

**LIQUIDITY, EQUITY PREMIUM AND PARTICIPATION**

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## LIQUIDITY, EQUITY PREMIUM AND PARTICIPATION

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I use price dispersion to model liquidity. Buyers may be rationed at the low price. An asset is more liquid if it is used relatively more in low price transactions and the probability that it will buy at the low price is relatively high. In the equilibrium of interest government bonds are more liquid than stocks. Agents with a relatively stable demand are willing to pay a high "liquidity premium" for holding bonds and they specialize in bonds. In equilibrium only a fraction of households (those with relatively unstable demand) hold stocks and the equity premium may be large.

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## 1. INTRODUCTION

The idea that liquidity is important for assets returns is not new. Recently, McGrattan and Prescott (2003) have argued that short term US government securities provide liquidity and are therefore overpriced. Amihud (2002) and Cochrane (2003) argued that some stocks are over-priced because they provide liquidity. For a survey of the recent literature see Amihud, Mendelson and Pederson (2005).

While many economists will agree that liquidity is important there is much less consensus on what exactly it means. The following are some dictionary definitions of liquidity: (a) The quality of being readily convertible into cash, (b) The ability or ease with which assets can be converted into cash, (c) The degree to which an asset or security can be sold or bought without affecting the asset's price, (d) Liquidity is characterized by a high level of trading activity.

The ease of converting an asset to cash may be measured by the average time it takes to do that. Houses for example are illiquid because they are typically on the market for a long time. But this measure is problematic: There are firms who will buy your house immediately at a price that is below the "market price".

Here I define liquidity in terms of a model with price dispersion. In the model sellers of goods choose price tags. Prices may be low or high and must specify the asset that is acceptable as means of payment. An asset is liquid if it is accepted by low price sellers and promises a high probability of making a buy at the low price. I focus on acceptability by low price sellers because only at the low price buyers may be rationed. Although there is no asset market in the model, we may interpret the "ease" at which the asset is exchanged for low price goods as the "ease" of exchanging the asset for "cash" at the buying price. I elaborate on this interpretation at the end of the paper.

An important byproduct is a partial resolution of two puzzles: the equity premium puzzle and the participation puzzle. The equity premium puzzle originated with Mehra

and Prescott (1985) who found a large difference between the average rate of return on equity and the average rate of return on Treasury bills. This paper led to a large literature surveyed by Kocherlakota (1996) who concluded that the equity premium is still a puzzle. Recently, Barro (2006) has argued for the importance of catastrophic events, previously suggested by Rietz (1988). Our explanation here borrows from Mankiw and Zeldes (1991), Attanasio, Bank and Tanner (2002) and Vissing-Jorgensen and Attanasio (2003) who observed that only a fraction of households actually hold stocks and the consumption of stockholders behave differently from the consumption of households who do not hold stocks. In particular, the standard deviation of the rate of consumption change is much higher (about 50% - 100%) for stockholders. They show that if we restrict the sample to stockholders (or households with the characteristics of stockholders) we do much better in terms of explaining the equity premium puzzle. The remaining question is why only a relatively small fraction of households hold stocks? This is the participation puzzle.

A related paper by Ait-Sahalia, Parker, and Yogo (2004) distinguishes between the consumption of basic goods and luxury goods. They find that the consumption of luxury goods is much more correlated with the stock market and argue that for the very rich, the equity premium is much less of a puzzle. But the equity premium is still a puzzle for the not so rich. They estimate a coefficient of risk aversion between 50 and 173 when using Personal Consumption Expenditures of non-durables and services. But when using their data on the consumption of luxury items the estimated coefficient is 7. These estimates leave the participation puzzle unanswered: Why only the "rich" hold stocks?

Here I address both puzzles. I use a flexible price version of Prescott (1975) "hotels" model: The Uncertain and Sequential Trade (UST) model in Eden (1990, 1994) and Lucas and Woodford (1993). I also use the ideas in Dana (1998) who considers a rigid price version of the model with heterogeneous agents.

Our approach borrows from the discussion of cashless economy in Woodford (2003). Woodford considers a technological advanced economy in which payments can be made in any asset. In Woodford's model money serves only as a unit of account and is priced correctly as an Arrow-Debreu security. Here we also allow payments in all assets but in our model, markets are incomplete.

Our approach is also related to the random matching models pioneered by Kiyotaki and Wright (1993). In both the random matching models and the UST model uncertainty about trading opportunities plays a key role. In the random matching models agents are uncertain about whether they will meet someone that they can actually trade with. But whenever a meeting takes place it is bilateral. In the UST model sellers are also uncertain about the arrival of trading partners but whenever a meeting occurs there are a large number of agents on both sides of the market. As a result there is a difference between the assumed price determination mechanisms. In the random matching models prices are either fixed or are determined by bargaining (as in Trejos and Wright [1995] and Shi [1995]). In the UST model prices clear markets that open.

Eden (2006) is a related paper. It assumes two types of government bonds and focus on seigniorage issues. Here I have stocks and government bonds. This is a substantial difference because as we shall see stocks and bonds are not symmetric.

## 2. OVERVIEW

Two types of risk-neutral people are born each period. Generations overlap. There are firms that last forever. There are two assets: Government bonds and stocks.

Sellers choose price tags that specify the terms of their offer to sell goods. A price tag may be in terms of bonds (the amount of bonds per unit of the consumption good) or in terms of stocks but not in terms of both assets. Price tags may be different across units.

There are two prices. Sellers can sell at the low price regardless of the state of aggregate demand (with probability 1) and at the high price only if demand is high.

In equilibrium there must be a match between the portfolio held by the agents who buy in the low aggregate demand state and the low price tags. If the agents that buy in the low aggregate demand state hold bonds then the low price tags must be in terms of bonds because otherwise the sellers cannot guarantee the making of a sale. (Recall that in equilibrium the sellers can sell at the low price with probability 1).

In the model buyers can always exchange assets for goods. The question is at what price? An asset is liquid if sellers use it primarily for specifying low price tags. Our focus is on the "acceptability" of the asset by low price sellers because buyers may be rationed at the low price.

To make a connection with the dictionary definitions of liquidity we may think in terms of a cash-in-advance model: The buyers first exchange their assets for cash and then use the cash to buy goods. The price of goods in terms of cash is constant (say 1 dollar per unit) but the dollar price of the asset fluctuates. If you sell the asset at a high price you end up paying a small amount of the asset per unit of the good and in terms of the asset you pay a low price. If you sell the asset at a low price you pay in terms of the asset a high price for the good. Following this interpretation, the more liquid asset has a higher probability of being sold for a high cash price. I elaborate on this interpretation at the end of the paper but in the model itself there is no cash-in-advance constraint and assets are exchanged directly for goods.

Agents with unstable demand buy only in the high aggregate demand state, and agent with a stable demand buy in both states and in particular, they buy in the low aggregate demand state. It is shown that in equilibrium agents with stable demand choose to specialize in bonds and as a result low price sellers choose to specify their price tags in terms of bonds. Thus bonds are more liquid than stocks.

The main characteristics of the equilibrium of interest are thus: (a) the stable demand type strictly prefers the more liquid asset and (b) bonds are more liquid than stocks. The reason for (a) is that the stable demand type has a high probability of finding the good at the low price and therefore he is willing to pay a high "liquidity premium" for holding the more liquid asset. We get (b) because we focus on a steady state equilibrium in which the firm accepts bonds. Since the firm represents stockholders, stockholders must be willing to hold both assets. This and (a) imply that when the two assets are not equally liquid, the more liquid asset must be bonds. It is also shown that the two assets will in general have different liquidity.

### 3. THE MODEL

I use an overlapping generations model. Two types of people are born each period. They live for two periods, work in the first and, if they want, consume in the second. A type  $h$  agent supplies in the first period of his life  $\lambda^h$  units of labor (or more generally inputs) inelastically. A type 1 that is born at time  $t$  will want to consume with probability 1 and his utility function is:  $U^1(C_{t+1}) = C_{t+1}$ , where  $C$  is his second period consumption. A type 2 wants to consume with probability  $\pi$ . His utility function is:  $U^2(C_{t+1}) = \theta_{t+1} C_{t+1}$ , where  $\theta$  is a random variable that may take the realization  $\theta = 1$  with probability  $\pi$  and  $\theta = 0$  otherwise. Both types maximize expected utility. The number of type 1 agents is the same as the number of type 2 agents. A single agent represents each type.

There is a firm that lasts forever.

There are two assets: Government bonds and stocks. At time  $t$ , a unit of bond promises  $R = 1 + r$  units of bonds at the end of the period. Stocks are shares in the ownership of the firm. Stocks promise dividends at the end of the period. We may think

of bonds as interest bearing money and stocks as dividend bearing money. I will sometime use dollar for a unit of bonds.

As in Abel (1985), it is assumed that if the type 2 old agents do not want to consume they leave their assets to type 2 young agents as accidental bequest. An alternative formulation may assume that agents derive utility from bequest as in Barro (1974), but the weight they assign to the utility of future generations is random. Unlike Diamond and Dybvig (1983) here it is important that the demand of type 2 agents is correlated with aggregate demand. An alternative formulation may assume a random tax instead of a taste shock and a household consisting to two types: Type 1 is responsible for basic consumption and always wants to buy. Type 2 is responsible for paying taxes and spends the money only if the government does not tax it (with probability  $\pi$ ) on luxuries as in Ait-Sahalia, Parker, and Yogo (2004). Another possible scenario may assume that problems in the credit market (of the type we are currently experiencing) occasionally arise. When agents cannot get a loan they spend less on durables. I do not think that these more realistic alternative formulations will change the main results. I will stick with the simpler formulation.

The aggregate state of the economy is a description of the portfolios held by the old agents after the distribution of dividends and interest payments but before the beginning of trade in the goods market. The aggregate state is denoted by  $y = (M^1, M^2, S^1, S^2)$ , where  $M^h$  is the amount of bonds held by a type  $h$  old agent and  $S^h$  is the amount of stocks held by type  $h$  old agent.

The representative firm uses  $L$  units of labor to produce  $L^\alpha$  units of consumption where  $0 < \alpha < 1$  is the return to scale parameter. Production takes place before the realization of the taste shock  $\theta$ . The price of labor is  $W$  dollars.

Trade occurs in a sequential manner. All agents who want to consume form an imaginary line. They then arrive at the market place one by one according to their place in the line. Upon arrival they see all prices, buy at the cheapest available offer and then



disappear. Their place in line is determined by a lottery that treats all agents symmetrically. When  $\theta = 0$  only type 1 agents are in the line. When  $\theta = 1$  both types are in the line and in any segment of the line there is an equal number of agents from both types.

From the firm's point of view, demand arrives sequentially in batches. A first batch arrives with probability 1 and a second batch arrives if  $\theta = 1$  with probability  $\pi$ .

Figure 1 describes the sequence of events within the period.

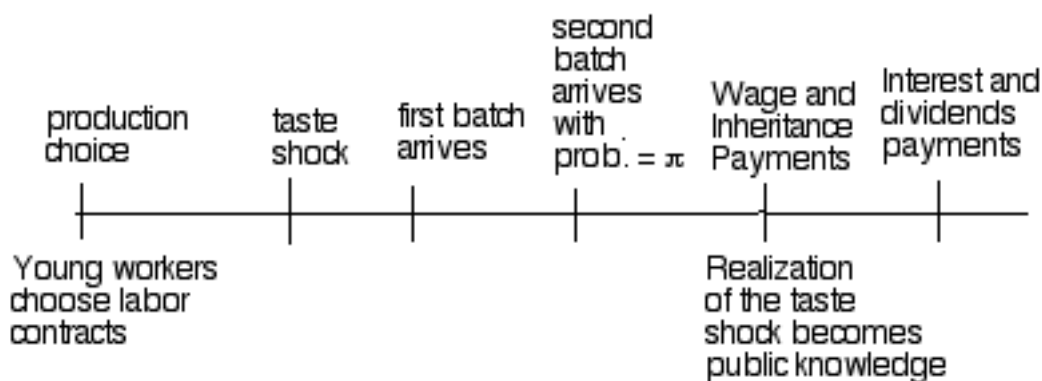


Figure 1

The firm is a price-taker. It knows that it can sell at the low price of  $p_1(y)$  units of bonds per unit to the first batch. It can sell at the higher price of  $p_2(y)$  units of bonds per unit to the second batch if it arrives. The firm can also choose to sell for stocks. It can sell for  $p_1^*(y)$  stocks per unit to the first batch and for  $p_2^*(y)$  stocks per unit to the second if it arrives. The firm chooses how much to sell to the first batch (and how much of it to sell for bonds) before it knows whether a second batch will arrive or not.

It may be helpful to think of sellers that put a price tag on each unit that they offer for sale. A price tag may specify the cost of the unit in terms of bonds or in terms of stocks (but not in term of both). Price tags may be different across units.

The choice of the payment asset is analogous to the choice of the payment currency in the international trade literature.<sup>1</sup> Here it may also be useful to think of an alternative cash-in-advance scenario. We may assume that the firm has many retail outlets for its output and issues "money" that is similar to chips issued by casinos. The retailers sell the good for "money" at a constant price of 1 "dollar" (chip) per unit. Buyers first go to the firm's "bank" and change their asset for "money". Buyers in the first batch get  $\frac{1}{p_1}$  "dollars" per unit of bonds and  $\frac{1}{p_1^*}$  "dollars" per stock. After the firm has observed that a certain amount of bonds and stocks were exchanged at these prices it changes the assets "dollar" prices and starts exchanging the assets for the second batch prices:  $\frac{1}{p_2}$  "dollars" per unit of bonds and  $\frac{1}{p_2^*}$  "dollars" per stock. As we shall see asset prices fall when the second batch arrives. This description seems more realistic but it is easier to write the model in terms of price tags on goods and I shall stick to that.

It is convenient to assume two hypothetical markets. The first market opens with certainty at the prices:  $p_1(y)$ ,  $p_1^*(y)$ . The second market opens only if  $\theta=1$  (with probability  $\pi$ ) at the prices:  $p_2(y)$ ,  $p_2^*(y)$ . The firm knows that it can make a sale with probability 1 in the first market and with probability  $\pi$  in the second market. It supplies  $x_1$  units to the first market and  $x_2$  units to the second market. It also chooses the fraction of the supply to market  $s$  that is offered for bonds,  $\psi_s$ , and the fraction that is offered for stocks,  $1 - \psi_s$ .

At the end of the period the government gives the firm a lump sum transfer of  $g_s$  units of bonds if exactly  $s$  markets open. There is also a lump sum tax on bequest of  $\tau$  units of bonds.

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<sup>1</sup> In a recent paper Devereux and Engel (2003) distinguish between pricing in terms of the producer currency (PCP) and pricing in terms of the consumer currency (LCP). They show that the implications of risk for foreign trade are highly sensitive to the choice of currency at which prices are set.

There are markets for contingent claims on bonds and on stocks. These markets are not complete. It is assumed that young agents and the firm can bet in this market on the realization of the taste shock of the current old. The environment does not permit the old people to bet on their own shock: After the realization of their own taste shock they meet one firm (out of many identical firms) buy goods and disappear before the realization of the shock becomes public knowledge. They do not hang around until everyone can verify the realization of the taste shock. For the same reason young people cannot bet on their own taste shocks ( $\theta_{t+1}$ ).

The price of a claim on a dollar that will be delivered if exactly  $s$  markets open is  $\Pi_s$ . The price of a claim on the ownership of the firm that will be delivered if exactly  $s$  markets open is:  $\Omega_s$ . The price of the firm in market  $s$ ,  $q_s$ , is related to these prices by the following "no-arbitrage" conditions:

$$(1) \quad q_1 = \Omega_1 + \Omega_2 ; \Omega_2 = \Pi_2 q_2 ; p_s = q_s p_s^*$$

To see the first equality note that the price of (unconditional ownership of) the firm before the beginning of trade is:  $\Omega_1 + \Omega_2$ . Since no new information is revealed by the arrival of the first batch of buyers, this is also the price of the firm in the first market. To understand the second equality note that if  $\Omega_2 > \Pi_2 q_2$ , then one can make money by selling a claim on the firm in state of demand 2 and buying  $q_2$  dollars in state of demand 2. Similarly, when  $\Omega_2 < \Pi_2 q_2$  one can make money by buying a claim on the firm in state of demand 2 and selling a claim on  $q_2$  dollars in state of demand 2. The third equality requires purchasing power parity. It says that the price in stocks after conversion to dollars must equal the dollar price.

The firm solves the following problem:

$$\begin{aligned}
(2) \quad & \max_{x_s, \psi_s, L} \Pi_1 p_1 \psi_1 x_1 + \Pi_2 (p_1 \psi_1 x_1 + p_2 \psi_2 x_2) \\
& + \Pi_1 q_1 p_1^* (1 - \psi_1) x_1 + \Pi_2 q_2 [p_1^* (1 - \psi_1) x_1 + p_2^* (1 - \psi_2) x_2] \\
& - WL + \Pi_1 g_1 + \Pi_2 g_2 \\
& \text{s.t. } x_1 + x_2 = L^\alpha, 0 \leq \psi_s \leq 1 \text{ and } x_s \geq 0.
\end{aligned}$$

The first two terms in (2) are the value of the revenues from goods offered for bonds:  $p_1 \psi_1 x_1$  is the revenue from goods supplied to the first market and  $p_2 \psi_2 x_2$  is the contingent revenue from goods supplied to the second market. The revenues in state  $s$  are multiplied by the price of a dollar in state  $s$ ,  $\Pi_s$ , so that the total revenues are in terms of "current" dollars (dollars delivered regardless of the state). The second two terms are the value of the revenues from goods offered for stocks:  $p_1^* (1 - \psi_1) x_1$  is the stock revenues in the first market and  $p_2^* (1 - \psi_2) x_2$  is the (contingent) stocks revenues in the second market. To convert these revenues to current dollars we multiply by  $\Pi_s q_s$ . The next term is labor cost and the last two terms are the value of the contingent transfer payments. The constraint  $x_1 + x_2 = L^\alpha$  says that the amount allocated to the two markets must equal total output.

The workers' problems: Agents form expectations about the probability that each asset will be accepted as payment for goods.

They expect that if they did not make a buy in the first market they will be able to use both assets to buy in the second market.

Their expectations with respect to acceptance in the first market are given by two parameters:  $m(y)$  and  $n(y)$ . In the state of high demand the probability of buying in the first market with bonds (stocks) is  $m$  ( $n$ ). In the state of low demand agents expect that they will be able to buy with bonds (stocks) if  $m > 0$  ( $n > 0$ ).

When exactly  $s$  markets open, a unit of bonds will buy on average  $z_s(y)$  units of consumption where

$$(3) \quad z_1(y) = \frac{1}{p_1(y)} \text{ if } m(y) > 0 \text{ and } \frac{1}{p_2(y)} \text{ otherwise;}$$

$$z_2(y) = \frac{m(y)}{p_1(y)} + \frac{1-m(y)}{p_2(y)}.$$

Similarly, the expected purchasing power of the ownership of the firm (a unit of stocks) is:

$$(4) \quad z_1^*(y) = \frac{1}{p_1^*(y)} \text{ if } n(y) > 0 \text{ and } \frac{1}{p_2^*(y)} \text{ otherwise;}$$

$$z_2^*(y) = \frac{n(y)}{p_1^*(y)} + \frac{1-n(y)}{p_2^*(y)}.$$

A labor contract is a vector  $(a_1, a_2, b_1, b_2)$  where  $b_s$  is the amount of (before interest payment) dollars that will be delivered if exactly  $s$  markets open and  $a_s$  is the amount of (before dividends payment) stocks that will be delivered if exactly  $s$  markets open.

The firm let worker  $h$  choose a labor contract out of the following budget constraint:

$$(5) \quad \Omega_1 a_1^h + \Omega_2 a_2^h + \Pi_1 b_1^h + \Pi_2 b_2^h = W \lambda^h.$$

Worker expects that the firm will distribute at the end of the period  $D_s(y)$  units of bonds as dividends if exactly  $s$  markets open. They also expect that the aggregate state in the next period will be  $y_s = y_s(y)$  if exactly  $s$  markets open.

Worker 2: A contract  $(a_1^2, a_2^2, b_1^2, b_2^2)$  owned by a type 2 worker born at t promises:

$a_2^2 [z_2^*(y_2) + D_2(y)z_2(y_2)] + b_2^2 R(y)z_2(y_2)$  units of consumption (at t+1) if  $\theta_t = 1$  and  $\theta_{t+1} = 1$  and  $a_1^2 [z_2^*(y_1) + D_1(y)z_2(y_1)] + b_1^2 R(y)z_2(y_1)$  units of consumption if  $\theta_t = 0$  and  $\theta_{t+1} = 1$ . A type 2 worker chooses  $(a_1^2, a_2^2, b_1^2, b_2^2)$  to maximize his expected consumption given that he wants to consume ( $\theta_{t+1} = 1$ ). He thus solves:

$$(6) \quad \max_{a_s^2, b_s^2} \pi a_2^2 [z_2^*(y_2) + D_2(y)z_2(y_2)] + (1 - \pi) a_1^2 [z_2^*(y_1) + D_1(y)z_2(y_1)] \\ + \pi b_2^2 R z_2(y_2) + (1 - \pi) b_1^2 R z_2(y_1) \quad \text{s.t. (5) .}$$

Worker 1: A type 1 agent always wants to consume. He therefore uses the unconditional expected purchasing power of a unit of bonds ( $Z$ ) and a unit of stocks ( $Z^*$ ):

$$(7) \quad Z(y) = \pi z_2(y) + (1 - \pi) z_1(y) ; Z^*(y) = \pi z_2^*(y) + (1 - \pi) z_1^*(y)$$

A type 1 worker will get on average:  $a_2^1 [Z^*(y_2) + D_2(y)Z(y_2)] + b_2^1 R(y)Z(y_2)$  units of consumption if  $\theta_t = 1$  and  $a_1^1 [Z^*(y_1) + D_1(y)Z(y_1)] + b_1^1 R(y)Z(y_1)$  if  $\theta_t = 0$ . A type 1 worker will thus solve:

$$(8) \quad \max_{a_s^1, b_s^1} \pi a_2^1 [Z^*(y_2) + D_2(y)Z(y_2)] + (1 - \pi) a_1^1 [Z^*(y_1) + D_1(y)Z(y_1)] \\ + \pi b_2^1 R Z(y_2) + (1 - \pi) b_1^1 R Z(y_1) \quad \text{s.t. (5) .}$$

Since the firm is willing to accept bonds as payment for goods, I assume that stockholders are indifferent between the two assets:

(9) If worker h chooses  $a_1^h > 0$  or  $a_2^h > 0$ , then there exists an interior solution to the problem of worker h:  $(a_1^h > 0, a_2^h > 0, b_1^h > 0, b_2^h > 0)$

Market clearing conditions: Equity market clearing requires:

$$(10) \quad a_2^1 + a_2^2 = 1; \quad a_1^1 + a_1^2 + S^2 = 1.$$

Condition (10) says that the claims on the ownership of the firm plus bequest of stocks must sum to 1.

The labor market clearing condition is:

$$(11) \quad L = \lambda^1 + \lambda^2.$$

Goods market clearing conditions are:

$$(12) \quad p_1^*(1 - \psi_1)x_1 = S^1; \quad p_1\psi_1x_1 = M^1; \quad p_2^*(1 - \psi_2)x_2 = S^2; \quad p_2\psi_2x_2 = M^2.$$

Note that the supplies to the first market must equal the minimum demand. Since only type 1 agents buy in the low demand state, we require that the value of the goods offered in the first market for stocks,  $p_1^*(1 - \psi_1)x_1$ , is equal to the amount of stocks held by type 1 buyers,  $S^1$ . Similarly the value of goods offered in the first market for bonds,  $p_1\psi_1x_1$ , must equal the amount of bonds,  $M^1$ , held by type 1 buyers. When demand is high some buyers from both types could not make a buy in the first market and the buyers who were rationed hold  $S^2$  stocks and  $M^2$  bonds. The purchasing power that could not make a buy in the first market will buy in the second market.

The probabilities  $m$  ( $n$ ) are given by the fraction of bonds (stocks) that make a buy in the first market:

$$(13) \quad m = \frac{M^1}{M^1 + M^2}; \quad n = \frac{S^1}{S^1 + S^2}.$$

Note that it is possible to have  $M^1 < S^1$  and  $m > n$ . What is important for liquidity is the use of the asset in the first market relative to its supply and not relative to the other asset.

Dividends distribution and next period state: At the end of the period, the firm distributes the bonds it has as dividends. In the low demand state the firm's bonds revenue is:  $M^1$ . The firm must pay the workers a total of  $b_1^1 + b_1^2$  units of bonds. The amount of bonds the firm has after the end of trade is:  $M^1 - (b_1^1 + b_1^2)$  units. These bonds earn interest and there is a transfer from the government. Therefore at the beginning of next period the firm will distribute:

$$(14) \quad D_1(y) = R(y)[M^1 - (b_1^1 + b_1^2)] + g_1,$$

units of bonds if only one market opens. When two markets open the firm's bond revenues are  $M^1 + M^2$  and its bond wage bill is  $b_2^1 + b_2^2$ . It will distribute:

$$(15) \quad D_2(y) = R(y)[(M^1 + M^2) - (b_2^1 + b_2^2)] + g_2$$

units of bonds if both markets open.

I now turn to describe the next period state if exactly  $s$  markets open in the current period:  $y_s(y) = [M_s^1(y), M_s^2(y), S_s^1(y), S_s^2(y)]$ .

The next period holding of bonds if in the current period only one market opens is:

$$(16) \quad M_1^1 = R b_1^1 + a_1^1 D_1; \quad M_1^2 = R b_1^2 + (a_1^2 + S^2) D_1 + R M^2 - \tau$$



The next period holding of stocks if only one market opens in the current period is:

$$(17) \quad S_1^1 = a_1^1; \quad S_1^2 = a_1^2 + S^2$$

The portfolios of the buyers in the next period when both markets open in the current period are:

$$(18) \quad M_2^1 = Rb_2^1 + a_2^1 D_2; \quad M_2^2 = Rb_2^2 + a_2^2 D_2; \quad S_2^1 = a_2^1; \quad S_2^2 = a_2^2.$$

We can now define equilibrium as follows.

Equilibrium is a policy choice  $(g_1, g_2, \tau)$  and a vector of functions

$$(R, W, \Pi_1, \Pi_2, \Omega_1, \Omega_2, q_1, q_2, p_1, p_2, p_1^*, p_2^*, L, x_1, x_2, \psi_1, \psi_2, z_1, z_2, z_1^*, z_2^*, Z, Z^*, D_1, D_2, a_1^1, a_1^2, b_1^1, b_1^2, a_2^1, a_2^2, b_2^1, b_2^2, n, m, y_1, y_2)$$

such that (a) all the functions are from the state  $y = (M^1, M^2, S^1, S^2)$  to the real line;

(b) given  $(R, W, \Pi_1, \Pi_2, \Omega_1, \Omega_2, q_1, q_2, p_1, p_2, p_1^*, p_2^*), (L, x_1, x_2, \psi_1, \psi_2)$  solves the firm's problem

(2); (c) given  $(R, W, \Pi_1, \Pi_2, \Omega_1, \Omega_2, z_1, z_2, z_1^*, z_2^*, Z, Z^*, D_1, D_2), (a_1^2, a_2^2, b_1^2, b_2^2)$  is a solution to

(6) and  $(a_1^1, a_2^1, b_1^1, b_2^1)$  is a solution to (8); (d) the conditions in (1)-(18) are satisfied.

Steady state: I now assume that the government pursues a policy of price stability. Since the price of the firm affects goods prices, here this can be achieved when dividend payments and wealth distribution remain constant over time. I therefore consider a steady-state equilibrium with the following additional properties:

$$(19) \quad D_1 = D_2 = D; \quad y_1 = y_2 = y; \quad \Pi_1 = \pi; \quad \Pi_2 = 1 - \pi;$$

$$q_1 = q_2 = q; \quad \Omega_1 = (1 - \pi)q; \quad \Omega_2 = \pi q; \quad p_s = qp_s^*$$

In the steady state dividends and the portfolios of the old agents do not change over time, contingent claims are priced in an actuarially fair manner, the price of the firm does not change during the trading process, good prices are constant and "purchasing power parity" holds.

Under (19), the first order conditions to the firm's problem (2) require:

$$(20) \quad p_1 = \pi p_2 = \frac{W}{\alpha L^{\alpha-1}},$$

where  $W / \alpha L^{\alpha-1}$  is the marginal cost.

I now turn to the workers' problems in the steady state.

Claim 1: In the steady state: (a) workers who buy contingent claims on stocks are indifferent between alternative combinations of contingent claims on stocks (if the worker buys  $a_1^h > 0$  or  $a_2^h > 0$  then any choice of  $a_1^h$  and  $a_2^h$  that satisfies the budget constraint is a solution to the worker's problem); (b) workers who buy contingent claims on bonds are indifferent between alternative combinations of contingent claims on bonds.

The proof is in the Appendix. Claim 1 does not rule out the possibility that workers may strictly prefer bonds to stocks but a worker that chooses to hold claims on the ownership of the firm, may own claims that will be delivered in one state only or in both states. Similarly, a worker may hold claims on bonds in one state only or in both states. To get a steady state, I assume that type 2 workers buy claims for delivery in state 2 only and type 1 workers buy the same amount of claims in both state of demand. Thus,

$$(21) \quad b_2^2 = b > 0; b_1^2 = 0; a_2^2 = a > 0; a_1^2 = 0; \quad b_1^1 = b_2^1 = B, a_1^1 = a_2^1 = A$$

Using this simplified notation, we can write the budget constraints (5) as:

$$(22) \quad Aq + B = W\lambda^1 \text{ for type 1 and } aq + b = \frac{w\lambda^2}{\pi} \text{ for type 2.}$$

In the steady state the type 2 old agent holds  $S^2$  stocks and the young type 2 worker chooses  $a = S^2$ . Therefore, the fraction of the firm owned by type 2 agents does not change over time: They may get it as a wage payment when  $\theta = 1$  or as a bequest when  $\theta = 0$ .

To get a steady state the lump sum transfers and bequest tax must satisfy:

$$(23) \quad D = R(M^1 - B) + g_1 = R(M^1 + M^2 - B - b) + g_2;$$

$$\tau = (R-1)M^2 + S^2D = rM^2 + S^2D$$

The portfolios in the steady state satisfy:

$$(24) \quad M^1 = AD + BR; M^2 = aD + bR; S^1 = A; S^2 = a.$$

Conditions (23) and (24) imply the following government's "budget constraint":

$$(25) \quad g_2 = -rM; g_1 = -rM + \tau,$$

where  $M = M^1 + M^2$ . Thus in the high demand state the government finances the interest payments by a lump sum tax on the firm. When demand is low, interest payments are financed by both a tax on the firm and a bequest tax.

I now turn to specify the type specific rates of return in the steady state. It is easier to work with the purchasing power of a dollar worth of stocks:  $v_s = z_s^*/q$ . Using  $p_s = q p_s^*$  yields:

$$(26) \quad v_1 = \frac{1}{p_1} \text{ if } n > 0 \text{ and } \frac{1}{p_2} \text{ otherwise; } v_2 = \frac{n}{p_1} + \frac{1-n}{p_2};$$

$$V = \pi v_2 + (1-\pi)v_1$$

I use  $R_e^h$  ( $R_b^h$ ) to denote the expected real rate of return on equity (bonds) to a type h worker:

$$(27) \quad R_e^2 = \pi \frac{qv_2 + Dz_2}{q}; R_e^1 = \frac{qV + DZ}{q}; R_b^2 = \pi R_{z_2}; R_b^1 = RZ$$

To derive (27) note that the ownership of the firm promises on average  $qv_2 + Dz_2$  units of consumption to a type 2 agent who wants to consume and  $qV + DZ$  units of consumption to a type 1 agent.

Since (9) says that stocks cannot be strictly preferred to bonds and we must have:

$$(28) \quad \frac{R_e^1}{R_b^1} \leq \frac{R_e^2}{R_b^2} = 1; \text{ or } 1 = \frac{R_e^1}{R_b^1} > \frac{R_e^2}{R_b^2}$$

Under the first alternative type 1 prefers bonds and type 2 is indifferent between the two assets. Under the second alternative type 1 is indifferent between the two assets and type 2 prefers bonds.

Using (27) and (28) leads to:

$$(29) \quad q = \frac{Dz_2}{R_{z_2} - v_2} \text{ if } \frac{R_e^1}{R_b^1} \leq \frac{R_e^2}{R_b^2} = 1 \text{ and } q = \frac{DZ}{RZ - V} \text{ if } 1 = \frac{R_e^1}{R_b^1} > \frac{R_e^2}{R_b^2}.$$

In (29) the value of the firm is determined from the point of view of the stockholders. In the special case

$\frac{R_e^1}{R_b^1} = \frac{R_e^2}{R_b^2} = 1$ ,  $q = \frac{Dz_2}{Rz_2 - v_2} = \frac{DZ}{RZ - V}$ . As will be shown later, this special case requires:

$$v_2/z_2 = V/Z \text{ and } m = n.$$

Since the risk neutral workers maximize expected real return we also require that when bonds earn on average a higher rate of return the worker will specialize in bonds:

$$(30) \quad A = 0 \text{ if } \frac{R_e^1}{R_b^1} < \frac{R_e^2}{R_b^2} = 1 \text{ and } a = 0 \text{ if } 1 = \frac{R_e^1}{R_b^1} > \frac{R_e^2}{R_b^2}.$$

We can now define steady-state equilibrium as a policy choice  $(g_1, g_2, \tau)$  and a vector of scalars

$$(R, W, q, p_1, p_2, L, x_1, x_2, \psi_1, \psi_2, z_1, z_2, Z, v_1, v_2, V, D, R_e^1, R_b^1, R_e^2, R_b^2, a, A, b, B, y, n, m)$$

that satisfies:  $x_1 + x_2 = L^\alpha$ , (3), (7), (11), (13), (20), (22)-(30) and the market clearing conditions:

$$(31) \quad a + A = 1;$$

$$(32) \quad p_1(1 - \psi_1)x_1 = qS^1; p_1\psi_1x_1 = M^1; p_2(1 - \psi_2)x_2 = qS^2; p_2\psi_2x_2 = M^2$$

Condition (31) is the equity market clearing condition. The goods market clearing conditions (32) is (12) written in dollar terms.

I assume that whenever an agent is indifferent between the two assets he holds strictly positive amounts from both. This rules out an equilibrium that can be sustained only for very special choice of parameters. Under this assumption I now show the following Proposition.

Proposition 1: We can have at most two types of steady state equilibria: (a) equilibrium in which type 1 specializes in bonds and type 2 holds both assets ( $n = 0$  and  $0 < m < 1$ ) or (b) equilibrium in which both types hold both assets in exactly the same proportions ( $0 < m = n < 1$ ).

The proof is in the Appendix. The intuition is as follows. When  $n = m$ , both assets are equally liquid and are therefore perfect substitutes. When  $n \neq m$ , there is a difference in liquidity between the two assets. Since the stable demand type has a higher probability of finding the good at the low price he is willing to pay a relatively high "liquidity premium" for holding the more liquid asset. Therefore, if type 2 is willing to hold both assets type 1 specializes in the more liquid asset. Since (9) rules out specialization in stocks, it follows that type 1 specializes in bonds and bonds are more liquid.<sup>2</sup>

In what follows, I focus on alternative (a) in which type 1 specializes in bonds.

Proposition 2: Under alternative (a) stocks earn a premium:

$$\frac{D}{q} > R - 1 = r.$$

Proof: Note that under (a), (29) implies:  $\frac{D}{q} = R - \frac{v_2}{z_2}$ . Alternative (a) also implies

$$n = 0 < m \text{ and } \frac{v_2}{z_2} < 1. \text{ Therefore, } \frac{D}{q} > r. \square$$

---

<sup>2</sup> We can have another equilibrium solution if we relax the assumption that the firm accepts both assets as payments for goods (and therefore stockholders must be willing to hold both assets). In this case we can have equilibrium in which bonds are valueless and only stocks are accepted as payments for goods ( $\psi_s = 0$ ).

The intuition is as follows. Since stocks are not accepted in the first market, they will be held only if they promise a higher rate of return on average. The premium on stocks is enough to compensate type 2 agents for their relative illiquidity but it is too small for compensating type 1 agents who choose to specialize in bonds.

I now turn to a numerical example.

Solution: It is easier to think of  $D$  as the policy choice variable and assume that  $(g_1, g_2, \tau)$  are chosen to satisfy (23).

I normalize by assuming  $\lambda^1 + \lambda^2 = 1$  and  $W = 1$ . Substituting  $L = 1$  in (20) yields:

$$(33) \quad p_1 = \frac{1}{\alpha} ; p_2 = \frac{1}{\pi\alpha}$$

We now use  $n = 0$ , (3), (26), (29) and (33) to get:

$$(34) \quad q = \frac{Dz_2}{Rz_2 - v_2} = \frac{D(m + (1-m)\pi)}{R[(m + (1-m)\pi)] - \pi}$$

We use (13),  $A = 0$ ,  $a = 1$ , (22) and (24) to get:

$$(35) \quad m = \frac{BR}{BR + D + bR} = \frac{R\lambda^1}{R\lambda^1 + D + R[(R\lambda^2/\pi) - q]}$$

We use (30) and (32) to get:

$$(36) \quad x_1 + x_2 = \frac{BR}{p_1} + \frac{q + D + bR}{p_2} = \alpha R + \pi\alpha(D - qr) = 1$$

Treating  $D$  as a policy choice, we now have three equations, (34)-(36), and three unknowns:  $q$ ,  $m$ ,  $R$ .

The special case  $D=0$ : In this case (34) implies that the price of the firm is  $q=0$  and there is only one asset in the economy: government bonds. Substituting  $D=q=0$  in (36) leads to  $R = \frac{1}{\alpha}$ . To build intuition it may be useful to consider the case in which  $\pi=1$  and there is no demand uncertainty. In this case only one market opens. The price of goods is the first market price in (33):  $\frac{1}{\alpha} \geq 1$ . But the total wage payment is 1. To clear the good market the government must transfer money to the buyers. Here the transfer is in the form of interest payments. When  $\alpha = 0.96$  the interest rate should be about 4%.

The case of  $R=1$ : This is a case of minimal government intervention. Substituting  $R=1$  in (23) and (25) leads to:  $g_2 = 0$  and  $g_1 = \tau = D$ . Thus the government intervenes only in the low demand state: It transfers the bequest tax revenues to the firm that is owned by the same people that pay the bequest tax. Substituting  $R=1$  in (36) leads to  $D = \frac{1-\alpha}{\pi\alpha}$ .

When  $\alpha = 0.96$  and  $\pi = 0.9$ , we get:  $D = 0.046$ .

I now turn to discuss changes in the "policy variable"  $D$ .

Changing  $D$ : Figure 2 describes the equilibrium levels of the interest rate ( $r$ ), the rate of return on equity ( $\frac{D}{q}$ ) and the difference between them (the equity premium) as a function of  $D$ . I use the parameters:  $\lambda^1 = \lambda^2 = \frac{1}{2}$ ,  $\pi = 0.9$  and  $\alpha = 0.96$ . As we can see the interest rate decreases with  $D$  but the return on equity hardly changes with  $D$ . As a result the equity premium increases with  $D$ . The negative relationship between the interest on bonds and  $D$  can be seen with the help of the "government budget constraint", (25). Starting from  $D=0$ , an increase in  $D$  implies less taxes on the firm and as a result the interest payment transfers is lower.



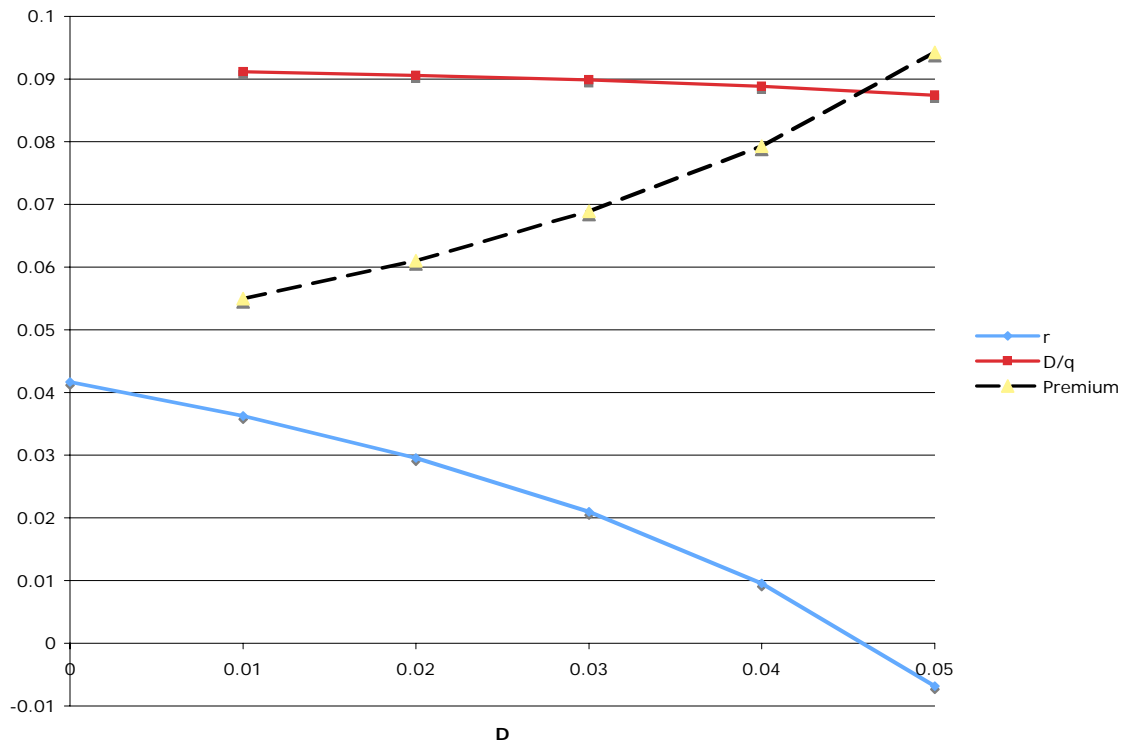


Figure 2:  $r$ ,  $\frac{D}{q}$  and  $\frac{D}{q} - r$  as a function of  $D$

$$(\alpha = 0.96, \pi = 0.9, \lambda^1 = \lambda^2 = \frac{1}{2})$$

Figure 3 describes total bond holdings before the beginning of trade ( $M = M^1 + M^2$ ), the amount of bonds held by type 1 buyer ( $M^1$ ), the amount of bonds held by type 2 buyer ( $M^2$ ) and the fraction held by type 1 ( $m = M^1/M$ ). An increase in  $D$  reduces  $M^2$  but it has only a modest negative effect on  $M^1$ . The negative effect on  $M^1 = BR = R/2$  is due to the negative effect of  $D$  on  $R$ . Note also that the aggregate bond holdings  $M$  declines with  $D$  and the fraction of  $M$  held by type 1 buyers increases with  $D$ .

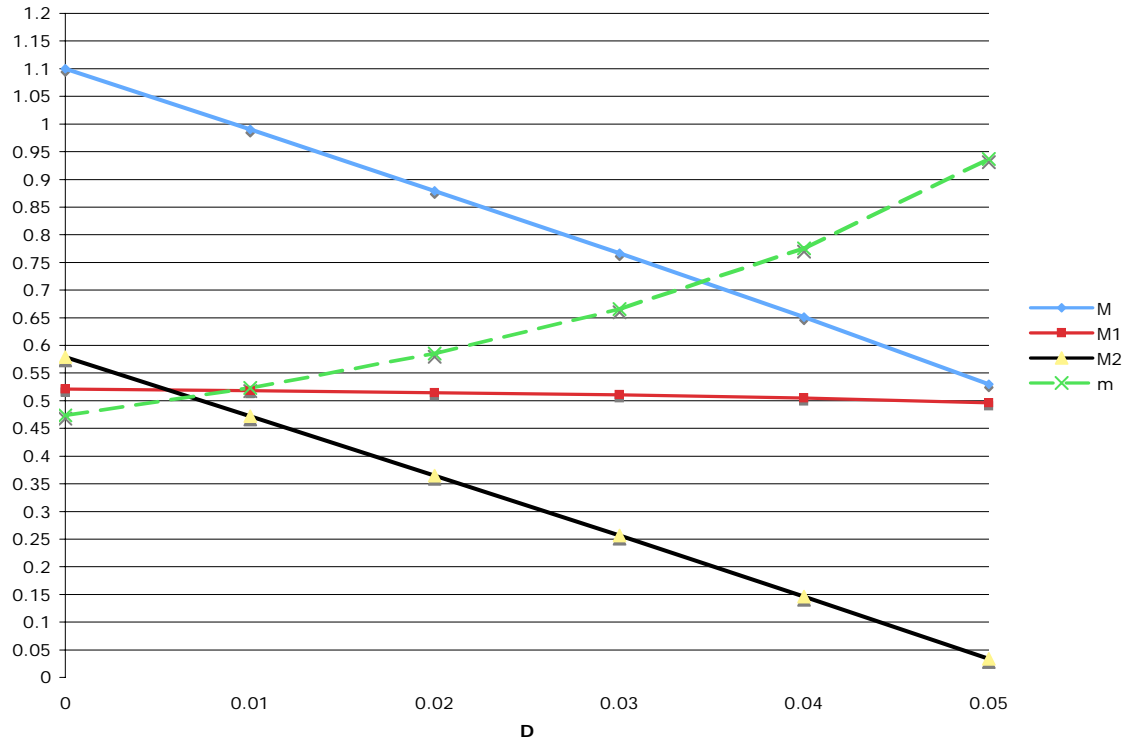


Figure 3:  $M, M^1, M^2, m$  as a function of  $D$  ( $\alpha = 0.96, \pi = 0.9, \lambda^1 = \lambda^2 = \frac{1}{2}$ )

Figure 4 describes  $M, q$  and  $M + q$  as a function of  $D$ . An increase in  $D$  leads to an increase in  $q$ . It also leads to a decrease in  $M$  by almost the same amount and as a result  $M + q$  does not change. This is analogous to the case of currency substitution discussed in the literature. Note that since prices do not depend on  $D$  in our model, the measure of money that is consistent with the quantity theory is  $M + q$ .

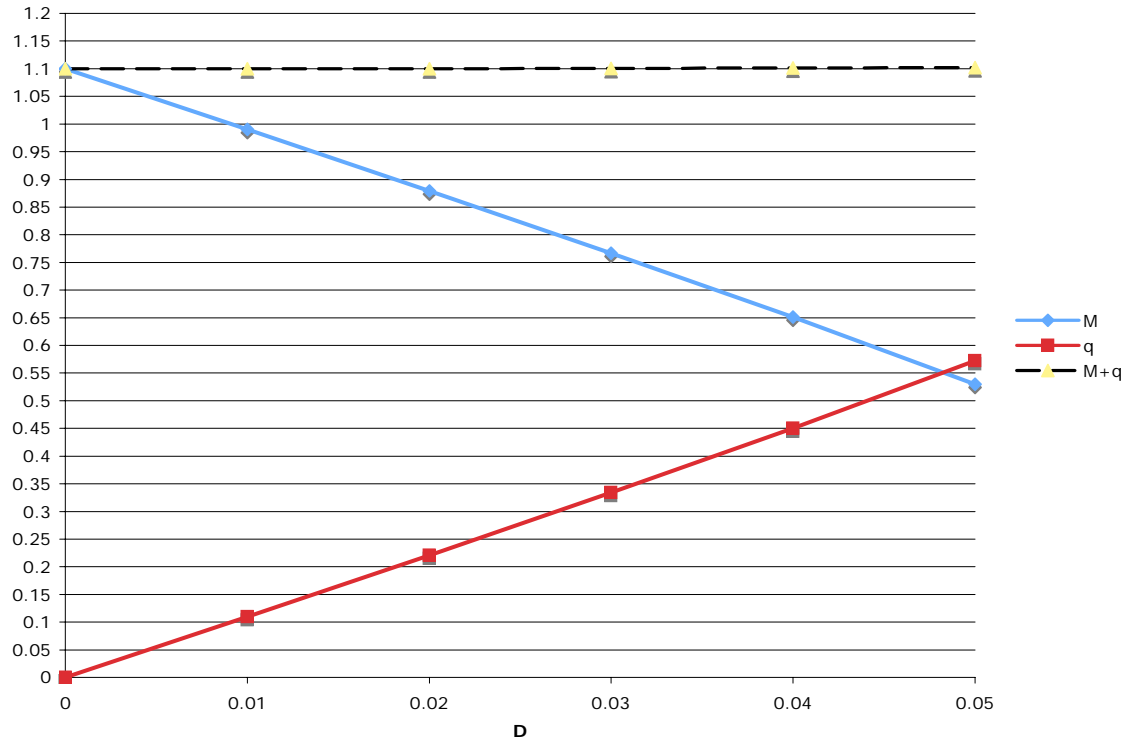


Figure 4:  $M, q$  and  $M + q$  as a function of  $D$  ( $\alpha = 0.96, \pi = 0.9, \lambda^1 = \lambda^2 = \frac{1}{2}$ )

Figure 5 describes the welfare of both types as a function of  $D$ . When  $D$  increases the interest decreases and as a result type 1 agents who strictly prefer bonds become worse off (only slightly in our numerical example). Type 2 agents are indifferent between the two assets and their welfare is not affected by the substitution of bonds for stocks.

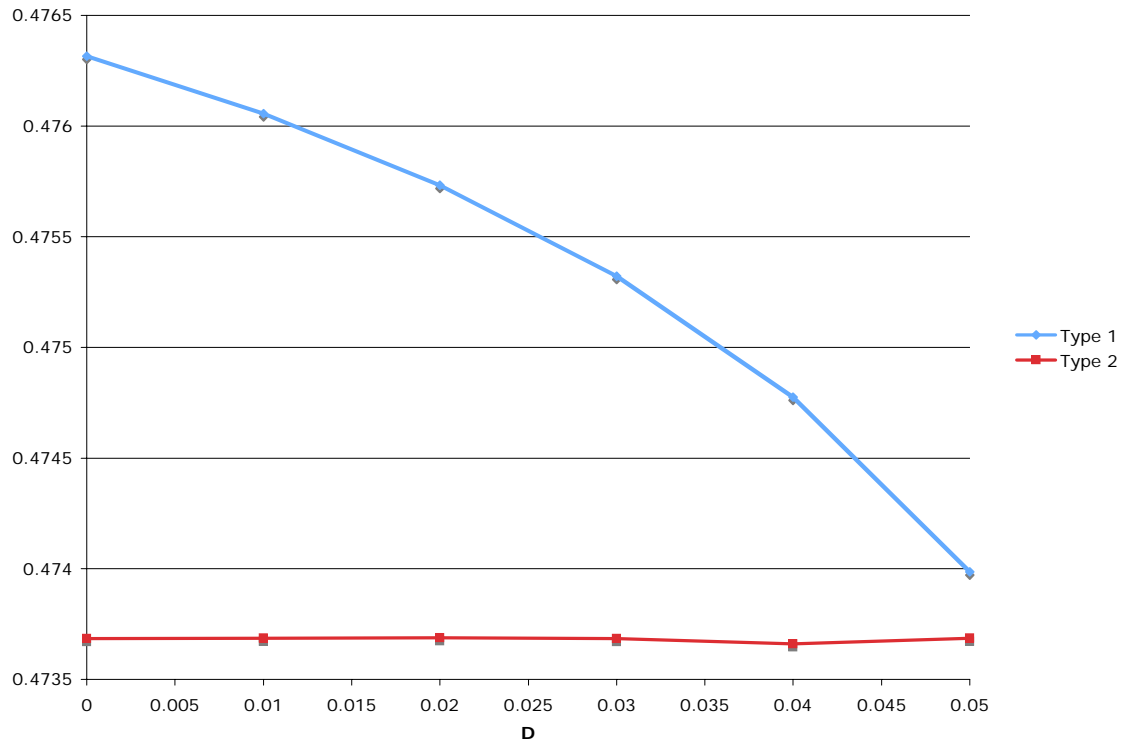


Figure 5: Welfare as a function of  $D$  ( $\alpha = 0.96$ ,  $\pi = 0.9$ ,  $\lambda^1 = \lambda^2 = \frac{1}{2}$ )

### Can the model accounts for the observed equity premium?

Mehra and Prescott (1985) observed an average rate of return on stocks of about 7% and an average real interest on short-term government bonds of less than 1%. They used data from 1890-1979 and their findings imply an equity premium of about 6%.

Mankiw and Zeldes (1991) calculated an 8% equity premium for the period 1948-1988.

As can be seen from Figure 2, our model can account for these findings under the assumptions:  $\alpha = 0.96$ ,  $\pi = 0.9$ ,  $\lambda^1 = \lambda^2 = \frac{1}{2}$  and  $D = 0.04$ . Can we defend this choice of parameters?

At the end of their paper Eeckhout and Jovanovic (1992, page 1299) provide a mini survey of the empirical estimates of the elasticity of output with respect to inputs. They cite estimates of  $\alpha$  in the range 0.95-0.99. Their own estimate is in the range:

0.94 - 0.99. Our choice of  $\alpha = 0.96$  is in this range. As can be seen from Figure 6, the equity premium does not change much with changes in  $\alpha$ , but the rates of return are highly sensitive to the choice of  $\alpha$ . The interest rate declines with  $\alpha$  because an increase in  $\alpha$  reduces the amount of taxes that is required to maintain constant dividends. This and the "government budget constraint" (25) lead to a decline in interest payments.

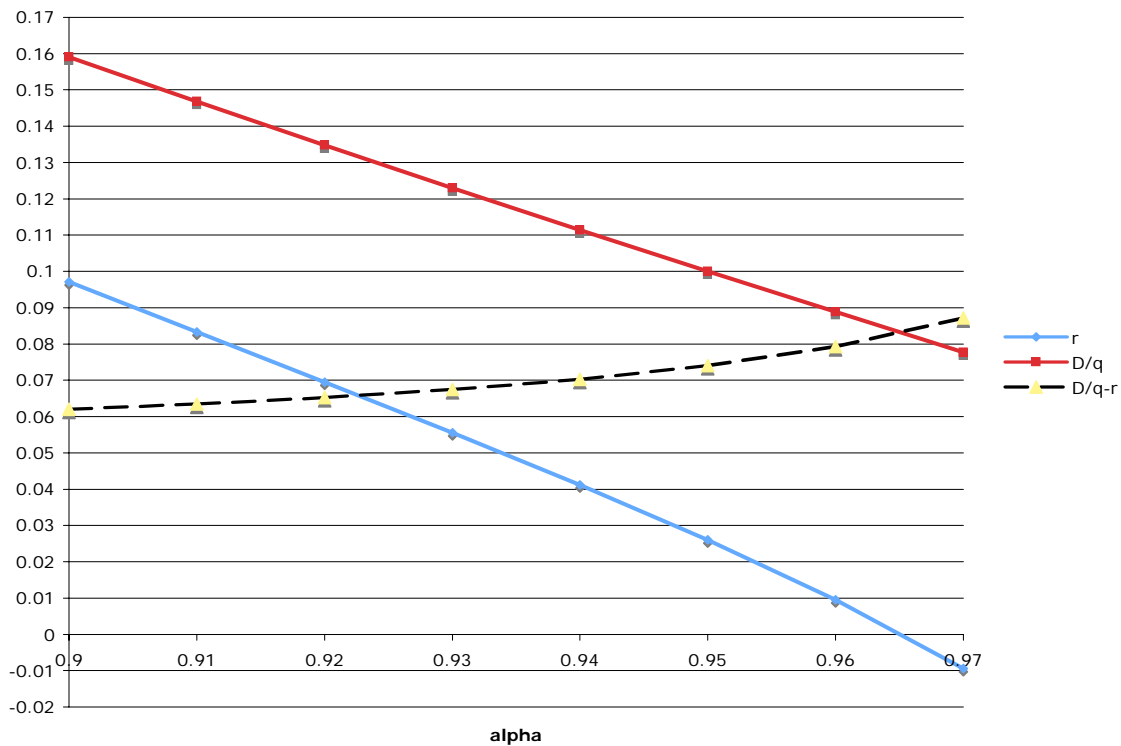


Figure 6:  $r$ ,  $\frac{D}{q}$  and  $\frac{D}{q} - r$  as a function of  $\alpha$  ( $D = 0.04, \pi = 0.9, \lambda^1 = \lambda^2 = \frac{1}{2}$ )

Corporate profits after tax were somewhat less than 6% of GDP during the period 1947-2007. In our model the firm represents large corporations that are publicly traded and their stocks are relatively liquid. The after tax profits of large corporations that are publicly traded is less than 6%. We chose 4% ( $D = 0.04$ ).

Mankiw and Zeldes (1991) used a survey of 2998 US families in 1984 Panel Study of Income Dynamics (PSID). They found that only 27.6% of households hold

stocks. Some stockholders own small amounts of stock. Only 23.2% of the sample holds equity in excess of 1000 dollars and only 11.9% holds equity in excess of 10,000 dollars. The fraction of stockholders increases with labor income and education. Mankiw and Zeldes report that in their sample, stockholders earn 38% of disposable income.

Since in our equilibrium type 2 workers are indifferent between stocks and bonds the fraction that actually hold stocks is not determined by the model. It is possible that all type 2 workers choose to hold stocks and it is also possible that only a fraction of them hold stocks provided that the total amount held is one. But the fraction of type 2 workers must be greater than the fraction observed in the data. This suggests  $\lambda^2 > 0.38$  ( $\lambda^1 < 0.62$ ). Our baseline specification of  $\lambda^1 = \lambda^2 = \frac{1}{2}$  is consistent with this restriction.

Figure 7 computes the equilibrium rates of returns for different  $\lambda^h$ , assuming  $\alpha = 0.96$ ,  $\pi = 0.9$ ,  $D = 0.04$ . An increase in  $\lambda^1$  has almost no effect on  $r$  but increases the rate of return on equity and the equity premium.

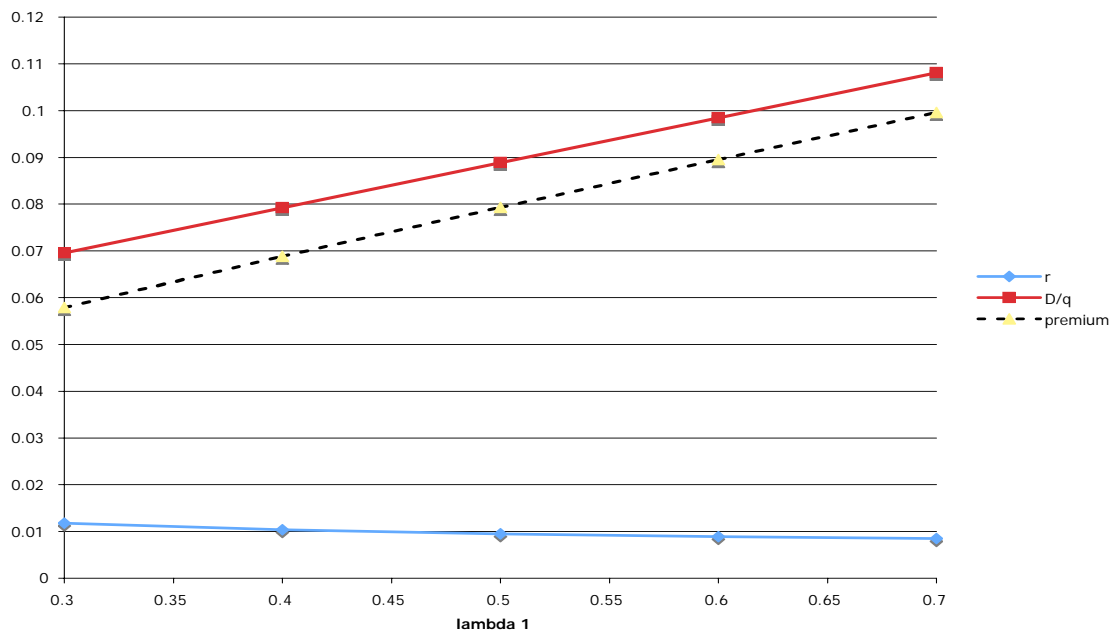


Figure 7:  $r$ ,  $\frac{D}{q}$  and  $\frac{D}{q} - r$  as a function of  $\lambda^1$  ( $\alpha = 0.96$ ,  $\pi = 0.9$ ,  $D = 0.04$ )

Figure 8 describes the rates of return on equity and bonds as a function of  $\pi$ . A close fit for the observed rates of return is obtained when  $\pi = 0.92$ . In this case,  $r = 0.9\%$ ,  $D/q = 0.79\%$  and the difference between the two is: 7%. Note that a decrease in  $\pi$  leads to an increase in  $D/q$  and a decrease in  $q$ . This may be viewed as "flight for quality" in response to an increase in uncertainty.

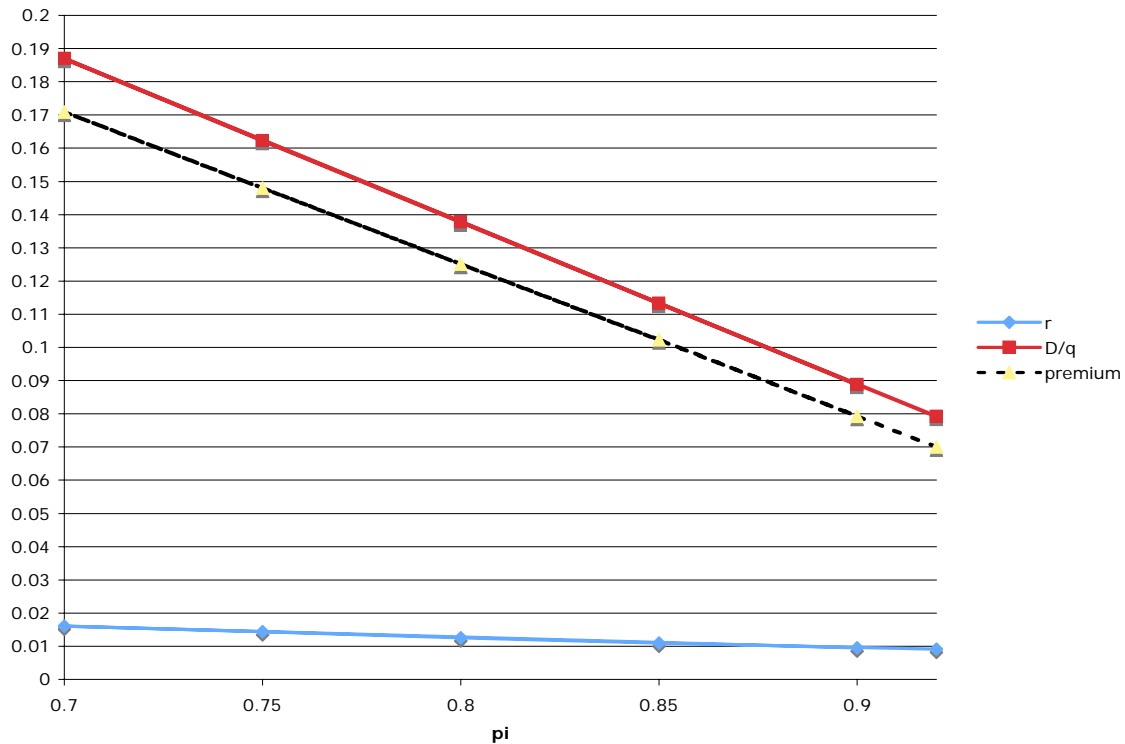


Figure 8:  $r$ ,  $D/q$  and  $D/q - r$  as a function of  $\pi$  ( $\alpha = 0.96$ ,  $\lambda^1 = \lambda^2 = 1/2$ ,  $D = 0.04$ )

Figure 9 describes the share of stocks in total wealth  $\frac{q}{M+q}$  and the share of bonds in total wealth  $\frac{M}{M+q}$ , where  $M = M^1 + M^2$  is total bonds holdings. Our model does not distinguish between cash, short-term bonds and long-term bonds. This is a problem when trying to compare Figure 9 to data. However if we define the liquid asset in our model as cash and short-term bonds we may get a lower bound on  $\pi$ . In the 1994 wealth supplement to PSID the ratio of the value of stocks to the value of

(cash + bonds + stocks) is 30%.<sup>3</sup> The ratio of the value of stocks to the value of (liquid assets + stocks) should be higher because not all bonds are short term. This implies:

$\frac{q}{M+q} > 0.3$ . As Figure 9 shows, this constraint is satisfied when  $\pi > 0.85$ .

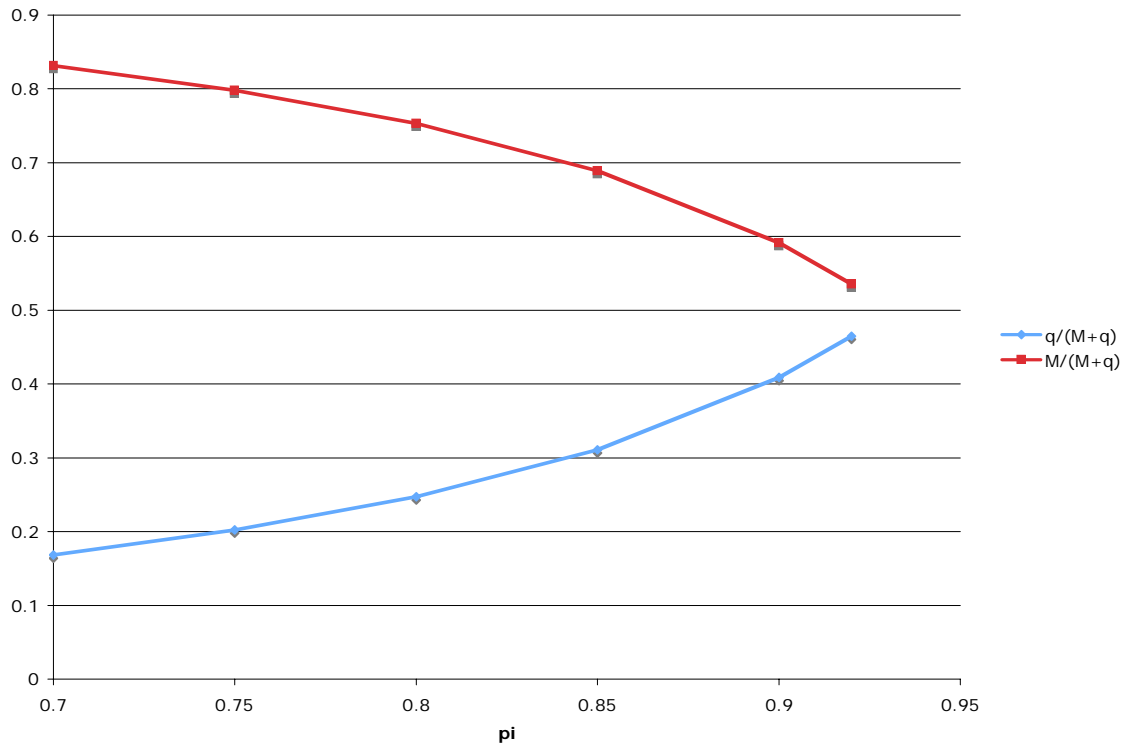


Figure 9: The average portfolio as a function of  $\pi$

$$(\alpha = 0.96, \lambda^1 = \lambda^2 = 1/2, D = 0.04)$$

Back to "casino" money: To make the connection between the model and the dictionary definitions of liquidity cited in the introduction, I now assume that there exists "money" that does not earn interest and represents an obligation of the firm to sell one unit of the good for 1 "dollar". Our "money" is like chips in the casino. Buyers do not hold it for any real length of time: They exchange their assets for chips before they enter the goods

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<sup>3</sup> I am indebted to Matt Chambers and Don Schlagenhauf for the data used in these computations.



market and then use them to buy goods. Our "money" thus serves as a unit of account and has no other function. Note that our use of "dollars" here is distinct from the use of dollars as a unit of government bonds in the rest of the paper.

As before, we assume that active buyers form a line. They then go to a bank that is owned by the firm and change assets for "money". They proceed to the goods market and exchange their "money" for goods. Transactions in the bank are costless.

The buyer does not face uncertainty about the terms in which he can exchange "money" for goods but he faces uncertainty about the terms in which he can exchange assets for "money". Buyers that arrive early will typically get more "money" for their assets than buyers who arrive late.

From the firm's point of view the amount of bonds and stocks that will arrive at its bank is random. We assume that the bank exchanges the first  $M^1$  units of bonds for  $\frac{1}{p_1}$  "dollars" per unit. If additional  $M^2$  bonds arrive the bank exchange them for  $\frac{1}{p_2}$  "dollars" per unit. Similarly, the bank exchanges the first  $S^1$  stocks that arrive for  $\frac{1}{p_1^*}$  "dollars" per unit and it exchanges the second  $S^2$  stocks for  $\frac{1}{p_2^*}$  "dollars" per unit.

In this formulation the "dollar" price of goods is constant and the prices of assets fluctuate. There is also a "cash-in-advance" constraint. But the constraint here does not affect the agents' choice of assets which they actually hold.

I now turn to discuss two common measures of liquidity.

The ratio of buying to selling price: This ratio is sometimes used to define the liquidity of the asset. I now use this measure of liquidity to compare bonds and stocks.

In the equilibrium of interest, the firm sells stocks to type 2 workers for  $q$  units of bonds that are worth  $\frac{1}{p_1^*} = \frac{q}{p_1}$  "dollars" (chips). Whenever the firm buys stocks it pays

$\frac{1}{p_2^*} = \frac{q}{p_2}$  "dollars". The ratio of the buying to the selling price is:  $s^e = \frac{p_1}{p_2} < 1$ .

The firm sells bonds to the worker for  $\frac{1}{p_1}$  "dollars" per unit. In the low demand state it also buys bonds at this price. In the high demand state it buys a fraction  $m$  of the bonds for  $\frac{1}{p_1}$  "dollars" per unit and a fraction  $1 - m$  of the bonds for  $\frac{1}{p_2}$  "dollars" per unit. The average ratio of the buying to the selling price in the high demand state is:

$$s^b = m + (1 - m) \frac{p_1}{p_2} > s^e. \text{ Thus in the state of high demand the buying to the selling price}$$

ratio is higher for bonds and in this sense bonds are relatively liquid.

Velocity: How often do the assets exchange hands in a market transaction? Stocks are held by type 2 and they are exchanged in a market transaction with probability  $\pi$ . The average time between transactions is  $\frac{1}{\pi}$ . Bonds held by type 1 are exchanged every period while bonds held by type 2 are exchanged on average every  $\frac{1}{\pi}$  periods. On average bonds are exchanged every  $m + (1 - m) \frac{1}{\pi} < \frac{1}{\pi}$ . Thus the average time between transactions is shorter for bonds and bonds' velocity is higher on average.

Risk aversion: It is shown in the Appendix that increasing risk aversion reduces the demand for stocks in the equilibrium of interest. It is also shown that if type 2 chooses to hold stocks, then type 1 chooses to specialize in bonds. This suggests that under risk aversion the equity premium will be higher for the same choice of parameters but still in equilibrium, only type 2 will hold stocks.

#### 4. CONCLUSIONS

We used price dispersion to model liquidity. We focused on the acceptability by sellers who offer to sell goods at a low price assuming that you can always use assets to buy goods at the high price. In the equilibrium of interest stocks are less liquid than bonds: They cannot buy goods at the low price and they change hands in a market

transaction relatively infrequently. To discuss conventional definitions of liquidity we may assume that assets are exchanged for "cash" which is then used to buy goods. The ratio of the buying to selling "cash" price is relatively low for stocks.

In the model, sellers choose price tags that specify the price in terms of either government bonds or stocks. There are two markets. The first market opens with probability 1. The second market opens only if aggregate demand is high, with probability  $\pi$ . Prices in the first market are lower than in the second market.

The liquidity of an asset is measured by its use in the first market relative to its use in the second market. An agent who holds a liquid asset has a higher chance of making a buy in the first market. Thus here we focus on acceptability in the first low price market. This is different from the money search model in Kiyotaki and Wright (1993). In their model there is a single exogenously given price and in any given period a buyer can buy the good if he meets a seller that accepts his money. Here you could always make a buy with either bonds or stocks. The question is at what price.

We have two types of people. Type 1 always wants to consume and type 2 wants to consume only if he gets a taste shock. In equilibrium the price tags of sellers in the first market must match the portfolio chosen by type 1 agents: If type 1 buyers hold only bonds a seller who posts a price in terms of stocks will not be able to sell in the low demand state.

Since type 1 buys also in the low demand state the probability that he will find the good at the low price (in the first market) is relatively high. Therefore, type 1 agents are willing to pay a relatively high liquidity premium for holding the more liquid asset. Type 2 agents are indifferent between the two assets.

Bonds and stocks are not symmetric in our model because agents who hold stocks must be willing to hold bonds but bondholders may strictly prefer bonds to stocks. This requirement emerges from the assumptions that the firm is willing to accept bonds as payment for goods and the firm represents the preferences of its stockholders.

Our calibration exercise suggests that the model can account for the rates of returns estimated by Mehra and Prescott (1985). It can also account for the observation that only a fraction of the population holds stocks.

We focused on a steady-state equilibrium. The analysis of non-steady state equilibrium will certainly provide important insights. I expect that we will get a positive correlation of consumption with the return on the stock market. I also expect that this correlation will be higher for type 2's consumption (luxury goods).

## APPENDIX

Proof of Claim 1: The first order conditions for the problem (6) are:

$$(A1) \quad \pi [z_2^*(y_2) + D_2(y)z_2(y_2)] - \lambda\Omega_2 \leq 0 \text{ with equality if } a_2^2 > 0;$$

$$(A2) \quad (1 - \pi) [z_2^*(y_1) + D_1(y)z_2(y_1)] - \lambda\Omega_1 \leq 0 \text{ with equality if } a_1^2 > 0;$$

$$(A3) \quad \pi R z_2(y_2) - \lambda\Pi_2 \leq 0 \text{ with equality if } b_2^2 > 0;$$

$$(A4) \quad (1 - \pi) R z_2(y_1) - \lambda\Pi_1 \leq 0 \text{ with equality if } b_1^2 > 0;$$

where  $\lambda$  is the Lagrangian multiplier.

In the steady state (A1) and (A2) can be written as:

$$(A5) \quad z_2^* + D z_2 - \lambda q \leq 0 \text{ with equality if } a_2^2 > 0 \text{ or } a_1^2 > 0.$$

Therefore if  $a_2^2 > 0$  then the first order condition for any  $a_1^2 > 0$  is satisfied and vice versa.

This implies (a) for type 2. The argument for type 1 is the same.

To show (b) note that in the steady state (A3) and (A4) can be written as:

$$(A6) \quad R z_2 - \lambda \leq 0 \text{ with equality if } b_2^2 > 0 \text{ or } b_1^2 > 0.$$

This implies (b) for type 2. The argument for type 1 is the same.  $\square$

Proof of Proposition 1: Note that (27) implies:

$$(A7) \quad \frac{R_e^1}{R_b^1} = \frac{1}{R} \left( \frac{V}{Z} + \frac{D}{q} \right); \quad \frac{R_e^2}{R_b^2} = \frac{1}{R} \left( \frac{v_2}{z_2} + \frac{D}{q} \right)$$

I now use (A7) to characterize the relationship between these type specific ratios and (m, n).

$$\text{Lemma 1: (A) } \frac{R_e^1}{R_b^1} < \frac{R_e^2}{R_b^2} \text{ if } m > n = 0; \text{ (B) } \frac{R_e^1}{R_b^1} = \frac{R_e^2}{R_b^2} \text{ if } m = n > 0;$$

$$\text{(C) } \frac{R_e^1}{R_b^1} > \frac{R_e^2}{R_b^2} \text{ if } n > m = 0; \text{ (D) } \frac{R_e^1}{R_b^1} > \frac{R_e^2}{R_b^2} \text{ if } m > n > 0; \text{ (E) } \frac{R_e^1}{R_b^1} < \frac{R_e^2}{R_b^2} \text{ if } n > m > 0.$$

Proof: In what follows I use the definitions (3), (7), (26) and

$p_2 > p_1$ . This inequality is implied by (20).

To show (A) note that when  $m > n = 0$  we must have:  $v_2 = V = v_1 < z_2 < Z < z_1$ . It follows that  $\frac{V}{Z} < \frac{V}{z_2} = \frac{v_2}{z_2}$  and therefore (A7) leads to (A).

To show (B) note that when  $n = m$ , we must have:  $\frac{V}{Z} = \frac{v_2}{z_2} = 1$ .

To show (C) note that when  $n > 0$  and  $m = 0$ , we must have:

$z_2 = Z = z_1 < v_2 < V < v_1$ . In this case  $\frac{V}{Z} > \frac{v_2}{Z} = \frac{v_2}{z_2}$  and (A7) leads to (C).

To show (D) note that when  $0 < n < m$ ,  $v_1 = z_1$  and  $v_2 < z_2$ . In this case:

$$(A8) \quad \frac{V}{Z} = \frac{\pi v_2 + (1 - \pi)v_1}{\pi z_2 + (1 - \pi)z_1} = \frac{\pi v_2 + (1 - \pi)z_1}{\pi z_2 + (1 - \pi)z_1} > \frac{v_2}{z_2}$$

and this leads to (D). When  $0 < m < n$ ,  $v_1 = z_1$  and  $v_2 > z_2$ . In this case the inequality in (A8) is reversed and (A7) leads to (E).  $\square$

We now turn to see which of the alternatives (A) - (E) in the Lemma are consistent with (28). Under (A) in the Lemma  $m > n = 0$ , type 1 specializes in bonds and the rates of return ratios are consistent with (28). Under (B) both types hold both assets and the rates of returns ratios are also consistent with (28). Alternatives (C) - (E) are not consistent with (28).

(C) in the Lemma assumes  $m = 0$  but the rates of return ratios implies that if (28) is satisfied, type 1 holds both assets (note that we do not allow the case in which the agent is indifferent between the two assets and choose to specialize). (D) in the Lemma assumes  $m > n > 0$  but (28) implies in this case that type 2 specializes in bonds. Similarly (E) assumes that both  $n$  and  $m$  are strictly positive but (28) implies that type 1 specializes in bonds.

We have thus shown that only alternatives (A) and (B) in the Lemma are consistent with (28). These two alternatives correspond to alternatives (a) and (b) in Proposition 1.  $\square$

The effect of risk aversion on the demand for stocks:

Here I allow for a more general power utility function, and consider the workers' problem under the assumptions that the parameters  $(q, D, p_1, p_2)$  are given. Under risk aversion a type 2 agent may want to hedge against the possibility of bequest. This can be done by buying claims for delivery when  $\theta_t = 1$ . I therefore assume that as in the case of risk neutrality, type 2 buys contingent claims and type 1 buys claims that will be delivered regardless of the state of demand.

It is shown that risk aversion reduces the demand for stocks. I also show that if type 2 chooses to hold stocks, then type 1 chooses to specialize in bonds (a corner solution).

We can write the problem of a type 2 agent in the steady-state as:

$$(A9) \quad \max_{a,b} mU\left(\frac{b}{p_1} + \frac{a(q+D)}{p_2}\right) + (1-m)U\left(\frac{b+a(q+D)}{p_2}\right) \text{ s.t. } aq + b = x = W\lambda^2/\pi$$

The first order condition for an interior solution to this problem is:

$$(A10) \quad g(a; U, x) = m \frac{U'(C_1^2(a))}{U'(C_2^2(a))} \left( \frac{q+D}{p_2} - \frac{q}{p_1} \right) + (1-m) \left( \frac{D}{p_2} \right) = 0$$

$$\text{where } C_1^2(a) = \frac{x - qa}{p_1} + \frac{a(q+D)}{p_2} \text{ and } C_2^2(a) = \frac{x + aD}{p_2}.$$

The first order condition (A10) can be satisfied only if:

$$(A11) \quad \frac{q+D}{p_2} - \frac{q}{p_1} < 0,$$

and this is therefore assumed. Under (A11),  $C_1^2(a)$  is a decreasing function and  $C_2^2(a)$  is an increasing function. Assuming  $U'' \leq 0$ ,  $g(a; U, x)$  is decreasing in  $a$ .

I now assume:  $U(C) = \frac{C^\gamma}{\gamma}$ . In this case  $\gamma - 1 \geq 0$  is the coefficient of relative risk

aversion. Using (33) and assuming  $b > 0$ , we get:

$$(A12) \quad R(a;x) = \frac{C_1^2(a)}{C_2^2(a)} = \frac{1}{x+aD} \left( \frac{x-qa}{\pi} + a(q+D) \right) > 1.$$

Note that under (A11) and (33),  $R(a;x)$  is strictly decreasing in  $a$  and  $R(0,x) = \frac{1}{\pi}$  for all

$x$ . I now define:

$$(A13) \quad g(a;\gamma,x) = m(R(a;x))^{\gamma-1} \left( \frac{q+D}{p_2} - \frac{q}{p_1} \right) + (1-m) \left( \frac{D}{p_2} \right)$$

Since  $R(a;x) > 1$  and  $\gamma-1 \geq 0$ ,  $g(a;\gamma,x)$  is decreasing in  $\gamma$ . It follows that an increase in the risk aversion parameter leads to a decrease in the demand for stocks. Figure A1 illustrates this point. When  $\gamma = 2$ , type 2 will choose  $a(2)$  which is greater than his choice for the case  $\gamma = 3$ , denoted by  $a(3)$ .

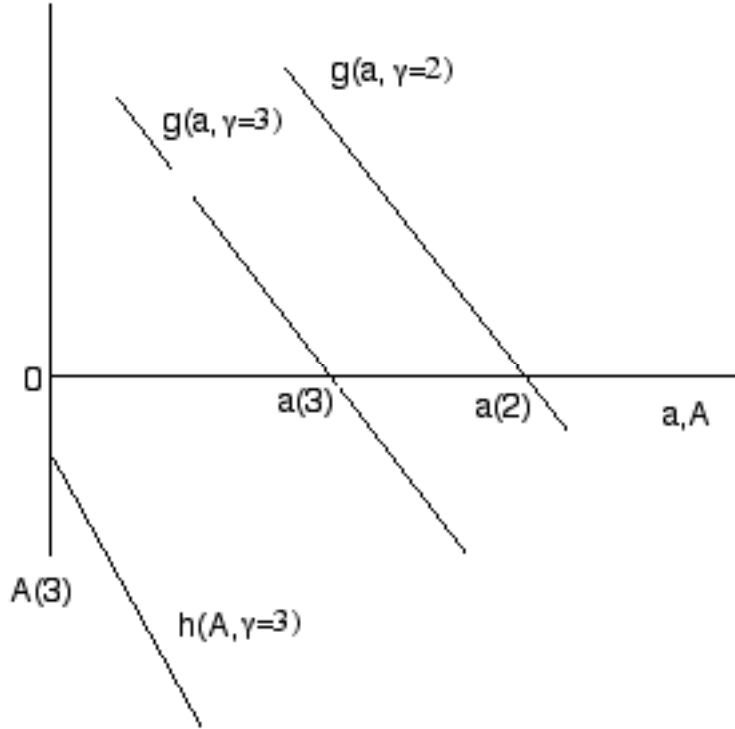


Figure A1

A type 1 agent solves the following problem:

$$(A14) \quad \max_{A,B} \pi \left[ mU \left( \frac{B}{p_1} + \frac{A(q+D)}{p_2} \right) + (1-m)U \left( \frac{B+A(q+D)}{p_2} \right) \right] + (1-\pi)U \left( \frac{B}{p_1} + \frac{A(q+D)}{p_2} \right)$$



$$\text{s.t. } B + Aq = x = W\lambda^1.$$

Assuming the power utility function, the first order condition for this problem is:

$$(A15) \quad h(A; \gamma, x) = (1 - \pi + \pi m)(R(A; x))^{\gamma-1} \left( \frac{-q}{p_1} + \frac{(q+D)}{p_2} \right) + \pi(1-m) \left( \frac{D}{p_2} \right) \leq 0$$

with equality if  $A > 0$ .

Note that the coefficient on the negative term (A11) is now larger and therefore:

$h(A; \gamma, x) < g(a; \gamma, x)$  for all  $\gamma$  and  $x$ . Therefore  $A(\gamma, x) < a(\gamma, x)$  for all  $\gamma$  and  $x$ , where  $A(\gamma, x)$  is the demand of type 1 and  $a(\gamma, x)$  is the demand of type 2.

Since  $R(0; x) = 1/\pi$  for all  $x$  and since  $R(a)$  is strictly decreasing, it follows that  $h(0; \gamma, x^1) < g(a; \gamma, x^2)$  for all  $(a > 0, x^1, x^2)$ . This implies that if  $g(a; \gamma, x^2) = 0$  then  $h(0; \gamma, x^1) < 0$ . Since the solution to each problem is unique, it follows that the only equilibrium solution possible is with  $a = 1$  and  $A = 0$ . This result does not require the assumption that the income of both types is the same. In Figure A1, the demand of type 1 agent when  $\gamma = 3$  is  $A(3) = 0$ .

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