A SIMPLE MODEL OF INEQUALITY, OCCUPATIONAL CHOICE, AND DEVELOPMENT

by

Maitreesh Ghatak and Nien-Huei Jiang

Working Paper No. 00-W07

January 2000

DEPARTMENT OF ECONOMICS
VANDERBILT UNIVERSITY
NASHVILLE, TN 37235

www.vanderbilt.edu/econ

A Simple Model of Inequality, Occupational Choice, and Development.

Maitreesh Ghatak[1] and Nien-Huei Jiang[2]

January, 2000.

Abstract : This paper analyzes a simple and tractable model of occupational choice in the presence of credit market imperfections. We examine the relative roles of parameters governing technology and transaction costs, and history in terms of the initial wealth distribution in determining the long term wealth distribution and level of income of an economy. The possibility of the existence of cycles, and the role of lotteries and redistributive policies in achieving greater e±ciency are examined.

1 Introduction

A well known implication of neoclassical growth theory is that economies that have similar preferences and technologies converges to the same steady state per capita income.[3] In contrast, in development economics we frequently encounter the idea of poverty traps: poor individuals and economies tend to remain poor because they start poor.[4] It is based on various positive feedback mechanisms that potentially lead to the existence of multiple steady state equilibria, in which case initial conditions govern which equilibrium will be reached. There are two main channels through which these mechanisms work - through productivity or preferences, or through incentives.[5] For example, if higher nutrition raises productivity of workers then poorer workers will be less productive, and get paid less in the labor market, which in turn will keep them poor.[6] Alternatively, because threats of punishment work less well against the poor, they

[1]Department of Economics, University of Chicago, Chicago IL 60637. mghatak@midway.uchicago.edu.

[2]Department of Economics, University of Chicago, Chicago IL 60637. nnjiang@midway.uchicago.edu.

[3]This goes back to the classic paper of Solow (1956). For a recent discussion see Barro and Sala-i-Martin (1995).

[4]See for example Nurkse (1953).

[5]See Banerjee and Newman (1994).

[6]See Dasgupta and Ray (1986) for a formal analysis. Similar wealth e®ects can exist with respect to preference parameters such as the propensity to save or supply labor.

may face greater borrowing constraints. This in turn may prevent them from adopting efficient technologies or choose profitable occupations and hence they remain poor. At the aggregate level, this has the implication that unlike in neoclassical growth models, two economies that are identical in terms of all parameters may end up with different levels of per capita incomes in the steady state if initially they have different proportions of people that are credit rationed.[7] In this paper we focus on this particular channel working through borrowing constraints and ask the following question : under what conditions governing parameters relating to technology and preference do initial conditions matter and how?

To answer this question we analyze a simple model of occupational choice in the presence of capital market imperfections similar to the well known model of Banerjee and Newman (1993). They show that the current distribution of wealth determines the proportion of credit constrained individuals in the economy, which in turn may affect equilibrium returns to various occupations in a way that affects the future wealth distribution through intergenerational transfers or savings. As a result, the initial distribution of wealth may affect the steady-state equilibrium of the economy. A novel feature of this model is that the transition of the state variable (namely, the wealth distribution) for the economy as a whole is non-linear, although for a single dynasty the relevant state variable (namely, its wealth) follows a linear stationary Markov process. The reason is, the current distribution of wealth affects the returns to various occupations which determines next period's wealth and so on. As the authors point out, as a result of this property the standard techniques to establish existence, uniqueness and global stability of the long-term distribution of wealth for standard linear models cannot be applied and hysteresis, cycles and other types of non-linear behavior can occur. They therefore restrict their analysis to some examples that show that multiple stationary wealth distributions may exist.

There are two main differences between the current model and that of Banerjee and Newman. First, we have a simpler occupational structure as a result one needs no more information about the wealth distribution than the proportion of people whose wealth is below the level needed to start an enterprise. Second, in the stochastic version of the model (see section 3) every agent has some probability of entering the economy with no wealth, and similarly, every dynasty has some probability of being able to overcome credit constraints if it receives a sequence of positive shocks. These simplifications allow us to characterize all the steady-state equilibria given various configurations of parameters governing technology and preferences. The goal is to understand better the relative roles of parameters and history (relating to the initial distribution of wealth) in governing the steady-state equilibrium of an economy, and to characterize how the initial distribution of wealth affects wages and total output (when it does). On the other hand, we lose some of the richness of the Banerjee-Newman model which allows for alternative institutional forms associated with the modern technology that differ in

[7] See for example, Banerjee and Newman (1993, 1994), Galor and Zeira (1993), and Piketty (1996).

terms of agency costs.

2 The Model

2.1 Demographics and Preferences

Consider an economy inhabited by infinitely-lived dynasties represented by successive generations of agents who live for one-period. The population is large and its size is normalized to 1. There is no population growth. There are two goods in the economy, labor, and some final output which can serve both as a consumption good and a capital good. In period t a dynasty i is endowed with 1 unit of labor and an initial wealth $a_{i,t}$. It earns income by supplying labor and capital and the resulting income $y_{i,t}$ is divided at the end of the period between consumption $c_{i,t}$ and savings, or bequest to the next generation, $b_{i,t}$. Therefore,

$$a_{i,t+1} = b_{i,t}.$$

We assume that the current generation saves a constant fraction s of its income and leaves it as bequest:

$$a_{i,t+1} = sy_{i,t}.$$

We also assume that all agents are risk-neutral .

In period t wealth is distributed according to the probability measure $\lambda_t(\cdot)$, and for convenience, we define

$$G_t(a) \equiv \lambda_t((-1, a)).$$

Function G_t is very similar to the distribution function except that it does not include the measure at point a.

2.2 Production Technologies

There are two production technologies both of which are deterministic. One uses no capital and one unit of labor to produce $\underline{w}$ units of output. This will be described as a subsistence (or agricultural) technology. The other uses $I > 0$ units of capital and two units of labor (one unit of supervisory labor and one unit of ordinary labor) to produce y units of output. One supervisor (or entrepreneur) can perfectly monitor one worker spending her entire labor endowment. This will be described as an entrepreneurial (or industrial) technology.[8] We

[8] In contrast in the Banerjee and Newman (1993) model apart from these two types of technologies there is a third one which involves some capital and one unit of labor ("self employment"). Also, in their model both the entrepreneurial and the self-employment technologies have stochastic returns. See section 3 for further discussion on this.

assume that this technology is superior in the sense that the net output of using this technology is greater than were two units of labor using the subsistence technology. That is,

$$y - rI > 2\underline{w} \qquad \text{(Assumption 1)}$$

where $r(\geq 1)$ is the exogenously given gross interest rate.

2.3 Occupations

There are three possible occupations open to an individual who has inherited wealth $a_{i,t}$.

(a) Subsistence : The agent earns some income by using her labor endowment to produce $\underline{w}$ with the subsistence technology. She puts her inherited wealth in the bank, which yields $ra_{i,t}$. Therefore her income is

$$y^S_{i,t} = \underline{w} + ra_{i,t}.$$

(b) Worker : The agent works for an entrepreneur for wage income w_t (which is determined endogenously). She puts her inherited wealth in the bank, which yields $ra_{i,t}$. Therefore her income is

$$y^W_{i,t} = w_t + ra_{i,t}.$$

(c) Entrepreneur : The agent invests an amount I to start a firm and hires 1 worker to produce an output y with certainty. Her job is to monitor the worker. The agent's income as an entrepreneur is the output of the project less wage and capital costs:

$$y^E_{i,t} = y - w_t + r(a_{i,t} - I).$$

2.4 Credit and Labor Markets

The supply of credit is perfectly elastic at the risk-free rate r. The credit market is imperfect in that if an individual's wealth is below a certain minimum level, $\underline{a}$, she would not get a loan no matter how high the interest rate she offers. Following Banerjee and Newman (1993) a simple way to generate this feature of the credit market is as follows : a borrower may run away with the money in order to avoid repaying her debt, $r(I - a_{i,t})$, but the cost of this action is that she gets caught with some probability π and then has to pay a fixed non-monetary cost due to imprisonment or social sanctions of F. It is assumed that even if a borrower gets caught trying to avoid repaying her debt, she gets to consume her profits. Therefore, the higher is an individual's wealth the lower is her incentive to run away. Thus only those get loans whose wealth satisfies the incentive compatibility constraint (ICC):

$$\begin{aligned}(y - w_t) - r(I - a_{i,t}) &\geq (y - w_t) - \pi F \\ \text{or, } a_{i,t} &\geq I - d\end{aligned}$$

where $d \equiv \frac{\pi F}{r}$. For simplicity, we set $d = 0$ so that the credit market does not operate at all.

The wage rate at which entrepreneurs are indifferent between working as wage laborers or hiring workers is given by

$$
\begin{aligned}
y - \overline{w} + r(a_{i,t} - I) &= \overline{w} + r a_{i,t} \\
\text{or, } \overline{w} &= \frac{y - rI}{2}.
\end{aligned}
$$

By Assumption 1, $\underline{w} < \overline{w}$. Below we show that to ensure labor market equilibrium, the wage rate w must lie in the interval $[\underline{w}, \overline{w}]$. Hence the occupation of entrepreneurship earns more than any other occupation for all wages (and strictly so for all $w < \overline{w}$). Given the features of the credit market, only those individuals who own enough capital ($a \geq I$) can become entrepreneurs even though everybody else would like to do so. We are going to refer to those individuals whose wealth is less than I as capital-constrained, or simply, poor.

Notice that the ICC tells us what fraction of the population is capital-constrained, namely, $G_t(I)$. For $w_t < \underline{w}$, labor supply is zero, but for $w_t = \underline{w}$ labor supply jumps to $G_t(I)$ and as w_t goes above $\underline{w}$, the supply of labor grows until the wage rate is high enough such that entrepreneurs are indifferent between working as wage laborers or hiring workers. That wage is given by

$$
\begin{aligned}
y - \overline{w} + r(a_{i,t} - I) &= \overline{w} + r a_{i,t} \\
\text{or, } \overline{w} &= \frac{y - rI}{2}.
\end{aligned}
$$

Notice that Assumption 1 implies

$$\overline{w} > \underline{w}.$$

Now we are ready to write down the supply curve of labor:

$$
\begin{aligned}
0 \text{ if } w_t &< \underline{w} \\
[0, G_t(I)] \text{ if } w_t &= \underline{w} \\
G_t(I) \text{ if } w_t &\in (\underline{w}, \overline{w}) \\
[G_t(I), 1] \text{ if } w_t &= \overline{w} \\
1 \text{ if } w_t &> \overline{w}.
\end{aligned}
$$

Conversely, to derive the demand curve for labor we notice that for $w_t > \overline{w}$ there is no demand for labor; as w_t falls to $\overline{w}$, the demand for labor jumps to any value between 0 and $1 - G_t(I)$. When $w_t < \overline{w}$ the demand for labor is at a maximum, $1 - G_t(I)$ and continues to remain so. Therefore the demand for labor is:

$$
\begin{cases}
0 & \text{if } w_t > \overline{w} \\
[0; 1 - G_t(I)] & \text{if } w_t = \overline{w} \\
1 - G_t(I) & \text{if } w_t < \overline{w}.
\end{cases}
$$

With labor supply and demand, we can easily get the equilibrium wage rate in period t:

$$
w_t^* = \begin{cases}
\overline{w} & \text{if } G_t(I) < \frac{1}{2} \\
[\underline{w}; \overline{w}] & \text{if } G_t(I) = \frac{1}{2} \\
\underline{w} & \text{if } G_t(I) > \frac{1}{2}.
\end{cases}
$$

Since each entrepreneur hires exact one worker, if there are more people who are capital-constrained (unconstrained), then the competition for entrepreneurs (workers) among them will drive the equilibrium wage rate down (up) to its lower (upper) bound. When $G_t(I) = \frac{1}{2}$, the equilibrium wage rate is indeterminate, and throughout this paper, we are going to assume that the wage rate is equal to $\overline{w}$ in this case.

Notice that on one hand, the equilibrium wage rate depends on the current wealth distribution but on the other hand, it also in°uences next period's wealth distribution through savings behavior of currently active agents.

2.5 Dynamics of Individual Wealth

Consider the factors governing dynasty i's bequest. First of all, the initial wealth level of an agent determines her capital income and her occupational choice. Secondly, the current wage rate is determined by the economy wide wealth distribution. With the knowledge of an individual's occupational choice and that the wage rate can take only two values ($\underline{w}$ and $\overline{w}$), we can write down the di®erence equations describing the evolution of a dynasty i's wealth as:

$$
\begin{aligned}
a_{i;t+1}(a_{i;t} \mid w_t = \underline{w}) &= s[ra_{i;t} + \underline{w}] && \text{if } a_{i;t} < I \\
&= s[r(a_{i;t} - I) + y - \underline{w}] && \text{if } a_{i;t} \geq I \\
a_{i;t+1}(a_{i;t} \mid w_t = \overline{w}) &= s[ra_{i;t} + \overline{w}] && \forall a_{i;t}.
\end{aligned}
$$

Figure 1 shows what these di®erence equations look like. Notice that there are two regimes of wealth transitions corresponding to the two wage levels. When the wage rate is low, an agent who is capital-constrained can only choose between being a worker and engaging in subsistence and in either case, her labor income is $\underline{w}$. A fraction s of the sum of her labor income and her capital income $ra_{i;t}$ is left for her next generation. An agent who is not credit constrained will strictly prefer to be an entrepreneur and her total income will be $r(a_{i;t} - I) + y - \underline{w}$. When the wage rate is high, nobody will engage in subsistence and all agents will be indi®erent between being entrepreneurs and workers.

We assume that it is not possible for a dynasty to get arbitrarily rich over time by merely saving a constant fraction of its income and earning interest income:

$$sr < 1 \qquad \text{(Assumption 2)}$$

2.6 Stationary Wealth Distributions and Wages

In this section, we examine the long run behavior of this economy. The first question we address is will the wealth distribution always converge to a stationary distribution? If the difference equations governing the wealth transitions are stationary, it would be easy to prove the existence of a stationary wealth distribution. However the fact that these difference equations depend on the wage levels raises the possibility that the process may not be stationary. In particular, the concern here is that the wage rate may change infinitely often. The following lemma rules out this possibility.

Lemma 1: The wage rate can change at most once.

Proof: Notice that the difference equations are order-preserving. That is, $a_{i,t+1} > a_{j,t+1}$ if and only if $a_{i,t} > a_{j,t}$. Therefore, in order to study the wage dynamics, we can only look at the wealth dynamics of the dynasty which has the median wealth. Define $a_t^m \equiv \max\{a : G_t(a) \leq \frac{1}{2}\}$. Note that a_t^m is well-defined because $G(\cdot)$ is continuous from below according to our definition. Then $a_t^m \geq I \iff G_t(I) \leq \frac{1}{2}$ which implies $w_t = \overline{w}$. Similarly, $a_t^m < I \iff G_t(I) > \frac{1}{2}$ which implies $w_t = \underline{w}$. Now if $w_t = \underline{w}$ and $w_{t+1} = \overline{w}$, then we must have $a_t^m < I$ and $a_{t+1}^m \geq I$. This implies $s(ra_t^m + \underline{w}) \geq I \Longrightarrow s\underline{w} \geq (1 - sr)I \Longrightarrow s(ra + w) \geq I$ for all $a \geq I$ and $w = \{\underline{w}, \overline{w}\}$. That is, once the high wage rate is reached, there will not be any downward mobility and hence the high wage will prevail forever. If $w_t = \overline{w}$ and $w_{t+1} = \underline{w}$, then we must have $a_t^m \geq I$ and $a_{t+1}^m < I$. This implies $s(ra_t^m + \overline{w}) < I \Longrightarrow s\overline{w} < (1 - sr)I \Longrightarrow s(ra + w) < I$ for all $a < I$ and $w = \{\underline{w}, \overline{w}\}$. That is, there will not be any upward mobility and once the low wage rate is reached, it will prevail forever. Therefore we can conclude that starting with any initial distribution of wealth, the wage rate can change at most once.■

According to Lemma 1, although we have two regimes of the wealth transition process, only one of them will prevail in the long run. Together with Assumption 2 which implies there exists a stationary point for each difference equation, we immediately have:

Proposition 1: Given any initial wealth distribution, there exists a unique stationary wealth distribution to which it converges.

Next we turn to characterize the stationary wealth distribution. In particular, we are interested in the following questions : how does the stationary distribution depend on various parameters and the initial wealth distribution? What is the level of income of the economy and is there any role for policy that will raise it?

Let $a^J(w)$ be the stationary point of the difference equation describing the wealth transition of a dynasty engaged in occupation J (where $J = S; W; E$ denote the three occupations : subsistence, worker and entrepreneur) when the wage rate is w. Then we have

$$
\begin{aligned}
a^S(w) &= \frac{s\underline{w}}{1-sr} \text{ for all } w: \\
a^W(\underline{w}) &= \frac{s\underline{w}}{1-sr} \\
a^E(\underline{w}) &= \frac{s(y-rI-\underline{w})}{1-sr} \\
a^W(\overline{w}) = a^E(\overline{w}) &= \frac{s(y-rI)}{2(1-sr)}:
\end{aligned}
$$

By Assumption 1, $a^E(\underline{w}) > a^E(\overline{w}) = a^W(\overline{w}) > a^W(\underline{w})$:

Comparing the values of these threshold levels of wealth with I we can completely characterize the long run outcome (in terms of the stationary distribution of wealth, the equilibrium wage rate and the level of net output) of the economy.

Proposition 2 : The initial distribution of wealth matters in determining the stationary distribution of wealth and the long run equilibrium wage rate if and only if

$$s(y-\underline{w}) \geq I \geq \frac{s\underline{w}}{1-sr}:$$

Otherwise the economy converges to a high wage equilibrium (if $I < \frac{s\underline{w}}{1-sr}$) or a subsistence equilibrium (if $I > s(y-\underline{w})$) irrespective of initial conditions.

Proof : The proof consists of the following two steps.

Step 1. The following four cases characterize the steady state equilibrium of the economy corresponding to various parameter values:

Case 1. $I > s(y-\underline{w}) \iff I > a^E(\underline{w})$: This is a situation where the steady-state wealth of the entrepreneurial class cannot finance the operation of the industrial technology even when wages are as low as possible. The only equilibrium in this economy is therefore one where everyone is engaged in subsistence production irrespective of the initial wealth distribution G_0. As a result the stationary wealth distribution displays no inequality.

Case 2. $s(y-\underline{w}) \geq I > \frac{sy}{2-sr} \iff a^E(\underline{w}) \geq I > a^E(\overline{w}) = a^W(\overline{w}) > a^W(\underline{w})$: The condition that $a^E(\underline{w}) \geq I$ implies $s[r(a-I)+y-\underline{w}] \geq I \ \forall a \geq I$. It says when the wage rate is low, offsprings of individuals who are able to start an enterprise in the current period will also be able to do so in the next period, i.e., there is no downward mobility. Similarly, $I > a^W(\underline{w})$ implies $s(ra+\underline{w}) < I \ \forall a < I$, which means there is no upward mobility when the wage is low. If the economy starts out with the low wage rate ($G_0(I) > \frac{1}{2}$), there will not be any mobility in either direction. This implies that the wage rate will always be equal to $\underline{w}$; the wealth of those dynasties whose are initially capital-constrained will converge to $a^W(\underline{w})$; the wealth of those that are not will converge to $a^E(\underline{w})$; and there will be $1 - G_0(I)$ firms operating in each period. Now suppose the economy starts out with the high wage rate ($G_0(I) \le \frac{1}{2}$). The condition, $I > a^E(\overline{w}) = a^W(\overline{w})$, implies $\overline{w}$ is not sustainable. There exists a finite τ such that $w_\tau = \overline{w}$ and $w_{\tau+1} = \underline{w}$. Thereafter the story follows the one described in the previous paragraph if we take $G_{\tau+1}(\cdot)$ as the initial wealth distribution in the new low wage regime. And of course, $G_{\tau+1}$ depends on G_0.

Case 3. $\frac{sy}{2-sr} \ge I > \frac{s\underline{w}}{1-sr} \iff a^E(\underline{w}) > a^E(\overline{w}) = a^W(\overline{w}) \ge I > a^W(\underline{w})$: Again, since $a^E(\underline{w}) > I > a^W(\underline{w})$, there is no upward or downward mobility when wage rate is low. Therefore if the economy starts out at low wage rate ($G_0(I) > \frac{1}{2}$), the story is the same as in Case 2. However, the condition, $a^E(\overline{w}) = a^W(\overline{w}) \ge I$, implies $s(ra + \frac{y-rI}{2}) \ge I \; \forall a \ge I$. Hence when the wage rate is high, people who are not capital-constrained will remain unconstrained, i.e., there is no downward mobility. Therefore if the economy starts out with $G_0(I) \le \frac{1}{2}$, the high wage $\overline{w}$ will last forever. As a result, every dynasty's wealth converges to $a^E(\overline{w})$:

Case 4. $\frac{s\underline{w}}{1-sr} \ge I \iff a^W(\underline{w}) \ge I$: The high-wage equilibrium will result irrespective of G_0 because even when wages are low, the steady-state wealth level of the working class permits them to start a firm. As a result the unique stationary wealth distribution displays no inequality.

Step 2. Next we show that the sets of parameter values that correspond to the four cases analyzed above are mutually exclusive and exhaustive with respect to the set of all admissible parameter values (i.e., those satisfying Assumptions 1 and 2).

Suppose $\frac{s\underline{w}}{1-sr} \le I$. This inequality implies $\frac{2-sr}{1-sr}\underline{w} \le 2\underline{w} + Ir$: As a result, Assumption 1 which guarantees $y > 2\underline{w} + Ir$; also implies $y > \frac{2-sr}{1-sr}\underline{w}$; i.e., $\frac{y}{2-sr} > \frac{\underline{w}}{1-sr}$: The last inequality in turn implies, upon rearranging, $s(y - \underline{w}) > \frac{sy}{2-sr}$ and $\frac{sy}{2-sr} > \frac{s\underline{w}}{1-sr}$. Thus we have the following inequality which is derived from Assumptions 1 and 2:

$$s(y - \underline{w}) > \frac{sy}{2-sr} > \frac{s\underline{w}}{1-sr} \tag{1}$$

which holds so long as $\frac{s\underline{w}}{1-sr} \le I$: If instead, $I < \frac{s\underline{w}}{1-sr}$ then Case 4 always applies. That is, the only possible equilibrium is the high wage equilibrium. ■

Figure 2 summarizes the four cases. Let us define the total income of the economy, the sum of wage and profit income, as :

$$Y = G(I)w + \{1 - G(I)\}\{y - w - Ir\}$$

The following result compares the equilibria in terms of total income.

Proposition 3 : The total income of the economy under a high wage equilibrium exceeds that under a low wage equilibrium, which in turn exceeds that under a subsistence equilibrium. For parameter values for which initial conditions matter, the greater is the fraction of the population who are initially poor, the lower is total income.

Proof : Under a subsistence equilibrium, total income is $Y = \underline{w}$. In a low wage equilibrium, total income is $Y = \{y - lr\}\{1 - G(l)\} - \{1 - 2G(l)\}\underline{w}$. Finally, in a high wage equilibrium, total income is $Y = \frac{y - lr}{2}$. Since $y - lr > \underline{w}$ by Assumption 2, and under a low wage equilibrium $G(l) \geq \frac{1}{2}$

$$\frac{\{y - lr\}}{2} \geq \{y - lr\}\{1 - G(l)\} - \{1 - 2G(l)\}\underline{w} > \underline{w}.$$

Proposition 2 shows that for the parameter values $s(y - \underline{w}) \geq l \geq \frac{s\underline{w}}{1 - sr}$ (corresponding to cases 2 and 3), if $G_0(l) > \frac{1}{2}$ then the economy converges to a low wage equilibrium where only $1 - G_0(l)$ firms operate. Hence the second part of the proposition follows. ∎

Hence the main conclusion is that the same model, with the same parameters is capable of generating very different steady states which can be ranked in terms of output depending on properties of the initial distribution of wealth. Other things given, the likelihood of this happening depends on the size of the set-up cost (l) relative to the productivity differential between the modern and the subsistence technology ($y - \underline{w}$). If the size of the set-up cost is too large or too small then characteristics of the initial wealth distribution does not matter. In the former case the economy collapses to subsistence and in the latter case, a high wage equilibrium with the efficient number of firms. For intermediate values of the set-up cost, the characteristics of the initial wealth distribution matters. In particular, the greater is fraction of the population that are capital constrained initially, the lower is the number of firms using the modern technology in the long run equilibrium, and hence the lower is total income. To the extent greater equality of the distribution of wealth reduces the fraction of the population who are capital constrained, both greater equity and greater efficiency (in terms of total income) is achieved. In addition, given the values of the technological parameters, the higher is the value of s the greater is the likelihood that the initial distribution of wealth matters in determining the stationary distribution of wealth and the long run equilibrium wage rate.[9]

Let us now examine the assumptions underlying this model that generates these results. There are two departures in the current model from the standard Arrow-Debreu framework. First, the credit market does not operate because of enforcement costs. Second, the production set is non-convex. Both labor and capital are indivisible in both technologies, and moreover, any convex combination of the two different technologies also yield no output.[10]

[9] This follows from the fact that given l the size of the interval $\left[\frac{s\underline{w}}{1-sr}, s(y - \underline{w})\right]$ is increasing in s for $y - \underline{w}\frac{2-sr}{1-sr} > 0$ (which is a necessary condition for initial conditions to matter).

[10] For example, no capital and half a unit of labor does not produce any output under the subsistence tech-

Suppose first that the credit market is perfect but the production set is non-convex, as before. Then the economy will end up in the unique equilibrium where everyone can borrow an amount $I - a$ to become an entrepreneur. Consider someone with wealth a. If he hires a worker at wage w and supervises him with his own labor endowment then his profit is $r(a - I) + y - w$ whereas if he joins the labor force he earns $ra + w$. He will prefer to become an entrepreneur if $r(a - I) + y - w \geq ra + w$; or, $\frac{y - rI}{2} \geq w$. But since this expression does not depend on a, if this holds then someone with wealth 0 and someone with wealth $a > I$ will both want to become entrepreneur. The only possible equilibrium is where one is indifferent between borrowing and hiring other person to work for you or being a worker yourself, i.e., $w = \overline{w}$. But this will mean that the stationary distribution is degenerate with everyone having the same wealth level $a = \frac{s(y - rI)}{2(1 - sr)}$ and the total income of the economy is the highest possible.

Suppose conversely, that the production set is convex, but that the credit market is still absent. In particular, assume that people can't even deposit money to earn interest income. Let the production technology be described by the following constant returns to scale production function $Y = F(L; K)$ where :

(a) all agents are still endowed with one unit of labor that is perfectly divisible

(b) the wealth distribution satisfies:

$$\exists\, \varepsilon > 0 \text{ such that } G_0(\varepsilon) = 0.$$

This economy is exactly like a closed economy version of the Solow model where instead of saving a fraction of their income individuals leave a fraction of their income to their offsprings. There we know that output per capita and capital per worker all converge monotonically to the same steady state so long as the parameters are the same. Indeed, if labor can flow across production units then capital per worker will be equalized at every point of time, and incomes and steady state dynastic wealths will also converge over time.[11]

An important feature of this model is that one shot redistributive policies can raise the total income of the economy permanently for parameter values for which the initial wealth distribution matters for the long term performance of the economy. For simplicity assume that the policy is implemented after the economy has settled down in a steady state equilibrium. Suppose the government taxes bequests of rich dynasties and redistributes the revenue (so that the government budget is balanced) to poorer dynasties whose wealth is less than I with the

nology. Similarly, $\frac{I}{2}$ units of capital, and half a unit of supervisory labor and ordinary labor each produce no output under the entrepreneurial technology, or, if capital can be thrown away, $\underline{w}$ units of output at best. But by Assumption 1 $\frac{y}{2} > \frac{y - rI}{2} > \underline{w}$. Finally, a rich individual cannot start two enterprises even if she has the money to do so.

[11] Piketty (1996) analyzes a model similar to that of Banerjee and Newman (1993) where instead of the wage rate, the interest rate is endogenized. His model has a convex production technology with respect to capital, but there is no labor market so that everybody has to be self employed. This non-convexity, together with moral hazard leading to credit market imperfections imply that multiple steady state equilibria are possible.

goal of making as many individuals to be able start their own enterprises as possible. Naturally this policy will have no effect when the economy is in a high wage or subsistence equilibrium because everyone has equal wealth to start with. For the case of low wage equilibrium, it can have an effect. Consider case 3. The policy moves everyone's wealth closer to the mean whereas whether the wealth of the median person is greater than or less than I determines whether there is a high or a low wage equilibrium. Starting from a low wage equilibrium, if the mean is greater than I then such a redistributive policy will push the economy towards a high wage equilibrium. Even if the mean is less than I in which case the high wage equilibrium cannot be achieved, the policy will increase the number of enterprises that are operated and hence raise total income. Similarly, in Case 2 such a policy will increase the number of enterprises that are operated and hence raise total income. However, the implication of this exercise is not to support any redistributive policy to increase total income, rather only those that increase the number of enterprises operating in the economy. For example in case 3, if the mean wealth level is less than I; then a complete redistribution will push the economy to subsistence.

Next we consider the role of lotteries in improving efficiency. A lottery is a well-known device to solve problems of non-convexity, which we argued is a necessary condition for inefficiency in this model. The motivation of introducing lotteries in this model is the risk-attitude of individuals. When the wage is low, individuals whose initial wealth is less than but close to I are risk-loving because

$$
\begin{aligned}
y_{i;t} &= ra_{i;t} + \underline{w} && \text{if} \quad a_{i;t} < I \\
y_{i;t} &= ra_{i;t} + \underline{w} + (y - rI - 2\underline{w}) && \text{if} \quad a_{i;t} \geq I;
\end{aligned}
$$

Therefore they have an incentive to buy lotteries, even those that have unfair odds.[12] Since this would facilitate upward mobility and increase the number of enterprises that are operated in the economy, the introduction of lotteries will improve efficiency if the economy starts off with a low wage equilibrium. Suppose the economy is in a low wage steady state where each poor individual has a wealth level of $a^W(\underline{w}) = \frac{s\underline{w}}{1-sr}$: Consider a lottery that requires the buyer to pay $a^W(\underline{w})$; and offers the winner I with probability $\frac{a^W(\underline{w})}{I}$ and 0 with probability $1 - \frac{a^W(\underline{w})}{I}$: Since we have a low wage equilibrium, we have $G(I) > \frac{1}{2}$: Also the population size is assumed to be large, and $a^W(\underline{w}) > 0$: Therefore such a lottery would always increase the total number of enterprises, and hence improve efficiency.

[12] Rotating saving and credit associations (roscas) are a frequently observed informal credit institution (Besley, Coate and Loury, 1993 provide a formal analysis) that can be interpreted as a lottery to overcome problems of indivisible investments. See Lehnert (1998) for a general treatment of lotteries in models of growth in the presence of credit rationing.

3 Extension : Stochastic Model with Mobility

An important feature of the model in Section 2 is that the incomes of all agents, and the bequests of their progeny are all deterministic. This is unsatisfactory for two reasons. First, the long-run wealth distribution has all probability mass concentrated on two points (for a low wage equilibrium) or one point (the high wage equilibrium or the subsistence equilibrium). As a result, there is no mobility across classes. Second, and more importantly, we want to check the robustness of the result that an economy may have multiple steady-state distributions of wealth in the presence of random shocks. As Banerjee and Newman (1994) point out, if agents are subjected to low-probability idiosyncratic shocks and the support of the distribution of shocks is large enough, the economy may still converge to a unique steady-state distribution of wealth in some models that have fixed costs and credit rationing, such as Galor and Zeira (1993).

Suppose there is an idiosyncratic i.i.d. shock on every individual's saving rate. In each period, each individual's saving propensity could be high ($\bar{s}$) with probability p or low ($\underline{s}$) with probability $1-p$. We assume that $p\bar{s}+(1-p)\underline{s}=s$ so that the total wealth of next generation in this economy is the same as the economy without saving shocks. If $\bar{s}$ ($\underline{s}$) is high (low) enough, we'll have upward (downward) mobility which is absent in the stationary distributions discussed in Section 2.[13] We make the following assumption regarding the support of the distribution of the saving rate:

$$\begin{aligned} &(1) \quad \bar{s} \in \left(\frac{l}{\underline{w}+rl}, \frac{1}{r}\right) \\ &(2) \quad \underline{s} = 0 \end{aligned} \qquad \text{(Assumption 3)}$$

The first part of Assumption 3 implies $\frac{\bar{s}\underline{w}}{1-r\bar{s}} > l$, that is if every member of dynasty i receives $\bar{s}$, then even though the wage is always low, eventually it becomes unconstrained. Therefore, this assumption ensures there is upward mobility in this economy. The upper bound of $\bar{s}$ is assumed for a technical reason. The second part of the assumption implies that each newborn agent will have a strictly positive probability entering the economy with no wealth no matter how wealthy her ancestors were. This assumption makes it possible to find the stationary distribution because almost all wealth levels are transient under the resulting transition functions to which now we turn.

[13] An alternative way to introduce random shocks in the model would be to let production be stochastic (as in Banerjee and Newman, 1993). However, given our assumptions about the production technology and preferences, the contractual form of payment to workers will be indeterminate (for example, wage contracts or profit/output sharing contracts will be equivalent). This is unsatisfactory since the specific contractual form will be crucial in driving the extent upward and downward mobility in the model.

When $w_t = \underline{w}$, we have

$$
\begin{aligned}
a_{i;t+1}(a_{i;t} \mid w_t = \underline{w}) &= \bar{s}[ra_{i;t} + \underline{w}] && \text{with probability } p && \text{if } a_{i;t} < I \\
&= 0 && \text{with probability } 1 - p && \\
a_{i;t+1}(a_{i;t} \mid w_t = \underline{w}) &= \bar{s}[r(a_{i;t} - I) + y - \underline{w}] && \text{with probability } p && \text{if } a_{i;t} \geq I \\
&= 0 && \text{with probablity } 1 - p. &&
\end{aligned}
$$

When $w_t = \overline{w}$, we have

$$
\begin{aligned}
a_{i;t+1}(a_{i;t} \mid w_t = \overline{w}) &= \bar{s}[ra_{i;t} + \overline{w}] && \text{with probability } p && \forall\, a_{i;t} \\
&= 0 && \text{with probabily } 1 - p. &&
\end{aligned}
$$

Figure 3 displays these transition functions. As before, we want to know whether this process is stationary and this involves two steps. First we need to show that in the long run wages are always low or always high, and second, given the long run wage rate, every initial wealth distribution will converge to the stationary wealth distribution corresponding to that wage rate.

Proposition 4: When the saving rate is subject to random shocks the wealth transition process is stationary under Assumption 3, and given any initial wealth distribution, it will converge to a stationary distribution.

Proof : See the appendix.

Knowing that there is only one wage rate which will prevail in the long run, we can treat them separately and find the corresponding stationary distributions. Let A^w be the ergodic set and λ^w be the stationary wealth distribution under the transition functions when wage rate is w. Given our assumption that every dynasty is subject to a low savings shock with some probability every period, every dynasty has a positive probability of starting with wealth 0. Hence at any date its wealth can only be some level that is generated from an initial wealth of 0 and a series of high saving shocks (e.g, $\bar{s}\overline{w}; \bar{s}(r\bar{s}\overline{w} + \overline{w})$,..in the high wage equilibrium). The probability of a dynasty's wealth not assuming any of these values is the same as the probability of never getting hit by a low savings shock, which is zero. Therefore the support of the stationary distribution is the set of these values $A^w = \{a_j^w\}_{j=1}^{\infty}$ (where $w = \underline{w}$ or $\overline{w}$) which is countable and infinite. Also λ^w can be represented by a sequence $\{\lambda_j^w\}_{j=1}^{\infty}$ where λ_j^w is the probability mass on wealth level a_j^w.

We show in the proof of Proposition 4 that $\bar{s} > \frac{I}{\underline{w}+rI}$ (Assumption 3) implies that there exists an integer m such that in the low wage regime, if a dynasty i starts at no wealth, it can become an entrepreneur if it receives m consecutive periods of high savings shocks. Therefore the smaller is m, the easier it is for people to have upward mobility. When $w = \underline{w}$, it is easy to verify that

$$
\begin{aligned}
a_j^{\underline{w}} &= 0 && \text{for } j = 1 \\
a_j^{\underline{w}} &= \sum_{i=0}^{j-2} (r\bar{s})^i (\bar{s}\underline{w}) && \text{for } j = 2, 3, \ldots, (m+1) \\
a_j^{\underline{w}} &= \sum_{i=0}^{j-2} (r\bar{s})^i (\bar{s}\underline{w}) + \bar{s}(y - rI - 2\underline{w}) && \text{for } j = (m+2), \ldots
\end{aligned}
$$

The calculation of $\lambda_j^{\underline{w}}$ is straight forward.

$$\lambda_j^{\underline{w}} = p^{j-1}(1-p) \quad \text{for } j = 1, 2, \ldots.$$

Therefore, the stationary distribution will look like Figure 4(a).

With the set $A^{\underline{w}}$ and the distribution $\lambda^{\underline{w}}$, we have to make sure that they are consistent with our assumption that the wage rate is low. The following condition is necessary for the existence of a low wage stationary equilibrium:

$$\sum_{j=1} p^{j-1}(1-p) > \frac{1}{2}$$

or, $p^m < \frac{1}{2}$. The value of p^m is critical since it is the chance of receiving m consecutive high saving shocks. If $p^m \geq \frac{1}{2}$, for each dynasty, there is more than half a chance that it is not capital-constrained in m periods which is inconsistent with low wage stationary equilibrium. We can also see that the smaller m is, the more difficult that this condition is satisfied because upward movement is easier.

When $w = \bar{w}$,

$$
\begin{aligned}
a_j^{\bar{w}} &= 0 && \text{for } j = 1 \\
a_j^{\bar{w}} &= \sum_{i=0}^{j-2} (r\bar{s})^i (\bar{s}\bar{w}) && \text{for } j = 2, 3, \ldots.
\end{aligned}
$$

Similar to m, we can find an integer n the interpretation of which is exactly the same as that of m except that in this case, the prevailing wage rate is high. Since $\bar{w} > \underline{w}$; $n \le m$ and therefore $p^n \geq p^m$. The stationary wealth distribution $\lambda^{\bar{w}}$ is

$$\lambda_j^{\bar{w}} = p^{j-1}(1-p) \quad \text{for } j = 1, 2, \ldots.$$

Figure 4(b) shows the stationary distribution.

The necessary condition for the high wage stationary distribution is:

$$
\begin{aligned}
&\sum_{j=1} p^{j-1}(1-p) \le \frac{1}{2} \\
\Longrightarrow\quad & p^n \geq \frac{1}{2}
\end{aligned}
$$

If $p^n < \frac{1}{2}$, then for each dynasty, there is more than half a chance that it is capital-constrained in n periods which is inconsistent with a high wage stationary equilibrium.

Depending on the values of p^m and p^n, we have the following cases. (See Figure 5.)

Case 1. $p^n \geq p^m \geq \frac{1}{2}$
Unique Equilibrium : High Wage

Since the necessary condition for a low wage equilibrium is not satisfied by Proposition 4, we must have a unique high wage equilibrium. The reason is that upward mobility is easy under both wage levels. Independent of the initial wealth distribution, the number of firms operating in the long run is equal to $1/2$.

Case 2. $\frac{1}{2} > p^n \geq p^m$
Unique Equilibrium : Low Wage

Contrary to the previous case, the necessary condition for a high wage equilibrium is not satisfied so we must have a unique low wage equilibrium. Independent of the initial wealth distribution, the number of firms operating in the long run is equal to $1 - \sum_{j=1}^{m} p^{j-1}(1-p)$.

Case 3. $p^n \geq \frac{1}{2} > p^m$
High or Low Wage Equilibrium, Initial Distribution Matters

In this case both necessary conditions are satisfied. Which equilibrium this economy will end up depends on its initial wealth distribution. If the economy starts out with many rich people, since the high wage rate could last for many periods some of the originally poor will have accumulated enough wealth by the time that most of the originally rich have been hit by a low savings shock. Therefore the stationary distribution is more likely to be the one associated with the high wage. Conversely, if the economy starts out with many poor individuals the low wage rate would last for a long time. Then only a few lucky individuals will be able to accumulate enough wealth before hit by a low savings shock. Therefore the low wage equilibrium results.

Let us examine the role of the specific restrictions on $\bar{s}$ made in Assumption 3 for the above result. If $\bar{s} \leq \frac{I}{\underline{w}+rI}$ then in a low wage equilibrium a dynasty that starts with zero wealth will not be able to accumulate enough wealth to start an enterprise in finite time. Given that a fraction $(1-p)$ of entrepreneurs will become poor every period, the economy will eventually converge to a subsistence equilibrium. But if $\bar{s} > \frac{I}{\bar{w}+rI}$ then a high wage equilibrium will exist if the condition mentioned in case 3 is also satisfied (namely, n is not too large).[14] The economy will converge to the high wage equilibrium if it starts out with many rich people, and to the subsistence equilibrium otherwise.

On the other hand, we require $\bar{s} \leq \frac{1}{r}$ to prove Proposition 4 to make sure that past wages

[14] In both cases the wage dynamics is stationary in the long run and so Proposition 3 applies.

get lower weight than current wages in the distribution dynamics. After several periods, the wealth level of poor individuals will be clustered at some finite number of wealth levels. From this point on, if the wage rate is high to start with, all these wealth levels will shift to the right (unless hit by a low savings shock). The reason is, past wages are discounted by the factor $\bar{s}r$ 1 and so the distribution dynamics is dominated by the current wage. Hence, the wage rate will always remember high. If the wage rate is low, then the opposite happens. If instead $\bar{s} > \frac{1}{r}$ the result will be to put more weight on past wages relative to current wages. As a result, wages may change infinitely often and the wealth transition process will not be stationary. In the appendix we provide an example where

$$\begin{aligned} w_{3t} &= w_{3t+1} = \underline{w} \\ w_{3t+2} &= \overline{w} \end{aligned}$$

for all $t = 0; 1; 2; :::$. In this example, when the wage rate is high, the richest among the poor stay poor because past wages (which are low) play a dominant role. Since there is only downward mobility but no upward mobility the wage rate switches from high to low. In period $3t + 1$ even though the wage is low, the richest dynasties among the poor become rich since the high wage they experienced two periods ago is weighted by $(\bar{s}r)^2$: The parameter configuration we assume ensures that there is more upward mobility than downward mobility so that the wage rate becomes high again. This process will go on forever and the economy will display cycles. It should be noted that under the parameter conditions assumed in the appendix, a stationary distribution (with either high or low wages) always exists and under some initial distributions the economy will converge to it. But if $\bar{s} > \frac{1}{r}$; our example shows that there also exists initial distributions under which the economy never converges.

4 Conclusion

In this paper we analyzed a simple dynamic model of occupational choice in the presence of credit market imperfections similar to Banerjee and Newman (1993) where wealth inequality and returns to various occupations are endogenous. We examined conditions under which multiple steady state equilibria exist and characterized how initial conditions affect which equilibrium the economy converges to. We conclude with two caveats both of which suggest directions for future research. First, while the simplicity of the model makes the working of the dynamic mechanism of interaction of the wealth distribution and returns to various occupations transparent, it also precludes making any general conclusions regarding this class of non-linear models (as Banerjee and Newman's original model also illustrates). Second, the relationship between credit market imperfections, incentives and inequality is likely be more complex than what is suggested in this simple model illustrating the possibility of poverty traps (see for example, Ghatak, Morelli and Sjostrom, 1999).

5 Appendix

Proof of Proposition 4

Step 1. Assumption 3 implies $\bar{s}(ra + w) > I \ \forall a \geq I$ for $w = \underline{w}$ or $\bar{w}$. Therefore a dynasty that is not poor will remain so in the next period unless it is subject to a low savings shock ($\underline{s} = 0$). Knowing this the wage dynamics will remain the same we change the transition function for entrepreneurs when $w_t = \underline{w}$ to the following artificial transition function for analytical convenience:

$$\begin{aligned} a_{i,t+1}(a_{i,t} \mid w_t = \underline{w}) &= \bar{s}[ra_{i,t} + \underline{w}] \quad \text{with probability } p \\ &= 0 \quad \text{with probablity } 1 - p. \end{aligned}$$

The reason we can do this is we are interested only in the wage dynamics at this point, and since the wage only depends on the fraction of rich people, we can alter the transition function so long as this fraction does not change. The particular transition function we use will not change the fraction of rich people because under it, once a dynasty is rich it will remain so unless it is hit by $\underline{s}$. We will assume this transition function throughout this proof.

Step 2. Since $\bar{s} > \frac{I}{\underline{w}+rI}$, there exists an m such that

$$\begin{aligned} \bar{s}\sum_{i=0}^{m-2}(r\bar{s})^i\underline{w} &< I \\ \text{and} \quad \bar{s}\sum_{i=0}^{m-1}(r\bar{s})^i\underline{w} &\geq I. \end{aligned}$$

When $\bar{s}r < 1$ this follows from the fact $\frac{\bar{s}\underline{w}}{1-\bar{s}r} > I$ by Assumption 3 and that $\lim_{n\to\infty}\bar{s}\sum_{i=0}^{n-1}(r\bar{s})^i\underline{w} = \bar{s}\underline{w}\frac{1-(\bar{s}r)^n}{1-\bar{s}r} = \frac{\bar{s}\underline{w}}{1-\bar{s}r} > I$. When $\bar{s}r = 1$ this follows from the fact that $\lim_{n\to\infty}\bar{s}\underline{w}\sum_{i=0}^{n-1}(r\bar{s})^i = \infty$. Therefore for dynasty which receives m consecutive periods of $\bar{s}$, it will be unconstrained regardless of its initial wealth and wage rates during these m periods.

Step 3. Let $W_t^m \equiv (w_{t-m+1}, \ldots, w_t)$ be the m-period wage history of the economy till time t. Let k_i be how many periods ago that dynasty i was last hit by a low saving shock. Then

$$\begin{aligned} a_{i,t+1}(k_i) &= 0 && \text{when } k_i = 1 \\ &= \bar{s}[\sum_{j=0}^{k_i-2}(r\bar{s})^j w_{t-j}] && \text{when } k_i = 2, 3, \ldots, m+1 \\ &> \bar{s}[\sum_{j=0}^{k_i-2}(r\bar{s})^j w_{t-j}] && \text{when } k_i > m+1. \end{aligned}$$

Next we turn to the characterization of the entire wealth distribution at period $t+1$. Notice that dynasties which have the same k ($1 \leq k \leq m+1$) will have the same wealth level

$\overline{s}[\sum_{j=0}^{k-2}(r\overline{s})^j w_{t-j}]$, denoted by $a_{t+1}(k)$ thereafter. Then $\lambda_{t+1}(\{a_{t+1}(k)\})$ is just the fraction of population whose last time of receiving $\underline{s}$ was k periods ago. So we have

$$
\begin{aligned}
\lambda_{t+1}(\{0\}) &= 1 - p \\
\lambda_{t+1}(\{a_{t+1}(k)\}) &= p^{k-1}(1-p) \qquad \forall k = 2, 3, \ldots, m+1 \\
\lambda_{t+1}((a_{t+1}(m+1), \infty)) &= 1 - \sum_{j=0}^{m} p^j(1-p) = p^{m+1}.
\end{aligned}
$$

By step 2 of the proof, $a_{t+1}(m+1) = \overline{s}[\sum_{j=0}^{m-1}(r\overline{s})^j w_{t-j}] \geq l$, so whether $G_{t+1}(l)$ is less than one half will depend on $a_{t+1}(k) ? l; k = 2, 3, \ldots, m$. In other words, w_{t+1} only depends on W_t^m.

Step 4. We can also characterize λ_{t+2} as

$$
\begin{aligned}
\lambda_{t+2}(\{0\}) &= 1 - p \\
\lambda_{t+2}(\{a_{t+2}(k)\}) &= p^{k-1}(1-p) \qquad \forall k = 2, 3, \ldots, m+1 \\
\lambda_{t+2}((a_{t+2}(m+1), \infty)) &= 1 - \sum_{j=0}^{m} p^j(1-p) = p^{m+1}.
\end{aligned}
$$

Now we want to compare λ_{t+2} with λ_{t+1}. Let $D_{t+1}^{t+2}(k); k = 2, 3, \ldots, m+1$ be the difference between $a_{t+2}(k)$ and $a_{t+1}(k)$. Then

$$
\begin{aligned}
D_{t+1}^{t+2}(k) &= \overline{s}[\sum_{j=0}^{k-2}(r\overline{s})^j w_{t-j+1}] - \overline{s}[\sum_{j=0}^{k-2}(r\overline{s})^j w_{t-j}] \\
&= \overline{s}\{w_{t+1} - (1 - r\overline{s})\sum_{j=0}^{k-3}(r\overline{s})^j w_{t-j} - (r\overline{s})^{k-2} w_{t-k+2}\}.
\end{aligned}
$$

Since $1 - r\overline{s} \geq 0$ (Assumption 3), we have

$$
\overline{s}\{w_{t+1} - (1 - r\overline{s})\sum_{j=0}^{k-3}(r\overline{s})^j \overline{w} - (r\overline{s})^{k-2}\overline{w}\} \leq D_{t+1}^{t+2}(k) \leq \overline{s}\{w_{t+1} - (1 - r\overline{s})\sum_{j=0}^{k-3}(r\overline{s})^j \underline{w} - (r\overline{s})^{k-2}\underline{w}\}
$$

$$
\Rightarrow \quad \overline{s}(w_{t+1} - \overline{w}) \leq D_{t+1}^{t+2}(k) \leq \overline{s}(w_{t+1} - \underline{w}).
$$

Therefore if $w_{t+1} = \overline{w}$, we have $D_{t+1}^{t+2}(k) \geq 0 \; \forall k = 2, 3, \ldots, m+1$. This implies $G_{t+2}(l) \leq G_{t+1}(l)$ and because $G_{t+1}(l) \leq \frac{1}{2}$, we have $G_{t+2}(l) \leq \frac{1}{2}$. Hence $w_{t+2} = \overline{w}$. On the other hand, if $w_{t+1} = \underline{w}$, we have $D_{t+1}^{t+2}(k) \leq 0 \; \forall k = 2, 3, \ldots, m+1$. This implies $G_{t+2}(l) \geq G_{t+1}(l)$ and because $G_{t+1}(l) > \frac{1}{2}$, we have $G_{t+2}(l) > \frac{1}{2}$. Hence $w_{t+2} = \underline{w}$ as well.

To conclude, we have

$$
w_{t+2}(W_{t+1}^m) = w_{t+1}.
$$

Therefore starting from period $m+1$, the wage rates of this economy are always high or always low. In either case, the wage dynamics is stationary. Given this, showing the convergence is straight forward. ∎

An Example of a Cycle

Given $\bar{s} > \frac{1}{r}$, consider the following parameter configuration:

$$p^4 < \frac{1}{2} < p^3$$

$$\bar{s}\left\{(r\bar{s})^2\underline{w} + (r\bar{s})\bar{w} + \underline{w}\right\} < I < \bar{s}\min\left\{(r\bar{s})^2\bar{w} + (r\bar{s})\underline{w} + \underline{w}, (r\bar{s})^3\underline{w} + (r\bar{s})^2\underline{w} + (r\bar{s})\bar{w} + \underline{w}\right\}.$$

Suppose the initial wealth distribution is

$$\begin{aligned}
\lambda_0(\{0\}) &= 1 - p \\
\lambda_0(\{\bar{s}\bar{w}\}) &= p(1 - p) \\
\lambda_0(\{\bar{s}[(r\bar{s})\underline{w} + \bar{w}]\}) &= p^2(1 - p) \\
\lambda_0\left(\left\{\bar{s}\left[(r\bar{s})^2\underline{w} + (r\bar{s})\underline{w} + \bar{w}\right]\right\}\right) &= p^3(1 - p) \\
\lambda_0([I, \infty)) &= p^4.
\end{aligned}$$

Given our assumptions on the parameters, calculating the entire wage and wealth distribution dynamics is straightforward. Under λ_0, $w_0 = \underline{w}$ under our assumptions. Then we can characterize λ_1 by

$$\begin{aligned}
\lambda_1(\{0\}) &= 1 - p \\
\lambda_1(\{\bar{s}\underline{w}\}) &= p(1 - p) \\
\lambda_1(\{\bar{s}[(r\bar{s})\bar{w} + \underline{w}]\}) &= p^2(1 - p) \\
\lambda_1\left(\left\{\bar{s}\left[(r\bar{s})^2\underline{w} + (r\bar{s})\bar{w} + \underline{w}\right]\right\}\right) &= p^3(1 - p) \\
\lambda_1([I, \infty)) &= p^4.
\end{aligned}$$

It follows that under this distribution, $w_1 = \underline{w}$. Again, we can characterize λ_2 by

$$\begin{aligned}
\lambda_2(\{0\}) &= 1 - p \\
\lambda_2(\{\bar{s}\underline{w}\}) &= p(1 - p) \\
\lambda_2(\{\bar{s}[(r\bar{s})\underline{w} + \underline{w}]\}) &= p^2(1 - p) \\
\lambda_2([I, \infty)) &= p^3.
\end{aligned}$$

Then the wage rate w_2 will switch to $\bar{w}$. It is straightforward to verify that $\lambda_{3t+i}(.)$ for all $t = 1, 2, ..$ and $i = 0, 1, 2$ will have the same form as $\lambda_i(.)$. Therefore we have,

$$\begin{aligned}
w_{3t} &= w_{3t+1} = \underline{w} \\
w_{3t+2} &= \bar{w}
\end{aligned}$$

for all $t = 0, 1, 2, ..$.

References

[1] Banerjee, A.V. and A. Newman (1993) : \Occupational Choice and the Process of Development." Journal of Political Economy 101.

[2] Banerjee, A.V. and A. Newman (1994) : \Poverty, Incentives, and Development" American Economic Review Papers and Proceedings 84.

[3] Barro, R. and X. Sala-I-Martin (1995) : Economic Growth. McGraw-Hill, New York.

[4] Besley, T., S. Coate and G. Loury (1993) : \The Economics of Rotating Saving and Credit Associations", American Economic Review 83.

[5] Dasgupta, P. and D. Ray (1986) : \Inequality as a Determinant of Malnutrition and Underemployment : Theory" Economic Journal, 96.

[6] Galor, O. and J. Zeira (1993) : \Income Distribution and Macroeconomics." Review of Economic Studies 60.

[7] Ghatak, M., M. Morelli and T. Sjostrom (1999) : \Dynamic Incentives, Occupational Choice and the American Dream", Mimeo. University of Chicago.

[8] Lehnert, A. (1998) : \Asset pooling, credit rationing, and growth", Mimeo. Federal Reserve Board, Washington DC.

[9] Nurkse, R. (1953) : Problems of Capital Formation in Underdeveloped Countries. New York : Oxford University Press.

[10] Piketty, T. (1997) : \The Dynamics of Wealth Distribution and the Interest Rate with Credit Rationing", Review of Economic Studies 64.

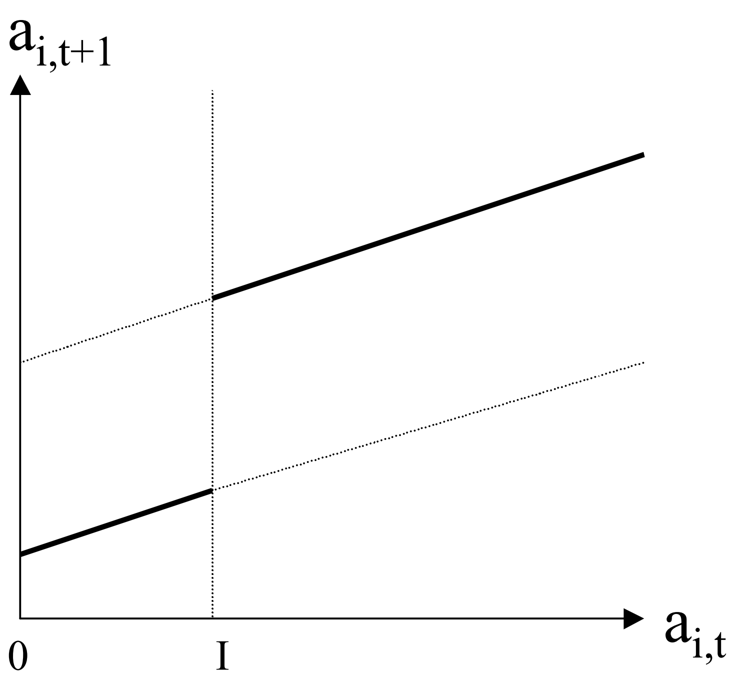

Fig. 1(a)
($w_t = \underline{w}$)

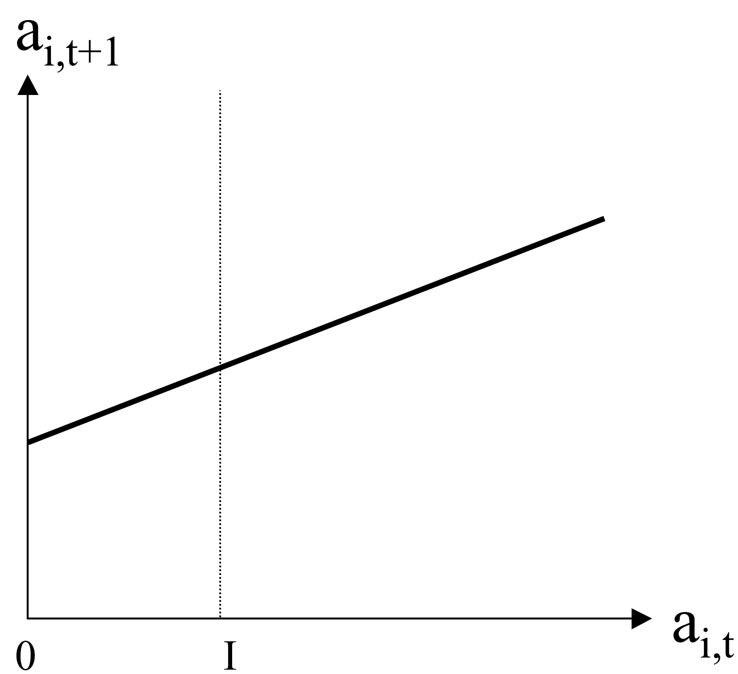

Fig. 1(b)
($w_t = \overline{w}$)

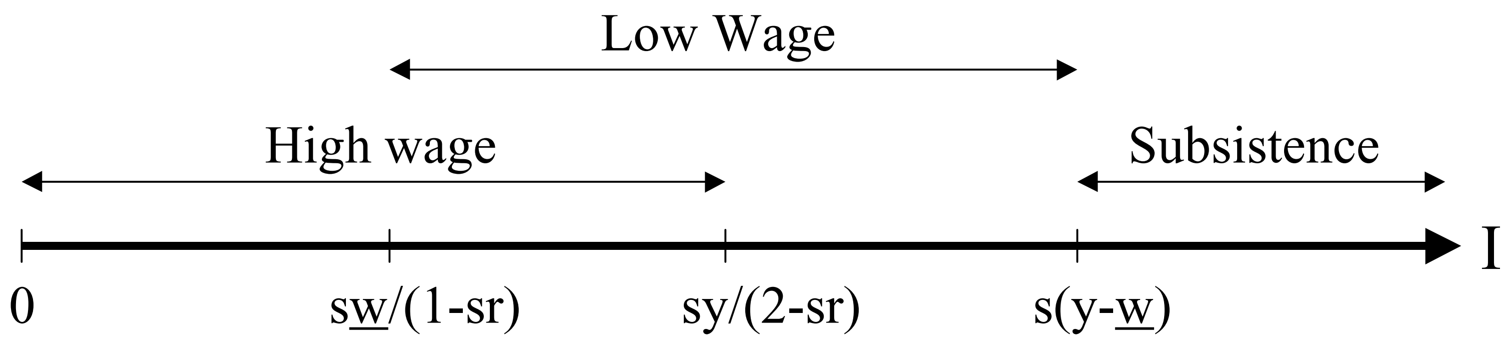

Fig. 2

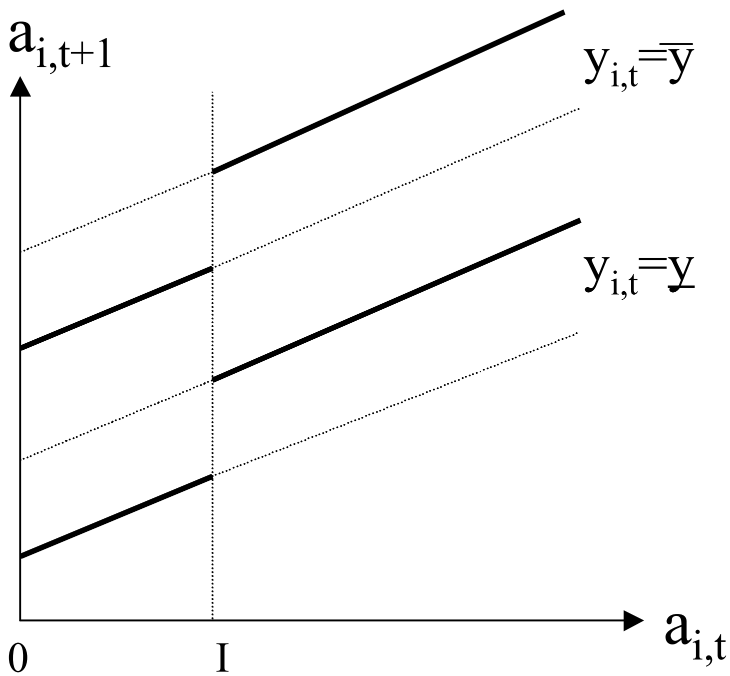

Fig. 3(a)
($\sigma_t=\underline{\sigma}$)

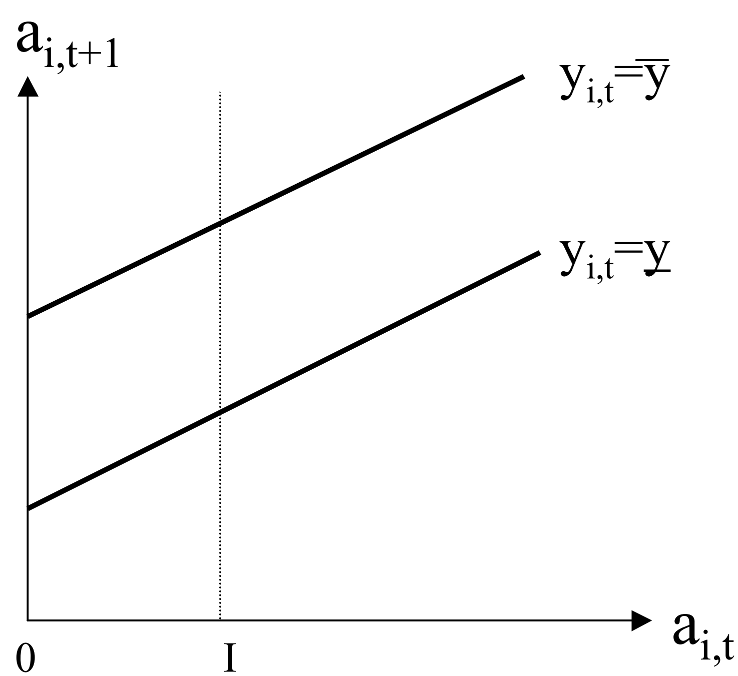

Fig. 3(b)
($\sigma_t=\overline{\sigma}$)

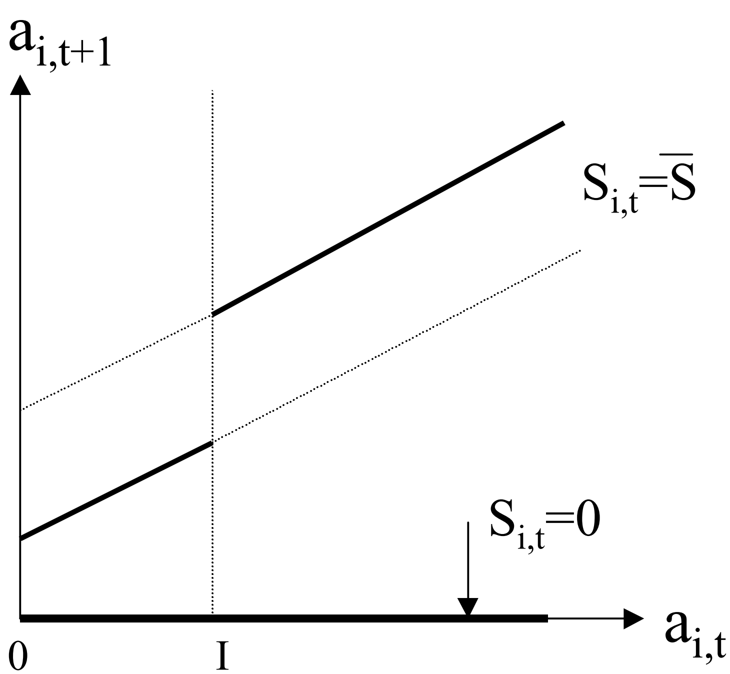

Fig. 4(a)
($w_t=\underline{w}$)

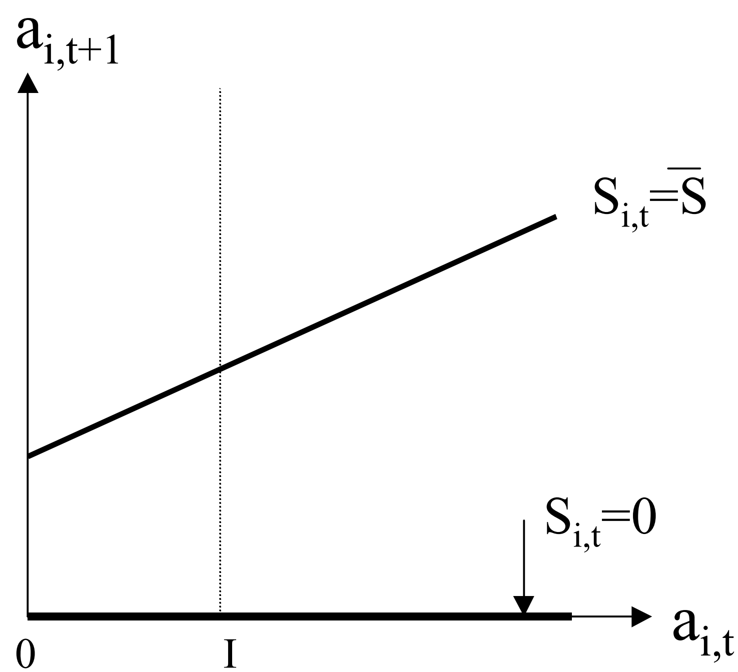

Fig. 4(b)
($w_t=\overline{w}$)

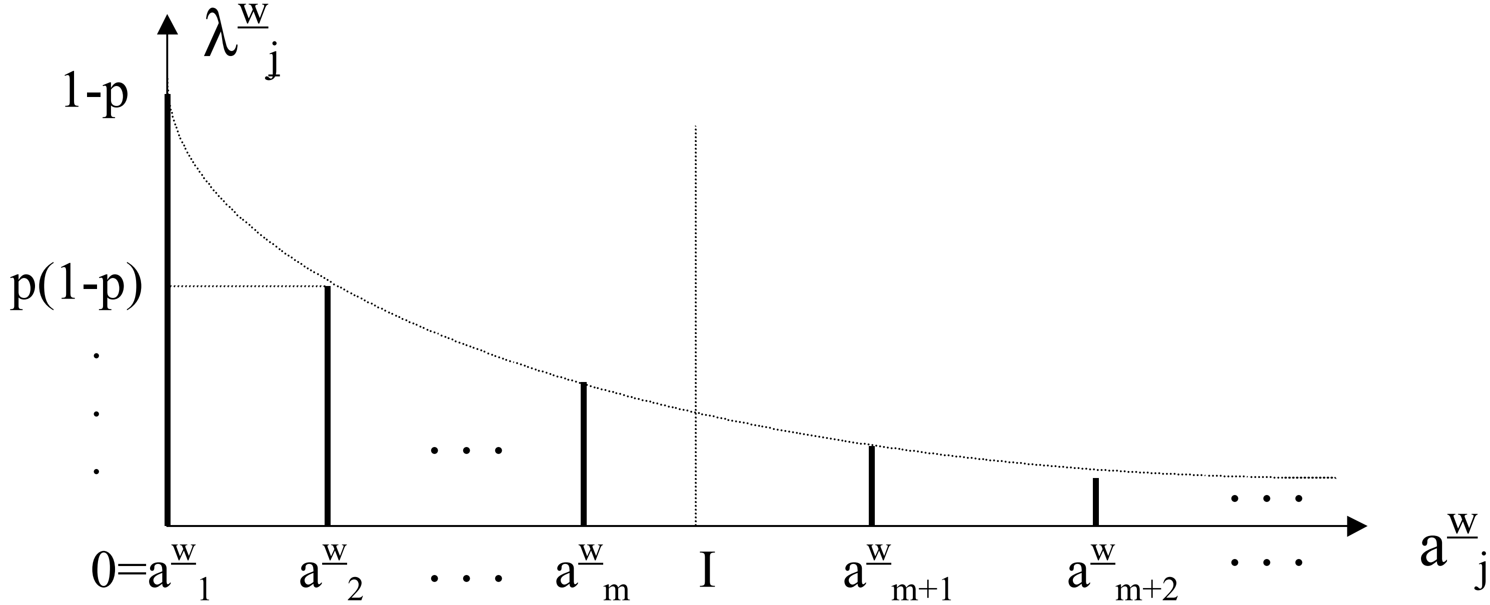

Fig. 5(a) (Low Wage Stationary Equilibrium)

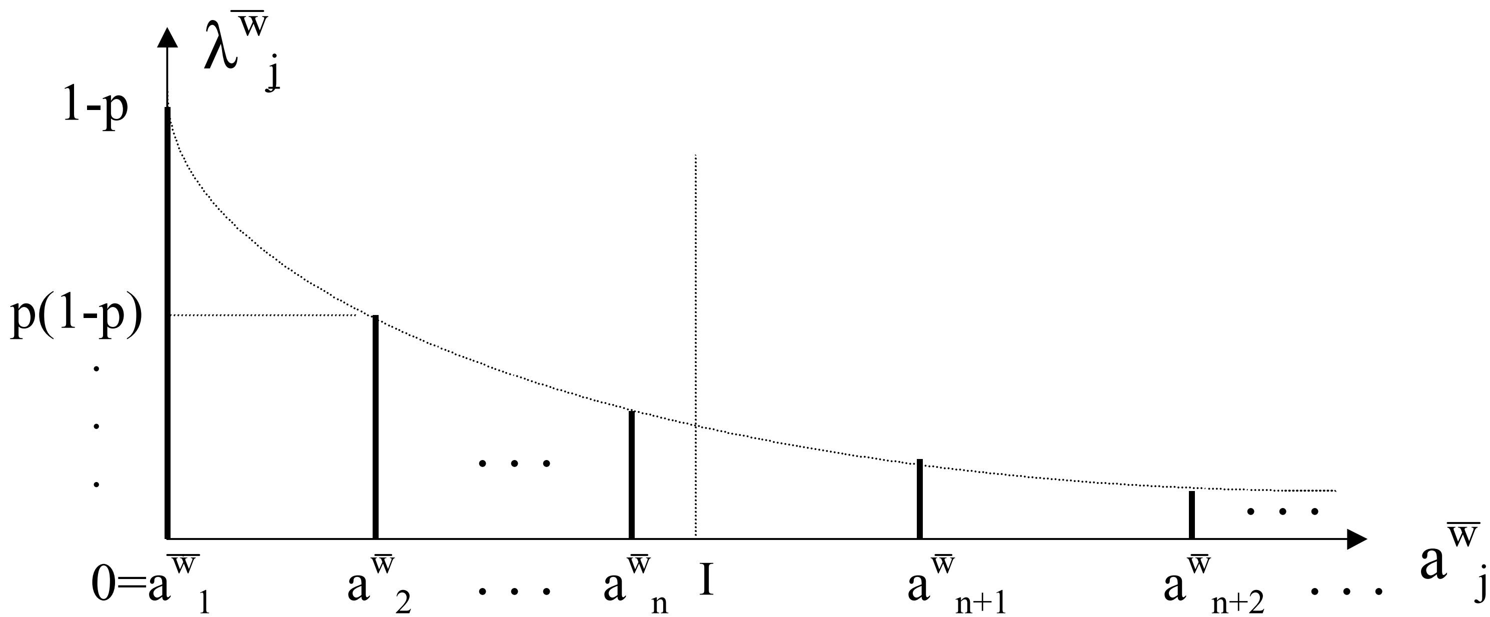

Fig. 5(b) (High Wage Stationary Equilibrium)

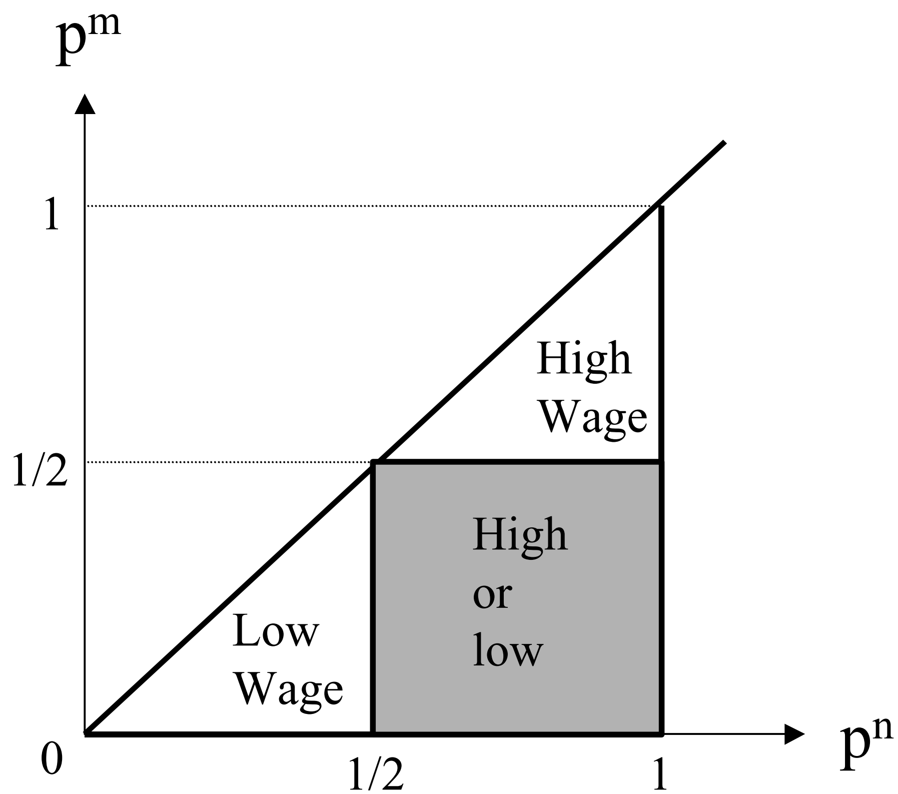

Fig. 6

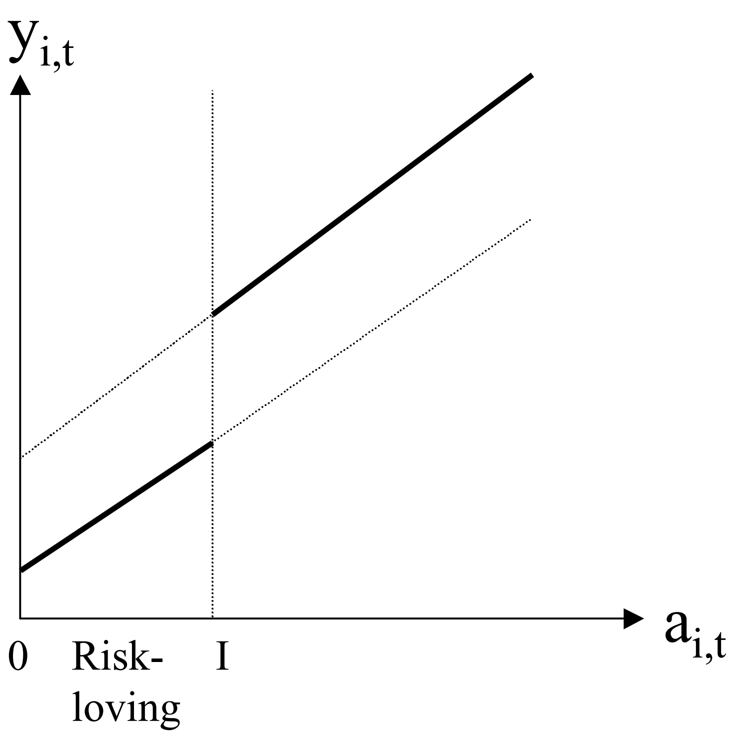

Fig. 7

($w_t = \underline{w}$)

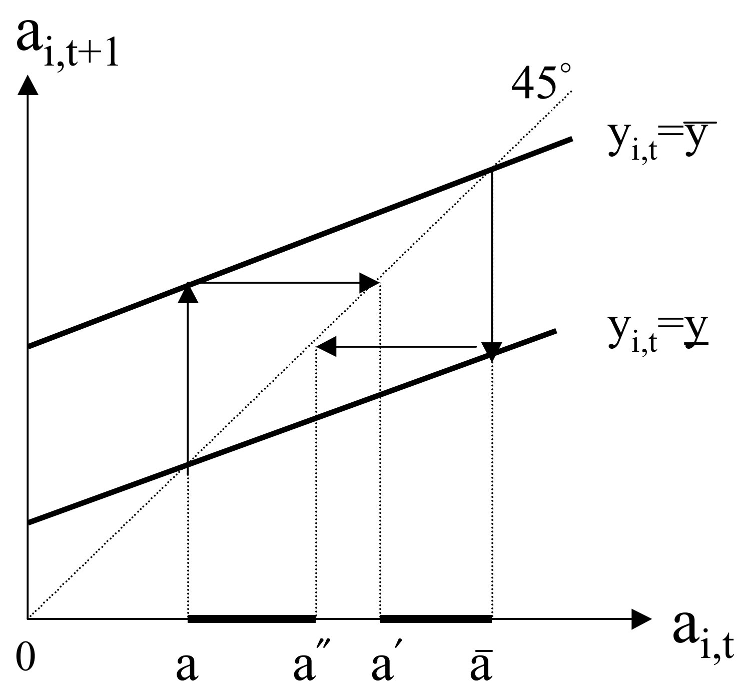

Fig. 8

($\sigma_t = \overline{\sigma}$)