Teacherly Response-ability:

A Relational Ethics in Mathematics Education

By

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In one of the many books that was foundational to this dissertation, Karen Barad's (2007) acknowledgments include these lines:

Re-membering and re-cognizing do not take care of, or satisfy, or in any other way reduce one's responsibilities; rather, like all intra-actions, they extend the entanglements and responsibilities of which one is a part. The past is never finished. It cannot be wrapped up like a package, or a scrapbook, or an acknowledgment; we never leave it and it never leaves us behind. (p. ix)

In this spirit, written acknowledgments will always be insufficient. I could name so many names of people who have championed, challenged, and accompanied me, and there are more whose names I do not know. Here, then, is just a selection of the entanglements wound through this dissertation, in the form of people to whom I hope this work continues to be responsible.

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Chapter 1

INTRODUCTION

Our world's in turmoil right now and I think it's not the kids' fault. It's like media and all this stuff. So anyways, these kids, they need somebody to validate the fact that they're alive and breathing and present, their thoughts are important. So, I think part of my job is to do that.

—Interview with Franck, 10/2018

The racism and oppression that pervade US public education not only systematically deny learning opportunities to students of color, but also constitute a form of spirit-murder for dark students who learn that they are often not welcome nor are they seen as human in schools that have been designed and maintained to exclude them (Love, 2019). Mathematics education plays a central role in this process, enacting a slow violence through repeated micro- and macro-aggressions that "convince[s] people that they are no longer mathematical" (R. Gutiérrez, 2018, p. 2). Against this backdrop of dehumanizing structures, norms, and policies, what is a mathematics teacher—who, as a participant in the institution of schooling, is inextricably implicated within these injustices —who cares about the humanity, wellbeing, and success of their students to do?

To think about responsibility in the context of spirit-murder and slow violence, consider Shotwell's (2016) description of two farmers who take on the responsibility of caring for radioactive cattle in the Fukushima exclusion zone after the 2011 tsunami-triggered nuclear reactor meltdown: she terms their labor a form of "care-as-protest" (p. 133). These farmers

contradict government orders for any remaining livestock to be slaughtered and burned, and in doing so, refuse to ignore that lives—animal and human—affected by disaster are still lives. Shotwell suggests that in the context of the environmental destruction, rampant inequality, and global suffering caused by intersecting systems of oppression rooted in settler colonialism and heteropatriarchal racial capitalism, we all already live in the wake of many disasters. Thus, caring when the system encourages us not to care, about or for what and whom supposedly do not deserve our care, is itself a form of protest. If public schooling and mathematics education are violent enterprises especially for students of color, and if mathematics teachers are complicit within it, then they/we "already live in this world thoroughly connected with all of the suffering" (Shotwell, 2016, p. 135). So the question, to paraphrase Shotwell, is not whether mathematics teachers who are concerned with justice should move to the educational equivalent of a nuclear exclusion zone to challenge mathematical violence, but rather, given that they are already in exclusion zones, how they should care-as-protest.

This dissertation describes how two veteran mathematics teachers care-as-protest against mathematical violence through the ethical stances they take in their Algebra 1 classrooms. I draw on feminist philosophers—specifically feminist new materialist theories of ethics and affect theory—and poststructural methodologies to interpret ethnographic data collected from one school year of observing how these two focal teachers teach, collaborate, sensemake, interrogate, and interact. Findings from this analysis challenge three commonsensical ideas shared by conceptions of ambitious and culturally responsive mathematics teaching: that teachers should gather knowledge about students as the foundation for building relationships; that teachers should set and reinforce equitable sociomathematical norms; and that creating a more just future requires teachers to have a clear vision of that future. Although there is some truth to these

axioms, I argue that instead, taking *response-ability* as an ethical stance towards relational work better accounts for the complex interactions, affects, histories, and contexts in which mathematics education is enmeshed. In this chapter, I discuss the purpose and significance of this dissertation and provide an overview of how this dissertation is organized.

Intent of Research

In describing the Fukushima cattle farmers, Shotwell (2016) poses this question prompted by their labor: "under conditions of oppression and exploitation, how might we enact practices of freedom that can shape worlds we currently cannot imagine?" (p. 18). This is a question about "right relation," a concept I borrow from Indigenous scholars¹ to encapsulate ethical ways of being with, towards, around, and for others. Much of the enterprise of mathematics education—given its own traditions and cultures but also its interconnectedness with other oppressive institutions—runs counter to the values and dreams of teachers, students, and families who interact with it, and yet mathematics teachers must nevertheless work within it. So how can mathematics teachers be in right relation with their students, first and foremost, but also with mathematics education and with themselves? This is surely an unanswerable question, given its complexity and specificity, but Shotwell offers a pragmatic path forward, drawing on Haraway's (2016) exhortation to "stay with the trouble" (p. 2): "each of us, however situated, could do what we can—recognizing that what we can do, on its own, will never be enough" (p. 127). Therefore,

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¹ In accordance with dominant academic practice, I have tried to trace a single source to cite for the use of the phrase "right relation." It is and has been commonly used by Indigenous people in describing an approach—or response—to the interdependence and relationality of all living and non-living beings, and as I understand it, is a way of life rather than an academic concept. The phrase first appears in my notes around the time I was listening to and reading the work of scholars such as Manulani Aluli-Meyer, Megan Bang, Talia London, Jenell Navarro, and Eve Tuck, and I am especially grateful to Shawn Wilson's *Research is Ceremony* (2008) and Rochelle Gutiérrez's *Living Mathematx* (2017) for informing my ongoing understandings.

in this dissertation, I seek to investigate the "what we can" that two veteran mathematics teachers do, situated as they are within "complex webs of suffering" (2016, p. 5).

In contrast to the prevailing prescriptiveness of much educational research, I do not aim to make claims about how all mathematics teachers ought to act and I recommend no list of practices that should be generalized and taken up by other teachers. Instead, I examine some possibilities for what right relation for mathematics teachers could be, knowing that these possibilities are inevitably imperfect, with the intent that illustrating these possibilities may illuminate other possibilities. In other words, I offer you a story, and with it, this provocation from Wilson's (2008) depiction of how stories are used by Indigenous elders: "it was up to the listener to piece together a lesson from the story and to apply the pieces where they fit to help in the current problem" (p. 27).

This may seem like a meager claim for a research project that has taken nearly three years already, and indeed, I intend it to be a humble claim. I intend it to be a humble claim in the way that what my focal teachers do in their classrooms is humble but not insignificant. Just as they are focused on their classrooms rather than, say, systemic advocacy, I am focused on only two of the hundreds of thousands of mathematics classrooms across the country. Yet what they do matters deeply for them and the students, families, colleagues, and others they engage with through their teaching. Likewise, I hope that what I am doing can matter for: aspiring mathematics teachers who read education news and learn education history with horror and yet believe that there is important work to be done as teachers; practicing mathematics teachers who are mired in the mess and yet committed to showing up every day with and for their students; and students, families, and community members advocating for expansive visions and radical imaginings of what mathematics education can be.

In this dissertation, I bring together education research on care and culturally relevant, responsive, and sustaining pedagogies that speak to possible teacher-student relationships within mathematics education. Drawing on feminist philosophers, I build on conceptions of relational work as ongoing and uncertain rather than as something that can be checked off on a rubric. By studying two teachers with shared practices but different social locations and commitments, I illustrate the variance and texture within similar ethical stances. Furthermore, this dissertation contributes to nascent uses of poststructural theory and methodology in empirical mathematics education research, which I believe can highlight different aspects of and approaches to mathematics education than are currently foregrounded in the field. It also brings an ethics of response-ability from its current applications in science, engineering, and international higher education into the field of K-12 mathematics education research.

Organization of This Dissertation

In Chapter 2, I outline how extant research conceptualizes right relation in mathematics education. Drawing on critical scholars, I argue that mathematics education is currently characterized largely by relations of violence: what R. Gutiérrez (2018) calls a slow violence that convinces people that they are not mathematical and that mathematics is not for them, and what Martin and colleagues (2019) describe as physical, symbolic, and epistemological violences that are inherent to mathematics education. I explain how care scholars—and among them, those who study mathematical care and politicized care—varyingly understand the characteristics of nonviolent and/or restorative relations between teachers and students, and how scholars of culturally-relevant, -responsive, and -sustaining pedagogies offer proofs of concept by describing characteristics of individual teachers who have navigated the violence of mathematics education to establish restorative relationships with their students. Then, I acknowledge the limitations of

these literatures for theorizing right relation in mathematics education and preview how an ethics of *response-ability* might address these limitations. I close by explaining the concepts of *teachers'ethical stance* and *relational work*, which I will use in my analyses.

In Chapter 3, I describe the methods of inquiry for this dissertation, beginning with my approach towards research and how it is influenced by my social location. I explain the process and artifacts of ethnographic data collection as I shadowed two veteran mathematics teachers, Franck To and Clark Zapatero,² over the 2018-2019 school year, and describe how my "outsider-insider" relationship with these focal teachers shaped not only my data collection but also my data analysis. I take a poststructural approach to integrating empiricism and interpretivism in data analysis, assembling a bricolage (Kincheloe, 2001) of analytic methods that integrates techniques from discourse analysis, narrative inquiry, and traditional grounded theory. I organize these methods under the analytical lens of what A. Y. Jackson and Mazzei (2013) call "thinking with theory" and read the empirical data alongside theoretical texts.

In Chapters 4, 5, and 6, I use the findings of these analyses to probe three truisms about what constitutes right relation in mathematics education. I seek not to contradict them, but to add nuance to how practitioners and researchers discuss them. Barad contrasts critique with "reading diffractively for patterns of differences that make a difference" (Dolphijn & Van der Tuin, 2012, p. 49). Therefore, these chapters integrate philosophical and explorations of the focal phenomenon, mathematics teachers' ethical stances on relational work, to identify differences that make a difference between Franck's and Clark's ethical stances and other common approaches. In each chapter, I demonstrate how Franck's and Clark's ethical stances point towards and make concrete elements of an ethics of response-ability in mathematics education.

² All participant and site names in this dissertation are pseudonyms.

First, in Chapter 4, I examine the premise that mathematics teachers must know their students in order to build strong relationships that can then be leveraged for mathematical learning. Through questioning whether it is even ethically possible to know students and considering how Franck and Clark engage in processes of knowing, I claim that what teachers know about students matters less than the interactions that occur as they seek to know, because these interactions create possibilities for subsequent student and teacher response. In an ethics of response-ability, creating possibilities for further response holds promise for rendering others capable and thereby, for living more justly.

In Chapter 5, I examine the premise that teachers should explicitly set and reinforce equitable sociomathematical norms, particularly around participation. By articulating the limitations of norm-setting and what is made possible by an absence of explicit norms—which, to be clear, is not the same as an absence of norms—I explore how Franck and Clark construct open normativities: processes that expand the possibility of more possibilities. I argue that these open normativities, as established through their implementation of pedagogical practices such as notice-and-wonder activities, daily random grouping, and classroom behavior policies normalize heterogeneity and creativity and thereby create possibilities for greater flourishing.

Chapter 6 delves into how history and affect interact with Franck, Clark, their students, and the infrastructures of mathematics education. In this chapter, I use affect theory and the construct of mathematical ghosts to trace how in-the-moment reactions both signal what we take to be normal and can move us away from violent pasts, pressing against the premise that education should be focused on the future. I illustrate how attending to the affective inheritances of the past takes a stance on who and what matters in ways that are essential for "setting in place a different future" (Gordon, 2008, p. 66).

Finally, I return to *response-ability*, now informed by the empirical findings of the previous three chapters. I discuss what an ethics of response-ability in mathematics education means for relational work, including student agency, and contrast it with common conceptions of responsiveness and responsibility. I argue that taking a diffractive view of the axioms challenged in Chapters 4-6 matters, because without it, we may not only not be able to establish nonviolent and restorative relations to set in place a future that differs from the predominant violence of mathematics education, but our well-intentioned efforts may actually make such relations impossible. I conclude with limitations of this dissertation and implications for researchers and practitioners.

Chapter 2

BACKGROUND AND FRAMEWORKS

This dissertation is premised on the question of right relation: what kind of relation should mathematics teachers have—with the students, mathematics, histories, institution of schooling, and other participants in their work—given that they already exist in Shotwell's exclusion zones? In this chapter, I draw on the research literature to argue that first, violence is the relation that characterizes much of contemporary mathematics education, and second, although care theory and theories of culture-rich pedagogies prescribe nonviolent and restorative relations as a solution, they are limited by treating mathematics education as a context for human relations rather than as an equal player in creating possible relations. Next, I preview the feminist new materialist proposition of response-ability as an ethical approach to the inextricable entanglements between teachers, students, mathematics, and the non-human and more-thanhuman actors in existence. As I will describe in the methods chapter, it was the inquiry and analyses described in Chapters 3-6 that led me to response-ability, rather than response-ability serving to launch the inquiry and analyses. However, I summarize the theoretical ideas underlying response-ability in this chapter as a touchstone in which to ground the following findings chapters; response-ability offers, among other conceptual resources, ways of understanding agency and justice that I will explore in the findings chapters. Finally, I explain how I conceptualize teachers' ethical stances on relational work for the empirical investigations in this dissertation.

Prior Work

First, I establish violence as the defining relation of contemporary mathematics education for most students and especially for students who are already targeted by oppressive social structures such as racism. Next, I read literature on care theory and culturally-relevant, - responsive, and -sustaining pedagogies because they represent two prominent strands of the educational research about what teacher-student relationships should be. I begin with literature on care, because care theorists focus on a specific and valuable type of relationship between teacher and student—a caring relationship—although they differ in how they define care and the scope of care. Then, I turn to literature which, for the sake of concision, I call culture-rich pedagogies. This literature, like care theory, generally emphasizes relationships, but focuses more strongly on those students who are most marginalized by the racism and other oppressive systems that pervade education in the United States. Although scholars of culture-rich pedagogies rarely explicitly label their work as "ethics," they nevertheless make strong claims about how teachers should be and act in relation to their students. Finally, I identify limitations of how care theory and culture-rich pedagogies literatures conceptualize right relation.

Violence in Mathematics Education

The culture of Western mathematics has traditionally revered abstract thinking, deductive reasoning, and objectification (e.g., Bishop, 1990), centering what Wagner and Shahjahan (2015) call a "mind-supremacist epistemology" and narrowly defining the possibilities for mathematical brilliance (e.g., Mendick, 2005). This form of mathematics valorizes rationality in prescribing ways of thinking and being, such as performing fast, accurate, and efficient calculations instead of, for example, prioritizing creative solutions that acknowledge many possible ways of arriving at them. Mathematics education in the United States has typically sought to socialize students

into this understanding of mathematics, positioning the teacher and even more so, the textbook as the ultimate authority on who is correct and how students will spend time in class (Herbel-Eisenmann, 2007), and framing mind-numbing rote skills practice as "diligence." Consequently, students' mathematics classroom experiences are frequently discouraging, dehumanizing, and rarely reflective of what those who love doing mathematics love about it: its potential for creating beauty, joy, and flourishing.

Common practices in mathematics education such as tracking, assigning repetitive procedural homework, and valuing speed over reflection and rule following over rule breaking contribute to what Dernikos and colleagues (2020) call "affective violence" in schools, which "humiliates, shames, misrecognizes, [and] trivializes" students (p. 18). R. Gutiérrez (2018) adds that by separating mathematics from politics or values or ethics, mathematics education constitutes a "slow violence" (p. 3) that dehumanizes students over years. Situated within the "state-sanctioned violence" of schooling (e.g., Yoon, 2019, p. 421), mathematics education also reflects the oppressive systems that structure schooling and society and stratifying students' access to mathematical learning opportunities; these nested layers of violence are particularly salient for students who have historically been marginalized due to racism and other forms of oppression. Martin and colleagues (2019), for example, describe the "physical, symbolic, and epistemological" violence (p. 32) that mathematics education enacts upon Black children, threatening their lives, regulating and punishing their behavior, denying their brilliance, and constructing them as inferior. Bullock and Meiners (2019) call mathematics education "an agent of the carceral state" (p. 339) for the role it plays in pushing students—especially the "Black, Brown, queer, migrant, poor" (p. 344) students who are already targeted by other institutions within the prison-industrial complex—out of higher mathematics and out of school, serving as an organizing logic to reify social hierarchies, and promoting racist narratives through statistics and algorithms. Finally, mathematics education has also been conceptualized as a "White institutional space" (Battey & Leyva, 2016; Martin, 2013) where the discourses, organizational logics, material artifacts, available identities, and other elements mirror and manufacture White supremacist ideologies and racial hierarchies.

As a result, mathematics education communicates to most people—and especially to Black, Latinx, and Indigenous people; people who identify as women and girls; people whose ways of being and knowing and whose home, cultural, and linguistic practices are dismissed as diverging from dominant mathematical practices—that they do not matter. Love (2019) argues that "dark people have never truly mattered in this country except as property and labor" (p. 7), framing mattering—and the quest to matter—as the difference between survival and thriving. Although industries and governments may seek to monetize or weaponize people's quantitative reasoning skills (e.g., Basile & Lopez, 2015; Chesky & Wolfmeyer, 2015) through what Martin (2019) calls "nationalist, militaristic, and xenophobic appropriations of mathematics education" (p. 467), the survival eked out when mathematics education turns people into capital (de Roock & Baildon, 2019) is far from thriving. Even worse, the exclusionary culture of mathematics education *convinces* people that they do not matter, by excluding them and their ideas from being considered smart or valued (Louie, 2017) and denying them their "undisputed dignity" (Anna Julia Cooper, as cited in Love, 2019).

Mathematics Teachers

Mathematics teachers, then, enact a specific kind of power and authority in their institutional roles. They are often responsible for implementing the gatekeeping practices of mathematics education, such as deciding whether students pass or fail, who has access to

advanced courses, and who can be successful on college admissions exams (Martin et al., 2010; Stinson, 2004). They are also responsible for enacting the practices of mathematics education described above in their classrooms. As instruments or at least as messengers, mathematics teachers have historically served as a vehicle for the epistemic and emotional violence that convinces students that they are not mathematical, not smart, or that they do not matter. Furthermore, mathematics teachers inhabit this power and authority from racialized and gendered bodies that are themselves laden with history—which are often, given the disproportionate Whiteness and womanhood of teachers compared to students in United States public schools, histories of oppression (e.g., Coloma, 2012; Leonardo & Boas, 2013). Given the weight of this context and how oppression has been ideologically justified and naturalized, continuing a fairly traditional exercise of power and authority would be commonsense to them (Kumashiro, 2004).

Many mathematics teachers are eager to interrupt the ongoing perpetuation of oppression from their positions as gatekeepers and architects of students' mathematical experiences (see, for example, Kokka's [2018] work with STEM teacher activists, but also that most teachers would claim to care about and want the best for their students). They engage in well-studied mathematical practices, participation practices, and relational practices aimed achieving equity in the classroom (Spencer et al., 2017). However, resisting the abundant social forces that reproduce the status quo is a monumental task. For example, Louie (2019) has demonstrated that even teachers who value students' mathematical agency can subscribe to dominant ideologies that fix students in hierarchical positions and thus undermine students' mathematical agency.

Mathematics teachers are inextricably complicit within the violent nature of mathematics education. Even if they were to perfectly implement every researched equitable instructional

practice and effectively shift their authority to students, they cannot singlehandedly overturn historical, structural, and symbolic systems of oppression, nor dissolve the material consequences of these systems. In that context, then, how can teachers and students in mathematics classrooms be in a relation that accounts for historical and existing violence without continuing violence? Next, I turn to literature on care theory and culture-rich pedagogies for their takes on the ethics of right relation in the context of oppressive systems.

Care Theory

Care is almost universally taken for granted as being desirable in educational contexts, and many scholars have sought to expand common perceptions of caring beyond a teacher's "personality trait or a warm, fuzzy feeling" shared by teacher and students (Bartell, 2011, p. 66). DeNicolo and colleagues (2017) offer an extensive review of the research literature concluding that care is essential to students' experience of school belonging, particularly for students from historically marginalized communities. They specify, however, that they are referring to critically conscious and authentic care, in contrast to what Valenzuela (1999) called aesthetic care: a superficial care, or care for objects and ideas rather than people. Valenzuela argues that aesthetic care, rather than authentic care, predominates in educational settings in the contemporary United States, and Toshalis (2012) adds that aesthetic care is often "sentimental and emotive" (p. 4), functioning to "provid[e] comfort for the care-giver, positio[n] the teacher as superior, and suppl[y] explanations that release the teacher from complicity in any perceived deficit" (p. 27). Attending to care as these scholars do draws attention to students' and teachers' affective experiences in learning, not just their intellectual experiences.

Perhaps the most often-cited care theorist in education, Noddings (1992) calls for an ethic of caring as a moral orientation to teaching. She conceptualizes caring as a relation rather than a

unidirectional caring for. Care must be attentive to students' actual needs, not just to what teachers believe students' needs are, and it must be received by students, for it to count as care. Hackenberg (2010) builds on Noddings to conceptualize "mathematical caring relations" as those that support mathematical learning through attending to both cognitive and affective features of interaction, focusing on student learning as the reason for caring. Similarly, Jaber (2016) claims that care in STEM education requires teachers to interpret students' affect as part of their sensemaking, and Krist and Suárez (2018) define epistemic care as integrating fidelity to persons—committing to students' growth and well-being—alongside fidelity to knowledge in learning science. These disciplinary theories of care assume that relational interactions between teachers and students control students' access to disciplinary learning, such as by positioning students as more or less mathematical and shaping the mathematics identities available to them (e.g., Battey, 2013; Hand & Gresalfi, 2015; Na'ilah Suad Nasir, 2002). They have comparatively little to say, however, about whether these caring relations are sufficient for overcoming or eliminating such consequences given the situatedness of mathematics classrooms within a society characterized by inequitable and oppressive power relations.

In this vein, Noddings is frequently criticized for not addressing politics and power in her theorizations of care: for proposing relations that are nonviolent but not necessarily restorative. To remedy this gap, Noblit (1993) defines caring as "the ethical use of power" (p. 24), and Tronto (1993) argues that care both illuminates relations of power—by revealing who cares for whom—and can play a role in changing relations of power—by changing assumptions about dependence, autonomy, needs, and vulnerability. She gives the example of an office worker and the janitor in their office; the office worker may not feel vulnerable compared to the janitor, who is more likely to suffer the vulnerabilities of having a low-paying job that requires less formal

education and holds fewer opportunities for advancement, but if the janitor "stopped [working], the office worker's vulnerability would be exposed" (p. 135).

Women of color have long written about the political nature of care, linking care for marginalized students to justice. Making a case for native language teaching for linguistic minority students, Bartolomé (2008a) notes that love is always both political and ideological. Therefore, a teacher's cariño for students must be driven by political and ideological clarity about "the unequal power relations among cultures," both broadly and within the structures of mathematics education, "that result in unequal status and treatment in society and schools" (p. 14). In Beauboeuf-Lafontant's (2002) study of Black women teachers, she identifies political clarity as central to an ethic of womanist caring, which brings into teaching:

the agency that each of us has to treat others as our own; the obligation we have to understand as fully as we can the world around us; and the responsibility we have to make sure that our actions contribute to the larger human goal of freedom for all (p. 84). Similarly, Rolón-Dow (2005) builds on Valenzuela and other Lat/Crit scholars to argue that a transformative praxis of teaching calls for intimate connections between teachers and students that are grounded in a historical understanding of students' lives, race-conscious pedagogy, and attention to both individual and institutional forms of care. McKinney de Royston and colleagues (2017) provide a concrete example of what this kind of politicized caring could look like: interactional smoothness made possible by, among other things, communal bonds between teacher and students. These scholars contend that right relation between mathematics teachers

unflinching in its embrace of students' historicized selves (K. D. Gutiérrez & Jurow, 2016)—who

and students, in the context of racism and intersecting systems of oppression, must be

they are in the context of time—and in its critical analysis of and resistance to hegemonic power structures. Yet, the structures of mathematics education often makes this embrace impossible.

Culture-Rich Pedagogies

Although some of the literature on culturally relevant, responsive, and sustaining pedagogies treats culture and race interchangeably or otherwise depoliticizes race (Milner, 2017), the foundational texts and most highly-cited scholars in this field take racism and race-related forms of oppression, such as raciolinguistic discrimination based on conflating racialized bodies with linguistic deficiency (Alim et al., 2020; Flores & Rosa, 2015), to be axiomatic. Scholars working in this tradition assume that students' experiences of school and the possible relationships between teachers and students are indelibly colored by racism, which shapes curriculum, pedagogy, institutional practices (e.g., Lewis & Diamond, 2015), and interactions in schools. As a result, right relation—in the form of equity or racial justice—relies on teachers' cultural competence and critical consciousness as well as their ability to build affirming relationships with students (Ladson-Billings, 1995). Cultural competence, in fact, can be considered a prerequisite to caring for marginalized students and developing meaningful teacher-student relationships (Howard & Rodriguez-Minkoff, 2017).

As in the care literature, the culture-rich pedagogies literature often identifies characteristics of teachers labeled as exemplary (e.g., Gay, 2000; Ladson-Billings, 1995) or articulates dispositions, either of teachers labeled exemplary or that teachers should develop in order to be culturally responsive (e.g., Aguirre & Zavala, 2013; Parker et al., 2017; Villegas & Lucas, 2002; Warren, 2018). Within mathematics education, scholars have also given examples meant to illustrate the existence of strong teacher-student relationships: Averill (2012) describes teachers attending to students' thoughts and feelings, sense of community, spiritual well-being,

and physical health through caring practices; Bonner and Adams (2012) describe a teacher setting high expectations, providing material essentials for students, and maintaining commitments to students outside of the mathematics classroom; and Nicol and colleagues (2013) describe differences in Aboriginal and non-Aboriginal teachers' approaches to building relationships. Maloney and Matthews (2020) find that teachers' empathetic care, for mathematics students of color, is linked more strongly to a sense of connectedness and perception of mathematics being valuable than transactional care. These proofs of concept effectively illustrate that some teachers' relationship-building practices meet researchers' criteria for being culturally relevant or responsive: that instead of mirroring the violent relations characteristic of much mathematics education, these teachers have constructed restorative relations that account for the hegemony of dominant schooling practices and how oppressive systems shape teachers' and students' lives.

Limitations of the Care and Culture-Rich Pedagogies Literatures

Conceptualizations of politicized care and of teacher-student relationships within culturerich pedagogical frameworks demand restorative relations instead of the violent relations that
often characterize mathematics education despite teachers' best intentions. Some scholars have
explored these relations specifically in the context of mathematics education; Bartell (2011)
conceptualizes "caring with awareness" as a practice of racial, cultural, political, and academic
caring. Caring with awareness is a nonviolent and restorative relation and therefore has the
potential to mitigate the academic, social, cultural, and political consequences of mathematical
violence. Bartell's emphasis, however, is on students' mathematical learning rather than
theorizing an ethics of relational work, and an ethical theory requires attention to several
additional questions. First, should students' mathematical learning even be the primary goal of

relational work? Or are there situations in which students' mathematical learning might be subordinated to other ethical concerns?

Second, when the capacity for relationship-building is located in individual teachers and their characteristics, dispositions, or actions, as it often is in both care theory and culture-rich pedagogical frameworks, what role are students expected to play in meaningful teacher-student relationships; do they simply receive or refuse what their teachers do, or do they play a more active role in co-constructing the relation? Are teachers' characteristics, dispositions, and actions necessary, sufficient, both, or neither? Can strong relationships be planned for, as seems to be the implication of teacher education models aiming to prepare teachers to teach in culture-rich ways? Are there universalities to such relationships, or are they so specific and particular to individual teachers and students that any commonalities might only be coincidental?

Furthermore, the existing literature tends to treat mathematics education as a context in which teacher-student relations occur. As such, it does not explicitly account for how the specific histories, cultures, and practices of mathematics education shape both what constitutes right relation between mathematics teachers and students and what is possible within mathematics education; in other words, it does not treat mathematics education as an equal actor. An ethical theory of relational work, then, would address these concerns about agency and power by putting a stake in the ground about 1) what constitutes right relation; 2) in the broader social, cultural, historical, and political context of what Shotwell calls exclusion zones; 3) given that the particular cultures and traditions of mathematics education—as a purportedly neutral, apolitical, and universal endeavor—make it difficult to account for affects and contexts; and 4) with teachers as important contributors to but not determinants of relations, because teachers are not the only ones with agency. For such a theory, I turn to feminist new materialist theories of ethics.

As mentioned in the introduction to this chapter, my interest in these theories emerged in concurrence with the empirical analyses I report in subsequent chapters, rather than serving as an antecedent, but I introduce them now so that I can draw on the conceptual resources they offer as I explain my findings.

Response-ability: A Feminist New Materialist Approach to Ethics

To think about ethics, agency, and power in right relation, I turn to feminist new materialists like Barad, Haraway, and those who have interpreted and extended their work. The literature on care and culture-rich pedagogies described earlier assumes agentic individuals between whom relationships are very important. By contrast, the relational ontology that undergirds feminist new materialism takes entanglement as its premise: relations are unavoidable. We are all entangled in relations with other humans, non-humans and more-thanhumans, with immaterial discourses and material infrastructures, with the past and the future. Entanglement, however, "is not simply to be intertwined with another, as in the joining of separate entities, but to lack an independent, self-contained existence" (Barad, 2007, p. ix). In other words, individuals exist only insofar as much as they come to exist through intra-acting³ with entities who are never entirely separate from them. And, as existence emerges through these (relational) intra-actions, so do realities: a creative process referred to in feminist theory as worlding (e.g., Barad, 2007; Haraway, 2016; Stewart, 2007). From the perspective of mathematics teachers, then, the question of right relation is a question about what world they are creating with and for their students.

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³ Barad coins the term intra-acting to distinguish conceptually between interactions that happen between discrete individuals and intra-actions that happen among those who are always and already entangled.

In a feminist new materialist relational ontology, agency is not about individual choice or even individual action. Instead, "agency is about response-ability, about the possibilities of mutual response" (Dolphijn & Van der Tuin, 2012, p. 55). Cavarero draws on Levinas to argue that human beings "are, of necessity, exposed to one another in our vulnerability and singularity, and that our political situation consists in part in learning how to best handle—and to honor this constant and necessary exposure" (as cited in Butler, 2005, p. 31). Response-ability lies in this handling and honoring of exposure: what responses are possible as we intra-act, and how should we respond? The "should" in this question makes response-ability a question of ethics or, a way of being—but an ethics that is "not about right responses to a radically exteriorized other, but about responsibility and accountability for the lively relationalities of becoming" (Dolphijn & Van der Tuin, 2012, p. 69). Haraway notes that response-ability is trickier—and riskier—than ethics in which the right action can be calculated based on predetermined rules or formulas, because response-ability depends on "multidirectional relationship, in which always more than one responsive entity is in the process of becoming" (as cited in Reardon et al., 2015, p. 34). Instead of being an ethics based on universal principles, then, response-ability is relational, responsible, and situated.

Tronto (1993) notes that responsibility has traditionally been theorized as arising out of obligation or emerging through reciprocity, which recalls common conceptions of cultural responsiveness whereby teachers have a duty to respond to students' cultural backgrounds. By contrast, grounding responsibility in vulnerability expands its scope: rather than being responsible because we are bound to a particular duty or particular debt, we are responsible because we, like every other being, are vulnerable. But vulnerability is not equally distributed; some beings, in some contexts, are more vulnerable than others. Consequently, responsibility

becomes a political and cultural practice of caring. Responsibility illustrates who and what we care for, care about, and how. As Barad (2007, p. 394) writes:

We are accountable for and to not only specific patterns of marks on bodies—that is, the differential patterns of mattering of the world of which we are a part—but also the exclusions that we participate in enacting. Therefore accountability and responsibility must be thought in terms of what matters and what is excluded from mattering.

Therefore, response-ability is not just any response, but response that "entails an ongoing responsiveness to the entanglements of self and other, here and there, now and then" (Barad, 2007, p. 394), and specifically, response that takes a position on who and what matters.

Taking such a position "requires the risk of being for some worlds rather than others and helping to compose those worlds with others" (Haraway, 2016, p. 179). It is neither a neutral nor a purely reactive practice. For Barad, this being-for and helping-to-compose is the foundation of ethics: "ethics is about mattering, about taking account of the entangled materializations of which we are part, including new configurations, new subjectivities, new possibilities. Even the smallest cuts [in the fabric of the world] matter" (Dolphijn & Van der Tuin, 2012, p. 69). By this, Barad means that every cut made in intra-action—the boundaries produced and the differences marked—matters, insignificant as it may seem on the surface. We are ethically responsible, then, for the cuts and consequently the mattering we participate in. In the context of mathematics education as violence, these cuts and this mattering is key to thriving for dark people (Love, 2019) and therefore to justice.

Much as there are no fixed rules for right behavior in an ethic of response-ability, however, there can be no fixed vision of justice, because justice "is not accomplished by remaking the entire world around a pre-figured sense of what justice looks like... justice

imagines a world in the making between past and future" (Reardon et al., 2015, pp. 24, 29). Of the indeterminacy of justice, Barad (2007, p. x) writes:

There are no solutions; there is only the ongoing practice of being open and alive to each meeting, each intra-action, so that we might use our ability to respond, our responsibility, to help awaken, to breathe life into ever new possibilities for living justly.

The ongoing responsiveness of response-ability, then, or the collaborative worlding that we participate in in intra-action, must be in service of "living and dying well together" (Haraway, 2016, p. 29) and of "rendering each other capable" (Despret, 2004, 2016; as cited in Bozalek & Zembylas, 2020, p. 27). In other words, living justly is a matter of constant responding to cultivate further response-ability, in ourselves and in those humans, non-humans, more-than-humans, discourses, and infrastructures we intra-act with.

Living justly, every cut mattering, constantly cultivating further response-ability—this may seem like a tall order given the deeply-rooted histories and patterns of oppression pervading the immaterial discourses and material infrastructures we intra-act with. It is. An ethics of response-ability does not offer a simple, magical solution ("just be or act this way and we will achieve justice"), because simple magical solutions cannot account for the complexity of entanglements nor for the simultaneous significance and insignificance of any individual or action. Instead, Shotwell (2016) concludes that "all there is, while things perpetually fall apart, is the possibility of acting from where we are" (p. 4). She draws on Haraway's work to frame this possibility, engendered by our situatedness and meager though it may seem to those seeking grander and more universal theories, as "partial, finite, adequate, modest, limited—and yet worth working on, with, and for" (p. 5).

Response-ability in Education

Within education research, some scholars have considered response-ability in the ethics of conducting education research (e.g., Clark/Keefe & Haines, 2019; Daelman et al., 2020; Mayes & Wolfe, 2020) and others have theorized about the role of response-ability in knowing (Stengel, 2004) and in the ethics and aims of mathematics education (Atweh, 2011; Atweh & Brady, 2009). Kayumova and colleagues (2019) illustrate the potential of conceptualizing response-ability as a goal of STEM education for marginalized students, supporting their "ability to respond instead of having to 'yield' to 'others,' and [be] positioned as authorities on their own spaces and ways of being" (p. 219) in contrast to deficit-based "empowerment" narratives. Little empirical research, however, has investigated practices of response-ability in K-12 education (for an exception, see Weldemariam's [2020] study on the contribution of theatrical pedagogies to pre-school children's response-ability and becoming-with bees).

To link response-ability as a theoretical idea to the practice of education, I look to the work of Bozalek and colleagues, who have sought to conceptualize and practice "response-able pedagogy" in higher education institutions around the world (Bozalek et al., 2018; Bozalek & Zembylas, 2020; Tronto, 2020), and scholars (including Barad) at the University of California Santa Cruz, who worked with graduate students interested in response-able science (Reardon et al., 2015). Bozalek and Zembylas (2020) define response-able pedagogies as "ethico-political practices which incorporate a relational ontology into teaching and learning activities" (p. 28), and identify four core relational practices that characterize such pedagogies: attentiveness, curiosity, responsibility, and being rendered capable. From the perspective of a teacher, then,

to be response-able actually requires altering one's power as "the responsible party" in a number of important ways. It demands that one be more observant and curious... It

requires not only a patience to engage with the world more broadly, but the courage to face uncertain, imprecise, hidden and complex realities that are in flux and that are all implicated in contemporary practices of power, care and knowledge production and transmission. (Tronto, 2020, p. 159)

Reardon and colleagues (2015) similarly emphasize the importance of attentiveness and humility—but also, action—in response-able relation, noting that to enact an ethics of response-ability, "it is necessary to become modest and do the hard work of attending to the specificities of one's situation (Haraway, 1997)" (p. 24).

Response-ability in Mathematics Education

Barad and Haraway stress that entanglement does not just link humans, and that the non-human and more-than-human contribute as much as humans to the worldings that unfold. As participants in Algebra 1 classrooms in one of the largest school districts in the United States, for example, the teachers and students in this dissertation are entangled with the material and discursive realities of infrastructures such as: public schooling and education reform; urbanization, ghettoization and gentrification; the theft of Indigenous land, Spanish missions, and racial capitalism in North America; global migration, citizenship, and immigration policy; and, perhaps most saliently to this dissertation, mathematics and mathematics education.

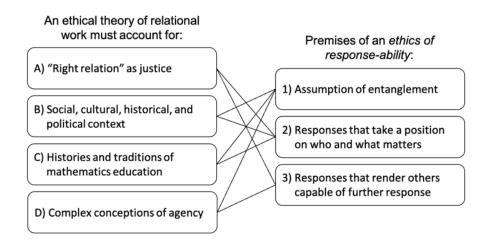
Boylan (2017) draws on Barad's work to constitute mathematics as an "agentic material-discursive practice that... work[s] on and with human and other-than-human existences" (p. 5); as the histories, cultures, and practices of mathematics education intra-act with teachers and students, they can constrain the possibilities for response-ability or enable it. For example, traditions of valorizing speed and accuracy over creativity and reflection shift teachers' attention to some forms of mathematical thinking and hence some students over others, making cuts about

who matters that reify existing social and mathematical hierarchies. Similarly, a tendency to privilege abstraction and universality in determining the utility of mathematical tools cuts against taking seriously the specificities of one's situation.

An ethics of response-ability in mathematics education, then, may be summarized in three premises. First, it assumes the entanglement not only of mathematics students and teachers, but also their entanglement with mathematics education itself and with other infrastructures as well. This premise accounts for social, cultural, historical, and political contexts in mathematics education and beyond, as well as creates a complex conception of agency. Second, following an ethics of response-ability in mathematics education means that, in constantly intra-acting, each actor must aspire to constant responses that take a position on who and what matters. This premise makes claims about what constitutes justice in the context of mathematics education and beyond. And third, these responses must render others capable of further response; this premise makes claims about what constitutes justice given a complex conception of agency. These premises are depicted in Figure 1.

Figure 1

Premises of an Ethics of Response-ability



Conceptual Framework

Relational Work

In this dissertation, I will refer to Haraway's modest and hard work of attending to specificities as relational work. Within mathematics education literature, "relational work" has traditionally been used to describe the relationships between teachers, students, and content (e.g., Grossman et al., 2007; Lampert, 2012), and teacher educators have sought to cultivate "relational practices," such as attending to student thinking and building trust, that teachers can adjust to suit different student populations (Ball & Forzani, 2009). Philip (2019), however, critiques this approach towards relational work as universalist and as overlooking the teacher's "positionality and the ideologies that construct difference and social hierarchy" (p. 4). Instead, Philip highlights the self-knowledge and critical knowledge required in relational work, as well as its aspirational dimension in "prefiguring"—again, an ongoing pre-figuring rather than working towards an already-prefigured world—a more just future. Bang and Vossoughi (2016) also link relational work to justice, suggesting that "privileging subject-object relations over and above subjectsubject relations may help to account for some of the ways in which some forms of epistemological hierarchy are reproduced and structural inequalities are left intact" (p. 180). In other words, power dynamics can be evaluated and challenged, if need be, by constantly reassessing the relations between human, non-human, and more-than-human others, the norms that govern such relations, and whether these norms ought to exist (Culbertson, 2013).

"Attending to the specificities of one's situation" (Reardon et al., 2015, p. 24), then, is relational work because attending to the specificities of our situatedness under the premise of entanglement means attending to relations: who and what we are intra-acting with, and how we are responsible in those intra-actions. Because it is inextricable from "values, ethics and affect"

(Boylan, 2017, p. 4), relational work is always being done in classrooms. It may be done uncritically, in ways that perpetuate existing injustices and the default dehumanization of mathematics education, or deliberately, in service of interrupting a hegemonic and exclusive status quo. In line with the feminist new materialist perspective, the justice-orientation advocated by Philip, Bang, Vossoughi, Boylan, and R. Gutiérrez—who cites the Indigenous concepts of In La'kech, reciprocity, and nepantla to bring learning and relationships with more-than-human persons into a practice she calls mathematx (2017)—elevates relational work from the instrumentality implied by traditional uses of relational work in mathematics education research to an ethical level. Attending to right relation in mathematics education is no longer just about compelling students to learn more mathematics, but rather about co-existing in complex and often-violent social and ecological structures.

Teachers' Ethical Stance

In this dissertation, I focus on teachers' perspectives on relational work, even though I have argued that one of the advantages of response-ability as an ethical theory is that it acknowledges the agency of students, mathematics education, and other infrastructures in worlding. This is in part a decision of expediency; I have neither the data nor the space in this dissertation to account more fully for other actors' co-construction of response-ability. It is also, however, a deliberate choice to foreground the "do what we can" (Shotwell, 2016, p. 127) of teachers, whose agency cannot be understated given their centrality to the worlding of mathematics education, and yet whose agency is often overattributed by those who would scapegoat teachers for failing to reverse systemic oppression (Chazan et al., 2016; Kumashiro, 2012). What teachers can do is both limited and significant, and it is worth focusing on because of teachers' particular influence in co-constructing students' experiences of mathematics

education. Additionally, I center what it means for teachers to be in right relation with students—as opposed to right relation with families, administrators, mathematics education itself, or other intra-actors—because the violent relations that characterize mathematics education so often recruit teachers into dehumanizing students, and because of the intimacy of what Franck calls "living with somebody five hours a week" (*fieldnotes*, 10/2018).

To explore how teachers engage their response-ability, I use the concept of teachers' ethical stance. The research literature has described teachers' approaches to their practice using several constructs that appear, on the surface, to be similar to the construct of an ethical stance. For example, teachers are often asked to submit written statements of their teaching *philosophies* as part of the hiring process, and terms like beliefs, vision, and values are also used to describe what matters to teachers. An extensive literature on teacher beliefs, much of which is grounded in cognitive and social psychology, has been critiqued for being essentialist and for making simplistic assumptions about the relationship between teachers' beliefs and their practices (Buehl & Beck, 2014). Scholars have argued that teachers' beliefs often do not align with their practice in general (Ball & Forzani, 2009; Cochran-Smith et al., 2016), in teaching mathematics (Thompson, 1992), and in multicultural education and issues of equity (Hollins & Guzman, 2005; Picower, 2009; Tatto, 1996). Teachers' beliefs and their visions often represent "images of ideal classroom practice" (Hammerness, 2006, p. 1) rather than what is actually enacted. Analyses of teachers' ideologies (Bartolomé, 2008b; Flores & Rosa, 2015; Philip, 2011) likewise center mental frameworks, albeit privileging cognitive explanations that are linked to broader societal norms and narratives about power and not just idiosyncratic ideas.

Teaching, however, "is more than a body of knowledge... it is a way of being and feeling, historically, in relation with others" (Zembylas, 2005, p. 468). Tate (2007) postulates that

teachers' academic responsibilities may be visible in their method, or what they choose to do, but relational responsibilities become evident only in their manner, or how they choose to do it.

Consequently, I focus on teachers' *stance*, which differs from concepts like beliefs, philosophies, or vision in that it is extrapolated from teachers' enactments. It is an etic, post hoc description of teachers' approach rather than being an emic, a priori prescription of what teachers aspire or intend to do. In other words, *stance* emerges from teachers' embodied and affective actions, which may or may not align with their cognitive beliefs, visions, values, philosophies, or ideologies.

Because my description of teachers' *stance* is constructed from both observational and interview data, however, it captures both the performance of what Franck calls "teacherly responsibility" and the sense they make of it. Existing research highlights that teachers actively construct understandings of their work based on their social context and the conceptual resources that are available to them (e.g., Coburn, 2001; Hall & Horn, 2012; Horn & Kane, 2015). The stories they tell about their work illustrate "how people create the world in which they find themselves, participating in a history that is of their own making" (Pereira & Doecke, 2016, p. 545). That said, an ethical stance does not need to be articulated, or even articulable, to be enacted. Teachers' actions may be inconsistent with their sensemaking, reflecting instead their aspirations and/or the narratives readily available to them (e.g., Chen, 2020; Louie, 2019; Yoon, 2016). Nevertheless, sensemaking provides one window into one component of teachers' ethical stance, particularly when triangulated with data from researcher observation and students' perceptions; what teachers can and do articulate offers insight into how they are experiencing their navigation of entanglements.

What is to Come

In other words, this dissertation explores response-ability as an ethical relation between humans and the specific infrastructure of mathematics education in response to systems of oppression: a mathematico-ethico-political investigation, if you will. I argue that Franck and Clark approach relational work as ongoing, expansive, and formed by affective inheritances, and in doing so, tender alternatives to taken-for-granted conceptions of agency and power in mathematics education. Response-ability as an ethical stance is, as Haraway suggests, modest; on a day-to-day level, it creates what Weis and Fine (2012) call "sweet spots of refuge," where students may experience a version of mathematics education that is less dehumanizing than what they otherwise could have expected. Franck and Clark, to paraphrase Shotwell, do what they can from where they are, given their situatedness as mathematics teachers in their specific public school, with their specific students, in their specific city, on the specific land that they are on. At the same time, however, this doing-what-they-can opens up possibilities for transforming claims about who and what matters, for rendering others capable and cultivating further response-ability: for living justly.

Chapter 3

METHODS

This chapter contains a description of the research context, data collection, and data analyses that formed this dissertation. But, because "all qualitative researchers are philosophers" (Denzin & Lincoln, 2011, p. 12, as cited in Kim, 2015), it also contains a philosophical stance on what research is and how to do research. I have attempted to synthesize and articulate the process of engaging this dissertation in a form that acknowledges the complexity and not just complicatedness of being a novice researcher; at the same time, I hope I have included each of the elements that readers expect from a methods chapter. Since a dissertation is supposed to be an assessment of what I have learned in graduate school, I begin with a detour into what I have learned about research and about myself as a researcher, and how these onto-epistemological perspectives are reflected in this dissertation.

Stance on Research

My experience with formal—as recognized by academic institutions—research has been an iterative process of disillusionment and discovery; as an undergraduate, I declared a major in psychology because I was interested in human behavior but found laboratory studies attributing behavior solely to individual differences to omit important social/societal influences. I decided to also major in organizational behavior so I could study how systems and institutions enabled and constrained human behavior, and then took a nearly-decade long break from formal research to pursue practice as a means of creating more immediate, material change than I thought theory could accomplish. Taking graduate courses with Department of Teaching and Learning faculty at

Vanderbilt continued my social science training in qualitative methods, but I felt compelled to also read (post-positivist, post-qualitative) postmodern, poststructural, and posthumanist perspectives that allowed me to explore my discomfort with 1) the role of the researcher as an observer and interpreter of an objective reality and 2) coding as a means of neatly classifying and sorting that which often has complex and blurry boundaries, especially when categories, no matter how repeatedly revised, never seemed to capture what most viscerally intrigued me (e.g., MacLure, 2013). That said, pursuing formal academic research requires, at least in this moment, some sort of claim to systematicity and methodology, and so this dissertation represents where I am in making sense of what I think I understand and what I yearn to understand about research; as Lather and St. Pierre (2013) write, "we always bring tradition with us into the new, and it is very difficult to think outside our training, which, in spite of our best efforts, normalizes our thinking and doing" (p. 630).

Over the history of this dissertation, I dove into texts about variants of ethnography (e.g., Erickson & Mohatt, 1982; Noblit et al., 2004); phenomenology (e.g., Moustakas, 1997; van Manen, 2014); autoethnography (e.g., C. Ellis, 2004; Forber-Pratt, 2015); narrative inquiry (e.g., Clandinin & Connelly, 2000; Gilligan et al., 2006; Kim, 2015); "post" methods (e.g., A. Y. Jackson & Mazzei, 2012; Lather, 2016; MacLure, 2013; Springgay & Truman, 2018); and feminist new materialist inquiries (e.g., Bozalek & Zembylas, 2017; Clark/Keefe & Haines, 2019; Ringrose & Renold, 2014), among others, to find a methodological fit for the type of work that felt worth doing. I had not intended to write a dissertation as philosophical as this one is, but it seemed impossible to adhere strictly to conventions of social science research that positioned empirical data as representing the world as it exists "at arm's length" from the analyst (MacLure, 2013, p. 168), or to conventions of phenomenology that assumed the possibility of bracketing

oneself off from the data (e.g., van Manen, 2014). In "continually recycling thought/action/reflection/writing in ways that pointed to new theoretical directions," or what Hermes (1998) calls research as "situated response" (p. 157), I found "theorization, like experimentation, [to be] a material practice" (Barad, 2007, p. 55) in which I, as a researcher, engaged materially and dynamically with Franck, Clark, the legacy of mathematics education, and my own past and present. As a result, I take up what Denzin and Lincoln call "bricolage," piecing together analytic methods from different traditions to be "fitted to the specifics of a complex situation" (2011, p. 4, as cited in Kim, 2015, Chapter 7). Bricolage, as a form of disciplined interdisciplinarity, allows for the integration of empirical and interpretive modes of inquiry (Kincheloe, 2001).

In this onto-epistemological investigation about what research is and how I can do it, I am heartened by what Todd (2008) writes, of her own educational research drawing on Levinasian theories of learning: that learning is not about applying (ethical) principles but instead, it is an orientation towards creating our own response. I also remember Wilson's (2008) observation that "if research doesn't change you as a person, then you aren't doing it right" (p. 83). Given the context in which this statement was written, I take it to mean that not only should the people you interact with and the things you "learn" over the course of research change you, but also, the process of doing research itself. Thus, I view this dissertation as an exercise in my learning, not only about mathematics teachers' ethical stances on relational work, but on research. As a result, it is perhaps more of an ontological and epistemological exploration of research than one might expect of a published social science research paper with (at least a narrative of) methodological coherence. Ultimately, one position I do feel confident about is that research, for me, is about being in right relation: with myself, by doing work that meets my

internal ethico-onto-epistemological standards as well as external standards of what constitutes research; with the teachers and students I engage with through research; with the histories and contemporary realities of mathematics education; with our situatedness within complex and overlapping structures that are often oppressive. Next, I articulate what I think this means about being a researcher, for me.

Being a Researcher

As with all research, though mostly acknowledged by qualitative researchers, the researcher serves as a primary instrument in this study (Spradley, 1980), and my moral commitments serve as a guideline for ethical ethnography "in the absence of a foundational truth" (Noblit et al., 2004, p. 23). Instead of attempting, impossibly, to minimize my role in this research, I aim to embrace strong objectivity in the tradition of feminist epistemologists like Sandra Harding: through an acknowledgment of the partiality of my perspective (Haraway, 1991) and ongoing and critical reflection on how my social situatedness shapes the knowledge I have access to and how I am accountable to the communities affected by my research (Fine, 2006). Although it would be impossible to describe—or even be aware of—all the elements of my positionality that were relevant to this particular research project, here I offer a few that were especially salient throughout data collection and analysis, and then describe how this positionality holds me accountable in research.

Professionally, I have spent over a decade working in high schools and with high school students, first as a mathematics teacher and then as a teacher educator supporting pre-service and novice high school mathematics teachers. Almost all of my professional experience has been in rural and urban schools marked for additional funding through Title I and where almost all the students were students of color, although this stands in marked contrast to my personal

experience of education, where I was always one of the only students of color in a predominantly white, middle-class suburban public school district. Due to the legacy of longstanding racism, segregation, and inequitable funding in the U.S. public school system, this means that most of the hundreds of high school mathematics teachers I have observed have been frustrated by their inability to create the conditions for their students to attain dominant forms of academic achievement, have struggled to balance this frustration with their ideals and aspirations, and have felt pressure from policies, administrators, families, colleagues, and the general public to do something differently from whatever it was that they were doing, regardless of what that whatever was. As a result, many resorted to logics of discipline, control, and management to impose a sense of order on the turbulent task of being a teacher today.

Because of how much time I have spent in particular kinds of mathematics classrooms, walking into high school mathematics classrooms in one of the largest school districts in the United States felt in some ways like going home, albeit—as with many homes—an uncomfortable home, where tension and history are ignored for the sake of getting on with the business of living/learning. I will say more about my experiences as a mathematics *student* in Chapter 6, but the ways I have experienced mathematics education as a student, teacher, teacher educator, and researcher, in conjunction with the traditions of mathematics education in the United States, have led me to expect rote, procedural interactions, a serious tone, and a miasma of negative affects—apathy, disengagement, anxiety, frustration, and resentment, among others—to be present more often than sophisticated mathematical conversations among students and affects such as joy, delight, playfulness, leisure, and affection. However, I believe that it is not only theoretically possible for the latter to be true of mathematics education, but that there are already existence proofs of such classrooms. Therefore, I approach my research aware that

there is lived truth to narratives about mathematics education as being hegemonic and dehumanizing (e.g., Greer & Mukhopadhyay, 2012; R. Gutiérrez, 2018; Lawler, 2016) and that these narratives are also complicated; as Gordon (2008) writes, the circulations and functioning of power are "never as transparently clear as the names we give to them imply" (p. 3).

I have worked mostly east of the Mississippi River, in school communities with few or no Asian Americans. Because I identify and present as a woman of East Asian descent, I am accustomed to students overtly and covertly questioning my presence when I visit schools, treating me as an object of curiosity or assuming I must be family to the only Asian American teacher in the building. At Banneker, however, there were several teachers in the building who identified as being of East or Southeast Asian descent, and this rendered my presence relatively unremarkable. Only once—to my knowledge—did anyone assume that I was related to someone else in the building. For me, my unremarkability was remarkable, routinely reminding me that the meaning of social identities is fluid and context-dependent. Practically, this made me more comfortable during data collection because I was not constantly on the alert for racialized comments; as Chiseri-Strater (1996) suggests, it also undoubtedly shaped what I perceived and was attuned to in this study. My background as the eldest child of immigrants, who literally and metaphorically interpreted the U.S. public schooling system for my family, as well as my racialization as Other in the U.S. system of white supremacy, afforded me particular access to what participants were willing to share with me while foreclosing other access.

Accountability

Of course, research is not a purely solipsistic enterprise, particularly if one believes, as I do, that learning and knowing are not singular pursuits of rational man,⁴ but rather embedded in a

⁴ Gender normativity intended, given the patriarchal histories of philosophy and the social sciences.

collaborative becoming. Informed by Wilson (2008), Patel (2016), and others, I engage in research not to produce knowledge but rather as a form of relation. I am thus accountable to building relationships through my research, and the written output of my research (this dissertation), then, must be faithful first and foremost to the people who constructed it alongside me and not to data as an authoritative representation of reality. Therefore, I take a teacher solidarity stance (Philip, Martinez, et al., 2016) that acknowledges my participation in a field of research that often positions teachers as the problem (e.g., Kumashiro, 2012; Pittard, 2015), recognizes teaching as a complex practice, and seeks to approach interpretations with humility. I feel honored that Franck and Clark invited me into their practice in the ways that they did and seek to take seriously the moral and ethical import of work that makes me "responsible for the world [I am] producing when [I] interpret and critique" (Noblit et al., 2004, p. 24).

Although I intend to honor my relationship with Franck and Clark with whatever I write in this dissertation and in the future such that I can be in right relation with them, this does not mean that I explicitly seek to present them in the best possible light. My goal is to write something that feels authentic in the context of our relationship and that rings true to them. What I learned of their teaching practice—what we co-created through this research project—is complex, despite only being partial, and their teaching practice is, by their own admission, imperfect. It may seem that the data I present in this dissertation portrays them as so-called great teachers; this is because I aim to highlight what a deep investigation of their practice has to contribute to researchers' and practitioners' understanding of teaching. It is also because I believe there is something to be learned from their ethical stance on relational work, especially when read in conversation with commonsensical claims about building relationships with students, and not because I have no evidence of their (or my, as a researcher) imperfections and limitations.

My accountability as a researcher extends beyond teachers, of course, and also to the students and administrators who participated in creating the material configurations of mathematics classrooms in which I collected data; while it would be impossible to identify, let alone member-check this dissertation, with every single contributor, I seek to craft an account that would at least be recognizable to those who were most directly involved: the students in Franck's and Clark's classrooms. I also am beholden to the colleagues, mentors, and advisors who supported and shaped my engagement in the doing of this dissertation, not just in the sense of acknowledging my gratitude for their input, but in the sense of making every effort to take seriously, engage deeply with, and learn from their perspectives on research (as well as on mathematics education and teacher learning, of course), even and especially as they diverge from mine and from each other's. And, for me, doing research requires that I be answerable to my ancestors and immediate family members who survived colonialism/imperialism, poverty, martial law, and political violence without the luxury of (literacy or) reading philosophical texts to contemplate right relation: that although the words and language I use to express my ideas may be foreign to them, the spirit of my work can resonate with what it means to go on living in a world that sometimes seems not to care very much about you.

Having made a good-faith attempt at summarizing what I currently believe I have learned about research and about myself as a researcher through writing this dissertation, I now turn to somewhat-more conventional descriptions of the research context, data collection, and analytic methods for this dissertation. In the following section, I first describe Banneker High School, its mathematics department, and the collaborative structures available to its mathematics teachers.

Next, I describe my focal teachers, Franck and Clark, both as individuals and as a pair, and how I positioned myself and was positioned in interacting with them. Then, I outline the data I

collected and explain the analytic processes that generated the findings I will report in the subsequent chapters of this dissertation.

Research Context

School Site: Banneker High School

In the 2018-2019 school year, Banneker High School was a 9th through 12th grade public high school in one of the largest school districts in the Western United States. The physical campus of Banneker High School was composed of two buildings encircled by a very tall, ivycovered fence and separated by a large half-covered courtyard that serves as lunchroom, student commons, and physical education classroom. Within each building, classrooms were spacious enough to comfortably accommodate typical classes of 30-35 students. There was little student work posted in the hallways, but there were often large student-created posters advertising upcoming school functions or athletic events, district-issued posters supporting the rights of immigrant students, and course listings for the local community college where some juniors and seniors took classes. Single-occupancy all-gender bathrooms were visible on each floor, in addition to bathrooms labeled girls and boys, and these were the bathrooms that teachers used. Although Banneker struck me as one of the cleaner, neater schools I have visited, even within the same district, Franck and Clark both alluded to its physical condition as being less than desirable; Franck contrasted Banneker, for example, to the high school he had attended, saying that his alma mater "even smelled nice" (fieldnotes, 11/2018).

Like at many schools in this district, over 90% of Banneker's approximately 850 students were identified as Latinx (the others are mostly identified as Black, Asian, and White) and over 90% of its students were identified as being eligible for free and reduced-price lunch, according

to the most recent Civil Rights Data Collection survey (2017-2018). According to an accreditation compliance report published by Banneker administrators in the year prior to data collection, fewer than 10% of students were labeled by the district as Limited English Proficient, and fewer than 10% of students were labeled as being from English-only homes; the vast majority of students were labeled as Reclassified English Proficient, suggesting that they entered the school district as Limited English Proficient but tested out of the program prior to enrolling at Banneker. Fewer than 5% of students were identified as being students with disabilities. Over 35% of students at Banneker were enrolled in at least one AP course. In the three years prior to data collection, fewer than one percent of students were suspended.

Banneker High School opened in the mid-2000s; the land and building were donated by a local institution whose leaders wanted to create a magnet school where students could explore relevant interests and intern with the institution. By 2018-2019, Banneker had no longer been affiliated with this institution for several years, but it remained an interest-based magnet school for which families must have submitted an application. Selection was based on lottery, but the district did require that students who apply to magnet high schools be considered "Algebra 1 ready" based on the recommendation of their middle school mathematics teachers. According to the teachers I talked to and the surveys that students informally filled out in one Algebra 1 class on the first day of school, the majority of Banneker freshmen attended the neighborhood middle school down the street and lived within walking distance or a short public transit ride away. A student I met on the city bus during the first week of school said that she liked Banneker because it was safe and focused on learning, but she did not like that it lacked athletic facilities. Her experience of Banneker aligned with 2017-2018 results from a district-wide student survey in which Banneker students most differed from the average district high schooler in their

perceptions of adult expectations for college attendance (higher at Banneker), physical safety (higher), student respect for adults (higher), acceptance of queer students (higher), extracurricular opportunities (lower), connectedness to the school (lower), and fair grading policies (lower).

By contrast, Banneker teachers seemed significantly more satisfied with their work environment than high school teachers across the district. On the same survey, their ratings of Banneker exceeded the district average by more than five—and often close to ten—percentage points on nearly all the indicators for the school's general academic focus, teachers' college-related preparation and expectations of students, students' access to college-preparation opportunities, teachers' opportunities for their own instructional improvement, parent engagement, availability of resources, school climate, discipline, student behavior, and teaching satisfaction. My observations of Banneker teachers supported the idea that Banneker was an enjoyable place to work as an adult; teachers frequently popped into one another's classrooms between passing periods and during non-teaching periods to greet and chat with each other, and several teachers' relationships with the maintenance staff appeared to be characterized by raucous joking and warm inquiries after one another's children and spouses by name.

The mathematics department at Banneker in the 2018-2019 school year consisted of eight teachers, six of whom also belonged to an external professional development organization that I will call PDO. PDO provided rich professional learning opportunities—including monthly professional development meetings, paid travel to and attendance at regional and national conferences, financial partnership with teachers' schools to allow for additional collaborative planning periods during the school day, and a supplemental stipend—to develop the teacher leadership of experienced mathematics teachers. The high concentration of PDO teachers at

Banneker was deliberately cultivated over time. Franck and Clark had applied to PDO after teaching at Banneker for several years; impressed with the quality of professional community they experienced at PDO, they recruited other PDO teachers to apply whenever positions opened up in Banneker's mathematics department. Due in part to the influence of these focal teachers and in part to being impressed with what PDO offered to teachers and partner schools, Banneker administrators were eager to hire additional PDO teachers over the years. In the year prior to this study, Banneker had four PDO teachers and hired two more over the summer.

The two new Banneker mathematics teachers from PDO, Abigail and Veronica, also taught two periods of Algebra 1 in addition to their other mathematics courses, just like Franck and Clark. The four of them constituted a "self-selected personal learning community (PLC)," which was a structure for teacher professional development at Banneker. Teachers could choose to participate in PLCs around a set of interests, and Franck, Clark, Abigail, and Veronica chose to define themselves as an Algebra 1 PLC. Banneker students were dismissed early approximately one afternoon every three weeks so that teachers could meet in their PLC groups to pursue self-directed professional development agendas, as well as participating in department meetings and whole-school faculty meetings. Additionally, PDO's financial support bought PDO teachers an extra planning period during the school day, and Franck, Clark, and Abigail were able to schedule one of their planning periods to be shared. They used this time primarily for Algebra 1-related collaborative planning. As a result of these structures, Franck and Clark both benefited from more planning time, more collaboration time, and more time for collaboration with likeminded colleagues, than many mathematics teachers at similar schools.

During the 2018-2019 school year, Clark was officially the chair of the mathematics department at Banneker, although he and Franck both told me that this was because Franck does

not "like to put [his] name on anything" (*interview from broader research study, 5/2018*); the two of them generally made department-related decisions together, including teaching assignments. Over their careers, Franck had taught every high-school mathematics course and Clark had taught everything except Geometry, but in the year of this study, Franck taught Algebra 1 and Advanced Math, and Clark taught Algebra 1 and AP Calculus. Banneker's principal strongly trusted Clark and Franck, and they described having extensive moral, professional, and financial—if not always instructional—support from school administration. For example, because of their high status within Banneker, Franck and Clark had less trouble acquiring material resources than teachers in other schools; in the summer leading up to this study, their principal purchased additional whiteboards and mounted them on the walls of Franck's and Clark's classrooms so that they no longer had to use mobile easels for student groupwork. As a result, the mathematics department seemed to make decisions about curriculum, assessment, and remediation remarkably unencumbered by the constraints of district initiatives and standardized tests that are common in many urban public schools.

Focal Teachers: Franck and Clark

I first met Franck To and Clark Zapatero in the spring of 2018 while working as a graduate assistant on a research project studying PDO as a learning environment.⁶ I immediately

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⁵ This is a course that Franck and Clark developed together as their PDO project and with the encouragement of their school administration. Advanced Math combined the traditional curricula of Algebra 2 and Precalculus into a single year so that students who started high school by taking Algebra 1 as freshmen could still have the opportunity to take an AP mathematics class by their senior year; without this Advanced Algebra option, students at Banneker would typically take Algebra 1 as freshmen, Geometry as sophomores, Algebra 2 as juniors, and Precalculus or Statistics as seniors. Franck and Clark intend for Advanced Math to replace both Algebra 2 and Precalculus in the school mathematics sequence over the next few years, so that all students would take Algebra 1, Geometry, Advanced Math, and then an elective fourth year mathematics course, rather than for it to be a form of tracking.

⁶ Supporting Instructional Growth in Mathematics (SIGMa): Enhancing Urban Secondary Teachers' Professional Learning through Formative Feedback (NSF-DRL Award No. 1620920). Principal

noticed their enthusiasm for professional learning and their openness to working with—at the time—relative strangers; they requested several times to participate in the larger research study, and Clark invited us to film his classroom the very first class period the very first time we visited Banneker. In Clark's classroom that day, and in Franck's classroom the next day, my fellow graduate assistant and I were struck by the warmth and rapport we perceived between teachers and students, and simultaneously, by the amount and quality of student collaborative groupwork. Classes at Banneker were on a block schedule and therefore two hours long on most days; in both Clark and Franck's classrooms, students spent the majority of the two hours discussing rich mathematical tasks and working through problems in small groups at whiteboards and seemed comfortable both working and joking with their teacher and peers.

Franck and Clark both joked frequently with their classes and with individual students (and students joked back), spoke to students in affectionate tones, and publicly expressed care and appreciation for students. Many students seemed comfortable talking to their classmates in small groups and sharing ideas with the whole group; the awkward pauses common in some classrooms when teachers ask for volunteers were rare in Franck and Clark's classrooms, and although some students certainly spoke more than others, it rarely felt like a small number of students were dominating the whole-group conversation. During groupwork, student conversations occasionally veered into jokes or stories unrelated to class, but typically returned to the mathematical task after a few exchanges and without intervention from the teacher. During passing periods, nutrition period, and lunch, students and former students frequently stopped by Franck's and Clark's classrooms to say hello or share updates about their lives.

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Investigator: Ilana S. Horn. Initially approved by Vanderbilt Institutional Review Board in July 2017 and renewed in August 2018.

Franck and Clark had been highly recommended to our research team as possible participants in the broader study on the strength of both their teaching practice and their enthusiasm for professional development. Both of them regularly sought out feedback and professional development through online mathematics teacher communities, through PDO, and through regional and national conferences. In the few years immediately preceding this study, they had also begun to think about what it meant to be teacher leaders at their schools, in the PDO community, in online spaces, and beyond. As a result, they had started presenting about their course materials and pedagogical practices at PDO and at regional mathematics conferences, which contributed to their reputations within PDO and at Banneker as teacher leaders. Another PDO teacher confessed that he had "total man crushes on those two, so I think I'm a fanboy for them, and I've never even actually seen them teach. But I literally tell all of my [middle school] kids to go to their high school" (interview from broader research study, 3/2018).

Franck and Clark talked about their teaching practice with great enthusiasm and frequency; sometimes, it was as if they could not resist. After observing Franck's class and before their co-planning period on the first day of school, for example, Clark told Franck that he was too exhausted to think. "Tapping out," he said, hanging his head; "I'm done." Less than ten seconds later, however, he was talking about teaching again: "Your first day was so different from mine. You trained them to do all the things... how to do warmup and class discussion... mine was like how to do productive struggle. What are you going to do tomorrow?" (classroom observation, 8/2018). Their incessant talking about teaching and familiarity with talking to each other about teaching created ample opportunities for me to collect data on their sensemaking, commitments, and in vivo theories while minimizing interruptions to their flow. When I did need

to clarify or request additional details, their eagerness to talk shop made it easy for me to ask questions informally and receive elaborated answers.

Franck and Clark had a strong personal and professional relationship: they both grew up in the metropolitan area (although neither of them attended public high schools in the district), met as college roommates at a nearby university, and entered the teaching profession together. They student taught and found jobs at the same schools for their entirety of their approximately 15-year careers. They were the best men at each other's weddings, and when I first met Franck's wife Sonia, who also taught at Banneker, she said to me, "You know they're a package deal, right?" Many of the people in their professional life seemed to treat them as a "package deal;" at PDO, program leaders and other teachers seldom mentioned one name without the other, and Franck introduced himself during PDO presentations as "the Franck in Franck and Clark." "Franck and Clark" appeared on the course materials they used in class, and during the first week of school, their former students clamored to order hoodies emblazoned with "Franck and Clark." Perhaps contributing to this impression, they often finished each other's sentences, and appeared, in conversations, to read each other's minds.

Their close personal and professional relationship was probably cause and consequence of similarities in their pedagogical and ethical commitments, yet their different personalities, priorities, and perspectives played out as they discussed (and often disagreed about) elements of their teaching practice. For example, Franck and Clark seemed to agree that Clark was "better" at thinking about curriculum, while Franck was the "implementation expert." Franck routinely shared personal information about himself with his students and had photographs of his children and former students featured prominently in his classroom, whereas Clark shared less information with students about his life outside of school, and the only posters on his classroom

walls (aside from an M.C. Escher print) were of his alma mater's athletics teams. Franck typically wore dress shirts and slacks, although he sometimes wore math-related t-shirts, and Clark often wore jogger pants and uncollared shirts. Clark would "try anything," in terms of classroom practices he felt were promising, whereas in collaborative meetings, Franck was more likely to take a stance of "you should do it and then you can convince me to do it" and "I'm not there yet" (*collaborative meeting*, 8/2018).

Franck's and Clark's different social locations illustrated variance and texture in how their similar ethical stances were enacted; they also influenced the specificities of what mattered to them and why. Franck presented and identified as an Asian American man and as an immigrant whose parents had been too busy working to play a significant role in his education; he grew up and still lived in what he described as similar neighborhoods to the ones that most Banneker students came from. Clark, on the other hand, identified as a White man whose mother was also a mathematics teacher and who was one of the reasons he wanted to become a mathematics teacher, and he explicitly noted the differences between his socioeconomic background and that of most of his students. Although the findings in this dissertation were generated from analyzing data in which both Franck and Clark participated, I also highlight important differences in their practices and illustrate claims with findings from the teacher for whom it was a particular point of emphasis. In the chapters that follow, I write about two teachers not as a unit but as individuals who share certain elements of their ethical stances. This is not a comparative case study, so I do not detail the specifics of how they are similar and different with respect to each finding that I report, but instead report data from the teacher that is most evocative of the findings generated from analyzing both of their data.

"As an Outsider and as an Insider:" Franck and Clark and Grace

Fine and Weis (1996) assert that qualitative researchers "coproduce the narratives we presume to 'collect" (p. 263), highlighting how researchers themselves contribute to what research participants say. As an example of this, Clark told me towards the end of the school year that he had "known" who I was at the beginning of data collection both because he knew Lani's work and because he had followed me on Twitter, and he suspected that this may have influenced what he shared with me early in our relationship but that it mattered less now that he had a different basis for knowing me (*personal communication*, 6/2019). Therefore, I sought to attend to what Emerson and colleagues (2011) call the "consequential presence" or "reactive effects" of a researcher (p. 4), and to my unique role as a co-participant in this research project.

Franck and Clark both welcomed me readily into their classrooms. They both occasionally paused to talk to me in the middle of class when they were circulating between small groups, usually to share enthusiasm or concern about something a student said or wrote mathematically, sometimes to hand me their keys and ask me to run extra photocopies of a worksheet, and sometimes to offer me a snack. After most class periods, they engaged me in conversation as soon as students had left the classroom, commenting on something that happened in the class that has just finished or asking if I noticed anything interesting. Because of this, I felt comfortable asking them questions about a particular moment as soon as class is over. At first, I consciously tried not to express judgment (even positive judgment) unless explicitly prompted to do so. But over time, I shared more of my own opinions and perspectives in response to Franck's and Clark's comments (which I interpreted as being both lighthearted and pointed) about my not doing so and out of a sense of reciprocity given that they were sharing with me.

Early in data collection, Clark commented that my fellow graduate assistant and I were the first people to observe an entire class period of his for at least the past four years (this account was disputed by PDO leaders, but Clark and later Franck both made this claim multiple times throughout data collection). He regularly emailed me after my visits to follow up on a conversation we had or to let me know that he enjoyed a conversation. Although Franck and Clark had significantly more teaching experience and expertise than I did, I felt that they respected my experience as being a different form of expertise and that they were happy to have a thought partner other than each other; they both routinely asked questions about what I had seen in other classrooms or what I thought about particular pedagogical strategies.

A little more than halfway into data collection, Clark emailed me to say that "I think you are playing your own little role in making our team so strong and I appreciate you as an outsider and as an insider" (*personal communication*, 2/2019). After receiving it, I wrote in a research memo that:

It makes me feel happy, and relieved/reassured/grateful that he sees my presence as valuable rather than being an imposition or even just neutral. I don't know if Franck would ever say anything like this-- he doesn't strike me as the type-- but I have also noted his interest in talking to me rather than planning when I'm around (and although I initially thought this might be procrastination, it can't just be that, I don't think) and asking questions that only I would know the answer to (e.g., his comment that other than students I'm the person who has spent the most time in his classroom, ever).

I was positioned "as an outsider and as an insider" as a researcher rather than as a fellow teacher but as a colleague with shared interest in mathematics teaching, and as someone who was only present at Banneker (and on the West Coast) for one week each month but who knew the specific

students and dynamics of their Algebra 1 classes—and therefore could discuss them—better than anyone other than Franck and Clark themselves. Because the work of teaching is so personal, regularly observing more of Franck's and Clark's teaching practice than any other adult positioned me to know them in a uniquely intimate—and of course, limited—way. So, although I was an outsider to Banneker and to Franck and Clark's longstanding relationship in many ways, we also established a new collaborative relationship through this study in which I was an insider (Herrenkohl et al., 2010). And, although they were outsiders to the academy and formal processes of conducting research, they shaped what and how I inquired into their practice, as I will explain further throughout this chapter.

Over time, my relationship with Franck and Clark shifted from purely pedagogical to increasingly interweaving the professional and personal. In addition to "data," I left Banneker visits with homegrown avocados, parenting advice, and the urge to text one of my best friends about the sense of wonder I felt from being reminded, by their teaching and sensemaking about teaching, of "the awe factor of interacting with so many lives, each of which is the most important and central to the students themselves but only a small part of your life [as a teacher]" (personal communication, 3/2019). In giving feedback on an early analysis for this dissertation, a mentor asked how it spoke to "making research or making friends" (Yoon, personal communication, 6/2018); since the official close of data collection, Franck, Clark, and I have continued to email and text regularly. If I were hewing closely to the conventions of traditional social science research, I might claim that I tried to keep the personal relationship separate from my research and analysis but taking a feminist new materialist approach to research suggests the more humble claim that I am simply reporting on the world that I observed and participated in because my observation and participation was part of what made that world possible.

Data Collection

Although ethnomethodology can be characterized by a "quest for invariant properties of social action" (Fontana & Frey, 2000, p. 649) that contradicts my approach to research, I am nevertheless drawn (perhaps by the training that Lather and St. Pierre mentioned) to ethnomethodology's close attention to sensemaking and action in the seemingly mundane details of ordinary lives (Heritage, 2013). Consequently, my data collection was designed to record how Franck and Clark talked about and did the day-to-day realities of relational work. For example, because the focus of this dissertation is on *teachers* ethical stance, and not students' experience of what they do as a result, my attention, when planning and conducting data collection, was primarily on teachers' actions and sensemaking. I sought multiple data sources for what would traditionally be called triangulation, because although:

neither polyvocality nor self-reflexivity nor methodological-theoretical disclosure—and, in fact, nothing—can ensure epistemological validity, the use of these discursive strategies more accurately conveys the dialogic way that ethnographers learn. (Chiseri-Strater, 1996, p. 129)

Therefore, I have tried to include notes and observations about students' experiences where possible, and interviewed students in order to collect data about their perceptions of what Franck and Clark were doing. However, this dissertation cannot speak to how students experienced Franck's and Clark's pedagogical or relational practices. Table 1 provides a summary of the main sources of data and the purpose of each source.

Table 1Full Data Corpus

	Data collected	Taken as evidence of
Formal teacher interviews	Building Relationships Interviews (n = 4, October 2018 and March 2019)	 Franck's and Clark's expressed commitments to and philosophies about students and about building relationships with students
	Roster Interviews ($n = 9$; February to June 2019)	 What Franck and Clark knew about their students Stories about Mr. Montoya
	Student Interview Reflection Interview (n = 1; January 2019)	 Franck's and Clark's aspirations for relationship-building and classroom climate What Franck and Clark knew about their students
Formal student Interviews	Student interviews (n = 17; November 2018 to May 2019)	 Students' perceptions of Franck and Clark and their classrooms Additional stories about Franck's and Clark's interactions with students Stories about Mr. Montoya
Classroom observations	Fieldnotes, handouts, artifacts (n = 67; August 2018 to May 2019)	 Franck's and Clark's interactions with and reactions to students Their pedagogical practices
Collaborative meetings	 Department meetings (n = 3) Planning periods (n = 8) Other (n = 6) 	 Franck's and Clark's sensemaking about students and pedagogy Stories about Mr. Montoya
Additional data	 Informal teacher interviews, conversations, and phone calls Field jottings from before/after school, transitions, lunches, PD sessions at Banneker and at PDO Results from teacher-given student surveys at the end of the first semester and the end of the year 	Corroboration, disconfirmation, or complication of hypotheses and interpretations resulting from other evidence

Data Collection Procedure

Lather and St. Pierre (2013) point out a limitation of conducting social science research as much of the academy currently expects it to be done:

The doer exists before the deed, so the researcher can (and must for IRBs) write a research proposal that outlines the doing before she begins. The assumption is that there is actually a *beginning*, an origin, that she is not always already becoming in entanglement. (p. 630)

Data collection for this dissertation may be assumed to have begun in August 2018, at the start of the 2018-2019 school year, or in February 2018 when I first met Franck and Clark and began observing their classrooms for the broader research study. Perhaps, however, it began when I started teaching high school mathematics in 2007 and consequently began noticing, professionally, things about mathematics students and mathematics teachers and mathematics classrooms that I previously had not been attuned to. Or, even earlier, when Franck and Clark were hired at a high school down the street from Banneker and started teaching mathematics in ways that would lead to the ethical stances I observed during the 2018-2019 school year. I could continue, but suffice to say that in this section, I will limit my reporting to the period of data collection covered by Vanderbilt's IRB.

From August 2018 to May 2019, I visited Banneker for approximately one week each month. Due to block scheduling, classes at Banneker did not meet every day, so during each data collection visit, I prioritized observing one of each of Franck's and Clark's Algebra 1 classes (their focal class periods) every time they met that week. Additionally, I observed Franck's and Clark's collaborative meetings and followed them during their non-teaching times: individual planning periods, lunch, and transitions. When these priorities allowed gaps in my data collection schedule, I occasionally observed Abigail's and Veronica's classrooms and/or collected data for the broader research study through which I had originally been introduced to Franck and Clark. After school, I occasionally interviewed students. I also followed Franck and Clark to

professional development sessions at Banneker, when I was invited, and at PDO on Saturdays when I was in town. When I was not actively collecting data at Banneker, I re-transcribed segments of professionally transcribed recordings to catch transcription errors and to supplement my fieldnotes of transcribed conversations with memory notes about body language, tone, and context that were re-activated by listening to the conversation again; reviewed fieldnotes; and wrote reflective, in-process memos to highlight what I was thinking, feeling, and wondering, particularly when I was struck by resonances, connections, and theoretical implications between what I had observed and either what I had previously observed or texts I had read (Emerson et al., 2011; Miles et al., 2014). I also constructed analytic memos to record emergent themes, methodological insights, and directions for further fieldwork and analysis. Table 2 illustrates a sample week in data collection, and more detailed descriptions of each type of data follow.

Classroom Observations

I observed a total of 67 class periods at Banneker for over 95 hours of classroom observations. When I observed, I typically sat at an unused student desk towards the back corner of Clark's classroom or, because all of his student desks were usually full, at an extra "teacher desk" in the back of Franck's classroom. I took detailed and timestamped descriptive fieldnotes on my laptop during class; when I was fortunate, my fellow graduate student researcher took simultaneous fieldnotes in a shared Google document. Although I tried to describe features of classroom interaction that would be visible to anyone observing the classroom, fieldnotes inevitably reflect "researchers' deeper assumptions about social life and how to understand it" (Emerson et al., 2011, p. 14). In my fieldnotes, I typed near-scripts of what students and teachers said, treating discursive language as reflecting meanings about their interactions with each other,

Table 2Sample Week in Data Collection

Day	Data Collection Schedule		
Monday	1. Clark's other Algebra 1 class (1 hour)		
	2. Clark's focal class (1 hour)		
	Franck's focal class (1 hour)		
	Abigail's class (1 hour)		
	Veronica's class (1 hour)		
	Lunch with teachers		
	Informal conversation during nutrition period		
	8. Collaborative meeting during planning period		
Tuesday	1. Clark's focal class (2 hours)		
	Franck's focal class (2 hours)		
	3. Lunch with teachers		
	Informal conversation during nutrition period		
	5. After school student interview		
Wednesday	1. Lunch with teachers		
	2. Collaborative meeting and/or teacher interviews during planning period (2 hours)		
Thursday	1. Clark's focal class (2 hours)		
	2. Franck's focal class (2 hours)		
	3. Lunch with teachers		
	4. Informal conversation during nutrition period		
	5. After school student interview		
Friday	1. Lunch with teachers		
	2. Collaborative meeting and/or teacher interview during planning period (2 hours)		
Saturday	1. Professional development sessions at PDO		
	2. Lunch with teachers		

Note: Other research activities during these weeks included cleaning up fieldnotes and transcribing audio recordings; planning interview questions and future data collection; and collecting data for the broader research study, including observing and videorecording additional classes at Banneker and at other schools. In addition to research activities, I occasionally ran errands for Franck and Clark, such as making copies.

with mathematics content, and with the institution of schooling, among other things; marked students' locations at their seats and when they stood to use whiteboards to jog my memory of the classroom's physical layout and relative proximity among teacher and students during verbal exchanges recorded in the near-scripts; recorded body language, facial expressions, tone, and other signals of emotion as well as I could and especially during moments I thought might be

significant for later analysis, such as when I perceived a breach in classroom norms; and tracked my own visceral reactions, thoughts, and questions. I also collected classroom artifacts such as photographs of student work and digital copies of students' task papers.

During classroom observations, I tried to be relatively unobtrusive in that I did not proactively engage with students about their classwork and did not position myself as someone they should ask for help. I responded to students without redirecting them if they addressed me first and participated if the focal teacher included me in a conversation with a student, but otherwise did not insert myself. My perception was that many students were at least moderately comfortable with my presence; they allowed me to overhear fairly personal information about them and their families both in class and in their informal conversations with teachers, greeted me when they entered the classroom, and asked me whether I could make them "YouTube famous" instead of writing a book that nobody would read.

Teacher Interviews

I treated Franck's and Clark's talk as being the most informative data source for evidence of their sensemaking. Given their near-constant talking about teaching with each other, with their colleagues, and with me, "in some cases I found the interviewing process to be merely way of blocking off some time and space for a conversation that was ongoing... it was not usually necessary to formally 'ask' questions" (Hermes, 1998, p. 160). However, I conducted three types of more formal semistructured interviews with Franck and Clark: Building Relationship Interviews, an interview in which they reflected on student interviews, and Roster Interviews.

Table 3 summarizes these three formal interview types and provides sample questions from each.

Table 3 *Teacher Interview Topics, Timing, and Prompts*

	Timing	Format	Sample prompts
Building Relationships Interview	October 2018 (~3 hours)	Franck and Clark interviewed separately	 How do you think about relationships in general? What do you notice about your students? What do you do with what you notice? What do you want your students to know about you? How do you relationships with students change over time?
	March 2019 (~3 hours)	Franck and Clark interviewed separately	 Describe the kind of relationship you aspire to have with your students. How do you know what kind of relationship students want to have with you? Do you ever push harder than you think a student might want? Are there students who remind you of yourself? Are there particular students who gravitate towards you?
Student Interview Reflection Interview	February 2019 (2 hours)	Franck and Clark interviewed together after being presented with anonymized data I collected from interviewing students	 Is this what you would expect students to say? Do any of these comments surprise you? Is trust something you explicitly think about? How do you think your curriculum and your personalities shape your relationships with students? Are there other questions you'd like me to ask students?
Roster Interviews	February-June 2019 (>3 hours)	Franck (n = 5) and Clark (n = 4) interviewed separately, mostly during morning commute phone calls	Tell me about [student name]

Building Relationships Interviews

First, in October 2018 and again in March 2019, for a total of about 6 hours of interviews, I asked Franck and Clark separately how they thought about their relationships with students. I used a small set of predetermined initial questions and then followed the flow of conversation by asking for clarification or elaboration or identifying additional questions based on their answers. In March, I sought to follow up on and clarify some themes that had emerged over the past few months of data collection. Not until I later sat down to analyze and compare the two interviews did I realize just how similar both Franck's and Clark's responses in their March interviews were to what they had said in their October interviews, speaking to a consistency in their approach to relational work. These interviews were central to the analyses presented in Chapter 4.

Student Interview Reflection Interview

Second, in February 2019, I presented Franck and Clark together with anonymized excerpts from the interviews I had been doing with their students (these excerpts are provided in Appendix A). I mixed comments from Franck's and Clark's students first because I noticed strong similarities and few differences in what their respective students were telling me, and second because I was curious whether they would be able to guess whose students had made each comment. For about 2 hours, I invited Franck and Clark to tell me what they thought of these student interview excerpts, whether anything surprised them, and how deliberate they were about the specific adjectives that students repeatedly mentioned, like trust and comfort.

Roster Interviews

Finally, throughout the spring of 2019, I asked Franck and Clark to individually tell me about the students on their rosters, in alphabetical order by last name. Most of these 9 (5 with Franck; 4 with Clark) interviews occurred during their morning commutes and, because of how

long teachers spent describing each student (Franck averaged 5 minutes per student and Clark averaged 10 minutes per student, totaling over 3 hours), we discussed close to half the students in their focal class periods (Franck: 20 of 34 students; Clark: 9 of 26 students) in the time that we had. This approach had limitations for the completeness of data collected, of course; it is possible that what Franck or Clark would have told me about the students we did not discuss would have differed significantly from what they told me about the students we did discuss. Furthermore, as I will explore further in Chapter 4, the extended time period over which these interviews were conducted (February through June) precluded my ability to make claims about what Franck and Clark knew about their students at any given point in time.

In addition to these more formal interviews, Franck and Clark and I had many informal conversations in which we verbally replayed (Horn, 2010) the class periods I had just observed, discussed particular students and/or student interactions, and addressed similar topics those addressed in the formal interviews. Some of these conversations included stimulated recall "interviews" in which I prompted them to tell me about their goals and decisions during an episode I had observed, using quotes or fieldnotes as a prompt (Calderhead, 1981). Sometimes, these informal conversations occurred in transitions during the school day or at lunch and I reconstructed them in my field jottings (see below) in as much detail as I could remember as soon as I had a reasonable opportunity to do so after the conversation. Other times, I was able to audiorecord them—when I felt that informing Franck or Clark that I was about to start recording would not unduly disrupt their train of thought or shift the tone of the conversation—and later had the recordings professionally transcribed.

Beginning roughly at the end of the first semester, first Franck and then Clark began calling me on their morning commutes when I was not visiting Banneker to continue

conversations we had not had time for during my visit or to share a recollection or reflection they thought I would be interested in. During these phone calls, I took notes as we spoke, typing near-scripts of what they and I said and also writing down follow-up questions I had, many of which I did not have a chance to ask. When possible, I also audiorecorded the calls for later transcription. These morning commute calls, along with text messages and meandering chats with which Franck, for example, passed the time while waiting for Sonia at the end of the school day, illustrated the shifting terrain of our relationship over the course of the school year and created many opportunities for me to ask follow-up questions and gather additional evidence of how Franck and Clark approached and made sense of relational work. I arbitrarily cut off the data I am including in these analyses at the end of the 2018-2019 school year out of respect for Franck's and Clark's privacy, the boundaries of the research project they had originally agreed to, and the IRB. For the sake of simplicity, I refer data and quotes from both the formal interviews and informal conversations as "interviews" throughout this dissertation.

Collaborative Meetings

Another source of teachers' sensemaking was their talk in collaborative meetings; I attended 25 hours of formal and informal collaborative meetings. Most of these meetings were from the planning period that Franck, Clark, and Abigail shared, in which they most frequently collaborated to plan upcoming Algebra 1 lessons. Other collaborative meetings included school-based mathematics department meetings and whole-faculty meetings, and collaborative meetings in which all the Algebra 1 teachers met to review and debrief video taken of their classrooms as part of the broader research study. My participation and data collection varied depending on the type of meeting and participants; in the shared planning period, I most often sat near the collaborating teachers and took fieldnotes similar to the fieldnotes I took during classrooms

observations, typing near-scripts of their conversations and taking photographs if they worked out an idea on the whiteboard. When I could do so without interrupting the flow of conversation and when I thought it might be particularly relevant to this dissertation—as in the case of the January collaborative meeting where Franck, Clark, and Abigail discussed Mr. Montoya—I audiorecorded the meeting and later had it professionally transcribed. In department and school meetings with Banneker teachers who were less familiar with me and my research than the Algebra 1 teachers were, I took notes by hand and typed them up later; in the video-based debrief meetings, which were also video recorded and later transcribed for the broader research study, I typically facilitated the conversation with the assistance of a colleague on the broader research team who took notes.

In addition to these formal collaborative meetings, Franck and Clark often entered each other's classrooms when they were not teaching to ask a question or share a piece of information and then ended up talking at length about whatever was on their minds about teaching; these conversations were, unsurprisingly, of great interest to me. In these conversations, particularly later in the school year, I often—but not always—participated more actively, depending on the topic of conversation, whether I had questions, whether I was explicitly invited to participate, and other factors. To record them, I likewise typed near-script notes, reconstructed the conversation from memory in field jottings recorded at a later time, and/or audiorecorded as the circumstances made possible.

Student Interviews

I interviewed students from Franck's and Clark's focal Algebra 1 class periods to elicit their perceptions of Franck and Clark as teachers and their perceptions of their experiences in Franck's and Clark's mathematics classrooms. As described above, my intent was to inform my

understanding of Franck's and Clark's ethical stances about relational work rather than to create a detailed representation of students' experiences. As such, I used student interview data primarily as a "check" on what teachers told me rather than starting with student interviews for any analysis. For example, if Franck and Clark had said they strived to make students comfortable, but the students I interviewed had unanimously reported feeling uncomfortable in class, that would have shaped my interpretation of Franck's and Clark's sensemaking.

At the beginning of the school year, I invited all students in their focal class periods to complete consent forms for individual interviews and described my research as studying their teachers. From the subset of students who returned affirmative consent, I selected individual students who were physically seated closest to me each day that I intended to conduct an interview; Franck's and Clark's use of daily random grouping afforded some level of randomness to my selection process, and selecting students based on physical proximity allowed me to ask students for interviews without disrupting class or attracting undue attention. Given Franck's and Clark's consideration of not singling students out in front of their peers, it felt important to me to respect that practice and be discreet in inviting students to interviews. That said, although I did not ask for students' grades out of respect for their privacy and to avoid activating biases of my own, my perception based on classroom observations and Franck's and Clark's comments is that I oversampled students who would be considered successful, in their classes if not necessarily in school overall. This is not surprising given both who would initially return an affirmative consent form for a one-on-one interview with a relative stranger and who would agree to an interview when asked in the moment (some students did postpone or decline my requests to interview them despite having returned affirmative consent forms earlier in the school year).

Most student interviews occurred after school (Banneker students mostly walked or took public transit and thus were less limited by the constraints of, say, a school-bus schedule), although some took place during lunch or students' free periods when I was not observing a class. Each interview lasted approximately ten minutes and was semistructured; I asked students to tell me about their mathematics classes and, based on themes that emerged from conversations with Franck and Clark and from my readings of research literature about caring and teacherstudent relationships, whether they felt understood, cared about, and/or challenged by their teacher. An interview guide and the list of students interviewed are presented in Appendix B. In retrospect, I wish I had asked students more about themselves. Although I was curious, I was reluctant to ask questions that were not obviously linked to their teachers, given how I had described the purpose of my research study, and this led me to err on the side of gathering too little information. I suspect that most of the students I interviewed would have been willing to tell me more about themselves, and that this could have offered 1) to me, additional stories to contextualize my understanding of their reported experiences of mathematics class and 2) to them, a different perspective on why an adult and "researcher" would want to talk to them.

Additional Data

As Emerson and colleagues (2011) suggest, I wrote field jottings each day after leaving Banneker. In these jottings, I recorded things that I had observed or heard in hallway conversations and transitional moments that I: 1) already knew would be important for my analyses but did not fit into other fieldnotes, such as when I left the building with Franck and Sonia at the end of the day and he called out to a student we passed to ask her about music practice; 2) thought would probably not be directly useful for data analysis but that would provide me valuable context and/or inform my ongoing engagement with Banneker teachers and

students, such as information about their participation in the nationwide teacher strikes in January 2019; and 3) did not know yet would be important, but wrote down nonetheless as a form of externalized memory, such as initial notes about Mr. Montoya before he emerged as a significant presence in this research.

One recurring source of these field jottings was lunch. Franck's classroom was often the location for informal group lunches attended regularly by some combination of Franck, Sonia, Clark, Veronica, Abigail, and other Banneker teachers; although there was never any plan for teachers to eat lunch in Franck's classroom, lunch often attracted an increasing number of participants over the course of the 20-minute lunch period. Conversations during lunch ranged from ruminations *about* lunch (perhaps the most popular topic) to intense discussions of educational research, and often provided me the opportunity to learn more context about, for example, the history of Banneker and local gentrification, teachers' personal histories, and particular students. Because these lunch gatherings were casual and included people who had not formally consented to participate in my research, I did not take notes in the moment and never audiorecorded, no matter how relevant the conversation. That said, participating in these lunches undoubtedly shaped my awareness of what was on Franck's and Clark's minds during any given day and, over time, the types of topics they gravitated towards discussing and the ways they engaged in conversation.

I also collected additional data over the course of my visits to Banneker that informed my analyses. For example, as part of my outsider-insider relationship with Franck and Clark, Franck asked me to help them revise the end-of-semester surveys they gave their students every year using what I knew from research about student surveys; the results of previous years' surveys had been "uplifting" more so than "constructive" in suggesting about what they might change

about their practice (fieldnotes, 12/2018). They presented this to me as a win-win situation in that I could ask questions I was interested in, and my creation of the survey would save them time. We created the revised survey together in a collaborative meeting, based on resources I shared, and Franck and Clark sent me links to students' completed (anonymous) responses after they administered the survey. Additionally, Franck sent me students' responses to a supplemental "teacher evaluation" survey he had administered at the same time, and Clark sent me a voice memo he recorded reflecting on his students' responses. Such data did not constitute the core of any of my analyses, but it nevertheless informed my perceptions of Franck and Clark and their teaching practice, contributed to my relationship with Franck and Clark, and complicated easy conclusions I might otherwise have drawn. For example, Clark named his voice memo file "tough evaluations," and wondered whether students perceived the reported easiness of his class as being created, at least in part, by his careful curricular and pedagogical designs to make Algebra 1 feel accessible and easy without reducing mathematical rigor, or whether they saw it as entirely due to their prior experiences or their individual aptitudes. Although I did not formally code this voice memo in any of the analyses I report later in this chapter, the vulnerability and uncertainty I heard in this voice memo corroborated and magnified my sense of Clark (and Franck, for other reasons) as a teacher who was not afraid to be vulnerable and who embraced dwelling in uncertainty. This undoubtedly shaped the interpretations I made in formal analyses.

Data Analysis

As with many dissertations, I collected far more data than was possible to analyze and report in a single dissertation, yet not always enough data to analyze important questions that emerged. And, as with much research, I conducted more analyses than ultimately yielded

meaningful findings. In this section, I first offer a brief summary of my overall approach to data analysis, and then describe the analyses that most directly contributed to the chapters that follow.

Although I fear I have said too much about this already, I again register my discontent with paradigms of research that assume the researcher develops a singular research question based on reading prior literature, determines in advance the data that will be allowed to speak to the research question, and analyzes the data as if it exists distinct from and distinguishable from theory (and from the analyst) rather than always already entangled with intuitions about, understandings of, and beings in the world. As a result, I find resonance with what A. Y. Jackson and Mazzei (2013) call "thinking with theory:" the data I collected informed the theory I read—in terms of what I read, how I read it, and the meaning I made of it—and the theory I read informed how I approached the data. A. Y. Jackson and Mazzei refer to analysis as "plugging one text into another" (p. 269), in which both theory and data are treated as texts. The analytic process, then, requires "approach[ing] the data with the analytical questions informed by the key concepts that [are] plugged into the data and in turn, back into the theory" (p. 265).

Likewise, MacLure (2013) highlights several poststructuralist concerns with the assumptions underlying coding as an analytic method in qualitative research: that relations between entities are fixed and hierarchical; that a rational analyst at a remove is best positioned to discover those relations; that abstraction matters more than intimate experiences; and that research coding offers something distinct from "that which is *already* coded by language, ideology, and the symbolic order [emphasis original]" (p. 170). She notes, however, that in the process of coding, a researcher can become "a live conduit wherein the materiality of things, the struggle for concepts, one's 'shared entanglement'... can be *felt* [emphasis original]" (p. 174). Therefore, she does not recommend abandoning coding, but rather, dwelling in the complexities

of data rather than seeking to come "out" of the data with dry, impersonal, abstract categories. Accordingly, I approached coding not as a way of classifying and sorting but as a way of foraging for what poststructural methodologists like A. Y. Jackson, Mazzei, and MacLure call hotspots, moments of wonder, and lines of flight to pursue through finer-grained analyses and thinking with theory.

The "Hospitality" Analysis

Because "thinking with theory" can sound vague, I begin by providing an example of how I did so. In this section, I describe how data collection, data analysis, and reading theoretical texts overlapped chronologically and conceptually in what I call the Hospitality Analysis, illustrating how theory and data constitute each other. It also illustrates how Franck's and Clark's (in this case, Clark's) interests and inquiries shaped my data analysis; as part of being in right relation with them, I wanted my analyses to produce findings they would find meaningful and useful in their ongoing thinking about teaching, rather than simply fancifying what they already knew with academic language or pontificating about abstract ideas that would be irrelevant to their daily practice.

During one of my Roster Interviews with Clark, which was conducted over the phone, he said the following about Jonathan:

If you came and talked to him, he was inviting. That's actually a really interesting thing to say. He was inviting in those interactions. Because some students are not inviting in those interactions and that's probably one of the things I struggle the most with... how do I convince you that this is okay, it's safe, it's productive, it's all the good things it could be. Do you know what I mean? Jonathan was inviting. I think he saw the value in our interactions. (*fieldnotes*, 6/2019)

Towards the end of this conversation, Clark asked me, "Are you recording this? Can you send me back some notes because I think some of this stuff is hitting me... I think I'm learning from Jonathan right now." I emailed Clark the near-script I had typed during the call of what he had said about Jonathan. He wrote back a list of questions he was wondering, such as "How do I encourage/convince them to invite me into their space in an authentic way that isn't just me invading because I have authority to do so?" and "How does the content and the lesson affect their willingness to be inviting?" Also in this email, he asked whether I'd heard other educators talking about invitingness as a concept.

Because this exchange occurred at the end of the formal 2018-2019 data collection year, I had begun systematically reviewing fieldnotes and conducting exploratory analyses at this point. Informed by what I was thinking and reading at the time, I wrote back:

I don't know if I've seen many people use the language of "inviting" students in, but I really like it. I think sometimes I see a one-off thing about "inviting students into mathematical practice" or "inviting students into participate" but I wonder if you're describing a little more than that... almost inviting into a relationship or inviting into a particular kind of relation?

Clark responded by articulating what he believed was different about his idea. Among other things, he wrote:

A teacher might say that they invite students into their rooms or into a lesson but this is more student centered. This is about the student's space and the decisions they make about inviting the teacher in. This assumes that the students have space... Student spaces are also complex and students may invite me into one room of the house and not another.

He also cited particular students—who he knew I had observed all year—to create analogies for different ways students could invite teachers in, or not (for more details, see Chapter 4).

Clark's inversion of the host-guest paradigm in his use of a house metaphor felt unusual to me given how I was accustomed to seeing hospitality used as a metaphor in education: with the teacher inviting students into some place or something. His analogies recalled for me the phrase "conditional hospitality," which I had encountered in prior research. Re-reading Sonu and Bellino (2018) and Derrida (2000) reminded me of concepts such as "stranger-making," inclusion, and "refuge," as well as locating these ideas in the context of how reciprocity, responsibility, and power ought to inform ethical relations. I wrote back to Clark:

There's postmodern scholars who write about notions of "conditional hospitality" as they relate to migration and citizenship, and the idea that some "guests" are only welcome in particular societies if they meet particular standards, and their invitation can be revoked if they are perceived to fall short of those standards... I could see that extending to teachers who make students feel differently welcome based on whether they're being "good students" and following the rules, doing what the teacher expects them to, etc. And what you're describing instead sounds like a space in which the host/guest boundary is much blurrier...

I continued by wondering "about the boundaries of the boundaries," posing questions of my own about "how firmly a closed door [in a student's "house"] is ever closed."

Clark and I exchanged several more emails in this vein, and Clark broadened his house metaphor to visualize a neighborhood under construction. During this period, I read additional texts such as Shirazi (2018), Mayo (2004), and Hutchinson (2004), as well as blog posts (e.g., Dingle, 2018) and tweets (e.g., Cupp, 2019), to continue thinking with theory. These texts

offered analytic concepts such as "hierarchies of membership," home, "difficult relations," and "belonging," which illuminated the contours of what Clark was describing, provided me with prompts I posed to him to extend our shared thinking, and recruited additional literature into the conversation. Reading Ruitenberg's (2015) analysis of Derrida's and other scholars' work on hospitality gave me language to frame this dilemma as an ethical aporia, or tension, encountered by teachers who have power, seek not to impose that power, yet must use that power responsibly. I wrote an analytic memo (Miles et al., 2014) to explicitly link my exchange with Clark with these texts, to highlight what I found analytically significant and what I felt would be meaningful to practitioners, and to identify directions for further thinking with theory. Excerpts from this memo are presented in Appendix C.

In this example of thinking with theory, Clark, his students, I, other teachers, philosophers, and educational researchers co-constructed a conceptualization of teachers' relational work as navigating an asymmetrical power relation that must simultaneously respect students' autonomy and teachers' positioning as those who have already made a home, however uneasy a home, in a world—that of mathematics, and that of the institution of schooling—that is still becoming accessible to students. This conceptualization inverted commonsensical ways of thinking about the directionality of obligation, in which students are expected to attend to and meet the conditions their teachers set. It was co-constructed through a process whereby questions and observations about teaching practice surfaced analytic concepts that generated further questions and observations about teaching practice, deepening Clark's and my respective and shared understandings of both theory and practice.

Niccolini (2016), who, like A. Y. Jackson and Mazzei, draws on poststructural and posthumanist theories in educational research, recommends treating empirical data as "an event

to think with." She quotes ethnographer Phillip Vannini as suggesting that data "enliven rather than report, to render rather than to represent, to resonate rather than validate, to rupture and reimagine" (2015, p. 15, as cited in Niccolini, 2016, p. 236). This is the charge I take up in this dissertation, heeding poststructuralist methodological advice to seek connections and multiplicity rather than essences or singular meanings in the data. I aim to do what Stengel (*personal communication*, *n.d.*) calls "philosophizing with data:" to offer one possible interpretation of the data I collected and to convince readers that this interpretation is plausible but more importantly, useful, rather than to claim that the interpretation I present is the only interpretation or the truth.

In the following sections, I describe four additional analyses that informed the writing of this dissertation and attempt to convince readers that they were conducted with sufficient methodological deliberation that they produce plausible and useful findings. With regards to the dissertation as a whole, these analyses provided a means of data reduction: selecting only the most relevant data to analyze more closely. I applied a bricolage (Kim, 2015; Kincheloe, 2001) of micro-level analytic methods, including inductive coding, discourse analysis, and using a listening guide, to further reduce and analyze data. For the sake of expository clarity, I will narrate each analysis as if it followed a purely linear logic of inquiry, even though the analyses often unfolded in messy and meandering ways. A summary of these analyses, along with the theoretical texts that spoke most strongly to them, is presented in Table 4.

The "Building Relationships" Analysis

This analysis sought to determine how Franck and Clark described building relationships with students; although my analysis was corroborated by what I had observed in classroom observations and what I learned from in student interviews, my analytic focus was on Franck's and Clark's *stances* and so the primary data sources were the two Building Relationships

Table 4Summary of Analyses

Analysis	Key analytic question	Focal data	Micro-analytic methods	Key texts	Findings
Building relationships	How do Franck and Clark describe the processes of relationship-building that they engage in?	☐ Classroom observations ☐ Teacher interviews ☐ Building relationships ☐ Student interview reflection ☐ Roster ☐ Collaborative meetings ☐ Student interviews	Listening guide	Butler (2005)	Chapter 4
Mr. Montoya	Why did Mr. Montoya keep coming up; how did he influence this year's relational work?	☐ Classroom observations ☐ Teacher interviews ☐ Building relationships ☐ Student interview reflection ☐ Roster ☐ Collaborative meetings ☐ Student interviews	Discourse analysis	Gordon (2008)	Chapter 6
Roster	What do Franck and Clark know about their students?	☐ Classroom observations ☐ Teacher interviews ☐ Building relationships ☐ Student interview reflection ☑ Roster ☐ Collaborative meetings ☐ Student interviews	Inductive coding	Ruitenberg (2015); Barad (2007)	Chapter 4
Vibe	What is a "chill vibe" as it characterizes Franck, Clark, and their classrooms?	 ☑ Classroom observations ☑ Teacher interviews ☐ Building relationships ☑ Student interview reflection ☐ Roster ☑ Collaborative meetings ☑ Student interviews 	Inductive coding; narrative analysis	Stewart (2010), Shotwell (2016)	Chapter 5 Chapter 6

Interviews I conducted with each of them. Accordingly, I opted for analytic methods grounded in narrative inquiry, which offer systematic ways of attending to the narratives in people's talk and in the stories they tell. Although this analysis was conducted in multiple phases over time as I clarified and refined my analytic questions and identified additional data that could contribute substantially to my understanding of Franck's and Clark's approach-as-they-articulated-it, I narrate the following phases as if they occurred linearly and sequentially.

Phase 1: Data Reduction

First, I sought to identify the data that would be most useful in elucidating Franck's and Clark's ethical stances towards the process of building relationships, in addition to the four formal Building Relationships Interviews. During data collection, and sometimes again as I reviewed it, I wrote in-process memos (Emerson et al., 2011) to summarize the topics, themes, and key ideas in data sources that struck me as having high potential for this analysis; these were most commonly teacher interviews. These in-process memos supported my decision to include two more interviews. First, I added an informal interview with Franck in which he, mostly unprompted, continued discussing similar themes to those he had raised in his formal Building Relationships Interview several days prior. Second, I added an informal interview with Clark in which we continued discussing the house metaphor I described in the Hospitality Analysis.

I created a table identifying themes and comparing quotes about these themes from the transcripts of all the included interviews; my intent was not to conduct a systematic comparative case study but to do a preliminary analysis that could focus my analytic questions and attention. For example, I noticed that both Franck and Clark discussed relationship-building as a highly situational and individualized process; they both described it as something that was both inherent to the job of teaching, in their perceptions, and enjoyable for them personally; they both

addressed the bifurcation of self while inhabiting the institutional role of being a teacher; they both highlighted the necessity of building relationships with every student, and not just those they liked or who sought them out or who "needed" a strong relationship with their teacher in order to be motivated in class. This process also determined that the richest data sources for this analysis were Franck's second Building Relationships Interview and his follow-up informal interview and Clark's first Building Relationships Interview and his house metaphor interview. These were the data sources I focused on for the extended analysis in Phase 2.

Phase 2: Using the Listening Guide for Detailed Analysis

In this phase, I drew on Carol Gilligan's work which centers listening as a way to understand the "structure of another person's inner world" and as a form of research relationship (Kiegelmann, 2009, sec. 2). To listen for curiosity, in contrast to the common researcher stance of listening for assessment or listening for evaluation, Gilligan suggests listening for plots, stories, voices, omissions, and repetitions in data, and writing "I" poems as a form of data reduction. She also recommends explicit attention to the researcher's reactions as a form of responsiveness, rather than choosing a relationship of non-responsiveness. For the focal transcripts, I followed the four-step process outlined by Gilligan and colleagues (2006). First, I identified the stories Franck and Clark told, the voices they used and how voices shifted within each transcript, omissions and repetitions, and my relationship with and reaction to what they were saying. Second, I pulled out all the "I" statements to create "I" poems, keeping a few words before and after each occurrence of "I" for context. As I was doing this, I noticed that there were many "you" statements, which sometimes referred to me, Grace, but other times were the general "you," so I created "you" poems as well and color-coded them for the "you" being referred to in each statement. Third, I highlighted the contrapuntal voices: for example, I identified voices of

conviction and voices of vulnerability and the transitions between them, which illuminated the sometimes-tension between professional responsibility and being a human who inhabits this professional responsibility, and I identified who teachers set themselves against in their stories and descriptions. After each of these steps, I wrote analytic notes summarizing emergent observations, insights, and questions. Finally, I composed analytic narratives that synthesized what had been revealed about Franck's and Clark's ethical stances on relational work by the three prior steps. In Appendix D, I have included sample excerpts from the listening guide analysis for Franck's second Building Relationships Interview. Findings from this analysis can be found in Chapter 4.

The "Mr. Montoya" Analysis

This analysis sought to examine the presence of Mr. Montoya, an eighth-grade mathematics teacher at one of Banneker's feeder middle schools, in the data. He was mentioned frequently in both student and teacher talk and seemed to often elicit strong reactions. Franck used curse words to describe him in addition to other strong language like "abuse" and "damage;" students seemed to have either very positive or very negative (sometimes both) experiences with him and brought them up to me and to their teachers unsolicited; and Franck, Clark, and Abigail dedicated about 15 minutes to talking about him and his effect on students while they were supposed to be collaboratively planning the introduction of their next unit in January. I wondered whether Mr. Montoya might serve as a useful analytic foil for determining what Franck and Claw saw as their ethical responsibilities as teachers, or what type of teacher they felt they should be, with regards to the type of experience that mathematics teachers create for their students. Mentions of him, I suspected, might have served as opportunities for Franck and Clark to air their individual ideas and for shared sensemaking. Furthermore, I was struck by

the shadow that Mr. Montoya cast over students' and thereby teachers' experiences in ninth grade even though he was no longer physically present, and how he started to haunt me as a researcher as well. I found myself alert for mentions of Mr. Montoya and wondering about Mr. Montoya: wondering whether teachers or students were alluding to him without naming him, wondering what he intended with some of the pedagogies students and teachers described, etc.

Phase 1: Data Reduction

To identify the relevant data for this analysis, I used NVivo software to find and extract episodes where Mr. Montoya was mentioned by name, either in conversation or in my observer notes wondering whether he was being alluded to. I identified 26 unique events between October 2018 and June 2019 where he was mentioned: seven teacher interviews, seven (out of 17 total) student interviews, four classroom observations, four collaborative meetings, and four other field jottings. I coded the resulting references first using *in vivo* codes that mirrored participants' language and then grouped these codes into six themes (Miles et al., 2014); these themes are displayed in Table 5.

After coding, I wrote an analytic memo summarizing my impressions from this process, which included the following excerpt:

In the Student IV Reflection, Franck links Mr. Montoya to a writing teacher who "broke students down and built them back up" as a power trip. In the 1/10 co-planning conversation, he links him to a teacher who won't let students bring backpacks into the classroom. In that conversation, teachers also extrapolate to possible long-term effects that Mr. Montoya might have on students and talk about becoming "Montoyas in the world" as a stand-in for "people who demand very narrow performances of mathematics from others." ... [These data] seem to suggest that Montoya is becoming an idea that

Table 5

Phase 1 Codebook for "Mr. Montoya" Analysis

Code	Explanation	References
Affect	Explicit emotion words	
Anger, hatred		10
Discomfort, discouragement		2
Fear		6
Motivation, pride		4
How teachers respond	Descriptions of what teachers do or what they think they need to do or should do in reaction to Mr. Montoya	4
Label	Using Montoya as a label i.e. "Montoya kids"	10
Lasting impact		
Damage	Descriptions of Mr. Montoya's influence on students that were characterized as "damage"	18
Learning	Descriptions of what students learned academically or mathematically from Mr. Montoya's class	15
Pedagogy	Descriptions of what Mr. Montoya did in his class; most of these instances overlap with characterizations of his pedagogy as "damage"	9
What mathematics is	Conceptions of what mathematics is, how to be good at mathematics, how mathematics should be done, who is good at mathematics; most of these instances were teachers contrasting their own perspectives with their inferences of Mr. Montoya's perspective	22

haunts these teachers-- that casts a shadow over their teaching of mathematics and how they think about the teaching of mathematics-- and not just an actual person in students' pasts. The mini-RQ for this phase would be "what does Mr. Montoya represent?" And part of the answer is linked to Franck's concern about teachers who wield too much power or wield power inappropriately (the writing teacher, the backpack teacher, G, and the activist teacher).

This analytic memo highlighted teachers' use of Mr. Montoya as a representative for ways of teaching mathematics and treating students that they found objectionable.

Phase 2: Analyzing Two Key Events

Next, I decided to concentrate on two of the extracted events where Mr. Montoya was discussed at the greatest length: Franck's October Building Relationships Interview, in which he answers one of my questions about building relationships with an extended tangent about Mr. Montoya and then brings him back up 20 minutes later to prove a point; and the January collaborative meeting in which Franck, Clark, Abigail, and I discussed Mr. Montoya for about 15 minutes. To investigate the impression I had that teachers felt strongly about Mr. Montoya and what he represented, I started by looking for signals of affect as evidenced through discourse and through any evidence of body language or personal perception I had recorded in the fieldnotes of these events. Drawing on methods of discourse analysis, I listened to the audiorecordings of these two conversations several times to mark the rhythms, volume, stress, and pitch of talk (e.g., Jordan & Henderson, 1995). I coded the transcripts for words denoting feelings (e.g., abuse, hate, obsessed, cry, sobbing, uncomfortable) and movement (e.g., make me, spinning, stare) (Miles et al., 2014). I used these affective cues to break the conversation into seven episodes and synthesized them into a table that summarized the atmospheric flows and shifts throughout the conversation (see Table 7 in Chapter 6). I also looked for the stories being told about Mr. Montoya (e.g., he damaged these kids; he would not respect Banneker Algebra 1 teachers' opinions), about students (e.g., if they are accustomed to memorizing conventions they will struggle in our classes; they learned how to manipulate him), about teachers themselves (e.g.,), and about mathematics education (e.g., the best part of mathematics class is figuring out something you didn't know before, not applying procedures to get correct answers; why an equation works is more important than whether it's written in the correct order) (e.g., Clandinin & Connelly, 2000; Kim, 2015).

Using the method of thinking with theory illustrated in the hospitality analysis earlier in this chapter, I read these affective signals and stories in concert with Gordon's (2008) text about ghosts and how the past remains present in the future as a call for justice. I also drew on theories of haunting as inheritance (Bellamy, 2020) and haunting in relation to futures (Yoon & Chen, 2021) and affect theorists (e.g., Jaggar, 1989; Stewart, 2007). Attending to MacLure's (2013) call to pay attention to "the gaps and intervals that we make as we cut and code the flow of difference are possible openings for wonder" (p. 181) drew my attention to my own participation in the January collaborative meeting, which surprised me because I had not previously understood my participation in the way that the analysis made apparent. The emergence of this finding, and other findings about the presence of Mr. Montoya's ghost in Franck's and Clark's classrooms, form the core of Chapter 6.

The "Roster" Analysis

I devised the Roster Interviews and analysis to learn more about what Franck and Clark "know" about their students, given both the axiom—which I take to be commonsensical based on my experiences as a teacher, teacher educator, and educational researcher—that "knowing one's students" is essential for teachers striving to build strong relationships with their students, and Franck's early mention of a mental "file folder" that he has for each of his students (fieldnotes, 10/2018). I was curious about was "in" the file folder, so to speak, as a way of making more concrete what it might mean for teachers to "know" their students. I did not expect that what Franck and Clark knew about their respective students would indicate what every teacher would know or should know, or even that what they knew was "enough" to know, nor did I expect that what they reported knowing would comprise the full set of things they actually "knew," but I did expect that what they reported knowing would offer a glimpse of what was

salient, relationally, to them. I describe this analysis linearly, in three distinct phases, although questions and insights that emerged from each "phase" informed the analyses of the other phases.

Phase 1: Iteratively Coding for "What"

I began the analysis by open coding the Roster Interview transcripts for the types of information Franck and Clark provided about their students, using NVivo to organize the coding process. I coded at the level of a unit of information, which was sometimes a single adjective (e.g., "polite"), sometimes a phrase, and sometimes a story meant to convey to me an example or an impression of what the student was like. Specifically, I started by classifying topics, like "family background," "extracurricular interests," and "how they act in math class." During my initial coding, I decided to allow rare instances of double-coding (Miles et al., 2014) and to leave some generic adjectives (e.g., good, great) uncoded if I could not discern, from context, whether it meant that a student was skillful, kind, a successful student, liked by the teacher, not concerning, something else, or that the teacher was simply done talking about that student. I used the constant comparative method (Charmaz, 2006) to revisit and refine codes through multiple passes (Strauss & Corbin, 1990). For example, I decided that the code for "characteristics" should include both personality traits such as "shy" and physical descriptors such as "has braces." I found that the codes for "math participation" and "how they interact with peers" exhibited some overlap, but were distinguishable by classifying more general statements such as "works well with others" and "gives help" under the latter code, along with indicators of social status (e.g., "she's popular with the boys"), and limiting the former code to descriptions of individual participation in mathematics class (e.g. "always volunteers to share") and descriptions of interactions with others in the specific context of a mathematics pedagogy (e.g., "he didn't love the group setting... but he was paying attention to what [his group members] were doing").

As I revised my codes, I routinely used NVivo to recode and check the codes at which particular units of information had been coded until I no longer found units that felt unclear or as if they could be coded at more codes than they already were. Codes with examples are located in Appendix E. Next, I calculated simple summary statistics about how often particular codes appeared, which highlighted what was most salient to Franck and Clark respectively and together. Although their top categories were the same, their code patterns also revealed interesting differences, as I will report in more detail in Chapter 4.

Phase 2: Exploring "Weird" and "Interesting"

Based on this coding, I decided to perform a follow-up query for the terms "weird" and "interesting" because of their relative frequency (65 references); I was curious what kinds of things Franck and Clark found weird or interesting about their students. I followed a similar process as in Phase 1, using NVivo to first extract and then code units of information from the Roster Interviews that used these key words. Reading through the results made clear that "weird" and "interesting" served as euphemisms or signals for contradictions between what the teacher expected and what he observed, so I thematically coded (Miles et al., 2014) the results for contradictions. I wrote an analytic memo that included the following note, lightly edited for readability:

The nature of these contradictions makes me think about how rarely we hold space for students to be both/and (vocal and quiet, getting into trouble with lighters yet quiet, "little guy" yet vocal, good at worksheets yet freezes in challenge, not good at math yet helps others, weird yet makes friends), or how we don't have the language to discuss that beyond saying it's weird or interesting or surprising. Teachers' openness to counterintuitive information means they're not demanding absolute consistency from

students (quiet in one setting, quiet in all settings), which reminds me of the Franck listening guide analysis and reading Butler (2005). However, the inconsistency between expectations and observations surprises Franck and Clark, as indicated by the use of "weird" and the non-evaluative "interesting," even though it probably shouldn't, because none of us are always consistent.

This memo summarized my perception that teachers made assumptions but also remained open—at least in the available evidence, which is certainly not to say that they always were or would be—to being surprised and to collecting counterintuitive information rather than falling into confirmation bias.

Phase 3: Iterative Coding for "How," "Where," and "What They Did"

Emergent questions and hypotheses from the content-based coding in Phases 1 and 2 prompted me to additionally code the data for three things, using similar processes. First, I open-coded for modes of talk. For any units of information that were not provided as straightforward descriptions, I looked for how teachers communicated what they knew about their students.

These codes were eventually synthesized into three modes of talk: analogies to other students, wonderings—including conjectures and hypotheticals, such as "maybe if she were in a different class...", and concrete stories. I wrote an analytic memo synthesizing teachers' wonderings by rewriting them in increasingly general language until the following three questions emerged that could account for most of their wonderings: 1) how would this student behave differently under different conditions, including what I might want to do differently? 2) What is motivating particular behaviors from this student? And 3) What is my responsibility to this student's autonomy vs. their learning?

Second, I coded for how teachers knew what they knew: whether from personal observation, talking to another teacher or to a family member, artifacts such as prior year's report cards, etc. Third, I coded for what teachers did with what they knew. Not all units of information in the data were accompanied by reports of how teachers knew or what they did with what they knew, so there were fewer codes in this phase than in Phase 1. Conducting this coding reminded me of instances in other data sources, such as the Building Relationships Interviews, where Franck and/or Clark described how they gathered information about students and what they did with the information they gathered, and classroom observations where I observed them engaging in some of the types of interactions they described; data from these sources supplemented the coding results in the findings I report in Chapter 4.

The "Vibe" Analysis

The idea for this dissertation arose when I—with my particular histories, expectations, and biases—was struck by how I felt in Franck's and Clark's classrooms when I first visited, and how different I felt from how I felt in other classrooms even of teachers reputed to be skillful. I, and a fellow graduate student researcher who shared in these initial observations and perceptions, struggled to put words to why their classrooms felt different, but we conjectured that their classrooms felt more relaxed, casual, and therefore human than the classrooms characterized by techno-rational, professional, and instrumental logics (e.g., Garner et al., 2017; Mehta, 2013), that we were both used to. Our impression of the "feel" in Franck's and Clark's classrooms seemed to be buttressed by students repeatedly describing Franck and Clark and their classes as being "chill" or having a "chill vibe," often in contrast to other mathematics classes or teachers they had experienced, and by Franck and Clark themselves saying that they sought to create particular "vibes" in their classrooms. In this analysis, then, I sought ways to capture and

characterize the "feel" of Franck's and Clark's classrooms, at least as it was experienced by some people some of the time, by attending to affect. As with the other analyses I have described, this one meandered and I have presented it linearly in distinct phases for the sake of accessibility.

Phase 1: Coding for Atmosphere

By the end of the 2018-2019 school year, "chill vibe" had become a bit of an outsiderinsider joke between Franck, Clark, and me because I had mentioned its frequency in students' and their comments and my accompanying inability to pin down exactly what it meant; they found it both self-explanatory and inexplicable. To identify the focal data for this analysis, I designed an NVivo search using the keywords "chill," "vibe," "comfort/ discomfort/ uncomfortable," "authentic/genuine," "fun," "trust," and "care." The additional four search terms were identified based on their popularity in teacher interviews and in student interviews (in part because I explicitly asked students whether they felt cared about by their mathematics teacher). An undergraduate research assistant provided invaluable assistance in performing and cleaning up the NVivo queries, adding several search terms she felt were relevant based on her reading of the results: "annoy," "cool," "dad," "enjoy," "honest," "open," "respect," and "welcome." After reading through the NVivo results, I determined that the search terms most relevant to this particular analysis were "chill," "vibe," and "comfort/discomfort/uncomfortable." I then open-coded the results of these searches to parse how Franck and Clark made sense of "comfort" as a concept. Three primary themes emerged: 1) Franck's and Clark's perceptions of the functions of comfort—what it does for students and teachers in a mathematics classroom; 2) their aspirations around comfort—what they wanted and why, and 3) their conjectures about factors that contributed to comfort.

Phase 2: Vignettes

Over the course of data collection and general analysis, I wrote vignettes—what Emerson and colleagues (2011) call "fieldnote tales"—to summarize moments from classroom observations that struck me, when:

a charge passes through the body and lingers for a little while as an irritation, confusion, judgement, thrill, or musing. However it strikes us, its significance jumps. Its visceral force keys a search to make sense of it, to incorporate it into an order of meaning. But it lives first as an actual charge... (Stewart, 2007, p. 39)

The moments I wrote into vignettes often stemmed from teacher-student interactions that caused my ears to prickle or my back to straighten because they were, in some way, not what I would have expected, or because they excited me for reasons I may or may not have been able to name right away. Emerson and colleagues (2011) describe fieldnote tales as being "temporary and conditional narratives" (p. 120) that reflect researchers' interpretations of which details are significant in an event at the moment of writing, and I occasionally rewrote vignettes as different elements became more salient based on other simultaneous analyses; a sample vignette is in Appendix F. As stories that I was telling as a researcher, they reflected the affects I perceived as a participant—although usually not the primary participant—in a particular situation. Thus, these vignettes not only served as a form of data reduction, but also helped me identify nascent analytic ideas. For example, the collected vignettes highlighted the significance of teachers' immediate reactions in shaping how an event—such as a mistake or something that might be treated as a behavioral infraction in another situation—would unfold. As a result, I turned to texts such as Ahmed (2004), Berlant (2011), Jaggar (1989), Stewart (2007), and Stewart (2010) to think with theory about 1) affect in embodied reactions as a carrier of our aspirations and

histories and thus as a signal of what we consider ordinary; and 2) the atmospheres and atmospheric attunements ("an alerted sense that something is happening," [Stewart, 2010, p. 4]) constructed by ordinary affects. Findings from this phase of analysis informed Chapters 5 and 6.

Phase 3: Pedagogical Practices

The processes of engaging in, and the results from, the two analytic phases I have just described pointed repeatedly to Franck's and Clark's belief that their pedagogical practices were central to the creation and sustenance of the "chill vibe" that made (at least some) students feel "comfortable" in their mathematics classrooms. Franck and Clark, as part of Banneker's Algebra 1 team, had identified three pedagogical practices as central to their instruction for the 2018-2019 school year: notice-and-wonder activities, (daily) visibly random grouping, and whiteboarding (using large-scale, public, vertical whiteboards for groupwork). As a result, I returned to data where Franck and Clark discussed and enacted these pedagogical practices: teacher interviews and classroom observations. I reread this data in conversation with the conjectures I had developed in Phase 1 and Phase 2 and with Shotwell's (2016) theoretical concept of open normativities, which illustrated the potential that Franck's and Clark's pedagogical practices had, as they enacted them, to contribute to the development of open normativities. In this iteration of thinking with theory, I traced the role of comfort in contributing to open normativities, and how Franck and Clark linked their pedagogical practices to the creation of comfort. Findings about the possible consequences of their pedagogical practices are reported in Chapter 5.

Reading Across Analyses

Finally, to develop the storyline for this dissertation, I synthesized key insights from each analysis into a comparative table (see Table 6). Reading across them suggested that Franck's and

Table 6Summary of Analytic Findings

Analysis	Key ideas		
Hospitality	 Redefining the conditions of hospitality: a teacher's interactions with students should not be conditioned on particular behaviors, and instead requires engaging on students' terms Inverting the direction of hospitality: students have the agency and autonomy to invite teachers into their space or not 		
Building Relationships	 Honoring unknowability: We cannot demand that students present a coherent self-narrative Opening oneself up is an ethical resource: Risk-taking and vulnerability make us more human; it's the alternative to paralysis 		
Mr. Montoya	 The past lives in the present and the future: contending with mathematical ghosts who call for justice is one way to acknowledge and address not just students' individual prior experiences but also teachers' location in a history of mathematics education that has traditionally been violent and unjust Working through rather than repeating domination (Zembylas, 2006): Franck and Clark react to what Mr. Montoya did and continue to examine their pedagogical practices in this light 		
Roster	 Ongoing: the constant practice of seeking to know students matters more than the factual knowledge that is acquired Non-intrusive: constantly seeking to know is a non-intrusive practice; it comes from attentively observing and from laying foundations, not from directly asking or demanding that students make themselves known Knowing allows continued knowing: Franck and Clark did not seek to leverage what they knew for instrumental reasons like motivating students to perform better in class, but as a fundamental human engagement 		
Vibe	 The right balance of comfort makes learning possible: Franck and Clark did not want students to always be fully comfortable in every way, because some forms of comfort and discomfort both support learning Attending to comfort requires paying attention to reactions: comfort cannot always be planned for up front, although particular pedagogical practices make it more or less possible 		

Clark's ethical stances on relational work could be characterized by what I will call ongoingness, expansiveness, and attunement to affective inheritances. These analyses spoke to student agency and teacher power and challenged conventional notions about building relationships in mathematics education. For a final round of thinking with theory, then, I brought the results of these analyses, as a further reduced representation of my data, into conversation with feminist

new materialists' theories of subjectivity and, therefore, ethics. Reading Shotwell (2016), Haraway (2016), and Barad (2007) directed me to *response-ability* as a theoretical construct that had explanatory power for findings that centered ongoingness, expansiveness, and attunement to affective inheritances as an ethical stance in mathematics teachers' relational work.

As I hope has been apparent throughout the discussion of data collection and analyses, my conversations with Franck and Clark offered near-constant opportunities for member-checking as I reflected back to them what I had heard them say, shared my perceptions of their teaching practices, and occasionally presented them with concrete data I had collected in order to elicit their reactions. They regularly asked me how the dissertation was going and what I was learning, and I did my best to respond with whatever I knew at the time. However, I also conducted more formal member-checking of the overall analytic findings during the process of writing the dissertation by sharing emerging ideas and draft chapters with Franck and Clark; among other things, Clark wrote back, "I love the overall approach and the phrasing of responseability. That term captures something that I see as core to what we are doing and how my class operates. I'm good with everything I saw about myself" (personal communication, 2/2021).

Data Reporting and What to Expect

To balance reliability and readability in this dissertation, which mostly foregrounds semantic meanings rather than meanings communicated by tone, pitch, prosody, and other qualities of talk, I have made some choices in how I report the data in the following chapters. First, I offer participant quotes abundantly, rather than paraphrasing or summarizing, in order to preserve their narrative voices. I report participant quotes as close to verbatim as I can, either based on the near-scripts I typed in my fieldnotes or based on a combination of professional transcription of audio recordings and re-listening to the audio to check for meaning. I omit filler

words such as "um" and "uh," use ellipses where I have omitted more than filler words, use dashes to represent extended pauses or to signal topical shifts, use capital letters to indicate stressed words, and add explanatory details in brackets where I believe they are essential to understanding the intent or meaning of the quote. The vignettes in this dissertation are not taken word-for-word from my fieldnotes, but rather, reconstructed to fit the space and intention of the vignette in the text. Where I report word-for-word field note excerpts, as I sometimes to do communicate direct quotations captured in the moment, I include the citation in parentheses.

In the next three chapters, I illustrate how ongoingness, expansiveness, and attunement to affective inheritances characterize Franck's and Clark's ethical stances on relational work. I open each chapter with a vignette, heeding Stewart's (2010) call for descriptive detours:

Rather than rush to answer the general question—"what's going on?"—we might give pause to wedge into that question a speculative curiosity—a descriptive detour into the necessarily compositional and generative nature of a present moment caught in the throes of emergences and wanings of all kinds (p. 8)

I invite readers to dwell in these vignettes for a moment and feel their own speculative curiosities before continuing into the chapter, where I introduce truisms about what constitutes right relation in mathematics education and identify questions or limitations that accompany these truisms. In each chapter, I then report how my analyses of Franck's and Clark's ethical stances on relational work spoke to these questions or limitations and discuss how these findings both illustrate and inform the conceptualization of an ethics of response-ability in mathematics education.

Chapter 4

KNOWING FOR RESPONSE-ABILITY

March 26, 2019, from an interview with Franck:

"Lucas? Lucas Marco? Brought me those two [Lego characters] and a figurine of Thanos. Weird. Interesting, right? He pulls out—on my birthday he goes, "Mr. To, do you like Avengers?" I was like sure, yeah. So, he gave me this Thanos.

[My child] Elliot—this is funny because I was going to show Lucas this [pulls out phone and shows me a photograph]—Elliot last night slept next to Thanos. [In class the other day] I was like "Elliot cut his face, had to go to urgent care." So Lucas pulls this out of his backpack. He's like, "Don't think this is weird or something, I have a lot of these," but then I was like, "This is weird!" But I think, in a sense I thought about it, well, a kid does not have many worldly possessions, and he values these things obviously, and to give it away to me on my birthday, it's meaningful I think on many levels.

And Lucas used to frequent my classroom a lot more at lunch and ask me the weirdest questions or give me the weirdest comments like, "Hey, you know, my mom, like--" Just weird. It's out of left field. But I never say, "Okay, this is weird." I always respond, and I'll always ask a follow-up question and whatnot. And I don't know, maybe he doesn't have a lot of adult friends or interactions, and so he values asking the same things.

...His math skills, he thinks differently. His work, it's really weird. It's not—what do you call it—it's not linear. It's circular. It's very interesting. But he consistently gets good grades on my tests and quizzes so hey, if it works for you, buddy, I'm not gonna be the guy that says it doesn't work.

Yeah. I like the kid a lot. I like him. He can make friends, which is really interesting. Like, "you're such a weird kid," but he makes friends in my class. He's a really slow talker that's like—he's almost a introvert, but brings something to the table that kids like him."

March 26, 2019, from a researcher memo:

This data makes me wonder about the ethical import of what teachers know about their students. It feels related to the idea of meeting students' needs and considering whether a student needs a parent figure, or a place to hang out, or a relationship with an adult (see notes about Lucas). This idea speaks to me of a sense of accountability or responsibility or answerability (whichever word I end up using): what is called for, from me, by others because of who I am in relation to who they are? ... I also sense a respect and humility I find really beautiful in the idea of "following students lead" and not "forcing them to like me," or an honoring of student agency in a way that doesn't press students into a relation they're uncomfortable with.

March 27, 2019, from classroom fieldnotes:

As he's circulating, Franck stops by group 4 and shows Lucas a photo on his phone. It's the photo he showed me yesterday, of his son sleeping next to the Thanos toy that Lucas gave Franck for his birthday. Lucas says "aww!" and I can see the large smile on his face from where I'm sitting all the way across the room. "I can't give him the other stuff because he'll swallow it," Franck says, referring to the Lego characters, "but thanks."

March 29, 2019, from an interview with Lucas:

Grace: Pretend I never met Mr. To. If you had to describe him, how would you describe him?

Lucas: He's a great guy. He helps. He isn't like other teachers, serious. He's a really expressive person and he has a lot of methods. He shows us some things of his personal life that help describe who he is. He's very nice.

Grace: Can you give me an example?

Lucas: Like his wedding video. It helps show how he was during that moment and how emotional he got and sometimes when he tells us about his children it helps kind of give a description about him and how he is as a father.

Grace: How does that make you feel?

Lucas: It makes me feel comfortable around him, really trust in being near him. I feel confident that he's going to help me learn throughout the year.

Throughout commonsense notions about teaching and the education research literature—especially the literature I reviewed in Chapter 2 describing the violent relations of mathematics education and the restorative relations proposed by care theory and culture-rich pedagogies—getting to know students and knowing students is taken to be a foundation of relational work. Doing so, the thinking goes, allows teachers to better build trust, adapt their curriculum to match students' interests and funds of knowledge, and ensure student learning. However, what it means to know students, and therefore how a teacher gets to know students, is neither as straightforward nor as simple as some educators make it seem when they focus on the acquisition of factual knowledge: what students like, where they live, who they love, etc. In fact, educational philosophers' conceptions of subjectivity, hospitality, and power raise questions about whether it is even possible for teachers to know their students, and whether it is coercive to try. Of course, the idea that it matters for teachers to know their students is compelling, especially because most

of us who have attended or worked in schools have felt the visceral and powerful difference it can make in both teachers' and students' experiences. In that case, what *does* it mean to know students, if not merely the acquisition of factual knowledge?

I begin this chapter by summarizing some of the prevailing assumptions about why knowing students is important and how teachers should know their students. Next, I outline some of the ethical reasons that knowing students may be impossible. Then, I identify the data and methods of inquiry from Chapter 3 that contributed most to what I report in this chapter. I present an analysis of Franck's and Clark's approaches to relationship-building and their sensemaking about it to offer some possibilities for what it can mean to know students, beyond factual knowledge, while navigating the ethical dilemmas I have laid out. I suggest that their ethical stances towards relational work take knowing to be an ongoing practice of attention, rather than the accumulation of factual knowledge, and as a practice of inviting further response, rather than collecting information to be leveraged for academic learning. I return to feminist new materialist theorizations of response-ability to argue that this approach towards knowing takes a stance on who and what matters. Finally, I close with implications for practitioners and researchers.

The Importance of Knowing One's Students

The importance of knowing one's students is a truism in teaching; getting to know students is a foundational step in establishing trusting relationships, and in common parlance, "knowing your students" is often assumed to lead to a form of pedagogical intimacy that supports student learning. For example, McCaughtry (2005) draws on the work of Hargreaves and others to challenge conceptions of pedagogical content knowledge that ignore students' "emotional and social experiences" (p. 381), arguing that this type of knowledge is essential for making pedagogical decisions. Knowing students is a key element in Noddings's (1992)

theorization of care, and Ayers (2004) writes that knowing students well is essential to teaching towards freedom. An extensive research literature recognizes the importance of teachers knowing their students; here, I briefly offer a selection of examples to illustrate that it is a widely accepted and examined concern for educators.

Research focused on students of color strongly emphasizes the role of teachers' knowledge of their students, linking this knowledge to the positive student-teacher relationships that can serve as a protective factor against the ways that racism plays out in school (e.g., Berry, 2008; Ladson-Billings, 1995). Within mathematics education, researchers highlight the importance of teachers' awareness of students' identities as mathematicians and as racialized cultural beings for establishing trusting relationships and creating positive mathematical learning experiences (e.g., Aguirre et al., 2013; Hand & Gresalfi, 2015; Martin, 2012; Na'ilah Suad Nasir, 2002). Parker and colleagues (2017) note that mathematics teachers must know their students' cultures in order to avoid dominant and oppressive mathematics education practices, and Gay (2007) specifies that knowledge of students' cultures is necessary to create cultural congruity in the classroom. Building on Valenzuela (1999) and other Lat/Crit scholars, Rolón-Dow (2005) notes that teachers' knowledge of students needs to be grounded in a historical understanding of students' lives in addition to the present moment. Teachers' knowledge of students' cultural and linguistic backgrounds and funds of knowledge, specifically, can create a sense of belonging; it can also reposition students, rather than racializing and marginalizing structures, at the center of teaching and learning (DeNicolo et al., 2017). In her theorization of caring with awareness, Bartell (2011) summarizes this as knowing students "mathematically, racially, culturally, and politically" (p. 65).

This research could be read as arguing that knowing one's students matters because teachers can leverage that knowledge for academic influence; Gehlbach and colleagues (2007), for example, find that students' grades improve when teachers are made aware of similarities between themselves and their students. Indeed, some educators may explicitly argue that the primary purpose of building teacher-student relationships is to support student learning. Another read, however, would be that what teachers know about their students, and how much they know, matters less than students' perception of being known by their teacher. From this angle, mere similarity seems to have less effect; in the aforementioned study, Gehlbach and colleagues found little effect from students being made aware of similarities between themselves and their teachers. Other scholars, however, report more positive results when students feel a connection to their teachers. Redding's (2019) review of the teacher-student racial-match literature, for example, suggests that one reason students benefit from having same-race teachers is because they feel culturally understood. Phillippo (2012) finds that students mostly appreciate "teacher personalism," which she defines as supportive one-on-one relationships. Drawing on the psychological literature about belonging and the developmental role of student-teacher relationships, Chhuon and Wallace (2014) and their colleagues have conceptualized students' sense of being known as comprised of the instrumental support, benefit of the doubt treatment, knowledge through observation, and meeting of the minds experiences they perceive from their teacher. They link being known to students' valuing of mathematics, expectancy of success in mathematics, and decreased reporting of experiencing racialized negative perceptions of them as mathematicians by others (Wallace & Munter, 2018). Regardless of whether researchers claim that knowing one's students is important because it enables teachers to teach more effectively or

because it enables students to feel like they belong in school, the literature seems to agree that teaching and learning are improved when teachers know their students.

In terms of how teachers come to know students, commonly proposed strategies include surveying students, assigning mathematics autobiographies, visiting institutions in students' neighborhoods, and designing lessons that reference students' lived experiences (e.g., Aguirre & Zavala, 2013; Emdin, 2017; Towers et al., 2017; Zeichner et al., 2016). These strategies are, for the most part, discrete actions that can be completed and checked off a list, which shortchange the depth of knowing implied by the descriptions above. This likely reflects the difficulty of writing about something as complex and particular as teacher-student relationships, and researchers' endeavors to provide concrete recommendations for practitioners who crave concrete recommendations, rather than limitations of scholars' conceptions of teacher-student relationships. Nevertheless, it points to a need for teacher-facing advice that is specific and actionable.

The Impossibility of Ethically Knowing One's Students

Despite the purported advantages to knowing one's students, however, philosophers have written about the impossibility of ever truly knowing another person, and therefore the ethical dilemmas inherent in attempting to do so. For example, Butler (2005) draws on Levinas, Foucault, Cavarero, and other philosophers to argue that because so much of who we are is shaped by inaccessible stories, histories, and "norms that precede and exceed" us (p. 17), we can never fully know who we are. And if we cannot definitively answer who we are, then we cannot reasonably demand of others that they present a coherent self-narrative either. In Margonis's (2007) words, "I—as a teacher—encounter in the face of the student an infinite and unknowable other... more basic than any posited unity stands the radical difference of the other" (p. 62).

Ruitenberg (2015) writes that the central injustice of education—which is amply evident in mathematics education specifically—occurs when children are told that the world is not for them, and that a teacher's job is to unlock the world for students. Conventional thinking suggests that unlocking the world for students might be easier if teachers know their interests and motivations and can thus find relevant avenues and opportunities to welcome students to an academic or mathematical world that teachers already inhabit. But Amin and Vithal (2015) warn about the dangers of knowing: what teachers know about their students, particularly students experiencing oppressive circumstances, frequently feeds stereotypes and low expectations. Furthermore, students' unknowability, to themselves and to the teacher, means that one cannot ethically demand to know students, no matter how much one might like to, no matter how much "easier" it might make teaching to have unfettered access to knowledge of students that could be used to more effectively influence them. There are three reasons for this: conditionality, coercion, and change.

First, expecting to know students imposes a condition on the teacher-student relationship by suggesting that a teacher is only able to teach effectively if students are willing to share (or surrender) something personal with their teachers: an extracurricular interest that teachers can integrate into the curriculum, perhaps, or a loyalty from being made to feel like they belong. Theories of hospitality in education, however, suggest that the teacher-student relationship is "unapologetically asymmetrical" in that teachers have a responsibility to extend what they are offering—access to the discipline of mathematics, or to higher-level mathematics courses that serve as a gatekeeper for future academic opportunities—rather than requiring students to meet certain conditions to maintain their invitation (Ruitenberg, 2015, p. 40). Teachers' responsibility stems from the fact that they already have access to the worlds that they are unlocking for

students, so the obligation is not mutual. Students are under no obligation to accept that which teachers offer, yet teachers must continue to offer it: "a student's lack of engagement or active misbehavior does not change my position as host" (Ruitenberg, 2015, p. 41). Or, in other words, how teachers engage with students should not rely, ethically speaking, on either whether students follow classroom rules or allow themselves to be "known."

Second, expecting to know students underscores teachers' power, especially when students from marginalized communities with a historically fraught relationship with schooling are asked to make themselves legible to teachers, who in their institutional roles represent the dominant culture. Students may not feel able to decline requests from an authority figure (Derrida, 2000) and may feel that teachers are threatening their privacy or agency. Phillippo (2012) uses language from student interviewees to describe this sense of being invaded as teachers "trying to know me" or "trying to know my business." This is not to say, of course, that teachers should not ask their students questions, give surveys, assign mathematical autobiographies, visit students' neighborhoods, or other strategies commonly recommended in teacher education literature. It is instead an acknowledgment that these requests cannot be expectations or demands—especially when they are unidirectional, but even if teachers offer themselves up in the process—because teachers must address themselves to students without "seek[ing] to reduce the independent nature of the Other's existence through domination, identification, understanding, or even care" (Todd, 2008, p. 177).

Finally, people are constantly changing, particularly in adolescence. "Who a student is" is fluid and ever-shifting, calling into question which part or version of a student, at which moment in time, is really "known" when teachers claim to know students, and what can be done with such a knowledge that is always partial (S. Wilson, 2008). Although conditionality, coercion, and

change pose ethical and epistemic challenges to the possibility of knowing one's students, it would be a mistake to dismiss the potential for meaningful human connection that comes from teachers feeling as though they know their students and from students feeling known by their teachers. What should it mean, then, for teachers to know their students given the constraints of conditionality, coercion, and change? Next, I remind readers of the empirical data and analyses that I use to inform a possible response to this question.

Methods (A Brief Reprise)

As discussed in Chapter 3, I selected Franck and Clark for this study in part because of their preexisting reputation for having good relationships with students. When I initially observed them, I was struck by a sense of warmth and friendliness in their interactions with students. Although neither the teachers nor I ever explicitly asked students whether they felt known by their teachers, I did ask the students I interviewed (see page 62 for a description of the student interviews) whether they felt like their teacher "understands or 'gets'" them, and all responded affirmatively. As a result of these observations and the prevailing wisdom about the importance of knowing one's students, I sought to investigate what it is that Franck and Clark "know" about their students and how they come to know it through the Roster Interviews, Roster Analysis, Building Relationships Interviews, and Building Relationships Analysis described in Chapter 3.

Although the feminist new materialist assumption of entanglement (see Chapter 2) contends that teachers are not independent subjects who know their students as discrete objects, eliciting teachers' reported knowing nevertheless provides a snapshot of what is legible about an entangled phenomenon (teachers, students, researcher, and context intra-acting) within one subset of the phenomenon at one moment in time. The Roster Interviews spanned several

months, capturing what was legible at several moments in time as well as reflections from Franck and Clark on how what was legible would have also differed had I conducted the Roster Interviews all at once and/or at a different point in the school year. Because I did not, I cannot claim that what Franck and Clark reported knowing about their students in the Roster Interviews was all they knew about their students, or even the most important things they knew about their students, nor can I claim that what they did not share with me was not known to them. What I report in the following sections, then, takes the Roster Analysis as an anchor but overlays findings from the Building Relationships Analysis to make a case for Franck's and Clark's knowing as an ongoing practice of attention that cultivates further response.

Findings

In this section, I present the what, how, and why of Franck's and Clark's knowing about their students. I argue that they take an ethical stance of "being open and alive to each meeting" and that this stance "breathe[s] life into ever new possibilities for living justly" (Barad, 2007, p. x) by cultivating both their ability to respond to students and students' ability to respond to them. First, I summarize what teachers "knew" about their students and compare that to what is recommended in the extant literature because doing so raises a question that the subsequent findings sections then address: if what Franck and Clark knew about their students diverged from what the literature recommends, then what was it that they did "know" that contributed to the strong teacher-student relationships that I observed and that students reported?

In the second findings section, I examine how Franck and Clark knew what they knew: through constantly "assessing and interacting" (*Franck, interview, 10/2018*). Next, I describe what they do with what they know, claiming that this constant assessing and interacting creates possibilities for both student and teacher response and, significantly, honors students' agency in

deciding how and when to invite their teachers in. Building on these findings, I then revisit feminist new materialist theorizations of knowing and mattering. I return to the vignette that opened this chapter as a case of how Franck's and Lucas's intra-action with mathematics education illustrates an ethics of response-ability. Finally, I conclude with implications for practitioners and researchers.

What Teachers Knew

When I asked Franck and Clark to tell me about their students, the most commonly coded references were simple adjectival descriptors, such as "funny," "outgoing," and "polite." Almost all of their responses began with such descriptors before then elaborating on what they meant or telling a relevant story. Aside from these simple adjectival descriptors, however, Franck and Clark both overwhelmingly described how students engaged in their Algebra 1 classes: how students encountered mathematics content, their mathematics abilities, and how they interacted with their peers in class. Although they mentioned facts about their students' lives outside of the mathematics classroom, such as extracurricular interests, family compositions, or grades in other classes, these facts were mentioned far less often.

Franck and Clark described how students engaged with mathematics content and students' mathematics abilities in great detail. They explained students' willingness to ask questions and their persistence in the face of difficulty. They noted which students grew particularly enthusiastic about notice-and-wonder activities, which students were hesitant to show their thinking on paper, which students were prone to making particular mistakes ("they miss a minus sign or don't carry the one," *Franck*, 3/2019), which students sought extra work, which students seemed to reach conclusions more quickly ("she'll stand there for a while... I'm not sure if it's her thought process or like showing she's struggling," *Franck*, 2/2019), which

students struggled to articulate what they were struggling with, and which students demonstrated non-linear or creative mathematical reasoning. They remarked on who thought the class was too easy and which students were being challenged at just the right levels. Both Franck and Clark recalled students' grades on assessments from memory, citing them as evidence of students' mathematical abilities, academic consistency, and/or exceptional circumstances, and used hypotheticals grounded in an assumption of carefully scaffolded tasks to communicate what students were likely to be able to achieve on their own ("whether they're going to get #7," *Franck*, 2/2019).

Finally, Franck and Clark described how students interacted with their peers in mathematics class: which students frequently helped others; which students asked for help; which students listened attentively to others; which students would defend their ideas ("he won't back down, so if you're butting heads in a group over math... he'll quietly be defiant... he's strong," *Franck*, 3/2019); which students specialized in explaining their thought processes; and which students were especially adept at "drawing other people out" (*Clark*, 6/2019). They also noted which students seemed to participate in consistent ways regardless of who else was in their group and which students' participation seemed heavily influenced by group composition. Teachers also discussed students' mathematical histories, such as their middle school mathematics experiences, and their level of confidence in mathematics.

Outside of mathematics, teachers mentioned students' families—such as siblings who also attended Banneker, or interactions teachers had had with their parents; their extracurricular interests—both school-affiliated, such as basketball team membership, and informal, such as hobbies; their general social status in school—often using shorthand terms like "popular" or "doesn't have many friends;" and their specific social dynamics—such as listing other students

they hung out with, used to hang out with, or had dated. Franck also reported students' grades in other classes as a way of gauging whether they were under- or over-performing in his class compared to what he perceived as their general academic abilities. These non-mathematical categories, however, occurred far less frequently than the mathematical categories; Clark mentioned these for fewer than half his students, and although Franck mentioned at least one of these categories for almost all his students, he did so with far fewer references and far more briefly than he discussed their performance in math class.

Their Knowing in Relation to the Literature

As a set, these findings about Franck's and Clark's attention to who students were in mathematics class recall Ball's (1997) invocation of the importance of "knowing particular students at specific moments in real, face-to-face mathematics teaching" (p. 732). Ball concentrates on knowing what students know or understand mathematically so that teachers can ascertain how to support students' learning, or their development of mathematical competence. Franck and Clark, on the other hand, also emphasized students' emotional and social experiences of mathematics: their personalities in mathematics class, their affects in mathematics class, and their intra-actions with particular forms of mathematics and with their peers while doing mathematics together. The literature suggests that some of the ways in which Franck and Clark referred to their students' mathematical abilities, such as saying someone is a "C student" or "smart" or "low," may have implications for those students' status and for how they treat those students (e.g., Horn, 2007; Louie, 2018; van Es, 2011). But, by attending to more than just students' mathematical abilities or even their mathematical thinking and reasoning, Franck and Clark affirmed that their students are humans intra-acting with mathematics, and not just differently-competent participants in the enterprise of mathematics.

When asked to tell me about their students one by one, Franck and Clark both focused on students' individual performances in math class rather than their racial identities or the broader cultural contexts of their students' lives. There were some exceptions; Franck, for example, offered the specific national origin of a few of his students' families, especially when they matched the national origin of his spouse's family. And, in other conversations, Franck and Clark demonstrated awareness of their students' racial and cultural contexts. Among other things, Franck described some of the challenges that Banneker students faced due to poverty and immigration status, connecting them to his own experiences growing up in a similar neighborhood just a few miles away, and said that he made an effort to attend every quinceñeara he was invited to; he also joked with students in class about being available to DJ their quinceñearas. Likewise, Clark discussed the impact of systemic racism and political decisions on his students' lives. However, an analyst looking for evidence that they deeply knew students' racial and cultural backgrounds and associated funds of knowledge in the Roster Interviews may have been disappointed. Rather than speculating about why Franck and Clark did not report students' racial and cultural backgrounds in the Roster Interviews, however, or about whether this affected their relationships with their students, I remind the reader that they appeared to have strong relationships with their students based on my observations, the observations of their colleagues and administrators, and what students shared with me, and now turn to what—other than the factual knowledge they did or did not "know" about their students—may have contributed to these relationships.

How They Knew It

Ongoing Practices of Attention

What Franck and Clark told me they "knew" about students could not have been collected via surveys or math autobiography. Rather, this knowing required an ongoing practice of attention: of observing students closely and over time in their classrooms. Furthermore, their observation of students could not be a passive observation—like mine mostly was—but rather required observation during active interaction. As Franck said:

Every day I'm gathering information about the students, assessing. And I'm adding to that [mental] folder and I'm really good at remembering things about the students. Because they're human beings... All those little things help me to interact. Assessing and interacting. (*interview*, 10/2018)

Similarly, Clark said that he was always "listening to [students]. I'm just observing them and keeping tabs on everything that's going on. And then I can see how they are... that's how I get to know them, and what are their preferences? Where are their choices?" (*interview*, 3/2019). This assessing and interacting, observing and keeping tabs, was a defining characteristic of how Franck and Clark engaged in relational work with their students.

Consequently, both teachers noted that their responses to my questions were shaped by the time of year; I conducted the Roster Interviews in the spring, and Franck and Clark suspected they would have responded differently had I asked them to tell me about individual students in the fall. When asked to hypothesize how he would have responded differently, Franck said:

I probably know more about [students] individually at the beginning of the year, and I've erased all that, because I've replaced it with math... those things I value so much more at

the beginning of the year because I don't know much more about them, but then now, it's like, I just get to business. (*interview*, 3/2019)

Franck noted that at the beginning of the year, he had asked students to fill out cards with their contact information and some autobiographical facts, including the middle school they had attended and their hobbies. By spring, however, he remembered mostly "something that's more relevant. And I might have forgotten that thing about their personal life" (*interview*, 3/2019). Over time and intra-action, students became differently legible to him, in that he "knew" differently about who they were. Interestingly, Franck described the autobiographical facts as knowing "more" about his students than what he knew after eight months of intra-action, perhaps testifying to the ready availability of conventional conceptualizations of knowing, and knowing students, as being based on the acquisition of factual information. For him, however, the autobiographical facts mattered primarily as fodder for "breaking the ice" with students (*interview*, 3/2019) and laying a foundation for future interactions, rather than being the end-all be-all of what teachers ought to know about their students.

Clark echoed the idea that factual information mattered less than ongoing intra-action in getting to know his students:

Do I know a lot about my students? And I think the answer is no. And in traditional sense I don't know what their favorite video game is and that kind of stuff. And I've said this before, I try to get to know their personality. I definitely pay really close attention to all my interactions, so I think that's really important, and so whenever I'm interacting with a student I'm always thinking and really closely paying attention to how are they responding to all the things I'm doing. So I might beginning of the year, mid-year, and all the way to the end of the year, I might test out some things and just sort of see. Is this

something that makes them smile when I say it? Is this something that makes them laugh when I say it? Is this something that makes them cringe when I say it? (*interview*, 4/2019) For Clark, knowing his students was about continuing to try out different ways of interacting with students and adjusting accordingly, "all the way to the end of the year," not about the facts he accumulated. Not only did Clark suggest that there was never a point at which students were sufficiently "known" that he could stop testing things out, but also, he claimed that this practice allowed him to "get to know someone without knowing facts about them" (*interview*, 10/2018). The information that both Franck and Clark gathered about their students—what made them smile, what made them laugh, what made them cringe—was ancillary to the process of gathering the information: actually making students smile, making them laugh, making them cringe.

From Their Social Locations

Because knowing depended on intra-action, in Franck's and Clark's stances, who they were mattered for how they knew. Clark noted that his intra-actions with students—and how he knew students or how he got to know students—could not have been the same during the school year that I observed as they would have been earlier in his career:

I'm not the same person that I was 10 years ago or five years ago. And so, I don't have 15 years' worth of data on interacting with students, because I only have the relationships I can have with them now as the person that I am now. Not the same type of relationships I could have with them when I was 26. (*interview*, 7/2019)

Likewise, Franck, described how he brought himself to his intra-actions with students:

Every student has something to offer. So do you. And you don't consciously think about it. But if you think about it the same way you think about your students, maybe you can invoke more of whatever you bring to the table. Not necessarily in a teaching way, but

more of in a human way. I don't think that this is discussed very often [in education]. What do you have to bring to the table? (*interview*, 3/2019)

For both Franck and Clark, how they "assessed and interacted" or "tested things out" with students depended not just on what they knew about their students and how their students responded to them, but also on who they were at a particular moment in time, as an accumulation of their own personal histories and experiences.

For example, a few days after hearing about Emdin's (2017) book entitled For White Folks Who Teach in the Hood... and the Rest of Y'all Too, Franck told me that his version of that book would be called "[For the] Immigrant Asian That Grew Up in the Inner City That Went to Magnet School That Went to [Local University] That Didn't Become an Engineer and Became a Teacher and Married a [his spouse's ethnicity, which is shared by many of their students] Who Went to the Same School" (interview, 3/2019). He saw these identities and experiences as being central to what he brought to his intra-actions with students. He said:

You know what also helps is I grew up in this area—not this area, but this kind of environment. I live in this area. I also stay up to date with current—do you call it current events? Because kids don't have current events; they're inventing their life. I think they walked in [to class one day] and I was playing Baby Shark and they were just looking at me like, whaaaaaat? Yeah, I know Baby Shark. (*fieldnotes*, 10/2018)

Franck found that his own geographical and social locations seeded a foundation for ongoing intra-actions because they created points of possible connection for students and opened up possibilities for further intra-action. Because what teachers observe and how they react are inevitably shaped by who they are, Franck's and Clark's ways of knowing students—which require attentive observation and active interaction—were deeply situated and personal.

Why They Knew: Inviting Further Response

Students' laughing, smiling, and cringing—the responses that Franck and Clark prompted through their ongoing intra-actions—made it possible for Franck and Clark to assess and interact even further: to respond to students' responses. In this section, I begin by describing how knowing fed Franck's and Clark's abilities to continually engage with students. Because they had more institutional power than students did, they saw it as their responsibility to tilt intra-actions towards further knowing by initiating conversations and by opening themselves up, and then by pushing the envelope and going deeper. Doing so created opportunities for students to respond in ways that signaled how they wanted their teacher to respond next. Some students contributed to a further opening up, but others refused; Clark drew on the metaphor of hospitality to conceptualize how he could honor students' agency in this context. I argue that Franck's and Clark's knowing, as inviting further response, made matter the humanity of their students and themselves as people and not just as learners and doers of mathematics.

Breaking Ice and Opening Up for Students' Responses

Franck described it as his responsibility to initiate the deepening of relationships: his relative power as a teacher, "because [as a teacher] you have the stage all the time" (*interview*, 3/2019), created an ethical obligation to make possible students' ability to respond in ways that deepened relationships:

That's part of your responsibility, to at least break the ice with everybody... but then you also have to find ways to give them the stage to tell you about themselves. (*interview*, 3/2019)

Franck saw this responsibility to "break the ice" as part of his fundamental obligation to every single student by nature of his role as teacher, regardless of how students responded:

I've never not taken an opportunity to get to know a kid. Like if you're gonna ask me a question or if you're gonna come ask me something, I'm gonna stop what I'm doing and respond to you... I will not discriminate on any kid... They have a choice, but I don't have a choice. If they approach me, I'm gonna open myself up or at least give them the opportunity. (*fieldnotes*, 3/2019)

In contrast to a teacher who might see it as students' responsibility to seek out the connection they desired, Franck felt that his institutional authority required him to initiate and open himself up to conversations and with them, possibilities for connection.

By "opening myself up," too, Franck made himself more legible to students, increasing their knowing of him and thus their possibilities for response. He frequently mentioned his family in class, especially by mentioning what they were planning to do over the weekend on Fridays or what they had done over the weekend on Mondays, and used the curriculum to humanize himself as well. During preliminary data collection, I watched his students complete a task in which they calculated how long it would take for a person to quit smoking cigarettes if he gradually reduced his consumption each month, and then how much he might be able to spend on an engagement ring with the money saved from not purchasing cigarettes. When introducing this task, Franck asked students to raise their hand if they knew anyone who smoked cigarettes almost every student raised their hand—and said that his own father smoked for a long time. After students finished the task, Franck said, "Can I let you in on a little secret? Promise you're not going to judge me? The task was actually about me." Students gasped, and he added, "I used to smoke, but when I met my wife, well, then my girlfriend, she was like, 'that's not going to work." A student called out, "so that was how much her ring was?" Franck shook his head no; "the numbers were fabricated to make it easy for you, but the situation was real" (fieldnotes,

3/2018). In an interview, Franck offered this example: "the other day we did a wedding task. I show them my wedding video that was five minutes long. So then they're like whoa, he's a human being" (fieldnotes, 10/2018). When I interviewed his students several months later, multiple students mentioned the wedding video as an example of why he was "nice" or "a great guy;" it, and the cigarette task and other tasks like them, provided fodder that students could draw on for follow-up questions and conversations if they chose, or simply to feel more connected to their teacher.

For the most part, Clark was more guarded about his entanglements outside of Banneker than Franck, in part because he speculated that some information could make some students feel excluded even as it made others feel connected. For example, throughout his career, students had routinely asked him whether he drank alcohol:

I am going to get asked seven times every year whether or not I drink alcohol... I also always think about how does that affect, how does that alienate, how might that alienate a kid? To find that information out, what are the benefits, if you know I drink alcohol, what does that do, how does that change your perception of me, and what does it do for every kid in the room? (*fieldnotes*, 7/2019)

Clark sought to respond in a way that did not alienate the students who might have "thought he was more wholesome than that [and not] really want to know that about him:" that did not close off possibilities in further intra-actions. That said, Clark was also very forthcoming about other interests and details, such as his identity as a father. He announced to the class that he might have to look at his phone for text messages from his spouse on a day when their toddler was ill, and as the birth of their second child approached, he repeatedly alerted students that he might have to

leave school anytime. One of his students, Claudia, told me that this made her feel understood and welcome, saying that, "he's a dad, so he knows how we'll feel" (*fieldnotes*, 2/2019).

Although they did so to different extents and in different ways, both Franck and Clark took up opportunities to reveal their outside-of-Banneker lives to students when they had the stage, making it possible for students to know them as people—not just in their institutional roles as teachers—and thus respond to them as people. In doing so, they staked a claim on who and what matters: their and their students' humanity as people and not just mathematicians. Franck said:

You're in a profession where it's human. I always go back to this. Human, right? Human interaction is part of your job. So maybe you interact every day with another human, there has to be a lot of privacy involved still, but there has to be walls that are broken down... we are forced to be in this relationship that breaks some private boundaries. (fieldnotes, 3/2019)

By breaking some private boundaries—opening themselves up to students beyond what would be strictly required within their institutional roles as mathematics teachers—Franck and Clark made their humanity more legible to students, generating different possibilities for connection and response.

Students' Responding by Caring

One way that students responded to Franck and Clark was by caring about and for them—at least, in Franck's and Clark's perceptions. Readers may recall that in Chapter 2, I noted some of the limitations of care theory for conceptualizing how mathematics teachers can engage in right relation; here, I use the word care not to characterize their overall relationships with students nor to imply it as the reason why Franck and Clark seek to know their students, but

because it is the word that they used to tell me about how they perceived students' responding to their ongoing practices of attention and opening up. Franck, for example, described how often former students came back to visit him in his classroom:

We get to know kids and they grow and they tell stories. And I think that's fun; it's important. Not every kid's going to come back [to visit], but I know when they leave, they felt it was a warm place to be in. Like, they care about you. They care about what happens to you. (*fieldnotes*, 10/2018)

During my year of observations, I saw several of Franck's former students return to visit him, some of whom were still enrolled at Banneker, and others of whom had long since graduated. These visits could be seen as a continuation of the ongoing intra-actions that began when students were in Franck's classroom, as students continued to respond—even when they were technically no longer students—to the openings that Franck had sown.

Towards the end of the school year, Clark called me to share an incident that had happened in his focal class period; he had asked some students to put their phones away and they had responded by furtively starting a three-way phone call with each other, and after engaging with them about it in class, Clark decided to accompany them down to the counselor's office.

Although he had much to say about that decision, here I focus on what he said about what happened when he returned to class without the students who had made the phone calls:

[Other students] definitely wanted to talk about what happened and be reassuring to me that I was doing things right, and I thought that was pretty interesting. And I definitely felt, after talking to them for fifteen minutes, like they were caring for me... it was very obvious that that word came up, that they were caring for me... I will remember this for a long time. I was pretty happy about it; I thought it was cool that that [feeling] has been

created regardless of everything else, and I talked to them about what I thought and why I acted the way I did... I [had been] as mad as I get [but] they were like "yeah, no, didn't really feel that." (*fieldnotes*, 5/2019)

Clark's ongoing intra-actions with students—and specifically, the effort he made to ensure that these intra-actions opened up rather than foreclosed possibilities for students' responses—made this moment possible: multiple students responded to an unexpected event (the only time in the school year when Clark asked students to leave his classroom) in a way that made Clark feel reassured and cared for, suggesting a mutuality in the teacher-student relationship that extends beyond that which might be expected under traditional notions of the teacher's near-absolute authority and responsibility.

Teachers' Responding by Pushing the Envelope and Going Deeper

The possibilities for connection and response created by Franck's and Clark's ongoing practices of knowing, however, did not just apply to students. As they were describing their students, Franck and Clark also described how knowing shaped their social and pedagogical interactions: their responses as teachers. Clark, for example, noted that Evelyn was "creative and clever in her thinking... [so she is] one of the students I watch for when it's time to think of something, and I'd kind of check in and be like 'Hey, did you think of it?'" (*fieldnotes*, 5/2019). Franck said that Daniela "respond[ed] to interactions" despite being generally quiet, "and so if I approached her group and they're having a conversation, I'll make a funny comment like 'Is Daniela talking trash again?' Or something like that... she'll be surprised and like 'What!' And then she starts laughing" (*fieldnotes*, 3/2019). Franck found Nadia to be a useful bellwether student due to her negative prior experiences with mathematics: "when I think about my instruction and planning and effectiveness as a teacher, she's the one I use" (*fieldnotes*, 3/2019).

These examples showed how Franck and Clark adjusted their responses based on what they knew about students.

As Franck and Clark engaged in ongoing practices of attention, students' legibility nourished further intra-action. Franck described an iterative process of deepening relationships over time in a way that takes its cues from students:

I'll keep on asking you questions, like "How's it going? Are you struggling or not, what's going on? How's your other class going?" And stuff like that. And I mean, if you want me to ask deeper questions, then you're gonna give me deeper answers. And then sometimes then they'll ask me a question about me and I'll answer. They're kids, they're like, "Why did you become a teacher?" Well, it's funny that you ask. I have a good answer; I have a long answer for you. (*fieldnotes*, 3/2019)

Clark read students' responses, such as the amount and timing of eye contact they made and the types of conversations they initiated, to determine how he should respond to them in the future:

There's a difference between a smile and a nod and just a nod... How did they respond at any moment? And then keep that in my memory bank as I go onto the next interaction with them. This student responds positively to this, but not to that... But just trying a little bit to keep pushing the envelope and then at the same time pulling back when necessary because I've noted that the reactions I'm getting aren't positive in this, this, this, and this area. (*fieldnotes*, 4/2019)

Franck and Clark calibrated their efforts according to what they sensed students wanted; if students gave deeper answers, Franck asked deeper questions. When students simply nodded, Clark took this as an indication that he should pull back. Knowing their students enabled Franck

and Clark to respond to how students took up the possibilities for further connection that they offered.

But Not Going Overboard

When students seemed disinterested in further connection, Franck dialed down his efforts. He recounted responding to students who volunteered to tutor or help in his classroom by offering them supplemental reading or showing them mathematical toys. He contrasted these stories with those of students who used to, but no longer, stop by during lunch and students who repeatedly gave terse answers to his questions: "if [student interest] is there, I will foster it, but I can't force a kid to like me... I'm not gonna go overboard... that's fine. I still talk to them" (*interview, 3/2019*). In such instances, Franck remained approachable, continuing to break the ice and create opportunities for deeper interactions should those students change their minds, but was careful not to appear coercive by "forcing" a student to act as if they liked him more than they did.

Clark framed "not going overboard" using the metaphor of hospitality. Commonplace narratives for inclusive and equitable mathematics classrooms often center idealized notions of hospitality that may seem benevolent but in fact reinforce oppressive power dynamics. When well-intentioned teachers "invite" students into the otherwise-exclusionary domain of mathematics or into "warm and welcoming" spaces, for example, such that students can feel "comfortable" and "at home," the teacher is cast as gracious host and the student as grateful guest. Furthermore, when teachers inhabit a host position that is made possible by oppressive dynamics—because they have become mathematics teachers by succeeding in a system that violently denies the mathematicalness, brilliance, and humanity of those marginalized by Eurocentrism and white supremacy—it becomes easy for them to overlook that what they are

trying to give (access to mathematics) was taken from somebody (Ruitenberg, 2015, citing Mireille Rosello). As a result, hospitality serves as a fraught metaphor for mathematics education.

Clark, however, however, inverted the metaphor, positioning students as hosts and himself as the guest. He described Mark, a student who was "out on the porch with lemonade for me all the time," seeking Clark's attention through mathematical questions about assignments and non-mathematical references to shared interests like video games. Clark contrasted Mark to Victoria, a student who routinely invited him into non-academic chatter during groupwork and rarely completed all the assigned tasks during a particular class period. Victoria frequently:

would even humor some help with a problem. But it was clear that I wasn't really invited to press her, to try to get her to complete things or follow through. She would invite me to help with #3 but she wanted me to leave #4 to her to not do. And here's the thing—I accepted that. She could shut the door on me after #3 without me pushing further into her space. (*email*, 6/2019)

Respecting Victoria's agency to keep out what she may have perceived as invasions meant respecting her refusal and seeing in her refusal not the opposition that is often assumed of students who do not comply with teachers' instructions, but rather, opportunities for examining the operations of individual and structural power and for imagining different forms of engagement (e.g., Tuck & Yang, 2014; T. S. Wilson, 2020). Clark framed his relationship with Victoria as a meditation on freedom and on the boundaries of what a teacher could do: "when we think about student freedom," he wrote in the same email, his authority as a teacher did not extend to "permission to barge into [students'] space without their invitation."

For example, during one classroom observation in March, I saw Clark make a Justin Timberlake joke to Victoria after reading something she had written on her paper, to which she responded by looking at her paper more closely; remind her to put her phone away and let her know that the task was "super doable if you give it a shot," to which she responded by putting her phone in her pocket and picking up her pencil; sit down with her group and ask her to share a notice from the notice-and-wonder activity, which she did (and then Clark shared a notice of his own with the group); and then tell her, in response to her asking if she could use the restroom:

I've been watching you. You don't seem to want to do the work today... I'm just wondering how I can help you pass the class; I check in with you a lot but you seem to be in the same place you were last time I checked in... I want you to pass the class and I think you'd prefer to pass the class... (fieldnotes, 3/2019)

Victoria did not go to the restroom during that class period. Up until then, and during other class periods I observed, Clark repeatedly prompted Victoria to work on the mathematical task and interpreted her responses as an indication of how she wanted him to engage with her further. In the terms of Clark's metaphor, he knocked on the door repeatedly, took the invitations Victoria extended, but then allowed Victoria to shut the door when she was no longer interested in engaging with him.

Discussion

By breaking the ice and opening themselves up for students' responses, by pushing the envelope and going deeper, but by not going overboard, Franck and Clark invited further response as they engaged in their ongoing practice of knowing: assessing and interacting and making students smile or laugh or cringe—not just accumulating factual knowledge. Franck and Clark invited further response from students, but also created possibilities for themselves to

continue to respond, suggesting that knowing students does not just matter instrumentally, such that knowing can be leveraged for academic learning, but ethically, because it takes a stance on who and what matters: teachers' and students' humanity and their connection as people, not just as learners and doers of mathematics. In this section, I unpack these claims and link them back to the literature.

First, engaging in ongoing practices of attention instead of treating knowledge about students as something to be acquired—or what Dooyeweer (1997, as cited in Amin & Vithal [2015]) distinguishes as knowing as an activity versus knowing as a commodity—addresses some of the ethical and epistemic impossibilities of truly "knowing" students. For one, observing intra-actions is a form of unsolicited knowing, which sidesteps the possibility of coercing students into feeling as though they must reveal information about themselves that they may not feel comfortable revealing (Amin & Vithal, 2015). For another, continuing to collect information and revise one's knowing about a student allows room for change and for paradoxes. Both Franck and Clark remarked on inconsistencies that they found "weird" or "interesting" about students, such as when a student was quiet in one classroom setting but vocal in another, displayed confidence with some forms of difficult mathematics but not others, or was socially awkward yet made many friends. Their openness to (what was to them) counterintuitive information meant they were not demanding absolute consistency from students (Butler, 2005).

Researchers studying restorative relations for students who have been historically marginalized in school on the basis of their racial, cultural, and/or linguistic backgrounds have presented compelling arguments for students' need for belonging, as have psychologists grounding their work in fundamental human needs (see Juvonen [2006] for a review), and argued that students experience greater belonging when teachers know them well. Belonging, however,

is not fixed; a student can feel as though they belong in a particular classroom at some times but not others, and the same circumstances can lead some students to feel belonging while others simultaneously feel excluded. In her reflection on what it means to belong in a country, Kerubo (2020) draws on her native Gusii language to say that "belonging, *oboamate* suggests, is an art that's actively earned through a reciprocal series of initiations and assessments" (sec. 2). This approach to belonging—although it may imply that belonging is an achievable state—corresponds with the approach that Franck and Clark take to knowing: through ongoing intraaction, teachers can know about their students and students can feel belonging, and this knowing and belonging are perpetually tentative, fluid, and dynamic processes.

Second, Franck's and Clark's stances on relational work treated knowing as inherently ethical, rather than as instrumental. Nowhere in the Building Relationships Interviews did Franck or Clark link their relationships with students to students' mathematical learning. They did not even offer a correlation without implying causation or instrumentality. Although it was certainly possible that Franck and Clark found that their relationships with students supported students' mathematical learning, they both made clear that they saw their relational work with students as an ethical obligation *because students were humans they intra-acted with*. Rather than placing a condition on their relationship, such that if students reciprocated by sharing information about themselves, teachers could help them learn more mathematics, this stance treated knowing as a matter of how teachers and students co-exist with each other. As Franck said, "if you're living with somebody five hours a week, you should enjoy their company" (*fieldnotes, 10/2018*).

His non-discriminatory graciousness opened him up to feelings of disappointment or betrayal: "sometimes it's hurtful when you realize [students] don't like you" (*fieldnotes*, 3/2019). But Butler (2005) notes that "this susceptibility is an ethical resource precisely because it

establishes our vulnerability" (p. 91). In Chapter 2, I wrote that feminist new materialist theorists suggest that ethics lies in how we handle and honor our exposure to others, and whether we take this exposure as an opportunity to respond in ways that render us and others more capable of further response. Next, I return to Barad to more explicitly link her conceptualization of knowing with this understanding of ethics.

Knowing as Response-ability

Barad's conceptualization of knowing, which stems from diffracting feminist theory and quantum physics through each other,⁷ hinges on the difference between uncertainty and indeterminacy. To explain, Barad contrasts two models of matter. Heisenberg's model assumes what Barad calls a knower-known binary: a detached observer stands outside the electron-apparatus configuration and makes observations. In some cases, they may not be able to observe the thing they seek to observe, resulting in uncertainty about what that thing is. By contrast, in Bohr's model—which is the one Barad takes up—"knowing is a distributed practice that includes the larger material arrangement" (Barad, 2007, p. 342): what is known about a

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⁷ There is far more depth and theory to it than would be suitable for this dissertation. To summarize as briefly as I can, Barad describes how electrons sent through a double-slit apparatus produce a wave pattern and not the particle pattern that might be expected from what we know about the properties of electrons based on studying single electrons. Upon observing this, Heisenberg proposed that we cannot know whether these electrons are waves or particles, because changing the apparatus so that we could observe the electrons "in transit" through the slits, so to speak, disturbs the electrons in such a way that we would no longer be observing the phenomenon we originally set out to observe. This inability to know is an epistemological dilemma. Bohr, however, suggested that the real dilemma is ontological, and what is at stake is indeterminacy, not uncertainty. It is not that we cannot know whether electrons are waves or particles because we cannot observe them appropriately, but rather, that electrons are not waves or particles on their own; they only behave like waves or like particles when they are part of particular apparatuses that allow particular observations. In other words, the electron-apparatus-observer phenomenon determines whether the electrons will produce a wave pattern or a particle pattern, and "there are not inherent properties and there are not inherent boundaries of things that we want to call entities"—either the electron, or the apparatus, or the observer—"before the measurement intra-action" (Dolphijn & Van der Tuin, 2012, p. 62).

phenomenon is known as *part* of that phenomenon. And, because phenomena are ever-becoming and ever-reconfiguring, what is known is only ever known about a specific configuration of the phenomenon. Therefore, what a thing *is*, so to speak, cannot exist apart from the context of everything else with which it is intra-acting. In this model, "practices of knowing and being are not isolable; they are mutually implicated. We don't obtain knowledge by standing outside the world; we know because we are *of* the world" (p. 185, emphasis original). Knowing, then, is an infinite and inescapable process, one that occurs constantly as we intra-act, without any beginning or end. In fact, Lange (2018) cites Maturana's claim that "knowing *is* the process of life... connecting what may have seemed disconnected" (p. 293, emphasis original).

As a result of shifting from knowing requiring a knower-known binary to knowing being entangled in material configurations, Barad's (2007) conceptualization of knowing is inseparable from ethics. If knowing is intrinsic to being entangled in and intra-acting in phenomena, then knowing is direct engagement—it is a material practice, not just a mediated or representational practice—in which "part of the world *becomes* differentially intelligible to another part of the world" (p. 342, emphasis added). In other words, knowing is when the intra-actors involved in a phenomenon become more or less or differently comprehensible to one another. This differential comprehensibility has ethical implications:

Knowing is not a matter of mere differential responsiveness in the sense of simply having different responses to different stimuli. Knowing requires differential accountability to what matters and is excluded from mattering. That is, what is required is differential responsiveness that is accountable to marks on bodies. (p. 380)

As intra-actors respond, become, and know together, their worldings make claims about who and what matters. By responding to their students' emotional and social experiences of mathematics,

their questions about Franck and Clark, their personalities, their smiles and laughs and cringes, and their invitations, Franck and Clark took the stance that people—both students and they themselves—matter as people, and not just as learners and doers of mathematics. Particularly in the context of mathematical violence, a stance on human mattering can be a step towards living justly.

This conceptualization of knowing, as entangled and ethical, suggests that "knowing one's students" is not an epistemic task which, when accomplished by the teacher-as-knower, produces a "knowledge" that can inform ethical judgment. Simply put, teachers cannot meaningfully acquire standalone knowledge about their students through activities or interactions and then apply that knowledge towards building a relationship. Instead, teachers' knowing can only exist in relation to their intra-actions with particular students in a particular time and place and context. It is better understood as a component of the total knowing—or legibility—within an entangled phenomenon than as an independent phenomenon in and of itself. As a result, feminist new materialist knowing does not demand that students make themselves known and is also able to account for their perpetual dynamism, addressing the concerns raised earlier about the impossibility of ethically knowing one's students. Instead, knowing is itself understood to be an inherently ethical relation. From this perspective, teachers' knowing—alongside students, more so than of students—is not merely a means for producing academic learning; it is an enactment of teachers' ethical stance on who and what matters. And, if it renders others capable, teachers' knowing alongside students has the potential to engender response-ability and thus move towards living justly. Consider the following example.

Lucas

I opened this chapter with some notes about Lucas, a student in Franck's class who Franck found a little "weird," a little unintelligible. Franck might not have sought Lucas out as a friend if they had been peers. Nevertheless, Franck continued to seek intelligibility, to strengthen their relationship, to "always ask a follow-up question." Franck did not know that Lucas liked the Avengers in order to compel Lucas's interest in Avengers-based curricular tasks, and Lucas did not know that Franck had children in order to learn better from him. Instead, Franck and Lucas were entangled in intra-action that elicited mutual vulnerability and mutual response. They learned more about each other as their entanglements shifted; as their entanglements shifted they learned more about each other. More importantly, through asking follow-up questions, acknowledging a joke he did not find funny, sharing the photograph of his child with the toy Thanos, and other responses, Franck communicated to Lucas that his comments and quirks and nonlinear thinking were all appreciated: that he mattered. Through gifting toys, coming in at lunch, starting conversations, and other responses, Lucas communicated to Franck that he mattered. Knowing, between Franck and Lucas, was a move towards living justly.

Conclusion

"Knowing is response-able relation," Stengel (2004, p. 139) writes; knowing, as the shifting intelligibility produced through intra-action, makes possible different responses and those responses, in turn, make claims about who and what matters. As they intra-acted with students, Franck and Clark sought to create opportunities to respond to students and for students to respond to them, in part by opening themselves up and going deeper but not going overboard. This intensifying of mutual intelligibility—while also honoring students' agency to refuse—affirmed their and students' shared humanity and their complex personhood outside of the

narrow roles that the infrastructures of mathematics education have prescribed for them. In doing so, teachers and students also mutually rendered each other capable of further response-ability.

This analysis of what Franck and Clark know about their students, how they know it, and why it matters contributes to researchers' conceptualizations of knowing as complex, ongoing, and uncertain, and as inextricable from being or doing. Philip (2019), for example, uses the concept of improvisation to describe the "interactional and responsive creativity" (p. 4) with which teachers co-construct meanings with students, suggesting that improvisation can be a form of theory-building that enables teachers to envision new possibilities. Taking knowing as response-able relation adds explicit attention to the envisioning of new possibilities specifically for *student* response, and how teachers' intentional practices of knowing create opportunities for students to exercise agency. To this end, this analysis also offers up specific practices of attention that teachers may consider in their own contexts: breaking the ice, opening themselves up, pushing the envelope, respecting students' authority to keep out invasions, etc. In the next chapter, I turn to how Franck's and Clark's pedagogical practices lay the foundation for these interactional practices by expanding possibilities for knowing and being.

Chapter 5

OPENING NORMATIVITIES THROUGH ATMOSPHERIC COMFORT

On the second day of school, Clark's Algebra I students were working in groups of three on a version of the classic river-crossing problem (see Appendix G). In this version, four hikers who traveled at different speeds needed to cross a bridge that could only hold two hikers at a time, and needed to do so before they were attacked by a bear. It was dark, and the hikers only had one flashlight between the four of them, so at least one person would need to bring the flashlight back across the bridge each time someone crossed safely to the other side.

The day before, students had started but struggled to finish the problem. Today, Clark invited students to figure out how quickly the four hikers could cross the bridge if it were daytime and no flashlight were necessary. As students talked in their groups, he walked around the classroom, listening in on their conversations. After a few minutes, Clark addressed the whole class: "A lot of you told me the answer," he said, "but I'm going to give a bonus point to Evelyn because she just blew my mind. Evelyn just made this problem so much harder which I LOVE because I LOVE hard problems." Clark asked Evelyn to explain to the whole class what she had been thinking.

"The slowest hiker could start walking," Evelyn suggested, "and the others could go one at a time and they would all pass her before she gets to the other side. And the bridge would still only have two people on it because the faster people wait until the other person is done before they go."

"Whoa!" exclaimed a student. "And I don't even know the answer anymore!" Clark enthused. Another student shouted out, "it's simple!" Several students jumped in, speaking loudly and over one another. Around the room I saw students smiling, students pounding on their desks with their fists, a student or two beginning to rise out of their seats, and almost all students physically turned toward at least one of the voices speaking. Over the din, Clark loudly called on a student who had been raising her hand energetically, "Luisa, I want to hear what you think about this." As Luisa explained her thinking, at least one more student said out loud, "Wow. I just got it." "Yeaaaaaaaah," Clark added, "This is so fun!"

"Oh man," Clark said, his voice finally calming down. "I can't wait to tell first period this because they didn't think of that. Evelyn just took it to the next level. I thought the answer was going to be 10 because you send the two slowest people together, but now, wow. Okay, back to nighttime and the flashlight. I'm going to give you ten minutes, ready set go."

Clark walked over to where Sammie and I were sitting at the back of the classroom. "Mind. Blown." he said, opening his eyes wide and making an explosive gesture with his hands. "Wow."

In supposing that the slowest hiker could be paired with every other hiker on the same trip, instead of only with one other hiker at a time, Evelyn generated a novel possibility for how the classic bridge problem could be interpreted—one that surprised both Clark and her peers. In other words, after struggling to solve the problem as written on the previous day, she "modified the constraints of [her] problems to make solutions possible (solvable) but also interesting (nontrivial)" (Jasien, 2020, p. 34). Evelyn's playful exploration, along with her creativity and enthusiasm, paralleled how professional mathematicians often engage with mathematics. Her mathematical insight was embraced by her peers and her teacher, rather than being dismissed as irrelevant or tangential to "getting the answer," so to speak, for the bridge problem. These elements are rarely present in even the most ambitious and equitable mathematics classrooms, due to both the limited opportunities that students have to exercise meaningful intellectual authority and the social risks of doing so (Agarwal, 2020; Horn, 2017). Additionally, Evelyn's initiative-taking may be especially rare on the second day of school when many teachers are still reviewing classroom rules and many students are still assessing the academic and social norms of an unfamiliar configuration of teacher, peers, and mathematics education.

In this chapter, I examine the association between the vibrant burst of energy in this episode with Evelyn's solution and Franck's and Clark's pedagogical practices, which I conjecture has to do with the comfortable atmosphere created (and creating) in Clark's classroom from the very first day of school. To do so, I begin with some context about how comfort has been conceptualized in mathematics education research. Then, I turn to Stewart's (2010) concept of an atmosphere and the Vibe Analysis described in Chapter 3. I claim that the complex conception of comfort that characterized Franck's and Clark's ethical stances expanded

possibilities for students' knowing and being. Consequently, I build on feminist philosophy to argue for cultivating open normativities, rather than setting norms, in mathematics education.

Relevant Literature

The experience of mathematics education is, for many people, characterized by discomfort and anxiety (Dowker et al., 2016). Discomfort has typically been viewed in one of two ways in education literature: as a problem, because students learn better when they feel safe and belonging (see, for example, the literature review in Chapter 4); or as productive, because dissonance is necessary for learning. Boler's (1999) pedagogy of discomfort, for example, encourages learners to engage with the anger, fear, and other uncomfortable emotions that often arise when challenged to think differently about race and sexual orientation, with the goal of "willingly inhabit[ing] a more ambiguous and flexible sense of self" (p. 176). Falter and Barnes (2020) note that teacher education literature uses the concept of a "comfort zone" widely and colloquially but claim it has rarely been theorized (for an exception, see Brown [2008]). That said, teacher educators have long chronicled the role of productive friction in teachers' learning (Ward et al., 2011) and the role that teachers' discomfort plays in their learning about race and racism (e.g., Burns, 2017; Hollins & Guzman, 2005; Matias, 2015; Picower, 2009).

In mathematics education research specifically, Rattan and colleagues (2012) found that receiving comfort-oriented feedback (e.g., "it's okay, not everyone can be good at math") demotivates mathematics students. Jaber (2016), however, claims that mathematics teachers' recognition of and response to students' discomfort—presumably in ways other than by giving comfort-oriented feedback—rather than preempting or ignoring it, is a crucial support for students' mathematics learning. Perhaps the most popular comfort-related concept in mathematics education research is that of *productive struggle*, which is widely touted as an

important contributor to mathematics learning (National Council of Teachers of Mathematics, 2014; Warshauer, 2015). According to this concept, students should engage with mathematics that is challenging enough that they become frustrated, but also be supported such that they are able to sensemake and problem-solve without giving up or being "rescued" (NCTM, 2014, p. 48). The prevalence of struggle in mathematics education—whether productive or not—makes mathematics learning risky: students face constant threats to their academic and/or social status (Horn, 2017), and this incessant riskiness is part of what makes mathematics education discomforting. To help students manage this risk, researchers and practitioners concerned with ambitious and equitable mathematics teaching often recommend strategies such as norm-setting and groupwork.

Norm-setting

For decades, mathematics education researchers have emphasized the importance of classroom and sociomathematical norms in shaping students' experiences of learning mathematics (e.g., Chazan et al., 2016; Franke et al., 2007; Yackel & Cobb, 1996) and the identities they form as mathematics learners (e.g., Aguirre et al., 2013; Hand & Gresalfi, 2015). In the face of gendered, racialized, and oppressive norms within school mathematics (de Freitas & Zolkower, 2009; Leyva, 2017), researchers suggest that the establishment of equitable classroom norms (e.g., Bartell et al., 2017; Kazemi & Stipek, 2001), particularly those that bridge students' interactional norms and disciplinary norms (Engle et al., 2014), can support students' mathematical participation, agency, belonging, and learning (e.g., Boaler & Greeno, 2000; Franke et al., 2015; Horn, 2012; Lampert, 2001). For example, eliciting students' ideas without any reaction from the teacher is sometimes encouraged by those who value how such a norm can normalize non-judgmentality in normally highly evaluative mathematics classrooms

(e.g., Dunleavy, 2015). In an ideal world, perhaps mathematics teachers could establish norms that ensure that students only experience "productive" forms of discomfort. However, given the intra-actions between mathematics teachers, the traditions and cultures of mathematics education, and mathematics students who participate in co-constructing classroom norms (e.g., Hand, 2010), this ideal world is unlikely to exist.

Groupwork

Groupwork is often touted as being able to open up forms of mathematical and social participation that differ from those available in classrooms where students work almost entirely independently. I do not provide a full review of groupwork as a pedagogical practice here because it has been amply studied (e.g., Cohen & Lotan, 1997; Staples, 2007). For example, Horn (2012) notes that ideally-implemented groupwork has the potential to mitigate some of the risks and violent relations that characterize traditionally-organized mathematics classrooms: the privileging of quick recall and accurate calculation as the most valued ways of doing mathematics; the enthroning of teacher and textbook as ultimate mathematical authority; the relegating of students to passive roles; and the sustaining of marginalization in mathematics classrooms.

Horn's conception specifies ideally-implemented groupwork because not all groupwork will address these concerns and in fact, some forms of groupwork may exacerbate them.

Groupwork can be exclusionary and reify existing status hierarchies (e.g., Engle et al., 2014; Esmonde, 2009; Esmonde & Langer-Osuna, 2013; Shah & Lewis, 2019), and it can also be constraining when students end up working independently in parallel, competing with their groupmates, or being confined to overly structured roles (e.g., Barron, 2003). In these cases, groupwork restricts students to ways of knowing and being that are typical of mathematics

education as a marginalizing and dehumanizing enterprise. As another example, some mathematics teachers assign individual students within groups to play specific roles in order to ensure that every student both has the opportunity to participate and is held accountable for participating (e.g., Boaler & Staples, 2008; Cohen & Lotan, 1997). Group roles, however, have also been linked to reinforced hierarchies based on gender and other status indicators (e.g., Langer-Osuna, 2011; Sengupta-Irving & Vossoughi, 2019). The simultaneous potential for groupwork to disrupt *and* reinscribe violent relations in mathematics education suggests that it may produce forms of discomfort that are both amenable *and* antithetical to student learning.

Discomfort, clearly, is not always good nor always bad, and neither norm-setting nor groupwork nor other strategies of ambitious and equitable mathematics teaching are likely to be able to guarantee that students are always just the right amount of uncomfortable. What, then, is an ethical approach for a teacher to take towards student comfort in mathematics education?

Next, I explain how I understand comfort in this chapter, reprise my methods for data collection and analysis, and present findings from Franck's and Clark's classrooms that illustrate a complex conception of comfort. In the discussion, I link this complex conception of comfort to open normativities as a contrast to the norm-setting I have just described.

Atmospheric Comfort

As described in Chapter 3, the analysis reported in this chapter stemmed from an inquiry into the "vibe"—and specifically, the "chill vibe"—of Franck's and Clark's classrooms. To conceptualize "vibe," I draw on Stewart's (2010) exposition of atmospheres as affective force fields that make certain things possible. As force fields—and not simply contexts—atmospheres contain a "capacity to affect and be affected that pushes a present into composition… a labor to make matter particular ways of living or living through" (p. 14). In other words, the vibe of a

classroom both is created by and creates how students, teachers, and the infrastructures of mathematics education intra-act and affect each other. Specifically, I investigate comfort, as a proxy for "chill vibe," because the easygoing, laidback, and happy energies that I observed (and that teachers and students reported) contradicted sharply with how students often feel in mathematics classrooms: tense, anxious, judged. I seek to determine, in Franck's and Clark's classrooms, what made atmospheric comfort possible—how atmospheres "accrue, endure, fade or snap" (Stewart, 2010, p. 2)—and what it made possible: which "potentialit[ies] [were turned] into a threshold to the real" (p. 4). Stewart calls an atmosphere a "proliferative condition" in that "it not only allows, but spawns, the production of different worlds, experiences, conditions, dreams, imaginaries and moments of hyperactivity, down time, interruption, flow, friction, eruption, and still lifes" (p. 8). So, in my analysis, I try to trace what "particular ways of living or living through" were made to matter by the specific comforts and discomforts that students experienced in Franck's and Clark's classrooms.

Methods (A Brief Reprise)

When I first visited Franck's and Clark's classrooms in spring 2018, before I had even begun to think about this dissertation, I was struck by the atmosphere in their classrooms; in my fieldnotes, I used descriptors like "warm," "friendly," and "casual." Their classrooms felt subjectively different from most math classrooms I have observed—even classrooms where similar pedagogical practices, like notice-and-wonder activities, groupwork, and inquiry-driven scenario-based tasks were used—which made me want to learn more. So, I spent the first week of the 2018-2019 school year with them, looking for how they established the norms of interaction that had made me sit up straighter and lean forward, curiosity piqued, during those initial classroom observations. However, in those observations I did not see them formally set

any norms or classroom rules or engage in many of the culture-building and get-to-know you activities that teachers often do during the first week of school, which turned my attention to things that could not be listed on a poster, a lesson plan, or a syllabus: how their interactions with students laid the foundation for the atmosphere that they would co-create over the course of the school year. In the Vibe Analysis reported in Chapter 3, I brought together classroom fieldnotes and teacher interviews as my primary data; student interviews, collaborative meetings, classroom artifacts, and teacher-given student surveys as supplemental data; and theoretical texts about atmospheres and normativities. Next, I report findings from this assemblage of empirical and interpretive sources.

Findings

In this section, I begin by describing Franck's and Clark's use of two pedagogical practices: notice-and-wonder activities and daily random grouping. Then, I explain how Franck and Clark attended to students' comfort through the use of these pedagogical practices. Next, I consider why they did so: what was made possible and what was made to matter by this atmospheric comfort. Finally, I provide an example of how Clark's approach to a classroom beverage policy likewise contributed to an atmospheric comfort that also made possible, and made to matter, a proliferation of possibilities for students' mathematical and social participation.

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⁸ Both Franck and Clark did do some of this; Clark, for example, told students that his "one rule" was that he, not the bell, dismissed students at the end of class, and Franck asked students to brainstorm how they thought they would be expected to act in his class but did not follow up except to say, "You all know how to act; this is proof." They both also printed name tents for students with boxes on the back where students could write comments at the end of each class the first week, and they wrote responses back to students most days that first week (*classroom fieldnotes*, 8/2018). Franck expressed impatience with even these few activities, saying he was eager to get to "real work," although as was discussed in the previous chapter, he valued and later drew on information that students shared during these first week activities. Clark explained his philosophy about the first week of school by saying the most important thing was for teachers to show their personality, because "kids want to know who you are," and that didn't require any specialized activities (*collaborative meeting*, 8/2018).

Franck and Clark expressed similar values underlying their use of notice-and-wonder activities and daily random grouping, and I observed a similar laxity in their approaches to students' consumption of food and drink in their classrooms, and also, to their own consumption. However, I draw more heavily on data from Clark's classroom than from Franck's in this chapter because Clark's hands, to use his own words, "just aren't as tight on the wheel" as Franck's (*fieldnotes*, 7/2019). He gave this as a reason that his "classroom management" (also his words) was not as strong as Franck's, but it also accounts for why I identified more and more clearly articulated examples of heterogeneity and dynamism in his classroom and sensemaking than in Franck's. In any event, I argue that both Franck and Clark attended to students' comfort in such a way that broadened the scope of what was considered acceptable—or even desirable—mathematical and social participation.

Pedagogical Practices

Franck and Clark centered several pedagogical practices that are widely recommended for ambitious and equitable mathematics teaching, including but not limited to notice-and-wonder activities and groupwork. However, as is common for teachers, their particular implementations differed from how such practices have been described in research literature and practitioner professional development due to what Marshall and Horn (in preparation) call contextually-situated goals. Therefore, I begin with brief descriptions of how Franck and Clark used these practices to provide context for the analytic findings that follow.

Notice-and-wonder Activities

Franck and Clark (and the other Algebra 1 teachers at Banneker) regularly began their lessons with a notice-and-wonder activity. Popularized by Max Ray-Riek and Annie Fetter at the Math Forum, the activity aims to elicit students' participation and prepare them for the main

mathematical task of the lesson by prompting them to make observations and express curiosity about something that is accessible to all students. Franck and Clark typically showed short YouTube videos or images downloaded from the internet that were connected, either topically or mathematically, to the mathematical task that students would be working on that day. They asked students to silently notice and wonder about the video or image for a few minutes and then engaged in a ritual of small-group and whole-group sharing: students would introduce themselves to their groupmates⁹ and then share something they had noticed or wondered for a few minutes, and then the teacher would ask students to share with their whole class either something they themselves had said or something one of their groupmates had said.

Notice-and-wonder activities are used in many mathematics teachers' classrooms, and one could imagine how they might be used in ways that constrain students' participation: by requiring that notices and wonders be mathematical, for example, or insisting that each student share something out loud with the whole class in order to hear everyone's voice, or collecting and grading students' written notices and wonders. Franck did sometimes encourage students to come up with "at least two notices and at least two wonders" and, while circulating to check in with students as they worked independently, made recommendations like "maybe something mathematical?" (classroom fieldnotes, 3/2019), but he never enforced these suggestions; from my observations, it seemed unclear how many students followed them. Franck and Clark both occasionally referred to notice-and-wonder activities as being separate from "the math"

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⁹ Franck and Clark asked students to share their names during every notice-and-wonder activity because they randomly changed students' groups every day (more on this in the next section). Franck and Clark believed that students should introduce (or re-introduce, as the case may be) themselves to their new groupmates each day as a way of acquainting themselves with their new configuration. Although Franck and Clark continued to instruct students to "say your name" before sharing their notices and wonders with their groups as late as February, I observed that only some students continued to do so (*classroom fieldnotes*, 2/2019).

(professional development fieldnotes, 3/2019; fieldnotes, 6/2019), suggesting that they valued the accessibility of notice-and-wonder activities more so than their role in preparing students conceptually and linguistically for the remainder of the lesson (K. Jackson et al., 2013).

Daily Random Grouping

Franck's and Clark's students worked in small groups for the majority of every class period, but their groupwork was distinguished from commonly recommended practice by two features: daily and visibly random assignment and a relative lack of structure. First, both Franck and Clark assigned students to sit and work in new groups of three every day, using a method that not only resulted in random assignments but also made clear to students that the assignments were random rather than deliberate. Visibly random grouping, in contrast to teacher-assigned or student-selected groups, has been shown to support students' mathematical learning, mathematical interdependence, enthusiasm and engagement, and to mitigate social barriers to doing mathematics (Cohen & Lotan, 1997; Liljedahl, 2005, 2014)). Many teachers, however, use visibly random methods to reassign groups every week or every few weeks. Although both Franck and Clark had experimented with weekly or unit-based random grouping in the past, they were committed to *daily* random grouping during the 2018-2019 school year as a means of "always tinkering" with what worked most effectively with their students (*Clark, interview, 7/2019*).

Franck implemented daily random grouping by handing students numbered colored index cards when they entered his classroom door at the beginning of every class period; he held these cards face down and conspicuously shuffled them from time to time to assure students that he was not choosing which student receives which card. The cards corresponded to groups of desks with blue, red, or green number cards taped on them; the student who received the red card with

a 3 on it at the door would find the desk with a red 3, which was grouped with desks labeled with a blue 3 and a green 3. Clark used a modified deck of playing cards that contained only three of each type of card and nine different types of cards to correspond with the nine groups of desks in his classroom. He also visibly shuffled the cards before handing them out to students face down.

Second, Franck and Clark did not use group roles, sentence starters, or discussion protocols, all of which are often recommended structures in mathematics classrooms (e.g., Cohen & Lotan, 1997; Smith & Stein, 2011). During some lessons, Franck and Clark did ask group members to use different color markers as they worked on whiteboards together and to take turns completing each step of a problem (e.g., *Clark, classroom fieldnotes, 10/2018*) or take responsibility for different parts of a multi-representational problem, such as words, equations, and tables/graphs (e.g., *Franck, classroom fieldnotes, 8/2018*). However, Clark alleged that structure was simply not his "style" (*collaborative meeting, 2/2018*). That said, an affordance of leaving groupwork relatively unstructured was that students could participate in heterogeneous ways within the same lesson instead of having to limit their participation to the role they had been assigned, normalizing the practice of responding to different group members in different situations in different ways.

How Franck and Clark Attended to Comfort

Franck and Clark attended carefully to students' comfort in their classrooms, simultaneously working to create a "chill vibe" while probing the nuances and limitations of how their pedagogical practices produced comfort. When I shared aggregated and anonymized student reflections with Franck and Clark in a collaborative meeting, for example, Franck turned to me and said, "I think that is what we strive to do, to make it comfortable" (*collaborative meeting*, 2/2019). However, they did not just seek for all students to be comfortable in all ways

all the time, nor did they try to dictate when and how students should be comfortable or uncomfortable by, for example, designating certain moments for "productive struggle."

Clark articulated the grounding assumption of this goal: that students—and the class as a whole—are more likely to flourish when they feel comfortable.

Then everything—a lot of things just become really naturally positive. Like when you're comfortable, and you're in a good mood, and you're allowed to behave the way you want to behave... All of those things, that's where good vibes come from. (*interview*, 10/2018)

Franck and Clark believed that students' comfort would "naturally" enable them to respond more positively to mathematics, to each other, and to their teachers.

However, Clark clarified that "there's a lot of different comfort zones [that students have]:"

Mathematically, I want to find a way to challenge you to meet me halfway between what you are super comfortable with and what you already know, and what I know and what I'm presenting. So I want to do that, but then relationship-wise and personality-wise, I think I am more like 'I want you to stay in your comfort zone.' And I don't want to press you too hard out of that zone to where I am making things uncomfortable in any kind of way. (*fieldnotes*, 4/2019)

By distinguishing between mathematical, relationship, and personality comfort zones, Clark was able to differentiate his press of students to create some mathematical discomfort—presumably that which would be conducive to learning—while maintaining social comfort. Although Clark did not specify why he wanted students to remain socially comfortable, it could have been because he believed that social comfort was conducive to learning, or because, as has been

discussed in previous chapters, it was part of what Franck calls "living with somebody for five hours a week" (*fieldnotes*, 10/2018).

That said, Clark questioned the extent to which students' social comfort should be a cornerstone of his ethical stance on relational work:

I default to not making students uncomfortable most of the time. And if that means that they don't grow as much as I was hoping they would grow from day 1 in my class to day 180 or whatever it is, I'm willing to sacrifice that so that they didn't feel uncomfortable.

And I don't know if that's a good choice or not, right? So I'm trying to think. (*fieldnotes*, 4/2019)

He recognized that because mathematical and social comfort are difficult to disentangle, just as students' mathematical and social participation are difficult to disentangle, prioritizing students' social comfort might also detract from the types of discomfort that lead to growth.

For example, Clark acknowledged that his commitment to daily random grouping challenged students who did not like being in groups at all, contradicting his desire to not make students uncomfortable: "when kids are already uncomfortable in a group of three, if you're in a group of three, you're at a deficit all the time. It sucks and I'm not sure what to do about that" (fieldnotes, 3/2019). Similarly, Franck gave a specific student's experience as an example of how someone might be "at a deficit all the time":

Cristina is really quiet. Really hard to get her to open up. Every day I'm like, "How's it going, Cristina? Why are you so quiet today, Cristina?" And she's like, "What are you talking about? I'm always quiet... it's 'cause I don't know anyone in my group." And I was thinking to myself, for people like her, I'm not so sure if this [daily] random grouping is helping in any way. Because she always does the work fine. She is always the

one that gets it in the group most of the time. She'll ask me if she needs help, and she's actually a lot more vocal than a lot of kids [during groupwork] except for that she's quiet in general. (*fieldnotes*, 3/2019)

Franck wondered about Cristina's comfort in a classroom so reliant on daily random grouping, which prompted his uncertainty about whether daily random grouping was worth the benefits that he and Clark perceived. I turn to these possible benefits next, but first, note that Franck and Clark treated students' comfort as both a function of classroom groupwork practices and as an agential factor in students' intertwined mathematical and social participation. Consequently, they carefully considered and reconsidered how students' comfort might affect their responses to the other people and to the mathematics in their intra-actions each day.

What Dis/Comfort Made Possible

The "chill vibe" of Franck's and Clark's classrooms, or the atmospheric comfort, spawned (to use Stewart's word) a proliferation of possibilities for mathematical and social participation beyond the expectations of traditional mathematics classrooms: that students must raise their hands to speak only when they have a correct answer or a clarifying question and they must avoid any behavior that could be perceived as off task. Instead, students were encouraged to contribute to the process of ongoing intra-action—not just to mathematical learning—and to experiment with new ways of interacting with mathematics and with their peers.

Contributing to the Process

Through their particular uses of notice-and-wonder activities and daily random grouping,
Franck and Clark communicated to students that their ideas were important contributions to the
version of mathematics education they were creating together, regardless of what those ideas

were. In a reflection on aggregated student interview findings, Franck and Clark discussed why they asked students to share what students found comfortable to share:

Franck: You are not particularly soliciting an answer, you're soliciting their thinking.

Whatever [students] say is correct, even if it's wrong. "It's okay, just tell me what you think. You can't think wrong, even if the answer is wrong."

. . .

Clark: I would rather not you contribute to the result but contribute to the process. The process is just as important as contributing the result...

Franck: I don't think in our classrooms anyone really values finishing first and getting the right answer.

Clark (in agreement): No, no.

Franck: If you can get there, get the right answer, cool. But when you're called for a notice-and-wonder, there's no wrong answer.

(collaborative meeting, 2/2019)

Keeping student participation low stakes created opportunities for students to contribute to class discussion in such a way that just about any contribution they could make would be affirmed. In Franck's and Clark's classrooms, the notice-and-wonder activity was not just a strategy for increasing participation, but also a way to underscore what mattered: contributing to collective mathematical experiences, more so than students' speed or accuracy in solving problems.

For example, Clark used the example of a specific student, Enrique, to describe how notice-and-wonder enabled students to engage meaningfully in class regardless of their academic challenges:

The notice-and-wonders made the class for him. He was a kid who he had 1000 ideas and thoughts going on in his head and they weren't necessarily all mathematical and he didn't necessarily love math or even love academic things, but he was super thoughtful and full of thoughts and ideas. Almost, like, bursting with them... the notice-and-wonders improved his overall learning because it kept him more engaged than he might otherwise have been in the whole class because he had opportunities as outlets for the kind of thinking and expressing himself that he really wanted to do. Then when the math started, he wasn't quite so included, but he would want to continue with that type of behavior into the math lesson and it would help him. (*fieldnotes*, 6/2019)

Because notice-and-wonder activities made space for open sharing, with no ideas disparaged for being off-task or irrelevant, Clark contended that notice-and-wonder activities encouraged Enrique—and students like him—to feel comfortable participating in the rest of class as well.

Both Clark and Franck often briefly indulged students' non-mathematical notices and wonders by following up with comments and questions that could be considered tangential to the day's mathematics lesson before moving to the next notice or the next wonder, rather than evaluating students' contributions for their relevance or productivity for mathematical learning. For example, within four minutes of Franck encouraging students to have mathematical notices and wonders and a minimum of two each, he also asked students what social networks they were using (the mathematical task for that day's lesson involved calculating exponential growth among a celebrity's Twitter followers), responded to their responses by saying he was "too old for Snapchat," gave "I wonder how many followers they have" as an example of a mathematical wonder, and answered a student's good-humored reply ("zero") by saying "I wonder how many haters there are in this class. But that's okay, because haters gonna hate. They hate us 'cuz they

ain't us," which prompted smiles from students—including the student who had said zero—in response (classroom fieldnotes, 3/2019). Franck's comment about haters might be seen as a contradiction to his value for the open-endedness of notice-and-wonder activities ("whatever you say is correct, even if it's wrong," collaborative meeting, 2/2019), but actually illustrates the nuance and complexity of expanding students' ways of knowing and being in contrast to merely establishing and maintaining norms the teacher perceives to be desirable, such as non-evaluativeness in a usually evaluative context. Franck clearly evaluated the "zero" reply, indicating his distaste for it, but in a way that also affirmed the vibe of the classroom as one in which students and teachers were welcome to be as mathematical, pop culture-savvy, witty, or simultaneously all of the above as they felt comfortable being.

Furthermore, Franck's responses invited students to respond further, without regard for whether students' further responses would be mathematical. Not only did this signal value for students' contributions to their ongoing intra-actions as an entangled teacher-students-mathematics education phenomenon, but it also signaled that those contributions mattered for more than their potential to support academic learning. By creating opportunities for low-stakes participation and participative momentum through repeated response, Franck and Clark made matter students' contributions to "living with somebody for five hours a week" and not just their mathematical thinking.

More Ways of Knowing and Being

The students in Franck's and Clark's classrooms "lived," so to speak, not just with their teachers but also with each other. To that end, the atmospheric comfort and specifically, what Franck called "the beauty in random grouping every day" (*collaborative meeting*, 5/2019) supported students in constantly figuring out new ways of interacting academically and socially.

Students were able to hear other ways of thinking as they worked with different peers on the mathematical tasks and could sometimes help others and sometimes receive help depending on who they were grouped with:

You're going to be somewhere [else] the next day, and you can't rely on that person that you relied on the other day to give you that answer, and some days you're going to end up in a group where you're going to have to be the one [who leads the group mathematically], or no one's going to be the one... (*Clark, collaborative meeting,* 10/2018)

Franck reasoned that students "have to learn to adjust to that equally," which not only would support their transition to "the real world... [where] some people help you and some people won't" (*collaborative meeting*, 5/2019), but also provided students with opportunities to practice new ways of responding in such a situation. They could develop different types of what Sengupta-Irving (2014) calls workships—affinities "forged through shared mathematical activity" (p. 32)—with different peers.

They had opportunities to develop friendships, too. Clark encouraged students to use their random group assignments to get to know each other socially, too. At the end of October, he said in class: "In a perfect world, you'd all walk by each other in the quad and say 'Hey! You're my friend from Mr. Zapatero's class!' And I'd be so happy in that world." This was important to both Clark and Franck because students at Banneker had attended multiple middle schools across the city, so most freshmen did not know each other early in the school year. Indeed, when interviewed in the spring, several of their students reported appreciating the opportunity to work with different people through math class. Rosa said that "you get to know everybody," and that now that the school year was almost over, she would most remember "all the friends I met in

[Franck's] class" (*fieldnotes*, 4/2019). Similarly, Zoe answered a question about a typical day in math class by describing the constant changes in grouping:

You get to know other people more, and if you're shy you need to interact with other people, which is better... we just don't stick with one group and I think that's a fun way to get to know each other more. You interact with the whole class, not only the same people every day. (*fieldnotes*, 5/2019)

For Rosa and Zoe, the chance to respond differently to different groupmates on different days mattered, and made their mathematics experience feel friendlier and more human.

Of course, not every student appreciated the dynamism of daily random grouping as much as Rosa and Zoe. Luisa, for example, explained to Clark how her mathematical engagement varied based on who she was grouped with that day, and Clark observed that she likely was not alone:

She's told me before that she tries to work with everybody, but sometimes she can sort of tell that certain interactions aren't super productive. Then she'll shut down a little bit in that group. I imagine a lot of kids would say that. Actually, a lot of kids have said that to me in this class and in different classes. I think it's one of the things about [daily] visibly random grouping—that is one of the drawbacks. But [Luisa] tries, and she wants to get along with people, I think. (*fieldnotes*, 5/2019)

Even though Luisa sometimes "shut down a little bit," closing off possibilities for flourishing, she continued to try "to work with everybody... to get along with people." Clark took this as evidence that the potentially negative consequences of daily random grouping—students occasionally shutting down—co-existed and even intra-acted with the potential benefits—

students being able to interact in more ways, and perhaps ways that enabled more flourishing, with more people—rather than one clearly outweighing the other.

As a result, Franck and Clark both remained in favor of daily random grouping—rather than always seating students with their friends, always separating them from their friends, or some teacher-controlled combination of the two—at least throughout the 2018-2019 school year, using the rationale that "if visibly random grouping gives you a 'bad' [air quotes] combination one day, it's just one day, you get through it, and tomorrow will be better" (Clark, fieldnotes, 8/2018). Furthermore, Clark was heartened by students who surprised him by responding to daily random grouping as an expansion of possible ways of knowing and being. A student named Isabelle, who Clark had previously described to me as "so shy" and frequently asking to sit with her friend Jessica, wrote about liking random grouping in her student survey at the end of the first semester. Reading Isabelle's response made Clark wonder if it was more so that Jessica wanted to sit with Isabelle, because it seemed that Isabelle appreciated the opportunity to engage with new people (classroom fieldnotes, 1/2019). It was precisely this type of experience—a student participating mathematically and socially in ways that their teachers (and maybe they themselves) would not have predicted—that Franck and Clark valued. They did not aim to produce a single best-case scenario, in which all students participated at some mathematical and social peak, and then reproduce it every day, but rather, to normalize heterogeneity among and within students in how they participated mathematically and socially, even if that meant occasional non-ideal participations.

In other words, changing groups every day reduced the likelihood of students' mathematical and social behavior ossifying over time. It created opportunities for students to try on engagements with different peers, new ways of engaging with the same peers, and new ways

of working together. Coupled with an emphasis on students' contributions to being together, sometimes mathematically and sometimes less so, it made possible more expansive ways of knowing and being and more possibilities for ongoing responses between teachers, students, and mathematics education in Franck's and Clark's classrooms. Because this section has focused on pedagogical practices up until now, I next turn to an example of how atmospheric comfort was also created by Clark's approach to classroom behavior policies and similarly served to normalize a proliferation of possibilities in mathematics education.

Classroom Beverage Policy

As far as I could tell, Banneker did not have an official school policy on food and drink outside of the lunchroom, leaving individual teachers to set and enforce their own expectations. Many schools and individual teachers prohibit eating and drinking in class, reasoning that it can distract students from their learning, lead to crumbs and spills, or make visible inequalities in the types and amounts of nourishment that students have access to. However, allowing students to eat and drink in class can also support students' physical comfort (especially for students who have medical reasons to regularly intake nutrients and liquids), boost concentration, ¹⁰ and create a "chill vibe" by increasing students' emotional comfort and minimizing conflicts instigated by enforcing food-and-drink related rules. Clark's stance, however, was not as simple as choosing the right rule: allowing food or disallowing food. Instead, he saw such rules as an opportunity to attend to atmospheric comfort and thereby cultivate possibilities for response-ability.

¹⁰ This dissertation, for example, could not have been completed without a large quantity of peanut M&Ms.

During a transitional moment on the third day of school, Antonio pulled a bottle of Gatorade out of his backpack, took a swig, and then closed the bottle and put it back in his backpack. Clark inhaled sharply and loudly, drawing students' attention. After a beat, he said:

Whew. This is a tough one for me. I had an incident happen last year [Clark begins to pace back and forth, while still facing Antonio] and it was something I never ever thought would happen. Students were drinking what I thought was Gatorade, but it turned out to be something else. And they got in trouble, but I never once thought this would happen, and it did, and now— [Clark trails off, and pauses; the room is quieter than it's been all morning, and students appear to be paying rapt attention I feel like haven't thought this policy all the way through yet, and I haven't finished thinking about it. Because when I was in college, I always brought something to drink in math class, and now I always have something to drink [Clark picks up a full, 32-ounce, reusable water bottle sitting on his desk]. But I think the policy this year has to be only water. And if you mix alcohol with water [Clark pauses, and shakes his head 'no'], you're crazy... and it's crazy that I have to do that, but [last year's incident is] fresh. It might take me a few years to recover from that. So I think the rule will be just water for now, but who knows, I may change it halfway through the year. Okay? Does that make sense? (Adapted from classroom fieldnotes, 8/2018)

Antonio nodded, and said "yeah" in a serious tone, loudly enough for the whole class to hear.

Clark continued with the lesson, introducing a notice-and-wonder activity by projecting an image of a fishing boat with birds circling above it.

After about one minute, Clark reminded students of the classwork tracking system he had recently introduced: students had a sheet of paper on which they were to record moments they

felt like they had done or said something worthwhile in class, and Clark would collect these sheets periodically for review. "Antonio, give yourself a point because I didn't let you drink your Gatorade. What did you notice and what did you wonder? Please don't be scared to share," Clark said, giving instructions for students to talk to their groups and beginning to circulate around the classroom. As students burst into a low murmur of conversation, Antonio exclaimed to his group, incredulously, "Those are birds? I thought they were jets!" Clark came running over to Antonio's group. "What? What did you say?" he asked, smiling. Antonio repeated that he thought the birds were jets, and Clark laughed heartily. "That's awesome," he said, and moved on to the next group. To me, Clark's intervention felt exuberant and perhaps a little exaggerated, as if he wanted to reassure Antonio that there were no hard feelings about Antonio's sip of Gatorade leading to Clark's issuance of this tentative classroom beverage policy. From my perspective as an observer, this exchange between Clark and Antonio reestablished the easy camaraderie that had been the tone of class prior to Clark's sighting of the Gatorade.

In his reaction to Antonio's Gatorade, Clark communicated that he valued students' autonomy and bodily needs; that it would be unfair to drink in class himself if he prohibited students from doing so; that he expected students to follow laws about legal alcohol consumption; and that policies should be both reasoned and flexible. In the context of explaining why he was able to avoid power struggles with students, Clark explained his approach to classroom policies this way:

I think I'm reasonable. It's going to be fair, what I'm doing is going to make sense. If you can't go to the bathroom, it's not because I don't let people go to the bathroom and it's

¹¹ For what it is worth, I observed students drinking beverages other than water in Clark's class in October and thereafter. I did not observe any further mentions of last year's incident or any comments about what students were drinking, from either Clark or his students.

just a weird policy that takes over my room. It's because I'm noticing you have a tendency to blank blank blank... everything is pretty negotiated in my class (collaborative meeting, 12/2018)

This notion of negotiation made Clark's issuance of a classroom beverage policy feel like an expression of vulnerability and a request for understanding and cooperation, rather than an assertion of his authority as a teacher to set rules. Later, Clark elaborated on this notion of negotiation, saying that he had been "trying to decide in the moment whether or not I was going to allow [Gatorade]... I just literally negotiated, but I had a strong case" (*fieldnotes*, 7/2019). Had Antonio responded with reasons he should be allowed to drink Gatorade, Clark suggested to me, he may not have instituted a water-only policy at all.

In many cases, the creation of classroom policies involves an exercise of teacher authority and the expectation of student acquiescence, but Clark instead treated it as an opportunity for response-ability: for him to respond to a situation as it arose in their particular entanglement (rather than responding to an abstract possibility), for students to respond to his response (by negotiating or not), and for him to then respond to their responses. Allowing water demonstrated a concern for students' autonomy and their physical comfort, and Clark's hesitation around setting a blanket rule normalized the idea that students' comfort with classroom policies mattered to him too. Engaging in a lighthearted interaction with Antonio immediately after singling Antonio out for what could have been interpreted as a transgression re-centered Antonio as someone who could contribute to the class in "awesome" ways. Furthermore, by explicitly acknowledging that a water-only policy was both arbitrary and also important to him, Clark conveyed that part of his and students' entanglements as people in this Algebra 1 classroom

could entail respecting that which is important to each other out of a shared care for each other's ways of being, even if they were somewhat arbitrary.

Discussion

In this chapter, I described how Franck's and Clark's uses of particular pedagogical practices—notice-and-wonder activities and daily random grouping—illustrate their attention to student comfort. I have also claimed that the atmospheric comfort in their classrooms made it possible for students to contribute and to experiment with their contributions in ways that expanded their mathematical and social possibilities for being. Next, I extend this claim by drawing on Shotwell's (2016) conception of open normativities to suggest that not only can atmospheric comfort support students' learning, but also, that this expansion is a step towards students and teachers rendering each other capable: towards response-ability and living justly.

Open Normativities

Shotwell (2016) notes that the word "normativity" is commonly used in the context of oppression: heteronormativity, for example, describes the valorization and imposition of particular sexual practices that are defined as normal in order to control those who might deviate from such norms. Grounding her case in queer theory and existential philosophy, however, Shotwell argues that normativity should instead be understood more broadly as a "process [emphasis added] by which people claim that a given way of being is good, beautiful, or to be endorsed" (p. 143). Although normativity may be—and often is—used to oppress, taking a stance on what is good is not necessarily in and of itself oppressive. In fact, normativity is inevitable as long as people have values and seek to live according to their values; central to Barad's (2007) "ethico-onto-epistemology" is the idea that "values are integral to the nature of

knowing and being" (p. 37). In the case of this dissertation, for example, the ongoing enactment of a stance on the "right relation" between teachers and students would be, according to Shotwell's definition, an expression of normativity.

As a result, Shotwell submits that rather than trying to avoid normativity, we should concern ourselves with what is being normalized. Drawing on Simone de Beauvoir's ethics of ambiguity, in which acting morally requires "holding in view how one's actions open or close down the possibilities for others to unfurl their possibilities" (2016, p. 131), Shotwell advocates for normalizing that which, when normalized, opens up more flourishing for more people. In Barad's (2007) words, "intra-acting responsibly as part of the world means taking account of the entangled phenomena that are intrinsic to the world's vitality and being responsive to the possibilities that might help us flourish (p. 396)." Oliver (2001) specifies that this responsiveness, and thus flourishing, is contingent on responding "to our environment and other people in ways that open up rather than close off the possibility of response" (p. 15)—or, response-ability. To revisit Haraway's (2016) language, then, the kind of world we should be for and help to compose if we care about justice is one which proliferates possibilities for people to not just survive, but thrive (Love, 2019). Shotwell (2016) refers the process of being for and helping to compose opportunities for response-ability and thriving as seeking open normativities because they create "richer contexts for knowing and being" (p. 154) instead of limiting the likelihood of composing more just futures.

Opening normativities, however, is not the same as simply establishing norms that diverge from existing oppressive norms. Esmonde (2014), for example, provides two cases of how actions intended to open up more flourishing for more people did little to substantively expand students' perspectives. In these cases, mathematics teachers taught participatory

mathematics lessons about wealth inequity and resource distribution in an attempt to broaden students' understanding of and perhaps inspire their commitments to social justice. Their students, however, ended up drawing hegemonic conclusions that defended disparities. The possibilities for students' "knowing and being" that were unfurled by these lessons were, effectively, very similar possibilities to those that had already existed, except perhaps that now students could offer mathematical justifications for their ideas. Therefore, Shotwell (2016) cautions against narratives that "swap out one restrictive norm for another" and call instead for those "that expand the space of what can be pursued, endorsed, and so on" (p. 154).

In the context of Franck's and Clark's pedagogical practices, one could imagine teachers setting norms and implementing strategies intended to ensure equitable student participation: reminding students to share the proverbial floor and giving students tokens to track the number of times they speak in class, for example, or expecting all students to be mathematical leaders and periodically rotating the "team captain" role among group members. Or, teachers might communicate a norm that "we welcome all ideas, however half-baked," in order to encourage problem-posing, but de facto norms are always co-constructed between teachers, students, and the infrastructures of mathematics education, and the riskiness of sharing "half-baked" ideas cannot be mitigated simply by fiat. By contrast, Franck's and Clark's ethical stances assumed that the expanded ways of knowing and being that they desired—for mathematical learning but also for co-existing—were made possible by good vibes, and not by norms that dictate which ways of knowing and being are acceptable.

Transforming norms by seeking open normativities through attending to atmospheric comfort, as a way of transforming mathematics education, is neither about surrendering authority to students nor about imposing what teachers imagine would be more just. It requires more than

new norms; it requires "hold[ing] in view other people's projects in enacting our own" (Shotwell, 2016, p. 131). In other words, teachers must hold students' agency on par with their own imaginations of what a transformative mathematics education should look like. They must engage in processes that consistently not only accept but also nourish the unfurling of possibilities they may not anticipate or even desire, insofar as these possibilities "forge more capacious and diverse ways of being" (p. 154), and they must respond encouragingly as students choose whether and how to take up these possibilities each time. That said, Shotwell also reminds us that not all norms or ways of being ought to be normalized, that "not everything deserves a future... norms that flatten complexity and close down flourishing for others [should be] rejected" (p. 155). So, mathematics teachers' intra-actions with mathematics education must also tip the scales specifically towards possibilities that generate more flourishing and more response-ability for more people, especially their students and themselves.

I opened this chapter with a story about how Evelyn blew Clark's mind, so to speak, with a novel and unanticipated mathematical solution. While I do not have sufficient data to make claims about Evelyn's sense of self as a mathematician, the opening vignette suggests that at least, in this moment, she was able to make meaning of mathematics and take on the intellectual authority to share her creative solution with her teacher and peers. And although I cannot know how Evelyn came up with her solution, I can (and do) claim that the possibility of Evelyn's solution in Clark's Algebra 1 class—that she would share it in the way that she did and that Clark would take it up in the way that he did and that other students would respond as they did—both created and was created by the comfortable vibe being established as she, Clark, the other students in the class, and the mathematical task all intra-acted. This vibe, as a force field that "pushes a present into composition" (Stewart, 2010, p. 14), made matter different and more

energizing ways of knowing and being than are typical in traditional mathematics classrooms. Through Evelyn's solution, students learned that Clark, despite (or perhaps in) his institutional role as a mathematics teacher, was an excitable human who did not have all the answers, and that he would encourage their creativity and their enthusiasm. ¹² Clark did not merely affirm Evelyn's solution, nor did he hold Evelyn up as an exemplar of mathematical smartness in a way that might make students who did not think of her solution feel self-conscious. Instead, he cheered the solution a way that communicated his authentic delight while also encouraging other students to respond in ways that were authentic to them—shouting, pounding on desks, sharing follow-up comments—normalizing "capacious and diverse ways of being" (Shotwell, 2016, p. 154) that celebrate complexity and enable different kinds of flourishing.

Conclusion

Franck's and Clark's ethical stances treat relational work as expansive, in that it can and should open normativities by normalizing diverse and thriving ways of knowing and being. They are careful to avoid paternalism in deciding which ways of knowing and being constitute "more possibilities" or "more flourishing" or "more thriving" for their students, in contrast to oft-repeated narratives in educational reform that treat guaranteeing the possibility of college for low-income Black and Brown students, especially, as a reason to constrain their behavior in school. Instead, they implement their pedagogical practices—like notice-and-wonder activities, groupwork, and behavior policies—in ways that invite complexity and preserve opportunities for students to explore multiple possibilities for mathematical and social participation. In its

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¹² And, when he came over to where we were sitting to share his awe with my fellow graduate student researcher and me, students learned that we—who would be watching them and writing down what they said and did over the course of the school year—were people who also cared about such things.

complexity, their relational work resists being constrained by fixed norms—equitable as those norms may aim to be—and instead insists on repeated responses that, in each response, make possible further response. This reliance on and aspiration towards response-ability recommends further against universalist best practices approaches to mathematics education (for other critiques, see Dutro & Cartun, 2016; Philip et al., 2019), and towards "the hard work of attending to the specificities of one's situation" (Haraway, as cited in Reardon et al., 2015, p. 24). In the next chapter, I turn to how attending to the specificities of one's situation is an inherently affective endeavor.

Chapter 6

ATTUNING TO MATHEMATICAL GHOSTS

October 23, 2018: In an interview, Franck tells me that he's figured out who "this fucking Mr. Montoya" is: a mathematics teacher who taught at least a quarter of the students in his focal class period last year. "Mr. Montoya teaches at the neighborhood magnet school," Franck tells me. "He believes in public abuse of these kids. So what do they do? Well, some of the quieter kids [said], 'he would make me go in front of the class and do a problem and do it right. If I didn't do it right, he wouldn't let me sit down. He'll make me do this, this, and this.' Even though they're really good, they know how to solve an equation, they know how to graph lines, [they] HATE math. I'm sorry—hate their math experience in the eighth grade. And I ask them, 'how do you feel about Mr. Montoya.' They're like, 'well, I hate him.' ... I feel like that's the damage he did to these kids."

October 24, 2018: During groupwork, Luisa calls out to Clark: "Mister! Are you sure you don't know Mr. Montoya?" Clark shakes his head: "I don't think so." Luisa continues, "I heard you say 'cool beans' and I was like, he says that all the time too."

December 4, 2018: The Algebra I teachers use very colorful language to complain about Mr. Montoya during a collaborative meeting. They joke that Veronica should go confront him. Franck says he and Abigail will back her up from outside the classroom, making alternating cheering and ducking gestures as if to suggest that he might hide behind the door while watching.

January 9, 2019: Mark and Adriana are breezing through a worksheet in Clark's class, chatting as they work. Mark says something to Adriana about being lucky that Mr. Montoya got jury duty. I wonder what this is referring to.

January 10, 2019: Clark, Franck, and Abigail are drawing visual patterns on Franck's whiteboard, planning the beginning of their sequences and series unit. Veronica, who is teaching her own class during this time, pops her head in and says, "I've had Montoya's kid in my room, spinning his wheels out of control, trying to remember some kind of formula," before leaving again. Several minutes later, Clark confesses, "I just think so much about this Montoya guy... I have a negative whatever of him just because of the way that his students describe his whatever." Franck agrees: "After school one day, four kids came to me and sat and just talked about Mr. Montoya the whole time. It's weird." Abigail notes that, "They seem a little obsessed." I mention that I've heard a lot about him too, especially when I'm interviewing students, and Abigail seems surprised. "You hear about him? He has really impacted their concept of themselves in math class." Franck, Clark, and I tell Abigail some of the stories we've heard about Mr. Montoya. "That makes me just want to cry hearing about it," she says, "I would've been that kid that's sobbing in the back." Clark affectionately calls her a dork, and Abigail continues, "that stresses me out. I don't like that at all... [in college] I thought I wasn't smart because I didn't get it fast enough."

This reminds me of how indignant and saddened I had felt earlier that week when Dylan told me about his experience in Mr. Montoya's class; "he's so grateful this year that nobody makes him feel stupid," I say. Abigail responds, "Now I'm angry."

February 5, 2019: I am sharing aggregated, anonymized findings from my interviews with students with Franck and Clark. We're talking about how students can trust their teachers and also can feel trusted by their teachers. Clark wonders whether what students are telling me reflects a contrast with their previous year's math teachers. I immediately feel Mr. Montoya's shadow over the conversation, even though nobody has said his name out loud.

February 6, 2019: David tells me that he likes math, and I ask whether he has always liked math. "Not in 8th grade because of my teacher," David says. I know from previous conversations that David had Mr. Montoya last year, and my ears perk up. David continues: "He would always pick on me because I don't know the content very well. He would just be like okay, he'd ask a question and he would just choose someone randomly, like what's the answer to this, and he would just stare at you until you give the answer." "How did that make you feel?" I ask. "Uncomfortable," David responds.

February 28-March 28, 2019:At my request, Franck is systematically telling me about each student on his roster. More than once, he says someone "was a Montoya kid" and I wonder what that descriptor means to him. Just that the student had Mr. Montoya last year, or are there particular traits or behaviors associated with it? Sometimes Franck is more specific. He tells me that Nadia "had Mr. Montoya and he would literally make her cry... from what I've heard [from Nadia's mother] it was all negative and she didn't want to come to school." Franck says he "can tell Mr. Montoya has hurt [James] in some way... he wasn't picked on like Nadia, so he was kind of falling through, getting a C [in contrast to the straight As James has gotten in Franck's class], and I don't think he enjoyed math."

April 26, 2019: Clark explains to me how he has been teaching multiple forms of linear equations. I mention that the way students were talking about it in interviews made me wonder whether they saw different forms as having different functionality or if they felt it was a matter of preference. "That's interesting. That is definitely Montoya's kids," Clark guesses (correctly). "They told me. They were like, quote unquote, [he wanted them to solve equations] 'the most efficient way."

This dissertation opened with the question of what it means for mathematics teachers to be in right relation with their students, given the historical and ongoing violence of mathematics education as an enterprise. Feminist new materialists argue that in an entangled world where we are always intra-acting, right relation is not about determining the right principles to follow or the right actions to undertake or the right practices to engage in, but rather, about response-

ability: handling and honoring our vulnerability to one another, by responding in ways that take a stance on who and what matters, by rendering each other more capable of further response. In this ethics, right relation between mathematics teachers and students is not merely about learning mathematics or even developing identities as mathematicians or even expanding what and who count as mathematical, but also about co-existing complexly with humans, non-humans, more-than-humans, discourses, and infrastructures, many of which "precede and exceed" each of us (Butler, 2005, p. 17).

By centering response, an ethics of response-ability highlights the importance of reactions, which may not always have time to be deliberated. Reactions are an "ordinary affect" that reflect what we consider to be good and normal (Stewart, 2007)—often what has been normalized to us, but also possibly what we aspire to normalize. In Berlant's (2011) words:

Our sense of reciprocity with the world as it appears, our sense of what a person should do and expect, our sense of who we are as a continuous scene of action, shape what becomes our visceral intuition about how to manage living. (p. 52)

Ordinary affects accrue into habits over time, reactions burrow their way into our muscles and bones, learned ways of knowing and being become "impervious to intellectual argument" (Jaggar, 1989, p. 159). Therefore, living justly as a means of resisting and transforming mathematical violence requires not just thinking differently or wanting something new to be true, but responding differently in light of the oppressive systems we find ourselves complicit in.

Haraway (2016) calls this "mak[ing] trouble, stir[ring] up potent response to devastating events" (p. 1). She contrasts stirring up potent *response* to the act of trying to build new, alternative worlds:

In urgent times, many of us are tempted to address trouble in terms of making an imagined future safe, of stopping something from happening that looms in the future, of clearing away the present and the past in order to make futures for coming generations.

(p. 1)

Education narratives are often affectively oriented towards the future and towards the promise of a better future, particularly for students who experience marginalization in the present (Dernikos et al., 2020). In mathematics, for example, scholars have sought to replace the limited and exploitative "college and career" futures projected by neoliberal discourses of STEM education (de Roock & Baildon, 2019) with equitable futures (Boaler & Staples, 2008), emancipatory futures as a "right to the city" (Bullock, 2017, p. 637), Nepatlerx futures (R. Gutiérrez, 2017), and Black fugitive futures (Vakil & Ayers, 2019), among others. Seductive though they may be, completely alternative futures are an impossible dream due to the traces that this oppressive world leaves even on our imaginations (Yoon & Chen, 2021): because what has happened before and what is happening now can never be truly cleared away. Instead, Haraway's (2016)potent response requires "learning to be truly present... as mortal critters entwined in myriad unfinished configurations of places, times, matters, meanings" (p. 1).

In other words, transforming mathematics education is not about attempting to create a perfect, unoppressive future for mathematics education, no matter how collaboratively or responsibly undertaken the attempt may be. Rather, it is about responding in the present in ways that account for the past. Said another way, creating a better future requires focusing on the past because "to live an ethical life is to be held accountable to history" (Hong, 2020, p. 201). Consequently, this chapter is about "intensely inhabiting specific bodies and places as the means to cultivate the capacity to respond to worldly urgencies with each other" (Haraway, 2016, p. 7).

It is about how affective responses ground us in the present by tying us to the past, and specifically, how Franck and Clark engage in what Stewart (2020) calls "teaching affectively... the exercise of a capacity to respond" (p. 33). I begin by unpacking the relationship between affect and time using the constructs of inheritances and mathematical ghosts, describe the incipience of this chapter, and then illustrate how Franck's and Clark's (and my) reactions in relational work are an accounting for history that cultivates response-ability. I conclude with a discussion of how reactions can be tuned over time.

Affective Inheritances

Although affect is often interpreted as being synonymous with emotion, I use affect in the tradition of feminist affect theorists like Ahmed, Berlant, Jaggar, and Stewart, as the "relational capacities that belong to the doings of bodies" (Seigworth & Gregg, 2010, p. 9). Affect reflects the potential of bodies to move and be moved by the intra-actions of apparatuses, including the more-than-human social dynamics that regulate their intra-actions. Rather than belonging to any individual body or being intrinsic to any body, affect is a more-than-human force in and of itself; it intra-acts by circulating among bodies, and it "shifts, surges, and subsides according to relational and historical contexts" (Chen, 2020, p. 156). Affect is, on the one hand, intensely present—it manifests in "contact zone[s] where... flows of power literally take place (Stewart, 2007, p. 3)—but on the other hand, what bodies come into contact with is also the past. I think about this by drawing on theater director and racial healing practitioner Bellamy's (2020) idea of inheritance.

After the video of Ahmaud Arbery's murder was widely circulated in spring 2020, and after Amy Cooper called the police on Christian Cooper for requesting that she obey leash laws a few weeks later, Bellamy wrote about how racial inheritance is embedded in our bodies. White

racial inheritance—a legacy of exercising power over Black people—is performed in reaction to perceived slights or out of a desire to feel control, she argues, and this racial inheritance endangers Black lives: neither Arbery's murderers nor Amy Cooper may have anticipated the specific ways in which they would threaten Black men, but seeing a Black man jogging and seeing a Black man birdwatching, respectively, sparked reactions that evoked histories of white individuals extrajudicially sentencing Black people to death or state-sanctioned violence. "What are you carrying dormant in your body that springs up when confronted with Black joy, Black power, Black brilliance, Black Blackness in the world?" Bellamy asks, and "How can you train your body to react differently?" (2020, para. 17). Training white bodies to respond differently from what their racial inheritance provokes is a way of creating a more just future: one in which Black lives are not the collateral damage of white thirst for recognition or power.

And so, we can imagine a mathematical inheritance that springs up—perhaps in the bodies of teachers who have themselves successfully navigated K-16 mathematics education—that commits a slower, less graphic, but still historied and embodied violence: one that leads otherwise-confident adults to wince at the prospect of dividing a restaurant check among friends, to cringe at the sight of their third-grader's math homework, or to put on crisply-ironed clothes and straighten their backs as they walk into a math classroom for parent conferences. Perhaps this mathematical inheritance is what prompts a teacher to grimace when a student expands 2(x + y) as 2x + y months after the distributive property was taught, to sigh when a student finishes only two of the ten assigned exercises, to tap their marker impatiently against the whiteboard tray when students meet what the teacher thought was a simple question with an excruciating minute of stone-cold silence, or to raise an eyebrow when a "D student" aces a unit test. Confronting this mathematical inheritance requires responding in ways that render others capable instead of

defaulting to those ordinary affects that support the status quo, thereby taking a stance on students—and students' humanity, their resistance, their creativity, their inconsistencies—mattering more than a pacing guide or curriculum or assessment.

For example, in traditional performances of mathematics education, students who do not meet participation expectations are typically viewed as off task, disengaged, or noncompliant. As a researcher who has observed in hundreds of mathematics classrooms, I have seen many teachers react to students this way, and many students accustomed to this treatment. As a result, when I saw Franck say to Ricardo, after giving him more than six minutes to think of notices and wonders independently and with his small group and after Ricardo says he does not have any wonders, "Oh, you didn't have enough time. My bad" (classroom fieldnotes, 8/2018), my "thinking [was] stopped, shocked, as it were, into a configuration or conjecture that crystallize[d] the social gist of a... mundane event" (Gordon, 2008, p. 65). Franck could have reacted by implying that Ricardo did not have a wonder to share with the class because he had been off task during independent or small group work time, which would have recited and reinforced a positioning of Ricardo—who had failed Clark's Algebra 1 class the previous year—as deficient. Without missing a beat, however, Franck conceded that he may not have given Ricardo enough time to develop an answer; Ricardo's non-answer was not solely Ricardo's fault, but rather, a product of the intra-action between teacher, students, and mathematics education. Franck publicly took the blame and moved on, demonstrating a "willingness to privilege the humanity [he] shares with [his] students in a way that challenges boundaries of us/them" and thus making connection possible (Dutro & Bien, 2014, p. 20).

Haraway (2016) praises Despret's work for being remarkably attuned to:

critters rendering each other capable of unexpected feats in *actual* encounters [emphasis added]... not to what critters are supposed to be able to do, by nature or education, but to what beings evoke from and with each other that was truly not there before. (p. 7)

It may seem like hyperbole to describe Franck's response to Ricardo as a "feat" because, on the one hand, it is so ordinary to acknowledge that Ricardo does not have an answer and move on. Yet, it is unexpected because mathematics teachers routinely use their authority to suspend activity when students do not participate as expected: to "hold students accountable," to "not let them off the *hook*," to "not let anyone get *away* with" something or "*slide* by." In that context, the collective exhalation, the collective continuation, the collective *movement* evoked by Franck's response creates potential for a less violent mathematics education and opens up possibilities for living justly.

Mathematical Ghosts

In this chapter, I use the construct of mathematical ghosts as a way of exploring affective inheritances: who or what haunts students, teachers, and researchers in mathematics classrooms, and how they respond—to each other in the present, as a means of responding to the past—as a result. Students who have been repeatedly subject to the slow violence of mathematics education carry trauma resulting from their prior experiences and also from the "ongoing, accruing impact and consequences of social malignancies such as racism" (Dutro & Bien, 2014, p. 23). In mathematics classrooms, "the past haunts the present" (Gordon, 2008, p. viii); mathematical ghosts are an apparition of students' mathematical trauma that spark affective reactions in the present. If this is the case, how ought mathematics teachers attend to the lasting impact of traumas perpetuated on students by hegemonic forms of mathematics teaching and learning within an already oppressive society?

Many scholars have probed the presence of ghosts in public schooling (e.g., Ewing, 2018; Lawrence-Lightfoot, 2003), and I follow Gordon's (2008) framing of ghosts as "one way in which abusive systems of power make themselves known and their impacts felt in daily life, especially when they are supposedly over and done with" (p. xvi). In Gordon's conceptualization, ghosts call out for justice: for the damage that has been done by abusive systems of power to be addressed, not because it can be undone, but in order to "en[d] this history and se[t] in place a different future" (p. 66). Otherwise, traumatizing histories repeat themselves; "indeed, oppression itself can be seen as the repetition, throughout many levels of society, of harmful citational practices" (Kumashiro, 2000, p. 41) that remake dehumanization. In mathematics, then, ghosts might call out for transformative ways of teaching and learning that are no longer traumatizing or violent.

How do ghosts call, and how does one listen when they do? Gordon (2008) suggests that ghosts are a symptom of what's missing; they represent both a loss and a future possibility. They appear in those "singular yet repetitive instances... when the over-and-done-with comes alive, when what's in your blind spot comes into view" (p. xvi). In those instances, ghosts are "leading us somewhere... [calling for] something to be done," because if we are haunted then we "are already involved, implicated, in one way or another" (p. 205). Mathematics teachers, students, and researchers do not just cognitively know that mathematics education has historically been violent, but they feel it as they intra-act in mathematical spaces. They are reminded of their own histories of mathematics education and they are called to bear witness, from their own locations, to how others across time and place have experienced mathematics education. Those who see hegemonic practices of teaching and learning mathematics as violent, then, are called by

mathematical ghosts to do something; greeting ghosts requires attending to their insistence on a future that is more just than the past (Yoon & Chen, 2021).

Shotwell (2016) argues that "our response to the past constitutes the conditions of possibility for the present, and that understanding ourselves as relationally coconstituted," or what Barad would call entangled, "offers us something helpful for both remembering the past and responding to the present" (p. 139). In other words, the seemingly-instinctive reactions we have to being haunted—our mathematical inheritances and the ordinary affects they produce—can reproduce the mathematical harms of the past and they can also open up the possibility of more just futures if they 1) are accountable to the past by bearing witness to what has come before and 2) compose intra-actions that render others capable of further response. In this chapter, I use the ghost of Mr. Montoya to illustrate how affective inheritances manifest in Franck's and Clark's classrooms, and how Franck's and Clark's reactions are movements towards "ending this history and setting in place a different future" (Gordon, 2008, p. 66). I argue that these movements cultivate response-ability—render each other capable—for teachers, students, and researchers.

Methods (A Brief Reprise)

During my time at Banneker, I was struck by the frequency and force with which both teachers and students named an eighth-grade teacher who had previously taught many of the students I was observing: Mr. Montoya. Mr. Montoya's ghost first appeared to me in October, when, in an interview about building relationships with students, Franck repeatedly cursed Mr. Montoya for "the damage that he did to these kids." Mr. Montoya's ghost provoked strong affects throughout the school year, illustrating the "living effects, seething and lingering, of what seems over and done with" (Gordon, 2008, p. 195). Despite no longer being in his class, students

brought him up unsolicited when I asked them about their *current* mathematics classes, saying he was "rude," "really bad," and "he would pick on me." He haunted teachers' interviews and collaborative meetings: Franck called his methods "public abuse," Clark mentioned "kids that hate him," and Abigail said that hearing them talk about him made her both "want to cry" and "fight."

As a researcher, I felt constantly alert to Mr. Montoya's name or the mere possibility that someone might be referring to him. For example, when students described their eighth grade mathematics teachers anonymously to me in interviews, I sought to confirm the identity of that eighth grade teacher by asking them or their teacher whenever I thought I could do so without revealing information that the student expected me to keep confidential.¹³ I also noticed that because students and Franck and Veronica spoke with such (mostly but not always negativelyladen) vehemence about Mr. Montoya, I felt myself rising to substantiate their claims in conversations with Clark and Abigail, who had heard less firsthand information directly from students about him; in the January collaborative meeting, for example, I offered unattributed examples of things students had said to me about him in interviews, along with my own reactions and frustrations with what he, to me, symbolized. Perhaps the stories I heard about Mr. Montoya reminded me of how my own students had been scarred by their prior experiences with traditional forms of mathematics education and how much of my teaching practice—and subsequent enrollment in graduate school—had been driven by a desire to undo and a pressure to show them (and myself) what else was possible in mathematics education. As an instantiation of both the individualized and structural trauma perpetuated in mathematics classrooms, Mr.

¹³ In several cases, I already knew from prior conversations who their eighth-grade teacher had been, and in others, I found out later in conversations that were disconnected from my interviews with these students.

Montoya's ghost made concrete how students, teachers, and researchers are haunted by histories of oppressive mathematics education.

To greet the ghost of Mr. Montoya, I attended to the "flow and arrest of thoughts" (Gordon, 2008, p. 65 emphasis original) in students' and teachers' talk to identify Mr. Montoya's presence, looking for "how a person translates his or her experience of historical trauma across time and space" (Zembylas, 2006, p. 315). I sought out thinking with theory as an entangled method that presses against forms of research seeking to classify and determine truths after data collection is "complete," instead honoring the ambiguous and emergent nature of any possible "truth." After I completed the analyses described in Chapter 3, I composed narratives that articulated the inheritances, affects, reactions and possibilities in each episode: ghost stories, so to speak. In the ghost stories that follow, I draw more heavily on Franck's data because he was freer with his censure of Mr. Montoya, making affect easier to discern. As a result, Mr. Montoya becomes a bit of a bogeyman in this chapter; I remind readers that this story is an analysis of the enduring affects that the real Mr. Montoya and millions of anonymous "Mr. Montoyas" leave behind in mathematics education, as made material in the data, rather than an indictment of a specific person.

Findings

In this section, I describe how students', teachers', and researchers' affective reactions bring the past into the present, accounting for the traces left on us by mathematical inheritances. I begin with how the ghost of Mr. Montoya haunted Franck, in the stories he told about his relationships with students, and how Mr. Montoya haunted me, as a researcher and participant in what was supposed to be a lesson planning meeting with Franck, Clark, and Abigail. Next, I recount how Franck and Clark took students' affects in class to be an indicator of the

conditionality and coercion that pervade many mathematics classrooms, and how they responded by examining their pedagogical practices accordingly. These instances illustrate how responses to mathematical ghosts can open up movement towards more just forms of mathematics education. Then, I revisit additional components of the vignette that opened this chapter, along with Lucas's story from the previous chapter, to explore some of the concrete ways that attuning to mathematical ghosts carries the potential for rendering others capable. Through these examples, I claim that affective reactions can cultivate response-ability, or not, and in doing so, make possible particular futures.

"This Fucking Mr. Montoya... He Believes in Public Abuse of These Kids"

By late October, when I first heard about Mr. Montoya, Franck had already been provoked by his existence; Mr. Montoya's ghost had already haunted Franck's students, his classroom, and by extension, him. Franck used a forceful tone and strong word choice in telling me about Mr. Montoya ("abuse," "hate," "damage"), testifying to Mr. Montoya's capacity to affect students and to their limited capacity to respond beyond experiencing antipathy; any resistance that students may have demonstrated was missing from the narrative (although I did hear, in other contexts, about students who skipped school and even transferred to another school in order to avoid Mr. Montoya). Franck's juxtaposition of students' mathematical abilities as being "really good" ("they know how to solve an equation, they know how to graph lines") with their hatred of what they experienced of mathematics in eighth grade recognized the cruelty in making people dislike something they are good at instead of letting them take pleasure in being successful. Listening, I felt like Franck was trying to impress upon me how horrible Mr. Montoya was—to bear witness to what Oliver (2001) calls a "psychoanalytic truth" that cannot

be seen using visual faculties alone—because I had not yet had the opportunity to hear directly from students about their experiences.

These affective reactions represented a break from the past: from mathematical inheritances that take Mr. Montoya's methods (cold-calling on students, shaming students for anything other than immediate correct responses when he called on them, insisting that students memorize procedures and conventions without regard for understanding) to be normal and even normative. They made possible the ending of this history and setting in place of a different future. As Franck said,

Otherwise, of course they're going to go to Mr. Montoya's class and learn the damn math and be out of there. But what do we want these kids—what do you want for these kids? Do you want them to learn the math and get out, or do you want them to have a good experience while learning the math? Just have a good experience and then who cares about the math? [I ask him what he wants] ...How do you train new teachers to the human aspect of teaching? ... Otherwise, you're here for a paycheck. (*fieldnotes*, 10/2018)

By contrasting "the damn math" and being "here for a paycheck" to "the human aspect of teaching," Franck asserted that learning mathematics could conflict with treating students humanely, but that it did not have to. Instead, teaching could center on students' humanity.

When I asked Franck to tell me about each student in the spring, several of his responses were characterized by a sense of righting wrongs. Of Nadia, who "didn't want to come to school" when she had Mr. Montoya, Franck said, "[Her experience] is different now. Whatever math that she is learning or not learning, I'm pretty sure it's going to stick more than it has in the past." Of James, who had been "hurt" and "falling through" in Mr. Montoya's class, Franck said,

"So he's learned something in my class, which is really cool... hopefully he's enjoying it more. He seems like he's having a good time" (*interview*, 3/2019). These redemption narratives described students' affective movement away from negative perceptions of mathematics and of themselves as mathematical beings, and towards possibilities for futures with more flourishing.

"I Just Think So Much About This Montoya Guy"

In January, during the planning period that Franck, Clark, and Abigail shared, Clark initiated a roughly 15-minute conversation about Mr. Montoya that was most likely directly prompted by Veronica's quick visitation but indirectly prompted by months of being haunted by Mr. Montoya's ghost. Table 7 summarizes the conversation in topical episodes and identifies some of the affects circulating during each episode. The overall atmosphere of this conversation was somewhat tense, in direct contrast to the more genial and relaxed tone of the planning period conversation preceding and following our discussion of Mr. Montoya. It is worth noting, of course, that teachers' affective responses in this conversation may have been constrained by what Zembylas (2005) calls emotional regimes: normative expectations about which emotions may be expressed under which circumstances by which people. As Jaggar (1989) writes, "a woman may cry in the face of disaster, and a man of color may gesticulate, but a white man merely sets his jaw" (p. 157)¹⁴, which reflected surprisingly well Abigail's consternation, Franck's animosity, and Clark's intellectualizing throughout the conversation. Rather than discussing the full transcript, I limit my exposition here to how the affective inheritances in this conversation moved

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¹⁴ Jaggar sets women as a group separate from people of color, leaving unclear how I might theorize my own reactions as a participant in this conversation; fortunately, women of color scholars such as Ahmed (2014), hooks (2000), Lorde (1984), Matias (2016), and Yamada (1981) have written amply about the racialization-and-gendering of affective normativity that disciplines the affective experiences and expressions of women of color, suggesting that I may have felt greater pressure for my expressed emotional reactions to mirror those of the people around me regardless of how I actually felt.

 Table 7

 Summary of January Co-Planning Conversation

Episode	Length	Summary	Affects
#1 "I just	lmls,	Clark says he's been thinking about Mr.	Uncertainty,
think so much about this	19 turns of talk	Montoya. Abigail wonders if he and Franck	surprise,
	(ToT)	are obsessed; she expresses surprise that I've heard a lot about Mr. Montoya too.	skepticism, curiosity, validation
Montoya guy" #2 Mr.	$\frac{(101)}{3\text{m}12\text{s}},$	Clark says his fear is that Mr. Montoya's	Anxiety,
Montoya is so	9 ToT	philosophy is so different from theirs that it	wistfulness,
different from	7 101	sets students up "for just a difficult time."	petulance
us		Grace gives examples of differences between	Permina
		what she's seen in his classroom and what	
		she's heard about Mr. Montoya's classroom.	
#3 Are there	1m40s,	Clark asks whether there are any positives	Aggrievement,
positives	15 ToT	about Mr. Montoya. Grace says that some	equivocation,
about what he		students said they felt smart in his class, but	resoluteness,
does?		others felt stupid, and the students who felt	foreboding
		smart may have a hard time empathizing	
		with others. Franck says that the latter "become Montoyas in the world."	
#4 "We	1m47s,	Clark suggests talking to Mr. Montoya.	Fear, conciliation,
should go talk	20 ToT	Franck, Abigail, and Grace turn this into a	combativeness,
to him"	20 101	joke.	drive, dampening
#5 "No, he	2m2s,	Franck describes how students manipulate	Uncanniness,
messes with	15 ToT	Mr. Montoya and also what else they've said	spookiness, disgust,
kids' minds"		about him.	awe, appreciation
#6 Mr.	2m14s,	Franck explains that Mr. Montoya's policies	Irritation, fear,
Montoya is	20 ToT	feel the harshest for the students who need	resistance, injustice,
"punishing		harsh policies the least, and wonders whether	reflexivity
the wrong		his policies have the same effect. Clark	
kids"	1 5 (assures him that his do not.	C - 1
#7 "It's sad"	1m56s, 11 ToT	Clark says that what Mr. Montoya does is sad. Grace says that a student she	Sadness, anger, indignation,
	11 101	interviewed is grateful that this year, he's not	arousal,
		made to feel stupid for thinking slowly,	galvanizing,
		unlike in Mr. Montoya's class. Abigail wants	incitement
		to cry and then becomes angry.	
#8 "That's	2m3s,	Clark and Abigail play out the consequences,	Exasperation,
going to	13 ToT	in their classes and beyond, for students who	irritation,
happen to		were successful in Mr. Montoya's class.	intellectualizing,
them"		Clark says the conversation was cathartic.	de-escalation, relief

me—as a researcher, and as an "outsider-insider" thought partner to Franck, Clark, and Abigail—to respond, as an example of how Mr. Montoya's ghost made the past present not just for students and teachers but for anyone attuned to the possibility of being haunted.

As I described in Chapter 3, by January, I was participating more actively in Franck's and Clark's collaborative conversations and stating more opinions than I had been at the beginning of the year, when I limited my participation primarily to asking questions and giving straightforward answers when asked directly for my perspective. Volunteering more of myself felt like the response-able thing to do, given the trust and generosity with which they were sharing their practice, ideas, and vulnerabilities with me; it seemed like one of the few offerings I could make towards mitigating the inevitable awkward asymmetry of a researcher-participant, observer-observed relationship. Until I conducted this analysis, however, I had not realized the extent of the role I played in moving this particular conversation along. In the moment, I had felt like I was reflecting affects already in circulation. Analysis, however, made clear that I had been recruited into movement specifically against Mr. Montoya, in reaction to Mr. Montoya's ghost, as a result of my own mathematical and affective inheritances.

In this conversation, I did not simply mirror back what teachers said or directly answer the questions they asked me. I provided interpretations and commentary after reporting what students had told me (e.g., "It's such a basic thing. He's so grateful this year that nobody makes him feel stupid if he needs more time. That shouldn't be something unusual") and added negative implications to what other teachers said (e.g., "Well, I feel like it makes your jobs harder...;" "Well, and just for being a member of a classroom..."). In episode 3, I presented positive comments that students had made about Mr. Montoya, as requested, but marked them with caveats ("that very traditional kind of teaching, *if* you are good at it, it does make you feel really

good about yourself [emphasis added];" "I've heard 'I felt really smart'... [but] when those kids become math teachers, they have a hard time empathizing"). And, instead of naming the effects of Mr. Montoya's pedagogies in the present, I described possible future consequences ("when those kids become math teachers...") as if to distance myself from the specific students I was observing that year and feign that my only concern was based in my role as someone who works with prospective teachers.

Upon reflection, however, it seems undeniable that my concern was also grounded in my own experiences as a mathematics student; my K-16 mathematics teachers were much more like Mr. Montoya than like Franck and Clark, and I "learned" mathematics largely by rote. Fortunately—for my performance and consequent acceptance by my teachers, if not for my understanding or appreciation of mathematics—I was skilled at memorizing and regurgitating formulas, conventions, and procedures, and I always tested easily into the highest-track of mathematics available to me. I took AP Calculus BC as a junior in high school, and what I remember from class that year is arduously plugging through every released AP exam from 1971 to 2000 while my teacher played Seinfeld episodes and joked with the students who shared his affinity for the show; by May, I had essentially come to recognize every possible AP test question and scored a 5 on the exam. I was so disinterested in this version of mathematics that I reveled in never having to take another mathematics class, until my undergraduate advisor counseled me to take Calculus III to demonstrate quantitative competence in case I wanted to go to graduate school in the future. The limits of my minimal conceptual comprehension—I could not have defined a function if asked—quickly made themselves apparent and I still remember how wretched I felt after grossly failing my first midterm (a 43% after the curve).

This mathematical inheritance shaped my work as a high school mathematics teacher, driving me to target mathematical understanding rather than mathematical performance, and made it impossible for me to even pretend any sort of objectivity in this collaborative meeting conversation. Instead, I responded to the call Franck had issued to me in October—to bear witness to the effects Mr. Montoya had had on students, both as an eighth-grade teacher in Banneker's neighborhood and as a representative of a version of mathematics education that reduced students to performing mechanics—because Mr. Montoya had affected me too. Hearing students tell me, in interviews, how petrified they had felt in Mr. Montoya's class summoned a sense of still being stuck in AP Calculus myself and made me yearn for all of us to be freed. Therefore, I felt compelled to report to Franck and Clark and Abigail what students had shared about how Mr. Montoya had affected them such that their negative experiences did not go unrecorded, as so much mathematical violence generally is, taken as it is to be normal and even necessary to mathematics education (e.g., "productive struggle").

Perhaps this conversation seemed like an appropriate moment to address our collective mathematical inheritances because I perceived an opportunity for students' testimonies to move us, and more specifically, to move their Algebra 1 teachers towards pedagogical choices that could reckon with and dislodge histories of harm. My intra-actions with Franck, Clark, and Abigail had given me more hope for a more mathematically just future than other teachers I had personally observed, and so movement away from violent mathematics pedagogies felt not only possible but already happening and likely to continue. And so, I responded to the presence of Mr. Montoya's ghost—who was also the ghost of my own experiences of mathematical trauma—by being moved to end this history and set in place a different future: one where mathematics

students, teachers, and researchers are able to render each other more capable, not just of mathematics learning, but also of responding to each other in healing and humanizing ways.

Forcing Students to Talk

Niccolini (2016) writes that "learning and teaching are affectively charged events... affect moves knowledge" (p. 230). As a more-than-human force intra-acting with students, teachers, and mathematics education, affect shifts what is taught and what is learned by shifting what and who moves. Sometimes (often), in mathematics classrooms, students do not immediately produce correct answers. When Mr. Montoya reacted by insisting that students stand at the whiteboard (as Franck fumed about in October) or by staring at them until they did produce a correct answer (as David described in February), students were literally immobilized—both the students at the front of the class and the students left in their seats, waiting. Feeling "picked on," as David did, positioned students as objects of manipulation, "having to 'yield' to 'others'" in contrast to being "able to respond... positioned as authorities on their own spaces and ways of being," and positioned Mr. Montoya as the only person with a "right to respond" (Kayumova et al., 2019, p. 223)—in this case, to adjudicate who was allowed to move and when. Without the possibility of collective response-ability, the agency in the student-teacher-mathematics-affect apparatus was asymmetrical, unbalanced, "out of joint" (Derrida, as cited in Yoon & Chen, 2021).

By contrast, consider this example of how Clark reacted when he noticed that the same students kept volunteering to answer his questions on the second day of school: "We're getting the same people raising their hands all the time," he observed out loud, "that's all right. I'll talk to the rest of you later" (*classroom fieldnotes*, 8/2018). Clark could have reacted to this phenomenon—common, in many mathematics classrooms—in many other ways. In similar

situations, I have heard teachers say things like: "I want to hear from someone I haven't heard from yet;" "If you've already spoken, put your hand down; "I'm going to wait for a hand I haven't seen yet;" "Please be aware of how much space you're taking up;" or start calling randomly on students to respond. Such reactions might also immobilize students: they might make students who have been volunteering feel transgressive, guilty, or unappreciated; they might make students who have not been volunteering feel called out, put on the spot, or intransigent. They might also make students who have not been volunteering feel motivated or grateful—my point here is not to argue that Clark's reaction was the best reaction, but to highlight what was made possible by acknowledging and affirming students' authority in choosing how they would like to participate on the second day of school. With his response, Clark communicated that he was aware that some voices were being heard more than others, that students who were not volunteering might have good reasons to not want to volunteer, that he would not force anyone to participate in ways they did not feel comfortable participating, and that what students were thinking still mattered even if they did not want to say it in front of the whole class.

The next day, Clark said to students:

If you don't want to share [with the whole class], you can raise your hand and share with me as I'm walking around... you don't have to share in front of the class to get points... I don't want you to share for points; I want you to share for love. (classroom fieldnotes, 8/2018)

In this way, Clark sought to make possible futures in which students volunteered to share ideas in class not for instrumental reasons such as assuaging him or earning credit, but because they felt positively moved—eager, willing, inspired—to share their ideas. In such futures, students'

responses could replicate patterns of flourishing and proliferation rather than patterns of conditionality or coercion.

Conditionality and coercion—and the desire to avoid them—weighed heavily on Franck, too, as he contemplated their role both in traditional versions of mathematics education and in what he understood to be prescribed by those advocating for ambitious and equitable mathematics education. Many scholars and practitioners concerned with equity in mathematics education argue that setting in place a different future, given the pervasiveness of initiate-respond-evaluate discourse (e.g., Greeno, 1998) and patterns of inequitable participation (e.g., Cohen et al., 1999) within mathematics classrooms, requires encouraging every student to speak in class. Franck responded heatedly to this idea; we had the following exchange in an interview, prompted by my asking him to elaborate on what he had said when Veronica had recommended a professional development book during a casual lunch with colleagues a few days prior:

Franck: Sometimes, I'm sick of all these righteous people. Everybody is so righteous these days.

Grace: In terms of telling people what to think and believe?

Franck: Like, "all you've been doing is pushing kids out. You're not 'agency-ing,'
whatever the fuck that is." I don't know what agency-ing means. But somehow I
feel like I'm not doing it.

Grace [seeking to reassure and perhaps mollify?]: Well, you are, but haven't used that word.

Franck: It makes me feel bad. I am not creating a classroom climate that is conducive for agency-ing. But I'm just saying. That's how I feel. Right?

(interview, 3/2019)

I was surprised to hear Franck say that he "felt bad" about his teaching practice; throughout our data collection, he struck me as confident without being arrogant, thoughtful about the uncertainties and complexities of teaching, aware of specific habits or practices he wanted to improve but also proud of what he did and who he was as a teacher, and always ready to patiently instruct me on what he did and why.

His perturbation—disgust, frustration, guilt—reflected affective inheritances saddled with mathematical violence and antidotal aspirations, both of which positioned students' agency as teachers' responsibility rather than as the possibility of mutual response in intra-action. Yet, it also made possible his critical interrogation of the purposes and consequences of student participation:

I'm not sure if I'm sold on the fact that we need to make all the students in our class talk to each other... I'll listen to the kids, but I know [student name] or certain kids in my class, they don't want to talk out loud and I don't want to force them to. The philosophical questions I'm trying to reconcile are: what people are doing in their classrooms that's deemed as equity and access [e.g. making students talk], to me it just seems annoying and you're forcing it upon the kids. (*interview*, 2/2019)

He questioned how forcing students to talk with equitable intentions differs from forcing students to talk as an assertion of teacher authority, the way Mr. Montoya apparently did, given that the strategies used were often similar (e.g., cold-calling using "equity sticks" or other forms of randomization). In another conversation, I mentioned how a student I interviewed had said she appreciated being pushed to talk because it helped build her courage and asked whether that would be a reason to "force," so to speak, students to talk. In response, Franck:

... asked what the purpose would be, especially since he's already made them share with their groups. Is it for him, because he wants to hear something good? If so, he should go through all the students' responses and pick out the best ones to share out loud. Is it for the rest of the class—does the rest of the class really care what they have to say? Is it for them? He says everything he does has to have a purpose, and he hasn't yet thought of a good purpose for randomly calling on students. If he thinks something will benefit that student or if it'll benefit the whole class then he'll do it, but if it's just for his own benefit, then it's not worth it. (*fieldnotes*, 3/2019).

The reaction that Franck had to feeling as though he was supposed to be randomly calling on students for the sake of equity and agency moved him to interrogate a practice that is widely accepted (albeit for sometimes contradictory reasons) in mathematics education and to examine what it would make possible in terms of students' ability to respond.

Perhaps because of his concerns with "forcing" students to talk in front of the whole class, I observed that Franck, and also Clark, spent the vast majority of class periods circulating among small groups, ¹⁵ and that most of this circulation time was spent working with students at wall-mounted whiteboards. This created opportunities for literal movement as students moved between their desks and whiteboards and between their group's whiteboard and other groups' whiteboards as they sought or offered help, and as teachers moved around the classroom and between groups. It also made possible different types of affective movement than would be possible if teachers remained at the front of the classroom and students remained in their seats

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¹⁵ The sample lessons analyzed in Ehrenfeld and Horn (2020) show that Franck and Clark spent 87% and 85% of their class periods, respectively, monitoring while students worked in groups; additionally, Franck and Clark each had more discrete interactions with students during this time (74 and 40 respectively, representing 96% and 88% of time in the monitoring phase spent interacting) than the other teachers in the study.

except when commanded to perform in front of the class, affording privacy and a relative intimacy as teachers and small groups of students interacted with mathematics, and as the overall murmur of the classroom provided sonic cover for teachers and individual students to respond to each other during check-ins (*classroom fieldnotes, recurring*). If haunting signals phenomena that are "pregnant with unfulfilled possibility for change" (Gordon, 2008, p. 183), then the specific manifestations of Mr. Montoya's ghost, as a representative of the mathematical inheritances present in Franck's and Clark's classrooms, reminded Franck and Clark of the somethings-to-be-done in response to the slow violence of a system that often adopts a dehumanizing, one-size-fits-all approach to mathematics education (Basile & Lopez, 2015).

The ghost of Mr. Montoya, both as the afterlife of the real Mr. Montoya and as a stand-in for oppressive histories of mathematics education, made himself felt in students', teachers' and my memories, in our feelings about ourselves and mathematics and ourselves in relation to mathematics, and in our visceral reactions. Perhaps Mr. Montoya's ghost sensed the relational work that Franck and Clark did to account for mathematical inheritances that haunt and, in response, made contact. He transported Luisa back to her eighth grade classroom when she heard Clark say "cool beans," made Dylan appreciate how good it felt to be "good at something like math" this year in Franck's class (interview, 1/2019), and invaded both teachers' collaborative meetings and my conversations with students and teachers, prompting fury and frustration and sorrow, among other emotions. Mr. Montoya's ghost animated Franck's and Clark's reactions to students and energized their commitments to relational work. He piqued my indignation and my learning about what mathematics education has been and can be, while reminding me that I can never be a dispassionate observer in mathematics classrooms.

Gordon (2008) cites Zora Neale Hurston to suggest that "ghosts hate new things precisely because once the conditions that call them up and keep them alive have been removed, their reason for being and their power to haunt are severely restricted" (p. xix). When they appear, they are due "a hospitable reception" that is nevertheless "partia[1] to the living. Because ultimately haunting is about how to transform a shadow of a life into an undiminished life whose shadows touch softly in the spirit of a peaceful reconciliation" (p. 208). Or, in Haraway's words, stirring up potent response in order to cultivate the capacity to respond. Franck's and Clark's attention to mathematical inheritances, and to how mathematical inheritances shaped and should shape their intra-actions with students, prompted them to respond to students, to taken-forgranted pedagogical practices, and to the apparition of Mr. Montoya's ghost in ways that made possible further response from students who were able to speak or not speak in class, experience meaningful mathematics learning, and connect with their teachers.

Not all ghosts are as transparent as Mr. Montoya's, though, nor are we equally attuned to all the inheritances we carry. Even so, our affective reactions still converse with them. For example, to return to an example from a previous chapter, Franck did not know much about Lucas's background or history, beyond some speculation that he might crave relationships with adults despite having ideas about what such relationships entail that differed greatly from what Franck considered normal. Yet, for Lucas, the ways that Franck responded to him—accepting his gifts, listening to his "weird" stories and asking sincere follow-up questions, showing him a photo of Elliot playing with Thanos—must have recalled previous relationships with teachers or other adults, either that Lucas had experienced or in their absent presence. If Franck responded to Lucas in ways that Lucas was accustomed to, he may have reinforced whatever habits Lucas had developed around interacting with teachers. If Franck responded to Lucas in ways that Lucas has

never experienced from someone in the institutional role of a teacher, he may have opened up different possibilities for Lucas's future relationships with teachers. Either way, the intra-action between Franck and Lucas was clearly unfamiliar to Franck, and so it already made possible new ways of knowing and being and responding for Franck. Franck's and Lucas's reactions to each other, together, rendered each other capable of further response, making possible a worlding that could enable flourishing for both of them.

Discussion

In this chapter, I have illustrated how affective reactions can be a way of cultivating response-ability (or not), because reactions in the present tie us to the past. They reveal how we account for the past: what we take of the past to be ordinary, normal, and desirable, and what we have inherited that we aspire not to reproduce. Or, as Berlant (2011) puts it, "intuition [reactions] is where affect meets history, in all of its chaos, normative ideology, and embodied practices of discipline and invention" (p. 52). By accounting for the past, reactions make possible particular futures. As Clark said, "a lot of [relational work] is in the reactions. Like, how you react to a kid's personality is how they learn whether or not they're allowed to be themselves, and express themselves, and be comfortable" (*fieldnotes*, 10/2018). Through reactions, students and teachers world futures together. Clark, and I would add Franck, sought to react to students' reactions by "acknowledg[ing] it, and respect[ing] it, and maybe interact[ing] with it in any way... Some positive, and some reassurance, like what they just did is okay, in some space and time" (*fieldnotes*, 10/2018).

Although Franck's and Clark's reactions in their classrooms, in collaborative meetings, and in their conversations with me may have seemed spontaneous or instinctive, they were undoubtedly tuned—developed as affective habits—over time. Both spoke at length about how

they had been different teachers earlier in their careers: they taught from textbooks, spent more class time on basic skills practice, relied on their youth to connect with students, needed to be stricter in some ways because of their youth, attended students' extracurricular events outside of school hours, had more time to interact informally with students because they were more willing to taking planning and grading work home, etc. It would be misleading to imply that the development of affective habits that cultivate response-ability is a natural consequence of experience, however, because certainly not every experienced teacher's reactions render students more capable of further flourishing.

Even Franck and Clark overlooked aspects of their mathematical inheritances because, as I wrote a few paragraphs ago, we are not equally attuned to all the inheritances we carry. For example, their colleague Veronica was highly attuned to gendered interactions in mathematics classrooms and a history of mathematics education as privileging masculinity (e.g., Leyva, 2017; Mendick, 2005). In a collaborative meeting, she brought up how students identified as girls could name both other girls and students identified as boys on a survey, but boys only named other boys. To address this, she consciously initiated conversations with girls when she approached groups with more boys than girls (*collaborative meeting*, 10/2018). She was also attuned to racism in different ways, mentioning what she was learning from reading books like Morris's (2016) *Pushout*. As a result, collaborating with Veronica became a way for Franck and Clark to tune their reactions; they joked about how they were learning not to say "you guys" to mixed-gender groups and Clark told her that "because you're here, I'm actually accountable to have those values be visible in my classroom... maybe those things I was overlooking before because there was nobody holding me accountable to that" (*collaborative meeting*, 2/2019).

Just as nonviolent reactions to Black jogging, Black birdwatching, or Black joy are not nearly sufficient for racial justice, mathematics teachers' reactions in and of themselves are insufficient for addressing and repairing inherited legacies of mathematical violence. That said, confronting the mathematical inheritances that are made viscerally and absently present by ghosts such as that of Mr. Montoya requires tuning reactions to, as often as possible, communicate that students matter as humans with individual and structural histories and not just as learners in the moment. As Shotwell's (2016) doing what one can, and Haraway's hard work of attending to specificities, such tuning makes possible affective movement that sets in place a different future by cultivating response-ability: rendering others capable.

Chapter 7

DISCUSSION

This dissertation sought to examine how Franck and Clark made sense of and enacted right relations in the context of mathematical violence and oppressive systems. By thinking with theory using ethnographically-collected data and poststructuralist and feminist new materialist texts, I found that Franck and Clark emphasized knowing instead of knowledge, open normativities instead of norm-setting, and histories instead of futures. This suggests that their ethical stances on relational work reflect a form of *response-ability*. In this chapter, I summarize the findings and their practical implications using Berlant's (2006) concept of cruel optimism, underscore the link between my findings and response-ability, consider what this link indicates about conceptualizing agency in unsettling the slow violence of mathematics education, and finally offer some limitations and additional implications of this research.

The Cruelty of Common Stances on Relational Work

This dissertation began with the premise that we are all already living in exclusion zones; as teachers, students, and researchers, we are both carrying and complicit in the violent histories and presents of mathematics education. In Chapter 4, I explored what Franck and Clark know about their students, in the context of what it is possible and ethical for mathematics teachers to know. In Chapter 5, I examined what is made possible by the atmospheric comfort in Franck's and Clark's mathematics classrooms, and how their pedagogical practices made that atmospheric comfort possible. In Chapter 6, I heeded what Mr. Montoya's ghost had to say about mathematical inheritances and the present role of the past in making futures. Now, I summarize

the findings from these chapters using Berlant's (2006) relation of cruel optimism, which has previously been applied to investigations of happiness, courage, and empathy in teaching (Boler et al., 2019).

"A relation of cruel optimism exists when something you desire is actually an obstacle to your flourishing... when the object that draws your attachment actively impedes the aim that brought you to it initially," Berlant writes (2011, p. 1). Stengel (2017) argues that "teachers today are stuck in a situation of cruel optimism" (p. 126) due to the exclusion zones we all already live in, where what brings teachers to teaching— whether their love of learning, their commitment to children, their passion for justice, their conviction in education as a public good, or other ideals— is often impossible to sustain given the contemporary conditions of the work of teaching— where neoliberal and technorational logics anesthetize, dehumanize, and marginalize the students and teachers involved. In this context, the findings from Chapters 4-6 suggest that commonly taken-for-granted stances on relational work may be cruel in that pursuing them may actually make their aims impossible.

In Chapter 4, for example, I found that Franck's and Clark's ethical stances on relational work took knowing their students to be an ongoing practice of attention and an ongoing practice of inviting further response. I conjectured that these stances made matter students' and teachers' humanity as complex persons, not just as learners and doers of mathematics, because what mattered was the response-ability of getting to know and not the product of knowing. By contrast, treating the process of getting to know students as an accumulation of factual knowledge to be leveraged for academic learning— as is sometimes implied by literature emphasizing students' and teachers' similarities or recommending finite strategies such as surveys and contextualized lesson plans— is susceptible to both ethical and practical pitfalls.

Treating teaching as being conditional upon knowing students, coercing students into self-disclosure, or ignoring students' complexity and change, for example, may create barriers to students' feeling known, trusting of, and connected to their teachers.

In Chapter 5, I found that Franck's and Clark's ethical stances on relational work expanded possibilities for mathematical and social participation, specifically, and for heterogeneous ways of knowing and being, more broadly. The atmospheric comfort made possible by their pedagogical and relational practices cultivated open normativities, creating opportunities for different responses, some of which Franck and Clark did not expect and which students themselves may not have predicted either. By contrast, establishing and enforcing a particular set of norms— even if those norms are intended to make mathematics more accessible or equalize participation— limits students' opportunities for co-creation. The opportunities that result from explicitly setting equitable norms may be more expansive than those possible in a traditional mathematics classroom where students' participation is rigidly scripted. But, explicit norms nonetheless constrain students' ways of knowing and being and concomitantly constrain what teachers, students, and the infrastructures of mathematics education can create together.

In Chapter 6, I found that Franck's and Clark's ethical stances on relational work greeted mathematical inheritances by creating affective movement that set in place futures that were different from the past. Tuning their reactions and their stances on student participation, as one example, moved students, Franck and Clark themselves, and me through the individual and structural traumas of mathematics education and towards a less violent future. By contrast, focusing solely on preparing students for an idealized future carries the risk of ignoring legacies that participants in mathematics education cannot escape—legacies that continue to haunt their intra-actions. Such a focus also risks treating students as vehicles for the perpetuation of

mathematics education, rather than as complex persons with affects and reactions and dreams of their own. The cruelty of focusing on the future, then— as with the cruelty of seeking to know students and setting equitable norms— makes impossible the nonviolent, restorative, and humanizing relations in mathematics education that represent a step towards living justly.

Calling these teaching truisms cruel is not a move to dismiss them. Ideas like getting to know students, setting equitable norms, and focusing on the future are truisms because they contain truths; thoughtful applications of them can certainly lay the groundwork for meaningful relationships and fulfilling experiences of teaching, and additionally, they are commonsensical, concrete, and provide a level of clarity and purpose that are often difficult to find given the uncertainties of teaching. However, I bring up their cruelty in the spirit of diffraction, or identifying "patterns of difference that make a difference" between existing truisms and the alternative approaches suggested by Franck's and Clark's response-able ethical stances, because these patterns draw attention to the "history of interaction, interference, reinforcement, [and] difference" that constitute the complexities of teaching (Dolphijn & Van der Tuin, 2012, pp. 49, 51). Delving into the ethical consequences of commonsensical notions, which are often overlooked in favor of simplistic slogans, points towards the contribution of *response-ability* as a way of understanding the ethics of relational work in mathematics education.

Response-ability in Mathematics Education

An ethics of response-ability, again, first posits that actors aspire to intra-acting in ways that take a position on who and what matters; given the dehumanizing and violent histories of mathematics education, an ethics of response-ability in mathematics education requires acting in ways that make matter the complex personhood, humanity, and dignity of each student, teacher, and participant. Second, an ethics of response-ability calls for responding in ways that render

others capable of further response. This means that mathematics teachers must wield their institutional authority in ways that expand possibilities for students' (and their own) response and that create conditions under which students (and themselves) are likely to pursue those expanded possibilities while keeping in mind that they are accountable specifically to possibilities for complexity and flourishing— not to all possibilities equally.

An ethics of response-ability, in contrast to theories of ethics driven by fixed rules or universal principles, accounts for the intensely and intimately situated nature of being, as well as for the dynamism and constant intra-action of mathematics students, mathematics teachers, and the infrastructures of mathematics education. It shifts the focus slightly from what is emphasized by theories of teachers' responsibility or responsiveness: teachers' unilateral obligations to students and especially students from historically marginalized backgrounds. Instead, responseability focuses on the mutual and ongoing co-construction of experiences of mathematics education, which enlist not only students and teachers as human actors but also more-than-human actors such as discourses and histories and affects. As a result, an ethics of responseability refrains from laying the burden of justice on teachers and instead, honors the agency that students and teachers (as well as more-than-humans) have in worlding what exists.

Response-ability and Agency

Agency— and specifically, the goal of respecting or even developing student agency— is a popular concept in mathematics education research and practice, especially among those who are concerned with equity and justice. Louie (2019) suggests that within mathematics education research, scholars often define student agency by drawing on three primary traditions. According to the disciplinary engagement literature, students can exercise conceptual agency by making meaning of mathematics and assuming the intellectual authority to produce, not just consume,

knowledge (e.g., Cobb et al., 2009; Sengupta-Irving, 2016). In literature foregrounding mathematics identities, students can develop a sense of self as a competent mathematical person (e.g., Aguirre et al., 2013). And, in work rooted in Freirean pedagogy, students can come to see themselves, through learning mathematics, as subjects capable of transforming the world (e.g., Gutstein, 2007). Although each of these forms of agency can be meaningful, however, teachers seeking to cultivate student agency must also be aware of several common pitfalls.

First, Miller and colleagues (2018) caution that when the concept of student agency is misapplied, "students will be treated as agentive in constructing useful knowledge, only to the extent that they construct expected/canonical knowledge products" (p. 1065). In other words, teachers may attempt to design learning opportunities where students can exercise intellectual authority but constrain the extent to which students' exercises of intellectual authority are validated. Students may not be given choices that are substantively meaningful (e.g., deciding which of two very similar problems to complete), or the choices that students make may be treated as unreasoned or ill-advised. In such cases, what students are offered is a "pseudoagency" (Miller et al., 2018, p. 1065) that can be considered, perhaps, a cruel agency in that, in aiming for student agency, it actually limits the ways in which students are able to exercise agency.

Relatedly, Kayumova and colleagues (2019) argue that popular notions of "empowerment," whereby teachers seek to grant agency or power to otherwise-disenfranchised students, actually uphold deficit-based and colonizing narratives. Such notions position students as passive, incompetent, or in need of "training" or "education," reifying existing hierarchies with teachers at the top and students at the bottom. They also contribute to bootstrapping narratives that claim students can overcome their oppressive circumstances purely by exercising agency, which fail to account for the extensiveness of systemic injustices. These approaches to

student agency are likely to perpetuate rather than subvert the violent relations of mathematics education.

Third, scholars have warned against superficial attempts to upend the teacher-knows-best tradition in mathematics education. Lindquist (2010) notes that "seductions of piety about our 'student-centeredness'" (p. 181) "can become a way of abandoning one's responsibility as an educator" (Ruitenberg, 2015, p. 91). Even Freire (1970/2000), often hailed as a model of student-centered pedagogy, warns that if he were to provide too little direction as a teacher,

I would leave the students by themselves, and it would be to fall into a kind of irresponsibility. At this moment, afraid of assuming authority, I lose authority. Authority is necessary to the educational process as well as necessary to the freedom of the students and my own... if I fall with this kind of irresponsibility, instead of generating freedom, I generate license, and then I don't accomplish my responsibility of teaching. (p. 180)

In contrasting *license*, as a state of unrestricted latitude, with *freedom* as an emancipation or liberation from oppression, Freire notes that freedom in an educational context necessitates responsibility from the teacher. Otherwise, those left entirely unguided are likely to continue, despite their best efforts to resist, existing relations of subjugation. Although student agency may be a worthwhile desire in mathematics education, these scholars' concerns about false agency,

Doing so requires that mathematics teachers take a nontraditional stance to power, neither wielding it authoritatively nor (pretending to) cede it to students entirely. Franck and Clark wrestled with their stances on power, telling stories that demonstrated their uncertainty. Franck, for example, described a teacher who "built an army of kids;" even though he did not know her intentions, and he may have agreed with some of her perspectives, he expressed doubt about

saviors, and license must be heeded as well.

situations where beloved teachers "have so much power over these kids. Sometimes it's not always good... I'm not sure" (*interview*, 3/2019). Clark asked rhetorically how he could push Sofia to participate in ways that he felt could contribute to the overall flourishing of the class, given Sofia's particular strengths, without making her feel as if "there's a deficit in what she's doing or there's anything wrong with what she's doing and that she needs to do more of something that she doesn't really want to do." He said:

I just don't want to be the reason why she does things... I want every kid to be just a little bit influenced by me and a lot influenced by themselves and what they're seeing and thinking and realizing... Some kids maybe they would... make themselves comfortable doing whatever I suggest of them... Obviously then I have to be careful because now my influence is higher, and I don't want to turn them into me. (*interview*, 4/2019).

An ethics of response-ability does not answer the question of how Clark could push or even whether he should, but it does put forward a centering principle for how Clark could engage with Sofia and how Franck could make sense of the power gained by having strong relationships with students: considering whether a particular push, in a particular moment, could render students capable of more expansive response to each other and to mathematics education in ways that account for the past.

Limitations

This is, of course, only one of many stories that could be told from the analyses I conducted or the data I collected, let alone from Franck's and Clark's classrooms or about mathematics teachers' ethical stances on relational work, which are undoubtedly wildly diverse. I have approached this inquiry from the perspective of teachers as one element of an intra-acting phenomenon, even though there are important related inquiries to be taken from students'

perspective, because there are teachers who wonder how to exercise the agency and authority they have—what they can do—within broken systems. The findings here, and an ethics of response-ability, suggests that there are things teachers can do—such as engaging in ongoing practices of inviting further response and cultivating open normativities and attending to mathematical ghosts—while being mindful not to expect particular results from their doing: not even mathematical learning, because although the relational work I describe here takes into account mathematics teachers' responsibilities to students' mathematical learning, it is first and foremost a stance on how to co-exist in complex webs of suffering. Furthermore, I hope this story serves as a reminder that students' agency is not limited to merely assenting to, contesting, or refusing what their teachers offer, because their responses often can and should present something new for the teacher to take up and then respond to.

I imagine this dissertation raises questions about how Franck and Clark are able to take the ethical stances that they take. Although I would oppose the temptation to look for dispositions that teacher education programs might try to select for or to "train," a clearer portrait of contributors to Franck's and Clark's ethical stances could illustrate conditions that make it more or less likely for mathematics teachers to take up an ethics of response-ability. For example, data I have collected but not yet thoroughly analyzed suggests the importance of humility and authenticity in how Franck and Clark approach their teaching, recognizing the limits of their work not because there are still specific pedagogical practices they are trying to master (although they did both talk about specific strategies they wanted to try in the future and described their teaching as not yet being where they wanted) but because they and their work—and any human and any human's work— is inherently limited in its ability to mitigate the suffering that students, teachers, and researchers experience as a result of the violence of

mathematics education. Clark, for example, talked about avoiding a white savior narrative, and Franck discussed how his humanness—his frustration, his anger, his sleep deprivation as a parent of young children—sometimes precluded how he would ideally like to act with students.

Franck and Clark both spoke about authenticity and "genuinity" in not "faking" interactions (Franck, interview, 3/2018), and about the importance of being comfortable with themselves and their choices. Humility and a commitment to authenticity seemed to underscore many of the stances that Franck and Clark took towards their intra-actions with students and mathematics education. I prefer to think of humility and authenticity not as innate or unique characteristics of Franck and Clark, but rather, characteristics of being in right relation with oneself that is made possible by the conditions of their work and lives. Consider, for example, the school and administrative contexts I described in Chapter 3, where Franck and Clark were well-respected and therefore trusted and supported at Banneker, in conjunction with the professional community they experienced at PDO and with other PDO teachers in the Banneker mathematics department, and the external validation of their competence that they received as a result. Had Franck and Clark been beleaguered by the distrust, deprofessionalization, and accountability pressures that assail many mathematics teachers in similar institutional positions (e.g., Horn, 2018; Mehta, 2013; Santoro, 2018), they may have found it more daunting or simply exhausting to consistently open themselves up to students, to questioning what they thought they knew, and to responding to possibilities they had not anticipated. In that sense, although Franck's and Clark's ethical stances on relational work illustrate one way that mathematics teachers can and do respond to living in exclusion zones, the particular exclusion zones they respond to may not resemble the exclusion zones that other mathematics teachers find themselves navigating.

Implications

For teachers and aspiring teachers who want to know how to be in a broken system,

Franck and Clark may serve as an example of how two teachers make sense of and enact what
they feel to be right given who they are, who their students are, and the context they work in.

They create opportunities for students to respond and for themselves to respond to students,
cultivate a "vibe" that expands possibilities, and tune their reactions to account for the influence
of the past in making possible a better future; these are all things that any teacher can do from
wherever they are situated, although the specifics will, of course, vary. Again, I do not claim that
what Franck and Clark do is "best practice," but rather, that what they do may provide fodder for
sparking imaginations and opening up possibilities about the narratives, metaphors, and
pedagogical practices we take for granted in mathematics education.

In that vein, I hope that this dissertation also serves as a reminder of the humanness of teaching despite increasing calls for standardization, routinization, and "science" in teacher education (V. Ellis et al., 2019). I suggest that mathematics education researchers and teacher educators make room for nuance when recommending that teachers get to know their students and set equitable norms. In doing so, we may end up offering similar strategies, but for different reasons and/or with more fully articulated ethical commitments, making possible more learning and mutual vulnerability for us and those who intra-act with our work.

Furthermore, the findings from this dissertation suggest that ethical approaches to relational work in mathematics education must beware instrumentality, whereby relationships are built or practices are implemented purely or primarily for the sake of student learning. Although mathematics teachers are beholden to mathematics education and to students' learning in their institutional roles, they are also accountable to the other people and infrastructures they intra-act

with as humans sharing a troubled planet. What makes relational work ethical in mathematics teaching is not that different from what makes relational work ethical overall, although the specific infrastructures of mathematics education present specific challenges and considerations.

I propose that feminist new material theories, including affect theories, and poststructural methodologies can further our collective understandings of these specific challenges and considerations in mathematics education. Their focus on the corporeal instantiations, contributors, and consequences of societal narratives, structural norms, and historical legacies draws attention to how pasts are made present and how futures can become different from the past, since the material world is where we come into contact with "flows of power" (Stewart, 2007, p. 3). Specifically, entanglement reminds us that we are always already intra-acting, and so there are always pasts to account for and there is always agency in intra-action; neither mathematics teachers, mathematics students, mathematics education research and researchers, nor their experiences of mathematics education "begin" when they enter a particular classroom.

Finally, the findings from this dissertation also suggest that disrupting the slow violence and dehumanizing conditions of mathematics education cannot be achieved solely by changing conceptions of what mathematics is (making mathematics more human) or who we take to be capable of mathematics (treating students more humanely). Even though both of those are critical components, this dissertation highlights the ongoing, daily, response-by-response nature of protesting injustices: micro-contestations, so to speak, of hegemonic forms of mathematics education (e.g., Philip, Olivares-Pasillas, et al., 2016; Sengupta-Irving et al., 2020). This response-by-response engagement in intra-action is a process of opening up (to make more legible, to expand), connecting (to bear witness, to make matter), and reconciling the status quo with a future that is not yet realized (to be moved, to set in place).

Conclusion

In this dissertation, I have offered you a story about two veteran mathematics teachers doing "what we can" from their situatedness within complex webs of suffering. I have illustrated their ethical stances on knowing, open normativities, and the past as approaches to the relational work that they do with students and with infrastructures of mathematics education. Through these findings, I argue that our entanglements— with each other and with mathematics education— are an opportunity to care, as a form of protest, about right relation. Doing so is one way to heed mathematical ghosts and thereby create more just futures: futures in which we render each other capable of responses and, specifically, further responses that take ethical stances on who and what matters. In other words, what we do in mathematics education may not be able to reverse settler colonialism or end racism or protect the planet, but we can tune our reactions and engage in intra-actions to proliferate possibilities for living justly.

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Appendix A

EXCERPTS USED IN STUDENT INTERVIEW REFLECTION INTERVIEW

- "He's cool, easy to talk to. He's very happy usually." "He's a chill person you can talk to." "I see him around, he's always like oh what's up." "He likes talking to kids." "If you talk to him he won't be mean." "He could care about you as a student if you're doing well academically or if you're having a tough day"
- "It's a very open environment, so you'll feel comfortable speaking up, saying your ideas."
 "Usually in math you don't want to take the class, you don't want to be here, but it's good to be here because it doesn't feel like too much work, it feels comfortable, and that feels very important"
- "If you already understand it he tries to help others and tries to get equality in how you learn... he always tries to help however he can." "Most teachers would give up but he tries to come up with new ways and shares it with us so he cares about us and how we learn."
- "So in some situations I do feel like he should be more **serious**, especially when kids are disrespecting him that's kind of serious...It's not directly disrespecting him but feeling like he's not going to say anything to them."
- "So it's pretty fun... he isn't really that serious about things so he lets you, he likes, I don't know. He lets you sort of be you here? As long as you're not disturbing and annoying others, get your work done." "He has a **fun** kind of vibe to him. Here you're not isolated and just thinking about math the whole time. He presents his perspective of what he thinks."

- "I do have trouble finding the answers and I just ask him and he'll help me with anything I need. He's very **patient**, he doesn't rush us, he waits for people to understand, he always asks if anyone needs help" "The way he teaches I can feel it that he cares. He's patient and doesn't rush us and asks if we have questions." "I feel comfortable because he's always there to help us with the work, he's always checking and going around to see if you need help; Let's say if I'm just there at my table just not doing anything, thinking really hard, he'll notice that, and then he'll be walking around to see if we have any progress. And then maybe at first he'll tell our teammates to help and then if we still don't get it he'll help us personally."
- "Then you have a great connection with your teacher. Cuz some teachers you might have them for more than one year and they still don't know your name and he already knows most of our names." "The way he randomly picks on you. The way he **trusts** you." "It makes me feel like he trusts in us to finish our work and be responsible."
- "Every time we do something and he knows that we did it good, he congratulates us and pats us on the back, stuff like that."

Appendix B

STUDENT INTERVIEW GUIDE

- 1. Tell me about your most recent math class (period). What happened, what did you do, what did you say, what did you think, how did you feel... Describe your math class to someone who's not in it.
- 2. What is Mr. like as a teacher? If I had never met him, how would you describe him?
 - a. What makes you say he is ?
 - b. Can you tell me about a (positive/negative) time? How did that experience make you feel?
 - c. Is this typical of teachers at Banneker?
- 3. Do you feel like he (understands / cares about / challenges / likes / trusts you)?
 - a. How do you know?
 - b. Example? Why did that make you feel like he understands you?
- 4. Do you have a story about him you can tel lme?
- 5. *If students say something critical:* Would you feel comfortable telling him that? Why/not?

Examples of follow-up prompts:

- Ask if feelings are reciprocal (e.g., you said he trusts you; do you trust him?)
- Do you think other students experience him the way you do?
- How does it feel to be doing easy work?
- One thing I felt the other day was X. Has this ever happened to you?

Table B1

Interviewed Students

Name	Teacher	Date
Adriana, Arielle, and	Clark	12.03.2018
Barbara (together)		
Amy	Franck	12.04.2018
Claudia	Clark	02.06.2019
Courtney	Franck	03.27.2019
Dylan	Franck	01.07.2019
Jonathan	Clark	02.06.2019
Lucas	Franck	03.29.2019
Lucina	Clark	02.06.2019
Luisa	Clark	11.30.2018
Malik	Clark	02.06.2019
Nadia	Franck	03.27.2019
Rafael	Franck	11.30.2018
Rosa	Franck	04.29.2019
Thomas	Clark	01.09.2019
Valeria	Franck	12.03.2018
Zoe	Franck	05.01.2019

Appendix C

EXCERPTS FROM ANALYTIC MEMO IN THE "HOSPITALITY" ANALYSIS

... My read of Derrida (see 6/19 call notes) raised the notion of hospitality as an aspiration and an ongoing ethical charge, but also of the host always having power over the guest and needing to have that power in order to set boundaries and especially if one is to safeguard home as refuge (if hospitality is unconditional, then both a persecutor and the persecuted would have equal claim to hospitality). Traditional views of teaching/schooling might suggest that the teacher is the host and the students are the guests, and teachers are the ones who place conditions on a student's welcome and also police the boundaries to ensure that students are safe (which maintains the teacher's role as authority/power).

What's different about what Clark is suggesting in his email chain is that he centers the student's space or the student's "home" and attends to whether he is invited into it, not whether they are invited into his space or his home.

... Clark seems to be referring both a physical space (like the cubic feet of "personal space" near a student, e.g. Mark with lemonade on his front porch) and intellectual space (like helping or pressing with a problem, e.g. Victoria being willing to accept that for some problems but not others). In addition to what Clark can do in terms of setting rules for the community and making sure someone isn't "setting up huge homes with sprawling lawns and racing their Mercedes through the streets" (extending the analogy to being a developer in charge of a new community), the student also has the autonomy and authority and power to put up walls/barriers

to keep out assaults or invasions that they're protecting themselves from, some of which might come from the teacher.

... As Mayo writes, home is sometimes home because it's not someone else's home—
others (people and ideas) have been excluded from the boundaries, which is what makes home
feel familiar and comforting. Her call to "give up home in order to engage in difficult relations"
is a call to engage in the ethics of hospitality and of hosting—even though doing so still doesn't
erase the power dynamic between host and guest—rather than sequestering oneself. It seems like
Clark does this when he acknowledges students' space and students' homes (he is giving up what
traditional views of teaching would consider HIS home in order to allow for their homes) and
part of doing so means that he's open to the classroom space changing (e.g. his tentativeness
around introducing whiteboards at the beginning of the year, having different "vibes" in different
class periods) per Shirazi's quote citing Ruitenberg: hospitality, in contrast to inclusion, stipulates
that "the arrival of the guest may change the space into which the guest is received" (p. 99).

... I think this challenges conventional notions of belonging as unconditional belonging, which makes me think of @mrs_jcupp's tweet about being valued as being bigger than belonging; I can appreciate and feel valued in a space even if I don't necessarily belong there, because I don't need to belong in every single space I'm in (e.g. when Hutchinson writes about being a white faculty member excluded from PoC affinity spaces; she doesn't need to belong everywhere and that doesn't mean she's not valuable).

... Ruitenberg defines Derrida's hospitality as "a gift given by a host who is aware of their indebtedness to the guest" in contrast to "conceptions of hospitality based on reciprocity or exchange, in which the guest incurs a debt by accepting hospitality." This feels important because of how it shifts the lens through which one thinks about power, and inverts conventional

ways of thinking about the directionality of obligation—after all, one cannot be a teacher without students (this parallels Clark's inversion of space belonging to students who can choose whether or not to invite him in, rather than the physical or intellectual space belonging to him as a teacher who then invites students in).

... Wondering if I can contrast (or bring into conversation) the idea of an ethic of hospitality with Noddings' ethic of care (while bringing in other care theorists, especially critical care) and Butler's ethic of accountability (per the analysis I did with Franck for the advisory board meeting). What do these ethical/poststructural perspectives offer us in understanding relationality/relational work/"right relation" that other perspectives (e.g. traditional psychological approaches, humanizing approaches, culturally responsive/sustaining approaches, sociocultural approaches, math ed/complex instruction/status approaches...) do not?

Appendix D

"BUILDING RELATIONSHIPS" ANALYSIS: LISTENING GUIDE EXCERPTS

In this Appendix, I provide examples of each step of the listening guide analysis using Franck's second Building Relationships Interview. For the sake of space, each section is abbreviated to include just enough examples to illustrate how the analytic method was used and does not contain the full list of stories, voices, etc. identified within each step. I have lightly edited the original analytic text for readability and context.

Step 1: Listening For Plot

Stories:

- Student who "needed a dad" in his first year of teaching, in contrast to student "who didn't need me"
- Student who babysat at his child's birthday party
- Teacher who was manipulative although maybe with good intentions

Places where the voice falls off or shifts:

• This transcript feels pretty consistent and pretty confident in the recitation of stories about former students that all illustrate the same concept: I respond to their needs... there's a simplicity to this narrative, belied in part by the "magic" of some relationships just "clicking" (those are my words)... the one exception is "sometimes it's hurtful when you realize they don't like you." This paragraph seems more vulnerable and interrupts the confident recitation of students who either demonstrated strong interest in a relationship

and Franck found ways to reciprocate, or students who simply were not interested and Franck was fine with that.

Omissions and repetitions:

- One thing that could be in here but isn't is the idea of students wanting more than he can provide; he acknowledges "I don't have a lot of time" (and again in contrast to being 26 when he didn't "have a family at home") but other than that doesn't tell any stories that would suggest regret or a sense of insufficiency.
- Things that are repeated a lot are examples of students who volunteer to do whatever (tutor, babysit, check supplies), examples of students who became teachers, and animations of his own voice both for things he would say and wouldn't say (e.g. "hey how can I help you" or "hey let's be friends")

Reflexive account:

- This reminds me of the non-school settings in which I've seen Franck interact: with the nurse in the elevator at his doctor's appointment, with the worker at the boba shop, etc. It feels true to what I hear him say about enjoying human interactions. This is a way in which we are different; when I was a teacher, I tended to let students take the lead on the closeness of relationship they wanted rather than repeatedly seeking out even the students who did not appear interested in a closer relationship.
- I'm struck by a respect and humility (where was I just reading about humility? Most likely in relation to "teaching with ignorance" and the "unknowability of the other," I think) I find really beautiful in the "following students' lead" and not "forcing them to like me," or an honoring of student agency in a way that doesn't press students into a relation they're uncomfortable with— which strikes me as being akin to how he won't

cold call on a shy student and takes that student into consideration when planning, and different from savior teachers who know what's best and right for their students including how their students should feel about class and learning and their teacher.

Step 2: Creating an "I" Poem

"I" poem:

I was in my first year of teaching...
I felt like, okay,
I don't have a family at home,
I'll go to your cross country meet and watch it.
I'll take you home.
I was able to do that,
I care about [student name redacted]
I saw that she needed a dad.
I'm okay with that, right?
And I treat her like a child.

"You" poem (**bold: to another teacher**; gray: to me, Grace; *italics: to student*; <u>underline:</u> <u>student voice</u>; plain text: you as a proxy for me, Franck):

a kid doesn't want you to be their parent

You need a ride home? I'll take you home.
Why are you cutting your classes?
Let me talk to you about it.
[student name redacted] was like, "Yes, To, will you come to my thing?
Hey, can I talk to you?
Hey, will you represent my parent at ... "
And you invest yourself into them.

it's like ... you know. how can I help you?

Analytic note from "I" and "you" poems:

• This also makes me think of the tension between the nice houses and rocket scientist salaries he wants to have vs. the work he's choosing to do, saying it has to be "worth it,"

and suggests that he finds meaning in the relationships he builds with his students even though he's not actively seeking them out—it just feels like part of his responsibility (that presumably would not be part of a rocket scientist's responsibility).

Step 3: Contrapuntal Voices

Franck sets himself against:

- 1) his former self, who would drive students home, text them, take them out to eat...
- 2) teachers who are manipulative/thrive on power
- 3) teachers who try to force a relationship
- 4) teachers who are only about learning and then going home
- 5) teachers who do TFA for two years and find themselves and then leave

Step 4: Composing an Analysis (sample paragraphs)

Franck seems to see relational work as a process that starts with him laying a substrate for engagement. He describes opening up about himself and his family to show students that he cares about more than just teaching mathematics and creating opportunities for students to communicate who they are beyond students of mathematics: questionnaires, asking how their other classes are going, opening up his classroom for students to hang out in. He reads how students respond to this substrate; some give him deep answers and ask him questions, while others "shut [him] off," which he takes as an indication that they are not interested in a relationship beyond teaching and learning mathematics. Franck describes this process matter-of-factly and with a voice of conviction, as if he is giving me instructions on how to do it: he represents the process as an orderly sequence of "stages;" uses prescriptive second-person

language like "you should," "you have to" and "you need to;" provides copious examples of the type of questions he asks students, using assertive language like "I'd be like" and using (sometimes implicit) conditional statements like "[if] I see... I'll show you" and "if I asked... and you answered a certain way... then...;" and offers illustrative proofs-by-example of specific students and how they engage with him outside of just teaching and learning mathematics (e.g., a student who volunteers to babysit at his child's birthday party; a student who stops by after school to ask if he needs any help in his classroom).

Laying the substrate and then responding to how students respond is part of his "teacherly responsibility," and it is an ongoing process that is both work—he talks about exerting effort ("you have to..." "you gotta...")—and not work—he repeats that the process is "organic," repeats that it takes the "right timing, the right needs, and the right personalities," and gives the analogy of being "like a marriage," which suggests (at least to me) that there is both an instinctive, affective component to relational work and a deliberate, intentional component. This description strikes me as being akin to a bat constantly emitting chirps to echolocate; even when a student "shuts [him] off," Franck will "keep on asking [them] questions" so that "there's a lot of opportunities" for them to engage. If he "consistently see[s] it not happening," however, Franck says, "Okay, I get it, I'm not going to go overboard."

Franck's constant sensing of the extent to which a student wants to engage seems to respect students' agency in setting the terms of their relationship, and to respect the dynamism of a relationship in that he does not express disappointment or betrayal or resignation when a student who has previously been eager to engage becomes less so...

Appendix E

THE "ROSTER" ANALYSIS CODEBOOK EXCERPTS

Table E1What Is In The File Folder

Code	Examples	References
Characteristic	Nice; funny; outgoing	74
Math participation	"not afraid to ask questions"; "he'll do notice and wonders"; "she shows up and tries every day"	72
Math abilities	"she's probably going to end up with a C if she keeps on doing what she's doing"; "in his haste to do it he makes mistakes"; "definitely one of the better mathematicians in the class"	70
How they interact with peers	"she actually took Javier under her wing"; "it's hard for him to argue with a girl"; "there are kids who are good at sensing how other kids are she doesn't judge them"; "he won't back down, so if you're butting heads in a group over math"	55
General academic abilities	"he failed most of his classes"; "she's valedictorian"; "pretty good student across the board"	30
Backstory	history, context, something a teacher might not know from simply having this student in class	29
Their experience in my class	"he's probably not learning much math"; "it's instilling the confidence that she lost along the way"; "seems like he's having a good time"; "I think she likes being in my class"	27
One-on-one interaction with teacher	"used to show up to my room at lunch"; "I told her sister"; "during my fourth period I always walk in [to her science class] and she's always like hi"; "will stay at lunch sometimes to talk about things"	22
Family	"I knew his sister, she graduated a few years ago"; "I spoke to her mom a couple times"; "her cousins were great students in my class for multiple years"; "he's the youngest in the family"	18
Extracurricular interests	"is into shoes"; "likes to draw"; "likes video games"; "she always talks about the music she likes"	17
Who they hang out with	"now he's hanging out with some other girls from my third period"; "was with Oliver for a while now she's with another boy"; "is good friends with Cynthia"; "always wants to sit next to David"	12

Code	Examples	References
Social status	"pretty popular with the girls"; "too cool for school but he's too school for cool"; "socially awkward and not very well accepted"	11
Math confidence	"her lack of confidence"; "confidence in mathematics has really come back"; "thinks fairly highly of herself, has a pretty good identity around that"	9
General participation	"will ask me questions that are most of the time irrelevant to math but he wants to be involved"; "notice and wonder gave him opportunities to express himself"; "when he jokes it's real surprising to me"	8
Source of motivation	"pressure at home from his sister"; "not being able to consistently challenge her makes it harder to keep her engaged"	4

Table E2 *Most Frequently Referenced Codes by Teacher*

Franck		Clark			
Code	References	Students	Code	References	Students
Math abilities	52	95%	Characteristics	38	100%
Math participation	50	95%	Math participation	22	100%
How they interact with peers	43	80%	Math abilities	17	89%
Characteristics	36	80%	How they interact with peers	12	67%

Table E3 *Most Different Codes Between Teachers*

Code	Franck		Clark	
	References	Students	References	Students
Social status	11	40%	0	0%
Family	17	45%	1	11%
Backstory	27	60%	2	27%
General academic abilities	27	55%	3	33%
Extracurricular interests	14	45%	3	22%

Appendix F

"THAT'S ENOUGH, NOAH" VIGNETTE

From the first day of Algebra 1, Noah stood out as a student who appeared to have a lot of energy. His eagerness to engage with everything Clark said, even comments to which Clark expected no response, meant that his voice often and loudly rang through the classroom. Clark sprinkled his speech with jokes, and although most students seemed to either miss or ignore these jokes, Noah acted as if he enjoyed them deeply. For example, after asking a student to explain why addition did not make sense as a strategy for thinking about a particular problem, Clark said, "Okay, good. That's helpful. Anyone want to add to that? I don't mean ADD to that, but..." As Clark invited further contributions about the inappropriateness of addition in this problem, Noah was the only student who laughed loudly.

Often, when Noah spoke, what he said was funny but directly responding to what he said would turn the conversation away from the purported focus of class. His commentary felt particularly salient because students like Luisa, another student who talked often and loudly in class, regularly engaged with the substance of Noah's comments. As an observer, I often found myself smiling behind my laptop while also feeling secretly glad that I was not the teacher who had to figure out how to respond publicly to Noah. From the beginning, Clark acknowledged Noah's enthusiasm and communicated that he appreciated Noah as a person, although I wondered whether he truly appreciated the frequency of Noah's contributions. On the second day of school, Clark and Noah had this exchange:

Clark asked Angela to explain her problem solution to the whole class, and then expressed excitement over her creativity.

Noah, loudly enough for the whole class to hear: But I got-- wh-- whoaaaaaa--

Clark, to the whole class: And I don't even know the answer anymore! Now that's your homework. I'm kidding, but do you get [what Angela suggested]?

Noah: It's simple! It's going to take them less than 8 minutes. I mean, think about it--Clark interrupted him: First of all, I DID think about it. And it won't be LESS than 8--

Noah interrupted him: I never said LESS, but it's going to take about 8 minutes.

Other students jumped into the conversation, and Clark paused to listen.

Later, Clark asked students to write down what they noticed and wondered about a photograph of a large ship that had many birds flying over it and a large orca in the water underneath it. Noah immediately called out, "what's the orca doing there?" and Clark responded, "that's a great wonder; go ahead and write that down on your paper." After several seconds, during which the room was mostly silent aside from the sound of pencils scratching on paper, Clark said quietly, but loudly enough for the whole class to hear, "So like Noah said, your notice can be 'I notice there's an orca' and your wonder can be 'I wonder what the orca is doing there'."

Clark gave students a minute to finish writing, and then a few minutes to share what they had written with their groups. Then, he asked for volunteers to share with the whole class. Noah was the fourth student to raise his hand and share. He took a deep breath and said, among a long list of other things, "It's a fishing boat! Come on, the birds wouldn't just surround the boat for no reason; it's a fishing boat!" Luisa, who was sitting across from Noah, interrupted him and said, loudly, that he should not make assumptions. Luisa and Noah bantered back and forth in a conversation about assumptions that attracted the class's attention while distracting them from

what the teacher had asked. After a few exchanges, Luisa said, excitedly, "My boy! You can't judge the picture without knowing the whole story! Maybe it's an Arctic cruise!"

Clark, loudly: Quiet please!

Noah continued to talk, but in a slightly more subdued voice.

Clark: Noah, I said quiet please.

Noah stopped talking. Clark paused for a beat, and then said, "Give yourself a point," referring to their classwide system of students tracking how they themselves contributed to class either publicly in their small groups. "Luisa, sometimes it is a cruise, but this time it really is a fishing boat." He paused for another beat. "Luisa, don't yell 'my boy'." Some other students chuckled.

Clark: I'm going to show you guys--

Noah, interrupting Clark: What about the--

Clark, interrupting Noah: Okay, that's enough Noah. I appreciate it. You're the best.

Noah, quietly: Thanks.

Although Noah and Luisa were arguing about the photograph they were supposed to be discussing, Clark felt that their argument did not exemplify the type of classroom interaction he wanted in the moment. Clark exerted his authority as a teacher to interrupt the interaction, instructed Noah to give himself a point, acknowledged Luisa's concern, clarified the nature of the boat in the photograph, addressed her use of 'my boy' almost as an afterthought, and then publicly expressed his appreciation for Noah's enthusiasm while also telling him to stop.

From my seat at the next group of desks, I was struck by how many other ways this interaction could have gone— and would have gone in many of the classrooms I have observed.

Clark publicly calling out Noah for continuing to respond to Luisa after he asked for quiet could

have felt harsher and more like a disciplinary measure, but Clark followed the public callout with a "point" that recognized Noah's contribution to the class discussion. Reprimanding Luisa's choice of words could have felt like policing her language or attacking her contribution, but instead communicated Clark's distaste for that particular phrasing while creating a lighthearted moment. Noah could have shut down, but he tried to participate again, and his "thanks" when Clark asked him to be quiet again seemed like an acknowledgment of Clark's attention and an agreement to the rules. Instead, however, when Clark cut off Noah's impending question, his tone of voice (to my ears) did not sound frustrated with Noah or disappointed in him, and he immediately followed up with an acknowledgment ("I appreciate it. You're the best.") that felt, to me, like a way of reassuring Noah that Noah was still a valued member of the class even if some of his behaviors were not desirable in the moment.

The next day, Clark appeared to grow more comfortable constraining Noah's participation in a good-humored way. He asked students to notice and wonder about an image of a surfer being chased by what appears to be an extremely large shark, and then asked them to compare it to another image showing a diver next to a much smaller shark, with the diver marked as being 5 feet tall and the shark marked as being 20 feet long. Clark asked the class for reactions. Several students shouted out responses without raising their hands, and Clark seemed to be okay with this form of interaction; he repeated several students' statements.

Noah, also calling out: Think about it! Think about it!

Clark: You keep telling me to think about it and I already have. Julio?

Julio shares a factoid about the size of blue whales as a point of comparison.

Clark: Noah?

Noah offers several rapid, wordy sentences about how something the size of that shark "can't just eat algae."

Clark: Your argument isn't really meshing, but that's okay I appreciate you anyway.

Anyone else want to tell me why they think the shark is bigger than 75 feet?

Noah: What if it's just our imagination?

Clark: Okay, stop. That's enough. [slightly louder] Anyone else?

Later, Clark and I discussed how other teachers set behavioral expectations in their classes more explicitly than he did. Clark told me about another Banneker teacher using an activity where students are asked to "list 5 things that annoy [the teacher]," and said that nothing really annoyed him. "I like all the kids," he said; "sometimes they do things that aren't going to help the class, but it doesn't annoy me... kids are funny. It's just that I also know what will go better for the class." Rather than having classroom rules and expectations— such as that students must always raise their hands before speaking— that are universally applied and that then need to be enforced to be effective, Clark was willing to tolerate Noah's enthusiasm in some instances and not others, based on whether Clark perceived his comment to be contributing to or interfering with a particular whole-class moment.

Clark's reactions to Noah, and his sensemaking about his approach to behavior that other teachers might consider "annoying," divorced student behavior from a teacher's personal preference and demonstrated clarity on the purpose of constraining behavior. The absence of explicit rules mitigated the opportunity for teachers' responses to students to be experienced as arbitrary, as when teachers inevitably allow some students to call out in some circumstances while insisting that others raise their hands before speaking. Reacting in this way also created opportunities for Clark to publicly appreciate Noah's way of participating, which normalized

first, Noah's enthusiasm as a welcome contribution, and second, that Clark would use his authority as a teacher to ensure that Noah's enthusiasm did not impose on other students' contributing. In doing so, Clark opened up possibilities for Noah's behavior, in contrast to a class where Noah may have had to sit on his hands or ration his participation tokens to share ideas. Simultaneously, he communicated to other students that they are, to quote Clark, "allowed to be themselves, and express themselves, and be comfortable," thereby opening possibilities for student flourishing that would not be present in a classroom where the teacher enforced stricter constraints on what constituted appropriate student behavior.

Appendix G

THE BRIDGE PROBLEM

Name: Date: Period:

The Bridge: Disney Version

(adapted from Nigel Coldwell - "Crossing the Bridge")

One afternoon this summer] Clark, Franck, Abigail, and Veronica went hiking in the mountains. They left from the parking lot, crossed a rickety bridge over a deep canyon, and hiked off into the mountains. The bridge was pretty scary. It was missing sections and could only hold two of the hikers at a time.



Miles into the backcountry they were attacked by Brer Fox and Brer Bear tied them up with ropes and left them in an abandoned shed. Veronica was able to get loose from the ropes and untied the rest of the crew. They started heading back to the parking lot, unsure of where the the fox and bear might be. Franck had a bite on his leg from Brer Fox and Abigail had been hit on the head by Brer Bear's club. They moved slowly and soon they could see the fox and bear behind them across the valley.

They made it back to the bridge. By then it was almost completely dark. Clark estimated that if they could get everyone safely across the bridge in *less than 20 minutes* they could get back to the parking lot and escape.



The bridge could only hold 2 of them at a time but now there was an extra complication. It was dark and they only had one flashlight. It was too risky to cross without a flashlight. Could they all make it across in time?!