IMPLEMENTING THE FRIEDMAN RULE BY A GOVERNMENT LOAN PROGRAM: AN OVERLAPPING GENERATIONS MODEL

by

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PROGRAM: AN OVERLAPPING GENERATIONS MODEL

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The welfare gains from adopting a zero nominal interest policy depend on the implementation details. Here I argue that implementing the Friedman rule by a government loan program may be better than implementing it by collecting taxes, even when lump sum taxes are possible. The government loan program will crowd out lending and borrowing and other money substitutes. Since money can be costlessly created the resources spent on creating money substitutes are a "social waste". Moving from an economy with strictly positive nominal interest rate to an economy with zero nominal interest rate will increase consumption by the amount of resources spent on lending and borrowing. But in general welfare will increase by more than that because consumption smoothing is better under zero nominal interest rate.

1. INTRODUCTION

There is a difference between the private and the social point of views about the cost of defaults. From a private bank point of view, defaults on loans are a loss of money and therefore the bank invests resources in checking collaterals and persecuting borrowers who do not pay their loans. From the social point of view defaults on loans are transfers of money from lenders to borrowers. A government that gives loans to young people may allow for the possibility that people with low permanent income will not pay the loan in full and in a sense will "partially default". The government may view these "partial defaults" as part of its income redistribution policy. Thus from the private point of view, defaults are costly but from the social point of view they are a way of redistributing income and may be regarded as beneficial. Since there is a difference between the private and the social point of view there may be grounds for a government loan program.

Here I explore the connection between this argument for a government loan program and the Friedman rule. This connection can be stated in the general terms used by Friedman (1969) who argued that since the cost of creating real balances is zero, the optimal policy is to satiate individuals with liquidity. In an overlapping generations model with imperfect bonds market, a government loan program will allow young agents to have sufficient liquidity.

A more explicit model is in Aiyagari, Braun and Eckstein (ABE, 1998). They use a cash-in-advance model of the type studied by Lucas and Stokey (1987) to argue that a reduction in the nominal interest rate will increase the amount of real balances and reduce the amount of credit. In their model there are cash and credit goods: cash goods are sold only for cash while credit goods are sold for costly credit. The distinction between credit good and cash good is endogenous. When the nominal interest rate declines more goods are bought for cash and resources used for the production of credit are saved. When the

nominal interest rate is zero, all goods are "cash goods" and no resources are spent for creating credit goods. ABE show that in their model the welfare cost of inflation is the area under the demand for money curve as in Baily (1956) and Lucas (2000).

Following Lucas and Stokey (1983, 1987) ABE assume that transactions in the bonds market are costless. Thus a reduction in the nominal interest reduces the role of credit within the period but does not reduce the role of credit in smoothing consumption between periods. Here I allow money to replace between periods credit, in the same way that it replaces within period credit.

I use an overlapping generations, Baumol-Tobin type model. There are two assets: Indexed private bonds and real balances (money). Private loan contracts are incomplete long-term contracts that allows for renegotiation as in Hart and Moore (1988). The standard incomplete loan contract is a typical savings account type contract: Balances accumulate interest automatically until a trip to the bank is made and the renegotiation option is exercised. There is a fixed cost of renegotiation: The cost of a trip to the bank. Real balances are modeled here as a potentially better contract that allows for costless renegotiation: Changes in the evolution of real balances do not require a trip to the bank. We thus assume that the government has a better renegotiation technology. Since smoothing consumption requires changes in asset holdings, money with its costless renegotiation feature has an advantage in smoothing consumption. This advantage is often not fully realized because there is a rate of return differential between the two assets. It will be fully realized in Friedman rule equilibrium with no rate of return differential.

Our private bonds and money are therefore not equivalent in the sense of Kocherlakota (2007). In his paper Kocherlakota assumes that private and public sectors have the same collection power and that money has no distinct transaction advantage over bonds. He concludes that "In reality, the collection power of the private and public sector may well differ, and money almost certainly does provide liquidity benefits that

loans do not". In view of the equivalence theorem in his paper he argues that the issue of collection power is especially important for understanding the impact of government financing decisions. Here I relax both assumptions. I argue that the government has a better "collection technology" and a better "renegotiation technology" that make money more liquid than bonds. The two advantages of the government over the private sector are related. Because of poor collection technology, the bank has an incentive to check collaterals and the ability of the borrower to pay. I will argue that this is a large part of the cost of the trip to the bank that must occur whenever an agent wants to get an additional loan.

Private bonds here are analogous to gold that require resources to dig it, store it and carry it. Will money crowd out private bonds in a Friedman-rule zero nominal interest rate equilibrium? The answer is in the positive when restricting attention to a steady-state equilibrium in an infinite horizon economy: There is no reason to go to the bank if you can write checks or pay in debit cards everywhere and your money in the checking account earns the same interest as bonds.

This is not always the case in a more realistic overlapping generations model. If we reduce the money supply by levying taxes, young agents who reach adulthood with no money may want to borrow money to pay the taxes and to smooth consumption. This requires trips to the bank (both by lenders and borrowers) and such trips are wasteful from the social point of view. I argue that implementing the Friedman rule by a government loan program may solve the need of young agents to borrow money from private individuals and will therefore save resources that are spent on unnecessary trips to the bank.

The government can use its tax collection system to implement the loan program in the following way. It gives a loan to young adults. Each year the borrower makes a loan payment that depends on his current income. People who consistently make low income will not pay the loan in full: They will "partially default". We now increase

consumption taxes and use the revenues to finance "partial defaults" by "poor" people and a reduction in the top marginal income tax rates.

This is a move towards a proportional consumption tax system with a rebate, in the form of "partial default", to low permanent-income people. Altig, Auerbach, Kotlikoff, Smetters and Walliser (2001) estimate that a switch to a proportional consumption tax would increase the steady state income by 9%. But low-income individuals will suffer a significant loss as growth fails to compensate for the decline in tax progressivity. Castaneda, Diaz-Gimenez and Rios-Rull (1998) estimated the moving to a proportional tax system in the US will increase output by 4.4% at the expense of a large increase in wealth inequality. Here the "partial default" by poor people mitigates the adverse effects on wealth distribution.

In addition to facilitating the move toward a flat consumption tax, a government loan program will facilitate consumption smoothing and crowd out many activities in the banking, rental, insurance and (wholesale and retail) trade industries. These sectors will contract and free resources for socially beneficial activities.

There is a vast literature on the Friedman rule and the welfare cost of inflation. Maybe the closest to this paper are general equilibrium versions of the Baumol-Tobin model. A partial list includes Jovanovic (1982), Romer (1986), Chatterjee and Corbae (1992) and Alvarez, Atkeson and Kehoe (2002). These versions differ in important details and in the type of question asked. Here I focus on the implementation details of the Friedman rule. The literature on the tax implementation of the Friedman rule is of-course also relevant. This literature is concerned with the optimality of the Friedman rule when lump sum taxes are not possible and only distortive taxes are available. A partial list includes Phelps (1973), Kimbrough (1986), Correia and Teles (1996), Chari, Christiano and Kehoe (1996) and da Costa and Werning (2007). I argue here that a loan program implementation may lead to outcome that is better than a tax implementation of the Friedman rule even when lump sum taxes are possible. The discussion on the

government's collection technology uses the literature on tax evasion and administration costs surveyed by Slemrod and Yitzhaki (2002).

Since lending and borrowing are viewed here as money substitutes, I start by incorporating money substitutes in Baily's original framework. An explicit multi-period analysis will follow.

2. BAILY (1956) WITH MONEY SUBSTITUTES

The literature distinguishes between the "transaction" role of money and the "store of value" role of money. I now argue that when the nominal interest declines, money will crowd out substitutes for the "transaction" function of money. In the next section I discuss the "store of value" function.

Baily (1956) treats money as a "good" with zero cost of production and argues that the welfare cost of inflation is "a reasonably straightforward extension of the welfare cost of an excise tax".[1] He explicitly rules out substitutes for money by assuming that "Bank deposits are not used as money or are negligible in amount" (page 94). Here I argue that once we allow for substitutes, there is no clear relationship between the welfare cost and the area under the demand for money curve.[2] To simplify, I use an analogous single period "real" (non-monetary) economy. A monetary version is in Eden (2005, chapter 3.3).

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[1] Baily analyzed the welfare gains from reducing the steady state inflation rate to zero and the steady state nominal interest rate to the level of the real interest rate. Lucas (2000) argued that much of the welfare gains are obtained when reducing the nominal interest rate from the real interest rate level to zero (and reducing the inflation rate from zero to the appropriate negative level). There are important differences between the two and it may be useful distinguish between the "welfare cost of inflation" and "the welfare cost of the nominal interest rate". Here I focus on the nominal interest concept.

[2] ABE assume that there is no substitution between goods and this may explain the difference in results.

The economy lasts for one period. There are two goods: X and Y. We may think of X as "liquidity services" and Y as consumption good (corn). The representative agent has a large endowment of Y. His utility function is: $U(x) + y$, where lower case letters denote quantities consumed. It is assumed that there is a satiation level z such that $U'(z) = 0$, $U'(x) > 0$ for $x < z$, $U'(x) < 0$ for $x > z$ and $U'' < 0$ everywhere.

The representative agent can produce "liquidity services" X by using money and paying an inflation tax of τ units of Y per unit of X. He can evade the tax payment by using "money substitutes". As in ABE the marginal cost of producing "liquidity services" by the credit technology is increasing with the amount produced. It is assumed that the cost of producing x units of "liquidity services" by the "money substitutes" (credit) technology is $C(x)$ units of Y where the cost function $C(x)$ has standard properties: $C(0) = 0$, $C' > 0$ and $C'' > 0$.

I borrow the terms from the tax evasion literature and use "legal production" for the money technology and "illegal (black market) production" for the "money substitute" technology. I use x_1 (x_2) to denote the quantity of X produced legally (illegally) and $x = x_1 + x_2$ to denote total production. The representative agent gets a lump sum transfer of g units of Y. This lump sum tax is equal to the per agent tax revenues. Since there are many identical agents in the economy, the individual agent cannot influence the size of g.

The representative agent solves the following maximization problem:

(1) $$\max_{x_1, x_2} U(x_1 + x_2) - C(x_2) - \tau x_1 + g$$

Assuming $x_2 > 0$, the first order conditions are:

(2) $$C'(x_2) = U'(x = x_1 + x_2) \le \tau \text{ with equality if } x_1 > 0.$$

Figure 1 illustrates, using $x(\tau)$ and $x_2(\tau)$ to denote the solution to (2). The Figure can be used to show that the total amount produced $x(\tau)$ is a decreasing function but the amount produced illegally $x_2(\tau)$ is an increasing function. It follows that the amount produced legally $x_1(\tau) = x(\tau) - x_2(\tau)$ is a decreasing function. When $\tau \geq \tau^*$ and the tax is above the intersection of the two curves, the amount produced is $x(\tau^*) = x_2(\tau^*)$ and the amount produced legally is zero.
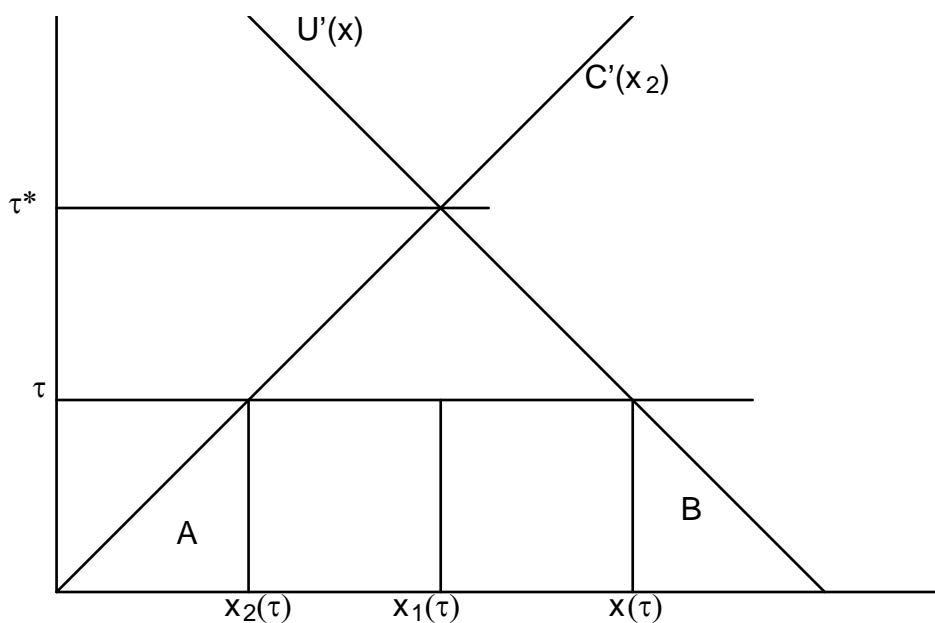

Figure 1: The solution to $x(\tau)$, $x_2(\tau)$ and $x_1(\tau) = x(\tau) - x_2(\tau)$

The welfare cost of the tax is:

$$(3) \qquad\qquad U(z) - U\big(x(\tau)\big) + C\big(x_2(\tau)\big)$$

The first component, $U(z) - U\big(x(\tau)\big)$, is the area B in Figure 1. The second cost saving component, $C\big(x_2(\tau)\big)$, is area A. It follows that:

<u>Claim 1</u>: The welfare cost of inflation is greater than the resources wasted on creating money substitutes (area A in Figure 1).

How is the welfare cost related to the area under the demand for money curve? To answer this question we add to Figure 1 the demand for "legal production", $m^d = x_1(\tau)$, that may be estimated by an econometrician who observes many points in the (x_1, τ) plane. The area under the demand for money (legal production) curve in Figure 2 is: $b + c$. The true welfare cost is the area: $a + c + d$.
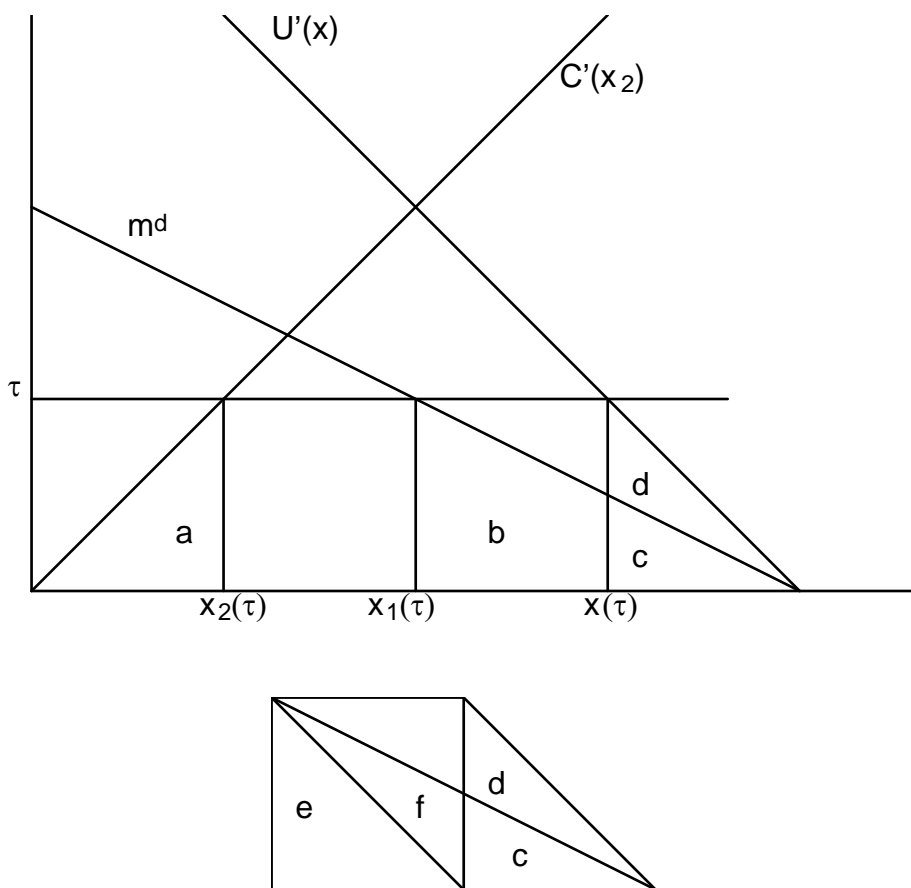

Figure 2: The true welfare cost $(a + c + d)$ and the area under the demand for money curve $(b + c)$. The lower graph establishes: $b - (d + c) = e + f - (d + c) = e - c$. When $e = a$, the true welfare cost is larger than the area under the demand for money curve.

With the help of the lower graph in Figure 2 we show that the area under the demand for money curve may be less than the true welfare loss. The area $e + f$ in the lower graph is equal to the area b in the main graph. The areas d and c are the same in both graphs. Since $d = f$ and $a = e$, it follows that the difference between the true welfare loss and the area under the demand curve is c. Thus in this case, the area under the demand for money curve is less than the true welfare loss. It is also possible to draw the marginal cost curve in a way that will reverse this result by making the area a small. Thus the area under the demand for money curve may be larger or smaller than the true welfare cost.

Claim 1 can be used for measuring a lower bound of the welfare cost. Aiyagari, Braun and Eckstein (1998) estimated the cost of creating demand deposits and credit cards and argued that this is about 0.5% of GDP. Humphrey, Pulley, and Vesala (2000) estimate the social cost of a nation's payment system as 3% of its GDP. They argue that from the social point of view, debit card transactions are much cheaper than checks transactions and more generally electronic payment is cheaper than paper based payment (excluding cash which is assumed to have zero cost). But typically agents do not see the true social cost of making a payment because most banks do not charge for transactions if a minimum balance is held. In addition agents may prefer to make the payment by checks rather than by debit card because the floating period for checks is longer. It seems that the discrepancy between the social and the private cost will be eliminated once the Friedman rule of zero nominal interest rate is adopted. In a world with zero nominal interest rates banks are expected to charge per transaction rather than requiring minimum balances and individuals will not care about the length of the float period. We may therefore expect that the cost of the payment system will go down with the nominal interest rate.

Bergman, Gulborg and Segendorf (2007) found that the cost of cash and card payments in Sweeden are about 0.4% of GDP. They found that credit card payments are

expensive relative to debit card payments. We may expect that at a zero nominal interest rate, debit cards will replace credit cards.

3. THE MODEL

I now turn to the consumption-smoothing (asset) motive for holding money. I assume a single good overlapping generations economy. A new generation is born each period. Each generation lives for T+1 periods. I start with the case in which all agents are identical. The representative agent consumes and receives endowment income starting from the second period of his life: in periods $t = 1,...,T$. The income at age t is Y_t units of the consumption good. I focus on the steady state in which only the age of the agent matters and therefore the calendar time at which the agent was born is suppressed.

There are two assets: real balances (indexed money) and indexed bonds. A unit of real balances represents an obligation of the government to exchange it for one unit of consumption.[3] An indexed bond represents an obligation by private agents to exchange it for a unit of consumption or a unit of real balances.

It may help to think in terms of two banks. There is a government run bank that offers interest-bearing checking (debit card) accounts. The interest on these checking accounts may be negative and is roughly equal to the negative of the rate of inflation in a fully articulated monetary model. There is also a private bank that offers interest-bearing saving accounts. The real interest rate on money (checking accounts) is r_m and the real interest rate on indexed bonds (saving accounts) is $r \geq r_m$.

The representative agent may borrow up to L units of real balances from the government (or the central bank) in the first period of his life. The amount that is actually

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[3] This assumption is made for simplicity. It is different from the assumption used by standard models of fiat money. But our assumption is not unlike the obligation of the government to accept money as tax payment.

borrowed is denoted by $d \leq L$. The agent pays the principal at the end of his life and

during his life he pays interest of rd per period.

It costs the government $\alpha_g > 0$ units to process a non-standard loan choice. It is

possible to save this cost by electing the default (standard) option: $d = L$. In this case, the

maximum amount of the loan is deposited directly to the agent's checking account. The

cost of processing the loan is: $tc(d,L) = \alpha_g > 0$ if $d \neq L$ and $tc(d,L) = 0$ otherwise. The

amount of real balances in the agent's account at age zero is:

(4) $$m_0 = d - tc(d,L)$$

As in the Baumol-Tobin model, there is also a fixed cost of α units for going to

the private bank (the bond market). I use b_t to denote the real value of bonds held at age

t. When the agent does not go to the bank at age t his bond holdings will be

$b_t = (1 + r)b_{t-1}$. This is the default option. The agent may change the evolution of the

bonds balances if he goes to the bank. I thus assume that the cost α occurs whenever

$b_t \neq (1 + r)b_{t-1}$ and therefore at age t the agent pays the cost:

(5) $\quad TC_t = TC(b_{t-1}, b_t) = \alpha$ if $b_t \neq (1 + r)b_{t-1}$ and $TC_t = 0$ otherwise.

Several assumptions are implicit in (5). A trip to the bank will not be needed of-

course in a world in which private contracts can be perfectly enforced and agents can pay

in personal IOUs. We may therefore think of banks as specializing in enforcing loan

contracts. An important part of the "trip to the bank" cost is the cost of checking

collaterals and approving new loans. Whenever someone gets an additional loan, there is

another person who increases the amount in his savings account. In (5) we assume that

the fixed cost of making a new loan is distributed equally between the lenders and the

borrower that changed the evolution of their bonds holdings. The main results will not change if only the borrowers pay the cost.

The specification (5) assumes that contracts are incomplete. If complete contracts were possible the agent would have written a detailed instruction to the bank that specifies how much money to transfer from his savings account to his checking account at any given date and will never go to the bank. One reason for contract incompleteness is the need to check collaterals when new loans are made. But the most obvious reason is the complexity of complete contracts.

The agent starts period t with $(1+r_m)m_{t-1}$ units of real balances and $(1+r)b_{t-1}$ units of indexed bonds. He gets his endowment income and a transfer from the government of g_t units. He pays interest on the initial loan and if he goes to the bank he pays the fixed cost. He chooses consumption, c_t and his end of trade portfolio, b_t and m_t subject to the following constraint:

(6)
$$b_t + m_t = Y_t + (1+r_m)m_{t-1} + (1+r)b_{t-1} - rd - c_t - TC_t + g_t$$

At the end of his life the agent returns the principal:

(7)
$$m_T = d.$$

The representative agent's utility function is given by the strictly monotone and strictly quasi-concave function: $U(c_1,...,c_T)$. His maximization problem is:

(8)
$$\max_{d,c_t,b_t,m_t} U(c_1,...,c_T) \text{ s.t. (4)-(7) and } b_0 = b_T = 0; \ c_t, m_t \geq 0$$

The population grows at the rate n. The number of age t agents is a fraction γ_t of the number of age 1 agents. Thus,

(9)
$$\gamma_0 = 1+n; \quad \gamma_1 = 1; \quad \gamma_{t-1} = (1+n)\gamma_t$$

The market clearing conditions are:

(10)
$$\sum_{t=1}^{T} \gamma_t c_t = -\gamma_0 tc(d,L) + \sum_{t=1}^{T} \gamma_t (Y_t - TC_t)$$

(11)
$$\sum_{t=1}^{T} \gamma_t b_t = 0$$

Equation (10) requires the clearing of the goods market and equation (11) requires the clearing of the private bonds market. Here there is no "money market": Under the market clearing conditions, the government may play the role of a central clearing institution and deliver on its promise to exchange a unit of real balances for a unit of the consumption good. A Walrasian auctioneer can do it as well.

Note also that here we do not have a cash-in-advance constraint. A cash-in-advance constraint assumes that during a period of a given length, the trader cannot use his income (or part of it) to finance consumption. Here an agent that wants to consume his income in each period can do it without holding money. Indeed if everyone wants to consume his income, money will not be used. See Jovanovic (1982) for further discussion.

A steady state equilibrium is a vector $(d,c_1,...,c_T,b_1,...,b_T,m_1,...,m_T;g_1,...,g_T;r,r_m)$ such that (a) given $(g_1,...,g_T;r,r_m)$, the magnitudes $(d,c_1,...,c_T,b_1,...,b_T,m_1,...,m_T)$ solve (8) and (b) the market clearing conditions (10) - (11) are satisfied.

To evaluate welfare I consider the problem of a social planner who can distribute at each point in time x units of consumption and wish to maximize the utility of the representative consumer in the steady state. The planner's problem is:

(12) $$V(x) = \max_{c_t} U(c_1,...,c_T) \text{ s.t. } \sum_{t=1}^{T} \gamma_t c_t = x,$$

where $V(x)$ is the maximum steady state utility that the planner can achieve.

I now assume that L is sufficiently large and show the following Proposition.

Proposition 1: There exists equilibrium with $r = r_m = n$. In this equilibrium: (a) $d = L$, (b) credit is not used ($b_t = 0$ for all t); (c) $\sum_{t=1}^{T}(1+n)^{1-t} g_t = 0$ and (d) the utility of the representative agent is: $V\left(Y = \sum_{t=1}^{T} \gamma_t Y_t\right)$.

In the proposed equilibrium the real rate of return on bonds is the same as the real rate of return on money and therefore the nominal interest rate is zero. I refer to this equilibrium as the "Friedman rule equilibrium". Part (a) says that agents will choose the default option in the government loan program. Part (b) says that agents will save the trips to the private bank and will hold no bonds. Part (c) says that the present value of the transfer payments must be zero. Part (d) says that the Friedman rule allocation is efficient.

Welfare cost: I now turn to compare welfare between two equilibria: The "initial" equilibrium with strictly positive nominal interest rate and the Friedman rule equilibrium with zero nominal interest rate. To simplify I assume that in the initial equilibrium the government chooses the limit L to satisfy the demand for loan: $d = L$.

I use $(c_1^i,...,c_T^i;b_1^i,...,b_T^i)$ to denote the consumption and bonds holdings in the "initial" equilibrium and $(c_1^F,...,c_T^F)$ to denote the consumption in the Friedman rule equilibrium. It is assumed that $b_t^i \neq 0$ for some t so that in the initial equilibrium bonds are used. Part (b) in Proposition 1 and the market clearing condition (10) imply that under the Friedman rule aggregate consumption at time t is higher by aggregate spending on trading in the bonds market:

$$(13) \qquad \sum_{t=1}^{T}\gamma_t(c_t^F - c_t^i) = \sum_{t=1}^{T}\gamma_t TC(b_{t-1}^i,b_t^i) = TC^i$$

I now argue that in addition to increasing aggregate consumption by TC^i, the adoption of the Friedman rule will also improve consumption smoothing (the allocation of consumption over the life cycle).

I assume that as in the original Baumol-Tobin model, money may be used to smooth consumption between trips to the bank. In our discrete time formulation, money is used if the agent does not go every period to the bank and hold a strictly positive amount of money between trips to the bank. Formally, money is used in the initial equilibrium if there are ages h and k $(h < k)$ such that (a) the representative agent chooses to go to the bank at ages h and k; (b) he do not go to the bank at ages $h < t < k$ and therefore $b_t = (1+r)^{t-h}b_h$ for $h \le t < k$ and $b_k \neq (1+r)^{k-h}b_h$; (c) he holds a strictly positive amount of money at some point during the above period: There exists an age index $h < j < k$ such that $m_j > 0$. I use $Y = \sum_{t=1}^{T}\gamma_t Y_t$ and show the following Claim.

<u>Claim 2</u>: $U(c_1^i,...,c_T^i) \le V(Y - TC^i)$ with strict inequality if money is used in the initial equilibrium.

Part (d) in Proposition 1 and Claim 2 imply:

<u>Claim 3</u>: $U(c_1^F,...,c_T^F) - U(c_1^i,...,c_T^i) \geq V(Y) - V(Y - TC^i)$ with strict inequality if money is used in the initial equilibrium.

This says that from the planner's point of view, the utility gain from adopting the Friedman rule is greater than the utility gain from saving the resources spent on "going to the bank". Similarly to Claim 1, it implies that the saving of resources is a lower bound on the welfare gain.

<u>Examples</u>: To illustrate the working of the model, I now assume that $Y_t = 1$ when t is even and $Y_t = 0$ when t is odd. I assume the special case: $U(c_1,...,c_T) = \sum_{t=1}^{T} \beta^t u(c_t)$, where $u(c_t) = \ln(c_t), \beta = 0.96$ and $T = 10$. There is no population growth ($n = 0$) and $r_m \leq r = 0$. The limit on the initial loan, L, is not binding.

Assuming that money is always held, the first order conditions for the consumer's problem (8) require:

(14) $$u'(c_1) = (\beta R_m)^{j-1} u'(c_j),$$

where $R_m = 1 + r_m \leq 1$ is the gross real rate of return on money. To derive (14) note that if the agent reduces his consumption at $t = 1$ by "one unit" he will suffer a loss of utility equal to: $u'(c_1)$ utils. He can use the accumulated unit of real balance to increase his consumption at age j by $(R_m)^{j-1}$ units and derive a benefit of $(\beta R_m)^{j-1} u'(c_j)$ utils. Since at the optimum such a deviation does not increase the level of the objective function we must have that the pain is greater than the gain: $u'(c_1) \geq (\beta R_m)^{j-1} u'(c_j)$. When money is always held, the consumer can also increase consumption at $t = 1$ and reduce it at age j. This will lead to $u'(c_1) \leq (\beta R_m)^{j-1} u'(c_j)$. Both inequalities lead to (14).

For the log utility function (14) implies:

$$(15) \qquad\qquad c_j = (\beta R_m)^{j-1} c_1$$

I computed the equilibrium path of consumption and money holdings on Excel in the following way. I choose c_1 arbitrarily and then compute consumption at all ages according to (15). I keep guessing until the market clearing condition (10) is satisfied. I then work the financing. I start with a guess about the size of the initial loan and then use (6) to figure out the evolution of real balances assuming that the inflation tax payment $g_t = -r_m m_{t-1}$ is paid back to the consumer at age t as a lump sum. I search for the minimum size of the initial loan that satisfies: $m_t \geq 0$ for all t.

Figure 3 illustrates the Friedman equilibrium with $r = r_m = n = 0$. Since $\beta R_m < 1$, consumption declines with age. The consumer starts with $c_1 = 0.597$ and reduces his consumption until it reaches 0.4 at the end of his life. To finance it the consumer takes a 0.75 units loan from the government at date zero. The amount of money in his checking account fluctuates with income, but the moving two periods average declines until age 5 and then increases reaching 0.75 at the end of his life. At this point he pays his debt to the government.
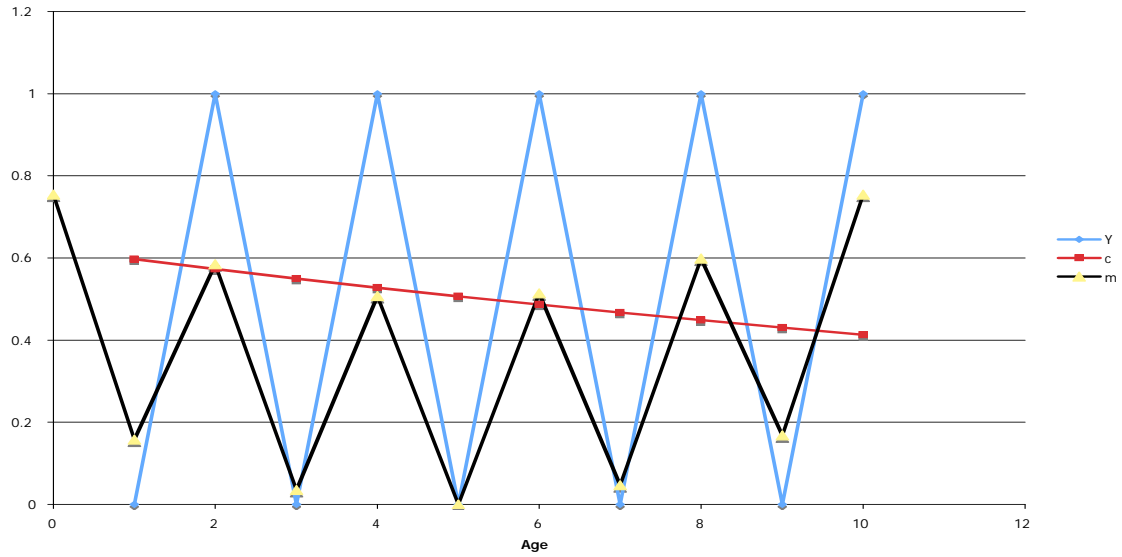
Figure 3: The evolution of income, consumption and real balances in the first best equilibrium; $\beta = 0.96$, $T = 10$, $R_m = R = 1$.

Figure 4 computes equilibrium for the case $R_m = 0.98$ assuming no trips to the bank. Consumption declines faster than under the Friedman rule. It starts at the level of 0.65 and ends at 0.37. It is financed by a 0.88 units loan.
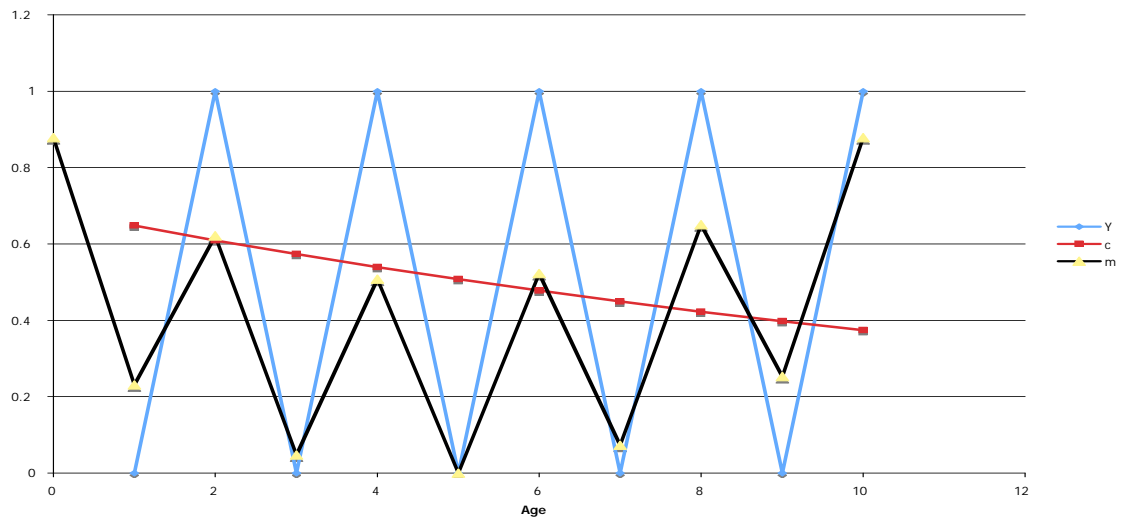

Figure 4: $\beta = 0.96$, $T = 10$, $R = 1$, $R_m = 0.98$.

To achieve the Friedman rule level of welfare we must increase consumption by 0.17% each period. This is a measure of the welfare cost of a nominal interest of 2% ($R_m = 0.98$). The welfare cost of a nominal interest of 5% ($R_m = 0.95$) is much larger and is equal to 1.07% of consumption.

I now allow trips to the bank at the cost of 0.1 per trip. To make such trips worthwhile, I assume that total lifetime income of 5 units is received at the end of the consumer's life in period 10. The consumer makes the trips to the bank to reduce inflation tax payments.

Figure 5 assumes $R_m = 0.98$ and two trips to the bank at age 5 and age 10. Total consumption is now 4.8 reflecting the cost of the trips to the bank. From the individual point of view, this cost is roughly equal to the reduction in the inflation tax payments. From the social point of view this is a waste of resources.

The consumer starts by borrowing from the government 2.276 units of real balances. He then borrows 2.165 units at age 5. The cost of the trips to the bank is close to 4% of consumption. To achieve the first best we need to increase consumption by about 4.3%.
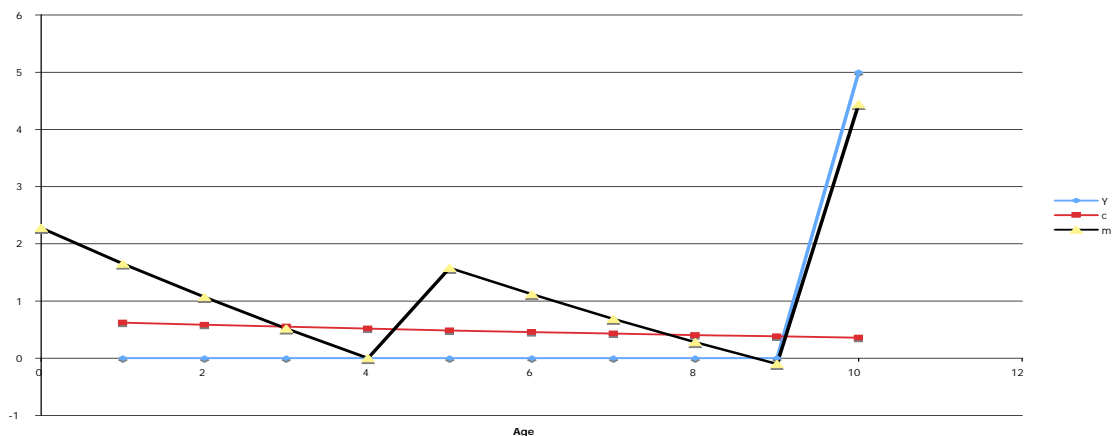

Figure 5: Income is received only at T; $R = 1$, $R_m = 0.98$; $d = 2.276$; At age 5 he borrows from the private bank 2.165 units; Cost of going to the bank = 0.1 per trip.

4. COLLECTION TECHNOLOGY

I now turn to the realistic case in which the government must pay administrative costs for collecting loan payments reflecting the cost of enforcing contracts and verifying income. This more realistic case is relevant for the question of whether the government has an advantage over the banking sector in collecting loan payments.

As was said in the introduction there is a different between the private and the social cost of default. From the private bank's point of view, bankruptcies are loss of money and therefore the bank spend resources on trying to minimize the probability of bankruptcy by verifying collaterals and the ability of the borrower to pay. From the social point of view, bankruptcies are transfers from borrowers to lenders. A government loan program may therefore save the resources spent on making private loans.

In addition, the government has a tax collection system in place and can audit income (see Holmstrom and Tirole [1998]). This suggests that we should consider the government loan program as part of the tax system. I start with the analysis of the potential effect of the loan program on the resources spent on auditing income by the government and the resources spent by agents on reporting (and underreporting) income.

As before, I assume that agents live for T+1 periods but now there are N types of agents per generation, indexed by: $h = 1,...,N$. One agent of each type is born in every period. Thus, there is no population growth and the distribution of the agents' characteristics is the same for all generations. As in the previous case, only the age matters. I will talk about agent (of type) h without specifying the date of his birth.

I distinguish between reported and actual income. Let $y_t^h = (Y_1^h,...,Y_t^h)$ denote actual income up to time t and let $y_t^h(\varepsilon^h) = (Y_1^h - \varepsilon_1^h,...,Y_t^h - \varepsilon_t^h)$ denote reported income up to time t, where $\varepsilon^h = (\varepsilon_1^h,...,\varepsilon_T^h) \geq 0$ is the vector of underreported magnitudes.

The government provides credit at all ages. At age $t \geq 0$ the agent can get a loan of Δ_t^h units subject to the constraint:

(16) $$\Delta_0^h \leq L_0; \quad \Delta_t^h \leq L_t\left(d_t^h, y_t^h(\varepsilon^h)\right),$$

where the limit $L_t \geq 0$ depends on the debt to the government at age t (d_t^h to be defined shortly) and on reported income history and $\Delta_t^h < 0$ is allowed. Note that since the government collects taxes (and audit income) it has information about past incomes.[4]

The default option is: $\Delta_t^h = L_t\left(d_t^h, y_t^h(\varepsilon^h)\right)$. If the agent chooses the default option the maximum amount, $\Delta_t^h = L_t\left(d_t^h, y_t^h(\varepsilon^h)\right)$, will be automatically deposited his checking account. A trip to the government loan office is necessary whenever this default option is not elected. The cost of a trip to the loan office is α_g.[5] At age t the agent pays the cost:

(17) $$tc(\Delta_t^h, L_t) = \alpha_g > 0 \text{ if } \Delta_t^h \neq L_t \text{ and } tc(\Delta_t^h, L_t) = 0 \text{ otherwise.}$$

The loan payments are contingent on the accumulated debt and reported income history. A t years old consumer who owes the government d_{t-1}^h units and reported $y_t^h(\varepsilon)$ will pay:

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[4] The formulation in (16) is less general than the mechanism design approach. This approach assumes that the individual declares his type subject to incentive compatibility constraints. It seems that declaring a type is rather complicated. The government can do it by asking you questions about your income and taste and this will require tedious and long forms. This approach is not consistent with our incomplete contract environment.

[5] The cost of a trip to the government loan office may be different from the cost of a trip to the bank. Both reflect the cost of deviating from the "default" option. But as was said before, deviating from the "default option" may require checking collaterals in the case of the bank and writing a different loan contract. In the case of the government this is literally a time and transportation cost.

(18)
$$p_t\left(d_{t-1}^h, y_t^h(\varepsilon^h)\right) \le d_{t-1}^h.$$

The debt to the government evolves according to:

(19)
$$d_0^h = \Delta_0^h; \quad d_t^h = (1+r)\left[d_{t-1}^h - p_t\left(d_{t-1}^h, y_t^h(\varepsilon)\right)\right] + \Delta_t^h$$

The payments are an automatic deduction from the individual's checking account. The net payments $p_t\left(d_{t-1}^h, y_t^h(\varepsilon)\right) + \Delta_{t-1}^h$ are more flexible because Δ_{t-1}^h are choice variables subject to the constraints in (16) and (17).

The government imposes income and consumption taxes. The consumption tax rate is $\kappa \ge 0$. The income tax payment depends on reported history and is given by:

(20)
$$\theta_t\left(y_t^h(\varepsilon)\right).$$

It may be convenient to keep tab on the present value of the tax payments to the government:

(21)
$$J_0^h = 0 \; ; \; J_t^h = (1+r)J_{t-1}^h + \theta_t\left(y_t^h(\varepsilon^h)\right)$$

It is assumed that there is a limit $\bar{J}$ to the amount of accumulated income tax payments: $\theta_t\left(y_t^h(\varepsilon^h)\right) \le \bar{J} - J_{t-1}^h$. This limit may be large and not binding.

There are costs of collecting taxes. Mayshar (1991) assumes that the taxpayer devotes effort to paying taxes and sheltering income. Alt (1983) argues that collecting consumption taxes from businesses is simpler than collecting taxes from individuals. This is especially true for value added tax because of the incentives for self-policing. Here I abstract from the cost of collecting consumption taxes and focus on the cost of collecting

income contingent (taxes and loan) payments from individuals, due to the cost of auditing income, reporting income and underreporting income.

I assume that the government spends e per period for auditing income. Given these expenditures, consumer h who chooses the underreporting vector ε^h spends $\{E_t^h(\varepsilon^h,e)\}_{t=1}^{T}$ units to shelter income and hire accountants. The cost $E_t^h(\varepsilon^h,e)$ may also reflect the cost of avoiding detection when cheating. I simplify the analysis by assuming that when the amount of resources spent is $E_t^h(\varepsilon^h,e)$ the probability of getting caught is zero. When the amount of resources spent is less than that the probability of getting caught is higher than zero and the penalty is prohibitive so that all agents choose to spend the required amount.

There is a cost of reporting income truthfully: $E_t^h(0,e) > 0$. The marginal cost of underreporting income (the additional cost of reporting income at age t by one unit less) is positive and increasing: $MC_t^h(\varepsilon^h,e) = \dfrac{\partial E_t^h(\varepsilon^h,e)}{\partial \varepsilon_t^h} > 0,\ \dfrac{\partial MC_t^h(\varepsilon^h,e)}{\partial \varepsilon_t^h} > 0$.

Reporting income is not required if the individual does not owe anything to the government. Formally, you do not have to report income if $I_t\left(d_{t-1}^h, J_{t-1}^h\right) = 0$, where the index function is defined by:

(22) $\qquad I_t\left(d_{t-1}^h, J_{t-1}^h\right) = 0$ if $J_{t-1}^h = \overline{J}$ and $d_{t-1}^h = 0$ and $I_t\left(d_{t-1}^h, J_{t-1}^h\right) = 1$ otherwise.

As before, the cost of private lending and borrowing is given by (5): the cost of a trip to the bank. The consumer's asset evolution equation when choosing the underreporting vector ε is:

(23) $\qquad b_t^h + m_t^h = Y_t^h + (1+r_m)\, m_{t-1}^h + (1+r)\, b_{t-1}^h + \Delta_t^h - \theta_t\left(y_t^h(\varepsilon^h)\right)$

$\qquad\qquad - (1+\kappa)\, c_t^h - TC(b_{t-1}^h, b_t^h) - tc(\Delta_t^h, L_t) - p_t\left(d_{t-1}^h, y_t^h(\varepsilon)\right) - I_t\left(d_{t-1}^h, J_{t-1}^h\right) E_t^h(\varepsilon^h,e)$

The consumer's problem is now:

(24) $$\max_{\Delta^h_t, c^h_t, m^h_t, b^h_t, \varepsilon^h_t} U(c^h_1,...,c^h_T) \text{ s.t. } (5), (16)\text{-}(23) \text{ and } c^h_t, m^h_t \geq 0.$$

In addition to the auditing expenses the government spends G units per period. The market clearing conditions are now:

(25) $$G + e + \sum_{h=1}^{N}\sum_{t=1}^{T} c^h_t = \sum_{h=1}^{N}\sum_{t=1}^{T}\left(Y^h_t - TC(b^h_{t-1}, b^h_t) - tc(\Delta^h_t, L_t) - I_t\left(d^h_{t-1}, J^h_{t-1}\right)E^h_t(\varepsilon^h, e)\right)$$

(26) $$\sum_{h=1}^{N}\sum_{t=1}^{T} b^h_t = 0$$

In equilibrium, consumers solve (24) and the market-clearing-conditions (25) and (26) are satisfied.

The analysis of the equilibrium choice of the reporting strategy ε^h and the government choice of the auditing expenditures, e is beyond the scope of this paper. Here I discuss the effect of the loan program on the choice of ε^h given e. I therefore assume that there exists a Friedman rule equilibrium with $r = r_m = n = 0$ and strictly positive reported incomes ($\varepsilon^h_t < Y^h_t$). I also assume that in this equilibrium there is a positive correlation between reported income and actual income.

From an accounting point of view, loans are on the expenses side and loan payments are on the revenues side. Using this convention, I show that the government budget must be balanced.

<u>Claim 4</u>: In a Friedman rule equilibrium with $r = r_m = n = 0$ we must have:

$$(27) \qquad G + e + \sum_{h=1}^{N} \sum_{t=0}^{T} \Delta_t^h = \sum_{h=1}^{N} \sum_{t=1}^{T} \left[\theta\big(y_t^h(\varepsilon^h) \big) + p_t(d_{t-1}^h, y_t^h) + \kappa c_t^h \right].$$

Note that unlike the market for private loans, the "market" for government loans does not have to clear: In general, $\sum_{h=1}^{N} \sum_{t=0}^{T} \Delta_t^h$ may be different from $\sum_{h=1}^{N} \sum_{t=1}^{T} p_t(d_{t-1}^h, y_t^h)$.

I now turn to characterize equilibrium. I evaluate the equilibrium outcome by the following planner's problem:

$$(28) \qquad V(x) = \max_{c_t^h} \sum_{h=1}^{N} \omega^h U^h(c_1^h, ..., c_T^h) \qquad \text{s.t. } G + \sum_{h=1}^{N} \sum_{t=1}^{T} c_t^h = x,$$

where $\omega^h > 0$ are the weights that the planner assigns to type h. I assume that L_t and $\overline{J}$ are sufficiently large so that agents always have to report income: $I_t\big(d_{t-1}^h, J_{t-1}^h \big) = 1$. Under these assumptions, I now show the following Proposition.

<u>Proposition 2</u>: (a) in a Friedman rule equilibrium with $r = r_m = n = 0$: (a) the consumer cannot do better than elect the default option $\Delta_t^h = L_t$ and the constraint $m_t^h \geq 0$ is not binding; (b) private credit is not used ($b_t = 0$ for all t); (c) We can write the budget constraints (23) in the following present value form:

$$(29) \qquad (1 + \kappa) \sum_{t=1}^{T} c_t^h = d_T^h + \sum_{t=1}^{T} \left[Y_t^h - \theta_t\big(y_t^h(\varepsilon^h) \big) - E_t^h(\varepsilon^h, e) \right];$$

(d) there are weights $\omega^h > 0$ such that welfare in equilibrium is:

$$V\left(-G - e + \sum_{h=1}^{N} \sum_{t=1}^{T} \left[Y_t^h - E_t^h(\varepsilon^h, e) \right] \right).$$

Part (a) says that the agents choose the default option. Part (b) says that the government loan program completely crowds out the operation of the private bank. Part (c) expresses the budget constraint in present value form. Part (d) says that the Friedman rule equilibrium is efficient given the resources spent on auditing and reporting income.

Given that the agent chooses $\Delta_t^h = L_t$, we can write the loan payments as a function of reported income only: $p_t\left(y_t^h(\varepsilon^h)\right)$. Writing the budget constraints in the present value form (29) allows for the following observations: (a) for "rich" people who pay their loan in full ($d_T^h = 0$) the loan program does not create incentives to underreport income; (b) for "poor" people who partially default ($d_T^h > 0$) the budget constraint (29) depends on the sequence of total payments: $\left\{P_t\left(y_t^h(\varepsilon^h)\right) = \theta_t\left(y_t^h(\varepsilon^h)\right) + p_t\left(y_t^h(\varepsilon^h)\right)\right\}_{t=1}^T$ and therefore changes in the functions $\theta_t\left(y_t^h(\varepsilon^h)\right)$ and $p_t\left(y_t^h(\varepsilon^h)\right)$ are neutral if they do not change the total payment function $P_t\left(y_t^h(\varepsilon^h)\right)$. In other words relatively poor people, who do not pay their loan in full will treat loan payments as income tax.

Consumption tax with "loan defaults": To economize on the collection costs of income contingent payments, $e + \sum_{h=1}^N \sum_{t=1}^T I_t\left(d_{t-1}^h, J_{t-1}^h\right) E_t^h(\varepsilon^h, e)$, without an adverse effect on wealth distribution, we may use consumption tax with rebate to "poor" people in the form of "loan defaults": $d_T^h > 0$ for some h.

I now consider an example in which a consumption tax is used both to finance "partial default" by the "poor" and a reduction in the income tax paid by the "rich". In this example there is no adverse wealth redistribution effect.

Two types of agents are born in each period: "poor" and "rich". I assume $r = r_m = 0$, $\Delta_0^P = \Delta_0^R = L_0 = L$ and $\Delta_t^h = L_t = 0$ for $t > 0$, where the superscripts P and R denotes "poor" and "rich". Thus the government makes loans at $t = 0$ only. The loan is equal to half of the total earnings of the poor. Loan payments are a flat rate of 25% of current income so that the "poor" do not pay it in full. The rich pay their loan in full:

(30)
$$L = 0.5 \sum_{t=1}^{T} Y_t^p < 0.25 \sum_{t=1}^{T} Y_t^R.$$

A consumption tax of 25% is used to finance the "partial default" by the "poor" and to finance a reduction in the top marginal income tax rate that is paid by the "rich".

The loan program has no effect on the distribution of wealth: The consumption tax paid by the "poor" finances their "partial default" and the consumption tax paid by the "rich" finances the reduction in their income tax payments. I now turn to compute a lower bound on the size of the loan L that can be financed by a 25% consumption tax rate.

In the IRS web site I find that in 2003 the average tax rate for the "poor" (bottom 50% of the income distribution) was only 3% and their adjusted gross income was: $Y^P = 879,735$ million dollars. In general we may expect that the lifetime earnings of the bottom 50% is larger than the earnings of the bottom 50% in any given year: $\sum_{t=1}^{T} Y_t^p > Y^p$.

To illustrate this point I assume that agents work for two periods only. Poor agents earn 10 in the first period and 20 in the second period of their life. Rich agents earn 10 and 30. There are 4 agents living in any given period (young-poor, young-rich, old-poor, old-rich). The young are at the bottom 50% of the income distribution and they make a total of 20. But the total lifetime earnings of the poor is 30.

To calculate a lower bound on the size of the loan I assume that Y^P is a proxy for $\sum_{t=1}^{T} Y_t^p$.

I assume that over the life cycle consumption of the poor is roughly equal to their income. In 2003 the number of 20 years old was: $N = 4,118,942$.[6] Assuming that half of those are "poor", a lower bound on the amount of the loan per 20 years old is:

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[6] U.S Census Bureau, International data base. Table 094 (Midyear Population by age and sex) www.census.gov/cgi-bin/idbagg. We obtained the number of 20 years old by dividing the 20-24 category by 5.

$$L = \frac{0.5Y^p}{(\frac{1}{2})N} = 213{,}582 \text{ dollars that is about } 222{,}000 \text{ in terms of 2007 dollars.}$$

A more realistic example may assume three types of agents: A quarter of the population earns consistently above the median; A quarter earns consistently below the median and half are 50% of the time below the median and 50% of the time above the median. Assume further that only those who are consistently below the median do not pay their loan. In this case only a quarter of the population will "partially default" on their loan and the necessary consumption tax is only 12.5%.

6. DISCUSSION

This paper leaves many questions unanswered. I now discuss some of the more obvious ones.

Design of the loan limits: A full characterization of the socially optimal loan contract is not attempted here. It seems that at $t > 0$, the loan limits $L_t\left(d_t^h; y_t^h(\varepsilon^h)\right)$ should be monotonically increasing in reported income: $y_t^h(\varepsilon^h)$. This reduces the incentives to under report income (for agents who are liquidity constrained) and may lower the amount of resources spent on underreporting. In addition, high reported income is likely to predict high future reported income. Therefore people with high reported income are more likely to pay their loan in full.

The effect of the loan program on labor supply: When the loan limits are increasing in reported income then the loan program may increase the labor supply of highly productive people who are liquidity constrained. To make this argument I start with the case in which L_t does not depend on reported income and consider the case where the agent plans to report sufficiently high income and to pay his loan in full. In this case, the

present value of the loan payments will not increase if the agent increases his labor supply and reports the additional income. There are also no first order wealth effects because the present value of the loan payments is equal to the amount of the loan and therefore the loan program is likely to have no effect on the labor supply choice of the agent. When L_t increases in reported income, productive people who are liquidity constrained will actually work more after the adoption of the loan program, because high income allows for larger loans in the future. The effect of a monotonically increasing limit function L_t on labor supply is not unlike the effect of the desire to accumulate assets that can be served as collaterals in the analysis of Campbell and Hercowitz (2004).

Those who do not pay their loan in full will treat the loan payments as income tax payments. In the absence of a change in productivity, this increase in the marginal tax rate and wealth will reduce the labor supply of the "poor". But the adoption of the Friedman rule is likely to increase labor productivity. Some, for example, will use the loan to get higher education or to accumulate other types of human capital and some may buy a more reliable car and will come to work on time.

Time spent on executing transactions is expected to go down with the adoption of the Friedman rule. Agents will therefore have more time to allocate between socially productive labor and leisure. This works in the direction of increasing labor supply.

For these reasons the effect of the loan program on the labor supply of the "poor" cannot be ascertained. The effect on the labor supply of the rich is likely to be positive because of the increase in productivity, the increase in the time available for socially productive activities and the incentive provided by loan limits functions that are increasing in reported income.

Cap on accumulated income tax payments: Relying on consumption tax will minimize the cost of auditing and reporting income. But to minimize the distortion in the labor market one may choose income tax with a relatively low cap $\overline{J}$ in addition to the

consumption tax. Under this scheme an individual will stop paying income tax when his accumulated income tax payments has reached $\bar{J}$. This type of income tax will not distort the labor supply decisions of relatively rich people who plan to reach the cap because for those people, an increase in earnings does not change the present value of income tax payments. By itself, a relatively small cap $\bar{J}$ is regressive. But it will allow for the increase in loan limits and poor agents will get a rebate in the forms of "loan defaults".

Business loans: Evans and Jovanovic (1989) found that wealthier people are more likely to become entrepreneurs because they are less liquidity constrained. This is the apparent rationale used by the U.S. Small Business Administration to provide loans and loan guarantees to small business for start-up and expansion. Similar programs are present in other countries. Our loan program relaxes liquidity constraints and may allow for poor people to become entrepreneurs. It may also incorporate or substitute existing programs aimed at facilitating start-ups.

The literature that discusses the moral hazard and adverse selection problems in the market for loans (Stiglitz and Weiss [1981] and Holmstrom and Tirole [1997, 1998] for example) suggests that increasing the fraction of the project that can be self-financed mitigates these problems and is likely to be beneficial from the social point of view. This suggests that the possibility of using part of the government loan for a business project rather than for consumption smoothing, will mitigate the moral hazard and adverse selection problems in the market for business loans.

On the other hand a government loan program that allows for "partial default" may create incentives to undertake projects with negative Expected Net Present Value. This is because the government takes part of the losses: If as a result of failure the permanent income of the entrepreneur drops, the entrepreneur may not pay part of his loan to the government. This problem is also present when making private loans that are not fully collateralized as in Stiglitz and Weiss (1981). It suggests that the government

should limit the amount of the loan and spread it over the life cycle so that individuals will not take large bets on their permanent income.

<u>Social security in reverse</u>: The main objective of the social security system is to reduce the poverty level among the elderly. It has the following elements: (a) you pay during the working age and get the benefits in retirement; (b) the system transfers resources from the "rich" to the "poor". The objective of the loan program discussed here is to facilitate consumption smoothing. A loan program that allows for "partial default" may transfer resources from the "rich" to the "poor" but here you get the benefits at a young age and pay later.

It is sometimes argued that we need social security because many people lack "self-control". The lack of self-control is certainly a problem for children. Traditional economics assumes that all children become adults and once they become adults they gain self-control. Behavioral economics argues that some people never become adults in this sense.

It seems that self-control is endogenous to some degree. Parents that are over protective will tend to raise children that are less independent. Similarly, governments that are over protective may encourage myopic behavior.

The loan program discussed here is likely to encourage behavior that looks like "self control". For example, in the present system a young adult who drives an expensive car transmits a signal about the wealth of his parents or his own ability to make money. This signal may help him with the other sex. Once everyone gets a loan, such a behavior may signal the lack of self-control.

In any case, self-control is likely to be a problem and the loan program should address these problems by making loans in various stages (say, a certain amount at age 20, an additional amount at age 30 and so on) and by offering options to buy annuities and other commitment devices.

7. CONCLUDING REMARKS

The question of the welfare cost of inflation is not a "straightforward extension of the welfare cost of an excise tax". In the excise tax case, there is no ambiguity about the implementation of a zero tax policy. In the monetary case, zero seigniorage policy can be done by imposing taxes or by offering loans to young individuals.

The literature has focused on the first tax alternative in an infinite horizon model. Here I focus on the second loan alternative and use an overlapping generations model.

A loan that is paid in full does not distort behavior and its effects are similar to the effects of a lump sum tax. Therefore, in an infinite horizon model with perfect enforcement, the government may implement the Friedman rule by offering loans at $t = 0$ and then burn the interest payments it receives. In a steady-state equilibrium, this is equivalent to the implementation of the Friedman rule by burning the receipts of lump sum taxes.

In an overlapping generations model, implementing the Friedman rule by a government loan program will be better than implementing it by lump sum taxes if the government has better collection technology. We argued that the government collection technology is better for two reasons: (a) it has a tax collection system in place and can audit income; (b) it may view loan defaults as transfers of income rather than a loss of income.

The argument for government intervention in the market for consumption loans is straightforward. From the private bank point of view a "default" is a loss of an asset. From the social point of view a "default" is a transfer from the "rich" to the "poor". Thus there is a difference between the social and the private costs of defaults and the standard reason for government intervention applies. This argument does not apply when the loan is used to finance a business project.

A government loan that is intended to finance consumption may be used to finance business projects. Is this a problem? The literature initiated by Stiglitz and Weiss (1981) and Holmstrom and Tirole (1997, 1998) suggests that increasing the fraction of the project that can be self-financed will mitigate the moral hazard and adverse selection problems that infects the market for business loans. By the same reasoning, an agent who gets a government loan to finance consumption and chooses to use part of it to finance a business project will also mitigate the problems. It thus seems that the possibility that the government consumption loan will be used to finance business projects is not a serious concern.

We should note the tension between government intervention in the loan market and free market philosophy. Most economists, Friedman being a prime example, believe that the private sector is in general more efficient than the government. But they still favor the government over the private sector in its money creation role. Here I extend the traditional "money creation role" of the government.[7]

We considered a Baumol-Tobin type model with two assets: indexed private bonds and indexed interest-bearing money. The difference between the two assets is in the ease in which one can affect their evolution. A trip to the bank is required for

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[7] It may be argued that the US government and the Fed are already involved in the loans market. For example, On Friday Dec. 7 2007 Reuter had the following news report. "President Bush announced a plan on Thursday that aims to help more than half of the two million homeowners who took out adjustable-rate subprime loans with payments due to move sharply higher soon. The move, which would offer some of them a five-year mortgage-rate freeze was hammered out with help from Treasury Department officials in an attempt to slow the wave of home loan foreclosures that has threatened to knock the U.S. economy into recession and rattled investors worldwide". The move has sparked a debate about the role of government and the incentives problems that may arise when supporting people who make irresponsible choices. I am not against bailing out poor borrowers: The above loan program allows for "partial default" when permanent income is low. But it seems that doing it in an orderly fashion is better: It saves resources on selection and monitoring. And it does not breed mistakes that will occur whenever people speculate on the government response to a crisis.

changing the evolution of bonds but the evolution of money can be changed without a trip to the bank. This gives money an advantage in smoothing consumption.

The cost of trips to the bank may be trivial for lenders who hold a positive amount in their savings account or its modern counterparts. But this cost is not trivial for borrowers who may need to provide new collaterals and negotiate new loan contracts. On a more fundamental level the advantage of money over bonds arises because of the cost of writing contracts. We have interpreted the Baumol-Tobin cost of a trip to the bank as the cost of deviating from the default option in which balances in saving accounts accumulates interest automatically.

Both assets may be used when the rate of return on bonds is higher than the rate of return on money. But only money will be used when the two assets promise the same rate of return. This is the case of zero nominal interest or the Friedman rule. In the simple version of the model, the Friedman rule allocation is efficient. Moving from an economy with strictly positive nominal interest rate to an economy with zero nominal interest rate will increase consumption by the amount of resources spent on trips to the bank. But in general welfare will increase by more than that because consumption smoothing is better under zero nominal interest rate.

In the version of the model that recognizes reporting (underreporting) costs, the allocation of the Friedman rule is efficient given the resources spent on reporting and auditing income.

The adoption of the government loan program may also facilitate the adoption of a consumption tax system that will drastically cut on the resources devoted to reporting and auditing income. The literature that followed Hall and Rabushka (1995) estimates large gains from moving to a flat consumption tax but is worried about the adverse wealth redistribution effects. An income contingent loan that allows for "partial default" by people who make consistently low income (the truly "poor") is a natural addition to this system.

A pure consumption tax system does not require that individuals will report their income. Only business will do that. A consumption tax system supplemented by loans does require that individuals report their income. Individuals with low permanent income will spend resources for underreporting income but individuals with high permanent income will return their loan in full and will report their income truthfully.

It is thus entirely possible that implementing the Friedman rule by a loan program will allow for improvements in the tax system, rather than imposing an additional burden on it. In addition, implementing the Friedman rule by a government loan program will crowd out activities in many private sectors, especially in banking, rental, insurance and trade. The resources saved can be used to produce products that are valued from the social point of view.

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APPENDIX

<u>Proof of Proposition 1</u>: To show (a) note that if the agent chooses $d < L$ he will pay

processing costs: $\alpha_g > 0$. This cannot be optimal because by increasing the loan size to

$d = L$, he can save the processing cost and increase his consumption at $t = 1$ by $(1 + r)\alpha_g$

without changing his consumption at $t > 1$. To see that this strategy is indeed feasible,

note that if he follows it he will have an additional $L - d$ units in his checking account for

all $t > 1$. Since he earns the interest r on balances held in the checking account and can

pay the additional interest payment $r(L - d)$ and return the additional principal at age T.

To show (b), I consider a plan $\{d, c_t, b_t, m_t\}$ that satisfies all the constraint in (8) and uses costly credit that leads to $TC_t > 0$ for some t. Under the Friedman rule the plan $\{d^* = L, c_t^* = c_t + TC_t, b_t^* = 0, m_t^* = m_t + b_t\}$ is also feasible. Since the "star" alternative has more consumption at each date the original plan cannot be optimal. We have thus shown that trading in the bonds market cannot be optimal under the Friedman rule.

We now substitute $b_t = 0$ in the asset evolution equation (6). By forward substitution we then get a budget constraint that can be expressed in present values terms:

$$(A1) \qquad \sum_{t=1}^{T} (1+n)^{1-t} c_t = \sum_{t=1}^{T} (1+n)^{1-t} (Y_t + g_t)$$

The budget constraint (A1) coincides with the market clearing condition (10) if and only if $\sum_{t=1}^{T} (1+n)^{1-t} g_t = 0$. Existence therefore requires that the present value of the transfer payments is zero. We have thus shown (c).

To show (d) note that the representative consumer's problem in a Friedman rule equilibrium is:

$$(A2) \qquad \max_{c_t} U(c_1, ..., c_T) \text{ s.t. (A1).}$$

The first order conditions for the problem (A2) require:

$$(A3) \qquad \frac{U_t}{U_{t+1}} = 1 + n \text{ for all t,}$$

where $U_t = \partial U / \partial c_t$ denotes the partial derivative of U with respect to consumption at age t. These are also the first order conditions for the planner's problem (12) and therefore the equilibrium outcome solves the planner's problem. $\square$

<u>Proof of Claim 2</u>: When money is used in the initial equilibrium, there are indices $h < j < k$ such that $m_j > 0$, $b_t = (1+r)^{t-h} b_h$ for $h \le t < k$ and $b_k \ne (1+r)^{k-h} b_h$. The first order conditions for the consumer's problem must be in this case:

(A4)
$$U_j(c_1^i, ..., c_T^i) = (1+r_m)^{k-j} U_k(c_1^i, ..., c_T^i);$$

(A5)
$$U_h(c_1^i, ..., c_T^i) = (1+r)^{k-j} U_k(c_1^i, ..., c_T^i)$$

To derive (A4) consider the following deviation from the optimal choice of the consumer in the initial equilibrium. The consumer reduces his consumption at age j by a unit, use it to increase his real balances and spend the accumulated amount of $(1+r_m)^{k-j}$ units at age k. The utility cost of reducing consumption at time j by a unit is $U_j(c_1^i, ..., c_T^i)$. The utility gain of increasing consumption at age k by $(1+r_m)^{k-j}$ units is: $(1+r_m)^{k-j} U_k(c_1^i, ..., c_T^i)$. Since at the optimum the agent cannot benefit from this deviation we must have:

(A6)
$$U_j(c_1^i, ..., c_T^i) \ge (1+r_m)^{k-j} U_k(c_1^i, ..., c_T^i)$$

Similarly, since $m_j > 0$ he can increase consumption at age j by a unit and reduce consumption at age k by $(1+r_m)^{k-j}$ units. Since at the optimum he cannot benefit from this deviation we must have:

(A7)
$$U_j(c_1^i, ..., c_T^i) \le (1+r_m)^{k-j} U_k(c_1^i, ..., c_T^i)$$

Conditions (A6) and (A7) imply (A4). The first order conditions to the planner's problem (12) require (A3). Since in the initial economy, $r_m < r$ the first order conditions (A4) and

(A5) imply a violation of (A3). Therefore a planner that can distribute $Y - TC^i$ units of consumption can improve on the initial equilibrium. $\square$

Proof of Claim 4: After forward substitution of (23) we get the budget constraint in terms of present values:

(A8)
$$\Delta_0^h + \sum_{t=1}^{T} \left[Y_t^h - \theta_t \left(y_t^h (\varepsilon^h) \right) + \Delta_t^h - p(d_{t-1}^h, Y_t^h) - (1+\kappa)c_t^h - TC_t(b_{t-1}^h, b_t^h) - I_t \left(d_{t-1}^h, J_{t-1}^h \right) E_t^h (\varepsilon^h, e) - (r - r_m)m_{t-1}^h \right] = 0$$

Summing over h leads to:

(A9)
$$\sum_{h=1}^{N} \left(\Delta_0^h + \sum_{t=1}^{T} \left[Y_t^h - \theta_t \left(y_t^h (\varepsilon^h) \right) + \Delta_t^h - p(d_{t-1}^h, Y_t^h) - (1+\kappa)c_t^h - TC_t(b_{t-1}^h, b_t^h) - I_t \left(d_{t-1}^h, J_{t-1}^h \right) E_t^h (\varepsilon^h, e) - (r - r_m)m_{t-1}^h \right] \right) = 0$$

Substituting $r = r_m$, $TC_t(b_{t-1}^h, b_t^h) = 0$ and the good market clearing condition in (A9) leads to:

(A10)
$$G + e + \sum_{h=1}^{N} \left(\Delta_0^h - \sum_{t=1}^{T} \left[\theta_t \left(y_t^h (\varepsilon^h) \right) - \Delta_t^h + p(d_{t-1}^h, Y_t^h) + \kappa c_t^h \right] \right) = 0,$$

and to (27). $\square$

Proof of Proposition 2: To show part (a) I assume that the consumer chooses at age t, $\Delta_t^h < L_t$ and pays α_g to go to the loan office. Since $r = r_m$, the consumer can take the

additional amount $x = L_t - \Delta_t^h$, hold it and pay it back whenever the government asks for it. In this way he will save the cost α_g.[8] Therefore, $\Delta_t^h < L_t$ cannot be optimal.

Part (b) was shown in the proof of Proposition 1. Part (c) can be shown by forward substitution. I now write the consumer's problem as:

(A11)
$$\max_{c_t^h} U^h(c_1^h,...,c_T^h)$$
$$\text{s.t. } (1+\kappa)\sum_{t=1}^{T} c_t^h = d_T^h + \sum_{t=1}^{T}\left[Y_t^h - \theta_t\left(y_t^h(\varepsilon^h)\right) - E_t^h(\varepsilon^h,e)\right]$$

The first order conditions for (A11) require:

(A12)
$$\frac{U_t^h}{U_1^h} = 1, \text{ for all } t, \text{ where } U_t^h = \frac{\partial U^h(c_1^h,...,c_T^h)}{\partial c_t^h}.$$

The first order conditions to the planner's problem (28) require that (A12) holds for all h and

(A13)
$$\omega^h U_t^h(c_1^h,...,c_T^h) = \omega^1 U_t^1(c_1^1,...,c_T^1).$$

To show part (d) note that the equilibrium allocation satisfies (A12) and we can choose weights ω^h such that (A13) is satisfied. □

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[8] The assumption that $\bar{J}$ is large and therefore the agent always have to report income is required here. When $\bar{J}$ is not large a consumer may choose to pay his debt early and get out of the obligation to file income tax return every year.