OPTIMUM TARIFFS AND RETALIATION:
HOW COUNTRY NUMBERS MATTER

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Optimum Tariffs and Retaliation:
How Country Numbers Matter[1]

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ABSTRACT: This paper identifies a new terms-of-trade externality that is exercised through tariff setting. A North-South model of international trade is introduced in which the number of countries in each region can be varied. As the number of countries in one region is increased, each government there competes more aggressively with the others in its region, by lowering its tariff, to attract imports from the other region. In doing so, all countries in a region exert a negative terms-of-trade externality on each other, collectively undermining their own terms of trade and welfare. This externality can increase efficiency if the numbers of countries in both regions are increased simultaneously.

KEYWORDS. Comparative statics, efficiency, North-South, tariff war, terms of trade.

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1. Introduction

This paper investigates the relationship between the number of countries in the world economy and the outcome of a tariff war. The model is placed in a familiar setting where trade takes place between the North and the South. The North is relatively abundant in capital and the South is relatively abundant in labor. This much is standard in Heckscher-Ohlin (H-O) trade theory with protectionism. The novel feature of the model is that there may be any number of countries in the North and any number of countries in the South and that these numbers can be varied. There is a potentially large number of countries in each region but some countries are autarkic by assumption; they set prohibitive tariffs. The number of countries in a region is increased - from the perspective of the world economy - by allowing additional countries to set non-prohibitive best-response tariffs in equilibrium. The idea behind varying the number of countries is to understand how the structure of the world economy under protectionism affects efficiency and equity.[3]

The issue of country numbers and protectionism has gone largely overlooked in the past literature. In the standard model there would be just one country in each region, equivalent to Johnson's (1953-4) classic model of a tariff war. Recent developments in the methods of monotone comparative statics are applied in the present paper to yield a simple characterization of a tariff war with many countries. Comparative statics are then undertaken by varying the number of countries in one region at a time and in both regions simultaneously.[4]

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[3]Under our approach, an increase in country numbers implies an increase in the factor endowments that are available for use in the world economy, albeit shielded to some (endogenously determined) extent by protection; think of a sequence of islands that successively open to trade. An alternative approach would be to vary the number of countries while holding underlying factor endowments constant, in effect redrawing the boarders across a given territory. The reason I chose not to adopt this alternative approach will be discussed in due course.

[4]Methods of monotone comparative statics have been developed to address questions of how the number of players affects the equilibrium outcome (Topkis 1998): how the number of firms in the market affects production levels under Cournot competition (Amir and Lambson 2000); how the number of traders affect the volume of trade in a strategic market game (Amir and Bloch 2009). The key innovation in the present paper with respect to the literature on monotone comparative statics is that, while in the earlier settings players (firms and traders respectively) can choose quantities traded directly, in our international trade setting players (i.e. governments) can only influence quantities traded indirectly using tariffs. Recent research has applied methods of monotone comparative statics in other areas of international trade; for example, Costinot (forthcoming) has used a related approach to develop an elementary theory of comparative advantage.

In our framework, a new tariff externality is revealed whereby an increase of the number of countries in a region leads these countries to compete more aggressively with each other for imports using tariffs. Relative to a standard tariff war in which there is no other country like it, each country sets its tariff 'too low' and this tendency increases monotonically with the number of countries in the region. The effects of this externality motivate results that contrast with the past literature on tariff wars and offer new testable predictions.[5]

In order to compare and contrast the results of the present paper with the previous literature, let us review how optimum tariffs are determined. In the standard two-country model of a tariff war, a country with monopoly/monopsony power on world markets can improve its terms of trade and hence welfare using trade restrictions. When a country imposes or increases a tariff on an importable good this increases the local price, bringing about a reduction in local demand and a reciprocal reduction in excess demand for that good on the world market. This in turn leads to a fall in the world price of the good, thereby improving the importing country's terms of trade. This terms-of-trade improvement comes at the cost of reduced trade volume. The optimum tariff is the tariff that maximizes welfare by balancing these two effects. A tariff war is thus characterized by a Nash equilibrium of the one-shot tariff game in which each country sets its optimum, or 'best-response,' tariff.[6]

The first main result of the paper provides a simple characterization of a tariff war

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[5]As in a classic tariff war, countries cannot coordinate their tariffs by assumption. Therefore, each country's tariff can also be thought of as being 'too low' in the sense that it is below the joint-optimum for the region.

[6]Most of the literature on tariff wars follows Johnson (1953) in using models of just two countries. Gorman (1958) shows that Johnson's model generalizes since its preference structure is a special case of the 'Gorman polar form.' Tower (1975) analyzes a trade war under alternative strategic instruments (quotas). Otani (1980) establishes general conditions for existence of Nash equilibrium in a tariff war. Optimum tariffs have been studied in models of more than two countries using numerical simulations and in models of preferential trade agreements (which usually use models of three countries). However, these literatures take country numbers as a given; they do not study the relationship investigated in the present paper between the underlying structure of the world economy in terms of country numbers and the outcome of a tariff war. Gros (1987) studies a setting where trade is based on monopolistic competition and all countries are identical to each other; he shows that optimum tariffs may be positive even when countries are made very small. Kennan and Riezman (1988) study how country size affects the outcome of a tariff war where the sizes of two countries are varied by varying their endowments of goods. Syropoulos (2002) identifies the proximate link between goods production and underlying factor endowments, which then enables him to investigate the variation of country size by undertaking radial expansions of a country's production possibility frontier.

in our model. The first step in establishing this result is to obtain conditions under which the tariff game between countries in one region is supermodular. To put this in equivalent terms more familiar from trade theory, tariffs of countries *within* a region can be identified as strategic complements; if one country lowers its tariff then it is a best response for all other countries in that region to lower their tariffs as well. This property, coupled with standard H-O assumptions, gives rise to an equilibrium in which all countries in a region (simultaneously and without communicating) set their tariffs at the same rate. Thus we are able to characterize a 'Northern tariff reaction function' and a 'Southern tariff reaction function'. The second step is to show that the tariff game *across* regions is submodular; tariffs across regions are shown to be strategic substitutes. This result can be regarded as a generalization of the standard two-country-model result. Finally, general equilibrium occurs at the intersection of the Northern and Southern tariff reaction functions.[7]

We can then perform comparative statics by varying the numbers of countries in each region and seeing how this affects equilibrium tariffs across the regions. Using the standard two-country model as a benchmark, say we increase the number of countries in one region, the South for example, holding the number of countries in the other region constant. This corresponds to an increase of the world-economy's total endowment of labor relative to capital. The second main result of the paper is that an increase in the number of countries in the South worsens their terms of trade and hence welfare, and improves the terms of trade and hence welfare of the countries in the North. The intuition for the tariff externality at work is as follows. Say there is an increase in overall Southern demand for imports from the North, whether due to a tariff reduction by a Southern country that is already engaged in trade or by one that we are introducing to the world economy through a tariff reduction from prohibitive levels. In any case it is a best response for all other countries in that region to try to restore import share by lowering their tariffs as well.

It is this result that contrasts most markedly with the previous literature. The *policy-induced terms-of-trade effect* of an increase in world factor endowments is the opposite in the present model to that of a standard two-country model as set out by Syropoulos (2002).

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[7]The general structure of the model allows for an 'ordering' of equilibria; this will be explained when the tariff game is analyzed.

For comparison, let us consider the same increase in the world economy's endowment of labor relative to capital in the standard two-country model that we just considered for our model.[8] Of course, under free trade, the direct effect on world prices of a given change in underlying factor endowments would be the same in the two models. Syropoulos (2002) establishes the opposite policy-induced terms-of-trade effect to ours; increasing the world relative endowment of labor by increasing the size of the labor abundant country improves its terms of trade and hence welfare. Correspondingly, the terms of trade and welfare of the other country is worsened.

Our second result has a simple corollary which provides an interesting twist on a long-running debate in the literature on optimal-tariffs; that of whether big countries 'win' tariff wars. Syropoulos (2002) establishes that under relatively weak conditions a limit exists on relative country size beyond which the big country does win a tariff war in the sense that it is better off when both countries set optimum tariffs than under free trade. The corollary to our second result is that if the asymmetry between the number of countries grows sufficiently large then the countries in the small region must unambiguously 'win' a tariff war. The intuition is easy to see. Consider a situation where there is a single country in one region and a very large number of countries in the other region. Then in equilibrium the single country sets a positive best-response tariff and the countries in the other region adopt a tariff that approaches free trade. By the Prisoner's Dilemma structure of the underlying tariff game, the country that is alone in its region must be better off than if it had adopted free trade.

We will also show, in the paper's third main result, that the tariff externality can increase efficiency if both regions are increased in size simultaneously; the number of countries is increased simultaneously while holding the ratio of countries across regions constant. Optimum tariffs in both regions decrease monotonically and, in the limit as country numbers in both regions become large, trade volumes and welfare levels converge to efficient/free trade levels.

This result is reminiscent of the one shown by Bond and Syropoulos (1996) in their model of CU formation. In the symmetric version of their model, they consider world in

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[8]Note that in our model all countries are of identical size, so an increase in the world relative endowment of a given factor can only be brought about through an increase of the number of countries in the region that is abundant in that factor.

which a given number of countries is shared between an increasing number of CUs. They push this exercise to the point where each CU is infinitesimally small relative to the rest of the world. This is different from our replication exercise but shares the feature that each trading unit - in their case a CU and in our case a country - becomes small relative to the rest of the world. In contrast to our result, countries in their model continue to set positive optimum tariffs; each CU continues to export a horizontally differentiated product over which it retains a degree of market power even as the CU becomes small. The feature of the outcome that their result shares with ours is that world welfare converges to the free trade level as, in the limit, each country's trade share becomes insignificant.

There are clear parallels between a change in country numbers as analyzed in the present paper and the process of CU formation. The difference is that here a change in country numbers changes underlying world factor endowments while under CU formation world factor endowments are held constant. Under CU formation, governments agree to remove mutual tariffs and to coordinate on the setting of a common external tariff. From the perspective of trade-policy-making this is observationally equivalent to the merging of the countries that form the CU into a single country, thus in effect reducing the number of countries. This suggests that a change in country numbers could have been analyzed in the present paper as isomorphic to CU formation. In effect, this would involve changing country numbers while at the same time keeping the world endowment of factors fixed, requiring a combination of our analysis and that of Syropoulos (to make each of the countries in the expanded region smaller until the original world factor endowments had been restored). Combining the framework of the present paper with that of Syropoulos is beyond the scope of the present paper but presents an interesting agenda for future research.[9]

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[9]Bagwell and Staiger's (1997) 'competing exporters' model offers another useful way to see the parallels between CU formation and the effect of changing country numbers considered in the present paper. Their model, like ours, has two regions with identical countries in each that can set optimum tariffs. Also, like in our model, the countries within a region do not trade with each other but 'compete' to export a good to the other region and import a different good from the other region. To facilitate their consideration of CU formation and how it affects the scope of multilateral trade liberalization, they use a more restrictive model structure than ours. The key differences are as follows: their regions are symmetric to each other, yet the effects of CU formation that they identify are similar to those identified in our third result on increasing the number of countries in both regions simultaneously; tariffs are independent across regions in their model whereas in our framework in general they are strategic substitutes, giving rise to an effect that is important for our welfare analysis; demand is linear in their framework whereas in ours it may be nonlinear, allowing income effects of tariff-setting to be taken into account.

The tariff externality that we will study is supported by recent empirical work in international trade with protectionism, and in turn the present study may contribute towards future attempts to better identify optimum tariff setting in the data. Broda, Limão and Weinstein (2008) find a positive relationship between inverse export supply elasticities and import tariffs, thus providing empirical support for the fundamental relationship that motivates optimum tariff setting.[10] Potentially complementary to their findings, the model of the present paper offers a new testable prediction. The factor of proportionality between inverse export supply elasticities and import tariffs that is identified empirically by Broda et al is lower than that predicted by the standard two-country model. Taking account of country numbers may offer a resolution. Interpreted within the context of our model, a lower optimum tariff arises as a result of more intense competition in tariff setting by a larger number of similar countries. This could be tested using an extension to the empirical methodology developed by Broda et al. that takes account of the number of countries exporting a particular good.

The tariff complementarity between countries within a region identified in the present paper is based on the 'deflection' of imports; if one country raises its tariff then imports are deflected to other countries in the region, giving them an incentive to raise their tariffs as well. Recent empirical work by Bown and Crowley (2006, 2007) provides evidence of such trade deflection consistent with the externality identified in the present paper. Their work focuses entirely on trade among Northern countries. The present framework offers a way to extend this line of empirical work to take account of the deflection of trade between North and South. Additional ideas for testing the predictions of the model against the data are discussed in the conclusions.

The paper proceeds as follows. In the next section, we will set up the basic model. We will then analyze the properties of this model in Section 3, laying the foundations we will need to characterize equilibrium in Section 4 and carry out comparative statics (a variation in the number of countries), thus identifying the tariff externality. In Section 5, the behavior of the general model is illustrated using two standard examples; assumptions introduced in Section 4 are shown to hold for the examples of Section 5 under certain

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[10]Our analysis will focus instead on the (inverse) relationship between the import demand elasticities and the optimum tariff; the equivalence of these relationships is easily demonstrated.

restrictions to preferences and the 'degree of comparative advantage'. These will be explained fully in due course. The efficiency and equity implications of the framework are examined further using a simple example as well. Conclusions and links to the literature are drawn in Section 6. The proofs of all results can be found in the Appendix.

2. A North-South Model with Many Countries

We will extend a standard general model of international trade. In the standard model there are two countries, each of which produces two goods using two factors. The two goods, manufactures and primary products, are homogeneous and are intensive in capital and labor respectively. Technology is the same across countries. Our model is identical except that there may be more than two countries.[11]

The set of all countries in the world economy, N, is divided into two subsets: The North, denoted by A, consists of the set of countries $A = \{1, 2, ..., a, ..., x\}$; the South, denoted by B, consists of the set of countries $B = \{x + 1, x + 2, ..., b, ..., x + y\}$. There are x countries in the North and y countries in the South. To put the model in a familiar context we will fix $x < y$, although this is not essential to the analysis. Following notation from the matching literature, a will be the representative Northern country and b will be the representative Southern country. Where convenient, we will use i to refer to any country in N. As mentioned in the introduction, there is assumed to be a (potentially large) number of countries outside the world economy operating in autarky by assumption. Thus when we vary x and y we are moving countries between autarky and the world economy, and we are effectively bringing productive factors, capital and labor, into and out of the world economy.

Manufactures will be labeled as good 1 and primary products good 2. Unless otherwise stated, a country will be denoted by a superscript and a good will be denoted by a subscript. Country i has an inelastically supplied vector of factor inputs $\mathbf{v}^i = \{k^i, l^i\}$.

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[11]With the assumption that the good produced in the manufacturing sector is homogeneous, the model departs from much of the recent research in international trade. (It is standard to assume that the primary product is homogeneous.) The aim is to focus attention specifically on variation in outcomes due to variation in country numbers, abstracting from issues of market structure. In the conclusions we will discuss possible extension to a framework in which the manufacturing sector produces differentiated products.

Each country $i \in A$ is endowed with one unit of capital and β units of labor, where $\beta \in [0, 1]$. Each country $i \in B$ is endowed with one unit of labor and β units of capital. In Section 5, a simple formalization of the 'degree of comparative advantage' will be developed based on the parameter β. For now, the point of emphasis is that all countries are normalized to be the same size in the sense that $k^i + l^i = 1 + \beta$ for all $i \in N$ while Northern (Southern) countries have a relatively large endowment of capital (labor). For consistency with the normalization over the size of countries, let us assume that goods enter preferences symmetrically for all countries $i \in N$.

2.1. A Single Country

In this subsection we will define a tariff distorted equilibrium for country i, taking tariffs and world prices as given. Each country has a population of citizens whose mass is normalized to unity; citizens behave as price-takers in their dual roles as consumers and producers.

Technology exhibits constant returns to scale. All markets are perfectly competitive and there are no domestic distortions. The utility function is strictly quasi-concave in goods 1 and 2 respectively. Tastes are identical and homothetic. So we can formulate the analysis in per-capita terms without loss of generality. Consequently, we can fix the utility level at μ^i and solve for the minimum level of expenditure required to meet that level. Let p_j^i be the domestic price of good $j \in \{1, 2\}$ in country i. Denote with $E^i \left(p_1^i, p_2^i, \mu^i \right)$ and $R^i \left(p_1^i, p_2^i, \mathbf{v}^i \right)$ country i's expenditure and revenue functions respectively. Both functions are assumed to be twice continuously differentiable in their arguments. So the compensated (Hicksian) demand function for good j is $c_j^i = E_{p_j^i}^i \left(= \partial E^i / \partial p_j^i \right)$, where $\partial c_j^i / \partial p_j^i = E_{p_j^i p_j^i}^i \leq 0$ by the concavity of the expenditure function in prices. The derivation of the supply function for good j from the revenue function is analogous.

Let p^i be the domestic relative price of country i's importable measured in terms of its exportable and let q^i be the world relative price of the same good.[12] Let t^i be country i's ad valorem import tariff and assume that this is the only available policy instrument

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[12]Under the trade patterns that will emerge in equilibrium, domestic relative prices may be denoted $p^a = p_2^a / p_1^a$ and $p^b = p_1^b / p_2^b$ respectively; world relative prices are denoted $q^a = 1/q^b = q_2/q_1$.

and let $\tau^i \equiv (1 + t^i)$.[13] Then we can express the relationship between domestic and world relative prices for country i as

$$p^i = \tau^i q^i, \ i \in A, B. \tag{2.1}$$

Let m^i denote country i's imports. Country i's import demand function is as follows:

$$m^i = E^i_{p^i} - R^i_{p^i}, \ i \in A, B. \tag{2.2}$$

This equation defines country i's import demand function in terms of excess demand. Since country endowments remain fixed throughout, we can suppress $\mathbf{v}^i$ from national revenue functions. Thus, the budget constraint for country i may be written as follows:

$$E^i \left(p^i, \mu^i \right) = R^i \left(p^i \right) + \left(\tau^i - 1 \right) q^i m^i, \ i \in A, B. \tag{2.3}$$

Each country's expenditure is equal to its income, which is derived from output plus tariff revenue. Taking τ^i and q^i as given, a (tariff distorted) competitive equilibrium is established in the domestic economy using equations (2.1)-(2.3).

2.2. The World Market

We will now define equilibrium in the world market in terms of a symmetric *general tariff game*; the tariff game played by all countries. Let us begin by introducing some notation. The vector of all tariffs, $\boldsymbol{\tau} = \left(\tau^1, ..., \tau^a, ..., \tau^x, \tau^{x+1}, ..., \tau^b, ..., \tau^{x+y} \right)$, can be partitioned into tariffs set by Northern countries, denoted by $\boldsymbol{\tau}^a$, and tariffs set by Southern countries, denoted by $\boldsymbol{\tau}^b$; $\boldsymbol{\tau}^a = (\tau^1, ..., \tau^a, ..., \tau^x)$ and $\boldsymbol{\tau}^b = \left(\tau^{x+1}, ..., \tau^b, ..., \tau^{x+y} \right)$ respectively. It will also be helpful to have notation for tariffs set by all countries in A *except* country a, which we will denote by $\boldsymbol{\tau}^{-a}$, and tariffs set by all countries in B *except* country b, denoted by $\boldsymbol{\tau}^{-b}$.

The extensive form of the symmetric general tariff game is as follows. First, each country i (simultaneously and without communicating - this is the sense in which the game is symmetric) chooses its import tariff τ^i. Then, given world prices $\mathbf{q} = \{q_1, q_2\}$ and tariffs, perfect competition in production takes place. Next, the representative consumer

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[13]Throughout the paper, for brevity, we will refer to τ^i as a 'tariff' although strictly it is a 'tariff factor' that takes a value $\tau^i \geq 1$ when $t^i \geq 0$. We will restrict attention to non-negative tariffs since, given the structure of our model, best-response tariffs will always be non-negative.

in each country chooses consumption to maximize budget constraints. This yields the usual indirect utility functions and excess demands. Then, conditional on tariffs, $\boldsymbol{\tau}$, markets clear and world prices $\mathbf{q}$ are determined. These world prices will of course depend on tariffs, i.e. $\mathbf{q} = \mathbf{q}(\boldsymbol{\tau})$, as will tariff revenues. If equilibrium prices are unique, given tariffs, then the mapping $\mathbf{q}(.)$ is one-to-one. Then the indirect utility function for country i, which we will denote with $u^i(\cdot)$, can be written as a function only of tariffs: $u^i(\mathbf{q}(\boldsymbol{\tau}), \tau^i)$. The corresponding expenditure function is obtained by inversion of $u^i(\cdot)$.[14]

World prices are determined in the usual way through the world market clearing condition. By the H-O Theorem and our assumptions which conform to an H-O framework we know that, in free-trade equilibrium, the countries in A will import good 2 while the countries in B will import good 1. It follows that the same pattern of trade will be maintained if at least one country from each region sets non-prohibitive tariffs.[15] Consequently, we have the following simple expressions for the aggregate value of imports to each region: $q_2 M^A = \sum_{i \in A} q_2 m^i$; $q_1 M^B = \sum_{i \in B} q_1 m^i$. Let good 1 be the numeraire, $q_1 = 1$, and let $q_2 = q$. Then we may write the world market clearing condition as follows:

$$qM^A = M^B. \tag{2.4}$$

The function for each element m^i is continuous in q^i and τ^i so the world market clearing condition implicitly defines a mapping from the vector of tariffs to the vector of world prices.

2.3. Equilibrium and Efficiency

In a *trading equilibrium*,

$$M^A > 0 \text{ and } M^B > 0.$$

If the equilibrium is not a trading equilibrium then it is *autarkic*.

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[14]Equivalently, as we shall see, indirect utility may be written as a function of tariffs and domestic prices. Transfers are not allowed between countries. While transfers do happen in practice, their size is generally constrained to be small by extraneous factors having to do with political viability.

[15]Perverse patterns of trade may be possible under extreme tariff settings where some countries subsidize imports. While such possibilities may be interesting, they will not have a bearing on the type of symmetric equilibrium we consider so we will rule them out.

We can now define a *trading equilibrium in tariffs* as a vector of tariffs

$$\overline{\boldsymbol{\tau}} = \left(\overline{\tau}^1, ..., \overline{\tau}^x, \overline{\tau}^{x+1}, ..., \overline{\tau}^{x+y} \right),$$

such that

(i) for any country $a \in A$, $\overline{\tau}^a$ maximizes $u^a \left(\mathbf{q} \left(\tau^a, \overline{\boldsymbol{\tau}}^{-a}, \overline{\boldsymbol{\tau}}^b \right), \tau^a \right)$;

(ii) for any country $b \in B$, $\overline{\tau}^b$ maximizes $u^b \left(\mathbf{q} \left(\tau^b, \overline{\boldsymbol{\tau}}^{-b}, \overline{\boldsymbol{\tau}}^a \right), \tau^b \right)$.

Standard arguments can be used to prove existence of an equilibrium, which is just a Nash equilibrium of the symmetric general tariff game.

Following standard definitions, *world welfare* is the sum of all national welfares, and *world efficiency* is given by

$$\max_{\boldsymbol{\tau}} \sum_{i \in A} u^i \left(\mathbf{q} \left(\boldsymbol{\tau} \right), \tau^i \right) + \sum_{i \in B} u^i \left(\mathbf{q} \left(\boldsymbol{\tau} \right), \tau^i \right).$$

Since there are no domestic distortions in the model, world efficiency corresponds to world free trade.

3. Analysis of the Model

Our approach will be to characterize the reaction functions of the representative countries, a and b, from which we will be able to define regional reaction functions for the North and the South. The aim will then be to analyze how each regional reaction function is affected by a change in x and y. In this section we prepare the ground by examining the economic properties of the model, from which we will be able to define concretely the payoffs to the general tariff game.

3.1. A Single Country

Analysis of the single country model follows Syropoulos (2002). First, we need some notation. Country i's expenditure share on its importable may be written as $\sigma_E^i \equiv p^i E_{p^i}^i / E^i$ and the share of revenue contributed by production of good j as $\sigma_R^i \equiv p^i R_{p^i}^i / R^i$. By the homotheticity of consumer preferences, we may write $E^i \left(p^i, \mu^i \right)$ as $e^i \mu^i$ where

$e^i = e(p^i)$. Also, let $T^i \equiv (\tau^i - 1)/(\tau^i)$, where $T^i \in [0, 1)$. Then from equations (2.1)-(2.3), the solution for country i's indirect utility function is as follows:

$$u^i = u^i\left(p^i, T^i\right) = \mu^i = \frac{R^i\left(1 - T^i\sigma^i_R\right)}{e^i\left(1 - T^i\sigma^i_E\right)}, \quad i \in A, B. \tag{3.1}$$

In (3.1), diversification of production requires that $0 < 1 - T^i\sigma^i_R \leq 1$.

And the solution for m^i is as follows:

$$m^i = m^i\left(p^i, T^i\right) = \frac{R^i\left(\sigma^i_E - \sigma^i_R\right)}{p^i\left(1 - T^i\sigma^i_E\right)}, \quad i \in A, B. \tag{3.2}$$

Since $1 - T^i\sigma^i_E > 0$, (3.2) implies that, as long as revenue and domestic relative prices are finite in equilibrium, if $\sigma^i_E - \sigma^i_R > 0$ then $m^i > 0$. In words, to import good j country i's expenditure share on good j must exceed its revenue share.

3.2. Tariffs, Terms of Trade and Welfare

The aim of this subsection is to characterize the effect of a tariff change on the terms of trade and welfare. The approach is adapted from Syropoulos (2002) to the present many-country environment. Let $q^a = q$ and $q^b = 1/q$ represent the respective terms of trade of countries a and b in the world market. We will adopt the convention of "hat algebra" that a "^" over a variable denotes a proportional change. So $\hat{q} < (>) 0$ is an improvement in the Northern (Southern) terms of trade and $\hat{m}^a$ and $\hat{m}^b$ are proportional changes in imports of countries a and b respectively. The relationship between the terms of trade and welfare may then be established by totally differentiating (2.3) for countries a and b respectively:

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\begin{aligned}
d\mu^a &= \frac{q^a m^a}{E^a_{\mu^a}}\left(-\hat{q} + (\tau^a - 1)\hat{m}^a\right); \\
d\mu^b &= \frac{q^b m^b}{E^b_{\mu^b}}\left(\hat{q} + (\tau^b - 1)\hat{m}^b\right).
\end{aligned}
\tag{3.3}
$$

The two terms in brackets capture the terms-of-trade and volume-of-trade effects of tariff changes on welfare respectively. The optimum tariff is the tariff that just balances these two effects: the value of τ^i that sets $d\mu^i = 0$. To obtain a reduced form for the right-hand-sides of these expressions, we now need to solve for $\hat{q}$, $\hat{m}^a$ and $\hat{m}^b$.

Let us start by solving for $\hat{q}$. To do so, denote by s^i the *share of total imports to its region received by country i*; this share is given by $s^i = m^i / M^I$, where, if $i \in A$ then $I = A$ and if $i \in B$ then $I = B$. Totally differentiating the world market clearing condition, (2.4), we obtain

$$\hat{q} + s^1 \hat{m}^1 + ... + s^a \hat{m}^a + ... + s^x \hat{m}^x \tag{3.4}$$
$$= s^{x+1} \hat{m}^{x+1} + ... + s^b \hat{m}^b + ... + s^{x+y} \hat{m}^{x+y}.$$

From this solution we see that in order to obtain a relationship between $\hat{q}$ and individual country tariffs we need to solve for each $\hat{m}^i$, $i \in N$.

To solve for $\hat{m}^i$, totally differentiate the definition of imports, (2.2), and use (2.1) to yield

$$\hat{m}^i = \frac{p^i E^i_{p^i p^i} - p^i R^i_{p^i p^i}}{E^i_{p^i} - R^i_{p^i}} \hat{p}^i + \frac{1}{\tau^i q^i m^i} \left(\frac{p^i E^i_{p^i \mu^i}}{E^i_{\mu^i}} \right) E^i_{\mu^i} d\mu^i.$$

Next note that the compensated price elasticity of import demand takes the form

$$\eta^i = -\left(p^i E^i_{p^i p^i} - p^i R^i_{p^i p^i} \right) / \left(E^i_{p^i} - R^i_p \right) > 0.$$

Using η^i, and recalling that $\sigma^i_E \equiv p^i E^i_{p^i} / E^i$, we can then write the total derivative of (3.2) for countries a and b respectively as follows:

$$\hat{m}^a = -\frac{\eta^a}{1 - T^a \sigma^a_E} \hat{p}^a - \frac{\sigma^a_E / \tau^a}{1 - T^a \sigma^a_E} \hat{q}; \tag{3.5}$$

$$\hat{m}^b = -\frac{\eta^b}{1 - T^b \sigma^b_E} \hat{p}^b + \frac{\sigma^b_E / \tau^b}{1 - T^b \sigma^b_E} \hat{q}. \tag{3.6}$$

The first term on the right hand side measures the effect of a change in domestic prices on imports through substitution of goods in consumption and production. The second term is an income effect. If a country's terms of trade improve then that country imports more from the other region due to the fact that its income has risen and both goods are normal.

Denote by ε^i the *direct (Marshallian) price elasticity of import demand*; by definition, $\varepsilon^i \equiv -\hat{m}^i / \hat{p}^i \, (> 0)$.[16] Letting $\gamma^i \equiv \eta^i / \left(1 - T^i \sigma^i_E \right)$ and $\phi^i \equiv \left(\sigma^i_E / \tau^i \right) / \left(1 - T^i \sigma^i_E \right)$ we can

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[16]To avoid the possibility of unnecessary confusion, note that this definition of ε^i depends on the definition of m^i in (3.2) as $m^i \left(p^i, \tau^i \right)$; it would also be correct if we wrote $m^i \equiv m^i \left(q, \tau^i \right)$, but not if $m^i \equiv m^i \left(p^i, q \right)$.

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decompose ε^i as follows: $\varepsilon^i = \gamma^i + \phi^i$. We can then use these expressions to rewrite (3.5) and (3.6):

$$\hat{m}^a = -\varepsilon^a \hat{q} - \phi^a \hat{\tau}^a; \tag{3.7}$$

$$\hat{m}^b = \varepsilon^b \hat{q} - \phi^b \hat{\tau}^b. \tag{3.8}$$

These expressions capture, for the representative Northern and Southern country respectively, the terms-of-trade effect and direct tariff effect on imports.

We can now establish the link between tariffs and the terms of trade. Using (3.7) and (3.8) in (3.4), and rearranging, we obtain

$$\hat{q} = s^{x+1} \frac{\phi^{x+1}}{\Delta} \hat{\tau}^{x+1} + ... + s^b \frac{\phi^b}{\Delta} \hat{\tau}^b + ... + s^{x+y} \frac{\phi^{x+y}}{\Delta} \hat{\tau}^{x+y} \tag{3.9}$$
$$-s^1 \frac{\phi^1}{\Delta} \hat{\tau}^1 - ... - s^a \frac{\phi^a}{\Delta} \hat{\tau}^a - ... - s^x \frac{\phi^x}{\Delta} \hat{\tau}^x,$$

where $\Delta \equiv \left(\sum_{i \in A} s^i \varepsilon^i + \sum_{i \in B} s^i \varepsilon^i - 1 \right)$. We assume that $\Delta > 0$; the Marshall-Lerner condition for world market stability holds. Equation (3.9) is obviously not a closed form solution for $\hat{q}$ since each s^i depends on q^i. But since all right-hand-side variables are continuous in their arguments, we can use this expression to evaluate the impact of a change in τ^i on $\hat{q}$.

By the implicit function theorem, we have the familiar property that an increase in a country's tariff rate causes its terms of trade to improve. This property carries over from the standard two-country model. But (3.9) also shows how each country's power to influence the terms of trade using tariffs is proportional to its share of imports to the region. In the standard two country model this share would be fixed at one. Here it is determined endogenously. As mentioned above, in symmetric equilibrium, the import share of each country will be inversely proportional to the number of countries in the region.

This feature of the model, that a country's power to influence its terms of trade is proportional to its share of imports to the region, is the source of the tariff externality between countries within a region. Later we will see that if one country in a region lowers its tariff then (in the absence of coordination) all other countries in that region have an incentive to lower their tariffs as well. But from (3.9) we can already see how this would lead them to collectively undermine their own terms of trade.

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We can also work out the impact of tariffs on domestic relative prices. Using (2.1) to determine p^i and then differentiating with respect to tariffs and substituting for $\hat{q}$ using (3.9) yields

$$\hat{p}^a = \left(1 - s^a \frac{\phi^a}{\Delta}\right) \hat{\tau}^a - \sum_{i \in A \setminus a} s^i \frac{\phi^i}{\Delta} \hat{\tau}^i + \sum_{i \in B} s^i \frac{\phi^i}{\Delta} \hat{\tau}^i; \tag{3.10}$$

$$\hat{p}^b = \left(1 - s^b \frac{\phi^b}{\Delta}\right) \hat{\tau}^b - \sum_{i \in B \setminus b} s^i \frac{\phi^i}{\Delta} \hat{\tau}^i + \sum_{i \in A} s^i \frac{\phi^i}{\Delta} \hat{\tau}^i. \tag{3.11}$$

From these equations we see that, in the absence of Meltzer's tariff paradox, $\hat{p}^a$ depends positively on τ^a and τ^i, $i \in B$, but negatively on τ^i, $i \in A \setminus a$. The effect of τ^a on $\hat{p}^a$ operates directly through (2.1). The effect of τ^i, $i \in B$, operates by bringing about an increase in $\hat{q}$; see (3.9). The effect of τ^i for $i \in A \setminus a$ operates by reducing $\hat{q}$; again see (3.9). The effects of tariffs on $\hat{p}^b$ are analogous.

By substituting (3.7) and (3.9) into the first equation in (3.3), we can derive a decomposition of the change in welfare for country a as a function of tariff changes across all countries:

$$d\mu^a = \frac{q^a m^a}{E_{\mu^a}^a} \left[\left(s^a - \left(\tau^a - 1\right) \Delta^a\right) \frac{\phi^a}{\Delta} \hat{\tau}^a \right. \tag{3.12}$$

$$\left. + \left(1 + \left(\tau^a - 1\right) \varepsilon^a\right) \left(\sum_{i \in A \setminus a} s^i \frac{\phi^i}{\Delta} \hat{\tau}^i - \sum_{i \in B} s^i \frac{\phi^i}{\Delta} \hat{\tau}^i \right) \right].$$

Analogously for country b,

$$d\mu^b = \frac{q^b m^b}{E_{\mu^b}^b} \left[\left(s^b - \left(\tau^b - 1\right) \Delta^b\right) \frac{\phi^b}{\Delta} \hat{\tau}^b \right. \tag{3.13}$$

$$\left. + \left(1 + \left(\tau^b - 1\right) \varepsilon^b\right) \left(\sum_{i \in B \setminus b} s^i \frac{\phi^i}{\Delta} \hat{\tau}^i - \sum_{i \in A} s^i \frac{\phi^i}{\Delta} \hat{\tau}^i \right) \right].$$

Country a's best-response tariff function can be obtained by setting the first term inside the square brackets of (3.12) equal to zero and solving for τ^a; this is the problem solved in Proposition 1 below. (This is equivalent to calculating $du^a/d\tau^a$, then setting the result equal to zero and solving for τ^a.) Country b's best-response tariff function can be obtained in an equivalent way using (3.13).

Equations (3.12) and (3.13) show the impact of the tariff externality that we identify between the countries in a region on welfare. For example, equation (3.12) shows that country a would gain from coordination of tariffs between all countries in A (since, for all $i \in A \backslash a$, τ^i enters with a positive coefficient); conversely, country a's welfare is undermined by any failure to coordinate. This feature will be evident in the characterization of equilibrium that follows.

4. Characterization of Equilibrium

In principle, the general tariff game described here is quite complex since many countries play strategies against one another simultaneously (and without communication). Following Amir and Bloch (2009), we can simplify the problem by characterizing the tariff game played by countries in one region, say the North, taking as given the tariffs of countries in the other region.[17] After having provided a simple characterization for the equilibrium of that game, it will then be straightforward to show that this characterization of equilibrium holds for both regions simultaneously, thus yielding an equilibrium of the general tariff game.

The formal procedure is as follows. For given parameters, x and y, characterize equilibrium of the (symmetric) *tariff game of the North*, $\Gamma\left(\tau^b\right)$, taking as given the tariff vector τ^b of countries in the South (where τ^b is fixed arbitrarily). In this way, obtain an equilibrium tariff vector $\overline{\tau}^a$. Then characterize equilibrium for the (symmetric) *tariff game of the South*, $\Gamma\left(\tau^a\right)$, to obtain the equilibrium tariff vector, $\overline{\tau}^b$, taking as given the tariff vector τ^a. Since τ^a is arbitrarily chosen, we may use the equilibrium tariff vector $\overline{\tau}^a$ when obtaining $\overline{\tau}^b$. In that case, $\overline{\tau}^b$ must be a best response to $\overline{\tau}^a$. Finally, check that $\overline{\tau}^a$ is a best response to $\overline{\tau}^b$. If so, $\left(\overline{\tau}^a, \overline{\tau}^b\right)$ are a pair of mutual best-response tariff vectors and, by the usual definition, we have a Nash equilibrium of the general tariff game.

We will restrict attention to proving existence of a *symmetric Nash equilibrium* of the general tariff game; an equilibrium in which all countries in the North set the *same*

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[17]In the symmetric Cournot oligopoly analyzed by Amir and Lambson (2000), the optimal quantity choice of an oligopolist depends on the total output of the $(n-1)$ remaining firms. Amir and Bloch extend this approach to a market-game setting in which there are two sides to the market, where a 'side of the market' in their model is similar to a region in ours.

equilibrium tariff, $\overline{\tau}^a$, and all countries in the South set the *same* equilibrium tariff, $\overline{\tau}^b$ (but where $\overline{\tau}^a \neq \overline{\tau}^b$ if the number of countries in the North is different from the number in the South).[18] To characterize this equilibrium, we follow the same procedure as above but with the following additional restriction. In the tariff game of the North, $\Gamma\left(\boldsymbol{\tau}^b\right)$, instead of allowing all tariffs in the vector $\boldsymbol{\tau}^b$ to be set at different levels, we fix them at the *same level*, τ^b, for all countries in the South (where τ^b is chosen arbitrarily but fixed at a non-prohibitive level). In this case, the tariff game of the North is denoted $\Gamma\left(\tau^b\right)$. The aim is then to prove existence of an equilibrium in which all Northern governments set their tariffs at the same level, $\overline{\tau}^a$. We then characterize the symmetric equilibrium of the tariff game of the South, $\Gamma\left(\tau^a\right)$, using $\overline{\tau}^a$. Thus we obtain $\overline{\tau}^b$ that is a best response to $\overline{\tau}^a$. If we are able to show that $\left(\overline{\tau}^a, \overline{\tau}^b\right)$ are a pair of mutual best responses then we have characterized a symmetric Nash equilibrium of the general tariff game.[19]

The following notation will be helpful in the determination of equilibrium conditions. Let $\theta^i \equiv \hat{\varepsilon}^i / \hat{p}^i$ and let $\zeta^i \equiv \hat{\varepsilon}^i / \hat{\tau}^i$. We now introduce three assumptions: the first governs θ^i; the second governs ζ^i; the third governs the relationship between ε^i and θ^i. These will be imposed as standing assumptions throughout the analysis.

A1. $\theta^i > 0$ for all $i \in A, B$.

A1 was first introduced by Corden (1984) and later adopted by Syropoulos (2002). The assumption that $\theta^i > 0$ is sufficient, in a conventional two-country model, to ensure the quasi-concavity of country i's payoff function in its own tariff. The present many-country setting calls for a stronger condition to achieve the same objective, which will be obtained by the conjunction of A1 and A3.

A2. $\zeta^i \geq 0$.

A2 is standard in the literature. This assumption constrains the (partial) effect of a

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[18]There may exist interesting asymmetric equilibria of this game but we will not study them in the present paper.

[19]Note that there is no further loss of generality, beyond the fact that we restrict attention to a symmetric equilibrium, in the assumption that all countries in the South set the same tariff τ^b in $\Gamma\left(\tau^b\right)$. The reason is that τ^b is introduced *after* the first order condition of government a's objective function is found.

country's tariff increase on its own-price elasticity of import demand.[20]

A3. $\varepsilon^i \geq \theta^i$ for all $i \in A, B$.

This stronger condition on θ^i, which in conjunction with A1 implies that $\varepsilon^i \geq \theta^i > 0$, is required to bound the change in a country's own price elasticity of import demand due to a tariff change by some other country $j \neq i$ in the same region. As suggested above, its main role is to ensure the quasi-concavity of country i's payoff function in its own tariff.

It would have been preferable to impose conditions on the primitives of the model rather than on the derived objects ε^i, θ^i and ζ^i. But to do so would require us to impose conditions on third derivatives of the expenditure function and revenue function, conditions for which economic theory gives us no guidance. Our approach will instead be to first establish, in the present section, the usefulness of A1-A3, as well as A4 which we will introduce below. Then, in Section 5 we show that A1-A4 are satisfied under reasonable conditions for two standard examples, one based on Cobb-Douglas preferences and the other on constant elasticity of substitution (C.E.S.).

4.1. Characterization of Equilibrium for The Tariff Game of the North

We will now characterize country a's *best-response tariff function*, $\tilde{\tau}^a \left(\boldsymbol{\tau}^{-a}, \tau^b \right)$. Let

$$\Delta^a \equiv \sum_{i \in A \backslash a} s^i \varepsilon^i + \sum_{i \in B} s^i \varepsilon^i - 1,$$

(and equivalently for b, let $\Delta^b \equiv \sum_{i \in B \backslash b} s^i \varepsilon^i + \sum_{i \in A} s^i \varepsilon^i - 1$). Then we have the following simple characterization of $\tilde{\tau}^a \left(\boldsymbol{\tau}^{-a}, \tau^b \right)$.

Proposition 1. *In the game* $\Gamma \left(\tau^b \right)$*, the best-response tariff function for country* a*,* $\tilde{\tau}^a \left(\boldsymbol{\tau}^{-a}, \tau^b \right)$*, is the solution to the equation*

$$\tau^a = \frac{s^a}{\Delta^a} + 1. \tag{4.1}$$

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[20]Syropoulos' (2002) Lemma 1 shows that $\zeta^i \gtreqless 0$ as $\sigma^i \gtreqless 1$, where σ^i is the elasticity of substitution in consumption. If C.E.S. preferences are written as $u^i = \left((1 - \alpha) \left(x_1^i \right)^\chi + \alpha \left(x_2^i \right)^\chi \right)^{\frac{1}{\chi}}$, then $1 \geq \chi \geq 0$ implies $\sigma^i = 1/(1 - \chi) \geq 1$.

We can see by inspection that Proposition 1 generalizes the solution for the best-response tariff in a setting where there are just two countries with one in each region. In that case, by assumption, $s^a = 1$, $s^i = 0$ for $i \in A \backslash a$, $s^b = 1$, and $s^i = 0$ for $i \in B \backslash b$. Then $\Delta^a = \varepsilon^b - 1$ and we would obtain the familiar expression for the best-response tariff; $\tilde{\tau}^a = \varepsilon^b / (\varepsilon^b - 1)$.

From Proposition 1 we get an indication of how the tariff externality between countries in the North could affect the best-response tariff of country a. In a symmetrical equilibrium for the tariff game of the North, s^i must be the same for all $i \in A$ and must be declining in the number of countries in A. Since, by Proposition 1, the best-response tariff must satisfy $\tau^a = s^a / \Delta^a + 1$, we see that a sufficient condition for $\tilde{\tau}^a$ to decrease with the number of countries in the North would be that $d\Delta^a / d\tau^i \leq 0$. The impetus for a reduction in $\tilde{\tau}^a$ would come from a reduction in s^a as country numbers were increased. This in turn would be reinforced by a consequent increase in Δ^a. Thus we have the rationale behind the introduction of our fourth assumption:[21]

A4. $d\Delta^a / d\tau^j \leq 0$, $a, j \in A$.

An equivalent restriction is imposed on Δ^b. As with A1-A3, A4 is a strong restriction imposed on derived objects. However, the conditions for fulfillment of this assumption have a natural economic interpretation which we will now consider.

By inspection of its definition, we see that Δ^a changes due to a change in τ^j as a result of two effects; one through the impact on elasticities and the other through the impact on import shares. These two effects may pull in opposite directions. The *elasticity effect* of an increase in τ^j may be positive and go against A4; inspection of (3.10) shows that $\hat{\tau}^j > 0$ results in $\hat{p}^j > 0$ and, since $\theta^j > 0$ by A1, this has a positive impact on ε^j. The effect of an increase in τ^j on the elasticities of countries in B is also positive; on the other hand the effect of an increase in τ^j on the elasticities of countries in $i \in A \backslash j$ is negative which does go in the direction of A4. The *share effect* of an increase in τ^j is negative and clearly goes in the direction of A4; the direct effect of an increase in τ^j is to reduce M^A.[22] For A4 to be satisfied requires that the (negative) share effect dominates

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[21]The structural form of $d\Delta^a / d\tau^j$ is presented in equation (5.3) below.

[22]In the standard two country model where there is only one country in a region, a tariff increase

the elasticity effect if it is positive.

We will now obtain a simple characterizations of how τ^a changes due to changes in tariffs of countries in the same region.

Lemma 1. *In the game* $\Gamma\left(\tau^b\right)$, $\widetilde{\tau}^a$ *is an increasing function of* $\boldsymbol{\tau}^{-a}$.

As shown in the proof, A4 provides a sufficient condition for Lemma 1 to hold. To see the intuition behind the result, take for example a tariff reduction by a country in $A\backslash a$. If a tariff in the vector $\boldsymbol{\tau}^{-a}$ is lowered then this results in a fall in import share of good 2 to country a and a corresponding loss of welfare. In order to offset the fall in import share, it is a best response for country a to lower its tariff as well. It follows that, for countries $1, ..., a, ..., x$ in the set A, the tariff game $\Gamma\left(\tau^b\right)$ is *supermodular*; tariffs are strategic complements.

Of course, both the initial reduction in $\boldsymbol{\tau}^{-a}$ and the subsequent lowering of τ^a have the effect of reducing country a's terms of trade and hence income. In principle the income effect on the elasticity of demand for imports may be large enough to cause a to try to offset the loss in terms of trade and hence income by increasing its tariff. The role of A4 is to bound this income effect.

The next result shows that if the $\Gamma\left(\tau^b\right)$ game is supermodular then the Nash equilibria of the tariff game of the North are symmetric; all countries in A set the same tariff in equilibrium.

Proposition 2. *For any number of countries in the North, x, all Nash equilibria of the game* $\Gamma\left(\tau^b\right)$ *are symmetric; all countries in the North set the same tariff in equilibrium.*

Symmetry of Nash equilibrium follows from the property that each country's tariff is an increasing function of all others (in A) coupled with the fact that all countries in A have identical endowments and technology. Of course, a similar result holds for the countries in B. We denote such a 'common' equilibrium tariff by $\overline{\tau}^a$ and refer to it as a *Northern equilibrium tariff*. Analogously, we will refer to a common equilibrium tariff in the South, $\overline{\tau}^b$, as a *Southern equilibrium tariff*.

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results in a reduction of imports to that region. This property extends naturally to the present model as shown in the proof of Proposition 1.

Even with the restrictions imposed by A1-A4, it is not possible to establish uniqueness of equilibrium in the $\Gamma\left(\tau^b\right)$ game. The reason is easy to see. Since tariff best-response functions are continuous, uniqueness would be guaranteed if it were possible to prove that $\partial\tau^a/\partial\tau^j < 1$, $j \in A\backslash a$, at any equilibrium point. As can be seen from the expression in the proof of Lemma 1, A4 ensures $\partial\tau^a/\partial\tau^j > 0$ but it does not ensure that $\partial\tau^a/\partial\tau^j < 1$. However, while there may be multiple equilibria, from any particular equilibrium we are able to obtain clear-cut predictions about the direction of change in $\overline{\tau}^a$ that would result from a change in τ^b and x, and it is these comparative statics properties that are of interest to us. Therefore, we will undertake comparative statics on a particular equilibrium while keeping in mind that equilibrium may not be unique.[23]

We will now undertake comparative statics on an equilibrium of the $\Gamma\left(\tau^b\right)$ game by varying τ^b, x and y respectively. While τ^b may obviously be varied continuously, it is natural to think of changes in x as being discrete. We add to x by assuming that one (or more) of the closed economies in the North adopts an open trade regime whereby it sets a non-prohibitive best-response tariff.[24] We add to y similarly. On the other hand, the effect of a change in x or a change in y can be analyzed most conveniently by treating these variables as if they can be varied continuously. Treating x and y as continous has the advantage that we can then use calculus to evaluate the effects of changes in these variables on equilibrium tariffs using the condition derived in Proposition 1. So we will proceed by treating x and y as continuous variables.[25] In the following, it will be helpful to note that, since all countries in the North set the same tariff, $\overline{\tau}^a$, the values for θ^i will be the same for all $i \in A$ and will be denoted by θ^a. Since all countries in the

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[23]The property that there may be multiple equilibria seems to be related to the idea, familiar in the literature on monotone comparative statics, that there is an 'ordering of equilibria,' from which predictions about monotonicity of comparative statics can be made only on the extremal equilibria. The ordering of equilibria arises from multiple equilibria that may arise from multiple crossing points of two given (upward sloping) reaction functions. In the present setting we have something different; the possibility of multiple (regional) reaction functions. A second difference is that our regional reaction functions are downward sloping and, for conditions we will derive below, any two reaction functions can cross only once. It follows that, for the equilibrium point given by any two downward sloping regional reaction functions, the comparative statics properties must be the same in terms of monotonicity of comparative statics. While the focus of our interest here is on the comparative statics properties of a given equilibrium, the issue of multiplicity of equilibrium revealed here seems to warrant further investigation in future research.

[24]It is well known that, for ad valorem tariffs in a two country tariff war, there is a continuum of Nash equilibria of a tariff game in which all countries set prohibitive tariffs and no trade takes place; see Dixit (1987). The same result would hold in the present many-country setting.

[25]The same results could be obtained, but in a more cumbersome way, by treating x and y as discrete.

South are assumed to set τ^b, the variable θ^b can be defined analogously. The next result characterizes the effect of a change in τ^b, x and y on $\overline{\tau}^a$.

Proposition 3. *Any Northern equilibrium tariff, $\overline{\tau}^a$, is decreasing in τ^b. If θ^b is sufficiently small, then $\overline{\tau}^a$ is decreasing in x. The effect of a change in y on $\overline{\tau}^a$ is ambiguous.*

As will be shown in Section 5, θ^i is small if countries in the North and South differ considerably in comparative advantage; that is, if β is restricted to be small.[26] We can think of this result as characterizing a 'Northern tariff reaction function.' The Northern tariff reaction function is downward sloping in $\left(\tau^a, \tau^b\right)$ space. Thus Proposition 3 extends a familiar result in the standard two-country model to the present setting. We will see in the next section that this implies Northern and Southern equilibrium tariffs are strategic substitutes.

From Proposition 3 we see that the intuition, outlined after Proposition 1, for how country a's tariff should respond to an increase in the number of countries in a region is only partially confirmed. The intuition outlined after Proposition 1 focused on the *direct effect* of a change in x on $\overline{\tau}^a$. According to this effect, $\overline{\tau}^a$ is decreasing in x due to the property of the model that (from equilibrium) country a's response to an increase in import demand by other countries in A is to lower its own tariff. As outlined above, it does not matter whether import demand in A increases because one (or more) existing country in $A\backslash a$ lowers its tariff or because an additional country is added to A and that country's imports are positive. But there is also an *indirect effect*; a change in x affects the world market clearing price, which in turn affects the elasticities of import demand functions. The interaction of the direct and indirect effects are captured by the first derivative of $\overline{\tau}^a$ with respect to x. Here we present a simpler expression that provides the essential information needed to sign $\partial\overline{\tau}^a/\partial x$:[27]

$$sign\left\{\frac{\partial\overline{\tau}^a}{\partial x}\right\} = sign\left\{-\left(\varepsilon^a + \left(\varepsilon^b - 1\right) + (x-1)\,\varepsilon^a\theta^a - x\varepsilon^b\theta^b\right)\right\}.$$

The first two terms capture the direct effect of a change in x on $\overline{\tau}^a$. The last two terms capture the indirect effect, which operates through θ^a and θ^b. The fact that the final term

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[26]By showing that θ^i can be restricted using β, we will show that this restriction on θ^i is consistent with assumptions A3 and A4.

[27]The full expression for $\partial\overline{\tau}^a/\partial x$ is presented in the proof or Proposition 3; observe that, as shown in the proof, at any interior symmetric equilibrium with optimum tariff intervention, $\varepsilon^b - 1 > 0$.

is negative goes against the result and this is why θ^b must be sufficiently small in order for the result to hold. With this restriction in place, the direct effect dominates and the effect of an increase in x on $\overline{\tau}^a$ may be represented as a downward/inward shift of the Northern tariff reaction function.

The effect of a change in y on $\overline{\tau}^a$ operates entirely through the indirect effect:

$$sign\left\{\frac{\partial \overline{\tau}^a}{\partial y} = sign\left\{(x-1)\,\varepsilon^a\theta^a - x\varepsilon^b\theta^b\right\}\right\}.$$

The indirect effect of an increase in y on $\overline{\tau}^a$ operates in the opposite direction to an increase in x. The reason is that, at constant q, an increase in x (y) raises Northern (Southern) demand for Southern (Northern) exports; $dq/dx > 0$ $(dq/dy < 0)$. From this expression we see that if θ^a and θ^b are small then the effect of a change in y on $\overline{\tau}^a$, while ambiguous, is small. We will make use of this property in the next result.

This concludes our characterization of equilibrium for the tariff game of the North. The characterization extends directly to equilibrium for the tariff game of the South. We may now proceed to characterize equilibrium in both regions simultaneously.

4.2. Characterization of Equilibrium for the General Tariff Game

In this section we consider the outcome of the general tariff game in which each and every country in N simultaneously and without communicating sets a best-response tariff. Given the structure of the model and the sequence of events set out in the extensive form, we know that the general tariff game has at least one Nash equilibrium. Using Propositions 1-3, we are now able to provide a characterization in the paper's first main result.

Proposition 4. *There exists a symmetric equilibrium of the general tariff game. If θ^a and θ^b are sufficiently small then at such an equilibrium, any Northern equilibrium tariff, $\overline{\tau}^a$, is decreasing in $\overline{\tau}^b$ and decreasing in x; any Southern equilibrium tariff, $\overline{\tau}^b$, is decreasing in $\overline{\tau}^a$, and decreasing in y; the effect of an increase in y (x) on $\overline{\tau}^a$ $\left(\overline{\tau}^b\right)$ is ambiguous but small.*

Proposition 4 extends Proposition 3 to equilibrium of the general tariff game. The assumption that θ^a and θ^b are sufficiently small plays two roles here. The first, as explained

after the statement of Proposition 3, is to bound the indirect effect of a change in y and x on $\bar{\tau}^a$ and $\bar{\tau}^b$ respectively. The second is required to ensure that the tariff reaction functions have slope greater than -1. Proposition 4 establishes that any reaction function crosses a reaction function of the other region only once. Thus, the above result shows that the logic of equilibrium existence in the standard two-country model extends to our two-region model. In this sense, we have now established that our two-region model behaves like a two-country model since all countries in a region set the same tariff.[28]

Proposition 4 also extends the results of Proposition 3 that relate to the effects of a change in the Southern common tariff and the number of countries in the South on $\bar{\tau}^a$. The result that tariffs across regions are strategic substitutes relies on A2. This property is extended from the standard two-country model. A2 implies that if countries in the other region raise their common tariff then the marginal welfare brought about by a tariff increase by any country falls because their trade partners' import demand functions become more elastic. The new property of equilibrium is that an increase in the number of countries in a region brings about a decrease of the common tariff in that region and the effect on the tariff in the other region is ambiguous but small. By (3.9), the terms-of-trade implications are unambiguously unfavorable to countries in the region that becomes larger and favorable to the other region.

4.3. Welfare Effects of Increasing Countries in One Region

Now, in the paper's second main result, let us consider the welfare effect on equilibrium of changes in the number of countries in one region.

Proposition 5. *If θ^a and θ^b are sufficiently small then, from any trading equilibrium, an increase (decrease) in the number of countries in a region unambiguously worsens (improves) the welfare of any country in that region and improves (worsens) the welfare of a country in the other region.*

We can understand Proposition 5 by focusing on country a while noting that the

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[28]Syropoulos (2002) adopts an alternative approach; he establishes that the Hessian matrix given by the equilibrium intersection of the two countries' reaction functions is negative semi-definite. It follows that equilibrium must be stable and hence unique. That approach cannot be taken here since it would involve finding the inverse of a non-symmetric unbounded $n \times n$ matrix.

same logic extends to country b. An increase of y has the effect of increasing imports by all countries in B of good 1, bringing about a reduction in $\overline{\tau}_b$; the effect on $\overline{\tau}_a$ is ambiguous but small. The decrease in $\overline{\tau}_b$ improves country a's terms of trade; from equation (3.9), $\hat{q}$ is negative. To see that $d\mu^a/dy > 0$, recall that the term on the first line of (3.12) is equal to zero, as implied by country a's first order condition. An increase in x has the opposite effect, bringing about a reduction in $\overline{\tau}_a$ and an ambiguous but small change in $\overline{\tau}_b$, with commensurate deterioration of Northern terms of trade and welfare.

4.4. Increase of Countries in Both Regions

So far we have seen that an increase in the number of countries within a region tends to increase the extent of the tariff externality between countries in that region. This in turn leads them to reduce their tariff, thus collectively undermining their terms of trade and welfare. In this subsection we show that the tariff externality does not necessarily lead to a worsening of welfare, providing that the adverse terms-of-trade effect can be neutralized. This happens if the numbers of countries in both regions are increased simultaneously. The tariff externality then works to bring about a tariff reduction everywhere but without an associated deterioration of terms of trade in one region versus the other.

We now formalize a *replication* of the world economy; a simultaneous increase in the number of countries in both regions such that the ratio of countries in the North versus the South is held constant. Let r represent the ratio of countries in the South to the North; $r \equiv y/x$. Now if we fix r then we can study the replication of the world economy. Since the response of the economy in equilibrium to replication is monotonic, we can define replication simply in terms of an increase in x. The effects of replication are presented in the paper's third main result.

Proposition 6. *Assume an initial symmetric equilibrium of the general tariff game. As the economy is replicated, $\overline{\tau}^a$ and $\overline{\tau}^b$ decrease monotonically. And as the number of replications becomes large equilibrium import volumes M^A and M^B converge towards efficient levels; the symmetric equilibrium of the general tariff game converges towards world efficiency/free trade.*

From the results that have already been established, the implications for world welfare

of replication follow naturally. By Proposition 6, as the international trading economy is replicated, import volumes increase monotonically as equilibrium tariffs fall in both regions. Eventually, as x becomes large, equilibrium tariffs tend towards zero, and in the limit the outcome of world efficiency (free trade) is attained. Note that efficiency may not increase for increases of x over an intermediate range; for example, note from (3.12) that a reduction of $\bar{\tau}^b$ tends to increase Northern welfare but a reduction of $\bar{\tau}^a$ tends to have the opposite effect. Clearer cut welfare results from replication can be obtained by putting additional structure on the model, as will be shown in Proposition 9.

5. Examples

First we will look at a Cobb-Douglas example. We will show that in order to satisfy A4 a restriction must be introduced to ensure that the degree of comparative advantage is sufficiently high. We then show that for A4 to be satisfied under C.E.S. also requires a restriction on preferences that ensures they are 'sufficiently close' to Cobb-Douglas preferences in a well-known sense. Finally, we will solve for a 'toy model' in which preferences are symmetric Cobb-Douglas and factor endowments are 'on the boundary;' the North only has capital while the South only has labor. The solution is appealing in its simplicity and useful for the additional insights it yields regarding replication of the world economy.

5.1. Cobb-Douglas

Here preferences are homothetic symmetric Cobb-Douglas, giving rise to the following form for the expenditure function:

$$E^i = 2 \left(p_1^i \right)^{1/2} \left(p_2^i \right)^{1/2} \mu^i, \; i \in N$$

Turning to the production side, the simplest possible functional form is assumed in which there is a one-to-one proportional relationship between the input of capital (labor) and the output of good 1 (good 2). Recall from Section 2 the structure of factor endowments across countries and that all factors are supplied inelastically. This production structure is rich enough to yield price elasticities of import demand, ε^i, that can vary with price, p^i, yet simple enough to maintain tractability. In particular, we will see that ε^is

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vary under these assumptions in a way that can be seen to depend in a natural way on the fundamentals of the model. We will refer to this combined specification of preferences and production structure as the *Cobb-Douglas example.*

The fundamental that we will be most interested in is the *degree of comparative advantage*, which will be determined by the size of the parameter β. If β is high (low) on the unit interval then each country has a low (high) degree of comparative advantage in the sense that (all else equal) there is a relatively small (large) difference between its output of its importable and any trade partner's output of the same good.

The restriction on the degree of comparative advantage operates partly through its effect on the determination of the price elasticity of import demand, ε^i. To see how the two are linked, it will be helpful to have a decomposition of ε^i into its component parts. To facilitate this, use the definition of the elasticity of demand, denoted η_D^i, to write $\eta_D^i = p^i E_{pp}^i / E_p^i$, and use the definition of the elasticity of supply, denoted η_S^i, to write $\eta_S^i = p^i R_{pp}^i / R_p^i$. Then we may write:

$$
\begin{aligned}
\eta^i &= \frac{\sigma_E^i \eta_D^i \left(1 - T^i \sigma_R^i\right) + \sigma_R^i \eta_S^i \left(1 - T^i \sigma_E^i\right)}{\sigma_E^i - \sigma_R^i}; \\
\phi^i &= \frac{\sigma_E^i / \tau^i}{1 - T^i \sigma_E^i}; \\
\varepsilon^i &= \eta^i / \left(1 - T^i \sigma_E^i\right) + \phi^i.
\end{aligned}
$$

For the Cobb-Douglas example, we have

$$
\eta^i = \frac{\beta p^i + \tau^i}{2\left(1 - \beta p^i\right)\tau^i} \text{ and } \phi^i = \frac{1}{2\tau^i \left(1 - \frac{1}{2}T^i\right)},
$$

from which

$$
\varepsilon^i = \frac{1}{1 - \beta p^i}. \tag{5.1}
$$

From this solution for ε^i, we then have

$$
\theta^i = \frac{1}{1 - \beta p^i} - 1 \text{ and } \zeta^i = 0. \tag{5.2}
$$

We can immediately see that A1-A3 are satisfied for any p^i.

Assumption A4 is more difficult to satisfy, and requires a restriction on the value of β. Nevertheless, for any range of finite prices, p^i, it is always possible to find a positive

value of $\beta' > 0$ such that A4 is satisfied for all $\beta \in [0, \beta']$. This restriction, $\beta \in [0, \beta']$, we refer to as the *restriction on the degree of comparative advantage*. The following result formalizes the role of this restriction in ensuring that A4 is satisfied under the Cobb-Douglas example.

Proposition 7. *Assume the Cobb-Douglas example and let $x \in [2, \infty)$, $y \in [2, \infty)$. Then there exists a range $\beta \in [0, \beta']$ for which A4 is satisfied.*

We can get an intuitive sense of the proof by writing $d\Delta^a/d\tau^j$ in full:

$$\frac{d\Delta^a}{d\tau^j} \equiv \sum_{i \in A \setminus a} s^i \theta^i \varepsilon^i \frac{dp^i}{d\tau^j} + \sum_{i \in B} s^i \theta^i \varepsilon^i \frac{dp^i}{d\tau^j} + s^j \zeta^j \frac{\varepsilon^j}{\tau^j} + \sum_{i \in A \setminus a} \varepsilon^i \frac{ds^i}{d\tau^j}, \tag{5.3}$$

$j \neq a \in A$. The proof of the proposition relies on the fact that if $\beta = 0$ then $\theta^i = 0$, $i \in N$, and $\sum_{i \in A \setminus a} \varepsilon^i ds^i/d\tau^j < 0$, so $d\Delta^a/d\tau^j < 0$. (Recall that $\zeta^j = 0$ under Cobb-Douglas.) Note that $ds^j/d\tau^j > 0$ but $ds^i/d\tau^j < 0$ for $i \neq j$. The proof of the proposition shows that the sum of the indirect effects (through $ds^i/d\tau^j$) dominate. Then by continuity of the terms on the right-hand-side, it must be the case that $d\Delta^a/d\tau^j \leq 0$ for some positive range of β.[29]

Numerical simulations indicate that the restriction of β to $\beta \in [0, \beta']$ may rule out a significant range of situations in which countries in the two regions are similar to each other. For example, letting $x = 2$ and $y = 4$, A4 is satisfied only if $\beta \in [0, \beta']$ where $\beta' = 0.1$. Nevertheless, there does not appear to be a problem with computing Nash equilibrium tariffs for $\beta \in [\beta', 1]$.

5.2. C.E.S.

We assume that underlying preferences are C.E.S., giving rise to the following form for the expenditure function:

$$E^i = \left(2^{\rho - 1} \left(p_1^i\right)^\rho + 2^{\rho - 1} \left(p_2^i\right)^\rho\right)^{\frac{1}{\rho}} \mu^i.$$

---

[29]It is well known that if $\beta = 0$ and there is only one country in each region then optimum tariffs under Cobb-Douglas are infinitely high; see the worked example below. One way to get around this problem would be to bound β away from zero. A simpler approach is to bound country numbers away from one in each region.

Production is the same as for the above Cobb-Douglas example. We will refer to this combined specification of preferences and production structure as the *C.E.S. example.*

We can now solve for ε^i as we did for the Cobb-Douglas example above. First we have the components:

$$\eta^i = \frac{(1-\rho)\,(p^i)^\rho\,(\beta p^i + \tau^i)}{(1 + (p^i)^\rho)\,((p^i)^\rho - \beta p^i)\,\tau^i} \quad \text{and}$$

$$\phi^i = \frac{2^{\rho-1}\,(p^i)^\rho}{\left(2^{\rho-1}\,(1 + (p^i)^\rho)\right)\tau^i\left(1 - \frac{2^{\rho-1}(p^i)^\rho(\tau^i-1)}{(2^{\rho-1}(1+(p^i)^\rho))\tau^i}\right)}$$

from which

$$\varepsilon^i = \frac{(p^i)^\rho\left((p^i)^\rho - \rho + \tau^i - \rho\left(\beta p^i + \tau^i - 1\right)\right)}{\left((p^i)^\rho - \beta p^i\right)\left((p^i)^\rho + \tau^i\right)}.$$

From the solution for ε^i we have

$$\theta^i = \frac{(1-\rho)\,\beta p^i}{(p^i)^\rho - \beta p^i} + \frac{\rho\tau^i}{(p^i)^\rho + \tau^i} - \frac{\rho\,(1-\rho)\,(\beta p^i + \tau^i)}{(p^i)^\rho - \rho + \tau^i - \rho\,(\beta p^i + \tau^i - 1)},$$

$$\zeta^i = \frac{\left((p^i)^\rho - \beta p^i\right)\rho\tau^i}{\left((p^i)^\rho + \tau^i\right)\left(\beta\rho p^i - (1-\rho)\,\tau^i - (p^i)^\rho\right)}.$$

If $\rho \leq 0$ then assumptions A1 and A2 are always satisfied; these expressions for θ^i and ζ^i can be signed appropriately when we take into account the direction of trade flows. Restrictions on ρ are required in order to ensure that A3 is satisfied. The approach we will adopt is simply to impose a restriction that ensures the C.E.S. preferences are 'sufficiently close' to the Cobb-Douglas preferences specified above. We can then use the conditions on β identified in Proposition 7 to ensure that A4 holds as well.

Proposition 8. *Assume the C.E.S. example and let $x \in [2, \infty)$, $y \in [2, \infty)$. Then, for some $\rho \in [\rho', 0]$, there exists a range $\beta \in [0, \beta']$ for which A1-A4 are satisfied.*

This result demonstrates that preferences must be sufficiently close to Cobb-Douglas and the degree of comparative advantage must be sufficiently large in order for our assumptions to hold. The result is appealing because it implies that the analysis does not depend on the 'knife edge' properties of the Cobb-Douglas production function. Apparently, for a range or representative country numbers, if $\beta = 0$ then no restriction is required on C.E.S. preferences. However, the higher is β the more C.E.S preferences must

be restricted in order to bring them closer to Cobb-Douglas.[30] The difficulty arises that as the absolute value of ρ is made larger and goods become more highly substitutable, θ^i becomes larger and eventually becomes greater than ε^i for any given p^i; in that case A3 cannot be satisfied for any equilibrium prices and our results would break down.[31]

5.3. Cobb-Douglas Preferences with Boundary Endowments

Sharper predictions about the distributional implications of replication can be obtained by restricting the model further. We now solve the model under the further restriction that $\beta = 0$; each country in A is endowed only with a unit of capital (no labor) and each country in B is endowed only with a unit of labor (no capital). We will also assume the following *symmetrical Cobb-Douglas preferences*:

$$u^i = \Pi_{j \in \{1,2\}} \left(z^i_j \right)^{1/2}, \ i \in N.$$

where z^i_j represents the Marshallian demand of good j in country i. Then the (unique symmetric) equilibrium tariffs may be obtained in the usual way using either the indirect utility function or the expenditure function. They are as follows:

$$\overline{\tau}^a = \frac{1}{x-1}; \ \overline{\tau}^b = \frac{1}{y-1}.$$

Solving for terms of trade q using (2.4), we obtain the following reduced form expression:

$$q(x,y) = q\left(\overline{\tau}^a(x), \overline{\tau}^b(y)\right) = \frac{x(x-1)(2y-1)}{y(y-1)(2x-1)}.$$

Using this expression and equilibrium tariffs in the indirect utility function, we have

$$
\begin{aligned}
u^a(x,y) &= u^a\left(q\left(\overline{\tau}^a(x), \overline{\tau}^b(y)\right), \overline{\tau}^a(x)\right) \quad &(5.4)\\
&= \left(\frac{x}{2x-1}\right)^{\frac{1}{2}} \left(\frac{y(y-1)}{x(2y-1)}\right)^{\frac{1}{2}}.
\end{aligned}
$$

---

[30]Note that if the underlying C.E.S. utility function is of the form stated in footnote 22, with the additional restriction that $\alpha = \frac{1}{2}$, then $\rho = \chi/(\chi-1)$ and $\sigma^i = 1 - \rho$; our restriction that $\sigma^i \geq 0$ amounts to a restriction that $\rho \leq 0$, which is equivalent to a restriction that $0 \leq \chi \leq 1$. Since under Cobb-Douglas preferences $\sigma^i = 1$, we can bring C.E.S. preferences closer to Cobb-Douglas preferences through a restriction on (the absolute value of) ρ, reflecting a restriction on the degree of substitutability between goods.

[31]A4 fails either if, for given $\beta \in (0,1)$, ρ is sufficiently large or if, given ρ, β is sufficiently large. For example, simulation results indicate that if $\rho^* = -1$, which implies $\sigma^i = 2$, then $\beta' = 0.01$.

Analogously,

$$
\begin{aligned}
u^b\left(x, y\right) &= u^b\left(q\left(\bar{\tau}^a\left(x\right), \bar{\tau}^b\left(y\right)\right), \bar{\tau}^b\left(y\right)\right) \\
&= \left(\frac{y}{2y-1}\right)^{\frac{1}{2}}\left(\frac{x\left(x-1\right)}{y\left(2x-1\right)}\right)^{\frac{1}{2}}.
\end{aligned}
$$

Once we have explicit solutions for optimal tariffs, it is convenient to use the indirect utility functions to measure welfare effects of replication since they embody income effects (through the Marshallian demands).

5.4. Increase of Countries in Both Regions

The following result obtains sharper predictions about replication of the world economy based on the example of the present section.

Proposition 9. *Let $x \geq 2$ and $y \geq 2$ and assume an initial trading equilibrium.*

World efficiency implications of replication: The higher is x then the higher is world welfare, and world welfare is maximized as $x \to \infty$.

Distributional implications of replication: There exists a value $r' > 1$ such that if $r = r'$ then a given increase in x will leave $u^a\left(x, rx\right)$ unchanged; if $r < r'$ then an increase in x will bring about an increase of $u^a\left(x, rx\right)$ and if $r > r'$ then an increase in x will bring about a decrease of $u^a\left(x, rx\right)$.

From the results that have already been established, the implications for world welfare of replication follow naturally. By Proposition 6, as the international trading economy is replicated, export volumes increase monotonically as equilibrium tariffs fall. Trade flows increase monotonically from one region to the other and world welfare increases monotonically as well. Eventually, as x becomes large, equilibrium tariffs $\bar{\tau}^a = 1/\left(x-1\right)$ and $\bar{\tau}^b = 1/\left(rx-1\right)$ tend towards zero, and in the limit the outcome of world efficiency (free trade) is attained.

The distributional implications are more surprising but they can be understood as follows. The higher the value of r, the more scarce is capital and hence the more scarce are manufactures relative labor and hence primary products. Country a is able to exploit this scarcity because the import elasticity of demand for its export is relatively low in

equilibrium. Consequently, country a sets a relatively high tariff in equilibrium compared to country b, and as a result $u^a(x, rx) > u^b(rx, x)$. If r is relatively high then the terms-of-trade effects of its relatively high tariff may be sufficient to ensure that $u^a(x, rx)$ is above its free trade level.

As x is increased this reduces the relative scarcity of country a's export, reducing a's equilibrium tariff. This has two effects on $u^a(x, rx)$. The static efficiency gains of tariff reduction increase $u^a(x, rx)$. On the other hand, the tariff reduction may also reduce a's terms of trade, reducing $u^a(x, rx)$. As x is increased, $u^a(x, rx)$ converges to its efficient (free trade) level. If at low levels of x, $r < r'$ then $u^a(x, rx)$ converges to its efficient free trade level from below; the static efficiency effects of tariff reduction dominate. But, if $r > r'$ then $u^a(x, rx)$ converges to its free trade level from above; the terms-of-trade effects dominate (while $u^b(x, rx)$ converges to its efficient/free trade level from below).

6. Conclusions

The theory of international trade with protectionism has traditionally tended to focus on models in which there are just two countries. Where more than two countries have been allowed for in the past literature, their number has been fixed and so it has not been possible to address questions of how the structure of the world economy affected the efficiency and equity of world trade. In this paper we have developed a model in which there are many countries and countries may differ in terms of their underlying factor endowments, with the government of each setting tariffs strategically. We couched this in familiar terms, where the North (South) is relatively abundant in capital (labor) and hence exports manufactures (primary products) to the South (North). Within this framework, we established a tariff externality between countries in a region through which the level of protectionism depends negatively on the number of countries in the region. As we increased the number of countries in the South, for example, this lead to a strategic lowering of Southern tariffs and a consequent worsening of Southern terms of trade and welfare. On the other hand, the tariff externality could be shown to have beneficial effects providing that the adverse terms-of-trade effects were not too strong. To show this possibility, we focused on a simultaneous increase in the number of countries in the North

and South holding the relative size of the regions constant. This was shown to increase efficiency.

The present paper has taken a first step in the application of methods of monotone comparative statics to the problem of optimum tariff setting by many countries. While useful insights have been obtained from this approach, a number of interesting questions remain to be answered in future research. First, through the course of our analysis, we found it convenient to impose restrictions on the price elasticities of import demand and on how these would respond to a change in prices. Future work should seek to impose restrictions instead on the underlying primitives. As noted in the body of the paper, the difficulty of this exercise lies in the fact that the implied restrictions would be on third derivatives of the underlying utility functions, for which economic theory gives us no guidance. Nevertheless, it may still be instructive to establish whether any such restrictions exist that would give rise to the results presented in the present analysis, and whether systematic predictions can be made when the restrictions fail. Also, while the present analysis has undertaken comparative statics on a given equilibrium, another interesting agenda for future research would be to examine the conditions on existence and uniqueness of equilibrium.

In the introduction it was suggested that, following Broda et al (2008), it should be possible to see whether the number of countries has explanatory power in determining the (inverse) relationship between import demand elasticities and tariffs. Further developing ideas for that line of research, it would be interesting to test the model's prediction, based on Proposition 4, that trade interventions set by the North on imports from the South are more restrictive than the reverse because there are more countries in the South than the North.[32] Of course, received wisdom would point in the other direction, based on the observation that the process of multilateral tariff reductions in the postwar period have brought down tariffs significantly more in the North than in the South. But this overlooks the fact that such tariff reductions have focused on manufacturing goods, and relatively little progress has been made in agriculture, where Northern tariffs on imports from the South are still relatively high.

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[32]Given cross-regional variation in tariffs and size, and within-region variation in tariffs, the pattern of tariff correlation described in Proposition 4 are identified in principle.

One of the reasons this type of question has remained open is because of the difficulties with obtaining reliable estimates of overall trade restrictiveness that incorporate non-tariff barriers as well as tariffs on a comparable basis. Progress has been made in this direction with the development of a methodological approach to estimating an overall *trade restrictiveness index*, beginning with Anderson and Neary (1994) and followed by a sequence of other papers; this literature is comprehensively surveyed by Anderson and Neary (2005). Recently there has been further progress with the implementation of this methodology by Kee, Nicita and Olarreaga (2009). One finding by Kee et al (2009) is that poor countries face more restrictive trade barriers on their exports than do rich countries. It would be interesting to establish the extent to which this pattern of protectionism is due to the fact that relatively high tariffs are levied by rich countries on poor country agricultural exports, and whether poor countries levy relatively low tariffs on manufactures from rich countries, as predicted by our framework.

A serious short-coming of our model from an empirical standpoint is that it cannot motivate within-region trade while in practice most trade takes place between countries in the North. In addition, Kee et al (2009) attribute their finding that poor countries impose more restrictive trade policies to the fact that they impose more restrictive trade policies on each others' goods. Thus, before the predictions of our model could be tested fully, the underlying framework of the model would need to be extended to allow for within-region trade and tariff-setting. Our model could be extended in two possible ways that have been developed in the literature to motivate within-region trade: one would be, following Helpman and Krugman (1985), to adopt increasing returns technology in the manufacturing sector; a second would be, following Davis (1997), to incorporate into a standard Heckscher-Ohlin-Vanek framework differences in techniques of production across sectors as well as suitable cross-country differences in factor endowments. Both extensions would make it possible to motivate within-region trade while at the same time motivating protectionism.

A second point to take account of in testing the empirical predictions of our model is that it is a static model and does not immediately predict tariff patterns that have been brought about through multilateral trading arrangements. Two approaches are possible here. One is to adopt the approach of Broda et al (2008) and restrict attention entirely to

countries that have not engaged in multilateral tariff reductions. Another would be, using the framework of Bagwell and Staiger (1999), to incorporate the possibility of bargaining over tariffs into the multi-country model of the present paper. One conjecture is that, while average world tariffs would be lower in any trade agreement, our static model prediction that tariffs would be lower in the South than in the North would be preserved.

To make the policy-making environment more sophisticated it would be possible, following Baldwin (1987) and Grossman and Helpman (1994), to introduce distributional/political concerns to governments' objective functions. A focus of that literature has been on the extent to which political/distributional concerns cause governments to set tariffs 'too high' relative to the single country optimum. On the other hand, we established above that, by failing to internalize the tariff setting effect of others' in the same region, each country sets its tariff 'too low' relative to the optimum. Thus, the two effects tend to pull in opposite directions. Following Goldberg and Maggi (1999), future empirical work could try to get a handle on which effect dominates.

A. Appendix

Proof of Proposition 1. We will work with the case of country a. The case of country b is analogous. Rewrite (4.1), the first order condition for government a's problem, as

$$
\begin{aligned}
\psi_{\tau^a}^a &= s^a - (\tau^a - 1)\,\Delta^a \\
&= s^a - (\tau^a - 1)\left(\sum_{i \in A \backslash a} s^i \varepsilon^i + \sum_{i \in B} s^i \varepsilon^i - 1\right) = 0.
\end{aligned}
$$

To prove the result, it is sufficient to show that a's objective function is quasi-concave; that is, $\partial \psi_{\tau^a}^a / \partial \tau^a < 0$ whenever $\psi_{\tau^a}^a = 0$. Using $ds^i/d\tau^j$ to denote the total derivative of s^i w.r.t. τ^j, we have

$$
\begin{aligned}
\frac{\partial \psi_{\tau^a}^a}{\partial \tau^a} &= \frac{ds^a}{d\tau^a} - \left(\sum_{i \in A \backslash a} s^i \varepsilon^i + \sum_{i \in B} s^i \varepsilon^i - 1\right) \\
&\quad - (\tau^a - 1)\left(\sum_{i \in A \backslash a} s^i \theta^i \frac{\varepsilon^i}{p^i}\frac{dp^i}{d\tau^a} + \sum_{i \in B} s^i \theta^i \frac{\varepsilon^i}{p^i}\frac{dp^i}{d\tau^a} \right. \\
&\quad \left. + \sum_{i \in A \backslash a} \varepsilon^i \frac{ds^i}{d\tau^a} + \sum_{i \in B} \varepsilon^i \frac{ds^i}{d\tau^a}\right).
\end{aligned}
$$

We will show that $\partial \psi_{\tau^a}^a / \partial \tau^a < 0$ by signing each of the three terms on the right-hand-side.

First we establish that $ds^a / d\tau^a < 0$. To see this, totally differentiate the expression for s^a and then divide the result by s^a to get

$$
\hat{s}^a = (1 - s^a)\,\hat{m}^a - \sum_{i \in A \backslash a} s^i \hat{m}^i.
$$

Since $d\tau^i = 0$ for all $\tau^i \neq \tau^a$, substitution of (3.9) into (3.7) simplifies to

$$
\begin{aligned}
\hat{m}^a &= (s^a \varepsilon^a - \Delta)\frac{\phi^a}{\Delta}\frac{d\tau^a}{\tau^a} \\
&= -\left(\sum_{i \in A \backslash a} s^i \varepsilon^i + \sum_{i \in B} s^i \varepsilon^i - 1\right)\frac{\phi^a}{\Delta}\frac{d\tau^a}{\tau^a} < 0.
\end{aligned}
$$

Analogously,

$$
\hat{m}^i = \varepsilon^i s^a \frac{\phi^a}{\Delta}\frac{d\tau^a}{\tau^a} > 0, \ \forall i \in A \backslash a.
$$

So $\hat{s}^a < 0$. Then $ds^a/d\tau^a = s^a \hat{s}^a/d\tau^a < 0$ as required.

To sign the second term (on the first line) of $\partial \psi^a_{\tau^a}/\partial \tau^a$, note that at interior solutions optimum tariff intervention by country a requires $\sum_{i \in B} s^i \varepsilon^i - 1 > 0$ (which implies $\Delta > 1$; this generalizes the standard condition in a two-country model that $\varepsilon^b - 1 > 0$ where b would be the sole element of the set B and s^b would necessarily be equal to 1). Therefore, the first line on the right-hand-side of $\partial \psi^a_{\tau^a}/\partial \tau^a$ is negative.

To sign the third term we need to sign the sum of the terms in the second set of brackets (second and third lines). First we use (3.10) to write

$$s^i \theta^i \frac{\varepsilon^i}{p_i} \frac{dp^i}{d\tau^a} = -s^i \theta^i \varepsilon^i s^a \frac{\phi^a}{\Delta} \frac{1}{\tau^a}, \ i \in A\backslash a$$

and (3.11) to write

$$s^i \theta^i \frac{\varepsilon^i}{p_i} \frac{dp^i}{d\tau^a} = s^i \theta^i \varepsilon^i s^a \frac{\phi^a}{\Delta} \frac{1}{\tau^a}, \ i \in B.$$

Next, note that since all countries in B set the same tariff, τ^b, it must be that s^i is the same for all $i \in B$; by symmetry of countries in B, each country in B must respond symmetrically to a change in τ^a and so $ds^i/d\tau^a = 0$ for all $i \in B$.

The final step in signing the third term is to show that

$$\frac{ds^i}{d\tau^a} = s^i \left(\varepsilon^i + \left(\sum_{j \in B} s^j \varepsilon^j - 1 \right) \right) s^a \frac{\phi^a}{\Delta} \frac{1}{\tau^a} > 0, \ i \in A\backslash a.$$

If this is the case then the third term simplifies to

$$- (\tau^a - 1) \left(\sum_{i \in A\backslash a} \left(\left(\varepsilon^i - \theta^i \right) s^i \varepsilon^i + s^i \left(\sum_{j \in B} s^j \varepsilon^j - 1 \right) \right) + \sum_{i \in B} s^i \theta^i \varepsilon^i \right) s^a \frac{\phi^a}{\Delta} \frac{1}{\tau^a} < 0 \quad \text{(A.1)}$$

as required since, by A1 and A3, $\varepsilon^i - \theta^i \geq 0$.

It remains only to establish the above expression for $ds^i/d\tau^a$. Totally differentiating

the expression for s^i, we have

$$
\begin{aligned}
\hat{s}^i &= \left(1 - s^i\right) \hat{m}^i - s^a \hat{m}^a - \sum_{j \in A \setminus \{a \cup i\}} s^j \hat{m}^j \\
&= \left(1 - s^i\right) \varepsilon^i s^a \frac{\phi^a}{\Delta} \frac{d\tau^a}{\tau^a} \\
&\quad + \left(\sum_{j \in A \setminus a} s^j \varepsilon^j + \sum_{j \in B} s^j \varepsilon^j - 1\right) s^a \frac{\phi^a}{\Delta} \frac{d\tau^a}{\tau^a} \\
&\quad - \sum_{j \in A \setminus \{a \cup i\}} s^j \varepsilon^j s^a \frac{\phi^a}{\Delta} \frac{d\tau^a}{\tau^a} \\
&= \left(\varepsilon^i + \left(\sum_{j \in B} s^j \varepsilon^j - 1\right)\right) s^a \frac{\phi^a}{\Delta} \frac{d\tau^a}{\tau^a} > 0.
\end{aligned}
$$

The result follows. $\square$

Proof of Lemma 1. We evaluate the response of $\widetilde{\tau}^a$ to a change in any of the tariffs in $\boldsymbol{\tau}^{-a}$. The aim is to show that $\partial \widetilde{\tau}^a \left(\cdot\right) / \partial \tau^j > 0$, $j \in A \setminus a$. By the implicit function theorem,

$$
\frac{\partial \widetilde{\tau}^a \left(\cdot\right)}{\partial \tau^j} = -\frac{\partial \psi_{\tau^a}^a / \partial \tau^j}{\partial \psi_{\tau^a}^a / \partial \tau^a} = \frac{-ds^a / d\tau^j + \left(\tau^a - 1\right)\left(d\Delta^a / d\tau^j\right)}{\partial \psi_{\tau^a}^a / \partial \tau^a}.
$$

(Recall that $\psi_{\tau^a}^a$ is introduced in the proof of Proposition 1 to denote the first order condition of government A.) From the proof of Proposition 1, $\partial \psi_{\tau^a}^a / \partial \tau^a < 0$ at any equilibrium point. It is easy to show, using the approach set out in the proof of Proposition 1, that $ds^a / d\tau^j > 0$. And by A4, $d\Delta^a / d\tau^j \leq 0$. $\square$

Proof of Proposition 2. Suppose to the contrary that we have an equilibrium in which countries $1, 2 \in A$ set tariffs $\overline{\tau}^1$ and $\overline{\tau}^2$ in such a way that $\overline{\tau}^2 > \overline{\tau}^1$ with the tariffs of all other countries in A fixed at some (equal) level $\overline{\tau}^a \in \mathbb{R}_+$. (We maintain the assumption that all tariffs in B are fixed at some equal level $\tau^b \in \mathbb{R}_+$.) By Lemma 1, we have that $\overline{\tau}^1$ is a strictly increasing function of $\overline{\tau}^2$ and also that $\overline{\tau}^2$ is a strictly increasing function of $\overline{\tau}^1$. Formally, we can represent this as $\overline{\tau}^1 = \varphi\left(\overline{\tau}^2\right)$ and $\overline{\tau}^2 = \varphi\left(\overline{\tau}^1\right)$, where $\varphi\left(\cdot\right)$ is a strictly increasing function. Then substituting $\varphi\left(\overline{\tau}^1\right)$ for $\overline{\tau}^2$ and $\varphi\left(\overline{\tau}^2\right)$ for $\overline{\tau}^1$ in the above inequality we have $\varphi\left(\overline{\tau}^1\right) > \varphi\left(\overline{\tau}^2\right)$. But because $\varphi\left(\cdot\right)$ is a strictly increasing function, this implies that $\overline{\tau}^1 > \overline{\tau}^2$; a contradiction. $\square$

Proof of Proposition 3. First we prove that $\partial \overline{\tau}^a / \partial \tau^b \leq 0$. The approach will be to analyze a tariff change by any single country in B, say country $x + 1$, from some (non-

prohibitive) common tariff level τ^b and use the result to infer the effect of a change in the common tariff τ^b. By Proposition 2 we know that all countries in A set the same tariff $\overline{\tau}^a$, so there will be no loss of generality in looking at the response of $\overline{\tau}^a$ to τ^{x+1}. If we can show that there is a monotonic (weakly) negative response of $\overline{\tau}^a$ to τ^{x+1} then we can infer that there will be a monotonic (weakly) negative response of $\overline{\tau}^a$ to an increase of any common tariff, τ^b.

By the implicit function theorem,

$$\frac{\partial \overline{\tau}^a}{\partial \tau^{x+1}} = -\frac{\partial \psi^a_{\tau^a}/\partial \tau^{x+1}}{\partial \psi^a_{\tau^a}/\partial \tau^a} = \frac{-ds^a/d\tau^{x+1} + (\tau^a - 1)\left(d\Delta^a/d\tau^{x+1}\right)}{\partial \psi^a_{\tau^a}/\partial \tau^a}.$$

Since, by Proposition 2, all countries in A set the same tariff, $\overline{\tau}^a$, it must be that $ds^a/d\tau^{x+1} = 0$. By Proposition 1, $\partial \psi^a_{\tau^a}/\partial \tau^a < 0$. It remains to show that $d\Delta^a/d\tau^{x+1} \geq 0$. It will then follow that $\partial \overline{\tau}^a/\partial \tau^{x+1} \leq 0$ as required.

Writing out $d\Delta^a/d\tau^{x+1}$, we have

$$\begin{aligned}
\frac{d\Delta^a}{d\tau^{x+1}} &= \sum_{i \in A \setminus a} s^i \theta^i \frac{\varepsilon^i}{p_i} \frac{dp^i}{d\tau^{x+1}} + \sum_{i \in B} s^i \theta^i \frac{\varepsilon^i}{p_i} \frac{dp^i}{d\tau^{x+1}} + s^{x+1} \frac{d\varepsilon^{x+1}}{d\tau^{x+1}} \\
&\quad + \sum_{i \in A \setminus a} \varepsilon^i \frac{ds^i}{d\tau^{x+1}} + \sum_{i \in B} \varepsilon^i \frac{ds^i}{d\tau^{x+1}}
\end{aligned}$$

We can simplify this expression using the symmetry properties of the equilibrium; as for θ^i, the value of s^i and the value of ε^i is the same for each and every $i \in A$ and will be denoted by s^a and ε^a respectively. Likewise, since by assumption all countries in B set a common tariff τ^b, the value of s^i and the value of ε^i is the same for each and every $i \in B$ and will be denoted by s^b and ε^b respectively. In symmetrical equilibrium all $ds^i/d\tau^{x+1} = 0$ for $i \in A \setminus a$. Also, since we are taking $d\tau^{x+1}$ as representative of a change in the common tariff τ^b, and since $ds^i/d\tau^b = 0$ for a change in the common tariff τ^b, we can impose $ds^i/d\tau^{x+1} = 0$ here. Using these properties plus (3.10) the above equation simplifies as follows:

$$\frac{d\Delta^a}{d\tau^{x+1}} = \left((1 - s^a) \theta^a \varepsilon^a s^b \frac{\phi^b}{\Delta} + s^b \theta^b \varepsilon^b \left(1 - \frac{\phi^b}{\Delta} \right) + s^b \zeta^b \varepsilon^b \right) \frac{1}{\tau^b}. \tag{A.2}$$

The fact that (at any interior solution) optimum tariff intervention implies $\sum_{i \in B} s^i \varepsilon^i - 1 = \varepsilon^b - 1 > 0$ in turn implies that $\left(1 - \phi^b/\Delta \right) > 0$. And by A1-A3, $\varepsilon^a \geq \theta^a \geq 0$, $\varepsilon^b \geq \theta^b \geq 0$, and $\zeta^b \geq 0$. So $d\Delta^a/d\tau^{x+1} \geq 0$ as required.

We now prove that, providing θ^b is sufficiently small, $\overline{\tau}^a$ is decreasing in x. In symmetric equilibrium, where $s^i = s^a = 1/x$ for all $i \in A$ and $s^i = s^b = 1/y$ for all $i \in B$. Then by Proposition 1,

$$\overline{\tau}^a = \frac{s^a}{(x-1)\, s^a \varepsilon^a + y s^b \varepsilon^b - 1} + 1 = \frac{1}{(x-1)\, \varepsilon^a + x\, (\varepsilon^b - 1)} + 1.$$

Differentiating $\overline{\tau}^a$ w.r.t. x,

$$\frac{\partial \overline{\tau}^a}{\partial x} = -\frac{\left(\varepsilon^a + (x-1)\, d\varepsilon^a/dx + \left(\varepsilon^b - 1\right) + x d\varepsilon^b/dx\right)}{\left((x-1)\, \varepsilon^a + x\, (\varepsilon^b - 1)\right)^2},$$

where

$$\frac{d\varepsilon^a}{dx} = \theta^a \frac{\varepsilon^a}{p^a} \frac{\partial p^a}{\partial q} \frac{\partial q}{\partial x} \text{ and } \frac{d\varepsilon^b}{dx} = \theta^b \frac{\varepsilon^b}{p^b} \frac{\partial p^b}{\partial q} \frac{\partial q}{\partial x}.$$

To sign $\partial \overline{\tau}^a/\partial x$ as negative, we only need to sign the numerator: $\varepsilon^a > 0$ by A1; $d\varepsilon^a/dx > 0$ since $\partial q/\partial x > 0$ (because at constant q an increasing in x raises Northern demand for Southern exports) and by (2.1), A1 and A3, all other terms in the expression for $d\varepsilon^a/dx$ are positive; recall from above that $\varepsilon^b - 1 > 0$; although $d\varepsilon^b/dx < 0$, if θ^b is small then the absolute magnitude of $d\varepsilon^b/dx$ is small (where $d\varepsilon^b/dx < 0$ because $\partial p^b/\partial q < 0$).

Finally, we prove that the effect of a change in y on $\overline{\tau}^a$ is ambiguous. Differentiating $\overline{\tau}^a$ w.r.t. y,

$$\frac{\partial \overline{\tau}^a}{\partial y} = -\frac{\left((x-1)\, d\varepsilon^a/dy + x d\varepsilon^b/dy\right)}{\left((x-1)\, \varepsilon^a + x\, (\varepsilon^b - 1)\right)^2},$$

where

$$\frac{d\varepsilon^a}{dy} = \theta^a \frac{\varepsilon^a}{p^a} \frac{\partial p^a}{\partial q} \frac{\partial q}{\partial y} \text{ and } \frac{d\varepsilon^b}{dx} = \theta^b \frac{\varepsilon^b}{p^b} \frac{\partial p^b}{\partial q} \frac{\partial q}{\partial y}.$$

Using the same reasoning as set out above, we see that $d\varepsilon^a/dy < 0$ while $d\varepsilon^a/dy > 0$; the result follows. $\square$

Proof of Proposition 4. The proof focuses on the first part of the result, concerning the Northern equilibrium tariff. The second part of the result follows by analogy. In any symmetric trading equilibrium, all countries in A set the same tariff, $\overline{\tau}^a$. Let τ^b be the *common tariff* set by *all* $i \in B$ and moreover denote by $\partial \tau^b$ a *small change in* τ^b *by all* $i \in B$. If we can establish that $\partial \overline{\tau}^a/\partial \tau^b > -1$, then a standard fixed-point argument may be applied to establish that the Northern reaction function intersects any given Southern reaction function only once.

It remains to establish that $\partial \bar{\tau}^a / \partial \tau^b > -1$ providing θ^i and ζ^i are sufficiently small. By the implicit function theorem, $\partial \bar{\tau}^a / \partial \tau^b$ is given by

$$\frac{\partial \bar{\tau}^a}{\partial \tau^b} = -\frac{\sum_{i \in B} \partial \psi_{\tau^a}^a / \partial \tau^i}{\partial \psi_{\tau^a}^a / \partial \tau^a} = \frac{\sum_{i \in B} \left(-ds^a / d\tau^i + (\tau^a - 1) \left(d\Delta^a / d\tau^i \right) \right)}{\partial \psi_{\tau^a}^a / \partial \tau^a}.$$

We will now examine the numerator and the denominator as θ^i and ζ^i are made to be small. First recall that $-ds^a / d\tau^i = 0$ in symmetric equilibrium of $\Gamma \left(\tau^b \right)$. Next note that each term in the expression for $d\Delta^a / d\tau^i$ is a function either of θ^i or of ζ^i; see (A.2). Hence $d\Delta^a / d\tau^i$ can be made sufficiently small by bounding the size of θ^i and of ζ^i. Note from (A.1) that $\partial \psi_{\tau^a}^a / \partial \tau^a$ does not approach zero as θ^i and ζ^i are made to be small. The first part of the result follows.

The proof that an increase in y has an ambiguous effect on $\bar{\tau}^a$ involves a conjunction of the "direct effect" demonstrated above and the "indirect effect" demonstrated in Proposition 3. Let us take the direct effect first; an increase in y brings about an increase of $\bar{\tau}^a$. Given a symmetric equilibrium of the game $\Gamma \left(\tau^a \right)$, an increase in y brings about a fall in $\bar{\tau}^b$ in the same way that an increase in x brings about a fall in $\bar{\tau}^a$. And a fall in $\bar{\tau}^b$ brings about an increase in $\bar{\tau}^a$ as just demonstrated. But as just noted, the size of this effect is made small by bounding θ^i and ζ^i.

Now let us examine the indirect effect; as demonstrated in Proposition 3, the effect of an increase in y on $\bar{\tau}^a$ is ambiguous but this effect too is made small by bounding θ^i. The result follows. $\square$

Proof of Proposition 5. We will analyze an increase in y on $\hat{q}$ and $d\mu^a$. The effects of a decrease in y and change in x on $d\mu^a$ follow by analogy. All effects on $d\mu^b$ operate in reverse.

Consider the introduction of a new country to the set B. Denote the new element as $x+y+1$ and assume that this country initially sets prohibitive tariffs. Then by Proposition 2 it has an incentive to reduce its tariff to the level of the Southern equilibrium tariff, $\bar{\tau}^b$, and by Lemma 2 this gives all other countries in B, $x+1$ to $x+y$, an incentive to reduce their tariffs. Use Proposition 3 to determine the overall (direct and indirect) change in $\bar{\tau}^b$ of an increase in y by 1; specifically, sum the effect on $\bar{\tau}^b$ of a sequence of changes dy that sum to 1; this sequence brings about a monotonic reduction of $\bar{\tau}^b$. Also by Proposition 3, there is an ambiguous effect of the change in y on $\bar{\tau}^a$; however, the magnitude of this effect

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is relatively small if θ^b is small. Since all countries in B reduce tariffs and tariff changes in A are small, by (3.12), $d\mu^a$ is positive (recall that country a's first-order-condition implies that the term on the first line of (3.12) is equal to zero). $\square$

Proof of Proposition 6. By Proposition 4, an increase in x brings about a reduction in $\bar{\tau}^a$ and an increase in y brings about a reduction in $\bar{\tau}^b$. As the economy is replicated, both x and y are increased in proportion and so both $\bar{\tau}^a$ and $\bar{\tau}^b$ fall. Since tariffs are the only distortions in the world economy, both m^a and m^b converge to their efficient levels as all tariffs converge to zero. And since in symmetric equilibrium m^a is representative of all m^i in M^A, and m^b is representative of all m^i in M^B, aggregate import volumes must converge to efficient levels. $\square$

Proof of Proposition 7. The part that is not obvious is to show that $d\Delta^a/d\tau^j > 0$ for $\beta = 0$. If $\beta = 0$ then, by (5.2), $\theta^i = 0$ for all $i \in N$. Consequently, the first two terms on the right hand side of (5.3) - the equation for $d\Delta^a/d\tau^j$ - are equal to zero. The third term is equal to zero for Cobb-Douglas; again see (5.2).

It remains to sign as negative the final right-hand-side term of (5.3);

$$\sum_{i \in A\backslash a} \varepsilon^i \frac{ds^i}{d\tau^j} \; j \in A\backslash a.$$

First note from (5.1) that, for $\beta = 0$, it is the case that $\varepsilon^i = 1$, all $i \in N$. So it only remains to establish that $\sum_{i \in A\backslash a} ds^i/d\tau^j < 0$. Now, since

$$\frac{ds^j}{d\tau^j} < 0 \text{ while } \frac{ds^i}{d\tau^j} > 0, \, i \neq j$$

the aim is to show that $ds^j/d\tau^j < -\sum_{i \in A\backslash\{a\cup j\}} ds^i/d\tau^j \, (< 0)$.

From the proof of Proposition 1, we know that

$$\hat{s}^j = \left(1 - s^j\right)\hat{m}^j - \sum_{i \in A\backslash j} s^i \hat{m}^i < 0.$$

Substituting for $\hat{m}^j$ and $\hat{m}^i$ by using (3.9) in (3.7) yields

$$
\begin{aligned}
\hat{s}^j \left(\tau^j \right) &= -\left(1 - s^j \right) \left(\sum_{i \in A \setminus j} \varepsilon^i s^i + \sum_{i \in B} \varepsilon^i s^i - 1 \right) \frac{\phi^j}{\Delta} \frac{d\tau^j}{\tau^j} \\
&\quad - \sum_{i \in A \setminus j} \varepsilon^i s^i s^j \frac{\phi^j}{\Delta} \frac{d\tau^j}{\tau^j} \\
&= -\left(\sum_{i \in A \setminus j} \varepsilon^i s^i + \sum_{i \in B} \varepsilon^i s^i - 1 \right) \frac{\phi^j}{\Delta} \frac{d\tau^j}{\tau^j} + s^j \left(\sum_{i \in B} \varepsilon^i s^i - 1 \right) \frac{\phi^j}{\Delta} \frac{d\tau^j}{\tau^j} \\
&= -\sum_{i \in A \setminus j} \varepsilon^i s^i \frac{\phi^j}{\Delta} \frac{d\tau^j}{\tau^j} - \left(1 - s^j \right) \left(\sum_{i \in B} \varepsilon^i s^i - 1 \right) \frac{\phi^j}{\Delta} \frac{d\tau^j}{\tau^j}
\end{aligned}
$$

Then

$$
\frac{ds^j}{d\tau^j} = s^j \hat{s}^j \left(\tau^j \right) \frac{1}{d\tau^j} = -\left(\sum_{i \in A \setminus j} \varepsilon^i s^i + \left(1 - s^j \right) \left(\sum_{i \in B} s^i \varepsilon^i - 1 \right) \right) s^j \frac{\phi^j}{\Delta} \frac{1}{\tau^j}.
$$

Now we can compare $ds^j / d\tau^j$ to $-\sum_{i \in A \setminus \{a \cup j\}} ds^i / d\tau^j$, where

$$
\frac{ds^i}{d\tau^j} = s^i \left(\varepsilon^i + \sum_{i \in B} s^i \varepsilon^i - 1 \right) s^j \frac{\phi^j}{\Delta} \frac{1}{\tau^j},
$$

so that

$$
\begin{aligned}
-\sum_{i \in A \setminus \{a \cup j\}} ds^i / d\tau^j &= -\sum_{i \in A \setminus \{a \cup j\}} \left(\varepsilon^i s^i + s^i \left(\sum_{i \in B} s^i \varepsilon^i - 1 \right) \right) \frac{\phi^j}{\Delta} \frac{d\tau^j}{\tau^j} \\
&= -\left(\sum_{i \in A \setminus \{a \cup j\}} \varepsilon^i s^i + \left(1 - s^j - s^a \right) \left(\sum_{i \in B} s^i \varepsilon^i - 1 \right) \right) \frac{\phi^j}{\Delta} \frac{d\tau^j}{\tau^j} \\
&> ds^j / d\tau^j.
\end{aligned}
$$

as required. $\square$

Proof of Proposition 8. By l'Hôpital's rule, the expenditure function

$$
E^i = \left(\alpha \left(p_1^i \right)^\rho + \left(1 - \alpha \right) \left(p_2^i \right)^\rho \right)^{\frac{1}{\rho}} \mu^i
$$

converges to $E^i = \left(p_1^i \right)^\alpha \left(p_2^i \right)^{(1-\alpha)} \mu^i$ as $\rho \to 0$. Since Propositions 1 - 8 were established for $\rho = 0$, and since all variables are continuous in their arguments, there must exist a range of $\rho < 0$ for which Propositions 1 - 8 continue to hold. The result follows. $\square$

Proof of Proposition 9. Efficiency implications of replication. In a trading equilibrium, by symmetry of the equilibrium world welfare is given by

$$xu^a(x, rx) + rxu^b(x, rx) =$$

$$x \left(\frac{x}{2x-1}\right)^{\frac{1}{2}} \left(\frac{rx(rx-1)}{x(2rx-1)}\right)^{\frac{1}{2}} + rx \left(\frac{rx}{2rx-1}\right)^{\frac{1}{2}} \left(\frac{x(x-1)}{rx(2x-1)}\right)^{\frac{1}{2}}$$

We will show that the above expression is globally increasing in x. Differentiating, we get

$$\frac{d\left(xu^a(x, rx) + rxu^b(rx, x)\right)}{dx} = \frac{1}{2x}\left(\Theta(x; r) + \Xi(x; r) + \Phi(x; r)\right)$$

where

$$\Theta(x; r) = \left(\frac{x}{2x-1}\right)^{\frac{3}{2}} (4x - 3)\left(\frac{r(rx-1)}{2rx-1}\right)^{\frac{1}{2}},$$

$$\Xi(x; r) = \frac{2r^2x^3 \left(\frac{x}{2x-1}\right)^{\frac{1}{2}}\left(\frac{r(rx-1)}{2rx-1}\right)^{\frac{1}{2}}}{(rx-1)(2rx-1)^2} + \frac{r(-x)^{\frac{3}{2}}x\left(\frac{r(rx-1)}{2rx-1}\right)}{(1-2x)^{\frac{1}{2}}(rx-1)(2rx-1)^2}$$

and

$$\Phi(x; r) = \frac{\left(\frac{x-1}{2rx-r}\right)\left(\frac{rx}{2rx-1}\right)^{\frac{3}{2}}(2x(r(x(4x-5)+2)+4-3x)-3)}{(x-1)(2x-1)}.$$

The result is established by verifying that $\Theta(x; r) > 0$, $\Xi(x; r) > 0$ and $\Phi(x; r, \alpha) > 0$ for all feasible x and r.

We will show the existence of a value $r' > 1$ for which $du^a(rx)/dx \gtreqless 0$ for $r \lesseqgtr r'$. To do so, first observe that

$$\frac{du^a(x, rx)}{dx} = \frac{du^a(x, rx)}{dx} + r\frac{du^a(x, rx)}{dx}$$

$$= \frac{r(2rx(1+x-rx)-1)}{2(1-2x)^{\frac{3}{2}}(-x)^{\frac{1}{2}}(2rx-1)^2\left(\frac{r(rx-1)}{2rx-1}\right)^{\frac{1}{2}}}$$

We can see that $du^a(rx)/dx$ is monotonically decreasing in r. Now if we fix $r = 1$ we find that

$$\frac{du^a(rx)}{dx} = \frac{2x-1}{2(1-2x)^{\frac{7}{2}}(-x)^{\frac{1}{2}}\left(\frac{x-1}{2x-1}\right)} > 0$$

So there must exist a value $r' > 1$ for which $du^a(x, rx)/dx = 0$. $\square$

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