PUTTING THE "TH" IN TENTHS: THE ROLE OF LABELING DECIMALS IN REVEALING PLACE VALUE STRUCTURE

By

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CHAPTER I

INTRODUCTION

We have an innate, basic ability to discriminate quantity (Dehaene, 1997), as well as an intuitive understanding of ratio (Sophian, 2000). Even with these advantages, representing quantity and relationships symbolically is a difficult task and undoubtedly contributes to mathematical understanding. As the National Council of Teachers of Mathematics (1989) pointed out, mathematical symbols must have meanings attached to them for mathematics to be meaningful to children. Attending to the language we use to describe mathematical symbols may be one way to change how meaningful or meaningless they are. In the current study, we examined the role of formal labels on children's understanding of mathematical symbols used to represent fractional quantities.

The Power of Providing Labels

Previous research suggests language may play a critical role in learning and understanding across a variety of domains (Chesney et al., 2012; Fuson & Kwon, 1991; Fyfe, McNeil, & Rittle-Johnson, 2015; Miura, Okamoto, Vlahovic-Stetic, Kim, & Han, 1999; Paik & Mix, 2003). Labels in particular have been shown to act as a powerful cognitive tool, recruiting processes that support categorization and relational thinking. For example, providing shared labels encourages children to treat objects similarly and categorize (e.g., Gelman & Markman, 1986; Graham, Kilbreath, & Welder, 2004). Further, children attribute characteristics of ambiguous objects based on their categorical label, rather than relying on perceptual features of the objects (Gelman & Markman, 1986). In addition to supporting categorization, providing shared labels that have a relational meaning enables children to map related sets of objects (Waxman & Gelman, 1986). Four-year-olds typically fail difficult match to sample tasks that vary across multiple dimensions (e.g., matching light-dark-light squares with little-big-little circles). However, children who learned to use the symmetric relational label “even” were able to solve the task correctly by generalizing an abstract rule inferred from the meaning of the label.

While the use of shared labels has been shown to elicit cognitive processes that support categorization and relational thinking, little is known about their role in making inferences about the structure of mathematics problems. Looking for and making use of structure is one of eight mathematical practice standards outlined by the Common Core State Mathematics Standards (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). For example, mathematically proficient students are able to recognize the relationship between place value location and the value of a digit (e.g., for multidigit numbers such as 248, the leftmost digit matters most).

Several indirect pieces of evidence suggest that labels may play a role in children’s mathematics understanding. First, findings from a recent study provide some evidence for the role of shared labels in revealing the mathematical structure of repeating patterns (Fyfe et al., 2015). Four- to five-year-olds solved repeating pattern problems and were exposed to either formal, abstract shared labels (e.g., A-B-B-A-B-B) or informal, concrete labels (e.g., blue-red-red-blue-red-red). Children in the formal labels condition solved more pattern problems correctly compared to children in the informal labels condition. The abstract pattern problems required children to make the same kind of pattern as a model pattern by recreating the part that repeats using new materials. Formal labels provided language that was shared across both patterns and may have helped reveal the relation or structure across patterns. Indeed, children who correctly adopted the formal labels language while describing their own patterns solved more problems correctly.

Second, some researchers argue that language supports fraction magnitude understanding (Miura, Okamoto, Vlahovic-Stetic, Kim, & Han, 1999; Paik & Mix, 2003) and whole number place value (Fuson & Kwon, 1991). English fraction labels lack information related to the relational magnitudes they represent. In comparison, East Asian languages use verbal names for fractions that explicitly represent part-whole relations, which may contribute to conceptual understanding of fraction magnitudes. Cross-cultural research compared how Korean, Croatian, and U. S. children performed on a fraction-identification task prior to receiving formal instruction on fractions (Miura et al., 1999). Korean children significantly outperformed Croatian and U.S. children, suggesting differences in fraction labels impacted performance. Additionally, when English-speaking children were provided with fraction names that revealed part-whole relations in a similar way as Korean fraction labels, they outperformed Korean children on a similar fraction-identification task (Paik & Mix, 2003). Evidence also exists for the role of language in understanding fractions written in decimal notation (Mazzocco & Devlin, 2008). Children's failure to correctly name decimals using place value labels (e.g., naming 0.5 as five tenths) was an indicator of math learning disability. However, correctly naming decimals did not guarantee accurate conceptual knowledge of fractions represented in multiple ways (fractions, visual proportions, and decimals).

Cross-cultural comparisons have also been made between the labels Eastern versus Western languages use for whole numbers. Whole number labels in Korean and other Asian languages are based on a system that explicitly reflects base-10 structure (e.g., 326 is read as "three *hundred* two *ten* six (*one*)"). In contrast, English whole number labels are irregular and lack information about base-10 structure (e.g., 12 is read as "twelve"). Korean and American kindergartners and first graders were asked to show multidigit numbers between 11 and 42 using base ten blocks (tens blocks were ten unit blocks long and marked to show ones units; Miura, Kim, & Okamoto, 1988). Korean children showed multidigit numbers using tens and ones blocks, whereas American children rarely used tens blocks and instead counted out single ones blocks. The authors interpreted their findings as evidence for the influence of number labels on children's cognitive representations of numbers. Other evidence comes from comparing Korean and American elementary school children's understanding of multidigit addition and subtraction. Fuson & Kwon (1991; 1992) have shown that Korean elementary school children experience little difficulty solving multidigit addition and subtraction problems that require borrowing or carrying. They argued that the base-10 structure of Korean number labels make recomposition strategies easier and potentially more available (e.g., $7 + 6$ is recomposed as $7 + (3 + 3) = (7 + 3) + 3 = 10 + 3 = 13$). American elementary children, however, experience substantial difficulties with these types of problems (see Fuson, 1990 for a review).

Thus, providing children with language that carries meaningful information and can be shared across multiple instances may be one way to support thinking that reveals the mathematical structure of problems. However, there is no direct, experimental evidence on the impact of labels on mathematics learning during problem solving. The current study tested the impact of labeling decimals on revealing place value structure.

Labels and Decimal Knowledge

How would you say the decimal 0.25? Most adults would name this decimal using informal "point" language (i.e., point two five or oh point two five). In contrast, when children learn to name decimals they are taught to use formal place value labels (i.e., twenty five hundredths). Teachers might also use decomposed place value labels by naming each place value separately (i.e., two tenths and five hundredths). The way we describe or label decimals may impact how children make sense of these numbers. The fractional amounts decimals represent are non-intuitive, and as a result, these symbols are often meaningless. In an effort to understand, children often treat decimals like numbers they have lots of experience with – whole numbers. Common, systematic errors reflect these misunderstandings (e.g., Stafylidou & Vosniadou, 2004). For example, when children are asked to compare decimal magnitudes they often think 0.25 is greater than 0.9 because 25 is greater than 9 (Resnick et al.,1998).

Labeling decimals using formal, decomposed place value labels might help children understand decimal magnitudes for at least three reasons. First, these labels could help reveal place value structure, which is one critical concept for making sense of decimal magnitudes. For numbers greater than 1, the value of the place increases by multiples of 10. The inverse of this relationship is true for numbers less than 1 (as you move to the right, the value of the place is divided by 10). Thus, the tenths place represents 1 divided by 10, and the hundredths place represents 1 divided by 100. Decomposed place value labels assign each digit with an associated value or magnitude, which may be especially helpful in the context of comparing decimal magnitudes. These labels provide a shared place value label, which may encourage children to compare digits that correspond to the place value that impacts a decimal's magnitude the most. For example, labeling 0.25 as two tenths and five hundredths and 0.5 as five tenths may encourage comparison of the shared tenths place value digits. Providing shared labels invites comparisons that can help children notice similarities that reflect more rule-based regularities as opposed to shallow, concrete similarities (Gentner & Medina, 1998). Further, providing shared labels promotes relational thinking potentially by revealing the mathematical structure of problems (Fyfe et al., 2015).

Second, decomposed place value labels may help children distinguish decimals from whole numbers. Given children's tendency to commit whole number errors while comparing decimal magnitudes, distinct place value labels may prevent children from treating decimals as a unified whole number. For example, when comparing 0.25 and 0.5 distinct place value labels may encourage children to compare place value digits instead of comparing 25 and 5.

Third, place value labels match the rule for naming unit fractions greater than three (i.e., adding "th" or "ths" after the name of the number; four*ths*, fif*ths*, six*ths*, etc.). Thus, the use of place value labels may help children understand one tenth is 1/10 and one hundredth is 1/100. Despite these benefits, subtle differences in place value labels may limit their utility. Tenths and hundredths are highly similar to the familiar, well-practiced place value labels tens and hundreds. This similarity may make it difficult for children to distinguish between these place value labels, influencing children to think hundredths are bigger than tenths because hundreds are bigger than tens. Further, if children don’t have a strong understanding of how much tenths and hundredths are worth based on previous instruction or experience, these place value labels may not be meaningful.

There are also several compelling reasons to predict that informal point labels will aid or harm thinking. In comparison to formal place value labels, informal “point” labels that reflect familiar language adults use may be less confusing and activate partial understanding of decimal magnitudes children acquire during everyday experiences. Children are exposed to these labels for decimals in everyday environments in which we often label decimal amounts, such as announcing running event times at sporting events, advertisements about radio stations, reading thermometers, and discussing weight. Children may acquire a partial understanding of decimal magnitudes by being exposed to adults’ use of these labels. Mix et al. (2014) found that children as young as 3 years showed surprising understandings of multidigit place values on simple tasks focusing on mappings between spoken number names to written numerals, dots, or block representations. The authors argued that these partial understandings of the place value system were likely acquired through statistical learning processes that occur in everyday environments rich with multidigit numerals and verbal number names. If children develop these partial understandings in a similar way with decimal magnitudes, labeling decimals using informal, familiar labels should activate this knowledge and provide an entryway into building understanding.

However, using informal labels may harm thinking by activating whole number misconceptions. Labeling digits using only their number names may encourage children to treat decimals like whole numbers. Activating misconceptions has been shown to hinder problem-solving performance (McNeil & Alibali, 2005), in part because children perseverate on using incorrect strategies (Fyfe, Rittle-Johnson & DeCaro, 2012). Thus, informal labels may encourage children to commit whole number errors and interfere with children's ability to accurately compare decimal magnitudes.

Current Study

In the current study, children solved decimal fraction magnitude problems in the context of a decimal card game (i.e., which decimal is greater?) and number line estimation task. We focused on decimal fraction magnitude because of the important role fraction magnitude knowledge plays in mathematics education (NMAP, 2008) and the serious learning challenges they pose (Siegler, Fazio, Bailey, & Zhou, 2013). Fraction magnitudes can be presented in common form (e.g., 1/2; fractions) or using decimal notation (e.g., 0.5; decimal fractions). In both the United States and United Kingdom fifth graders' fraction competence predicted their later algebra success and overall mathematics achievement in high school (Siegler et al., 2012). Furthermore, sixth graders' fraction knowledge predicted gains in mathematical achievement from sixth grade to seventh grade (Bailey, Hoard, Nugent, & Geary, 2012). However, many students struggle to understand fraction magnitudes despite receiving significant amounts of instruction starting in Grades 3 or 4. Results from the 2008 National Assessment of Educational Progress found that only 50% of eighth graders correctly ordered the magnitudes of three fractions (Martin, Strutchens, & Elliott, 2007). Even adults show limited fraction knowledge. U.S. community college students correctly answered only 70% of fraction magnitude comparison problems, where chance was 50% correct (Schneider & Siegler, 2010). Difficulties understanding fractions also exist for decimal fractions. Over half of fifth graders consistently chose 0.274 as being larger than 0.83 (Rittle-Johnson, Siegler, & Alibali, 2001), and less than 30% of eleventh graders were able to express the magnitude of 0.029 as a fraction (Kloosterman, 2010).

We examined the influence of naming decimals using formal, decomposed place value labels compared to informal, everyday decimal labels or no labels on children's problem-solving performance. We hypothesized that formal labels would facilitate problem-solving performance by revealing place value structure and reducing whole number errors. Additionally, we predicted that the advantages of informal labels would be counteracted by the activation of whole number misconceptions that interfere with problem solving. Therefore, we predicted children in the informal labels condition would perform similarly to the no labels condition.

CHAPTER II

EXPERIMENT

Method

Participants

Consent was obtained from 122 third- and fourth-grade students from three suburban parochial schools and one public school aftercare program. A pretest was given to identify children who did not already demonstrate a high level of decimal magnitude knowledge so that differences between conditions could be detected. Children scoring below 75% on this pretest were selected for the study and randomly assigned to experimental condition. Thirteen children did not meet the pretest-in criteria. Data from four children were excluded because they had diagnosed learning disabilities and one child who was a pilot participant. The final sample (N = 104; 56% female) consisted of 63 third-graders and 41 fourth-graders. The average age was 9.6 years (range 7.2-11.2). Approximately 26% of participants were ethnic minorities (13% African-American, 7% Asian, 6% Hispanic). These demographic characteristics did not differ significantly by condition. There were grade differences for time spent covering content on decimals and in exposure to formal place value labels. Fourth-grade teachers reported spending more time an average covering decimal content in class than third-grade teachers (a few weeks vs. zero to a few days). Fourth-grade teachers also reported that more of their students on average were familiar with naming decimals using place value labels compared to third-grade teachers (most students vs. less than a few students).

Design

Children completed a brief pretest and participated in a single individual session. Children were randomly assigned to one of three conditions: formal labels (*n* = 35; *n* = 21 third-graders, *n* = 14 fourth-graders), informal labels (*n* = 34; *n* = 21 third-graders, *n* = 13 fourth-graders), or control (*n* = 35; *n* = 21 third-graders, *n* = 14 fourth-graders). The only difference between conditions was the labels the experimenter and children used to name decimals during the decimal game and number line task.

Materials

Pretest

An abbreviated version of a validated assessment measured children's correct decimal magnitude knowledge as well as common misconceptions (Durkin & Rittle-Johnson, 2015). The assessment included items from four subscales: magnitude comparisons (n = 7), role of zero (n = 2), density (n = 3), and number line items (n = 3). Two measures of decimal misconceptions were also included. The first measured the proportion of responses that corresponded to either whole number, role of zero, or fraction misconceptions. The second measured how students generally thought of decimal magnitude problems using two versions of a "hidden decimal task" (Resnick et al., 1989). The original version of the item asked students to identify whether 0.X or 0.XXXX was greater (the Xs represent numbers covered by pieces of paper) or if it was impossible to know. A new version of the item was developed that provided the value of the tenths place (0.8 or 0.2XX) to reveal how competing strategies (length of digits versus comparing tenths) influence responses.

Decimal comparison game

The decimal labels manipulation occurred while children played a decimal magnitude comparison game (e.g., which decimal is greater?). The game had the same rules as the familiar card game War. Children played the game with the experimenter using a deck of decimal cards, and the player with the greater decimal won each round. During game play, children compared the magnitudes of 40 pairs of decimals. Numbers in each pair ranged from 0 to 1 and had either one or two digits. Pairs were designed to reveal different levels of understanding based on previous research that has identified common errors children make when comparing decimal magnitudes (Desmet, Grégoire, & Mussolin, 2010; Durkin & Rittle-Johnson, 2015; Resnick et al., 1998). Comparisons fell into three different categories and focused on errors that arise from treating decimals like whole numbers (see Table 2 for examples of each comparison type).

Congruent and incongruent comparisons

The first category included 17 pairs that required children to make either congruent or incongruent comparisons. Congruent comparisons can be solved correctly by comparing decimals as whole numbers (e.g., 0.68 and 0.2; $n = 7$), whereas incongruent comparisons cannot be solved correctly using whole number rules (e.g., 0.51 and 0.8; $n = 10$).

Role of zero comparisons

The second category included 18 pairs with one decimal that had a zero in either the tenths or hundredths place. Common errors related to whole number errors involve misunderstanding the role of zero. For example, children often ignore a leading zero (e.g., 0.04 is the same amount as 0.4) and think a trailing zero changes a decimal's magnitude (e.g., 0.40 is greater than 0.4). Eleven of these pairs had identical non-zero digits (e.g., 0.40 and 0.4 or 0.09 and 0.9). The remaining 7 pairs had different non-zero digits and a zero in the tenths place only (e.g., 0.07 and 0.1 or 0.8 and 0.02), so competing strategies of either comparing the digit values or comparing the length of the decimals could be used.

Benchmark comparisons

The third category included 5 pairs in which children compared a decimal to a familiar 0 or 1 benchmark. Pairs were administered in a random order, and all children compared the pairs in the same order. Proportion correct across all comparison problems and by comparison type was calculated for each child.

Children read aloud the decimal labels printed on the cards before choosing the greater decimal (see Figure 1 for example materials for each condition). The printed labels were removed halfway through game play to give children an opportunity to practice generating the decimal labels on their own with feedback from the experimenter. Children received a score for the proportion of target labels they correctly generated on their first try.

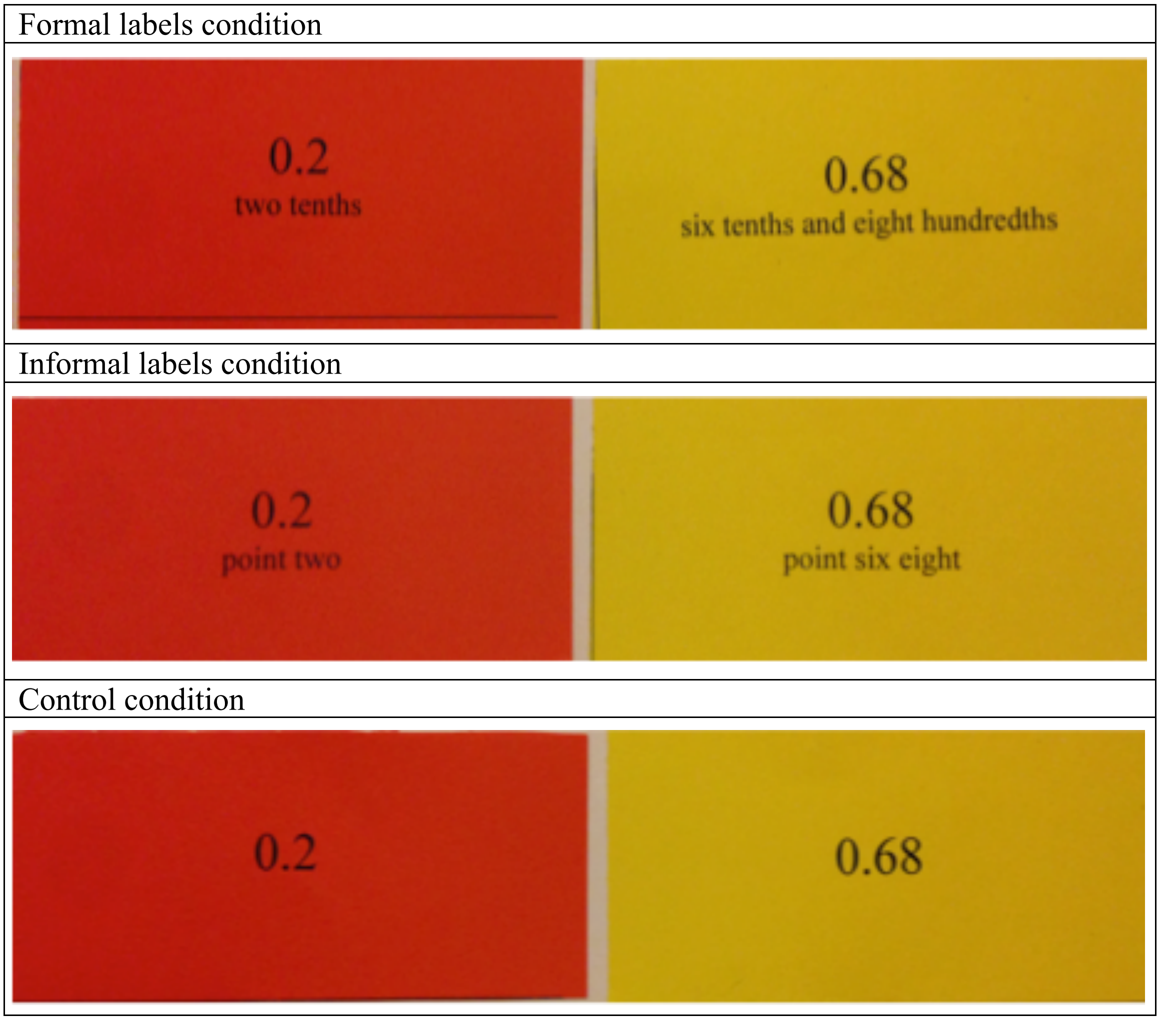

Figure 1: Sample Decimal Cards from the Decimal Comparison Game by Condition

Decimal number line estimation

To measure magnitude knowledge, a 0-1 decimal number line task was created (18 trials). Children were instructed to name each decimal according to their assigned label condition before placing the decimal on the number line. The decimals were taken from previous work that used pencil-and-paper number line tasks and included decimals with one or two digits (Rittle-Johnson, Siegler, & Alibali, 2001). The decimals used were

0.2, 0.09, 0.40, 0.87, 0.07, 0.9, 0.10, 0.63, 0.16, 0.6, 0.02, 0.5, 0.80, 0.14, 0.08, 0.3, 0.46, and 0.70.

Percent absolute errors (PAE) were calculated for number line estimations according to the formula: (|Child's Answer – Correct Answer|)/Numerical Range (Siegler & Thompson, 2014). For example, if a child was locating 0.6 on a 0-1 number line and marked the location corresponding to 0.67, PAE would be 7%, as calculated by (|0.67 – 0.60|)/1. More accurate estimates reflect *lower* PAEs. Each child received an average PAE score across all 18 trials. Pilot work suggested that some children solve this task by treating the 0 to 1 number line as a 0 to 100 number line and placing decimals accordingly. This is a useful strategy for locating hundredths but leads to inaccurate estimations for tenths. Therefore, we calculated an average PAE score for each child separately for hundredths trials (e.g., 0.46; n = 5) and for tenths trials (e.g., 0.2; n = 5). An average PAE score was also calculated for the remaining 8 trials that included decimals with a zero in the tenths or hundredths place (e.g., 0.40 and 0.09) because children often experience confusion about the role of zero.

Decimal labels screening and manipulation check

At the beginning and end of each session, children were asked to name decimals with tenths and/or hundredths place values (i.e., 0.6, 0.07, and 0.53). The screening measure was used to identify individual differences in children's prior knowledge of decimal labels, as well as to provide a manipulation check that children were able to generate decimal labels learned during the labels manipulation after a brief delay.

Decimal comparison transfer

Decimal comparison transfer items included decimal comparison problems that children were not exposed to during the decimal labels manipulation. Items were adapted from previous work measuring children's decimal magnitude knowledge (Durkin & Rittle-Johnson, 2015; Rittle-Johnson, Siegler, & Alibali, 2001). Children completed 10 decimal magnitude comparison problems with decimals that included digits in either the thousandths or ones places. For these comparison problems children chose the decimal that was greater or decided if they were the same amount. Half of the problems were role of zero comparisons in which one of the decimals in a pair had a zero in the tenths, hundredths, or thousandths place (e.g., 0.9 and 0.901, 1.09 and 1.9, or 3.3 and 3.300).

Place value identification

Two items assessed children's place value knowledge as used in Rittle-Johnson, Siegler, & Alibali (2001). These items were administered towards the end of the experimental session after the labels manipulation had occurred to determine if using formal decomposed place value labels helped children understand place value concepts. One item presented the number 413.728 and asked how much the 2 was worth from a list of 5 choices: 0.2, 2 tenths, 2 hundredths, 2 tens, or 2 hundreds. The second item asked how many tenths were in 30 hundredths.

Additional measures given, but not used

Several additional items were included for exploratory purposes. All items were administered after the labels manipulation had occurred. Due to poor performance and difficulty in interpreting students' responses and few condition differences, results are not reported for these measures.

Transfer number line

Transfer number line items included 3 multiple-choice problems. For these problems, one decimal was marked on a 0-10 number line and children identified the location of an unmarked decimal from a list of 4 choices. Overall, children answered approximately 1 of the 3 items correctly (M = 31% correct, SD = 24%).

Misconceptions

The two versions of a "hidden decimal task" (Resnick et al., 1989) from the pretest were used to measure misconceptions about how children generally think about decimal magnitude comparison problems. Only 13% of children correctly answered the version of the task with all digits hidden, with no differences between conditions. Instead, 67% of children chose the decimal with more digits as being larger. Performance was better on the other item with the tenths digit shown but is difficult to interpret (Loehr & Rittle-Johnson, 2015). More children in the formal labels condition solved this item correctly (43%) compared to children in the informal labels condition (29%) and no labels condition (14%), χ^2 (2, N = 104) = 6.97, p = .03.

Density

The concept of decimal density was also measured to determine if providing labels influenced children to treat decimals discretely as opposed to understanding the continuous nature of their magnitudes. For these problems, children were asked to say a number that comes between two decimals (e.g., 0.4 and 0.5; n = 4; Durkin & Rittle-Johnson, 2015). Performance was low on these items (M = 33%, SD = 30%).

Encoding

Four items assessed children's encoding of problem structures by asking children to reproduce decimal comparison problems from memory (e.g., 0.08 ☐ 0.62 or 0.56 ☐ 240.0). Children received 1 point for correctly reproducing each problem without any errors. Children correctly reproduced about half of the problems (*M* = 56%, *SD* = 27%).

Procedure

Children completed the written pretest in their classrooms in one 15-minute session. The one-on-one experimental session occurred at least one day after the pretest and lasted approximately 40 minutes. All experimental tasks were presented on a laptop computer with the exception of the decimal game. All children completed the tasks in the same, fixed order. Table 1 presents tasks in the order of presentation and identifies the tasks during which the labels manipulation occurred. The individual sessions were video and audio taped.

Table 1: Task Order Administration and Condition Manipulation Information

Order	Task	Labels Manipulation
1	Decimal labels screening	No
2	Comparison game	Yes
3	Number line estimation	Yes
4	Density	No
5	Place Value	No
6	Memory	No
7	Comparison transfer	No
8	Number line transfer	No
9	Hidden decimal	No
10	Decimal labels manipulation check	No

Data Analysis

To examine children's performance on the primary outcome measures (performance on decimal comparison game, decimal number line estimation accuracy, and decimal comparison transfer), a series of ANCOVAs with condition as a between-subject variable were performed. Specifically, condition was dummy coded with formal labels and informal labels entered into the models, and no labels as the reference group. In all models, children's age, grade, and their score on the pretest were included as covariates. Preliminary analyses revealed no interactions with age, grade, or pretest scores so these interaction terms were not retained in the final models.

Results

Pretest

At pretest, children who were included in the study answered a minority of problems correctly (M = 32% correct, SD = 14%). In addition, fourth-graders exhibited greater decimal magnitude knowledge than third-graders, $F(1, 102) = 31.9$, $p < .01$, $\eta p^2 = .24$), answering about two more items correctly on average (M = 41% correct, SD = 16% vs. M = 26% correct, SD = 10%). As expected, there were no differences in knowledge by condition at pretest, $F < 1$, ns.

Decimal Labels Screening and Manipulation Check

None of the children generated formal, decomposed place value labels prior to the naming manipulation. As expected, the majority of children (77%) provided informal labels on at least one of the three decimals they were asked to name. At the end of the session, all children in both label conditions provided the respective labels they were exposed to during the labels manipulation.

Decimal Comparison Game Performance

There were no significant effects between any of the label type conditions for children's overall performance on decimal magnitude comparison problems (see Table 2), *F's* < 2.5. Children in all three conditions performed significantly above chance (33%), $t(103) = 7.05$, $p < .001$, but condition differences were not reliable. Comparison types were designed with common errors in mind, and some comparisons could be solved correctly using whole number strategies. Thus, we compared performance on the three comparison types. Table 2 presents means and standard deviations by condition for each comparison type.

Table 2: Summary of Performance on Main Measures by Condition

		Formal Labels		Informal Labels		No Labels	
Task	Examples	*M*	*SD*	*M*	*SD*	*M*	*SD*
Proportion of correct responses							
Comparison game		.53	(.25)	.44	(.18)	.47	(.20)
Benchmark (n = 5)	0 vs. 0.1 1 vs. 0.45	.74	(.28)	.71	(.27)	.71	(.28)
Congruent (n = 7)	0.68 vs. 0.2	.91	(.26)	.96	(.18)	.94	(.21)
Incongruent (n = 10)	0.51 vs. 0.8	.31	(.43)	.14	(.31)	.16	(.35)
Role of zero same digits (n = 11)	0.09 vs. 0.9 0.40 vs. 0.4	.43	(.38)	.20	(.34)	.24	(.39)
Role of zero different digits (n = 7)	0.07 vs. 0.1 0.03 vs. 0.4	.45	(.38)	.53	(.28)	.62	(.21)
RZ congruent (n=4)	0.03 vs. 0.4	.55	(.42)	.79	(.37)	.91	(.38)
RZ incongruent (n=3)	0.07 vs. 0.1	.31	(.41)	.18	(.34)	.24	(.38)
Percent absolute error[a]							
Number line estimation		.20	(.12)	.18	(.06)	.16	(.04)
Tenths	0.6	.30	(.16)	.36	(.13)	.35	(.13)
Hundredths	0.46	.13	(.10)	.10	(.08)	.09	(.05)
Role of zero	0.40, 0.09	.18	(.17)	.11	(.11)	.08	(.05)
Proportion of correct responses							
Comparison transfer		.48	(.16)	.39	(.13)	.43	(.12)
Congruent and incongruent	0.453 vs. 0.21 0.86 vs. 0.827	.42	(.19)	.39	(.13)	.38	(.12)
Role of zero	0.9 vs. 0.901 0.37 vs. 0.072	.54	(.18)	.39	(.18)	.47	(.21)

[a]Lower percent absolute errors reflect more accurate estimates

Benchmark comparisons

Children's percent correct on benchmark comparisons was similar in the formal labels condition (*M* = 74%, *SD* = 28%), informal labels condition (*M* = 71%, *SD* = 27%), and no labels condition, (*M* = 71%, *SD* = 28%). There were no significant condition effects, *F's* < 1. Across conditions children were more successful at comparing a decimal to a 0 benchmark (n = 2) than to a benchmark of 1 (n = 3; *M* = 98%, *SD* = 13% vs. *M* = 55%, *SD* = 46%, respectively).

Congruent and incongruent comparisons

As expected, children across all three conditions were successful on congruent comparison problems that could be solved correctly using a whole number strategy (*M* = 94% correct, *SD* = 22%). There were no significant effects of label type, *F's* < 1. However, for incongruent comparisons in which a whole number strategy produced an incorrect answer (e.g., 0.51 and 0.8), children's percent correct with formal labels was highest (*M* = 31% correct, *SD* = 43%), lower with no labels (*M* = 16% correct, *SD* = 35%), and lowest with informal labels (*M* = 14% correct, *SD* = 31%). There was a marginal effect of formal labels relative to no labels, $F(1, 98) = 2.89$, $p = .09$, $\eta_p^2 = .03$. There was no significant effect of informal labels relative to no labels, $p = .91$. A follow-up analysis revealed a marginal effect of formal labels relative to informal labels, $F(1, 98) = 3.24$, $p = .08$, $\eta_p^2 = .03$. Children in the formal labels condition performed at chance (33%), $t(34) = -.26$, $p = .80$, but children in the informal labels and no labels conditions performed significantly below chance, $t(33) = -3.56$, $p = .001$ and $t(34) = -2.91$, $p = .006$, respectively. These results suggest children in the informal and no labels conditions were more likely to choose the decimal with greater digit values using whole number rules.

Role of zero comparisons

Children's percent correct on role of zero comparisons with identical non-zero digit values (e.g., 0.40 and 0.4 or 0.09 and 0.9) was highest with formal labels (*M* = 43%, *SD* = 38%), lower with no labels (*M* = 24%, *SD* = 39%), and lowest with informal labels

(M = 20%, SD = 34%). There was a significant effect of formal labels relative to no labels, $F(1, 98) = 4.94$, $p = .03$, $\eta_p^2 = .05$. There was no significant effect of informal labels relative to no labels, $p = .72$. A follow-up analysis revealed a significant effect of formal labels relative to informal labels, $F(1, 98) = 6.55$, $p = .01$, $\eta_p^2 = .06$.

Children's correct percent on role of zero comparisons with different non-zero digits (e.g., 0.07 and 0.1) was highest with no labels (M = 62%, SD = 21%), lower with informal labels (M =53%, SD = 28%), and lowest with formal labels (M = 45%, SD = 38%). There was a significant, negative effect of formal labels relative to no labels, $F(1, 98) = 7.50$, $p = .01$, $\eta_p^2 = .07$. There was no significant effect of informal labels relative to no labels, $p = .21$. A follow-up analysis revealed no significant effect between the two label types, $p = .15$. To understand the negative effect of formal labels relative to no labels, we examined performance on problems where the correct answer can be achieved using whole number rules. On 4 of these 7 problems, ignoring a zero and choosing the decimal with the greater digit will result in the correct answer. There was a significant, negative effect of formal labels relative to no labels, $F(1, 98) = 17.64$, $p < .01$, $\eta_p^2 = .15$. There was no significant effect of informal labels relative to no labels, $p = .20$. A follow-up analysis revealed a significant, negative effect of formal labels relative to informal labels, $F(1, 98) = 8.25$, $p = .01$, $\eta_p^2 = .08$. Thus, children in the informal- and no-labels conditions seemed to be utilizing this strategy, whereas children in the formal labels condition did not ignore zeroes but seemed to be experiencing confusion about the values of the tenths and hundredths places.

Decimal comparison game performance summary

Children in all three conditions were successful at benchmark and congruent comparisons. Performance on more difficult incongruent and role of zero comparisons revealed some positive but mixed effects of providing formal labels. Performance on incongruent comparison problems suggested that formal labels facilitated performance somewhat, in part by discouraging children from systematically using a whole number strategy. Formal labels also led to higher performance on role of zero comparisons, but only when there was no competing digit value information (i.e., only for problems that had identical non-zero digits). For role of zero comparisons that had different non-zero digits, formal labels harmed performance by discouraging children from using whole number strategies that led to the correct answer.

Number Line Estimation

For children's overall average PAE there was an unexpected significant, negative effect of formal labels relative to no labels, $F(1, 98) = 5.22$, $p = .03$, $\eta_p^2 = .05$. There was no significant effect of informal labels relative to no labels, $p = .31$, and no significant effect between the two label types, $p = .21$. Because we suspected children were likely to use a 0-100 number line strategy (i.e., treating the 0-1 number line as a 0-100 number line and locating decimals accordingly), we examined performance on trials in which this strategy would lead to accurate vs. inaccurate estimations. Table 2 presents children's average PAE across trials and for each trial type by condition.

Tenths trials

There were no significant effects between any of the three label type conditions for children's average PAE on the most difficult tenths trials, *p's* > .10.

Hundredths trials

There was a marginal, negative effect of formal labels relative to no labels for hundredths trials, $F(1, 98) = 3.45$, $p = .07$, $\eta_p^2 = .03$. There was no significant effect of informal labels relative to no labels, $p = .77$, and no significant effect between the two label types, $p = .13$.

Role of zero trials

For role of zero trials there was a significant, negative effect of formal labels relative to no labels, $F(1, 98) = 12.56$, $p < .01$, $\eta_p^2 = .11$. There was no significant effect of informal labels relative to no labels, $p = .22$. A follow-up analysis revealed a significant, negative effect of formal labels relative to informal labels, $F(1, 98) = 5.16$, $p = .03$, $\eta_p^2 = .05$.

Number line estimation summary

In general, formal labels impeded children's ability to accurately estimate the location of decimals on a 0-1 number line. This negative effect of formal labels was strongest for role of zero trials and was not present for the most difficult tenths trials.

Transfer

Children in all three conditions performed significantly above chance on transfer magnitude comparison problems (33%), $t(103) = 7.43$, $p < .001$ (see Table 2). There was no significant effect of either label type relative to no labels, *p's* > .14. A follow-up analysis revealed a significant effect of formal labels relative to informal labels, $F(1, 98) = 5.16$, $p = .03$, $\eta_p^2 = .05$.

We also examined performance on congruent and incongruent comparison problems and role of zero comparison problems. For congruent and incongruent comparisons, there were no effects between any conditions, *p's* < .31. For role of zero comparisons, there was no significant effect of formal labels relative to no labels, $p = .23$. There was a marginal, negative effect of informal labels relative to no labels, $F(1, 98) = 3.62$, $p = .06$, $\eta_p^2 = .04$. A follow-up analysis revealed a significant effect of formal labels relative to informal labels, $F(1, 98) = 9.60$, $p < .01$, $\eta_p^2 = .09$.

Place Value

We used two items to assess children's place value knowledge to determine if using formal decomposed place value labels helped children understand place value concepts. The first item presented the number 413.728 and asked how much the 2 was worth from a list of 5 choices: 0.2, 2 tenths, 2 hundredths, 2 tens, or 2 hundreds. The second item asked how many tenths were in 30 hundredths.

Despite exposure to formal place value labels, only a quarter of the children in the formal labels condition were able to use the learned labels to correctly identify the hundredths place value (26%). A similar percentage of children in the informal labels condition (18%) and the no labels condition (6%) were successful on this item, χ^2 (2, $N = 104$) = 5.18, $p = .08$. A distractor analysis showed that children applied place value rules for whole numbers to place values less than one. Using this strategy would lead to identifying the tenths place as ones and the hundredths place as tens (or tenths). Indeed, 57% of children incorrectly chose tenths and 24% of children chose tens as the correct answer. More children in the informal labels condition (12 of 34 [35%]) and no labels conditions (10 of 35 [29%]) incorrectly chose tens compared to 3 of 35 children (1%) in the formal labels condition, χ^2 (2, N = 104) = 7.34, p = .03. Importantly, children in the formal labels condition were able to differentiate tens from tenths. We also included hundreds as a distractor to determine if children would distinguish hundredths from hundreds, but only 1 child in each condition chose this option.

Children were much more successful at determining how many tenths were in 30 hundredths. More children in the formal labels condition answered this item correctly (69%) compared to children in the informal labels condition (38%) and no labels condition (43%), χ^2 (2, N = 104) = 7.43, p = .02. Thus, while formal labels seemed to reveal place value structure, as evidenced by understanding the relationship between tenths and hundredths, children still struggled to understand place values less than one.

CHAPTER III

DISCUSSION

The language we use can act as a powerful cognitive tool. For example, providing shared labels encourages children to categorize (e.g., Gelman & Markman, 1986; Graham et al., 2004), as well as match sets of objects based on their relations instead of perceptual features (Waxman & Gelman, 1986). While shared labels have been shown to support categorization and relational thinking, less is known about their role in making inferences about the structure of mathematics problems. Several indirect pieces of evidence suggest that labels play a role in mathematics understanding (Cheseney et al., 2012; Fuson & Kwon, 1991, 1992; Fyfe et al, 2015) and fraction understanding in particular (Miura et al., 1999; Paik & Mix, 2003). The current study is the first to provide experimental evidence for the effect of providing formal versus informal or no labels for understanding decimal fractions.

We found that naming decimals using formal, decomposed place value labels had mixed effects on decimal magnitude problem solving performance. Children who learned to name decimals using formal labels (e.g., "two tenths and five hundredths") compared to informal labels (e.g., "point two five") or no labels were better able to solve decimal magnitude problems that required understanding the role of zero and pace value structure. In particular, they solved slightly more incongruent magnitude comparison problems correctly (e.g., Which decimal is greater, 0.51 or 0.8?), solved more role of zero magnitude comparison problems correctly with decimals that had identical non-zero digits (e.g., Which decimal is greater, 0.4 or 0.40? 0.09 or 0.9?), and were better able to determine the relationship between tenths and hundredths. In part, they were less influenced by whole number strategies compared to children in the informal and no labels conditions.

However, there were unexpected negative effects of formal labels compared to informal and no labels. Their performance was lower on role of zero magnitude comparison problems and number line estimation problems that required explicit understanding of tenths and hundredths place values. This decrement in performance may reflect a shift away from an incorrect strategy that sometimes yields the correct answer to more varied strategies that yield more random performance. These problems included decimals with a non-zero digit in either the tenths or hundredths place, which isolated each place value. Children in the informal labels and no labels conditions seemed to rely on whole number strategies that sometimes led to correct solutions, whereas children in the formal labels condition may have been attempting a variety of correct and incorrect strategies that led to more random performance. One possibility is that children in the formal labels condition were attending to place value but did not have a reliable understanding of these place value magnitudes and alternated between using correct and incorrect strategies.

Overall, findings from the current study suggest formal place value labels helped children understand limited aspects of decimal magnitudes, but did not result in successful problem solving in several contexts. These findings align with research showing that children who provided correct labels for decimals did not always demonstrate accurate understanding of decimal magnitudes (Mazzocco & Devlin, 2008).

Middle-school students with a wide range of math ability were asked to name decimals and complete a ranking proportions test (i.e., rank order proportions presented as decimals, common fractions, and pictures from smallest to biggest and identify equivalent ratios). Most children in Grade 6 provided correct formal place value labels for decimals, and in Grade 8, most children who passed the ranking proportions test passed the naming test in Grade 6. However, many students who failed the ranking proportion test in Grade 8, passed the naming test in Grade 6. Thus, some students who passed the naming test failed to accurately rank order proportions.

Although formal labels did not always lead to accurate decimal magnitude knowledge, formal labels seemed to impact performance by revealing place value structure. We hypothesized that shared place value labels might highlight place value structure by assigning each fractional digit a distinct label with an associated value or magnitude. The negative effects found for formal place value labels suggested children understood that each place value had an associated magnitude but were confused about magnitudes of each place value. Distinct place value labels also seemed to help children distinguish decimals from whole numbers. Indeed, children in the formal labels condition were less likely to rely on whole numbers strategies for incongruent comparison problems. These findings align with past work showing that shared labels may help reveal the mathematical structure of problems (Fyfe et al., 2015).

Some features of formal place value labels may raise concern about their utility, especially for younger children. Informal review of an evidence-based mathematics curriculum found that third-grade teachers were discouraged from introducing decimals using unfamiliar tenths and hundredths place value names. The argument was that students should wait to learn the place value labels in Grade 4. However, young children learn whole number labels long before attaching a magnitude meaning to the number words or numerals (Wynn, 1997). Additionally, children as young as 3 years showed some competence in understanding multidigit number names long before receiving any kind of formal place value instruction (Mix et al., 2014). Indeed, children in the current study experienced little difficulty learning to name decimals using unfamiliar formal place value labels. All children in the formal labels condition generated a correct decomposed place value label at the end of the experimental session. However, children demonstrated little knowledge of place value concepts for place values less than one. Children in the formal labels condition were only slightly more likely to correctly identify the hundredths place compared to children in the informal and no labels conditions. Instead, children in all conditions inappropriately applied whole number place value rules to fractional place values.

It is also possible that children experienced confusion between these less familiar place value labels and highly similar, well-practiced place value labels for tens and hundreds. Results from the place value identification item demonstrated that only children in the formal labels condition differentiated place value labels to the left of the decimal point from place value labels to the right of the decimal point. Another possibility is that children were influenced by the magnitude relationship between highly similar sounding place value labels tens and hundreds. Because hundreds are bigger than tens, children may have been influenced to think hundredths are being bigger than tenths. However, performance on magnitude comparison problems did not suggest they were systematically using this incorrect strategy.

Instead of using formal place value labels, most adults label decimals such as 0.25 by saying, “point two five.” Using labels that reflect this common, everyday language may activate contextual knowledge children acquire through everyday experiences with decimal amounts. Previous research has shown that children may develop partial understandings of multidigit numbers in this way (Mix et al., 2014). In that study, children completed simple tasks that focused on mappings between spoken number names to written numerals, dots, or block representations. In contrast, the tasks used in the current study may not have been suitable for revealing the kinds of knowledge children may have about everyday decimal amounts associated with informal labels.

However, using informal labels could activate whole number misconceptions. Activating misconceptions hinders problem-solving performance (McNeil & Alibali, 2005), in part because children perseverate on using incorrect strategies (Fyfe, Rittle-Johnson & DeCaro, 2012). For example, children often have misconceptions about the meaning of the equal sign that result in common, systematic errors while solving equations with operations on both sides of the equal sign (e.g., 3 + 7 = 4 +__). Instead of understanding the equal sign as meaning “the same as,” children often interpret the symbol as meaning “get the answer” or “the total.” These misunderstandings are thought to arise based on previous exposure to arithmetic problems with an “operations = answer” format (e.g., 3 + 7 = 10). McNeil & Alibali (2005) conducted an experiment with undergraduates in which two groups of students solved equations with operations on both sides of the equal sign. One group completed several tasks designed to activate an operational meaning of the equal sign (reflecting the common misconception children experience). A control group completed filler tasks that did not focus on the equal sign.

Findings suggested activation of an operational meaning of the equal sign interfered with students' performance while solving equations with operations on both sides of the equal sign. There is limited evidence in the current study that whole number misconceptions were activated by informal labels. Children in the informal labels condition performed somewhat worse than children in the no labels condition on transfer role of zero magnitude comparisons. However, in general informal language did not seem to activate misconceptions more so than no labels.

In conclusion, findings from the current study provide some evidence for how the language teachers and students use impacts problem-solving success. A subtle change in how teachers label decimals during classroom instruction could enhance classroom discussions about place value structure that may help children learn about decimal magnitudes. We have shown that labeling decimals using decomposed place value labels helps reveal place value structure by highlighting the role of zeroes as placeholders. These results extend previous research on the role of language in mathematics learning, and more specifically the use of labels to reveal the mathematical structure of problems (Fyfe et al., 2015; Miura et al., 1999; Paik & Mix, 2003). Identifying mathematically meaningful labels may be a powerful first step in the process of impacting students' problem-solving behavior and understanding.

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APPENDIX

Measures

Pretest

For each pair, circle the decimal that is greater:

1)	0.24	0.049
2)	0.3	0.92
3)	0.561	0.17
4)	0.87	0.835
5)	0.429	0.7

Write a number that comes between:

6) 0.3 and 1.0 ______

7) 0.5 and 0.52 ______

8) 0.5 and 0.6 ______

9) Circle all the numbers that are worth the same amount as 0.04

a) 0.4
b) 0.40
c) 0.004
d) 4
e) none of the above

10) Circle the number that is **greater** than 0.36

a) 0.4
b) 0.360
c) 0.2
d) 0.279

11) Circle the number that is **less** than 0.52

a) 0.6
b) 0.5
c) 0.567
d) 1.4

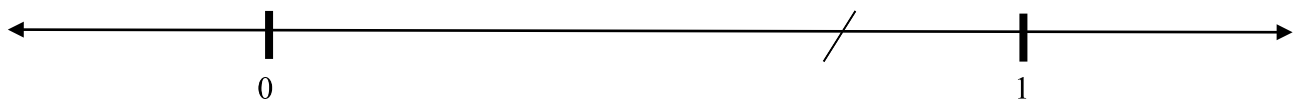
12) 0.26 is ______________ 0.260

a) greater than
b) less than
c) the same as

What number tells about where the slash is on the number line? Circle the answer.

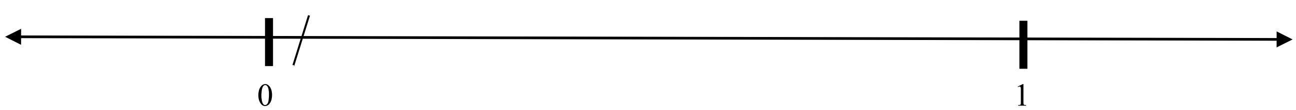
13)

a) 0.76
b) 0.3
c) 0.08
d) 0.401



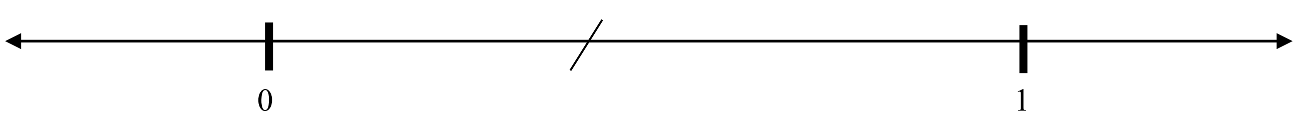
14)

a) 0.534
b) 0.5
c) 0.032
d) 0.80



15)

a) 0.189
b) 0.4
c) 0.05
d) 0.87



Please wait for directions before answering these questions.

16) Which of these numbers is greater?

a) 0.□ b) 0.□□□□ c) Can't tell

17) Which of these numbers is greater?

a) 0.8 b) 0.2□□ c) Can't tell

Decimal Comparison Game

1)	0.68	0.2
2)	0.40	0.4
3)	0	0.02
4)	0.04	0.5
5)	0.07	0.1
6)	0.80	0.8
7)	0.51	0.8
8)	0.9	0.72
9)	0.2	0.20
10)	0.35	0.6
11)	0.5	0.94
12)	0.6	0.60
13)	0.29	0.4
14)	0.7	0.03
15)	0.3	0.05
16)	0.86	1
17)	0.7	0.87
18)	0	0.1
19)	0.09	0.9
20)	0.3	0.13
21)	0.8	0.98
22)	0.10	0.1
23)	0.06	0.6
24)	0.03	0.4
25)	0.2	0.08
26)	0.61	0.6
27)	0.7	0.54
28)	0.45	1
29)	0.36	0.4
30)	0.50	0.5
31)	0.82	0.9
32)	0.90	0.9
33)	0.5	0.25
34)	0.8	0.02
35)	0.7	0.70
36)	0.79	0.2
37)	1	0.01
38)	0.3	0.22
39)	0.1	0.14
40)	0.3	0.30

Decimal Number Line Estimation

1) 0.2
2) 0.09
3) 0.40
4) 0.87
5) 0.07
6) 0.9
7) 0.10
8) 0.63
9) 0.16
10) 0.6
11) 0.02
12) 0.5
13) 0.80
14) 0.14
15) 0.08
16) 0.3
17) 0.46
18) 0.70

Density

Write a number that comes between:

1) 0.3 and 1.0 ______

2) 0.4 and 0.42 ______

3) 0.4 and 0.5 ______

4) 0.82 and 0.83 ______

Place Value

1) How much is the **2** worth in **413.728?**

a. .2
b. 2 tenths
c. 2 hundredths
d. 2 tens
e. 2 hundreds

2) How many tenths are in 30 hundredths? ________________

Memory

Practice: 70 ___ 0.45

1) 0.30 ___ 0.030

2) 55.5 ___ 0.550

3) 0.08 ___ 0.62

4) 0.56 ___ 240.0

Decimal Comparison Transfer

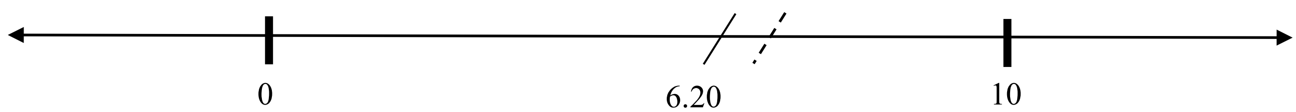
1)	0.37	0.072
2)	0.98	1.25
3)	0.453	0.21
4)	0.86	0.827
5)	2.2	0.22
6)	0.820	0.82
7)	1.09	1.9
8)	0.7	0.429
9)	3.3	3.300
10)	0.9	0.901

Decimal Number Line Transfer

This number line goes from 0 to 10
One number is already marked on the number line. Circle the number that tells about where the dashed slash is on the number line.

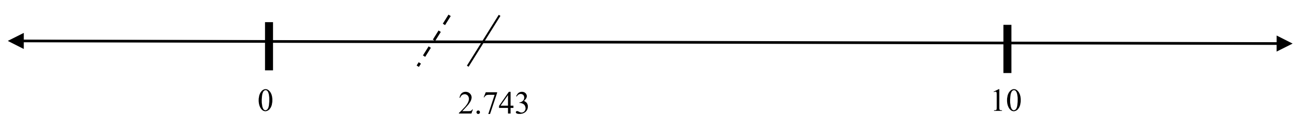
1)

a) 6.173
b) 6.8
c) 6.05
d) 0.45



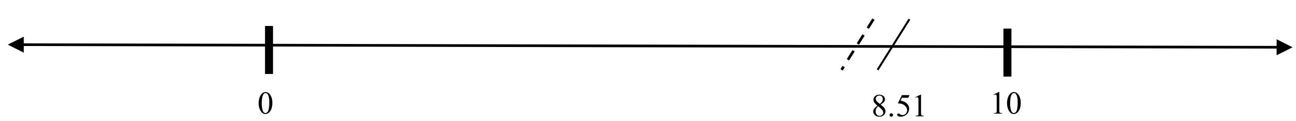
2)

a) 2.814
b) 0.2
c) 2.9
d) 2.09



3)

a) 8.147
b) 8.6
c) 0.8
d) 8.510



Misconceptions

Which of these numbers is greater?

a) 0.□□□□ b) 0.□ c) Can't tell

Which of these numbers is greater?

b) 0.2□□ b) 0.8 c) Can't tell