joy because it allows for the fulfillment and satisfaction of the full range of their personhood and in doing so, allows for them to engage in mathematics more deeply and meaningfully. This could involve attending to students' mental health in the math classroom, structuring lessons to encourage students to freely move and learn math with their bodies, curating a culture of mutual relational care and collaboration, providing intentional opportunities for students to develop sociopolitical consciousness and empathetic problem-solving social justice activism, or supporting their curious exploration of mathematics puzzles and problems. As noted above, attending to our full humanity involves satisfying a complex entanglement of our mental, physical, relational, societal, and intellectual well-being. The literature below is synthesized through the lens of these facets of students' well-being.

Mental Well-being

Acknowledging and attending to students emotional states and mental well-being requires attention to students phycological and mental health (Milner, 2021; Milner et al., 2021) and healing from trauma, especially trauma caused by educational institutions and systems (Kokka, 2019; McGee & Stovall, 2015). This fundamentally resists the dehumanizing view of mathematics as objective and detached from emotion. Gutiérrez claims that "by attending to emotions, individuals would be encouraged to be more in tune with themselves and less likely to succumb to pressures to ignore their senses and 'just pretend' in order to do school mathematics" (Gutiérrez, 2018a, p. 5). This requires a shift in institutional practices and allocations of resources such as responsive, mental-health programs, humanizing and community-oriented pedagogical practices, culturally relevant curriculum, and the removal of police from schools (McGee & Stovall, 2015; Morris, 2019). This also requires specific attention to the formation of caring relationships which is critical to students' academic well-being and their emotional and psychological healing and well-being (Morris, 2019).

Physical Well-being

Gutiérrez attends to inviting students' liberated bodies into the mathematical learning space, resisting the ideological oppression that mathematical is strictly a logical or mental endeavor by honoring students' bodies through the incorporation of movement, voice, touch, intuition, and vision into learning environments (Gutiérrez, 2018a). Embodied cognition in mathematics supports this in proposing that learning cannot be fully understood by only attending to social or contextual factors, but that we as a field must also attend to learning that is "embodied within a shared biological and physical context, and to examine the ways in which this embodiment helps to determine the nature of mathematical understanding and thinking" (Núñez et al., 1999, p. 46). The inclusion of students' bodies as integral to the learning experience also requires us to remove harmful practices that seek to control students' bodies like physical, militant compliance and tracking (Martin et al., 2019; Wells, 2023).

Relational Well-being

Social interactions within a learning community define the ways that learning occurs in that space. Therefore, to design for joyful learning, it is crucial that we consider the ways that relationships within the learning community support more humanized learning opportunities. The criticality of relational intimacy in mathematics is emphasized by Su (2020) who summarizes his book, *Mathematics for Human Flourishing*, by focusing on the "love one might have for another human being through and because of mathematics" and claiming that love "is the greatest human desire, for it serves all the other human desires - for exploration, meaning, play, beauty, permanence, truth, struggle, power, justice, freedom, community - and love is served by them" (Su, 2020, p. 205). Rather than promoting individual academic gain, foundationally, more humanized learning is designed by teachers who intentionally foster positive, supportive, and collaborative relationships within a community of learners who demonstrate responsibility for one another in academic collaboration and social construction of knowledge (Gay, 2010; Joseph et al., 2019; Ladson-Billings, 1995b). This co-construction of mathematical ideas requires us to attend to how students are positioned with respect to each other and to mathematical practices (Lave & Wenger, 1991; Sfard, 1998) and reframe diversity of thought as an asset to collective knowledge construction (Cohen & Lotan, 1995). This "involves recognizing hierarchies in classrooms and society and shifting the role of authority from teacher/text to other students" (Gutiérrez, 2018a, p. 5). Thus, co-construction of knowledge and opening up of mathematics to explore potential otherwise unconsidered mathematics concepts and relationships requires a redistribution of power that makes clear that everyone's voice and intellect is valuable (Jaworski, 2012; Joseph et al., 2019). Gholson and Martin (2014) attend to the complex social nature of mathematics schooling for Black girls in their study that investigated the classroom culture designed and developed by Ms. Robinson and her students. This classroom was designed to support active learning, ownership, autonomy, music, rigor, choice, relational intimacy, community, communication/voice, collaboration, and membership. They note that social dynamics and networks were instrumental in shaping mathematical learning opportunities and thus identity development with respect to these Black girls' academic, mathematical, and racial identities in that the social roles of students determined the extent and the dynamics of their participation (Gholson & Martin, 2014). This development of community and deep supportive relationships in mathematics allows for students to face more interesting and complex problems, come up with more diverse solutions, construct viable arguments, and critique the reasoning of others (considered crucial to math participation by Common Core Math Practice Standards) with joy (Parks, 2020).

Storytelling is also a significant component to allowing space for students to value one another in humanizing pedagogies. Singh makes the bold claim that "our best voices are those we use when we tell stories" (Singh, 2021, p. 21) because "the emotion - the essential humanity - of mathematics, lies in its stories" (Singh, 2021, p. 78). Not only does storytelling, as Singh indicates, provide affective ways of making meaning of the world it also reminds us that our history, in this case, the history of mathematics is a humane and collective endeavor and that without stories and their associated humaneness, there is no hope for mathematics to be anything except an "abyss of dried-up ideas" (Singh, 2021, p. 151). Wager and Parks emphasize the importance of storytelling as a way to motivate mathematical concepts in play through problem posing and storytelling that focuses on mathematics concepts and thinking. They propose that this has the potential, with careful observation and support, to support children's mathematical play by initiating introductions to concepts that may emerge later to allow for students to take up these ideas more readily and eagerly (Wager & Parks, 2014). Storytelling also works to overcome implicit bias of minoritized groups in mathematics if there is a lack of stories told about mathematics history, those who influence it, as well as our own personal experiences (Su, 2020). Kumashiro relates this storytelling to anti-oppressive education which he claims involves looking beyond what it is we teach and learn to expand our curriculum to include the stories typically untold that add to the contributions and practices of mathematics. This challenges the social norms that inform what we have come to perceive and teach as objective "truths" in mathematics and instead affirms students' identities, knowledge construction, and ways of engaging with mathematics (Kumashiro, 2001).

Societal Well-being

Applying mathematics to solving “real-world” problems including those that involves students’ community and social activism is one of the most well-researched components of reformed mathematics education (Aronson & Laughter, 2016; Gutstein, 2012; Kokka, 2019; Muñiz, 2019). Contextualized mathematics that is embedded in students lived experiences has the potential to stimulate curiosity and offer deep challenge that stimulates deeper, serious, and enjoyable engagement with mathematics, but these “real world” contexts can fall short of being authentic in their connection to students’ lives or they can lack fruitful generation of rich mathematical thinking (Jaworski, 2012). Authenticity of social connections to mathematics curricula requires critical reflection and action toward social justice and community activism that are genuine and relevant to students’ lives (Aronson & Laughter, 2016). Indeed positive relationships between students, teachers, families, and communities are often utilized to build a pedagogical, curricular, social, and academic anchor for students in school (Martin et al., 2019; Milner et al., 2021; Moll et al., 1992). Centering of communities and developing intimate relationships with them through mathematics is also deeply connected to intentionally shifting power dynamics between academic communities and social communities, especially communities of color because of the white-dominate exclusivity that has traditionally characterized mathematics learning. This is accomplished using inclusive curricula and activities to support the analysis of cultures represented and explicitly addressing oppressive systems both in the classroom as well as in society in the pursuit of social justice for all (Aronson & Laughter, 2016, p. 167). For example Teaching Mathematics for Social Justice (TMfSJ) positions students as doers of mathematics especially with respect to “using mathematics in school, as an educational practice to analyze and affect society” (Gutstein, 2012, p. 23). Contextualizing mathematics involves a complex design of students’ engagement in “a deep tackling of rigour alongside joyful engagement in mathematical argument that relates concepts and real world issues” (Jaworski, 2012, p. 21). In one of her five strategies for creating joy in mathematics learning, Parks proposes that offering problems that are meaningful for students provides the feelings of wonder and pride in mathematical problem solving which are potential sources of joy for students (Parks, 2020).

It is important to note the emphasis on reflective practices for the sake of community activism in these and other equity-based pedagogical models that by situating addressing bias and developing sociopolitical consciousness as central to disrupting inequities both inside and outside of schools by collaborating with families and local communities (Ladson-Billings, 1995b; Milner et al., 2021; Muñiz, 2019). In mathematical reflection and social action, it is important that teachers develop specific knowledge about mathematics, connect that knowledge with an understanding of their own privileges, the students they serve, and the social justice goals that they hold (Gutiérrez, 2018a). Joseph and Jordan-Taylor (2016), in their historical analysis of Black mathematics education, and the framework for Black Liberatory Mathematics (Martin et al., 2019) both emphasize that mastery of mathematical content is important, but it is also significant for teachers to deeply understand and be responsive to the social realities and identities of Black students in order to empower students to become agents of social change to oppressive systems and practices with mathematics. This is important for leading students in similar reflective and action-oriented work. Therefore, the concept of community activism goes beyond simply contextualizing mathematics in an aspect of students' lives to make meaning of the mathematics. Rather, it involves supporting, advocating for and having hope in the prosperity of a community – requiring empathy and care for a collective of people and their well-being. Justice and empathy are deeply intertwined in that "those who pursue justice in mathematics cultivate the virtues of empathy for the marginalized and concern for the oppressed… and builds in us a willingness to challenge the status quo" (Su, 2020, p. 161). Thus, not only is pursuing justice through mathematics meeting a basic need for our own flourishing, building up our community, improving society, but it is also making students more empathetic problem solvers who seek out opportunities for resistance for the sake of others and themselves.

Intellectual Well-being

Humanizing mathematics most often supports students' intellectual well-being through creative exploration in freedom of expression, co-creation of learning, productive exploration, and inquiry (Gay, 2010; Moll et al., 1992). Creative exploration in mathematics education is typically characterized by agency and autonomy through the "freedom or independence to move, play, think, or create…. when children exhibited confidence and determination in their actions" (Hunter-Doniger, 2021, p. 24). This freedom in exploration allows for creativity and play in both finding and solving problems by leaning into their natural curiosities. Allowing students to explore opportunities for what Morris calls "extended epistemologies - gives students' a chance to center their experiences, tell stories, dance, sing, write, meditate, and play their knowledge into a curriculum that recognizes and values these expressions as commensurate with other 'data' they will learn in school" (Morris, 2019, p. 127). Abolitionist Teaching "is built on the creativity, imagination, boldness, ingenuity, and rebellious spirit and methods of abolitionists to demand and fight for an educational system where all students are thriving, not simply surviving" (Love & Muhammad, 2020, p. 695). Thus, acting against injustices is coupled with fun and freedom and intellectual exploration is an expression of that freedom (Morris, 2019, p. 141). Research about humanizing mathematics importantly poses potential future research about creating space for play in mathematics – "a place for [Black girls] to be happy, gregarious, social, and 'goofy' (Joseph et al., 2019, p. 149) with evidence that the participants in the study were more engaged in the mathematics learning when allowed space for this playful creativity. This requires teacher to children the freedom to make choices in their learning including "how to spend our time, who to spend it with, what materials we can use, and how we hold our bodies" (Parks, 2020, p. 62). Much of the research about embracing supporting students' mathematical exploration, knowledge construction, and interest involves addressing the natural conflict between child-centered design and procedural compliance-focused structures of education (Hunter-Doniger, 2021; Lerkkanen et al., 2016; Paris & Lung, 2008).

Jaworski (2012) claims that reawakening and supporting joy and rigor together requires not only allowing for mathematical autonomy but also promoting students' mathematical inquiry, students' propensity to "ask questions and seek answers; to recognize problems and seek solutions; to wonder, imagine, invent and explore" (Jaworski, 2012, p. 27). Notably when designing tasks and learning activities, this requires that teachers also embrace multiple uncertain solution pathways, resisting the anxiety of mathematical perfection, embracing curiosity and failure in tandem and acknowledge that unpolished problem solving in “collective irresolution” is mandatory for teaching and learning mathematics (Singh, 2021, p. 35). This is characteristic of puzzles in that players tackle mathematical problems in a specific context, generate their own solutions and then defend those solutions in light of challenge from their peers that can generate the necessity for argumentation (Jaworski, 2012). This inquiry can also be seen in “creative power”, characterized by “stuff-making” and “sense-making” power (Su, 2020). One mechanism by which creative power is encouraged is through developing and allowing for multiple representations and developing multiple ways to understand a mathematical problem so that you have the power to choose which solution makes the problem solving easier based on the tools available (Su, 2020). The development of multiple representations encourages perspective taking and revision of mathematics ideas and the iterative process of producing and iterating strategies towards increasingly sophisticated mathematical representations, reasoning, and understandings (Enyedy, 2003, p. 555; Lehrer et al., 2002; Sengupta-Irving & Enyedy, 2015).

Conclusion

We can center students’ full humanity in mathematical learning spaces by implementing or attending to a wide range of pedagogical practices or facets of models that support students’ mental, physical, relational, societal, and intellectual well-being. Various pedagogical frameworks that prioritize humanizing mathematics, like Rehumanizing Mathematics (Goffney et al., 2018; Gutiérrez, 2018b), capture the full range of students’ humanity and thus provide insights about the various ways that we could potentially support students multidimensional well-being. Taking up these potential avenues for supporting students’ well-being, lays a foundation for students’ joyful mathematical engagement. In the following chapters, I build on this foundation to investigate what mathematical joy might look like for this group of 6th grade student and the unique conditions that could support their mathematical joy.

Chapter 3: Methods

Study Overview

This is an empirical, ethnographic study of mathematical joy. This study is situated in a year-long Research Practice Partnership with 6th grade mathematics teacher and 21 students in her 3rd block class. Through this collaboration, the teacher and I sought to learn about the students in this classroom and to design tasks to support their unique joyful mathematical engagement. We did this through an iterative design approach involving three phases of data collection and three analytic cycles (Figure 1) to build on a series of research questions that ultimately sought to answer:

1. What are some characteristics of joyful mathematical engagement?
2. What conditions support students' joyful mathematical engagement in a formal math classroom?

The first phase of data collection focused on getting to know the context of the school, the classroom, and the lives, preferences, and personalities of this group of students, and using that information to design mathematics activities that Mrs. Fry and I predicted students would enjoy. The second phase narrowed our scope on four focal students who were treated as exemplary cases of students due to their varied forms of engagement and affective responses. In this second phase, we listened carefully to these students' stories of their mathematical experiences and explored how they experienced and demonstrated affect in mathematics. The third phase of data collection focused on designing a joyful mathematics unit that was responsive to these students and resisted the unique forms of dehumanizing mathematics they experienced. Data collected across these three phases was analyzed using grounded theory approach with qualitative data (Charmaz, 2001; Emerson et al., 2011; Emerson & Mayer, 2002), case study analysis (Dyson & Genishi, 2005), and open-ended abductive postcoding analysis (Parks, 2021) depending on the phase of study and research questions of interest. Each of these activities, data collected therein, the timeline for data collection procedures, and the analytic methods are described in detail below and preceding the respective findings chapters.

Research Context

This study does not explore the "universal biological experience" of mathematical joy, but instead focuses on the "local particulars of [an] abstract social phenomenon" (Dyson & Genishi, 2005, p. 3), the mathematical joy of 21 students situated in the context of a specific 6th grade mathematics classroom. Therefore, this methods section and each of the following findings chapters begin with a rich description of the context and subject of analytic focus. This is intended to paint a picture of the metaphorical landscape of this site and probe potential influences on students' mathematical learning and their joy. A detailed description of the site of study is intended to help the reader draw meaning from the phenomenon described herein as that meaning critically depends on the context (Dyson & Genishi, 2005). In this broad overview of the methods of this research, I begin by describing the relevant background information that extends "beyond the local talk and its immediate settings" (Goodwin & Duranti, 1992, p. 8) to paint a picture of what I could see at the horizon of this ethnographic landscape: at the school district level and school level. I also address my own positionality as a researcher.

District

This study is situated in a large urban public school district in the Southeastern United States comprised of zoned schools, charter schools, specialty schools, alternative learning centers, and magnet schools. Families can enroll their children in their zoned school to which they are automatically assigned or apply to alternative options through a school choice process. This school district has a reputation of being underfunded and low-performing with respect to academic standards and testing. In 2022, approximately one-third of middle school students at the state level had test scores that demonstrated they meet or exceed grade level expectations for mathematics. In contrast approximately one-fifth of middle school students' test scores in this district demonstrated that they meet or exceed grade level expectations for mathematics.

School

This study focuses on students' attending Porter Academic Magnet Middle School, which enrolled 5th to 8th grade students and was part of the school choice option in this district. The optional choice school application is available to every family in the district who can navigate transporting their child to their choice school. Admission into the school of your choice is dependent on seat availability and, in the case of magnet schools, grades and testing requirements. The school building is well maintained, clean, organized, functional and the landscaping outside is well kept and regularly tended to. This building physically demonstrates the intentionality and resources of the district, administrators, teachers, and parents that are investing in this institution. In the lobby of the Porter there was almost always a donation box for school supplies or non-perishable goods, coats or other material goods based on the season. The lobby of the school displays the 120+ year history of Porter, including its history as a Black school on the same physical campus.

During the year this study was conducted, the racial makeup of the school was 10.6% Asian, 19.3% Black or African American, 6.7% Hispanic/Latino, .2% Native Hawaiian or Other Pacific Islander, and 63% White, which is markedly different than the racial make-up the school district and even the county with less proportion of students being Black and Hispanic/Latino and more proportion of students being White and Asian (Table 1). Prior to desegregation, which wasn't enacted fully in this district until the 1980's, this school was historically a Black school and the only Black school in the neighborhood. In the late 1800's this school added high school grade levels as Black citizens demanded higher grade levels of formal public education beyond grammar school. After multiple transformations of grade levels throughout the early to mid 1900's it was closed in 1969 to follow desegregation orders and became a middle school. In 1983 it was converted into a magnet school with claims from the district that this was in order to continue the city's desegregation efforts. This historic record of the district's segregation of Black students through Porter has undeniably had an influence on the state of the magnet school today. One result of this history is the continued racial segregation of students in the district based on accessibility to choice schooling which has led to the gentrification of Porter today as a primarily white institution.

	Asian	Black or African American	Hispanic/Latino	Native Hawaiian or Other Pacific Islander	White

County (n=708,144)	4%	27%	11%	.1%	57%
District (n=82,127)	3.76%	39.4%	32.13%	.25%	24.27%
Porter (n=695)	10.6%	19.3%	6.7%	.2%	63%
3rd Block (n=25)	12%	12%	4%	0%	72%

Table 1: Racial make-up of county, school district, school, and 3rd block class

The incongruency of racial proportions of Porter as compared to the district was concerning for this study, especially with its focus on rehumanizing mathematics for Black, Indigenous, and Latinx students. Two pieces of information encouraged me to move forward in this work in this context. First, my partnership with Mrs. Fry and our friendship allowed for challenging, controversial, and political conversations to occur with trust and vulnerability. This is crucial in work around equity especially in relation to emotion and affect. As a graduate student I would not have the time to form this kind of trusting, long-term relationship with a different teacher at a different school. Second, I observed how the culture of power and academic hierarchies at Porter were deeply systemic, embedded in the practices of the school, and had a daily effect on students' lives in Mrs. Fry's math class, especially for students of color. Mrs. Fry was reflective about these systems, admitted their presence and negative effects, and was eager to create change in her own practice. Therefore, I saw this school and these classes as the site for a potentially fruitful pilot study. My relationship with Mrs. Fry allowed for genuine collaboration and the work that we did together sought to make space for joy for students learning mathematics under her care.

Classroom, Teacher, and Students

This study depended on a Research Practice Partnership (Farrell et al., 2021) with a 6th grade teacher, Mrs. Fry, a 13-year veteran mathematics teacher teaching 6th grade math at Porter Academic Magnet Middle School and a close friend of mine for nearly 10 years. By partnering with Mrs. Fry, I had the opportunity to learn from the students in her 3rd block 6th grade math class. There were 25 students, aged 11-12, in this class; 21 of those students were consented and assented to be part of this study. Shortly after beginning baseline data collection for this study, it was apparent that both joy and mathematics looked very different through the eyes of each child. Thus, we narrowed the scope of the study to develop deep and thoughtful relationships with four focal students, Star, Ruqaiya, Isaac, and

Thomas. The teacher and students will be described in more detailed as relevant in the introductions to the respective findings chapters.

Researcher

Due to the ethnographic nature of this study and my positionality as a white, adult, female, former middle school mathematics teacher, and friend of Mrs. Fry, it was necessary for me to carefully attend to the nature and boundaries of my role as a researcher (Dyson & Genishi, 2005) . This was not only for the integrity of the research, but also for the long-term care of my friendship with Mrs. Fry. Mrs. Fry and I often talked about what we each valued in our research relationship, what we both could bring to the partnership, and how the power dynamics between researcher and practitioner bled into even our close friendship as we worked together. The closeness of this long-term friendship allowed for us to attend to and shift the power relationships that often arise in research practice partnerships (Henrick et al., 2017; Teeters & Jurow, 2019). For example, just as this study was taking shape in 2022, we were taking our daughters trick-or-treating together as we do with our families every year for Halloween and casually discussing the research while we walked. As we rounded the corner of the block back home with our daughters dragging overflowing buckets of candy, Mrs. Fry mentioned that she sometimes got nervous when I was in her classroom because she was unsure if she could sit down at her desk during her 3rd block for fear of underperforming as a teacher in my eyes. When I scoffed and said that of course she could be herself and take care of herself she said, “I know you’ve only been out of the class for a few years, but I’m not sure how removed you are, really”. Despite our years of friendship, this power dynamic of evaluative researcher and performing practitioner was evident. In our next research practice partnership meeting, we addressed this directly and decided that if either of us ever felt an imbalance of power in this space that we would discuss and collectively resolve the source of it as immediately as possible. We both adhered to this agreement and had many similar discussions throughout the study.

During data collection, I made attempts to shift (or at least complicate) power dynamics between myself and students while in the classroom. Students recognized and were curious about my positionality in the classroom, often asking if I was a teacher, student, student-teacher, administrator, or someone’s mom. My goal as I collected data was to trouble the notion that I might be in a teacher role in the classroom and instead become "an unhelpful but attentive adult friend of children" (Dyson & Genishi, 2005, p. 52). Each day that I came to the classroom, I wore casual but not too casual clothing (jeans and a sweater or t-shirt) in order to be unnoticed, approachable, but recognizable as an adult. In the first 3 months of baseline observations, I largely sat on the periphery of the class during lessons and only interacted with students before or after the class.

As I began conducting case studies and focusing on forming relationships with Star, Isaac, Ruqaiya, and Thomas specifically, I sat with student groups whenever there were chairs available. I was present, but I was not often very mathematically or practically helpful. As a former mathematics teacher this was a difficult role for me to take. In one of my first one-on-one encounters with Star, she turned her back to me and blocked me from speaking to her after I asked if I could help her with a problem that she seemed to be struggling with. It occurred to me later that she immediately perceived me as a teacher and therefore was resistant to engaging in a relationship with me. It took time and patience to undo that interaction, and after that moment I realized that I had to intentionally shift my role through my interactions with students. When students would ask mathematical questions, I made sure to ask questions in return, point to a potential resource, or ask the question to another student nearby. When asked practical questions like requests for materials or permission to use the restroom, I would either relay the questions to their small group or say something to the effect of "yeah, I'm not sure, I think I'd have to ask your teacher that" depending on the situation.

After a few weeks sitting with students, I began to be invited to engage in mathematical or social activities like play card games or Jeopardy. The first time I played a Jeopardy review game, I intentionally chose to simply record answers for my team. Unfortunately, my team won, and I could sense the audible and visible frustration at the unfairness of having me on a team. Thankfully this group of 6th graders was quick to forgive, consistently friendly, and eager to chat. After spending 2-3 times per week in the class for almost five months, I shadowed the class for their entire day at school. At this point I felt comfortable sitting among students with the permission of their teachers, but attempted to do so in a way that didn't center myself in the context. For example, in social studies I sat in a student chair in the back of the room, took notes in a small notebook rather than a computer, and during social emotional learning time participated in the activity about student leadership as if I was a student but only because students invited me into the activity. At lunch that day, I ate my packed lunch at the student lunch tables within earshot and eyesight of two groups of students from Ms Fry's class, but wasn't directly a part of their conversations.

The collective effort of these actions seemed to be effective in making me ambiguous but also friendly. At the beginning of class one day a few months into the study, I was setting up cameras at the side of the room and I overheard two students talking about how they missed me being in class that day. A third student quickly corrected them saying that I was there just not sitting at their table yet and they both smiled at this news and at me when I joined their table. I also have a treasured voice memo that I was recording as I got into my car to leave one day where you can hear Star yelling across the front parking lot of the school to get my attention and happily waving good-bye to me. In this voice memo you can hear my disposition change as I felt so cared for and seen by these students. Despite these friendly, familiar, and endearing encounters, my ambiguity was evident. Toward the end of the study, a student that I sat with regularly was utterly bewildered after finding out from Mrs. Fry that I was the mother of three children and told me, "I can't tell if you're really old or really young!"

Analytically, this research was also shaped by the close research practice partnership with Mrs. Fry and my own experiences in learning, teaching, and researching mathematics. Mrs. Fry and I had many formal meetings and information conversations about the sociomathematical norms of the classroom, students' engagement in specific tasks, students' lives and stories, the joys and conflicts of teaching, and more. This information shaped my analytic lens as I observed students engaging in mathematics and informed me as to the external influences that may be at play with respect to students' mathematical joy. In each of the following set of findings, I attend to specific potential influences from the research practice partnership by highlighting Mrs. Fry's perspective on each analytic finding whenever relevant. Empirically, joy's multifaceted nature made researching mathematical joy challenging because positive emotion isn't always indicative of joy and joy doesn't necessitate positive emotion. Only through personal engagement and interaction in mathematics with students could I begin to feel and describe the satisfaction and purpose born from the coupling of personal mathematical well-being and collective mathematical benevolence at the heart of mathematical joy.

Finally, throughout this work it was important for me to attended to my own feelings and reactions based on my personal history with formal mathematics education. I did this primarily by reflecting in voice memos immediately following interactions with students and teachers. Through these voice memos, I could hear my own affect, describe it, and attend to it as I conducted research. Analytically, I took up methodological practices that embraced my own joy and experiences as significant analytic findings like ethnographic open-ended abductive postcoding analysis. By taking affective cues from both myself and from students as significant findings, providing rich descriptions of students' joyful interactions through multimodal interaction analysis (Norris, 2004; Parks & Schmeichel, 2014), and making connections across data, I provided a landscape of joyful mathematical engagement in this context that attended both to its characteristics and the conditions that supported it.

Data Collection Activities

I sought to answer these research questions by collecting data across a variety of modalities incrementally and iteratively. I used the data outlined below to learn about the context of the classroom, the stories of four focal students, the process of designing for mathematical joy, and affective student experiences in this class. Data collected for this study included fieldnotes, voice memos, audio recordings, transcripts, video recordings, and artifacts collected from classroom observations, individual and group interviews, and Research Practice Partnership meetings.

Classroom Observations

I conducted classroom observations of Mrs. Fry's 3rd block class in three ways. For the duration of the study, I observed Mrs. Fry's 3rd block math class 2-3 times per week. I expanded the scope of this observation to shadow this same group of students for the duration of their school day midway through the study. Three times during the semester, once during each phase of data collection, I observed a full mathematics unit, which lasted for 4-7 days. Depending on the phase of the project, these observations were intended to better understand teacher practices, the classroom culture, the ways that children were engaging mathematics, and appearances of affect. These observations were recorded through ethnographic fieldnotes (Emerson et al., 2011), voice memos immediately following observations, and video/audio recordings of three math units.

Fieldnotes were guided by a template that was developed by the Playful Learning Project (Figure 2) to capture teachers' and students' interactions with tools, each other, and mathematical ideas. This template provided space to take notes about: 1) activities, materials, physical environment, and routines; 2) teacher responsivity, scaffolding, and positivity/joy; and 3) children's participation, agency and sense of belonging. This template was used for classroom observations throughout each of the three phases of the study.

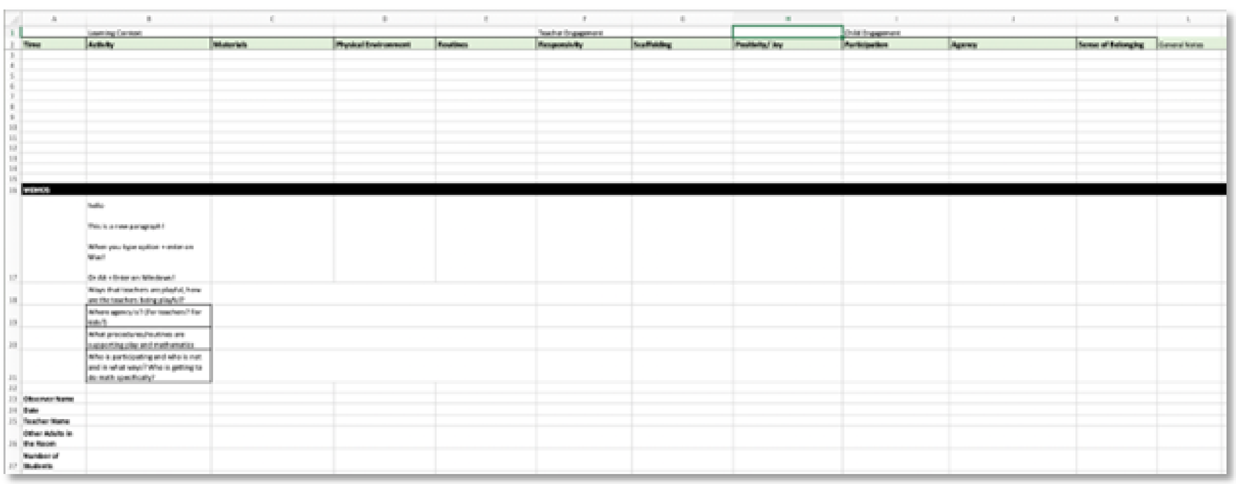


Figure 2: Observation Fieldnotes Template

For one day of the study, I expanded the setting of classroom observations to include this group of students' entire day at school, shadowing them from bell to bell. Fieldnotes during this time were recorded in a small, lined notebook rather than on my computer. This was intended to minimize my appearance and counter the portrayal as an evaluative adult both for teachers and for students as administrators in this context often recorded their observations on computers. For these fieldnotes, I focused on students' affect and interactions between students, paying particular attention to how affect and interactions changed as students moved through their day.

Voice memos were recorded after most classroom observations to provide context and nuance about the observation, notes about moments that stood out as significant, and real-time personal reflection about the observations. Voice memos were transcribed following the study.

Interactions with students and between students were recorded via audio when necessary during the regular weekly observations. Students were video and audio recorded as a whole group as well as in small groups during three math units throughout the semester, one unit in each phase of data collection. A Swivl camera tracked the teacher's interactions and whole group engagement in the mathematics lessons. 360-degree GoPro cameras were placed at each small group table to record students' engagement in the mathematics lessons, with one another, and with the teacher.

Student Artifacts

Artifacts of student work were collected three times during classroom observations. In Phase 1, I collected students' Math Autobiographies, throughout Phase 2 and 3 I collected students' Mathematical Moments Reflection Journal Entries, and at the end of Phase 3 I collected student work and reflection from the Data Analysis Rubric. These three artifacts are detailed below.

Illustrated Math Autobiographies

At the beginning of this study, Mrs. Fry and I conjectured about her students' beliefs about mathematics, how to engage in it, and what brings them joy. We largely made informed assumptions based on our observations. We wanted to find a way for students to express their beliefs about and relationships with math in a way that was fun and engaging. We therefore created an activity that was completed during the first week of the semester, based on a document taken from the work of Peg Cagle of the Park City Math Institute. Students were directed to fill out the activity as honestly as possible and to use any form of expression they would like to fill in each box. Students were encouraged to write sentences or bullet points, draw or color, explain with words or simply demonstrate through images. This assignment had the highest completion rate of any take-home assignment given throughout academic year to that date.

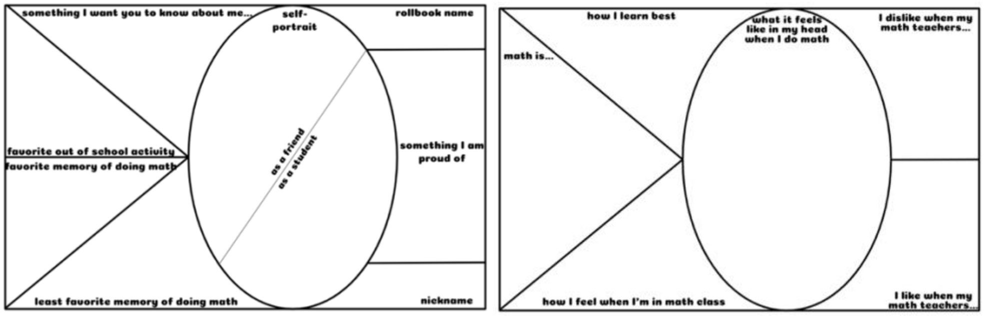


Figure 3: Math Autobiography assignment completed by students at the beginning of the study.

Student Mathematical Moments Reflection Journal Entries

After the Phase 1 of the study, Mrs. Fry and I asked students to complete a reflection about their mathematical learning. Students filled out a google form (https://forms.gle/RtNNRZSaWNxBMLGU9) to reflect on some of the new mathematics activities. These questions were based on Lisa Bejarano's weekly warm-ups template and collaboratively adapted for our work. The reflections that we received from students were fruitful, honest, and productive for better understanding what mathematical experiences students had throughout that week. As a result, we decided to formalize and continue these reflections through weekly Math Moments Reflection Journals. All students were asked to complete a regular journal entry for the remainder of the study. Students received a blank journal that they decorated and personalized to their liking, and asked to write about mathematical moments in their life using the prompts provided (Figure 4). We also provided students with a polaroid camera in class that they could use to take photo of math moments they wanted to write about. These journal entries took no more than 10 minutes and students were asked to do them once per week or more if they would like. These journals informed interviews with the four focal students, the design phase of the study, and were used as textual evidence of students' mathematical voice, beliefs, ideologies, and experiences.

Math Moments Journal

Tell a story of any math moment and how that moment made you feel. If you want to, you can take a picture of these moments and send it to [redacted]. I'll print it out for you to keep in your journal! These moments could be in math class, at school, at home, on the bus (whenever!) with your friends, with your family, with your teacher, or by yourself (with whomever!). You can draw/color pictures, write bullet points or stories or poems, decorate - Have fun with it!

Tell a story of a math moment….

- that you enjoyed
- that you didn't enjoy
- that challenged you
- in which you noticed something that helped you solve a problem
- in which you made a mistake and how you learned from it
- in which you used a strategy that was new
- that you were proud of.

Figure 4: Mathematical Moments prompt given to students in their journals

Data Analysis and Statistics Unit Rubric

Based on our data and analysis from Phases 1 and 2, Mrs. Fry and I created a rubric for students to complete throughout the data and analysis unit designed for in Phase 3. More details about this design process and use of this rubric are provided in Chapter _____. The rubric we created outlined each mathematical content goal that students' were required to learn during this unit and space for students to mark the date that content was learned, evidence of students' learning, notes, and peer signature. At the end of this rubric was also a space for giving five shout-outs to other students, and five reflection questions including, "I used to this… but now I think…", "Something I want Mrs. Fry to know is…", "Something that surprised me this week was…", "Something I wish we did more of in math class", and "Something I wish we did less of in math class".

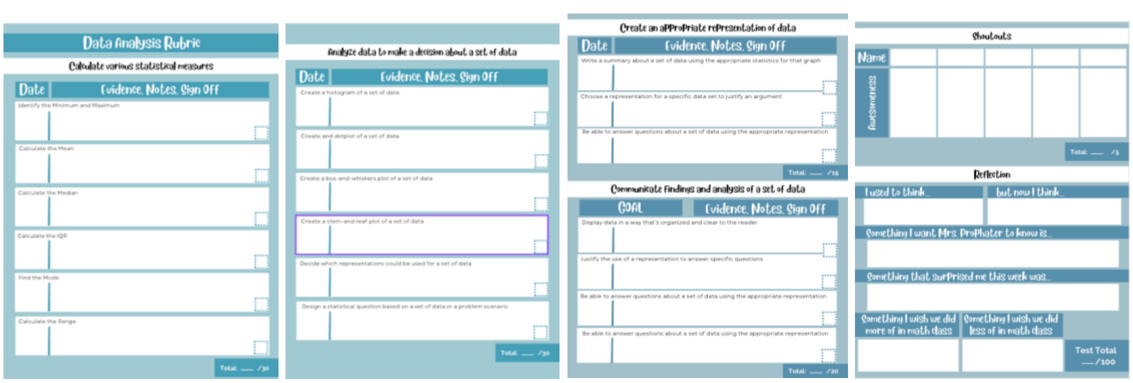


Figure 5: Data Analysis Rubric completed at the end of Phase 3

Research Practice Partnership Meetings

Mrs. Fry and I engaged in regular Research Practice Partnership meetings throughout every phase of the study, 18 in total with nine in Phase 1, five in Phase 2, and four in Phase 3. During these meetings we discussed theoretical and research-based information about mathematical joy, how students are engaging in the mathematics in this context, and co-design some learning activities that were intended to support students' joy. During these sessions we talked about the literature about more humane forms of mathematics like Su's book *Mathematics for Human Flourishing (Su, 2020),* Lockhart's book *A Mathematician's Lament* (Lockhart, 2009), Gutiérrez's framework for rehumanizing mathematics (Goffney et al., 2018; Gutiérrez, 2018b), and Amy Park's article *Creating Joy in Pk-Grade 2 Mathematics Classrooms* (Parks, 2020). We spent significant time reflecting on our own mathematical beliefs and identities and the mathematical beliefs and identities of her students. As the study progressed, we spent increasingly more time discussing the ways children engaged in mathematics with joy in this class and designing for ways to support mathematical joy. All sessions were audio recorded and transcribed. I also recorded a researcher voice memo after each session to provide context, nuance, and real-time reflection about to the audio record. This data was used to investigate the complexities and development of the Research Practice Partnership, to document Mrs. Fry's knowledge about her own students and how they are engaging in mathematics in her classroom, and the design decisions that we made collaboratively to support students' joy.

Student Interviews

The four selected focal students were individually interviewed twice during Phase 2 of data collection with a focus on their ideas about math and about themselves. These interviews were informed by their ongoing journal entries, artifacts from the class, and the researcher's classroom observations to stimulate their recall of particular events and allow the conversation to occur around a shared experience (Tobin, 2019). Interviews with students were semi structured and ask questions to better understand how students talk about enjoyment, about mathematics, and their own identity with respect to mathematics. The first interview was conducted two months into the study and after conducting classroom observation of a week-long unit in math class. This interview was conducted in a place of the student's choosing that they considered their "favorite place in school". The questions focused on activities that students enjoyed both inside and outside of math class, how they experience "doing math" inside and outside of school both in enjoyable ways and in not so enjoyable ways, and how they perceive the ways that other people do math in their math class. Each set of interview questions was written to uniquely respond to what each student wrote in their math autobiography. Follow up questions for this interview were largely based on my observations of their work in math class in the first two months of the study. The second interview was conducted three and a half months into the study, after I observed the second units of math lessons. These questions focused on students' relationships with other people in the class, how people interact with each other around mathematics, the purpose of math and school math, and assessments and competition in math class. These interviews were tailored to students based on my observations of their joyful experiences with math over the bath 3 months as well as their mathematical moments journal entries.

At the end of Phase 3 one group interview was conducted with every consented student in the class in groups of 4-5 students per group. The interviews were scheduled following the final mathematics unit that we designed to support students' joy. All interviews were conducted while sitting in a circle on the schools' front lawn outside in the Spring sunshine. These interviews were semi structured and focused on students' experiences in and reactions to the responsive rehumanizing mathematics lessons. In this interview, I asked students to ask me interview questions, respond to prompts about the new class norms instituted in Phase 3, discuss the rubric that they completed as part of the unit, respond to quotes from anonymous students' journal entries about how other people felt when doing these lessons, and discuss how it felt to give and get shout outs as part of this unit. These interviews lasted approximately 30 minutes.

Data Collection Timeline

The data collection timeline began with the context of the classroom, then to students' stories in mathematics, and finally to investigate joy or lack of joy that students experienced in this class. This required that I first build my understanding of this community and these students through observation, then develop trusting relationships with the teacher and students in this context through interaction, and lastly co-design and implementing responsive designs of math lessons intended to support mathematical joy in this context.

Phase 1: Observation				
Timing	Focus	Activity	Frequency	Data Collected
January and February	Getting to know the context and students	Classroom Observations	Twice weekly	Ethnographic fieldnotes
				Voice memo
			Distributive property unit: 4 consecutive days in February	Whole class video
				Small Group video
				Fieldnotes
				Voice Memos
		RPP meetings	10 total: 4 prior to the semester starting, 6 during the semester	Audio Recording
				Transcript
				Voice Memo
				Meeting Notes
		Illustrated Math Autobiographies	First week of semester	Student artifacts
Phase 2: Interaction				
Timing	Focus	Activity		Data Collected
February-May	Developing trust and relationships	Classroom Observation	2-3 times weekly	Audio Recording
				Ethnographic fieldnotes
				Voice Memos
			Geometry Lesson: 5 consecutive lessons in March	Whole class video
				Small Group video
				Fieldnotes
				Voice Memos
		Math Moments Journals	Once weekly	Student artifacts
		Student Interviews	Two interviews: Once monthly	Audio Recording and Transcripts
				Voice Memo
Phase 3: Design				
Timing	Focus	Activity		Data Collected
April and May	Co-designing for responsive rehumanizing mathematics	RPP meetings	Three prior to Data Analysis Unit	Audio Recording
				Transcripts
				Voice Memo
			Once after Data Analysis Unit	Audio Recording and Transcript
				Voice Memo
		Classroom Observation		Video and Audio Recording

			Data Analysis Unit: 7 days	Voice Memo
		Group Student Interviews	Once after Data Analysis Unit	Audio Recording
				Voice Memo
		Math Moments Journals	Once weekly	Student artifacts

Phase 1 focused on getting to know this mathematical learning community primarily through observation and research practice partnership meetings (RPP). RPP meetings occurred 10 times through the semester: 4 prior to the semester's start and 6 after the semester began. For the first two months of the study, I conducted observations of the classroom environment twice weekly. In these observations, I paid specific attention to the context in which students were engaging in mathematics and how it shaped the ways that students were interacting with and talking about mathematics. Through these observations and through the research practice partnership meetings, I began to get to know the students in this class. It was through these observations in concert with the RPP meetings, that I began to understand the ways that students were engaging with mathematics and how their humanity (their emotions, the participation) intersected with the ways they did mathematics in this classroom context.

The goal of Phase 2 was to interact with students to form relationships and develop trust with them within the context of this math classroom. After making some observations and listening carefully in RPP meetings to learn how this classroom functions and what students experience within it in Phase 1, I spent February through May pursuing deeper relationships with the four focal students. I engaged in observations 2-3 times weekly and had the opportunity to observe and film the four focal students for 5 days of a Geometry unit about area and surface area. I also shadowed this group of students through a whole school day to see how they interact in other academic and social spaces. During classroom observations, I was more engaged in the lessons by sitting with students and participating in lessons when the opportunity arose. During Phase 2, I also conducted two individual interviews with these students, in March and in April. Mrs. Fry and I also collected students' reflection from their math moments journals once per week to hear from them more about what brings them joy, how they engage in mathematics and with what affect. Through these closer interactions, I began to learn more about who these students were in this space and in their lives more holistically. These relationships and the things I learn about students from this time informed Phase 3 as we attempted to design math lessons in ways that supported students' joy.

In April and May, for Phase 3, Mrs. Fry and I co-designed a rehumanizing math unit to center and support students' joy. This was responsive to what we knew about students' full humanity, how they engage in mathematics, the ways they have experienced dehumanizing mathematics learning, and the potential ways that they experience and express joy. Veronica and I met for three RPP co-design and planning meetings to plan for this 7 day math unit about Data Analysis. Throughout these lessons, I conducted daily classroom observations recording with fieldnotes and voice memos, captured whole group and small group video. Immediately following this unit, I learned from students' perspective about the ways that they experienced these lessons from their math moments reflection journal entries and through group interviews with every consented student in the class. In the week following this unit, I also interviewed Mrs. Fry about her experience teaching this unit and engaging in the study more broadly.

Data Analysis

This analysis addresses the following research questions:

3) What are some characteristics of joyful mathematical engagement?
    a. What does mathematical joy look like, sound like, feel like, and how does it look different for different students?
    b. How do students describe joyful mathematical engagement?
    c. When does joy occur during mathematical engagement and are there other affective states that often accompany joy?
4) What conditions support students' joyful mathematical engagement in a formal math classroom?
    a. What is the role of the task in supporting mathematical joy?
    b. What is the role of others in students' mathematical joy?
    c. What conditions seem to undermine or prevent joy when engaging in mathematics?

This analysis involved looking at different pieces of the corpus of data using unique analytic methods to answer the research questions in increasing depth over time. These research questions are answered ethnographically through three levels of data analysis using grounded theory approach with qualitative data (Charmaz, 2001), case study analysis (Dyson & Genishi, 2005; Emerson & Mayer, 2002), and open-ended abductive postcoding analysis (Parks, 2021). Open coding of qualitative data investigated themes across the data collected in Phase 1 to determine how students in this space engaged with mathematics, what they enjoyed both inside and outside of mathematics class, and what affect was observed when students engaged in mathematics. Analysis of case studies focused on four focal students' mathematical joy (or lack of mathematical joy) as it is situated in the cultural, political, and personal context of this classroom. And open-ended abductive postcoding analysis (Parks, 2021) was used to capture affective moments of joy or lack of joy throughout the final Data and Statistics unit designed in Phase 3 of data collection and track the conditions that supported those moments back through data from Phase 1, 2, and 3.

The two sets of research questions about joyful mathematics characteristics and conditions were answered incrementally by adapting the research questions at each level to match the relevant sources of data and focus of analysis. In open qualitative coding will answer research questions about the context and students in the class more generally (qualitative open coding). In the case studies the research questions will focus on the four focal students. In the abductive post coding level of analysis the research questions will focus on the design and implementation of potentially joyful mathematics experiences for these students. The final story that is incrementally constructed about students' mathematical joy through each level of analysis will answer the research questions with greater depth and nuance than would be possible without this incremental approach.

Research Question 1: What are some ***characteristics*** of joyful mathematical engagement for these students?		
Analytic Methods	Research Questions	Data Collection Activity
Qualitative Open Coding	What are some characteristics of ***mathematical engagement*** for these students? a. What are the sociomathematical norms of this classroom?	Phase 1 RPP Meetings Phase 1 Classroom Observations Phase 1 Illustrated Math Autobiographies
Case Studies	What are some characteristics of ***affective mathematical engagement***? a. What does affect look like, sound like, feel like for these four students?	Phase 2 Classroom Observations in Geometry Unit

Abductive Postcoding	What are some characteristics of ***joyful mathematical engagement***? a. What does mathematical joy look like, sound like, feel like, and how does it look different for different students? b. When does joy occur during mathematical engagement and are there other affective states that often accompany joy?	Phase 3 Classroom Observations – Data and Statistics Unit

Research Question 2: What ***conditions*** support students' joyful mathematical engagement?		
Analytic Methods	Analytic Focus	Data Collection Activity
Qualitative Open Coding	What conditions support ***students' joy***?	Phase 1 Illustrated Math Autobiographies Phase 1 Classroom Observations – Distributive property Unit
Case Studies	What conditions potentially ***supported or hindered students' joyful mathematical engagement?*** a. What tasks, interactions, or experiences explain students' affect in mathematics? b. What conditions seem to support or suppress students' joy when engaging in mathematics?	Phase 1 and Phase 2 RPP Planning Meetings Phase 2 Planning Period Observations Phase 1 and Phase 2 Classroom Observations – weekly and in Geometry Unit Phase 2 Math Moments Journals Phase 2 Student Interviews
Abductive Postcoding	In what ways did ***designing for responsive resistance*** (in this case with mathematical reciprocity and play) ***support students' joyful mathematical engagement*** in a formal math classroom?	Phase 1, 2 RPP Meetings, Observations, Interviews, Artifacts Phase 3 Classroom Observations – Data Analysis Unit Phase 3

Each of the analytics methods and the research questions asked at each level of analysis will be provided in detail in the introduction to each relevant chapter.

Chapter 4: Baseline Understanding of Sociomathematical Norms and Enjoyment

Introduction

The goal of this chapter is to detail initial pursuits toward a deeper understanding of the characteristics of and the conditions that support students' joyful mathematical engagement in this space. In the first phase of this study, it was first necessary to understand what it meant to be doing mathematics in this space, what kinds of tasks and activities student enjoyed doing more generally, and what affect student demonstrated when students engaged in mathematics in ways they enjoyed. Therefore, in this chapter the research questions are more closely defined as:

1. What are some characteristics of mathematical engagement for these students?
    a. What are the sociomathematical norms of this classroom?
2. What conditions support students' joy?
    a. What kinds of tasks do students enjoy?

These questions were answered through an analysis of research practice partnership meetings with the teacher, research observation fieldnotes and voicememos, and student artifacts.

Research Context

Teacher

In the year prior to this study, I received many phone calls from Mrs. Fry about her exhaustion with the tensions and stressors of being a teacher in the post-COVID world of education. Mrs. Fry admitted openly that she was "done" with the field of education and needed a way to be reflective about her practices so that she could continue in her career as a teacher. Her success as an educator and relatively lengthy career wasn't enough to inspire her to continue working in this environment. Indeed, she claimed "I'm good at this job and should keep doing it.... And I want to keep doing it, [but] I have to change how I'm doing [it]". The frustration with her career was in part rooted in the systems in which she worked and the resulting lack of well-being for her students. As someone who cared deeply for her students, she was motivated to engage in this study because of the lack of joy and overall anxiety that she saw in her students' mathematical learning. In the initial stages of this study Mrs. Fry expressed that her goal for herself was to find time to create, to connect to students' real lives, and to have a new lens to be intentional about how we design mathematical learning opportunities for students.

Because of Mrs. Fry's knowledge about my passion for mathematical play and mathematics education and our friendship and collaboration as mathematics teachers in the past, she reached out to me asking about how to form a math play area in her classroom. Mrs. Fry was pursuing a kind of playful mathematics learning that was unique for the context in which she worked; one that was intentionally upending the norms of traditional formal mathematics learning that she acknowledged was systemic in her classroom. By reaching out for information from someone researching mathematics education, Mrs. Fry pursued research as the leading activity; seeking tools, designs, opportunities, and philosophical ideas that could shift mathematical engagement to be something more positive in this space. In the initial planning meeting for this study, as we reflected on her teaching practice and the goals for this partnership, she repeatedly noted that a crucial component of this work would need to be to attend to students' personal well-being and their full humanity: not just putting their favorite tv characters into a word problem or making a test review activity about their favorite sport. She was therefore coming to the work intending to gain a deeper understanding as a teacher of the intimate connection between designing for rehumanizing mathematics and seeing students' joy in mathematics.

Despite Mrs. Fry's desire for a joyful mathematical experience for her students, there was an obvious tension between her conviction about what mathematics learning should be and the kinds of mathematics that students engaged in currently in her class. The system in which she worked and the context in which these students learned mathematics unsurprisingly shaped the ways students engaged in math. For example, Mrs. Fry saw the enormous pressures of state testing as widely influential to the ways students engaged in mathematics. In expressing her frustration with the system of formal mathematics learning in schools, Mrs. Fry said, "[state testing] is crap. Yeah, those questions are nothing like real thinking". She desired for students to engage "real thinking" that focused on the process of doing mathematics rather than the product of test taking. Despite this conviction, she acknowledged the tension that the product would, for her and her students, be the only indicator of their success to the powers that be.

“I think that, for me, it's interesting to have kids love the process instead of the end result... [but] at the end of the day, I still get graded on [students’ performance on state testing]. Yeah, it's still an end result. Like, no matter how much in my spirit I want to have fun with it. I'm like, not allowed. You still have [state testing] at the end.”

When pressed about how much agency she has in this space to “have fun with it” and not focus on the testing, in frustration, she admitted:

> *“the answer's none. Like honestly, like just for the sake of transparency, in order to do my job well, I can't do anything they tell me to do... [for example] my scope and sequence … is mandated by a group of people who have never met my students or maybe even talked to a sixth grader in the last 10 years of their life... you‘re not trusted to be a professional in your field. I am legitimately told this week you teach this thing but use data to decide what you're going to do. But you still need to do the thing I told you to do [on the scope and sequence]”*

This pressure to analyze students’ learning and progress based on “data,” while also being responsive to students learning, while simultaneously tracking daily with the scope and sequence to ensure that you cover all math content by the end of year testing (which determined Mrs. Fry’s professional teaching evaluation) created incommensurable cognitive dissonance in Mrs. Fry’s work. It often appeared in this class, as in most math classes, that there were boundaries to the extent to which she could design for the math learning that she was convinced students should be engaging. In Mrs. Fry’s words: “my conviction doesn’t change my target”. The products that demonstrated her students’ success and her own success as a teacher were her ultimate target and therefore it was challenging to focus instead on the process of doing mathematics.

Students

In this study, I learned from students in Mrs. Fry's 3rd block 6th grade math class. There were 25 students, aged 11-12, in this class and 21 of those students were consented and assented to be part of this study. Everyday these students came to math class having already attended either Social Studies or Science (on a rotating basis), Study Hall, Recess, Lunch, English Language Arts, and Related Studies (a quarterly schedule of computer programming, orchestra, drama, art, band, and physical education). Besides their Related Arts class, which was still grouped by grade level, this group of students had the same schedule together every day. As a result, these students spent a lot of time together and knew each other extraordinarily well. Before and after nearly every class, students were giggling together, playing games, telling secrets, sharing snacks, and talking about the pleasantries and issues of their lives. Students were generally kind and respectful to one another with very few outward evidence of disputes or relational tension.

Analytic Method: Qualitative Open Coding

Investigating the characteristics of and conditions that support mathematical joy is complex because it is an intermingling of mathematical engagement and enjoyment. Therefore, it was necessary to ask both how mathematical engagement was characterized in this space and about what things students' enjoyed. This analysis began with an ethnographic triangulation of data collected from the perspective of myself, Mrs. Fry, and students, and laid the foundation for understanding the sociomathematical norms and students' enjoyment in this space. Therefore, fieldnotes from my class observations, researcher voice memos after observations and meetings, audio discussion with Mrs. Fry during research practice partnership meetings, students' illustrated math autobiographies, and students' math journals provided information about how this space was set up to communicate what it meant to engage in mathematics and what kinds of things students' enjoyed more generally. All fieldnotes, voice memos, audio/transcripts, and artifacts from Phase 1 of data collection activities were open coded. Open coding of these documents in a grounded theory approach (Charmaz, 2001; Emerson et al., 2011) involved reading the data chronologically "line-by-line to identify and formulate any and all ideas, themes, or issues they suggest"

(Emerson et al., 2011, p. 172). Working chronologically within this set of data allowed for me to analytically “take in, for the first time in a relatively concentrated time stretch, everything that [I] was able to observe and record” (Emerson et al., 2011, p. 174) during our initial research activities that informed our first designs for supporting students’ mathematical joy.

Discovering Sociomathematical Norms

Focused open coding of the data from Phase 1 sought to describe the sociomathematical norms, “normative aspects of mathematical discussions that are specific to students' mathematical activity” (Yackel & Cobb, 1996, p. 458), of this classroom space. This was done by triangulating data from multiple sources: from myself, from students, and from Mrs. Fry. This created a descriptive piece of what sociocultural mathematical norms were operating in this space. Artifacts from students’ illustrated mathematical autobiographies, quotes students from audio recorded in class observations, and student journal entries reinforced the themes that emerged from observations and reflective conversations about classroom culture between myself and Mrs. Fry. Due to the nature of our research practice partnership, Mrs. Fry was a critical guide to my understanding of the sociocultural mathematical norms of this classroom. Students experience was not only about what I observed Mrs. Fry doing in her class, but it was also about a set of dynamics that transcended the classroom. Some of this information was evident to me and much of it was shared with me based on Mrs. Fry’s experiences. She was a key informant to the contextual factors that helped to produce the norms that operated in this cultural space and because of our close partnership, I ultimately could not disentangle what I observed in the classroom and the framework that Mrs. Fry provided me in our early conversations during Phase 1 of this study.

Moments that made apparent how sociomathematical norms were operating in this space were compiled in a data log and used to generate a theoretical memo (Emerson et al., 2011; Strauss & Corbin, 1990). This theoretical memo was then coded for themes about characterizing mathematics. This coding revealed four sociomathematical norms in this space: mathematical engagement was product oriented, competitive, rooted in compliance, a complex balance of enjoyable and stressful.

Students' mathematical autobiographies were open coded separately for perceptions about mathematics. To understand students' relationship with mathematics, I coded responses to three math-specific prompts 1. "math is...", 2. "what it feels like in my head when I do math", and 3. "how I feel when I'm in math class". Out of 94 codes identifying places where students responded about doing mathematics, 67 of the 94 codes were about how students see themselves when they do mathematics and 27 of the items talked about what mathematics is. For example, students might finish the sentence "Math is..." by saying "Math is fun and boring" which I interpreted as "I am having fun when doing math" or "I feel bored when doing math" while some students said "math is equations and problems we solve with numbers". These two kinds of references to mathematics were coded as "I am... when I do math" and "math is..." respectively. The 27 items coded as "math is…" were discussed descriptively in relation to the focused coding above. The 67 items coded as "I am … when I do math" were categorized as positive, neutral, and negative to get a sense of students affect about themselves when they do math and some of the themes that emerged from students' positive or negative feeling of themselves when doing math. The themes that emerged about this context and these students based on these forms of data were further discussed and member checked with Mrs. Fry in our RPP meetings to confirm or develop more nuance. This information was then incorporated to the theoretical code memo concerning the sociomathematical norms of this space.

Designing for and Looking for Students' Enjoyment

To design for students' joy in mathematics, it was necessary to be responsive to students' perceptions of doing mathematics and students' experiences and preferences with joy by asking what kinds of tasks students enjoy. Because of Mrs. Fry's initial concern that her classroom was not necessarily characterized by joy, we didn't look for joyful task engagement in her math class during Phase 1. Instead, we relied on information from students' mathematical autobiographies. Mrs. Fry and I used this information to design a unit about the distributive property. The goal of this unit was to create an atmosphere in which we would have a greater potential see students' joy or some affect as they engage in mathematics. Students' widely varied opinions made it impossible to design tasks that directly correlated with activities that every student enjoyed. Instead, we drew on a research-based practices and pedagogies as an initial starting point (insert cites here).

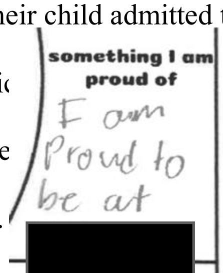
Mrs. Fry emphasized that engaging in mathematics with joy in her classroom would require resisting the performative product-focused norms and culture that was unique to the school and the students. Because of this desire for long-term resistance and pushing back on dehumanizing practices and the focus on joy, we watched Gutiérrez's 2018 talk titled "Rehumanizing Mathematics: A Vision for the Future" (Gutiérrez, 2018b). We decided in our first design meetings of this study that that this framework was something we could attempt to take up through our design work, but that working in this context would mean designing in the midst of an often harmful and denigrating system of formal mathematics education.

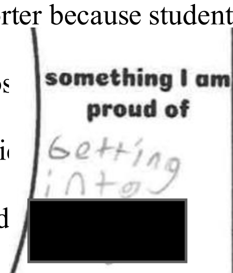
Therefore, the items coded "conditions for enjoyment" were categorized as one or more of the 8 dimensions for Rehumanizing Mathematics: participation/positioning, cultures/histories, windows/mirrors, living practice, creation, broadening mathematics, body/emotions, and ownership. We prioritized the top 4 of those 8 categories as we designed the distributive property unit to support these categories of rehumanizing mathematics in ways that were responsive to students. Many items were categories by multiple dimensions of rehumanizing mathematics. For example "playing softball on the softball team" could be categorized by body/emotion and participation/positioning because it involved both physical movement and participating in a team sport. Our designs for the distributive property unit focused on the four most frequently coded categories and attempted to design activities that were responsive to the ways many students found enjoyment in those categories.

RQ1: Characterizing Mathematical Engagement

Mathematical Engagement was Product Oriented

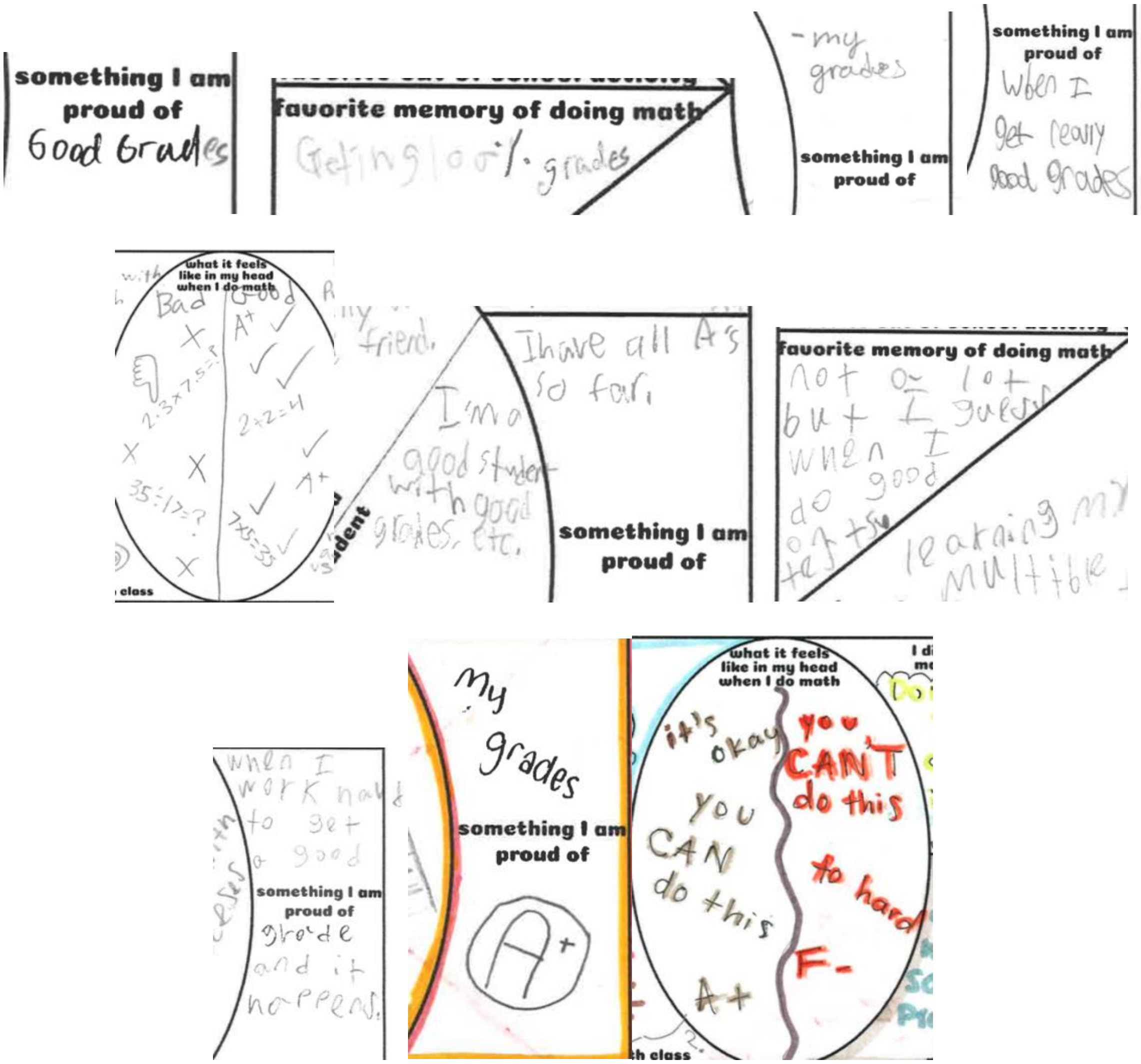
As noted, Porter was an academic magnet middle school which required minimum state academic testing requirements for admission. Additionally, due to the high demand of admission into Porter, a lottery system was employed to select amongst eligible students. To be entered into the lottery, elementary students wanting admission to Porter had to have an academic average of 80 or higher for all subjects with no failing grades in the Spring semester before admission. They had to also receive scores of Met Expectations or Exceeded Expectations in English/Language Arts (ELA) and Math OR receive state stanine scores that total 14 or higher during the previous year. The requirements were rigorous, especially given that the middle schools in the same zoned district have only around 5% proficiency across subject areas and a 10-15% suspension rate. Therefore, as an academic magnet school Porter had a reputation for high achievement, often winning state and national awards. There was significant incentive for families having their child admitted to Porter because students who matriculated from this middle school were automati… e mos… chool in the district. Every student sitting in this 6th grade … s uni… e in this school largely because of their performance on tests. … dged … natical autobiographies by saying that admission into Porter was a proud accomplishment for them.



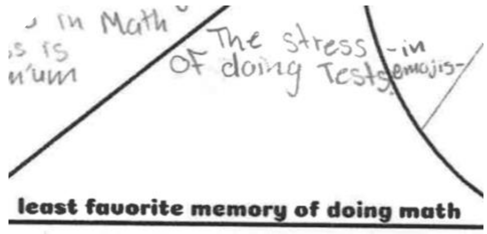


As with most formal math classrooms, this 6th grade classroom demonstrated the importance of becoming proficient in math standards that were mandated by the district as evidenced by good grades on tests. In Mrs. Fry’s classroom, the district appointed standards and objectives, vocabulary words, and the daily agenda were always updated on the back whiteboard. The files in the back of the room were updated with students’ missed work (from absences) or retake opportunities for tests. These grades on assessments which were conducted individually and determined almost exclusively based on correct answers to standards-based math questions.

In students’ math autobiographies, students described mathematics in a wide variety of ways. Some students described mathematics by the process of doing mathematics like “engaging in numeric problem solving”, “mental exercises”, “puzzles”, or as a “universal language”. But the most frequent codes used were for students describing their math engagement were about getting good grades, testing, and getting correct answers.



It was evident from students' math autobiographies that students generally took pride in their good grades. They associated good grades with being a good student, with the ability to do math, with hard work, and with correct answers. This was often associated not just with good grades, but with perfect grades with students noting they were proud of A's or 100%. Students also demonstrated the connection between these good grades, testing, and correct answers and the resulting stress they felt.



It was apparent not only from students' autobiographies but also from observations that students feel significant pressure about tests. Often in class, students redirected conversations about mathematics to the testing of mathematics. For example, during an explorative activity they would say things like, "I hope the test is this easy" or ask Mrs. Fry whether these questions would be on the test. The centrality of testing was demonstrated by students and reinforced by Mrs. Fry. During a conversation with a student before class one day, the student questioningly looked at some interesting math lesson materials that were laid out in the class, asking what activity they were doing that day. Mrs. Fry responded, "That's for next week. You're taking the test today. Yeah, that's normally how we do school. I teach you a thing and then we have a test on it." In casual conversations before or after classes students readily admitted the stress they felt over an upcoming test or project or uncompleted assignment.

When asked directly to describe what she believed students thought mathematics was, Mrs. Fry said "I think [for them] doing math is boxing in an answer." This is not unusual for formal math classes, but Mrs. Fry emphasized that this was particularly important characterization for these students because of the importance they have placed on achievement via testing in their lives. Mrs. Fry and her grade level math team try to push back on this narrative by having the first assignment in 6th grade math to be one that doesn't have a single right answer. Mrs. Fry indicated that this assignment was really uncomfortable for students, but begins to address the tendency of these students to believe that right-answer-getting is all that mathematics is.

In an attempt to promote more process-oriented mathematics over product-oriented mathematics, Mrs. Fry incorporated discussion and group/partner work in her classes. At the entrance to the classroom a set of math discussion questions were posted with prompts like, "describe a mistake you made and what you learned from it" or "explain how you challenged yourself today." However, while there was significant discussion in the class, Mrs. Fry admitted that it was often was rooted in making sure that students got the right answers: "[discussion is] not necessarily ideas more than it [was] explanations. I think a lot of my students' dialogue is like, 'ask for help or tell them how you got there'; not so much like, 'what do you think?'" In the places procedurally structured by Mrs. Fry to encourage more social and process-oriented mathematics, the tasks sometimes reinforced these ideas that mathematics should look like correct answer getting.

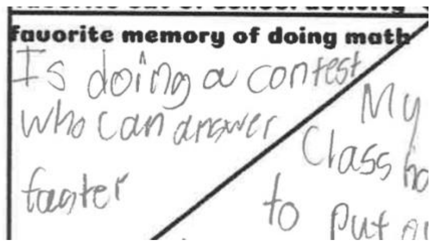
Mrs. Fry's goal for her students was "even just falling in love with the process of learning something instead of having the right answer at the end." Mrs. Fry related this desire for right answers and good grades to enormous anxiety for students:

> *So my motivation and thinking about who my students are, is just like, the majority of my students are just anxiety riddled, little perfectionist, who only see learning as a grade at the end. And that's not genuine learning... they're not willing to take a risk or get joy from learning something from failure... a lot of my motivation is just the like, social emotional needs of my kids. Like, you are 11, you do not need to be sick to your stomach over a test grade.... I would just like kids to like learning and not be so worried about.. the question when I'm in the middle of teaching a lesson, 'so on the test, will you blah, blah, blah' - You are 11 [years old]!*

Mrs. Fry blamed this largely on the structure of school and the perception of school-based mathematics saying "I mean, the design of school is for that. And math specifically is very right or wrong - black or white."

Mathematical Engagement was Competitive

It was apparent that students' stress around testing was not merely related to their own goals to have the right answers all the time, but to have *more* right answers than their peers. In interviews, all students quickly were able to identify students in their class who were "good at math", often someone other than themselves. Students frequently and openly compared themselves to one another in this space. They often initially attributed being "good at math" to grades, but when pressed if they actually knew the student's grades they would shift to attributing being good at math to always getting the correct answers, being faster, or being smarter than anyone else. Some procedures that appeared to be intended to manage classroom behavior and assess progress in the class ended up reinforcing an emphasis on speed and comparison of speed to between peers. When individuals or groups were finished with an activity, they would sometimes be asked to stand or give some hand motion to communicate that they were done. The small act was noticed by many students in the classroom and reinforced the comparison of speed in pursuit of final products and correct answers. The handful of students who typically "won" these contests seemed to really enjoy them. For example, when completing a scavenger hunt, Quantavious smiled broadly and excitedly ran around the class trying to answer all the questions before his peers. In his math autobiography, he was clear that this was his favorite kind of activity.



Mrs. Fry admitted that there was an "unspoken hierarchy" that led directly to individualism and competition when doing mathematics in this space. Mrs. Fry attributed this hierarchy both to the school environment and to her own choices as a teacher. The hierarchy was developed and encouraged from the moment of admission to Porter because "they're at this school, because they're the smartest." Students arrived at Porter convinced of the importance of test grades, and they took significant pride in the idea that their admission indicated that their test scores were not just good but they were higher than other students in the district. Mrs. Fry saw and acknowledged the emphasis on testing, grades, and the potentially negative impact that has on students, but she also often characterized students as having a fixed ability in mathematics describing some students consistently as "high", "strong" or "gifted". Notably, approximately 70% of the students at Porter were designated by the school district as "gifted and talented" and 18 out of the 25 students in Mrs. Fry's 3rd block class were designated as "gifted and talented" in at least 1 subject area.

The "unspoken hierarchy" of the class most evident in Mrs. Fry's assignment of table groups. Each day students sat at their assigned seat in table groups of four. The desks in these groups were each labeled with a number 1-4. Students were assigned a different number three times during the semester without being told what each number indicated when reassigned. The first time was based on demographics so that groups were heterogeneous with respect to race, ethnicity, and gender. The second was based on their previous year's TCAP scores. And the third assignment was based on their semester grade in Mrs. Fry class. The students assigned as a 1s were the 6 students in the highest quartile of the class. The students assigned as a 4 were the 6 students in the lowest quartile of the class. Mrs. Fry said that this was her attempt to "I make them work with a variety of people" and "engage with everybody". She strategically assigned roles based on students' numbers so that the kids who were both the lowest and highest scoring had various roles (like scribe or reporter) and so it wasn't always the students with the highest grades doing all the mathematical thinking. Mrs. Fry's goal in this practice was that kids were learning from one another and there was a (highest scoring) child at each table that could be a resource to help others. Mrs. Fry admitted that even though students were not privy to the knowledge of how they were assigned numbers, she assumed that students would see the pattern. Regardless, she rationalized the decision saying, "for some of these classes, there's only maybe two kids in each class that in any other room wouldn't be the [1] which is part of why I don't feel bad about doing it, because you're all doing great. And then it changes quarterly… [for example] Thomas, who I think he might have gone from a 1 to a 2, he's still got an A, he just wasn't the top six." This grade-based hierarchy had obvious impacts on engagement. Students, either consciously or subconsciously, would frequently defer to the "1" student to check their answers after completing an activity or physically and academically ostracize a "4" student who they didn't see as a mathematical resource. For example, Thomas, a 1, was often asked by other students to check their answers against his and in moments where he didn't know the answer he automatically went to Mrs. Fry for help rather than asking his peers. He was seen as a significant mathematical resource. In contrast, Star, a 4, was occasionally laughed at for her mathematical reasoning and in discussion groups she was rarely asked for her reasoning (unless prompted by Mrs. Fry) and sometimes even physically excluded from discussions. More detail of these examples will be provided in the next chapter.

Math activities and lessons almost always involved discovery of some key idea and discussion with peers, as Mrs. Fry stated that students should explore mathematical ideas and construct their own understanding of mathematical content. Mrs. Fry encouraged tasks that involved exploration and were discussion-based by choosing intentional resources and participation structures from the Illustrative Mathematics curriculum, Desmos tasks and explorations, and Kagan Collaborative practice activities. Mrs. Fry attempted to fight against traditional individualistic competitive norms of doing mathematics by providing space for student discussion and having students work daily in table groups as discussed above. But, because of external pressures to move efficiently through content and because of competition between students, discussion was often wrought with power inequalities and oriented around efficient correct answer-getting as the end goal.

Although students were generally pleasant and agreeable toward one another, one of Mrs. Fry's motivations for engaging in this study was the anxiety that students felt to perform in competition with one another. When discussing her hopes for the ways she wished students would engage in mathematics in this school, she expressed her frustration with the enactment of this particular sociomathematical norm:

> *I think that's the thing, especially in my school is like, there, a lot of them, their entire identity is wrapped up in being the fastest, smartest, most right all the time. And that's not learning at all. That's just like regurgitation.*

Students' performance in mathematics in this space not only characterized their identity, but it also defined their positionality in comparison to other students as the "fastest, smartest, and most right all the time." Students' anxiety of constantly not measuring up to the performances of their peers or always trying to maintain their positionality of being "the best" effected their willingness to take risks. But taking risks in mathematics class significantly required students' agency, which was challenging in this context because math was largely characterized as an act of compliance.

Mathematical Engagement Required Compliance

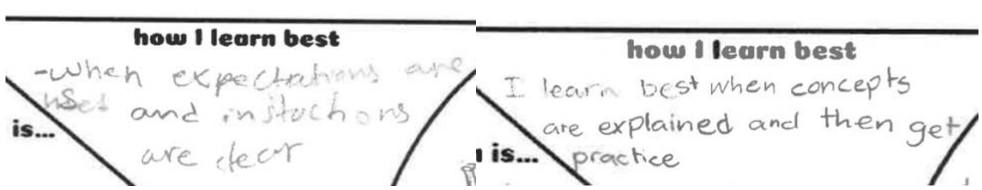
When proposing this study, the public school district asked for justification for choosing Porter as the site of this study saying "most, if not almost all students at Porter will find joy in learning." Mrs. Fry addressed this mischaracterization as a conflation between compliance and joy saying:

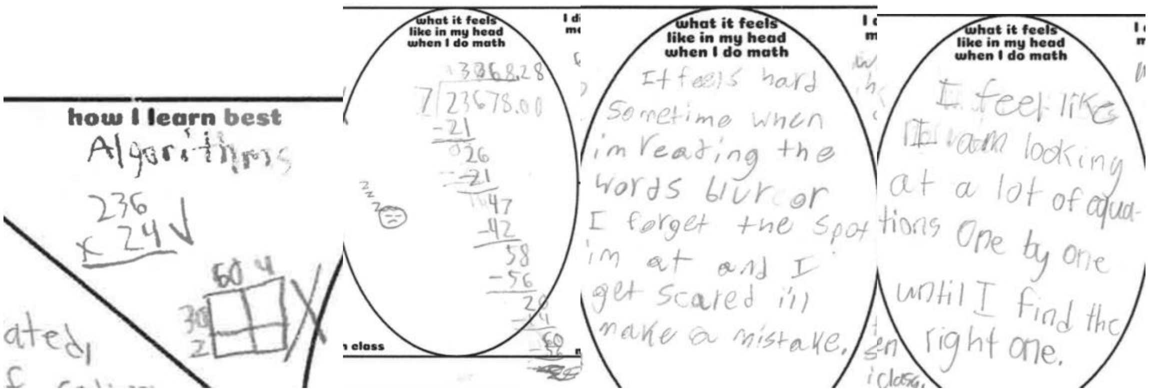
> *People had a lot to say about doing [this study] at Porter. But I think like the problem is this is the school where [students] don't [play and have joy]. Because the kids will just sit*

and do it. They're compliant but compliant is not joy… that's just obedience that's not like love of learning.

Mrs. Fry described this kind of compliance as getting in the way of students getting to do “real math.” In her view, students generally would simply fall in line into whatever the system asked of them and so they never got to experience “real math” because they are willing to just “do the worksheet.” She described that for the students in her class “it's very much like a rule following. Just tell me, just tell me, and I'll do it. And then I know I'm good.” Relatedly, after reading A Mathematician’s Lament (Lockhart, 2009), Mrs. Fry admitted her own belief that mathematics is beautiful because of its straightforward outcomes. She said, “I will confess that I don't see math - like the thing that makes me love math and my understanding of what math is, is that it is a set of rules. And if I do the things you tell me to do, I will get the outcome and it works out.” She went so far as to be concerned that giving students agency in choosing the ways in which they engage in mathematics would give them more anxiety. Doing mathematics to them seemed to mean following the rules and solving numeric problems, complying with the mathematics procedures quickly and precisely and to do mathematics in another way may cause significant frustration and uncertainty.

Students’ math autobiographies reinforced this procedural perspective of engaging in mathematics as they often described doing or knowing mathematics as a set of algorithms or preferred to learn by listening and then practicing. They expressed fear of making mistakes and described seeking some externally defined correct answer rather than freedom to explore, take risks, or have ownership over the mathematics.





This procedural compliance was not only true for mathematical algorithms, but also for behavioral practices and procedures. Students were typically early to class and would happily chat with their peers. As class began, students sat at their assigned seats calmly, and worked nearly silently for 5-10 minutes on their warm-up. When asked to pair up and discuss an activity, all students did, even if reluctantly, without exception. When given the option to engage in a math scavenger hunt around the room or do test review questions, each child chose one or the other and worked and diligently and persistently. It was rare to see a child off task, and if they were it was almost always by reading a book for fun or completing an assignment for another class. I never saw a student on the cell phone or on a random website when their computers were out for a math activity. In the semester I was there, in any context both inside and outside of Mrs. Fry's math class I didn't see a single student receive any form of formal punishment like a detention or in-school suspension. That wasn't to say that children were never punished or that they never did anything wrong, but only to say that the occurrence was so rare that it was never evident to me. There was one moment in which Johnsen, a boy in Mrs. Fry's class, was playfully wrestling with another student before class began. When Mrs. Fry walked in from the hallway (where she was monitoring and welcoming students into class), Johnsen was reprimanded by Mrs. Fry. He immediately stopped what he was doing, hung his head, and was asked to sit on the side of the class while completing his work for the duration of the class period. He appeared to remain on task, albeit isolated, for that class period and students returned to their normal routine.

Notably, the school provided students with a significant amount of freedom relative to some traditional, structured forms of public education. Children walked freely through the hallways between classes and chatted to their friends, peers, and teachers as they moved rather than being forced to walk silently or in single file. Each grade level was housed in a specific hallway, but students moved throughout the building to their Related Arts classes, outside to recess, and to lunch. Children didn't have a school uniform and were only required to adhere to a dress code that stated that private parts must be covered. But teachers actively reinforced students' compliance. In grade level teacher meetings, it was evident that students' compliance made the job easier to some degree for other teachers and therefore was encouraged. In one meeting, the teachers discussed students requesting to buy items for math play spaces or soft spaces for their math class to make it more like their literature class. One teacher suggested that they didn't have the time or energy to create "nice cute work spaces" and another echoed that students are fine and should just sit in their desks. While Mrs. Fry spoke of students' compliance as counter to what she hoped would characterize mathematical engagement in her class, she also admitted that students went along with her even when they didn't want to because of the relationship she had with them. Her care for them led students to trust that she had their best interest at heart and so they complied with her instructions, regardless of any potential for their own agency.

Mathematical Engagement was Both Fun and Stressful

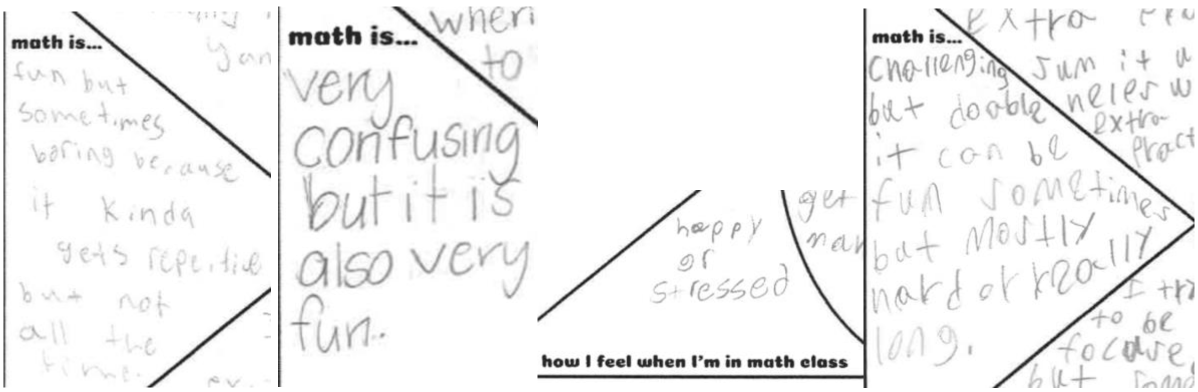
Mrs. Fry was able to make mathematics meaningful to students and convince them to comply consistently even when it seemed to induce significant anxiety. There was a sense of intentionality, care, and stability to this classroom that indicated Mrs. Fry's attempt to support students' joy and endurance in mathematics even in the midst of harmful systemic structures. This mathematics classroom was a loving and meaningful place in which students engaged in math. At the end of every day at dismissal, Mrs. Fry dismissed class by saying "Bye! Love you!" As Mrs. Fry and I met in her classroom, it was hard to miss the physical indicators that Mrs. Fry was a teacher who deeply cared about her students and was well loved. Behind her desk was a wall and storage cabinet covered in wealth of cards and photos both from previous students, drawings and notes from current students, pictures of Mrs. Fry and her loved ones, and cute quippy teacher sayings. Mrs. Fry was deeply invested in her students and knew them well. Between lockers in the hallways, the walls were covered in recent student work from various classes. Some math work displayed from Mrs. Fry's class demonstrated linear relationships about Pokémon cards, bakery sales, lululemon and fast-food purchases. Mathematics was connected to students lives and made meaningful to them in this space.

This tension between potentially harmful outcomes of the sociomathematical norms above and students' enjoyment in mathematics learning in this space was apparent in students' math autobiographies. While students demonstrated the prevalence of sociomathematical norms outlined above, the items coded about how students saw themselves when they do mathematics provided nuance into the ways that students took up these norms. For students, mathematical engagement was situated in tension between mathematics that was fun, powerful, and interesting for them and mathematics that was stressful or boring because of practices that felt necessary within their system of formal education. Of the 67 moments where students identified how they characterize themselves when they do math, four were neutral descriptors (such as "I feel okay when I do math"), 21 were negative descriptors (such as "Math is mostly stress"), and 42 were positive descriptors ("Math makes me feel powerful" or "Math is interesting"). The most common positive code was "having fun" (9 out of 42 total codes) and the most common negative code was "stressed" (9 out of 21 codes).

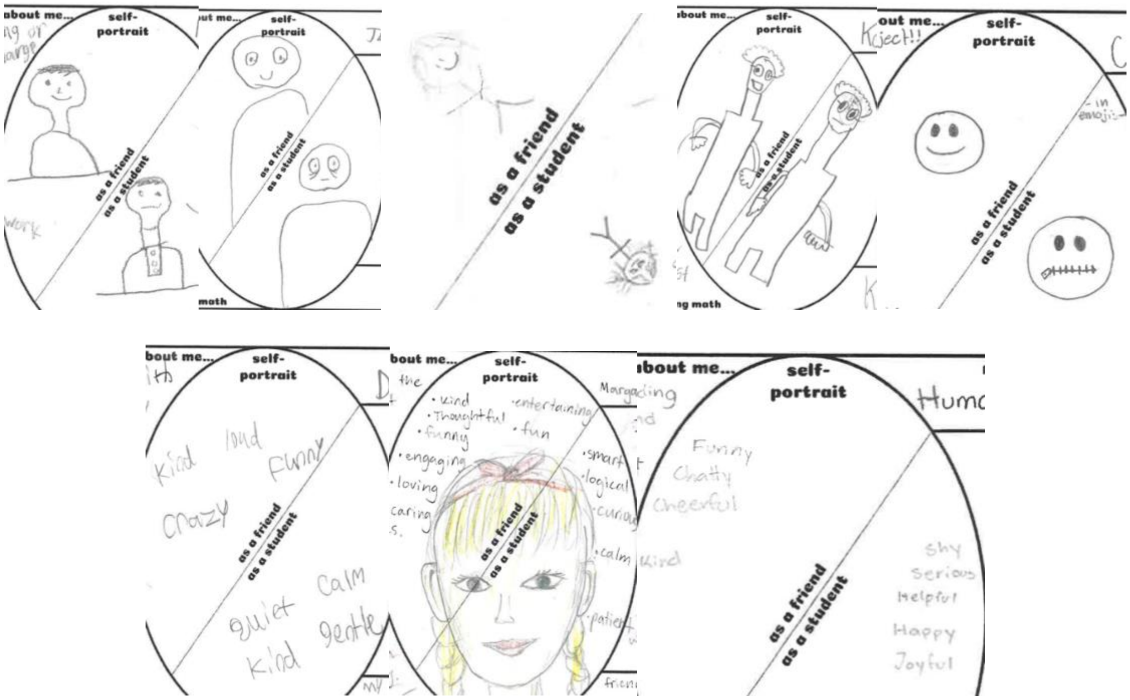
Positive	Neutral	Negative
having fun	okay	stressed
confident	concentrating	scared
refreshed	[neutral face]	confused
motivated		bored
powerful		in pain
growing		chaotic
challenged		tired

Table 2: Examples of codes used for "I feel… when I do math"

Students expressed significant enjoyment in the collaborative games, in activities that involved movement, in clear instructions from Mrs. Fry, in her jokes and stories, or in feeling successful. Although students experienced anxiety and stress, they seemed generally positive about mathematics; more positive than they were negative. Not only that, but some students' illustrated math autobiographies described math as both positive and negative. Five of the 21 students in this 3rd block class filled out the "math is…" section by saying that math is "fun" along with some other negative descriptor like boring or hard.



Regardless, Mrs. Fry felt a lack of joy in her classroom and therefore sought out this collaboration. Her observations and instincts were confirmed by my own observations and by students' depictions of themselves in their math autobiographies. While they described math as fun, they also drew dichotomous self-portraits of themselves as a friend as compared to themselves as a student. In these self-portraits, students often represented themselves as a student with sad or emotionless faces or with dark circles under their eyes. Students also described their personalities as noticeably different: calmer, gentler, or more shy. In contrast, they drew themselves as a friend with broad smiles and described themselves as funny, crazy, or cheerful. They appeared to be adjusting who they were outside of this context to match the expectations of them as a math learner.



RQ2: Rehumanizing Conditions with the Potential to Support Students' Joy

To address research question 2 and explore ways to support students' mathematical joy, it was critical to understand the ways that students understood how to engage in mathematics and to be responsive to the ways that students found joy in their lives, both inside and outside of math class. To begin to understand students' joy in this context, I asked: What kinds of things/tasks/activities do students enjoy? Coding students' illustrated mathematics autobiographies for "conditions for enjoyment" unsurprisingly revealed that there was a wide variety of tasks and activities that students enjoyed. This was diverse group of students that enjoyed reading in hammocks, hanging out with their friends, playing softball, listening to music, ballet, playing games with their families and everything in between. Students' widely varied experiences and preferences made it impossible to design tasks that directly correlated with specific activities that every student enjoyed.

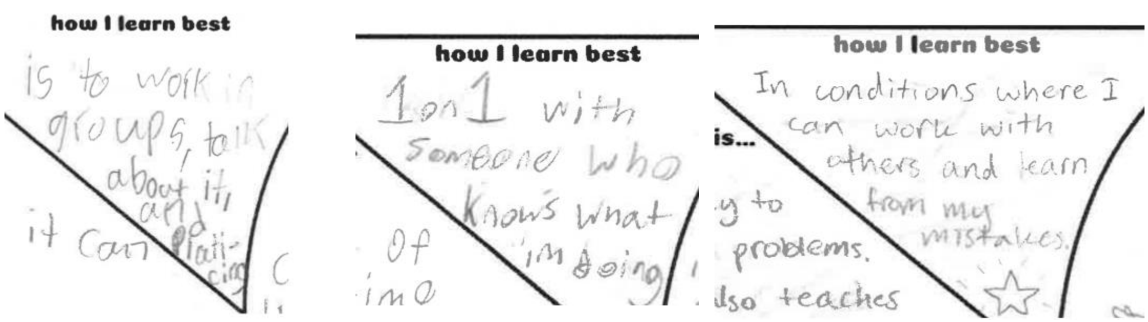
Instead, because of our desire to resist potentially dehumanizing forms of mathematics as described above, these items were coded for conditions for enjoyment were categorized by the eight dimensions of Rochelle Gutiérrez's framework for Rehumanizing Mathematics: participation/positioning, cultures/histories, windows/mirrors, living practice, creation, broadening mathematics, body/emotions, and ownership. Each item identified as a condition for students' enjoyment was labeled as one or more category of rehumanizing mathematics. This was to shed light on the ways that students positively experienced the world and mathematics, and incorporate design features into the math class that were responsive to students' humanity. Using this information, we turned to the children to learn from them about their relationship with mathematics and what they enjoy in the world and in mathematics for that information to inform design. The four most prevalent dimensions of rehumanizing mathematics were: participation/positioning, body/emotions, cultures/histories, and ownership.

Participation and Positioning

Gutiérrez says that "rehumanizing mathematics with respect to participation and positionality involves recognizing hierarchies in classrooms" (Gutiérrez, 2018a, p. 5) such that students attend to one another rather than seeking the approval of the teacher. In their illustrated mathematical autobiographies, this was the most frequently coded dimension (almost twice as often as the next highest category). This was in large part due to children expressing a desire for play and group work.

Students frequently mentioned enjoyment of play. This was categorized as participation and positionality because of the inherent nature of play to be child-centered, social, spontaneous and agentic, thus upending the cultural hierarchies of traditional teacher-centered, individualistic mathematics classrooms. Students in this class expressed their enjoyment of tasks that involved play, game play, and social play with friends and family. They enjoy playing outside, playing specific games in math class like Sum 'Em Up or gimkit, playing games with friends, or playing on teams. Almost all the items coded involved some kind of game-based play.

With the exception of one student who preferred individual work, students expressed enjoyment of group work. They preferred to work in groups, work with their friends, learn from one another, and be able to make mistakes in this context. Students referenced talking in groups, working one-on-one in pairs with someone that they could trust and rely on mathematically, and learning math from and with their classmates.

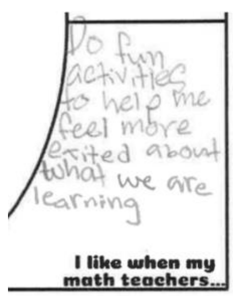


Body and Emotions

The rehumanzing mathematics dimension of body and emotions is characterized by departing from the perspective of mathematics being purely logical or cognitive and instead inviting all parts of students' physical and emotional humanity into the learning context (Gutiérrez, 2018a, p. 5). For example, learning mathematics could involve vision, movement of their bodies, touch, intuition and estimation, affect and emotions, etc.

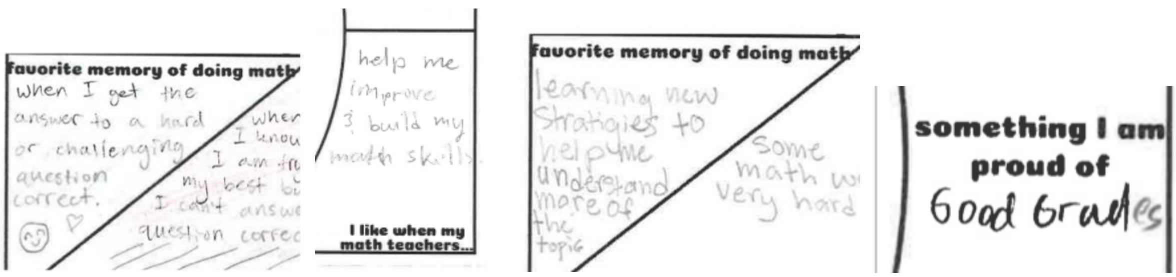
Students mostly enjoyed physical activities, but a few students also mentioned enjoying time for quiet and rest, visual representations, and simply time to have fun. Students enjoyed moving their bodies in many forms of physical activity and especially with respect to playing sports or being outdoors. Students mentioned basketball, swimming, dancing, tennis, skateboarding, softball, soccer, football, and others. In class, students often discussed what sports games were happening that night or weekend, how practice was the previous day, or what kinds of sports their friends and siblings played. Students were often dressed in athletic gear like tennis skirts or basketball shorts and wearing t-shirts from various middle school sports teams. Students also mentioned a physical activity that they enjoying in math class. For example, "passports" was a practice-based task in which students had to get up and move around the room to solve mathematical problems, collecting various passport stamps from locations around the world as they went. When engaging in movement based tasks , students were noticeably more excited, talkative, and energetic as they moved around the room.

Some students also talked about the importance of being able to have fun and be excited about what they were learning in math class – attending to students' emotionality into the learning space. Having fun in math class this was most often seen as the responsibility of Mrs. Fry. Students liked when she was sarcastic, made fun activities for them, or told funny jokes to make them laugh.

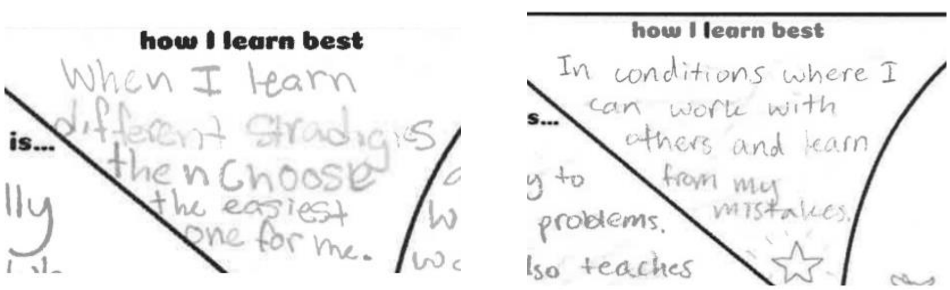


Ownership

Students taking ownership of mathematics was indicted when students showed evidence of doing mathematics as "something one does for oneself, not just for others" (Gutiérrez, 2018a, p. 5). This was a complicated dimension to address because students' pride in their own learning of mathematics was often grade-based which was positive reinforcement given by an outside party. Students could reasonably have been pursuing deeper understanding of mathematics, wanting to improve their math skills, or wanting to get good grades because they were trying to meet the expectations of Mrs. Fry, their parents, or other outside figure. Motivation was difficult to disentangle, therefore in students' math autobiographies were coded as “ownership” if students demonstrated they felt joy, pride, or satisfaction in their success or growth of learning mathematics, even if that success or growth was indicated by good grades. Because of the context in which these students learn, as described above, students often expressed enjoyment in feeling successful in mathematics by getting correct answers, improving mathematics skills and understanding, and getting good grades.



Students also expressed a desire to have some autonomy in their mathematics learning by choosing their own mathematical strategies in ways they deemed fit and adapt and learning from their own mistakes.



Cultures and Histories

Gutiérrez characterizes the culture and histories dimension of rehumanizing mathematics as “acknowledging students’ funds of knowledge, diverse algorithms, history of mathematics, and ethnomathematics” (Gutiérrez, 2018a, p. 5). This dimension was widely varied in students’ illustrated mathematical autobiographies. In coding for this dimension, I captured any moment that provided insight into who students were outside of this class based on their preferred activities or indicators of cultural upbringings. For this group of students, this involved listening to music or going to live shows, playing games, hanging out with friends and family, telling stories, and working with animals in some way.

Students demonstrated appreciation and enjoyment of drawing on their cultural identities in math class. Students perked up and listened carefully to specific songs when Mrs. Fry played their class's music playlist during transitions or work time. Mrs. Fry intentionally created tasks that drew on things students enjoyed outside of math class. For example: the graph hanging in the hallway reflected pieces of their outside worlds with data graphed about sales of Pokémon cards, lululemon outfits, burgers, and baked goods.

Attempting to design for students' mathematical joy

Our initial attempt to witness students' mathematical joy in this 6th grade academic magnet mathematics classroom involved setting up conditions that we believed to be optimal for students to experience mathematical joy. Based on the data collected and analyzed above, we designed a 4-day math unit about the distributive property that incorporated research-based practices that proposed to being joy into math lessons; and supported these students' unique joy by designing responsively with dimensions of rehumanizing mathematics.

Parks postulates that "children experience joy in well-designed mathematics classrooms" and provides five research-based practices for bringing joy into PreK-Grade 2 math lessons (Parks, 2020). These five practices are: creating space for play, allowing children to make choices, offer problems that allow for exploration, relax a little about time on task, and create caring relationships (Parks, 2020). We placed Parks' practices alongside those from Gutierrez (2018) because of the appeal of Rehumanizing Mathematics that encapsulated a broad range of students' mental, physical, relational, societal, and intellectual well-being as outlined in Chapter 2. Using this framework, we co-designed a math unit about the distributive property that incorporated explorative tasks, involved discussion and partner work, play, choice and agency, music playlists created by students, and more.

Immediately after the first lesson of the distributive property unit I asked Mrs. Fry if she thought that the redesigns made a difference in the ways students did math. In her response, she didn't mention any form of affect or emotion, but she noted increased participation. Similarly, in my voice memo about the first day of the unit, I noticed that students grasped the mathematical content and students participated more and for longer periods of time, but I didn't note anything about increased affect or joy. After the second day, in the voice memo, I noted that the lesson "didn't necessarily feel distinctly joyful". There wasn't a lot of smiles, laughter, or even movement. Students generally sat at their desks, there was a lot of chatter as kids were actively talking to one another about mathematics, every kid was participating, and there was no one sitting on the sidelines not doing math. After the last day of the unit, I reflected that "there's a lot of talk, but I'm not sure I would characterize it as student joy." Moments recorded that did attune to affect almost exclusively noted negative affect. After the first day of lessons, Mrs. Fry and I reflected on the students' participation and engagement and discussed students' anxiety, competitive performance, and lack of feelings of belonging in this learning space. These reflections were not what you would expect after finishing the first day of lessons designed to supports students' joy. Students' reflection about the week's lesson supported these observations.

Students completed a set of reflection questions at the end of the week. Students were given five prompts: "I used to think… but now I think…", "I wish Mrs. Fry knew", "One thing that surprised me this week was…", "Something I wish we did more of in math class is…" "Something I wish we did less of in math class is…". In these reflections five students noted that math this week was "fun" or "great" in some way. Eight students said they wanted to do more partner work and/or stations like we did this week, but seven students said they'd like to do less partner work and stations like we did this week. Student's responses to these intentionally designed lessons were largely mixed. Some students enjoyed it while others did not.

In a week’s worth of fieldnotes, memos, and reflections, the only moment which I talked about affect, was when I had a personal on-on-one moment with a students and noted that they smiled and were frustrated. By the end of this unit, I was actively wresting with the question, “How do we know when we're looking at students joy?” The affect evident only in this personal exchange made it evident that a deeper relationship and more narrow focus on student interaction and stories was necessary to investigate joy. This set of lessons pushed the research in a new direction, one that required a closer focus on students: talking to them about what they think joy is, noticing the small moments of affect as they engage in mathematics, and learning their stories to make sense of their mathematical affect in this space.

Conclusion

Ethnographic open coding of the data from Phase 1 revealed that students largely engaged in mathematics in this space in ways that were product-focused, competitive, compliant, and that students largely enjoyed doing mathematics in this space. Students’ mathematical autobiographies gave a picture of students’ lives, preferences and beliefs about mathematics. These artifacts showed that the four most prevalent conditions for students’ enjoyment with respect to the dimensions of rehumanizing mathematics were: participation/positioning, body/emotions, cultures/histories, and ownership. Designing to support students’ joy in this mathematical learning environment based on responsive rehumanizing mathematics and research-based pedagogical practices to support students’ joy revealed little apparent change in students affect but increased participation and engagement. These findings urged this study to focus more narrowly on specific students and their unique affective mathematical learning experiences while learning mathematics in this space.

Chapter 5: Students' Affective Experiences with Mathematics

This chapter focuses on case studies of four students who provided a window into the local particulars of how we can support mathematical joy in this math classroom. The conceptual framework, which operationalized our definition of mathematical joy in conjunction with frameworks of humanizing mathematics, made clear that there was potential to design for students' mathematical joy by centering students' full humanity in mathematical learning spaces by satisfying their mental, physical, relational, societal, and intellectual well-being. Doing so resists prevalent and persistent forms of dehumanizing mathematics that restrict students' well-being and thus suppress students' joy. The lack of joy in this classroom indicated a degree of students' unfulfilled well-being. Mrs. Fry and I wondered what was hindering mathematical joy in this space, in what ways students were feeling uniquely dehumanized in this space, and what kinds of well-being were going unsatisfied in this math classroom.

It was evident to us from the first few weeks of the study that researching joy in mathematics required relationality and a fine-tuned look into the personal experiences and interactions of students in this math class. Therefore, I spent three months observing and forming relationships with the four focal students and learning from them what conditions had the potential to support or hinder their unique mathematical joy. I did this with the goal to report back to Mrs. Fry and at the end of the semester, and collaboratively design a math unit to intentionally support students' mathematical joy (detailed in Chapter 6). After three months, I came to Mrs. Fry to member check my preliminary findings based on my in situ analysis of students affective mathematical engagement. We both observed significant stress and anxiety for students based on competition and comparison to other students. This aligned with the observation that students seemed to engage in math with more joy when engaging in tasks that necessitated students' mutual, social, relational, playful engagement – the anthesis of competition and hierarchical comparison. Three potential conditions for supporting students' mathematical joy stood out to me and Mrs. Fry: 1. Humility, vulnerability, and needing one another; 2. Confident generosity, feeling like a resource to people around you, and looking outside of yourself to help your peers with and through mathematics, and 3. Social games, play, surprise, interest, and challenge. It was apparent to us while working with these students that tasks that intentionally supported these kinds of mathematical engagement would support students' mathematical joy because it pushed back on the competitive sociomathematical norms that were causing significant stress and anxiety for students. In this chapter, I'm calling the coupling of these first two themes, *mathematical reciprocity*. Mathematical reciprocity is the mutual sharing of mathematical ideas and resources such that students need one another and look outside of themselves to intentionally help one another. We intentionally designed the final math unit (detailed in Chapter 6) to necessitate or support a disposition of mathematical reciprocity (designing for conditions 1 and 2) in as many ways as possible through game based or playful tasks (designing for condition 3). In this chapter, we will take a close look through multimodal interaction analysis to investigate what interactional and affective cues Mrs. Fry and I were seeing that led us to this conclusion. The interaction analysis adds nuance to the stories of each four students' expression of joy and serves as an exploration of the themes identified through observation and interviews to see whether and how those themes played out in moment-to-moment interaction.

Led by my definition of mathematical joy (characterizing math joy as a positionality toward mathematics that animates engagement leading to satisfaction), I looked to students to learn how specific mathematical tasks and interactions led to their animated engagement *or disengagement*. Moments in which students demonstrated this animated, affective intensity that led to their mathematical engagement or disengagement pointed me to potential facets of well-being or benevolence that were satisfied or dissatisfied for them. Through a collective understanding of each student's unique mathematical engagement, demonstrated affect, learned histories, and experiences within the cultural, political, and personal context of this classroom, I made sense of these moments to better understand what unique ways children were experiencing dehumanization in this math class.

By intentionally looking to students' affective reactions, I built an understanding of what conditions might support or hinder students' mathematical joy. Therefore, in this chapter, I refine my focus with respect to the two primary research questions:

1. What are some characteristics of affective mathematical engagement? What does affect look like, sound like, feel like for these four students?
2. What conditions support or hinder students' joyful mathematical engagement and participation?
    a. To what extent do these align with the themes that emerged from earlier observations?

This case study analysis focuses on students' affect because I was seeking moments of intensity that indicated a positionality toward mathematics that animated or suppressed mathematical engagement and participation. Affect here is defined as "an intensity in and around bodies (Papacharissi, 2015; Ringrose & Renold, 2014) that nudges them, often preconsciously, in certain directions of thought and action" (Parks, 2021, p. 123). By demonstrating affective intensity that motivates mathematical engagement or disengagement, students revealed a stance toward mathematical tasks and interactions and indicates that some human desire or need is being satisfied or dissatisfied for them respectively. This satisfaction or dissatisfaction is at the core of humanizing and dehumanizing mathematics respectively as it pointed to ways that students' full humanity had been uplifted or oppressed. This chapter seeks to understand what these students experience as dehumanizing rather than make assumption about their stance towards interactions and tasks and designing with only broad generalizations in mind. Parts of students' humanity that are or are not being satisfied could in turn point the research toward what potentially supports or prevents students from engaging in mathematics with joy. These affective episodes that follow are a snapshot of student desires and thus are not sufficient to make conclusions about why the given tasks, interactions, materials, or local context leads to this reaction. Thus, what I learned about students (their stories, experiences, interactions, personalities, relationships, desires, etc) by ethnographically getting to know them over the course of 4 months was leveraged to make sense of these episodes of students' affect in mathematics.

Research question 1 is an inquiry about local moments of activity and interaction, asking: What are some characteristics of affective mathematical engagement? What does affect look like, sound like, feel like for these four students? Video record of each students' engagement in their Geometry unit was analyzed to look for moments of affect when students were engaging in mathematics. The intensity in and around the individual and the resulting action that students took toward or away from engaging in mathematics defined these moments as significant. An affective moment could be anything from a student rolling her eyes dramatically, giggling, or burying their face in their hands, so long as the moment nudged the student toward (even momentary) engagement or disengagement in mathematics. These moments were recorded in a data log. For each moment, I gave a rich description of the affective moment (what it looked like, sounded like, felt like), and noted the context (the task and the interactions between the students and other people during this moment).

Research question 2 focuses on making sense of students' affective moments in mathematics. What events or experiences explain these students' affect in mathematics? For each of the focal students, I kept a data log containing student information collected from research practice partnership meetings, classroom observations, illustrated math autobiographies, Math Moments Journals, and interviews. After locating a significant affective moment, I used this data log to then make sense of these moments to offer an explanation of what events or experiences informed students' affect. This sought to discover what the intensity revealed about students' basic human desires that were being satisfied or unsatisfied in this mathematics class. We postulated that, by designing for activities that met these needs for students, they might experience more mathematical joy. Therefore, these affective moments and the cultural meaning drawn from them informed the design of the Data and Statistics unit (detailed and analyzed in Chapter 6) to support students joyful mathematical learning.

Research Context

This chapter focuses in on the lives, interactions, and mathematical engagements of four students: Star, Isaac, Thomas, and Ruqaiya. Episodes containing affective moments for these children were found during their geometry unit. Therefore, details of this research context now focus on the four focal students and these geometry tasks specifically.

Selecting Focal Students

Four focal students, Star, Isaac, Thomas, and Ruqaiya were intentionally chosen “constructed cases” that provided an “angled vision” on the phenomenon of mathematical joy in this setting (Dyson & Genishi, 2005, p. 12). The four students appeared to engage with mathematics in unique ways: their participation and grades in the class varied widely, and they varied demographically. Two students were male and two were female, one girl was Black (Star), another girl was Bangladeshi-American (Ruqaiya), one boy was white (Isaac), and the other boy was Asian-American (Thomas). These students therefore provided a window into the landscape of the varied experiences in this classroom that directly influenced our ability to design for students’ mathematical joy. Their stories and experiences in this context provided insights into the phenomenon of mathematical joy or lack thereof by representing a small piece of a context that was “overflowing with human experiences and human stories” (Dyson & Genishi, 2005, p. 12).

These students were selected during the first phase of the study based a triangulation of data that was collected from the perspective of the researcher through my fieldnotes from weekly observations, the teacher through interviews and Research Practice Partnership meetings, and the students through their Mathematical Autobiographies. Leading up to and during the week of distributive property lessons detailed in Chapter 5, I made note of four students that seemed to me to be interesting, uniquely representative cases of affective mathematical engagement in this class. Having identified my potential list of case studies, I talked with Mrs. Fry, and asked her which students in her class she felt would be interesting to focus on for a case study. She noted that if we could design lessons in which Star, Thomas, and Isaac had joy in mathematics then we would be doing something right. Ruqaiya was not on Mrs. Fry’s initial list, but in other conversations she mentioned that Ruqaiya was a student who took her some time to get to know.

As mentioned in the previous chapter, five of the 21 students in Mrs. Fry’s 3rd block class filled out the “math is…” section of their illustrated mathematical autobiography by saying that math is “fun” along with some other negative descriptor like boring or hard. Isaac, Star, and Thomas were three of these five students. Ruqaiya also described math as “Interesting and can be fun,” and when asked how she feels when she’s in math class she simply wrote “Happy” in a pleasant curly script. But, as described below, my first interaction with Ruqaiya revealed her immense fears and anxieties around mathematics in this space. Each of these students found math fun, but each of them also keenly hits on the apparent juxtaposition of joy and formal mathematics learning.

Introducing Star

Star was a Black female student who stood out in my baseline observations as social and friendly, but largely withdrawn and often actively pushed to the outskirts of participation in mathematics. As a typical example, in one activity, students were asked to consider an Estimation 180 problem (https://estimation180.com/) and were randomly assigned to one of four corners of the room to talk in groups about their estimations. In Star’s small group, six students chatted and discussed their potential estimations for how long a roll of bubble wrap might be. Star stood up from her desk, went to the corner of the room she was assigned to, but stood off from the group slightly and didn’t contribute her ideas. She wasn’t invited into the conversation by her peers at any point, and after a few minutes the circle of students physically closed themselves off from Star. She was left standing as an outsider until the activity concluded. Mrs. Fry. described Star as “standing out” in the class. This was in large part because other students sometimes didn’t want to work with her. When discussing Star in groupwork, Mrs. Fry said, “[students] know she's not a source in this room. Yeah – so… she thinks she's not a source.”

Star sometimes chose to disengage based on a variety of factors, and students sometimes chose not to work with her because she was not perceived as being a source of mathematical knowledge. During the semester, Mrs. Fry had a student come to her personally to ask to change partners after working with Star because she was seen at best as unhelpful and at worst as a burden. As a result, Mrs. Fry and I discussed Star as an evident example of a child who lacked belonging in this space, which was a detriment to her engaging in math with joy. Regarding Star’s experience within the classroom culture, Ms. Fry said:

> *Like the culture of this space makes her feel like she's here [motions her hand low to the ground]. Yeah. Like she can't engage in the way that she needs to and so it's just piling on to her lack of feeling like she's not belonging or wanted. And this might be a safe space physically, but emotionally - I cannot imagine that it is not hard to be surrounded by hyper-achiever, over-organized … she's not that low on the totem pole in a normal world. But in this world, she stands out, and it sucks.*

Therefore, in this class, Star was frequently characterized as not being engaged or successful in mathematics learning.

Mrs. Fry often talked about Star not completing retakes or turning in homework. She didn't pass her first math test until the end of January. She was labeled as a number "4" in the class meaning her grades were in the lowest quartile of the class. In an interview, Star claimed that she was only at Porter because of her ELA scores, meaning that she wasn't proficient in math on state testing and she was well aware of it. During my classroom observations, I heard Star consistently speak negatively about her own ability to do mathematics. During one observation, Star and Isaac were doing a word problem and Star wrote "10x-2." This was incorrect but contextually reasonable. In response, Isaac said, "it's wrong" and Star responded, "I know". Isaac then asked why she wrote it and she said "I'm just really bad at math" and hung her head. These interactions surface the dehumanizing, exclusionary nature of formal mathematics for Star especially with respect to forms of participation, relationships with other students, grading or assessments of performance, and the frustration Star demonstrated at the hands of her interactions with other people around mathematics.

In contrast, Star enjoyed English Language Arts (ELA) because she liked the task, people at school and in her family told her that she's good at it, and she'd been formally recognized as being successful in it based on her test scores and admission to Porter. During her first interview, I was surprised (because of her perceived lack of success in the subject) when Star also talked about enjoying math. She specifically like doing Numbered Heads together because it involved "just doing work together. And then when you get it right, people give you compliments and stuff." She claimed to only really like math when it was social, when she could work with other people, and build her confidence by receiving encouragement, help, and experiencing success alongside her peers.

Introducing Thomas

Thomas was an Asian-American student who was chosen as a case study because, while seemingly extraordinarily successful in mathematics class, he was often stern, serious, and quiet during mathematical activities. However, outside of math class, he was bouncy, chatty, playful, and very social with his friends. He had a group of close friends that he felt comfortable around, but was mostly reserved around adults. Mrs. Fry told me that there were many days where he didn't speak to her at all, and during my interview with Thomas he spoke into his hoodie for almost the entirety of the interview. When asked for a second interview he said that he would rather not be interviewed.

Thomas valued high grades and he would retake tests even if he got a 92%. He was also very hard on himself when he didn't fully understand a lesson or got a problem wrong. Isaac, a classmate, even described Thomas as "scolding" himself if he got an answer wrong even by a few decimals. Mrs. Fry also taught Thomas' older sister and remembered her success in mathematics and remarked on her high achievement at the most prestigious high school in the district. Thomas also noted that one of his other older sisters was taking Calculus. Similarly to his sisters, he was perceived in this space by most student to be one of the best math students.

He was often named by his peers as a student who was "good at math" as people assumed that he had the highest grade in the class even though he was a #2 for part of the year, meaning that he was in the 2nd quartile of grades. Students in the class often treated him as resources for checking if their answers were correct and he was regularly found telling other students how to solve their problems or how to solve them more quickly or efficiently. As a result, he rarely asked for help from other students or from Mrs. Fry. During a test review activity one day, he went to Mrs. Fry with a question but seemed extraordinarily flushed and embarrassed. This embarrassment increased as Mrs. Fry encouraged him to talk to his tablemates because when he did, they loudly told him that they would have been able to help him if he just asked them. Similarly, during his interview, I was struck by what felt like fear when responding to questions. I noted that he seemed to be concerned that he wasn't going to say the right thing and that he appeared to think that the interview questions were evaluative. He would squirm or seem uncomfortable when answering an interview question and talked into his elbow or hoodie when he did answer.

In his interview, Thomas attributed his hesitancy to speak to other people with him being shy. He acknowledged that he felt really good in math class when he got answers correct, but that he didn't raise his hand often because he was shy. This demeanor seemed to be primarily reserved for math class because, as mentioned above, he was boisterous, bouncy, smily, and social as soon as class was dismissed. He was open about preferring to work by himself in math class but would work with other people if forced to. He placed a lot of significance on speed. He was very regularly the first student to stand up or raise their hand to indicate they had finished a problem.

Thomas enjoyed playing games in math class, especially games like Gimkit and Kahoot. During his interview he claimed that he liked games because he "usually gets first place" due to good "strategy". When asked what makes games fun, he said that he just liked "competitive games" and when I followed up to ask if they are fun regardless of the outcome he said, "I don't really lose that much, but I think winning is a lot of fun".

Introducing Ruqaiya

Ruqaiya was a Muslim daughter of immigrant parents who moved to the United States from Bangladesh. She was chosen for this case study because she seemed to largely enjoy math class but was openly riddled with anxiety about school. One of my first one-on-one interactions with Ruqaiya went like this:

Ruqaiya: I'm probably not cut out for the test.
Madison: Why not? Whatcha Feeling?
Ruqaiya: Anxiety. Same as always.
Madison: Do you feel that a lot in this class particularly?
Ruqaiya: Yeah. It's every class.
Madison: Just every class all day long?
Ruqaiya: Pretty much.
Madison: That feels kind of overwhelming
Ruqaiya: My life is overwhelming!
Madison: It's just full of anxiety?
Ruqaiya: Pretty much
Madison: Why do you think that is?
Ruqaiya: Stress

Madison: Is it especially with relation to like tests and stuff like that?

Ruqaiya: It's really just focused around school.

Madison: It does feel like there's a lot of anxiety at Porter - with you and all around

Ruqaiya: It is, like specific - you know how your parents at least expect you to have a certain score in school?

Madison: yeah

Ruqaiya: Let's just say - my life

Madison: yeah, you feel a lot of pressure?

Ruqaiya: Always.

Later in this conversation with Ruqaiya, she expressed a desire for freedom from this pressure and anxiety in schooling especially the pressure from her parents. She personally pursued this freedom by escaping into fiction mythology books. She prided herself on being a "mythology geek". In every classroom video, she had a large chapter book sitting on her desk, usually in the mythology fiction genre. Ruqaiya was also playful and social with me in the classroom space. After our first interaction transcribed above, the next day Ruqaiya eagerly talked with me, played itsy-bitsy spider in an attempt to "scare" me, and invited me to play math jeopardy with her team in a review game. My relationship with her was quick to develop, she was eager to talk, and open about her feelings and perspectives about mathematics.

Ruqaiya perceived that her love for fiction books, desire for freedom, and her playfulness were in direct opposition to her parents' wishes for how she spends her time. During an interview sitting in the mythology section of the school library, Ruqaiya said of her parents:

> *Especially with Asian parents, you're going to have to be more proficient in studies like mathematics, especially because they want you to be more advanced... So I get really pushed really hard to do that nonfiction section over there [points away from the mythology section and toward the nonfiction section of the library]. And I have to study a lot, which is why I'm good at math. But I also like math because it's simple.*

Ruqaiya's parents wanted her to pursue a career in medicine, and she knew that mathematics was useful in pursuing a future in a scientific field. When asked how she felt about math immediately following a conversation about her parents' desire for her to pursue a career in medicine she said, "Math is good too. I'll need it, especially when I grow older. Like the more you get into science, the more advanced you have to be in mathematics." She was expected to study hard and perform exceptionally well in mathematics and science specifically.

She felt pressure to perform academically from her family, but also demonstrated immense love for her family. She talked openly and excitedly about her family. Her parents were immigrants from Bangladesh and her grandparents still lived there. She loved speaking Bengali and reading and quoting the Quran and Hadith. She eagerly shared the Bengali names for her family members and taught me a few Bengali words saying that "the word 'feel' in English it's like your emotions, but in Arabic, "feel" means elephant." Ruqaiya also shared about her family's culture in the math classroom. During one math lesson, students were creating combo meals with images of food to learn about the distributive property. Ruqaiya expressed playful frustration with being asked to talk about food because she was fasting for a Muslim holiday during the day. She daily wore traditional Muslim attire: a head covering and outerwear that covered everything except her face and hands. She also always wore a medical mask around her nose and mouth so that typically only her eyes showed. She shared with me both her discomfort in wearing this attire on a hot day as well as the comfort it gave her. This comfort was in part by concealing her relationship to her triplet brothers who were in the same school.

Regardless of the pressure to succeed in mathematics for the sake of a future career in medicine, Ruqaiya had a multifaceted perspective of what mathematics was for her. She saw it as an interesting puzzle but also straight forward, useful and practical but also changing and dynamic. In Ruqaiya's mathematical autobiography she described what it feels like in her head when she does math as a mental puzzle in which she "tries to put the pieces together". She believed herself to be good at math because she studied hard and because it "clicks in [her] mind, …like a puzzle". But she called her favorite kind of math "flat math" which she described as math that is "like problems straight out… like numbers and equations rather than pictures". She had little interest in mathematics represented by models or pictures. She was practical about math's utility claiming that, "you're still going to need [math] for everything in your daily life" including paying bills, taxes, shopping, writing books, gardening, cooking and baking, and meteorology.

With this level-headed and utilitarian view of mathematics, Ruqaiya saw math as something that was always growing and dynamic. She explained that her parents viewed Mathematics as a more important subject than something like History because “They don't really consider history that high much, because, I mean, history is history. It's not going to change, people. But with mathematics, there's always something new to learn in science and math.” This dynamic view of mathematics appeared to be hugely influential in how Ruqaiya interacted with mathematics. She saw mathematics as playful “because everything we know now, it really just started with people playing around with things like the concept of 0, decimals. Those-- weren't they all started by someone playing around with stuff?” She understood this view of mathematics because of its roots in Muslim history. She explained that in a book she had at home called the “1001 Inventions of the Enduring Muslim Legacy,” she learned about al-Khwarizmi and the origins of Algebra, astrolabes, and camera obscura. She explained that she didn’t necessarily feel a personal, cultural connection to mathematics because of this history, but she thought it was wildly interesting because it “had a pull”.

Introducing Isaac

Isaac was a white, male student who stood out as someone who didn’t conform to traditional middle school social norms in this space, was honest about his experiences in school, and expressed a desire for meaningful and playful learning experiences. Isaac wore his hair long and sometimes painted his nails pink. He liked “retro” ’80s music and had an ’80s themed birthday party before Stranger Things made it cool. He often expressed not having a large close social group but was kind and friendly to anyone that I saw him work with. He had a handful of people, primarily girls, in the class that he enjoyed spending time with and ate lunch with them daily.

While outwardly he seemed content with the daily on-goings of school, when interviewed, he readily admitted that couldn’t think of any place in the school that he could show me where he enjoyed being. At the beginning of the first interview, I asked each student to show me a place that they enjoyed so we could walk there, and he claimed to not have one because he didn’t like being at school. Instead, we sat outside because he noted that he liked being outdoors on his mathematical autobiography. In a parent meeting, his dad and Mrs. Fry confirmed this tendency to get-by in a context that he largely didn’t enjoy as they discussed that Isaac seemed to “suffer the fools while here.”

He resisted doing school-based activities that wasted time or didn't hold meaning. He expressed disdain for teachers that didn't have lessons planned or who planned activities that to him seemed meaningless, even if they appeared to be fun. For example, his science teacher gave them a project to build a roller coaster when they learned about momentum, and he saw that as a waste of time. Isaac was practical in his ideas for reforming math education into something more enjoyable, saying:

> *All the stuff that goes beyond pencil and paper in math would be expensive to do, chaotic, I'm sure, because most of the kids would probably think that it's just playtime. I don't know. I don't feel like it should because people are clearly learning enough as is. And if this is inexpensive for them, and it's not very chaotic, then I don't see a reason to fix something that's not broken.*

In a similar gesture, when asked to pick a pseudonym, he was one of the only students who said that he didn't care at all and would rather me just pick one for him. At the time, even though his peers were really enjoying picking out their pseudonym, it was evident to me that he didn't feel the need to conform to his peers' ideas of fun or enjoyment. I noted in my reflection that it seemed he was always saying to himself, "I don't need to play that game."

Mrs. Fry described him by saying "he like really just kind of a little bit above it all. But that presents itself as not engaging. And then not engaging looks like nervousness, but it's more like you're not worth my time." Isaac demonstrated very little affect generally, both inside and outside of math class. He participated, raised his hand to ask questions, completed all his work, but his head often rested on his hand and his facial expressions were almost always neutral. Similarly to Mrs. Fry, in my notes, I mentioned Isaac being a "low intensity" child and I often associated his disengagement with an indication of his anxiety. In one of my first baseline observations of this class, I watched students play a group online math game about inequalities from Gimkit. This game was highly competitive and dependent on speed and accuracy. As students played, I noticed Isaac was having trouble typing the less than sign (<) on his keyboard when necessary to get a question marked correct. After getting frustrated with not knowing how to type the inequality symbol, he closed his computer and exited the game completely rather than raising his hand for help or asking one of his tablemates about how to type this symbol. He returned to the game after seeming to collect himself for a few minutes. He again was given a similar question and sat back, frustrated at this inability to answer these kinds of questions correctly. I asked if he was having trouble typing something and he said yes and explained that he wanted to type a less than symbol but didn't know how. I taught him how to find it on his keyboard. He smiled, quickly typed in the correct answer, and moved onto the next question. At the time, I attributed his disengagement to anxiety about not being the correct answer, but in learning more about Isaac it was possible that his mathematical disengagement was also a result of his social isolation or his tendency to not excitedly engage in work that he deemed to be meaningless.

Games and play were significant in Isaac's life. In his mathematical autobiography he mentioned playing soccer which he played alongside his brother. In an interview, when asked to name an activity that he enjoyed outside of school, Isaac focused heavily on video games like Mario, Zelda, and Smash Brothers. When asked what kinds of activities he enjoyed in math class he also mentioned playful game-based activities like Passports and Escape Game. Most of the games and activities that Isaac expressed enjoying in math class could be played individually and were largely explorative and non-competitive.

This is significant because, Isaac expressed in his interview that in math class he preferred to work individually because he didn't like to compare himself to other "smarter" people. He admitted that this comparison was a creation of his own mind saying:

> *If I'm paired up with someone that's smarter than me, even though they're nice people, they've only been good to me since I've met them, they're not judging me at all, I'm like, 'dude, stop judging me. I get that I got the problem wrong. Leave me alone.' But they're not doing anything…there's a self-comparison to other people that are better at that than me.*

Isaac didn't enjoy school activities that involved competition or comparison, like PE or mathematics. He felt this comparison even while simultaneously feeling no ill-will by his classmates. He felt a sense of isolation from other student that he deemed to be better than himself at mathematics even though he was largely friendly to others and also treated by others with kindness. Relatedly, he equated being "good at math" with getting good grades, but also being good at "education" – for example, completing and turning in your work on time. He reasonably (albeit incorrectly) believed that since he typically got 70s on his grades and he also put a lot of effort into getting his work completed and turned in, that this was an indication to him that he was bad at math. When asked if he enjoyed math he quickly responded “no” and claimed that he didn’t enjoy it because he was bad at it.

Geometry Unit Tasks

The affective moments for each of the students in this chapter were analyzed from video recordings of students' small groups from a four-day set of lessons that started the students’ geometry unit. The goal of these lessons was for students to learn about finding the area of parallelograms, triangles, composite shapes, and to find the surface areas of 3-dimensional objects with nets. Over the four days the students completed several different activities to explore these ideas. In the descriptions of the affective moments that follow in the next chapters, the relevant activities from these five days of lessons will be described in detail.

Analytic Methods: Case Studies and Multimodal Interaction Analysis

The case studies were analyzed on two levels: 1. Looking for affective episodes while students were engaging in mathematics and 2. Making sense of students’ affect through students’ stories. Throughout student-level video recorded from the Geometry unit, I used multimodal interaction analysis (Norris, 2004; Parks & Schmeichel, 2014), looking for moments of affect for these four students as they engaged in mathematics. I drew from positioning theory (Davies & Harré, 1990; Harré & Van Langenhove, 1998; Parks & Schmeichel, 2014) to describe the positive or negative stance that students took to the mathematics in relation to their interactions with peers and with the mathematics. Students’ spoken and embodied discourse conveyed an affiliation or stance toward mathematical engagement and demonstrated their exercised agency in expressing satisfaction or dissatisfaction in relation to the interaction with and around mathematics (Holland, 2001). These potentially small affective indicators were critical to attend to given the culture of compliance in this space and the limited agency students expressed in disengaging from mathematics. Therefore, moments of affect were identified for their intensity in and around students’ bodies and the apparent nudging of students in certain directions of engagement or disengagement in the mathematics. I considered it my upmost responsibility in this analysis to avoid deficit-oriented descriptions of marginalized children and to humbly interpret affect and response in these moments noting the potential influence of cultural norms of nuances (Parks & Schmeichel, 2014) and avoiding egocentric descriptions of affective reactions (Holland, 2001). Affective moments could be intentional or unintentional. Moments of affect were recorded in a separate affective data log. This data log recorded a rich description of the affective moment (what it looked like, sounded like, felt like), the task the student was engaging in when this affective moment occurred, and the interactions with other people during this moment. Rich descriptions of affective episodes sought to answer: "What are some characteristics of affective mathematical engagement? What does affect look like, sound like, feel like for these four students?" The episodes described here each demonstrate a shift in students' apparent positionality toward mathematics through students' engagement *and* their disengagement following different affective moments that represented what we saw in their desire for mathematical reciprocity. Each of the episodes described below chronologically included the first set of affective moments for each student in the geometry unit. The episode that includes both Thomas and Isaac working as partners was chosen because it demonstrated this shift for both students and also provided a glimpse into both interactional sides of an affective episode and the potential reasoning behind students' wildly different reactions to the same mathematical task.

In the second phase of this study, by focusing on these students, I learned about their lives, their mathematical experiences and perspectives and the things that brought them joy. This was done primarily through observation of students in classroom observations, through video record of student interactions during math class, journal entries, and interviews. To tell each student's story, a data log was generated for each student in which I recorded every piece of information about that student from any data collected in Phase 1 and Phase 2. This student story data log was created through chronological ethnographic analysis of fieldnotes, voice memos, audio and transcripts from research practice partnership meetings, mathematical autobiographies, audio and transcripts from interviews, video recording of students engaging in mathematics, and artifacts from students' math moments journals. After recording episodes of affect while engaging in mathematics, students' data log was referenced to make sense of and tell the story of potential reasons why this affect occurred. This was necessary to understand what experiences or events led to students' affective reactions to learning mathematics and what conditions supported students' affective mathematical engagement and participation. Ethnographic analysis was utilized on these data sources "as a tool for discovering cultural meaning" (Spradley, 2016, p. 92) of mathematical affect in this space. Learning about these students and the responsive affect that they demonstrated, revealed the human desires that were present in this space and therefore provided insights about the ways mathematics could be more humanizing and engaged in with more joy. This information informed mine and Mrs. Fry's collaborative design of joyful mathematical learning experiences in Phase 3 of the study.

RQ1: Students' Affective Mathematical Engagement

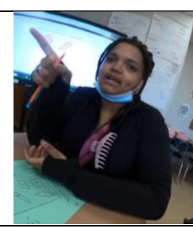
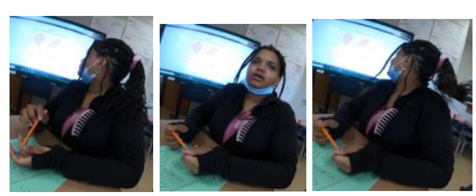
The following is a detailed description of three episodes that each center affective moments of Star, Ruqaiya, Isaac, and Thomas. As we will see in these three episodes, in unique ways, students revealed that *mathematical reciprocity*, a mutual, benevolent sharing of mathematical ideas and resources, was a potential support for students' mathematical joy in this space. Students' positive and negative affective reactions in these episodes revealed the ways that mathematical reciprocity acted in opposition to the competitive and hierarchal sociomathematical norms in this space. Based on students' reactions, Mrs. Fry and I posited that resisting the dehumanization that students seemed to feel as a result of the competitive and hierarchal sociomathematical norms was necessary in design so that each student could uniquely experience more humanizing mathematics and thus experience more mathematical joy. These three episodes emerged from varied tasks, interactions, and student histories with mathematics. In the affective episodes below, I describe students' engagement and interactions during a given task around an affective moment. I then contextualize students' actions, interactions, and reactions within the students' broader experiences and story to make-sense of their affect in that moment. Based on this cultural meaning making, I demonstrate themes that Mrs. Fry and I saw across students' mathematical engagement in the to make decisions about designing for these students' mathematical joy.

Star

Over the course of the first two days of the geometry unit, Star's positionality toward mathematical engagement shifted from animated and social engagement to withdrawn and reluctant engagement and then back to playful and positive engagement. In the following episode of affect, Star demonstrates affect that nudges her away from mathematics through eye rolls, scoffs, sad frowns, and requests to go home. But she demonstrates affect in which she was drawn into animated mathematical engagement through smiles, laugher, playful joking, sing-song vocalization, encouragement, and celebration. In the following episode we see how over the course of 2 lessons, Star's affect reveals her desire for respect and encouragement of her peers through which she could gain confidence as a mutual knower and doing of mathematics.

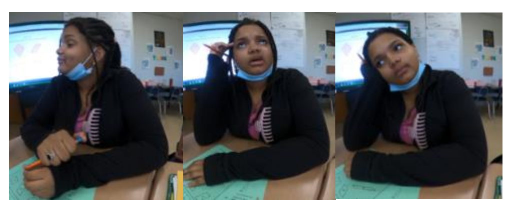
Affective Episode

On Day 1 of this geometry unit, the lesson began with a small group discussion between 4 students in each group that included an activity called Which One Doesn't Belong (Danielson, 2023). An image of 4 quadrilaterals was shown on the board and students had to select which one they believed didn't belong and why. In her group of four, Star was the first to propose an answer and reasoning to her groupmates, Isaac, Andrea, and Nick. In claiming that the kite didn't belong, Star was met with laughter and derision from her tablemates:

Line	Time	Speaker	Discourse	
1	6:10	Star	the first one [indicating the kite]	
2	6:13	Andrea	The first one?	
3	6:15	Star	Yeah	
4	6:17	Andrea	[nods] yeah, the first one because it's the only shape that's	
5	6:19	Star	That doesn't [wags finger] have equal sides	
6	6:21	Nick	No, that's not true [laughs]	
		Isaac	No, that's not true	
		Andrea	[shakes her head, smiles and laughs] No, that not	
7	6:22	Star	Wait [looks behind her to board showing quadrilaterals] I mean. Wait. [looks back at Andrea]	
8	6:25	Isaac	The one that doesn't have paralle – the one that doesn't have a set of parallel lines	
9	6:25	Star	Yes. Yes [pointing and shaking finger at Isaac. Looking behind her at quadrilaterals]	

10	6:27	Star	Yes it does. Yes it does [looking at Isaac]	
11	6:30	Andrea	No it doesn't [Star looks back to quadrilaterals]	
12	6:31	Isaac	No it doesn't	
13	6:32	Andrea	No it doesn't [smiles and laughs] It CONNECTS [gestures hands to make wrists touch and fingers point upward like the downward vertex of the kite. Moves hands downward so they intersect. Laughs. Looks toward Nick and smiles.]	
14	6:33	Isaac	Girly. No it doesn't.	
15	6:36	Nick	Woa- [holds hand up to Isaac as it to gesture to stop]	
16	6:38	Star	Okay. Okay. [turns back to table from board]	

The first quadrilateral provided is a kite that appeared to have two sets of equal sides and no parallel sides. Nick and Andrea, two peers sitting across the table from Star, both laugh at Star for proposing that the kite doesn't belong because it has equal sides (line 6). Isaac corrects Star's statement by saying that the kite doesn't have parallel sides (line 8) and the group then confuses Star's agreement with Isaac by thinking she is arguing that they do in fact have parallel sides (lines 8-14). As a result, they laugh at her again, and dismiss her by calling her "girly" (line 14). She turns back to the table and concedes their correction without any of the group members asking questions of each other's reasoning or engaging in a sense of understanding of one another's thinking (lines 16). In this 28 second, 16-line exchange Star's partners say "no" to her 7 different times. Nick continues the discussion saying:

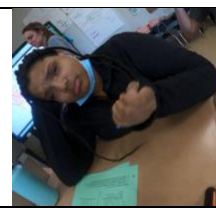
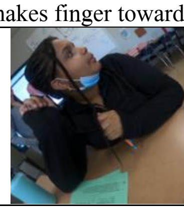
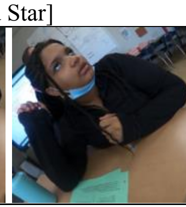
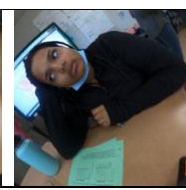
Line	Time	Speaker	Discourse	
18	6:39	Nick	See. My reason for that is that it's the only one that has a corner at the top. [laughs]	
19	6:42	Star	Okay [sounding resigned. Shakes head from side to side. Blinks slowly.] I'm so much better at ELA than math. [Tucks hair behind ears. Rolls eyes. Rests head on hand.]	

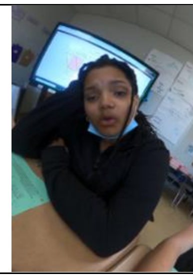
Star's affect noticeably shifts from adamant and confident to disengaged and resigned as she rolls her eyes and rests her head on her hand (line 19).

This small group discussion is followed by a whole group discussion led by Mrs. Fry launching their new Geometry unit. Throughout this Star participates, raises her hand, answers questions, complets the tasks, but **often scoffs saying "ugh", rolls her eyes, or levels her hand next to her face as if to motion "duh" even in response to raising her own hand and getting an answer correct in the whole group**

discussion. Mrs. Fry directs students to find the areas of all the other examples on quadrilaterals on their papers in their groups. Star immediately starts on an example by counting unit blocks inside the shape. Shortly after, Mrs. Fry comes over to their table talking to Isaac, Andrea, and Nick about the strategies or questions they had. In this group interaction, Mrs. Fry doesn't engage with Star. **Star then puts her head on her hand and then eventually lays her head down on her desk.** Even while consistently participating, getting answers correct, and completing her work, Star progressively affectively distances herself from the group's tasks. This culminates in Star being ignored by Mrs. Fry when she comes over to talk to the group - not being pursued for her mathematical ideas or questions by Mrs. Fry or by her group mates.

After completing this task in their groups, Mrs. Fry tells the class to go back to their small groups and talk about what was similar and what was different from the strategies they just heard and the strategies they talked about in their group. Star was the first to speak up:

Line	Time	Speaker	Discourse
20	24:06	Star	They are similar because in the end they make the same shape. [Looks as if she's in pain as she answers. Throws one hand up to shoulder height]. I don't know. 
21		Andrea	[mouths "yes" and affirmatively shakes finger toward Star]
22		Star	YOUR TURN ISAAC [looks intently at Isaac. Rolls eyes. Rests head back on hand]   
23	24:18	Isaac	Uh. I think they're similar because they both made boxes to fill in the holes and I think they're different because they made boxes differently.
24	24:27	Star	Okay [Rolls eyes. Curls Lip. Lays head down on desk]
25	24:29	Andrea	Um. I think they're different because they -
26	24:32	Star	I think YOU'RE different [raises head and speaks directly to Andrea]
27	24:33	Andrea	Are you saying that because I'm Hispanic? [smiles and points at Star]
28	24:35	Nick	Excuse- Uh [open's mouth in shock]
29	24:36	Star	Ugh. Shut Up. [lays her head back on her crossed arms on desk]
30	24:36	Andrea	Okay. So I think they're different because Gabriel's strategy using the outside and Daysha's strategy uses the inside.

31	24:50	Star	I wanna go home [whimpering]	  

By the end of this activity, Star is frustrated and rolls her eyes even when she gets an answer correct or after other students give their explanations (lines 20-24). She playfully draws on social connection with Andrea, as two close friends and as girls of color, as a way to continue to engage (line 26 and 27), but despite attempts to consistently engage and interact, she whimpers, “I want to go home” (line 31). Star’s disposition appears detached and lethargic throughout the rest of the Day 1 lesson and into Day 2, keeping her head on her arms and sleepily stared down at her desk or around at her classmates.

On Day 2 of these lessons, students practiced using area formulas in an activity called *Numbered Heads Together*. In this activity students did a problem independently on their white board, checked their solutions as a whole group, and made corrections and checked for each other’s understanding. Then a similar problem was given and a random student was selected by spinning a wheel. That student then stood to do the problem independently and checked their solution with other students of that same number across the whole class.

For the first problem, students are given a triangle on the board and asked to find the area. After completing the problem individually on their white boards, Mrs. Fry says “Heads Together” and students are expected to stand together and compare answers on their whiteboard in their small groups. Star completes the problem but said “oh” squinting her eyes from behind her white board as she looked around and realized that her answer was different from the answer on her group mates’ white boards. After reminding Nick to add units onto his answer, Andrea prompted Star asking, “what’d you get?” Star admitted that she got 20 and Andrea reminded Star that she needed to divide by 2. Star picked up her whiteboard, erased her work with her fingers and said sarcastically “yeaaaah”. She firmly put her whiteboard back on her desk with a sad look on her face.

In the second problem, Star works through the problem even though it was only Nick that stands and completes the problem for the whole group. After finishing her work, she immediately turns her whiteboard over to hide her answer, but when Mrs. Fry confirmed that she answer was 33 **she celebrated saying "yes!" adamantly.** In the third problem she worked out the multiplication of the base and height of the triangle, but took time to mentally calculate 72 divided by 2. She wasn't finished when time was called and the rest of her table mates revealed the answer (36 inches squared) on their white boards.

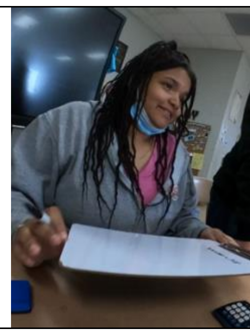
Line	Time	Speaker	Discourse	
32	4:00	Star	I DEFINITELY got 36	
33	4:02	Andrea	Deeeeefinately [under her breath]	
34	4:04	Star	Yeah I did. I just didn't-	
35	4:05	Andrea	You just didn't register it in your brain yet [smiles, extends hand toward Star]	
36		Star	No I just didn't um [looks around at other group members]	
37	4:10	Nick	Write it down [smiles at Star]	
		Andrea	Write it down [smiles at Star	
38	4:11	Star	[shaking finger up and down 3 times] Yeah. Yeah. [grins at Nick and Andrea]	

Star here appears to feels successful as evident by the celebration when getting the answer correct. In this exchange she doesn't have the final answer written down, but she does have "72÷2" written down. Andrea gives her the benefit of the doubt by using the word "yet", smiling gently to Star, and extending her hand out toward her (line 35). Andrea and Nick both smile at Star and complete her sentence attempting to explain why she didn't have 36 on her whiteboard. They collectively assume that she knows the mathematics, but she just didn't write the answer down (line 37). This interaction is characterized by kindness coupled with the exchange of mathematical ideas and a sense of mutuality in mathematical knowing and doing.

Andrea then reminds Star that there's a calculator available in the middle resource bin at the table if she needs it for the next problems. Star picks one up, smiling at the calculator as she does. Andrea's number is then called for her to complete a problem independently. **Star encourages Andrea by softly cheering "Let's go, Andrea. Let's go! Yipee. I'm so excited for you" and "You're too smart" winking and**

pursing her lips at Andrea. She later cheers again for her saying “Let’s go, Andrea! Baddie!” in a sing-song voice. After Andrea completes the problem, a new problem is posted on the board and **Star exclaims “Oh, okay then!” grinning while looking toward the whiteboard**. After comparing her answer to Nick, Isaac, and Andrea, she smiles and says “I’m so smart”. In building her confidence in playful mutuality with her groupmates, she takes on the identity that she’s previously prescribed only to Andrea - “smart”.

Star worked through problems talking to her table mates in a **playful British accent and drawing silly shapes on her whiteboard** between problems. When she got a problem incorrect, a partner validated her reasoning:

Line	Time	Speaker	Discourse	
39	10:44	Isaac	I got 74	
40	10:45	Star	So cool. I got 148. [smiling and chuckling]	
41	10:49	Isaac	How?	
42	10:50	Star	Um.	
43	10:52	Isaac	You did the multiplication wrong	
44	10:54	Star	How’d I do the multiplication wrong when I used a calculator? [looks at Isaac incredulously]. But okay [erases whiteboard]	
45	10:59	Madison	I don’t know if that’s true. Can you write it again. I don’t think you did the multiplication wrong.	
46	11:05	Star	[writes on whiteboard]. Oops. Wrong thing.	 
47	11:07	Nick	No. [typing numbers into calculator] You did the multiplication right. You just forgot to divide by 2.	
48	11:11	Star	Yeah. [smiles down at whiteboard. scrunches up nose playfully]. Why do I always forget to do that?	
49	11:14	Andrea	So smart	
50	11:15	Star	Too smart. Big brain [grinning]	

As Star gets the answer incorrect (lines 39-41), rather than asking what she did or inquiring about her mathematical thinking Isaac makes an assumption about her incorrectness (line 43). Star pushes back on this comment with incredulity, looking at him with skepticism and a sense of frustration. But her affect changes again to smiling and playfully scrunching her nose (line 48) after I step in and give her appropriate mathematical credit. Nick backs me up by saying that she was right, but she just "forgot" to divide by 2. His comment indicates that, from his perspective, Star's ideas and engagement have mathematical value as he indicates that she knew how to do it, but she "just forgot" (line 47). The word "just" in Nick's response holds a lot of weight because it communicates that this mistake wasn't significant, and it doesn't indicate anything about her mathematical ability. Andrea and then Star both echo this encouragement saying "so smart" (line 49) and "too smart. Big brain" (line 50). Star's grin at the end of this is indicative of her positive positionality towards these mathematical interactions.

Making Sense of Star's Affect

Star's eagerness and engagement at the beginning of this lesson decreased progressively as she was diminished mathematically in three separate moments in this short 20-minute series of interactions. But she reengaged in response to interactions with her peers in Numbered Heads Together. Before this interaction began on Day 1, leading up to the Which One Doesn't Belong discussion, Star was actively and energetically engaged in the discussion prompts. She was interested in her peers' responses, looked around at her peers, and responded frequently. For example, in the first small group discussion prompt, students discussed what they would do on an extra day of Spring Break. When Isaac said he would play video games, Star interrupted him twice to ask what kinds of video games he liked to play. Students were them prompted to discuss in their small groups their goals for the semester. Star said that her goal was to be "good at math" and no one protested to this being a reasonable goal for Star. The lack of resistance to this being her goal implied that her peers agreed that "not good at math" was an appropriate label for Star. Star was the first to offer potential answer and reasoning in the Which One Doesn't Belong activity. But her peers' laughter in response and disregard to her answer resulted in a relatively quick shift in stance toward her peers and the mathematics indicated by Star rolling her eyes.

Rolling her eyes was a common affective indicator for Star when she was annoyed or felt degraded. For example, she demonstrated this toward Mrs. Fry after being required to come to study hall for help when she was failing the class and was thanked by Mrs. Fry for coming as she left. She seemed to do this when she was frustrated or feeling aversive to the interactions that she was having with the people around her. Shortly after this eye roll, Star echoed the sentiment of her being “bad at math” by saying, “I’m so much better at ELA than math.” Lastly, toward the end of this exchanged she was ignored by both Mrs. Fry and her peers as they didn’t engage with her or her potential mathematical insights. After a few short minutes of these interactions, it was evident based on her physical and verbal discourse that she was no longer comfortable being in math class, saying “I want to go home,” but this noticeably shifted when she began a group-based task the next day.

In Numbered Heads Together on Day 2, all students were expected to help one another when their teammates got an answer incorrect so that their table would be successful when the random number was called. It was in the group’s best interest to make sure that everyone succeeded. In these problems, Star was helped by her peers, and treated with mutuality. As a result, Star immediately cheered on her tablemates, smiled, grinned, and identified herself as “smart”. In stark contrast to the affective moment above, Star was playful, silly, and leaned into this mathematical activity.

As noted in her introduction, Star was not considered by Mrs. Fry or by her peers to be participatory or competent in mathematics. This was evident in her peers closing her off from group discussions, her numeric ranking in her small group, Mrs. Fry’s concern for her grade and her completion of assignments, and by her own self-identification as “bad at math.”

Over the semester, Mrs. Fry incorporated increasingly more group work in which discussion and collaboration by every student was required for the group to be mathematically successful in the task. In these kinds of tasks, we saw an increased level of participation and enjoyment for Star. She talked more, completed more problems, and was more successful in the tasks. Throughout the semester, in moments where Star was actively and excitedly participating in math class, it was during activities that involved group work that required her participation for the group to have success.

It was evident that Star knew that she was not seen as a mathematical resource in this class, but that she thrived in mathematical interactions that required mutual mathematical give and take with her peers. When I asked who she pictured in her class as good at math Star, identified Andrea “because when

I need help, she always is helping me. And I realize she gets all her stuff right". For Star, being successful in mathematics was not solely due to correct answer getting ("getting stuff right") but also a willingness to help the people around you ("she's always helping me"). Star enjoyed mathematics in tasks and activities in which she could be social, playful, silly with her friends and her peers and in which she was benevolently helped by other people and in which her own ideas were seen as valuable and respected. The free, creative exploration and expression of her ideas thrived when she was encouraged and valued intellectually and socially with benevolence by her peers.

Therefore, if we were to design activities that supported Star's unique mathematical joy we might design activities that necessitated students' social interactions that gave freedom of movement and discourse to allow for play, humor, and silliness. These tasks would also require that students' look out for each other, encourage each other, and as a result share mathematical ideas and resources that are seen as valuable and respected mutually.

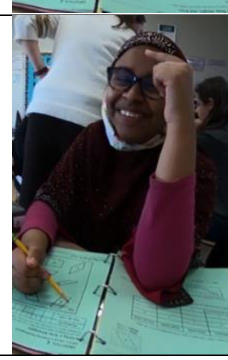
Ruqaiya

Over the course of the first two days of the geometry unit, Ruqaiya's positionality toward mathematical engagement shifts from playful and giddy to calm and downcast. In the following episode of affect, she demonstrates affect that nudges her into animated mathematical engagement with broad smiles, giggles, sparkles in her eyes, and pulling her mask down for conversation. She demonstrates affect in which she pushes away from mathematical engagement by crossing her arms, sitting calmly, puffing air out of her mouth, and vocally expressing dissatisfaction in her own participation. In the following episode we see how over the course of 2 lessons, Ruqaiya's affect reveals her desire to face mathematical puzzles or problems by collecting interesting, unique, and novel solutions from and with her peers through which she has the opportunity to playfully push the boundaries of what's possible with mathematics.

Affective episode

On the first day of the geometry unit, students are discovering the formula for the area of a parallelogram by exploring various parallelograms on paper, each drawn with inset grid lines. Students are challenged to find unique ways to calculate the area of each parallelogram. For example, students could count the squares in the grid inside the shape, break up the parallelograms into pieces they knew how to find the area of (like rectangles and triangles), or break up the parallelograms and rearrange the pieces to make rectangles. Students were told that they could work individually, in the groups, in partners, but weren't given specific roles or guidelines for working together if they chose to. Ruqaiya is working with a partner who didn't consent to be part of the study. Therefore, her partner's contribution is edited out of the transcript.

Ruqaiya unsuccessfully tries to find the area of some of the parallelograms independently. She puts her head in her hand, sets her pencil down, and says, "I forgot everything… Formulas - they give me headaches." The following interaction proceeds:

Line	Time	Speaker	Discourse	
1	3:57	Madison	how's it going?	
2	4:04	Partner	[Admits to being confused]	
3	4:06	Ruqaiya	[Looks up from head propped in head. Smiles. Speaks in an bemused and exasperated voice] If there's one thing that gives me more headaches than a hot car it's things I don't get	
4	4:13	Madison	What don't ya get? What's going on?	
5	4:17	Ruqaiya	I know formulas, but I feel like my brain exploded over Spring Break and I can't remember a single thing	
6	4:30	Partner	[Her partner then explains that you could take parts of the boxes and rearrange them to complete the boxes and make a rectangle]	

7	4:40	Ruqaiya	[puts her pencil down forcefully. Grins] You are not serious!	
8	…		We discuss different strategies for counting whole boxes and making whole boxes out of pieces from each side of the parallelogram	
9	5:03	Ruqaiya	We would just count the ones that are actually whole. Okay 1 2 3 4 5 6 7 8 No 8 9 10 11 12 and then you have like 6 other ones that don't work [squints, giggles, and smiles broadly]	 

Mrs. Fry leads a discussion where students share some of their possible strategies for finding the areas of these parallelograms. **Ruqaiya smiles for most of this discussion**. **She giggles while her partner explains their unique strategy.**

Ruqaiya and her partner then, with the prompting of Mrs. Fry, discuss the similarities and differences between all the strategies that were shared during the whole group discussion and two additional possible strategies described on their activity page. Ruqaiya then explains how this breaking apart and putting together strategy wouldn't work for other kinds of shapes.

Line	Time	Speaker	Discourse	
10	15:58	Ruqaiya	Let's see about. I mean if it was a really really really really really messed up shapes like this [she draws an irregular 7-sided convex shape on her paper] it probably wouldn't work as well. Just a fact. Because depending on the shape you'd have to cut it up several times just to get the pieces to fit	 
11	…		[partner responds]	
12	16:13	Ruqaiya	If you were clever enough to try to find it.	

Ruqaiya is now familiar with the new strategy of breaking apart and putting back together to find their area. She smiles and seems to be enjoying pushing the boundaries of what's possible with these strategies by changing the shapes' regularity, dimensions, and properties (line 10).

After some more discussion and continuing to work through the exploration, Mrs. Fry shows the class a video of another strategy for calculating the area of a parallelogram from Desmos but prefaced with "Are you ready? You think you're ready, but you're not. My mind was blown." She played a video of a long skinny very tilted parallelogram getting cut horizontally into pieces and rearranged to make a tall skinny rectangle.

Line	Time	Speaker	Discourse	
13	21:46	Ruqaiya	[watching the video on the whiteboard] Are you serious? [speaks in an excited whisper, giggles, holds her hand to her mouth to suppress laughter]	
14	21:53	Mrs. Fry	In your table what did they do?	
15	21:55	Ruqaiya	[Turns back to partner, smiling and giggling] They just cut it into triangles!	
16			[Partners asks about how to know how many triangles to cut the parallelogram into]	
17	22:15	Ruqaiya	Look at it! You could just go like. You could just cut it into triangles, and then stack the triangles. I don't think it would work very well for a circle though. I can tell you that right now	

Ruqaiya seems thrilled by this new possibility (line 15). As she familiarizes herself with this new strategy, she again pushes the boundary of using it with a circle rather than a parallelogram (line 17).

Mrs. Fry then started explaining other strategies on the board. There was an obvious change in Ruqaiya's affect from when students were sharing strategies to when only Mrs. Fry was talking. When Ruqaiya is sharing her strategies or listening to other students share their strategies, specifically around mathematical problems that she finds puzzling, she smiles, giggles, laughs, teases, and tests the boundaries. **When Mrs. Fry was talking Ruqaiya is calm, arms crossed, and silent.**

Students are then directed to fill out a table with the area they found of each parallelogram as well as the base and height dimensions for each parallelogram. They then look at the table and observe the pattern of the base, height, and area to find the formula for calculating area of a parallelogram. During this activity, Ruqaiya works calmly and demonstrates very little affect.

On the second day of these lessons, students are tasked with practicing using the area formula to solve different area problems for parallelograms and then take notes on what they discovered. During these activities, Ruqaiya shows very little affect. She is productive but not affective. In a Desmos Notes activity, a problem asks about a triangle being half the area of the parallelogram. She gets this problem correct, but she looks frustrated about it. As her and her partner get the answer correct, her partner celebrates and calls themselves “smart”, but **Ruqaiya with a downcast tone says, “I memorize too much” and blows out a puff of air.**

Making Sense of Ruqaiya’s Affect

Throughout these activities, Ruqaiya demonstrated excitement and curiosity when sharing and being challenged by new strategies especially with mathematical problems that involved some kind of puzzling or unknown. These moments of intensity toward and around mathematics centered around puzzles and challenges that she was unsure how to solve, collecting alternative strategies from her peers that she hadn't initially thought of, and then playing with these newly collected mathematical ideas. She smiled and giggled while coming up with new strategies and then listening to her partner share them with the class. She was thrilled by new strategies that were shared by her peers or her teacher like cutting the shapes into triangles horizontally instead of vertically. After listening to and understanding each new strategy she pushed the boundaries of what might be possible using that strategy. Each time, she proposed a new shape that she thought the strategy could not be used for – for example, an irregular convex shape or a circle. In contrast, she was calm, arms crossed, even sometimes openly frustrated when given or required to provide a single, individual procedural solution. For example, when Mrs. Fry was lecturing, she sat calmly and silently with her arms crossed. When the formula for the area of a parallelogram was revealed, she didn't react at all. Similarly, she got a correct answer in Desmos the next day and she seems sad or potentially disappointed saying, "I memorize too much."

Each moment of affect that pushed her toward engagement in mathematics in the episode above was centered around mathematical problems or puzzles that she resolved by collecting strategies from peers around her. She readily and excitedly admitted when she was confused or when she didn't know how to proceed. She became familiar with other people's mathematical solutions that helped resolve the mathematical problems they were wrestling with. She became increasingly giddy the more strategies she collected from other people. Just as Ruqaiya reveled in the mystery, narratives, characters, plot lines, and climaxes of her beloved mythology fiction books, she also seemed to thrive in mathematical mystery, the building of solutions, and the variety of mathematical narratives that could be told in a single mathematical quandary. She leaned into mathematics with apparent satisfaction when she had the opportunity investigate her peers' multifaceted ideas about mathematical puzzles.

As discussed in her introduction, Ruqaiya felt a pull toward the playful, puzzling nature of mathematics. This was evident in the interest she showed in multiple strategies for solving the same problem, but also in the playful boundary pushing that she engaged in after becoming familiar with each solution. Almost instantly when she became familiar with a peers' strategy, she would play with how far that strategy could go – posing that it would be difficult to break apart and put convex shapes back together or to cut circles into triangles. This mathematical boundary pushing echoed her boundary pushing against schooling more generally. Even though her parents would have preferred her to be reading in the non-fiction textbook section of the library and she wasn't allowed to read at home because she was required to do homework, her locker was jam-packed with checked-out mythology library books, and she carried a book around with her constantly at school. As she found freedom in stories from the anxiety, testing and school she playfully pushed this boundary to its limits. In a similar was, in the face of mathematical problems she seemed to find joy in collecting multiple strategies - pushing back against the product-oriented sociocultural norms of this space – and playfully pushing the boundaries of how far those solutions could take her.

Therefore, if we were to design mathematical activities that supported Ruqaiya's unique mathematical joy, we might design tasks and activities that were open ended and encouraged a variety of possible solutions that were shared and explored among peers. These problems would be challenging but not impossible for Ruqaiya to allow her to feel successful and also puzzled. Importantly, these tasks would have avenues for boundary pushing and mathematical rule breaking as students explored various mathematical content.

Thomas and Isaac

This episode is a complex entanglement of interactions between Thomas and Isaac on Day 4 of the geometry unit through an activity called Rally Coach. This episode provides a glimpse into both interactional sides of an affective episode and the potential reasoning behind students' wildly different reactions to the same mathematical task. In this partnership there was an obvious unspoken hierarchy with Thomas positioned as more mathematically capable and Isaac positioned as less mathematically capable. As a result, Isaac exhibits negative affect by shutting down in response to being scolded and corrected by Thomas. But Isaac exhibits positive affect through smiles and laughter after Thomas indicates mutuality between them and leans into mathematical engagement by pushing back against the unspoken hierarchy between himself and Thomas. Thomas demonstrates negative affect in this episode through frustration with the tensions between the task structure and his own inclination to work quickly and independently as well as scolding (both of himself and of Isaac) for getting answer incorrect. But Thomas demonstrates positive affect, by softening his demeanor and thanking Isaac when he acknowledges a mathematical need with some vulnerability and receives Isaac's help. Through these affective cues, Thomas reveals his hesitancy to work collaboratively and learn from Isaac at the expense of efficiency and precision and his frustration with falling short of perfection when relying solely on his own mathematical reasonings. But this episode also demonstrates a conscious or unconscious desire for support from his peers when he recognizes his own need. In conjunction, Isaac reveals his need to have his voice heard and respected in this space.

Affective Episode

On Day 4 of the geometry lessons, Isaac and Thomas are working as partners on an activity called Rally Coach solving a set of 12 problems on a worksheet finding the area of irregular polygons as composite shapes. Each student folds their paper in half vertically so that half of the questions are on one side of the paper and half are on the other side of the paper. Students take turns solving problems with one student solving the problem and the other student playing the role of "coach". Only the problem solver is allowed to hold a pencil during each turn, and the coach is only supposed to ask questions to the problem solver. The partner holding the pencil reads the question and then explains what they think their first step should be. They then ask their partner, "do you agree?" before writing this step. The partner either agrees or asks a question and they determine the next correct step in the solution together. The problem solver then writes that step and continues the same process for every step of their solution. After completing a problem, the students trade roles, flip over their paper, and complete the next problem. This continues until all the problems given are completed. Students are given a list of numbers at the bottom of each page of questions so they can check if their solutions to the randomized answer key.

At the start of this activity, Thomas is the problem solver for the first problem. This isn't discussed between Isaac and Thomas, but Thomas is already holding his pencil when Mrs. Fry say "go". Thomas draws a line to divide the composite shape into two triangles, uses the calculator to find the area of each rectangle, adds then together, writes down and circles his answer. This process takes 40 seconds. Isaac looks over Thomas' arm which is extended out between Isaac and Thomas' paper**, Isaac says, "yeaaaah" in a somewhat uncertain tone.** Thomas shrugs and flips over his paper indicating that it's Isaac's turn to complete the next problem.

On the second problem, Isaac is now the problem solver and Thomas is the coach. Isaac attempts to follow the Rally Coach procedures by asking Thomas, "do you agree, if we draw the lines like that?" after dividing up his composite shape. Thomas still has his pencil in his hand and responds, "you want know something easier that you could do?" Isaac responds affirmatively and (even through Mrs. Fry interrupts them reminding them of their roles as coach and problem solver) Thomas leans over to Isaac's paper erases his work and tells him a different solution to use. Isaac responds saying, "“Oh, okay, yeah. That is easier" and picks up his calculator to calculate the final solution.

For the third problem, Thomas is supposed to be the problem solver and Isaac is the coach. Thomas solves the problem out loud while Isaac follows each step he makes on his own paper. After Thomas looks slightly uncertain (looking at the problem again and then looking over Isaac's shoulder to his paper), Isaac asserts, "I would like to do triangles because triangles are just easier for me. You can do it however you want. That's just what I would do." The two work collaboratively bouncing questions and corrections back and forth to finish this problem, each with pencils in their hand. Mrs. Fry walks over and asks, "Whose working it and whose coaching it?" **Isaac quickly puts his down pencil, looks Mrs. Fry in the eye, smiles and apologizes as he points to Thomas. Both students smile and laugh.**

To begin the 4th problem, Isaac is supposed to be the problem solver and Thomas is the coach. Isaac looks at the problem and begins debating out loud how best to solve it while Thomas works on the same problem on his own paper:

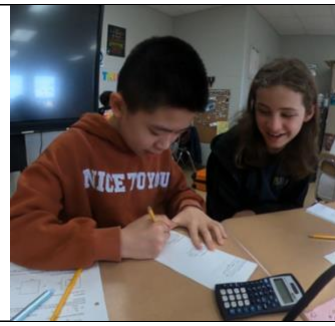
Line	Time	Speaker	Discourse	
1	4:27	Isaac	Okay. I think I'm going to do two triangles and a rectangle. [drawing on paper]	
		Thomas	[Looks at problem #5 on his own paper. Puts pencil down. Picks up calculator and begins typing]	
2	4:36	Isaac	Well. Hmmm. No, I'm not doing that. [Erases]	
3	4:40	Thomas	[writes down answer on his own paper]	

		Isaac	Hmmm. I don't know how we should split it. [looks over at Thomas' paper]	
4	4:45	Thomas	[Moves Isaac's hands away from his own paper and begins drawing on Isaac's paper] We could just do two times [incoherent] minus this thing which would be 12	
5	4:50	Isaac	Yeah. That's smarter. [pauses and thinks. Counting on his fingers]	
		Thomas	[erases and writes again on his paper]	
6	4:58	Isaac	No that's 16. They're both 16.	
7	5:02	Thomas	No they're not both 16.	
8	5:03	Isaac	Yeah because 6, 12, 18, 24. [raising a finger after each interval] Oh! [raising his hand slightly and leans back] My bad. My bad.	
9	5:06	Thomas	[reaches over to Isaac's paper, gestures to Isaac's paper with pencil] and then 12 because [picks up calculator and types] 5 12 minus 24. 4 88.	
10	5:13	Thomas	[Notices Mrs. Fry walking over, quickly looks up at her] Wait I'm not supposed to say anything. [Burys head in hand and slowly drops pencil]	
11	5:15	Mrs. Fry	Every time I've walked over here you've both had a pencil in your hand.	
12	5:18	Isaac	Sorry [smiling]	
13	5:20	Mrs. Fry	This is like self-control challenge. Major.	
14	5:23	Isaac	Alright. So it [typing in calculator] wait.	
15	5:27	Thomas	[watches Mrs. Fry walk away and then immediately picks up pencil again to write on his paper]	
16	5:30	Isaac	Wait…. 16 times 32	
		Thomas	[unfolds paper to check answer on answer key]	
17	5:37	Thomas	You should get 488 -	
18	5:38	Isaac	- Minus [typing in calculator]	
19	5:40	Thomas	You should get 488	

Isaac makes a bid for Thomas' help by looking over at Thomas' paper and saying "Hmmm. I don't know how we should split it". Thomas then moves Isaac's hands away from his own paper and begins drawing on Isaac's paper and says, "We could just do two times (incoherent) minus this thing which would be 12." **Isaac calls Thomas' idea "smarter" and uses Thomas' solution to solve the problem. The two disagree on a few calculations, Isaac tries to correct Thomas on a calculation, but Thomas pushes back on his correction (lines 6 and 7) and within approximately 30 seconds Thomas tells Isaac the answer (line 9).** At this point, Mrs. Fry walks over and redirects the two to be following the procedures of Rally Coach, admitting that this requires significant self-control (lines 10 and 11). Thomas appears openly frustrated at this, stops writing on his own paper for around 15 seconds and then as soon as Mrs. Fry walks away he returns to solving the problem independently, telling Isaac the final answer again another two times. A hierarchical power dynamic is evident in this interaction (lines 1-5). Just as Thomas reveals the correct answer (line 9), Mrs. Fry walks up and Thomas demonstrates significant affective reaction, verbally calling himself out for disregarding the procedures of the task, burying his head in his hands and dropping his pencil (line 10). Since the start of the task, he's been not following the procedures by solving the problems independently and more quickly (lines 1, 3, 9, 15, 17 and 19) rather than only taking on the role of coach or positioning Isaac as a coach. Isaac doesn't push back on Thomas breaking procedure or blame him for it. In fact, he appears to demonstrate his understanding of this task as a collective endeavor by apologizing to Mrs. Fry (line 12) for their collective behavior.

Isaac finds a solution, but he does not get 488 on his calculator. Thomas responds by laughing, taking the calculator out of Isaac's hand, and typing in the numbers for him. After Isaac acknowledges that the number Thomas calculated was a possible solution by looking at the answer key, they move on to the next problem. It is evident from the partnership throughout these first four problems that Thomas and Isaac seem to be on unequal footing. Thomas is perceived as a resource for Isaac, but it isn't evident that the opposite is true. Thomas corrects Isaac, tells him the answer to problems multiple times, laughs when he gets the wrong answer, types in the calculator for him, and then directs him to put his pencil down. Isaac also positions Thomas as more mathematically capable, by calling his solutions "easier" and "smarter", readily erasing his answers and replacing them with Thomas' solutions and attempting to follow the procedures of Rally coach multiple times. Thomas hits on table, wags his finger, and smiles at Isaac saying, "You can put your pencil down now". Isaac responds by putting his pencil on the table and Thomas proceeds to work on the 5th problem, but in this exchange, Thomas demonstrates a need for Isaac:

Line	Time	Speaker	Discourse	
20	6:58	Thomas	What about the base of that triangle? [pushing paper toward Isaac]	
21	7:00	Isaac	[looks at paper] What?	
22	7:01	Thomas	The base of the triangle [points to paper with pencil]	
23	7:04	Isaac	That's not a triangle	

24	7:05	Thomas	That IS a triangle	
25	7:06	Isaac	No	
26	7:07	Thomas	What? [pulls paper back to his desk]	
27	7:08	Isaac	It's a trapezoid	
28	7:09	Thomas	Wait. Okay. That's it	
29		Isaac	[laughs]	

In this exchange, Thomas identifies what he thinks is a triangle in his composite shape (lines 20, 22, and 24). By Thomas making a bid for Isaac's help (line 20), Thomas opens himself up with some vulnerability to Isaac's correction. Isaac corrects Thomas (lines 23, 24, and 27), leans into his desk, and laughs. This laugh seems to not be in derision of Thomas, as Isaac holds Thomas in high regard mathematically, but rather at how silly the situation was that he corrected Thomas on identifying basic shapes. In this moment, he leans into the discussion.

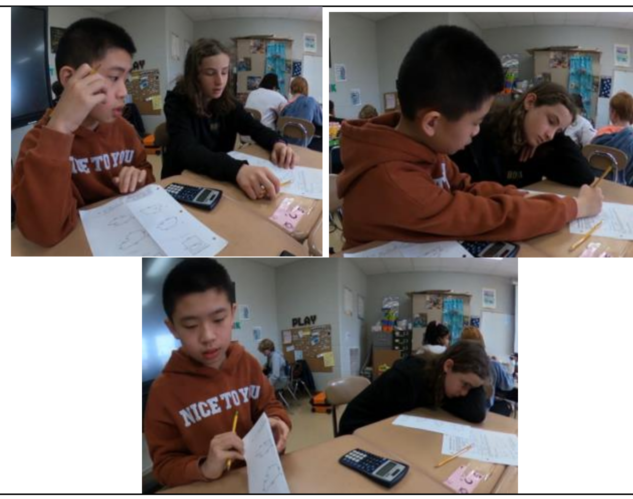
They continue working through the problems and only pause to listen to Mrs. Fry remind the whole class of the activity's rules. She asks them to assess for themselves if they were listening to their partners, if they were waiting to write down the next step until after their partner agrees, and if they were asking questions when they were the coach. She acknowledges that there was a lot of self-control in this activity. In the middle of this reminder, she pauses to say "Thomas" in a stage whisper to get him to stop solving the next problem on his own as she's speaking. She asked them to reflect and then continue.

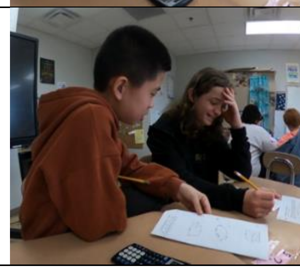
Line	Time	Speaker	Discourse	
30	9:18	Thomas	I just want to get this done.	
31	9:19	Isaac	[laughs]	
32	9:22	Thomas	Is it my turn or your turn?	
33	9:24	Isaac	It's mine [begins reading and working on problem]	
34	9:30	Thomas	[Leans over Isaac's arm with his pencil in his hand to mark on Isaac's paper] Remember what I told you. No! Do the – [incoherent].	
		Isaac	[doesn't respond]	
35	9:41	Thomas	Yes. It's the easiest way. You must do it the easiest way. [points at Isaac's paper] See you made a square.	
36	9:50	Isaac	Do you agree? [looking down at paper with a stoic look]	
37	9:52	Thomas	Okay. I'm going to sit here and do it. We do this side	

			now. [flips folded paper over] Oh my God. [looks over at Isaac paper, seemingly shocked]	 
38	9:55	Isaac	[laughs]	
39	9:59	Thomas	Okay it's time to make it a rectangle	

Thomas outrightly acknowledges the tension he feels with abiding by the procedures (line 30). It would require more time to take on the roles of rally coach and problem solver - to listen to Isaac and help him by asking questions rather than solving the problems quickly and concurrently. Thomas' priority is to simply get it done, the way the task is designed (as a worksheet), and Thomas' confidence in the content makes it significantly easier to work independently rather than collaboratively. But he quickly reaffirms that there are roles, and it should be someone's "turn" (line 32). It's Isaac's turn to be the problem solver, but Thomas quickly leans over with his pencil in hand to correct him, commanding him to "do it the easiest way" (lines 34 and 36). In response, Isaac is stoic, staring down at his paper and with apparent sarcasm remarks "do you agree?" after Thomas' reprimand (line 36). Thomas appears to ignore this comment and attempts to move onto the next problem (line 37). But he is caught off-guard, looking at the next problem with apparent shock and he says, "Oh my God" (line 37). Thomas is uncertain how to solve the next problem, and this seems to draw Isaac back into the task and he laughs (line 39). The two partners proceed through the activity working through the next problem now with Isaac as the coach and Thomas as the problem solver:

Line	Time	Speaker	Discourse	
39	9:59	Thomas	Okay it's time to make it a rectangle	
40	10:05	Thomas	So. It's...Wait. Wait. Wait. Wait. Wait.	
41	10:13	Thomas	So it 18 plus 5. [slightly pushes paper and calculator toward Isaac] What's 18 plus 5? [types into calculator] 23	
42	10:16	Isaac	[yawns and slowly looks toward Thomas] 23 [sleepily]	
43	10:20	Thomas	Is that the base?	
44	10:21	Isaac	I agree.	
45	10:23	Thomas	The height is 13. Do you agree?	
46	10:26	Isaac	Yes	
47	10:30	Thomas	[typing into calculator] It's 299. Do you agree?	
48	10:35	Isaac	[looks over at Thomas' paper while resting his head on his hand] Yeah. [nods without lifting his head]. (incoherent) Subtract it out	

49	10:41	Thomas	2. 5. Minus (incoherent) minus 16. (incoherent)
50	10:52	Isaac	What? [resting head on hand]
51	10:53	Thomas	What gets you 23 from 16? That's what this is [gestures to paper with pencil]
52	10:55	Isaac	Okay.
53	10:58	Thomas	What is this?
54	11:00	Isaac	[put hand to head and leans over to Thomas' table] I don't know
55	11:04	Thomas	So it's 16. 23 minus 16. [types into calculator] Okay this is 7. [writes on paper] 299 minus 52 [types in calculator] 2 point 7 centimeters squared [writes on paper]
56	11:25	Thomas	[opens paper to check answer key] Wait wait wait wait. It's not on there.
57	11:31	Isaac	[starts to write answer on paper. Flips over paper to also check answer key] No. [chuckles slightly in an exasperated tone]
58	11:34	Thomas	Maybe it's 2 point 4, but I don't know how.
59	11:35	Isaac	[sits upright] I'm confused on how you got the top rectangle. Like this part. Like, how did you get that? [shows paper to Thomas and points to problem] 
60	11:42	Thomas	[looks over to Isaac. Begins gesturing to Isaac's paper with his pencil] You do like this. Because the base is 7 and the heights is (incoherent) 42. You get it?
61	11:52	Isaac	Yeah [lays head down on desk and buries head in elbow]
62	12:05	Thomas	[turns back to his paper. Mmmmmm [frustratedly] I know it's 2 point 4, but how? Did I do my math wrong?
63	12:11	Thomas	Oh! I found I- oh wait nevermind. [looks around at other people's papers] How!? [exasperated] How but how? [orients paper and body towards Isaac's desk. Looks at Isaac]
64	12:29	Isaac	The height isn't 13. The height is 15.
65	12:32	Thomas	[shocked and seemingly frustrated, he jolts upward, hits his head with his hand, and furrows his brow] Oh. Oh my. Oh my. Oh my. Oh my God. [picks up calculator and starts typing] Oh my God. 18 times. Wait what what is that what is the length there 23. 23 times what? 15? 
66	12:46	Isaac	Mmmmhmm. Wait. Yes. Yes. Yes. Yes. Yes.
67	12:50	Thomas	Minus 52

68	12:52	Isaac	So 23 times 15 -	
		Thomas	It’s 2 93	
69	12:55	Thomas	Okay that’s the answer to (incoherent). Okay. I didn’t know that. Thank you. [turns to Isaac and smiles toward him, covers his mouth with his hand briefly]	 
70	12:57	Isaac	[giggles as he looks at and starts the next problem]	

In this exchange, Thomas appears to be stumped by this problem (lines 37 and 40) and he seeks out Isaac’s help for the first time in this activity (lines 41, 43, 45, 47, 51). Isaac responds to all of Thomas questions (lines 42, 44, 46, 48) starting with only affirmations of Thomas’ reasoning and leading to posing questions to Thomas (lines 50 and 59) and as he does this he gradually opens his body up more and more to Thomas (lines 48, 50, 54, 59). They get the problem wrong, and this seems to draw Isaac into the problem (line 59). But Thomas poses a quick procedural response and says, “You get it?” After this, Isaac again appears to close off to the problem and to Thomas, laying his head on his desk. After Thomas physically orients his body and paper toward Isaac and directly poses the questions “how!?” (line 63), Isaac poses a possible solution. As Thomas recognizes that Isaac is correct and he made a mistake, Thomas shows brief, open, and intense frustration with himself saying “Oh. Oh my. Oh my. Oh my. Oh my God” (line 65). The two continue collaboratively arrive at an answer that matches a possible solution on the answer key. After they find the correct answer, Thomas softens, turns to Isaac, smiles toward him, shyly covers his mouth, and says “Okay. I didn’t know that. Thank you.” (line 69). In response, Isaac turns over his paper, proceeds to the next problem, and giggles (line 70). Thomas then turns to Isaac, put his pencil on the table and said, “I’m just gonna let you do this one.” But within 10 seconds, he leans over to correct Isaac’s strategy. But in this moment, Isaac interrupts him.

Line	Time	Speaker	Discourse	
71	13:33	Thomas	You know you [incoherent] rectangle	 
72	13:37	Isaac	[holds up finger. Grins. Side-eyes Thomas] Hold on. Hold on. No. Hold on.	
73	13:40	Thomas	Okay [Rubs his hands together. Grins.] I trust your strategy.	
74	13:42	Isaac	Okay so 7, 8, 9. So this is 2, times 17. Which is 34. Do you agree?	
		Thomas	[pick up pencil to write on paper]	
75	13:43	Thomas	Yeah	

As Thomas attempts to again quickly correct Isaac and give him an alternative solution, but Isaac asserts himself saying, “Hold on. Hold on. No. Hold on” while side-eyeing Thomas and holding his figure up as if to ask Thomas to pause (line 72). While this may not appear to be an intense affective moment for Isaac, having seen Isaac work for months, this assertion toward Thomas was bold relative to Isaac’s typical disposition. It seems that the mutual collaboration in the previous problem in which Thomas needed Isaac’s help gave Isaac confidence to attempt his own solution and ask Thomas to wait to provide his own. Thomas concedes (line 73) and then Isaac states his first step, again pushing Thomas to take on the role of coach in the activity (line 74). Thomas responds but does so by again breaking the activities protocol, picking up his pencil.

Making Sense of Isaac’s Affect

For Isaac, during this task, two intense affective moments stood out: 1. When Isaac physically leans into the mathematical task and laughs after Thomas indicates that he needs Isaac’s mathematical input (lines 29 and 38) and 2. Isaac’s resistance to Thomas’ correction in the last problem (lines 71-75). These two moments are characterized by Isaac and Thomas each in their own ways partially dismantling the unspoken hierarchy evident here and the confidence built from sharing ideas with one another.

Thomas’ propensity for independent problem solving coupled with Isaac’s positioning as less capable mathematically led to an unequal partnership in this task. Thomas, while in the role of coach, corrected Isaac and gave him a correct answers or short cuts on all of the problems that he solved. In contrast, in the role of coach, Isaac mostly only responded when Thomas asked him a question and only toward the end did he pose question to Thomas (line 59) as the coach. Isaac also readily erased his answered and replaced them with Thomas' or called Thomas' solutions "easier" or "smarter" because of his own perspective of who he was as compared to Thomas. This reflects Isaac's tendency to compare himself negatively to other people in math class, as noted in his introduction. In our initial interview, when asked to picture someone in his class that is "good at math", he was quick to answer that Thomas was a student in his class who was "good at math" and (unprompted by me) he identified himself as "bad at math". He said that he was able to respond quickly to the question of who was good at math because he "thought about it a lot". He believed that Thomas was good at math because "he [could] always figure out the answer within a quick-- within a really fast amount of time. And when he [got] an answer wrong by a decimal or something, he'[d] scold himself for it". Therefore, this inequitable positioning was not only due to Isaac's tendency to always think of himself as lesser in mathematics, but there was significant emphasis on this dynamic here because of how Isaac positioned Thomas in his mind. This was reinforced by Thomas' resistance to working collaboratively with Isaac.

In this hierarchic partnership, Isaac continued to engage in the task but without much positive affect, especially when he was scolded by Thomas for not doing the problem the easiest way or when he got answers wrong. Notably, Isaac was drawn into the mathematics, smiled, and laughed while leaning over Thomas' work after providing information about a trapezoid and after Thomas expressed some uncertainty about how to proceed with a problem – indicating in both cases that he needed Isaac's help (line 29 and 37). Thomas' uncertainty led to a mutual exchange of ideas which resulted in Thomas thanking Isaac for his help and admitting that he didn't know how to do the problem. Immediately after, Isaac giggles while looking at the next problem and when he pursued solving it, he pushed back on Thomas' corrective behavior saying "hold on. hold on. hold on" and paused Thomas by holding a finger up to him. In this moment, potentially as a result of feeling like a mutual resource with Thomas, Isaac not only expressed a bit of ease and positive affect through his smile, but also an act of resistance to the hierarchy he felt with and through Thomas.

Therefore, if we were to design mathematical tasks and activities that support Isaac's unique mathematical joy, we might necessitate students' mutual sharing of mathematical expertise based on their need of one another in mathematical problem solving. This would require the development of expertise and then the wielding of that knowledge for the benefit of someone else.

Making Sense of Thomas' Affect

For Thomas, during this episode, three affective moments stood out: 1) his apparent frustration when caught and redirected by Mrs. Fry to use the Rally Coach procedures instead of independently solving each problem (lines 1-20), 2) his anxious reaction when he realized that he didn't get an answer correct (lines 39-65), and 3) his softening and "thank you" toward Isaac when he admitted vulnerability and accepted Isaac as a resource (66-70).

As noted in the introduction, Thomas had a propensity in math class for quick, competitive, correct answer-getting. This, along with the closed nature of the worksheet, compelled him to disregard the procedures of the task consistently even after being corrected twice by Mrs. Fry and encouraged by Isaac to take on more collaborative procedures and roles. The procedures of this activity were intended to promote students' exchange of ideas and collective problem solving - attempting to focus students on the process of problem solving by attending together to each step. But the worksheet-based task lent itself more readily to independent correct answer-getting. Thomas' intense reaction and apparent frustration when redirected to follow the collaborative procedures, his (very) momentary withdrawal as he put his pencil down, and his immediate return to independent problem solving revealed his hesitancy to learn from and with Isaac at the expense of correct answers or efficiency. He admits that his main goal in this task is not to learn from Isaac or to help Isaac, but to "just… get this done" (line 30).

In times where Thomas was working independently and he struggled, this positionality toward the mathematics led him (rather than to seek help) to be extraordinarily hard on himself. This aligns with Isaac's observation of Thomas who noted that he would scold himself even if he was only off by a few decimals. During this task he scolded himself numerous times saying things like, "did I do my math wrong? *How*!? … Oh! Oh my. Oh my. Oh my. Oh my god. Oh my god" (lines 62 and 65) after getting an answer incorrect and being corrected by Isaac. By wondering “how did I do *my* math wrong”, Thomas reveal his engagement with this as an independent task despite Mrs. Fry’s attempt to encourage collaboration and his anxiety and self-deprecation as a result of this isolation reveals the harm being done to Thomas. Regardless, Thomas seems to prefer working independently, that is until he realizes a gap in his own understanding and his inability to find a solution. In this moment, he communicates his need with some vulnerability to Thomas, makes a bid for his help by asking him questions. After completing this problem collaborative, Thomas softens, physically opens up to Isaac, and says thank you.

Notably, in other moments throughout the week, Thomas experienced frustration at an inability to solve a problem and this also led him to work with mutuality with his regular tablemates, Jorge and Peter. Together they work in a Desmos exploration about the area of triangles. Together they counted the base, height and the units inside each triangle and parallelogram to find their area. They asked questions of each other, listened to feedback, counted the units inside in the triangle in unison, and passed the computer back and forth between them. At one point, Thomas handed the computer to Peter smiling and saying, “I don’t know how to do this!” They entered a best estimate for the area and moved on to the next screen. On this screen they dragged a triangle around onto the area of a parallelogram with the same base and height and find that the area is exactly half of the area of the parallelogram. Steven looks excited and shocked, pushed back from the computer and claps.

This task in which Thomas could, as he smiled, express his own mathematical short-comings was a more open-ended and explorative task and didn’t lend itself to procedural correct answer-getting and speed. Therefore, Thomas more readily collaborated with his peers and demonstrated more expressive positive affect.

Therefore, to design mathematical tasks and activities that supported Thomas' unique mathematical joy, we would move away from closed tasks that foreground procedural answer-getting and speed (like worksheets). We would also intentionally incorporate ways in the tasks that students genuinely need one another and can have opportunities to socially turn to one another for help.

RQ2: Conditions that Support or Hinder Students' Joyful Mathematical Engagement

In these three episodes we saw students demonstrate a wide range of affective reactions to very different tasks and interactions that demonstrated a diversity of desires and needs in this mathematical classroom. Star's leaned into social interactions that gave freedom discourse to allow for play, humor, and silliness as well as tasks that required that students' look out for each other, encourage each other, and as a result share mathematical ideas and resources that are seen as valuable and respected mutually. It was evident that Ruqaiya loved a good, challenging mathematical puzzle that was offered through open ended exploration with a variety of possible solutions shared among peers. She also pursued avenues for boundary pushing and mathematical rule breaking to explore mathematical content. While Isaac in this class was largely neutral and lacking affect, he did push into and positively engage in tasks that foregrounded his own development of mathematical expertise and then the wielding of that knowledge for the benefit of someone else. And lastly, Thomas demonstrated expressive positive affect toward more open, explorative tasks intentionally incorporated ways that students genuinely need one another and socially collaborate.

These three episodes, along with the meaning made of them, demonstrate students' conscious or subconscious need or desire for a more fulfilled relational and intellectual well-being. Each child, while their affective reactions and conditions were wildly different, demonstrated that one potential support their all of their mathematical well-being was mutual, respectful, sharing relationships that encouraged intellectual growth and exploration – satisfying both relational and intellectual well-being. This reflected the themes that Mrs. Fry and I took up when we discussed designing for students' mathematical joy. Discovering this theme as we analytically pursued students was only possible because we relationally pursued students. The theme across these students' positive and negative reactions to mathematics was the mutual, benevolent sharing of mathematical ideas and resources. I'm calling this mathematical reciprocity. Mrs. Fry and I saw this throughout the semester and described it as humility, generosity, confident expertise, and vulnerability. We also saw evidence of students' desire games and play which was evident in these and in other affective episodes, so this certainly isn't a comprehensive description of all potential avenues of support joy for every student. Here, I highlight mathematical reciprocity as a design feature for joy in this space because it emerged as a commonality across all of these episodes.

Mathematical reciprocity is a potential design feature for mathematical joy because it is an answer to how to humanize mathematics uniquely for these students. Student's affect demonstrated moments when they pushed away from dehumanizing interactions or leaned into humanizing interactions. Star leaned into mathematical opportunities in which she was helped and encouraged mathematically by her peers and she distanced herself affectively when she disrespected or disparaged for her mathematical ideas. Ruqaiya was giddy and playfully pushed the boundaries of mathematical solutions when she had to opportunity to collect other people's potential mathematical reasonings, and she shut down when presented with closed, procedural problems. Thomas exhibited frustration in the tension between individual, product-oriented mathematics and harshly scolded himself for his failure to perfectly solve problems independently, but he expressed thankfulness when his needs were met with kindness by a peer. Isaac animatedly engaged when it was apparent that he was needed by Thomas and his mathematical ideas were treated as valuable to the learning space, and (after having some opportunities for feeling valued mathematically) he actively pushed back on moments that reinforced the hierarchy of mathematical knowing and doing. In each of these cases we see students harmed by interactions that demonstrate a lack of mutuality (or a hierarchical nature) of mathematical interactions. Traditional sociocultural norms of mathematics education could dehumanize students in a multitude of ways, as seen from the literature review. But these students' intense affective reactions center on moments of satisfied mathematical reciprocity or interactions that lacked mathematical reciprocity. This indicated that a common threat to these children's humanity even in their diverse mathematical experiences was the hierarchal, competitive sociomathematical norm in this space. Therefore, in the final math unit of their school year, in an attempt to design for their joy, Mrs. Fry and I designed a unit about data and statistics that, in as many ways as possible, necessitated students' mathematical reciprocity and in doing so resisted hierarchal, competitive sociomathematical norms.

Chapter 6: Student's Joyful Mathematical Learning

Based on the insights revealed by students in the case study, Mrs. Fry and I co-designed a set of lessons about Data and Statistics specifically aimed to support her students' mathematical joy. Through their affective reactions, Star, Isaac, Thomas, and Ruqaiya demonstrated a desire for mathematical reciprocity, a mutuality in sharing mathematical ideas and resources. This desire appeared to be in resistance to the hierarchical, competitive forms of mathematics typical of this cultural context. Therefore, we designed tasks that necessitated mathematical reciprocity in as many ways as possible to determine if students would, as a result, be more likely to experience mathematical joy.

In previous chapters research questions were adapted to appropriately fit the data collected in Phase 1 and Phase 2 and to incrementally develop an understanding of the context, students' affect, mathematical stories, and potential for joy. In this chapter, I expand these research questions and build on previous findings toward a deeper, more nuanced understanding of mathematical joy:

1. What are some characteristics of joyful mathematical engagement for these students? What does mathematical joy look like, sound like, feel like, and how does it look different for different students?
2. In what ways did designing for responsive resistance (in this case using mathematical reciprocity to resist competition and hierarchies) support students' joyful mathematical engagement in a formal math classroom?

Research Context

This chapter broadens the analytic scope to focus on mathematical engagement and interactions of consented students across this 6th grade class during a week of math lessons about data and statistics. These 21 students had just finished a week of standardized testing and were approximately 2 weeks from their summer break. Students engaged in lessons about data and statistics largely in small groups with 4-5 students per group as they would have throughout the school year.

In years past, the data and statistics unit for this class was conducted after the district and school had completed their end-of-year standardized testing. In the 6th grade at Porter, students completed an end-of-year project in which they learned about different representations of data, chose a topic of interest, wrote some statistical questions, collected data from their classmates, and created a display of their data. This kind of activity, one which showed more flexibility in structure and connected to students lives, seemed to lend itself well to designing and re-designing for mathematical joy. Additionally, with state testing completed, Mrs. Fry felt more at ease with making more significant changes to the tasks and curriculum without fear of significant institutional consequences for her and for her students. Therefore, at the end of Phase 2 of data collection, I brought some preliminary case study analysis to our Research Practice Partnership meeting to member check (Lincoln & Guba, 1985) this information with Mrs. Fry and to begin re-designing the Data and Statistics unit to support students' joy. These designs were based on what we learned about the potential avenues for supporting students' joy in this context from the case studies.

Designed Tasks: Mathematical Reciprocity and Complex Instruction

The design of the final data and statistics unit was based on the in situ ethnographic analysis of data conducted in Phase 1 and Phase 2 of this study. As described in the introduction to the last chapter, our designs focused specifically on supporting students' generosity, confidence to share, vulnerability, humility, looking out for one another, as well as students' engagement in play, puzzles, and open-ended explorative tasks. Because the multimodal interaction analysis in Chapter 5 described students' desire for mathematical reciprocity, I will focus our attention here on the ways that we designed for the first half of this list of design features that we pursued. This isn't to say that we didn't design for puzzles and play, just that it isn't the focus of this text. We, therefore, designed a set of tasks, explored whether and how class norms and procedures needed to be changed, and developed an assessment that students had to complete collaboratively in groups. These tasks were all intended to be more social and less competitive, encouraging and requiring students to lean on one another for growth and feedback.

The shift to a more cooperative task and assessment structure necessitated a shift in procedures and classroom norms. The desire to incorporate mathematical reciprocity brought us to incorporate Complex Instruction (Cohen et al., 1999) for structural consistency of tasks and norms throughout the math unit. Complex Instruction (CI) intentionally centers “cooperative learning as an alternative to tracking and ability grouping and as an appropriate and promising strategy for academically and linguistically heterogeneous classrooms” (Cohen et al., 1999, p. 80). In CI tasks, students work in small groups around activities and through norms that are intentionally structed to necessitate that students “talk and work together and *serve as resources for one another*” (Cohen et al., 1999, p. 80). This represented a shift but not a complete transformation of the prior classroom structure; students would still work in small groups as they did throughout the school year, but the CI structure could shift procedures and norms within the groups that students revealed were problematic, like a focus on competition and resulting hierarchies as well as conformity and compliance. For example, one of the norms insisted that students were expected to monitor that “everyone contributes and no one takes over – you have the responsibility to ask for help and offer help and pay attention to what other group members need” (Cohen & Lotan, 2014; Featherstone et al., 2011). Additionally, each of the group roles - recorder reporter, facilitator, reflection leader, and resource manager - focused on monitoring the group’s progress toward some common goal (Cohen & Lotan, 2014; Featherstone et al., 2011).

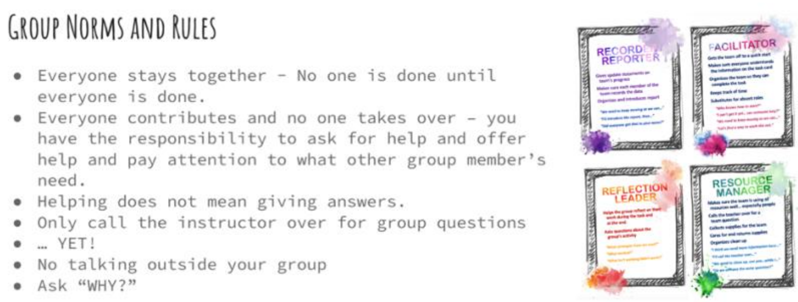


Figure 6: Complex Instruction Group Norms (left) and roles (right)

On the first day of the unit, students were introduced to these new norms and roles through a task called Broken Circle (Cohen & Lotan, 2014). The norms and roles were then used in every small-group task throughout the unit.

Broken Circles

We set the stage in the first task of the unit through a task called Broken Circles. This puzzling, playful, mathematical task introduced students to the CI group norms and new student roles. For the task, each small group was given 4 or 5 envelopes (depending on the number of students in the group) each with 2 to 3 pieces of a puzzle inside. In each group, the resource monitor distributed one envelope to each student. The object of Broken Circles was to put the puzzle pieces together in such a way that each member in the group ended up with a complete circle, although the puzzle pieces they were given would not allow them to accomplish this. Therefore, students needed to exchange puzzle pieces with other members of their group to complete the circle. Students were not allowed to talk, point, or give any hand signals. Each player had to put together their own circle and no one could show a player how to do it for them. Students also were not allowed to take a piece from another player. You could only give your pieces away, one at a time, to any other members of your group. While engaging in the task, students were expected to adhere to the new group norms and roles. The task wasn't complete until every person in the group had completed a circle. Critically, unbeknownst to students, one of the envelopes contained a complete circle at the beginning of the task. Therefore, for the group to finish the task, this student who received the envelope with the complete circle had to break apart their circle and give away their pieces to other students to help complete their circles. After this activity, students completed an individual, small-group, and whole-group reflection in which they were asked: What did you think this activity was all about? What things did your group do to help solve the problem? What things did your group do that made it harder? What things could your group do better in the future?

Data and Statistics Project

At the end of their 6th grade year, every student at Porter Academic Magnet completed a data and statistics project. Students were allowed to choose a topic of interest, write statistical questions for that topic, collect data from their peers, and then make a presentation using various mathematical representations (dotplots, box and whisker plots, histograms, and stem and leaf diagrams) of those data to answer questions. We tweaked this project and added activities in the days before it so that each student became an expert in one of the mathematical representations.

Expert Groups

Prior to starting the project, students were allowed to choose which kind of graph they wanted to become an expert in. Each table had 4 to 5 students and the standards required that students learn about histograms, dot plots, box-and-whisker plots, and stem and leaf diagrams. So, on the first day of the unit, in their small groups, students discussed and decided who wanted to be the expert on which of those graphs with each student choosing a unique representation. We designed this to provide an opportunity for each student to be a valued expert mathematical resource in their respective small groups during the project. For the following two days, students would divide into "expert" groups based on their chosen representation and complete activities to learn about either histograms, dot plots, box-and-whisker plots, or stem and leaf diagrams. These activities included Desmos activities, videos online, an activity called "Critique, Correct, Clarify (Help a Friend)" which used rough drafts of their kinds of graphs which they corrected as a group, and a culminating activity called Mystery Data.

Mystery Data

In their expert groups, students engaged in a Complex Instruction activity called Mystery Data. There were eight expert groups in the class; each mathematical representation had two expertise groups. Each of the 2 groups were given samples of data about the same variable for two different populations, but out of context. For example, one of the histogram groups got a set of numbers that were the fastest recorded speed of various land animals, while the other histogram group received a set of numbers that were the fastest recorded speed of various aquatic animals. However, groups simply received a list of numbers, so they didn't know what their set of numbers represented. Each student in the group was given a piece of their list of data, and the group had to work together to construct their list of data. They then each randomly chose 2 statistics to calculate about the data and had to help one another calculate the statistics they chose. Groups then worked together to represent each set of data on a poster as histograms, dot plots, box-and-whisker plots, or stem and leaf diagrams depending on their expert group. Using their graphs and statistics as a guide, groups then made up a story about this data and gave it a context.

While engaging in the task, Mrs. Fry randomly assigned roles in each group and displayed the CI norms on the board and on the task card. At each step of the task students were directed that if they made a suggestion about what to do, then someone else should write down the suggestion. After completing their graphs, students shared their invented stories and context for their graphs. Mrs. Fry then, with much anticipation, revealed the true context. Students then hung their posters around the room and engaged in a museum walk comparing each of the sets of graphs, comparing the fastest speed of a sample of land animals and aquatic animals. During the walk, students were given a stack of post-it notes with which they could offer thoughts about each representation or compare how the data looked between different kinds of representations. After the museum walk, Mrs. Fry led a discussion with students about what conclusions could or could not be made about the speed of land animals and aquatic animals.

JigSaw Data and Statistics Project

On the end of the third day, students went back to their original small groups of four or five students, with each child now being an expert in their graph. Students then could choose their topic of interest and write some statistical questions to pose to their peers about that topic. Students helped one another chose topics and write questions based on their interests like Taylor Swift, art, baking, sports, or music. After writing their questions, students used an online tool to collect data from their classmates and then used that data to make various graphs or calculate statistics about their topic. Students had to ask one another about how to make the graphs that they hadn't yet learned. Students who were the expert in each graph were expected to look around and help their classmates with that graph whenever needed. Students were not given a specific project or product that they had to complete. They were only directed to represent their data in some way that helped them complete the Data and Statistics Rubric.

Rubric

Learning about data and statistics was assessed through a rubric that students completed collaboratively throughout the unit. On this rubric, every standard for data analysis and statistics was listed along with a blank place for students to fill in the date completed, evidence of mastery and notes about each standard, and a box in which a peer could sign off that they had demonstrated they learned that standard. For each of these standards students were required prove to one of their peers that they could do that standard and have them sign off on it. Only students who had that signature already could sign off on someone else's. Mrs. Fry gave the first signature to each student after they finished their activities in their "expert groups." Mrs. Fry reinforced with students that the assessment was not about the project, but about learning the mathematics required to do the project well.

Figure 7: Data Analysis and Statistics Rubric

After Mrs. Fry introduced students to the rubric on the first day of the unit, students related the rubric and this assessment format back to the norms of Complex Instruction. Students discussed that they were responsible for helping one another and that no one at their group was done until everyone was done with the rubric because they could always be helping someone who needed a signature. Mrs. Fry conveyed the importance of academic integrity with the whole class and how they were responsible for being honest about their peers' performance on each standard. Students also completed a set of reflection questions and gave shoutouts to their peers at the end of the rubric.

Shoutouts

The unit began and ended with students giving one another shoutouts. On the first day of the unit, Mrs. Fry announced that that day's exit ticket would be to give a shoutout to someone else in the class about how they helped you, encouraged you, or did something mathematically awesome that day. Students wrote their shoutout to a peer on a sticky note and posted them at the front of the class. Mrs. Fry reinforced to students as they gave their shoutouts: “As we go through this next unit, be watching each other, looking around and seeing all of the awesome things that you classmates are doing in math.” On the rubric that students progressively completed throughout the unit, students were also provided with a place to give five “Shoutouts” to their peers. Each of these shoutouts that they gave to someone else added one point to their final grade on the rubric.

Analytic Methods: Abductive Postcoding Analysis

Open-ended abductive postcoding analysis (Parks, 2021) was used to capture affective moments of joy throughout the data and statistics math unit and trace the conditions that supported those moments to the design of the tasks based on the previous two analyses. Meaning was made of these joyful moments using student group interviews as well as the data collected and analysis conducted throughout the study.

This methodology served not only to support an investigation of joy, but also embodied joy itself as methodology (Parks, 2021, p. 123). In this final chapter, joy is not simply the topic of my research questions, but it is the procedural and analytical methodology by which I draw these conclusions. Taking up joy as methodology invites my own wonder, excitement, awe, enchantment (Geoghegan & Woodyer, 2014; Parks, 2021) by operationalizing analytic methods that feature “embracing passions, attending to everyday details, and freeing oneself to entangle with the world rather than attempting to observe it from a distance” (Parks, 2021, p. 123). In this analysis, I, therefore, was able to embrace my own perspectives of mathematical joy as the human instrument in qualitative research (Lincoln & Guba, 1985; Peredaryenko & Krauss, 2013).

Therefore, analytically I attended first to my own feelings of mathematical joy and my personal perceptions of students’ mathematical joy in the classroom context in order to identify moments of potential mathematical joy for students. Fieldnotes and voice memos were reviewed for moments (Parks, 2021) in which I felt, experienced, and observed mathematical joy throughout the class. The tasks in which students were engaged during these moments of joy were collected and described in a data log. In each task, the whole class Swivl video and personal reflections between myself and Mrs. Fry were reviewed to ascertain where my attention was throughout these tasks – focusing my analytic attention on the interactions in which I was experiencing mathematical joy either directly with or tangentially to students. I then focused on tasks identified by my own and by students' mathematical joy and selected case studies of two tasks so provide a close-up look at the phenomenon of interest.

Research question 1 poses that I determine characteristic of mathematical joy – what it looked like, sounded like, and felt like and what other potential affective states accompanied it for these students. Providing a single case study for each task allowed for descriptions of mathematical joy that were "intensive" (providing rich complete details with significant internal variance) and stressed "developmental factors" that evolved over time in each task that had the potential to significantly impact students engagement and joy as well as the context and relation of students to their environment (Flyvbjerg, 2011). I characterized the content and context of these joyful interactions with emergent codes, including things like play, debate, vulnerability, expression, joking, and encouragement. These two cases were chosen as an "information-oriented selection to maximize the utility of information from small samples and single cases. Cases are selected on the basis of expectations about their information content" (Flyvbjerg, 2011, p. 307). The expectation here was the potential for significant evidence of mathematical joy as demonstrated by mine and the students affect during this activity and my attention during the task.

These cases of episodes of mathematical joy were coded for kinds of student engagement across the activity. The linguistic nature of this coding exercise ignored (or at least limited) the "entanglements of language and matter, words, and things" (MacLure, 2013, pp. 170-171). Therefore, to capture and describe moments of joy in the data and statistics math unit it was necessary to rely on qualitative research "that is neither inductive—working from data to theory—nor deductive—working from theory to data, but rather 'abductive,' which is catching a moment of 'breakdown, surprise, bewilderment, or wonder" and reasoning through to explanations'" (Parks, 2021, p. 126). In each case, I abductively identified these moments of "breakdown" and examined these moments in close detail through multimodal interaction analysis (Flewitt et al., 2009; Norris, 2004; Parks & Schmeichel, 2014).This allowed for unique attention to be paid to multiple forms of interaction relevant to joyful affect, such as verbal discourse, gaze, facial expressions, gesture, posture, and sound. This provided insight about what joyful moments looked like, sounded like, and felt like in the moment.

Finally, I reviewed all completed analyses to make sense of connections and entanglements between design, interactions, bodies, emotions, materials, and theory that otherwise would go unnoticed. This was done by primarily drawing student voice from focus group interviews and utilizing my knowledge and data collected about the class throughout the study, which I related to these moments of stumble data. This open-ended abductive postcoding analysis allowed for me as the researcher to "chase connections across the story of this study rather than stopping at artificial edges" (Parks, 2021, p. 123). By taking affective cues from both myself and from students as significant findings, providing rich descriptions of joyful interactions, and making connections across data using diffractive analysis, I provided a picture of a landscape of joyful mathematical engagement in this context that attended both to its characteristics and the conditions that supported it.

By drawing from the practices of cultural geographies (Geoghegan & Woodyer, 2014; Wylie, 2010), this work aims to collapse "the macro, mezzo, and micro levels of analysis that are common in various modes of research and takes up a humble view of research impact" (Parks, 2021). The conclusions drawn here are thus not aimed at scalable, social change projects in education. Rather this work aims to highlight the small, seemingly insignificant interactions of 6th grade children as they engage in mathematics learning with joy to so as to engage in and "make the marginal visible" (Geoghegan & Woodyer, 2014, p. 225). The goal of this chapter is to humbly create something "scholarly and story-like" (Wylie, 2010, p. 212) so as to share in, feel, and elevate the potential of mathematical joy by describing it "so as to share its power, rather than be consumed with analytical debunking" (Geoghegan & Woodyer, 2014, p. 220). The following are two stories of students' mathematical engagement in two different math tasks. I will first provide descriptions of engagement and context of some positive affective moments across the class for each activity. Then I will zoom in on a single group engaging in this task to describe more specifically what their mathematical joy looked like, sounded like, felt like, as well as additional accompanying affective states.

RQ1: Characteristics of Joyful Math Engagement

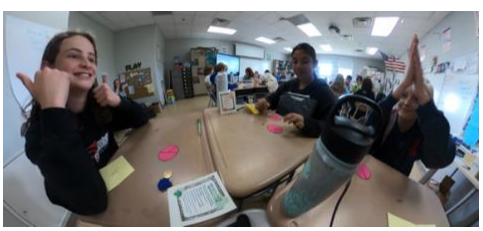
In looking through fieldnotes and voice memos from the data and statistics unit, I noted moments of students' apparent mathematical joy demonstrated through positive affect during the Broken Circles activity (Day 1), Mystery Data (Day 3), the jigsaw and data project (Day 4), Gallery Walk (Day 4), and when students gave one another shoutouts (Day 1 and on Rubric). For example, during Broken circles, I described specific students as "bouncing up and down" in their seats, clapping and "getting really excited". When students were working in their expert groups, I describe them as "genuinely smiling", "super giggly". During Mystery Data I made an extensive voice memo about all the positive affect, but my only fieldnote was "Up and moving too much to take notes, because this is SO MUCH FUN." During the Gallery walk I noted that "There's so much social chatter and giggles and play happening as students talk and walk around, but also a lot of math talk and critical/constructive feedback" and "Thomas is SO incredibly smiley - especially when they are at the histogram (his groups' graph)." Notably, not every task was joyful. My voice memo about the second day of the task when students started their expert groups I said that this day generally was "less joyful and a little bit more intense… there was lots of unknown and uncertainty" that made students uncomfortable as they started becoming experts in their chosen graph. The following sections detail students' engagement across and within small groups as they engaged in two tasks: Broken Circles and Mystery Data. These two tasks were chosen because I made voice memos about each of these tasks about both my own joy and students joy as students engaged in the task. Additionally Broken Circles was an introduction to the Complex Instruction norms and procedures (a pivotal component to designing for mathematical reciprocity) and Mystery Data was focused on learning new mathematical content that aligned with the 6th grade standards using these norms and procedures. Mystery Data was of particular interest because it lasted for an entire class period and because my own affect was so intense on this day. For each task, I show my reaction that drew me to look at students' engagement in these tasks more closely, provide some examples of students' interactions that appeared to evoke joy during these tasks across all small groups, fine-detailed multimodal interaction analysis for a single group engaging in the entirety of each task to provide a few example of some characteristics of joy that were demonstrated in this class students, and then students' reflections and context to make meaning of students' joyful engagement in each task across the class.

Broken Circles

The first activity of the data and analysis unit involved students developing awareness of and practice utilizing new norms and practices of Complex Instruction in an activity called Broken Circles (Cohen & Lotan, 2014). The specifics of the Broken Circles activity are explained in depth in the introduction to this chapter. Notably, this task was mathematical in that it required students to solve a puzzle involving four different circles divided into various sectors, but it didn't mathematically align with the data and statistics objectives of the unit. Our primary goal was for students to engage collectively to introduce norms around Complex Instruction in a way that was both mathematical and playful. After this activity, I created a voice memo in which I said,

> *Students seem to be taking up these activities with a lot of excitement. Like even in the first CI tasks that we did, Broken Circles, there was lots of like clapping and cheering when they got it right…I saw like quite a bit of joy. There was bouncing up and down in chairs and clapping… there was lots of positive affect, and students seem to enjoy working together… most groups really took up this idea of group work and watching out for each other and helping each other.*

Looking at student video of this task confirmed that students demonstrated significant positive affect across the class. This was remarkable given that the task required them to remain completely silent. In every group in the class there was at least one student who smiled, gave a thumbs up, giggled, clapped for their peers, joked with their group members, or expressed bemusement in the playful task at hand. In more than half of the groups, every student in the group exhibited some positive affective expression.



Isaac, Andrea, and Nick smile broadly, grin, clap, and celebrate collectively as they complete the Broken circles task

Johnsen, Quantavious, Ava, and London silently giggle as they unsuccessfully try to communicate which pieces to pass to one another.

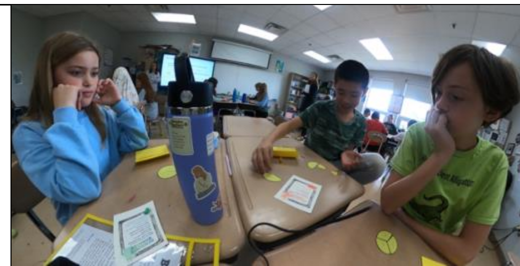
Lyra and Thomas grin as they convince Peter to break apart his completed circle so they can finish the puzzle collectively

All groups demonstrated some positive affect when they successfully solved the puzzle. Surprisingly, groups also demonstrated positive affect when they were frustrated, confused, unsuccessful, while also looking to one another for insights. Here, we enter into Lrya, Thomas, and Peter's group as they engage in broken circle to get a closer look at the interactions and conditions that made this task joyful for them throughout the entirety of the task. This group was chosen because I talked about this group in my voice memo and also discussed it with Mrs. Fry immediately following the class on Day 1.

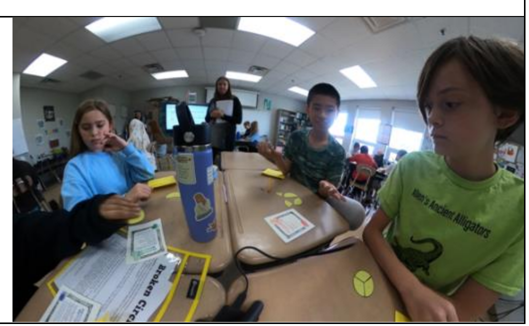
Lyra, Thomas and Peter with Broken Circles

Lyra (left), Thomas (center), Peter (right), and a fourth unconsented student (outside of scope of camera lens) are working in a small group on Broken Circles. This group had been sitting together in a group for approximately 2 months and were, therefore, familiar with working together. Mrs. Fry gives the directions for the activity and the resource manager at the table distributes the envelopes that contained some puzzles pieces to each student. Mrs. Fry says, "go" and everyone in the group opens their envelope to inspect what puzzle pieces they have been given. Peter's pieces (envelope A) immediately complete a circle. He successfully puts his circle together and sits stoically, occasionally glancing casually at his peers uncompleted puzzle pieces. The puzzling begins with Lyra and Thomas coordinating to successfully solve Lyra's circle:

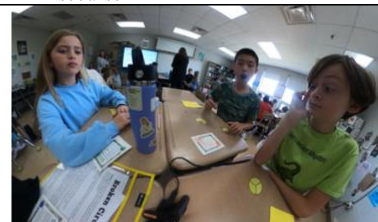
Turn	Time	Student	Discourse	
1	3:23	All	Receives envelopes from Resource Manager and takes puzzle pieces out of envelopes	
5	3:48	Thomas	Hands puzzle piece to Lyra	
		Lyra	Uses piece from Thomas to complete circle	
6	3:53	Thomas	Can't complete circle, looks at Peter's completed circle	
7	3:58	Thomas	Receives piece [smiles and shrugs] Unsuccessfully tries to assemble 4 pieces into circle. Looking questioningly over table. Looks in envelope for any missing pieces	

		Lyra	Watches piece get passed to Thomas	
8	4:12	Thomas	Looks inquisitively at peers table	
9	4:20	Thomas	Tries to assemble same pieces again [smiles]	
		Peter	Looks at Thomas' table	
10	4:26	Thomas	Look at Lyra [smiles, puts hands to mouth]	
		Lyra	Looks on and raises her eyebrows	

Lyra and Thomas are the first to exchange pieces (turn 5). As a result of this first trade Lyra completes her circles calmly without expression. Thomas is unable to complete a circle with the pieces he has and so begins a sequence of smiling, animatedly shrugging, giggling, and apparent bemusement at the puzzle throughout the entirety of the activity (turn 7). After this moment, half of the exchanges involve Thomas demonstrating some positive affect. Lyra then moves the puzzle forward by sacrificing her own completed circle.

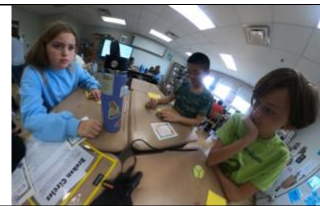
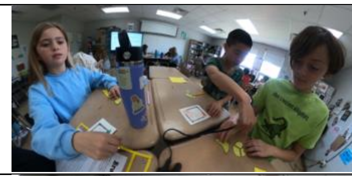
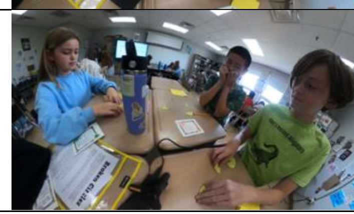
Turn	Time	Student	Discourse	
11	4:42	Lyra	Breaks apart completed circle and hands piece to peer who tries unsuccessfully to complete her circle	
12	4:45	Thomas	Looks at peer's uncompleted circle holds out hands and smiles	
		Lyra	Receives pieces back from peer	

After completing her circle, Lyra sits complacently with her completed circle for 50 seconds until breaking it apart to give a piece to the fourth student (line 11). After this she smiles slightly and then takes an active role in looking at other people's puzzle pieces and trying to give what she can away to solve the puzzle.

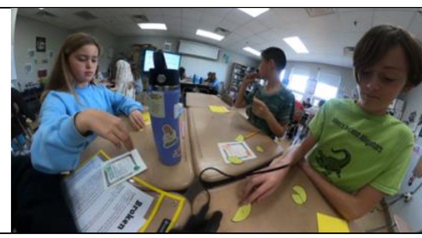
Turn	Time	Student	Discourse	
18	5:41	Thomas	Looks at peer's circle. Gasps, looking surprised.	
		Lyra	Looks at Peter's completed circle	
19	5:47	Thomas	Points at the empty space in his circle and looks to Peter's circle for the size needed to complete his.	
		Peter	Opens his hand as if to say "what?" up and shakes his head "no"	
20	5:50	Thomas	Shakes his fists up and down 3 times while smiling	
21	5:57	Thomas	Gives piece to peer, but it doesn't work. Smiling. Giggles. Puts hand to mouth.	 

Lyra begins to notice Peter's completed circle and looks at it intently (Line 18). Thomas also pays attention to Peter's completed circle, noting that he has a piece missing and that Peter's pieces looks like it would fit, but Peter disregards this request to break up his circle and tells Thomas "no" (line 20).

Thomas exhibits some playful frustration as a result (line 21). But the group continues to persuade Peter to invest in the group effort.

Turn	Time	Speaker	Discourse	
24	6:07	Lyra	Looks intently at Peter and leans toward him in her seat as she nudges her chair closer to the table.	
26	6:25	Lyra	Breaks up her completed circle and hands piece to Peter	
27		Peter	Shakes head "no" slightly and opens up his hand in response to receiving Lyra's pieces	
28	6:28	Thomas	Points and makes a circular motion over Peter's pieces indicating that he needs to break them up	
29	6:38	Lyra	Hands Peter second piece. Grins.	
		Peter	Breaks apart original completed circle.	
		Thomas	Smiles	
30	6:39	Peter	Opens one hand upward while trying unsuccessfully to put 5 pieces together	
		Thomas	Shrugs and smiles open handed	
		Lyra	Receives piece from peer	
31	6:42	Thomas	Smiles	
		Lyra	Look intently at Peter's puzzle	
		Peter	Smirks as he looks around to see who he can give one of his pieces to	
32	6:46	Peter	Gives piece to Thomas	
33	6:48	Thomas	Laughs silently, gives piece to Lyra	
34	6:50	Thomas	Puts his hands to his mouth to try to keep himself from laughing out loud. Giggles silently for 8 seconds.	

Both Lyra and Thomas attempt to convince Peter that he must break up his own circle for the good of the group by staring at him intently (line 24), breaking up their own completed circle and giving him one of their pieces (line 26), and gesturing overing his pieces to indicate breaking them up (line 28). After Peter finally concedes to break up his circle (line 29), both Lyra and Thomas smile as Peter begins to take part in their collective challenge (line 30). Peter grins for the first time in the activity (line 31) as he decides who to give his puzzle piece to. As they continue to wrestle together with the puzzle, Thomas begins uncontrollably giggling, placing his hands over his mouth (line 34). The four students continue to pass pieces back and forth to one another trying to complete the puzzle.

Turn	Time	Student	Discourse	
37	7:27	Thomas	Looks around at the rest of the table groups	
		Lyra	Gives piece to Peter	
		Peter	Tiniest little grin	

Suddenly, Thomas' smile and giggles fade as he looks around and realizes that every other group in the room has finished before them (37). But he returns to the task relatively quickly and they complete the puzzle, three and a half minutes after they started.

Turn	Time	Student	Discourse	
40	8:00	Thomas	Claps, looks around to the completed puzzles	
		Lyra	Smiles with a look of satisfaction, giggles silently	

As they finish, Thomas claps and smiles looking around at all of the completed puzzles and Lyra grins with some satisfaction gazing at her peers (line 40).

Characteristics of Lyra, Thomas, and Peter's Mathematical Joy

The final moment of this activity for these four students demonstrated collective celebration, but each student was drawn in and found joy in slightly different aspects of interaction and engagement throughout the task. In this activity, Thomas was consistently and gleefully baffled that his puzzle pieces never fit into a circle. He constantly looked around to his peers, giving and taking pieces to try to make the collective endeavor work. He also celebrated and appeared to relish all of the compiled puzzles as she clapped and looked around at every puzzle when they successfully completed the task. His playful positionality toward the task illuminates how the task was structured to be playful and noncompetitive by creating students' need for one another around a puzzle. One of the few times that Thomas' smile faded was when he looked around to the rest of the class and realized that everyone else has completed the task faster than them. As we say in Chapter 5, we saw that Thomas enjoyed "winning" and had a propensity for speed and correct answer getting especially as compared to other people in the class. But for four minutes of this task, he was relieved of that comparison and competition. In this time, he constantly smiled and seemed to really enjoy the challenge of the puzzle and working with his groupmates. But he quickly fell back into the habit of comparing himself competitively to his peers as he noticed that his group was the only group that hadn't finished the task yet. One of the reasons I focused on this group for this multimodal interaction analysis was because of this moment for Thomas. Mrs. Fry and I both noticed this happen in moment and described it as a moment of "panic" for Thomas. I was interested to see how this moment emerged over the course of the task.

Lyra was calm and temporarily disengaged from the task for nearly a minute after completing her circle following her first exchange with Thomas. But she leaned in, grinned, and actively engaged as she realized that her pieces were needed. She also astutely realized that Peter's pieces were needed and (through intent stares and an example of personal sacrifice) coaxed him to give up his completed circle. She smiled and seemed to find satisfaction in her ability to persuade Peter. She showed collective satisfaction in the work of her groupmates as she grinned around at her peers rather than at the completed puzzles.

Peter, who completed his circle initially with all of the pieces from his own envelope, was stoic and immoveable for the majority of the activity. He rarely looked around at other's circle pieces, and when he did it was without expression. But Peter grinned when he finally conceded and broke his initial completed circle apart. He, with some measure of apparent satisfaction, realized that his peers needed his pieces to be successful, broke his completed circle apart, and looked around to see who might need his puzzle pieces.

In this set of exchanges as students silently, animatedly, and collaboratively solved this puzzle, we see that positive affect emerged when students looked outside of themselves to be a resource for others or to see others as a resource. They did this by sacrificing their own accomplishments for the good of the group, being a resource to other people, debating and persuading of an equal collaborator, or demonstrating curiosity about and puzzling with interesting materials in cooperation with each other.

Mystery Data

On the third day of the Data and Statistics unit, students were grouped into "expert groups" in which they were together mastering how to make a specific kind of graph in a task called "Mystery Data." The day prior, students had also been in these groups completing various activities about their respective graph: dot plots, stem and leaf plots, histograms, and box-and-whisker plots. In their groups students were each expected to become an expert resource about their graph so that they could help their classmates learn about that graph when they returned back to their table groups to work on their projects.

After engaging with students in this task on Day 3 of the unit, I made a voice memo that exclaimed (loudly):

> *AHHH! That was so fun!! …That was a fun math class … there was movement… they were talking to each other, they were helping each other, they were becoming experts in something that they didn't know at the beginning of the class and learning about it and wanting to learn about it. It was fun. And there was math, and there was joy, and it was all wrapped up into the same thing… It doesn't mean that there weren't frustrating parts, and that they weren't stuck sometimes and that they didn't argue with each other. It wasn't game based, it wasn't competitive. There was no winner, it was looking out for each other…That's the joy.*

In every group, students engaged in this task with some measure of joy. Students expressed positive affect while debating potential storylines for the context of their graph, contesting and debating the accuracy of various mathematics definitions or procedures, collectively developing expertise and confidence for a purpose, telling jokes and singing songs, asking questions and learning from and with one another, fluidly switching back and forth between social mathematical engagement and social non-mathematical engagement, encouraging one another, dancing and freely moving their bodies around the table to work on their graph, celebrating one another, and with vulnerability needing and reflecting honestly with one another.


Jorge, Ruqaiya, and Peter smile, laugh and animatedly talk when checking each other's answers and being "experts" for each other


Daisey, London, Regg, and Nick make jokes and playfully banter during mathematical debate


Andrea and Izzy laugh as they debate which context to choose to match their data

Students' joyful expression in mathematics varied widely both with and across expert groups in the class. Here, we enter into Nick, Regg, Daisey, and London's group as they work together in an "expert group" based on their choice to become an expert in box-and-whisker plots. This level of analysis focuses on them throughout the entirety of the task to give space for observing what conditions supported their mathematical joy in the context of the activity. This activity was nearly an hour long. This group was chosen because I spent significant time with this group, as evidenced from my moments in the whole class Swivl video and was potentially attuning to their work when I animatedly described my own mathematical joy.

Regg, Nick, London, and Daisey with Mystery Data

At the beginning of the task, Mrs. Fry gives the group a poster on which to work, the task card for the assignment, the materials needed, and their rubric. They were given autonomy to accomplish the items on the task card on the poster in whatever way they felt necessary. In the first 10 minutes of the Mystery Data task, Regg, Nick, Daisey, and London all actively participate by calculating various statistical measures like mean, and mode, debating the accuracy of one another's calculations, and determining the goals of the task ahead of them. There is significant participation, but very little affect. There are several instances in which the four students debate about both the mathematical content, as well as the procedures of the task. For example, Nick disagrees with Regg's answer for mode because he miscounted the most frequent value; Daisey believes that each of them need to prove that they can do each math objective individually in order to get their rubric signed, whereas Regg and London believe that as long as they prove that they can do it as a group, then everyone should get signatures on their rubrics. These debates all resolve peaceably, and the group comes to a collective conclusion after each point of tension.

After working on the activity for 13 minutes, Mrs. Fry interrupts the group work and announces to the class

> *Okay, I need you think about the norms that we started on Monday. Those norms were and included, "no one is done until everyone is done." The divide and conquer mentality that is happening is robbing you of points, of understanding, of joy, of the good things that this is designed to do. You are meant to do this TOGETHER. The poster should be a mess because 3 of you are simultaneously doing something in the same space. Let go of cute. You aren't getting a grade on your swirlies. You're getting a grade on working together and doing the math and learning. No one is done until everyone is done. It is a lens shift. It is difficult. And I'm asking you to do it. Continue.*

After this speech, there is an immediate explosion of talk throughout the room. Regg, London, Nick, and Daisey work together on calculating each of the statistical measures that they've been assigned to calculate. Daisey and Regg stand up over the poster while London and Nick remained seated, Nick eating some Goldfish. Daisey immediately identifies the range of the data as the next measure to calculate and asks broadly to the group, "Is the range the biggest number minus the smallest number?" Both London and Regg answer affirmatively and London reads out the largest and smallest number in their data set. Daisey calculates the range mentally out loud in response. Regg suggests that someone needs to write the range somewhere on the poster while Daisey gets signed off by Mrs. Fry as a "range expert" on her rubric. Regg then walks off to get a calculator so they can calculate the mean.

Line	Time	Student	Discourse
1	15:51	Regg	Okay, I got the calculator. Who's got mean?
2	15:52	Nick	[while dumping Goldfish from the bag into his mouth, raises his right hand and wiggles his figures]
3	15:54	Regg	Wow. You're kind of MEAN. [places calculator on Nick's desk, smiles and laughs at his own joke]
4	15:55	Nick	What?! [feigning shock]
5	15:56	London	I would agree with that [casually glancing over at Nick, smiles at Regg]
6	15:57	Regg	It's a pun, okay! [smiling and giggling]

Regg interjects a mathematically themed joke into the conversation (line 3) and both Nick and London mirror his joking with their own jokes and playful banter (lines 4 and 5). Everyone in the group smiles and this seems to break down a barrier from active participation to playful participation. As the group continues to work through calculating mean, median, and interquartile range they begin to make increasingly more playful jokes with one another. For example, after signing off on her groupmates rubrics, Daisey makes a joke about Nick not putting his name on his paper. She says, "I don't see your name on this. Nope. I think it's mine," and Regg plays along saying, "Double the grade!" Regg, Daisey, and Nick all smile. These examples demonstrate that the humor shared among the group is interwoven into these mathematical tasks, as students make puns with mathematical words (line 1-6) or makes jokes about their rubric grades. But they also fluidly move between social and mathematical humor and joking. They laugh about cute nicknames for each other and make playful banter as they collectively calculate the statistical measures needed to create their box plot.

Nick is then tasked with finding the interquartile range (IQR) but admits that he doesn't know how to do it. After a discussion about IQR with me, the group decides that they need to take some time to calculate the IQR of their data together. Daisey finds the median, Regg calculates the first (Q1) and third (Q3) quartile by finding the *mean* of the top half of the data and the *mean* of the bottom half of the data respectively. Simultaneously, London and Nick draw a number line and begin creating the box and whisker plot. Regg tells them where to put the median and Q1 and Q3, but Regg's answers for Q1 and Q3 don't look right to Nick and London. Nick finds the median of the first half of the data instead of the mean and Regg accepts this correction saying:

Line	Time	Student	Discourse
7	32:46	Regg	So it's actually at 35 [referring to Q3, hovering over the poster]
8	32:49	London	Okay. So where does the box end? Here. Here [tries to take marker from Nick]
9	32:52	Nick	Hey! No. No. [smiling at London and refusing to give her the marker]
10	32:53	London	Yes
11	32:53	Nick	No
12	32:54	London	Yes
13	32:54	Nick	No [Grinning]
14	32:55	London	Yes [Smiling. Reaching toward marker in his hand]
15	32:55	Nick	No
16	32:56	London	Yes

17	32:56	Nick	No	
18	32:57	London	Yes	
19	32:57	Daisey	[stands up straight after bending over data announcing] Yeah it's 35	
20	32:58	Nick	TOLD YOU! Cause I'm BETTER like that.	
21	33:02	Regg	Yeah, I found the media-	

The playful negotiation of resources and roles is evident as London and Nick smile and ping-pong dialogue (lines 8-18) in an attempt to decide who will draw the first and third quartiles onto the box and whisker plot. This playfulness is embedded in the mathematical task of checking the accuracy Nick's calculation of Q3 by Daisey and Regg (lines 7 and 19-21). After affirmation from Daisey, Nick relishes in the success of his calculation (line 20), but Daisey continues to check the accuracy of the group's calculations so far:

Line	Time	Student	Discourse	
22	34:30	Daisey	[checks the accuracy of the minimum, Q1, median, Q3, and maximum on the number line, bending over and investigating the poster closely. Stands up when completed]	
23	34:46	Regg	Alright. Now we can find the interquartile range.	
24	34:48	Nick	[throws marker across table to London and smiles] If I do that…	
25		London	[giggles, smiles, nods] Yeah	
26		Daisey	[begins dancing in place]	
27	34:54	Regg	Okay, so who's gonna do the interquartile – Wait [points to Nick] Nick, Nicki, you have to put the interquartile range [gestures hand to box plot on poster] down.	
28	34:58	Daisey	[Dancing progressively gets bigger until she is around the table, spinning in a circle, kicking her feet, and bouncing up and down as she moves, and lands back standing at her original chair.]	
29	35:03	Nick	It's there [hands on hips, looking at poster, points with two fingers at the upper and lower quartiles of the box]	
30	35:07	Regg	Yeah, no. 12 to 35. What is it? You have to put it down.	
31	35:11	Nick	Put what down?	
32	35:13	Regg	The interquartile range.	

After successfully checking that the calculation is correct and that they are close enough to being accurately marked on the box plot with respect to the number line, Daisey confidently stands up and begins freely dancing around the table (lines 26-28). The dancing begins small with her hips slowly moving back and forth and her arms out to her side. But it progressively grows until she is spinning in circles moving around the table, kicking her feet out in front of her, and dancing herself all the way back to or original chair (line 28). All this happens while Nick and Regg debate about calculating IQR. The group continues working through calculating IQR (largely debating if it's a set of two number or a calculation of subtraction those two number) for this set of data while freely moving their bodies around. Regg turns to the neighboring groups to give them high-fives, holding his hand straight up in the air. He's significantly taller than the rest of his classmates so Nick has to stand on a chair to reach his (very) high-five. Soon after, Mrs. Fry comes over and asks the group if they understand a box and whicker plot.

Line	Time	Student	Discourse	
33	38:10	Mrs. Fry	If I started asking you box and whisker questions, could you all answer them?	
34	38:15	Regg	A little [smiles]	
		Nick	Maybe [smiles and shows his teeth looking uncertain, looks at Mrs. Fry and then looks around at groupmates]	
35	38:17	London	Yes	
		Daisey	Probably	
36	38:18	Mrs. Fry	Do you want me to try it and see if you can get the signature for box and whisker	
37	38:24	Nick	Maybe -	
38	38:25	Daisey	- Um. Let's talk together one more time	

Even in the vulnerable admission that they need more time and don't fully understand the graph they've been assigned (lines 34 and 35), both Nick and Regg smile as they sheepishly look to Mrs. Fry. And with the collective decision that they want to talk more together, the IQR debate continues. Daisey asks the group about how to calculate IQR and if it's only one number. London says, "so you subtract 35 and 12 because it the amount of number in here" [gesturing to the box]. Regg then excitedly relates the concept of "interquartile range" with the concept of "range" saying "oh! The range the 65. The interquartile range is 23." Nick gets upset saying that the range and IQR are two totally different things. London gestures strongly with her hands to the range and then to the interquartile range on the graph. Regg reaffirms and shakes his head. They debate about IQR animatedly for about 2 minutes until they are all able to say out loud how to calculate it. The debate ends when London writes, "Nick did this," circling a dent in the poster where Nick animatedly threw a marker down onto the poster earlier. He thinks she's talking about calculating IQR and claims it proudly – hitting his chest and saying, "yeah I did!" The group laughs as he realizes that she's circled the dent he made in the paper instead of IQR.

At the end of the lesson, Nick asks, "Wait, are we done now?" Regg replies, "yes, well we're experts."

Characteristics of London, Regg, Nick, and Daisey's Mathematical Joy

The fluidity of social and mathematical interaction in this episode makes it an interesting and complex space in which mathematical joy could emerge. This occurs for all four students largely through humor, debate, and movement.

Regg introduced humor into the activity around 15 minutes into the task by making a mathematical pun (line 3), and the activity ends with humor when London made a joke about Nick's contribution to the poster (both jokes interestingly directed at Nick). This episode was sprinkled with jokes and playful banter that drew humor into the task, which was possible in part because of the open, social, and collaborative nature of the task and the tendency for these sixth graders to bring humor into any social situation.

The Mystery Data task depended on students working together to calculate different statistical measures. They also had to decide which of those measures were needed to create the mathematical representation with which they were tasked to become an expert. Most of these statistical measures for this group only required small reminders and the group could work on them collectively, as seen in the beginning when they calculated range together. But when faced with a challenge, like in the task of calculating IQR, there was opportunity for significant vulnerability in admitting that they didn't know how to accomplish the task. They could also debate while toward a solution together (lines 7-21). These moments were characterized with excitement, laughter, and smiles even when this vulnerability was expressed to Mrs. Fry. Because they could fall back on their collective effort, the vulnerability of not knowing "yet" emerged with humor (line 34), rather than anxiety. This need to debate also emerged in London and Nick's negotiation over materials (lines 9-18) and in negotiation of roles and tasks (lines 23-32).

Students' joy in this task also emerged in and among students' ability to move freely. We see this as students worked in various positions around and over the poster (lines 3-7), in Daisey's dancing (lines 27-32), and Regg and Nick's jumping high-five. Students throughout this interaction were walking around the poster, walking across the class to get materials, hovering over or dancing around the poster, dancing, gesturing to one another, and moving their bodies freely.

RQ2: Joy through Responsive Resistance

Characteristics of student engagement were identified across these episodes in Broken Circles and Mystery Data. This included students' back and forth playful banter, willingness to be vulnerable, excitement to become an expert and share information with others, persuasive and peaceful debate. These descriptions of interactions were then groups by themes according to potential demonstrations of students' well-being. As students engaged in these math activities with joy, their participation represented their satisfied mental, physical, relational, societal, and intellectual well-being. Interestingly, manifestations of students' joy and animated engagement seemed to emerge primarily in forms that represent students' relational and intellectual well-being (as seen in the table below). But we also see other kinds of students' well-being emerge, like dancing and movement showing students' physical well-being. This is not to say that these descriptors of students' mathematical engagement only fit into one category. Indeed, students laughing at jokes together can serve as manifestations of both emotional and relational well-being.

Mental Well-being	Physical Well-being	Relational Well-being	Societal Well-being	Intellectual Well-being
Celebration	Dancing	Vulnerability	Purpose	Play
Expression	Movement	Connection	Developing expertise	Debate
Joking		Cooperation		Productive conflict
Self-reflection		Sacrifice		Persuasion
Surprise		Investment		Music
		Intentionality		Curiosity
		Social connection		Challenge
		Encouragement		Boundary setting/pushing
				Puzzling

This demonstrates a potential relationship between the intentional design for mathematical reciprocity as a form of resistance in this context, students' affective demonstrations of joy, and the characteristics of students' engagement that demonstrate their multifaceted well-being.

Students' Perceptions of Joyful Mathematical Engagement

But the relationship between joy and design wasn't limited to observation and coding. Students appeared to primarily experience these tasks as an opportunity to look out for each another and become mutual mathematical resources for one another. This demonstrated that, in these tasks in which they experienced joy (as evidenced from RQ1), it was evident to students that the designs foregrounded mathematical reciprocity.

After completing the Broken Circles puzzle students completed an individual written reflection, shared their reflections in their small group, and then voiced their small group discussion in a whole-group reflection. When reflecting about what they thought the goal of the task was, almost every student voiced that they believed that the goal was to work together, especially by observing other people and helping them. Daisey, Ashley, Gabrille and Abigail's group said, "I think this exercise was about … paying attention to yourself and to others", "working together" and "sharing different things and helping other people." This group also reflected that they liked "how [they] were constantly looking around to see they needed help" and when they "looked at the other person's circle and were able to see like, 'oh I have that piece'." Izzy, Regg, and Charlie's group said that the task went well because they were "working together and having patience", looking "at each other's pieces versus just our own." Regg provided nuance to this working together by saying that the task was not simply collaborating but about trust, "to see if you can trust them to do the right things" and "to see if your teammates were reliable." Andrea emphasized that something that would make this task better was by "observ[ing] more in the future." Students made evident through their interactions that this puzzle supported their joy and it was apparent in students' reflections that their primary focus was on one another and on the puzzle, rather than on their own individual success. "Observing one another," "working together," and "sharing different things to help each other" was central to the task, a task that support their joy.

After the unit was over and students were finishing working on their projects, I conducted focus group interviews with groups of 4-5 students in which students discussed the norms of Complex instruction, the rubric, and the various tasks they participated in over the previous week. Most students saw that the new norms of Complex Instruction had an influence on their interactions with peers and with the mathematics. Andrea and Star noted that the first norm, "no one is done until everyone is done," was the most enforced and the most noticeable. The result of students taking up this norm was that:

> *Andrea: You have to take breaks between each and every, like each thing you do and help someone else*
> *Star: it makes you slow down a little bit*
> *Madison: is that a good thing?*
> *Star: [nods and smiles] yeah*
> *Andrea: so you can focus*
> *Star: on more like what you're doing and stuff*
> *Andrea: and you can find like what you did wrong when you're trying to help somebody else.*

Daisey also recognized this relationship between this specific norm and the time that it allowed to develop mathematical understanding:

> *It was a bit more helpful because there were some moments like before in the year where my group would move on without me, so the "everyone stays together. No one is done until everyone is done." I like to understand things when I do math, so it helps to like have time for everyone to finish their work.*

Abigail also noted the relationship between the norms and the development of understanding by saying that the norms and activities required students to help one another and that this was helpful because students didn't rush to simply "give away the answers without anyone understanding it." Ashley related the development of understanding to the norm “everyone contributes and no one takes over,” saying that by giving people the time to build their own understanding they weren’t following prescribed methods like, “Okay, guys, we're gonna do this. We're gonna do this. We're gonna do this.” Instead, “you had to let other people get their ideas and it was more interesting that way.” These norms set up an environment in which students could engage in tasks such that it was necessary and expected to help one another. This provided time to develop personal understanding and the unique multifaceted development of understanding made for more interesting and fun mathematics.

But students agreed that having to rely on each other for understanding was a shift and a significant challenge, not only because they had to wrestle with mathematical ideas together, but also because they had to trust each other. Ashley emphasized that this was a concern that if she asked one of her groupmates as compared to Mrs. Fry, there was a chance that they would teach her something wrong. London and Ava related this to the norm “only call the instructor for group questions” because if they had a partner that couldn’t explain a topic well and they were having a hard time understanding it then it wasn’t a group question but they still didn’t understand the mathematics. But students also largely conveyed that becoming an expert in an idea, helping one another, and building understanding together made them more confident.

Reality Check

It was evident that students did, at times, experience mathematical joy in this data and statistics unit and that the intentional designs of the tasks were evident to students as they engaged in them. But these designs certainly did not create a utopia of formal mathematics education in this class. In interviewing students in focus groups after engaging in the tasks we designed, students expressed persistent concerns and frustrations with math and with schooling. Some students expressed a sincere dislike of the rubric saying things like, “I kind of want to bring my wrath upon the person who made [the rubric]. It’s like a tumor. You have to deal with it and it’s just annoying. I just feel like more school to do.” They also noted that the Complex Instruction norms didn’t feel all that different from what they normal did in class because it just involved, “being a decent human” which they were always expected to do. Additionally, because of previous experiences in formal math learning some students expressed their pervasive dislike of mathematics no matter the context saying things like, “It wasn’t exciting, it was just doing math. I just don’t like math that much. I’m not a math person. Math isn’t my thing.”

You could also sense in talking with students or reading students’ responses that these designs were pushing back against some sociomathematical norms of this class. While this was an intentional move on mine and Mrs. Fry’s part, for students, that pushing back and resistance was uncomfortable and beyond the scope of what they wanted to be doing in school. Isaac and Ruqaiya both expressed that because this task design took more time they felt frustrated because they just wanted to “get it done” and get their grade. They also felt frustrated that they didn’t understand the purpose of this kind of task because if the goal was just to do well on state testing, and state testing was over, they wondered what the point was. Ruqaiya astutely noted, “if we don’t know why, how can we do the how?” Some students understandably, especially at the end of the year, didn’t feel motivated to up-end social norms when it meant more effort, more time, and complex renegotiation of social norms and practices that were well established.

Conclusion

Because the cases studies in the previous chapter revealed ways that students were satisfied and dissatisfied, designing for mathematical reciprocity supported both humanizing mathematics and critically resisting dehumanizing mathematics for these students. In this space, children were resisting individual competitive performance and the resulting hierarchy of mathematical engagement. It often left them anxious, self-critical, and demeaned when they fell short some perceived expectation or comparison. Therefore, designing for their mathematical joy looked like intentionally finding ways that students could experience mutuality and reciprocity, without hierarchical competition. Designing for students’ mathematical reciprocity as a form of resistance to potentially dehumanizing competitive hierarchical sociomathematical norms in this math classroom, led to increased mathematical joy throughout the class.

This was evident through my own feelings of mathematical joy, students’ positive animated engagement in mathematics in these tasks, and through the characteristics of engagement that emerged throughout these tasks that manifested students' multifarious mathematical well-being. By providing mathematical tasks that supported students' mathematical reciprocity, students engaged in mathematics not only with giggles and dancing, but also with genuine, respectful debate, vulnerability, challenges, surprise, and puzzling. This is not to say that every moment of this week of data and statistics learning was joyful for these children or that any moment of these lessons was joyful for every child. But generally, based on a wide variability of factors, design, and cultural influences based on each students' unique story, students had the opportunity to experience more mathematical joy.

Conclusions

Summary

This study sought to investigate how to design for students' mathematical joy in a formal secondary math classroom. To do so, I explored some potential characteristics of mathematical joy and conditions that could support students' joyful mathematics learning in the context of a 6th grade math classroom. Through a research-practice partnership with Mrs. Fry at Porter Academic Magnet Middle School, I learned from and with 21 11 and 12-year-old students about their mathematical joy. This was done through three stages of data collection over the course of five months, and three levels of analysis that built incrementally and iteratively to, over time, construct a deeper and more nuanced understanding of mathematical joy in this space.

In the first level of analysis, detailed in Chapter 4, I sought a baseline understanding of students' mathematical engagement and students' enjoyment in mathematics by asking:

1. What are the sociomathematical norms of this classroom?
2. What conditions support students' joy? What kinds of tasks and interactions do students enjoy?

Through qualitative open coding, I found that mathematical knowing and doing in this math classroom was product-oriented, competitive, compliant, fun, but also stressful. Students enjoyed learning in this space largely due to positive relationships and believing that mathematics was meaningful and useful, but also experienced overwhelming stress and anxiety. Students' primary focus for math engagement in this classroom was personal academic achievement which, because of the culture of the school, led to significant competition between students and hierarchical ordering of students. In part because of these pressures to perform competitively and because of a caring, trusting relationship with Mrs. Fry, students readily complied with social and academic expectations. This compliance was paralleled in students' knowing and doing mathematics as students often expected and preferred procedural, algorithmic mathematical tasks. As we reflected on how to design math lessons for these students to intentionally support their joy, we decided that we needed to better understand what mathematical joy might mean for these students' uniquely.

Therefore, the second phase of the study, detailed in Chapter 5, focused on a case study of four students, Star, Ruqaiya, Isaac, and Thomas, who each uniquely provided a window into the local particulars of how we could support mathematical joy in this math classroom. Mathematical joy here was defined as a positionality toward mathematics that *animates engagement* and leads to feelings of satisfaction and purpose, born from mental, physical, relational, societal, and intellectual well-being. Because of the noticeable lack of joy in this space, I looked for both evidence of students' *animated engagement* indicating their apparent satisfaction, as well as students' *animated disengagement* indicating their dissatisfaction. Analytically, I looked for affect across small group videos of students engaging in math over the course of a week of geometry lessons. I then made sense of these affective episodes through students' stories. This revealed what conditions might have supported students' unique affective evidence of satisfaction or dissatisfaction in their mathematical interactions in this moment. This pointed me to potential facets of well-being that were met or unmet for them in this mathematics classroom, and thus what conditions might support these students' joy. This process sought to answer:

1. What are some potential characteristics of students' affective mathematical engagement? What does affect look like, sound like, feel like for these four students?
2. What conditions were potentially supporting or hindering students' joyful mathematical engagement?

Months of observations of the affective engagement of Star, Ruqaiya, Isaac, and Thomas taught me about students' conscious or subconscious desire for mathematical reciprocity: a mutual, benevolent sharing of mathematical ideas and resources in this math classroom. This desire for mathematical reciprocity spoke to students' aversion to sociomathematical norms of competition and the resulting hierarchies of student engagement. Therefore, we focused our efforts of humanizing mathematics to meet an apparent common desire for mathematical reciprocity and attended to students' relational and intellectual well-being in this classroom space.

Therefore, in the third phase of the study, detailed in Chapter 6, Mrs. Fry and I designed a unit about data and statistics that foregrounded the humanizing pedagogies of Complex Instruction which necessitated students' interactions with mathematical reciprocity. By looking at all students' interactions and engagement with mathematics across this unit and drawing from the wealth of data collected and analysis conducted up to this point, I asked:

1) What were *some* characteristics of these students' joyful mathematical engagement?
    a. What might mathematical joy look like, sound like, feel like, and how does it look different for different students in this space?
    b. When does joy occur during mathematical engagement and are there other affective states that often accompany joy?
    c. How do students describe joyful mathematical engagement?
2) In what ways did designing for responsive resistance support students' joyful mathematical engagement?

Post-coding abductive analysis demonstrated students' joyful mathematical engagement in various ways and to various degrees throughout the data and statistics unit. Not every student appeared to engage in mathematics joyfully for the entirety of the data and statistics unit. But this analysis demonstrated that students' animated engagement in these math tasks manifested some students' multifaceted mathematical well-being through their humor, dancing, debate, vulnerability, challenges, surprise, and puzzling.

Discussion

As stated, this work began with a starting point that mathematical is born from the coordination of mental, physical, relational, societal, and intellectual well-being. This definition was broad enough to capture the potential complexities and breadth of joy in this space and made clear that to support joy we must attend to the entangled aspects of our well-being. Our design work focused on designing for students' relational and intellectual well-being through mathematical reciprocity. But investigating these various dimensions of students' fulfilled or unfilled well-being required an intermingling of various procedural and analytic methodologies that, in coordination with one another, sought to attend to and collapse different influences on students' affect.

The literature review summarized some of the research in math education about the systemic societal norms and practices that have dehumanized students to various degrees and in various ways through traditional, formal math learning spaces. Then, in Chapter 4, open qualitative coding revealed themes about mathematical engagement and students' enjoyment in this math class that demonstrated the translation of some of these broader systemic norms in practice. By investigating the norms and practices of this classroom space, Chapter 4 demonstrated how some of these societal dehumanizing norms potentially came to bear on this classroom space. For example, the Literature Review highlighted how knowing and doing mathematics is often portrayed as performative and competitive, themes which also emerged in students' perceptions of mathematics engagement in this classroom. In Chapter 5, students demonstrated some of the ways that these aspects of classroom norms impacted the mathematical interactions of four specific students, Star, Ruqaiya, Thomas, and Isaac. Ethnographic case studies detailed in Chapter 5 showed subtle intricacies of students' diversity of reactions and affect that demonstrated not just an aversion to forms of hierarchical competition but leaning into moments that pushed back against this hierarchy. Examples of this push back could be seen when Thomas quietly said "Thank you" to Isaac after receiving help in Rally Coach, when Star cheered on her friends and groupmates when they were called on to solve an area problem, or when Ruqaiya giggled as she collected various kinds of solutions to an exploration of areas of parallelograms.

In future work, I hope to attend more carefully and intentionally to the broader systemic norms and practices that influence students' experiences uniquely based on their complex, multidimensional, racial, gendered identities. The current analysis relied on interactional cues, and subsequent designs primarily focused on students' relational and intellectual well-being. This admittedly disregarded potential impact of students' fulfilled or unfulfilled societal well-being that was influenced by broader aspects of structural inequities and expectations. By not attending to these influences on children's affective mathematical engagement, this work missed opportunities in analysis and design to attend to some of the dimensions of students' well-being such as their unique, embodied physical well-being in relation to their societal well-being.

Mrs. Fry and I took the generalized understanding of students' aversion to relational and intellectual dehumanization to design lessons that intentionally resisted dehumanizing competition and hierarchy by necessitating mathematical reciprocity. Chapter 6, then investigated the potential impact of these designs on students' engagement.

Limitations

Conceptually, this work is built on the work of humanizing mathematics and Black Feminist Theory, and thus must acknowledge and resist deep systemic oppressions experienced by Black and brown students at the hands of formal systems of education. By taking up these works as inspiration for design I hope to honor the work of the academics, poets, philosophers, feminists, and thinkers, especially those who are Black women, who have laid the foundation for experiencing joy by resisting dehumanization. This empirical study worked in a context with primarily white students, but intentionally learned from and with children of diverse ethnic, cultural and academic backgrounds to better understand their unique dehumanization and their unique joy in this context. But the critical foundation and inspiration of this work did not yet carry through to the analytic methodologies because of the focus on using multimodal interaction analysis to closely attend to affect. Expanding this work in the future will involve taking up more critical analytic techniques. It is my hope, that in continuing to do this work, research-practice partnerships can be born and develop into new opportunities for learning about mathematical joy from and with more diverse groups of students.

This empirical study is narrowly focused on the affective experiences of 21 students in a single math classroom. The goal of this study was to investigate local particulars of the social phenomenon of mathematical joy in the context of this 6th grade math classroom. This work does not, therefore, provide a list of dimensions of designing for mathematical joy (for example: mathematical reciprocity) nor a set of characteristics that define what joy looks like when engaging in mathematics (for example: joking, debating, dancing, or helping). Here, rather than generalizing to implications about designing for mathematical joy for all students, I remain within the scope of this project for designing for these students' unique mathematical joy but making some implications for future research and application.

Implications

Through open coding and case studies in combination with multimodal interaction analysis, I had the opportunity to represent minute and seemingly insignificant interactional affective moments with as much richness as possible through students' voice, gaze, bodies, and expressions. This mingling of methodologies, while limited and incomplete (lacking in more critical analytic lenses), I hope, paints of story of the complexity of mathematical joy in this learning space. For example, Ruqaiya found significant joy in collaborating with her peers and collecting many different solutions to mathematical puzzles and enjoyed pushing boundaries of mathematical procedures, but she also experienced significant anxiety and hindered joy in mathematical testing and performance because of the expectations of this high-achieving school as well as pressure from her parent, both immigrants from Bangladesh. It was necessary to make sense of Ruqaiya's interactions and affective reactions, described in detail through multimodal interaction analysis, through case study descriptions of her broader story. In the future I hope to continue this work to connect her story to the broader social and cultural narratives that she described having an impact on her joy in this space based on her multi-faceted identity.

I came into this work with preconceptions of what mathematical joy might be, that resistance could be the secret to joy in the face of oppressive systems, and that humanizing mathematics could somehow support students' mathematical joy (as outlined in the literature review and in conceptual framework). What is to be generalized from this study is that humanizing mathematics is indeed critical to designing for students' mathematical joy, but humanizing mathematics is only possible if students are given the space to be heard. Mrs. Fry, in her longing for her students' joy (even while working within an oppressive system) made space for her students to be heard through various forms of reformed pedagogical moves. For example, she provided opportunities for discussion, social relationships, meaning making, and exploration using curriculum like Illustrative Mathematics and Desmos as well as tasks like Estimation 180 and Which One Doesn't Belong. While students didn't always experience joy in these activities, it was only because of this condition in which we worked that it was possible for me and Mrs. Fry to carefully listen to students' reactions, their discussions, their affect. I expect that conducting this research in a traditional, silent, worksheet-based, desks-in-rows math class would have made this impossible. In listening and responding to student's reactions, we were able co-design humanizing mathematical learning opportunities for them and as a result witness a piece of mathematical joy. Thus, if teachers want to begin the work of humanizing mathematics for supporting students' joy they must, unsurprisingly, implement reformed mathematics educational practices that allow for students' to be heard both mathematically and affectively. It's only then that you can even begin to see multiple facets of students' fulfilled or unfulfilled well-being.

After having the opportunity to listen to children, it would be unrealistic and unsatisfying to say that teachers must then simply design for their students' well-being. The concept of supporting students' mathematical joy by designing for students' well-being is too broad to be practically applicable to any single classroom situation. There is such a wide range of opportunities to attend to well-being in math classrooms as evident from the humanizing mathematics conceptual framework. In this classroom, children demonstrated an aversion to individual competitive performance and the resulting hierarchy of mathematical engagement. They seemed to desire mathematical opportunities in which they could be mutual and social mathematical resources for one another. Therefore, supporting students' joy, for these children, looked like intentionally designing mathematical tasks and activities that supported mutuality and reciprocity, lessening opportunities for competition and comparison, and therefore designing for students increased relational and intellectual well-being. Supporting joy in any specific classroom necessitates paying close attention to how students express their complex human desires for their own well-being in their classroom and the ways they are seeking to satisfy their full humanity through and with mathematics. Therefore, the take-away here is *not* a design framework for mathematical joy but a refocusing on the centrality of attending to children: their stories, their pain, their relationships, their joy. By centering the design process on listening to children, caring for them, and finding opportunities to resist some of the ways that they are uniquely dehumanized in math classrooms we will make progress toward designing for their well-being and, thus, their joy in mathematics.

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