

**INFLATION TARGETING IN CANADA:
AN EXPECTED UTILITY ANALYSIS IN A DSGE MODEL WITH TRADE COSTS**

by

Hakan Yilmazkuday

Faculty Sponsor: Mario Crucini



Working Paper No. 08-WG02

May 2008

DEPARTMENT OF ECONOMICS
VANDERBILT UNIVERSITY
NASHVILLE, TN 37235

www.vanderbilt.edu/econ

Inflation Targeting in Canada: An Expected Loss Analysis in a DSGE Model with Trade Costs*

Hakan Yilmazkuday[†]

Vanderbilt University

April 2008

Abstract

This paper develops an open economy DSGE model with an emphasis on trade costs to evaluate the performance of the Bank of Canada in the Canadian inflation targeting experience. For model parametrization, the New-Keynesian Phillips curve for the Canadian economy, together with the monetary policy rule of the Bank of Canada, is estimated. When a utility-based expected loss function is considered, the Bank of Canada is found to be far from being optimal in its actions, independent of trade costs. When an *ad hoc* expected loss function considering the volatilities in inflation, output and interest rate is considered, it is found that the actions of the Bank of Canada are explained best when trade costs in fact exist but the Bank of Canada ignores them. In other words, the Bank of Canada can employ a better monetary policy by considering the existence of trade costs. Thus, trade costs play an important role in forming the monetary policy rules, which is ignored in the literature. Finally, given the *ad hoc* loss function, the actions of the Bank of Canada are best explained when 70% of weight is assigned to inflation, 15% of weight to interest rate and 15% of weight to output.

JEL Classification: E52, E58, F41

Key Words: DSGE Model, Monetary Policy Rule, Trade Costs, Inflation Targeting, Canada.

* I thank Kevin Huang, Mario Crucini and Eric Bond for their helpful comments and suggestions. All errors are my own responsibility.

[†] Department of Economics, Vanderbilt University, Nashville, TN, 37235, USA;
e-mail: hakan.yilmazkuday@vanderbilt.edu

I. INTRODUCTION

Inflation targeting regimes and monetary policy rules have been analyzed at length by researchers both from descriptive and prescriptive perspectives. As stated by Svensson (2003), from a descriptive perspective, research has examined the extent to which monetary policy rules proposed in the literature explain actual Central Bank behavior. This question has been extensively analyzed in the literature.¹

In this paper, we contribute to this literature by considering the period since 1991, in which the Bank of Canada has explicitly targeted inflation.² We introduce a dynamic stochastic general equilibrium (DSGE) model through a New-Keynesian perspective with an emphasis on trade costs to evaluate the performance of the Bank of Canada in the Canadian inflation targeting experience. A New-Keynesian Phillips curve, together with the monetary policy rule of the Bank of Canada, is estimated for the Canadian economy. Under a utility-based expected loss function, the Bank of Canada is far from being optimal in its actions, independent of trade costs. In contrast, under an *ad hoc* expected loss function, the actions of the Bank of Canada are explained best when trade costs in fact exist, but the Bank of Canada ignores them. Thus, trade costs play an important role in forming the monetary policy rules. We also show that, given the *ad hoc* loss function, the actions of the Bank of Canada are best explained when 70% of weight is assigned to inflation, 15% of weight to interest rate and 15% of weight to output.

We set up an open economy model with the home country and the rest of the world. In the model, there are three sets of agents: individuals, firms and the central bank. In particular,

¹ See Taylor (1993, 2000), Goodfriend and King (1997), Clarida et al. (1998, 1999, 2000, 2001), Judd and Rudebusch (1998), Ball (1999), Carlstrom and Fuerst (1999), McCallum and Nelson (1999, 2001), Walsh (1999), Nelson (2000), Erceg et al. (2000), Svensson (2000), Dib (2001, 2003), Sutherland (2001), Ghironi and Rebucci (2002), Obstfeld and Rogoff (2000, 2002), Parrado and Velasco (2002), Benigno and Benigno (2003), Devereux and Engel (2003), Laxton and Pesenti (2003), Woodford (2001, 2003), Ambler et al. (2004), Parrado (2004), Murchison et al. (2004), Christiano et al. (2005), Corsetti and Pesenti (2005), Gali and Monacelli (2005), Huang and Liu (2005), Yazgan and Yilmazkuday (2007), Yilmazkuday (2007), among many others.

² See Freedman, 2001; Dodge, 2002; Ragan, 2005, among others, for descriptive analyses of the Canadian inflation targeting. Also see Ravenna (2006) for the relation between business cycles and inflation targeting for the Canadian economy.

the individuals maximize their intertemporal lifetime expected utility function consisting of utility obtained from domestic (home) goods and imported goods, together with disutility coming from supplying labor. The production of goods requires labor input combined with technology. The model employs a Calvo price-setting process, in which firms are able to change their prices only with some probability, independent of other firms and the time elapsed since the last adjustment. It is also assumed that firms behave as monopolistic competitors. Imported goods are subject to a trade cost for both domestic individuals and foreign individuals. The main nuance of the model is the inclusion of trade costs which is important in terms of its implications on real exchange rates and the Law-of-One-Price.³

The micro-foundations of the individual-firm behavior results in the IS curve and the New-Keynesian Phillips curve. While the New-Keynesian Phillips curve takes into account the non-zero inflation target as the steady-state inflation (similar to the studies such as Kozicki and Tinsley, 2003; Ascari, 2004; Cogley and Sbordone, 2006; Amano et al., 2006, 2007; Bakhshi et al., 2007; Sbordone, 2007), the IS curve considers the effect of trade costs on the output, which is not the usual case in the literature.⁴ In particular, we find that the output decreases with trade costs. Moreover, an expected increase in trade costs has a negative effect on the expected change in output gap, *ceteris paribus*. For the monetary policy rule, we assume that the central bank manages a short-term nominal interest rate according to an open economy variant of the Taylor rule. Following Yazgan and Yilmazkuday (2007), we modify the monetary policy rule of Taylor (1993) and Clarida et al. (1998, 1999, and 2000) by keeping the inflation target in the final form of the rule.

Another contribution of this paper is the estimation of the New-Keynesian Phillips curve, together with the monetary policy rule for the Canadian economy, by using the Generalized

³ See Alessandria (2004), Caves et al. (1990), Crucini et al. (2005), Engel (1983), Engel and Rogers (1996), Krugman and Obstfeld (1991), Lutz (2004), Parsley and Wei (2000), Rogers and Jenkins (1995). Also see Obstfeld and Rogoff (2000) who show that trade cost are also important in explaining the six major puzzles in international macroeconomics.

⁴ See McCallum and Nelson (1999, 2001), Walsh (2003), Woodford (2003), Parrado (2004), Gali and Monacelli (2005), Yilmazkuday (2007), Lubik and Schorfheide (2007), among many others, that consider foreign output levels in the IS curve instead of trade costs.

Method of Moments (GMM). However, recently, GMM estimators have been criticized on the ground that inference, based on these estimators, is inconclusive. The related econometric literature indicates that there has been considerable evidence that asymptotic normality provides a poor approximation of the sampling distributions of GMM estimators. Particularly, the GMM estimator becomes heavily biased (in the same direction as the ordinary least squares estimator), and the distribution of the GMM estimator is quite far from the normal distribution (e.g. bimodal). Stock and Wright (2000) attribute this problem to “weak identification” or “weak instruments,” that is, instruments that are only weakly correlated with the included endogenous variables. Stock et al. (2002) and Dufour (2003) provide a comprehensive survey on weak identification in GMM estimation. In this paper, we address the problem of weak identification by using two different tests. The first of these tests is the Anderson and Roubin (1949) test (*AR*-test) in its general form presented by Kleibergen (2002). The second test is the *K*-test developed by Kleibergen (2002). These two tests are robust in the case of nonlinear models (see Dufour, 2003; Stock et al., 2002), and perhaps more importantly, they are robust even to excluded instruments (see Dufour, 2003). Since it is rarely possible to use all possible instruments, this latter property is quite important from an applied point of view (see Yazgan and Yilmazkuday, 2005, 2007).

By applying a simulation based on the estimated parameters, we find optimal monetary policy rules under different scenarios. In particular, following the lead of Ambler et al. (2004), Cayen et al. (2006), Murchison and Rennison (2006), Ortega and Rebei (2006), which give insights about the Bank of Canada’s policy-analysis models for the Canadian economy, we use the method of stochastic simulations to determine the vector of monetary policy rule parameters that minimizes the expected loss function, given the dynamics of the Canadian economy (i.e., the IS curve and the estimated New-Keynesian Phillips curve).⁵ Following Woodford (2003), we first consider a utility-based expected loss function and show that the Bank of Canada is far from being optimal in such a case, independent of trade costs.

⁵ Also see Tetlow and von zur Muehlen (1999), Erceg et al. (1998, 2000) as other studies on optimized monetary policy rules.

We then consider an *ad hoc* expected loss function and compare the calculated optimal monetary rules with the estimated monetary policy rule to obtain the weights assigned to inflation, output and interest rate volatilities, at which the percentage deviation of the expected loss from its optimal value takes its minimum value. We follow an optimistic approach and accept these calculated weights as the Bank of Canada's policy weights. Thus, instead of assigning specific weights to the mentioned variables in the loss function, we calculate them by simulation techniques.⁶ The simulation results show that the actions of the Bank of Canada are best explained when trade costs actually exist but the Bank of Canada ignores them. This is important, because trade costs are ignored in monetary policy models. Thus, we show that trade costs in fact should be considered when forming monetary policy rules by the central banks.

The rest of this paper is organized as follows: Section II introduces the New-Keynesian model and illustrates our modified specification of monetary policy developed to take into account the inflation targets. Section III provides model parametrization by estimating the model. Section IV depicts the results and comparisons of the simulation based on the Canadian economy. Section V concludes. The derivation of the model, together with the details of the data used, is given in the Appendices.

II. THE MODEL

As stated by Goodfriend and King (1997), the basic assumption that distinguishes the New-Keynesian models from other models is that the price-setting firms are explicitly modeled as monopolistic competitors. The reason for imperfect competition is emphasized as the source of the real output effect of money when prices are subject to costs of adjustment; *i.e.* the nominal price rigidity. In this context, extending a simpler version of Gali and

⁶ See Rotemberg and Woodford (1997); Woodford (1999); Batini and Nelson (2001); Smets (2003); Parrado (2004); Yilmazkuday (2007), among many others, for different types of loss functions considered in the literature.

Monacelli's (2005) model by introducing trade costs, we set up a continuum of goods model in which all goods are tradable, the representative individual holds assets (bonds), and the production of goods requires labor input. Time is discrete. The optimality conditions of the agents are derived in the Appendices, and the key equations are introduced in the text.

The log-linearized model is expressed in terms of four blocks of equations: aggregate demand (i.e., the IS curve), aggregate supply (i.e., the New-Keynesian Phillips curve), monetary policy rule, and stochastic processes. Lower case letters denote log variables; the subscripts H and F stand for domestically produced and imported variables, respectively; the superscript $*$ stands for the variables of the rest of the world; and lastly, a bar on a variable ($\bar{\cdot}$) stands for the target.

2.1. Definitions and Some Identities

We define the CPI as follows:

$$p_t \equiv (1 - \gamma)p_{H,t} + \gamma p_{F,t} \quad (1)$$

where $p_{H,t}$ is the (log) price index for domestically consumed home goods; $p_{F,t}$ is the (log) price index for imported goods; and γ is the share of domestic consumption allocated to imported goods. In other words, γ represents a natural index of openness. Both $p_{H,t}$ and $p_{F,t}$ are in domestic currency. The price index for imported goods is given by:

$$p_{F,t} = e_t + p_{F,t}^* + \tau_t \quad (2)$$

where e_t is the (log) nominal effective exchange rate; $p_{F,t}^*$ is the (log) price index for domestically consumed foreign goods at the source; and τ_t is the (log) gross trade cost,

which is an income received by the rest of the world.⁷ The autoregressive parameter, ρ_τ , appears in the evolution of trade costs as follows:

$$\tau_t = \rho_\tau \tau_{t-1} + \varepsilon_t^\tau \quad (3)$$

where $\rho_\tau \in [0,1]$ and ε_t^τ is assumed to be an independent and identically distributed (i.i.d.) shock with zero mean and variance σ_τ^2 .

If we define the (log) effective terms of trade as $s_t \equiv p_{F,t} - p_{H,t}$, we can write the CPI formula as:

$$\begin{aligned} p_t &\equiv (1-\gamma)p_{H,t} + \gamma p_{F,t} \\ &= p_{H,t} + \gamma s_t \end{aligned} \quad (4)$$

Thus, we can write the formula of CPI inflation as follows:

$$\pi_t = \pi_{H,t} + \gamma(s_t - s_{t-1}) \quad (5)$$

where $\pi_{H,t} = p_{H,t} - p_{H,t-1}$ is the inflation of home-produced goods.

By combining $s_t \equiv p_{F,t} - p_{H,t}$ and $p_{F,t} = e_t + p_{F,t}^* + \tau_t$, we can write:

$$s_t \equiv e_t + p_{F,t}^* + \tau_t - p_{H,t} \quad (6)$$

⁷ For future reference, $p_{H,t}^*$ is the (log) price index for the imported goods for the rest of the world, and $p_{F,t}^*$ is the (log) domestic price index for the rest of the world. We assume that the trade costs consist of transportation costs and transportation sector is owned by the rest of the world, so there is no transportation income received by the home country. Another interpretation of this would be to have iceberg trade costs. See Anderson and van Wincoop (2003) for a discussion of iceberg melt structure of economic geography literature and trade costs. Since we assume that the transportation costs are *not* good or source specific, the (log) gross trade cost directly enters the price index for imported goods.

We can define the (log) effective real exchange rate as:

$$q_t = e_t + p_t^* - p_t \quad (7)$$

By using Equations (1), (2) and (6), together with the symmetric versions of Equations (1) and (2) for the rest of the world, we can rewrite Equation (7) as follows:

$$q_t = (1 - \gamma - \gamma^*)s_t - (1 - 2\gamma^*)\tau_t \quad (8)$$

where γ^* is the share of foreign consumption allocated to goods imported from the home country. In a special case in which the home country is a small one (i.e., γ^* is a very small number), Equation (8) can be approximated as:

$$q_t \approx (1 - \gamma)s_t - \tau_t \quad (9)$$

Compared to the studies in the literature that ignore trade costs in open economy models, such as Parrado (2004), Gali and Monacelli (2005), Lubik and Schorfheide (2007), Yilmazkuday (2007), the presence of trade costs is important in Equations (8) and (9). In particular, as is shown empirically by Caves et al. (1990), Crucini et al. (2005), Engel (1983), Engel and Rogers (1996), Krugman and Obstfeld (1991), Lutz (2004), Parsley and Wei (2000), Rogers and Jenkins (1995), trade costs play a big role in the determination of real exchange rates. The implications of the trade costs in our model will be clearer when we introduce our aggregate demand below and, especially, when we make our simulation analysis in Section IV.

The uncovered interest parity condition is given by:

$$i_t = i_t^* + E_t[e_{t+1}] - e_t \quad (10)$$

where E is the expectation operator. The derivation of this condition is given in Appendix A. Equation (10) relates the movements of the interest rate differentials to the expected variations in the effective nominal exchange rate. Since $s_t \equiv e_t + p_{F,t}^* + \tau_t - p_{H,t}$, we can rewrite Equation (10) as follows:

$$s_t = \left(i_t^* - E_t[\pi_{F,t+1}^*] \right) - \left(i_t - E_t[\pi_{H,t+1}] \right) + E_t[s_{t+1} - \Delta\tau_{t+1}] \quad (11)$$

where $\Delta\tau_{t+1}$ is the change in trade cost from period t to $t+1$. Equation (11) shows the terms of trade between the home country and the rest of the world as a function of current interest rate differentials, expected future home inflation differentials and its own expectation for the next period together with the expected future change in trade cost. Here, the evolution of foreign interest rate shock is given by:

$$i_t^* = \rho_i^* i_{t-1}^* + \varepsilon_t^{i^*} \quad (12)$$

where $\rho_i^* \in [0,1]$, and $\varepsilon_t^{i^*}$ is assumed to be an independent and identically distributed (i.i.d.) shock with zero mean and variance $\sigma_{i^*}^2$.

2.2. Aggregate Demand

In the model, the aggregate demand is as follows:

$$y_t = E_t(y_{t+1}) - \left(i_t - E_t(\pi_{H,t+1}) \right) + E_t(\Delta\tau_{t+1}) \quad (13)$$

where y_t is the (log) output; i_t is nominal interest rate; and other variables have the same notation as before. In particular, Equation (13) represents an IS curve that considers the effect of trade costs on output, which is not the usual case in the literature. The derivation of Equation (13) is given in Appendix C.

From another point of view, Equation (13) represents an IS curve that relates the expected change in (log) output (i.e., $E_t(y_{t+1}) - y_t$) to the difference between the interest rate and the expected future domestic inflation (i.e., an approximate measure of real interest rate that becomes an exact measure of real interest rate when the terms of trade are constant across periods), and the expected change in trade costs.⁸ An increase in the difference between the expected inflation and the nominal interest rate decreases expected change in output gap, with a unit coefficient. Finally, an expected increase in the trade costs leads to a decrease in the expected change in (log) output. The latter is due to the intertemporal substitution of supply in response to a change in trade cost.

As is also shown in Appendix E, Equation (13) can be written in terms of output gap as follows:

$$x_t = E_t(x_{t+1}) - (i_t - E_t(\pi_{H,t+1})) + E_t(\Delta z_{t+1}) \quad (14)$$

where $x_t = y_t - \bar{y}_t$ is the output gap defined as the deviation of (log) domestic output y_t from the domestic natural level of output \bar{y}_t , defined as the one under flexible price equilibrium; and z_t is the (log) level of technology, which evolves according to:

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z \quad (15)$$

⁸ See Kerr and King (1996), and King (2000) for discussions on incorporating the role for future output gap in the IS curve with a unit coefficient.

where $\rho_z \in [0,1]$ and ε_t^z is assumed to be an i.i.d. shock with zero mean and variance σ_z^2 .

2.3. New-Keynesian Phillips Curve

The New-Keynesian Phillips curve in this economy is given by:

$$\pi_{H,t} = \lambda_\pi E_t[\pi_{H,t+1}] + \lambda_m (\varphi + \widehat{mc}_t) \quad (16)$$

where $\lambda_\pi = \frac{\beta\alpha}{1-(1-\alpha)(\bar{\Pi})}$, $\lambda_m = \frac{(1-\alpha)(1-\alpha\beta)}{1-(1-\alpha)(\bar{\Pi})}$, and $\varphi = 1 - \bar{\Pi}(1 - \bar{\pi})$. In particular, α is the probability that a firm does not change its price within a given period; β is the discount factor; and $\bar{\Pi} = \exp(\bar{\pi})$ is the exponential of trend inflation. Note that this expression reduces to a standard New-Keynesian Phillips curve when trend inflation is equal to zero (i.e., $\bar{\pi} = 0$ or $\bar{\Pi} = 1$). The derivation of Equation (16) is given in Appendix D.

2.4. Monetary Policy Rule

For the monetary policy rule, we assume that the central bank manages a short-term nominal interest rate according to an open economy variant of the Taylor rule. Following Taylor (1993) and Clarida et al. (1998, 1999, and 2000), consider the following monetary policy rule in the form:

$$\bar{i}_t = r + \bar{\pi} + \chi_\pi [E_t(\pi_{t+1}|\Omega_t) - \bar{\pi}] + \chi_x E_t(x_t|\Omega_t) \quad (17)$$

where \bar{i}_t denotes the target rate for nominal interest rate in period t ; Ω_t is the information set at the time the interest rate is set; π_{t+1} denotes CPI inflation one period ahead; $\bar{\pi}$ is the target for CPI inflation; x_t is the output gap in period t ; and r is the long-run equilibrium

real rate.⁹ As in Clarida et al. (2000), we assume that the real rate is stationary and is determined by non-monetary factors in the long run. Since we consider the monthly sample over the period 1996:1 to 2006:12, in which the annual inflation target range is exactly the same (i.e., 2%, the midpoint of a control range of 1% to 3%, according to the Bank of Canada, Macklem, 2002, and Coletti and Murchison, 2002) and the long-run interest rates are pretty much stable for the Canadian economy, to assume r and $\bar{\pi}$ are time invariants is reasonable in our analysis.

Similar policy rules to (17) have been used extensively in empirical research of several countries. However, most of these and previously mentioned studies consider a zero inflation target over the period of estimation. In this study, following the lead of Yazgan and Yilmazkuday (2007), we keep the inflation target in the monetary policy rule and modify Equation (17) as follows:

$$i_t = r + \bar{\pi} + \chi_\pi [\pi_{t+1} - \bar{\pi}] + \chi_x x_t + \psi_t \quad (18)$$

where i_t is the actual nominal interest rate, and $\psi_t = -\chi_\pi [\pi_{t+1} - E_t(\pi_{t+1}|\Omega_t)] - \chi_x [x_t - E_t(x_t|\Omega_t)] + \mu_t$. The term μ_t captures the difference between the desired and the actual nominal interest rate, i.e. $\mu_t = i_t - \bar{i}_t$.¹⁰ According to Clarida et al. (2000), this difference may result from three sources. First, the specification in Equation (18) assumes an adjustment of the actual overnight rates to its target level, and thus ignores, if any, the Bank of Canada's tendency to smooth changes in interest rates (we will address this issue below). Second, it treats all changes in interest rates over time as reflecting the Bank of Canada's systematic response to economic conditions. Specifically, it does not allow for any randomness in policy actions, other than that which is associated

⁹ It should be noted that r is an "approximate" real rate since the forecast horizon for the inflation rate will generally differ from the maturity of the short-term nominal rate used as a monetary policy instrument. As noted by Clarida et al. (2000), in practice, the presence of high correlation between the short-term rates at maturities associated with the target horizon (1 year) prevents this from being a problem.

¹⁰ We assume that μ_t is identically and independently distributed.

with misforecasts of the economy. Third, it assumes that the Bank of Canada has perfect control over the interest rates, i.e., it succeeds in keeping them at the desired level (e.g., through open market operations).

Interest rate smoothing is introduced into the model via the following partial adjustment mechanism (see Clarida et al., 1998, 2000):

$$i_t = (1 - \rho_i) \bar{i}_t + \rho_i i_{t-1} + v_t \quad (19)$$

where $\rho_i \in [0, 1]$ captures the degree of interest rate smoothing. Equation (19) postulates that in each period, the Bank of Canada adjusts the funds rate to eliminate a fraction $(1 - \rho_i)$ of the gap between its current target level and its past value. And, v_t is an independently and identically distributed error term. Substituting Equation (17) into Equation (19) yields:

$$i_t = (1 - \rho_i) (r + \bar{\pi} + \chi_\pi [\pi_{t+1} - \bar{\pi}] + \chi_x x_t) + \rho_i i_{t-1} + \varepsilon_t \quad (20)$$

where $\varepsilon_t = -(1 - \rho_i) \{ \chi_\pi [\pi_{t+1} - E_t(\pi_{t+1} | \Omega_t)] + \chi_x [x_t - E_t(x_t | \Omega_t)] \} + v_t$.

2.5. Utility-Based Welfare

Using the utility function specified in the model, we may define the utility-based welfare function as follows:¹¹

¹¹ The derivation of the utility-based welfare function is shown in Appendix F.

$$\begin{aligned}
E_t \sum_{k=0}^{\infty} \beta^{t+k} (U(C_{t+k}) - V(N_{t+k})) &= E_t \sum_{k=0}^{\infty} \beta^{t+k} (\log \zeta + \tau_{t+k} - \gamma) s_{t+k} \\
&\quad - \frac{1}{2} E_t \sum_{k=0}^{\infty} \beta^{t+k} \left((1 - \log \zeta - \tau_{t+k}) \frac{\theta}{\lambda_w} (\pi_{H,t+k})^2 + (x_{t+k})^2 \right) \quad (21) \\
&\quad + t.i.p + o(\|a^3\|)
\end{aligned}$$

where $\lambda_w = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$; $\zeta \equiv \theta/(\theta-1)$ is a markup as a result of market power; $\theta > 1$ is the price elasticity of demand faced by each monopolist; *t.i.p.* represents terms independent of policy; and finally, $o(\|a^3\|)$ represents terms that are equal to or higher than 3rd order. Note that the utility-based welfare function depends on the volatility in inflation and output gap as well as the trade costs and the terms of trade.

In our initial welfare analysis, we assume that the Bank of Canada is benevolent and thus uses Equation (21) as its objective function. We will relax this assumption later to consider an alternative *ad hoc* objective function. For now, we need parameter values of α , β , θ to calculate the value of the welfare function of Equation (21). We achieve this parametrization in the next section.

III. ESTIMATION AND MODEL PARAMETRIZATION

In this section, we separately estimate the monetary policy rule of the Bank of Canada and the New-Keynesian Phillips curve for the Canadian economy, by using continuous updating GMM. Our reason for individual GMM estimations is that joint GMM estimations can be hazardous according to Hayashi (2000, p.273). In particular, while a joint estimation theoretically provides asymptotic efficiency, it may suffer more from the small-sample bias in practice. Our estimation results will not only help us determine how our model explains the Canadian data, but they will also provide parameters for our simulation analysis in the next section. The data are described in Appendix H.

3.1. Estimation of the Monetary Policy Rule

Let \mathbf{z}_t be a vector of variables, within the central bank's information set at the time it chooses the interest rate (i.e. $\mathbf{z}_t \in \Omega_t$) that are orthogonal to ε_t . Possible elements of \mathbf{z}_t include any lagged variables that help to forecast inflation and output gap, as well as any contemporaneous variables that are uncorrelated with the current interest rate shock μ_t . In sum, we have the following orthogonality condition:

$$E_t \left[i_t - (1 - \rho_i) \{ r + \bar{\pi} + \chi_\pi [\pi_{t+1} - \bar{\pi}] + \chi_x x_t \} - \rho_i i_{t-1} \mid \mathbf{z}_t \right] = 0 \quad (22)$$

In Equation (22), the expected signs of $r, \rho, \chi_\pi, \chi_x$ are all positive. By using this orthogonality condition, we use continuous updating GMM to estimate the parameter vector $[r, \rho, \chi_\pi, \chi_x]$.¹² Since the econometric estimation procedure that we use here (GMM) requires that all the variables (including instruments) used in the estimation should be stationary, all of the variables are tested by using the Augmented Dickey-Fuller (ADF) tests. We find that the null of unit root is rejected in all variables, at least at the 10 percent significance level, when tests are applied at different lags.¹³ The results are illustrated in Table 1. The instruments we use for GMM estimation consist of twelve lags of home inflation, percentage change in M1 and three lags of output gap.¹⁴

¹² For continuous updating GMM estimators, we have modified the GAUSS code originally used by Stock and Wright (2000). All of our codes are available upon request. Gauss version 6.0 has been used.

¹³ These results are available upon request.

¹⁴ By choosing these instruments, we implicitly assume that these variables are strong instruments for predicting inflation and output gap.

Table 1

GMM Estimates of the Monetary Policy Rule

r	χ_{π}	χ_x	ρ_i
1.37	5.50	0.09	0.96
(0.8441)	(4.1913)	(0.0616)	(0.0258)
[0.0523]	[0.0946]	[0.0835]	[0.0000]

Notes: Standard errors calculated using the Delta method are in parentheses and p-values are in brackets. The sample size is 114 after considering data availability and instruments used which consist of twelve lags of home inflation, percentage change in M1 and three lags of output gap.

Table 1 reports the estimates of r , χ_{π} , χ_x and ρ_i . All of the estimates satisfy their expected signs.¹⁵ In particular, the estimate of the coefficient on the difference between expected and targeted inflation is around 5.50 for Canada. That is, if expected inflation were 1 percentage point above the target, the Bank of Canada would set the interest rate approximately 5.50 percent above its equilibrium value. This coefficient is significant at the 10% level when we use asymptotic normality as an approximation to the sampling distribution of GMM estimators.

The response of the Bank of Canada to the deviations of the expected output gap from its target (assumed to be zero) is around 0.09. In other words, holding other parameters

¹⁵ Although the comparison of these estimates with the existing literature is absurd due to the differences in model specifications and sample periods, see Ambler et al. (2004), Murchison et al. (2004), Cayen et al. (2006), Ortega and Rebei (2006), Lubik and Schorfheide (2007) for other monetary policy rule estimations of the Bank of Canada.

constant, one unit increase in output gap induces the Bank of Canada to increase the interest rates by 9 basis points. This coefficient is again significant at the 10% level. The equilibrium real interest rate is estimated as 1.37 percent and it is significant at the 10% level using normal asymptotics. The estimation results also indicate that the smoothing parameter is highly significant and equal to 0.96. This estimate implies that the Bank of Canada puts forth significant effort for smoothing interest rates.

Table 2 illustrates the test statistics for GMM estimation. The Hansen's J -statistic does not reject the null hypothesis that the overidentifying restrictions are satisfied at conventional significance levels.

Table 2

Test Statistics for GMM Estimation of the Monetary Policy Rule

<i>AR-stat</i>	<i>K-stat</i>	<i>J-stat</i>	<i>Adjusted R²</i>
$F(27;87)$	$\chi^2(27)$	$\chi^2(2)$	$\chi^2(25)$
0.74	19.87	2.39	15.53
[0.82]	[0.84]	[0.30]	[0.93]
			0.99

Notes: p-values are in brackets.

Despite their significance, one should be wary about GMM-based results that are obtained under the asymptotic normality of the sampling distributions obtained under conventional asymptotics. Under weak-identification asymptotics, the sampling distributions are quite far from being normally distributed. In this paper, we address the problem of weak

identification by using two different tests. The first of these tests is the Anderson and Roubin (1949) test (*AR*-test) in its general form presented by Kleibergen (2002). The second test is the *K*-test developed by Kleibergen (2002). These two tests are robust in the case of nonlinear models (see Stock et al., 2002; Dufour, 2003; Dufour and Taamouti, 2005, 2006), and perhaps more importantly, they are robust even to excluded instruments (see Dufour, 2003). Since it is rarely possible to use all possible instruments, this latter property is quite important from an applied point of view (see Yazgan and Yilmazkuday, 2005).

AR and *K*-test statistics are used to test the null hypothesis that:

$$H_0 : r = 1.37; \chi_\pi = 5.50; \chi_x = 0.09; \rho_i = 0.96$$

i.e. given the instruments that we used, whether the estimated parameters are compatible with the data or not.¹⁶ Since both of these tests are fully robust to weak instruments (see Stock et al., 2002, pp.522), a non-rejection of this null hypothesis means that our estimates are also “data-admissible” even under the case of weak instruments.

As is evident from Table 2, given the high *p*-value of the *AR*-test, our parameter estimates cannot be rejected.¹⁷ In other words, our GMM estimates of the Bank of Canada’s Reaction Function cannot be refuted by the Canadian data.

However, as argued by Kleibergen (2002), the deficiency of the *AR*-statistic is that its limiting distribution has a degree of freedom parameter equal to the number of instruments. Therefore the *AR*-statistic suffers from the problem of low power when the number of

¹⁶ As suggested by Kleibergen (2002), the *AR*-test and the *K*-test statistics are calculated by interpreting all data matrices in the test as residuals from the projection on exogenous variables.

¹⁷ The *AR*-statistics, under the above null hypothesis, has an exact Fisher distribution with *k* and *T-k* degrees of freedom (where *k* is the number of instruments, and *T* is the number of observations), given that the error terms are i.i.d. normal, and the instruments are strictly exogenous. *k* × *AR* statistics are asymptotically distributed chi-square with *k* degrees of freedom even without i.i.d. normal errors under standard regularity conditions (see Dufour and Jasiak, 2001, pp. 829, and Dufour 2003, pp.20).

instruments highly exceeds the number of parameters. Kleibergen proposed a statistic (K -statistic) that remedies the drawback of the AR -statistic. Kleibergen, unlike the AR -test, does not provide a finite sample theory, but instead shows that his K -statistics follows an asymptotic $\chi^2(G)$ distribution (where G is the number of endogenous regressors) under the null hypothesis in the absence of exogenous regressors. As can be seen from Table 2, our K -statistics provides a similar result to the AR -test.

Table 3

GMM Estimates of the New Keynesian Phillips Curve

α	β	λ_π	λ_m
0.99	0.99	1.09	0.01
(0.02)	(0.01)	(0.00)	(0.00)
[0.00]	[0.00]	[0.00]	[0.00]

Notes: Standard errors are in parentheses and p -values are in brackets. The sample size is 127 after considering data availability and instruments used, which consist of six lags of home inflation, six lags of the percentage change in terms of trade and two lags of percentage change in $M1$. The standard errors of λ_π and λ_m have been calculated by using the Delta method.

3.2. Estimation of the New-Keynesian Phillips Curve

We continue with the structural estimation of the New-Keynesian Phillips curve defined by Equation (16) where the expected signs of α and β are both positive. We use exactly the

same methodology that we used for the estimation of the monetary policy rule. The estimation results are illustrated in Table 3. The instruments we use for the GMM estimation consist of six lags of home inflation, six lags of the percentage change in terms of trade and two lags of percentage change in M1. As is evident, both estimates satisfy their expected signs.¹⁸ Finally, both *AR*-and *K*-statistics in Table 4 support our estimation results for the Phillips curve.

Table 4

Test Statistics for GMM Estimation of the New Keynesian Phillips Curve

<i>AR-stat</i>		<i>K-stat</i>	<i>J-stat</i>	<i>Adjusted R²</i>
$F(14;113)$	$\chi^2(14)$	$\chi^2(2)$	$\chi^2(13)$	
0.65	9.03	0.05	11.99	0.86
[0.82]	[0.83]	[0.98]	[0.53]	

Notes: p-values are in brackets.

3.3. Remaining Parameters

Before we continue with our simulation analysis, we must estimate the serial correlation parameters for productivity, trade costs and foreign interest rate as $(\rho_z; \rho_\tau; \rho_{i^*}) = (0.98; 0.97; 0.99)$. Moreover, the related standard deviations, which are used to determine the size of the shocks in our simulations next section, are estimated as $(\sigma_z; \sigma_\tau; \sigma_{i^*}) = (0.01; 0.09; 0.17)$. We set the share of domestic consumption allocated to imported goods to

¹⁸ Although the comparison of these estimates with the existing literature is absurd due to the differences in model specifications and sample periods, see Ambler et al. (2004), Murchison et al. (2004), Dufour et al. (2006), Lubik and Schorfheide (2007) for recent New-Keynesian Phillips curve estimations of the Canadian economy.

$\gamma = 0.36$, which is implied by Equation (34) as the mean ratio of the value of imports to the value of GDP over the sample period. Finally, we set the gross markup equal to $\zeta = 1.35$, which is equal to the average markup in the manufacturing sector in Canada, and thus, it is implied that price elasticity of demand faced by each monopolist is set as $\theta = 3.85$. Now, we have each parameter used in the utility-based welfare function (i.e., Equation (21)) and the model solution given in Appendix G. By using our model solution, we start our simulation analysis based on the Canadian economy in the next section.

IV. RESULTS AND COMPARISONS

In order to compare the expected loss implications of alternative monetary policy rules, a criterion is needed. We consider two alternative approaches that are highly accepted in the literature:

- Approach 1. Utility-based loss function
- Approach 2. *Ad hoc* loss function

While the utility-based loss function is obtained through the microfoundations of our model, the *ad hoc* loss function is assumed to depend on the volatility in inflation, output gap and interest rate. We provide the details of each approach in the following subsections.

4.1. Utility-Based Loss Function

The utility-based loss function implied by Equation (21) is as follows:

$$E_t \sum_{k=0}^{\infty} \beta^{t+k} L_{t+k}^{ub} = E_t \sum_{k=0}^{\infty} \beta^{t+k} \left(\frac{\theta(1 - \log \zeta - \tau_{t+k})(\pi_{H,t+k})^2}{2\lambda_w} + \frac{(x_{t+k})^2}{2} - (\log \zeta + \tau_{t+k} - \gamma)s_{t+k} \right) \quad (23)$$

Estimated policy function is evaluated relative to the optimal policy as follows:

- Exercise 1. Since a typical central bank determines its policy considering the dynamics of the economy (i.e., the IS curve and the New Keynesian Phillips curve), given

these dynamics, following the lead of Ambler et al. (2004), Cayen et al. (2006), Murchison and Rennison (2006), and Ortega and Rebei (2006), we search for the optimized monetary policy rules under possible types of shocks. In particular, we use the method of stochastic simulations to determine the vector of parameters that minimizes the expected loss function; i.e., for each possible combination of ρ_i , χ_π , χ_x and χ_s values in Equation (20), we calculate the expected loss value by Equation (23).

Exercise 2. We compare the performance of the estimated monetary policy rule of the Bank of Canada with the optimized monetary policy rule (obtained by Exercise 1) in terms of expected loss in the economy (i.e., Equation (23)).

In both exercises, we consider a combination of three possible types of shocks, namely a trade cost shock, a technology shock, a foreign interest rate shock. These shocks are determined by Equations (3), (12) and (15). We set the size of the shocks equal to one standard deviation of the relevant shock variables.¹⁹

The results of both exercises are given in Table 5 which compares optimal monetary policy rules and historical (i.e., estimated) monetary policy rules. Note that we have considered the cases of with and without trade cost to show the effect of trade costs. While the case with trade costs refers to the unrestricted version of our model, the case without trade costs refers to the restricted version of our model in which trade costs are ignored (i.e., $\tau_t = 0$ for all t). In any case, optimal χ_π and χ_x values are much higher compared to the estimates of historical monetary policy rule of the Bank of Canada. Nevertheless, ρ_i values are very close to each other. In other words, *given* the utility-based welfare function, the Bank of Canada places much lower weight upon inflation and output compared to the optimal monetary policy, while it gives approximately the same weight to smoothing the interest rate.

¹⁹ MATLAB version 7.1.0.246 R(14) Service Pack 3 has been used for the simulation. The codes are available upon request.

Table 5
Optimal vs. Historical Monetary Policy Rule

	χ_π	χ_x	ρ_i	Welfare Loss
Optimal MPR With Trade Costs	18.5	0.37	0.97	1.11×10^{-5}
Optimal MPR without Trade Costs	13.0	0.36	0.95	2.29×10^{-5}
Historical MPR with Trade Costs	5.5	0.09	0.96	12.76
Historical MPR without Trade Costs	5.5	0.09	0.96	13.65

When we compare the welfare loss values calculated by Equation (23), we see that the historical monetary policy rule is far from being optimal. In other words, the Bank of Canada's actions are not maximizing the welfare of the individuals in the economy according to our analysis.²⁰ This brings another possibility into the picture: What if the Bank of Canada has its own expected loss function rather than the utility-based loss function? We consider this possibility in the following subsection by considering an *ad hoc* loss function.

4.2. Ad Hoc Loss Function

Similar to Svensson (1997), Rotemberg and Woodford (1997), Rudebusch and Svensson (1998), Woodford (1999), Batini and Nelson (2001), Smets (2003), the *ad hoc* intertemporal loss function is assumed to depend on the deviations of the inflation and the

²⁰ When we compare the discounted (lifetime) value of the deviation of consumption between optimal and historical monetary policy rules, we see that the consumption implied by the historical rule deviates around 5% from the one implied by the optimal rule, in the case with trade costs. This deviation increases to around 9% in the case without trade costs.

output from their steady state values, and the volatility of the policy instrument. It can be demonstrated as follows:

$$E_t \sum_{k=0}^{\infty} \kappa^k L_{t+k}^{ah} \quad (24)$$

where κ is the discount factor of the central bank (which can be different from the consumer discount factor, β), and the period loss function, following Smets (2003), is given by:

$$L_t^{ah} = \psi_{\pi} (\pi_{H,t})^2 + (1 - \psi_{\pi}) (\psi_x (x_t)^2 + (1 - \psi_x) (\Delta i_t)^2) \quad (25)$$

where $0 \leq \psi_{\pi} \leq 1$ and $0 \leq \psi_x \leq 1$. While the inclusion of inflation and output into the loss function is almost standard, as Cayen et al. (2006) point out, the policy instrument may enter as an argument of the loss function for three different reasons: (i) big and unexpected changes to interest rates may cause problems for financial stability (Cukierman 1990; Smets 2003), (ii) the policy-makers may be concerned about hitting the lower nominal bound on interest rates (Rotemberg and Woodford 1997; Woodford 1999; Smets 2003), or (iii) in reality, the monetary authority (and other agents) may be uncertain about the nature and the persistence of the shocks at play in the economy at the time it must make a decision about its policy instrument.

Following Rudebusch and Svensson (1998), we consider the limiting case of the central bank discount factor satisfying $\kappa = 1$ in order to interpret the intertemporal loss function as the unconditional mean of the period loss function², which is equal to the sum of the unconditional variances of the goal variables:

$$E[L_t^{ah}] = \psi_{\pi} Var[\pi_{H,t}] + (1 - \psi_{\pi}) (\psi_x Var[x_t] + (1 - \psi_x) Var[\Delta i_t]) \quad (26)$$

Instead of assuming specific values as in the related empirical literature (see Batini and Nelson, 2001; Rudebusch and Svensson, 1998; Cayen et al., 2006), we consider different possible values for ψ_π and ψ_x in our analysis. In particular, we employ the following exercises:

Exercise 1. By considering all possible values for ψ_π and ψ_x , we analyze the performance of our estimated model (i.e., by using the estimated parameters of the New Keynesian Phillips curve and monetary policy rule) in terms of the expected loss function, after possible types of shocks.

Exercise 2. Since a typical central bank determines its policy considering the dynamics of the economy (i.e., the IS curve and the New Keynesian Phillips curve), given these dynamics, following the lead of Ambler et al. (2004), Cayen et al. (2006), Murchison and Rennison (2006), and Ortega and Rebei (2006), we search for the optimized monetary policy rules under possible types of shocks, again by considering all possible values for ψ_π and ψ_x .

Exercise 3. By considering the expected loss functions calculated by Exercise 1 and Exercise 2, we compare the performance of the estimated monetary policy rule of the Bank of Canada with the optimized monetary policy rule in terms of expected loss in the economy. By this comparison, we search for the weights assigned to inflation, output and interest rate volatilities in the loss function at which the Bank of Canada is most successful. We follow an optimistic approach and accept these calculated weights as the Bank of Canada's policy weights.

We depict the details of each exercise in the following subsections. In all exercises, we again consider three possible types of shocks, namely a negative foreign interest rate shock, a negative trade cost shock and a positive technology shock. We again set the size of the shocks equal to one standard deviation of the relevant shock variables.

4.2.1. Exercise 1

This subsection calculates the expected loss function given by Equation (26) considering the estimated model parameters in Section III (i.e., the estimated parameters of the New Keynesian Phillips curve and monetary policy rule) together with all possible ψ_π and ψ_x values. We also consider two cases: one with trade cost, the other without trade costs. The results are given in Figure 1 and Figure 2. As is evident, roughly speaking, the expected loss function decreases in ψ_π and increases in ψ_x for Figure 1, while it is slightly different for Figure 2. The intuition behind this result will be clearer by the following exercises.

Figure 1 - Expected Loss Values for Historical MPR in the presence of Trade Costs

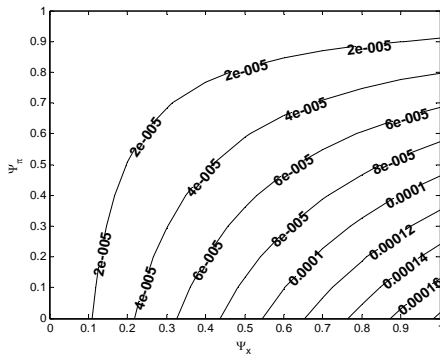
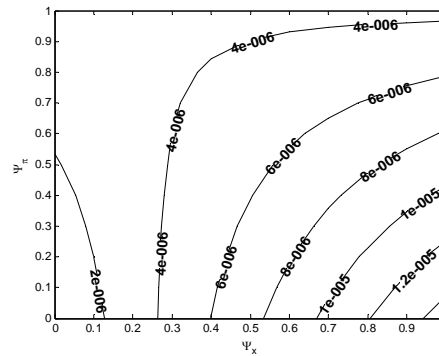


Figure 2 – Expected Loss Values for Historical MPR in the absence of Trade Costs



4.2.2. Exercise 2

This subsection searches for the optimized monetary policy rules (MPRs) with and without trade costs. As before, following the lead of Cayen et al. (2006), and Murchison and Rennison (2006), we use the method of stochastic simulations to determine the vector of parameters that minimizes the expected loss function. In particular, for each possible combination of ρ_i , χ_π , χ_x and χ_s values in Equation (20), we calculate the variance of inflation, the output gap, and the change in the level of the interest rate to find the minimized expected loss, after simultaneous shocks of technology, trade cost and foreign interest rate. We again consider all possible (ψ_π, ψ_x) pairs in our analysis. Our grid search

in the existence of trade costs results in the expected loss values in Figure 3 which are computed through Equation (20) by using the calculated optimal monetary policy coefficients given in Figures 4-6.

Figure 3 – Expected Loss Values for Optimal MPR in the presence of Trade Costs

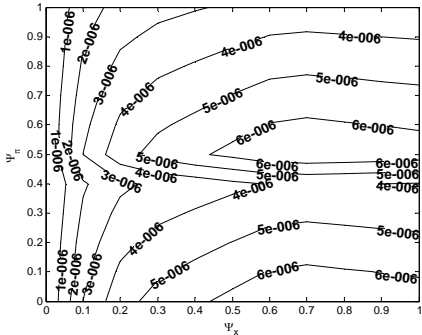


Figure 5 – Optimal χ_x in the presence of Trade Costs

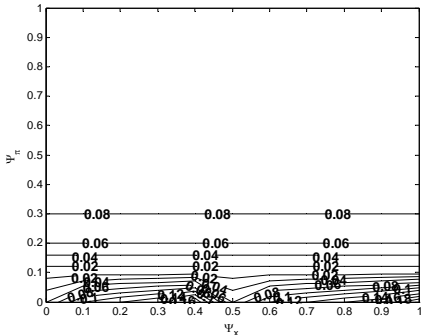


Figure 4 – Optimal χ_π in the presence of Trade Costs

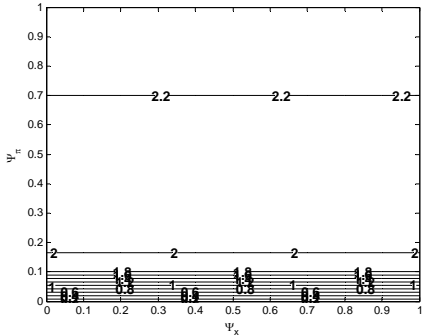
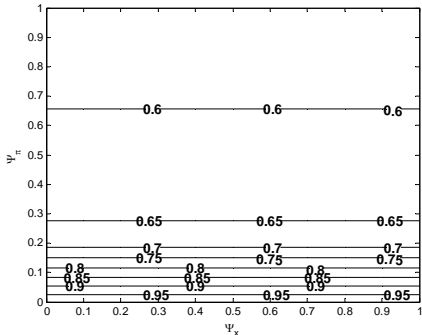


Figure 6 – Optimal ρ_i in the presence of Trade Costs



As is evident from Figure 3, the expected loss function under optimal policy rules increases in ψ_x while it takes its lowest value when we move toward $\psi_\pi = 1$. When we look at the optimal monetary policy rules under possible (ψ_π, ψ_x) pairs in Figures 4-6, we see that the optimal χ_π , χ_x and ρ_i take higher values when ψ_π decreases.

When we repeat the same analysis in the absence of trade costs, the effects of the inclusion of trade costs become clearer. The results are given in Figures 7-10.

Figure 7 – Expected Loss Values for Optimal MPR in the absence of Trade Costs

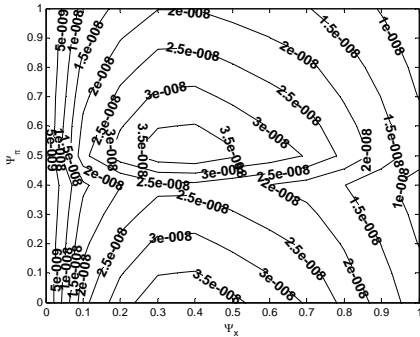


Figure 9 – Optimal χ_x in the absence of Trade Costs

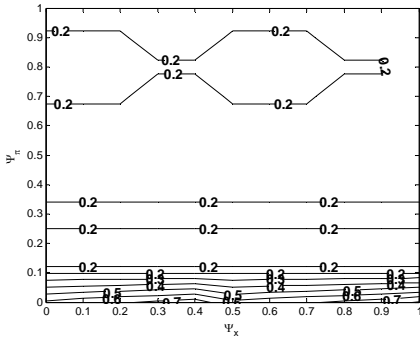


Figure 8 – Optimal χ_π in the absence of Trade Costs

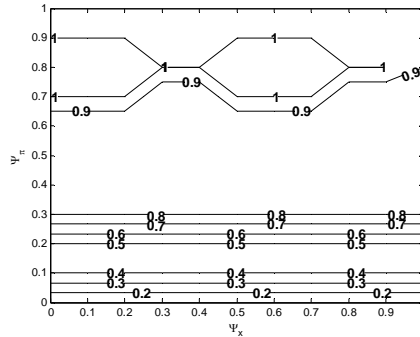
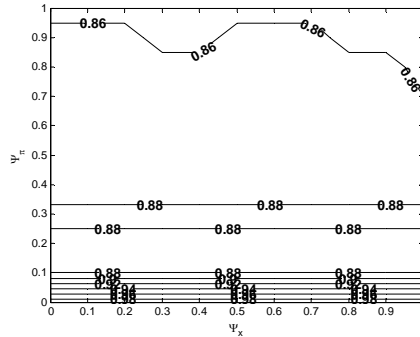


Figure 10 – Optimal ρ_i in the absence of Trade Costs



Figures 3-10 show that the loss function specification of the central bank (i.e., the (ψ_π, ψ_x) values) together with the inclusion of trade costs plays a big role in the determination of optimized MPRs. We use this information to compare the performance of estimated MPR with the optimized MPRs in the following exercise.

4.2.3. Exercise 3

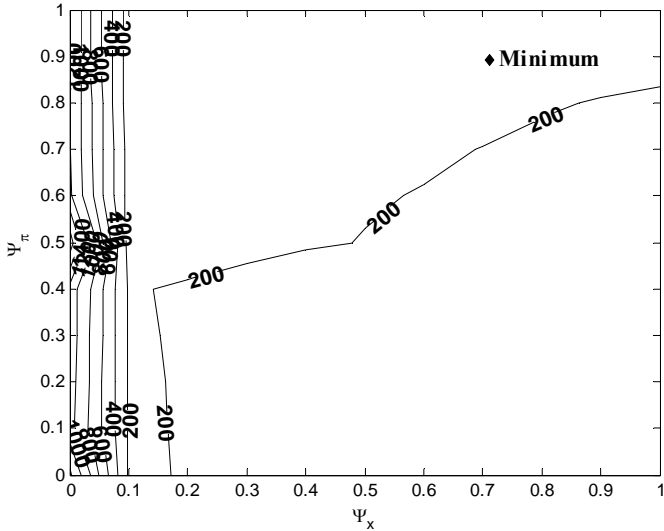
By considering the expected loss functions calculated by Exercise 1 and Exercise 2, this subsection compares the performance of the estimated (historical) monetary policy rule of the Bank of Canada with the performance of the optimized monetary policy rule in terms of expected loss in the economy, under all possible (ψ_π, ψ_x) pairs together with considering the effect of trade costs. By this comparison, we search for the weights assigned to

inflation, output and interest rate volatilities in the loss function by which the actions of the Bank of Canada are explained best.

In particular, we consider three different cases:

- Case 1. The presence of trade costs, i.e., the unrestricted version of our model.
- Case 2. The absence of trade costs, i.e., the restricted version of our model in which $\tau_t = 0$ for all t .
- Case 3. The hybrid case in which trade costs exist, but the Bank of Canada ignores them.

Figure 11 - Percentage Deviation from Optimal Expected Loss in the Presence of Trade Costs



For Case 1, we compare the expected loss values in Figure 1 and Figure 3. We make this comparison by calculating the percentage deviation of the expected loss under estimated monetary policy from the one under optimal monetary policy. The results are given in Figure 11. As is evident from Figure 11, the percentage deviation takes lower values towards $(\psi_\pi, \psi_x) = (0.9, 0.7)$ at which it reaches its minimum. According to these values,

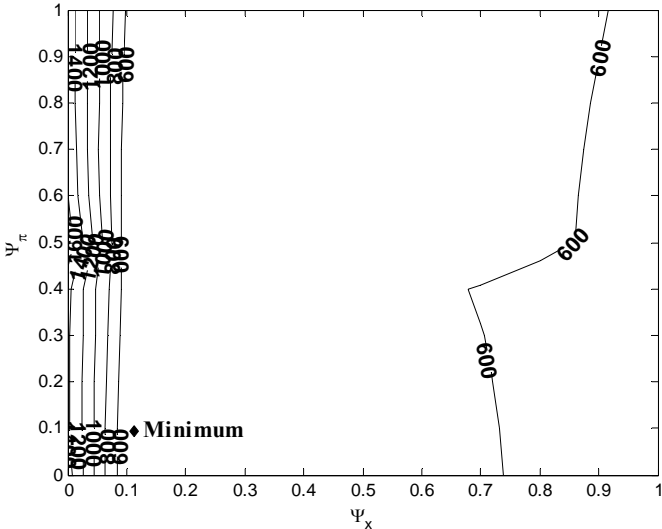
for Case 1, it follows that the Bank of Canada assigns 90% of weight to inflation, 7% of weight to output gap and 3% weight to interest rate in the loss function.

According to the calculated weights, the optimal MPR for Case 1 is implied as follows:

$$\chi_{\pi}^o = 2.2; \chi_x^o = 0.08; \rho_i^o = 0.57$$

Compared to the estimated/historical MPR in Table 1, the optimal $\chi_{\pi}^o = 2.2$ and $\rho_i^o = 0.57$ values are lower while the optimal $\chi_x^o = 0.08$; value is almost the same.

Figure 12 - Percentage Deviation from Optimal Expected Loss in the Absence of Trade Costs



For Case 2, we compare the expected loss values in Figure 2 and Figure 7. We make this comparison again by calculating the percentage deviation of the expected loss under the estimated monetary policy from the one under optimal monetary policy. The results are given in Figure 12. As is evident from Figure 12, the percentage deviation takes lower values toward $(\psi_{\pi}, \psi_x) = (0.1, 0.1)$ at which it reaches its minimum. According to these

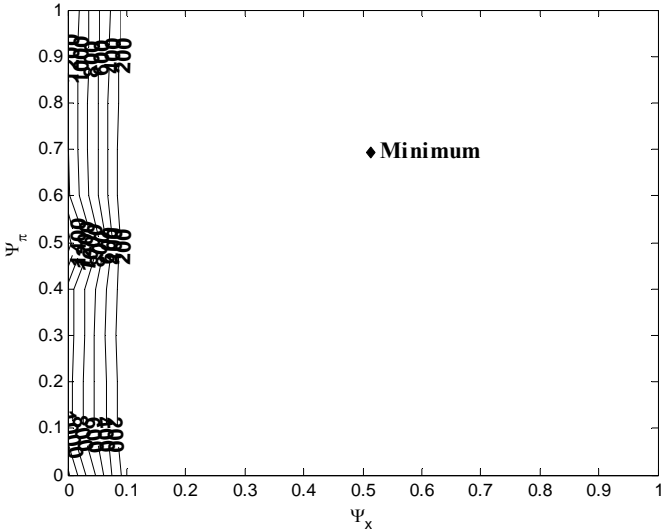
values, for Case 2, it is implied that the Bank of Canada assigns 10% of weight to inflation, 9% of weight to output gap and 81% weight to interest rate in the loss function.

According to the calculated weights, the optimal MPR for Case 2 is implied as follows:

$$\chi_{\pi}^o = 0.9; \chi_x^o = 0.27; \rho_i^o = 0.85$$

Compared to the estimated/historical MPR in Table 1, the optimal $\chi_{\pi}^o = 0.9$ and $\rho_i^o = 0.85$ values are lower while the optimal $\chi_x^o = 0.27$; is higher.

Figure 13 - Percentage Deviation from Optimal Expected Loss for the Hybrid Case



For Case 3, we compare the expected loss values in Figure 2 and Figure 3. We again make this comparison by calculating the percentage deviation of the expected loss under the estimated monetary policy from the one under optimal monetary policy. The results are given in Figure 13. As is evident from Figure 13, the percentage deviation takes lower values toward $(\psi_{\pi}, \psi_x) = (0.7, 0.5)$ at which it reaches its minimum. According to these

values, for Case 3, it is implied that the Bank of Canada assigns 70% of weight to inflation, 15% of weight to output gap and 15% weight to interest rate in the loss function.

According to the calculated weights, the optimal MPR for Case 3 is implied as follows:

$$\chi_{\pi}^o = 2.2; \chi_x^o = 0.08; \rho_i^o = 0.57$$

which is the same as in Case 1.

Table 6
Expected Loss Values

<i>Case</i>	<i>Monetary Policy Rule</i>		Percentage Deviation
	Estimated MPR	Optimized MPR	
Presence of Trade Costs	3.77×10^{-6}	3.44×10^{-6}	9%
Absence of Trade Costs	1.60×10^{-6}	2.34×10^{-8}	422%
Hybrid Case	4.40×10^{-6}	4.40×10^{-6}	0%

Notes: MPR stands for Monetary Policy Rule. Percentage deviation is defined as 100 times the log difference between the expected loss functions under estimated MPR and optimized MPR.

Now, we have to find a criterion to evaluate which case is more likely to represent the actions of the Bank of Canada. We achieve this by considering the percentage deviation of the historical monetary policy rule from the optimal monetary policy rule in terms of expected loss values for each case. The results are given in Table 6. As is evident, the

minimum percentage deviation is achieved by the Hybrid Case, which suggests that the actions of the Bank of Canada are explained best when trade costs in fact exist but the Bank of Canada ignores them.²¹ This is the most important result of this paper which shows that trade costs are in fact important in forming monetary policy, and thus, they must be taken into account; otherwise, the central bank authorities can easily deviate from the optimal monetary policy rules.

4.3. Impulse Response Functions

This subsection compares the impulse response functions under the estimated (historical) monetary policy with the ones under optimal monetary policy (both utility-based and ad hoc), after possible types of shocks. We consider the cases with trade costs in our analysis. The results under simultaneous shocks of technology, trade cost and foreign interest rate are given in Figures 14-17. We consider simultaneous shocks rather than individual shocks, because, according to our data, they are the possible shocks that the economy can experience in a typical period.

Figure 14 – Response of Output Gap

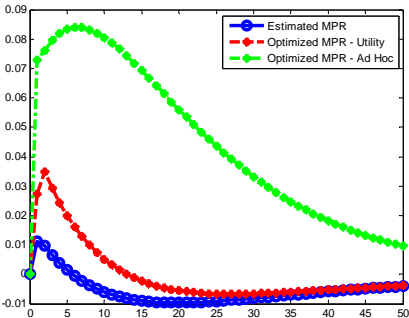
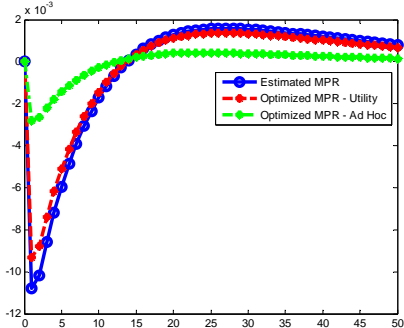


Figure 15 – Response of Inflation



²¹ When we compare the discounted (lifetime) value of the deviation of consumption between optimal and historical monetary policy rules, we see that the consumption implied by the historical rule deviates around 101% from the one implied by the optimal rule, in the presence of trade costs. The deviation is around 18% in the absence of trade costs. In the Hybrid Case, the deviation is calculated as 99%.

Figure 16 – Response of Nominal Interest

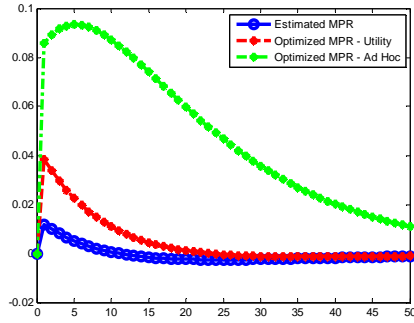


Figure 17 – Response of Real Exchange Rate

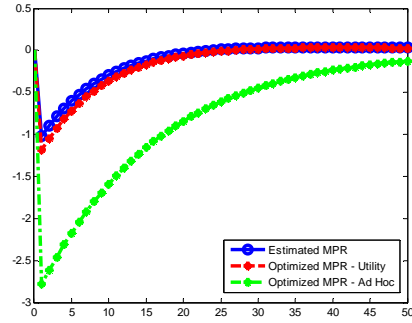


Figure 14 compares the response of output gap to three simultaneous shocks under estimated and optimal MPRs. As is evident, the volatility in output gap is best controlled under estimated MPR, while it is highest under optimized MPR found by the ad hoc expected loss function. Nevertheless, it is the opposite case for inflation when we consider Figure 15: the volatility in inflation is best controlled under optimized MPR found by the ad hoc expected loss function, while it is highest under estimated MPR. Similar comparisons can be made in Figures 16 and 17.

V. CONCLUSION

We have attempted to discover the performance of the Bank of Canada on the Canadian inflation targeting experience via a DSGE model with an emphasis on trade costs. Our log-linearized model is expressed in terms of four blocks of equations: aggregate demand (i.e., the IS curve), aggregate supply (i.e., the New-Keynesian Phillips curve), monetary policy rule and stochastic processes. For model parametrization, we estimate the New-Keynesian Phillips curve for the Canadian economy together with the monetary policy rule of the Bank of Canada.

By considering the dynamics of the Canadian economy (i.e., the New-Keynesian Phillips curve and the IS curve), we have calculated optimal monetary policy rules under different scenarios and compared them with the estimated monetary policy rule to evaluate the performance of the Bank of Canada. When we consider a utility-based expected loss function, we find that the actions of the Bank of Canada are far from being optimal.

When we consider an *ad hoc* expected loss function based on inflation, output and interest volatilities, we find the actions of the Bank of Canada are best explained when trade costs actually exist in the economy but the Bank of Canada ignores them. Thus, trade costs play an important role in the determination of monetary policy. Finally, we find that the Bank of Canada assigns 70% of weight to inflation, 15% of weight to interest rate and 15% of weight to output in its *ad hoc* loss function.

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APPENDICES

A. Individuals

We can make our analysis for a *representative* individual who has the following intertemporal lifetime utility function:

$$E_t \left[\sum_{k=0}^{\infty} \beta^k \{U(C_{t+k}) - V(N_{t+k})\} \right] \quad (27)$$

where $0 < \beta < 1$ is the discount factor; $U(C_t)$ is the utility out of consuming a composite index of C_t ; and $V(N_t)$ is the disutility out of working N_t hours. The composite consumption index, C_t , is defined by:

$$C_t = (C_{H,t})^{1-\gamma} (C_{F,t})^\gamma \quad (28)$$

Consumption sub-indexes, $C_{H,t}$ and $C_{F,t}$, are symmetric. These Dixit-Stiglitz type indices are defined by:

$$C_{H,t} = \left[\int_0^1 C_{H,t}(j)^{(\theta-1)/\theta} dj \right]^{\theta/(\theta-1)} \quad \text{and} \quad C_{F,t} = \left[\int_0^1 C_{F,t}(j)^{(\theta-1)/\theta} dj \right]^{\theta/(\theta-1)} \quad (29)$$

where $\theta > 1$ is the price elasticity of demand faced by each monopolist and $C_{H,t}(j)$ and $C_{F,t}(j)$ are the quantities purchased by home agents of domestic and imported goods, respectively. The optimality conditions result in:

$$C_{H,t}(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\theta} C_{H,t} \quad (30)$$

$$C_{F,t}(j) = \left[\frac{P_{F,t}(j)}{P_{F,t}} \right]^{-\theta} C_{F,t}$$

where

$$P_{H,t} = \left[\int_0^1 (P_{H,t}(j))^{1-\theta} dj \right]^{1/(1-\theta)} \quad (31)$$

and

$$P_{F,t} = \left[\int_0^1 \left([P_{F,t}(j)] \right)^{1-\theta} dj \right]^{1/(1-\theta)} \quad (32)$$

Similarly, the demand allocation for home and imported goods implies:

$$C_{H,t} = \frac{(1-\gamma)C_t P_t}{P_{H,t}} \quad (33)$$

and

$$C_{F,t} = \frac{\gamma P_t C_t}{P_{F,t}} \quad (34)$$

where $P_t = (P_{H,t})^{1-\gamma} (P_{F,t})^\gamma$ is the consumer price index (CPI).

The individual household constraint is given by:

$$\int_0^1 [P_{H,t}(j)C_{H,t}(j) + P_{F,t}(j)C_{F,t}(j)] dj + E_t [F_{t,t+1}B_{t+1}] = W_t N_t + B_t + T_t \quad (35)$$

where $F_{t,t+1}$ is the stochastic discount factor, B_{t+1} is the nominal payoff in period $t+1$ of the portfolio held at the end of period t , W_t is the hourly wage, and T_t is the lump sum transfers/taxes.

By using the optimal demand functions, we can rewrite (35) in terms of the composite good as follows:

$$P_t C_t + E_t [F_{t,t+1} B_{t+1}] = W_t N_t + B_t + T_t \quad (36)$$

The home agent's problem is to choose paths for consumption, portfolio and the output of good j . Therefore, the representative consumer maximizes her expected utility [equation (27)] subject to the budget constraint [equation (36)]. By FOC, we obtain:

$$\beta E_t \left[\frac{U_C(C_{t+1}) P_t}{U_C(C_t) P_{t+1}} \right] = \frac{1}{I_t} \quad (37)$$

where $I_t = 1/E_t [F_{t,t+1}]$ is the gross return on the portfolio. Equation (37) represents the traditional intertemporal Euler equation for total real consumption. We also obtain the labor supply decision of the individual as follows:

$$\frac{W_t}{P_t} = \frac{V_N(N_t)}{U_C(C_t)} \quad (38)$$

The problem is analogous for the rest of the world. The Euler equation for the rest of the world would thus be:

$$\beta E_t \left[\frac{u_C^*(C_{t+1}) P_t^* \Xi_t}{u_C^*(C_t^*) P_{t+1}^* \Xi_{t+1}} \right] = E_t [F_{t,t+1}] \quad (39)$$

where Ξ_t is the nominal effective exchange rate. By combining Equations (37) and (39), together with assuming $U(C) = \log C$, one can obtain:

$$C_t = C_t^* Q_t \quad (40)$$

for all t , where $Q_t = \Xi_t P_t^* / P_t$ is the real effective exchange rate. Under the assumption of complete international financial markets, by combining log-linearized version of Equations (37), (39) and (40) together with Equation (7) (the log linear version of $Q_t = \Xi_t P_t^* / P_t$), one can obtain:

$$i_t = i_t^* + E_t [e_{t+1}] - e_t \quad (41)$$

where $i_t = \log(I_t) = \log(1/(E_t[F_{t,t+1}]))$ and $i_t^* = \log(\Xi_t/(E_t[F_{t,t+1}\Xi_{t+1}]))$. Equation (41) is the uncovered interest parity condition given by Equation (10) in the text.

After introducing the micro-foundations of aggregate demand, we can now find a log-linearized equation for the IS curve. From now on, lower case variables will denote the log variables, and the capital letters without time subscript will denote steady-state values of the respective ratios.

B. Firms

We assume that the production function is as follows:

$$Y_t(j) = Z_t N_t(j) \quad (42)$$

where Z_t is an exogenous economy-wide productivity parameter; and N_t is labor input. Accordingly, the marginal cost of production is given by:

$$MC_t^n = (1 - \omega) \frac{W_t}{Z_t} \quad (43)$$

where ω is the employment subsidy. By also using Equation (38) together with assuming $U(C) = \log C$ and $V(N) = N$, we can write the *real* marginal cost as follows:²²

$$\begin{aligned} mc_t &= \log(1-\omega) + w_t - p_{H,t} - z_t \\ &= \log(1-\omega) + c_t + \gamma s_t - z_t \end{aligned} \quad (44)$$

Moreover, If we define the aggregate output in the home country as

$Y_t = \left[\int_0^1 Y_t(j)^{(\theta-1)/\theta} dj \right]^{\theta/(\theta-1)}$, labor market equilibrium implies:

$$N_t = \int_0^1 N_t(j) dj = \frac{Y_t A_t}{Z_t} \quad (45)$$

where $A_t = \int_0^1 \frac{Y_t(j)}{Y_t} dj$ of which equilibrium variations can be shown to be of second order in log terms. Thus, we can write:

$$y_t = z_t + n_t \quad (46)$$

C. Aggregate Demand

For all differentiated goods, market clearing implies:

$$Y_t(j) = C_{H,t}(j) + C_{H,t}^*(j) \quad (47)$$

By using Equation (30), we can rewrite it as follows:

$$Y_t(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\theta} C_{H,t}^A \quad (48)$$

where $C_{H,t}^A = C_{H,t} + C_{H,t}^*$ is the aggregate world demand for the goods produced in the home country. By using Equation (33) and the symmetric version of Equation (34) for the rest of the world, we can rewrite it as follows:

²² Balanced growth requires the relative risk aversion in consumption to be unity, and thus we set $U(C) = \log C$. Following the lead of Hansen (1985), we also assume that labor is indivisible, implying that the representative agent's utility is linear in labor hours so that $V(N) = N$.

$$Y_t(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\theta} \left((1-\gamma) \frac{P_t C_t}{P_{H,t}} + \gamma^* \frac{P_t^* C_t^*}{P_{H,t}^*} \right) \quad (49)$$

By using $Y_t = \left[\int_0^1 Y_t(j)^{(\theta-1)/\theta} dj \right]^{\theta/(\theta-1)}$, we can write:

$$\begin{aligned} Y_t &= \left((1-\gamma) \frac{P_t C_t}{P_{H,t}} + \gamma^* \frac{P_t^* C_t^*}{P_{H,t}^*} \right) \\ &= \left(\frac{P_t}{P_{H,t}} \right) C_t \left((1-\gamma) + \gamma^* \left(\frac{P_t^* P_{H,t}}{P_t P_{H,t}^*} \right) Q_t^{-1} \right) \end{aligned} \quad (50)$$

which implies that we can rewrite Equation (49) as follows:

$$Y_t(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\theta} Y_t \quad (51)$$

Log-linearizing Equation (50) around the steady-state together with using $s_t \equiv p_{F,t} - p_{H,t}$ and Equation (8) will transform it to the following expression:

$$y_t = c_t + \gamma s_t - \tau_t \quad (52)$$

By also using Equation (5) and the log-linearized version of Equation (37) (i.e., Euler), we can rewrite Equation (52) as follows:

$$y_t = E_t(y_{t+1}) - (i_t - E_t(\pi_{H,t+1})) + E_t(\Delta \tau_{t+1}) \quad (53)$$

D. The New-Keynesian Phillips Curve

Now, we have the equation of aggregate demand. In order to find the equation of aggregate supply, we have to analyze the producer part. Our derivation draws on Galí and Monacelli (2005) except for the fact that we consider the target inflation as the steady-state level of inflation.²³ The model employs a Calvo price-setting process, in which producers are able to change their prices only with some probability, independently of other producers and the

²³ For other closed- and open-economy specifications with non-zero steady-state inflation, see Ascari (2004), Bakhshi et al. (2007), Cogley and Sbordone (2006), Kozicki and Tinsley (2003), Amano et al. (2006, 2007), Sbordone (2007).

time elapsed since the last adjustment. It is assumed that producers behave as monopolistic competitors. Accordingly, each producer faces the following demand function:

$$Y_t(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\theta} C_{H,t}^A, \quad (54)$$

where $C_{H,t}^A = C_{H,t} + C_{H,t}^*$ is the aggregate world demand for the goods produced. Note that this expression is the same with Equation (48).

Assuming that each producer is free to set a new price at period t , it follows that the objective function can be written as:

$$\max_{\tilde{P}_{H,t}} E_t \left[\sum_{k=0}^{\infty} \alpha^k F_{t,t+k} \left\{ Y_{t+k} \left(\tilde{P}_{H,t} - MC_{t+k}^n \right) \right\} \right] \quad (55)$$

where $\tilde{P}_{H,t}$ is the new price chosen in period t , and α is the probability that producers maintain the same price of the previous period. The problem of producers is to maximize equation (55) subject to Equation (54). The first order necessary condition (FONC) of the firm for this maximization is:

$$E_t \left[\sum_{k=0}^{\infty} \alpha^k F_{t,t+k} \left\{ Y_{t+k} \left(\tilde{P}_{H,t} - \zeta MC_{t+k}^n \right) \right\} \right] = 0 \quad (56)$$

where $\zeta \equiv \theta/(\theta-1)$ is a markup as a result of market power. Using Equation (37), we can rewrite Equation (56) as follows:

$$E_t \left[\sum_{k=0}^{\infty} (\beta\alpha)^k \frac{Y_{t+k}}{C_{t+k}} \frac{P_{H,t-1}}{P_{t+k}} \left\{ \frac{\tilde{P}_{H,t}}{P_{H,t-1}} - \zeta \Pi_{t-1,t+k}^H MC_{t+k} \right\} \right] = 0 \quad (57)$$

where $\Pi_{t-1,t+k}^H = \frac{P_{H,t+k}}{P_{H,t-1}}$ and $MC_{t+k} = \frac{MC_{t+k}^n}{P_{H,t+k}}$. Log-linearizing equation Equation (57) around trend inflation $\bar{\Pi}$ together with balanced trade results in:

$$\tilde{p}_{H,t} = \varphi + p_{H,t-1} + \bar{\Pi} E_t \left[\sum_{k=0}^{\infty} (\beta\alpha)^k \pi_{H,t+k} \right] + \bar{\Pi} (1 - \beta\alpha) E_t \left[\sum_{k=0}^{\infty} (\beta\alpha)^k \widehat{mc}_{t+k} \right] \quad (58)$$

where $\varphi = 1 - \bar{\Pi}(1 - \bar{\pi})$ and $\bar{\pi} = \log \bar{\Pi}$ are constants; $\widehat{mc}_t = mc_t - mc$ is the log deviation of real marginal cost from its steady state value, $mc = -\log \zeta$. Equation (58) can be rewritten as:

$$\tilde{p}_{H,t} - p_{H,t-1} = (1 - \beta\alpha)\varphi + \beta\alpha E_t \left[\tilde{p}_{H,t} - p_{H,t-1} \right] + \bar{\Pi}\pi_{H,t} + \bar{\Pi}(1 - \beta\alpha)\widehat{mc}_t \quad (59)$$

In equilibrium, each producer that chooses a new price in period t will choose the same price and the same level of output. Then the (aggregate) price of domestic goods will obey:

$$P_{H,t} = \left[\alpha P_{H,t-1}^{1-\theta} + (1-\alpha)\tilde{P}_{H,t}^{1-\theta} \right]^{1/(1-\theta)} \quad (60)$$

which can be log-linearized as follows:

$$\pi_{H,t} = (1-\theta)\left(\tilde{p}_{H,t} - p_{H,t-1}\right) \quad (61)$$

Finally, by combining Equations (59) and (61), we obtain an expression for the New-Keynesian Phillips curve (Equation (16) in the main text):

$$\pi_{H,t} = \lambda_\pi E_t \left[\pi_{H,t+1} \right] + \lambda_m \left(\varphi + \widehat{mc}_t \right) \quad (62)$$

where $\lambda_\pi = \frac{\beta\alpha}{1-(1-\alpha)(\bar{\Pi})}$, $\lambda_m = \frac{(1-\alpha)(1-\alpha\beta)}{1-(1-\alpha)(\bar{\Pi})}$, and $\varphi = 1 - \bar{\Pi}(1 - \bar{\pi})$. Note that this expression reduces to zero-inflation steady state New-Keynesian Phillips curve when $\bar{\pi} = 0$ (i.e., $\bar{\Pi} = 1$).

E. Equilibrium Dynamics

We start with obtaining an expression for real marginal cost in terms of output. In particular, we can combine Equations (44) and (52) as follows:

$$mc_t = \log(1 - \omega) + y_t - z_t + \tau_t \quad (63)$$

By using the symmetric version of Equation (52) for the rest of the world, namely $y_t^* = c_t^* + \gamma^* s_t^* - \tau_t$, together with Equations (8), (40) and $s_t + s_t^* \equiv 2\tau_t$, we can also obtain:

$$y_t = y_t^* + s_t - \tau_t \quad (64)$$

As discussed in Rotemberg and Woodford (1999), under the assumption of a constant employment subsidy ω that neutralizes the distortion associated with firms' market power, it can be shown that the optimal monetary policy is the one that replicates the flexible price equilibrium allocation in a closed economy. That policy requires that real marginal costs

(and thus mark-ups) are stabilized at their steady state level, which in turn implies that domestic prices be fully stabilized. However, as shown by Gali and Monacelli (2005), there is an additional source of distortion in open economy models: the possibility of influencing the terms of trade in a way beneficial to domestic consumers. Nevertheless, an employment subsidy can be found that exactly offsets the combined effects of market power and the terms of trade distortions, thus rendering the flexible price equilibrium allocation optimal. In order to show this, consider the optimal allocation from the social planner's point of view: maximize Equation (27) subject to Equations (42), (45), (50) and (51). This optimization results in a constant level of employment, $N_t = 1$.

On the other hand, as in Gali and Monacelli (2005), flexible price equilibrium satisfies:

$$\frac{\theta - 1}{\theta} = \overline{MC}_t \quad (65)$$

where \overline{MC}_t stands for real marginal cost at flexible price equilibrium. If we combine Equations (38), (43), (65) with the optimal allocation of the social planner's problem (i.e., $N_t = 1$), we can obtain:

$$\frac{\theta - 1}{\theta} = 1 - \omega \quad (66)$$

which suggests that an employment subsidy can be found that exactly offsets the combined effects of market power and the terms of trade distortions.

After defining domestic natural level of output as the one satisfying flexible price equilibrium (i.e., Equation (63) with $mc_t = -\log \zeta$), we can write it as follows:

$$\bar{y}_t = -\log \zeta - \log(1 - \omega) + z_t - \tau_t \quad (67)$$

which can be rewritten by using Equation (66) as follows:

$$\bar{y}_t = z_t - \tau_t \quad (68)$$

Now, we can define output gap as the deviation of (log) domestic output (i.e., y_t) from the domestic natural level of output as follows:

$$x_t = y_t - \bar{y}_t \quad (69)$$

By using Equation (63), we can also write the (log) deviation of real marginal cost from its steady state in terms of output gap as $\widehat{mc}_t = x_t$, which implies that the New-Keynesian Phillips curve can be written in terms of output gap as follows:

$$\pi_{H,t} = \lambda_\pi E_t [\pi_{H,t+1}] + \lambda_m (\varphi + x_t) \quad (70)$$

By using Equations (53), (67) and (69), we can write the IS curve in terms of output gap as follows:

$$x_t = E_t (x_{t+1}) - (i_t - E_t (\pi_{H,t+1})) + \Delta z_{t+1} \quad (71)$$

F. Utility-Based Welfare

The period specific utility from consumption, $U(C_t)$, and disutility from working, $V(N_t)$, can be second-order approximated around their steady states as follows:

$$U(C_t) = c_t + t.i.p. + o(\|a^3\|) \quad (72)$$

and

$$V(N_t) = n_t + \frac{1}{2} n_t^2 + t.i.p. + o(\|a^3\|) \quad (73)$$

where *t.i.p.* represents terms independent of policy and $o(\|a^3\|)$ represents terms that are higher than 3rd order. We have used the steady state relation $V_N(N)N = U_C(C)C$ together with our assumptions $U(C) = \log C$ and $V(N) = N$ for Equations (72) and (73). By using Equation (52), its symmetric version for the rest of the world, $s_t + s_t^* \equiv 2\tau_t$, log version of Equation (40) and Equation (8), we can write the following expression for c_t :

$$c_t = (1-\gamma)y_t + \gamma y_t^* + (1-\gamma)\tau_t \quad (74)$$

Defining $\tilde{c}_t = c_t - \bar{c}_t$ as the deviation of (log) consumption from its flexible pricing equilibrium, we can write:

$$c_t = (1-\gamma)x_t + (1-\gamma)\bar{y}_t + \gamma\bar{y}_t^* + (1-\gamma)\bar{\tau}_t \quad (75)$$

which can be inserted into Equation (72). Related to Equation (73), after defining $\tilde{n}_t = n_t - \bar{n}_t$ as the deviation of (log) employment from its flexible pricing equilibrium, by using the log version of Equation (45), we can write:

$$n_t = x_t + a_t + \bar{y}_t - \bar{z}_t \quad (76)$$

where $a_t = \log \left(\frac{\int_0^1 Y_t(j) dj}{Y_t} \right) = \log \left(\frac{\int_0^1 P_{H,t}(j) dj}{P_{H,t}} \right)$ by using Equation (51) and we have used $\bar{a}_t = 0$ which is implied by the definition of flexible pricing. Then, by using Equations (75) and (76), we can write:

$$U(C_t) - V(N_t) = - \left(\gamma x_t + a_t + \frac{1}{2} (x_t + a_t + \bar{y}_t - \bar{z}_t)^2 \right) + t.i.p. \quad (77)$$

The following lemmas are helpful for our analysis.

Lemma 1. $a_t = \frac{\theta}{2} \text{var}_i(p_{H,t}(i)) + o(\|a^3\|)$.

Proof: See Galí and Monacelli (2005), p.732.

Lemma 2. $\sum_{t=0}^{\infty} \beta^t \text{var}_i(p_{H,t}(i)) = \frac{1}{\lambda_w} \sum_{t=0}^{\infty} \beta^t \pi_{H,t}^2$ where $\lambda_w = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$.

Proof: See Woodford (2003), Chapter 6.

According to our lemmas and Equations (12), (64), (69), (77), we can write the utility-based welfare function as follows:

$$\begin{aligned} E_t \sum_{k=0}^{\infty} \beta^{t+k} (U(C_{t+k}) - V(N_{t+k})) &= E_t \sum_{k=0}^{\infty} \beta^{t+k} (\log \zeta + \tau_{t+k} - \gamma) s_{t+k} \\ &\quad - \frac{1}{2} E_t \sum_{k=0}^{\infty} \beta^{t+k} \left((1 - \log \zeta - \tau_{t+k}) \frac{\theta}{\lambda_w} (\pi_{H,t+k})^2 + (x_{t+k})^2 \right) \\ &\quad + t.i.p + o(\|a^3\|) \end{aligned} \quad (78)$$

Note that the utility-based welfare function depends on the volatility in inflation and output gap as well as the trade costs and the terms of trade.

G. Model Solution

The dynamic system is given by the main Equations (5), (11), (14), (20), (70), by the exogenous shock Equations (3), (12), (15), by the definition of domestic inflation $\pi_{H,t} = p_{H,t} - p_{H,t-1}$ and by the definition of output gap $x_t = y_t - \bar{y}_t$ and Equation (67). For simplicity, after substituting $x_t = y_t - \bar{y}_t$ and Equation (67) into Equations (14), (20), (70) and after substituting Equation (5) into Equation (20), we can rewrite the equations used in the solution of the model as follows:

$$y_t + \tau_t - E_t(y_{t+1} + \tau_{t+1}) + (i_t - E_t(\pi_{H,t+1})) = 0 \quad (79)$$

$$\pi_{H,t} - \lambda_\pi E_t[\pi_{H,t+1}] - \lambda_m (y_t - z_t + \tau_t) = 0 \quad (80)$$

$$s_t - i_t^* + (i_t - E_t[\pi_{H,t+1}]) - E_t[s_{t+1} - \tau_{t+1} + \tau_t] = 0 \quad (81)$$

$$\begin{aligned} i_t - \rho_i i_{t-1} - (1 - \rho_i) \chi_\pi [E_t(\pi_{H,t+1})] - (1 - \rho_i) \chi_x [E_t(y_t - z_t + \tau_t)] \\ - \gamma (1 - \rho_i) \chi_\pi [E_t(s_{t+1} - s_t)] = 0 \end{aligned} \quad (82)$$

$$\pi_{H,t} - p_{H,t} + p_{H,t-1} = 0 \quad (83)$$

$$i_t^* = \rho_i^* i_{t-1}^* + \varepsilon_t^i \quad (84)$$

$$\tau_t = \rho_\tau \tau_{t-1} + \varepsilon_t^\tau \quad (85)$$

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z \quad (86)$$

where we have set all the constants equal to zero.²⁴ Following the lead of Uhlig (1997), the vector of endogenous state variables is $\mathbf{x}_t = [i_t \quad p_{H,t} \quad y_t \quad s_t]'$, the single vector of non-predetermined variable (jump variable) is $\mathbf{y}_t = [\pi_{H,t}]$ and the vector of shock variables is $\mathbf{z}_t = [i_t^* \quad \tau_t \quad z_t]'$. The model in matrix form is thus:

$$\mathbf{A}\mathbf{x}_t + \mathbf{B}\mathbf{x}_{t-1} + \mathbf{C}\mathbf{y}_t + \mathbf{D}\mathbf{z}_t = 0$$

$$E_t[\mathbf{F}\mathbf{x}_{t+1} + \mathbf{G}\mathbf{x}_t + \mathbf{H}\mathbf{x}_{t-1} + \mathbf{J}\mathbf{y}_{t+1} + \mathbf{K}\mathbf{y}_t + \mathbf{L}\mathbf{z}_{t+1} + \mathbf{M}\mathbf{z}_t] = 0 \quad (87)$$

$$\mathbf{z}_{t+1} = \mathbf{N}\mathbf{z}_t + \boldsymbol{\varepsilon}_{t+1}$$

In our case, we will rewrite Equation (83) in matrix form as follows:

$$\mathbf{A}\mathbf{x}_t + \mathbf{B}\mathbf{x}_{t-1} + \mathbf{C}\mathbf{y}_t + \mathbf{D}\mathbf{z}_t = 0 \quad (88)$$

²⁴ Setting all constants equal to zero doesn't affect our results at all

where $\mathbf{A} = [0 \ 1 \ 0 \ 0]$, $\mathbf{B} = [0 \ -1 \ 0 \ 0]$, $\mathbf{C} = [1]$, and $\mathbf{D} = [0 \ 0 \ 0 \ 0]$.

We can write Equations, (79),(80), (81) and (82) in matrix form as follows:

$$E_t [\mathbf{F}\mathbf{x}_{t+1} + \mathbf{G}\mathbf{x}_t + \mathbf{H}\mathbf{x}_{t-1} + \mathbf{J}\mathbf{y}_{t+1} + \mathbf{K}\mathbf{y}_t + \mathbf{L}\mathbf{z}_{t+1} + \mathbf{M}\mathbf{z}_t] = 0 \quad (89)$$

where

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -\gamma(1-\rho_i)\chi_\pi \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -\lambda_m & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & -(1-\rho_i)\chi_x & \gamma(1-\rho_i)\chi_\pi \end{bmatrix},$$

$$\mathbf{H} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\rho_i & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{J} = \begin{bmatrix} -1 \\ -\lambda_\pi \\ -1 \\ -(1-\rho_i)\chi_\pi \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix},$$

$$\mathbf{L} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{M} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\lambda_m & \lambda_m & 0 \\ -1 & -1 & 0 & 0 \\ 0 & -(1-\rho_i)\chi_x & (1-\rho_i)\chi_x & 0 \end{bmatrix}.$$

Finally, we can rewrite Equations (84), (85) and (86) in matrix form as follows:

$$\mathbf{z}_{t+1} = \mathbf{N}\mathbf{z}_t + \boldsymbol{\varepsilon}_{t+1} \quad (90)$$

where

$$\mathbf{N} = \begin{bmatrix} \rho_i^* & 0 & 0 \\ 0 & \rho_\tau & 0 \\ 0 & 0 & \rho_z \end{bmatrix}, \quad \boldsymbol{\varepsilon}_{t+1} = \begin{bmatrix} \boldsymbol{\varepsilon}_{t+1}^i \\ \boldsymbol{\varepsilon}_{t+1}^r \\ \boldsymbol{\varepsilon}_{t+1}^z \end{bmatrix}$$

H. Data Appendix

The data cover the monthly sample over the period 1996:1 to 2006:12. The data sources are the web page of the Bank of Canada (<http://www.bankofcanada.ca>), the online version of the International Financial Statistics (IFS), and the Energy Information Administration. The details are below.

1. For the data downloaded from the web page of the Bank of Canada:
 - ◆ 12 month growth rate in total CPI has been used for Canadian inflation.
 - ◆ Overnight rate has been used for Canadian short-term interest rate.
 - ◆ Canadian-dollar effective exchange rate index (CERI) has been used for Canadian effective terms of trade.
 - ◆ M1+ (gross) has been used for Canadian M1.
 - ◆ The inflation target has been set to the midpoint of the target range, which is equal to 2.

2. For the data downloaded from online IFS:
 - ◆ Industrial production series (IPS) has been used for Canadian output.
 - ◆ The output gap has been found by detrending Canadian IPS by using Hodrick–Prescott (HP) filter.²⁵
 - ◆ For foreign interest rate, government bond yield of the U.S. for 10 years has been used.

3. For the data downloaded from Energy Information Administration:
 - ◆ All Countries Spot Petroleum Price FOB Weighted by Estimated Export Volume (International Dollars per Barrel) has been used for trade costs.²⁶

²⁵ We use the definition of Khalaf and Kichian (2004) for the measure of output gap. That is, rather than detrending the log of IPS using the full sample, T , we proceed iteratively: to obtain the value of the gap at time t , we detrend IPS with the data ending in T . We then extend the sample by one more observation and re-estimate the trend. This is used to detrend IPS and yields a value for the gap at time $t+1$. This process is repeated until the end of the sample.

²⁶ We have also considered using the “Couriers and Messengers Services Price Index” downloaded from Statistics Canada as an alternative for trade costs. However, the data cover only the period from 2003 to 2006, which is much shorter than our sample period. Nevertheless, from 2003 to 2006, the correlation coefficient between “All Countries Spot Petroleum Price FOB Weighted by Estimated Export Volume” and “Couriers and Messengers Services Price Index” is around 0.95, which can be seen as an indicator of robustness of our analysis.