

LIQUIDITY PREMIUM AND INTERNATIONAL SEIGNIORAGE PAYMENTS

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Why do people hold dollar denominated assets when higher rate of return alternatives are available? Can a country collect seigniorage payments from other countries in the long run? Does the supplier of the international currency benefit from doing so? I provide qualitative answers to these related questions in terms of a model with price dispersion, heterogeneous agents and two government-backed assets (interest-bearing monies). In the steady state one of the assets is used primarily in low price transactions and earns a relatively low (measured) real rate of return. The stable demand country that issues the relatively liquid asset gets seigniorage but its welfare may be less than under autarky because trade increases the uncertainty about demand in the relevant markets and uncertainty sometimes leads to ex-post pricing mistakes and waste.

JEL codes: E42, G12

Key Words: Liquidity, Sequential Trade, International Currency, Currency Substitution, The Friedman Rule, Seigniorage.

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1. INTRODUCTION

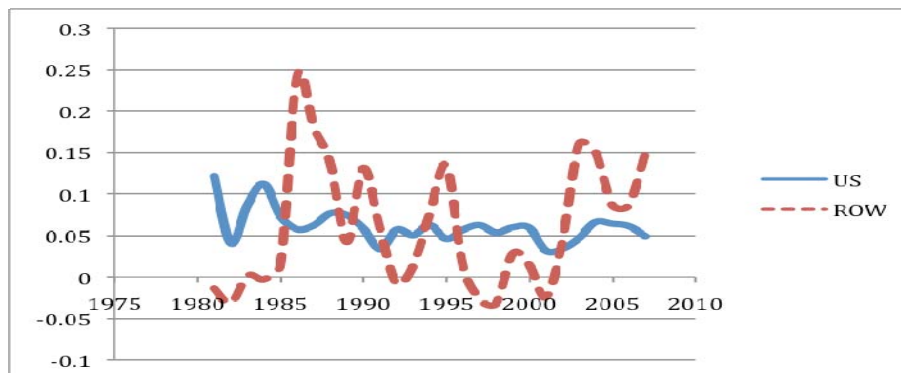
It seems that dollar denominated assets held by non-US residents earn a lower rate of return than comparable alternatives. In a recent study, Gourinchas and Rey (2005) found that during the post Bretton Woods era (1973 - 2004) the real rate of return on foreign bonds held by US residents was 4.05% while the real rate of return on US bonds held by foreigners was only 0.32%. Why are non-US residents willing to hold US bonds when bonds with higher rate of returns are available at home? Can the US get seigniorage payments in a long-run steady state equilibrium by selling bonds below their "correct price"?

Here I use an overlapping generations model with two government-backed assets to show that it is possible to get a steady state equilibrium in which bonds issued by the home country earn a relatively low rate of return and the home country gets seigniorage payments from the rest of the world. This may occur if nominal demand in the home country is more stable (predictable) than in the rest of the world, a condition that is likely to be met for countries like the US, the UK and possibly other G-7 countries.

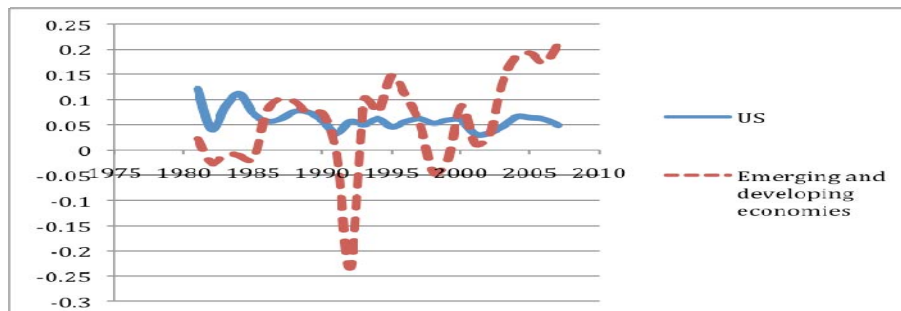
Aguiar and Gopinath (2007) found that emerging market experience substantial volatility in trend growth and that the average standard deviation of change in output in emerging markets is about twice as that of developed markets. They measure GDP in constant prices.

From the point of view of sellers who choose prices, predicting nominal rather than real demand is relevant. Figure 1 plots the rate of change in nominal GDP. Panel A plots the rates of change of GDP in the US and in the rest of the world (*ROW*). Panel B plots the rates of change in the US and in the group of 142 countries labeled by the IMF as emerging and developing economies. The Figure suggests that nominal demand is more predictable in the US than in the *ROW* when measured in dollar

terms. During the period 1980-2007, the standard deviation of the rate of growth in US nominal GDP was 2% while the comparable number for the *ROW* was 7.6%, almost 4 times larger.¹



A.



B.

Figure 1: Rates of change in nominal GDP (measured in current US dollars)

Here I study the implications of the difference in the predictability of demand between developed economies like the US and the UK and less developed (emerging markets) economies. I use a flexible price version of Prescott's (1975) "hotels" model: The uncertain and sequential trade (UST) model in Eden (1990, 1994, 2007)

¹ Based on the IMF World Economic and Financial Surveys (World Economic Outlook Database, April 2008). See <http://www.imf.org/external/pubs/ft/weo/2008/01/weodata/index.aspx>

and Lucas and Woodford (1993). Other versions of the Prescott model are in Bryant (1980), Rotemberg and Summers (1990) and in Dana (1998).

Similar to the cashless economy in Woodford (2003) the assets in the model are interest bearing government obligations that can be exchanged directly for goods. One possible interpretation may therefore assume that the assets in the model are government bonds. The other may assume that these are monies. In any case, both assets can serve as a unit of account.

In many papers on international trade there is an assumption about the units of account used and the results are typically sensitive to changes in this assumption. In a recent paper Devereux and Engel (2003) distinguish between pricing in terms of the producer currency (PCP) and pricing in terms of the consumer currency (LCP). They show that the implications of risk for foreign trade are highly sensitive to the choice of currency at which prices are set. Here sellers choose the pricing currency and this choice may vary across sellers and across units that belong to the same seller.

The questions asked here are similar to the questions in the random matching models pioneered by Jones (1976), Kiyotaki and Wright (1989) and in the articles that use this approach to discuss international currency (Matsuyama, Kiyotaki and Matsui [1993] and Wright And Trejos [2001], for example). In both the random matching models and the uncertain and sequential trade (UST) model uncertainty about trading opportunities plays a key role. In the random matching models agents are uncertain about whether they will meet someone that they can actually trade with. But whenever a meeting takes place it is bilateral. In the UST model sellers are also uncertain about the arrival of trading partners but whenever a meeting occurs there is a large number of agents on both sides of the market. As a result there is a difference between the assumed price determination mechanisms. In the random matching models prices are either fixed or are determined by bargaining (as in Trejos and Wright [1995] and Shi [1995]). In the UST model prices clear markets that open.

Maybe the closest to the present paper is Lagos and Rocheteau (2008) who allows for competing media of exchange. In their formulation the two competing media (money and capital) earn the same rate of return while here there is a difference in the rate of return between the two assets. To get a difference in rates of returns the matching literature focus on differences in acceptability that may arise for example when some assets are more susceptible to counterfeiting than others or when there is asymmetric information about asset returns. See Lagos (2008) for discussion and references. Here a difference in the measured rates of return arises because of agents' heterogeneity: different types value liquidity differently.

2. OVERVIEW

The paper uses a 2 period overlapping generations model with 2 types of agents and 2 countries. The model is first presented in a simplified version where there is no production. Agents get an endowment of a non-storable good in the first period of their life but want to consume only in the second period of their life. The 2 countries (US and ROW) are characterized by the type of agents living there. Each country has a government that backs an interest-bearing asset (dollars or shekels) used for transactions between young and old agents. The US is populated entirely by type 1 agents who always want to consume (in the second period). Type 2 agents live in the rest of the world (*ROW*) and wish to consume with probability π . Otherwise, with probability $1 - \pi$, they leave their asset as an accidental bequest.

To understand how this UST model works, it is easiest to think of 4 separate markets defined by price tags that specify the price and the currency of payment (for example, 2 dollars per unit or 6 shekels per unit). There are 2 markets in each of the two assets. Buyers arrive sequentially and a particular market opens when buyers that are willing to buy at the market's price tag arrive. Whether a particular market opens

or not therefore depends on the realization of demand and the composition of asset ownership. If type 1 old agents hold dollars and shekels, then the first market in each currency open with certainty. If type 2 agents hold both dollars and shekels, then the second market in each currency will open with probability π . If either type does not hold a particular asset, then the associated market will not open. For example, the paper studies a steady-state equilibrium where type 1 agents strictly prefer dollars. In this case, the first shekel market never opens because there are no shekel holding buyers who would show up with probability 1. (The second shekel market opens if type 2 buyers arrive, with probability π).

It is assumed that individual agents cannot affect the price tag offers in the four markets and cannot affect the probabilities that these markets will open. In equilibrium markets that open are cleared and after the end of trade there are no buyers who wanted to exchange their asset for goods and could not do it. But there may be sellers who wanted but could not sell some of their supply. And there may be buyers who could not make a buy at the cheaper price.

The young agent's problem is to maximize expected utility by choosing how to allocate his endowment across the 4 markets before the state of demand is known. To do this, the agent must form expectations about the real value of the proceeds from a sale in each market. It is useful to note that in order for positive amounts of goods to be allocated to the second market in each currency the price in these markets must be higher than in the first markets. The higher prices are required to compensate the seller for the possibility that the market will not open and he will not make a sale.

To understand the expected purchasing power of the proceeds from a sale is somewhat complicated, but very useful in comprehending the model. Uncertainty about the real value of a dollar or shekel stems from 2 sources. The first is the *current period* state of demand. The realization of demand affects the proceeds from goods allocated to the second market (in each currency) and therefore the future portfolio

holdings of next period's buyers (current period sellers) which in turn affects the probability that a dollar will get to buy in the first market at the lower price. This source of uncertainty disappears in the steady state where the portfolios do not change over time. The second source of uncertainty comes from the realization of demand in the *next period*. This is important to a type 1 agent because it determines how much of the consumption good he will get in the first markets at lower prices. If type 2 buyers show up (this is the high demand state), then type 1 buyers may be forced to purchase in the second markets at higher prices.

A steady state equilibrium in which both types hold both assets has a knife-edge property. I focus on an asymmetric steady state equilibrium in which one type specializes and hold one asset only while the other type hold both assets. As a result of the asymmetry, the assets in the model promise different probabilities of making a sale at the low price: The more liquid asset promises a higher probability.

In equilibrium the measured rate of return on the less liquid asset is higher than the measured rate of return on the more liquid asset. The equilibrium difference - the illiquidity premium - exactly compensates the unstable demand type and they hold both assets. The equilibrium illiquidity premium is not large enough to compensate the stable demand type and they specialize in the liquid asset.

I now turn to the details of the model, starting from an exchange economy.

3. AN EXCHANGE ECONOMY

I use an overlapping generations single good model. Two types of people are born each period. They live for two periods, get an endowment of a non-storable good in the first and, if they want they consume in the second. A type j agent gets an endowment of λ^j units of the good. A type 1 that is born at time t will want to consume with probability 1 and his utility function is: $U^1(C_{t+1}) = C_{t+1}$, where C is his

second period consumption. A type 2 wants to consume with probability π . His utility function is: $U^2(C_{t+1}) = \theta_{t+1} C_{t+1}$, where θ is a random variable that may take the realization $\theta = 1$ with probability π and $\theta = 0$ otherwise. Both types maximize expected utility. There is a unit measure of each type of agents. I start by assuming that type 1 agents reside in the US and type 2 agents reside in the rest of the world (*ROW*).

There are two interest bearing monies (or government bonds): dollars and shekels. A dollar promises $R = 1 + r$ dollars at the end of the period. A shekel promises $R^* = 1 + r^*$ shekels at the end of the period. Interest payments are financed by lump sum transfers: At the end of the period Type 1 receives g dollars and type 2 receives g^* shekels.

As in Abel (1985), it is assumed that if the type 2 old agents do not want to consume they leave their assets to type 2 young agents as accidental bequest. An alternative formulation may assume that agents derive utility from bequest as in Barro (1974), but the weight they assign to the utility of future generations is random. The main results will not change if this more general specification is employed.

The aggregate state of the economy is a description of the portfolios held by the old agents after the distribution of transfers and interest payments but before the beginning of trade in the goods market. The aggregate state is denoted by $y = (D^1, D^2, S^1, S^2)$, where D^j (S^j) is the amount of dollars (shekels) per type j old agent.

Trade occurs in a sequential manner. All agents who want to consume form an imaginary line and arrive at the market place one by one according to their place in the line. Upon arrival they see all prices, buy at the cheapest available offers and then disappear. Their place in line is determined by a lottery that treats all agents symmetrically. When $\theta = 0$ only type 1 agents are in the line. When $\theta = 1$ both types are in the line and in any segment of the line there is an equal number of agents from

both types. More generally, any segment of the line represents the population mix in the world economy.

Since agents are risk neutral there is no incentive for young agents to trade in contracts contingent on the current taste shock. There is an incentive to trade in contract contingent on the next period taste shock (type 2 agents would like to sell claims on consumption that will be delivered if demand in the next period is low and they do not want to consume). But the assumption that old agents disappear after the completion of trade and before the realization of demand is known prohibit this type of contracts. There may be a reason for a foreign exchange market. The need to trade in foreign exchange may emerge if agents get a transfer payment in a currency that they do not want to hold. To simplify, I will not have a foreign exchange market and require instead that in the steady state agents get a transfer in a currency that they are willing to hold.

The young agents try to sell their endowments for one or both assets. From the young agents' point of view, demand arrives sequentially in batches. I distinguish between dollar demand and shekel demand. The minimum dollar demand is the amount held by type 1 old agents. Therefore from the sellers' point of view a first batch of D^1 dollars arrives with certainty. A second batch of D^2 dollars arrives if $\theta=1$ with probability π . Similarly a first batch of S^1 shekels arrives with certainty and a second batch of S^2 shekels arrives if $\theta=1$ with probability π .

The representative seller is a price-taker. He knows that if $D^1 > 0$, he can sell at the low price of $p_1(y)$ dollars to (buyers in) the first batch. He can sell at the higher price of $p_2(y)$ dollars to the second batch if it arrives and $D^2 > 0$. The seller can sell for $p_1^*(y)$ shekels to the first batch if $S^1 > 0$ and for $p_2^*(y)$ shekels to the second if it arrives and $S^2 > 0$. The seller chooses how much to sell to the first batch of buyers before he knows whether a second batch will arrive or not.

It may be helpful to think of sellers that put a price tag on each unit that they offer for sale. A price tag may specify the cost of the unit in terms of dollars or in terms of shekels (but not in term of both). Price tags may be different across units.

International trade typically allows for such a price-tag choice. But a restaurant owner in the US will not be able to sell if he only accepts shekels. We may therefore think of the good in the model as perfectly tradable and assume that the amount spent by each buyer on tradable goods and the utility he derives from it do not depend on his consumption of non-traded goods. I elaborate on this interpretation later.

It is convenient to assume four hypothetical markets: a market for exchanging goods for dollars at the price $p_1(y)$ that opens if $D^1 > 0$; a market for exchanging goods for shekels at the price $p_1^*(y)$ that opens if $S^1 > 0$; a market for exchanging goods for dollars at the price $p_2(y)$ that opens if $\theta = 1$ and $D^2 > 0$ and a market for exchanging goods for shekels at the price $p_2^*(y)$ that opens if $\theta = 1$ and $S^2 > 0$. There are thus two dollar markets and two shekel markets. A market opens if buyers with its payment currency arrive.

To simplify, I assume that both types of buyers hold dollars and $D^1, D^2 > 0$. Under this assumption the first dollar market opens with certainty and the second dollar market opens with probability π . The seller knows that he can make a sale in any market that opens. Seller j supplies x_i^j units to the i th dollar market and x_i^{*j} to the i th shekel market ($i = 1, 2$). Figure 2 describes the sequence of events within the period.

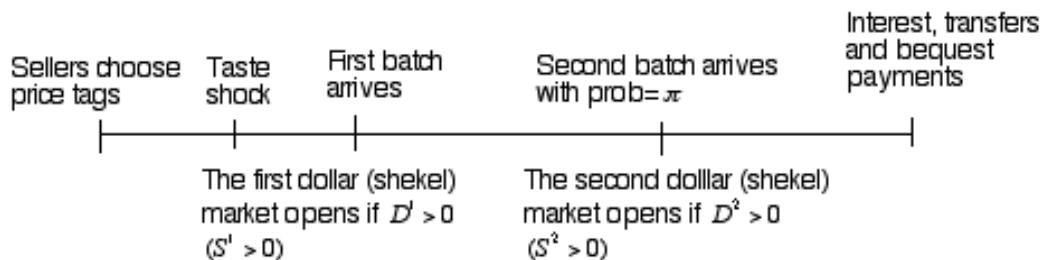


Figure 2

Sellers form expectations about the probability that each asset will be accepted in the next period as payment for goods. They assume that: (a) their own actions cannot affect these probabilities. (The state y is the average portfolio held by each buyer type and an individual agent is small and cannot affect the average portfolio); (b) In the state of low demand ($s = 1$) the first dollar (shekel) market will open if $D^1 > 0$ ($S^1 > 0$) and they will be able to buy in any market that opens (no rationing in the low demand state); (c) if they will not be able to buy in the first markets they will be able buy in the second markets and (d) the probability of buying in the first markets depends on the portfolios held by the buyers (described by the vector y) and the state of current demand but does not depend on calendar time.

To describe the sellers' expectations it is useful to define the fractions of the dollar supply and shekel supply that is held by type 1 buyers:

$$(1) \quad m(y) = \frac{D^1}{D^1 + D^2} ; m^*(y) = \frac{S^1}{S^1 + S^2}.$$

The sellers' expect that in the state of high demand ($s = 2$) the probability of buying in the first dollar (shekel) market is m (m^*). These expectations will be correct in equilibrium: In the high demand state when more money is chasing goods, only a fraction of the total purchasing power will be able to buy in the first markets.

In state of demand s , exactly s dollar markets open and a dollar will buy on average $z_s(y)$ units of consumption where

$$(2) \quad z_1(y) = \frac{1}{p_1(y)} \text{ if } m(y) > 0 \text{ and } \frac{1}{p_2(y)} \text{ if } m(y) = 0;$$

$$z_2(y) = \frac{m(y)}{p_1(y)} + \frac{1-m(y)}{p_2(y)}.$$

Similarly, the expected purchasing power of a shekel is:

$$(3) \quad z_1^*(y) = \frac{1}{p_1^*(y)} \text{ if } m^*(y) > 0 \text{ and } \frac{1}{p_2^*(y)} \text{ if } m^*(y) = 0;$$

$$z_2^*(y) = \frac{m^*(y)}{p_1^*(y)} + \frac{1 - m^*(y)}{p_2^*(y)}.$$

Sellers also form expectations about the average portfolios in the next period, $y_s(y)$, if the current demand state is s . I use

$$(4) \quad \bar{z}_2 = \pi z_2(y_2) + (1 - \pi)z_2(y_1); \quad \bar{z}_2^* = \pi z_2^*(y_2) + (1 - \pi)z_2^*(y_1)$$

to denote the expected value of next period purchasing power (z_2, z_2^*) before the state of current demand is known.

Seller 2 can get p_2 dollars per unit in the second dollar market (if it opens) that will become p_2R dollars after interest payments. If the (current) second markets open the relevant deflator is $z_2(y_2)$ and therefore the expected real price in the second dollar market is $p_2Rz_2(y_2)$ units of consumption. Seller 2 can also get p_1R dollars per unit in the first dollar market. Since in the first market he does not know the state of demand, the relevant deflator is \bar{z}_2 and the expected real price in this market is $p_1R\bar{z}_2$. The expected real prices in the shekel markets can be calculated in a similar way.

Seller 2 thus chooses his supplies to the four markets (x_i^2 and x_i^{*2}) by solving the following problem.

$$(5) \quad \max_{x_1^2, x_2^2, x_1^{*2}, x_2^{*2}} p_1 x_1^2 R \bar{z}_2 + p_1^* x_1^{*2} R \bar{z}_2^* + \pi \{ p_2 x_2^2 R z_2(y_2) + p_2^* x_2^{*2} R z_2^*(y_2) \}$$

s.t. $x_1^2 + x_2^2 + x_1^{*2} + x_2^{*2} = \lambda^2$ and non-negativity constraints.

The first two terms in the objective function are the expected consumption from supplying to the first markets. The rest is the expected consumption from supplying to the second markets. The constraint says that the total supplies to the four markets must equal the endowment.

I assume a solution in which the supplies to the dollar markets are strictly positive: $x_1^2, x_2^2 > 0$. The first order conditions for a solution of this type are:

$$(6) \quad p_1 R \bar{z}_2 \geq p_1^* R^* \bar{z}_2^* \quad \text{with equality if } x_1^{*2} > 0;$$

$$(7) \quad p_2 R z_2(y_2) \geq p_2^* R^* z_2^*(y_2) \quad \text{with equality if } x_2^{*2} > 0;$$

$$(8) \quad \bar{p}_1 \bar{z}_2 = \pi p_2 z_2(y_2)$$

The first two conditions say that the expected real price in the dollar markets must be greater than the expected real price in the shekel markets. The last equality says that the expected real price in the first dollar market must equal the expected real price in the second dollar market.

Seller 1 will always want to consume. He therefore uses different deflators of current nominal revenues. I start with the expected purchasing power of a dollar (Z) and a shekel (Z^*) held by a type 1 buyer who knows the state (y):

$$(9) \quad Z(y) = \pi z_2(y) + (1 - \pi) z_1(y) ; Z^*(y) = \pi z_2^*(y) + (1 - \pi) z_1^*(y)$$

The unconditional expectations are:

$$(10) \quad \bar{Z} = \pi Z(y_2) + (1 - \pi) Z(y_1) ; \bar{Z}^* = \pi Z^*(y_2) + (1 - \pi) Z^*(y_1)$$

Thus uncertainty about the purchasing power stems from two sources. The first source described by (9) is the state of demand in the next period. The second source

described by (10) is the state of demand in the current period that determines the next period portfolios. As we shall see the second source will disappear in the steady state when the portfolios of the buyers do not depend on the current state of demand.

Using (9) and (10), seller 1 solves:

$$(11) \quad \max_{x_1^1, x_1^{*1}} p_1 x_1^1 R \bar{Z} + p_1^* x_1^{*1} R^* \bar{Z}^* + \pi \{ p_2 x_2^1 R Z(y_2) + p_2^* x_2^{*1} R^* Z^*(y_2) \}$$

$$\text{s.t. } x_1^1 + x_2^1 + x_1^{*1} + x_2^{*1} = \lambda^1 \text{ and non-negativity constraints.}$$

The first order condition for a solution with strictly positive supplies to the dollar markets ($x_1^1, x_2^1 > 0$) are:

$$(12) \quad p_1 R \bar{Z} \geq p_1^* R^* \bar{Z}^* \text{ with equality if } x_1^{*1} > 0;$$

$$(13) \quad p_2 R Z(y_2) \geq p_2^* R^* Z^*(y_2) \text{ with equality if } x_2^{*1} > 0;$$

$$(14) \quad p_1 \bar{Z} = \pi p_2 Z(y_2)$$

The market clearing conditions are:

$$(15) \quad p_1(x_1^1 + x_1^2) = D^1; \quad p_1^*(x_1^{*1} + x_1^{*2}) = S^1; \quad p_2(x_2^1 + x_2^2) = D^2; \quad p_2^*(x_2^{*1} + x_2^{*2}) = S^2;$$

$$g = -rD; \quad g^* = -r^*S$$

where $r = R - 1$ and $r^* = R^* - 1$ are the interest rates, $D = D^1 + D^2 = 1$ is the dollar supply and $S = S^1 + S^2$ is the shekel supply.

Note that the supplies to the first markets must equal the minimum demand. Since only type 1 agents buy in the low demand state, the value of the goods offered in the first dollar market is equal to the amount of dollars held by type 1 buyers and the same holds for shekels. When demand is high some buyers from both types do not make a buy in the first market. These buyers hold a total of D^2 dollars and S^2

shekels. The purchasing power that could not make a buy in the first market buys in the second market so that markets that open are cleared.

The first D^1 dollars that arrive will buy in the first dollar market. In the high demand state, the probability of buying with dollars in the first market is the value of first market goods offered for dollars divided by the dollar supply. Similar statement holds for shekels and therefore the expectations in (1) are correct. Note that it is possible to have $D^1 < S^1$ and $m > m^*$. What is important for liquidity is the use of the asset in its first market relative to its supply and not relative to the other asset.

The next period state is $y_s(y) = (D_s^1(y), D_s^2(y), S_s^1(y), S_s^2(y))$ if the state of current demand is s where:

$$(16) \quad \begin{aligned} D_1^1 &= R p_1 x_1^1 + g ; D_1^2 = R(p_1 x_1^2 + D^2); D_2^1 = R(p_1 x_1^1 + p_2 x_2^1) + g ; \\ D_2^2 &= R(p_1 x_1^2 + p_2 x_2^2); S_1^1 = R^* p_1^* x_1^{*1} ; S_1^2 = R^*(p_1^* x_1^{*2} + S^2) + g^* ; \\ S_2^1 &= R^*(p_1^* x_1^{*1} + p_2^* x_2^{*1}); S_2^2 = R^*(p_1^* x_1^{*2} + p_2^* x_2^{*2}) + g^* \end{aligned}$$

In (16) the beginning of next period balances are equal to revenues + transfer payment + bequest + interest payments. When demand is low there are revenues from goods allocated to the first markets only and type 2 get bequest. When demand is high there are revenues in both markets.

Equilibrium is a policy choice (g, g^*, R, R^*) and a vector of functions

$(p_1, p_2, p_1^*, p_2^*, m, m^*, y_1, y_2, z_1, z_2, z_1^*, z_2^*, x_1^1, x_2^1, x_1^{*1}, x_2^{*1}, x_1^2, x_2^2, x_1^{*2}, x_2^{*2})$ such that all functions are from y to the real line and satisfy the conditions in (1)-(16).

In the steady state the portfolio held by the old agents remains constant over time $y_1 = y_2 = y$ and equilibrium is a vector of scalars rather than a vector of functions. I focus on a steady state equilibrium in which both currencies are used.

Since we assume that dollars are used this assumption implies that at least one shekel market may open.²

I now show the following Lemmas and Claims that characterize the steady state in which both currencies are used. All proofs are in the Appendix.

Lemma 1: $p_1 = \pi p_2$ and $p_1^* = \pi p_2^*$.

Lemma 2: If a seller is willing to supply a strictly positive amount to one shekel market, then he is also willing to supply a strictly positive amount to the other shekel market.

Claim 1: A steady state equilibrium with $S^1 > 0$ and $S^2 > 0$ requires $m = m^*$.

Claim 1 says that agents must hold a symmetric portfolio in equilibrium in which both types hold both assets. This is different from Karaken and Wallace (1981) who get many possible solutions for the case in which both assets are held.

The knife-edge property $m = m^*$ is not likely to hold. I therefore focus on a steady state equilibrium in which one type specializes and hold dollars only. A steady state in which type 2 specializes in dollars and type 1 holds both assets is not natural and violates the assumption that agents get transfer payments in a currency that they are willing to hold. I therefore focus on a steady state equilibrium in which $S^1 = 0$ and $S^2 > 0$. In this steady state, the first shekel market does not open and the second shekel market opens with probability π . Since the first shekel market does not open, the probability of buying at the cheaper price with shekels is zero while the probability of buying at the cheaper price with dollar is greater than zero. In this

² We can also have: (a) No trade equilibrium (none of the four markets ever open) and (b) Equilibrium in which only one currency is used (shekel markets never open).

sense, dollars are more liquid than shekels. I thus define liquidity by the probability of making a buy at the cheaper price. Other definitions of liquidity will be discussed in the conclusion section.

In a steady state in which type 2 agents hold shekels and type 1 do not it must be the case that type 2 supplies a strictly positive amount to the second shekel market

($x_2^{*2} > 0$) and (7) holds with equality. This leads to: $\frac{R}{R^*} = \frac{p_2 z_2^*}{p_2 z_2}$. Since $m^* = 0$, (3)

implies $p_2 z_2^* = 1$. The definition (2) implies $p_2 z_2 = m(p_2/p_1) + (1-m)$. We can now use Lemma 1 to substitute $1/\pi$ for p_2/p_1 and to get:

$$(17) \quad \frac{R}{R^*} = \frac{p_2 z_2^*}{p_2 z_2} = \frac{1}{p_2 z_2} = \frac{1}{m(p_2/p_1) + (1-m)} = \frac{\pi}{m + (1-m)\pi} < 1.$$

We now use (17) to solve for the illiquidity premium:

$$(18) \quad R^* - R = mR \left(\frac{1}{\pi} - 1 \right) > 0.$$

The premium in (18) compensates type 2 for the illiquidity of the shekel but it is not large enough to compensate type 1 agents who strictly prefer dollars. The reason is that type 1 agents value liquidity relatively more because they buy in both states and the advantage of the dollar is larger in the low demand state when they buy at the cheaper price with probability 1.

I now turn to solve for the steady state magnitudes. Since in the steady state type 1 agents do not hold shekels, we have: $x_1^{*1} = x_2^{*1} = 0$. A steady state in which the portfolio of type 1 does not change over time requires that they will supply to the first dollar market only. I therefore assume: $x_1^1 = \lambda^1, x_2^1 = 0$. Under these assumptions, the steady state portfolios $y = (D^1, S^1 = 0, D^2, S^2)$ are:

$$(19) \quad D^1 = Rp_1x_1^1 + g = Rp_1x_1^1 - rD; \quad S^2 = p_2^*x_2^{*2}; \quad D^2 = p_2x_2^2$$

The amount of dollars held in the steady state by a type 1 agent (D^1) is equal to his secured revenues from the first dollar market plus the transfer payment. The amount of dollars held by a type 2 agent (D^2) is equal to his revenues in the second dollar market. To see that this must be the case, note that since in the steady state the amount of dollars held by type 2 buyers does not depend on the state of demand, D^2 must satisfy: $R(p_1x_1^2 + p_2x_2^2) = R(p_1x_1^2 + D^2)$. The left hand side of this equation is the amount earned by a type 2 seller in the high demand state and the right hand side is the amount earned in the low demand state plus the value of the bequest. This leads to $D^2 = p_2x_2^2$. Similarly S^2 satisfies: $R^*p_2^*x_2^{*2} - r^*S^2 = R^*S^2 - r^*S^2$. Note that when demand is low type 2 get bequest that compensates for the lack of revenues in the second dollar market.

Note also that (16) requires that the holding of dollars by type 2 agents will be equal to their dollar revenues in the high demand state: $D^2 = R(p_1x_1^2 + p_2x_2^2)$. This and (19) implies: $-rD^2 = -rp_2x_2^2 = Rp_1x_1^2$. This says that the revenues in the first dollar market are used to pay seigniorage. Since $x_i^2 \geq 0$, it also implies that in the steady state $r \leq 0$. Thus in the steady state US government bonds yield negative real rate of return (but may still yield positive nominal returns). The real rate of return on shekels is higher and may be strictly positive. In the next section I introduce population growth to allow for positive real rate of return on the dollar.

I use $b = \frac{\lambda^2}{\lambda^1}$ to denote the relative size of country 2 and show the following Claim.

Claim 2: A steady state equilibrium that satisfies (19) exists if $r \leq 0$ and

$$1 \geq m \geq F(b, \pi, r) = \frac{\pi R - r - br}{\pi R - r + b}.$$

Under the condition $r \leq 0$, the function $F(b, \pi, r)$ is decreasing in b . This means that the larger the size of country 2 is, the smaller m may be. This is intuitive because we

cannot have a steady state in which a small country hold a large fraction of the world supply of assets. To get some quantitative idea of the constraint, we may consider the case in which $R=1$, $b=3$ and $\pi=0.95$. In this case the constraint is:

$$m \geq \frac{\pi}{\pi+b} = 0.24. \text{ We also note that } F(b, \pi, r) \text{ is decreasing in } r \text{ and increasing in } \pi.$$

As was said before in the steady state sellers in the *ROW* use their revenues in the first dollar market to pay seigniorage: $-rD^2 = Rp_1x_1^2$. Does this mean that the *ROW* loses from trade? To answer this question, I now turn to discuss welfare in the steady state.

Claim 3: Welfare in the US (W) and in the *ROW* (W^*) are given by:

$$(20) \quad \begin{aligned} W &= Zm = \left(\frac{R\lambda^1 m}{m+r} \right) \{ \pi(m + \pi(1-m)) + (1-\pi) \} \\ W^* &= \pi(\lambda^1 + \lambda^2 - z_2 m) = \pi(\lambda^1 + \lambda^2) - \pi \left(\frac{R\lambda^1 m}{m+r} \right) (m + \pi(1-m)) \\ W + W^* &= \pi(\lambda^1 + \lambda^2) + (1-\pi) \frac{R\lambda^1 m}{m+r} \end{aligned}$$

I start with the special case $R=1$. In this case there are no seigniorage payments and (20) is simplified to: $W = \lambda^1 \{ \pi(m + \pi(1-m)) + (1-\pi) \}$,

$$W^* = \pi(\lambda^1 + \lambda^2) - \lambda^1 \pi(m + \pi(1-m)) \text{ and } W + W^* = \pi(\lambda^1 + \lambda^2) + (1-\pi)\lambda^1. \text{ Since the}$$

sum $W + W^*$ does not depend on m , changes in m only redistribute welfare between the two countries. To satisfy the conditions in Claim 3, I allow m to vary between

$$m = \frac{\pi}{\pi+b} \text{ and } 1. \text{ } W^* \text{ is decreasing in } m \text{ and is maximized at the lower limit:}$$

$$m = \frac{\pi}{\pi+b}. \text{ Thus when } R=1, \text{ the } \textit{ROW} \text{ gains from trade. US welfare is increasing in}$$

m and is maximized under autarky when $m=1$. The intuition is as follows. When $R=1$, $x_1^2=0$ and sellers in the *ROW* supply only to the second dollar market. In the high demand state some buyers from the *ROW* buy at the low dollar price and some US buyers are forced to buy at the high dollar price. As a result, US exports at the low

price and imports at the high price. This term of trade effect leads to a welfare loss in the US and welfare gains in the *ROW* all relative to autarky.

In general, the World's aggregate expected consumption $W + W^*$ depends on both R and m and is increasing in the real value of the seigniorage payment. To show this claim, I use (A11) in the Appendix to compute the total supply to the first market.

$$(21) \quad x_1(R, m) = x_1^1 + x_1^2 = \frac{m}{p_1} = \frac{R\lambda^1 m}{m + r}$$

Substituting (21) in (20) we get $W + W^* = \pi(\lambda^1 + \lambda^2) + (1 - \pi)x_1$ which says that welfare is equal to expected aggregate consumption: In the high demand state the entire endowment is consumed and in the low demand state only the supply to the first market is consumed. Expected consumption is increasing in the supply to the first dollar market because only goods allocated to the first dollar market are consumed with certainty. When $r < 0$ and $0 < m < 1$, $x_1(R, m)$ is decreasing in both arguments. The World's expected consumption is therefore maximized when both r and m are low. The intuition is in the fact that when both are low the real seigniorage payment (equal to the amount of goods allocated by type 2 to the first market) is high and we are transferring resources to type 1 agents who are more "efficient" in consumption. Note that the real seigniorage payment $x_1^2(R, m) = \frac{R\lambda^1 m}{m + r} - \lambda^1 = \lambda^1 \left(\frac{Rm}{m + r} - 1 \right)$ is different from the nominal seigniorage payment $-rD^2 = -r(1 - m) = Rp_1 x_1^2$ but both are decreasing in r and m .

I now turn to take account of the fact that m is determined endogenously in our model. In general, the *ROW* is composed of many countries and therefore we may assume that it takes the real interest on US bonds as given. The *ROW* can then choose R^* and this determines the equilibrium supply of shekels as well as m . To simplify we assume that the *ROW* chooses $m = m(R)$ directly subject to the

constraint that guarantees the existence of equilibrium:

$$1 \geq m \geq F(b, \pi, r) = \frac{\pi R - r - br}{\pi R - r + b}.$$

In what follows I assume that the *ROW* chooses $m(R)$ to maximize welfare. This in general, leads to a decreasing reaction function: $m(R)$. The intuition is in the many models of currency substitution and the fact that the holding of US bonds by the *ROW* is $1 - m$. We can now write the supply to the first market as:

$$x_1(R) = \frac{Rm(R)}{m(R) + R - 1}. \text{ When } m'(R^2 - R) > 0 \text{ is small } x_1(R) \text{ is a decreasing function}$$

implying that $W + W^*$ declines with R . In this case we can also show that $W(R) = x_1(R)\{\pi(m(R) + \pi(1 - m(R))) + (1 - \pi)\}$ is a decreasing function and $W^*(R) = \pi(\lambda^1 + \lambda^2) - x_1(R)\pi(m(R) + \pi(1 - m(R)))$ is an increasing function.

Note that changes in R cause a redistribution of welfare between the two countries. Therefore any level of R leads to an allocation that is efficient in the sense that it solves the problem of maximizing the welfare in one country subject to a given level of welfare in the other country. This is different from standard Walrasian models in which one can focus on maximizing expected world's consumption and then allow for side payments to get the desired distribution of income. Here the so-called planner's problem cannot be separated in this way. The reason we get higher expected world's consumption when seigniorage payments go up is precisely because type 1 agents are more efficient in consumption. Of course one could give the planner more information than the agents in the economy and allow him to distribute aggregate consumption after the realization of the taste shock is known. But this more powerful planner is not useful for capturing the information constraints that arise as a result of the sequential nature of trade and the fact that agents must make irreversible decisions before they know the realization of demand. I will elaborate on this point shortly.

Figure 3 illustrates the effects of changes in R when $\pi = 0.95$ and $\lambda^1 = \lambda^2 = 0.5$. The reaction function $m(R)$ (on the right axis) is decreasing in accordance with the currency substitution intuition. Welfare (expected consumption)

in the US is weakly decreasing with R and attains its maximum level under autarky when $m = 1$. Welfare in the *ROW* is weakly increasing in R and attains its minimum level under autarky.

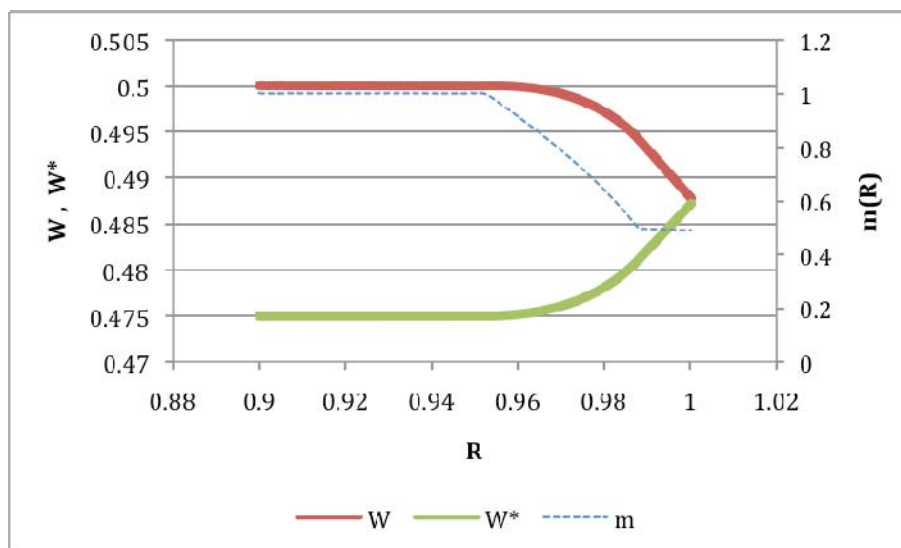


Figure 3: The *ROW* best reaction ($m(R)$, on the right axis) and welfare ($\pi = 0.95, \lambda^1 = \lambda^2 = 0.5$)

I now turn to a more realistic case in which low real interest rate distorts choices.

4. A PRODUCTION ECONOMY WITH STORAGE

I assume that instead of goods, young agents get an endowment of time that they can use to produce goods. To calibrate the model I introduce two additional realistic features: storage and population growth. Storage is introduced because national accounting treats unsold storable goods as an investment in inventories. Population (or productivity) growth is introduced to allow for a positive interest rate.

It is assumed that production and storage technologies are the same across types. Output is equal to labor input and the utility cost of production is

$v(L) = (\frac{1}{2})(L)^2$, where L is the amount of labor (= output). A young agent who does not sell his output can store it and get in the next period q units per unit stored. To simplify, I assume that a unit stored for two periods is worthless.

The population gross growth rate, Φ , is also the same across types. I start with the case of autarky in the *ROW*. I drop the asterisk and superscripts notation for the autarkic case when no distinction between the two types is necessary.

The utility function of the representative type 2 agent born at time t is: $\theta_{t+1}c_{t+1} - v(L_t)$. The amount of shekel supply per old agent is constant and is given by S . (Later I use S^2 to denote the amount of shekels per type 2 old agent). There is only one market that opens with probability π . The shekel price of the good if the market opens is p and $z = \frac{1}{p}$ is the expected purchasing power of a shekel. I assume that the rate of return on storage is less than the rate of return on bonds ($q \leq R$) and therefore the young agent tries to sell the good and store it only if he fails to make a sale. The young agent expects to get (pay) a lump sum of g shekels and chooses output by solving:

$$(22) \quad \max_L \pi(\pi p R z + (1 - \pi)q)L + g z - v(L)$$

The term $(\pi p R z + (1 - \pi)q)L$ is the expected real labor income conditional on wanting to consume ($\theta_{t+1} = 1$). The agent gets a real wage of $p R z = R$ when demand is high and a real wage of q otherwise. To get the unconditional expected real income we multiply by the probability that $\theta_{t+1} = 1$.

The first order conditions for an interior solution to (22) are:

$$(23) \quad \pi(\pi p R z + (1 - \pi)q) = \pi^2 R + (1 - \pi)\pi q = v'(L) = L$$

This says that the marginal cost L should equal the expected real wage $\pi^2 R + (1 - \pi)\pi q$. To get the intuition for the π^2 term I first consider the case of no storage: $q = 0$. In this case, the young agent will reap the benefits from his effort only if he sells (if $\theta_t = 1$) and if he wants to consume ($\theta_{t+1} = 1$). The probability that this joint event will occur is π^2 and therefore the real wage is $\pi^2 R$. When $q > 0$, the expected real wage can be written as $\pi^2(R - q) + \pi q$. The benefit from trying to sell is the difference in the real rates of return $(R - q)$ and this benefit will be realized with probability π^2 .

There are $\frac{1}{\Phi}$ old agents per young agent. Since S is the amount of shekels per old agent, the amount of shekels per young agent is $\frac{S}{\Phi}$ and the market clearing condition is:

$$(24) \quad pL = \frac{S}{\Phi}$$

To close the model we add a condition that determines g . For this purpose, note that $\frac{S}{\Phi}$ = total amount of shekels held by the old = total revenue if demand is high = before interest bequest if demand is low. The young agent will therefore hold $\frac{SR}{\Phi} + g$ after interest and transfers. In the steady state the young holds S shekels at the end of the period after interest and transfer payments. Therefore $\frac{SR}{\Phi} + g = S$ and $g = S \frac{\phi - r}{\Phi}$, where $\phi = \Phi - 1$ is the (net) population growth.

I now turn to discuss efficiency. To compute the expected consumption in the steady state note that the old agents will have inventories if they did not sell in the previous period. The stock of inventories per old agent in the steady state is $(1 - \pi)qL$ on average. The output currently produced is ΦL per old agent. The expected consumption in the steady state is therefore $\pi(\Phi L + (1 - \pi)qL)$ and a planner who maximize the steady state welfare solves:

$$(25) \quad \max_L \pi(\Phi + (1 - \pi)q)L - v(L)$$

The first order condition to the planner's problem (25) is:

$$(26) \quad \pi(\Phi + (1 - \pi)q) = v'(L)$$

The equilibrium outcome (23) coincides with the planner's solution (26) if:

$$(27) \quad R = \Phi/\pi$$

The policy choice (27) holds also for the case in which the utility function of the representative agent is $\beta\theta_{t+1}c_{t+1} - v(L_t)$ and does not depend on the magnitude of β . The intuition for the case $\Phi = 1$ and $q = 0$ is as follows. From the social point of view a unit produced will be consumed if $\theta_t = 1$ with probability π . From the individual's point of view the benefits from a unit produced will materialize with probability π^2 . When $R = 1$ and $\pi < 1$ there is a discrepancy between the social and the private point of views. When $R = 1/\pi$ the two points of views coincide. To get the intuition for the general case assume that the planner is considering a unit increase in the labor supply of all young agents (in the current and all future generations). When the old generation in a representative period does not want to consume the planner gets q units (per young agent) that will be consumed next period with probability π yielding an expected payoff of πq . This is also the expected payoff from the individual's point of view. But when the old generation wants to consume the planner gets Φ units while the individual gets R units with probability π . We therefore must choose $R = \Phi/\pi$ to equate the social and the individual payoffs in the high demand state.

Welfare in the steady state is:

$$(28) \quad \begin{aligned} W &= \pi(\Phi + (1 - \pi)q)L - v(L) \\ &= \pi(\Phi + (1 - \pi)q)(\pi^2 R + (1 - \pi)\pi q) - (\frac{1}{2})(\pi^2 R + (1 - \pi)\pi q)^2 \end{aligned}$$

Welfare attains its maximum level when $R = \frac{\Phi}{\pi}$ and $L = \pi\Phi + (1 - \pi)\pi q$. The maximum level of welfare is:

$$(29) \quad W^{\max} = (\frac{1}{2})(\pi\Phi + (1 - \pi)\pi q)^2$$

The analysis of autarky in the US is a special case with $\pi = 1$.

I now turn to the full integration case.

Trade under Full Integration: There are b young agents in the *ROW*, $\frac{1}{\Phi}$ old agents in the US and $\frac{b}{\Phi}$ old agents in the *ROW* all per young agent in the US. I will analyze a representative period in which the number of young agents in the US is 1, the number of old agents in the US is $\frac{1}{\Phi}$ and the number of old agents in the *ROW* is $\frac{b}{\Phi}$. As before, I assume that 100% of the US population is of type 1 and 100% of the *ROW* population is of type 2. This assumption will be relaxed later.

Motivated by the discussion in the exchange economy case, I assume that type 1 agents do not hold shekels and focus on a steady state where the portfolios of the representative old agents are described by: $(D^1 > 0, D^2 > 0, S^1 = 0, S^2 \geq 0)$. Since the number of old agents in the US is $\frac{1}{\Phi}$ and the number of old agents in the *ROW* is $\frac{b}{\Phi}$, the dollar supply is $\frac{b}{\Phi}$ and the shekel supply is $\frac{1}{\Phi}$ where $S = bS^2$ and $D = D^1 + bD^2$. I normalize by choosing $D = 1$ and use $m = D^1$ to denote the fraction of the dollar supply held by type 1 agents and $1 - m = bD^2$ to denote the fraction of the dollar supply held by type 2 agents. At the beginning of the second period of their life, the representative type 1 buyer gets a transfer of g dollars and the representative type 2 buyer gets a transfer of g^* shekels.

I assume that type 2 seller is willing to supply to all the four markets and choose the quantities supplied by solving:

$$(30) \max_{x_1^2, x_1^{*2}, x_2^2, x_2^{*2}} \pi \{ p_1 x_1^2 R z_2 + p_1^* x_1^{*2} R^* z_2^* + \pi (p_2 x_2^2 R z_2 + p_2^* x_2^{*2} R^* z_2^*) + (1 - \pi) q (x_2^2 + x_2^{*2}) \} \\ - v(L^2 = x_1^2 + x_2^2 + x_1^{*2} + x_2^{*2})$$

The first order conditions for an interior solution to this problem are:

$$(31) \quad p_1 R z_2 = p_1^* R^* z_2^* = \pi p_2 R z_2 + (1 - \pi) q = \pi p_2^* R^* z_2^* + (1 - \pi) q = \frac{v'(L^2)}{\pi}$$

Substituting $z_2 = \frac{m}{p_1} + \frac{1-m}{p_2}$ in $p_1 R z_2 = \pi p_2 R z_2 + (1 - \pi) q$ and using $P = \frac{p_1}{p_2}$ leads to³:

$$(32) \quad P = \pi + \frac{(1 - \pi) q}{p_2 R z_2} = \pi + \frac{(1 - \pi) q P}{R(m + (1 - m) P)}$$

I assume $0 \leq q < R\pi$ and show the following Lemma.

Lemma 3: There exists a unique solution, $\bar{P}(m, q)$, to (32) that satisfy:

(a) $\pi \leq \bar{P}(m, q) < 1$ and (b) $\bar{P}(m, q)$ is decreasing in m and increasing in q .

The intuition for part (a) is as follows. The price ratio is computed under the assumption that seller 2 is willing to supply to all markets and is inversely related to the required compensation for the risk of not selling. The required compensation is $P = \pi$ when $q = 0$ and storage is not possible. The ability to store unsold goods reduces the required compensation and therefore $P \geq \pi$. Since $q < R$ some

³ This also leads to the quadratic equation:

$(1 - m)RP^2 + (Rm - \pi R(1 - m) - (1 - \pi)q)P - \pi Rm = 0$. In the numerical solutions I use the positive solution to this quadratic equation.

compensation is required and therefore $P \leq 1$. Note also that when q goes up the value of unsold inventories is higher and less compensation is required. As a result the price ratio goes up when q goes up.

Lemma 3 and (31) leads to the following Claim.

Claim 4: (a) $p_1RZ \geq p_1Rz_2 = p_1^*R^*z_2^* > q$ and (b) $p_1RZ \geq \pi p_2RZ + (1 - \pi)q$ with strict inequality when $0 < m < 1$.

The surplus from participating in the first dollar market is: $p_1RZ - q$ for a type 1 agent and $p_1Rz_2 - q$ for a type 2 agent that wants to consume. Part (a) says that this surplus is positive for both types and is larger for type 1. Type 1 has a larger surplus because he will buy in both states and therefore has a higher chance of buying at the low price. This leads to $Z > z_2$ and to the first inequality in (a). The second inequality follows from the assumption $\pi R > q$. Part (b) says that type 1 prefers the first market. This is because the surplus from selling in the first market is larger for type 1 and therefore he requires a higher compensation for the risk of not selling.

Type 1 strictly prefers the first market when $m < 1$. When $m = 1$, $Z = z_2$ and type 1 is indifferent between the two markets. But in this case the second dollar market does not open and type 1 sells only in the first market. We may therefore say that in any case type 1 chooses to supply to the first market only and chooses $L = x_1^1$ by solving: $\max_L p_1LRZ - v(L)$. The first order condition for this problem is:

$$(33) \quad v'(L = x_1^1) = p_1RZ$$

Market clearing requires:

$$(34) \quad p_1(x_1^1 + bx_1^2) = \frac{m}{\Phi} ; p_1^*bx_1^{*2} = \frac{S^1}{\Phi} = 0 ; p_2bx_2^2 = \frac{1-m}{\Phi} ; p_2^*bx_2^{*2} = \frac{S}{\Phi}$$

$$g = \frac{\phi - r}{\Phi} ; g^* = \frac{S(\phi - r^*)}{\Phi}$$

In the steady state the portfolios of the representative agents do not change and therefore:

$$(35) \quad m = p_1 LR + g$$

To simplify, I assume that the exchange rate is fixed at the level of one dollar per shekel and $p_2 = p_2^*$.

I define a steady state equilibrium as a policy choice (g, g^*, R, S, R^*) and a vector $(p_1, p_2, m, m^* = 0, z_1, z_2, Z, L = x_1^1, x_1^2, x_2^2, x_1^{*2}, x_2^{*2})$ that satisfy (2), (9), (31), (33), (34) and (35).

Claim 5: When $\phi \geq r$ and $b \geq \frac{s}{\pi}$, there exists a unique steady state equilibrium. The steady state level of m is decreasing in b and r and is increasing in S .

I now compute the illiquidity premium in a way that is similar to the derivation of (18). This leads to:

$$\text{Claim 6: } R^* - R = mR \left(\frac{1}{P} - 1 \right).$$

Note that now P replaces π in the computation of the illiquidity premium.

I now turn to the comparison between free trade and autarky. I show that when there is full dollarization and no government transfers the US suffers from trade.

Claim 7: When $R = \Phi$ and $S^2 = 0$, welfare in the US is lower than under autarky.

The intuition is that demand uncertainty is "bad" in our model because it leads to ex-post "pricing mistakes" and waste (less than full capacity utilization). Since trade increases demand uncertainty the stable demand country suffers from trade.

In general if we start from autarky it is not possible to improve welfare in both countries. To show this claim, I now turn to the problem of a "world planner".

In a representative steady state period, the "world planner" chooses the amount of work by a young type 1 agent (L) and by a young type 2 agent (L^*). He then allocates the total output between the two types of old people giving c units to a type 1 old agent and c^* units to a type 2 old agent subject to the constraint: $c + bc^* = \Phi(L + bL^*)$. The quantities (c, c^*, L, L^*) are chosen prior to the realization of the current period demand shock. When the old type 2 agents do not want to consume (with probability $1 - \pi$) the planner let the young type 2 agent store the unused output of c^*/ϕ units (per young agent). On average a type 2 young agent will therefore consume $\pi(c^* + (1 - \pi)q(c^*/\phi))$ units and his expected utility is therefore $\pi(c^* + (1 - \pi)q(c^*/\phi)) - v(L^*)$. The planner wants to insure that it is larger than the minimum level x . The planner thus solves the following problem⁴.

$$(36) \quad \max_{c, c^*, L, L^*} c - v(L)$$

$$\text{s.t. } c + bc^* = \Phi(L + bL^*) \text{ and } \pi(c^* + (1 - \pi)q(c^*/\phi)) - v(L^*) \geq x.$$

The first order conditions for this problem are:

$$(37) \quad v'(L) = \Phi \text{ and } v'(L^*) = \pi(\Phi + (1 - \pi)q).$$

⁴ Note that the problem (36) is different from the problem of maximizing the sum of utilities: $\max_{c, c^*, L, L^*} c - v(L) + b\{\pi(c^* + (1 - \pi)q(c^*/\phi)) - v(L^*)\}$ s.t. $c + bc^* = \Phi(L + bL^*)$. As was said before, the solution to this problem is to give the entire consumption to type 1 agents who are more efficient in consumption.

These are the same as (26) that were derived for the autarky case. We have also shown that under the optimal monetary policy (27) the UST allocation satisfies (26) and hence (37). We have thus shown the following Claim.

Claim 8: The outcome under autarky with optimal policy in each country solves the World planner's problem (36).

The Claim says that from efficiency point of view there is no reason to trade. But as in the case of an exchange economy trade will occur because type 2 agents want to buy at the cheaper price.

I now turn to a calibration exercise.

5. NUMERICAL EXAMPLES WITH MIXED POPULATION

The above model cannot account for the fact that residents of the US hold foreign assets. To remedy this problem and to move towards a more realistic example, I consider the case in which both types of agents live in both countries but the population mix may be different across countries. I use γ to denote the fraction of the US population that are of type 1, $\mu = 1 - \gamma$ to denote the fraction of the US population that are of type 2. There are b young agents in the *ROW* per young agent in the US. As before, I assume that in the period we analyze, the number of young agents in the US is 1 and the number of young agents in the *ROW* is b . Asterisk notation are used for the *ROW*. To simplify, I assume that the transfer payment is paid to the majority in each country: Type 1 US residents get the dollar transfer and type 2 *ROW* residents get the shekel transfer. Under this assumption agents get transfer in terms of a currency that they are willing to hold and there is no need for a foreign exchange market. The details of the model are in Appendix B.

I start with the view that consumption in the model is an aggregate of storable goods and services. When a good is not sold the entire service component is wasted and also some of the storable component is wasted due to depreciation. For example, a meal is produced in a restaurant by using food ingredients and labor services (cooking, serving the meal). When a meal is not sold, the time of the workers is wasted but the food ingredients can be stored. Only a fraction of the food ingredients will be usable in the next day: The rest will get spoiled.

The fraction of storable goods in the consumption basket, denoted by η , turns out to be a key parameter in the calibration exercise. The percentage of world GDP from services during the period 1980-2004 was 63 on average.⁵ I therefore choose $\eta = 1 - 0.63 = 0.37$. A depreciation rate between 5% and 10% for capital is used in the literature. Here I use a depreciation rate of about 20% because inventories tend to depreciate more than capital. This leads to $q = 0.3$. The results did not change much when I used a depreciation rate of 10%.

The annual gross rate of change in the world's real GDP during the period 1980-2007 (using IMF WEO tables) was 3.3%. I therefore choose $\Phi = 1.033$. I also choose $b = 3$ which implies that the US GDP is 25% of the world GDP.

In our model posted prices do not change but transaction prices and nominal GDP do change. I choose parameters that match the standard deviations of nominal (measured in current dollars) GDP growth: These were 0.2 percent for the US and 0.76 percent for the *ROW*.⁶ The parameter π was chosen to get the average standard

⁵http://earthtrends.wri.org/searchable_db/index.php?action=select_countries&theme=5&variable_ID=216. See also

Development Data Group, The World Bank. 2007. 2007 World Development Indicators Online. Washington, DC: The World Bank. Available at: <http://go.worldbank.org/3JU2HA60D0>.

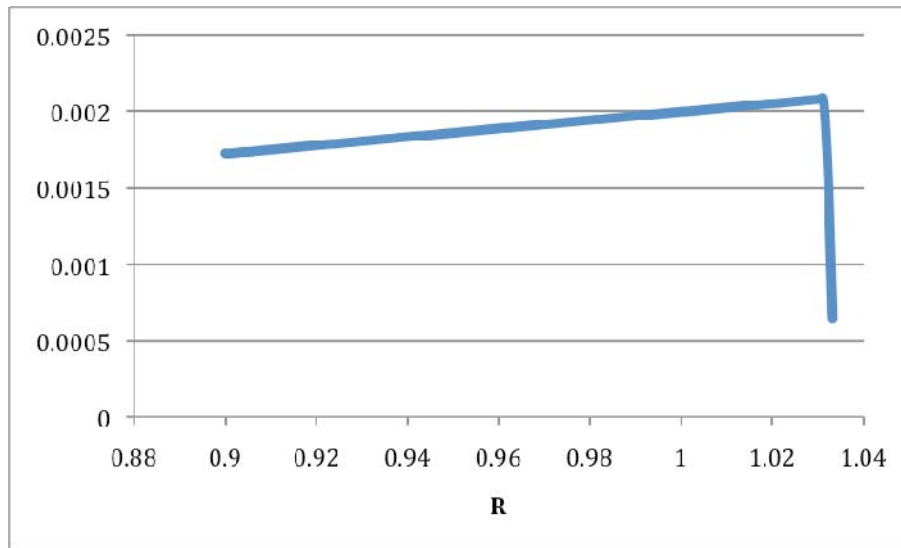
⁶ In an earlier version I introduce a perfectly anticipated inflation by increasing the supply of assets at a known rate. Surprise changes in asset supplies (that occur after the posting of prices) leads in general to uncertainty about nominal demand. This is captured in our model by the parameter π .

deviation (a lower π implies higher average) and the population mix parameter γ was chosen to get the observed ratio of the standard deviations of nominal output growth:

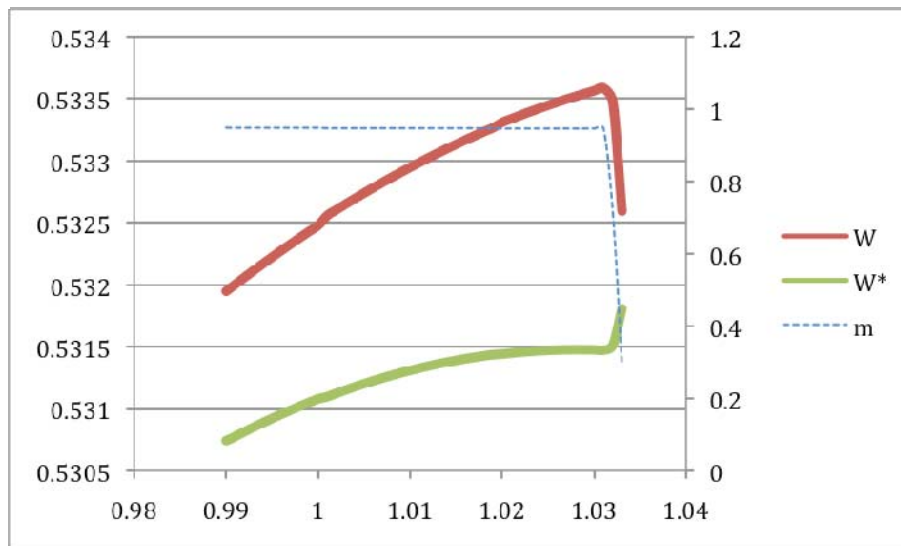
$$\frac{SD(G_{ROW})}{SD(G_{US})} = 3.8.$$

I assumed that the *ROW* takes R as given and chooses $m(R)$ to maximize welfare. The US moves first and chooses R to maximize welfare taking the reaction function $m(R)$ as given. The assumption that both countries maximize welfare is just one out of several possibilities to close the model. Indeed it may be more reasonable to assume that R is exogenously given at a level that is close to unity.

Figure 4 illustrates some of the equilibrium magnitudes for the above choice of parameters. Panel A is the liquidity premium as a function of R . The liquidity premium reaches a maximum at 0.2% and is an order of magnitude smaller than the observation made by Gourinchas and Rey. Panel B plots welfare in the two countries as well as the best reaction function $m(R)$. We see that $m(R)$ is a decreasing function in accordance with the currency substitution literature. When $R = 1.033$ there are no seigniorage payments and the *ROW* chooses the lowest m that is consistent with the existence of equilibrium (see the conditions in Claim B1 in the Appendix). When R is reduced the *ROW* reacts by increasing m and lowering the amount of dollars in their portfolio. Welfare in the *ROW* is increasing in R and welfare in the US is hump shaped reaching a maximum at $R = 1.031$ that is very close to the best policy under autarky ($R = 1.033$). By reducing the interest rate from $R = 1.033$ to $R = 1.031$ the US gains close to 0.2% in terms of welfare (average consumption).



A. Liquidity premium $R^* - R = R\left(\frac{1}{P} - 1\right)m(R)$ as a function of R



B. Welfare as a function of R . US welfare is maximized at $R = 1.031$

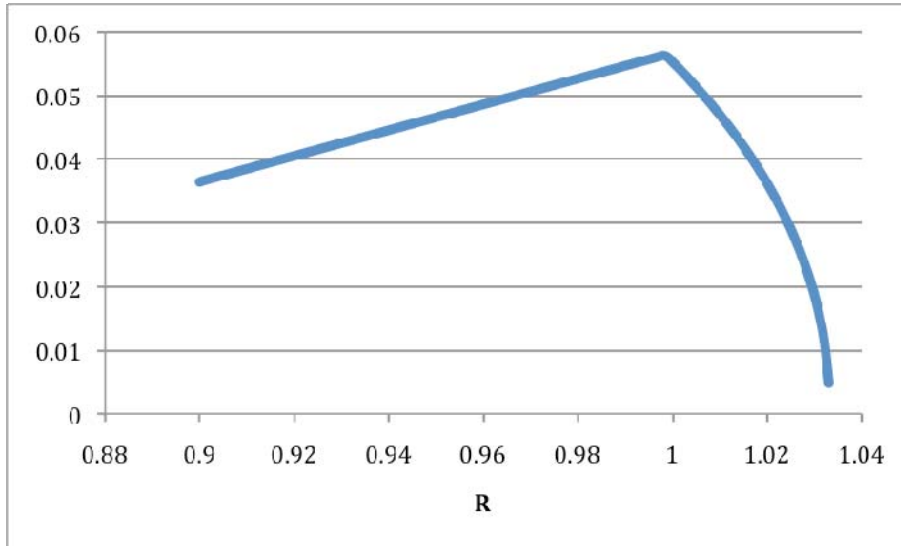
Figure 4: The effects of changing R using the best response of the ROW .

$(\eta = 0.37, q = 0.3, \pi = 0.997, \gamma = 0.65, \mu = 0.35, \gamma^* = 0.093, \mu^* = 0.907, b = 3, \Phi = 1.0332)$

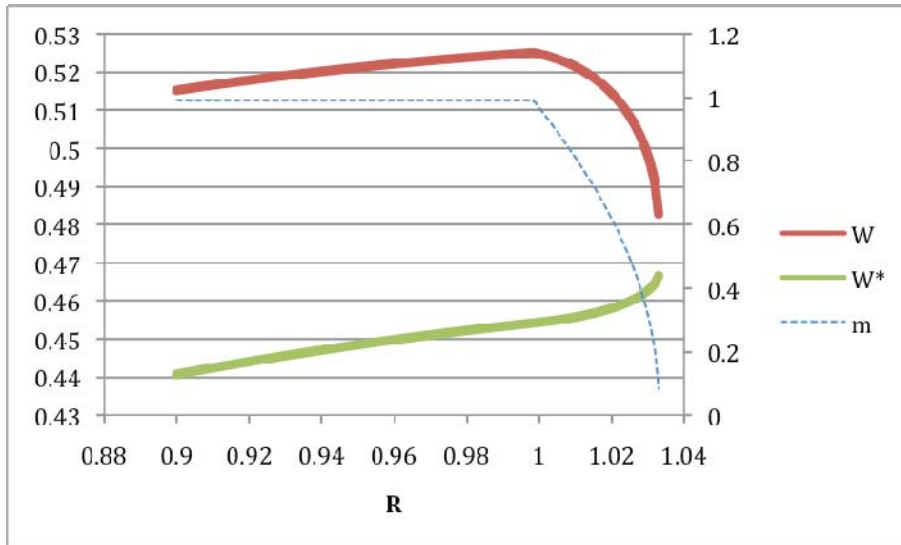
An alternative interpretation of the model may distinguish between traded and non-traded goods and assume that the good in the model is tradable. This will lead to a

higher η because most traded goods are storable.⁷ Assuming $\eta = 0.7$ and $q = 0.56$, leads to $(\pi = 0.974, \gamma = 0.72)$ and to a liquidity premium of 1.13%. Assuming $\eta = 0.9$ and $q = 0.72$ leads to $(\pi = 0.83, \gamma = 0.72)$ and to a liquidity premium of slightly more than 5%. Figure 5 illustrates the last example. The difference in the US welfare between choosing the optimal policy of $R = 1$ and choosing the no seigniorage policy of $R = 1.033$, is large and is equivalent to raising consumption by 8.7% on average.

⁷ I have not worked out the details of such a model but the main ingredients may be as follows. We may assume a utility function $U(y) + \theta c - v(L)$, where y denotes the consumption of non-traded goods and c is the consumption of traded goods. We may assume that non-traded goods can be bought with the local currency only in standard Walrasian markets that operate before the realization of the taste shock and the trade in the sequential markets for traded goods. In this setting there is no uncertainty about the amount that the buyer will spend on the non-traded goods and we can focus on the sequential market part of trade.



A. Liquidity premium $R^* - R = R\left(\frac{1}{P} - 1\right)m(R)$ as a function of R



B. Welfare as a function of R . US welfare is maximized at $R=1$

Figure 5: The effects of changing R using the best response of the

ROW

$(\eta = 0.9, q = 0.72, \pi = 0.83, \gamma = 0.72, \mu = 0.28, \gamma^* = 0.093, \mu^* = 0.907, b = 3, \Phi = 1.0332)$

CONCLUDING REMARKS

We used price dispersion to model liquidity and international seigniorage payments. In our model, an asset is relatively liquid if it is used relatively more in low price transactions.

In the model sellers accept payments in bonds as in Woodford (2003) cashless economy. The dictionary definitions of liquidity usually assume that assets are converted to cash. For example, one definition talks about the ability to convert the asset into cash quickly and without any price discount. To make a connection with the dictionary definition we may introduce to our model, sellers' backed "cash" that are similar to chips in casinos and assume that when buyers arrive at the store they convert their assets to chips that are then used to buy goods at the price of 1 chip per unit. Buyers who arrive early convert their dollar assets at the rate of $\frac{1}{p_1}$ chips per dollar. Buyers who arrive late convert their dollar assets at the rate of $\frac{1}{p_2}$ chips per dollar. Since we assume that the exchange rate is 1, buyers who arrive at stores that accept shekels can convert their shekel assets at the rate of $\frac{1}{p_2}$ chips per shekel regardless of their arrival order. Thus a shekel buys on average less chips than a dollar and some sellers are not willing to convert shekels to chips (and to accept shekels as payments for goods). In this sense it is not "easy" to convert shekels to chips and we may say that shekels are less liquid than dollars.

In the model, some sellers are willing to hold shekels because of the higher nominal rate of return. The distinction between real and nominal is not straightforward in our model. Inflation is usually measured by comparing posted prices in the current period to posted prices in the previous period. This measure is zero in our model because posted prices do not change over time. However, the expected real return is different from the nominal rate and depends on the seller's type. A type 1 seller who sells a unit of consumption in the first market for p_1 dollars

will get on average $p_1 R Z$ units of consumption while a type 2 agent that sell in the first market for the same dollar price will get on average $\pi p_1 R z_2$ units of consumption. In the steady state the real rate of return on the two assets is the same from the point of view of a type 2 seller: $\pi p_1 R z_2 = \pi p_1 R^* z_2^*$. Since shekels are less liquid ($z_2 > z_2^* = 1/p_2$), there is a difference in the nominal rates and $R < R^*$. In spite of the lower nominal return, the real rate of return on the dollar asset is higher than the real rate of return on the shekel asset from the point of view of a type 1 seller: $p_1 R Z > p_1 R^* (1/p_2)$. This is because a type 1 seller has a higher probability of buying in the first market.

In the model sellers choose the payment currency and there are no incentives to trade in a foreign exchange market. To check we may allow for trade in dollars for shekels at the exchange rate of 1 dollar per shekel (I assumed an exchange rate of 1 for convenience). Type 2 sellers are indifferent between selling for dollars to selling for shekels and therefore they will not want to trade in a foreign exchange market that opens before the realization of their taste shock. Type 1 sellers strictly prefer dollars and since after the end of trade in the goods markets they have only dollars they also do not have an incentive to trade before the realization of their taste shock. There is also no incentive to trade in the foreign exchange market after the realization of the taste shock and before trade in the goods market. This is because after the taste shock is known, both types of buyers are symmetric and are indifferent between dollars and shekels.⁸

The welfare implications of issuing an international currency were also discussed. It was argued that issuing an international currency that yields seigniorage payments is not a "free lunch". The downside of having an international currency is

⁸ A constant exchange rate requires constant assets supplies. In an earlier version of this paper I allow for the supplies of assets to change at different rates. The exchange rate changed at a rate equal the difference between the dollar and the shekel exchange rates without altering the main results of the paper.

that shocks to demand in the *ROW* will have an impact on US buyers. When demand is high some US buyers will import goods at a price that is higher than what they pay under autarky.

Will competition drive seigniorage payments to zero as was argued by Grubel (1969) and McKinnon (1969)? This question requires a model with many stable demand countries. At this stage we may conjecture that since international currency that yield seigniorage is not a "free lunch", competition will not drive seigniorage payments to zero.

The model can account for a modest liquidity premium (0.2%) when assuming a single good world economy in which all goods are tradable and only 37% of the goods are storable. The model can account for a higher liquidity premium if we interpret the good in the model as the tradable part of consumption that is relatively storable.

APPENDIX

Proof of Lemma 1: In the steady state, $\bar{z}_2 = z_2$, $\bar{z}_2^* = z_2^*$, $\bar{Z}^* = \pi z_1^* + (1 - \pi)z_2^*$ and $\bar{Z} = \pi z_1 + (1 - \pi)z_2$. The equality $p_1 = \pi p_2$ follows directly from (8) and (14) when using the steady state assumption.

To show that $p_1^* = \pi p_2^*$, assume that seller 1 is willing to accept both currencies in both markets. Then (12) and (13) hold with equality and imply: $\frac{R}{R^*} = \frac{p_1^*}{p_1} = \frac{p_2^*}{p_2}$. This

and $p_1 = \pi p_2$ leads to: $p_1^* = \pi p_2^*$. A similar argument can be made for the case in which seller 2 is willing to accept both currencies. \square

Proof of Lemma 2: Suppose that seller 2 is willing to supply a strictly positive amount to the first shekel market. Then (6) hold with equality and the steady state assumption implies: $p_1 R z_2 = p_1^* R^* z_2^*$. Using this and Lemma 1, leads to: $p_2 R z_2 = p_2^* R^* z_2^*$ and

therefore he must also be willing to supply a strictly positive amount to the second shekel market. A similar argument can be used to show that if he is willing to supply a strictly positive amount to the second shekel market he is also willing to supply a strictly positive amount to the first shekel market. The argument with respect to seller 1 is symmetric. \square

Proof of Claim 1: When $S^1 > 0$ and $S^2 > 0$, both types supply a strictly positive amount to one of the shekel markets and by Lemma 2 (12) and (6) hold with equality.

This leads to:

$$(A1) \quad \frac{R}{R^*} = \frac{p_1^* \bar{Z}}{p_1 Z}$$

$$(A2) \quad \frac{R}{R^*} = \frac{p_1^* z_2^*}{p_1 z_2}$$

Therefore both types will accept shekels only if:

$$(A3) \quad \frac{\bar{z}_2^*}{z_2^*} = \frac{\bar{Z}}{Z}$$

In the steady state, $\bar{z}_2 = z_2$, $\bar{z}_2^* = z_2^*$, $\bar{Z}^* = \pi z_1^* + (1 - \pi) z_2^*$ and $\bar{Z} = \pi z_1 + (1 - \pi) z_2$.

Therefore (A3) implies:

$$(A4) \quad \frac{z_2^*}{z_2} = \frac{\pi z_1^* + (1 - \pi) z_2^*}{\pi z_1 + (1 - \pi) z_2}$$

This condition will be satisfied for $0 < \pi < 1$ only if $\frac{z_2^*}{z_1^*} = \frac{z_2}{z_1}$. To see this we write

$z_1^* = k^* z_2^*$ and $z_1 = k z_2$, where k^*, k are constants. Then we can write (A4) as:

$$(A5) \quad 1 = \frac{\pi k^* + 1 - \pi}{\pi k + 1 - \pi}$$

This equality holds only if $k = k^*$. To show that (A4) requires $m = m^*$, I use Lemma 1 and the definitions (2) and (3) we get:

$$(A6) \quad \frac{z_2^*}{z_1^*} = m^* + \frac{(1 - m^*) p_1^*}{p_2^*} = m^* + \frac{1 - m^*}{\pi}; \quad \frac{z_2}{z_1} = m + \frac{(1 - m) p_1}{p_2} = m + \frac{1 - m}{\pi}$$

Therefore $\frac{z_2^*}{z_1^*} = \frac{z_2}{z_1}$ only if $m = m^*$. \square

Proof of Claim 2: I normalize by assuming $D^1 + D^2 = 1$. Since only $\frac{S^2}{P_2^*}$ is determined

in equilibrium I assume $p_2^* = p_2$.

I start by choosing (R, R^*) and $m = D^1$ that satisfies (17), $0 < m \leq 1$ and $0 < R \leq 1$. I then use (15) and (19) to solve for the following 9 unknowns:

$(p_1, p_2, x_1^1, x_1^2, x_1^{*2}, x_2^2, S^2, g, g^*)$. It is assumed that seller 1 supplies only to the first market:

$$(A7) \quad x_1^1 = \lambda^1$$

The dollar balances of type 2 buyer (19) must equal the revenues of type 2 seller in the high demand state:

$$(A8) \quad D^2 = 1 - m = p_2 x_2^2 = R(p_1 x_1^2 + p_2 x_2^2)$$

This implies:

$$(A9) \quad -r p_2 x_2^2 = -r(1 - m) = R p_1 x_1^2$$

Market clearing requires:

$$(A10) \quad p_1(x_1^1 + x_1^2) = m$$

Substituting (A9) in (A10) and using $x_1^1 = \lambda^1$ leads to:

$$(A11) \quad p_1 = \frac{m + r}{R \lambda^1}$$

Using Lemma 1 leads to:

$$(A12) \quad p_2 = \frac{m + r}{\pi R \lambda^1}$$

In the steady state (19) requires:

$$(A13) \quad 1 - m = p_2 x_2^2.$$

Substituting (A12) in (A13) leads to:

$$(A14) \quad x_2^2 = \frac{(1 - m) \pi R \lambda^1}{m + r}$$

Substituting (A11) in (A9) leads to:

$$(A15) \quad x_1^2 = \frac{-r(1 - m)}{R p_1} = \frac{-r(1 - m) \lambda^1}{m + r}$$

Using type 2 seller's budget constraint leads to:

$$(A16) \quad x_2^{*2} = \lambda^2 - x_1^2 - x_2^2 = \lambda^2 + \frac{(1-m)\lambda^1(r-\pi R)}{m+r}$$

$$(A17) \quad S^2 = p_2 x_2^{*2} = \frac{\lambda^2(m+r)}{\pi R \lambda^1} + \frac{(1-m)(r-\pi R)}{\pi R}$$

We can now use (15) to solve for g and g^* . We have thus solved for the 9 unknowns.

Note that since (A15) is positive we must have $r \leq 0$. Since (A16) is positive we must have: $b \geq \frac{(\pi R - r)(1-m)}{m+r} \geq 0$, where $b = \frac{\lambda^2}{\lambda^1}$. This leads to the condition in the Claim.

□

Proof of Claim 3: Using (2), (9), (A11) and (A12) leads to:

$$Z = \frac{(1-\pi)}{p_1} + \pi \left(\frac{m}{p_1} + \frac{\pi(1-m)}{p_1} \right) = \frac{R\lambda^1}{m+r} \{ \pi(m + \pi(1-m)) + (1-\pi) \}$$

and $W = Zm = \frac{R\lambda^1 m}{m+r} \{ \pi(m + \pi(1-m)) + (1-\pi) \}$. I now compute welfare in the *ROW*

by subtracting the expected consumption of *US* residents in the high demand state,

$z_2 m = m \left(\frac{m}{p_1} + \frac{\pi(1-m)}{p_1} \right)$, from total supply. Using (A11), this leads to:

$$W^* = \pi(\lambda^1 + \lambda^2 - z_2 m) = \pi(\lambda^1 + \lambda^2) - \frac{\pi R \lambda^1 m (m + \pi(1-m))}{m+r} = \pi(\lambda^1 + \lambda^2) - W + \frac{\pi R \lambda^1 m (1-\pi)}{m+r}$$

□

Proof of Lemma 3: We need to show that there exists a unique solution to:

$$(A18) \quad P = \pi + \frac{(1-\pi)qP}{R(m+(1-m)P)}$$

The *RHS* of (A18) is increasing in P . When $P=1$ and $q < R$, the *RHS* of (A18) is less than unity and when $P=0$ it is equal to π . As can be seen from Figure 1 there exists a unique solution $\pi \leq \bar{P} < 1$ to (A18). Figure A1 can also be used to show that \bar{P} is increasing in q and decreasing in m . □

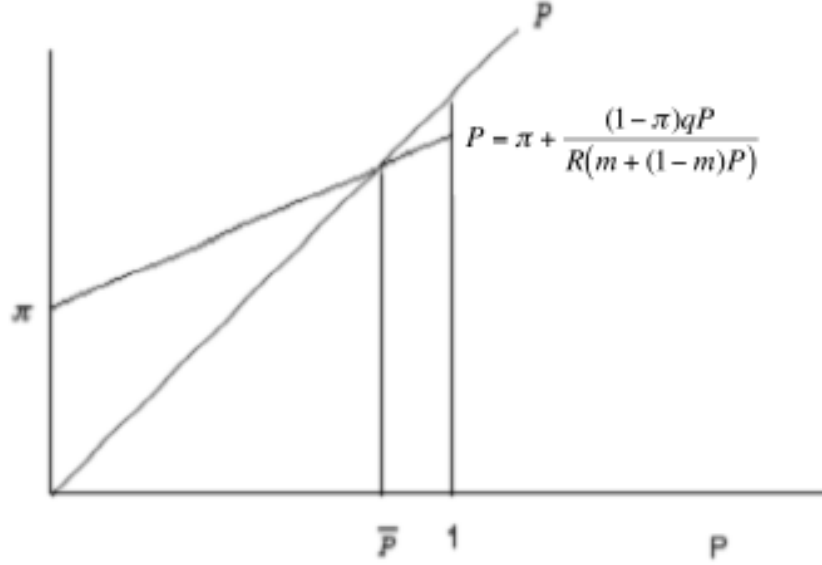


Figure A1

Proof of Claim 4: Since $Z \geq z_2$, we get the first inequality in part (a). To show the second inequality I use $z_2 = \frac{m}{p_1} + \frac{1-m}{p_2}$, Lemma 3 and the assumption $R\pi > q$ to get:

$p_1 R z_2 = R(m + (1-m)P) \geq RP \geq R\pi > q$. To show part (b) note that:

$$p_1 R z_2 = \pi p_2 R z_2 + (1-\pi)q \Rightarrow p_1 = \pi p_2 + \frac{(1-\pi)q}{R z_2} \geq \pi p_2 + \frac{(1-\pi)q}{R Z} \Rightarrow p_1 R Z \geq \pi p_2 R Z + (1-\pi)q$$

because $Z \geq z_2$. When $0 < m < 1$, $Z > z_2$ and we get strict inequalities. \square

Proof of Claim 5: I use (2), (9), (31) and (33) to solve for the labor supplies in the two countries:

$$(A19) \quad L = p_1 R Z = R\{\pi(m + (1-m)P) + (1-\pi)\}; \quad L^* = \pi R p_1 z_2 = \pi R(m + (1-m)P)$$

I treat S as the *ROW* policy choice and solve for m as a function of R and S .

For this purpose it is useful to aggregate the second dollar market and the second shekel market. The aggregate demand in these second markets is the dollar value of the portfolios of the type 2 buyers: $(1-m+S)/\Phi$. The aggregate supply is $b(L^* - k^*)$, where $k^* = x_1^2$ is the supply of a type 2 seller to the first dollar market. The market

clearing conditions for the first dollar market and the aggregate second market can now be written as:

$$(A20) \quad \Phi p_1(L + bk^*) = m; \quad \Phi p_2 b(L^* - k^*) = 1 - m + S$$

In the steady state the portfolio of the old does not change over time:

$$(A21) \quad m = p_1 LR + g = p_1 R^2 \{ \pi(m + (1-m)P) + (1-\pi) \} + g,$$

where I use (A19) to substitute for L . This leads to:

$$(A22) \quad p_1 = \frac{m-g}{LR} = \frac{m-g}{R^2 \{ \pi(m + (1-m)P) + (1-\pi) \}}$$

I now solve for p_1 using the second (aggregate) market clearing condition. I start by substituting $p_1 L = \frac{m-g}{R}$ in the first market clearing condition $p_1 L + p_1 b k^* = \frac{m}{\Phi}$. This leads to: $p_1 b k^* = \frac{m}{\Phi} - \frac{m-g}{R}$. (Note that since $p_1 b k^* \geq 0$, we must have $\phi \geq r$). I now substitute this, $p_1 = (P)p_2$ and $L^* = R\pi(m + (1-m)P)$ in the second market clearing condition $\Phi p_1 b(L^* - k^*) = P(1-m) + PS$ to get:

$$(A23) \quad p_1 = \frac{R(P(1-m) + PS + m) - (\Phi m - \phi + r)}{\Phi b R^2 \pi(m + (1-m)P)}$$

Equating (A22) and (A23) leads to:

$$(A24) \quad \frac{\Phi b \pi(m + (1-m)P)}{\pi(m + (1-m)P) + (1-\pi)} = \frac{\Phi R(P(1-m) + PS + m)}{\Phi m - \phi + r} - \Phi$$

Lemma 4: When $b \geq \frac{S}{\pi}$, there exists a unique solution \bar{m} to (A24). The steady state level of \bar{m} is decreasing in b and r and increasing in S .

Proof: The *RHS* of (A24) is decreasing in m and reaches a minimum of $\Phi PS \leq \Phi S$ when $m=1$. When $m > \frac{\phi-r}{\Phi}$ and $m - \frac{\phi-r}{\Phi}$ is small the *RHS* is large. The *LHS* of (A24) is increasing in m and when $m=1$ it is equal to $\Phi b \pi$. Therefore when $b \geq \frac{S}{\pi}$

and $m=1$, the *RHS* is less than the *LHS* and there exists a unique solution \bar{m} in

Figure A2.

Since the *LHS* is increasing in b , \bar{m} is decreasing in b . Since the *RHS* is decreasing in r \bar{m} is decreasing in r . Since the *RHS* is increasing in S \bar{m} is increasing in S . \square

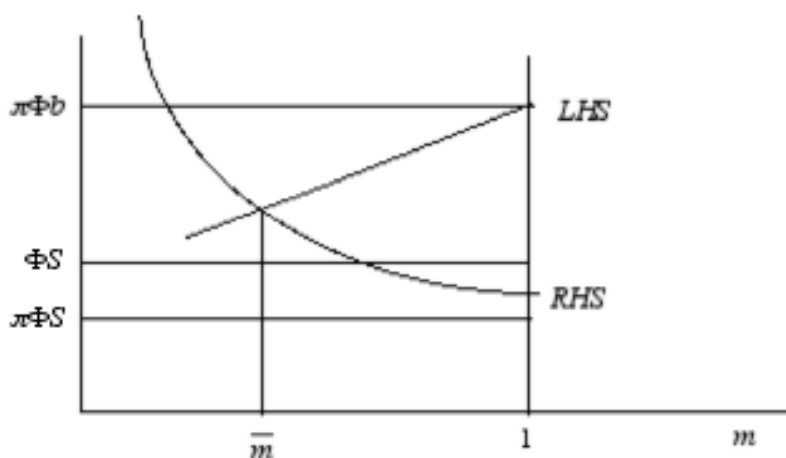


Figure A2

We can now use \bar{m} to solve for all the equilibrium magnitudes. \square

Proof of Claim 7: Labor supply in the US is: $L = p_1 RZ = R\{\pi(m + (1-m)P) + (1-\pi)\}$.

Using (29), the maximum welfare under autarky for the US is: $W^{\max} = (\frac{1}{2})(\Phi)^2$.

When $R = \Phi$ and $S^2 = 0$ there are no transfers and welfare can be written as:

$$(A25) \quad W = p_1 RZL - v(L) = (\frac{1}{2})L^2 = (\frac{1}{2})(\Phi)^2 \{\pi(m + (1-m)P) + (1-\pi)\}^2 \\ \leq \frac{1}{2} \leq W^{\max} = (\frac{1}{2})(\Phi)^2$$

The inequality follows from $P \leq 1$ (Lemma 3). Since (A25) holds with equality if $m = 1$ and with strict inequality otherwise, the US suffers from trade when $R = \Phi$ and there is full dollarization. \square

APPENDIX B: THE MIXED POPULATION CASE

Autarky: I start with the case of autarky and consider two assumptions about the transfer payment: (a) the transfer payment is the same for all agents and (b) the transfer payment is given to the more common type that has most of the votes. The

second assumption is considered here because it turns out to be computationally simpler once we go to the full integration case. Another alternative that was explored in previous drafts is that the *ROW* maximizes seigniorage payments. The results were qualitatively the same.

The number of type 1 young agents in the period we study is γ and the number of type 2 young agents is $\mu = 1 - \gamma$. There is a single asset: The dollar. The amount of dollars held in the steady state by a type j buyer is D^j . I assume $\gamma D^1 + \mu D^2 = 1$ and use: $m = \gamma D^1$; $1 - m = \mu D^2$. I start with the case in which only type 1 agents get the transfer payment and each gets $\frac{g}{\gamma}$ dollars where $g = \frac{\phi - r}{\Phi}$. The type 1 agent solves:

$$(B1) \quad \max_L p_1 L^1 R Z - v(L^1) + (\frac{g}{\gamma}) Z$$

The first order condition for this problem is:

$$(B2) \quad v'(L^1) = p_1 Z R$$

Using x_i to denote the supply of type 2 to market i , type 2 agent solves:

$$(B3) \quad \max_{x_1, x_2} \pi(p_1 x_1 R z_2 + \pi p_2 x_2 R z_2 + (1 - \pi) q x_2) - v(L^2 = x_1 + x_2)$$

The first order conditions for this problem are:

$$(B4) \quad \frac{v'(L^2)}{\pi} = p_1 R z_2 = \pi p_2 R z_2 + (1 - \pi) q$$

I assume that the policy variable R is given. I now enter a loop starting with an arbitrary choice of m and solve for the equilibrium magnitudes (including m) as a function of m and R .

I use (32) to solve for $P = \frac{p_1}{p_2}$ and (A19) to solve for L^i .

Market clearing requires:

$$(B5) \quad p_1(\gamma L^1 + \mu x_1) = \frac{m}{\Phi}; \quad \mu p_2 x_2 = \frac{1 - m}{\Phi};$$

The steady state condition is:

$$(B6) \quad D^1 = p_1 L^1 R + \frac{g}{\gamma}$$

Using (B6) and $m = \gamma D^1$ we get:

$$(B7) \quad p_1 = \frac{m - g}{\gamma L^1 R}$$

We can now solve for p_1 . Using the first market clearing condition in (B5) we solve for

$$(B8) \quad x_1 = \frac{m}{\mu p_1 \Phi} - \frac{\gamma}{\mu} L^1$$

The second market clearing condition in (B5) can be written as:

$$(B9) \quad m = 1 - \Phi \mu p_2 x_2 = 1 - \frac{\Phi \mu p_1 (L^2 - x_1)}{P}$$

We now use (B9) to solve for m and iterate until the solution to (B9) is equal to the initial value of m .

Using the steady state level of m we now compute D^j . Welfare is given by:

$$(B10) \quad W^1 = D^1 Z - v(L^1) ; W^2 = \pi D^2 z_2 + \pi(1 - \pi) q x_2 - v(L^2) ;$$

$$W = \gamma W^1 + (1 - \gamma) W^2$$

Figure B1 computes the steady state magnitudes for the case in which $q = 0.3$, $\pi = 0.997$ and $\gamma = 0.65$. As can be seen the steady state welfare of type 1 agents declines with R and the steady state welfare of type 2 agents increases with R . This is because a reduction in R leads to a redistribution of wealth from type 2 agents to type 1 agents who get all the seigniorage revenues. This effect dominates the distortion in the labor market. Aggregate welfare W is slightly increasing with R (only slightly, you cannot see it on the graph).

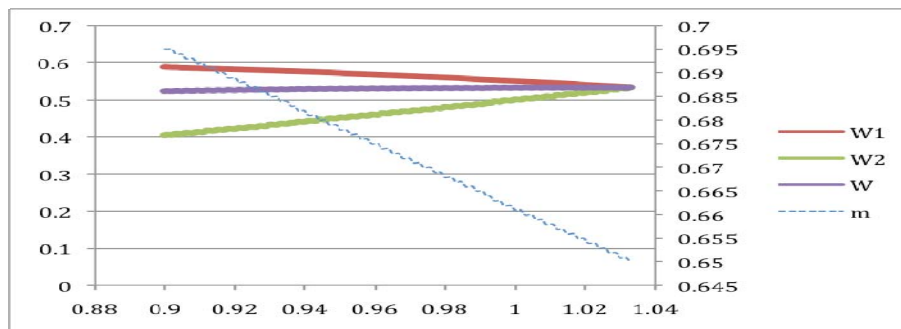


Figure B1: Autarky when only type 1 gets the transfer ($q = 0.3, \pi = 0.997, \gamma = 0.65$)

I now turn to the case in which the transfer payment is distributed equally among the agents in the model. In this case (B6) changes to:

$$(B6') \quad D^l = p_1 L^l R + g$$

and accordingly, (B7) changes to:

$$(B7') \quad p_1 = \frac{m - \gamma g}{\gamma L^l R}$$

As can be seen from Figure B2 the welfare of both types is increasing in R because of the labor market distortion.

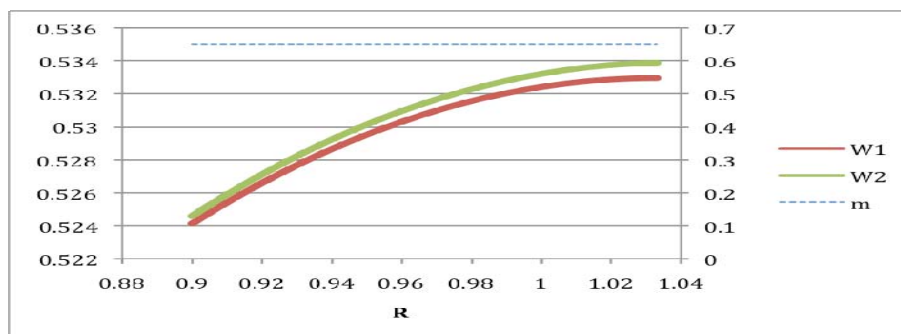


Figure B2: Autarky when both types get the transfer ($q = 0.3, \pi = 0.997, \gamma = 0.65$)

I now turn to the case of full integration.

Full integration

I use an asterisk for the *ROW* variables and assume the following demographics (for the representative period we study).

The number of young agents in the US is: 1

The number of young agents in the *ROW* is: b

γ = the fraction of type 1 agents in the US population

$\mu = 1 - \gamma$ = the fraction of type 2 agents in the US population

The number of type 1 young agents in the world is: $\Gamma = \gamma + \gamma^* b$

The number of type 2 young agents in the world is: $\Omega = \mu + \mu^* b$

The number of old agents is equal to the number of young agents divided by Φ . A type 1 buyer living in the US holds D^1 dollars. Since there are $\frac{\gamma}{\Phi}$ type 1 buyers who live in the US, $\frac{\gamma D^1}{\Phi}$ is the total amount of dollars held by old type 1 agents living in

the US. Similarly,

$\frac{\mu D^2}{\Phi}$ = the total amount of dollars held by old type 2 agents living in the US

$\frac{\gamma^* b D^{*1}}{\Phi}$ = the total amount of dollars held by old type 1 agents living in the ROW

$\frac{\mu^* b D^{*2}}{\Phi}$ = the total amount of dollars held by old type 2 agents living in the ROW

$\frac{D}{\Phi}$ = total amount of dollars held by the old, where

$$D = \gamma^* b D^{*1} + \gamma D^1 + \mu D^2 + \mu^* b D^{*2} = 1$$

$$m = \frac{\gamma^* b D^{*1} + \gamma D^1}{D} = \gamma^* b D^{*1} + \gamma D^1 = \text{fraction of the dollar supply held by type 1 old}$$

agents

$\frac{m}{\Phi}$ = total amount of dollars held by old type 1 agents

$\frac{\mu S^2}{\Phi}$ = the total amount of shekels held by old type 2 agents living in the US

$\frac{\mu^* b S^{*2}}{\Phi}$ = the total amount of shekels held by old type 2 agents living in the ROW

$\frac{S}{\Phi}$ = total amount of shekels held by the old (type 2), where $S = \mu^* b S^{*2} + \mu S^2$.

To simplify, I assume that transfers are paid to the majority in each country: Dollar transfers are paid to type 1 agents residing in the US and shekel transfers are paid to type 2 agents residing in the ROW.

Lemma B1: (a) The dollar transfer per type 1 young agent in country 1 is $\frac{g}{\gamma}$ where

$g = \frac{\phi - r}{\Phi}$; (b) The shekel transfer per type 2 young agent in country 2 is: $\frac{g^*}{\mu^* b}$ where

$$g^* = \frac{S(\phi - r^*)}{\Phi}.$$

Proof: $\frac{\mu^* b S^{*2}}{\Phi} + \frac{\mu S^2}{\Phi} = \frac{S}{\Phi}$ = total amount of shekels held by the old = total revenue if

demand is high = before interest bequest if demand is low. The young agents will

therefore hold a total of $\frac{S}{\Phi}R^* + g^*$ after interest and transfer. In the steady state the

holding of the young at the end of the period is a total of S shekels. Thus,

$\frac{S}{\Phi}R^* + g^* = S$. This leads to: $g^* = \frac{S(\phi - r^*)}{\Phi}$ where g^* is the total transfer of shekels.

When type 2 agents that reside in the *ROW* get the entire transfer each of them will get: $\frac{g^*}{\mu^* b}$ shekels.

Similarly for dollars $g = \frac{\phi - r}{\Phi}$ = the total transfer. Since only type 1 agents living in the US get the dollar transfer the transfer per agent is $\frac{g}{\gamma}$. \square

I assume that the parameters $(\pi, q, \gamma, \mu, \gamma^*, \mu^*, b)$ are given. I assume that the US chooses R and the *ROW* chooses R^* . The choice of R^* leads to m in the relevant range and as before I assume that the *ROW* chooses m directly. I now solve the equilibrium magnitudes for the choice of R and m .

I start by calculating P from (32). I then use P to calculate labor supplies for each type. These are:

$$(B11) \quad L^1 = p_1 R Z = R \{ \pi (m + (1 - m)P) + (1 - \pi) \}; \quad L^2 = \pi R p_1 z_2 = \pi R (m + (1 - m)P)$$

A type 1 seller supplies to the first market only. A type 2 seller supplies k units to the first market and $L^2 - k$ units to the second market. Assuming that the exchange rate is 1, the market clearing conditions are:

$$(B12) \quad p_1 (\Gamma L^1 + \Omega k) = \frac{m}{\Phi}; \quad p_2 \Omega (L^2 - k) = \frac{(1 - m) + S}{\Phi}$$

In the steady state the portfolio of the old does not change over time.

$$(B13) \quad D^1 = p_1 L^1 R + \frac{g}{\gamma}; \quad D^{*1} = p_1 L^1 R$$

This leads to:

$$(B14) \quad p_1 = \frac{D^{*1}}{L^1 R} = \frac{\gamma D^1 - g}{\gamma L^1 R}$$

We can now solve for D^{*1} :

$$(B15) \quad D^{*1} = D^1 - \frac{g}{\gamma}$$

I now use

$$(B16) \quad m = \gamma^* b D^{*1} + \gamma D^1$$

and (B15) to get:

$$(B17) \quad D^1 = \frac{m}{\Gamma} + \frac{\gamma^* b g}{\Gamma \gamma} ; D^{*1} = \frac{m}{\Gamma} + \frac{g}{\gamma} \left(\frac{\gamma^* b}{\Gamma} - 1 \right)$$

We can now use (B14) to solve for p_1 and $p_2 = p_1/p$. This allow us to compute welfare for type 1 agents given by:

$$(B18) \quad W^1 = D^1 Z - v(L^1) ; W^{*1} = D^{*1} Z - v(L^1)$$

From the first market clearing condition in (B12) we get:

$$(B19) \quad k = \frac{1}{\Omega} \left(\frac{m}{\Phi p_1} - \Gamma L^1 \right)$$

We can now use the second market clearing condition in (B12) to solve for

$$(B20) \quad S = \mu S^2 + \mu^* b S^{*2} = \Phi p_2 \Omega (L^2 - k) - (1 - m)$$

I use $\psi = \frac{S}{(1 - m) + S}$ to denote the fraction of second market output offered for

shekels. Type 2 agents who reside in the US will have $p_2 \psi (L^2 - k) R^*$ shekels if the second shekel market opens. If the second shekel market does not open they will get a bequest of $R^* \frac{S^2}{\Phi}$ shekels. To get a steady state holding of shekels I assume that type

2 agents enter an insurance agreement with the government. They give the

government whatever realization of shekel balances they have in exchange for the

mean: $S^2 = \pi p_2 \psi (L^2 - k) R^* + (1 - \pi) R^* \frac{S^2}{\Phi}$. This leads to:

$$(B21) \quad S^2 = \frac{\Phi \pi p_2 \psi (L^2 - k) R^*}{\Phi - (1 - \pi) R^*}$$

Similarly, type 2 agents residing in the *ROW* get

$S^{*2} = \pi p_2 \psi (L^2 - k) R^* + (1 - \pi) R^* \frac{S^{*2}}{\Phi} + \frac{g^*}{\mu^* b}$ which leads to:

$$(B22) \quad S^{*2} = \frac{\Phi \pi p_2 \psi (L^2 - k) R^* + \Phi \frac{g^*}{\mu^* b}}{\Phi - (1 - \pi) R^*}$$

In the steady state $D^2 = D^{*2}$ and $1 - m = \mu D^2 + \mu^* b D^{*2}$. This leads to:

$$(B23) \quad D^2 = D^{*2} = \frac{1 - m}{\Omega}$$

Welfare of type 2 agents are given by:

$$(B24) \quad \begin{aligned} W^2 &= \pi(D^2 z_2 + S^2 z_2^*) + (1 - \pi)\pi q(L^2 - k) - v(L^2) ; \\ W^{*2} &= \pi(D^{*2} z_2 + S^{*2} z_2^*) + (1 - \pi)\pi q(L^2 - k) - v(L^2) \end{aligned}$$

Welfare in the two countries are given by:

$$(B25) \quad W = \gamma W^1 + \mu W^2 ; \quad W^* = \gamma^* W^{*1} + \mu^* W^{*2}$$

This solution procedure assumes that the *ROW* can implement the choice of m by an appropriate choice of S . We now check whether this is indeed the case.

Claim B1: When $\phi > r$ and $S < \frac{\Omega\pi}{\Gamma}$, there exists a unique steady state equilibrium.

The steady state level of m is increasing in S .

Proof: Substituting (B17) in the steady state condition (B3) and using (B1) leads to:

$$(B26) \quad p_1 = \frac{D^{*1}}{LR} = \frac{m - g}{\Gamma LR} = \frac{m - g}{\Gamma R^2 \{ \pi(m + (1 - m)P) + (1 - \pi) \}}$$

From the second market clearing condition in (B2) to get $p_1 = \frac{PS + P(1 - m)}{\Phi\Omega(L^2 - k)}$.

Substituting (B11) and (B19) in this equation leads to:

$$(B27) \quad p_1 = \frac{PS + P + m(1 - P)}{\Phi\Omega\pi R(m + (1 - m)P) + \Phi\Gamma R \{ \pi(m + (1 - m)P) + (1 - \pi) \}}$$

Equating (B26) and (B27) leads to:

$$(B28) \quad \frac{\Phi\Omega\pi(m + (1 - m)P) + \Phi\Gamma \{ \pi(m + (1 - m)P) + (1 - \pi) \}}{\pi(m + (1 - m)P) + (1 - \pi)} = \frac{\Gamma R(PS + P + m(1 - P))}{m - g} \\ = \frac{\Phi\Gamma R(PS + P + m(1 - P))}{\Phi m - \phi + r}$$

Since P is decreasing in m , the *RHS* of (B28) is decreasing in m and reaches a minimum of $\Phi\Gamma(PS + 1)$ when $m = 1$. When $m > \frac{\phi - r}{\Phi}$ and $m - \frac{\phi - r}{\Phi}$ is small the

RHS is large. The *LHS* of (B28) is increasing in m and when $m = 1$ it is equal to $\Phi\Omega\pi + \Phi\Gamma$. Therefore when $S < \frac{\Omega\pi}{\Gamma P} < \frac{\Omega\pi}{\Gamma}$ there exists a solution \bar{m} to (B28).

Figure B3 illustrates. We can now go back and use the solution \bar{m} to solve for all the other steady state magnitudes.

Since P does not depend on S the *RHS* of (B28) is increasing in S and \bar{m} is increasing in S .

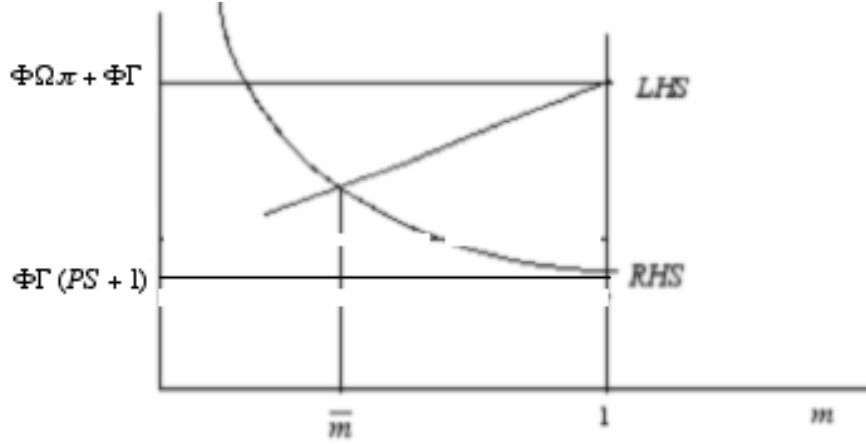


Figure B3

Rates of nominal GDP growth

The official statistics about nominal GDP treat unsold goods differently from unsold services. Typically, unsold goods are viewed as an investment in inventories while unsold services are valued as zero. This makes a big difference both for the average rate of growth and its variance.

I start with the assumption that goods that were not sold are valued at zero and use G_{ij} to denote the gross rate of change in nominal GDP when moving from state of demand i to state of demand j . Using these notations, the nominal rates of nominal GDP growth are:

$$(B29) \quad G_{LH} = \Phi \frac{p_1(\gamma L^1 + \mu k) + p_2 \mu (L^2 - k)}{p_1(\gamma L^1 + \mu k) + \eta p_1 \mu (L^2 - k)}; \quad G_{LL} = \Phi;$$

$$G_{HL} = \Phi \frac{p_1(\gamma L^1 + \mu k) + \eta p_1 \mu (L^2 - k)}{p_1(\gamma L^1 + \mu k) + p_2 \mu (L^2 - k)} = \frac{\Phi^2}{G_{LH}}; \quad G_{HH} = \Phi$$

The expected nominal rates conditioned on being in state of demand i are:

$$(B30) \quad G_L = \pi G_{LH} + (1 - \pi)\Phi ; G_H = (1 - \pi)G_{HL} + \pi\Phi = (1 - \pi)\frac{\Phi^2}{G_{LH}} + \pi\Phi$$

The unconditional expected rate of growth is:

$$(B31) \quad G = \pi G_H + (1 - \pi)G_L$$

The variance of the rate of growth is:

$$(B32) \quad \begin{aligned} Var = & \pi(1 - \pi)(G_{LH} - G)^2 + (1 - \pi)^2(G_{LL} - G)^2 \\ & + \pi^2(G_{HH} - G)^2 + \pi(1 - \pi)(G_{HL} - G)^2 \end{aligned}$$

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