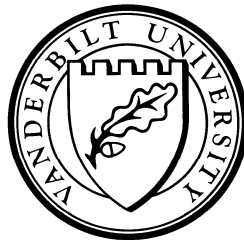


LIVING WITH A MONETARY SYSTEM INFECTED BY BUBBLES

by

Benjamin Eden



Working Paper No. 11-W19

September 2011

DEPARTMENT OF ECONOMICS
VANDERBILT UNIVERSITY
NASHVILLE, TN 37235

www.vanderbilt.edu/econ

LIVING WITH A MONETARY SYSTEM INFECTED BY BUBBLES

Benjamin Eden

Vanderbilt University

September 2011

I study the real effects of bubbles in a price-setting environment. Bubbles cause price dispersion and overinvestment in assets that are overvalued. And when they pop some goods are not sold and capacity is not fully utilized. I argue that a government monopoly on the creation of bubble assets is desirable but may be difficult to achieve. A non-linear tax on capital gains and a “high” interest rate policy can play a role in protecting the government’s monopoly on the creation of bubble assets.

Key Words: Bubbles, Money, Money Substitutes.

JEL codes: E31, E32, E42, E52.

1. INTRODUCTION

The recent financial crisis has led to a renewed interest in bubbles and their effect on the economy. There are many open questions. What are the real effects of bubbles? What regulations are required to mitigate the phenomena? Should the government or the central bank attempt to pop bubbles once they are identified? Should the government bailout assets that pop? Can a low interest rate policy lead to bubbles?

Here I discuss these questions in a price-setting environment. I also discuss the fundamental question of who can “over charge” for an asset he owns. To answer this question I use an insight in Tirole (1985) who studies the possibility that bubbles may emerge in overlapping generations models.¹ At the end of his seminal paper he makes the following observation: “In a sense I have been considering the demand for bubbles. The supply is virtually unlimited. For example I am always willing to pretend that a drawing I made when I was young is worth \$1000, say. However I doubt I will be successful in convincing others that they should invest in it. If I were famous, I might be able to do so.” (page 1093). Here I follow Tirole in assuming that only assets that are widely known can be overpriced. Otherwise, there is room for asymmetric information that may lead to a market for lemons problem.

Bubble assets that can pop cause price dispersion and may cause overinvestment in assets that are partially backed. The effect of pops is similar to the effect of a negative money shock. It is modeled here along the lines in Eden (1994) who uses a version of the Prescott (1975) hotels model. In the Prescott model, there is uncertainty about aggregate demand and typically not all goods are sold. Sellers in the model are indifferent between

¹ The question in a Walrasian setting is whether a Walrasian auctioneer can announce prices that are not strongly correlated with fundamentals but nevertheless clear markets. In their well-known working paper, Blanchard and Watson (1982) answer this question in the positive. Santos and Woodford (1997) show that it is more difficult to get bubbles when we get closer to an economy with infinitely lived agents because in such economies individuals will not want to hold the accumulated bubble wealth when it becomes large. This is less of a problem in an overlapping generations model with agents that have no bequest motive. Jovanovic (2007) argues that bubbles in prices of exhaustable resources are possible.

posting a low price with a relatively high probability of making a sale to posting a high price. Demand uncertainty in the model is “bad” for two reasons: It leads to price dispersion and in the case of low demand it leads to waste (some goods are not sold and capacity is not fully utilized). Since bubbles cause uncertainty about demand, they cause price dispersion and reduce welfare even in periods in which they do not pop. The loss of welfare in the periods in which bubbles pop is greater because of waste.

I make the analogy between fiat money and pure bubble assets and between commodity money and bubble assets with some fundamentals. The main difference between the two is that in the latter case, when the bubble asset is partially backed, we get overproduction of the underlying capital asset as an additional welfare cost.

The paper has three main sections. I start with an economy in which there is only one government created bubble asset: High-powered money. Unlike more conventional models, here money may pop, because the young lose faith in it and are no longer willing to accept it as payment for goods. When money pops, goods are not sold and capacity is not fully utilized. The optimal interest rate is increasing in the popping probability because the incentives to work are affected when the payment is done in terms of an asset that may pop.

The second part of the paper allows for privately created bubble assets with no fundamentals. I assume that to create a bubble asset one must commit to a low risk of asymmetric information: When the asset pops everyone has to know about it at the same time. Not all agents have access to the same commitment/transmission technology. In terms of Tirole’s metaphor, some agents are more “famous” than others. I assume that the government is the “best known” agent and has the “best” technology that allows a commitment to the lowest popping probability. Under this assumption, a government monopoly on the creation of bubble assets is a good idea. But implementation may be difficult if bubble assets are backed by some fundamentals. This case is analyzed in the third part of the paper.

2. A GOVERNMENT CREATED BUBBLE ASSET

I use an overlapping generations (OG) model in which agents live for two periods, work in the first and consume in the second. The utility function of the representative agent born at time t is: $\beta c_{t+1} - v(L_t)$, where L_t is the amount of labor he supplies in the first period of his life, c_{t+1} is his consumption in the second period of his life and $\beta > 0$ can be interpreted as a parameter that determines time preference or the value of leisure.

Money (or government bonds) is the only asset. The government pays interest on money financed by a lump sum tax of $-g$ units on old agents. The gross interest is a policy variable: $R = 1 + r$.

In the steady state the representative old agent holds M dollars before interest and tax payments. Money works with probability π . In the case of panic that occurs with probability $1 - \pi$, no one wants to accept money, there is no trade and the output produced by the young is wasted.

The panic is self-fulfilling. After the panic the money that the young refused to accept is indeed worthless. Eventually, a new government is elected and issue new money. The new money is not better than the old one. It can also pop with probability π . But for some reason that will not be modeled here, the agents have faith in the new money and not in the old one.²

We may distinguish between two cases. In the first the government reacts immediately after the old money pops and gives the new money to the current old. In this case, popping is neutral. The second case that will be analyzed here is when the government cannot react immediately and gives the new money in the next period to the

² Alternatively, we may assume that the loss of confidence lasts for some time and then confidence in money is restored.

next period's old agents. The market for goods opens only when money works. The sequence of events is in Figure 1.

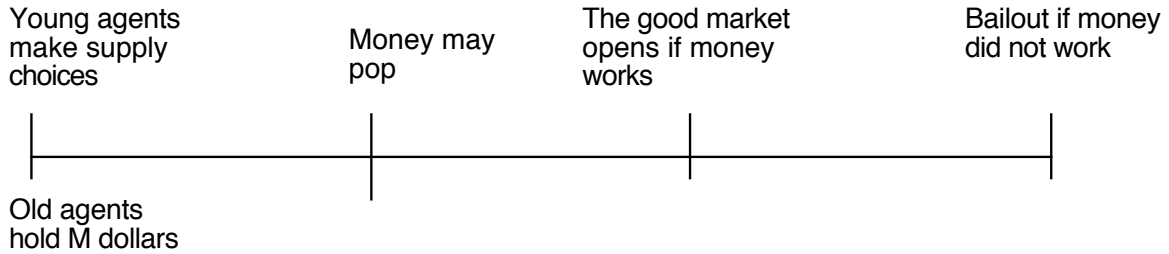


Figure 1: The sequence of events within the period

The dollar price of the good if the market opens is p (and in the steady state it does not change over time). I use $z = \pi^R/p$ to denote the expected purchasing power of a dollar held at the beginning of the period.

At the end of the period the young agent will have pL dollars in the no panic state. In the panic state, he will get a transfer of M dollars. The young agent chooses the amount of labor by solving:

$$(1) \quad \max_L \beta(\pi p L z + (1 - \pi) M z) - v(L)$$

The first order condition for an interior solution to this problem is:

$$(2) \quad \beta \pi p z = \beta \pi^2 R = v'(L)$$

These equations say that the marginal cost should equal the (expected discounted) real wage $\beta \pi^2 R$. The intuition for the π^2 term in the real wage is as follows. The young agent invests effort and will reap the benefits if he sells (if money works in the current period) and if money works when he is old. The probability that this joint event will occur is π^2 and therefore the real wage is $\beta \pi^2 R$.

Market clearing requires:

$$(3) \quad pL = M; \quad g = -rM$$

The first equation is the clearing of the goods market, when it opens. The second is a balanced budget requirement. Equilibrium outcome is described by the first order condition (2).

Average capacity utilization (ACU) is the ratio between expected consumption and output. In equilibrium: $ACU = \pi$. A social planner that can choose $ACU = 1$ may achieve the first best by solving: $\max_L \beta L - v(L)$. The Fed cannot attain the first best because it must use imperfect money that may pop. I therefore consider the second best problem of a less powerful social planner that takes $ACU = \pi$ as given and chooses labor by solving:

$$(4) \quad \max_L \pi \beta L - v(L)$$

The first order condition for the planner's problem is:

$$(5) \quad v'(L) = \pi \beta$$

The equilibrium outcome (2) coincides with the planner's solution (5) if:

$$(6) \quad R = \frac{1}{\pi}$$

Note that when $\pi < 1$, the optimal interest rate is different from the rate of population growth advocated by Samuelson (1958) and Diamond (1965).³ To understand the reason for the departure from the standard recommendation note that when $R = 1$, there is a difference between the return to effort from the social and the individual's point of view. From the social point of view a unit produced will be consumed if money works in the current period and therefore the social benefits from producing a unit occurs with probability π . From the individual's point of view the benefits from a unit produced occurs only if money works in both periods, with probability π^2 . When $R = 1$ and $\pi < 1$ there is thus a discrepancy between the social and the private point of views. When $R = \frac{1}{\pi}$ the real wage is at the optimal level and the social and the individual points of view coincide.

A taste shock interpretation: In the previous section money may not work because young agents may loose trust for no apparent reason. I now describe the case in which money may not work because old agents may choose not to consume.

I assume that the utility function of the representative agent is $\theta_{t+1}\beta c_{t+1} - v(L_t)$ where θ_{t+1} is an *iid* random variable that takes the realization 1 with probability π and zero otherwise. When the old agents do not want to consume they give their money holding as an accidental bequest to the young agents. The young agents' maximization problem is given by (1) and the analysis goes through leading to (6).

Thus, money may not work because the old may hoard it or because the young may loose trust. The model and the policy implications are the same in both cases.

³ In Friedman (1969), there is no interest on money and the rate of return on money is determined by the steady state rate of inflation. Unlike the result in Friedman, here the optimal rate of return on money does not depend on β .

The bailout issue: Should the government bailout the old? The answer here is a trivial yes. And the sooner the government can restore money by bailout - the better. The issue is not trivial when the private sector can also create bubble assets.

3. PRIVATELY CREATED FIAT BUBBLE ASSETS

I start with the case of privately created pure bubble assets that like fiat money have no fundamentals. These are a special and simpler case of the “overvalued assets” discussed in the next section. They may therefore serve as an introduction.

As in Blanchard (1979), bubble assets may pop. When a bubble pops, there may be asymmetric information and a market for lemons problem as in Akerlof (1970). I therefore require that when a bubble pops everyone will know about it at the same time. The bubble asset must therefore be “well known”. I assume that the creator of a bubble asset must also be “well known” and must have a special attribute called “fame”.⁴

Not everyone can be “famous”. At each point in time the number of agents (firms or individuals) who can have the special attribute (“fame”) is constant. After an asset pops, the agent who created it loses his “fame” but another agent takes his place and becomes “famous”. The agent with the newly acquired “fame” creates a new bubble asset.

This is modeled by assuming a fixed number of slots that can be used to create bubble assets. Whenever a bubble asset pops the slot becomes vacant and after some time, another agent\asset occupies it. For example, typically there are only few international currencies. The main international currencies used to be gold and silver

⁴ This is related to Stein (2010) who assumes that assets that are entirely riskless yield utility because they are easy to evaluate. Unlike Stein, here assets do not enter the utility function and are “easy to evaluate” even when they pop.

(with gold being more prominent). It is now the dollar and the euro (with the dollar being more prominent) and there are talks that this may change soon.⁵

I model the special attribute (“fame”, “credibility”, “status”) as a commitment and transmission technology. A “famous” agent can commit to the probability distribution of the rate of return on the asset that he creates and can transmit information about it to all agents. He can also commit to low risk of asymmetric information: When the bubble pops everyone will know about it at the same time.

In the real world, agents invest resources in acquiring “fame”. A firm may do it by investing in buildings, advertisement and overcapacity. The attribute “fame” can also be acquired by securitization: A security backed by many assets maybe easier to evaluate than the underline individual assets.⁶ Here I simplify and assume that the commitment\transmission technology is allocated by a lottery and a firm that wins the lottery gets it at no cost. I assume that the government has the best technology and can commit to the highest survival probability. For simplicity and unlike the assumption in the previous section, I assume that the survival probability of the government’s bubble is 1. Private agents (firms) can commit only to lower survival probabilities.

I assume n status slots indexed by $i = 1, \dots, n$. The government occupies the first slot. Private firms occupy slots $i > 1$. The survival of assets is determined by the number of “sun spots” that occur in the period. The probability that there will be $1 < s \leq n$ “sun spots” is Π_s . All the assets indexed $i > s$ pop and all the assets indexed $i \leq s$ survive when the state (the number of “sun spots”) is s . I use $q_i = \sum_{s \geq i} \Pi_s$ to denote the

⁵ There are other areas in which the number of “slots” is more or less constant and the identity of the occupier of the slot changes over time. In macroeconomics there are two main “slots”: Keynesian and Classical. New Keynesian replaced old Keynesian and real business cycle replaced monetarism. Also in religion there seems to be a relatively constant number of “slots”. Christianity replaced the pagan religion of the Greek and the Roman while Islam replaced the pagan religion of the Arabs.

⁶ It is often argued that it was very difficult to evaluate mortgage-backed securities. But this is still much easier than the evaluation of individual mortgages. Here I assume that everyone was aware of the risk associated with buying the securities. This is of course a modeling device.

probability that asset i will survive and rank slots by the survival probabilities:

$$1 = q_1 > q_2, \dots, > q_n > 0.$$

The occupier of slot $i > 1$ announces the nominal gross rate of growth in the survival state: g_i . When he announces a rate of growth $g_i > R/q_i$, his bubble asset will dominate money and agents will want to exchange their entire money holdings for his asset. I assume that the government follows a policy that insures valued money and does not allow the crowding out of money by private assets. This policy can be achieved for example, by imposing reserve requirements on assets that threaten to crowd out money⁷. As a result, the creator of the bubble operates under the constraint $q_i g_i \leq R$ and announces:

$$(7) \quad G_i = R/q_i = \arg \max q_i g_i \text{ s.t. } q_i g_i \leq R.$$

After a bubble asset $i > 1$ pops the value of the asset drops to zero and the value of the new firm that occupies the slot is I_i , where I_i is arbitrarily small (later it will be treated as zero). The price (dollar value) of asset $i > 1$ in state s at time t depends on its price at time $t - 1$ in the following way:

$$(8) \quad m_{it}^s(m_{it-1}) = \{ G_i m_{it-1} = \frac{R}{q_i} m_{it-1} \text{ if } i \leq s \text{ (with probability } q_i) \text{ and } I_i \text{ otherwise} \}.$$

I use $m_t^s(m_{t-1}) = (m_{2t}^s, \dots, m_{nt}^s)$ to describe the prices of privately created assets at time t in state s , where $m_{t-1} = (m_{2t-1}, \dots, m_{nt-1})$ is the beginning of period prices.

⁷ Thus, whenever $q_i g_i > R$, a fraction θ_i of the value of the asset must be held at the central bank in the form of reserves that pay no interest, where the fraction θ_i is determined by: $(1 - \theta_i) q_i g_i = R$. We may assume that the threat of reserve requirements or any other regulation induces firms to “stay under the radar” and satisfy the constraint in (7). Alternatively, we may interpret G_i as the nominal rate of growth net of the implicit tax imposed by the reserve requirement. The assumption that money has value because of government intervention is similar to the legal restrictions in Sargent and Wallace (1982). But here protecting money improves matters.

Government intervention: The government (and the central bank) can react immediately after the new asset prices are realized (and before the beginning of trade in the goods market). It may print (high-powered) money, collect lump sum taxes and engage in open market operations (exchange high-powered money for other bubble assets). Formally, the government commits to a vector of policy reaction functions $(M_1(m_t^s), \sigma_2(m_t^s), \dots, \sigma_n(m_t^s))$, where $M_1(m_t^s)$ is the post intervention amount of (high powered) money and $0 \leq \sigma_i(m_t^s) \leq 1$ is the fraction of asset $i > 1$ that the government will buy for money when the price of assets are m_t^s . The post intervention dollar value of asset $i > 1$ held by the representative old agent is:

$$(9) \quad M_i^s(m_{t-1}) = (1 - \sigma_i[m_t^s(m_{t-1})])m_{it}^s(m_{t-1})$$

Figure 2 illustrates the sequence of events within the period. At the beginning of the period young agents make supply and price choices and old agents trade in assets. The old generation then gets interest payments on money, the state (s) is revealed, the government reacts and trade in the goods market follows.

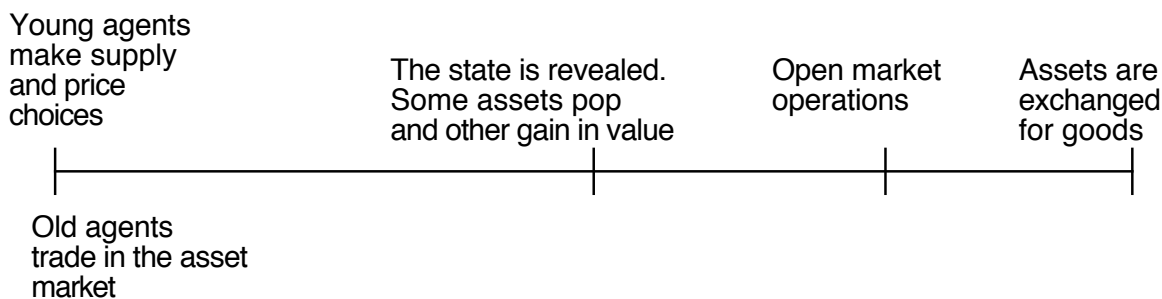


Figure 2: Sequence of events within the period

As in the Prescott model, the dollar prices of goods cannot be changed during the period and cheaper goods are sold first. There is no cash-in-advance constraint and assets are exchanged directly for goods. Thus a buyer that finds a unit at the price of P dollars can use P dollars worth of any asset to pay for it, where the dollar value of the assets in the goods market are determined by (8) after the state is observed.

The seller puts a price tag on each unit produced and these tags may be different across units. There are n cutoff prices ($P_{1t} < P_{2t} < \dots < P_{nt}$) where the cutoff price P_{it} clears a hypothetical market that will be described shortly. The seller expects that if he puts a price tag $P_{i-1t} < p \leq P_{it}$ he will sell the good with probability q_i . Therefore P_{it} dominates any price $P_{i-1t} < p < P_{it}$ and we may limit the price choice of the seller to the n cutoff prices.

Let x_{it} denote the number of units with a price tag P_{it} . Total production (labor supply) is:

$$(10) \quad L_t = \sum_{i=1}^n x_{it}$$

The expected consumption that the seller will get from a unit with a price tag P_{it} is $q_i P_{it} z_{t+1}^i$, where z_{t+1}^i is the expected purchasing power of a dollar at the beginning of next period, conditional on selling the unit ($i \leq s$). The seller chooses x_{it} by solving:

$$(11) \quad \max_{x_{it} \geq 0} \beta \left(\sum_{i=1}^n q_i x_{it} P_{it} z_{t+1}^i \right) - v \left(L_t = \sum_{i=1}^n x_{it} \right)$$

The first order conditions that an interior solution for this problem must satisfy are:

$$(12) \quad \beta q_i P_{it} z_{t+1}^i = \beta P_{1t} z_{t+1}^1 = v'(L_t)$$

Hypothetical Markets: I assume that the buyers form a line and arrive at the market sequentially according to their place in the line. Upon arrival, each buyer spends his entire portfolio of assets at the cheapest available price. From the sellers' point of view, the purchasing power that arrives, rather than the number of buyers, is relevant.

To simplify, I assume that the post intervention dollar value of the assets held by the representative old agent, $\sum_i M_i^s(m_{t-1})$ is increasing in s . The minimum amount that the old agents will spend is therefore $\Delta_1(m_{t-1}) = \min_s \sum_{i=1}^s M_i^s(m_{t-1}) = M_1^1(m_{t-1})$. The minimum

additional amount that will be spent if $s > 1$ is:

$$\Delta_2(m_{t-1}) = \min_{s>1} \sum_{i=1}^s M_i^s - \Delta_1 = M_1^2 + M_2^2 - \Delta_1. \text{ And in general,}$$

$$(13) \quad \Delta_i(m_{t-1}) = \min_{s>i-1} \sum_{j=1}^s M_j^s - \sum_{j=1}^{i-1} \Delta_j = \sum_{j=1}^i M_j^i - \sum_{j=1}^{i-1} \Delta_j.$$

Note that it is possible to compute $\Delta_i(m_{t-1})$ on the basis of information available at the beginning of the period because the government reaction functions are known.

The first Δ_1 dollars worth of assets that arrive buy in the first market (at the lowest price, P_{1t}). If $s > 1$ then a second batch of Δ_2 dollars worth will open the second market and buy at the price P_{2t} . If $s > 2$ then a third batch of purchasing power will arrive and buy in the third market and so on.⁸

⁸ I assumed that the state (number of sunspots) is known before the beginning of trade but young sellers cannot change their prices in response to the information about the state. In this sense, prices are rigid. This assumption is not necessary for the main results. We can assume that sunspots appear sequentially and no one knows when the process will stop. Asset i survives if the number of sunspot is: $s \geq i$. Sellers accept asset i immediately after observing $s \geq i$. As a result money will buy in the first market, the closest substitutes will buy in the second and so on. In this version, prices are not rigid because a seller that accepts asset i does not know whether market $i+1$ will open or not. See Eden (1990, 1994) for a UST model that insists on price flexibility.

The assumption that sellers accept all assets directly for goods can also be relaxed. We can impose a cash-in-advance constraint in a sequential trade model with flexible prices. In the first market the young gets all the high-powered money. Then if they observe a second sunspot ($s \geq 2$) they go to the asset market and exchange the money they have for asset 2. The old who sell asset 2 go immediately to the goods market and use the money they have to buy goods. This process continues until the old have sold their

In equilibrium markets that open are cleared:

$$(14) \quad P_{it} x_{it} = \Delta_i(m_{t-1})$$

The expected purchasing power of a dollar: I now calculate the expected purchasing power of a dollar held at the beginning of the period as a function of $(\Delta_1, \dots, \Delta_n)$. I use

$$\phi_i^s(m_{t-1}) = \Delta_i(m_{t-1}) \left(\sum_{j=1}^s \Delta_j(m_{t-1}) \right)^{-1}$$

to denote the probability that a dollar worth of an asset

will buy in market i when exactly s markets open. The expected purchasing power of a dollar held at the beginning of the period (before interest payments) is:

$$(15) \quad z(m_{t-1}, P_t) = R \sum_{s=1}^n \Pi_s \sum_{i=1}^s \frac{\phi_i^s(m_{t-1})}{P_{it}},$$

where $P_t = (P_{1t}, \dots, P_{nt})$ is the vector of current period prices (of goods not assets).

I use $P_{t+1}^s = (P_{1t+1}^s, \dots, P_{nt+1}^s)$ to denote expectations about next period prices if in the current period exactly s markets open. Using this notation, the expected purchasing power of a dollar next period if exactly s markets open is: $z(m_t^s, P_{t+1}^s)$. The expected purchasing power of a dollar if market i opens ($i \leq s$) is:

$$(16) \quad z_{t+1}^i = (\gamma_{qi}) \sum_{s=i}^n \Pi_s z(m_t^s, P_{t+1}^s)$$

where Π_s / γ_{qi} is the probability of state s conditional on $i \leq s$.

entire holding of asset 2. Everyone then waits and sees whether a third sunspot will appear. If it does, the young go to the asset market and exchange the money they hold for asset 3 and so on.

I now define equilibrium as follows. Equilibrium is a vector of functions $(m_i^s(m), \sigma_i[m^s(m)], M_i^s(m), P_i(m), x_i(m), \Delta_i(m), z(m), z^i(m); i, s = 1, \dots, n)$ from the beginning of the period asset prices m to R_+ such that:

$$(17) m_i^s(m) = \{G_i m_i \text{ if } i \leq s \text{ and } I_i \text{ otherwise}\}$$

$$(18) M_1^s(m) > 0$$

$$(19) 0 \leq \sigma_i[m^s(m)] \leq 1 \text{ and } M_i^s(m) = (1 - \sigma_i[m^s(m)])m_i^s(m), \text{ for } i > 1$$

$$(20) \beta q_i P_i(m) z^i(m) = \beta P_1(m) z^1(m) = v'(\sum_i x_i(m))$$

$$(21) \Delta_1(m) = M_1^1(m) \text{ and } \Delta_i(m) = \sum_{j=1}^i M_j^i(m) - \sum_{j=1}^{i-1} \Delta_j(m)$$

$$(22) P_i(m) x_i(m) = \Delta_i(m)$$

$$(23) z(m) = R \sum_{s=1}^n \Pi_s \sum_{i=1}^s \frac{\Delta_i(m)}{P_i(m)} \left(\sum_{j=1}^s \Delta_j(m) \right)^{-1}$$

$$(24) z^i(m) = (1/q_i) \sum_{s=i}^n \Pi_s z(m^s(m))$$

Equilibrium condition (17) is the beginning of next period asset prices; condition (18) requires that money is not crowded out; condition (19) defines the dollar value of assets held by the public after the open market operations; (20) are the first order conditions that an interior solution to the young agent's problem must satisfy; (21) defines the nominal demand for each of the hypothetical markets and (22) are market clearing conditions; (23) is the expected purchasing power of a dollar held at the beginning of the period and (24) is the expected purchasing power of a dollar if market i opens.

Price stability and a stable price distribution: In a standard single bubble economy, stable price level requires stable money supply. Here there is price dispersion and the objective of price stability may be interpreted as maintaining a constant expected purchasing power of a dollar (23). To do that the government will have to engage in open market operations

and taxes to control asset supplies that are analogous to the money supply in a single bubble asset economy.

To illustrate how this may be done, I turn now to show that the government can control the entire probability distribution of prices and can also reach a stable price distribution. I start by showing that the government can choose any vector of constant asset supplies.

Lemma: The government can choose any vector $\Delta = (\Delta_1, \dots, \Delta_n) \geq 0$ regardless of the asset prices m .

The proof is in the Appendix. It uses the assumption that the government can make lump sum transfers (taxes) and therefore controls the money supply. When the constraint $M_1^s(m) > 0$ is not binding, the use of lump sum transfers is enough to get the desired vector Δ . When the constraint is binding the government may use open market operations: It may buy assets for money and then, once the public has enough money it can tax it.

Since the government controls the vector Δ , there is equilibrium with stable price distribution.

Claim 1: There is an equilibrium that can be described by vectors of constants: $\Delta(m) = \Delta$, $z(m) = z$, $P_i(m) = P_i$ and $x_i(m) = x_i$ for all m .

I show this claim in the Appendix for the special case $v'(L) = L$ where the solution is:

$$(25) \quad q_i P_i = P_1 = (\beta R)^{-1} \left[\sum_{s=1}^n \Pi_s \sum_{i=1}^s q_i \Delta_i \left(\sum_{j=1}^s \Delta_j \right)^{-1} \right]^{-1} \sum_i q_i \Delta_i$$

$$(26) \quad x_i = \frac{q_i \Delta_i}{P_1}$$

$$(27) \quad z = \beta R^2 \left[\sum_{s=1}^n \Pi_s \sum_{i=1}^s q_i \Delta_i \left(\sum_{j=1}^s \Delta_j \right)^{-1} \right]^2 \left(\sum_i q_i \Delta_i \right)^{-1}$$

Welfare in the steady state: To maximize steady state welfare the government should eliminate demand uncertainty by eliminating all privately created bubble assets: The policy maker should choose $\Delta_i = 0$ for $i > 1$. This policy may be implemented by imposing a 100% reserve requirements on all privately created assets. It may run into difficulties when the bubble assets have some fundamental values, as in the next section.

4. BUBBLE ASSETS WITH PARTIAL BACKING

The introduction of capital into OG model raises two questions: (a) Will capital crowd out money? and (b) Can money play a useful role when capital is productive? The answer to the first question depends on the interest paid on money (or the rate of deflation in a complete monetary model). The answer to the second depends in general on whether or not money facilitates transactions. Here I argue that money may improve matters even in the absence of the transaction motive for holding it and even when capital is productive. The new role is in limiting the effects of bubbles. Thus, capital may work well as a store of value when it is priced “correctly”. But money may help when capital may be “over-priced”.

I model the case of housing in a way that is similar to gold and other types of commodity monies: housing yield some services and in addition can be used as a store of value. In the previous case, each of the n assets (slots) had no fundamental value and when the bubble bursts the value of the asset reverted to an arbitrarily low level. Here the asset has some fundamentals and when it pops it reverts to an initial value that may be relatively “large”.

Each slot $i > 1$ represents a housing type. As before, old agents own all the assets in the economy and want to exchange them for goods. But here they derive utility from housing. The stock of housing (measured in physical units such as square feet) type $i > 1$ at the beginning of period t is:

$$(28) \quad H_{it} = (1 - \delta)(H_{it-1} + f_i(y_{it-1})),$$

where H_{it-1} is the quantity of houses that the old bought when they were young (at $t - 1$), $0 < \delta < 1$ is the depreciation rate, y_{it-1} is the amount of labor that they invested (when young) in housing and $f_i(y_i)$ is a production function measuring investment in housing. I assume that $f_i'(0)$ is large and that there exists \bar{y}_i such that: $f_i'(y_i) > 0$ when $y_i < \bar{y}_i$ and $f_i'(y_i) \leq 0$ otherwise. There is thus a limit on the amount of houses that can be produced. I also assume that $\sum_{i>1} \bar{y}_i$ is small so that housing production does not completely crowd out goods production.⁹ The stock of type i housing yields services equivalent to $\gamma_i H_i$ units of consumption, where $\gamma_i > 0$ is the per unit services.¹⁰

The utility of the representative young agent at time t is:

$$(29) \quad \beta \left(c_{t+1} + \sum_{i>1} \gamma_i H_{it+1} \right) - v \left(L_t = \sum_i x_{it} + \sum_{i>1} y_{it} \right)$$

The sequence of events within the period is as follows. At the beginning of the period the young make labor supply and price decisions and the old trade in assets. Then the old realize the non-pecuniary services from the post-depreciation housing, get interest payments and pay lump sum taxes. A new price of housing is then realized. The

⁹ Another way to avoid corner solutions is to assume a utility function from goods consumption, $U(c)$, that satisfies the Inada condition: $U'(0) = \infty$. This more standard approach was not taken because a linear utility function is much simpler to work with.

¹⁰ Note that the analogy with gold is not complete. We may assume that agents derive pleasure from looking at their gold reserves and digging gold in a limited amount is possible. But gold does not depreciate.

government may respond to the new housing prices by buying and selling assets and by making lump sum transfers. At the end of the period, the old use the assets they have to buy goods. Figure 3 describes the sequence of events within the period.

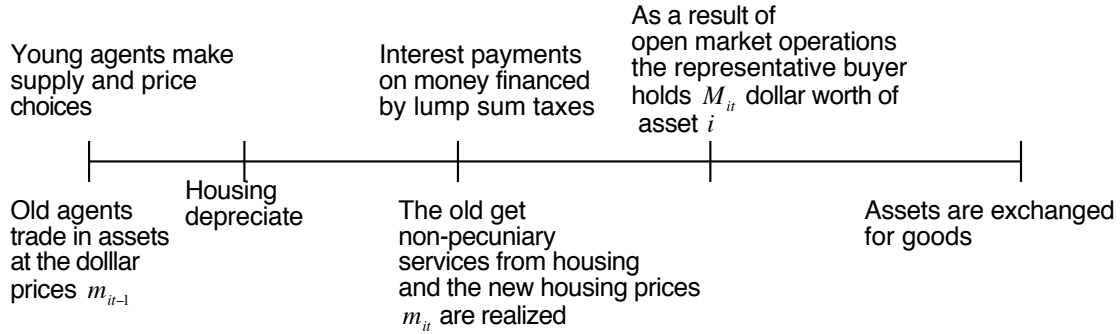


Figure 3: The sequence of events within the period

Trade in assets: The old at time t trade in assets before they get the housing services and before depreciation. The beginning of period price and quantity of housing type i is m_{it-1} and $H_{it-1} + f_i(y_{it-1})$. The relevant information at the beginning of the period is the quantities and prices of all housing types:

$m = \{(H_{2t-1} + f_2(y_{2t-1})), \dots, (H_{nt-1} + f_n(y_{nt-1})); m_{2t-1}, \dots, m_{nt-1}\}$. After the number of sunspots s is observed, housing prices change to:

$$(30) \quad m_i^s(m) = \{g_i(m)m_i \text{ if } i \leq s \text{ (with probability } q_i) \text{ and } I_i(m) \text{ otherwise}\}.$$

This is similar to (8) but here the initial value I_i may be large and both the initial value and the rate of growth in the survival state depend on the beginning of period housing prices.

I use $z^i(m), z^{ni}(m)$ to denote the expected purchasing power of a dollar when market i opens and when it does not open, respectively. The expected consumption that an old agent can get from investing a dollar in type i housing is:

$$(31) \quad (1-\delta) \left(q_i G_i(m) z^i(m) + (1-q_i) \frac{I_i(m)}{m_i} z^{ni}(m) + \frac{\gamma_i}{m_i} \right)$$

The expression in the bracket has two components:

$q_i G_i(m) z^i(m) + (1-q_i) \frac{I_i(m)}{m_i} z^{ni}(m)$ is the expected consumption from exchanging a dollar worth of housing for goods and $\frac{\gamma_i}{m_i}$ is the non-pecuniary services from the ownership of a dollar worth of housing.

As in the previous section, I assume that the government follows a policy of having valued money and does not allow the crowding out of money by private assets. As a result of (the threat of) intervention the expected real return of a dollar invested in housing is equal the expected real return on holding a dollar:

$$(32) \quad q_i G_i(m) z^i(m) + (1-q_i) \frac{I_i(m)}{m_i} z^{ni}(m) + \frac{\gamma_i}{m_i} = \frac{Rz(m)}{1-\delta}$$

The rate of nominal price change in the survival state is therefore:

$$(33) \quad G_i(m) = \frac{Rz(m)}{(1-\delta)q_i z^i(m)} - (1-q_i) \frac{I_i(m) z^{ni}(m)}{m_i q_i z^i(m)} - \frac{\gamma_i}{m_i q_i z^i(m)}$$

I assume a lower and upper bounds ($\underline{g} \geq 0, \bar{g} < \infty$) such that:

$$(34) \quad \underline{g} \leq G_i(m) \leq \bar{g} \text{ for all } m \text{ and } i.$$

The lower bound reflects the requirements that prices cannot be negative. The upper bound maybe the result of government policy that imposes a 100% capital gains

tax when the rate of capital gain is above a certain threshold. This policy will be discussed shortly.

The representative young agent observes the beginning of period housing prices m and uses it to form expectations about the purchasing power of a dollar in the next period, described by (23) and (24). The expected purchasing power from a unit of the consumption good supplied to market i is $q_i P_i z^i(m)$, and the expected purchasing power from a unit of housing produced at the beginning of the period is:

$$(35) \quad p_i^H(m) = \sum_s \Pi_s m_i^s(m) z^i(m^s(m))$$

The young agent solves:

$$(36) \quad \max_{y_i, x_i \geq 0} \beta \left(\sum_i q_i x_i P_i z^i(m) \right) + \beta \left(\sum_{i>1} f_i(y_i) p_i^H(m) \right) - v \left(\sum_i x_i + \sum_{i>1} y_i \right)$$

The first order conditions that an interior solution to (36) must satisfy are:

$$(37) \quad \beta q_i P_i z^i(m) = \beta f_i'(y_i) p_i^H(m) = v'$$

Since now the value of the assets that pop is not small we should modify the algorithm that computes the amount of purchasing power that will arrive at each of the hypothetical markets as follows. The post intervention dollar value of the assets held by the representative old agent in state s is: $\sum_{i=1}^n M_i^s(m_{t-1})$. As before, I assume that this amount

is increasing in s . The minimum amount that the old agents will spend is therefore

$$\Delta_1(m_{t-1}) = \min_s \sum_{i=1}^n M_i^s(m_{t-1}) = \sum_{i=1}^n M_i^1(m_{t-1}).$$

The minimum additional amount that will be

spent if $s > 1$ is: $\Delta_2(m_{t-1}) = \min_{s>1} \sum_{i=1}^n M_i^s - \Delta_1 = \sum_{i=1}^n M_i^2 - \Delta_1$. And in general,

$$(13') \quad \Delta_i(m_{t-1}) = \min_{s>i-1} \sum_{j=1}^n M_j^s - \sum_{j=1}^{i-1} \Delta_j = \sum_{j=1}^n M_j^i - \sum_{j=1}^{i-1} \Delta_j.$$

We can now modify the definition of equilibrium in the previous section as follows. Equilibrium is a vector of functions $(G_i(m), m_i^s(m), \sigma_i[m^s(m)], M_i^s(m), P_i(m), p_i^H(m), x_i(m), y_i(m), \Delta_i(m), z(m), z^i(m), z^{ni}(m); i, s = 1, \dots, n)$ from the beginning of period state m to R_+ that satisfy the following modified conditions:

$$(17') \quad m_i^s(m) = \{ G_i(m)m_i \text{ if } i \leq s \text{ (with probability } q_i) \text{ and } I_i(m) \text{ otherwise} \}.$$

$$(18') \quad M_1^s(m) > 0$$

$$(19') \quad 0 \leq \sigma_i[m^s(m)] \leq 1 \text{ and}$$

$$M_i^s(m) = \left(1 - \sigma_i[m^s(m)]\right) (1 - \delta)(H_{i-1} + f_i(y_{i-1}))m_i^s(m_i), \text{ for } i > 1$$

$$(20') \quad \beta q_i P_i(m) z^i(m) = \beta f_i'(y_i) p_i^H(m) = v' \left(\sum_i x_i(m) + \sum_{i>1} y_i(m) \right)$$

$$(21') \quad \Delta_1(m_{t-1}) = \sum_{i=1}^n M_i^1(m_{t-1}) \text{ and } \Delta_i(m_{t-1}) = \sum_{j=1}^n M_j^i - \sum_{j=1}^{i-1} \Delta_j$$

$$(22') \quad P_i(m) x_i(m) = \Delta_i(m)$$

$$(23') \quad z(m) = R \sum_{s=1}^n \Pi_s \sum_{i=1}^s \frac{\Delta_i(m)}{P_i(m)} \left(\sum_{j=1}^s \Delta_j(m) \right)^{-1}$$

$$(24') \quad z^i(m) = (\gamma_i) \sum_{s=i}^n \Pi_s z(m^s(m))$$

And the following additional conditions:

$$z^{ni}(m) = (\gamma_i) \sum_{s=1}^{i-1} \Pi_s z(m^s(m))$$

$$\underline{g} \leq G_i(m) = \frac{Rz(m)}{(1-\delta)q_i z^i(m)} - (1-q_i) \frac{I_i(m) z^{ni}(m)}{m_i q_i z^i(m)} - \frac{\gamma_i}{m_i q_i z^i(m)} \leq \bar{g} \text{ and}$$

$$p_i^H(m) = \sum_s \Pi_s m_i^s(m) z(m^s(m)).$$

I now turn to analyze the case in which the government follows a policy of stable expected purchasing power: $z(m) = z$ for all m . I also assume that in the case of price stability the initial values remain constant: $I_i(m) = I_i$ for all m .

Under these assumptions we can write (33) as:

$$(38) \quad G_i(m_i) = \frac{R}{q_i(1-\delta)} - \frac{\gamma_i}{q_i m_i z} - (1-q_i) \frac{I_i}{q_i m_i}$$

Note that:

$$(39) \quad G_i'(m_i) > 0, \quad G_i(m_i) \leq \frac{R}{q_i(1-\delta)}$$

$$\lim_{m_i \rightarrow \infty} G_i(m_i) = \frac{R}{q_i(1-\delta)}, \quad \lim_{m_i \rightarrow 0} G_i(m_i) = -\infty$$

To further analyze the behavior of prices, let

$$(40) \quad F_i = \gamma_i \sum_{\tau=1}^{\infty} \left(\frac{1-\delta}{1+r} \right)^{\tau} = \frac{\gamma_i(1-\delta)}{r+\delta},$$

denote the present value of the services that a unit of type i housing will deliver or its fundamental value. Using $A_i = I_i z / F_i$ to denote the real price of housing immediately after popping (zI_i) relative to its fundamental value, (38) implies:

$$(41) \quad G_i(I_i) = \frac{1}{q_i} \left(\frac{R}{(1-\delta)} - \frac{r+\delta}{A_i(1-\delta)} - 1 \right) + 1$$

This leads to:

$$(42) \quad G_i(I_i) > 1 \text{ if } zI_i > F_i, \quad G_i(I_i) = 1 \text{ if } zI_i = F_i \text{ and } G_i(I_i) < 1 \text{ if } zI_i < F_i$$

Since G_i is an increasing function, prices will increase with the age of the bubble when the initial price is higher than the fundamental value, and decrease when it is lower than the fundamental value. Figure 4 describes the price of housing (in logs) as a function of the age of the bubble.

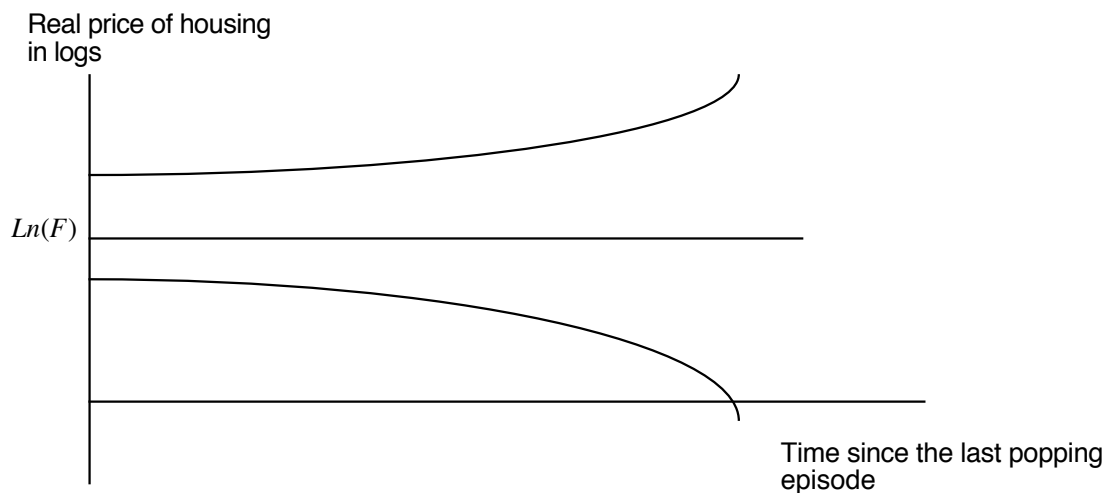


Figure 4

I rule out the case in which the real price of housing after popping is less than the fundamental value ($zI_i < F_i$) because in this case the price of the house may be negative when the “bubble” survives for a long time.

I also rule out the case in which $I_i z > F_i$ and $\frac{R}{q_i(1-\delta)} > \bar{g}$ because the rate of growth may get close to $\frac{R}{q_i(1-\delta)}$ (if the bubble survives for a long time) and may violate (34). I thus

require:

$$(43) \quad zI_i \geq F_i \quad \text{with equality when} \quad \frac{R}{q_i(1-\delta)} > \bar{g}.$$

Note that the policy variables (R, \bar{g}) can be used to eliminate bubbles. I now turn to argue for a policy of eliminating bubbles whenever they are clearly identified.

A planner's problem: In the steady state the amount of housing does not change and therefore:

$$(44) \quad f_i(y_i) = \frac{\delta H_i}{1 - \delta}$$

To maximize welfare in the steady state a planner will choose the production of goods and the stock of housing by solving:

$$(45) \quad \max_{x, H_i} \beta \left(x + \sum_{i>1} \gamma_i H_i \right) - v \left(x + \sum_{i>1} f_i^{-1} \left(\frac{\delta H_i}{1 - \delta} \right) \right) \quad \text{s.t.} \quad f_i^{-1} \left(\frac{\delta H_i}{1 - \delta} \right) < \bar{y}_i$$

To simplify, I assume that γ_i is small and the constraint is not binding. The first order conditions that an interior solution to (45) must satisfy are:

$$(46) \quad \beta = v' \quad \text{and} \quad \beta \gamma_i = \frac{v' f_i^{-1} \delta}{1 - \delta} = \frac{v' \delta}{f_i'(1 - \delta)}$$

This leads to:

$$(47) \quad f_i' = \frac{\delta}{\gamma_i(1 - \delta)}$$

Claim 2: The equilibrium outcome maximizes the steady state welfare if and only if the price of housing is equal to the fundamental value (no bubbles) and $R - 1 = r = 0$.

To show this Claim, note that when the price of housing is equal to the fundamental values, there is no demand uncertainty and only one market opens (with probability 1).

The real price in the goods market is $P_1 z = R$. In equilibrium we have an interior solution and the first order condition (48) can be written as:

$$(48) \quad \beta q_i P_1 z = \beta P_1 z = \beta R = \beta f'_i(y_i) p_i^H(m) = v'$$

Substituting the real price of housing $p_i^H(m) = I_i z = F_i$, and $R = 1$ in (48) leads to (47).

Claim 2 supports a policy of eliminating privately created bubbles and an interest rate that is equal to the rate of population growth. In our model, the government can eliminate bubbles by setting $\bar{g} = 1$, say by imposing a 100% capital gain tax. This result relies heavily on the assumption that the amount of services that one can get from housing does not change over time. A more general model may allow for growth in the amount of services yield (or dividend yield). This will produce a growth in the fundamental value (40) and a 100% capital gain tax may ruin the market for an asset that is “correctly priced”. For example, risky projects like oil and gas explorations should realize capital gains when oil is found and taxing it may reduce the incentive to explore. I therefore expect that the optimal second best policy will set \bar{g} at a high level that will allow a “reasonable” growth in the price of assets and will eliminate only bubbles with low probability of survival.

A second best problem: In section 2 we analyzed the case in which there is just one bubble asset and argued that when the popping probability of this asset is strictly positive, the optimal interest rate is higher than the rate of population growth ($R = \frac{1}{\pi} > 1$). I now examine this conclusion for the case in which money never pops but there are privately created bubble assets that may pop.

To focus on the choice of the interest rate and to allow for a comparison with section 2, I assume that the government does not use capital gain taxes to eliminate

bubbles (that is, it sets \bar{g} at a level that is too high to eliminate bubbles) and chooses a policy that leads to a constant $\Delta = (\Delta_1, \dots, \Delta_n)$ and a stable price distribution. (See Claim 1). It is shown in the Appendix that the fraction of output allocated to market i and average capacity utilization are:

$$(49) \quad \mu_i = x_i \left(\sum_j x_j \right)^{-1} = q_i \Delta_i \left(\sum_j q_j \Delta_j \right)^{-1}$$

$$(50) \quad ACU = \sum_{s=1}^n q_s \mu_s$$

The probability that a dollar will buy in market i is:

$$(51) \quad \eta_i = \sum_{s=i}^n \Pi_s \phi_i^s$$

The marginal cost and the expected real wage (defined as the expected present value of consumption per unit of labor) are:

$$(52) \quad v'(L) = \beta q_1 P_1 z = \beta R w$$

$$(53) \quad w = \sum_{i=1}^n q_i \eta_i$$

I now consider the problem of a planner who cannot change the fractions μ_i and average capacity utilization:

$$(54) \quad \max_L \beta L(ACU) - v(L)$$

The first order condition to this second best problem is:

$$(55) \quad v'(L) = \beta(ACU)$$

The equilibrium amount of labor (52) is equal to the planner's choice (55) if $Rw = ACU$,
or

$$(56) \quad R = \frac{ACU}{w}$$

In section 2 I assumed $n = 1$ and used $q_1 = \Pi_1 = \pi$. In this case, $ACU = q_1 \mu_1 = \pi \mu_1 = \pi$, $w = q_1 \eta_1 = \pi^2$ and $R = \frac{1}{\pi} \geq 1$. We can also have $w > ACU$ and an optimal interest rate that is less than unity. To see this possibility, I assume two markets: The first market opens with probability 1 and the second with probability $\frac{1}{2}$. The supply to each market is 1 units and therefore: $ACU = \frac{3}{4}$. I now compute: $\eta_1 = \frac{1}{2} + \frac{1}{2} \phi_1^2$, $\eta_2 = \frac{1}{2} \phi_2^2 = (\frac{1}{2})(1 - \phi_1^2)$ and $w = \eta_1 + \frac{1}{2} \eta_2 = \frac{1}{2} + \frac{1}{2} \phi_1^2 + (\frac{1}{4})(1 - \phi_1^2) = \frac{3}{4} + \frac{1}{4} \phi_1^2 > ACU$. Thus, the optimal interest rate may be above or below unity depending on the underline parameters.

A numerical example: To illustrate the working of the model and to judge its ability to account for rare financial crises, I consider now a complete numerical example. I assume $n = 2$, $\Delta_1 = 10$ and $1 - \Pi_1 = \Pi_2 = 0.98$. Thus the probability of a financial crisis is 2%. Figure 5 describes the results. On the horizontal axis we have Δ_2 and on the vertical left (primary) axis we have the fraction of goods' output allocated to the second market, μ_2 . The optimal R and average capacity utilization are on the right. The fraction of output that is lost in the case of a financial crisis (μ_2) is increasing in Δ_2 and average capacity utilization is decreasing in Δ_2 . The optimal R is less than unity but very close to it. When $\Delta_2 = 4$, for example, a financial crisis will reduce the nominal wealth of the representative agent from 14 to 10 and this will lead to a 28% reduction in consumption.

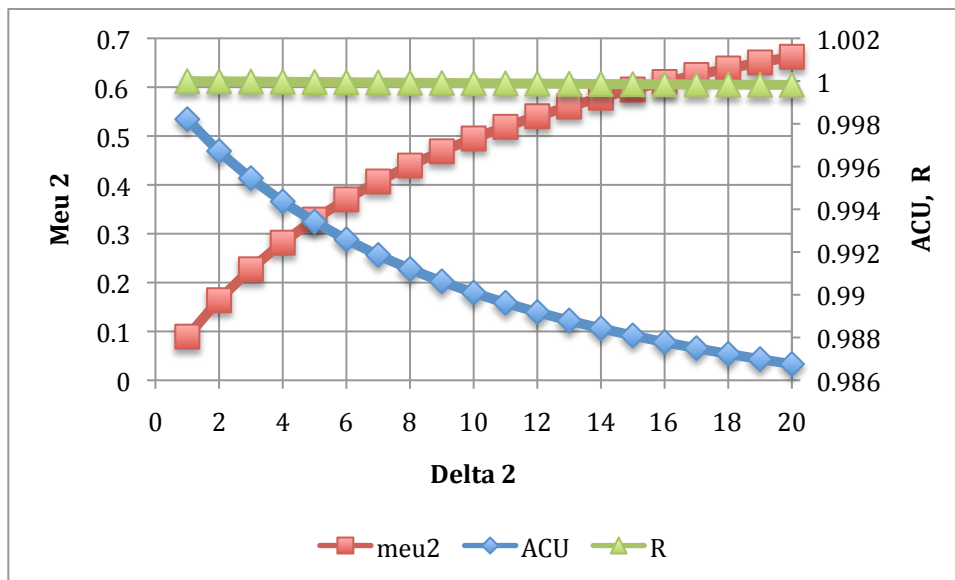


Figure 5: The fraction of output allocated to the second market (μ_2), average capacity utilization (ACU) and the optimal interest rate (R) as a function of Δ_2 when $\Delta_1=10$ and $1-\Pi_1=\Pi_2=0.98$

A financial crisis will not affect the average posted price $\mu_1 P_1 + \mu_2 P_2$ but will affect the average price per unit that is actually sold because in the state of financial crisis only the low priced goods are sold. In the steady state $q_2 P_2 = \Pi_2 P_2 = P_1$ and therefore the average transaction price will fall in a crisis to:

$$(57) \quad TPC = \frac{\mu_1 P_1}{\mu_1 P_1 + \mu_2 P_2} = \frac{\Pi_2 \mu_1}{\Pi_2 \mu_1 + \mu_2}$$

Figure 6 plots the transaction price in the crisis state (57) as a function of Δ_2 . As can be seen the average transaction price drops considerably as a result of the crisis. When $\Delta_2 = 10$ for example, average transaction price drops by 50%.

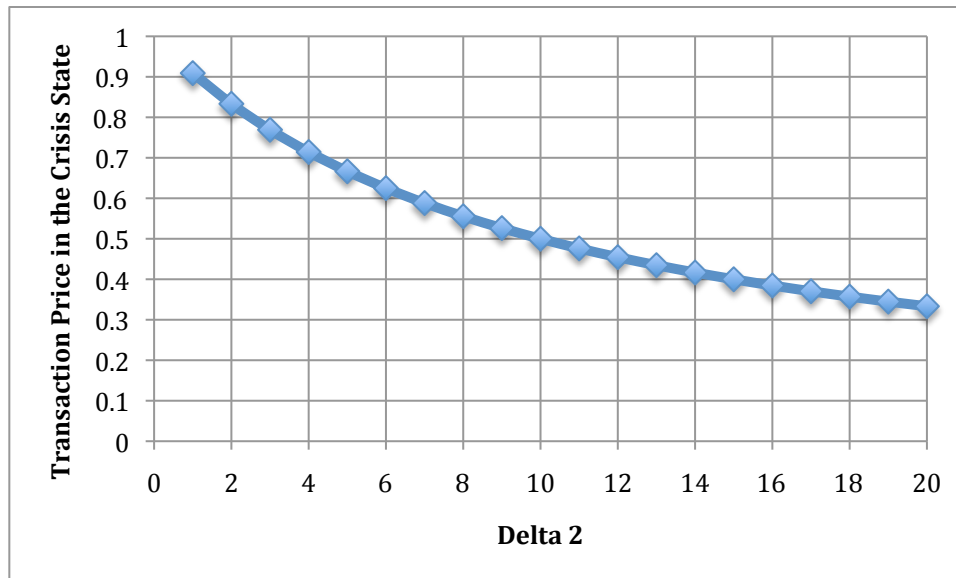


Figure 6: The transaction price in the crisis state (TPC) as a function of Δ_2 when $\Delta_1 = 10$ and $1 - \Pi_1 = \Pi_2 = 0.98$

This example is broadly consistent with the behavior of consumption and prices during the great depression years: 1929-1939. I treat the entire episode as one period. Both per capita M2 and per capita national wealth declined by 16% during the period.¹¹ This suggests $\Delta_2 = 1.9$. Under this assumption the numerical example implies a reduction in per capita consumption of 16% which is in line with the actual reduction in consumption during the period.¹²

The model also predicts a reduction of 16% in the average transaction price during the crisis. In 1939 the CPI was 19% lower than in 1929. If we assume that the CPI at the end of the period reflects the prices charged by the low price sellers, this is also close to the model's prediction.

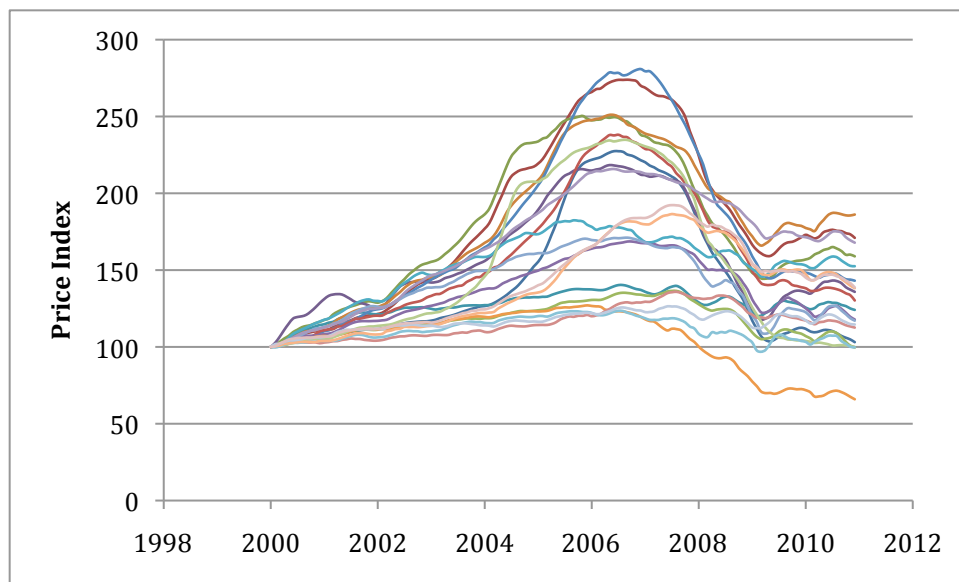
The recent housing cycle: Our model can generate variations in the rate of nominal price change, even if there are no changes in fundamentals. Since when assets pop (say as a

¹¹ Based on data from the Historical Statistics of the United States, average consumption during the period 1930-1939 was 18% below trend.

¹² Based on data from Robert Shiller's web page.

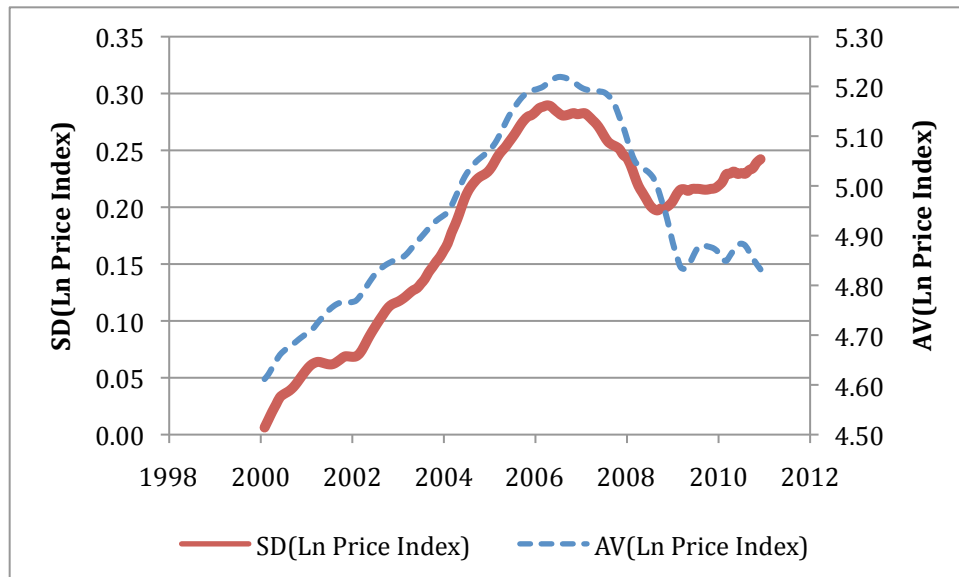
result of a low realization of the number of “sunspots” in our model), their prices revert to their initial values (I_t), assets that grew faster, fall harder and as a result we should observe a drop in the cross sectional standard deviation of housing prices during the bust period.

This seems broadly consistent with the recent experience in the housing market. Figure 7 illustrates. Figure 7A describes the Case-Shiller index of housing prices in 20 major metropolitan areas across the US.¹³ The index is set at 100 in the year 2000 for all the 20 metropolitan areas. It then increased for all observations reaching a level above 250 in 2006 in some cases (273 for Los Angeles CA and 278 for Miami FL). By Dec. 2010 the indices were much lower (170 for Los Angeles and 143 for Miami). As can be seen from Figure 7B the cross sectional standard deviation of the log of the Case-Shiller price indices fell considerably during the period 2006-2008 from about 0.3 to 0.2.



A. Price Indices for 20 metropolitan areas (1/1/2000=100)

¹³ Data from Robert Shiller’s web page.



B. The Cross Sectional Standard Deviation and the Mean of the log of the Price Indices

Figure 7: The Case-Shiller Price Indices for 20 Metropolitan areas

5. DISCUSSION

The literature on rational bubbles typically assumes that prices are determined by a Walrasian auctioneer and asks whether he (the auctioneer) can announce asset prices that are not strongly correlated with fundamentals but nevertheless clear markets. Here I assume that agents set prices. There are two advantages from adopting a price-setting environment. First we get the effects of pops on economic activity. As far as I know this is absent in the Walrasian approach to bubbles. Second, it leads to the focus on the question of who can over charge for an asset he owns.

In our model all assets are used to buy goods. Therefore symmetric information is important for preventing the risk of a market for lemons problem. In the model, privately created bubble assets have a risky rate of return but since all agents are informed about the popping of a bubble at the same time there is no risk of asymmetric information. The

agent that creates an overpriced asset must be able to convince all the agents in the economy that they will be informed about a popping event at the same time.

Does minimizing the risk of asymmetric information is equivalent to minimizing the probability of a pop? Although there is no necessary connection between the two, it may be the case that asymmetry in information can arise only when the asset pops and therefore a small popping probability implies a small risk of asymmetric information. It may therefore be the case that only agents with enough reputation to promise a high survival probability will be able to create bubble assets with small risk of asymmetric information. Under this assumption, a financial crisis must be a rare event.

Another issue that is often discussed in the literature is whether the value of assets can become too large relative to GDP. Unlike some “real” models of bubbles here the value of assets can never be larger than GDP. In each period all assets are exchanged for goods and an increase in the nominal value of assets will lead to an increase in nominal prices.

A related question is whether privately created bubble assets can crowd out money. I assume that the government uses open market operations to eliminate this possibility.

In the model, privately created bubble assets are “bad” from the social point of view. Even in periods in which bubbles do not pop, the mere potential of pops create price dispersion. And when bubbles pop some goods are not sold and capacity is not fully utilized.

The government can eliminate bubbles by imposing a capital gain tax. This solution is problematic once we allow for dividend (housing services) growth. In this case, capital gain taxes may ruin the market for assets that are “correctly priced”. The solution maybe in adopting a non-linear capital gain tax, imposing a high rate on assets that realized a high percentage increase in their value.

Should the government bailout agents who hold bubble assets that pop? If the government adopts a policy of buying assets that pop, it will increase capacity utilization in the short run but not necessarily in the long run. The adverse long run implications may occur because a bailout policy increase the survival probability and relaxes the constraint in (43). Thus, the possibility of a government bailout may allow for the formation of bubbles that otherwise will not be formed.

The broader regulations question has some similarity with the question of regulations designed to limit “money substitutes” and the desirability of a 100% reserve requirement. Hume (1752, p. 35) expressed “a doubt concerning the benefit of banks and paper-credit, which are so generally esteemed advantageous to every nation”. He seems to favor regulations against paper (inside) money and argue (on page 36) for a government run bank. Simons (1948, p. 79-80) argued for “Financial reform (banking reform primarily) aiming at sharp differentiation between money and private obligations” and for “Increasing concentration on the hands of the central government of the power to create money and effective money substitutes”.

The differentiation between money and private obligations requires the understanding of the nature of money. I think that if we could ask Henry Simons, he would stress the transactions role of money. But as was pointed out by Woodford (2003) this role has become less important in our technological advanced society.

Here the distinct feature of money is that it is a bubble. Discouraging “money substitutes” therefore requires regulations that limit the ability of the private sector to create bubble assets.

APPENDIX

Proof of the Lemma: The proof is by induction. The government can choose any

$\Delta_1 = M_1^1 > 0$, which is the amount of money when all privately created bubble assets pop,

by choosing the lump sum transfers (taxes) in this state. Assuming that it can choose

$$\sum_{j=1}^{i-1} \Delta_j, \text{ (20) and (21) imply that it can also choose } \sum_{j=1}^i M_j^i(m) \text{ and therefore it can choose:}$$

$$\Delta_i(m) = \sum_{j=1}^i M_j^i(m) - \sum_{j=1}^{i-1} \Delta_j(m). \quad \square$$

Proof of Claim 1: I assume that $z(m) = z$ is a constant and show that in this case we can solve for prices and quantities as a function of Δ . I then verify that $z(m) = z$ is indeed a constant.

When $z(m) = z$ is a constant, the first order condition (22) implies:

$$(A1) \quad q_i P_i = q_1 P_1$$

We can therefore write the market clearing conditions (24) as:

$$(A2) \quad x_i = \frac{q_i \Delta_i}{q_1 P_1}$$

Substituting (A2) in (12) leads to:

$$(A3) \quad L = (q_1 P_1)^{-1} \sum_i q_i \Delta_i$$

Using (A1) we can write (25) as:

$$(A4) \quad z = (q_1 P_1)^{-1} R \sum_{s=1}^n \Pi_s \sum_{i=1}^s q_i \Delta_i \left(\sum_{j=1}^s \Delta_j \right)^{-1}$$

Using (A4) and the first order condition (22) lead to:

$$(A5) \quad v'(L) = L = \beta q_1 P_1 z = \beta R \sum_{s=1}^n \Pi_s \sum_{i=1}^s q_i \Delta_i \left(\sum_{j=1}^s \Delta_j \right)^{-1}$$

Substituting (A3) in (A5) leads to the following equilibrium condition:

$$(A6) \quad P_1(\Delta) = \left[\beta R \sum_{s=1}^n \Pi_s \sum_{i=1}^s q_i \Delta_i \left(\sum_{j=1}^s \Delta_j \right)^{-1} \right]^{-1} \sum_i q_i \Delta_i$$

Using (A6) we write (A4) as:

$$(A7) \quad z(\Delta) = \beta \left[R \sum_{s=1}^n \Pi_s \sum_{i=1}^s q_i \Delta_i \left(\sum_{j=1}^s \Delta_j \right)^{-1} \right]^2 \left(\sum_i q_i \Delta_i \right)^{-1}$$

Since $\Delta(m) = \Delta$ for all m , (A6) and (A7) implies that (P_1, z) are constants. We can now use (A1) and (A2) to solve for (P_i, x_i) . We have thus shown that there exists a solution in which the values assigned by the equilibrium functions do not depend on m . \square

Average capacity utilization and the expected real wage in the steady state: The fraction of (goods) output allocated to market i can be derived from (A2). It is:

$$(A8) \quad \mu_i = x_i \left(\sum_j x_j \right)^{-1} = q_i \Delta_i \left(\sum_j q_j \Delta_j \right)^{-1}$$

Average capacity utilization is:

$$(A9) \quad ACU = \sum_{s=1}^n \Pi_s \sum_{i=1}^s \mu_i = \sum_{s=1}^n \Pi_s \sum_{i=1}^s q_i \Delta_i \left(\sum_j q_j \Delta_j \right)^{-1} = \sum_{s=1}^n q_s \mu_s$$

The probability that a dollar will buy in market i is:

$$(A10) \quad \eta_i = \sum_{s=i}^n \Pi_s \phi_i^s$$

The marginal cost is:

$$(A11) \quad v'(L) = \beta q_1 P_1 z = \beta R w$$

Using (A6) and (A7) leads to:

$$(A12) \quad w = \sum_{s=1}^n \Pi_s \sum_{i=1}^s q_i \Delta_i \left(\sum_{j=1}^s \Delta_j \right)^{-1} = \sum_{s=1}^n \Pi_s \sum_{i=1}^s q_i \phi_i^s = \sum_{i=1}^n q_i \eta_i$$

where w is the expected real wage (defined as the expected present value of consumption per unit of labor). When $n = 2$ we get: $\mu_1 = \Delta_1 (\Delta_1 + \Pi_2 \Delta_2)^{-1}$,
 $\mu_2 = \Pi_2 \Delta_2 (\Delta_1 + \Pi_2 \Delta_2)^{-1}$ and $ACU = \mu_1 + \Pi_2 \mu_2 = \Delta_1 (\Delta_1 + \Pi_2 \Delta_2)^{-1} + (\Pi_2)^2 \Delta_2 (\Delta_1 + \Pi_2 \Delta_2)^{-1}$
 I also get: $\eta_1 = \Pi_1 + \Pi_2 \phi_1^2 = \Pi_1 + \Pi_2 \frac{\Delta_1}{\Delta_1 + \Delta_2}$, $\eta_2 = \Pi_2 \frac{\Delta_2}{\Delta_1 + \Delta_2}$ and
 $w = \eta_1 + \Pi_2 \eta_2 = \Pi_1 + \Pi_2 \frac{\Delta_1}{\Delta_1 + \Delta_2} + (\Pi_2)^2 \frac{\Delta_2}{\Delta_1 + \Delta_2}$.

REFERENCES

- Akerlof, George A, 1970. "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism" The Quarterly Journal of Economics, MIT Press, vol. 84(3), pages 488-500, August.
- Blanchard Olivier J. "Speculative Bubbles, Crashes and Rational Expectations", Economics Letters 3 (1979) 387-389.
- _____ and Mark W. Watson "Bubbles, Rational Expectations and Financial Markets" NBER WP #945, July 1982.
- Diamond, Peter, A., 1965: "National Debt in a Neoclassical Growth Model" The American Economic Review, Vol. 55, No. 5, Part 1, 1126-1150.
- Eden, Benjamin. "Marginal Cost Pricing When Spot Markets are Complete" Journal of Political Economy, Dec.1990. Vol. 98, No.6,1293-1306.
- _____ "The Adjustment of Prices to Monetary Shocks When Trade is Uncertain and Sequential" Journal of Political Economy, Vol. 102, No.3, 493-509, June 1994.
- Friedman Milton, 1969: "The Optimum Quantity of Money." In *The Optimum Quantity of Money and Other Essays*. Chicago: Aldine, pp. 1-50.
- Hume, David, 1970: Writing on Economics, ed. Eugene Rotwein. Madison: University of Wisconsin Press.
- Jovanovic Boyan, "Bubbles in Prices of Exhaustible Resources" NBER Working Paper No. 13320 Issued in August 2007.
- Prescott, Edward. C., "Efficiency of the Natural Rate" The Journal of Political Economy, 83 (Dec. 1975): 1229-1236.
- Samuelson, Paul A., 1958: "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," Journal of Political Economy, 66 (6), 467-82.

- Santos Manuel S. and Michael Woodford., “Rational Asset Pricing Bubbles”
Econometrica, Vol. 65, No. 1 (Jan. 1997), pp. 19-57.
- Sargent, Thomas J. and Wallace, Neil, 1982: “The Real-Bills Doctrine versus the
Quantity Theory: A Reconsideration,” Journal of Political Economy, 90 (6), 1212–36.
- Simons, Henry, 1948: Economic Policy for a Free Society. Chicago, Ill.: University of
Chicago Press.
- Stein Jeremy C. “Monetary Policy and Financial-Stability Regulation”, mimeo (April
2010).
- Tirole Jean., “Asset Bubbles and Overlapping Generations” Econometrica, Vol. 53, No. 6
(Nov., 1985), pp. 1499-1528.
- Woodford Michael., Interest and Prices: Foundations of a Theory of Monetary Policy,
Princeton University Press, 2003.