

# INTERNATIONAL SEIGNIORAGE PAYMENTS

by

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What are the "liquidity services" provided by "over-priced" assets? How do international seigniorage payments affect the choice of monetary policies? Does a country gain when other hold its "over-priced" assets? These questions are analyzed here in a model with demand uncertainty (taste shocks) and sequential trade. It is shown that a country with a relatively stable demand may issue "over priced" debt and get seigniorage payments from countries with unstable demand. But this does not necessarily improve welfare in the stable demand country.

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## 1. INTRODUCTION

Cash may disappear in a technological advanced society. Will seigniorage payments disappear?

In Woodford (2003) cashless economy, money does not enter as an argument into the utility function and therefore interest must be paid on it. In his cashless economy markets are complete, money is "correctly priced" and the government does not get seigniorage payments. In the international context, McKinnon (1969, pp. 17-23) and Grubel (1969, pp. 269-72) argued that competition will drive international seigniorage payments to zero even in the absence of complete markets.

Here I examine the possibility that seigniorage payments between countries will occur even when paying interest on money is technically feasible and money does not enter as an argument in the utility function. Unlike Woodford (2003) I use a model in which markets are incomplete, trade occurs in a sequence of Walrasian markets and uncertainty about demand causes price dispersion.

Price dispersion allows for the distinction between the rate of return on the asset and its "liquidity". The rate of return depends on the asset and not on the individual who holds it. Liquidity is an individual specific attribute. It depends on the probability that you will be able to use the asset to buy at the low price when you want to consume.

For the sake of concreteness I assume two countries: The home country (US) and the foreign country (Japan or the rest of the world).

There are two assets: US government bonds and Japanese government bonds. These assets will be called dollars and yen for short.

In the model risk neutral sellers choose both the price and the asset that they are willing to accept. They may choose a low price or a high price and may state that they are willing to accept the equivalent amount in terms of any asset or just in terms of a particular asset. In the equilibrium we study high price sellers choose to accept both assets, but low price sellers choose to accept dollars only. Since dollars are generally accepted, in equilibrium its rate of return is lower than the rate of return on the yen. The difference between the yen rate of return and the dollar rate of return is a "liquidity premium". (You pay a "liquidity premium" for holding dollars or you get an "illiquidity premium" for holding yen).

When making portfolio choices, individuals take into account the probability that they will find the good at the low price. This is relevant because the liquidity advantage of the dollar is realized only when the individual finds the good at the low price. An individual who typically buys in the high demand state has a low chance of finding the good at the low price and will therefore require a relatively small "illeguidity premium" to hold yen. In equilibrium only the agents who typically buy in the high demand state will hold both assets.

I assume that the demand of the Japanese is erratic and plays the role of "aggregate demand shifter". The demand of the Americans is stable. Since the Americans are more likely to buy at the low aggregate demand state they are willing to pay a relatively high premium for holding the generally accepted asset. In equilibrium the dollar promises a lower rate of return but is more "liquid" than the yen. For the

Japanese the liquidity of the dollar exactly compensates for its lower rate of return and in equilibrium they hold both assets. The Americans are willing to pay a higher "liquidity premium" and accept dollars only.

As was mentioned above, price dispersion is required for our definition of "liquidity". In Prescott's (1975) "hotels" model there is price dispersion. Versions of the Prescott model have been studied by, among others, Bryant (1980), Rotemberg and Summers (1990), Dana (1998) and Deneckere and Peck (2005). Here I use a flexible price version: The uncertain and sequential trade (UST) model in Eden (1990, 1994) and Lucas and Woodford (1993).

## 2. THE MODEL

I consider a single good overlapping generations model. There are two countries. The demand in the home country (US) is stable (predictable). The demand in the foreign country (Japan) is unstable. I start with the case of autarky assuming a single asset.

### Autarky:

A new generation is born each period. Individuals live for two-periods. They work in the first period of their life and if they want they consume in the second.

The utility function for the representative agent born at  $t$  is:  $E\{\theta_{t+1}\beta c_{t+1} - A_t v(L_t)\}$ , where  $c_{t+1}$  is the amount of second period consumption,  $\beta$  is a discount factor and  $L_t$  is the amount of first period labor. Expectations are taken over two independently distributed random variables:  $\theta_{t+1}$  is a "taste shock" and  $A_t$  is a "productivity shock". The

taste shock is the driving force behind the results. It is assumed that  $\theta_{t+1}$  is an i.i.d random variable that can take the realizations 1 with probability  $\pi$  and 0 otherwise. The shock to technology plays a minor role. A unit of labor produces  $A_t$  units of output so that  $A_t L_t$  is output. The cost of supplying  $L$  depends on productivity because the home production alternative depends on it. To simplify, I assume that the gross rate of change in productivity  $\varepsilon = A_t/A_{t-1}$  is an i.i.d random variable with  $E(\varepsilon) = 1$ . Assuming an arbitrarily given mean  $E(\varepsilon)$  will not change the results. For simplicity I assume:  $v(L) = (\frac{1}{2})(L)^2$ .

The realization of the productivity shock  $A_t$  is known before the choice of the labor input but the taste shock  $\theta$  is known only after the choice of output. Output produced will be sold only when  $\theta = 1$ . As in Abel (1985), when the old generation experiences  $\theta = 0$  they transfer their balances to the young generation as an accidental bequest. An alternative formulation may assume that agents derive utility from bequest as in Barro (1974), but the weight they assign to the utility of future generations is random. The main results will not change if this more general specification is employed.

There is a single asset (government bonds) called yen. After receiving interest payments buyer  $h$  (an old agent) has  $M_t^h$  yen. He then gets a perfectly anticipated lump sum transfer of  $G_t$  yen. The average per-buyer post-interest payment balances are:  $M_t = (\frac{1}{N}) \sum_{h=1}^N M_t^h$ . The average post-transfer balances are:  $M_{t+1} = G_t + M_t$ . The deterministic rate of change in the asset supply is:  $M_{t+1}/M_t = 1 + \mu$ . In what follows I normalize:  $N = 1$ .

The representative young agent born at time  $t$  takes the yen prices of the consumption good  $(P_t, P_{t+1})$  and the yen amount of the transfer payment  $(G_{t+1})$  as given. If the old generation experiences  $\theta_t = 1$ , he sells his output and gets  $P_t A_t L_t$  yen for it. He then deposits the revenue in a government owned bank that pays interest at the nominal rate  $i$ .<sup>2</sup> His pre-transfer next period balances are thus:

$M_{t+1} = P_t A_t L_t (1 + i)$ . When  $\theta_t = 0$ , the old agent leave

$M_{t+1} = (M_t + G_t)(1 + i)$  as accidental bequest. The expected next period post-transfer balances are:

$$(1) \quad B_{t+1} = \pi [P_t A_t L_t (1 + i) + G_{t+1}] + (1 - \pi) (M_{t+1} + G_{t+1})$$

The worker will use these balances in the next period if  $\theta_{t+1} = 1$ . He therefore chooses  $L$  by solving:

$$(2) \quad \max_L - A_t v(L_t) + \pi \beta B_{t+1} E(1/P_{t+1}) \quad \text{s.t. (1)}.$$

The first order condition for this problem is:

$$(3) \quad A_t v'(L_t) = \pi^2 \beta A_t R_{t+1},$$

where  $R_{t+1} = E[P_t(1 + i)/P_{t+1}]$  is the gross real rate of interest. We may think of  $\pi^2 \beta A_t R$  as the (expected discounted) real wage. The term  $\pi^2$

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<sup>2</sup> We may think of a check or a debit card transaction in which the money is transferred directly from one interest paying account to another. Alternatively, we may assume that the government pays, in addition to the lump sum transfer, a proportional transfer of  $i$  yen per yen.

plays a role because a unit produced yields utility to the producer only if it is sold (with probability  $\pi$ ) and only if he will want to consume (also with probability  $\pi$ ). The probability of this joint event is  $\pi^2$ . The real wage is therefore  $\beta A_t R$  with probability  $\pi^2$  and zero otherwise. The first order condition (3) says that the marginal cost must equal the expected real wage.

We require market clearing when  $\theta_t = 1$ . That is,

$$(4) \quad P_t A_t L_t = M_t + G_t$$

I focus on an equilibrium in which inflation is constant and the nominal price of a unit of labor ( $A_t$  units of output) is proportional to the post-transfer asset supply. I thus assume a normalized price  $p$  such that:

$$(5) \quad P_t A_t = p M_t (1 + \mu)$$

Substituting (5) in the first order condition (3) leads to:

$$(6) \quad L = \pi^2 \beta R$$

where  $R = 1 + r = (1 + i)/(1 + \mu)$  is the now constant gross real rate of return. Note that labor supply does not depend on the realization of the productivity shock:  $A_t$ . This is due to the assumption that the cost of labor is proportional to productivity. An alternative is to allow for an income effect that will cancel the substitution effect.



Note also that the government can vary  $R$  (and  $L$ ) by varying  $\mu$  and  $i$ .

With the risk of repetition I now set the problem in magnitudes that are normalized by the post-transfer asset supply. This will become useful later when full integration is considered. A normalized yen (NY) is  $M_t(1 + \mu)$  regular yen. The nominal price of a unit of labor ( $A_t$  units of output) is  $p = P_t A_t / M_t(1 + \mu)$  NY. When the price of  $A_t$  units is half NY it means that you have to pay half of the post-transfer asset supply to get  $A_t$  units. Since the asset supply changes over time we must renormalize every period. A normalized yen (NY) in the current period that is carried to the next period is worth

$$\omega = M_t(1 + \mu) / M_{t+1}(1 + \mu) = (1 + \mu)^{-1} \text{ in terms of next period's NYs.}$$

A worker (young agent) who sells a unit of labor ( $A_t$  units of output) for  $p$  NYs will have in the next period  $p(1 + i)\omega = pR$  normalized yen. The expected real wage conditional on selling is  $pRZ$ , where  $Z$  is the expected purchasing power of a normalized yen. To define  $Z$  note that next period  $p$  normalized yen will buy  $A_{t+1}$  units and 1 normalized yen will buy  $A_{t+1}/p$  units. Since  $E_t(A_{t+1}) = A_t$ , the expected purchasing power of a yen is:

$$(7) \quad Z_t = \pi A_t / p.$$

When  $\theta_t = 1$  the worker sells his output ( $AL$  units) and gets on average  $\omega(pL)(1 + i)Z = (pL)RZ$  units of consumption in period  $t+1$ . In addition he gets a transfer payment of  $g$  normalized yen. This transfer payment will buy on average  $gZ$  units. His expected consumption when

$\theta_t = 1$  is therefore:  $(pL)RZ + gZ$  units. When  $\theta_t = 0$  the worker does not sell his output but receives a bequest. The value of the bequest plus the transfer payment is 1 (= the post-transfer asset supply). The worker's maximization problem is therefore:

$$(8) \quad \max_L \pi[(pL)R + g]\beta Z + (1 - \pi)\beta Z - Av(L),$$

The first order condition for (8) is:

$$(9) \quad Av'(L) = \beta\pi pRZ = \beta\pi^2 RA,$$

where the last equality uses (7). The market clears when demand is strictly positive ( $\theta = 1$ ):

$$(10) \quad pL = 1.$$

Note that the equilibrium conditions (9) and (10) are the same as (4) and (6) but their derivation does not require algebra.

The sum of (the buyer's and the seller's) period  $t$  utilities:

$$(11) \quad \text{Welfare} = A_t\{\beta\pi L_t - v(L_t)\} = A_t W_t,$$

where  $W_t = \beta\pi L_t - v(L_t)$  is welfare when  $A_t = 1$ . Since  $E(A_{t+1}) = A_t$ , (11) is also the expected utility of the representative young agent in a steady state where  $L$  does not change over time.

Substituting the equilibrium level of labor  $L = \beta\pi^2 R$  in (11), we get:  $W(\pi, R) = \beta\pi(\beta\pi^2 R) - v(\beta\pi^2 R)$ . When  $R \leq 1/\pi$ , this function is

decreasing in  $\pi$  and increasing in  $R$ . Maximum welfare for any given  $\pi$  is attained at  $R = 1/\pi$  and this is therefore the optimal policy.

A planner's problem: To gain some further insight, I now consider a planner who solves:

$$(12) \quad \max_L AW = A\{\pi\beta L - v(L)\}$$

The first order condition for this problem is:

$$(13) \quad v'(L) = \pi\beta$$

Since in equilibrium  $v'(L) = \beta\pi^2 R_t$  efficiency requires:

$$(14) \quad \beta\pi^2 R_t = \pi\beta \text{ or } R_t = 1/\pi$$

Note that when  $\pi < 1$ , efficiency requires a strictly positive interest rate ( $R > 1$ ). This result is similar to the well-known result by Friedman (1969) but here, as in other OG models, the optimal real interest rate does not depend on the discount factor. The argument for  $R > 1$  is however, analogous to Friedman's argument. When  $R = 1$  there is a difference between the social and the private value of a unit produced. From the social point of view, a unit produced will be consumed with probability  $\pi$ . Therefore, its social value is  $\pi\beta$ . From the individual's point of view a unit produced yields utility only if he sells it and only if he wants to consume. This joint event occurs with probability  $\pi^2$ . Therefore when  $R = 1$ , a unit produced is worth to the

individual only  $\beta\pi^2$  units of consumption.  $R > 1$  is required to correct for the difference between the social and the individual's point of view.

Note also that  $\bar{R}\pi = 1$  where  $\bar{R}$  is the optimal choice of  $R$ . This says that at the monetary authorities should compensate the seller for the risk of not making a sale.

Predictability: The taste shock plays an important role in the choice of monetary policy because its realization is not known at the time production decisions are made. To see this point note that if demand is known at the time production decisions are made, the seller will choose  $L = 0$  when  $\theta = 0$ . In this case we can write the seller's problem (8) as:  $\max_L \pi[(pL)R + g]\beta Z + (1 - \pi)\beta Z - \pi Av(L)$ . The first order conditions for this problem is:  $v'(L) = \beta\pi R$  and the optimal monetary policy is  $R = 1$  regardless of  $\pi$ . We will return to this point when discussing potential applications.

Since discounting does not play an important role in the analysis I assume, in what follows,  $\beta = 1$ . To illustrate, Table 1 calculates the equilibrium magnitudes for different values of  $R$  and  $\pi$ .

Table 1: Autarky ( $\beta = 1$ )

$\pi$	$R$	$L$	Welfare/A = W
1	1	1	0.5
0.9	$1/\pi$	$\pi$	0.405
0.9	1	0.81	0.401

monetary policy for a small open economy: I assume that in the US demand is stable and  $\theta = 1$  with probability 1. With a slight abuse of notation I assume that in Japan (the foreign country)  $\theta = 1$  with probability  $\pi < 1$ . I start by considering the choice of monetary policy under the assumption that Japan is small relative to the US. I assume further that there are no barriers to trade but American sellers accept dollars only.

Suppose that we start with the policies (14): In the home country  $R = 1$  and in the foreign country  $R^* = 1/\pi$ . To simplify I assume that  $\mu = i = 0$  and therefore regular dollar prices do not change over time. I also assume  $\beta = A = 1$ .

With these choices of monetary policies, Japanese sellers are indifferent between dollars and yen. If they choose to accept dollars they can always sell to Americans and get a real rate of return  $R = 1$ . If they choose to accept yen they will sell to Japanese with probability  $\pi$  but since  $R^* = 1/\pi$  the expected gross real return is also unity. Similarly, American sellers are indifferent between dollars and yen. We can thus have an equilibrium in which Japanese accept yen only and American accept dollars only. This is an equilibrium because sellers cannot benefit from changing their policies.

But the Japanese government can do better by adopting full dollarization. To see this point let  $\bar{L}^*$  denotes the labor supply of the Japanese when  $R^* = 1/\pi$  (thus  $\bar{L}^*$  solves [12]). Under autarky the maximum steady state utility level is:  $\pi \bar{L}^* - v(\bar{L}^*)$ . Under full dollarization the expected utility of the Japanese is:  
 $\pi \bar{L}^* [\pi + (1 - \pi)2] - v(\bar{L}^*) > \pi \bar{L}^* - v(\bar{L}^*)$ . To see this claim let  $P$  denotes the (constant) regular dollar price of a unit. Under dollarization, the young Japanese can always sell their output and get

$P\bar{L}^*$  dollars for it. When the old Japanese want to consume the young get only the revenue from selling their output. When the old do not want to consume the young get an accidental bequest in addition to the revenue from selling their output. The Japanese expected consumption under full dollarization is therefore:

$$\pi\{\pi(P\bar{L}^*/P) + (1 - \pi)(2P\bar{L}^*/P)\} = \pi\bar{L}^*[\pi + (1 - \pi)2].$$

Thus when  $R = 1$  Japan can improve on  $R^* = 1/\pi$  by adopting full dollarization. We have assumed that Japan is small relative to the US. I now relax this assumption and show that when  $R = 1$ , the adoption of the dollar by Japan will harm the US.

### 3. A FULLY INTEGRATED WORLD ECONOMY

I now allow for trade between the two countries under the assumption of costless transportation and travel.<sup>3</sup> Productivity in the home country is  $A_t$ . Productivity in the foreign country is:  $A_t^* = bA_t$ ,

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<sup>3</sup> A real version of this model that allows for transportation costs is in Eden (forthcoming). In the real version of the model I also distinguish between the case in which goods must be displayed on location before the beginning of trade to the case in which orders are placed first and delivery occurs later. Here I focus on the second delivery to order case. We may think, for example, of the market for resorts. Buyers from all over the world may make reservations on the internet. Those who make early reservations may get relatively cheap vacations. Other examples may be trade in intermediate goods. Ethier (1979) and Sanyal and Jones (1982) emphasize the fact that much of international trade is in intermediate inputs and not in final goods. But for simplicity, I keep the assumption that there is one final good produced by labor only.

where  $b > 0$  is a known constant. As before, I assume that the gross rate of change  $\varepsilon = A_{t+1}/A_t$  is i.i.d with  $E_t(\varepsilon) = 1$ .

The supply of dollars grows at the rate of  $\mu$  because of interest and transfer payments. While all holders of dollars get interest payments, only home country buyers get transfer payments.

After receiving interest payments the representative buyer in the home country holds  $m$  normalized dollars and the representative buyer in the foreign country holds  $1 - m$  normalized dollar, where a normalized dollar (ND) is the post transfer supply of dollars. I start with a steady state analysis in which  $m$  does not change over time.

Trade occurs sequentially. At the beginning of the period buyers who want to buy form a line. When  $\theta = 0$ , only US buyers want to consume and therefore only US buyers get in line. When  $\theta = 1$  buyers from both countries get in line. The place in the line is determined by a lottery that treats all buyers symmetrically. Since the number of buyers is large I assume that any segment of the line represents the population of active buyers.

Active buyers arrive at the market place one by one according to their place in the line and buy at the cheapest available price offer.

The amount that will be spent is  $m$  ND if only the home country buyers want to consume and  $1$  ND if all buyers want to consume. We say that the first  $m$  NDs buy in the first market at the price of  $p_1$  ND per  $A_t$  units. If  $\theta = 1$  an additional amount of  $1 - m$  NDs will arrive, open the second market and buy ( $A_t$  units) at the price  $p_2$ . The use of normalized prices assumes that the regular dollar prices of  $A_t$  units of output is proportional to the money supply:  $P_{st}A_t = p_s M_t (1 + \mu)$ .

When aggregate demand is low and only one market opens the probability of buying at the first market price is unity and the expected purchasing power of a normalized dollar is:  $A_t/p_1$ . When demand is high and two markets open the probability of buying at the first market price is  $m$  (= the fraction of dollars that will buy in the first market). When two markets open, the expected purchasing power of a normalized dollar is:  $mA_t/p_1 + (1-m)A_t/p_2$ . In what follows I use  $z_{st}$  to denote the expected purchasing power of a normalized yen if exactly  $s$  markets open:

$$(15) \quad z_{1t} = \frac{A_t}{p_1} \quad \text{and} \quad z_{2t} = A_t \left( \frac{m}{p_1} + \frac{1-m}{p_2} \right)$$

The unconditional expected purchasing power of a normalized dollar is:

$$(16) \quad Z_t = (1 - \pi)z_{1t} + \pi z_{2t} \quad , \quad \text{for a home country buyer and} \\ Z_t^* = \pi z_{2t} \quad , \quad \text{for a foreign country buyer.}$$

Note that a buyer in the home country will buy regardless of the realization of  $\theta$  and therefore  $Z$  is a weighted average of  $z_1$  and  $z_2$ . A foreign buyer will buy only if  $\theta = 1$ . In this case two markets will open and therefore  $Z^*$  is a weighted average between zero and  $z_2$ .

Sellers (workers) take prices as given. They know that they can sell (in the first market) at the price  $p_1$  with probability 1 and (in the second market) at the price  $p_2$  with probability  $\pi$ . I use  $k_s A_t$  to denote the supply of the home country seller to market  $s$ . The home country seller solves:



$$(17) \quad \max_{k_s} - Av(k_1+k_2) \\ + (1 - \pi)[(p_1k_1)R + g]Z + \pi[(p_1k_1 + p_2k_2)R + g]Z.$$

The first term in (17) is the cost of producing  $k_1 + k_2$  units. The last two terms are the expected consumption. When only one market opens the seller sells  $k_1A_t$  units and his revenues is  $p_1k_1$  ND which are invested at the gross real rate  $R$ . In addition he gets a transfer payment of  $g$  ND so his next period money balances are  $(p_1k_1)R + g$  NDs. When both markets open the seller's revenues are  $p_1k_1 + p_2k_2$  and his next period balances are  $(p_1k_1 + p_2k_2)R + g$ . To convert next period's balances to expected consumption we multiply by  $Z$ .

The representative young agent in the foreign country supplies  $k_s^*A_t^* = k_s^*bA_t$  units to market  $s$ . If he sells it he gets  $bp_s k_s^*$  ND. He therefore solves:

$$(18) \quad \max_{k_s^*} - bAv(k_1^* + k_2^*) \\ + (1 - \pi)[bR(p_1 k_1^*) + (1 - m)]Z^* + \pi bR(p_1 k_1^* + p_2 k_2^*)Z^*$$

Note that the expected purchasing power function is different ( $Z^*$  instead of  $Z$ ) and the foreign agent does not get a transfer payment from the government but in the low demand state he gets a bequest.

It is convenient to use:  $L = k_1 + k_2$  and  $L^* = k_1^* + k_2^*$  for the supply of labor. I focus here on a steady state in which the home country seller supplies to the first market only ( $L = k_1$ ) and the post transfer balances held by the buyer in the home country do not change over time and are given by:

$$(19) \quad m = Rp_1L + g$$

To state the first order condition for the problem (17) [(18)] note that the expected real revenue per unit of labor is  $Rp_1Z$  ( $bRp_1Z^*$ ) if the unit is supplied to the first market and  $\pi Rp_2Z$  ( $\pi bRp_2Z^*$ ) if it is supplied to the second market. At the optimum the marginal cost ( $AV'[L] = AL$ ) must equal the expected real wage:

$$(20) \quad AL = Rp_1Z = \pi Rp_2Z; \quad bAL^* = bRp_1Z^* = \pi bRp_2Z^*$$

A steady state equilibrium is a solution  $(L, L^*, k_1^*, p_1, p_2, m)$  to (19) - (20) and the market clearing conditions:

$$(21) \quad p_1(L + bk_1^*) = m; \quad p_2b(k_2^* = L^* - k_1^*) = 1 - m.$$

Claim 1: (a) There exists unique steady state equilibrium for the single-asset world, (b) An increase in the relative productivity of the foreigners (the parameter  $b$ ) reduces the steady state level of  $m$ .

The proof of this and all other claims is in Appendix A. The comparative static is intuitive: Since labor supplies do not depend on the parameter  $b$  the foreign country's share of income and wealth increases with its relative productivity.

Table 2 illustrates the steady state solutions for two values of  $\mu$ , assuming  $\pi = 0.9$ ,  $i = 0$ ,  $b = 1$ . The last two columns are the steady state welfare in each country computed by:

$$W = c - (\frac{1}{2})L^2, W^* = \pi(L + L^* - c_2) - (\frac{1}{2})(L^*)^2, \text{ where } c = (1 - \pi)c_1 + \pi c_2,$$

$$c_1 = m/p_1 \text{ and } c_2 = m[(m/p_1 + (1-m)/p_2)].$$

Table 2: The fully integrated single asset world

( $\pi = 0.9$ ;  $i = 0$ ,  $b = 1$ )

$\mu$ (R)	m	L	$L^*$	W	$W^*$
0 (R=1)	0.501	0.955	0.855	0.456	0.447
0.05 (R=1/1.05)	0.526	0.912	0.817	0.498	0.404

Comparing Tables 2 and 1 reveals that when  $\mu = 0$  both employment and welfare in the home country are higher under autarky. Buyers in the home country suffer from the price dispersion introduced by the foreigners because sometimes they cannot buy at the cheaper price. Imposing a moderate inflation tax works in the direction of compensating the home country.

Equilibrium selection: We assumed that the seller in country 1 (seller 1) specializes in the first market. To motivate this assumption, I assume small transportation costs of  $\tau$  normalized dollars per unit. (See the real version of this model in Eden [forthcoming] for a more comprehensive treatment). When buyers go on the internet they see the location of the seller and take transportation costs into account. A buyer will thus prefer to buy at  $p$  ND from a home country seller rather than at  $p^*$  ND from a foreign seller if:  $p \leq p^* + \tau$ .

I now consider an alternative in which seller 2 specializes in market 1 and seller 1 supplies to both markets. Seller 1 posts the

prices:  $p_1, p_2$ . Since he is indifferent between the two markets we must have:  $p_1 = \pi p_2$ . Seller 2 can guarantee the making of a sale only if he posts the price:  $p^* = p_1 - \tau$ . To see this point note that in the low demand state all buyers are from country 1 and they will prefer an offer of  $p_1$  from a seller in the home country to any offer  $p^* > p_1 - \tau$  from seller 2. Note also that seller 2 can get the price  $p_2 + \tau$  in the high demand state from buyers in country 2 who did not make a buy in market 1. But  $p_1 = \pi p_2$ , implies:  $p_1 - \tau < \pi(p_2 + \tau)$ . Therefore seller 2 strictly prefers market 2 and we cannot have equilibrium in which seller 2 specializes in market 1.

A similar argument can also be used to rule out equilibria in which both sellers supply to both markets. I could not rule out the case in which seller 2 specializes in market 2. This case is more complicated because the distribution of wealth changes over time. But I think that the main results will hold also in this more complicated case.

### 3.1 A TWO ASSETS WORLD

I now introduce an additional asset: the yen. I start by assuming that US sellers accept dollars only. Japanese accept both assets in the second market but only dollars in the first market. This assumption will be justified later where it will be shown that in equilibrium American strictly prefer dollars and Japanese are indifferent between the two assets.<sup>4</sup> Since dollars are accepted by all sellers dollars are more

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<sup>4</sup> The motivation for the assumption that Japanese accept only dollars in the first market is as follows. Since Americans do not hold yen, a

liquid. Therefore Japanese will hold both assets only if the rate of return on the yen is higher than the rate of return on the dollar.

The post-transfer, post-interest-payments supply of dollars (yen) at time  $t$  is  $M_t + G_t (M_t^* + G_t^*)$ . The rate of growth of the dollar supply (yen supply) is  $\mu$  ( $\mu^*$ ). The growth rates ( $\mu$ ,  $\mu^*$ ) are deterministic.

In the steady state, the yen and the dollar rates of return are constant and there exist normalized dollar prices,  $p_s$ , and normalized yen prices  $p_s^*$  such that:

$$(22) \quad P_{st}A_t = p_s(M_t + G_t); \quad P_{st}^*A_t = p_s^*(M_t^* + G_t^*).$$

The dollar price of yen ( $e_t$ ) is determined in a foreign exchange market that opens before the realization of the time  $t$  shocks. Since nothing happens between the selling of the goods and the opening of the foreign exchange market in the next period, we require:

$$(23) \quad P_{st}^* = P_{st}/e_{t+1}$$

Equations (22) and (23) lead to:

$$(24) \quad \frac{e_{t+1}}{e_t} = \frac{1 + \mu}{1 + \mu^*}$$

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seller who gets a yen offer will conclude that this must be a state of high demand. He may therefore not deliver at the low price claiming that he is stocked out and offer to deliver at the high price. It follows that a seller cannot commit in a credible time consistent manner to a low yen price.

Thus as in standard models, the rate of growth of the exchange rate depends on the ratio of the money supplies growth rates. Note that (24) implies that the dollar value of the yen supply is a constant fraction  $\alpha$  of the dollar supply:

$$(25) \quad \alpha = e_t (M_t^* + G_t^*) / (M_t + G_t) = e_{t-1} (M_{t-1}^* + G_{t-1}^*) / (M_{t-1} + G_{t-1}).$$

In the steady state Americans hold  $m$  normalized dollars. Japanese hold a portfolio of both assets that is worth  $1 - m + \alpha$  normalized dollars.

Taking the rate of return on the dollar as given, the Japanese central bank determines  $\alpha$  by an appropriate choice of the rate of return on the yen (a higher  $\alpha$  requires a higher rate of return on the yen). It is convenient however to treat  $\alpha$  as the policy choice variable and the yen return as an endogenous variable. An alternative that treats the yen rate of return as the policy choice variable will make no difference for the analysis.

The expected purchasing power of a normalized dollar is given by (16). Since yen can buy goods in the second market only, the unconditional expected purchasing power of a normalized yen is:

$$(26) \quad \begin{aligned} X &= A / p_2^* \text{ for a home country buyer and} \\ X^* &= \pi(A / p_2^*) \text{ for a foreign country buyer.} \end{aligned}$$

The first order conditions (20) describe the labor supply choices under the assumption that sellers accept dollars only. Here we add

conditions that justify the assumed choice of assets. We require that US sellers cannot benefit by selling in yen:

$$(27) \quad AL = Rp_1Z = \pi Rp_2Z \geq R^* p_1^* X = \pi R^* p_2^* X$$

And we require that Japanese sellers are indifferent between dollars and yen:

$$(28) \quad AL^* = Rp_1Z^* = \pi Rp_2Z^* = \pi R^* p_2^* X^*$$

Steady state equilibrium requires (19), the first order conditions (27) - (28) and the market clearing conditions

$$(29) \quad p_1(L + bk_1^*) = m ; p_2b(L^* - k_1^*) = 1 - m + \alpha,$$

where  $\alpha$  is the supply of yen in terms of normalized dollars. I require  $0 \leq m \leq 1$ . We now show (the proof is in the Appendix) the following Proposition.

Proposition 1: When  $\alpha \leq b$ , there exists a unique steady state

equilibrium for the two assets world with the following properties:

- (a)  $L \geq L^*$  and  $R^* \geq R$  with the inequalities being strict when  $\pi < 1$ ;
- (b) An increase in  $\alpha$  leads to a decrease in  $1 - m$ , and an increase in labor supplies in both countries;
- (c) US sellers strictly prefer dollars to the equivalent yen amount.

The condition  $\alpha \leq b$  is stronger than required to guarantee existence. When  $b = 1$ , it says that the dollar value of the yen supply is not greater than the dollar supply. The intuition for (a) - (c) is as follows. (a) Foreign workers may not want to consume and therefore have less incentive to work. The yen rate of return must be higher because lower price sellers do not accept it.

(b) When  $\alpha$  increases foreign agents substitute yen for dollars and  $1 - m$  goes down. As a result the dollar promises a higher chance of buying in the first market and a higher yen rate of return is required to compensate for the difference in liquidity. The higher rate of return on the yen leads to a higher expected real wage in Japan. The expected real wage in the US also goes up as a result of the increase in  $m$  and the increase in the probability that US buyers will buy at the cheaper price. The increase in the expected real wage leads to an increase in labor supply in both countries. (c) The "liquidity premium" on the dollar is sufficient to make Japanese sellers accept both assets. US sellers are willing to pay a higher "liquidity premium" on holding dollars because they buy in both states and the advantage of the dollar is larger in the low demand state (where the probability of buying at the cheaper price is unity for the dollar and zero for the yen).

Note that sellers make portfolio choices in the goods market. Since nothing happens between the end of trade in the goods market and the trade in foreign exchange, there are no transactions in the foreign exchange market.

Uncovered interest parity does not hold in our model. To see this note that Proposition 1 says:  $R = (1 + i)/(1 + \mu) < (1 + i^*)/(1 + \mu^*) = R^*$ . Using (24), this implies:  $(1 + i)/(1 + i^*) < (1 + \mu)/(1 + \mu^*) = e_{t+1}/e_t$ .



We can have for example,  $\mu = \mu^*$  and  $i^* > i$ . In this case, the exchange rate does not change over time but the nominal interest rate on the foreign asset is higher. We can also have both  $i^* > i$  and  $\mu > \mu^*$ . This is the "forward premium puzzle" found in the empirical literature, where the low interest rate currency tends to depreciate. (See Burnside, Eichenbaum, Kleshchelski and Rebelo [2006] for a recent discussion).

Are there arbitrage opportunities? The standard argument is that when there is an "over priced" asset one can make money by holding a negative amount of it. But this assumes that the speculator is not interested in consumption. If he is interested in consumption he may find that he has to buy at the high price with his relatively illiquid asset.

I now turn to an example. Table 3 computes the equilibrium magnitudes for various  $\mu$  and  $\alpha$  assuming  $i = i^* = 0$ ,  $\pi = 0.9$ ,  $b = 1$ . In the first four rows  $\mu = 0$  and  $\alpha$  takes four values:  $\alpha = 0, 0.1, 0.8, 1$ . Note that  $\alpha > 0$  requires  $R^* > 1$  ( $\mu^* < 0$ ). Furthermore, an increase in  $\alpha$  requires an increase in  $R^*$  because it reduces the probability that a dollar will buy in the second market and therefore increases the liquidity premium on the dollar. An increase in  $\alpha$  reduces welfare in the foreign country and increases welfare in the home country because it reduces the probability that Japanese buyers will buy at the low price.

When  $\mu > 0$ , increasing  $\alpha$  (and holding  $\mu$  constant) has an ambiguous effect on welfare. It reduces both the inflation tax paid by foreigners and the probability that a foreign buyer will buy at the low price. The first inflation tax effect works to improve welfare in the foreign country and reduce welfare in the home country. The second, term of trade effect, works in the opposite direction. The inflation tax

effect dominates when  $\mu$  is large. This can be seen in the last four rows of Table 3 when  $\mu = 0.1$ .

Increasing  $\mu$  (and holding  $\alpha$  constant) has also two effects on welfare. It increases the inflation tax collected from foreigners (when  $\alpha < 1$ ) and it creates a distortion in the labor supply choice. When  $\alpha$  is low the inflation tax effect dominates and therefore an increase in  $\mu$  increases welfare in the home country and reduces welfare in the foreign country. When  $\alpha$  is large the distortion effect dominates and an increase in  $\mu$  reduces welfare in both countries.

Figures 1 and 2 describe welfare in both countries as a function of  $\pi$ . The measure plotted is welfare relative to the no-taste shock case (Since in the no shock case,  $W = \frac{1}{2}$ , I plot  $2W, 2W^*$ ). This is done for  $\mu = 0$  and two values for  $\alpha$ :  $\alpha = 0.1$  and  $\alpha = 0.8$ . Note that a decrease in  $\pi$  has an adverse effect on welfare. The effect on welfare is more pronounced in Japan but has also a considerable effect on the US. Welfare in the US is lower and welfare in Japan is higher for small  $\alpha$ . This is special to the case  $\mu = 0$  when no inflation tax is imposed.

Table 3: The fully integrated world economy with two assets

( $\pi = 0.9$ ,  $i = i^* = 0$ ,  $b = 1$ )

$\mu$	$\alpha$	m	$\mu^*$	L	L*	W	W*
0	0	0.501		0.955	0.855	0.456	0.447
0	0.1	0.551	-0.06	0.960	0.860	0.460	0.443
0	0.8	0.900	-0.09	0.991	0.891	0.491	0.414
	1	1	-0.1	1	0.9	0.5	0.405
0.05	0	0.526		0.912	0.817	0.498	0.404
0.05	0.1	0.574	-0.01	0.916	0.821	0.495	0.407
0.05	0.8	0.905	-0.05	0.944	0.849	0.495	0.407
0.05	1	1	-0.05	0.952	0.857	0.499	0.404
0.1	0	0.549		0.872	0.781	0.531	0.366
0.1	0.1	0.594	0.03	0.876	0.785	0.522	0.375
0.1	0.8	0.910	-0.00	0.902	0.811	0.497	0.400
0.1	1	1	-0.01	0.909	0.818	0.496	0.402

\* The first two columns are the choice of the two policy-makers:  $\mu$ ,  $\alpha$ . We then have the following endogenous variables: the fraction of the post transfer dollar supply held by the buyers in the home country (m), the equilibrium rate of change in the yen supply ( $\mu^*$ ), labor supply in the home country (L), labor supply in the foreign country L\* and welfare in the two countries (W, W\*).

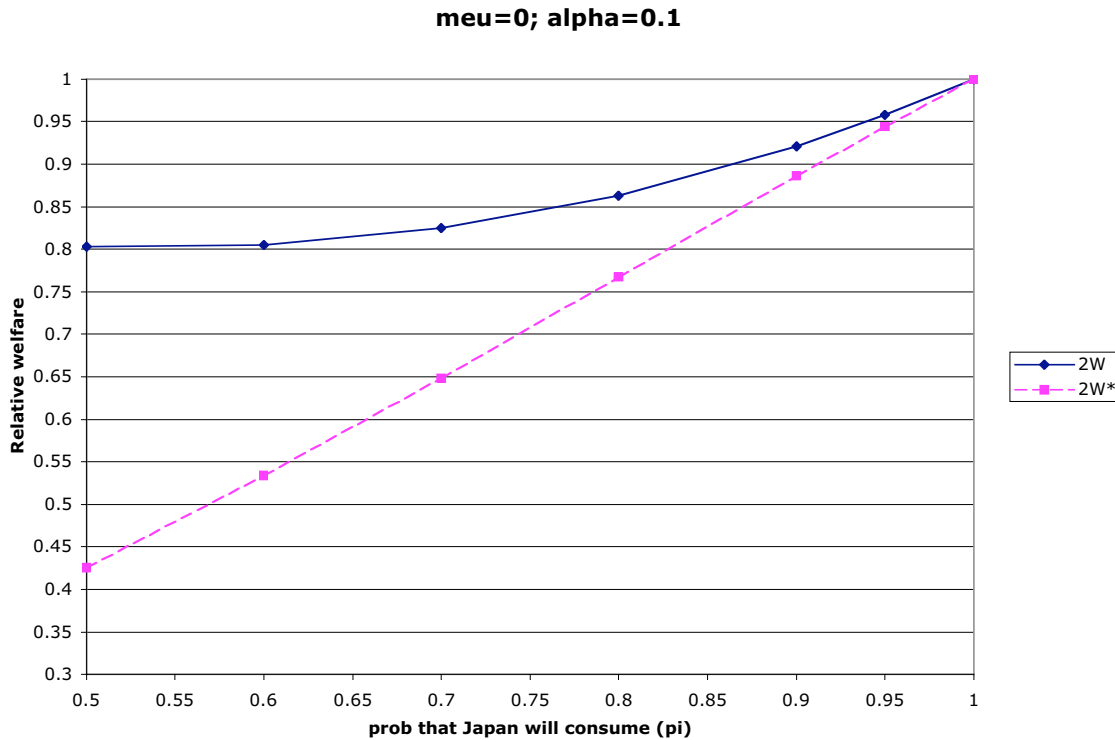


Figure 1: Welfare relative to the no-shock case ( $W/0.5, W^*/0.5$ ) when  $\mu=0$  and  $\alpha = 0.1$

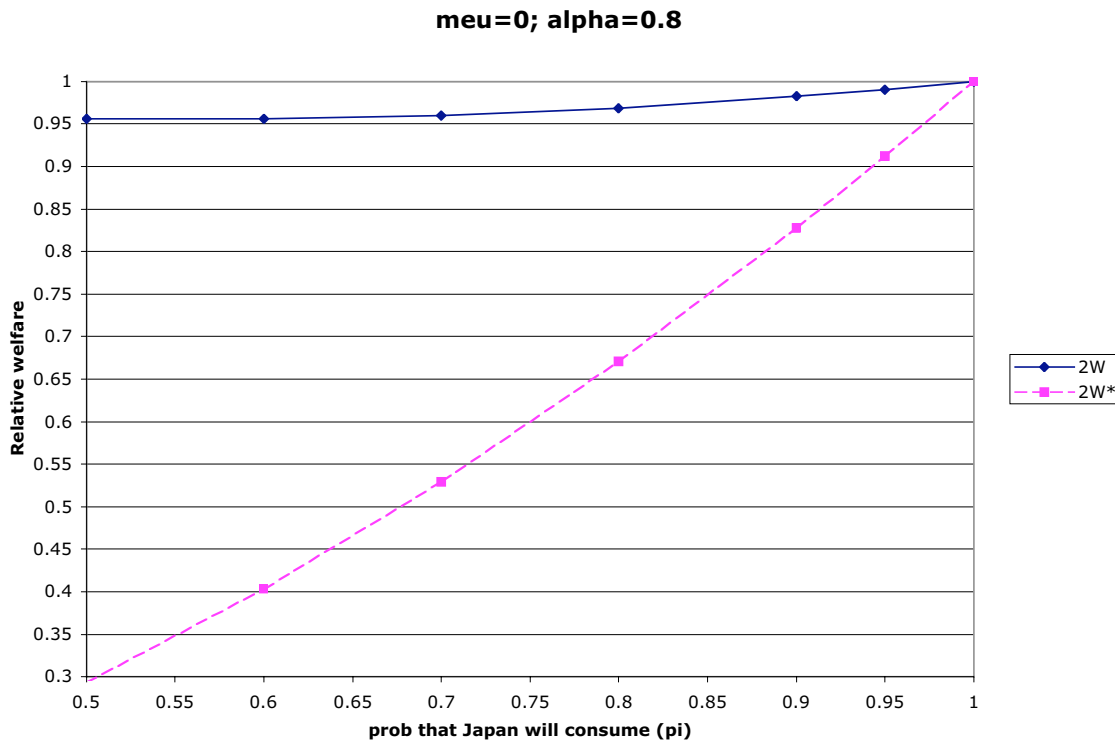


Figure 2: Welfare relative to the no-shock case ( $W/0.5, W^*/0.5$ ) when  $\mu=0$  and  $\alpha = 0.8$

Paying interest on the home asset: We have seen that under autarky changes in  $\mu$  and  $i$  that hold  $R$  constant are neutral. In a fully integrated world this is still true for the foreign country but does not exactly hold for the home country. In the Appendix (Lemma 1) I show that seigniorage in the home country is  $g = R(\mu - i)/(1 + i)^2$ . Therefore changing  $\mu$  and  $i$  while holding  $R$  constant will have real effects. In the examples I have worked out the effect is small because  $\mu - i$  is a good approximation for  $R$ . Since approximately only  $R$  matters I shall continue with the assumption that  $i = 0$ .

#### A Sequential policy game

I now turn to a brief description of a sequential game between the policy makers. Since there are 193 countries in the world I assume that the US moves first and chooses  $\mu$ , knowing the reaction function of the rest of the world. The rest of the world then chooses  $\alpha(\mu)$ .

Figure 3 illustrates the reaction function  $\alpha(\mu; \pi)$  of the representative foreign government for the US choice of  $\mu$ . This is done for two cases:  $\pi = 0.9$  and  $\pi = 0.95$ . The foreign country trade-off is between the terms of trade (the probability of buying at the low price) and the inflation tax. When  $\mu = 0$  there is no inflation tax and therefore the foreign country focus on the terms of trade which are best when  $\alpha = 0$ . When  $\mu$  is positive a higher  $\alpha$  means less inflation tax but also less favorable terms of trade. When  $\mu$  is sufficiently high the inflation tax dominates and the foreign country chooses  $\alpha = 1$ . Note that when  $\pi$  increases from 0.9 to 0.95 the term of trade effect becomes less important and the foreign government chooses higher  $\alpha$  for any

given  $\mu$  to avoid the inflation tax. In the limit case when  $\pi = 1$ , the foreign government will choose  $\alpha = 1$  regardless of  $\mu$ .

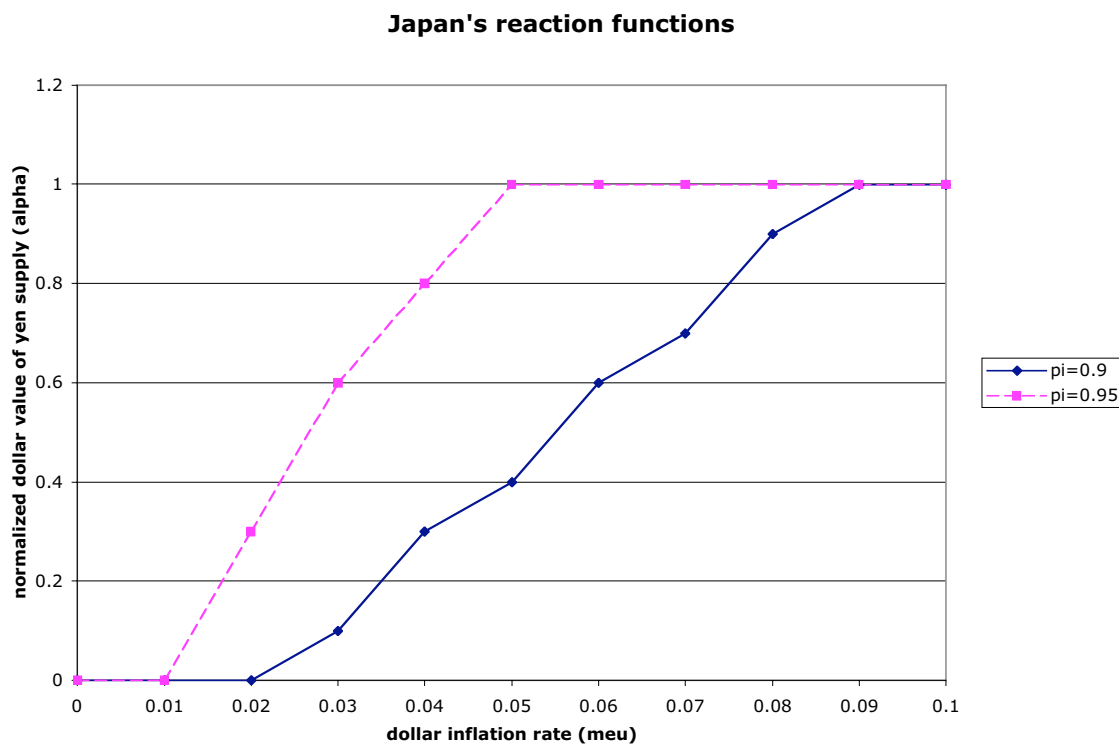


Figure 3:  $\alpha(\mu)$  for  $\pi = 0.9$  (the solid line) and  $\pi = 0.95$

Table 3 shows that choosing  $\mu = 0$  is not optimal from the US point of view. When  $\mu = 0$ , the rest of the world will choose  $\alpha(0) = 0$  and welfare in the US will be  $W = 0.456$ . The US can do better by choosing  $\mu = 0.1$  for example. In this case, the rest of the world will choose  $\alpha(0.1) = 1$  and the US welfare will be  $W = 0.496$ .

A more detailed calculations reveals that the optimal choice of the US is  $\mu = 0.08$  when  $\pi = 0.9$  and  $\mu = 0.05$  when  $\pi = 0.95$ . In the first case the optimal reaction is  $\alpha(0.08; 0.9) = 0.9$ . In the second the optimal reaction is  $\alpha(0.05; 0.95) = 1$ . It seems that the optimal  $\mu$  is "too high" relative to recent observations. We may increase  $\pi$  and get

a lower  $\mu$  but this will lead to  $\alpha = 1$ . It seems that a successful calibration of the model will require some modification. For example, we may add a "transaction motive" for holding money. This will increase the welfare cost of inflation and reduce the optimal  $\mu$ .

#### Net export and the real exchange rate

Table 3 shows that when the dollar inflation is low and there is partial or full dollarization in Japan, the US suffers from trade. This is because of an adverse effect on the terms of trade: as a result of trade US buyers are sometimes forced to import at the high price.

Table 4 uses Table 3 to illustrate the adverse effect on the terms of trade by calculating measures of net exports for the home country. Net export is measured here by the difference between output and consumption. The physical unit measure of net exports ( $x_s = L - c_s$ ) varies with the states of nature while the nominal measure ( $p_1L - m$ ) does not.<sup>5</sup>

When  $\mu = 0$ , the nominal measure is always zero. But the physical unit measure in the high demand ( $x_2$ ) is strictly positive and decreasing in  $\alpha$ . This occurs because in the high demand state, there is cross-

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<sup>5</sup> Some "real measures" that ignore the variations in the terms of trade may also remain constant over time and states. For example if we measure "real net export" by the dollar value of net export divided by the price charged by US sellers we will get  $L - m/p_1$  which does not vary over time and states. If we use a price index that is a weighted average of the prices quoted by foreign sellers and domestic sellers (say  $p = \delta p_1 + (1-\delta)p_2$  where  $\delta$  remains constant over time) we will also get a measure that does not vary over time and states.

hauling. The home country exports the good at the low price and pay the high price for some of its imports.

When  $\mu = 0.05$  the nominal measure of net export is negative and decreasing in absolute value with  $\alpha$ . This is the inflation tax imposed on foreigners. But the physical unit measure of export in the high demand state is positive reflecting the terms of trade effect.

When  $\mu = 0.1$  the inflation tax effect dominates and all measures of net exports are negative. Note that net export are decreasing with  $\mu$  but are not monotonic in  $\alpha$ . Again this is because of the two effects of increasing  $\alpha$ : The inflation tax effect and the terms of trade effect.

Note that in a steady state with low inflation rate (say  $0 < \mu \leq 0.05$  and  $\alpha < 1$  in Table 3) net export in the US are positive in the high demand state and buyers in the US will pay higher prices on average. Thus, a high demand state may be characterized by an increase in US CPI and an increase in net exports but not by a devaluation of the dollar: The rate of change of the exchange rate (24) is independent of the state of nature.

The volume of trade in our model may be measured by the absolute value of exports from the home country in the two states:  $|x_1| + |x_2|$ . Holding  $\mu$  constant the Table reveals a negative correlation between  $\alpha$  and  $|x_1| + |x_2|$ . This is consistent with the observation that the adoption of a common currency increases trade (Rose and Wincoop [2001]).

The last two columns in Table 4 calculate measures of the real exchange rates. The average price of consumption in state 2 (in terms of normalized dollars) is  $CPI_2 = mp_1 + (1 - m)p_2$  for an American and  $CPI_2^* = [(1-m)CPI_2 + \alpha p_2]/(1 - m + \alpha)$  for a Japanese. The ratio  $CPI_2^*/CPI_2$  is in the seventh column. It shows that increasing  $\alpha$  increases this



measure of the real exchange rate. The last column computes  $CPI_2^*/CPI$  where  $CPI = (p_1 + CPI_2)/2$  is the average across states price paid by the Americans.

Table 4: Net export for the home country ( $i = i^* = 0, \pi = 0.9$ )

$\mu$	$\alpha$	$x_1$	$x_2$	Ex	$p_1L - m$	$CPI_2^*/CPI_2$	$CPI_2^*/CPI$
0	0	0	0.047	0.043	0	1	1.027
0	0.1	0	0.043	0.039	0	1.011	1.035
0	0.8	0	0.010	0.009	0	1.088	1.094
0	1	0	0	0	0	1.111	1.111
0.05	0	-0.043	0.002	-0.002	-0.024	1	1.026
0.05	0.1	-0.035	0.005	0.001	-0.021	1.012	1.035
0.05	0.8	-0.005	0.004	0.003	-0.005	1.089	1.095
0.05	1	0	0	0	0	1.111	1.111
0.1	0	-0.078	-0.035	-0.040	-0.045	1	1.024
0.1	0.1	-0.064	-0.026	-0.030	-0.041	1.012	1.035
0.1	0.8	-0.009	-0.001	-0.002	-0.009	1.090	1.095
0.1	1	0	0	0	0	1.111	1.111

\* The first two columns are the policy choices ( $\mu, \alpha$ ). We then have real net export in the low demand state ( $x_1 = L - c_1$ ) and real net export in the high demand state ( $x_2 = L - c_2$ ). The column that follows calculates the expected real net export:  $Ex = (1 - \pi)x_1 + \pi x_2$ . The sixth column is the normalized dollar measure of net export:  $p_1L - m$ . The last two columns are measures of the real exchange rate.  $CPI_2^*$  is the average price paid by Japanese in state 2,  $CPI_2$  is the average price paid by Americans in state 2 and  $CPI$  is the unconditional average price paid by Americans, all in terms of normalized dollars.

#### 4. POTENTIAL APPLICATIONS

The model assumes two countries and (at most) two assets. In the real world we have many countries and many assets. Therefore the application is not straightforward. I will argue however that the analysis maybe relevant for the US and less stable countries that hold US government bonds: Japan and countries that are not in the G-7.

Stability is defined in our model by the predictability of demand. Predictability plays an important role in our model because producers must choose output before they know the realization of demand and output is wasted whenever demand is low. In a well-known paper, Clarida et al. (2000) claim that the improvement in monetary policy accounts for the relative stability of the business cycle in the Volcker-Greenspan era. Since "bad" monetary policy may lead to demand shocks, this view is consistent with the hypothesis that recently US demand fluctuates less than in the pre Volcker era. Kahn et al. (2002) advance the hypothesis that demand became more predictable (and the US economy more stable) because of the improvement in information technology. Since the US is leading in the IT revolution this suggests that the US demand became more predictable relative to the rest of the world. But unfortunately, I did not find direct evidence about the predictability of demand in the US relative to other countries.

Predictability of GDP may serve as a proxy for the predictability of demand. This is only a proxy because GDP in our model is determined by both technology and demand and for our purposes only the demand shocks matter. This may be a problem because the predictability of GDP may be higher for countries that do not innovate simply because

technology does not change much and is therefore easy to predict. But since we do not have a better proxy for demand I will use GDP.

Recently Stock and Watson (2003) estimated the predictability of GDP for the G-7 countries. Column (1) in Table 5 is a measure of the predictability for the entire sample: 1969 - 2002. The mean squared error is highest for Japan and Japan's GDP is therefore the hardest to predict. Column (2) is the estimate for the sub-period 69-83 and column (3) is the estimate for the sub-period 84-02. The following column is the ratio of column (2) to column (3). This shows that predictability improved in all of the G-7 countries. The largest improvement is in the UK and the smallest improvement is in Japan. Next we divide column (1) by the US measure. It shows that in the sample period Japan RMSE is 30% higher than the US while France's RMSE is 30% lower. The next column repeats the calculation for the 84-02 period. It shows that Japan's MSE is 90% higher while France's RMSE is only 7% lower. The last three columns report statistics about the rate of growth during the period 84-04: The average (Av.), Standard deviation (SD) and the coefficient of variation. Relative to the G-7, the US ranks highest in terms of the average rate of growth, lowest in terms of the coefficient of variation and in the middle in terms of the standard deviation.

Table 5: Predictability of quarterly GDP in the G-7 countries.

	Pseudo one-step ahead forecast Root Mean Squared Error <sup>1</sup>			Ratios			Rate of GDP growth: 84-04 <sup>2</sup>		
	(1) 69-02	(2) 69-83	(3) 84-02	$\frac{(2)}{(3)}$	$\frac{(1)}{US = 3.43}$	$\frac{(3)}{US = 2.27}$	Av.	SD	$\frac{SD}{Av}$
Japan	4.46	4.65	4.31	0.93	1.30	1.90	2.35	2.13	0.91
Germany	4.37	4.89	3.97	0.81	1.27	1.75	2.14	1.59	0.74
UK	4.16	6.40	2.41	0.38	1.21	1.06	2.72	1.47	0.54
Italy	3.65	5.16	2.47	0.48	1.06	1.09	1.95	1.31	0.67
US	3.43	4.92	2.27	0.46	1	1	3.3	1.52	0.46
Canada	3.35	4.09	2.78	0.68	0.98	1.22	3.06	1.97	0.64
France	2.39	2.73	2.12	0.78	0.70	0.93	2.05	1.20	0.59

<sup>1</sup> Based on Stock and Watson (2003, Table 2). They estimate a new autoregression for each forecast date using quarterly GDP growth (not detrended) and a moving window of eight years of data which ends one quarter before the quarter being forecasted; the entry is the squared root of the average squared forecast error over the indicated period.

<sup>2</sup> Using annual IMF data.

The relative predictability of the US was dramatically improved after 84. During the entire sample period the "other six" countries (the G-7 without the US) had on average a RMSE that was 9% higher than the RMSE of the US. Before 84 the RMSE of the "other six" was slightly lower than that of the US. After 84 it was 33% higher on average.

France's RMSE is consistently lower than that of the US. Should we think of France as the "stable demand country"? To sharpen this question we may think of a non-industrialized country with a recent history of low, flat and highly predictable GDP per capita. Do we expect that the debt of such a country will be "overpriced" as the debt of the stable demand country in the model? The possibility of economic disasters such as revolutions, wars and epidemics seems relevant for this question.

The importance of rare disasters has been debated since Rietz (1988) has proposed to use them as an explanation of the equity risk-premium puzzle. Recently Barro (2006) has reexamined Rietz's argument and found that the probability of a disaster is 1.5 - 2 percent per year with a distribution of decline in per capita GDP ranging between 15 percent and 64 percent. Barro's used his estimates to explain the equity risk premium puzzle and other puzzling observations about assets returns.

Judging from Barro (2006, Table 1) it seems that during the 20th century France and Germany are disaster prone countries while the US, Canada and the UK are not. Of course we do not expect France and Germany to go to war in the near future. But there are other problems. It seems that aging population is more of a problem for Europe than the US. And Europe seems to be less successful in integrating its immigrants.

It thus seems that the US and Japan are reasonable candidates for the countries in our model. An alternative is to assume that the unstable demand country in the model represents all countries that are not in the G-7. According to the IMF the world GDP is divided among the US (30%), the G-6 countries (G-7 minus the US; 34%) and other countries (36%).<sup>6</sup> The IMF Tables provide information about 181 countries. During the period 1984-2004 the average annual rate of change of real GDP in countries that do not belong to the G-7 was 3.6 with a standard deviation of 4.9. (The median rate of growth was 3.2 and the median standard deviation was 3.9). For the G-7 the average rate of growth was

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<sup>6</sup> IMF World Economic Outlook Database for September 2006.

<http://www.imf.org/external/pubs/ft/weo/2006/02/data/index.aspx>

2.5 with a standard deviation of 1.6. (The respective medians were: 2.4, 1.5). It thus seems that the countries that are not in the G-7 experienced much more fluctuations than the G-7 countries.<sup>7</sup>

Apparent seigniorage payments: Does the US get seigniorage payments from the ROW? In a recent article Gourinchas and Rey (GR, 2005) found strong evidence of sizeable excess returns of gross US assets over gross US liabilities. They found that during the period 1952 - 2004 the average annualized real rate of return on gross liabilities was 3.61% while the average annualized real rate of return on gross assets was 5.72%. The difference of 2.11% is considerable. This difference is especially large when looking at the post Bretton-Woods period: 1973 - 2004. The post Bretton-Woods average asset return is 6.82% while the corresponding total liability return is only 3.50%. The excess return in the post Bretton Woods era is thus 3.32%.

In Appendix B I provide some preliminary calculations of the seigniorage that the US may expect to receive from foreigners. The calculations assume risk neutrality, expected excess return equal to the post Bretton-Woods average (3.32%) and quantities at their 2004 levels. If we adopt the narrow definition of seigniorage (payments on cash) we get roughly 0.2% of US GDP. If we adopt a broad definition we get 2% of US GDP. The broad definition includes payments both to the US government

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<sup>7</sup> I also looked at a balanced sample of countries that have complete information. After eliminating the G-7 countries this yields a sample of 145 countries. The average growth rate and the average standard deviation for this sample are: 3.6 and 4.6. The medians are: 3.3 and 3.8.

and to US private agents. The US government may expect to collect about 0.7% of US GDP from securities and cash held by foreigners.

Rates of return and seigniorage: Appendix B shows that under risk neutrality the US government can get seigniorage if it sells bonds that promises an expected return that is less than the "world interest rate" which is equal to the highest expected rate of return that one can get. The reason is that under risk neutrality any asset with an expected rate of return less than the maximum available is "over-priced".

Since Japan holds US government bonds that promise an expected rate of return that is less than the maximum available alternative (equity or FDIs), Japan pays seigniorage to the US. This is true regardless of whether the best available alternative is in Japan or elsewhere.

Indeed US private agents who hold US government bonds also pay seigniorage to the US government. The paper does not explain why US private agents are willing to hold US government bonds (the equity premium puzzle) and it does not explain why Japanese private agents are willing to hold the government bonds of Japan. It does attempt to explain why Japanese agents are willing to hold US government bonds when higher expected return alternatives are available.

Portfolio choices: The analysis here shed light on the difference in the composition of US foreign assets and US foreign obligations. In 2004, fixed income securities were 9.6% of US foreign assets and 37.3% of US foreign liabilities (see Table 1 in Higgins et al. [2005]). The amount of fixed income securities on the liability side is about 5 times as

much as on the asset side. When looking at the share of equity plus foreign direct investment it is 58.2% on the asset side and 36.9% on the liability side. Thus the US holds relatively more equity and less fixed income securities on the asset side. This is consistent with the view that the US is "selling liquidity" to foreigners.

The model suggests that the fraction of US securities in the portfolio of less stable countries is relatively high. In terms of our model this will be the case if  $(1 - m)/(1 - m + \alpha)$  is a decreasing function of  $\pi$ . I have not been able to prove this intuitive result analytically. But this is the case in the examples I worked out.<sup>8</sup>

To examine the above hypothesis I looked at the value of foreign holdings of US securities by major investing countries in Report (2006). The report discusses the limitations of the data. One of the main problems is that a security held in a Swiss bank will be reported as Swiss-held even if an American actually owns it. They say that among the top 10 countries that hold US securities, five are financial centers: Belgium, the Cayman Islands, Luxemburg, Switzerland and the United Kingdom.

This maybe less of a problem for US treasury debt that is mostly held by foreign official institutions for which the identity of the

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<sup>8</sup> Figure 3 suggests that for any given US inflation rate, countries with higher  $\pi$  will choose higher  $\alpha$ . In the examples I worked out  $m$  is not sensitive to changes in  $\pi$  but is highly sensitive to changes in  $\alpha$ . So roughly speaking we expect that a country with high  $\pi$  will choose high  $\alpha$  and this will reduce  $1 - m$  and  $(1 - m)/(1 - m + \alpha)$ .



owner is clear.<sup>9</sup> I therefore looked at the ownership by country of US treasury debt (Table 16). The G-6 countries hold a total of 701 billion dollars worth of US treasury debt. Out of this Japan holds 572 billion dollars that are 82% of the total. Germany and the UK hold about 6% each. France 3% and Italy and Canada about 2% each. For comparison, China holds 277 billions which is about 40% of the total amount held by the G-6.

We thus see that Japan's holding of treasury debt is higher than the amount held by all the other members of the G-6. It seems that Japan is unstable relative to the G-7 but maybe stable relative to countries that are not in the G-7. Does Japan's holding of US treasury debt is large relative to countries that are not in the G-7? The answer depends on whether we use income or wealth to measure size.

The total GDP of the countries that are not in the G-7 is 36% of the world's GDP. Japan's GDP is about 11.5% of the world's GDP and together they account for 47.5% of the world's GDP. Japan's income share in this group is  $11.5/47.5 = 24\%$ . The countries that are not in the G-7 hold about 900 billion dollars worth of US treasury debt. Together with Japan they hold 1469 billion dollars worth. Japan's share in this total is 39%. Thus Japan's share in holding US treasury debt is higher than its income share.

The income share is not a good proxy for the wealth share. In the US people who are in the top 1% of the income distribution hold 16% of the wealth. People who are in the top 10% of the income distribution

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<sup>9</sup> Foreign official institutions hold 66% of the foreign holding of US treasury debt but only 8% of equity.

hold 50% of the wealth and people who are in the top 20% of the income distribution hold 63% of the wealth. See Diaz-Gimenez, Quadrini and Rios-Rull (Table 5, 1997). Since Japan is rich relative to countries that are not in the G-7 we may expect that its share in the wealth of this group of countries is higher than its income share.

Other implications: Our model has price dispersion and can therefore explain deviations from PPP. In particular, it is consistent with the observation that the US is cheap relative to the prediction of income-price regressions. See Balassa (1964), Samuelson (1964) and Rogoff (1996).

As in Rose and Wincoop (2001), the adoption of a common currency increases trade in our model. This does not hold in all models. Recently, Bacchetta and van Wincoop (2000) used a cash-in-advance model to analyze the implications of a monetary union and demand uncertainty that arises as a result of asset supply shocks. They find that exchange-rate stability is not necessarily associated with more trade. Devereux and Engel (2003) find that the implications of risk for foreign trade are highly sensitive to the choice of currency at which prices are set. In these models prices are rigid and firms satisfy demand. In the UST model used here prices can be changed during trade and sellers are not committed to satisfy demand (indeed, low price sellers are stocked out in the high demand state).

Americans work hard in our model because they are relatively certain about the prospect of enjoying the fruits of their labor. This is not unlike the tax explanation in Prescott (2004). Nothing will

change in our model if instead of a taste shock we assume that the Japanese government imposes a random tax on accumulated wealth.

Dollarization: Although the paper focus on a broad definition of money it has bearing on the issues of dollarization and currency unions that typically focus on narrow definitions of money. Fischer (1982) argues that countries choose to have national monies to avoid paying seigniorage to a foreign government. Here we showed that an unstable demand country may gain from full or partial dollarization even if it pays moderate seigniorage to the US.

Transaction costs and the ability to commit play a major role in Alesina and Barro (2001, 2002) analysis of dollarization and currency unions. They argue that seigniorage should be part of the overall negotiations. This may be feasible in the case of currency unions they consider. But here we discuss the holding of dollar denominated assets by agents from all (193) countries. Cooperation in this case is more difficult and therefore a sequential game in which the US moves first seems more appropriate.

## 5. CONCLUDING REMARKS

We extended previous UST models by allowing sellers to choose the assets that they will accept as payment for the goods they offer. The main contribution is in using price dispersion to model liquidity.

The idea that liquidity may play a role in explaining assets returns is of-course not new. Recently, McGrattan and Prescott (2003) argue that short term US government securities provide liquidity and are

therefore overpriced. Cochrane (2003) argued that some stocks are overpriced because they provide liquidity.

Our model illustrates the possibility that the apparent seigniorage paid to the US by the rest of the world may continue in a steady-state equilibrium if the US demand is and will continue to be relatively predictable. This is different from the steady-state analysis in Blanchard Giavazzi and Sa (2005) who follow the partial-equilibrium portfolio balance literature of Kouri (1982). In their model, the larger is the net debt, the larger is the steady state trade surplus. Here we can have trade deficit and debt in the steady state.

Our analysis has some common elements with Caballero et al. (2006). They attribute the increase in the importance of US assets to an unexpected reduction in the growth rate of European and Japanese output and (or) a collapse of the asset markets in the rest of the world.

Our approach is also related to the random matching models pioneered by Kiyotaki and Wright (1993). In both models uncertainty about trading opportunities plays a key role. In the random matching models agents are uncertain about whether they will meet someone that they can actually trade with. But whenever a meeting takes place it is bilateral. In the UST model sellers are also uncertain about the arrival of trading partners but whenever a meeting occurs there are a large number of agents on both sides of the market. As a result there is a difference between the assumed price determination mechanisms. In the random matching models prices are either fixed or are determined by bargaining (as in Trejos and Wright [1995] and Shi [1995]). In the UST model prices clear markets that open.

At the end of their paper Kiyotaki and Wright (1993) consider an economy with two currencies: red and blue. The red currency circulates with a higher probability and in equilibrium yields a lower rate of return. The high return asset is less acceptable or less liquid. Similarly, here the currency that promises the higher chance of buying at the low price yields a lower rate of return. The difference is that here the international currency is more liquid than the domestic currency. This is not natural if we think of bilateral meetings. It makes sense in our setup where trade is done on the internet and a broad definition of money is used.

Matsuyama, Kiyotaki and Matsui (1993), Zhou (1997), Wright and Trejos (2001) and Liu and Shi (2005) use the random matching approach to study international currency. Wright and Trejos (2001) show that there can be three distinct type of equilibria, where in every case monies circulate locally, and either one, both, or neither circulate internationally. The assumed matching process plays a key role in determining the type of equilibria possible. For example, in the absence of inflation tax equilibrium with two national monies and no international money exists if the two countries are similar and the probability of meeting a foreigner is low. In our model the key difference between the two countries is in the probability of the taste shock. The example in Table 4 suggests that in the absence of inflation tax it is not possible to get equilibrium with national monies only (unless  $\pi = 1$  and the two countries are completely symmetric).

The difference in the taste shock probability limits the applicability of Gresham's law. In our model we get a steady state equilibrium with two monies even when  $\mu \neq \mu^*$ . This is different from

Karekan and Wallace (1981). In their model, there is no difference between the currencies. As a result there is a continuum of equilibria that differ in the nominal exchange rates. At any given equilibrium, the nominal exchange rate is constant over time and therefore the currency whose supply grows at a faster rate will represent an increasing fraction of the currency portfolio held by agents.

Our overlapping generations model does not distinguish between money and bonds. The framework in Lagos and Wright (2005) may be a good way of doing it. In their framework, random matching occurs during the "day" and Walrasian auction occurs during the "night". We may replace the Walrasian auction that occurs during the night with sequential trade. That is, after interacting in a decentralized market with anonymous bilateral matching during the day, agents go on the internet and place orders as in our model. During the night it is easy to transfer funds from one account type to another and therefore we may assume that in fact everyone accepts bonds. Other useful extensions may include longer horizon agents with a smoothing of consumption motive and the introduction of physical capital.

#### APPENDIX A: PROOFS

##### Proof of Claim 1:

The first order conditions (20) imply:

$$(A1) \quad p_1 = \pi p_2.$$

We substitute (A1) in (15)-(16) to get:

$z_1 = A/p_1$  ;  $z_2 = mA/p_1 + (1-m)\pi A/p_1$  and

$$(A2) \quad z = (A/p_1)[1-\pi + \pi^2 + \pi(1-\pi)m]; \quad z^* = (A/p_1)[\pi^2 + \pi(1-\pi)m]$$

Substituting (A2) in (20) yields:

$$(A3) \quad L = R[1-\pi + \pi^2 + \pi(1-\pi)m] ; \quad L^* = R[\pi^2 + \pi(1-\pi)m]$$

From (19) we get:  $p_1L = (m - g)/R$ . Using (A3) leads to:

$$(A4) \quad p_1 = \frac{m - g}{R^2[1-\pi + \pi^2 + \pi(1-\pi)m]}$$

Substituting  $p_1L = (m - g)/R$  in the market clearing condition

$p_1(L + bk_1^*) = m$  leads to:  $p_1bk_1^* = m - (m - g)/R$ . We now substitute this

in the market clearing condition  $p_1b(L^* - k_1^*) = \pi(1 - m)$  to get:

$p_1bL^* = \pi(1 - m) + m - (m - g)/R$ . Using (A3) leads to:

$$(A5) \quad p_1 = \frac{\pi(1 - m) + m - \frac{m-g}{R}}{bR[\pi^2 + \pi(1-\pi)m]}$$

Equating (A4) to (A5) leads to:

$$(A6) \quad \frac{b[\pi^2 + \pi(1-\pi)m]}{R[1-\pi + \pi^2 + \pi(1-\pi)m]} = \frac{\pi(1 - m) + m - \frac{m-g}{R}}{m - g}$$

**Lemma 1:**  $g = R(\mu - i)/(1 + i)^2$ .

**Proof:** To show that we use the following steps:

$$M_{t+1} = (M_t + G_t)(1 + i)$$

$$M_{t+1}/M_t = 1 + \mu$$

$$(M_t + G_t)/M_t(1 + \mu) = 1/(1 + i)$$

$$\omega + G_t/M_t(1 + \mu) = 1/(1 + i)$$

$$g = G_t/M_t(1 + \mu) = 1/(1 + i) - \omega = (\mu - i)/(1 + \mu)(1 + i)$$

$$= R(\mu - i)/(1 + i)^2. \quad \square$$

Lemma 2: When  $m = 1$ , the right hand side of (A6) is:  $1/(1 - g) - 1/R < 0$

Proof: Using Lemma 1, we get:

$$1 - g = [(1 + i) - \omega(\mu - i)]/(1 + i). \text{ Substituting}$$

$$1/R = (1 + \mu)/(1 + i) \text{ leads to:}$$

$$1/(1 - g) - 1/R = [(1 + i) - \omega(\mu - i) - (1 + \mu)]/(1 + i)$$

$$= (1 + \omega)(i - \mu)/(1 + i) < 0 \text{ because } i < \mu. \quad \square$$

Lemma 3: There exists a unique solution to (A6) when

Proof: When  $L > 0$ , (19) implies  $m > g$ . When  $m - g$  is small (and positive) the right hand side (RHS) of (A6) is large. Using the Lemma and  $\mu > i$  we get that when  $m = 1$  the RHS is of (A6) is negative. The LHS of (A6) is increasing and when  $m = 1$  it is equal to  $\pi b/R$ . Therefore there exists a unique solution  $\bar{m}$  in Figure A1. An increase in  $b$  will shift the LHS curve up and reduce  $m$  (increase  $1 - m$ ).  $\square$



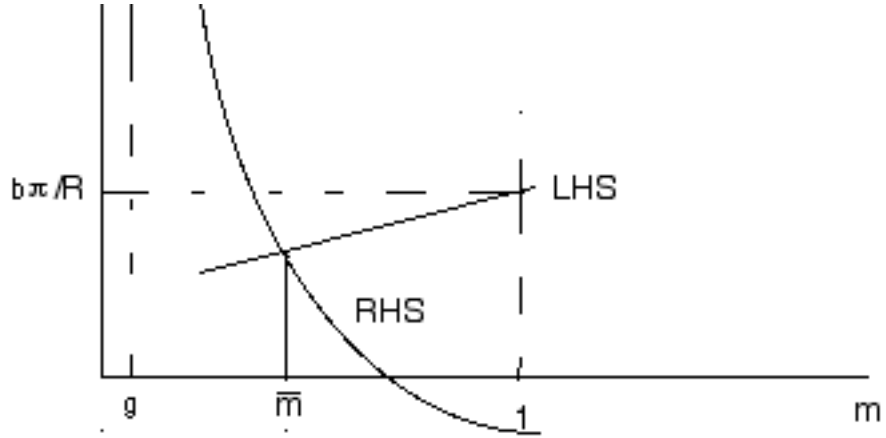


Figure A1

We now substitute the solution  $\bar{m}$  in (A3) to solve for the steady state magnitudes  $L$  and  $L^*$ . We proceed by solving for  $p_1$  from (A5) and  $p_2 = \pi p_1$ . To solve for  $k_1^*$  we use the market clearing condition:  
 $p_2 b(L^* - k_1^*) = 1 - m$ .  $\square$

Proof of Proposition 1:

I start by solving for the steady state level of  $m$ . As before we get:  $p_1 L = (m - g)/R$  from (19). Substituting this in the condition for clearing the first market,  $p_1(L + b k_1^*) = m$ , leads to:

$p_1 b k_1^* = m - (m - g)/R$ . We now substitute this in the second market clearing condition,  $p_1 b(L^* - k_1^*) = \pi(1 - m) + \pi\alpha$ , to get:

$p_1 b L^* = \pi(1 - m) + \pi\alpha + m - (m - g)/R$ . We also verify that equations (A3) and (A4) still hold. Using (A3) leads to:

$$(A7) \quad p_1 = \frac{\pi(1 - m) + \pi\alpha + m - \frac{m - g}{R}}{bR[\pi^2 + \pi(1 - \pi)m]}$$

Equating (A4) to (A7) leads to:

$$(A8) \quad \frac{b[\pi^2 + \pi(1-\pi)m]}{R[1-\pi + \pi^2 + \pi(1-\pi)m]} = \frac{\pi(1-m) + \pi\alpha + m - \frac{m-g}{R}}{m-g}$$

I now turn to show that there exists a unique solution to (A8). The left hand side of (A8) is the same as the LHS of (A6).

The RHS of (A8) is decreasing. Since  $\alpha \leq b$ , Lemma 2 implies:  $\pi[\alpha/(1-g) - b/R] < 0$  and  $b\pi/R > \pi\alpha/(1-g)$ . Thus, the RHS is less than the LHS when  $m = 1$  and there exists a unique solution,  $\bar{m}$  in Figure A2.

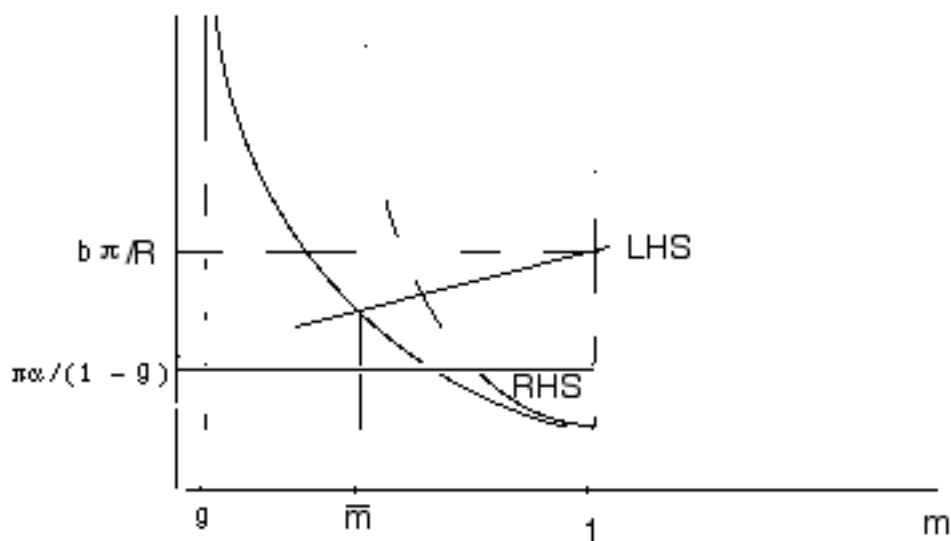


Figure A2

We now use the solution  $\bar{m}$  to solve for the steady state magnitudes. We have thus shown existence and uniqueness.

I now choose  $\omega^*$  so that the Japanese seller is indifferent between yen and dollars. From (26) and (28) we get:

$$(A9) \quad Rp_2Z^* = R^*p_2^*X^* = \pi AR^*$$

We use (A1) and (A2) to get:  $Rp_2Z^* = AR[\pi + (1-\pi)m]$ . Substituting this in (A9) leads to:

$$(A10) \quad R^*(m) = R \left( 1 + \frac{(1-\pi)m}{\pi} \right)$$

Condition (A10) implies that  $R^*(m)$  is an increasing function and  $R^* > R$ . Note that (A3) implies  $L \geq L^*$ . We have thus shown part (a).

To show (b) note that an increase in  $\alpha$  increases the RHS of (A8) for all  $m$  and therefore shifts the RHS curve in Figure B2 to the right. This leads to an increase in the steady state level of  $m$ . Note also that (A3) implies that  $L$  and  $L^*$  are monotonic in  $m$ . Therefore as  $\alpha$  grows and  $m$  grows and labor supplies in both countries grow.

I now turn to show that the US seller strictly prefers dollars. A US seller that sells for dollars will have the expected real wage:

$$(A11) \quad Rp_1Z = AR[1 - \pi + \pi^2 + \pi(1 - \pi)m]$$

The expected real wage when selling in yen is:

$$(A12) \quad R^* p_1^* X = R^* \pi p_2^* (A/p_2^*) = \pi AR^* = \pi AR \left( 1 + \frac{(1-\pi)m}{\pi} \right)$$

where the last equality uses (A10). Subtracting (A12) from (A11) leads to:

$$(A13) \quad Rp_1Z - R^* p_1^* X = AR(1 - m)(1 + \pi^2 - 2\pi) \geq 0.$$

When  $\pi < 0$ , this difference is strictly positive and decreasing in  $\pi$ . We have thus shown (c).  $\square$

#### APPENDIX B: SEIGNIORAGE PAYMENTS

The model has at most two assets and this limits the application. Here I show that the US government will get seigniorage whenever it sells an "over priced" asset. Under risk neutrality an asset is "over priced" if its expected rate of return is less than the "world interest rate" defined by the highest expected return that one can get in the world economy. I also provide preliminary calculations about the amount of seigniorage paid to US agents.

I assume a single good, two periods exchange economy with four assets: an international currency (the dollar), US (home country) government bonds, US private bonds and foreign bonds. I assume that the rates of return on these assets are exogenous and work out the implications of the assumption that budget constraints are satisfied and markets are cleared.

At time  $t$  the representative agent in the home (foreign) country gets an endowment of  $Y_t$  ( $Y_t^*$ ) units of the consumption good. The representative agent in the home country gets also a transfer payment from his government. The real value of the transfer payment is:  $G_t$ . There are no explicit taxes and no government in the foreign country. The transfers are financed by seigniorage revenues.

I use  $m$ ,  $b^g$ ,  $b^p$  and  $f$  to denote the real value of money, US government bonds, US private bonds and foreign bonds. The real rates of return these assets are:  $r^m$ ,  $r^g$ ,  $r^p$ ,  $r^f$ . It is assumed that:

$E(r^m) \leq E(r^p) \leq E(r^g) \leq E(r^f)$ , where  $E$  denotes expectations taken in the first period. The representative US consumer first period budget constraint is given by:

$$(B1) \quad f = Y_1 + G_1 - b^g - b^p - m - c_1$$

His second period consumption is:

$$(B2) \quad c_2 = Y_2 + G_2 + b^g(1 + r^g) + b^p(1 + r^p) + m(1 + r^m) + f(1 + r^f)$$

Substituting (B1) in (B2) yields:

$$(B3) \quad c_1(1 + r^f) + c_2 \\ = (Y_1 + G_1)(1 + r^f) + Y_2 + G_2 - b^g(r^f - r^g) - b^p(r^f - r^p) - m(r^f - r^m)$$

Similarly, for the representative foreign agent we have:

$$(B4) \quad c_1^*(1 + r^f) + c_2^* = Y_1^*(1 + r^f) + Y_2^* \\ - b^{g*}(r^f - r^g) - b^{p*}(r^f - r^p) - m^*(r^f - r^m)$$

Market clearing requires:

$$(B5) \quad c_1 + c_1^* = Y_1 + Y_1^*; \quad c_2 + c_2^* = Y_2 + Y_2^*; \\ b^p + b^{p*} = 0.$$

We can now add (B3) and (B4) to get:

$$\begin{aligned}
\text{(B6)} \quad & [b^g(r^f - r^g) + b^p(r^f - r^p) + m(r^f - r^m)] \\
& + [b^{g*}(r^f - r^g) + b^{p*}(r^f - r^p) + m^*(r^f - r^m)] \\
& = (Y_1 + G_1 + Y_1^*)(1 + r^f) + Y_2 + G_2 + Y_2^* - (C_1 + C_1^*)(1 + r^f) - C_2^*
\end{aligned}$$

Substituting the market-clearing conditions yields:

$$\begin{aligned}
\text{(B7)} \quad & [b^g(r^f - r^g) + m(r^f - r^m)] + [b^{g*}(r^f - r^g) + m^*(r^f - r^m)] \\
& = G_1(1 + r^f) + G_2
\end{aligned}$$

The right hand side of (B7) is the future value of the government transfers. The left hand side is the seigniorage revenue: The terms in the first bracket are the seigniorage paid by US consumers and the terms in the second bracket are the seigniorage paid by foreigners. Note that when  $r^f > r^g$ , seigniorage is paid on government bonds as well as on real balances.

The market clearing condition  $b^p + b^{p*} = 0$ , implies:

$$\text{(B8)} \quad b^p(r^f - r^p) = -b^{p*}(r^f - r^p)$$

Thus if  $b^{p*} > 0$  and  $r^f > r^p$ , foreigners pay "seigniorage" to private US agents. The term "seigniorage" may be appropriate because it reflects the ability of US private agents to create an asset that is "overpriced".

I focus on the expected value of the total seigniorage paid by foreigners:

$$\text{(B9)} \quad b^{g*}E(r^f - r^g) + m^*E(r^f - r^m) + b^{p*}E(r^f - r^p)$$

We may assume that all the transfers are made to the old generation. I now turn to a calibration exercise.

Calibration: As was said before, GR estimated that the average real rate of return on foreign assets held by US residents between 1973 - 2004 was 6.82%. The estimated real rate of return on US foreign liabilities in this period is: 3.50%. The estimated premium is thus 3.32%. The estimated premium for the entire sample (1952-2004) is 2.11%.

I start by using the post Bretton Woods data for forming expectations. I also assume that there is no difference between US government securities and US private securities and that the expected rate of inflation is 2%. I thus assume:

$$(B10) \quad E(r^f) = 6.82\%, \quad E(r^s) = E(r^p) = 3.50\%; \quad E(\pi^m) = -2\%;$$

The seigniorage paid by foreigners to US agents (government and private) requires an estimate of  $b^* = b^{s*} + b^{p*}$  and  $m^*$ . We can find recent levels of gross liabilities in a report prepared by the Department of the Treasury, Federal Reserve Bank of New York and Board of Governors of the Federal Reserve System (Report [2005]). In this report there are data on foreign holdings of U.S. securities for the years 2002 - 2004 (Table 1 on page 3). The total was 4338 billion dollars for 2002, 4979 billion dollars for 2003 and 6006 billion dollars for 2004. These estimates include Equities, government and corporate debt. Our analysis does not distinguish between debt and equity. I will therefore lump them together and use  $b^* = 6006$  billion for the current estimate on total US

securities held by foreigners. Multiplying this amount by the expected premium of 3.32% we arrive at a seigniorage figure of 199 billion which is 1.7% of 2004 US GDP. If we use the more moderate "liquidity premium" of 2.11% we get about 1% of 2004 GDP.

We add to that the amount that foreigners expect to pay on their holding of currency:  $m^*E(r^f - r^m)$ . The amount of cash held by foreigners in 2004 is estimated to be close to 333 billion dollars. Multiplying this by the premium  $r^f - r^m = 8.82\%$  we arrive at a seigniorage figure of about 29 billions dollars or 0.25% of US 2004 GDP. Adding this to the seigniorage on other assets we get a total close to 2% of US GDP. If we use GR estimates for the entire sample (1952 - 2004) we get a total seigniorage of about 1.3%. These are big numbers. They are close to Switzerland's entire GDP (which is roughly 2.2% of US GDP).

An alternative view may focus on seigniorage earned on Government securities and cash. GR finds that during the post Bretton Woods era (1973 - 2004) the real rate of return on foreign bonds held by US residents was 4.05% while the real rate of return on US bonds held by foreigners was only 0.32%. The excess return on bonds is thus 3.73%. The BEA estimate that about 1.5 billions dollar worth of US government securities were held by foreigners in 2004 (close to 13% of GDP). The expected seigniorage to the US government is close to 56 billions which is close to 0.5% of GDP. Adding to this the seigniorage on cash held by foreigners we arrive at a total of 0.7% of US GDP.



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