THE LINGUISTIC AND COGNITIVE PREDICTORS OF PATTERNING IN PRE-KINDERGARTEN

by

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Abstract

Patterning is recognizing and creating patterns, which entails identifying and comprehending the underlying structure and order of a sequence of objects or events. Patterning is important for math development because these skills are helpful for children to improve their critical thinking, analyze data, and make predictions. In this study, linguistic (phonological awareness) and cognitive (visual-spatial working memory, relational reasoning, and attention) abilities were tested at the beginning of pre-K to see whether they predict patterning at the end of pre-K. A total of 511 children participated in this study. Results showed that age at the end of pre-K, visual-spatial working memory, relational reasoning, and patterning at the beginning of pre-K were significant and unique predictors of patterning at the end of pre-K. Findings are discussed with respect to theories of patterning development.

Keywords: Patterning, phonological awareness, working memory, relational reasoning, attention, pre-K, mathematics

Dedication

I am dedicating this thesis to people who have meant and continue to mean so much to me. First and foremost, to my parents Emine Önder and Mustafa Önder for raising me as an independent, self-determined woman who stands on her own two feet, and for being my first support system to turn to in every decision I make.

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TABLE OF CONTENTS

LIST OF TABLES

CHAPTER I

Introduction

Patterning is defined as the ability to recognize and create patterns, which entails identifying and comprehending the underlying structure and order of a sequence of objects or events (Miller et al., 2016; Rittle-Johnson et al., 2013, 2019; Schmerold et al., 2016; Wijns et al., 2017). Three types of patterns are spatial structure patterns, repeating patterns, and growing patterns (Papic & Mulligan, 2007; Papic, 2015). Spatial structure refers to consistent relationships between different geometrical shapes, like triangles, squares, blocks, arrays, and grids. The number, size, arrangement, and spacing of the individual shapes varies to create the pattern (Papic & Mulligan, 2007). Repeating patterns have a structure that is cyclic and can be created by using a smaller section of the pattern over and over again. A simple example is red-white-red-white (ABAB)... (Liljedahl, 2004, pp. 26–27). Growing patterning involves a series of elements that increase or decrease in a systematic manner, such as 1, 2, 3, 4, … (Papic et al., 2011).

In patterning studies, manipulatives and laminated cards have been used to assess early childhood patterning abilities. Manipulatives allow young children to touch, move, and enter concepts more concretely and physically without needing to rely on spoken language to respond. For example, children are asked to match the patterns using laminated cards with cards of different colors or shapes, to complete the missing piece of the pattern, or to choose the missing pattern from among four alternative answer cards (Rittle-Johnson et al., 2019; Schmerold et al., 2016; Wijns et al., 2019, 2021).

Relationship Between Patterning and Math Achievement

Early patterning skills, cognitive flexibility, and working memory are unique predictors of math achievement in kindergarten and first grade (Lüken et al., 2012, 2014, as cited in Schmerold et al., 2016). Patterning skills are thought to be important for cognitive development because they help children develop their problem-solving skills as they learn to identify and apply patterns to solve problems in different contexts (Miller et al., 2016; Rittle-Johnson et al., 2013; Wijns et al., 2019). These skills are especially important for math development because these skills are helpful for children to improve their critical thinking, analyze data, make predictions, and so on (Rittle-Johnson et al., 2013; Wijns et al., 2019)

LeFevre et al. (2010) developed a model they named "Pathways to Mathematics," which consists of three cognitive pathways (quantitative, linguistic, and spatial attention) that contribute to different types of kindergarten math skills. Burr et al. (2022) tested the Pathways to Mathematics model (LeFevre et al., 2010), extending it to include patterning as a critical pathway to math development. In the new version of the Pathways to Mathematics model, Burr et al. (2022) measured the effects of children's repeating pattern abilities on children's later arithmetic abilities and found that patterning was an early and unique indicator of children's later math development in addition to the previously identified pathways.

Zippert et al. (2019) examined the relationship between the ability to recognize patterns and young children's overall math knowledge. They found that the ability to recognize patterns was a significant predictor of both general math knowledge and specific skills such as arithmetic and size comparison. Their results were consistent with previous studies and emphasized the importance of patterning in children's mathematical thinking. Zippert et al. (2019) also looked at children's reasoning and memory skills and found that patterning played a unique and crucial role in shaping young children's math abilities. Overall, there is consensus in the examined literature that patterning is a significant predictor of math achievement. Because patterning seems to be related to math achievement, examining the predictors of patterning in preschool children may be informative for telling us something about both patterning development and later math achievement.

Predictors of Patterning

There are limited studies investigating the relationship between cognitive and linguistic predictors of patterning skills. Given the limited literature on predictors of patterning and the relationship between patterning and early math skills, the current study includes both previously studied predictors of patterning, such as visual-spatial working memory and relational reasoning, as well as predictors not previously studied, but which have been shown to be related to early math development, such as attention (sustained attention and inhibition), phonological awareness, and biological sex. Potential predictors of patterning that were tested in the current study are discussed next.

Visual-Spatial Working Memory

Visual-spatial working memory (VSWM) refers to the ability to simultaneously hold and process information about the spatial and visual properties of objects and scenes in our environment (Mix & Cheng, 2012; Mix et al., 2016). Bull et al. (2008) reported strong correlational evidence which showed that pattern recognition and spatial abilities influence concurrent and later math achievement. The researchers also found that VSWM predicts math achievement through third grade. In addition, they found that in kindergarten, children's ability to remember and manipulate visual-spatial information is linked to various math skills such as counting, calculating, and understanding shapes (Bull et al., 2008). According to Mix and Cheng's research (2012), children who have stronger VSWM skills are more likely to identify and continue patterns successfully. In addition, according to Rittle-Johnson et al. (2019), some math problems, like addition problems that require carrying or mental arithmetic problems, require using VSWM.

In their study, Collins and Laski (2015) assessed the relationship between early pattern skills and other cognitive skills, including visual-spatial short-term memory, verbal short-term memory, verbal and visual-spatial working memory, and inhibition. They tested memory using the WISC-IV Digit Recall task and the Corsi Blocks task. Digit recall asks the child to repeat a series of numbers presented orally both forwards and backwards, with forwards recall measuring verbal short term memory and backwards recall measuring verbal working memory. Corsi Blocks, asks the child to touch a series of visually presented blocks arranged in a scattered series in forward and backward order to measure visual-spatial short-term memory and VSWM, respectively. Collins and Laski (2015) found a moderate positive correlation between patterning and both visual-spatial short-term memory and VSWM.

According to Miller et al. (2016), children use their executive function capacity to develop repeating pattern skills. Their research suggests that this is especially true for working memory capacity in preschool-aged children, as working memory capacity appears to be a more significant factor in children's patterning skills than age or relational reasoning. They found that having a higher working memory capacity can help children form patterns, making it easier for them to process and learn the different components of patterns. This increased understanding of patterns may also improve their awareness and comprehension of similarities in different repeating patterns. Other research (Toll et al., 2016) studying 5-year-old children's visual working memory and number sense showed that both abilities could predict their math performance in the long run. Moreover, the study also highlighted that there are reliable ways to identify 5-year-old children who are likely to struggle with math later on.

Relational Reasoning

Relational reasoning is the cognitive process of understanding relationships between concepts or objects (Miller et al., 2016). It entails the capacity to spot patterns, parallels, and discrepancies between objects and other visual stimuli, as well as patterns and parallels in verbal stimuli, and to apply this knowledge to infer relationships or solve problems. Researchers that have been effective in teaching patterning skills to first graders have hypothesized that, by doing so, they were helping children to develop relational reasoning, which in turn increased children's knowledge of arithmetic (Pasnak et al., 2016, as cited in Zippert et al., 2019).

To measure relational reasoning in young children, Miller et al. (2016) utilized a match-to-sample task, previously used by Kotoysky and Gentner (1996) and Son et al. (2011). This task required children to match picture cards that followed the same relational rules. The results of the study showed that relational knowledge was a significant predictor of patterning ability during the pretest but not a unique predictor of patterning during the posttest. This suggests that although relational knowledge cannot uniquely explain patterning ability, it is still an important aspect to consider in understanding preschoolers' patterning skills, as young children may use relational knowledge during patterning.

Sustained Attention and Inhibition

Two aspects of attention that are measured in young children are selective/sustained attention and executive attention/inhibition (Steele et al., 2012). Both sustained attention and inhibition have been shown to be related to math achievement in preschool children (Barnes et al., 2020). In this study, both sustained attention and inhibition were examined.

Sustained attention is the ability to focus on a task or activity for an extended period of time (Brueggemann & Gable, 2018). Sustained attention skills are important for early learning skills such as literacy and numeracy (Steele et al., 2012). Children with better sustained attention are also more adept at counting, non-symbolic quantification, and cardinality. (Brueggemann & Gable, 2018). Fishler et al. (2013) conducted a study that showed a positive association between sustained attention and arithmetic skills in 3–5-year-old children. Barnes et al. (2020) showed that difficulties in sustained attention in pre-k predicted significant difficulties in math and that difficulties in sustained attention were associated with the most severe difficulties in math at this age. In summary, although the relationship between sustained attention and math skills has been established, whether there is a relationship between sustained attention and patterning has not yet been tested.

Inhibition refers to the ability to control natural or automatic reactions in favor of those more appropriate to a task or situation (Miller et al., 2016; Schmerold et al., 2016). In terms of patterning, Collins and Laski (2015) found in their study with pre-K children that inhibition was related to patterning, but Miller et al. (2016) and Bock et al. (2015) reported that inhibition was not related to patterning. Schmerold et al. (2016) with first-grade children also did not find that inhibition was related to patterning. Differences in findings for the relationship between inhibition and patterning across studies might be related to the measures of inhibition that were used, the subjects' ages, and the patterning tasks used (Miller et al., 2016; Schmerold et al., 2016). Overall, there is mixed evidence for the association between inhibition and patterning, and more research is needed to establish whether inhibition is a predictor of patterning.

Phonological Awareness

Phonological awareness (PA), according to Marinus and Castles (2015, p. 663), is "an individual's awareness of the phonological structure, or sound structure, of spoken words." PA is a potential predictor of interest for patterning because of its relationship to later math achievement. Bradley and Bryant (1985) reported that early PA was related to math achievement evaluated three years later (as cited in Krajewski & Schneider, 2009). In addition, studies of early PA found significant correlations between PA and later math achievement in first grade (Alloway et al., 2005) and in second to fifth grade (Hecht et al., 2001, as cited in Krajewski & Schneider, 2009).

Biological Sex

Lüken & Sauzet (2021) reported that female students perform better on patterning tasks, and they hypothesized that this finding might be related to female students incorporating more patterning-like activities in their free gameplay. However, more research is needed on the association between sex and patterning.

Patterning In Young At-Risk Students

Studies suggest that math knowledge at school entry affects subsequent academic success (Claessens et al., 2009). One of the risk factors for low math knowledge at school entry is low socioeconomic status (SES; Jordan et al., 2009). Many studies show a relationship between low SES and early math achievement (see review in Barnes et al., 2016); however, not all low-income children have math difficulties either at the beginning or at the end of pre-K (Barnes et al., 2016). Little is known about patterning and what predicts patterning in low-income children coming into pre-K with very low math knowledge, which is the type of sample used in the present study.

Study Rationale

The literature above shows a relationship between patterning skills and math achievement. Studies have found a relationship between various cognitive predictors of patterning, such as VSWM, relational reasoning, and inhibition, but studies have not included other known cognitive and linguistic predictors of general math skills, such as PA and sustained attention to see whether they also predict patterning skills. Therefore, the goal of this study is to examine a set of previously tested as well as previously untested cognitive and linguistic abilities to see which ones are unique predictors of patterning, using the data from a large pre-K math intervention study.

Research Question

What linguistic and cognitive abilities assessed at the beginning of pre-K uniquely predict patterning performance at the end of pre-K, after accounting for the beginning of pre-K patterning skills, in a group of children coming into pre-K with very low math knowledge? The examined linguistic predictor is PA. Cognitive predictors are VSWM, relational reasoning, sustained attention, inhibition, and biological sex.

CHAPTER II

Method

The current study performs a secondary analysis of data collected and reported by Barnes et al. (2016). Barnes et al. (2016) examined two intervention approaches to address the academic and cognitive difficulties of low-income children entering pre-K classes with very low math knowledge. There are three different intervention groups: i) students receiving math and attention training, ii) students receiving math intervention only, and iii) business-as-usual students not receiving any additional intervention. Since Barnes et al. (2016) reported a main treatment effect on math outcomes at the end of pre-K, we included the intervention group assignment as a covariate in the analyses even though patterning was not a focus of the math intervention program.

Participants

This study was conducted in state pre-K programs in Texas and California. The students participating in the study had very low performance in math at the beginning of pre-K. There were 511 participants in total. 245 of the participants were female, while 266 of the participants were male. The mean age of the study participants was 5.088 years, with a standard deviation of 0.269.

Participants came from low-income families (see Barnes et al., 2016, for more details on the sample).

Measures

The measures included in the analyses for this study are originally described in Barnes et al. (2016 & 2020). Measures for the original study were collected at the beginning of pre-K, at the end of pre-K, and in kindergarten. This study includes predictors measured at the beginning of pre-K and the outcome of patterning measured at the end of pre-K.

Patterning

A patterning task was used to assess the children's ability to copy a repeating pattern using sets of objects which vary in color (Starkey & Klein, 2012). The set includes four resource sets consisting of picture cards and small objects. There are four duplication problems. In one of the duplication problems, the instructor shows the child a card featuring a picture of blocks of the same colors as the ABAB-designed model and asks the child to look at the pattern on the card. Finally, they ask the child to copy the model on the card with the real blocks given. The other three duplication problems follow a similar procedure, using cards with different arrangements pictured and different objects for the children to arrange. The task ends when the child responds incorrectly on two consecutive problems. The raw score was used in these analyses, and the highest score in this task was four.

Visual-spatial Working Memory (VSWM)

A VSWM task was used to assess working memory in preschool children (Bisanz et al., 2005, as cited in Barnes et al., 2020). The task asked children to replicate the sequence of jumps between lily pads made by a frog, starting with a span of a single jump and increasing to a total of seven jumps. Each span level consists of two trials. Following two failed trials at a particular span level, the task ended. A VSWM score was measured based on children's total accuracy. The task has an adequate level of internal consistency in 4- and 5-year-old children, measuring .70, and is moderately correlated with measures of phonological awareness, vocabulary, non-symbolic arithmetic, and number naming (LeFevre et al., 2010, as cited in Barnes et al., 2020). The test-retest reliability in this study was also adequate, measuring .70, meaning that it produces consistent results when administered to the same child on two different occasions (for more details, see Barnes et al., 2020).

Relational Reasoning

At the beginning of pre-K, Kaufman Brief Intelligence Test, Second Edition (KBIT-2; Kaufman & Kaufman, 2004, as cited in Barnes et al., 2020) Matrices subtest was given. This task is designed to evaluate nonverbal cognitive skills in people ranging from 4 years old to 90 years old. The subtest presents an examinee with a page containing one image at the top and a selection of images below. The images include concrete stimuli (people and objects) and more abstract stimuli (designs and symbols). The objective for the examinee is to choose the picture at the bottom that is related to the top picture. The Matrices subtest is reliable, with an internal consistency of .86 for ages 4-18. Scaled score was used to measure the relational reasoning in this study (for more details, see Barnes et al., 2016).

Sustained Attention and Inhibition

In this study, attention was measured using the Child-Attention Networks Test (Child-ANT; Rueda et al., 2005, as cited in Barnes et al., 2020). In the Child-ANT, congruent trials measure sustained attention while incongruent trials measure inhibition. The Child-Ant test is a computerized task that assesses vigilance and executive attention and is designed for preschool-aged children (Bauer & Zelazo, 2014, as cited in Barnes et al., 2020). The test has a high test-retest reliability of .92, meaning that it produces consistent results when administered to the same child on two different occasions. The test also has good convergent validity with another cognitive test called the Wechsler Preschool and Primary Scale of Intelligence-III (WPPSI-III Block Design), with a correlation of .60 (Zelazo & Bauer, 2013, as cited in Barnes et al., 2020).

The child must determine the direction of and catch the middle fish in a set of three swimming fish using a net and the "Z" or "?" key on a laptop. The child is provided with extensive practice before taking the test, both with cards and on the computer, and scaffolding is provided to help them learn the task. The test consists of four blocks of 17 trials, either cued or uncued, congruent or incongruent. During cued trials, bubbles appear on the screen before the fish, while during uncued trials, bubbles do not appear. Congruent trials have all the fish swimming in the same direction, while incongruent trials have the middle fish swimming in the opposite direction to the fish on either side. Accuracy on congruent trials was used as the measure of vigilance because it required sustained attention over time without the need to inhibit irrelevant information. The child had to press the button on the keyboard that matched the direction in which the middle fish was swimming. The accuracy of the child's responses on incongruent trials was used as a measure of inhibition, which reflects the ability to ignore irrelevant stimuli and focus on the target stimulus. Test-retest reliabilities for the composite score for congruent trials (a measure of sustained attention) and for the composite score for incongruent trials (a measure of inhibition) were both .80 in the original study. Response time data were not used because preschool children tend to have low accuracy rates (Davidson et al., 2006, for more details, see Barnes et al., 2020).

Phonological Awareness (PA)

The study used the PA subtest of the Test of Preschool Early Literacy (Lonigan et al., 2007, as cited in Barnes et al., 2020) or the Spanish Preschool Early Literacy Assessment (Lonigan, 2012, as cited in Barnes et al., 2020), which include tasks that involve elision and blending of sounds. Both tests have a high level of internal consistency, exceeding .89 (Goodrich & Lonigan, 2017; Lonigan et al., 2007, both as cited in Barnes et al., 2020). Raw scores were converted to z-scores for the sample (for more details, see Barnes et al., 2020).

Statistical Analyses

Correlations between the predictors, measured at the beginning of pre-K, and patterning at the end of pre-K were calculated and were estimated for multicollinearity. In addition, correlations were examined to see if sex was correlated with patterning at the end of pre-K to warrant its inclusion in the regression model. A multiple regression analysis was used to examine the relationships between patterning and VSWM, relational reasoning, sustained attention, inhibition, and PA after including the pretest of patterning. Age at the end of pre-K was also included in the regression model to account for any age-related variance in unstandardized scores. Also, intervention group assignment was included in the regression model as Barnes et al. (2016) reported significant differences between the groups on TEMA-3 scores at the end of pre-K.

CHAPTER III

Results

Table 1

Correlations of Predictors with Posttest Patterning Raw Score

Predictor	Pearson's Coefficient (r)
Patterning at beginning of pre-K	0.378**
Condition	-0.018
Age at end of pre-K	0.210**
Visual spatial WM at beginning of pre-K	0.295**

Phonological awareness at beginning of pre-K	0.160**
Relational Reasoning KBIT at beginning of pre-K	0.211**
Sustained Attention at beginning of pre-K	0.265**
Inhibition at beginning of pre-K	0.087*
Biological Sex	0.069

Note. WM = Working Memory, **p* < .05. ***p* < .001

Table 2

Multiple Regression Results

Predictor	β
Patterning at beginning of pre-K	.267**
Condition	.023
Age at end of pre-K	.115*
Visual Spatial WM at beginning of pre-K	.136*
Phonological awareness at beginning of pre-K	.016
Relational Reasoning at beginning of pre-K	.113*
Sustained Attention at beginning of pre-K	.086
Inhibition at beginning of pre-K	-.035

Note. Adjusted R^2 = .20; F (8,502) = 16.922, p < .001; WM = Working memory

**p* < .05. ***p* < .001

There were moderate, positive correlations between patterning at the end of pre-K with patterning at the beginning of pre-K (r (516) = .378 and p < .001), with VSWM (r (512) = .295, p < .001), and with sustained attention (r (514) = .265, p < .001). There were small, positive correlations between patterning at the end of pre-K with relational reasoning (r (514) = .211,

$p < .001$), age at the end of pre-K ($r(516) = .210$, $p < .001$), phonological awareness ($r(513) = .160$, $p < .001$), and inhibition ($r(514) = .087$, $p = .048$). There was no significant correlation between patterning at the end of pre-K and sex ($r(514) = .069$, $p = .118$), so sex was not included in the regression model.

The results of the multiple regression indicated a significant model ($F(8,502) = 16.922$, $p < .001$) and that the eight predictors included in the model explained 20% of the variance in patterning at the end of pre-K (adjusted $R^2 = .20$). The significant predictors were age at the end of pre-K (standardized $\beta = .115$, $p = .007$), VSWM (standardized $\beta = .136$, $p = .002$), relational reasoning (standardized $\beta = .113$, $p = .008$), and patterning at the beginning of pre-K (standardized $\beta = .267$, $p < 0.001$). The remaining predictors were not statistically significant.

CHAPTER IV

Discussion

The purpose of this study was to investigate which linguistic and cognitive abilities measured at the beginning of pre-K predicted patterning at the end of pre-K. The examined linguistic predictor was PA. The examined cognitive predictors were VSWM, relational reasoning, and sustained attention and inhibition. Although all predictors, except group assignment and sex, were significantly related to patterning at the end of pre-K, only age at the end of pre-K and VSWM, relational reasoning, and patterning at the beginning of pre-K were significantly and uniquely related to patterning at the end of pre-K. In the following section, the findings of this study are discussed in relation to extant literature.

Unsurprisingly, patterning ability at the beginning of pre-K was found to be a significant predictor of patterning ability at the end of pre-K. It is not uncommon for children to retain and further develop skills they possess at the beginning of pre-K, and in many studies of academic skill development, typically, the best predictor of future development of the same skill is the child's previous skill level. In their 2022 meta-analysis, Lin and Powell investigated the effects of initial math, reading, and cognitive skills on subsequent math performance. According to their analysis of more than 500,000 students from 250 studies, initial math calculation is unique predictor of later math performance. In the present study, initial patterning skills was one of the most important predictors of later patterning performance. Children's patterning ability coming into pre-K may be due to a combination of factors, including exposure to patterning skills outside of formal education, such as at home or in childcare during free play or organized activities. It is important to note that while some children may have a natural inclination towards patterning skills (Fox, 2005), all children can benefit from exposure and practice in developing these skills prior to entering school.

Another significant and unique predictor of patterning at the end of pre-K was relational reasoning. As described above, relational reasoning refers to a cognitive process of understanding relationships between objects. In a previous intervention study, relational reasoning was found to be significantly related to patterning at pretest; however, it was not a unique predictor of patterning at posttest (Miller et al., 2016). The findings of the current study are generally consistent with those of Miller et al. (2016) in that relational reasoning was found to be a unique predictor of patterning skills at the end of pre-K. The relationship between patterning and relational reasoning is reasonable in that relational reasoning tasks assess the child's ability to solve puzzles that require them to understand relationships between stimuli, and patterning requires the ability to understand the underlying relationship between items in order to extend a presented sequence of items.

While patterning and relational reasoning are exactly the same, they are related cognitive processes that involve the ability to identify and understand relationships between different elements, and they can both contribute to a child's ability to solve problems and make decisions

(Miller et al., 2016; Zippert et al., 2019). Although this study indicates a relationship between patterning and relational reasoning, there remains a need for more research to test whether different types of relational reasoning are related to different types of patterning in children at different ages.

In the current study, VSWM was found to be a significant predictor of patterning at the end of pre-K. VSWM is the ability to remember the location of visual information in space, temporarily. Recalling information about visual stimuli, especially their spatial relationships, is critical for patterning skills because the retained information is needed to predict the next step. This suggests that children with better VSWM may also have better patterning abilities. Our findings are similar to those of Collins & Laski (2015) who found a relationship between working memory and patterning using a different type of working memory task than the one used in the current study. The relationship between VSWM and patterning is complex and may be bidirectional, meaning that improvements in one skill may lead to improvements in the other or that improvements in one skill may be a result of improvements in the other. Some studies have shown that VSWM may be a precursor to the development of patterning abilities. For example, Mix and Cheng (2012) found that children with better VSWM tend to be better at recognizing and extending patterns. Moreover, Miller et al. (2016) found that having better VSWM capacity assists children in creating patterns, resulting in a simpler and more efficient method of comprehending and learning the various elements of patterns. Finally, Nutley et al. (2011) reported that training in patterning tasks could improve VSWM.

In this study, age at posttest was found to be a significant and unique predictor of patterning at the end of pre-K, suggesting that older children tended to have better patterning abilities. This finding is consistent with previous research on cognitive development, that the ability to recognize and extend patterns tends to improve with age during the preschool years (e.g., Laski & Siegler, 2014). In addition, previous research conducted with children aged 3-5 has shown that, as children age, the strategies they use for processing visual spatial information also changes (Lüken & Sauzet, 2021), which suggests that age-related changes in cognitive and visual-spatial abilities may contribute to the observed differences in patterning skills among pre-K children.

Phonological awareness showed a significant but small correlation with patterning (r = .16), but it was not found to be a significant unique predictor of patterning at the end of pre-K. even though there is conceptual overlap between PA and patterning, as both involve recognizing and manipulating structural elements, one in the visual domain, and the other in the verbal domain (Marinus and Castles, 2015). It is possible that the lack of PA as a unique predictor of patterning in the current study was due to the specific measures used to assess these skills. In this study, "Test of Preschool Early Literacy" and "Spanish Preschool Early Literacy Assessment," which involve the selection and comparison of sounds, were used. In contrast, in the Krajewski & Schneider (2009) study, participants heard a series of singular phonemes and had to match them with the appropriate word and choose the corresponding picture from among four pictures. And in the sound classification task of Bradley & Bryant, 1985, participants listened to a sequence of four words and were then asked to indicate which word did not rhyme with the others. In these two studies, which did not measure patterning, PA predicted math achievement. It may be useful to consider additional measurements of PA or different age groups in future research. It is important to mention that these other studies did not test the other cognitive skills that were tested in the current study and that were included in the multiple regression, so although phonological awareness at the beginning of the year was correlated with patterning at the end of the pre-K year, it was not uniquely predictive when in the regression model with all other predictors.

Consistent with the role of attentional processes in many cognitive tasks, and its relations to general math achievement (e.g., Barnes et al, 2020; Burr et al. 2022; Wijns et al., 2019; Zippert et al., 2019), sustained attention showed a significant moderate sized relationship to patterning and inhibition showed a significant small correlation to patterning. However, neither aspect of attention was uniquely related to patterning in the regression analysis. Attention is a complex construct that can be difficult to measure, and the measures (i.e., Child-ANT) used in this study may not measure those aspects of attention that may be related to patterning in young children. It may also be the case that when attention is tested along with other cognitive abilities, such as working memory, which themselves require attentional processes in addition to memory processes, attention is no longer uniquely predictive of patterning. Furthermore, it is possible that more complex patterning tasks than the ones used in the current study might also draw to a greater extent on sustained attention and inhibition.

The absence of a significant relationship between sex and patterning at the end of the pre-K period suggests that there may not be a sex-related difference in patterning skills in this age group. Lüken & Sauzet (2021) reported that girls might engage in different types of play than boys and that children's play activities can have an impact on their cognitive development. This suggests that there may be differences in the way girls and boys are taught to form patterns at school or home, which in turn affects their performance in related tasks. Additional research is needed to understand more fully whether there is a relationship between sex and patterning and, if so, to identify the specific factors that might contribute to any differences.

Limitations

Our predictors explained 20% of the variance in patterning at the end of pre-K. Thus, 80% of the variance in patterning at the end of pre-K remains to be explained. The inclusion of additional predictors, such as set-shifting, numerical ability, counting, or number competencies, may help explain the patterning skills at the end of pre-K further. For example, Miller et al. (2016) found that set-shifting was correlated with patterning at pretest in their study; they used the adapted measures of patterning from Rittle-Johnson et al. (2013). Additionally, Wijns et al. (2021) found a bidirectional relationship between patterning and numerical abilities between the ages of 4-5. They reported that patterning and numerical abilities of 4-year-old children predicted each other after one year. However, 5-year-old children's patterning skills predicted 6-year-old numerical ability, but 5-year-old numerical ability did not predict 6-year-old patterning skills.

More specifically, Wijns et al. (2021) established an association between counting skills and patterning abilities, counting skills involve recognizing and manipulating sets of objects or symbols (Carpenter et al., 2017; Wijns et al., 2021). Patterning skills also require identifying and understanding the underlying structure and order of an object or sequence of events. Patterning skills may naturally map onto counting skills. For example, in the red-blue-red-blue (ABAB) pattern, children need to duplicate, extend, or abstract one by one red and blue. Or in the red-red-blue-red-red-blue (AABAAB) pattern, children should complete the requested task in the form of two reds and one blue. As can be understood from these examples, counting skills might be a predictor of patterning skills, and more research is needed in this area.

Jordan et al. (2009) found that number competencies significantly and substantively predicted math achievement at the end of third grade. Although there is not a direct relationship between number competencies and patterning, both skills are significant predictors of later math achievement (Miller et al., 2016; Rittle-Johnson et al., 2013; Wijns et al., 2019), and number competencies may be a predictor of patterning. Future research can seek to evaluate the relationship between these abilities and patterning.

Another limitation of this study involves measurement of the patterning task. As described above, the patterning skills assessed in this study focused only on a single aspect of patterning and measuring the skill with only four problems may have limited our ability to measure broader patterning skills in this study. For example, Papic et al. (2011) measured repeating patterns, spatial structure, and growing patterns tasks, and included five problems with increasing difficulty levels in each model. Rittle-Johnson et al. (2013) created a construct map, including duplication, extension, abstraction, and unit recognition, and used a total of ten items. In another study (Kidd et al., 2014), researchers used symmetrical patterns. Using laptops and note cards, researchers asked participants to find the missing pieces (beginning, middle, or end) in each problem among the options offered. Finally, Miller et al. (2016) created individual item maps called Wright maps produced from Rasch models. In addition to duplication, they measured the extension and abstraction abilities of patterning skills. These studies all measured multiple kinds of patterning and had more problems for participants to complete. Because our study measured only duplication patterns and only included four problems, there may be issues related to the type of patterning measured and the reliability of that measurement.

Implications

Patterning at the beginning of pre-K, relational reasoning, VSWM, and age at the end of pre-K were significant and unique predictors of patterning at the end of pre-K. However, PA, attention, and sex were not significant predictors of the patterning at the end of pre-K. This study highlights the importance of exposure and practice in developing patterning skills and the need for additional research to measure how different types of relational reasoning and working memory might affect the development of early patterning skills. The findings of this study contribute to our understanding of the cognitive processes involved in patterning as well as the cognitive processes that do not seem to be important for patterning and may be useful for informing the development of interventions to improve patterning skills in young children.

References

Alloway, T. P., Gathercole, S. E., Adams, A.-M., Willis, C., Eaglen, R., & Lamont, E., (2005). Working memory and phonological awareness as predictors of progress towards early learning goals at school entry. *British Journal of Developmental Psychology*, *23*(3), 417–426. https://doi.org/10.1348/026151005X26804

Barnes, M. A., Raghubar, K. P., English, L., Williams, J. M., Taylor, H., & Landry, S. (2014). Longitudinal mediators of achievement in mathematics and reading in typical and atypical development. *Journal of Experimental Child Psychology*, *119*, 1–16. https://doi.org/10.1016/j.jecp.2013.09.006

Barnes, M. A., Klein, A., Swank, P., Starkey, P., McCandliss, B., Flynn, K., Zucker, T., Huang, C.-W., Fall, A.-M., & Roberts, G. (2016). Effects of tutorial interventions in mathematics and attention for low-performing preschool children. *Journal of Research on Educational Effectiveness*, *9*(4), 577–606. https://doi.org/10.1080/19345747.2016.1191575

Barnes, M. A., Clemens, N. H., Fall, A.-M., Roberts, G., Klein, A., Starkey, P., McCandliss, B., Zucker, T., & Flynn, K. (2020). Cognitive predictors of difficulties in math and reading in pre-Kindergarten children at high risk for learning disabilities. *Journal of Educational Psychology*, *112*(4), 685–700. https://doi.org/10.1037/edu0000404

Bergman Nutley, S., Söderqvist, S., Bryde, S., Thorell, L. B., Humphreys, K., & Klingberg, T. (2011). Gains in fluid intelligence after training non-verbal reasoning in 4-year-old children: A controlled, randomized study: Fluid intelligence gains after training nonverbal reasoning. *Developmental Science*, *14*(3), 591–601. https://doi.org/10.1111/j.1467-7687.2010.01022.x

Bock, A., Cartwright, K. B., Gonzalez, C., O'Brien, S., Robinson, M. F., Schmerold, K., Shriver, A., & Pasnak, R. (2015). The role of cognitive flexibility in pattern understanding. *Journal of Education and Human Development*, *4*(1). https://doi.org/10.15640/jehd.v4n1a3

Bock, A. M., Cartwright, K. B., McKnight, P. E., Patterson, A. B., Shriver, A. G., Leaf, B. M., Mohtasham, M. K., Vennergrund, K. C., & Pasnak, R. (2018). Patterning, reading, and executive functions. *Frontiers in Psychology*, *9*, 1802. https://doi.org/10.3389/fpsyg.2018.01802

Brueggemann, A., & Gable, S. (2018). Preschoolers' selective sustained attention and numeracy skills and knowledge. *Journal of Experimental Child Psychology*, *171*, 138–147. https://doi.org/10.1016/j.jecp.2018.02.001

Bull, R., Espy, K. A., & Wiebe, S. A. (2008). Short-term memory, working memory, and executive functioning in preschoolers: Longitudinal predictors of mathematical achievement at age 7 years. *Developmental Neuropsychology*, *33*(3), 205–228. https://doi.org/10.1080/87565640801982312

Burr, S. M. D. L., Xu, C., Douglas, H., LeFevre, J. A., & Susperreguy, M. I. (2022). Walking another pathway: The inclusion of patterning in the pathways to mathematics model. *Journal of Experimental Child Psychology*, *222*, 105478.

Carpenter, T. P., Franke, K. L., Johnson, N. C., Turrou, A. C., & Wager, A. A., (2017). Foundational Concepts of Counting and Number. *Young children's mathematics: Cognitively guided instruction in early childhood education.* (pp. 7-20). Heinemann.

Claessens, A., Duncan, G., & Engel, M. (2009). Kindergarten skills and fifth-grade achievement: Evidence from the ECLS-K. *Economics of Education Review*, *28*(4), 415–427. https://doi.org/10.1016/j.econedurev.2008.09.003

Collins, M. A., & Laski, E. V. (2015). Preschoolers' strategies for solving visual pattern tasks. *Early Childhood Research Quarterly*, *32*, 204–214. https://doi.org/10.1016/j.ecresq.2015.04.004

Fisher, A., Thiessen, E., Godwin, K., Kloos, H., & Dickerson, J. (2013). Assessing selective sustained attention in 3- to 5-year-old children: Evidence from a new paradigm. *Journal of Experimental Child Psychology*, *114*(2), 275–294. https://doi.org/10.1016/j.jecp.2012.07.006

Fox, J. (2005). Child-initiated mathematical patterning in the pre-compulsory years. *Reports - Research; Speeches/Meeting Papers*, *2*, 313–320.

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology*, *45*(3), 850–867. https://doi.org/10.1037/a0014939

Jordan, N. C., & Levine, S. C. (2009). Socioeconomic variation, number competence, and mathematics learning difficulties in young children. *Developmental disabilities research reviews*, *15*(1), 60-68.

Kidd, J. K., Pasnak, R., Gadzichowski, K. M., Gallington, D. A., McKnight, P., Boyer, C. E., & Carlson, A. (2014). Instructing first-grade children on patterning improves reading and mathematics. *Early Education and Development*, *25*(1), 134–151. https://doi.org/10.1080/10409289.2013.794448

Kotovsky, L., & Gentner, D. (1996). Comparison and categorization in the development of relational similarity. *Child Development*, *67*(6), 2797-2822. https://doi.org/10.2307/1131753

Krajewski, K., & Schneider, W. (2009). Exploring the impact of phonological awareness, visual–spatial working memory, and preschool quantity–number competencies on mathematics achievement in elementary school: Findings from a 3-year longitudinal study. *Journal of Experimental Child Psychology*, *103*(4), 516–531. https://doi.org/10.1016/j.jecp.2009.03.009

Lachance, J. A., & Mazzocco, M. M. M. (2006). A longitudinal analysis of sex differences in math and spatial skills in primary school age children. *Learning and Individual Differences*, *16*(3), 195–216. https://doi.org/10.1016/j.lindif.2005.12.001

Laski, E. V., & Siegler, R. S. (2014). Learning from number board games: You learn what you encode. *Developmental Psychology*, *50*(3), 853–864. https://doi.org/10.1037/a0034321

LeFevre, J.-A., Fast, L., Skwarchuk, S.-L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to Mathematics: Longitudinal predictors of performance: Pathways to Mathematics. *Child Development*, *81*(6), 1753–1767. https://doi.org/10.1111/j.1467-8624.2010.01508.x

Liljedahl, P. (2004). Repeating pattern or number pattern: The distinction is blurred. *Focus on Learning Problems in Mathematics*, *26*(3), 24–42.

Lin, X., & Powell, S. R. (2022). The roles of initial mathematics, reading, and cognitive skills in subsequent mathematics performance: A meta-analytic structural equation modeling approach. *Review of Educational Research*, *92*(2), 288-325.

Lüken, M. M., & Sauzet, O. (2021). Patterning strategies in early childhood: A mixed methods study examining 3- to 5-year-old children's patterning competencies. *Mathematical Thinking and Learning*, *23*(1), 28–48. https://doi.org/10.1080/10986065.2020.1719452

Marinus, E., & Castles, A. (2015). Precursors to reading: Phonological awareness and letter knowledge. In E. L. Bavin & L. R. Naigles (Eds.), *The Cambridge Handbook of Child Language* (2nd ed., pp. 661–680). Cambridge University Press. https://doi.org/10.1017/CBO9781316095829.030

Miller, M. R., Müller, U., Giesbrecht, G. F., Carpendale, J. I., & Kerns, K. A. (2013). The contribution of executive function and social understanding to preschoolers' letter and math skills. *Cognitive Development*, *28*(4), 331–349. https://doi.org/10.1016/j.cogdev.2012.10.005

Miller, M. R., Rittle-Johnson, B., Loehr, A. M., & Fyfe, E. R. (2016). The influence of relational knowledge and executive function on preschoolers' repeating pattern knowledge. *Journal of Cognition and Development*, *17*(1), 85–104. https://doi.org/10.1080/15248372.2015.1023307

Mix, K. S., & Cheng, Y.-L. (2012). The relation between space and math. In *Advances in Child Development and Behavior* (Vol. 42, pp. 197–243). Elsevier. https://doi.org/10.1016/B978-0-12-394388-0.00006-X

Mix, K. S., Levine, S. C., Cheng, Y.-L., Young, C., Hambrick, D. Z., Ping, R., & Konstantopoulos, S. (2016). Separate but correlated: The latent structure of space and mathematics across development. *Journal of Experimental Psychology: General*, *145*(9), 1206–1227. https://doi.org/10.1037/xge0000182

Mulligan, J., Oslington, G., & English, L. (2020). Supporting early mathematical development through a ‘pattern and structure’ intervention program. *ZDM*, *52*(4), 663–676. https://doi.org/10.1007/s11858-020-01147-9

Papic, M. (2015). An Early Mathematical Patterning Assessment: Identifying young Australian Indigenous children’s patterning skills. *Mathematics Education Research Journal*, *27*(4), 519–534. https://doi.org/10.1007/s13394-015-0149-8

Papic, M., & Mulligan, J. (2007). The growth of early mathematical patterning: An intervention study. *Mathematics: Essential research, essential practice*, 591-600.

Papic, M. M., Mulligan, J. T., & Mitchelmore, M. C. (2011). Assessing the development of preschoolers’ mathematical patterning. *Journal for Research in Mathematics Education*, *42*(3), 237–268. https://doi.org/10.5951/jresematheduc.42.3.0237

Pasnak, R., Schmerold, K. L., Robinson, M. F., Gadzichowski, K. M., Bock, A. M., O’Brien, S. E., Kidd, J. K., & Gallington, D. A. (2016). Understanding number sequences leads to understanding mathematics concepts. *The Journal of Educational Research*, *109*(6), 640–646. https://doi.org/10.1080/00220671.2015.1020911

Rittle-Johnson, B., Fyfe, E. R., Hofer, K. G., & Farran, D. C. (2017). Early math trajectories: Low-income children’s mathematics knowledge from ages 4 to 11. *Child Development*, *88*(5), 1727–1742. https://doi.org/10.1111/cdev.12662

Rittle-Johnson, B., Fyfe, E. R., McLean, L. E., & McEldoon, K. L. (2013). Emerging understanding of patterning in 4-year-olds. *Journal of Cognition and Development*, *14*(3), 376–396. https://doi.org/10.1080/15248372.2012.689897

Rittle-Johnson, B., Zippert, E. L., & Boice, K. L. (2019). The roles of patterning and spatial skills in early mathematics development. *Early Childhood Research Quarterly*, *46*, 166–178. https://doi.org/10.1016/j.ecresq.2018.03.006

Schmerold, K., Bock, A., Peterson, M., Leaf, B., Vennergrund, K., & Pasnak, R. (2016). The relations between patterning, executive function, and mathematics. *The Journal of Psychology*, *151*(2), 207–228. https://doi.org/10.1080/00223980.2016.1252708

Son, J. Y., Smith, L. B., & Goldstone, R. L. (2011). Connecting instances to promote children's relational reasoning. *Journal of experimental child psychology*, *108*(2), 260-277.

Starkey, P., & Klein, A. (2012). Scaling up the Implementation of a Pre-Kindergarten Mathematics Intervention in Public Preschool Programs. *Grantee Submission*.

Steele, A., Karmiloff-Smith, A., Cornish, K., & Scerif, G. (2012). The multiple subfunctions of attention: Differential developmental gateways to literacy and numeracy: attention subfunctions as gateways. *Child Development*, *83*(6), 2028–2041. https://doi.org/10.1111/j.1467-8624.2012.01809.x

Toll, S. W. M., Kroesbergen, E. H., & Van Luit, J. E. H. (2016). Visual working memory and number sense: Testing the double deficit hypothesis in mathematics. *British Journal of Educational Psychology*, *86*(3), 429–445. https://doi.org/10.1111/bjep.12116

Wijns, N., Torbeyns, J., De Smedt, B., & Verschaffel, L. (2017). Young children's patterning competencies and mathematical development: A review. In K. M. Robinson, H. P. Osana, & D. Kotsopoulos (Eds.), *Mathematical Learning and Cognition in Early Childhood* (pp. 139–161). Springer International Publishing. https://doi.org/10.1007/978-3-030-12895-1_9

Wijns, N., Torbeyns, J., Bakker, M., De Smedt, B., & Verschaffel, L. (2019). Four-year-olds' understanding of repeating and growing patterns and its association with early numerical ability. *Early Childhood Research Quarterly, 49*, 152–163. https://doi.org/10.1016/j.ecresq.2019.06.004

Wijns, N., Verschaffel, L., De Smedt, B., & Torbeyns, J. (2021). Associations between repeating patterning, growing patterning, and numerical ability: A longitudinal panel study in 4- to 6-year-olds. *Child Development*, *92*(4), 1354–1368. https://doi.org/10.1111/cdev.13490

Zippert, E. L., Clayback, K., & Rittle-Johnson, B. (2019). Not just iq: Patterning predicts preschoolers' math knowledge beyond fluid reasoning. *Journal of Cognition and Development*, *20*(5), 752–771. https://doi.org/10.1080/15248372.2019.1658587

Zippert, E. L., Douglas, A.-A., & Rittle-Johnson, B. (2020). Finding patterns in objects and numbers: Repeating patterning in pre-K predicts kindergarten mathematics knowledge. *Journal of Experimental Child Psychology*, *200*, 104965. https://doi.org/10.1016/j.jecp.2020.104965

Zippert, E. L., Douglas, A.-A., Tian, F., & Rittle-Johnson, B. (2021). Helping preschoolers learn math: The impact of emphasizing the patterns in objects and numbers. *Journal of Educational Psychology*, *113*(7), 1370–1386. https://doi.org/10.1037/edu0000656