Exploring the Development of Children's Ordinality Knowledge

By

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Dissertation

Submitted to the Faculty of the

Graduate School of Vanderbilt University

in partial fulfillment of the requirements

for the degree of

DOCTOR OF PHILOSOPHY

in

Psychology

May 12, 2023

Nashville, Tennessee

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For my dad, who never pressured me to do anything except follow my dreams

For my mom, whose endless love and support are the two biggest reasons I’m where I am today

For my sister, who is always there for me and always help me see the positive in everything

ACKNOWLEDGEMENTS

First, I need to thank my family. My parents and sister have always been my biggest supporters, and this accomplishment would not have been possible without their constant love, support, and encouragement. I can't begin to tell you how big a factor knowing you are always there for me has played in helping me on this journey. I would not be where I am today without you. Love you.

Next, I need to thank my amazing advisor Dr. Bethany Rittle-Johnson. I am forever grateful to you for opening your doors to me, for always challenging and pushing me, and most of all, for believing in me. It's a great feeling knowing I will always have you in my corner. I also need to thank my other wonderful mentors and committee members, Dr. Lisa Fazio, Dr. Marcia Barnes, Dr. Eric Wilkey, and Dr. Gavin Price. It's been a pleasure getting to know all of you, and I cannot thank you enough for all of your support and guidance throughout the years. Additionally, I'd like to thank Granty, Gerry, and Gareth for always finding time for me, for helping me see the bigger picture, and for all the encouraging words and guidance.

I would also like to thank Sara, Kenzy, Heidi, Chen, Roger, and Ismail for all their amazing work and dedication to this project. If not for their hard work, this project would not have been completed. I would also like to thank my fellow graduate students and members of the Children's Learning Lab for their support and for providing such a great environment. Finally, I need to thank all my friends for always being there for me.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

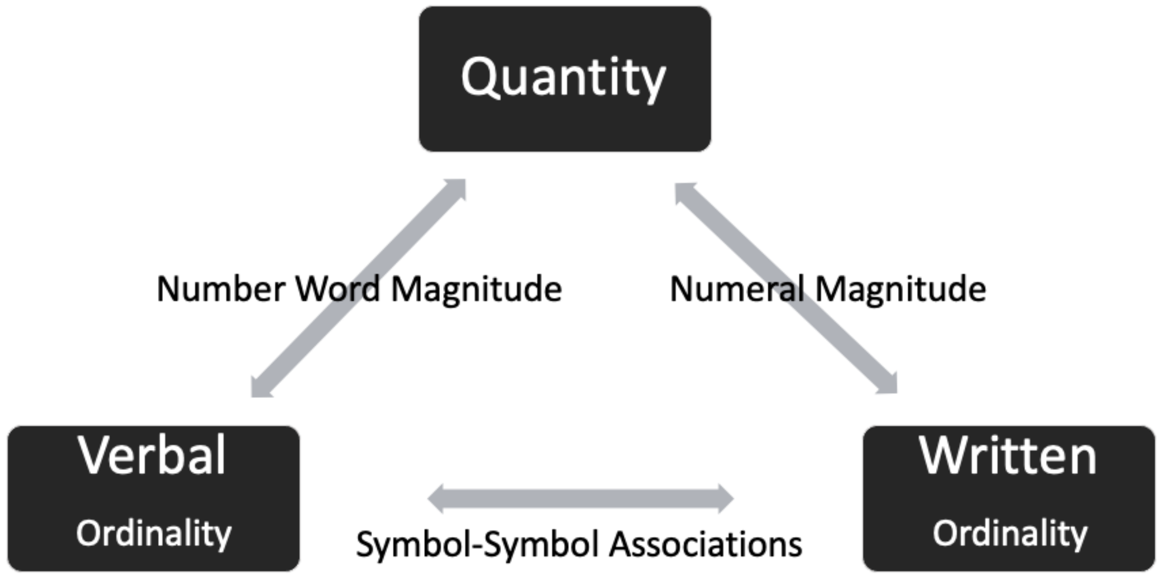
Exploring the Development of Children's Ordinality Knowledge

Numbers are everywhere. Whether telling the time, paying for groceries, or checking if your favorite sports team won last night, an understanding of numbers is critical to our success. Beyond these basic everyday interactions, number skills also play an important role in our academic achievement. Prekindergarten math skills have been found to be the single best predictor of later overall school achievement (Duncan et al., 2007). More specifically, numerical knowledge assessed at 4 years of age predicted mathematical knowledge at 15 years of age, after controlling for IQ, SES, reading comprehension, and working memory (Watts et al., 2014). Furthermore, mathematical achievement during the high school years is related to college degree attainment (Murnane et al., 1995), job quality and salary (Rivera-batiz, 1992), and choices when making health care decisions (Reyna et al., 2009). Thus, it is critical we understand how children develop their early number skills as they lay the foundation for later academic achievement and subsequent life success.

Put forth in Dehaene's (1992) triple-code model, humans have three codes by which we represent numbers (see Figure 1). First is the analog magnitude representation which involves our ability to approximate quantities. Second is the auditory verbal form which allows us to precisely represent numerical information through verbal representations (i.e., number words). Third is the visual number form which also allows for the precise representation of numerical information through written symbols (e.g., Arabic numerals). Our abilities as humans to work with numerical information relies on our understandings and integrations of these three forms of numerical codes, which solidify as children develop their early number skills.

Figure 1

Dehaene's Triple-Code Model



Note. Ordinality is within the Verbal and Written boxes because by definition it does not require a link to quantity, unlike number word and numeral magnitude knowledge, which does require a link to quantity.

Research on children's early number skill development has identified two understandings that are critical for children to develop. Magnitude knowledge for the quantities associated with number words and numerals is one major type of early number knowledge children must develop. When developing magnitude knowledge, children use their knowledge of the magnitude associated with verbal number words as a scaffold to develop their understanding of the magnitude associated with written Arabic numerals (e.g., Reynvoet & Sasanguie, 2016). Children's understanding of numerals may involve associating numerals with children's already developed understanding of number words, and that symbol-symbol associations may be a mechanism children rely on when developing their early written numeral skills.

A second critical early number skill children must develop is their understanding of the order of numbers, i.e., ordinality knowledge (Lyons et al., 2014). Ordinality knowledge does not require a link to quantity, as children can likely learn the order of some number sequences by rote (e.g., adjacent numeral sequences such as 4 5 6). Further, mature ordinality knowledge requires going beyond just understanding the order of adjacent number words and numerals to knowledge of non-adjacent number words (e.g., “five” is after “two”, but before “eight”) and numerals (e.g., 5 is after 2, but before 8; see Table 1). To our knowledge, ordinality has only been assessed in relation to numerals, not number words.

Table 1

Overview of focal early number skills

Number Skill	Definition	Example task	Begins to develop
Magnitude Knowledge	Knowledge for the quantities associated with number words and numerals.	Number Word Comparison: stating which of two or more number words is the largest. Numeral Comparison: selecting which of 2 or more numerals is the largest.	Number words: 4-years-old Numerals: 5-years-old
Ordinality Knowledge	Knowledge of the ordered number word and numeral sequence.	Order judgment: Are the numerals/number words 3-5-7/“three-five-seven” in correct order? Missing numeral task: What numeral is missing from this sequence? 1 3 4 Smallest to biggest numeral ordinality task: Pointing to numerals in order from smallest to biggest.	Number words: yet to be explored Numerals: 5-years-old
Shallow Ordinality	Only recognizing sequences of adjacent number words and numerals as in correct order.	Are the numerals/number words 2, 3, 4 in correct order?	Numerals: 5-years-old
Deeper Ordinality	Recognizing both adjacent and non-adjacent number word	Are the numerals/number words 3, 5, 7 in correct order?	Numerals: 6-years-old

	and numeral sequences as in correct order.

Note. Presented in order of acquisition (earliest at top).

1.1 Numeral Ordinality Knowledge

Children's numeral ordinality knowledge is related to their arithmetic achievement. An early cross-sectional study recruited a sample of Dutch students in grades 1 through 6 and explored associations between these students' arithmetic and basic numerical abilities, including children's numeral ordinality knowledge. Children's numeral ordinality knowledge consistently increased from grades 1 through 6 in its concurrent predictive power of students' arithmetic ability, and by the end of sixth grade, numeral ordinality knowledge was the strongest concurrent predictor of arithmetic ability – stronger than numeral magnitude knowledge (Lyons et al., 2014). Similarly, second-grade students' numeral ordinality knowledge concurrently predicted their numeral addition performance (Xu & LeFevre, 2021), as well as their overall numeral arithmetic ability (Sasanguie & Vos, 2018).

Taken together, these results suggest that an understanding of numeral ordinality is important for arithmetic achievement. However, despite the growing literature indicating associations between numeral ordinality knowledge and arithmetic, less is known about how children's numeral ordinality knowledge develops.

1.1.1 *Developing Numeral Ordinality Knowledge*

There are two commonly used tasks to assess children's numeral ordinality knowledge. First is the order judgment task, where children are presented with a sequence of adjacent (e.g., 3 4 5) , non-adjacent (e.g., 3 5 7), or out-of-order numerals (e.g., 3 5 4), and asked to judge if the numerals are in correct order (e.g., "You need to decide, as quickly as possible, whether or not the numbers are in the correct order from smallest to biggest"; Gilmore & Batchelor, 2021;

Lyons et al., 2014). Second is a numeral ordering task (i.e., smallest to biggest numeral ordinality task), where children are presented with three numerals (e.g., 3 4 5; 1 7 4), and asked to point to the numerals from smallest to biggest (Xu & LeFevre, 2021, 2016). In contrast to the numeral order judgment task, which is only a recognition task, in the smallest to biggest numeral ordinality task a participant must generate a response and not simply indicate yes or no. Although no past study has used both numeral ordinality tasks, I anticipate that performance might be lower on the smallest to biggest numeral ordinality task because it is a production task.

Recent research has begun to explore how children develop their understanding of numeral ordinality. Two related studies, that both used the numeral order judgment task, have revealed that children pass through phases of numeral ordinality understanding (Gilmore & Batchelor, 2021; Hutchison et al., 2022). As early as 5-years of age, children correctly identified adjacent sequences; however, it was not until 6-years of age that children demonstrated an understanding of non-adjacent sequences (Gilmore & Batchelor, 2021; Hutchison et al., 2022). Given that adjacent sequences directly correspond to the count sequence, only a shallow strategy of matching the numeral sequence to the count sequence is necessary to be successful on these trials. It is reasoned that children who fail to correctly recognize non-adjacent sequences do so because they are relying on a shallow strategy of matching the numeral sequence to the count-list. Thus, these children who have not yet developed this understanding of non-adjacent sequences can be said to possess a shallower understanding of ordinality (see Table 1).

On the other hand, children who correctly recognize non-adjacent sequences as in order can be said to possess a deeper understanding of ordinality (see Table 1). To be successful on these trials, children cannot rely on a shallow strategy of direct matching to the count-list.

Rather, children must possess a stronger understanding of the numeral sequence to recognize that just because a numeral is missing, does not mean the sequence is out of order.

Interestingly, across both studies, a bimodal response distribution was found when examining children's knowledge of non-adjacent numeral sequences. Hutchison et al. (2022) identified that it was between the end of kindergarten and end of first grade that this bimodal distribution appears, where a group of children began to correctly identify non-adjacent sequences as in-order. Using a slightly older sample, Gilmore & Batchelor (2021) identified that this bimodal distribution was present in 6- to 8-year-olds. Taken together, these findings indicate that beginning around 6 years of age and continuing through 8-years of age, the majority of the children in both studies fell into one of two groups, those who did or did not identify non-adjacent sequences (e.g., 2-4-6) as in correct ascending order. This pattern of results suggests that some conceptual shift is taking place that allows children to recognize non-adjacent numeral sequences as in order, and transitions these children from a shallow to deeper understanding of numeral ordinality.

To further support the notion that a conceptual shift occurs to transition children to recognizing non-adjacent sequences as in order, Hutchison et al. (2022) examined changes in the interrelations among adjacent, non-adjacent, and out-of-order trial types; and did so separately for the two groups of children who did or did not recognize non-adjacent sequences as in order. They found that for children who successfully recognized non-adjacent sequences as in order at the end of first grade, the correlations with accuracy on the other trial types (i.e., adjacent and out-of-order) was near-zero or negative at the beginning and end of kindergarten, but positive at the end of first grade. On the other hand, for children who did not recognize non-adjacent sequences as in order at the end of first grade, the correlations with accuracy on the other trial types remained negative or near-zero across the three time points. This sudden shift in the relations between performance on the non-adjacent trials and all other trial types for children who recognized non-adjacent sequences as in order, but not for the group of children who did not, provides further support to the notion that some conceptual shift occurred in this group of children. Hutchison et al. (2022) found that improvements in children's non-adjacent order judgments at the end of first grade was predicted by children's numeral magnitude knowledge at the end of kindergarten; suggesting children's numeral magnitude knowledge may play a role in the conceptual shift and development of deeper numeral ordinality knowledge.

Finally, an interesting, albeit initial, finding, is that an understanding of non-adjacent numeral sequences appears to be related to greater arithmetic ability, more so than an understanding of only adjacent numeral sequences. Despite similar performance on adjacent trials and no age differences, children who correctly recognized non-adjacent numeral sequences as in order had higher arithmetic scores than their peers who did not correctly recognize non-adjacent numeral sequences (Gilmore & Batchelor, 2021). Furthermore, In Lyons et al.'s (2014) study showing children's numeral ordinality knowledge becomes the greatest predictor of arithmetic ability by the end of grade school, numeral ordinality was assessed using both adjacent and non-adjacent trials, suggesting that students with higher scores were successful with non-adjacent as well as adjacent trials.

1.1.2 *Predictors of Numeral Ordinality Knowledge*

Xu and LeFevre (2021) explored potential knowledge that could contribute to children's developing numeral ordinality knowledge. They administered numeral magnitude comparison, missing numeral, smallest to biggest numeral ordinality, and numeral addition tasks to a sample of Grade 1 (*M* = 78 months) and Grade 2 (*M* = 90 months) middle-class Canadian students.

Tasks were administered twice with roughly 4-months between sessions. This study, as well as a study previously conducted by the same authors, are the only two studies to date that have used the smallest to biggest numeral ordinality task as opposed to the numeral order judgment task, and the task included both adjacent and non-adjacent sequences (Xu & LeFevre, 2016). On the missing numeral task, children stated the missing numeral in a sequence of adjacent numerals (e.g., 4 5 _ 7), and thus measures children's ordinality knowledge for adjacent numeral sequences.

Children's performance on the smallest to biggest numeral oridnality task significantly improved from grades 1 to 2. In both grades, children's numeral magnitude comparison performance was concurrently correlated with their smallest to biggest numeral oridnality task performance. However, it was only in grade 2, when children's scores on the smallest to biggest numeral ordinality task improved, that children's missing numeral task performance was concurrently related to their smallest to biggest numeral oridnality task performance. This raises the question of the role knowledge for adjacent numeral sequences has in completing the smallest to biggest numeral ordinality task. Children could be performing differently on adjacent versus non-adjacent sequences, making it important to explore if there are differences in difficulty between these types of sequences.

Overall, children in early elementary school possess varying levels of numeral ordinality knowledge. There may be a shift in understanding that transitions children from recognizing only adjacent sequences as in order to recognizing both adjacent and non-adjacent sequences as in order, but it is unknown if there is a similar shift when producing ordered sequences. Further, children's numeral magnitude knowledge is related to individual differences in numeral ordinality knowledge and may help support the development of this knowledge.

1.1 Current Study

Although the current state of the ordinality literature has provided some interesting insights into children's ordinality knowledge, I identified three gaps I aimed to explore in this study. Previous research has found that children's deeper numeral ordinality knowledge begins to develop in the first grade and continues through the first few years of formal schooling (i.e., until at least third grade), so I recruited a sample of kindergarten through third graders for this study.

First, I examined the development of children's number word ordinality knowledge. Surprisingly, I have not identified any past research exploring children's number word ordinality development, or the association between number word and numeral ordinality knowledge. However, one activity that is reflective of number word ordinality that children are encouraged to do is skip count (Frank, 1989). Skip counting is counting forward by a number other than one; for example, skip counting by two's ("two, four, six, eight, ten"). Although there is limited research on the frequency and interval's children skip count by, it can be reasoned that it is more common for children to skip count by 2's than by 3's, making it a possibility that children develop an understanding of familiar non-adjacent sequences that differ by 2 before they develop an understanding of non-adjacent sequences that differ by 3. An even stronger test of number word ordinality understanding might involve non-adjacent sequences that have unequal intervals between numerals (e.g., "two, four, seven"). I broadened our understanding of the development of children's number word ordinality knowledge by including adjacent and non-adjacent sequences that differ by 2, 3, or unequal intervals to explore if finer distinctions could be made when tracking the development of children's number word ordinality knowledge. I predicted that adjacent number word sequences would be easiest, and that sequences would get progressively harder for number word sequences that differ by 2, 3 and unequal intervals, respectively.

Second, I aimed to further our understanding into the learning trajectory associated with numeral ordinality knowledge. Previous work has not yet explored if there are finer distinctions for non-adjacent sequences, including equal versus unequal interval sequences. I predicted that adjacent numeral sequences would be easiest, and that sequences would get progressively harder for numeral sequences that differ by 2, 3 and unequal intervals (e.g., 2 4 7), respectively. Additionally, the two tasks used to assess numeral ordinality knowledge (order judgment and smallest to biggest numeral ordinality tasks) have not been compared, so I used both tasks. I predicted that the smallest to biggest numeral ordinality task would be more difficult because children had to produce the order, opposed to just recognizing order in the order judgment task, and chance performance was lower on this task (17% vs 50%).

Finally, I further explored predictors that may be associated with children's numeral ordinality knowledge. With previous research finding an association between children's numeral magnitude and numeral ordinality knowledge, I aimed to replicate this finding, as well as explore if children's number word magnitude knowledge was related to their numeral ordinality knowledge (Hutchison et al., 2022; Xu & LeFevre, 2021). Furthermore, with research suggesting children may use symbol-symbol mapping as a mechanism to help develop their written numeral skills, it could be that children are relying on their number word ordinality knowledge to help develop their numeral ordinality knowledge (Reynvoet & Sasanguie, 2016). Therefore, I also explored if children's number word ordinality knowledge was related to their numeral ordinality knowledge. I predicted that children's numeral magnitude and number word ordinality knowledge would be unique predictors of children's numeral ordinality knowledge, but that number word magnitude knowledge would not be a unique predictor.

CHAPTER 2

Method

2.1 Participants

This study was conducted virtually, so researchers met with participants on Zoom. Participants were recruited through a departmental database and one local private school. A total of 134 families consented to participate. However, 21 participants did not attend their session, 15 participants had too much parent involvement (e.g., parent told their child what a number was or gave additional unwarranted instructions on the ordinality tasks, such as "even if you skip numbers they can still be in order"), 3 were unable to properly load our experiment on their device, and one demonstrated dissenting behavior. These 40 participants were dropped so our sample with usable data consisted of 94 children (43 Female, 51 Male) ranging from 5.20 to 8.92 years of age (M = 6.89, SD = 0.95). Parents identified their children, with 82% identifying their child as White, 6% as Bi-racial, 5% as Asian, 4% as Black, 1% as Italian, and 1% chose not to respond. Four percent of participants had mothers who attended some college, 29% had mothers with a Bachelor's degree, 40% had mothers with a Master's degree, and 27% had mothers with a Doctorate. Ninety percent of the sample spoke only English in the home, while the remaining 10% spoke a language other than, or in addition to, English.

Some children were unable to complete all tasks. Two children did not complete the number word ordinality task because of dissenting behavior. Sixteen children did not complete the numeral ordinality tasks because they were using devices that were incompatible with our experiment software. See Table 2 for demographics for the subset of children with available data for the different tasks.

Table 2

Demographics for Full Sample and Samples with Available Data by Task

	N	Age	Maternal Education Level*
Full Sample	94 (43 Female)	6.89 (0.95)	5.85 (0.96)
Completed Number Word Ordinality task	92 (42 Female)	6.92 (0.94)	5.86 (0.97)
Completed Numeral Ordinality tasks	78 (36 Female)	6.96 (0.96)	5.88 (0.95)
Completed All Tasks	71 (33 Female)	7.01 (0.95)	5.90 (0.96)

Note: *Mother's highest level of education was coded as a single variable with 7 levels: 1 (less than high school), 2 (high school graduate), 3 (some college), 4 (2 year degree), 5 (4 year degree), 6 (Master's degree), and 7 (Doctorate). No mothers were in the categories of less than high school, high school graduate, or 2 year degree.

2.2 Measures

2.2.1 *Numeral Order Judgment Task*

The numeral order judgment task was based off Lyons et al. (2014) and Gilmore and Batchelor (2021). Children were shown triplets of single digits (e.g., 4 5 6) on their laptop screen, and asked "You need to decide, as quickly as possible, whether or not the numbers are in the correct order from smallest to biggest." Worth noting is that a different study using this task instructed children to "decide whether the three numbers are in correct order (from left to right)" (Hutchison et al., 2022). I decided to use "from smallest to biggest" to keep the instructions as similar to the smallest to biggest numeral ordinality task as possible.

Table 3

Overview of the tasks

Task	Trial Type	Example	Number of Trials
	Adjacent Sequences	1 2 3; one two three	4

Order Judgment Tasks (Numeral and Number Word)	Non-adjacent sequences that differ by 2	2 4 6; two four six	4
	Non-adjacent sequences that differ by 3	3 6 9; three six nine	4
	Non-adjacent unequal interval	2 4 7; two four seven	4
	Out-of-order distractors	2 7 4; two seven four	16
Smallest to Biggest Numeral Ordinality Task	Adjacent sequences	Click on the numerals 2 3 1 from smallest to biggest.	4
	Non-adjacent sequences that differ by 2	Click on the numerals 2 6 4 from smallest to biggest.	4
	Non-adjacent sequences that differ by 3	Click on the numerals 6 3 9 from smallest to biggest.	4
	Non-adjacent unequal interval	Click on the numerals 4 2 7 from smallest to biggest.	4
Magnitude Comparisons (Numeral and Number Word)	NA	2 | 5; Two | Five	26

Children were shown a total of 32 trials, with most trials matching those used in previous studies with the addition of non-adjacent unequal interval trials (Lyons et al., 2014; Gilmore & Batchelor, 2021; Hutchison et al., 2022). There were 5 trial types, as outlined in Table 3. Children received one point for every correct response. Out-of-order trials were made up of numerals used in each of the other sequence types and were included as distractor trials (so “no” was the appropriate answer half of the time), but these trials were not included in scoring.

Each trial began with the presentation of a fixation cross for 500 ms, followed by the presentation of a numeral sequence. Order of trial presentation was randomly generated for each participant through our experiment running software (Shevchenko, 2022). The sequence remained on screen until the participant clicked a green checkmark if they thought the numerals were in order, or a red “x” if they did not think the numerals were in order. There were two practice trials (1 2 3 and 1 3 2) with feedback to familiarize children to the task, as did Hutchison et al. (2022) and Lyons et al. (2014).

2.2.2 *Number Word Order Judgment Task*

The verbal number word order judgment task was the same as the numeral order judgment task, except that children were asked to judge if spoken number words were in correct order from smallest to biggest. The experimenter said a sequence of three number words, while the participant simultaneously saw the three number words written on the screen. The directions remained the same and children were asked to provide a verbal “yes/no” response. The same practice trials with feedback were given. Children received one point for every correct response.

2.2.3 *Smallest to Biggest Numeral Ordinality Task*

The smallest to biggest numeral ordinality task was based off Xu and LeFevre (2016; 2021) and asked children to indicate the order of three single-digit numerals from smallest to biggest. Unlike the numeral order judgment task, this was a production task. The directions were to “click on the numbers on screen in order from smallest to biggest”. The test trials were always out-of-order, and the same 16 out-of-order sequences used in the numeral order judgment task were used.

Three practice trials (1 3 2, 3 1 2, 2 1 3) with feedback were given. If a child struggled to understand the task instructions, additional guidance was given during the feedback trials. The additional guidance was to ask “Which number is the smallest? Which number comes after [insert first number]? Which number comes last in this sequence?” Additional guidance was needed often, partly due to the technological demands of clicking with their computer’s mousepad. Children received one point for every correct response.

2.2.4 *Numeral Comparison*

The numeral comparison task was adapted from Xu and LeFevre (2021) and used the same 26 trials. Because the task was moved to an online format, the response changed from touching the larger numeral on a tablet to clicking on the larger numeral using their mousepad. For each trial, children were presented with a pair of single-digit numbers (e.g., 4 7) on the screen and asked to "click on the number that is more". Children who failed to respond after 3 seconds were presented with the next trial. Half of the trials had a small distance between the digits (i.e., the difference ranged from 1 to 3), and the other half had a large distance (i.e., the difference ranged from 4 to 7). Children received one point for every correct response. Performance on this task was near ceiling (*M* = 25.75, *SD* = 0.59), so it was not a useful individual difference measure.

2.2.5 *Number Word Comparison*

The number word comparison task mirrored the numeral comparison task and used the same trials. The experimenter said to the child, ''I am going to say two numbers and you need to tell me which number is more". Children also saw the two number words on screen. Children received one point for every correct response. Performance on this task was near ceiling (*M* = 25.55, *SD* = 0.70), so it was not a useful individual difference measure.

2.2.6 *Number Word Reading*

This task was used as a control variable to assess whether children were able to read the number words when completing the number word magnitude and ordinality tasks. Children were shown the number words "one" through "nine" and asked to read and say the number word out loud. The number words were presented one at a time, in a random order, and remained on screen until the child either read the word or said they did not know the word. Children received one point for every word that was read correctly.

2.2.7 *Forward Digit Span*

The Forward Digit Span task from the Wechsler Intelligence Scale for Children, Fourth Edition was used as a control measure for short-term memory (Wechsler, 2003). The experimenter said "I'm going to say some numbers. Listen carefully, I can only say them one time. When I stop, you say them back to me in the same order. Just say what I say." The child then repeated the sequence. Sequences start with two numbers and went to nine numbers, with two trials per span. Testing ended when the child was wrong on both trials within a span. One point was awarded for each sequence children correctly repeated back to the experimenter.

2.2.8 *Demographics*

Parents of participating children were asked for basic demographic information about themselves and their child. This included the mothers' highest level of education, child's gender, racial and ethnic identity, and what language(s) were spoken in the home. For research exploring early childhood, mothers' education level has been found to accurately reflect the socioeconomic status of the family (Cheadle, 2008; Haveman & Wolfe, 1995; Reardon, 2011)

2.3 Procedure

The study was conducted virtually via one synchronous Zoom session, and sessions lasted around 30 minutes. The researcher worked with the participant with a parent or guardian present. The child completed seven tasks, all via Open Lab (Shevchenko, 2022). The two number word tasks were administered together, and the three numeral tasks were administered together. Presentation of the tasks was counterbalanced, such that participants completed either the number word tasks or numeral tasks first, and the order that the tasks were administered within each grouping was randomized. The final two tasks children completed were the number word reading and forward digit span tasks. As a thank you, the child was sent a "Junior Researcher" participation certificate, and the parent was sent documents containing tips on how to incorporate more number activities into the home environment and everyday activities.

2.4 Data Analysis

To examine the development of children's number word and numeral ordinality knowledge, item response theory (IRT) analyses were used to examine participants' ordinality knowledge (i.e., person scores) and item difficulty on the same scale. Unlike total scores, IRT considers the items a child is correct on, giving more credit when a child is correct on more challenging items. Therefore, IRT person scores can be said to be a more accurate representation of a child's ability level (Embretson & Reise, 2000). Person scores and item difficulty estimates were generated using a Rasch model, with a Laplace approximation and empirical Bayesian prediction method, appropriate for sample sizes of at least 50 with at least 10 items (Cho & Rabe-Hesketh, 2011). An assumption of Rasch models is unidimensionality of the construct being measured. To assess unidimensionality, first, a parallel analysis to examine how many factors may be underlying each construct was conducted. Parallel analysis is generally the most accurate method for determining the number of factors to extract in factor analysis (Horn, 1965; Humphreys & Montanelli Jr, 1975; Velicer et al., 2000; Zwick & Velicer, 1986). Parallel analysis is a Monte-Carlo-based simulation method that compares observed eigenvalues with those obtained from uncorrelated normal variables. A dimension is retained if an eigenvalue from the observed data is larger than the corresponding value from the random data. Because the current data contained binary responses, parallel analyses with tetrachoric correlations were conducted (Cho et al., 2009).

Then, an exploratory factor analysis (EFA) to identify how items clustered to each factor was conducted. If our data in each analysis was unidimensional, there should have been one dominant factor that items loaded to. Using tetrachoric correlations for binary responses, a series of EFAs were conducted, extracting one to three factors. Fit indices were compared across one- to three-factor models. Guidelines used to assess the goodness of model fit included a root-mean-square error of approximation index (RMSEA; Stieger & Lind, 1980) of < .06, a root-mean-square residual (RMSR) of < .08, and Tucker–Lewis index (TLI; Tucker & Lewis, 1973) > .95 (Hu & Bentler, 1999).

To address our third goal, a hierarchical regression model was used to test predictors of children's numeral ordinality knowledge. Due to near ceiling performance on the number word and numeral magnitude tasks, these tasks were dropped from our analyses. Thus, number word ordinality knowledge was the only variable of interest. Our design and hypotheses were preregistered and can be found at the following link: https://osf.io/w32fe.

CHAPTER 3

Results

3.1 Exploring the Learning Trajectory of Children's Number Word Ordinality Knowledge

3.1.1 *Descriptive Statistics*

Children performed best on adjacent trials. However, accuracy on the 3 non-adjacent trial types was fairly similar and there was no clear distinction in performance (see Table 4). Interestingly, number word order judgment total scores were positively correlated with age (r = 0.28, $p < .05$; see Table 5). Cronbach's alpha for the number word order judgment task was high, $\alpha = .92$.

3.1.2 *Item Response Theory Analysis*

The parallel analysis suggested two factors may be underlying this data (see Appendix B). Goodness of fit was evaluated for the one-, two-, and three-factor models according to the model-data fit indices described above (see Table 6). Goodness of fit was similar for all three models, however RMSEA and TIL values did not meet our criterion cutoff values of < 0.06, and > 0.95, respectively. Consistent with a unidimensional approach, almost all number word ordinality items loaded highly onto a single factor. Given that goodness-of-fit indices suggested that a one factor model was comparable to two- and three-factor models, and the limited number of test items that loaded uniquely on a second factor, a one-factor model provided an acceptable fit to our number word ordinality data, suggesting the data was unidimensional. See Figure 2 for the Wright Map which provides an overview of the distribution of participants' number word ordinality knowledge, and item difficulty levels, on the same scale.

Table 4

Mean Total Scores and Range of Scores on the Number Word Order Judgment Task, Numeral Order Judgment Task, and Smallest to Biggest Numeral Ordinality Task

	Number Word Order Judgment Task (n = 92)		Numeral Order Judgment Task (n = 80)		Smallest to Biggest Task (n = 84)	
	Mean Total Score (SD)	Range	Mean Total Score (SD)	Range	Mean Total Score (SD)	Range
All Trials	9.38/16 (5.72)	2 – 16	10.23/16 (5.91)	1 – 16	14.42/16 (2.21)	2 – 16
Adjacent Sequences	3.83/4 (0.46)	2 – 4	3.66/4 (0.65)	1 – 4	3.29/4 (0.90)	0 – 4
Non-Adjacent Sequences	5.55/12 (5.62)	0 – 12	6.56/12 (5.59)	0 – 12	11.13/12 (1.62)	2 – 12
Non-Adjacent Sequences Differ by 2	1.93/4 (1.93)	0 – 4	2.19/4 (1.92)	0 – 4	3.70/4 (0.69)	1 – 4
Non-Adjacent Sequences Differ by 3	1.83/4 (1.90)	0 – 4	2.14/4 (1.87)	0 – 4	3.69/4 (0.64)	1 – 4
Non-Adjacent Unequal Sequences	1.78/4 (1.86)	0 – 4	2.24/4 (1.89)	0 – 4	3.74/4 (0.62)	0 – 4

Table 5

Correlation matrix using total scores

	M (SD)	1	2	3	4	5	6	7	8
1. Numeral Ordinality: Order Judgment Total Score	10.25 (5.94)	1	0.26*	0.98***	0.75***	-0.04	-0.07	-	-
2. Numeral Ordinality: Smallest to Biggest Total Score	14.90 (1.41)	0.31**	1	0.46***	0.21	-0.18	-0.18	-	-
3. Numeral Ordinality: Combined Total Score	25.15 (6.51)	0.98***	0.5***	1	0.74***	-0.07	-0.11	-	-
4. Number Word Ordinality: Order Judgment Total Score	9.76 (5.71)	0.76***	0.24*	0.74***	1	-0.09	0.03	-	-
5. Forward Digit Span Total Score	8.34 (1.62)	0.1	-0.1	0.07	0.03	1	0.11	-	-
6. Number Word Reading Total Score	7.92 (2.74)	0.07	-0.1	0.05	0.16	0.28*	1	-	-
7. Child Age (Years)	7.01 (0.95)	0.32**	0.15	0.32**	0.28*	0.41***	0.48***	1	-
8. Maternal Education Level	5.90 (0.96)	0.16	0.17	0.18	-0.02	-0.01	-0.14	-0.03	1

*p < .05, **p < .01, ***p< .001

Note. Based on 71 participants who had data for all tasks. Correlations were extremely similar using IRT person scores instead of total scores (see Appendix A). Upper diagonal represents partial correlations controlling for age and maternal education level.

Table 6

EFA Results

Results	Fit Index		
	1-factor	2-factor	3-factor
Number Word Ordinality (n = 16)			
RMSEA	0.16 [0.14, 0.18]	0.15 [0.13, 0.17]	0.16 [0.14, 0.19]
RMSR	0.05	0.03	0.02
TLI	0.86	0.87	0.85
Numeral Ordinality (n = 32)			
RMSEA	0.10 [0.09, 0.11]	0.08 [0.07, 0.09]	0.08 [0.06, 0.09]
RMSR	0.13	0.07	0.06
TLI	0.78	0.85	0.86
Number Word and Numeral Ordinality (n = 48)			
RMSEA	0.20 [0.20, 0.21]	0.18 [0.18, 0.19]	0.18 [0.18, 0.19]
RMSR	0.1	0.08	0.07
TLI	0.30	0.44	0.42

Note. Guidelines used to assess the goodness of model fit included a root-mean-square error of approximation index (RMSEA) of < .06, a root-mean-square residual (RMSR) of < .08, and Tucker–Lewis index (TLI) > .95. Values in brackets show 90% confidence interval for RMSEA. EFA = exploratory factor analysis.

Figure 2

Wright Map showing distribution of participants' number word ordinality knowledge and number word ordinality item difficulties, on the same scale

Frequency of Participants' Knowledge (greater knowledge at top)	Estimate	Items (hardest at top)			
	5				
xxxxxxxxxxxxxxxxxxxxxxxxx					
	4				
	3				
xxxxxxxxxx		three, six, eight	two, five, eight_T1	four, six, nine	
		two, five, eight_T2	one, three, five	one, three, six	
	2				
xxxxxxx		two, five, seven	three, five, seven	one, four, seven	three, six, nine
		four, six, eight			
	1				
xx		two, four, six			
	0				
x					
	-1				
xx					
	-2				
xx					
	-3				
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx					
		four, five, six	seven, eight, nine		
	-4				
		two, three, four			
xxxxxxx		five, six, seven			
	-5				
x					
	-6				

Item Types
Non-Adjacent Unequal
Non-Adjacent that differ by 3
Non-Adjacent that differ by 2
Adjacent

Note. Participants at top have greatest knowledge and items at top are most difficult. T1 indicates Time 1 and T2 indicates T2 for the sequence that was used twice.

The Wright Map revealed a pattern of item difficulties where no clear distinction could be made between non-adjacent sequences that differed by 2, 3, or unequal intervals, with the exception that the trial "two-four-six" was easier than the other non-adjacent sequences (see Appendix E for item difficulty estimates). On the other hand, adjacent sequences were separate from non-adjacent sequences, and were much easier. Looking at the left side of the Wright Map in Figure 2, a bimodal distribution was found for children's number word ordinality knowledge. Children were likely to fall into one of two clusters. The first cluster was children who only succeeded on adjacent order judgments. The second cluster was children who also succeeded on most to all the non-adjacent order judgments.

3.2 Exploring the Learning Trajectory of Children's Numeral Ordinality Knowledge

3.2.1 *Descriptive Statistics*

See Table 4 for an overview of participant's' performance on the two numeral ordinality tasks, broken down by task type and trial type. Surprisingly, children performed better on the smallest to biggest numeral ordinality task compared to the numeral order judgment task, $t\ (70) = 6.91$, $p < .001$. On the numeral order judgment task, children performed best on adjacent compared to non-adjacent sequence judgments. The opposite pattern was seen on the smallest to biggest numeral ordinality task. Children performed worst on ordering adjacent compared to non-adjacent sequences. Finally, when looking at performance for the three types of non-adjacent sequences in each task separately, there was no clear distinction in mean total scores (see Table 4).

Finally, an interesting finding was that significant correlations were found between numeral order judgment and smallest to biggest numeral ordinality total scores ($r = 0.26$, $p < .05$), and age and numeral order judgment total scores ($r = 0.32$, $p < .01$). However, age was not significantly correlated with smallest to biggest numeral ordinality total scores (see Table 5). Cronbach's alpha for the two numeral ordinality tasks combined was high, α = .92. Cronbach's alpha for the numeral order judgment task, and smallest to biggest numeral ordinality task were α = .97, and α = .76, respectively.

3.2.2 *Item Response Theory Analysis*

The parallel analysis suggested three factors may be underlying this data (see Appendix C). Goodness of fit was evaluated for the one-, two-, and three-factor models according to the model-data fit indices described above (see Table 6). Goodness of fit was better for the two- and three-factor models compared to a one-factor model, however RMSEA and TIL values did not meet our criterion cutoff values of < 0.06, and > 0.95, respectively. When examining factor loadings, the majority of items loaded highly onto two factors. Numeral order judgment items loaded highly onto one factor, while smallest to biggest numeral ordinality task items loaded highly onto a second factor. Given that goodness-of-fit indices were comparable for two- and three-factor models, and because the majority of items loaded onto two factors, a two-factor model provided an acceptable fit to our numeral ordinality data. This suggests that our numeral oridnality data was not unidimensional. However, research has indicated that person scores and item difficulty estimates are robust to violations of unidimensionality within a unidimensional IRT approach, as was done in the current study (e.g., Rasch model; Crişan et al., 2017; Luecht & Miller, 1992).

The Wright Map (see Figure 3) indicated that for the numeral order judgment task, judging adjacent sequences as in-order was easier than judging non-adjacent sequences as in-order, but there was no clear distinction in the different types of non-adjacent sequence order judgments (see Appendix F for item difficulty estimates). For the smallest to biggest numeral ordinality task, the opposite pattern occurred. Ordering adjacent sequences was more challenging than ordering non-adjacent sequences. When comparing the tasks, non-adjacent order judgment trials were harder and separate from adjacent smallest to biggest, adjacent numeral order judgments, and non-adjacent smallest to biggest trials, which grouped together. However, adjacent smallest to biggest items were more challenging than both adjacent numeral order judgments and non-adjacent smallest to biggest items.

Lastly, participant's numeral ordinality knowledge produced a trimodal distribution. Children were likely to fall into one of three groups. The first group was children who did not succeed on any trials, except for a few children who succeeded on the easiest smallest to biggest numeral ordinality items. The second group was children who succeeded on all smallest to biggest numeral ordinality trials and adjacent order judgments; a third group also succeeded on non-adjacent order judgments.

Figure 3

Wright Map showing distribution of participants' numeral ordinality knowledge and numeral ordinality item difficulties, on the same scale

Frequency of Participants (greater knowledge at top)	Estimates	Items (hardest at top)			
	3				
xxxxxxxxxxxxxxx					
	2.5				
		2,5,8_T1	3,5,7	3,6,9	2,5,7
		1,4,7	1,3,5	2,4,6	
	2				
		4,6,8	1,3,6	3,6,8	
xxxxxx		2,5,8_T2	4,6,9		
	1.5				
xxxxxxxxxx					
	1				
xxxxxxx					
	0.5				
xxxx					
		6,7,5_SB			
	0				
		3,5,4_SB			
x		5,4,6_SB			
	-0.5				
x		8,6,7_SB	7,8,9	3,9,6_SB	
		2,3,4	6,8,4_SB	2,7,5_SB	
	-1				
x		4,5,6	5,6,7	4,1,7_SB	
		7,3,5_SB	5,8,2_SB	3,6,1_SB	
	-1.5				
xxxx		4,2,6_SB	5,9,7_SB	6,4,9_SB	8,3,6_SB
xxxxxxx		7,1,4_SB			
	-2				
xxxxxxxxxxxxxxxxx					
	-2.5				
xxxx					
	-3				
x					
	-3.5				

Item Type

Non-Adjacent Order Judgment
Non-Adjacent Smallest to Biggest
Adjacent Order Judgment
Adjacent Smallest to Biggest

Note. Participants at top have greatest knowledge and items at top are most difficult. T1 indicates Time 1 and T2 indicates T2 for the sequence that was used twice.

3.3 Exploratory Analysis: Comparing Number Word and Numeral Ordinality Knowledge

3.3.1 *Descriptive Statistics*

This was an exploratory analysis that explored the trajectory of children's overall ordinality knowledge, and compared performance on trials from all three ordinality measures that allowed for a more direct comparison of the item difficulties. Given that in previous analyses no clear distinctions were found between the non-adjacent sequences of different intervals, the three different non-adjacent sequences were collapsed into one category. See Table 7 for an overview of participant's' performance on the three ordinality tasks, broken down by task type and trial type.

When comparing children's performance on the tasks, a paired samples *t*-test found no significant differences in participants numeral order judgment total scores and number word order judgment total scores t (70) = 1.02, p = .31. Additionally, no difference was found for children's non-adjacent number word judgment total score and non-adjacent numeral judgment total score, t (70) = -1.24, p = 0.13. Cronbach's alpha for the three ordinality tasks combined was high, α = .96.

Table 7

Mean Total Scores for Analysis with All Items

	Mean Total Scores (SD)		
	Number Word Order Judgment	Numeral Order Judgment	Smallest to Biggest Numeral Ordinality
Overall	9.76/16 (5.71)	10.25/16 (5.94)	14.90/16 (1.41)
Adjacent Sequences	3.87/4 (0.38)	3.65/4 (0.68)	3.46/4 (0.71)
Non-Adjacent Sequences	5.88/12 (5.67)	6.61/12 (5.61)	11.44/12 (0.98)

Note. To be consistent with the IRT Analysis, the sample of 71 participants who had data on all three ordinality tasks was used to obtain mean total scores for this table.

3.3.2 *Item Response Theory Analysis*

The parallel analysis suggested six factors may be underlying the combined number word and numeral ordinality data (see Appendix D). To be consistent with our previous two analyses, and given this was an exploratory analysis, goodness of fit was evaluated for one-, two-, and three-factor models according to the model-data fit indices described above (see Table 6). Goodness of fit was better for the two- and three-factor models compared to a one-factor model, however RMSEA and TIL values did not meet our criterion cutoff values of < 0.06, and > 0.95, respectively. When examining factor loadings, the majority of items loaded highly onto three factors. Number word order judgment items, numeral order judgment items, and smallest to biggest numeral ordinality task items loaded highly to separate factors. Given that goodness-of-fit indices were comparable for two- and three-factor models, and because the majority of items loaded onto three factors, a three-factor model provided an acceptable fit to our combined number word and numeral ordinality data. This suggests that our combined number word and numeral ordinality data was not unidimensional.

Figure 4

Wright Map showing distribution of participants' overall ordinality knowledge and item difficulty estimates, on the same scale

Frequency of Participants' Knowledge (greater knowledge at top)	Estimates	Items (hardest at top)					
	4						
xxxxxxxx							
	3						
		three, six, eight	two, five, eight_T1	four, six, nine	one, three, six		
xxxxxxx		three, five, seven	three, six, nine	two, five, eight_T2	two, five, seven		
		one, three, five	one, four, seven				
		2,5,8_T1	3,6,9	2,5,7	four, six, eight	3,5,7	
	2						
xxxxxxx		1,3,5	1,4,7	1,3,6	two, four, six		
		2,4,6	4,6,8	3,6,8			
xxxxxxx		2,5,8_T2	4,6,9				
	1						
xx							
x							
xx							
	0						
xx							
xxx		6,7,5_SB					
xxxx							
	-1						
		3,5,4_SB	8,6,7_SB				
		7,8,9	5,4,6_SB				
		2,3,4	4,5,6	5,6,7			
		6,8,4_SB	2,7,5_SB				
	-2						
xxxx		7,3,5_SB	3,9,6_SB	5,8,2_SB			
xxxx		5,9,7_SB	4,1,7_SB	7,1,4_SB	3,6,1_SB	seven, eight, nine	
xxxxxxxxxxxxxx		4,2,6_SB	6,4,9_SB	8,3,6_SB	five, six, seven	four, five, six	two, three, four
	-3						
xxxxxx							
	-4						

Item Types		
Number Word Order Judgment	**Numeral Order Judgment**	**Smallest to Biggest**
Non-Adjacent	Non-Adjacent	Non-Adjacent
Adjacent	Adjacent	Adjacent

Note. Participants at top have greatest knowledge and items at top are most difficult.
T1 indicates Time 1 and T2 indicates T2 for the sequence that was used twice.

For both number word and numeral order judgments, adjacent sequences were easier than non-adjacent sequences (see Figure 4 for Wright Map; see Appendix G for item difficulty estimates). In contrast, for the smallest to biggest task numeral ordinality task, non-adjacent sequences were easier than adjacent sequences. The Wright Map also suggested that non-adjacent numeral order judgments were easier than non-adjacent number word order judgments; however, non-adjacent numeral and number word sequences were much more difficult than the other item types. Of the easier items, a clear progression was seen. Adjacent smallest to biggest items were most challenging, followed by adjacent numeral order judgments which were slightly easier. Then the easiest items were non-adjacent smallest to biggest and adjacent number word order judgments, with no clear distinction in difficulty between these two item types.

When examining the distribution of participant's overall ordinality knowledge, there was a wider spectrum of performance that did not fit neatly into different levels. Participant's knowledge was more broadly distrusted, opposed to the bi- and trimodal distributions observed for children's number word and numeral oridnality knowledge, respectively.

3.4 Examining Predictors of Children's Numeral Ordinality Knowledge

Analysis 4 explored predictors of children's numeral ordinality knowledge. A hierarchical regression model with children's numeral ordinality IRT person scores as the dependent variable was conducted (see Table 8 for regression overview).

Table 8

Hierarchical regression output using numeral and number word IRT person scores

	Step 1			Step 2		
Variable	Estimate	SE	*t*	Estimate	SE	*t*
Intercept	-6.26	1.79	-3.49**	-3.53	1.25	-2.83**
Child Age	0.89	0.27	3.24**	0.43	0.19	2.24*
Maternal Education Level	0.69	0.29	2.42*	0.69	0.19	3.56***
Forward Digit Span	-0.09	0.15	-0.59	-0.01	0.10	-0.08
Number Word Reading	-0.07	0.09	-0.80	-0.10	0.06	-1.67

Number Word Ordinality IRT Person Scores				0.40	0.04	8.92***
R^2	0.21					0.64
R^2_{adj}	0.16**					0.62***
ΔR^2_{adj}	0.16					0.46

Note. $N = 71$; $*p < .05$, $**p < .01$, $***p < .001$ Maternal education level was collapsed into three levels: 1. Bachelor's degree or less, 2. Master's degree, 3. Doctorate degree.

Step 1 included control variables, which accounted for 16% of the variability in children's numeral ordinality scores, F (4, 66) = 4.42 $p < .01$. Age and maternal education level were the only significant predictors (see Table 8). In step 2, children's number word ordinality IRT person scores was added. This model accounted for 62% of the variability in children's numeral ordinality scores, F (5, 65) = 23.67, $p < .001$. Thus, the addition of children's number word ordinality IRT person scores accounted for an additional 46% of the variability in children's numeral ordinality scores and was also a unique positive predictor of children's numeral ordinality knowledge. Age and maternal education remained significant predictors. Children's average combined total scores on the numeral ordinality tasks increased as maternal education increased from Bachelor's degree or less (M = 22.38/32, SD = 6.92), to Master's degree (M = 25.97/32, SD = 6.09), to Doctorate degree (M = 26.85/32, SD = 6.05).

Because previous research on children's ordinality knowledge has not used IRT before, the regression was replicated with children's total scores on the ordinality measures. The results were nearly identical to the analysis using the IRT person scores, except age was no longer a significant predictor in step 2 (see Table 9).

Table 9

Hierarchical regression output using numeral and number word accuracy scores.

	Step 1			Step 2		
Variable	Estimate	SE	*t*	Estimate	SE	*t*
Intercept	20.75	5.96	3.48***	9.56	4.27	2.24*
Child Age	2.77	0.91	3.05**	1.23	0.65	1.91
Maternal Education Level	2.05	0.95	2.16*	1.95	0.65	3.01**

Forward Digit Span	-0.22	0.49	-0.45	0.10	0.34	0.28
Number Word Reading	-0.22	0.30	-0.72	-0.35	0.21	-1.66
Number Word Ordinality IRT Person Scores				0.81	0.09	8.73***
R^2			0.19			0.63
R^2_{adj}			0.14**			0.60***
ΔR^2_{adj}			0.14			0.16

Note. N = 71; *p < .05, **p < .01, ***p< .001 Maternal education level was collapsed into three levels: 1. Bachelor's degree or less, 2. Master's degree, 3. Doctorate degree.

CHAPTER 4

Discussion

The current study contributes to the growing literature exploring children's ordinality knowledge. More specifically, I aimed to further our understanding into the learning trajectory of children's numeral and number word ordinality knowledge, as well as further explore predictors of children's numeral ordinality knowledge. In the following sections, I discuss how the findings from the current study relate to and expand upon what has previously been found in relation to children's ordinality knowledge, implications from our findings, and how the current findings setup future directions.

4. 1 Exploring the Development of Number Word Ordinality Knowledge

A novel contribution of the current study was to explore the development of children's number word ordinality knowledge. I expected that judging the order of adjacent sequences would be easier than judging the order of non-adjacent sequences given children's familiarity with the verbal count sequence and work exploring numeral ordinality knowledge (Gilmore & Batchelor, 2021; Hutchison et al., 2022).

Our results indicate three main takeaways. First, a bimodal distribution was found for children's number word ordinality knowledge. This suggests that a conceptual shift may occur within number word ordinality knowledge that transitions children from shallow to deeper knowledge. Second, adjacent sequences were easiest for children to identify as in order, suggesting that only being able to identify adjacent sequences as in order may be reflective of shallow number word ordinality knowledge. Third, there was no clear distinction in performance or difficulty on non-adjacent sequences that differ by 2, 3, or unequal intervals. The ability to recognize that these sequences are in order may indicate deeper number word ordinality knowledge.

The lack of distinctions between the non-adjacent sequences suggests that skip counting may not be playing as large of a role in the development of number word ordinality knowledge as hypothesized. However, an interesting observation from the Wright Map (see Figure 2) was that for non-adjacent sequences that differed by 2 it was the sequences that started with “two” and “four” that were easier, while sequences that started with “one” and “three” had difficulty levels similar to non-adjacent sequences that differ by 3 and unequal interval sequences. When skip counting, children almost always start from “zero” (e.g., children are rarely if ever asked to skip count three, five, seven), and if skip counting by two’s is more common, then children would have greater familiarity with sequences involving even numbers, and this is exactly what the Wright Map suggested (see Figure 2). Thus, teachers and parents could look to increase the frequency with which they skip count, and also the types of intervals they skip by, to potentially help improve children’s non-adjacent number word ordinality knowledge.

4.1.1 *Potential Mechanisms Underlying Number Word Order Judgments*

4.1.1.1 Magnitude Comparison Mechanism. Previous research on numeral ordinality knowledge has suggested children may be making successive magnitude comparisons when making order judgments (Hutchison et al., 2022; Xu & LeFevre, 2021). However, our results do not clearly suggest magnitude knowledge may be involved. Given the ceiling effect of our magnitude tasks, all children seemed to have high and similar magnitude knowledge. If accessing their magnitude knowledge is the main mechanism underlying order judgments, then I would expect to see similar performance on the different number word order judgments. However, there are large differences between non-adjacent and adjacent sequence judgments. In our magnitude comparison task, children did not have any time pressure, and reliable response times were not available because of difficulties obtaining data via our online experimental procedure. Past studies that found a relationship between magnitude and ordinality knowledge considered efficiency, not just accuracy, either using time pressure (e.g., two-minute limit) or response time data (Gilmore & Batchelor, 2021; Hutchison et al., 2022; Xu & LeFevre, 2021). Thus, perhaps a magnitude mechanism is still involved in number word order judgments, but it has more to do with the efficiency of accessing and comparing magnitudes.

4.1.1.2 Familiarity and Direct Recognition Mechanism. Another possible mechanism is one that relies on familiarity with the count sequence. Turconi et al., (2006) argued that given our familiarity and the high frequency with which we are shown adjacent numerals (e.g., 4 5), children may simply be directly accessing representations they have in their memory to make an order judgment. The high performance and ease of adjacent trials is easy to explain given the high exposure and frequency in which these number words are spoken together, making these sequences much more familiar to children. Children likely have excellent representations of adjacent sequences, making direct recognition easy. On the other hand, children are exposed to non-adjacent sequences less frequently, making them less familiar to children. To succeed on non-adjacent sequence judgments via familiarity and direct recognition, a child must have a deeper understanding of the count sequence, including a better grasp on more complex counting procedures like skip counting. This understanding may develop as children get older and have more experience working with the count sequence, a notion supported by the fact that there is a positive correlation between age and number word order judgment total scores.   Future work could look to include a measure of skip counting to explore this possibility of direct recognition and familiarity of the count sequence being one mechanism underlying number word order judgments. Stronger evidence would come from an experimental study that tested whether skip counting practice increased number word ordinality compared to a control condition.

4.2 Exploring the Development of Numeral Ordinality Knowledge

Children's numeral ordinality knowledge produced a trimodal distribution. In contrast to the bimodal distribution found when just the order judgment task has been used, this suggests that the inclusion of the smallest to biggest numeral ordinality task can provide additional information into the development and distribution of children's numeral ordinality knowledge (Hutchison et al., 2022; Gilmore & Batchelor, 2021).

In relation to the numeral order judgment task, our prediction that adjacent numeral sequences would be easier than non-adjacent sequences was supported and in line with past research (Gilmore & Batchelor, 2021; Hutchison et al., 2022). Furthermore, it is knowledge of adjacent sequences that may be reflective of shallow numeral ordinality knowledge. On the other hand, our results did not support our prediction that performance on non-adjacent sequences that differ by 2, 3, or unequal intervals would be distinct and increase in difficulty. These three trial types were of similar difficulty levels suggesting children come to the conclusion that sequences that skip numerals can still be in order all at once, no matter the interval between numerals (see Figure 3). However, these trials were associated with children who had higher numeral ordinality knowledge scores, suggesting it is an understanding of these trials that may be associated with deeper numeral ordinality knowledge.

When looking at the numeral smallest to biggest numeral ordinality task, our results did not support our hypothesis. A novel finding was the clear distinction that adjacent trials were harder than non-adjacent trials. Previous research using this task with first and second grade students did not explore total scores, but found no difference in processing time for adjacent and non-adjacent sequences (Xu & LeFevre, 2021). In addition, there was no clear distinction between non-adjacent sequences that differed by 2, 3 or unequal intervals, as these items had similar item difficulty levels.

When comparing the difficulty levels of the two tasks, once again I see a pattern of results contrary to my prediction. Specifically, when comparing non-adjacent sequences on the two tasks, it was judging the order of the non-adjacent sequences, opposed to producing the order of non-adjacent numerals, that was more challenging. Given the distinct differences in non-adjacent sequences between the two tasks, it suggests different task characteristics may be present in addition to ordinality knowledge. Differences in the potential mechanisms involved in these tasks could be impacting how children's ordinality knowledge is being assessed. Given that to our knowledge this was the first analysis to explore the difficulty of the different numeral ordinality tasks and items, a construct map was developed based off the Wright Map (see Figure 3) to reflect the construct of numeral ordinality knowledge (see Figure 5).

Figure 5

Numeral Ordinality Knowledge Construct Map

Deeper Numeral Ordinality Knowledge
- Strong representation of numeral sequence
- Very efficient accessing numeral magnitude

Non-Adjacent Order Judgments

Medium Numeral Ordinality Knowledge
- Improving representation of numeral sequence
- Improving efficiency accessing numeral magnitude

Adjacent Smallest to Biggest

Adjacent Order Judgments

Shallow Numeral Ordinality Knowledge
- Weak representation of numeral sequence
- Strong numeral magnitude representation but poor efficiency accessing

Non-Adjacent Smallest to Biggest

Note. On the left side of the arrow, greater knowledge is reflected at the top. On the right side of the arrow, harder items are at the top.

4.2.1 *Potential Mechanisms Underlying Numeral Ordinality Knowledge*

4.2.1.1 Serial Search Mechanism. Serial search involves children relating the numerals presented in the trial sequence to their stored representation of the numeral sequence (Jou, 1997, 2003; Lovelace & Snodgrass, 1971). For example, with sequence “3 5 4”, children will think to themselves “Which number do I come to first when working through the numeral sequence? Which number do I come to second?” Assuming a child has a strong representation of the numeral sequence, this search process will allow the child to properly judge, or produce, the order of numerals from smallest to biggest.

Unfortunately, no measures specifically aimed at assessing children’s representation for the numeral sequence were included. This makes it challenging to conclude the role serial search may be playing in children’s numeral ordinality knowledge. However, when examining item difficulties on the Wright Map (see Figure 3), it appears serial search may be involved in the smallest to biggest numeral ordinality task where adjacent and non-adjacent sequences are grouped together, with adjacent sequences being slightly more challenging. Serial search involves the same procedure no matter the numeral sequence given. If children are using this mechanism, it follows that performance would be similar for all sequence types. Additionally, serial search can explain why adjacent sequences are slightly more challenging. When working through the numeral sequence, children would be more likely to make errors when numerals are close together. This could lead to worse performance (i.e., more challenging) on adjacent sequences.

4.2.1.2 Magnitude Comparison Mechanism. A second potential mechanism children could rely on to judge or produce the order of numerals is successive magnitude comparisons (Hutchison et al., 2022; Xu & LeFevre, 2021). This is similar to serial search; however, it does not involve a full search process as children may be able to compare magnitudes of only the numerals provided in the trial sequence. Sticking with our example sequence "3 5 4", children would compare 3 to 5 and 4 and identify it is smallest, then compare 5 and 4 to determine 4 is smaller than 5. Previous research using accuracy on timed tasks, or response time, have found an association between children's numeral magnitude knowledge and numeral ordinality knowledge (Hutchison et al., 2022; Xu & LeFevre, 2021). However, Vogel et al., (2015) found no association between the distance effect for children's numeral magnitude and numeral order judgments. If children were relying on their numeral magnitude knowledge to make numeral order judgments, we would expect the distance effects between the two tasks to correlate. These findings indicate inconsistencies in the role magnitude knowledge may play in ordinality knowledge. However, these findings suggest that the association between numeral magnitude and ordinality knowledge may have to do with the efficiency in which children can access and process numeral magnitude representations. This conclusion is in line with Hutchison et al. (2022), who found that children's numeral magnitude comparison performance at the end of kindergarten predicted unique growth in performance on non-adjacent sequence judgments when completing the same numeral magnitude comparison and order judgment tasks as ours (some trials differed), but with a two-minute time limit.

In relation to numeral order judgments, because performance on our (untimed) numeral magnitude task was near ceiling, if participants were relying on their numeral magnitude knowledge, I would expect to see similar performance on all numeral order judgment trials since all participants demonstrated excellent numeral magnitude knowledge. Yet, despite high levels of numeral magnitude knowledge in our sample, large differences in difficulty between adjacent and non-adjacent sequences were still found, suggesting some other mechanism may be involved in numeral order judgments.

On the other hand, large differences in difficulty were not seen for adjacent and non-adjacent sequences on the smallest to biggest numeral ordinality task (see Figure 3). These items were grouped together on the Wright Map, suggesting children may be accessing their numeral magnitude knowledge to complete the smallest to biggest numeral ordinality task. Since our sample was close to ceiling on the magnitude task, if children are using their magnitude knowledge to help them complete a different task, it follows that items on that task would be of similar difficulty as children are relying on a mechanism in which they are proficient.

Further supporting the notion children may be accessing their numeral magnitude knowledge for the smallest to biggest numeral oridnality task is, that although grouped together, adjacent sequences were slightly more challenging than non-adjacent sequences trials (see Figure 3). When numerals are close together, there is more overlap in the magnitude representations associated with each number. This overlap has been found to result in longer processing times in magnitude comparisons when numerals are closer together (e.g., faster to respond that 5 is bigger than 3 than 5 is bigger than 4) and is termed the canonical distance effect (Dehaene, 1992). Children are thought to have a harder time deciding which numeral is larger given the similarity and overlap in magnitude representations. This overlap in activation can cause difficulty in processing numerals that are more similar in magnitude, and result in worse performance (i.e., more challenging) on trials where numerals are close together (i.e., adjacent trials).

The discrepancy in difficulty between adjacent and non-adjacent sequences within the two numeral ordinality tasks suggests that the demands of the tasks differ. One possibility is that the smallest to biggest numeral ordinality task is reflective of deeper numeral ordinality knowledge, and that the order judgment task has additional demands that mask this knowledge earlier on in children. Performance on the smallest to biggest numeral ordinality task may be possible at an earlier age due to children more intentionally producing the order of numerals based on magnitude knowledge. This production based off accessing magnitude knowledge may be easier for children than the mechanism that may be underlying order judgments, resulting in greater performance on the smallest to biggest numeral ordinality task, especially for non-adjacent sequences.

4.2.1.3 Familiarity and Direct Recognition Mechanism. Another potential mechanism underlying numeral ordinality knowledge involves a child's familiarity and knowledge of the numeral sequence (Gilmore & Batchelor, 2021; Hutchison et al., 2022). Although similar to serial search, direct recognition differs in that children are not searching through the numeral sequence one after the other to judge or produce order. Rather, they are able to represent the numeral sequence as a whole and conclude the order of the sequence as a singular unit.

Unfortunately, I do not have response time data, but given the longer processing time involved in serial search, one way to explore the difference between serial search and direct recognition could be to look at response time.

Relying on their familiarity of the numeral sequence, children may be able to properly judge, or produce, the order of numerals from smallest to biggest (Turconi et al., 2006). However, given that the numeral sequences presented in the smallest to biggest numeral ordinality task were all out of order (e.g., 3 5 4), it is unlikely that children are relying on a mechanism of familiarity and direct recognition to perform this task. If they were, then I would expect poor performance on this task. However, children performed very well on the smallest to biggest numeral ordinality task. Furthermore, the lack of a correlation between age and smallest to biggest total scores suggest some other mechanism, potentially serial search or magnitude comparison as discussed above, may be involved in completing the smallest to biggest numeral ordinality task. As children get older, they will likely have greater familiarity with the numeral sequence given greater exposure and experience working with numerals. Thus, if children were using familiarity and direct recognition as a mechanism on the smallest to biggest numeral ordinality task, I would expect age to correlate with performance.

A positive correlation was found between age and numeral order judgment total scores, suggesting familiarity and direct recognition may be involved in numeral order judgments. Specifically, older children who likely have stronger representations of the numeral sequence may be using this understanding to correctly identify non-adjacent sequences as in order, resulting in higher total scores. It is important to note that the correlation between age and numeral order judgment total scores could also be reflective of older children having better magnitude processing skills. However, with our magnitude tasks being near ceiling performance, the significant correlation between age and numeral order judgment scores points more to greater knowledge and familiarity of the numeral sequence being a mechanism involved in order judgments. Future research should include measures to assess if familiarity and direct recognition of the numeral sequence could be one mechanism underlying numeral order judgments.

The inconsistent correlations between age and the two numeral ordinality tasks further suggests the demands and mechanisms children rely on to complete the two tasks differ. While the smallest to biggest numeral ordinality task may involve children making magnitude comparisons, order judgments may rely more on a child's familiarity and knowledge of the numeral sequence, and their ability to directly recognize sequences as in-order based off greater exposure and familiarity with the numeral sequence. This is not to say that children may not involve their magnitude knowledge on order judgments, but that if they do, the more efficient magnitude processing that appears to be involved in those with deeper numeral ordinality knowledge is likely a result of greater familiarity with, and a stronger representation of, the numeral sequence. However, given that smallest to biggest numeral ordinality task items, which could rely on children's access to magnitude knowledge, are easier than non-adjacent order judgments and of similar difficulty to adjacent order judgments which are thought to be reflective of shallow numeral ordinality knowledge (see Figure 3), this suggests that familiarity and direct recognition may be the more prominent mechanism at play when examining numeral ordinality knowledge, and specifically, deeper numeral oridnality knowledge.

4.2.2 *Numeral Ordinality Knowledge Conclusions*

The discrepancy between conclusions regarding children's ordinality knowledge depending on the task used is similar to the debate that occurred in the cardinality literature when we first started to measure children's cardinality knowledge. Initial research exploring children's cardinality knowledge used the How-Many task, which asked children to count the number of items presented in a line, and then state how many items they counted. However, subsequent research revealed that the Give-N task, which asked a child to provide a certain number of items to a puppet, provided a better indication of a child's cardinality knowledge (Sarnecka & Carey, 2008). Given that research on ordinality is just emerging, perhaps the ordinality literature is dealing with a similar issue the cardinality literature dealt with; that different tasks are differentially measuring aspects of ordinality knowledge.

Our findings and assessment of the potential mechanisms underlying performance on the two tasks suggest that the numeral order judgment task may be the better indicator of numeral ordinality knowledge. By definition, numeral ordinality knowledge is our understanding of the numeral sequence. It does not require a link to magnitude knowledge, but this does not mean it cannot involve magnitude knowledge. However, if the goal of assessing numeral ordinality is to measure the quality of a child's representation of the numeral sequence, our findings suggest the numeral order judgment task may be a more appropriate measure of that representation, and a truer reflection of a child's level of ordinality knowledge.

4.3 Integrating Number Word and Numeral Ordinality Knowledge

Analyses three and four aimed to help us get a better sense of children's overall ordinality knowledge, and variables that may be associated with children's numeral ordinality knowledge. A novel contribution from the study was to explore the relationship between number word and numeral ordinality knowledge. In contrast to our prediction, children's number word ordinality knowledge was not greater than their numeral ordinality knowledge, and there was overlap in item difficulty for number word and numeral items (see Figure 4). This result suggests that children may not rely on their number word ordinality knowledge to help develop their numeral ordinality knowledge. Therefore symbol-symbol mapping may not be a mechanism involved in ordinality knowledge development. However, a strong association was still found between the two skills. This suggests that the acquisition of number word and numeral ordinality knowledge may occur together.

In addition to children's number word ordinality knowledge being a unique predictor of children's numeral ordinality knowledge, maternal education was a unique predictor in all four regression models (see Tables 8 and 9). Previous research has shown that more educated mothers engage in more number related activities in the home, resulting in greater symbolic number ability in their children; although, the highest level of maternal education in these studies was a Bachelor's degree (Muñez et al., 2021; Susperreguy et al., 2020). Therefore, it is interesting to still see an effect of maternal education in my highly educated sample where all but three mothers had at least a Bachelor's degree. Perhaps this is an indication that maternal education continues to play a factor beyond just a Bachelor's degree level. More educated mothers are more likely to create rich interactions with their children, although the highest level of maternal education in this study was also a Bachelor's degree (Vandermaas-Peeler et al., 2009). One possibility is that as maternal education increases, the richer the interactions are with their child. Mother's with more educational experience may have additional insights into how best to engage and promote their child's learning, resulting in greater symbolic number abilities. However, additional research would be needed to explore this notion. Ultimately, given the highly educated nature of my sample, it is likely that my findings may not generalize to children whose parents do not have at least a Bachelor's degree.

4.4 Limitations and Future Directions

Despite the promising initial findings from the current study, a few limitations limit the generalizability of the results. First, my sample consisted of children with highly educated mothers, with all but three mothers having at least a college degree, and 28% having a doctorate degree. More educated mothers likely have greater access to resources and more time to support their children's early number learning, which may be leading to greater symbolic number abilities for the children in my sample. Second, my sample was predominantly White. Future work should look to recruit a sample whose maternal education level, and racial and ethnic composition, are representative of the general population. Second, the nature of the study being online resulted in larger amounts of missing data. The study was conducted using an online experimental platform that was not compatible with all of our participants' devices, and the additional parental involvement required since the study was over Zoom sometimes resulted in the parent providing unwarranted help to their children on our tasks. Third, the lack of reaction time data or time pressure on our tasks meant I could not consider children's efficiency in processing. Finally, our numeral ordinality data, and number word and numeral ordinality combined data, appeared to be multidimensional. A Rasch model may not have been the most appropriate way to analyze these datasets and multidimensional approaches should be considered.

Future work could look to build on this study in a number of ways. First, a more sensitive or challenging measure of magnitude knowledge should be included. A timed task, or a magnitude task more comparable to the order judgment task could be used (e.g., find the largest of three numerals). Second, measures should be included to assess the potential mechanism of familiarity and direct recognition. For number words, a skip counting task could be given that includes both typical (e.g., skip counting starting from "zero") and atypical (e.g., skip counting starting from "one) sequences, and skipping by different intervals (e.g., by two's and three's). Similarly, next and previous number tasks, like the ones used in the Screener for Early Number Sense, could be given that ask children to state the number that comes two before or two after a given number word (Jordan et al., 2010). For numerals, similar to Sella et al.'s (2019) Direction, Order, and Space task, children could be asked to complete a number line that increases by intervals other than one (e.g., 2 4 _ 8) and that start with numerals other than 0 (e.g., 1 _ 7 10). Third, retrospective strategy reports could be implemented to help researchers understand how children are solving the ordinality tasks (e.g., Rittle-Johnson & Siegler, 1999). These reports could also help researchers understand the mechanisms separating children with shallow and deeper ordinality knowledge, and help provide insight into the conceptual shift that may be occurring. Finally, a training study using the smallest to biggest numeral ordinality task would be interesting to see if it improves children's deeper numeral ordinality knowledge. The structure of the smallest to biggest numeral ordinality task nicely allows for children to conclude that just because numerals are not adjacent, does not mean they are out of order. A positive correlation was found between total scores on the numeral order judgment and smallest to biggest numeral ordinality tasks suggesting there may be some overlap in knowledge to complete these tasks.

Conclusion

The current study helps further our understanding into the learning trajectory of children's ordinality knowledge. In particular, correctly judging non-adjacent sequences as in order was most difficult, and might be reflective of deeper ordinality knowledge. Furthermore, performance on the smallest to biggest numeral ordinality task appeared to be more reflective of shallow numeral ordinality knowledge, and overall performance was much better on this task compared to the numeral order judgment task. Finally, a strong association was found between children's number word and numeral ordinality knowledge, and children performed similarly on the two tasks, suggesting that children's level of proficiency with one may be a good indicator of their proficiency in the other. It is our hope that the findings from this study will help continue the push to understanding the development of children's ordinality knowledge, and ways to improve it in children who may be struggling with their number knowledge and mathematics achievement.

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Appendix A

Correlation Matrix Using IRT Person Scores

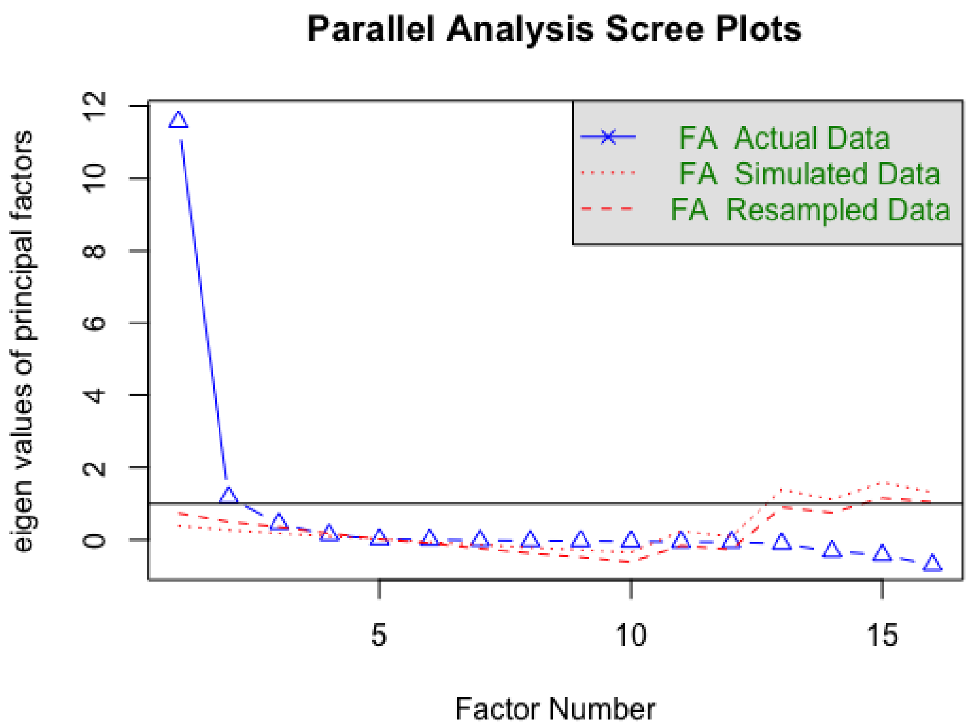
	M (SD)	1	2	3	4	5	6	7	8
1. Numeral Ordinality: Order Judgment IRT Person Score	-0.24 (3.54)	1	0.24*	0.96***	0.75***	-0.02	-0.03	-	-
2. Numeral Ordinality: Smallest to Biggest IRT Person Score	0.08 (0.78)	0.3*	1	0.47***	0.20	-0.18	-0.23	-	-
3. Numeral Ordinality: Order Judgment and Smallest to Biggest Combined Total Score	0.01 (1.98)	0.96***	0.51***	1	0.72***	-0.09	-0.12	-	-
4. Number Word Ordinality: Order Judgment Total Score	0.16 (3.47)	0.75***	0.23	0.73***	1	-0.08	0.04	-	-
5. Forward Digit Span Total Score	8.34 (1.62)	0.12	-0.1	0.06	0.05	1	0.11	-	-
6. Number Word Reading Total Score	7.92 (2.74)	0.1	-0.14	0.05	0.18	0.28*	1	-	-
7. Child Age (Years)	7.01 (0.95)	0.32**	0.15	0.34**	0.31**	0.41***	0.48***	1	-
8. Maternal Education Level	5.90 (0.96)	0.18	0.18	0.17	-0.03	-0.01	-0.14	-0.03	1

*p < .05, **p < .01, ***p< .001

Note. Upper diagonal represents partial correlations controlling for age and maternal education level.

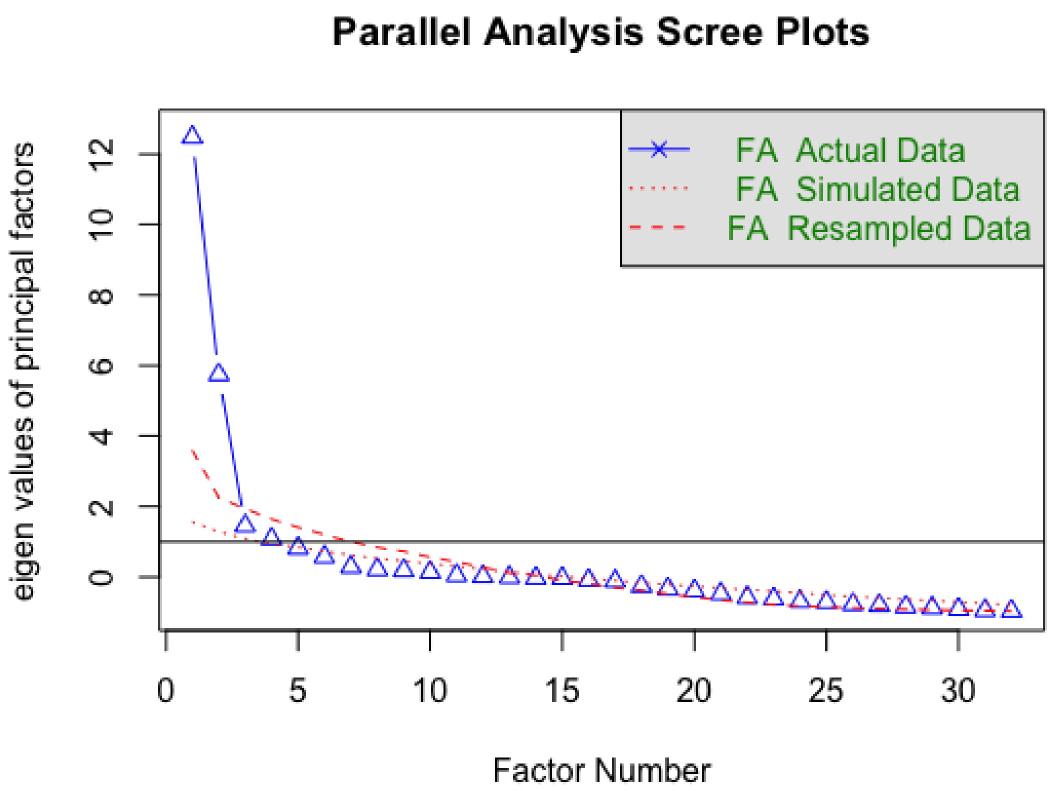
Appendix B

Scree Plot for Parallel Analysis of Number Word Ordinality Data



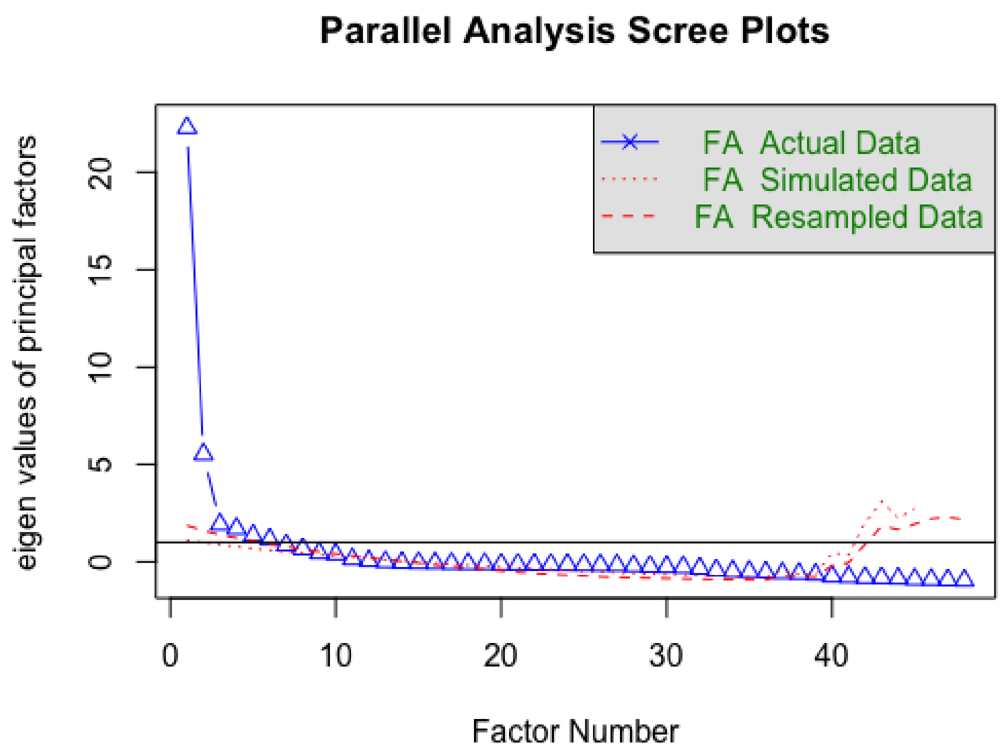
Appendix C

Scree Plot for Parallel Analysis of Numeral Ordinality Data



Appendix D

Scree Plot for Parallel Analysis of Number Word and Numeral Ordinality Data Combined



Appendix E

Item Difficulty Estimates and Item-Total Correlations from Number Word Ordinality IRT Analysis

Item	Sequence Type	Item Difficulty Estimates	SE	Item-Total Correlation
four, five, six	Adjacent	-3.93	0.65	0.07
seven, eight, nine	Adjacent	-3.93	0.65	0.14
five, six, seven	Adjacent	-4.84	0.79	0.10
two, three, four	Adjacent	-4.17	0.68	0.11
one, three, five	Non-adjacent 2	2.01	0.62	0.92
three, five, seven	Non-adjacent 2	1.57	0.62	0.96
four, six, eight	Non-adjacent 2	1.36	0.62	0.94
two, four, six	Non-adjacent 2	0.94	0.60	0.93
two, five, eight_T1	Non-adjacent 3	2.43	0.61	0.91
two, five, eight_T2	Non-adjacent 3	2.23	0.62	0.93
one, four, seven	Non-adjacent 3	1.57	0.62	0.98
three, six, nine	Non-adjacent 3	1.57	0.62	0.90
three, six, eight	Non-adjacent unequal	2.63	0.60	0.88
four, six, nine	Non-adjacent unequal	2.43	0.61	0.89
one, three, six	Non-adjacent unequal	2.01	0.62	0.87
two, five, seven	Non-adjacent unequal	1.79	0.62	0.93

Note. T1 = Trial 1, T2 = Trial 2. These values were obtained using the same sample of 92 participants used in the Number Word Ordinality Analysis (i.e., Goal 1) .

Appendix F

Item Difficulty Estimates and Item-Total Correlations from Numeral Ordinality IRT Analysis

Item	Task	Sequence Type	Item Difficulty Estimates	SE	Item-Total Correlation
7,8,9	Numeral Order Judgment	Adjacent	-0.77	0.43	0.33
2,3,4	Numeral Order Judgment	Adjacent	-0.93	0.45	0.27
4,5,6	Numeral Order Judgment	Adjacent	-1.10	0.47	0.25
5,6,7	Numeral Order Judgment	Adjacent	-1.10	0.47	0.13
3,5,7	Numeral Order Judgment	Non-adjacent 2	2.22	0.37	0.81
1,3,5	Numeral Order Judgment	Non-adjacent 2	2.03	0.37	0.86
2,4,6	Numeral Order Judgment	Non-adjacent 2	2.03	0.37	0.87
4,6,8	Numeral Order Judgment	Non-adjacent 2	1.93	0.37	0.89
2,5,8_T1	Numeral Order Judgment	Non-adjacent 3	2.42	0.37	0.84
3,6,9	Numeral Order Judgment	Non-adjacent 3	2.22	0.37	0.80
1,4,7	Numeral Order Judgment	Non-adjacent 3	2.12	0.37	0.86
2,5,8_T2	Numeral Order Judgment	Non-adjacent 3	1.83	0.36	0.89
2,5,7	Numeral Order Judgment	Non-adjacent Unequal	2.22	0.37	0.83
1,3,6	Numeral Order Judgment	Non-adjacent Unequal	1.93	0.37	0.82
3,6,8	Numeral Order Judgment	Non-adjacent Unequal	1.93	0.37	0.88
4,6,9	Numeral Order Judgment	Non-adjacent Unequal	1.74	0.36	0.92
6,7,5_SB	Smallest to Biggest Numeral Ordinality	Adjacent	0.20	0.38	0.16
3,5,4_SB	Smallest to Biggest Numeral Ordinality	Adjacent	-0.24	0.40	0.31
5,4,6_SB	Smallest to Biggest Numeral Ordinality	Adjacent	-0.49	0.41	0.18
8,6,7_SB	Smallest to Biggest Numeral Ordinality	Adjacent	-0.63	0.42	0.18
6,8,4_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent 2	-0.93	0.45	0.12
7,3,5_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent 2	-1.29	0.49	0.26
4,2,6_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent 2	-1.50	0.52	0.24

5,9,7_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent 2	-1.50	0.52	-0.04
3,9,6_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent 3	-0.77	0.43	0.37
4,1,7_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent 3	-1.10	0.47	-0.04
5,8,2_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent 3	-1.29	0.49	0.27
7,1,4_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent 3	-1.76	0.56	0.09
2,7,5_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent Unequal	-0.93	0.45	0.04
3,6,1_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent Unequal	-1.29	0.49	0.12
6,4,9_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent Unequal	-1.50	0.52	0.14
8,3,6_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent Unequal	-1.50	0.52	0.07

Note. T1 = Trial 1, T2 = Trial 2. These values were obtained using the same sample of 78 participants used in the Numeral Ordinality Analysis who had data for both numeral ordinality tasks (i.e., Goal 1).

Appendix G

Item Difficulty Estimates and Item-Total Correlations from Numeral Ordinality IRT Analysis

Item	Task	Sequence Type	Item Difficulty Estimate	SE	Item-Total Correlation
7,8,9	Numeral Order Judgment	Adjacent	-1.37	0.48	0.34
2,3,4	Numeral Order Judgment	Adjacent	-1.55	0.50	0.24
4,5,6	Numeral Order Judgment	Adjacent	-1.55	0.50	0.27
5,6,7	Numeral Order Judgment	Adjacent	-1.55	0.50	0.14
3,5,7	Numeral Order Judgment	Non-adjacent 2	2.04	0.43	0.83
1,3,5	Numeral Order Judgment	Non-adjacent 2	1.91	0.43	0.87
2,4,6	Numeral Order Judgment	Non-adjacent 2	1.78	0.43	0.86
4,6,8	Numeral Order Judgment	Non-adjacent 2	1.78	0.43	0.87
2,5,8_T1	Numeral Order Judgment	Non-adjacent 3	2.18	0.43	0.89
3,6,9	Numeral Order Judgment	Non-adjacent 3	2.18	0.43	0.79
1,4,7	Numeral Order Judgment	Non-adjacent 3	1.91	0.43	0.88
2,5,8_T2	Numeral Order Judgment	Non-adjacent 3	1.65	0.43	0.87
2,5,7	Numeral Order Judgment	Non-adjacent unequal	2.18	0.43	0.86
1,3,6	Numeral Order Judgment	Non-adjacent unequal	1.91	0.43	0.82
3,6,8	Numeral Order Judgment	Non-adjacent unequal	1.78	0.43	0.91
4,6,9	Numeral Order Judgment	Non-adjacent unequal	1.51	0.43	0.91
6,7,5_SB	Smallest to Biggest Numeral Ordinality	Adjacent	-0.35	0.43	0.18
3,5,4_SB	Smallest to Biggest Numeral Ordinality	Adjacent	-1.04	0.46	0.21
8,6,7_SB	Smallest to Biggest Numeral Ordinality	Adjacent	-1.20	0.47	0.15
5,4,6_SB	Smallest to Biggest Numeral Ordinality	Adjacent	-1.37	0.48	0.15
6,8,4_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent 2	-1.76	0.53	0.04
7,3,5_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent 2	-2.00	0.56	0.16

5,9,7_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent 2	-2.29	0.61	-0.08
4,2,6_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent 2	-2.64	0.68	0.25
3,9,6_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent 3	-2.00	0.56	0.31
5,8,2_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent 3	-2.00	0.56	0.22
4,1,7_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent 3	-2.29	0.61	-0.01
7,1,4_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent 3	-2.29	0.61	0.09
2,7,5_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent unequal	-1.76	0.53	0.04
3,6,1_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent unequal	-2.29	0.61	0.09
6,4,9_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent unequal	-2.64	0.68	0.01
8,3,6_SB	Smallest to Biggest Numeral Ordinality	Non-adjacent unequal	-2.64	0.68	0.01
seven, eight, nine	Number Word Order Judgment	Adjacent	-2.29	0.61	0.09
five, six, seven	Number Word Order Judgment	Adjacent	-2.64	0.68	0.09
four, five, six	Number Word Order Judgment	Adjacent	-2.64	0.68	0.01
two, three, four	Number Word Order Judgment	Adjacent	-2.64	0.68	0.01
three, five, seven	Number Word Order Judgment	Non-adjacent 2	2.45	0.43	0.87
one, three, five	Number Word Order Judgment	Non-adjacent 2	2.31	0.43	0.89
four, six, eight	Number Word Order Judgment	Non-adjacent 2	2.18	0.43	0.88
two, four, six	Number Word Order Judgment	Non-adjacent 2	1.91	0.43	0.87
two, five, eight_T1	Number Word Order Judgment	Non-adjacent 3	2.71	0.43	0.83
three, six, nine	Number Word Order Judgment	Non-adjacent 3	2.45	0.43	0.86
two, five, eight_T2	Number Word Order Judgment	Non-adjacent 3	2.45	0.43	0.87
one, four, seven	Number Word Order Judgment	Non-adjacent 3	2.31	0.43	0.89
three, six, eight	Number Word Order Judgment	Non-adjacent unequal	2.97	0.43	0.79
four, six, nine	Number Word Order Judgment	Non-adjacent unequal	2.71	0.43	0.83
one, three, six	Number Word Order Judgment	Non-adjacent unequal	2.71	0.43	0.83
two, five, seven	Number Word Order Judgment	Non-adjacent unequal	2.45	0.43	0.85

Note. T1 = Trial 1, T2 = Trial 2. These values were obtained using the same sample of 71 participants used in the exploratory analysis comparing number word and numeral ordinality knowledge.