FIGHT FIRE WITH FIRE:
A MODEL OF POLLUTION AND GROWTH WITH COOPERATIVE SETTLEMENT

by

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Abstract

This paper establishes a growth model where firms and residents in polluted areas bargain cooperatively to settle environmental concerns. While economic development affects the extent of the negotiation outcomes, the bargaining results also influence firms' incentive to undertake R&D and thus economic growth. Due to the opposing effects of production and matching technologies, an inverted-U relationship between pollution and growth is obtained. Contrasting to growth-promoting policies, policies that create barriers to firm entry or matching may reduce pollution without harming growth. Due to the opposing effects of thick-matching versus effective-discounting and pollution-externality, the decentralized outcome may involve over or under-pollution.

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1. Introduction

While the establishment of a dirty industry may create job opportunities and enhance tax revenues in the local economy, community residents would suffer from the accompanied degradation of the local environment. The conventional view, led by Meadows, Meadows and Randers (1972), emphasizes that there is no free lunch as pollution is an inevitable by-product of the process of economic advancement. A more recent view, best represented by the World Bank Development Report (1992) and Grossman and Krueger (1995), suggests however that there is an inverted-U relationship between environmental degradation and income.[1] That is, environmental concern is not the poor man's game: pollution increases with income at the early stage of economic development and decreases with income at the later stage. Given that environmental issues are central to public policy debates and in the popular press, the presence of these conflicting views gives birth to a growing literature since the mid-1990s.

There has been a large literature within the static framework with and without trade.[2] Recently, Howarth and Norgaard (1992) claim that sustainability of efficient allocation of economic resources and environmental services is a matter of intergenerational distribution, as environmental concern is usually the interest of future generations. This insight has led to the studies by John and Pecchenino (1994), Fisher and van Marrewijk (1994) and Jones and Manuelli (1995), all of whom adopt the overlapping generations framework investigating environmental development and regulating policies. Similar issues are revisited by Byrne (1995) and Stokey (1998) under infinite-horizon endogenous-growth setups in the presence of the

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[1] Typically, the indicators of pollution are classified into two categories: urban air pollution, measured by sulphur dioxide (SO_2) and suspended particles, and water pollution, measured by the oxygen regime, pathogenic contamination and heavy metals. An inverted-U relationship exists for several indicators, including smoke in cities and oxygen in rivers as well as fecal and arsenic metal contamination of rivers. There are, however, exceptions exhibiting an inverted-J relationship (inverted-U plus an increasing portion toward the high income end): SO_2 and mercury and nickel contamination of rivers.

[2] The reader is referred to references in Baumol and Oates (1988, chapters 13 and 14) and Copeland and Taylor (1994).

tradeoffs between economic growth and environmental quality maintenance. In most of the overlapping generations models, pollution control in a decentralized economy depends on a collective choice mechanism. Jones and Manuelli (1995) consider, in addition to collective choice, a voting mechanism to achieve a decentralized equilibrium. In the infinite-horizon endogenous-growth models, pollution is purely a by-product of goods production whose process is given to each individual and controlled entirely by the central planner. Typical environmental policies considered in this literature are direct regulation (Byrne 1995, Jones and Manuelli 1995 and Stokey 1998), Pigovian taxation (Fisher and van Marrewijk 1994, Jones and Manuelli 1995 and Stokey 1998) and a voucher system (Stokey 1998). Interestingly, John and Pecchenino (1994) and Jones and Manuelli (1995) find that a decentralized equilibrium is insufficiently polluted compared to the social optimum, while Byrne (1995) concludes that over-pollution arises in equilibrium.

In contrast with previous approaches, our paper proposes a new and realistic mechanism, namely *cooperative settlements*, allowing for negotiations between the polluters and pollutees.[3] Consider the petroleum industry that has been frequently opposed by environmentalists. Back to a decade or two ago, petroleum firms obtained local right-of-way via the political channel by dealing with local officers or leaders. In recent years, there has been increased community awareness that local residents, together with the legislators, have involved extensively in negotiating with those petroleum companies, i.e., *fighting fire with fire*. Examples include, to name but a few, Shell Canada Ltd. at Fort McMurray (Alberta, Canada) in 1992, Trans Mountain Pipe Line Co. Ltd. at Port Angeles (Washington State, U.S.A.) in 1999, as well as many cases in less-developed countries, such as BP in Colombia, Mobil in Peru and Arco in Ecuador, over

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[3] Shibata (1971) suggests that the bargaining framework may be relevant for public goods and goods imposing externalities. In particular, he uses diagrammatic analysis of bilateral monopoly to determine public expenditure and the distribution of taxes. Although our paper considers a very different bargaining method, his insight motivates our study.

the past decade. These negotiations have led to compensations of various forms, from cash payments to communal benefits, such as schools and clinics.[4]

We argue that the interplay between such endogenous settlements and economic growth may provide further insights toward understanding the process of environmental development and evaluating the effects of relevant public policies. Moreover, our framework allows us to compare a decentralized equilibrium allocation with the socially optimal allocation in a fashion that reconciles the conflicting findings in the literature. Specifically, our paper establishes a growth model in which firms and residents in polluted areas bargain cooperatively to settle their environmental concerns. While the stage of economic development affects the extent of the negotiation outcomes, the bargaining results feed back to influence firms' incentives to undertake R&D and thus the rate of economic growth. The primary benefit of employing this matching framework is not only to simplify the settlement of pollution compensation but to allow the extent of market (entry and matching) frictions, in addition to conventionally considered preference/technology parameters, to play a role in affecting the level of pollution and the rate of economic growth.

The main findings of the paper can be summarized as follows. First, in the case of exogenous R&D, pollution co-moves with income whereas a local tax or environmental regulation that discourages firm entry or matching can reduce the level of pollution. Second, with endogenous R&D, changes in the firm entry cost or the matching scaling factor result in a negative relationship between growth and pollution, whereas changes in the production technological parameters lead to a positive relationship. As a consequence, any growth-promoting policy (such as a production subsidy or a capital investment tax credit)

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[4] Wasserstrom and Reider (1998) provide a comprehensive report on how several petroleum companies negotiated with Latin American communities. They indicate that those companies usually hire community relations specialists, visiting and consulting communities before and during the environmental impact assessment, developing guidelines for contact with local residents, as well as working out cash payments, community support programs and deals to employ local residents.

will enhance economic growth at the expense of an increased level of pollution; yet, a policy that creates barriers to firm entry or matching or a policy that requires a minimum compensation ratio may reduce pollution without harming economic growth. Third, when technological advancements are more feasible at the early stage of economic development and environmental regulations become more a concern at the later stage, an inverted-U relationship between pollution and income growth is obtained. Finally, due to the opposing effects of thick matching versus effective discounting and pollution externality, the decentralized equilibrium may involve over or under-pollution.

To the end, it may be informative to contrast our paper with a few closely related studies, particularly John and Pecchenino (1994), Jones and Manuelli (1995), Byrne (1995) and Stokey (1998) . First, the self-regulating mechanism in the present paper is a consequence of a cooperative settlement via bargaining, rather than collective choice or majority voting. Second, previous work obtains the inverted-U relationship between environmental degradation and income (or economic growth) based on a direct tradeoff between environmental quality and final goods output through individual preferences. Thus, such a relationship depends crucially on the functional forms of utility (for example, the elasticity of marginal utility) and pollution generating activity.[5] In our paper, it is the endogenous cooperative settlement with endogenous investment in R&D, leading to such a non-monotonic relationship. Third, the finding of over-pollution in decentralized equilibrium by Byrne (1995) is due mainly to the assumption that individuals make no decision on pollution abatement and such decision is made by the social planner. In papers with a self-regulating mechanisms (John and Pecchenino 1994 and Jones and Manuelli 1995), the economy under a decentralized equilibrium is always insufficiently polluted in the absence of an accumulation of the pollution stock. This is because individuals fail to account for the positive effect of investment in physical capital on future generations' welfare and hence under-investment occurs. Thus, a reallocation from

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[5] For example, in the endogenous growth model of Stokey (1998, p.11), it is required that the intertemporal elasticity of substitution for the composite good be sufficiently low.

physical capital to pollution abatement investment results in an insufficiently polluted equilibrium. In our paper, we show that even without the accumulation of pollution stock or pollution abatement, it is possible to have an overly polluted decentralized equilibrium, depending crucially on the magnitudes of thick matching versus effective discounting and pollution externality effects.

The remainder of the paper is organized as follows. In Section 2, we outline the basic environment of the economy. Section 3 presents the matching technology and the rule of cooperative Nash bargaining. To gain more insight, we study the steady-state equilibrium in Section 4 in which the R&D effort is taken as exogenously given. We then examine the full equilibrium with endogenous R&D in Section 5, completely characterizing the determinants of the level of pollution and the rate of endogenous growth. Section 6 analyzes the main policy implications of the model and Section 7 summarizes the paper and provides some plausible avenues for future work.

2. The Model

To allow for endogenous cooperative settlement between the polluters and the pollutees, we adopt a simple continuous-time matching framework. The economy is populated with a continuum of firms and a continuum of residential regions of a given mass R where each has a continuum of residents of mass $n \geq 0$. To circumvent technical difficulties, we assume that matching is anonymous and that each agent is atomistic and behaves competitively, taking matching parameters as given. In this Section, we describe the behaviors of firms and residents and then specify their value functions in matched and unmatched states.

2.1 Firms

Firms are endowed with a production technology, which enables an output of a final good accompanied by pollution. There is a large number of such potential firms. Let q denote the R&D effort of a representative firm. Upon paying an entry (or setup) cost, each firm accumulates “productive knowledge” K at rate $\theta = \theta(q)$:

$$K(t) = k_o e^{\theta(q)t} \tag{1}$$

where $k_o > 0$ is the initial knowledge capital stock and $\theta' > 0$, $\theta'' < 0$ and $\theta^s \equiv \sup_q \theta(q) < \infty$ (i.e., θ is bounded). The setup costs may be thought of as establishing the background or basic structure of the particular production activity. The higher the R&D effort is, the more effectively a firm can accumulate product-specific knowledge. This setup of R&D-dependent knowledge growth is consistent with the innovation and endogenous growth literature [e.g., see Aghion and Howitt (1992) and Laing, Palivos and Wang (1997)].

Assume that the production technology is linear (constant-returns-to-scale) with a scaling parameter A > 0. Thus, the instantaneous flow of output at any point in time is AK. Utilizing (1), we can then specify the (capitalized) value of production (stock) to each firm from time t and on as: $Y(t) = \int_t^\infty AK(\tau) e^{-r(\tau-t)} d\tau = \frac{Ak_0}{r-\theta(q)} e^{\theta t}$, which is integrable under the Brock-Gale condition: $r > \theta^s$. Defining the value of production per effective unit as $y(t)=Y(t)/e^{\theta t}$, we therefore have:

$$y = \frac{Ak_0}{r-\theta(q)} \tag{2}$$

Note that the Brock-Gale condition also ensures the positiveness of the value of output (per effective unit). Thus, this value is increasing in the production scaling factor, the initial knowledge capital stock and the R&D effort, and decreasing in the discount rate. It is important to transform perpetually growing variables into stationary ones (in effective units) such that the conventional stationary bargaining theory can be applied.

As noted above, pollution is a joint output of the production of the final good. When a firm occupies a region to undertake production activity, it creates a pollution flow of p (to be endogenously determined in equilibrium), which in turn damages the residents in the occupied area. Thus, in order for the residents in this region to agree upon the establishment of the production site, a (one-time) pollution-compensation must be provided, which is a pure transfer from the firm to the residents. Denote the amount of the pollution-compensation to each resident (per effective unit) as x, the entry cost (per effective unit) as ν_0, and the cost of R&D (per effective unit) as c(q). Both the entry and the R&D costs are sunk. Then a representative firm's optimization problem is therefore given by:

$$\underset{q}{\mathrm{Max}}\ (y - nx^* - c(q)) \tag{3}$$

subject to the technology specified in (2) and the non-negativity constraint $y - nx^* - c(q) \geq \nu_0$. That is, for a given schedule of x = x*, each firm selects its R&D effort to maximize its value net of spending on pollution compensation and R&D effort. Of course, it is profitable for firms to be operative only if such net value exceeds the fixed entry cost; otherwise, the equilibrium degenerates and no firm would enter the production sector.

It is important to note, however, that the compensation schedule x is endogenously determined. Thus, this optimization problem is non-trivial, which will be accomplished in three steps. First, we will adopt a cooperative bargaining method to pin down the compensation offer function x*, depending on any particular list of matching parameters (contact rates and flow matches) and the R&D effort. Next, based on the compensation offer schedule obtained from the first step and the R&D effort, we can solve for endogenous matching parameters. Finally, we apply backward induction arguments to obtain the optimal level of R&D investment, using the compensation offer schedule and endogenous matching variables determined in previous steps. As we will show below, the recursive nature of the model enables us to use the backward solving technique, determining all the endogenous variables in equilibrium.

2.2. <u>Residents</u>

In any two-dimensional area, the firm site is one-dimensional (an interval). Thus, it is reasonable to imagine that there is a continuum of potential areas suitable for a firm to establish a factory. Consider now a polluted region. Given the level of pollution flows generated by the firm occupying this particular region, each resident suffers a pollution damage (per effective unit), denoted z. Since pollution is a by-product of the production, z can be measured as a fraction of the value of production. Let δ denote this fraction, which can be referred to as the unit pollution damage. Conventional studies regard δ as an exogenous constant. In our paper, we allow for cross-region pollution spillovers and capture this feature by assuming that the unit pollution damage is increasing in the amount of the society's pollutant generated, p. That is, pollution is a "public bad" in the sense that the entirety of the aggregate negative externality from production affects each resident in each polluted area. Obviously, when there is no pollution flow in the entire economy ($p = 0$), no pollution damage is created and so $\delta(0) = 0$. Moreover, we restrict our attention to the case where the total pollution damage does not exceed the value of final goods produced (i.e., $\delta \leq 1$). We thus have:

$$z = \delta(p)y \tag{4}$$

where $0 \leq \delta \leq 1$, $\delta(0) = 0$, $\delta' > 0$ and $\delta'' < 0$ (the strict concavity of δ is imposed to guarantee the second-order condition to hold).

Therefore, a representative resident's net value by allowing a firm to establish a production site in the neighborhood is $x - z$. When the pollution compensation is not enough to cover the pollution damage, the residents in the relevant region would not grant permission for the firm to undertake production activity in their community.[6] This latter case will involve no action and autarky emerges, in which the economic activity degenerates. Thus, we will be only interested in the case of $x > z$.

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[6] Implicitly, we consider a unanimous decision rule by the residents with regard to permission to set up a factory. Note, however, that with identical residents in an anonymous bargaining game, this is equivalent to the majority rule.

2.3. Value Functions

There are two separate states: matched (M) and unmatched (U). A region could be occupied (matched) or unoccupied (unmatched) and a firm could be operative (matched) or non-operative (unmatched). Imagine that there is a continuum of potential regions of mass R and a continuum of potential firms of mass V which have "entered" the market but are still at the pre-production non-operative stage. Let Π_i denote the value of a firm (per effective unit) in state i and J_i be the value of a resident (per effective unit) in state i (i = M, U). Denote the flow probability for a firm to locate a region as η and the flow probability for a region to be located a firm as μ. Consider the non-degenerate case with $y/n > x > z$.

We can now specify the values as follows:

$$\Pi_M = \max\ \{y - nx, 0\} = y - nx \tag{5}$$

$$\Pi_U = \int_t^{\infty} \eta \max\{y - nx, 0\} e^{-(r+\eta)(\tau - t)} d\tau = \tfrac{\eta}{\eta + r}(y - nx) \tag{6}$$

$$J_M = \max\{x - z, 0\} = x - z \tag{7}$$

$$J_U = \int_t^{\infty} \mu \max\{x - z, 0\} e^{-(r+\mu)(\tau - t)} d\tau = \tfrac{\mu}{\mu + r}(x - z)\,. \tag{8}$$

While the matched values are obvious, the unmatched values deserve further comments. In (6), the flow value for a firm to be operative (by locating in a region) is $\eta \max\{y - nx, 0\}$. To obtain the present value, this must be discounted by the effective discount rate, $r+\eta$, where the exogenous discount rate r is augmented by η as the latter measures the probability of exiting the unmatched state. The unmatched value of a representative resident in (8) follows a similar interpretation, where relevant contact rate is μ and the associated effective discount rate is $r+\mu$.

Obviously, the unmatched values depend positively on their corresponding contact rates and values of matching, and negatively on the discount rate. Moreover, it is easily seen that both matched values are higher than the associated unmatched values, as their multipliers, $\eta/(\eta+r)$ and $\mu/(\mu+r)$ are strictly less than unity. The latter point gives rise to a positive incentive for firms and residents to seek for a match.

2.4. The Flow of Pollutant

An immediate question arises is how to determine the pollution flows. A simple way to derive the society's pollution flows is to employ the constant-returns-to-scale random matching technology developed in Diamond (1982): $p = p_0 P(V, R)$, satisfying $P_V > 0$, $P_R > 0$, $P_{VV} < 0$, $P_{RR} < 0$, the boundary conditions, $P(0, R) = P(V, 0)=0$, and the Inada conditions. That is, an increase in either the mass of firms or the mass of potential regions results in more matches and hence larger pollution flows. The absence of either side of the parties yields no match and zero pollutant. The Inada conditions and the property of constant returns are imposed to ensure the existence of the steady-state equilibrium. Here, the matching scaling factor p_0 may be thought of as a indicator of the degree of social acceptability of environmental pollution or effectiveness for the government to coordinate social conflicts.[7]

This completes the setup of the basic environment of the economy. The structure of our framework is briefly outlined in Figure 1.

3. Matching and Bargaining

Given the masses and the contact rates of both sides of the matching party, the flow matches of firms and regions are ηV and μR, respectively. In the steady state, these two flow matches must be equal to each other, as well as to the society's pollution flows generated. Under the random matching technology specified in Section 2.4, we can express pollution market equilibrium as:

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[7] For instance, Baumol and Oates (1988) argue that the social acceptability of pollution may vary in different countries and thus lead to different environmental policies. They further propose that such social acceptability may be related to income class: "[pollution may] produce anguished cries from middle- and upper-income inhabitants of a potential site and yet be welcomed as a source of more remunerative jobs by residents whose earnings are low" (p. 191). In the benchmark model, we for simplicity assume that the social acceptability is an exogenous parameter.

$$\eta V = \mu R = p = p_0 P(V,R)\ . \tag{9}$$

Upon a meeting, each pair of region representative and firm undertakes cooperative Nash bargaining. Precisely, a firm and a region representative take their unmatched values (Π_U and J_U) as the threat points and given these, seek to split the surplus accrued from a successful matching. To a firm, the surplus is measured by $\Pi_M - \Pi_U$, whereas the surplus to a region representative is $J_M - J_U$. Consider a simple case where the relative bargaining power of each side of the matching party is constant (and parametrically given). Thus, each pair of the matching party desires to maximize the joint surplus: $(\Pi_M - \Pi_U)^{\beta}\,[n(J_M - J_U)]^{1-\beta}$, where β measures the share of the surplus going to the firm in negotiation and 1-β that to the residents in the selected region.[8] This implies the following Nash bargaining rule:

$$\frac{\Pi_M - \Pi_U}{\beta} = \frac{(J_M - J_U)n}{1-\beta} \tag{10}$$

Following Pissarides (1984) and Laing, Palivos and Wang (1995), we assume that each atomistic competitive firm takes Π_U as parametrically given. Substituting the value functions in (5), (7) and (8) into (10), we obtain the pollution compensation offer function:

$$x = \frac{(1-\beta)(r+\mu)(y-\Pi_U)/n + \beta r \delta(\mu R) y}{r + \mu(1-\beta)}\ . \tag{11}$$

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[8] The consideration of cooperative Nash bargaining is purely for the sake of simplicity. In the game constructed here with complete information, one could adopt other efficient but non-cooperative bargaining methods, such as the infinite alternating-offer framework by Rubinstein (1982), to reach similar conclusions. Moreover, by using inefficient bargaining methods, it would only imply fewer matches and give an additional dimension of social inefficiencies of decentralized equilibrium, leaving our main findings (such as pollution versus growth and economic welfare) unchanged.

This pollution compensation offer function is well-defined as long as the total output exceeds the unmatched value for firms: $y > \Pi_U$ (which, as discussed in Section 4 below, will hold in the steady state with free entry). Straightforward comparative statics yield:

Proposition 1: (Pollution Compensation Offer) *Under cooperative Nash bargaining, the (per capita) pollution compensation offer function can be written as* $x^* = X(y,\beta,\mu,r,n,\delta(\mu R),\Pi_U)$ *, satisfying:*

$$\frac{\partial X}{\partial y} > 0,\ \frac{\partial X}{\partial \beta} < 0,\ \frac{\partial X}{\partial \mu} > 0,\ \frac{\partial X}{\partial r} \gtrless 0,\ \frac{\partial X}{\partial n} < 0,\ \frac{\partial X}{\partial \delta(\mu R)} > 0,\ \frac{\partial X}{\partial \Pi_U} < 0 .$$

Obviously, an increase in R&D (q) that raises output (y) has a positive wealth effect on pollution compensation, while a higher bargaining share for firms (β) reduces their compensation offered to residents in polluted areas. An increase in the resident's flow contact rate (μ), on the one hand, strengthens the resident's relative bargaining power due to thick matching, which results in a higher pollution compensation to residents. On the other hand, a higher resident's contact rate raises the effective discount rate ($r+\mu(1-\beta)$), thus reducing the matching surplus and pollution compensation. It is easily seen from (11) that the thick matching effect always dominates the effective discounting effect. As a consequence, an increase in the resident's contact rate leads to a larger pollution compensation. Next, consider a higher pollution cost per unit of output (δ) that reflects an increased severity of pollution externalities and hence requires a higher compensation in order to maintain a fixed proportion of surplus to each side of the bargaining party. As the population in each region (n) increases, it is more costly to compensate the residents to operate the factory; to maintain a fixed proportion of surplus for firms, the per capita pollution compensation must be lower. Finally, a higher Π_U increases the firm's bargaining power, thus resulting in a higher pollution compensation to the residents.

4. Steady-State Equilibrium with Exogenous R&D Effort

In this Section, we focus on examining the steady-state equilibrium with a given level of the R&D effort. This step is important. As illustrated in Section 5 below, the steady-state contact rate schedules determined in this Section will be used to pin down the optimal R&D investment and the full steady-state equilibrium. To begin with, we utilize the recursive nature of the model, combining a transformed version of the steady-state matching condition with the equilibrium entry condition to determine the steady-state equilibrium contact rates, η and μ. We then substitute these equilibrium contact rates into the steady-state matching relationships to solve for the equilibrium level of pollution flows (p) and the endogenous population of potential firms (V).

By the constant-returns-to-scale property, steady-state matching (9) can be rewritten as:

$$\eta^{SS} = \mu(R/V) = p_0 P(1, \eta/\mu) , \tag{12}$$

which can be referred to as the SS (steady-state matching) locus. This locus can be plotted in (μ,η) space: it is downward sloping, with both axes as asymptotes (which is a consequence of the Inada conditions). Notably, this relationship is parallel to the concept of the Beveridge curve which illustrates the steady-state matching condition in the conventional space of the unmatched populations. When social acceptability is higher or government coordination becomes more effective (i.e., a higher value of p_0), the SS locus will shift outward.

We next turn to examining the property of equilibrium entry. In particular, we postulate that competitive firms continue to enter the market searching for a region to establish their production site until their ex ante (unmatched) value equates with the entry cost, v_o. That is, we have:

$$\Pi_U = \nu_0 . \tag{13}$$

Substituting (6) and (11) into (13) then yields the equilibrium entry condition, referred to as the EE (equilibrium entry) locus, which depends only on the two endogenous contact rates:

$$\eta^{EE} = \frac{\nu_0(r+\mu(1-\beta))}{\beta(y-\nu_0-n\delta(\mu R)y)}\,. \tag{14}$$

It is tedious but straightforward to show that the righthand side of (14) is strictly increasing and strictly concave in μ and thus the EE locus is upward sloping in (μ,η) space with a vertical intercept $\eta_o = v_o r/[\beta(y - v_0)]$ (see Appendix). Obviously, an increase in output (y) or in the firm's bargaining power (β), or a decrease in the mass of potential regions (R), the population of residents in each region (n) or the fixed entry cost (v_o) will shift the EE locus rightward.

We are now prepared to solve for the steady-state equilibrium in a recursive manner. First, combining the SS and EE loci, (12) and (14), the (unique) steady-state equilibrium values of the contact rates (μ^*, η^*) can be obtained (for a diagrammatic illustration, see Figure 2). The existence of the steady-state equilibrium contact rates is guaranteed by the Inada condition of the matching technology P and the monotone increasing property of the equilibrium entry condition. Next, utilizing these contact rates μ^* and η^*, the steady-state equilibrium level of pollution flows p^* and the steady-state equilibrium mass of potential firms V^* can be solved recursively using (9):

$$p^* = \mu^* R \tag{15}$$

$$V^* = \mu^* R/\eta^*. \tag{16}$$

It is clear that a higher resident's contact rate or mass of potential regions raises the level of pollution and potential firm entrants, whereas an increase in the firm's contact reduces the mass of potential firms.

Consider formally the concept of steady-state equilibrium,

Definition 1: *A non-degenerate steady-state equilibrium with exogenous R&D is a pollution compensation offer function* $x = X(y,\beta,\mu,r,n,\delta(\mu R),\Pi_U)$ *and a quadruple* $\{\mu^*, \eta^*, p^*, V^*\}$ *satisfying the following conditions:*

(i) *cooperative Nash bargain: (10);*

(ii) *steady state matching: (13), (15) and (16);*

(iii) *equilibrium entry: (14);*

(iv) *operative production:* $y - nx - c(q) \geq n_0$ *with y specified as in (2).*

Notably, part (iv) above ensures that in the steady-state, $y > \Pi_U$ and $(1-\delta n)y > \Pi_U$. The latter inequalities are required to guarantee that x-z is positive and that x is increasing in q, respectively.

By utilizing Figure 2, the properties of the EE and SS loci discussed above and equations (15) and (16), we have:

Proposition 2: (Steady-State Equilibrium Contact Rates) *In steady-state equilibrium, an increase in output (y) or firm's share of matching surplus (*β*), or a decrease in population density (n), the mass of potential regions (R) or the firm entry cost (*ν_0*) will raise resident's contact rate but suppress firm's contact rate. The matching scaling factor (*p_0*), on the other hand, has positive effects on both resident's and firm's steady-state equilibrium contact rates.*

Proposition 3: (Steady-State Equilibrium Level of Pollution Flows) *In steady-state equilibrium, an increase in output (y),firm's share of matching surplus (*β*) or the matching scaling factor (*p_0*), or a decrease in the population density (n) or the firm entry cost (*ν_0*) will raise the level of pollution flows.*

The effect of the scaling factor in Proposition 2 is straightforward: an increase in social acceptability of pollution or government coordination effectiveness raises both firm's and potential region's contact rates. The effect of a higher output, a greater firm's share of matching surplus or a lower firm entry cost is to encourage firm entry, thus raising the probability for a region to locate a firm but reducing that for a firm to locate a region. A lower mass of potential regions or population in each region increases firm competition for a production site and thereby enhances potential region's contact rates but suppresses firm's contact rates.

Since the level of pollution in the steady state is measured by $p^* = \mu^* R$, any autonomous changes except for R that raise the potential region's contact rate will generate more pollutants. Interestingly, the mass of potential regions has an ambiguous effect on pollution: while its direct effect is to raise pollution, the indirect effect through a reduced resident's contact rate is to decrease it. An important consequence of Proposition 3 is that the level of pollution flows p depends positively on output, the firm's bargaining power and the matching scaling parameter, but negatively on the population density and the firm entry cost.

The two propositions together therefore lead to some intriguing implications. First, pollution (measured by the society's pollutant, p) co-moves with income (measured by the value of output per effective unit, y) in the absence of endogenous R&D effort, due primarily to an increase in firm entry. Second, an increase in government coordination effectiveness (measured by p_0) as in socially planned, growth-oriented countries tends to produce more environmental problems. Moreover, if the degree of social acceptability of pollution (also measured by p_0) is higher in low income regions, then the low income regions may or may not be associated with higher levels of pollution, as a consequence of the conflicting effects of output and social acceptability. Finally, a local tax or environmental regulation that discourages firm entry or matching can reduce the level of pollution.

5. Steady-State Equilibrium with Endogenous R&D Effort

In Sections 3 and 4, we have derived the pollution compensation and contact rate schedules for any given level of R&D investment. We are now prepared to solve for the full equilibrium with endogenous

R&D effort (q) based on the optimization problem specified in (3).

A potential firm decides endogenous R&D effort taking as given its unmatched value (Π_U) and the region's contact rate (μ). In this optimization problem, the rate of growth of output is endogenous, depending positively on q. Utilizing (2), (11), (12) and (14), we can derive the first-order condition as:

$$\frac{Ak_0\beta r[1-n\delta(\mu R)]\theta'(q)}{[r+\mu(1-\beta)][r-\theta(q)]^2}=c'(q) \tag{17}$$

The second-order condition is guaranteed by: $(-\theta''(q))(r-\theta(q)) > (\theta'(q))^2$. We impose this stronger global condition (rather than a local condition around equilibrium q^*) such that the equilibrium level of R&D effort is unique. This claim can be easily verified as follows. Notice that the marginal benefit of R&D, the lefthand side of (17), is decreasing in μ. By Proposition 2, μ, in the steady state, depends positively on y and thus q [recall equation (2)]. Under the above condition, both the direct (via θ and θ') and indirect (via μ) effects of q are to reduce the marginal benefit of R&D. On the other hand, the marginal cost, the righthand side of (17), is strictly increasing in q. Thus, the uniqueness of equilibrium R&D determination is obtained.[9] In this case, we have:

Proposition 4: (Endogenous R&D Effort) *The R&D effort* $q^*=Q(Ak_0,\beta,n,R,\nu_0,p_0)$ *is decreasing in the matching scaling factor (p_o), but increasing the firm entry cost (ν_o). Moreover, if the direct effect of the technological parameters (A and k_o) dominates, a technological advancement encourages firms to undertake more R&D investment. The effect of firm's share of matching surplus (β), the population density (n) or the mass of potential regions (R) on the endogenous R&D effort is in general ambiguous.*

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[9] An example is in order. Consider a bounded function: $\theta'(q) = \theta_0 e^{-\alpha q}$. When $\theta_0 < r/3$, not only r - $\theta(q)$ is always positive but the sufficient condition for uniqueness is also satisfied.

From Proposition 2, an increase in the matching scaling factor or a decrease in the firm entry cost increase the resident's flow contact rate (μ). A higher resident's contact rate raises the effective discount rate ($r+\mu(1-\beta)$) as well as the level of pollution ($p = \mu R$) which subsequently increases the environmental damage due to negative pollution externalities (via δ). As a consequence, the net value of firm profit is lower, thus discouraging the R&D activity. Concerning the comparative-static results with respect to the firm's share of matching surplus, the population density or the mass of potential regions, each has a direct effect on the marginal benefit of R&D and an opposing indirect effect via its influence on the resident's contact rate, thus leading to ambiguity outcomes. However, one may expect that such a direct effect for technology advancement (i.e., the productivity effect) is likely to dominate, in particular if the flow matches are insensitive to the mass of firms so that the SS locus is very steep (i.e., μ becomes insensitive to technological changes).[10] In this case, improvements in production technology encourage investment in R&D.

Consider,

Definition 2: *A non-degenerate steady-state equilibrium with endogenous R&D is a pollution compensation offer function* $x^* = X(y,\beta,\mu,r,n,\delta(\mu R),\Pi_U)$ *, a R&D effort function* $q^* = Q(Ak_0,\beta,n,R,\nu_0,p_0)$ *and a quadruple {*μ^**,* η^**,* p^**,* V^**} satisfying the conditions (i)-(iv) in Definition 1 and additionally,*

(v) optimal R&D investment: (17).

Recall that the economy's rate of growth θ depends positively on firm's R&D effort. An important implication of Propositions 3 and 4 is that the relationship between the level of pollution and economic

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[10] More precisely, let $P(V,R) = [\phi V^{\sigma} + (1-\phi)R^{\sigma}]^{1/\sigma}$, where $\sigma < 0$. Then, flow matches are insensitive to the mass of firms if ϕ is sufficiently close to zero.

growth depends crucially on the underlying driving forces. Specifically, we can conclude:

Proposition 5: (Growth versus Pollution) *Under the circumstances specified in Propositions 1-3, changes in the matching scaling factor (p_0), the population density (n) or the firm entry cost (ν_0) result in a negative relationship between growth and pollution. Conversely, changes in the production technological parameters (A and k_0) lead to a positive relationship.*

One may assume that for an economy at the early stage of economic development, technological advancements are more feasible through imitation (i.e., international knowledge spillovers). For an economy at the later stage of development, technical progress becomes difficult. However, as the pollution problem becomes more severe, it is more costly to establish a new firm and matching becomes more restricted as a result of environmental regulation (via reductions in p_0). These entry/matching parameter changes thus turn out to be the dominant forces. From Proposition 4, we obtain, in a cross-country sense, a positive pollution-growth relationship at the early stage and a negative one at the later stage of economic development, consistent with empirical findings in Grossman and Krueger (1995).

This inverted-U relationship between environmental degradation and income (or economic growth) is also established in Stokey (1998).[11] Her finding is based on direct tradeoff between environmental quality and final good output through individual preferences. Thus, such a relationship depends crucially on the functional forms of utility (for example, the intertemporal elasticity of substitution for the final good) and pollution generating activity. In our paper, it is the endogenous cooperative settlement that interacts with endogenous investment in R&D, leading to such non-monotone relationship. As a consequence, the specification of individual preferences plays no role in driving the result. Our

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[11] Jones and Manuelli (1995) also find conditions for inverted-U and a sideways mirrored S relationships between pollution and income growth based on a similar trade-off along various transition stages in a framework with cross-generation collective choice and voting.

interpretation may thereby complement previous studies, suggesting an alternative channel through which environmental degradation and real income may be related in a non-monotone fashion.

6. Policy Implications

The results in Proposition 5 generate intriguing policy implications. On the one hand, any growth-promoting policy, such as a production subsidy or a capital investment tax credit, will enhance economic growth at the expense of an increased level of pollution. Nor can a Pigovian tax be implemented to correct this problem.[12] On the other hand, a policy that creates barriers to firm entry or matching (by increasing v_o or reducing p_o) may reduce pollution without harming economic growth. The latter type of policy is not of the Pigovian type - it can be implemented in any economy with market (entry and matching) frictions in which pollution compensation is settled by cooperative Nash bargains. For example, a direct regulation controlling the entry of dirty firms or an administrative red tape delaying the match between residents and firms may serve the purpose. Moreover, one may consider a policy that requires a minimum compensation to matching surplus ratio, which sets a ceiling for firm's share, β. Notice that from Proposition 2, a lower β discourages firm's entry and hence reduces the resident's contact rate as well as the equilibrium level of pollutant. Proposition 4 suggests that although a decrease in β has a direct effect to lower the R&D effort, it also creates a positive indirect effect via the contact rate (referred to as the endogenous matching effect). When this latter endogenous matching effect is strong, a reduction in β need not discourage firm's incentive to undertake R&D. Thus, such a policy may reduce pollution without retarding economic growth.

One may now conduct welfare analysis within our framework. First, in the optimal R&D decision

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[12] This is mainly due to the presence of matching externalities. In a conventional dynamic framework, Farzin (1996) suggests that with environmental stock externalities, Pigovian taxes must be modified; Mohtadi (1996) shows that a combination of optimal taxes and quality controls is welfare-improving upon an optimal tax policy; Marsiliani and Renström (2000) find that the time-inconsistency problem can arise when the social planner determines an optimal environment tax.

specified in (3), a social planner would consider the positive effect of R&D investment on the resident's contact rate. That is, the social planner's first-order condition with respect to q can be expressed as: $\frac{\partial y}{\partial q}\left(1-n\frac{\partial X}{\partial y}\right)-n\frac{\partial X}{\partial \mu}\frac{\partial \mu}{\partial q} = c'(q)$, which differs from the individual optimization by an additional term - the second term on the lefthand-side. Since a higher resident's contact rate raises the pollution compensation and thus lowers the marginal benefit of R&D investment (i.e., the second term on the lefthand-side is negative). Thus, the optimal level of R&D is lower, as does the level of pollution μR (recall Proposition 2). This implies that in the decentralized Nash equilibrium, there is excessive investment in R&D and the economy is overly polluted compared to a social planner's solution of the same maximization problem for R&D effort.

Second and more interestingly, a social planner may indeed go beyond the previous objective, considering the maximization of the welfare of the society. In our model equilibrium entry implies that firm's welfare in steady-state equilibrium is fixed and the mass of residents is given at nR. As a consequence, the representative resident's unmatched value at time 0 multiplied by their mass ($nJ_U(0)$) can be used to measure the society's welfare:

$$nJ_U(0) = \frac{(1-\beta)\mu}{r+\mu(1-\beta)}\left\{[1-n\delta(\mu R)]\frac{Ak_0}{r-\theta(q)} - v_0\right\}. \tag{18}$$

Straightforward comparative statics shows three separate effects of μ on $J_U(0)$: a thick matching effect (via $(1-\beta)\mu$ in the numerator of the first term), an effective discounting effect (via the effective discount rate in the denominator of the first term) and a pollution externality effect (via δ in the bracket). While the thick matching effect is to raise the welfare, both effective discounting and pollution externality effects tend to reduce it. Since individual firms fail to account for either effect (taking the contact rates as given), the decentralized Nash equilibrium could involve under-investment in R&D and under-pollution if the thick matching effect dominates the effective discounting and pollution externality effect. Thus, we conclude:

Proposition 6: (Equilibrium versus Optimum) *Under the circumstances specified in Propositions 1-3, the decentralized Nash equilibrium may be characterized by under-investment in R&D and under-pollution if the thick matching effect of matching is strong, whereas the reverse is true if the effective discounting and pollution externality effects are dominant.*

The contrast between decentralized equilibrium and social optimum has been examined by John and Pecchenino (1994), Jones and Manuelli (1995) and Byrne (1995). The finding of over-pollution in decentralized equilibrium by Byrne (1995) is due mainly to the assumption that individuals make no decision on pollution abatement and such decision is accounted by the social planner. In John and Pecchenino (1994) and Jones and Manuelli (1995), the economy under decentralized equilibrium is always insufficiently polluted in the absence of an accumulation of the pollution stock. This is because individuals fail to account for the positive effect of investment in physical capital on future generation's welfare and hence under-investment occurs.[13] Thus, a reallocation from physical capital to pollution abatement investment results in an insufficiently polluted equilibrium. In our paper, we show that even without accumulation of pollution stock or pollution abatement, it is possible to have an overly polluted decentralized equilibrium, depending crucially on the magnitudes of thick matching versus effective discounting and pollution externality effects.

7. Concluding Remarks

This paper establishes a growth model in which pollution is regarded as a public bad and firms and residents in polluted areas bargain cooperatively to settle their environmental concerns. While the stage of

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[13] The decision mechanism is also essential to the conclusions. For example, in an overlapping-generations framework with a long-lived government which centrally plans for the environmental policy, short-lived individuals overly pollute in equilibrium (see John, Pecchenino, Schimmelpfennig and Schreft 1995).

economic development affects the extent of the negotiation outcomes, the bargaining results feedback to influence firms' incentive to undertake R&D and thus the rate of economic growth. Our theory provides justification for the inverted-U relationship between environmental degradation and economic growth. We also suggest that a policy that creates barriers to firm entry or matching or a policy that requires a minimum compensation ratio may reduce pollution without harming economic growth. Due to the opposing effects of thick matching versus effective discounting and pollution externality, the decentralized equilibrium may be excessively or insufficiently polluted.

Since our model is simple enough, it is possible to parameterize the production and matching technologies as well as the bargaining power, the entry cost and the population size. By calibration exercises, we may generate the stage-dependent inverted-U relationships between pollution and income growth to match with empirical evidence presented in Grossman and Krueger (1995). Moreover, we may also perform simulation analysis to examine qualitatively and quantitatively the welfare implications of the environmental policies. It may be of particular interest to contrast the conventional Pigovian tax policy, as discussed in Bovenberg and de Mooij (1997), with environmental regulations that influence the entry or matching of potential firms.

Along the theoretical lines, it may be interesting to consider two production sectors (dirty and clean) where there are pollution spillovers from the dirty to the clean sector. Similar pollution compensation schedules may be derived based on cooperative settlement between firms from different sectors. Moreover, one may consider heterogeneous residents with regards to their preferences over a clean environment, which can be simply captured by different thread points in our bargaining framework. One may then examine by free migration whether endogenous segregation will occur in which clean-air lovers agglomerate. Finally, another possibility for future research is to generalize the setup of the aggregate negative pollution externality. In particular, one may consider explicitly a spatial structure and allow for distance-dependent pollution spillovers to study the spatial patterns of pollution and the distribution of population.

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Appendix

Proof of Proposition 1: Substitution of of (5), (7) and (8) into (10) gives:

$$(1-\beta)(y-nx-\pi_u)=\beta n(x-z)\frac{r}{r+\mu}$$

which reduces to (11). Straightforward differentiation then leads to the comparative static results with respect to y, n, δ and Π_U. For μ, we have:

$$\frac{\partial X}{\partial q}\propto(1-\delta n)y-\Pi_U$$

which is positive (the validity of this assumption is based on the free entry condition that $\Pi_U = v_o$ and the operative production condition that $y\text{-}nx\text{-}c(q) \geq v_o$ to be discussed in the Section 4 below). For β, we have:

$$\frac{\partial X}{\partial \beta}\propto-\left\{[(1-n\delta)y-\Pi_U]-(1-\beta)(\frac{\mu}{r+\mu})n\delta y\right\}$$

which is negative if the first term in the bracket dominates the second (i.e., the direct effect is dominant).||

Derivation of the EE Locus: Substitution of (6) into (13) leads to:

$$\frac{\eta}{r+\eta}(y-nx)=\nu_0$$

which can be combined with (11) to produce:

$$\frac{\eta}{r+\eta}\left(y-\frac{(r+\mu)(1-\beta)(y-\nu_0)+\beta nrz}{r+\mu(1-\beta)}\right)=\nu_0$$

Straightforward manipulation of the above expression gives the EE locus (14). Next, differentiating the EE locus with respect to μ and manipulating yields:

$$\frac{\partial\eta^{EE}}{\partial\mu}\propto(1-\beta)[y-v_0-n\delta(\mu R)y]+n\delta'(\mu R)yR[r+\mu(1-\beta)]>0$$

$$\frac{\partial^2\eta^{EE}}{\partial\mu^2}\propto-\left(\{2(1-\beta)-n\delta''(\mu R)yR^2[r+\mu(1-\beta)]\}[y-v_0-n\delta(\mu R)y]+2n\delta'(\mu R)yR[r+\mu(1-\beta)]\right)<0.$$

This implies that the EE locus is strictly increasing and strictly concave.||

Proof of Proposition 4: The maximization problem is $\underset{q}{\text{Max}}\,\{y - nx^* - c(q)\}$ s.t. $\{y - nx^* - c(q)\} \geq \nu_0$.

Notice first that

$$\begin{aligned}
&y - nx^* - c(q)\\
&= \frac{(r + \mu(1-\beta))\nu_0 - \beta r\pi_u + \beta ry(1 - n\delta(\mu R))}{r + \mu(1-\beta)} - c(q)\\
&= \frac{Ak_0\beta r(1 - n\delta(\mu R))}{(r + \mu(1-\beta))(r - \theta(q))} - c(q).
\end{aligned}$$

Differentiating this expression with respect to q gives:

$$\frac{\theta'(q)Ak_0\beta r(1 - n\delta(\mu R))}{(r + \mu(1-\beta))(r - \theta(q))^2} - c'(q) = 0$$

which yields (17).||

Proof of Proposition 6: First, from (11), (13) and the definition of z, we have:

$$\begin{aligned}
x - z &= \frac{(1-\beta)(r+\mu)(y - \pi_u) + \beta nr\delta y - nr\delta y - n\mu(1-\beta)\delta y}{n\left[r + \mu(1-\beta)\right]}\\
&= \frac{(1-\beta)(r+\mu)(y - \pi_u - n\delta y)}{n\left[r + \mu(1-\beta)\right]}\\
&= \frac{(1-\beta)(r+\mu)\left\{\left[1 - n\delta(\mu R)\right]y - \nu_0\right\}}{n\left[r + \mu(1-\beta)\right]}
\end{aligned}$$

Substituting this into (8) yields (18).||

Figure 1: The Basic Environment of the Economy

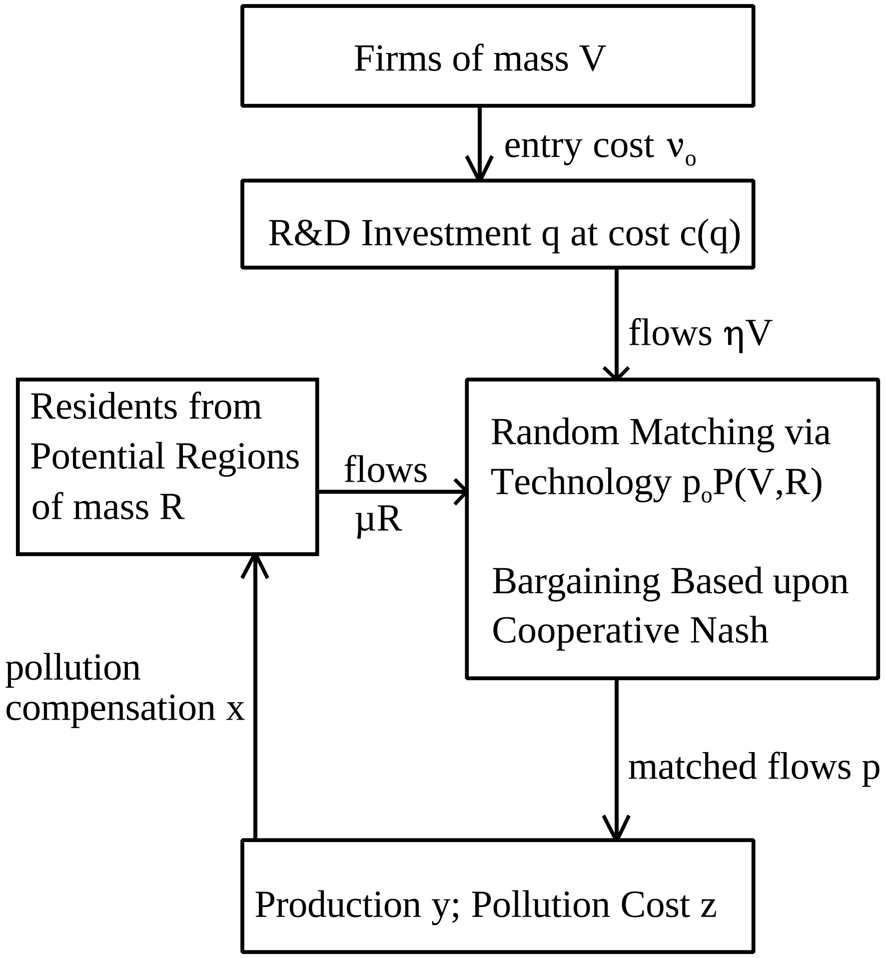

Figure 2: Determination of Steady-State Equilibrium Contact Rates