

AN ALTERNATIVE VIEW OF TAXATION AND ENDOGENOUS GROWTH

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Abstract

In this paper, a modification is made to the endogenous growth model studied by Lucas[2]. It is shown that if individuals derive utility from their level of human capital, then a tax on the return to physical capital can *raise* the equilibrium growth rate. Consumption taxation may increase the growth rate. If there is an externality in production of human capital, then it may be optimal to impose a capital *tax*, as opposed to a subsidy, to achieve the optimal growth rate. This may be a reason why existing estimates of the welfare costs of capital taxation may be overstated.

1 Introduction

The model of endogenous growth described by Lucas [2] has been instrumental in enhancing our understanding of the factors influencing growth economic activity. In this paper it is shown that one subtle change in the economic environment can give rise to economic behavior that is quite different from what one might ordinarily expect.

The model proposed by Lucas is one in which there are essentially two sectors. One sector produces the consumption good, and this can also be used to augment the stock of physical capital. The inputs into this sector are physical capital, and labor input, which consists of human capital multiplied

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by the time devoted to production. The other sector produces human capital. The inputs to producing human capital are the stock of human capital, and time devoted accumulating human capital. Agents derive utility from the discounted stream of consumption.

The proposed change to this model that is studied here is to assume that agents derive utility from consumption, but that they also receive utility from their stock of human capital or knowledge as well. The reason for this is that it appears that agents engage in at least some learning or skill acquisition for purposes that may not necessarily lead to increased income or output. For example, someone who earns a baccalaureate in the study of ancient Greek mythology, or the study of Latin, is likely to do so not because it will result in a much higher level of future income, but instead because they genuinely love the discipline or the knowledge derived from this education. After all, this college degree likely costs as much as a similar degree in business or engineering, but it is likely that these two latter degrees would be much more likely to raise the individual's future earning power. Similarly individuals may acquire skills in carpentry, photography, music or a variety of other specialties. These skills could be utilized to generate a higher income, or equally likely, they may just generate a higher level of satisfaction for the individual by themselves.¹

It could also be argued that there are other non-pecuniary reasons why individuals engage in learning of an academic variety. For example, it can serve as a social signal to others, which can have a return in terms of higher social status. This is an issue studied by Fershtman, Murphy and Weiss [1], who use a quite different model study the implications this issue has for growth.

With this in mind, it seems plausible to consider the economic implications of a model in which agents derive utility from holding human capital. In fact, it might seem inappropriate to argue the opposite: namely, that many types of education have little to offer beyond the ability to raise economic productivity of the worker.

Now it could be argued that learning in *any* activity raises the level of human capital because it somehow enhances the individual's capacity for the future acquisition of skills. However, it seems equally plausible that

¹Similarly, why do many individuals study ancient history if they didn't enjoy it? Also, why is there such a large tourism industry transporting people to site of bygone ruins of temples and battles, unless these people are "consuming" the history.

individuals also derive utility from having these skills directly, as opposed to indirectly through their enhanced labor productivity.

With this minor modification to the model of Lucas, it is shown that the behavior of the model is quite different from what one might ordinarily expect. In particular, it is shown that a tax on capital or labor can *raise* the optimal growth rate. In addition, if there is an externality in production of human capital, then in order to achieve the optimal growth rate it is appropriate to impose a capital (or labor) tax, rather than a subsidy to capital.

This paper is also related to the work Lucas [3], who finds that capital taxation may not have a substantially negative impact on the growth rate. Subsequent work by Stokey and Rebelo [5] supports these conclusions, even for a variety of specifications of the technology. The model of this paper shows that capital taxation may even result in an increase in the growth rate. Additionally, this may be one reason why existing the welfare costs of capital taxation in the existing literature may be overstated.

2 The Model

Consider an environment very similar to that of Lucas [2], and his notation will be utilized throughout. The preferences of agents are given as follows:

$$\int_0^{\infty} e^{-\rho t} \left(c_t^\phi + \eta h_t^\phi \right)^{(1-\sigma)/\phi} dt \quad (1)$$

where $\phi \leq 1, \sigma \geq 0$ and $\rho > 0$. Here c_t is the consumption of the market-produced good, while h_t denotes the individual's stock of human capital. The elasticity of substitution between the two goods denoted by $\left(\frac{1}{1-\phi} \right)$. Also, $(1/\sigma)$ is the intertemporal elasticity of substitution. Of course, here $\eta > 0$ measures an individual's relative utility derived from having human capital.² With $\eta = 0$, then the model reduces to the framework studied by Lucas. The resource constraints are given as follows:

$$\dot{K}_t + c_t = AK_t^\beta (u_t h_t)^{1-\beta} \quad (2)$$

²This might seem like there is now an externality in production. In the appendix another interpretation of the model is offered in which there are two consumption goods, produced different technologies, without any externalities.

and the equation governing human capital is given as follows:

$$\dot{h}_t = h_t \delta (1 - u_t). \quad (3)$$

Here K is the stock of physical capital, u is the quantity of time spent in the production sector, while $(1 - u)$ is the quantity of time spent accumulating human capital. For now, there are no externalities or population growth, which are present in Lucas' model, since these are not germane to the central point to be made here. The analysis here will focus exclusively on balanced growth paths. The Hamiltonian for this problem is as follows:

$$\mathcal{H} = (c^\phi + \eta h^\phi)^{(1-\sigma)/\phi} + \theta_1 [AK^\beta (uh)^{1-\beta} - c] + \theta_2 [h\delta(1 - u)].$$

where θ_1 and θ_2 are multipliers. The first-order conditions associated with optimal consumption (c) and work effort (u) are as follows:

$$\frac{(1 - \sigma)c^{\phi-1}}{(c^\phi + \eta h^\phi)^{\frac{\phi+\sigma-1}{\phi}}} = \theta_1 \quad (4)$$

and

$$\theta_1 A(1 - \beta)K^\beta (uh)^{-\beta} = \theta_2 \delta. \quad (5)$$

The co-state equations are given as follows

$$\dot{\theta}_1 = \rho\theta_1 - \theta_1 A\beta K^{\beta-1} (uh)^{1-\beta} \quad (6)$$

and

$$\dot{\theta}_2 = \rho\theta_2 - \theta_1 A(1 - \beta)K^\beta (uh)^{-\beta} u - \theta_2 \delta (1 - u) - \left[\frac{(1 - \sigma)\eta h^{\phi-1}}{(c^\phi + \eta h^\phi)^{\frac{\phi+\sigma-1}{\phi}}} \right]. \quad (7)$$

Equation (6) can be re-written as follows:

$$\left(\frac{\dot{\theta}_1}{\theta_1} \right) = \rho - A\beta \left(\frac{uh}{K} \right)^{1-\beta}. \quad (8)$$

Of course, equation (5) can be used to re-write equation (7) as follows:

$$\overset{\circ}{\theta}_2 = \rho\theta_2 - \theta_2\delta - \left[\frac{(1-\sigma)\eta h^{\phi-1}}{(c^\phi + \eta h^\phi)^{\frac{\phi+\sigma-1}{\phi}}} \right], \quad (9)$$

It is relatively easy to see that along a balanced growth path, the growth rates are determined as follows:

$$\kappa = \left(\overset{\circ}{c}/c \right) = \left(\overset{\circ}{K}/K \right) = \left(\overset{\circ}{h}/h \right) = \delta(1-u). \quad (10)$$

Since the capital stock (K) and human capital (h) have the same growth rate, it follows from this equation that

$$\left(\overset{\circ}{\theta}_2/\theta_2 \right) = \left(\overset{\circ}{\theta}_1/\theta_1 \right) = -\sigma\kappa \quad (11)$$

along a steady-state growth path. From equation (8) this helps pin down the capital-labor ratio as follows

$$A\beta \left(\frac{uh}{K} \right)^{1-\beta} = \rho + \sigma\kappa, \quad (12)$$

The resource constraint (2) can be written as

$$\left(\overset{\circ}{K}/K \right) + (c/K) = A \left(\frac{uh}{K} \right)^{1-\beta} = \frac{\rho + \sigma\kappa}{\beta},$$

where the last equality comes from equation (12). This can be re-written as follows

$$(c/K) = \frac{\rho + \sigma\kappa}{\beta} - \kappa = \frac{\rho + (\sigma - \beta)\kappa}{\beta}.$$

Using this last equation, and using equation (12) to solve for the ratio of physical to human capital (K/h), it is then possible to see that

$$\frac{c}{h} = \frac{c}{K} \frac{K}{h} = \left[\frac{\rho + (\sigma - \beta)\kappa}{\beta} \right] \left(\frac{\rho + \sigma\kappa}{A\beta} \right)^{\frac{-1}{1-\beta}} u. \quad (13)$$

Using equation (9), (10), (11), and (13) together with (5), the following expression can be derived

$$-\sigma\kappa = (\rho - \delta) - \left[\frac{(1 - \sigma)\eta h^{\phi-1}}{\theta_1 (c^\phi + \eta h^\phi)^{\frac{\phi+\sigma-1}{\phi}}} \right] \left[\frac{\delta}{A(1 - \beta)} \right] \left[\frac{K}{uh} \right]^{-\beta}.$$

Now using equations (4) and (5) to eliminate the multipliers (θ_1 and θ_2), this last expression can be written as

$$\sigma\kappa = (\delta - \rho) + \left[\frac{(1 - \sigma)\eta h^{\phi-1}}{(c^\phi + \eta h^\phi)^{\frac{\phi+\sigma-1}{\phi}}} \right] \left[\frac{(c^\phi + \eta h^\phi)^{\frac{\phi+\sigma-1}{\phi}}}{(1 - \sigma)c^{\phi-1}} \right] \left[\frac{\delta}{A(1 - \beta)} \right] \left[\frac{K}{uh} \right]^{-\beta}.$$

Now, using equation (12) to get rid of the capital-labor ratio (K/uh), and equation (13) eliminate (c/h), and collecting terms yields the following

$$\sigma\kappa = (\delta - \rho) + \eta \left[\left(\frac{\rho + (\sigma - \beta)\kappa}{\beta} \right) \left(\frac{\delta - \kappa}{\delta} \right) \right]^{1-\phi} \left[\frac{\delta}{A(1 - \beta)} \right] \left(\frac{\rho + \sigma\kappa}{A\beta} \right)^{\frac{\phi-1+\beta}{1-\beta}} \quad (14)$$

It is easy to see that the large term on the right side of this equation is indeed positive in an equilibrium. This means that it will be the case that the economy will exhibit higher growth if $\eta > 0$ than if $\eta = 0$. The more that agents care about human capital, the greater will be the stock that they hold, and consequently the higher will be the growth rate of the economy. Also, this model has the implication that if you were able to look across a variety of different economies or cultures, one should witness a higher growth rate in economies in which agents “care” about human capital the most, *ceteris paribus*.³

It also has to be recognized that the growth rate in these economies cannot be unrestricted, because it must be that the discounted utility has

³Although we have let human capital directly enter the utility function, one might have an alternative interpretation of this. Obviously some cultures put a higher premium or value on education. Consider classifying agents as being in different cultures depending upon their value of η (i.e. different cultures are characterized by their different values of this parameter). To the extent that individuals might value being a part of that culture, they would then place a direct intrinsic value on their own level of human capital.

to be finite, or that the relevant transversality conditions are satisfied. The restrictions that must be satisfied are as follows

$$\kappa < \left(\frac{\rho}{1 - \sigma} \right) \quad \text{if } \sigma \in (0, 1)$$

and

$$\kappa > \left(\frac{\rho}{1 - \sigma} \right) \quad \text{if } \sigma \in (1, \infty).$$

3 Taxation

It is then of interest to see how taxes can affect the above analysis. In particular, let us consider a tax on capital at the rate of τ_k and a tax on labor income at the rate of τ_l . To simplify the analysis, it will be convenient to levy the tax on the gross return to capital, with depreciation being 100% per period. After a tedious amount of algebra, it is possible to see that equation (14) now takes the following form

$$\sigma\kappa = (\delta - \rho) + \eta \left[\left(\frac{\rho + [\sigma - \beta(1 - \tau_k)]\kappa}{\beta(1 - \tau_k)} \right) \left(\frac{\delta - \kappa}{\delta} \right) \right]^{1-\phi} \left[\frac{\delta}{A(1 - \beta)(1 - \tau_l)} \right] \left[\frac{\rho + \sigma\kappa}{(1 - \tau_k)A\beta} \right]^{\frac{\phi-1+\beta}{1-\beta}}. \quad (15)$$

Of course, in the model of Lucas, $\eta = 0$, and so this equation reduces to

$$\sigma\kappa = \delta - \rho, \quad (16)$$

and hence a tax capital tax has no effect on the ultimate growth rate. The reason for this can be seen from equation (12), which has the marginal product of capital on the left side. A capital tax merely means that the capital-labor ratio is altered, so that there is a relatively higher ratio of human to physical capital. This results in an increased before-tax return to physical capital. However, the growth rate is unaffected.⁴

⁴Similarly, a constant labor tax does not influence the growth rate either if $\eta = 0$. This is because a tax on labor income in one period discourages work effort and encourages human capital accumulation in that period (i.e. u is lowered). But this is exactly offset by the fact that the tax on labor income in future periods discourages this accumulation of human capital.

It is then of interest to see how the equilibrium growth rate is affected by these tax rates if $\eta > 0$. Consider setting the parameters of the model as follows: $\beta = .3, \sigma = 2.5, A = 0.1, \delta = .1, \rho = .025$, and $\eta = 0.05$. By adopting a “small” value for η this analysis is not departing too much from the framework employed by Lucas, and therefore not giving too big a role for the utility derived from human capital.

Figure 1 shows how the capital tax rate influences the equilibrium growth rate. Clearly, raising this tax rate results in a higher economic growth rate. Holding everything else constant, lowering the value of ϕ results in both a lower growth rate, and a smaller increment to the growth rate from raising the tax rate (τ_k). In fact, if $\phi + \beta > 1$, it is possible to show analytically that the growth rate is increasing in this tax rate. However, numerical calculations reveal that the tax rate (τ_k) and the growth rate are positively related for a much wider set of parameter values.

It is important to remember that the only thing that is changing in this illustration is the capital tax rate. It is *not* the case that the capital tax rate is being raised while lowering the labor tax rate, as is considered in Lucas [3].

Equation (15) implies that the growth rate is increasing in the labor tax rate (τ_l) for all configurations of the parameter values. Figure 2 shows how the labor tax rate influences the equilibrium growth rate, for the same parameter values as those used for the first figure. Here the labor tax causes the individual to devote less time to producing output. Consequently, this implies that the agent will then substitute away from market consumption by lowering the value of u . This results in a higher rate of acquisition of human capital.

3.1 Comparison With the Existing Literature

This example illustrates that the impact of capital taxation on the growth rate depends critically on the nature of the technology. In particular, in this particular example, it depends upon the nature of the technology that determines human capital accumulation. The effect would be similar if physical capital was an ingredient, but of small importance, in producing human capital. The importance of the makeup of this technology for human capital, was an issue studied by Rebelo [4], as well as Stokey and Rebelo [5].

This model also implies that if agents were to suddenly “care more” about human capital, as measured by the value of η , then this would lead to a higher

growth rate. This is in contrast with the findings of Fershtman, Murphy and Weiss [1]. Within the context of a different model, they find that increased emphasis on the social status associated with certain types of human capital can reduce the growth rate, if it induces the “wrong” individuals (i.e. those with low abilities) to acquire human capital, and thereby lower the rewards to the “right” individuals (i.e. those with high abilities).

3.2 Consumption Taxation

It is also of interest to see how a consumption tax would affect the growth rate. Normally in this model (i.e. with $\eta = 0$), a consumption tax would not influence the growth rate because although it would discourage current consumption, this would be exactly offset by the fact that it would also discourage saving because future consumption would also be taxed.

In this model with $\eta > 0$, this feature is still present, but is overridden by another type of substitution. A tax on consumption would then cause the agents to substitute human capital for consumption for the consumption of goods.⁵ This causes the agent to produce more human capital and less physical capital or consumption (by lowering u), which would raise the growth rate.

4 The Impact of Externalities

It is then illuminating to investigate how the additional presence of externalities would influence this analysis. Throughout the remainder of the analysis, it will ease the notational burden to let the labor tax rate (τ_l) be zero.⁶ Suppose again that the preferences and technology are again given by equations (1) and (2) respectively. However, suppose initially that the technology for producing human capital (the counterpart to equation (3)) is given by the following:

$$\overset{\circ}{h}_t = (h_t^{1-\gamma} h_{a,t}^\gamma) \delta(1 - u_t). \quad (17)$$

⁵Or, as is explained in the Appendix, the substitution of the human capital intensive good, for the more physical capital intensive good.

⁶It is important to note that for all of these examples, an increase in the labor tax rate will not reduce the growth rate.

Here $h_{a,t}$ is the average level of human capital in the economy, and so the individual benefits from the human capital accumulated by other agents. Here γ is a measure of the degree of this external effect. Of course, the economy still displays the same constant-returns-to-scale features that are compatible with balanced growth. After an exhausting amount of algebra, it is possible to show that the *equilibrium* aggregate growth rate is given by the following equation:

$$(\sigma + \gamma) \kappa = (\delta - \rho) + \eta \left[\left(\frac{\rho + [\sigma - \beta(1 - \tau_k)] \kappa}{\beta(1 - \tau_k)} \right) \left(\frac{\delta - \kappa}{\delta} \right) \right]^{1-\phi} \left[\frac{\delta}{A(1 - \beta)} \right] \left[\frac{\rho + \sigma \kappa}{(1 - \tau_k) A \beta} \right]^{\frac{\phi-1+\beta}{1-\beta}} \quad (18)$$

It is possible to show that the *optimal* growth rate is again given by equation (15), with the tax rates set to zero. So, not surprisingly, the presence of the parameter γ only appears on the left side of equation (18), and so the actual growth rate derived from this expression will be lower than the optimal growth rate. It may then be necessary to raise the capital tax rate in order to achieve the optimal growth rate. Figure 3 shows the capital tax rate necessary to achieve the socially optimal growth rate, for different values of γ , and for the following parameter values: $\beta = .3, \sigma = 2.5, A = 0.1, \delta = .1, \rho = .025$, and $\eta = 0.05$. This illustration shows that to attain the optimal growth rate for even modest values of the externality, it may necessitate substantial values for the capital tax rate.

4.1 Other Types of Externalities

In the model of Lucas [2], he considers an externality in production, so it seems appropriate to consider these here as well. Suppose that the technology for producing the consumption good is as follows

$$AK_t^\beta (u_t h_t)^{1-\beta-\gamma} h_{a,t}^\gamma \quad (19)$$

where h_a denotes the *average* level of human capital, and γ is a measure of the size of the externality.⁷ This technology exhibits constant returns to

⁷This is slightly different than the externality analyzed by Lucas [2]. Instead, he uses the following technology for output: $AK_t^\beta (u_t h_t)^{1-\beta} h^\gamma$. The specification used by Lucas has the feature that under balanced growth with $\gamma > 0$, human capital must grow at

scale in human and physical capital. If $\gamma = 0$, then the technology is that shown in equation (2).

Now consider the case in which the externality is present, so $\gamma > 0$. It is easiest to initially consider the case in which $\eta = 0$. In this instance it is possible to show that the equilibrium growth rate takes the simple form

$$\sigma\kappa = (\delta - \rho) + \eta \left[\left(\frac{\rho + [\sigma - \beta(1 - \tau_k)]\kappa}{\beta(1 - \tau_k)} \right) \left(\frac{\delta - \kappa}{\delta} \right) \right]^{1-\phi} \times \left[\frac{\delta}{A(1 - \beta - \gamma)(1 - \tau_l)} \right] \left[\frac{\rho + \sigma\kappa}{(1 - \tau_k)A\beta} \right]^{\frac{\phi-1+\beta}{1-\beta}} \left(\frac{\delta - \kappa}{\delta} \right)^{\left(\frac{\gamma\phi}{1-\beta}\right)}$$

while the optimal growth rate (which will be labelled κ^*) is written as follows:

$$\sigma\kappa^* = (\delta - \rho) - \left[\frac{\gamma(\delta - \kappa^*)}{(1 - \beta - \gamma)} \right] + \eta \left[\left(\frac{\rho + [\sigma - \beta(1 - \tau_k)]\kappa^*}{\beta(1 - \tau_k)} \right) \left(\frac{\delta - \kappa^*}{\delta} \right) \right]^{1-\phi} \times \left[\frac{\delta}{A(1 - \beta)(1 - \tau_l)} \right] \left[\frac{\rho + \sigma\kappa^*}{(1 - \tau_k)A\beta} \right]^{\frac{\phi-1+\beta}{1-\beta}} \left(\frac{\delta - \kappa^*}{\delta} \right)^{\frac{\gamma\phi}{1-\beta}}$$

It is easy to see that even in the case where $\eta = 0$, the optimal growth rate is *less* than the equilibrium growth rate. The reason for this is that if $\gamma > 0$, the agent does not receive as high a marginal return from working to produce output, and as a result, he works less (reduces u). But because then $(1 - u)$ increases, equation (3) implies that the growth rate of the economy *increases*. Hence, the presence of the externality causes the optimal growth rate to be lower than the equilibrium growth rate.

Usually, presence of this type of externality would normally lead one to suggest that a subsidy to investment or capital accumulation would be in order to generate the optimal growth rate for the economy. However, if the preferences are given by equation (1) with $\eta > 0$, and the technology is given

by equation (19), which is qualitatively very similar to that employed by Lucas. However, this would then cause problems in the present model because, given the preferences specified by equation (1), the marginal rate of substitution between consumption and human capital would not be constant. Hence, the present analysis uses the technology given by equation (19), which is qualitatively very similar to that employed by Lucas.

by equation (19) with $\gamma > 0$, then to achieve the socially optimal growth rate, it may be detrimental to impose a capital tax. However, the reason why it is bad is the opposite from what one usually finds. In the present model a capital tax may *raise* the growth rate, but in the presence of the externality the optimal growth rate is *lower* than the equilibrium growth rate.

If $\gamma > 0$, then the equilibrium growth rate is elevated even higher *above* the optimal growth rate.

On the other hand, consider an alternative type of externality in the technology, in which output is determined by the following production function:

$$AK_t^\beta (u_t h_t)^{1-\beta-\gamma} (\bar{u} h)^\gamma. \quad (20)$$

Now it is possible to show that, if $\eta = 0$, the equilibrium growth rate and the optimal growth rate are identical, and are determined according to equation (16). If $\eta > 0$, then the equation determining the equilibrium growth rate is the following:

$$\begin{aligned} \sigma \kappa = & (\delta - \rho) + \eta \left[\left(\frac{\rho + [\sigma - \beta(1 - \tau_k)] \kappa}{\beta(1 - \tau_k)} \right) \left(\frac{\delta - \kappa}{\delta} \right) \right]^{1-\phi} \times \\ & \left[\frac{\delta}{A(1 - \beta - \gamma)} \right] \left[\frac{\rho + \sigma \kappa}{(1 - \tau_k) A \beta} \right]^{\frac{\phi-1+\beta}{1-\beta}} \end{aligned} \quad (21)$$

It is then interesting to characterize the optimal growth rate which would result from maximizing the utility function of each agent, subject to the resource constraint, while taking the externality into account. This growth rate is given by equation (15), with the tax rates set to zero.

Of course, these equations are identical if $\gamma = 0$, but otherwise this is not likely to be the case. Beginning from a situation where the tax rate is zero, and $\gamma = 0$, consider a rise in the value of γ . This would raise the equilibrium growth rate, as seen in equation (21), but does not influence equation (15). What is going on here is very similar to that of the previous example. If $\gamma > 0$, then the agent does not receive a sufficiently high reward to working, and consequently he will lower the value of u , but this will then result in a higher growth rate, because of the form of equation (3).

The issue here is that the growth rate in this model is determined by the capital-labor ratio (see equation (12)). By taxing physical capital, it is possible to achieve the *optimal* capital-labor ratio, which is then associated

with the optimal growth rate. Obviously when $\gamma > 0$ a tax on physical capital accomplishes this goal because otherwise agents would be choosing to hold too high a ratio of physical to human capital.

5 Final Remarks

The model studied above is one in which there are two capital stocks. It was demonstrated that when individuals derive utility from one of these stocks, then this will alter the impact of tax policies from what might traditionally expect.

Stokey and Rebelo[5] point out that there has been a substantial secular rise in the size of government in the US over the 20th century, which has been accompanied by commensurate increase in the tax distortions of the returns to capital and labor. Within the context of many models this would result in a lower equilibrium growth rate. Nevertheless, there appears to be no evidence that these increased distortions have generated a fall in growth rates. The model described above shows that human capital enters the utility function, that this would then result in an increase in the growth rate from raising these tax rates. This may be one reason why there is little empirical association between higher tax rates and the aggregate growth rate. This may also be reason why the welfare costs of capital taxation, as measured in the existing literature, may be overstated.

6 Appendix

In this section it is shown that the model described above could be interpreted as an economy without any externalities. Consider an economy populated by representative agents with preferences characterized by the following utility function

$$\int_0^{\infty} e^{-\rho t} \left(c_{1,t}^{\phi} + \eta c_{2,t}^{\phi} \right)^{(1-\sigma)/\phi} dt.$$

Obviously there are two types of consumption goods. The resource constraint for output is given as follows

$$\overset{\circ}{K} + c_{1,t} = A^* K_t^{\beta} (u_t h_{1t})^{1-\beta}.$$

Here h_{1t} is human capital used in the production of physical output. The equation governing human capital accumulation is still given as follows:

$$\dot{h} = h_t \delta (1 - u_t).$$

Now there is a Leontief technology governing how human capital is used. First of all, some fraction of it is used to produce the second consumption good

$$c_{2,t} = (2\alpha)h_t,$$

while the remainder is used to produce human capital to be used in the production of physical output

$$h_{1t} = (1 - \alpha)h_t.$$

Of course this Leontief technology can be thought of as the limit of a CES technology as the elasticity of substitution between the production of these two goods becomes infinite. This model, as described, does not have any externality. Now let $\alpha = 1/2$, $A = A^*(1 - \alpha)^{(1-\beta)}$, and this model becomes the one described in the body of the paper.

One might also think of the consumption good denoted $c_{2,t}$ as being a non-market consumption good, or the result of home-production. Then the agent is able to engage in substitution between these two goods.

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Figure 1

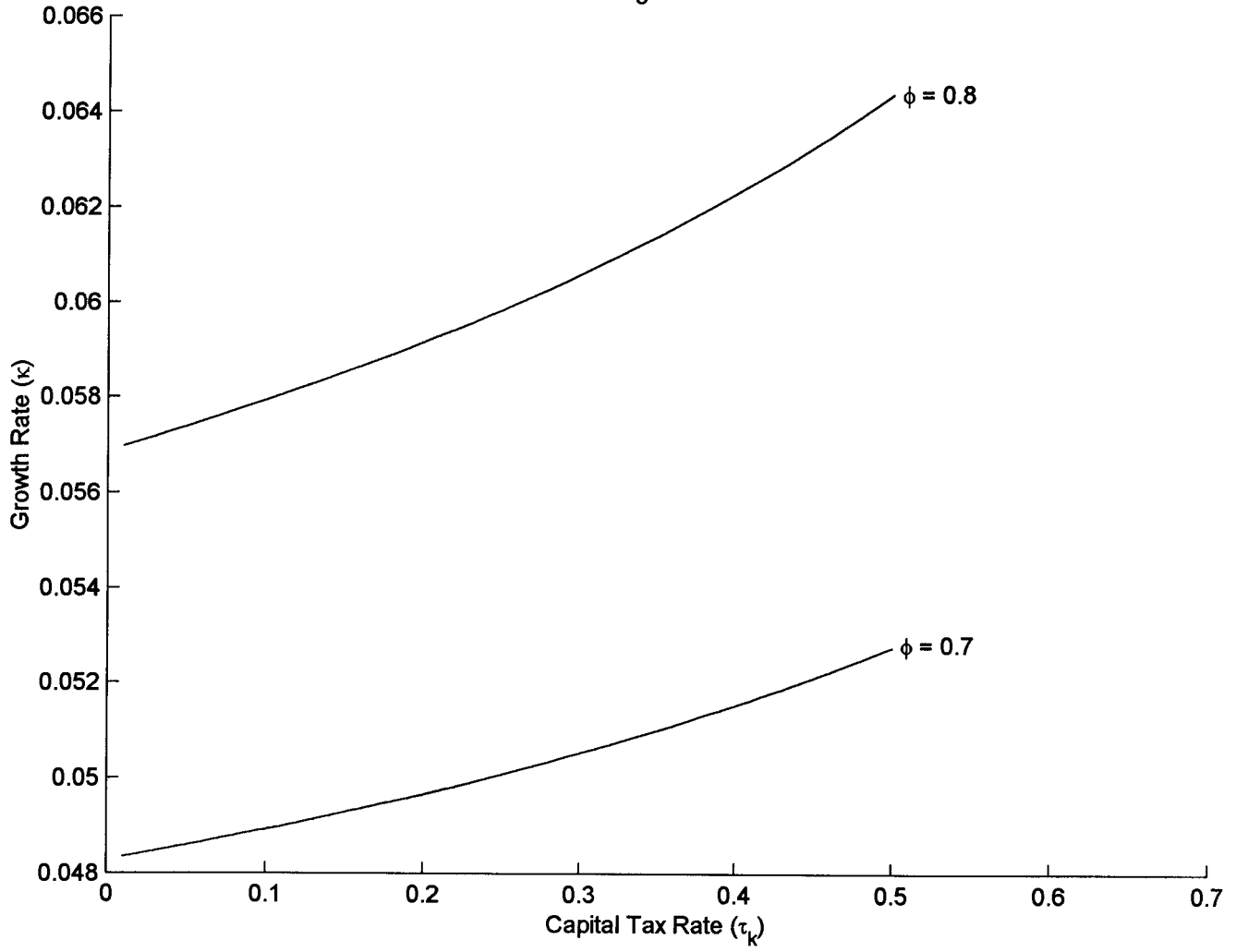


Figure 2

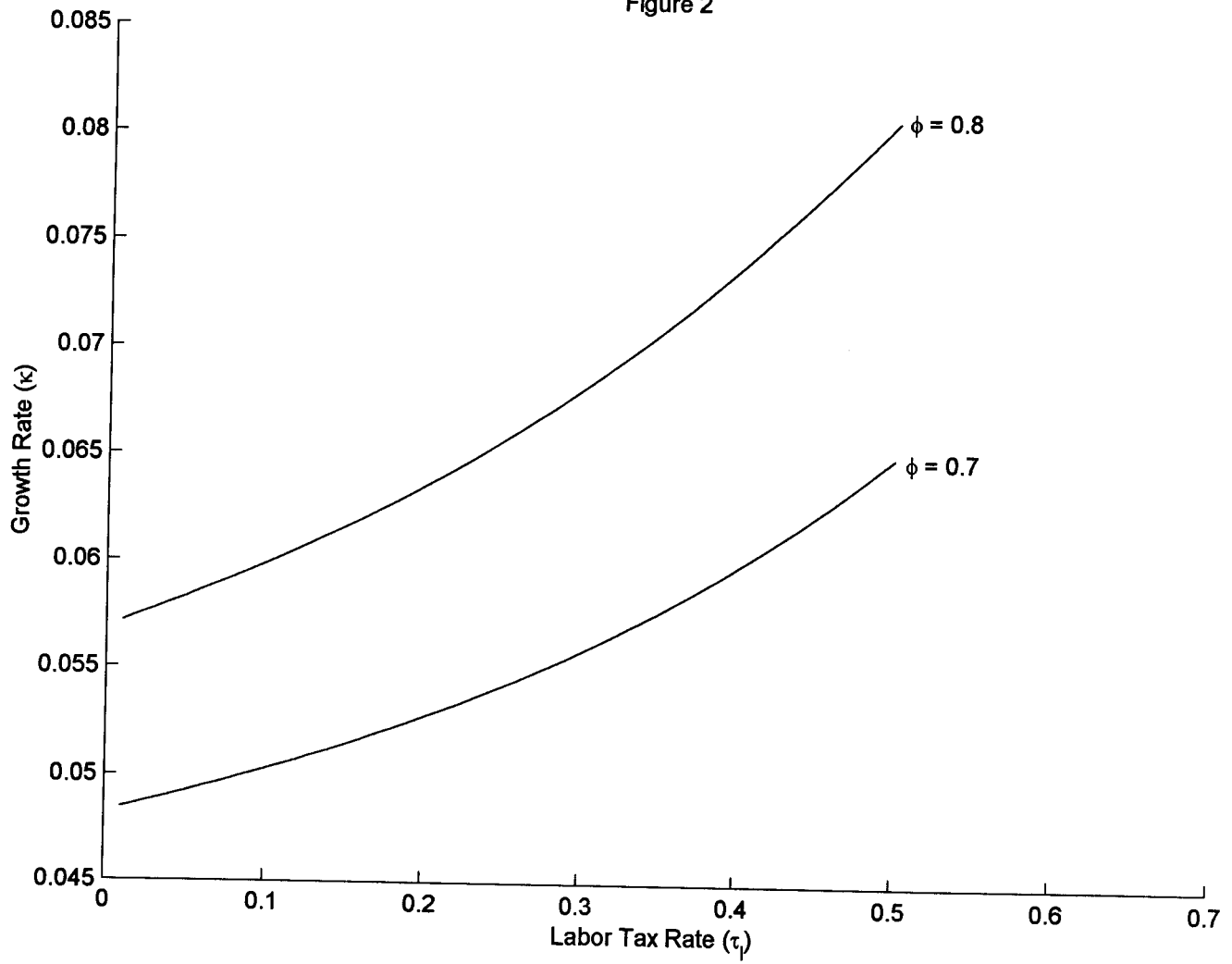


Figure 3

