A COMPARISON OF BAYESIAN NETWORK STRUCTURE LEARNING ALGORITHMS ON EMERGENCY DEPARTMENT AMBULANCE DIVERSION DATA

BY

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CHAPTER I

INTRODUCTION

Modeling using Bayesian methods can be a daunting task to pursue with limited training in either machine learning or Bayesian statistics. Algorithms specifically designed to learn the Bayesian network structure for a data set can provide access to those without this training. Many Bayesian network implementations exist, but are generally aimed at computer scientists and focus on teaching how the underlying algorithm is programmed. Few studies have examined “off the shelf” implementations of Bayesian network structure learning algorithms which could allow an individual with minimal Bayesian model training to build Bayesian networks.

The study has four aims: 1) to compare different Bayesian network structure learning different algorithms on real world emergency department ambulance diversion data, 2) to compare the machine-learned Bayesian network structures to an expert-created Bayesian network, and 3) to compare how well the different Bayesian network structures generalize to predict emergency department ambulance diversion up to twelve hours in advance.

CHAPTER II

BACKGROUND

This chapter provides the purpose of evaluating Bayesian network structure learning algorithm packages by using a real world data set of predicting emergency department diversion data. The Bayesian network (BN) section gives a brief overview of BNs and a high-level explanation of how they work. The discretization section explains the purpose of discretization and describes the two discretization methods implemented in the study. The BN structure learning algorithms section describes the motivation to use automated methods for learning BNs, and gives a brief explanation of each algorithm included in the study. The last section focuses on the emergency department processes, the need to predict ambulance diversion, and previous work related to ambulance diversion prediction.

Purpose

Medical personnel who have limited computer science and machine learning education frequently use "off the shelf" products to develop BN models rather than constructing the models themselves. These packages range from implementations developed by the academic community, which are often free or in the public domain, to commercial applications, costing thousands of dollars. While individual reviews for specific software packages are available, very few studies have compared the performance of “off the shelf” BN structure learning algorithms.

Bayesian Networks

BNs are machine learning methods which use Bayes' theorem to calculate the probability that an event will occur. BNs are not a "black box" that obscures the reasoning of how the probability is calculated, unlike artificial neural networks or support vector machines. The graphical nature of BNs allows a user to see how information flows through the network and what the relationships the model represents between the variables. BNs provide methods to decompose the joint probability tables, which contain the probability of every combination of events, into a compact structure (1). BNs capture the same information as the joint probability tables by modeling the conditional relationships among the variables.

Figure 1 shows the Chest Clinic BN, a classic example to demonstrate how BNs work (2; 3). The example is a prototypical medical diagnoses system to identify whether a patient has

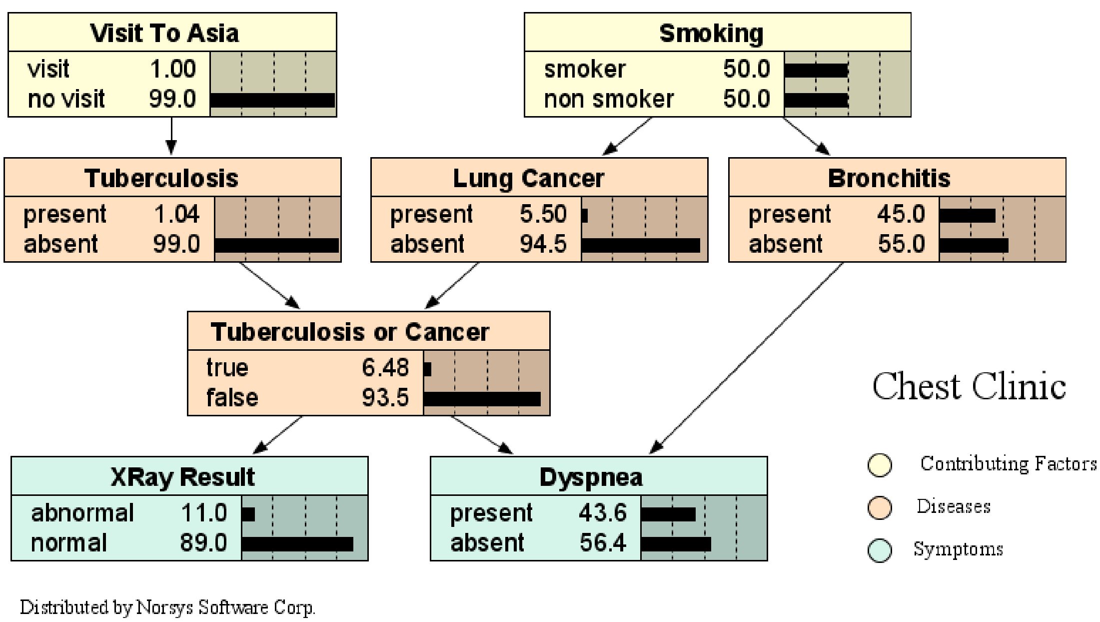

Figure 1 The Chest Clinic Bayesian Network without out any evidence provided

tuberculosis, lung cancer, or bronchitis. The arrows between the variables of the network show a hypothesized cause and effect relationship between the variables. For example, smoking can cause lung cancer or bronchitis, but does not cause tuberculosis. Likewise, visiting Asia can affect the probability of having tuberculosis, but does not have a direct effect on the probability of having lung cancer or bronchitis. The two nodes at the bottom of the network represent a diagnostic test and an observable symptom. The presence of one or more of these diseases influences both the chest x-ray test result and the patient exhibiting dyspnea (shortness of breath).

When new evidence is incorporated into a BN, the probabilities of the unobserved variables are updated. Four pieces of new knowledge could be acquired during a clinical visit for

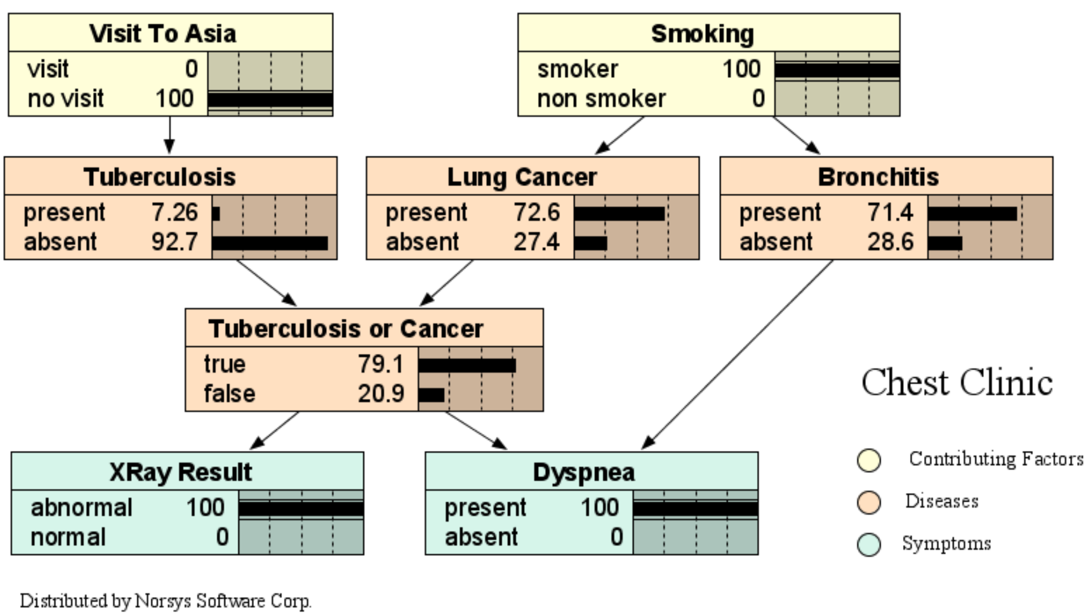

Figure 2 The Chest Clinic network provided with evidence whether the patient visited Asia, smokes, had an abnormal chest X-ray, or has shortness of breath.

the Chest Clinic BN: a chest x-ray, history of smoking, foreign travel by the patient, or the presence of dyspnea in the patient. The network in Figure 2 shows the Chest Clinic network instantiated with the previously mentioned evidence. The probability of the patient having each disease is updated based on the new knowledge.

The full joint probability table for the Chest Clinic network would contain every combination of all eight variables, which equal 256 different probabilities (2). Representing the joint probability tables as a BN reduces the number of probabilities to 36 conditional probabilities. Most real world BNs contain more variables with different numbers of possible states.

BNs use the concept of conditional independence to simplify "...both the structure of the model and the amount of computations needed to perform inference and learning ...” (4). Two variables are said to be conditionally independent when, given a variable, the two other variables do not affect each other. In probability notation,

$$P(a|b,c) = P(a|c)$$

.Variables a and b are said to be conditionally independent of each other given c. For example, the probability of having a heat stroke and a car overheating are conditionally independent given the day is hot. Having a heat stroke does not affect whether a car overheats or not, and vice versa. If the day is hot, then the probabilities increase for both heat stroke and car overheating. Figure 3 shows the graphical representation of this concept. A more detailed explanation of BNs and conditional independence can be found in (1; 4; 5). Pourret, Naïm, and Marcot provide a set of examples of how BNs have been applied to real world problems, including examples in clinical medicine (6).

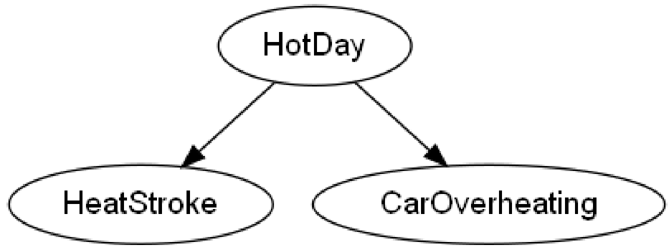

Figure 3 A DAG representation of conditional independence between heat stroke and a car overheating given it is a hot day.

Discretization

Discretization transforms continuous variables into discrete variables while retaining as much information as possible (7). Two basic methods of discretization are a) equal frequency and b) equal width.

a) Equal frequency discretization seeks to put an equal number of data points in each bin (7; 8). Figure 4 shows a data set discretized to three bins using equal frequency discretization. The algorithm sums up the number of data points in the data set and evenly distributes them among the bins. Notice the ranges of numbers in each bin are not the same, but each bin has the same number of data points. The equal frequency discretization in this instance is not affected by the lack of numbers between two and seven.

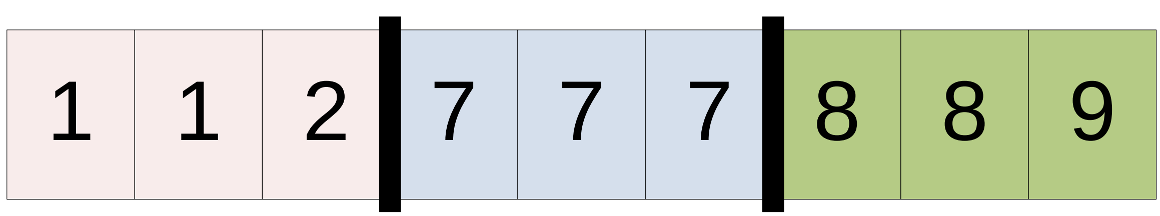

Figure 4 An example data set discretized to three bins using equal frequency discretization. The black bars separate the three bins.

b) Equal width discretization determines the bin ranges based on the range of values in the data set (7; 8). Figure 5 shows the same example data set as used with the frequency discretization example, but discretized with three bins using equal width discretization. The algorithm first calculates the width of each bin. For this example the width size equals (91+1)/3=3. The first bin contains all values less than three. The second bin would contain all values greater than three, but less than or equal to nine. In this particular data set, no values exist within the second bin's range. The final third bin contains all values greater than six, which 2/3 of the example data lies within the third bin's range. Over all, the final discretization for the example data set has two bins containing data.

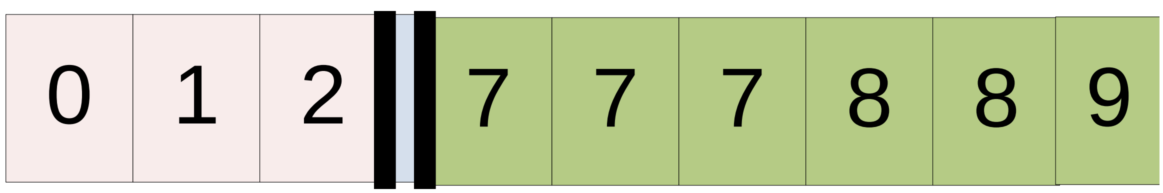

Figure 5 An example data set discretized to three bins using equal width discretization. The example only contains one separation, represented by the black bar, because the middle bin did not hold any data points from the example data set

Structure Learning Algorithms

The overall structure of a BN is a directed acyclic graph (DAG). The term DAG comes from graph theory and consists of vertices and edges. A DAG is a special form of a graph where all edges are directed and no cycles exist within the graph. Each of the vertices within the DAG is assigned a specific variable from the data set. The edges represent the relationships between the variables.

Searching over all possible DAGs, also called the search space for BN structure learning, for a set of variables to find the best structure is infeasible for all but the most trivial of problems. A DAG with 18 unlabeled has approximately 1.6 * 10^{43} (rounded to two significant figures) different possible configurations (9). This number does not account for assigning specific variables to each node. Finding the structure or the relationships between the variables, for a BN is an NP-Hard problem (4). A heuristic, or rule of thumb, is the common method to reduce the overall search space of an NP-hard problem (1; 4). The following sections provide descriptions of each structure learning algorithm included in the study and an overview of the heuristic used to search for BN structures.

Greedy Search

Greedy search uses a “greedy” heuristic to find a solution within a search space. The greedy heuristic selects the move that gives the search the most immediate gain without regard to the consequences later in the search process (10). Applying greedy search to BN structure learning, the greedy search starts with a fully disconnected BN. The greedy search then modifies the current BN by adding, removing, or reversing links while maintaining the DAG requirement of a BN (5). A comparison metric is supplied to the greedy search method to compare how well one structure performs against another one. The search continues until a specified number of moves occur without improving the metric.

Max-Min Hill Climbing Algorithm

The Max-Min Hill-Climbing (MMHC) algorithm is a hybrid method for learning Bayesian network structures combining constraint-based and search-and-score techniques (11). First, MMHC uses Max-Min Parents and Children (MMPC) to identify the parents and children of each variable in the data set. The Max-Min heuristic in MMPC seeks to maximize the minimum association between a variable and the target given the candidate parents and children. The parents and children information is used to constrain the greedy search (an edge can only be added if the parent-child link is identified by MMPC). The greedy search returns a DAG that maximizes the score of a selected evaluation metric.

Tabu Search

Tabu search is a heuristic optimization method which uses a short term memory to ensure the search explores new areas to prevent being stuck in a local optimum (12; 13). The exact implementation used in this study is a proprietary version created by BayesiaLab software creators, a commercial BN system. The tabu search starts with a BN structure without any links. The operations are the same as the greedy search described above, adding, removing, or reversing links to find the best next move. The tabu search differs from the greedy search by adding a short-term memory of the links added between moves. When a change to a link between two nodes is made, the link is stored in the short-term memory. Tabu search does not modify any links in the short term memory for a predetermined number of moves (13). The algorithm stops the search when a better network structure is not found for a set number of moves.

Augmented Naïve Bayesian Network

Naïve Bayesian networks (NB) make the assumption that all variables in the BNs are conditionally independent of each other, given the target variable (4; 5). Figure 6 shows the graphical representation of this relationship. All variables are children of the target variable. The NB structure creates a computationally efficient model requiring only the conditional probabilities for each child node given the target variable and the prior probabilities of the target node. While this assumption does not often hold for real life data sets, the NB has shown strong results as a predictor (5).

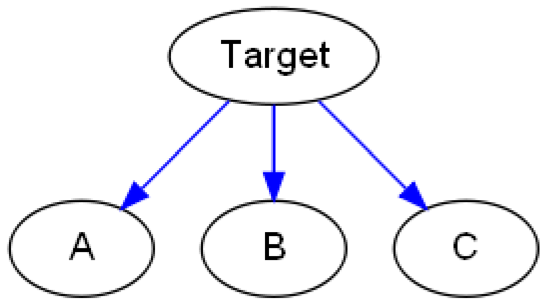

Figure 6 An example naive Bayesian network.

The augmented naïve Bayesian algorithm begins with a NB structure as shown in Figure 6, but relaxes the conditional independence assumption between the child variables. After creating the standard NB structure, the creators of BayesiaLab use a proprietary greedy search algorithm to find connections between the child nodes. Once the algorithm finishes the greedy search for relationships between the child variables the network could look as shown in Figure 7. The blue edges are the original edges from the initial NB structure. The red edges are examples of the potential edges that could be added during the greedy search (14).

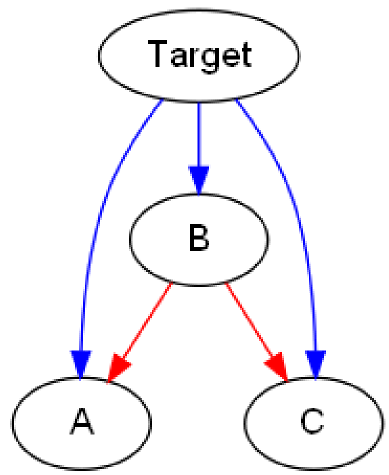

Figure 7 An example augmented Naive Bayesian network structure. The blue lines represent the links added in the naive Bayesian network phase and the red lines represent potential links added in the unsupervised search stage.

Augmented Markov Blanket

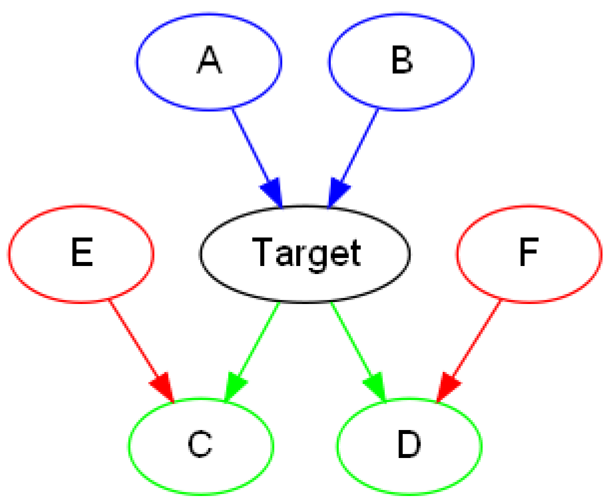

Figure 8 An example of the three relations within the Markov blanket of a variable: parent nodes(blue), child nodes (green), and parents of the children nodes (red).

The Markov blanket for a variable consists of all variables that make it conditionally independent of all other variables (1). Figure 8 shows the three types of variable relationships included in the Markov blanket: parents (blue), children (green), and children's parents (red).

Similar to the augmented naïve Bayesian algorithm described above, the augmented Markov blanket algorithm relaxes the condition that the connections only be made through the target variable, but allows connections between the parents, children, and children's parents of the target. First, the augmented Bayesian network identifies the Markov blanket for the target variables. Next, Bayesia's proprietary unsupervised greedy search identifies beneficial relationships between the other variables within the Markov blanket. Figure 9 shows the original Markov blanket edges in gray. The red and blue edges are examples of edges which could identified during the greedy search for relationships between the other variables (14).

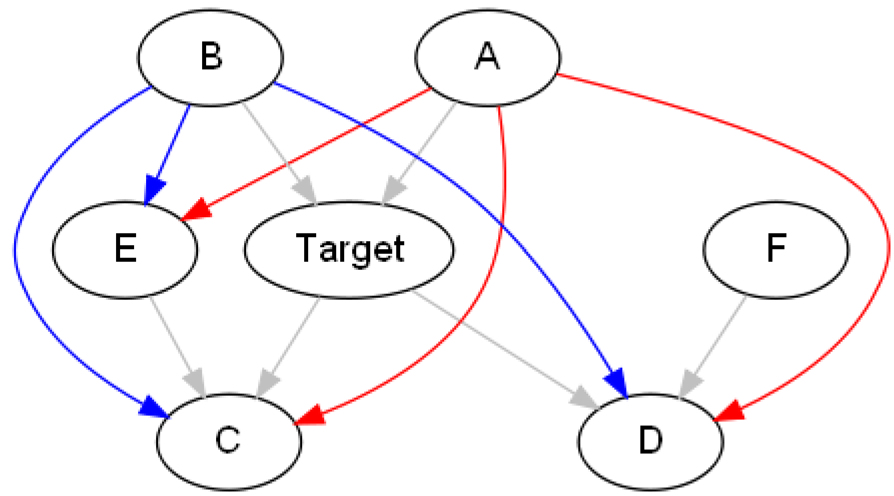

Figure 9 Example of how a Markov blanket model could be modified during the augmentation step. The solid lines indicate the original links in the Markov blanket. The dashed lines are potential relations learned during the unsupervised step

Semi-supervised Algorithm

The semi-supervised algorithm is an unpublished proprietary algorithm included in BayesiaLab. The semi-supervised algorithm applies BayesiaLab's Markov Blanket algorithm recursively (14).

Expert Learning

A common method used to develop BN structures is to employ the help of an expert in the field or use the current literature on the topic to determine the relationships between the variables (5). Some disadvantages of using only experts and literature are that only currently known relationships between the variables can be learned, the expert could bias the network, and it is a time consuming process.

Outcome Metrics

Selection of an outcome metric is an important part of the model building process. Each metric brings its own interpretation of what model is best, such as discriminatory ability or the amount of model complexity. Described below are three outcome metrics included in the study.

Area under the Receiver Operator Characteristic Curve

Receiver operator characteristic (ROC) curves are used as a statistical method to evaluate the discriminatory ability of a binary classifier. ROC curves have been used in machine learning, medicine (15), and biomedical informatics (16) as a method to evaluate classifiers. ROC curves plot the true positive rate of a classifier against the false positive rate to evaluate the classifier at different thresholds (15). Metz explains, "...ROC curves provide the most comprehensive description, because they indicate of all of the combinations of sensitivity and specificity that a diagnostic test is able to provide as the test's 'decision criterion' is varied (17)."

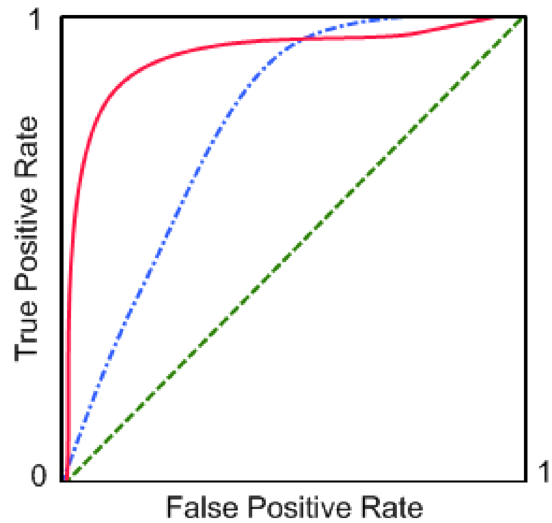

Figure 10 An example of three types of ROC curves. The diagonal green line show an ROC curve equivalent to random guessing. The red ROC curve has a large AUC, but the blue curve performs for situation where false positives are high.

Figure 10 shows three ROC curves. The closer to the upper left hand corner of the graph that the curve lies, the better it perform. If the ROC curve is a diagonal line, like the diagonal green ROC curve in Figure 10, the classifier performs equivalent to random guessing. If the ROC curve goes below the diagonal line then the classifier is guessing the opposite of the correct answer, which can be easily corrected (15).

The area under the ROC curve (AUC) compares two classifiers in a more generalized manner. The AUC is a commonly used index of ROC curve (16). Perfect AUC is equal to 1, having the curve in the far left corner; .5 is the equivalent of random guessing, or a diagonal ROC curve (15). Though the AUC of one classifier may be higher than another, the classifier with the higher AUC may not be the best performing classifier in all situations. Figure 10 displays an example of this. The red ROC curve has a higher AUC than blue ROC curve. If a high true positive rate (or sensitivity) is needed, the blue ROC curve is the better choice because it has a higher sensitivity (15). An example of when the blue ROC curve would be a better choice is an HIV test. Informing a HIV- patient they are HIV+ is a better choice than informing a HIV+ person they are HIV-. AUC does act as a simple quantifiable summary measure for ROC curve when the cost function is not known (15) .

Negative Log Likelihood

The likelihood of a model quantifies the difference between a model's hypothesized distribution and from the true distribution of the data set. The likelihood function for a model gives the likelihood the data set was created using the model's parameters (18). The likelihood is used to select the set of parameters which maximizes the likelihood of the model in relation to the data. This process is known as maximum likelihood estimation. Using the log of the likelihood function (LL) makes the calculation easier to compute since the product of multiple probabilities get subsequently smaller making it more susceptible to computational rounding errors. The logarithm transformation is a monotonically increasing function. The resulting LL is multiplied by negative one in order to make the result of the calculation positive. The equation below shows the probability for calculating the likelihood where N is the number of data points, d is the true classification of the data point, and θ are the hypothesized parameters for the model. The resulting equation is called the negative log likelihood (NLL). The NLL can only be used to compare models on the exact same data set, but is not limited to binary outcomes.

$$NLL = -log \prod_{n=1}^{N} p(d_n \mid \theta)$$

Akaike Information Criterion

Akaike information criterion (AIC) was developed in 1973 by Hirotugu Akaike as an extension of log likelihood for maximum likelihood estimation (MLE) (19). AIC penalizes a model for complexity, using the concept of Occam's razor, which can be summed up as the simplest explanation is best (20). Below is the equation for AIC where θ is the set of parameters for a model and *k* is the number of parameters. The penalty term *2k* causes the metric to favor less complex models to discourage selecting a model which does not generalize well or over fits the data set used to create the model. A criticism of information criterion which penalizes models for complexity is that they favor overly simplistic models (20). AIC, like the NLL, is not generalizable to compare models. AIC must be compared based on the same data set.

$$AIC(\theta) = 2 * NLL + 2k$$

Emergency Department Ambulance Diversion

Although time is not a critical factor for most medical care, some patients require immediate attention. In situations where the patient is having a heart attack or has been in a car accident, timeliness of care for these patients may significantly affect the potential for loss of life. To provide timely care most hospitals have an emergency department (ED) open 24 hours a day to offer care for patients in need of urgent care.

Asplin et al. developed a conceptual model that views the ED as an input-throughput-output model (21). Following is a description of input, throughput, and output processes at Vanderbilt University Medical Center's adult emergency department. A general and detailed examination of the emergency department as an input-throughput-output model can be found in (21).

Input (Arrival): Patients arrive at the ED either by ambulance, car, or foot. Patients in serious condition are taken directly to a treatment area. Patients not in a serious condition upon arrival are registered and wait to be triaged by a nurse or physician. The nurse or physician uses the criteria of the Emergency Severity Index v3 (ESI) to assign an acuity level (22). The ESI estimates the amount of ED resources consumed by the patient. Patients not in need of immediate medical attention are sent to the waiting room and added to a priority queue.

Throughput (Clinical/Treatment): Once in the clinical area patients are placed into one of nine different types of ED rooms based on the severity of their condition. The physician examines the patient to determine whether the patient needs laboratory, radiological, electrocardiogram tests, and/or a consultation.

Output (Discharge): When treatment has been completed the patient is discharged home, admitted to the hospital, or transferred to another medical care facility.

Overcrowding and Diversion

The Emergency Medical Treatment and Labor Act of 1986 (EMTALA) requires all hospitals with an ED to treat and stabilize patients without regard to their ability to pay (23). EMTALA has unfortunately created a significant financial burden for hospitals with EDs because of the financial loss incurred from patients lacking the ability to pay. The lack of compensation results in higher prices for those patients with the ability of pay, whether through insurance or privately. Between 1988 and 1998 the number of EDs in the United States decreased by 28% (23). In the same period, the number of ED encounters increased by 10%. In addition, the severity and complexity of patients has increased (24) due to the increasing age of the general population of the United States (25). Hospital administrators aim to keep hospital occupancy as close to 100% as possible and in attempt to increase revenue. A full hospital does not allow for unexpected increases in demand for inpatient beds. This results in ED patients waiting in the ED for an inpatient bed to become available (25).

The ED must divert ambulances when overcrowding reaches a critical point of a risk for patients currently in the ED or those in transit to the ED due to reduced ability to treat patients. When this critical point is reached the hospital then informs the ambulance dispatching authority the ED is going on ambulance diversion because of its inability to safely treat new incoming critically ill patients. The ambulance authority then sends ambulances to other hospitals in the area though the overcrowded ED may be the closest. If more than three hospitals in the area are in a state of ambulance diversion, then the diversion is lifted from all hospitals. Ambulance diversion leads to an increased amount of time before patients can be treated, which could increase the severity of their condition. Patients arriving by means other than ambulance continue to be treated. Once the hospital has reduced the amount of overcrowding, the hospital alerts the dispatching authority of the ability to accept new ambulance patients. Identifying the causes of overcrowding is a complex problem with many factors making diversion difficult to identify in advance. Two reviews of the problem of emergency department overcrowding can be found in Trzeziak and Rivers (25) and Hoot and Aronsky (26). Trzeziak and Rivers gives the motivation for predicting ambulance diversion in advance in order to create a system to provide EDs with a much needed early warning system to allow ED administrators to: " ...anticipate and prepare for overcrowding, rather than react to overcrowding after it has occurred (25)."

Previous work

Solberg et al. (27) gathered 74 experts to determine a generalized set of emergency department measures for planning, warning, or research. While measurements provide a standard way to evaluate emergency departments, these metrics are limited because they track activities over a period of time as explained in the limitations section of the paper.

Hoot et al. developed an ED simulation to forecast ED overcrowding 2, 4, 6, and 8 hours in advance (28). The simulation predicts seven measures related to ED overcrowding: waiting room count, average waiting time, ED occupancy level, average length of stay, number of patients waiting to be admitted, the average time a patient waits to be admitted, and the probability of ambulance diversion. Hoot et al. did a prospective study of the ForcastED simulation system with respect to the seven measures (29). ForcastED had an AUC of 0.93 predicting two hours in advance to 0.85 for predicting 8 hours in advance. One other study specifically included ED diversion as a prediction metric. Leegon et al. developed a Gaussian process for predicting ambulance diversion for up to two hours in advance. The Gaussian process had an AUC of 0.93 predicting diversion two hours in advance (30). Gaussian processes are not computationally feasible to implement in real world setting at this time.

CHAPTER III

METHODS

The following chapter explains the methodology used for evaluating the BN structure learning software packages. The setting section provides an overview of technologies implemented at Vanderbilt University Medical Center (VUMC). The data section describes the original database and feature selection process that selected the eighteen variables. A summary from the database specification for each selected variables is in the operational definition of variable. The procedures section explains the method discretization process, how the BN structures in each of the software packages were created, and the methods used for evaluation.

Setting

The adult emergency department (ED) at VUMC is a 45 bed Trauma level I, academic, urban ED. During 2008 the adult ED had more than 55,000 encounters. VUMC has an information technology infrastructure with computerized order entry (31) and a longitudinal electronic medical record system (32). In addition to the main hospital's information technologies, the ED has an advanced electronic whiteboard system (33) and computerized triage application (34); both developed internally at VUMC.

Data

A data set was extracted from a locally curated database designed for analyzing ED overcrowding. The database contained hospital operational information at five minute intervals with no missing values. All data point for a 2-year period from July 1, 2006 – June 30, 2008 were included in the study. The data set was subsequently divided into training/validation and test sets; each contained one year of data. ED ambulance diversion was selected as the reference standard for identifying overcrowding. The institutional guidelines for when the ED should go on diversion are 100% occupancy and more than 10 people in the waiting room.

The full curated database contained over 100 variables of operational information for different parts of the hospital, but focused on the ED and those areas of the hospital likely to affect the ED. The eighteen variables selected were based on a previous study. In summary, the study used logistic regression models for an initial screening of each variable in the full database. The variables were selected based on an AUC above a predefined threshold for each time point of prediction. All variables in the previous study performing above the defined AUC thresholds for identifying ambulance diversion 1, 2, 3, 4, 6, 8, and 12 hours in advance were included in the current study. The current ED diversion status was not included as a predictive variable to keep subjective variables to a minimum. The final data set used contained eighteen variables and the ED diversion status from the original database.

Operational definition of variables

An internal VUMC specification contained descriptions for each variable in the complete database. Below are summaries of the specification definition for each of the 18 variables and the target variable selected for the study.

1. VUH_ED_DIVERSION - a Boolean variable indicating the current diversion status of the ED. VUH_ED_DIVERSION is the reference standard variable for the study.
2. NO_OF_BED_REQUESTS_ED – an integer variable of the number of inpatient bed requests from the ED.
3. ED_NO_OF_WAITING_ROOM_PTS – an integer variable of the current number of patients in the waiting room.
4. ED_OCCUPANCY - a real variable ratio of number of patients in all of the ED to the number of available licensed beds. The value is calculated as:

$$\frac{ED\ No\ Of Patients}{ED\ Bed\ Hours}$$

5. ED_PTS_AVG_TIME_TO_ED_BED – a real variable calculating the average time for a patient to be placed in a ED bed. The equation below shows the method of calculation for the average time of placement into an ED bed. Where *i* is equal to patients who have been triaged and have not been placed in and ED bed.

$$\frac{\sum_i (\text{Current Time} - \text{Time Patient Was Triaged}_i)}{\text{Number Of Patients Triaged AND Not In An ED Bed}}$$

6. ED_NO_OF_CURRENT_PTS – an integer variable of the number of patients in the ED.
7. ED_NO_OF_NURSES – an integer variable of the number of nurses who worked in the ED during the 60 minutes prior to the current time.

8. NO_OF_BED_REQUESTS_TRANSFER - an integer variable of the number of admission requests from patients at other hospitals.
9. ED_VOL_HOSP_CAPACITY_ALL_BDS – a real variable calculating the ratio of ED volume to hospital capacity for all beds.

$$\frac{\mathit{The\ Number\ of\ new\ ED\ patients\ during\ the\ past\ 60\ mins}}{\mathit{Number\ of\ Available\ Hospital\ Beds\ at\ (Current\ Time - 60\ mins)}}$$

10. ED_VOL_HOSP_CAPACITY_ICU_BDS – a real value calculating the ratio of ED volume to available number of ICU Beds.

$$\frac{\mathit{The\ Number\ of\ new\ ED\ patients\ during\ the\ past\ 60\ mins}}{\mathit{Number\ OF\ ICU\ Beds\ availible\ eat\ (Current\ Time - 60\ mins)}}$$

11. ED_PTS_AVG_TIME_TO_DISCHARGE – a real variable of the average amount of time for both inpatient and outpatients to be discharged.
12. ED_INPTS_AVG_TIME_TO_DSCHRG - A real variable of the average time patients wait in the ED to be admitted to the hospital. Variable *i* in the equation below is each patient to be admitted with a bed assigned.

$$\frac{\sum_i (\text{Current Time} - \text{Bed Request Time}_i)}{\text{ED Patients To Be Admitted AND Has a Bed Assigned}}$$

13. ED_BED_ASSIGN_TIME_AVG - a real variable of the average time for a patient to be assigned a bed. Variable *i* is each patient who has a bed request made for them, but has not been assigned a bed.

$$\frac{\sum_i (\text{Current Time} - \text{Bed Request Time}_i)}{\text{ED Patients To Be Admitted AND Has Not Had A Bed Assigned}}$$

14. ED_INPTS_AVG_LOS_TIME - a real variable of the average length of stay for inpatients.
15. ED_PTS_AVG_LOS_TIME - a real variable of the average length of stay for both inpatients and outpatients in minutes.
16. NO_OF_BED_REQUESTS_PERIOP – an integer variable of the number of patients to be admitted to the hospital from the perioperative service.
17. TOTAL_NO_OF_SURGERY_PER_HR – an integer variable of the number of surgeries scheduled for the current day.
18. HOSP_AVAIL_BED_CAPACITY - an integer variable of the number of beds available in the hospital.
19. HOSP_DISCHARGE_POTENTIAL – an integer variable equal to the total number of occupied inpatient hospital beds minus the number of patients waiting to be discharged from the hospital.

Procedures

The BN structure learning algorithms were evaluated in two steps. The first step discretized the data as required for each software package; next, the discretized data were input into each of the BN structure learning algorithms.

Discretization

All implementations of the BN structure learning algorithms included in the study required the data to be discrete. Discretization was completed prior to BN structure learning to ensure the individual packages discretization methods were not the actual method being evaluated. Two unsupervised discretization algorithms were used in the study: equal distance and equal frequency. Equal distance and equal frequency discretization algorithms were implemented using a modified version of the Weka data mining libraries source code (35). The modifications were done to allow the Weka's implementation of the two discretization methods to be integrated into the testing framework used for evaluation.

A greedy search approach was used to determine the optimal number of bins for each variable. Each variable was evaluated with 2- 10 bins. Limiting the number of bins reduced the overall complexity possible for each network. A naïve Bayesian classifier implemented in the NeticaJ 4.03 API for Java (3) served as the testing model. The bin sizes and variables were compared using AUC calculated by PropROC 2.3.1 (36). The comparison was done using the data set for July 1, 2006 – June 30, 2007. Figure 11 shows how the data set was divided. The first 70% of the data set was used as training data to determine the parameters of the naïve Bayesian model. The validation data consisted of the remaining 30%. The validation data was used to evaluate each variable's performance when added to the naïve Bayesian model. The process was done for both discretization algorithms and resulted in the creation of two data sets for the study.

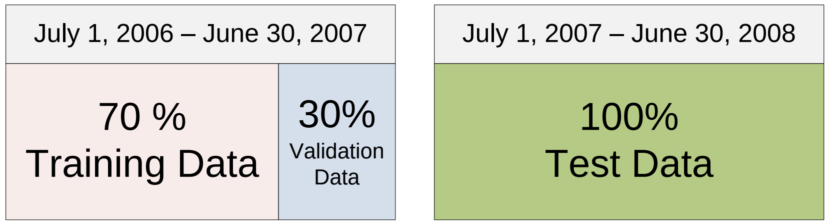

Figure 11 How the two years of data were divided into training, validation, and test data sets.

The algorithm evaluated each variable/bin combination's AUC result when the combination was the only child of the target variable (Figure 12 and Figure 13). The best performing variable/bin combination was added to the set of selected variables (Figure 14). The remaining bin sizes for the selected variable were discarded. The variable was fixed as child to the target variable. Only the top two performing bin sizes for each remaining variable were kept (Figure 15).

The remaining variable/bin combinations were each added as a child of the target node, one at a time, to the current set of selected variables (Figure 16). The best performing variable/bin combination was added to the final bin sizes (Figure 17). The unused bin size for the selected variable was then discarded (Figure 18). The process continued until all variables had a bin size chosen (Figure 19).

Once the final discretization ranges for each discretization algorithm were selected, both the training/validation and test data sets were discretized using each discretization method.

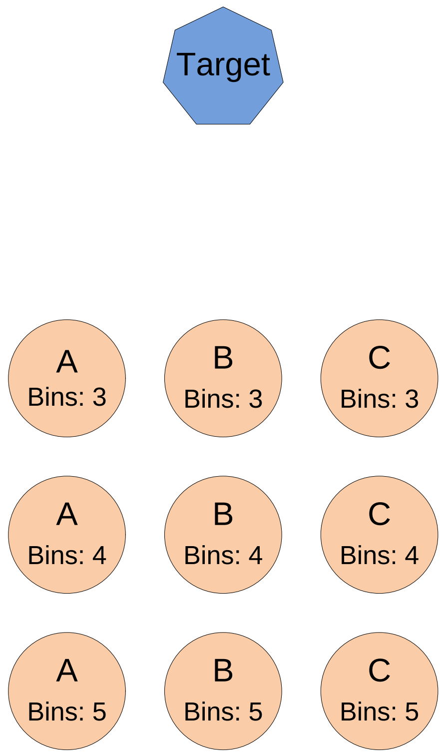

Figure 12 Target is the variable to be predicted. A, B, and C are the variables in the data set. In this example each variable have been discretized to have 3 – 5 bins.

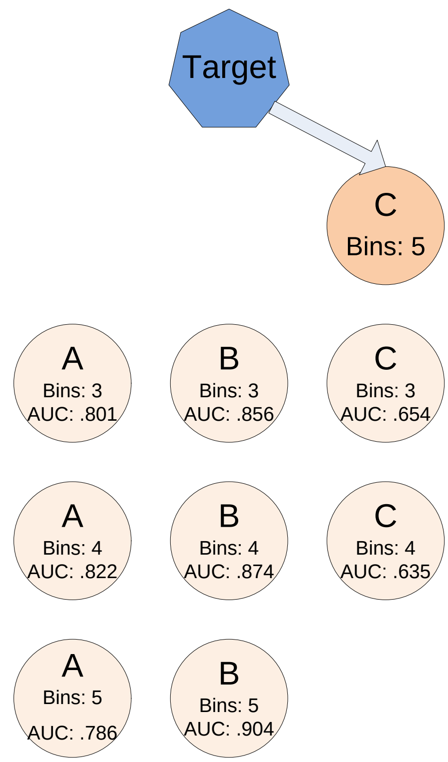

Figure 13 Each variable/bin combination were made the only child node of the target and evaluated.

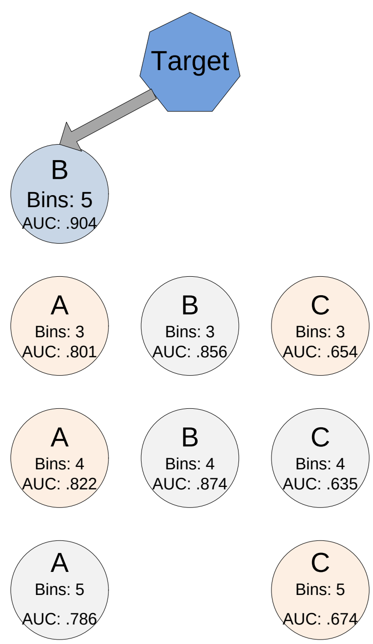

Figure 14 After all variables have been evaluated the best variable/bin combination was fixed as a child in the naïve Bayesian network.

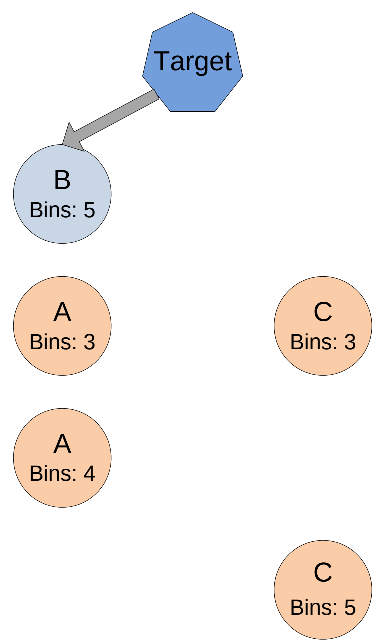

Figure 15 The remaining variables for B are removed. Only the top two performing bin sizes for A and C are kept.

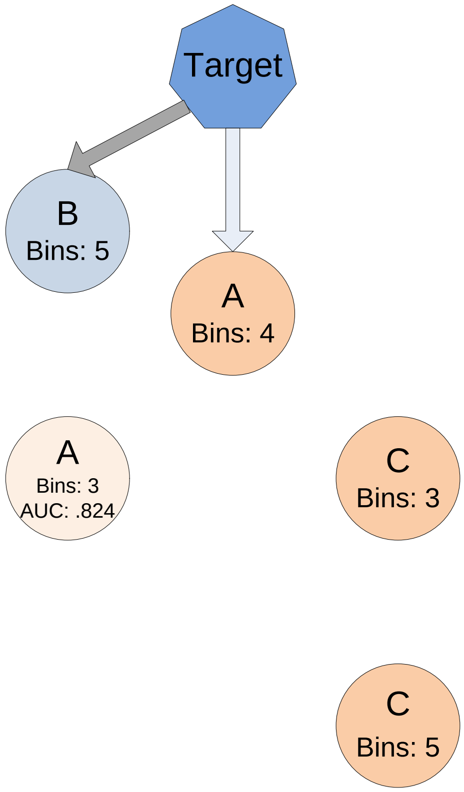

Figure 16 The remaining variable/bin size combinations were added one at a time to the current network and evaluated.

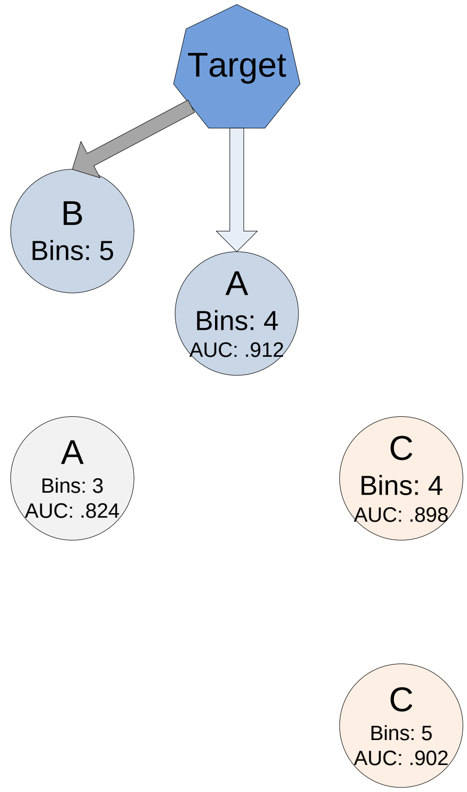

Figure 17 Once all variable/bin size combinations have been evaluated the best combination was selected and added to the network.

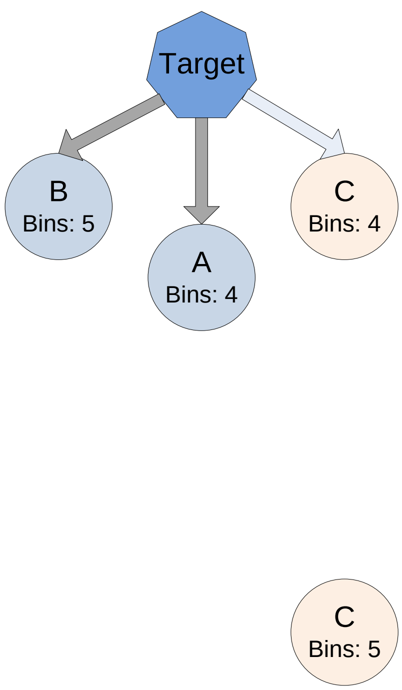

Figure 18 This process continued through the remaining variables.

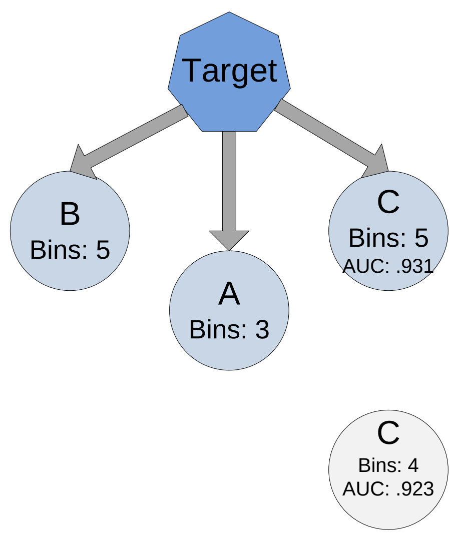

Figure 19 The algorithm terminated when all variables have had a bin size selected.

Network Creation

Table 1 Each structure learning algorithm, the package of the implementation, and the type of algorithm.

Algorithm	Software	Algorithm Type
Augmented Naïve Bayes	Bayesia ® BayesiaLab	Supervised
Augmented Markov Blanket	Bayesia ® BayesiaLab	Supervised
Greedy Search	Causal Explorer	Unsupervised
Max-Min Hill Climbing	Causal Explorer	Unsupervised
Semi-supervised	Bayesia ® BayesiaLab	Semi-supervised
Tabu Search	Bayesia ® BayesiaLab	Unsupervised

Six BN structure algorithms implemented by two Bayesian network applications, BayesiaLab and Causal Explorer, were included in the study. Bayesia® BayesiaLab 4.5.1 (37), is a commercially developed application for data mining and Bayesian network creation, and Causal Explorer 1.4 (38) is an academically developed toolbox for MatLab containing several BN structure algorithms. All Causal Explorer algorithms were executed in MatLab version 14 release 2008b (39). All included algorithms guaranteed a DAG would be created. The supervised algorithms focus on developing the network structure to predict a specific variable. The unsupervised algorithms develop a general structure based on relationships between all included variables. In addition to the machine learning methods, a previous, expert-developed Bayesian network was included for comparison. Table 1 lists the algorithms included in the study.

Each application developed Bayesian network structure was learned on the whole train/validation data set for predicting diversion one hour in advance. The default methodology for creating the Bayesian network structure was used for each algorithm. The MMHC algorithm was only supplied with a data set sampled every 15 minutes rather than 5 minutes to reduce the amount of computation time required for generating a DAG.

The expert-developed BN was previously created with the same data set for predicting ambulance diversion one hour in advance as supplied to the machine learning algorithms. All algorithms used data discretized the same way.

Evaluation

Each resulting Bayesian network structure was recreated in the BN application Norsys Netica 4.02 (3). Each BN structure was trained and evaluated on data to predict ambulance diversion at 1, 2, 4, 6, 8, and 12 hours in advance. The network parameters were learned using the Norsys® NeticaJ 4.03 API for Java from the entire training/validation set using the counting method. The counting method calculates the BN parameters by counting the number of times each conditional probability occurs in the data set. The counting method was selected over the Expectation-Maximization (EM) algorithm because the data set did not contain missing values and it takes more time to obtain the same result as counting method (40).

Outcome Measures

The AUC and its 95% confidence interval (CI), negative log likelihood (NLL), and Akaike Information Criterion (AIC) were calculated on the test data set not used during discretization or model creation. NLL and AIC were selected as additional methods of evaluation because of their use in model selection.

CHAPTER IV

RESULTS

Data

Descriptive Statistics

The study's training/validation set contained 104,975 observations and the emergency department was on ambulance diversion 23.2% of the time. The test set, which contained leap year day for 2008, contained 105,265 observations. The ED was on ambulance diversion 22.3% of the time during the test set. Table 2 statistics for each variable in train/validation and test data sets. Table 2 gives the descriptive statistics for the train/validation data sets prior to discretization.

Table 2 statistics for each variable in train/validation and test data sets.

Variable Name	Training Mean	Training Standard Deviation	Test Mean	Test Standard Deviation
NO_OF_BED_REQUESTS_ED	10.26	7.05	11.93	7.64
ED_NO_OF_WAITING_ROOM_PTS	4.55	5.50	4.39	5.31
ED_OCCUPANCY	0.80	0.19	0.82	0.22
ED_PTS_AVG_TIME_TO_ED_BED	26.84	37.56	26.51	38.55
ED_NO_OF_CURRENT_PTS	45.97	13.85	45.99	14.73
NO_OF_BED_REQUESTS_PERIOP	4.28	5.12	3.71	4.82
TOTAL_NO_OF_SURGERY_PER_HR	62.77	35.13	61.12	33.90
ED_NO_OF_NURSES	12.96	1.87	12.96	1.87
HOSP_AVAIL_BED_CAPACITY	21.77	38.78	33.09	35.84
NO_OF_BED_REQUESTS_TRANSFER	26.08	12.30	22.98	10.24
ED_VOL_HOSP_CAPACITY_ALL_BDS	0.17	0.13	0.16	0.13
ED_VOL_HOSP_CAPACITY_ICU_BDS	0.38	0.30	0.38	0.32
ED_PTS_AVG_TIME_TO_DISCHARGE	415.67	263.29	423.74	269.01
ED_INPTS_AVG_TIME_TO_DSCHRG	456.95	274.61	461.56	279.12
HOSP_DISCHARGE_POTENTIAL	0.03	0.03	0.04	0.05
ED_BED_ASSIGN_TIME_AVG	443.81	285.61	459.31	280.76
ED_INPTS_AVG_LOS_TIME	552.14	255.47	607.22	289.98

Discretization

Table 3 shows the order each equal frequency discretized variable was selected. Table 4 shows the same information as the equal frequency ordering, but for each equal width discretized variable. Appendix A contains the bin ranges for each method and the AUCs for the first pass of discretization where each variable/bin combination was made the sole child of the target variable.

Table 3 shows the order and number of bins for each variable when equal frequency discretization was used.

Order Added	Variable Name	Bin Size
1	ED_NO_OF_NURSES	2
2	ED_NO_OF_WAITING_ROOM_PTS	3
3	ED_INPTS_AVG_LOS_TIME	4
4	HOSP_AVAIL_BED_CAPACITY	4
5	TOTAL_NO_OF_SURGERY_PER_HR	8
6	ED_PTS_AVG_TIME_TO_DISCHARGE	4
7	NO_OF_BED_REQUESTS_PERIOP	3
8	NO_OF_BED_REQUESTS_TRANSFER	2
9	ED_OCCUPANCY	4
10	ED_BED_ASSIGN_TIME_AVG	5
11	ED_INPTS_AVG_TIME_TO_DSCHRG	4
12	ED_VOL_HOSP_CAPACITY_ALL_BDS	2
13	ED_PTS_AVG_LOS_TIME	2
14	NO_OF_BED_REQUESTS_ED	3
15	ED_NO_OF_CURRENT_PTS	10
16	ED_VOL_HOSP_CAPACITY_ICU_BDS	2
17	HOSP_DISCHARGE_POTENTIAL	4
18	ED_PTS_AVG_TIME_TO_ED_BED	10

Table 4 shows the order and number of bins for each variable when equal width discretization was used.

Order Added	Variable Name	Bin Size
1	ED_PTS_AVG_TIME_TO_ED_BED	5
2	ED_NO_OF_WAITING_ROOM_PTS	10
3	ED_INPTS_AVG_LOS_TIME	2
4	HOSP_AVAIL_BED_CAPACITY	3
5	TOTAL_NO_OF_SURGERY_PER_HR	3
6	ED_PTS_AVG_TIME_TO_DISCHARGE	2
7	NO_OF_BED_REQUESTS_PERIOP	2
8	NO_OF_BED_REQUESTS_TRANSFER	7
9	ED_OCCUPANCY	3
10	ED_BED_ASSIGN_TIME_AVG	2
11	ED_INPTS_AVG_TIME_TO_DSCHRG	2
12	ED_VOL_HOSP_CAPACITY_ALL_BDS	2
13	ED_PTS_AVG_LOS_TIME	4
14	NO_OF_BED_REQUESTS_ed	2
15	ED_NO_OF_CURRENT_PTS	3
16	ED_VOL_HOSP_CAPACITY_ICU_BDS	3
17	HOSP_DISCHARGE_POTENTIAL	3
18	ED_NO_OF_NURSES	3

Algorithm Comparison

Table 5 gives an overview of the best performing machine learned BN structures. Table 6gives the same overview as

Table 5, but includes the expert-developed network for comparison. Figure 20 plots the AUC for each BN structure data set for predicting diversion one hour advanced, the data set the structure was developed. gives the AUC and 95% CI for the data graphed in Figure 20. Table **7** shows the overall complexity for each network created for each algorithm, using each discretization algorithm. All algorithms included 18 variables plus the diversion status except for the augmented Markov Blanket which had 16 nodes and the expert-developed network which had 11 in addition to the ambulance diversion status. Diagrams of all Bayesian network structures created using the equal frequency discretized data set can be found in Appendix B and the network structures created using the equal width data set can be found in Appendix C. The complete tables of results for all prediction times can be found in Appendix D.

Table 5 The best performing algorithms for each of the selected metrics. The letter attached to the end of the algorithm indicates whether it was the network where equal frequency (F) discretization was used or equal width (W) discretization. Where Area Under ROC has multiple networks list, the networks 95% Confidence Intervals overlapped. The table only includes the structure learning algorithms.

Prediction Time	Area Under ROC	Negative Log Likliehood	Akaike Information Criterion
1 Hour	MMHC-F	MMHC-F	Semi-Supervised-W
2 Hours	MMHC-F	MMHC-F	Tabu-W
4 Hours	Tabu-W Augmented Markov Blanket-W	Tabu-W	Tabu-W
6 Hours	Tabu-W Augmented Naïve Bayes-W	Tabu-W	Tabu-W
8 Hours	Augmented Naïve Bayes-W Augmented Markov Blanket-W	Augmented Markov Blanket-W	Augmented Markov Blanket-W
12 Hours	Augmented Naïve Bayes-W	Semi-Supervised-F	Augmented Markov Blanket-W

Table 6 contains the best BN structures, including the expert network for each of the selected metrics. The letter attached to the end of the algorithm indicates whether the network was learned on the equal frequency (F) discretized data or equal width (W) discretized data. Where Area under the ROC has multiple networks list, the networks 95% Confidence Intervals overlapped. The table only both machine learned structures and the expert developed network

Prediction Time	Area Under ROC	Negative Log Likliehood	Akaike Information Criterion
1 Hour	MMHC-F Expert Created -F	MMHC-F MMHC-F	Expert Created -W
2 Hours	Expert Created -F MMHC-F	Expert Created-W	Expert Created -W
4 Hours	Expert Created -F	Tabu-W	Expert Created -W
6 Hours	Expert Created-F	Tabu-W	Expert Created -F
8 Hours	Expert Created -F Augmented Naïve Bayes-W Augmented Markov Blanket-W	Augmented Markov Blanket-W	Expert Created -F
12 Hours	Augmented Naïve	Semi_Supervised-F	Expert Created -F

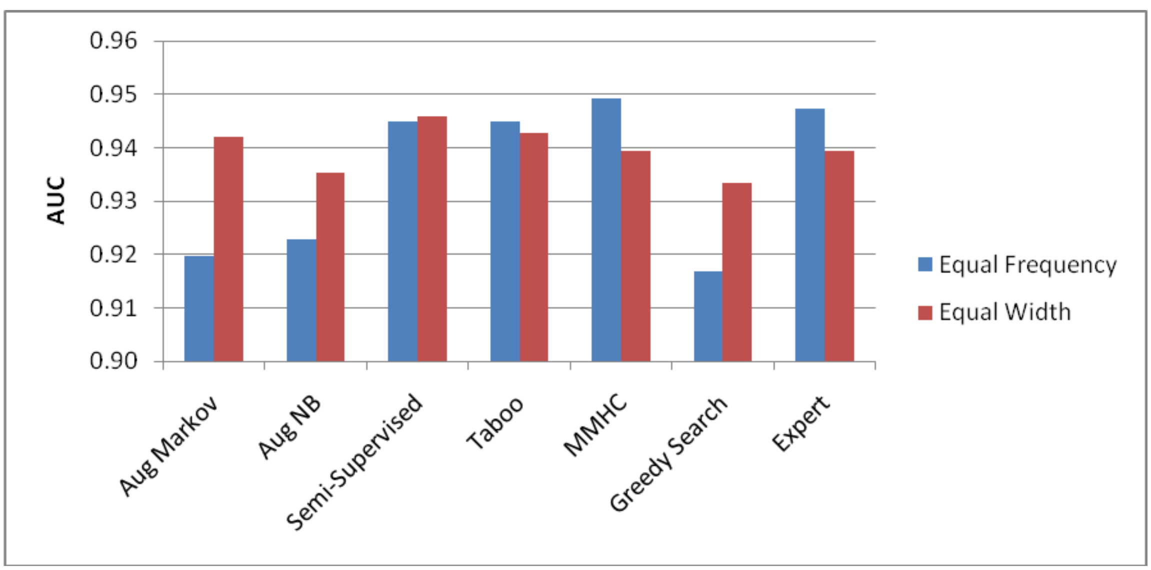

Figure 20 AUC performance of each learned BN structure on the data set for predicting diversion one hour ahead.

Table 7 AUC and 95% CI for each algorithm on the predicting diversion one hour in advance data set

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Equal Frequency Discretized			
Algorithms	**AUC**	**Confidence Interval**	
Aug Markov	0.9198	0.918	0.922
Aug NB	0.9229	0.921	0.925
Semi-Supervised	0.9449	0.943	0.946
Taboo	0.9449	0.944	0.946
MMHC	0.9491	0.948	0.950
Greedy Search	0.9168	0.915	0.919
Expert	0.9473	0.946	0.949

Equal Width Discretized			
Algorithms	**AUC**	**Confidence Interval**	
Aug Markov	0.9421	0.941	0.944
Aug NB	0.9352	0.934	0.937
Semi-Supervised	0.9460	0.945	0.947
Taboo	0.9427	0.941	0.944
MMHC	0.9395	0.938	0.941
Greedy Search	0.9334	0.932	0.935
Expert	0.9395	0.938	0.941

Table 8 The number of links and the resulting number of probability tables for each created network structure

Algorithms	Equal Frequency Discretized Networks		Equal Width Discretized Networks	
	Number of Links Between Nodes	Total Number of Conditional Probabilities	Number of Links Between Nodes	Total Number of Conditional Probabilities
Aug Markov	58	8069	53	3083
Aug NB	66	8994	64	4382
Semi-Supervised	68	13730	57	3331
Taboo	63	6853	56	3397
MMHC	53	5244	60	8430
Greedy Search	76	215966	82	32994
Expert	19	365	19	653

CHAPTER V

DISCUSSION

Six different BN structure learning algorithms were used to create two DAGs learned from a data set to predict diversion one hour in advance. Each algorithm created a DAG for both of the discretization methods used. The AUC, NLL, and AIC were calculated as an output metrics. In this chapter we discuss the quality of the data set, how the machine learned DAGs performed in relation to each other, compare the machine learned structures to the expert-developed network, and evaluate how well the DAGs generalized predicting ambulance diversion to predict up to twelve hours ahead.

The Data Set

The train/validation and test data set both contained over 100,000 data points, neither of which contained any missing values. The reference standard of ED diversion was not perfect. The institutional guidelines state the ED is to go on ambulance diversion when the occupancy is greater than or equal 100% and there are 10 or more patients in the waiting room. In the train/validation data set the ED was on diversion 80% of the time when these criteria were met and 78% of the time for the test set.

Structure Learning Algorithm Comparison

Table 6 lists the best performing algorithms for each of the selected metrics. The letter attached to the end of the algorithm indicates whether the BN structure was learned from the equal frequency (F) or equal width (W) discretization data set. When the AUC field in the table has multiple BNs, this indicates their 95% CI overlap. Except for the BN structures learned from the augmented Markov blanket algorithm, all machine learning algorithms included each of the 18 variables in the structure. Both the structures learned by the augmented Markov blanket algorithm on the equal width and equal frequency data sets excluded the ED_VOL_HOSP_CAPACITY_ALL_BDS and ED_VOL_HOSP_CAPACITY_ICU_BDS variables. These two variables not being included in the network indicates they did not lie within the identified Markov Blanket. This implies the nodes did not have a direct effect on the target variable. Their inclusion in the other graphs indicates ED_VOL_HOSP_CAPACITY_ALL_BDS and ED_VOL_HOSP_CAPACITY_ICU_BDS had some effect on at least one of the other variables within the network.

Notice in Figure 20 the best performing networks for predicting diversion 1, 2, and 4 hours in advance favored unsupervised algorithms. Since all structures were learned on the data set for predicting diversion one hour in advance, the unsupervised algorithms seem to identify the relationships between all variables in the data set. This could indicate the variables which affect diversion at 1, 2, and 4 hours in advance are closely related.

The reason the targeted algorithms performed better when predicting diversion after 4 hours may be that the relationships between the variables predicting diversion in advance change. For the two included supervised algorithms, the overall structures would not likely change. These naïve targeted structures develop a skeleton structure based on the target variable before looking for relationships between any of the other variables.

For example, the augmented naïve Bayesian structure first creates a naïve Bayesian network. This initial network structure would be the same regardless of which diversion prediction data set was used. The only relationships learned that would be dependent on the relationships between the variables at the specified amount of time in advance, but the underlying structure would be the same.

Comparison Structure Algorithms and the Expert-Created Network

The expert-developed BN was among the best performing networks for both AUC and AIC up to 8 hours in advance. Eight hour and above the relationships between the variables could have changed. The expert network used only 11 variables compared to the machine learned structures which used between 16 and 18 variables. The expert-developed network also contained fewer links than the BNs learned by the structure learning algorithms and had fewer conditional probability tables. The lower complexity alone would be a reason why the expert-developed network performed best using AIC. The expert-developed network incurred a much smaller penalty for complexity compared to the machine learned structures.

Though the expert-developed model was simpler, MMHC performed just as well on the data sets to predict diversion one and two hours in advance. If discriminative ability is the primary concern, MMHC makes an excellent choice because a human is involved in the preprocessing of the data, as would also be required to human develop a BN. The user is only required to allow MMHC develop the DAG, but the many links make it difficult to learn relations.

Discretization Methods Comparison

The BN complexity was a result of discretization showing it to be a factor affecting the performance of the learned BNs. Each BN structure learning algorithm created two structures, one for each discretization method. When predicting diversion one hour in advance, the simpler network created by the algorithm consistently performed better or was within the 95% CI for earlier predicting networks. The equal width discretization tended to result in the less complex networks. Of the machine learned network structures MMHC was the only algorithm which created a simpler network using the frequency discretized data set. Overall, the results showed discretization was a factor in BN creation because networks with fewer conditional probabilities performed better than their more complex counterparts for all, but prediction times less than diversion eight hours. Equal width discretization led to simpler networks for all, except MMHC.

Limitations

The study was limited by the number of data sets used for evaluation. Before any general conclusions could further be made, a broader number of data sets would be needed. In addition to the use of a single data set, a limited number of discretization methods were used prior to model building. The simpler models performed best between the two, showing discretization affected the results.

CHAPTER V

FUTURE WORK

Further exploration in both the comparison of BN structure learning packages along with developing a BN for predicting ambulance diversion in advanced are needed.

Bayesian Network Structure Learning Packages

Future work evaluating BN structure learning packages could be expanded in several ways. The next step in future work is to include a larger number of algorithms for comparison. The results should be based on more data sets each discretized using a more expansive number of techniques.

Emergency Department Ambulance Diversion

For ambulance diversion, the network should be learned on each of the time points the network is expected to predict rather than learning just one network for all times. Another possibility would be to use a score metric like the BDeu score to learn a network structure containing all desired prediction times in a single network. Learning on the additional times could provide valuable information regarding the relationships between these elements. Manual modification of the machine learned BN structures by experts could result in less complex models. Bayesian models which specifically integrate the temporal relationships between the variables with regard to time, such as dynamic Bayesian networks, hidden Markov models, or Kalman filters should be examined.

CHAPTER VI

CONCLUSION

The results showed the BN algorithms ability to learn models with strong discrimination. Most of the machine learned structures were able to perform as well as the expert-developed network. The study showed on the curated data set BN structure algorithms can create models with strong discriminatory power. These "off the shelf" implementations of BN structure learning algorithms can provide those with entry level machine learning knowledge the ability to develop BN models.

APPENDIX A

DISCRETIZATION RESULTS

Table 9 The selected equal frequency discretization ranges for each variable.

Equal Frequency Discretization											
Variable Names	**Ranges**										
ED_NO_OF_CURRENT_PTS	−∞	27.5	34	37.5	41.5	45.5	49.5	53.5	58.5	65.5	∞
ED_PTS_AVG_TIME_TO_ED_BED	−∞	0.5	5.5	12.5	22.5	33.5	46.5	60.5	77.5	101.5	∞
TOTAL_NO_OF_SURGERY_PER_HR			−∞	9.5	14.5	71.5	79.5	84.5	89.5	94.5	∞
ED_BED_ASSIGN_TIME_AVG						−∞	162.5	357.5	524.5	693.5	∞
HOSP_DISCHARGE_POTENTIAL							−∞	0.005	0.010	0.045	∞
ED_OCCUPANCY							−∞	0.675	0.845	0.955	∞
ED_INPTS_AVG_TIME_TO_DSCHRG							−∞	238.5	462.5	658.5	∞
ED_PTS_AVG_TIME_TO_DISCHARGE							−∞	202.5	418.5	612.5	∞
ED_INPTS_AVG_LOS_TIME							−∞	347.5	539.5	733.5	∞
HOSP_AVAIL_BED_CAPACITY							−∞	-12.5	14.5	43.5	∞
NO_OF_BED_REQUESTS_ED								−∞	6.5	14.5	∞
NO_OF_BED_REQUESTS_PERIOP								−∞	0.5	5.5	∞
ED_NO_OF_WAITING_ROOM_PTS								−∞	1.5	5.5	∞
NO_OF_BED_REQUESTS_TRANSFER									−∞	26.5	∞
ED_VOL_HOSP_CAPACITY_ALL_BDS									−∞	0.135	∞
ED_PTS_AVG_LOS_TIME									−∞	401.5	∞
ED_VOL_HOSP_CAPACITY_ICU_BDS									−∞	0.305	∞
ED_NO_OF_NURSES									−∞	12.950	∞

Table 10 The AUC for each variable/bin size combination for equal frequency discretization when added a sole child node of VUH_ED_DIVERSION.

	AUC's From First Pass of Frequency Discretization								
	Frequency Bin Sizes								
Variable Names	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
ED_NO_OF_CURRENT_PTS	0.745	0.755	0.757	0.758	0.757	0.759	0.759	0.759	0.761
ED_PTS_AVG_TIME_TO_ED_BED	0.721	0.708	0.708	0.709	0.710	0.710	0.711	0.711	0.711
TOTAL_NO_OF_SURGERY_PER_HR	0.676	0.671	0.663	0.675	0.684	0.648	0.686	0.684	0.677
ED_BED_ASSIGN_TIME_AVG	0.549	0.564	0.577	0.573	0.539	0.553	0.560	0.563	0.557
HOSP_DISCHARGE_POTENTIAL	0.538	0.555	0.545	0.522	0.533	0.532	0.543	0.539	0.536
ED_OCCUPANCY	0.723	0.716	0.720	0.714	0.711	0.715	0.716	0.712	0.711
ED_INPTS_AVG_TIME_TO_DSCHRG	0.567	0.575	0.589	0.586	0.585	0.561	0.568	0.572	0.575
ED_PTS_AVG_TIME_TO_DISCHARGE	0.569	0.572	0.579	0.579	0.577	0.578	0.563	0.567	0.569
ED_INPTS_AVG_LOS_TIME	0.521	0.529	0.531	0.516	0.519	0.524	0.512	0.516	0.519
HOSP_AVAIL_BED_CAPACITY	0.571	0.580	0.574	0.572	0.563	0.567	0.561	0.559	0.557
NO_OF_BED_REQUESTS_ED	0.615	0.614	0.603	0.607	0.605	0.610	0.610	0.609	0.608
NO_OF_BED_REQUESTS_PERIOP	0.761	0.647	0.647	0.645	0.645	0.646	0.646	0.646	0.647
ED_NO_OF_WAITING_ROOM_PTS	0.738	0.723	0.715	0.717	0.717	0.717	0.717	0.717	0.717
NO_OF_BED_REQUESTS_TRANSFER	0.553	0.553	0.545	0.552	0.551	0.554	0.546	0.546	0.547
ED_VOL_HOSP_CAPACITY_ALL_BDS	0.621	0.601	0.605	0.603	0.603	0.599	0.602	0.600	0.603
ED_PTS_AVG_LOS_TIME	0.557	0.540	0.540	0.538	0.541	0.546	0.546	0.542	0.545
ED_VOL_HOSP_CAPACITY_ICU_BDS	0.636	0.629	0.632	0.630	0.628	0.628	0.629	0.629	0.628
ED_NO_OF_NURSES	0.802	0.780	0.779	0.776	0.775	0.777	0.777	0.776	0.776

Table 11 The selected equal width discretization ranges for each variable.

Equal Width Discretization											
Variable Names	**Ranges**										
ED_NO_OF_WAITING_ROOM_PTS	−∞	3.40	6.80	10.20	13.60	17.00	20.40	23.80	27.20	30.60	∞
NO_OF_BED_REQUESTS_TRANSFER				−∞	10.57	20.14	29.71	39.29	48.86	58.43	∞
ED_PTS_AVG_TIME_TO_ED_BED						−∞	64.60	129.20	193.80	258.40	∞
ED_PTS_AVG_LOS_TIME							−∞	371.00	635.00	899.00	∞
HOSP_AVAIL_BED_CAPACITY								−∞	0.00	82.00	∞
TOTAL_NO_OF_SURGERY_PER_HR								−∞	39.33	76.67	∞
ED_NO_OF_CURRENT_PTS								−∞	36.33	63.67	∞
ED_VOL_HOSP_CAPACITY_ICU_BDS								−∞	1.22	2.45	∞
HOSP_DISCHARGE_POTENTIAL								−∞	0.06	0.11	∞
ED_OCCUPANCY								−∞	0.50	0.89	∞
ED_NO_OF_NURSES								−∞	12.00	14.00	∞
NO_OF_BED_REQUESTS_ED									−∞	20.50	∞
ED_BED_ASSIGN_TIME_AVG									−∞	876.50	∞
ED_INPTS_AVG_TIME_TO_DSCHRG									−∞	780.00	∞
ED_VOL_HOSP_CAPACITY_ALL_BDS									−∞	0.64	∞
ED_PTS_AVG_TIME_TO_DISCHARGE									−∞	746.50	∞
NO_OF_BED_REQUESTS_PERIOP									−∞	17.50	∞
ED_INPTS_AVG_LOS_TIME									−∞	896.00	∞

Table 12 The AUC for each variable/bin size combination for equal width discretization when added a sole child node of VUH_ED_DIVERSION.

	AUC's From First Pass of Width Discretization								
	Width Bin Sizes								
Variable Names	2	3	4	5	6	7	8	9	10
ED_NO_OF_CURRENT_PTS	0.769	0.766	0.760	0.765	0.765	0.755	0.757	0.760	0.760
ED_PTS_AVG_TIME_TO_ED_BED	0.674	0.593	0.708	0.882	0.876	0.866	0.806	0.788	0.783
TOTAL_NO_OF_SURGERY_PER_HR	0.639	0.676	0.639	0.648	0.658	0.668	0.626	0.632	0.616
ED_BED_ASSIGN_TIME_AVG	0.662	0.600	0.541	0.562	0.552	0.558	0.560	0.545	0.557
HOSP_DISCHARGE_POTENTIAL	0.553	0.558	0.518	0.525	0.516	0.520	0.520	0.521	0.521
ED_OCCUPANCY	0.732	0.739	0.701	0.707	0.720	0.708	0.708	0.704	0.705
ED_INPTS_AVG_TIME_TO_DSCHRG	0.661	0.574	0.579	0.567	0.569	0.577	0.572	0.570	0.570
ED_PTS_AVG_TIME_TO_DISCHARGE	0.651	0.591	0.573	0.562	0.567	0.571	0.568	0.570	0.569
ED_INPTS_AVG_LOS_TIME	0.527	0.530	0.518	0.519	0.514	0.521	0.513	0.520	0.511
HOSP_AVAIL_BED_CAPACITY	0.500	0.579	0.552	0.555	0.544	0.574	0.565	0.556	0.551
NO_OF_BED_REQUESTS_ED	0.633	0.606	0.611	0.597	0.605	0.601	0.599	0.594	0.600
NO_OF_BED_REQUESTS_PERIOP	0.847	0.828	0.638	0.654	0.645	0.638	0.636	0.646	0.646
ED_NO_OF_WAITING_ROOM_PTS	0.825	0.667	0.675	0.697	0.717	0.718	0.725	0.726	0.726
NO_OF_BED_REQUESTS_TRANSFER	0.544	0.545	0.543	0.542	0.539	0.558	0.545	0.552	0.543
ED_VOL_HOSP_CAPACITY_ALL_BDS	0.627	0.700	0.603	0.599	0.601	0.602	0.606	0.608	0.606
ED_PTS_AVG_LOS_TIME	0.500	0.543	0.571	0.530	0.540	0.554	0.554	0.543	0.555
ED_VOL_HOSP_CAPACITY_ICU_BDS	0.728	0.804	0.759	0.611	0.643	0.641	0.630	0.627	0.636
ED_NO_OF_NURSES	0.802	0.780	0.777	0.776	0.777	0.777	0.776	0.776	0.776

APPENDIX B

FREQUENCY NETWORK STRUCTURES

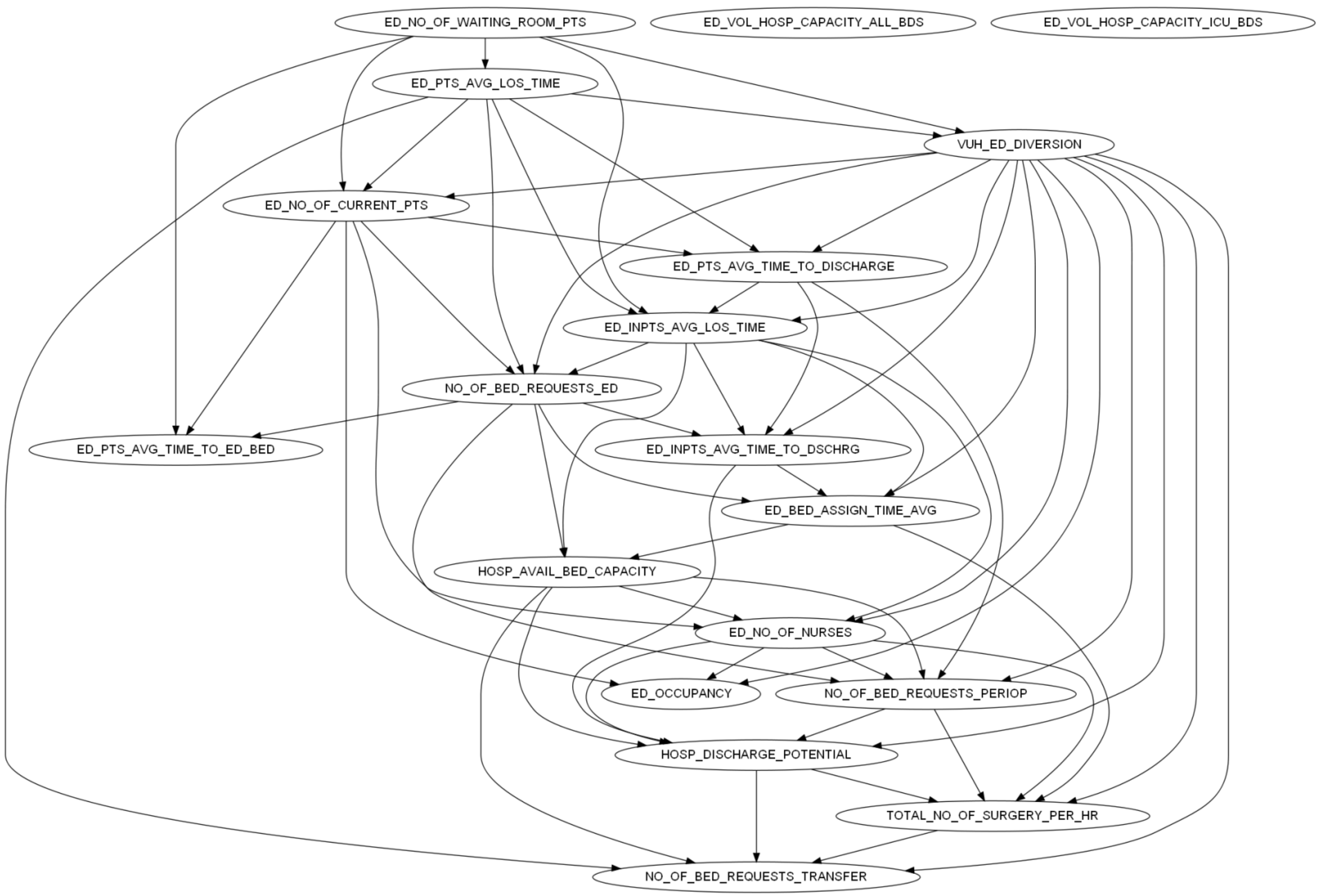


Figure 21 Augmented Markov Blanket

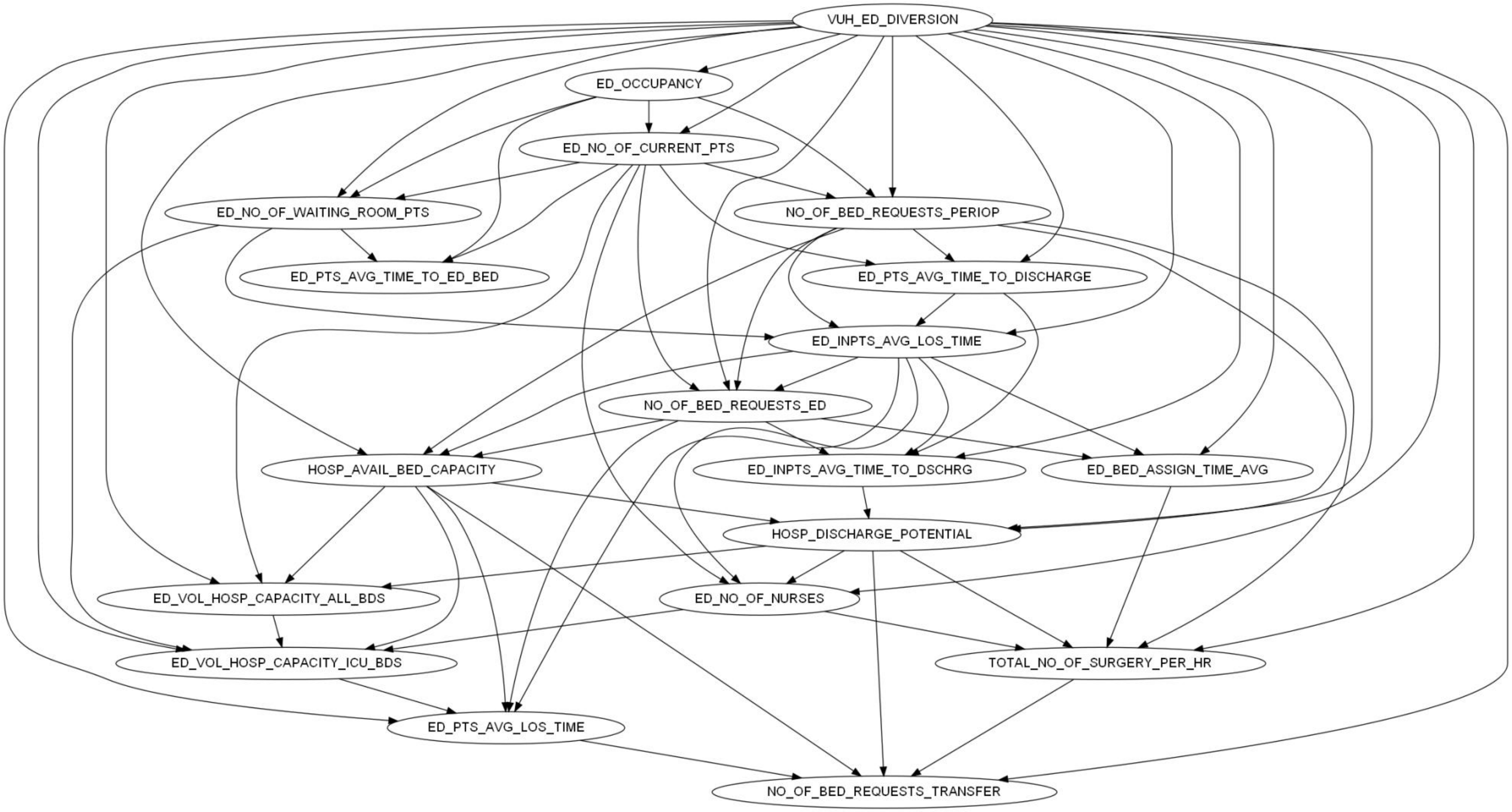


Figure 22 Augmented Naive Bayesian Network

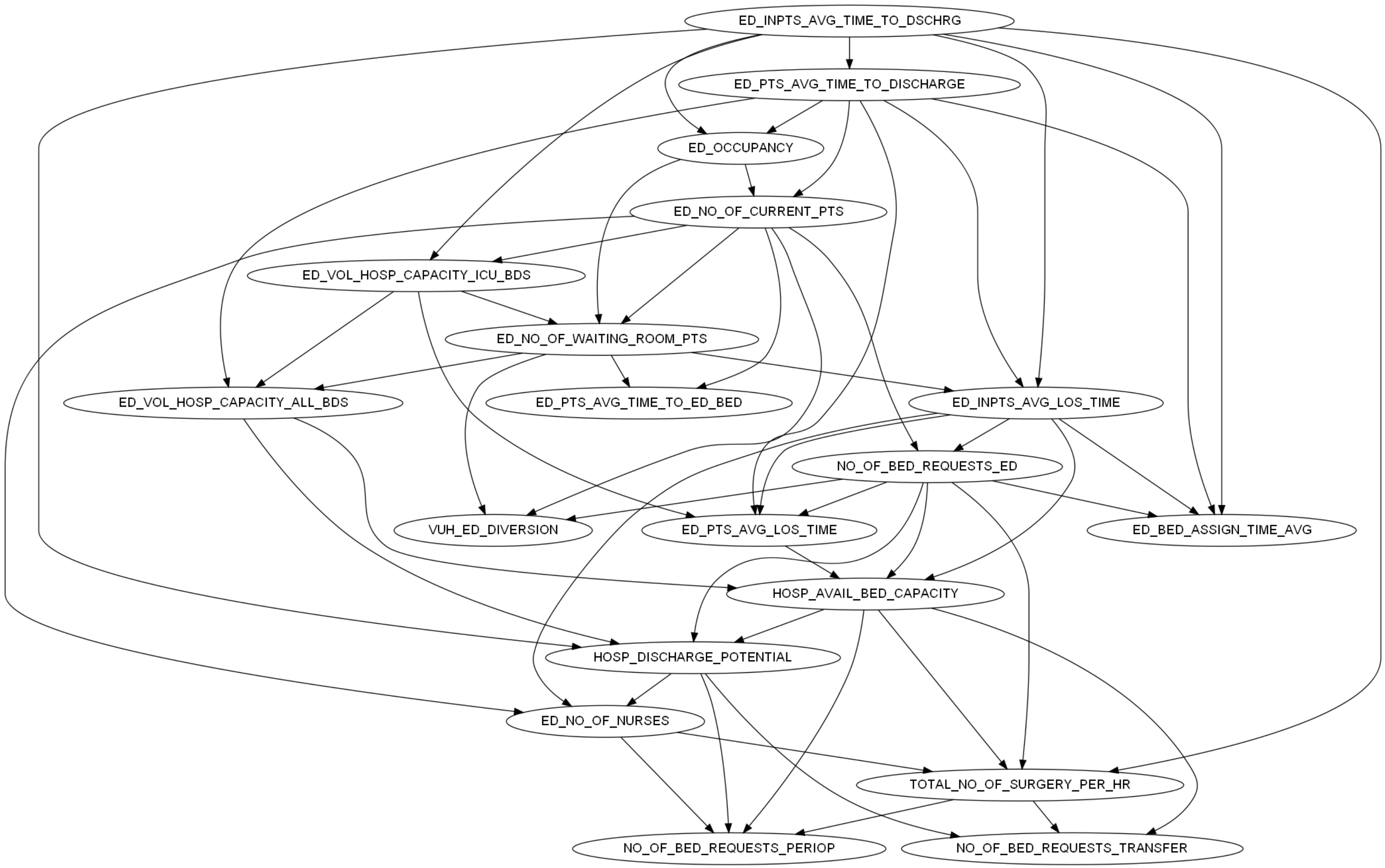


Figure 23 MMHC Learned Structure

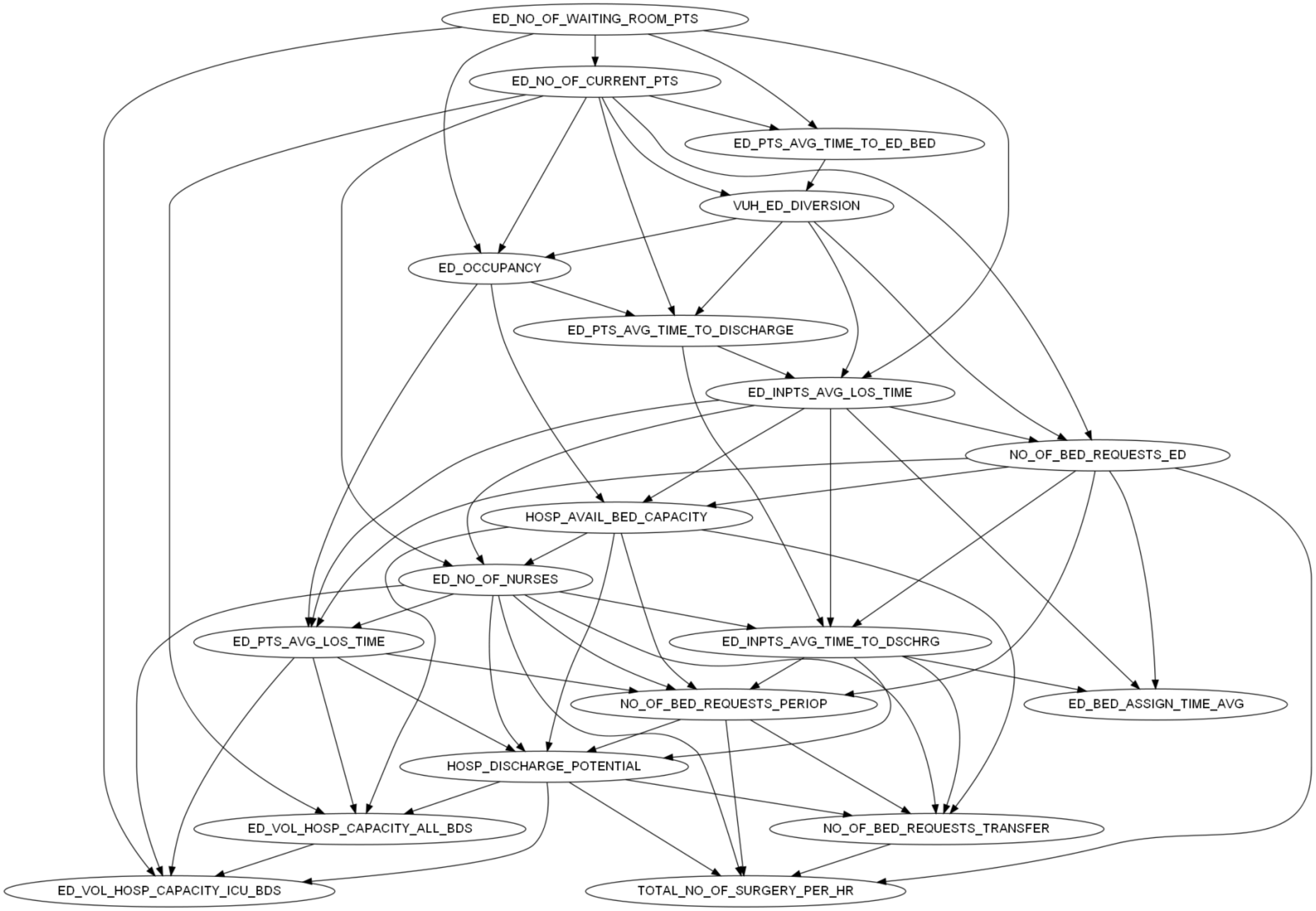


Figure 24 Taboo Learned Network

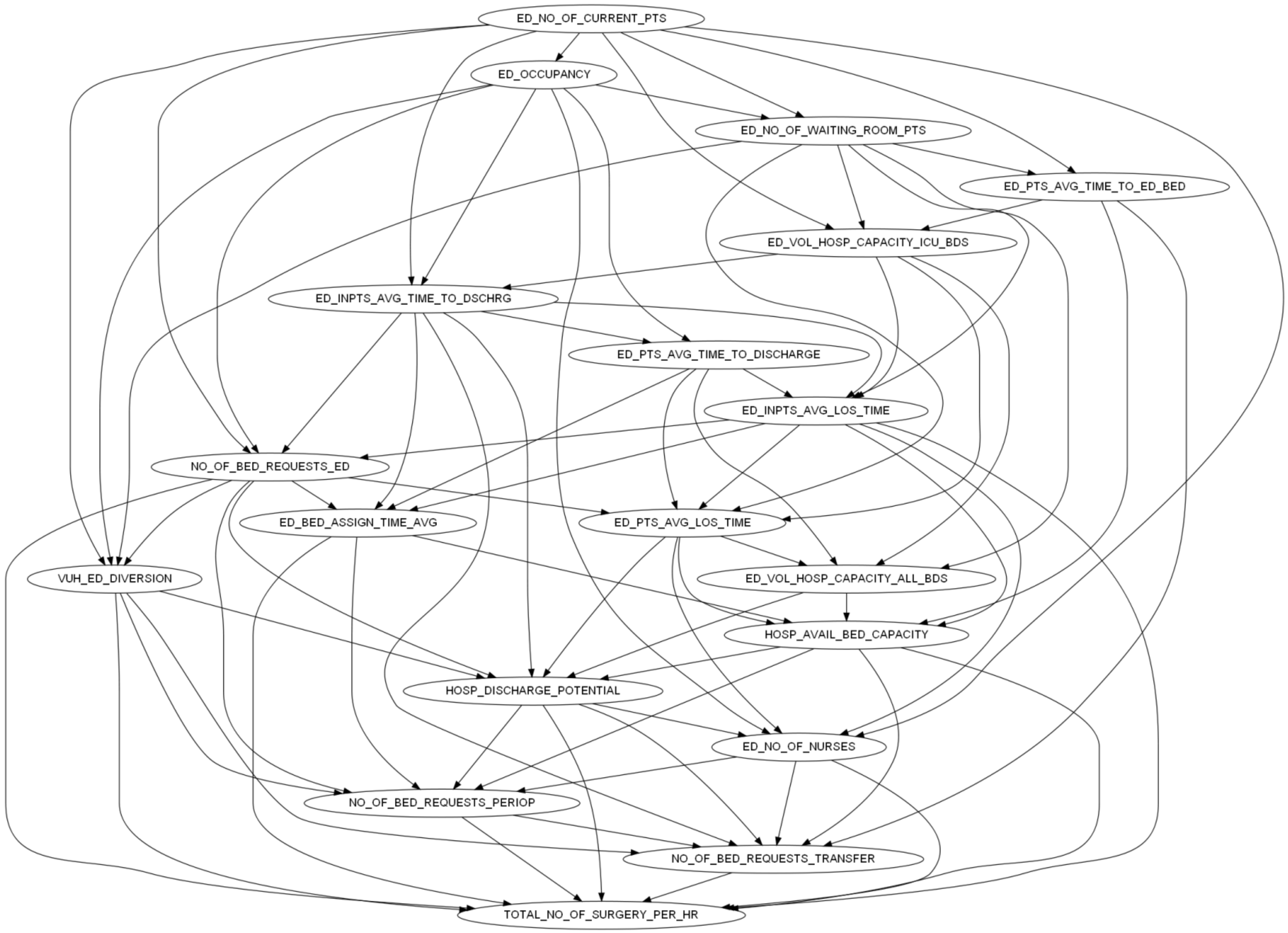


Figure 25 Greedy Search

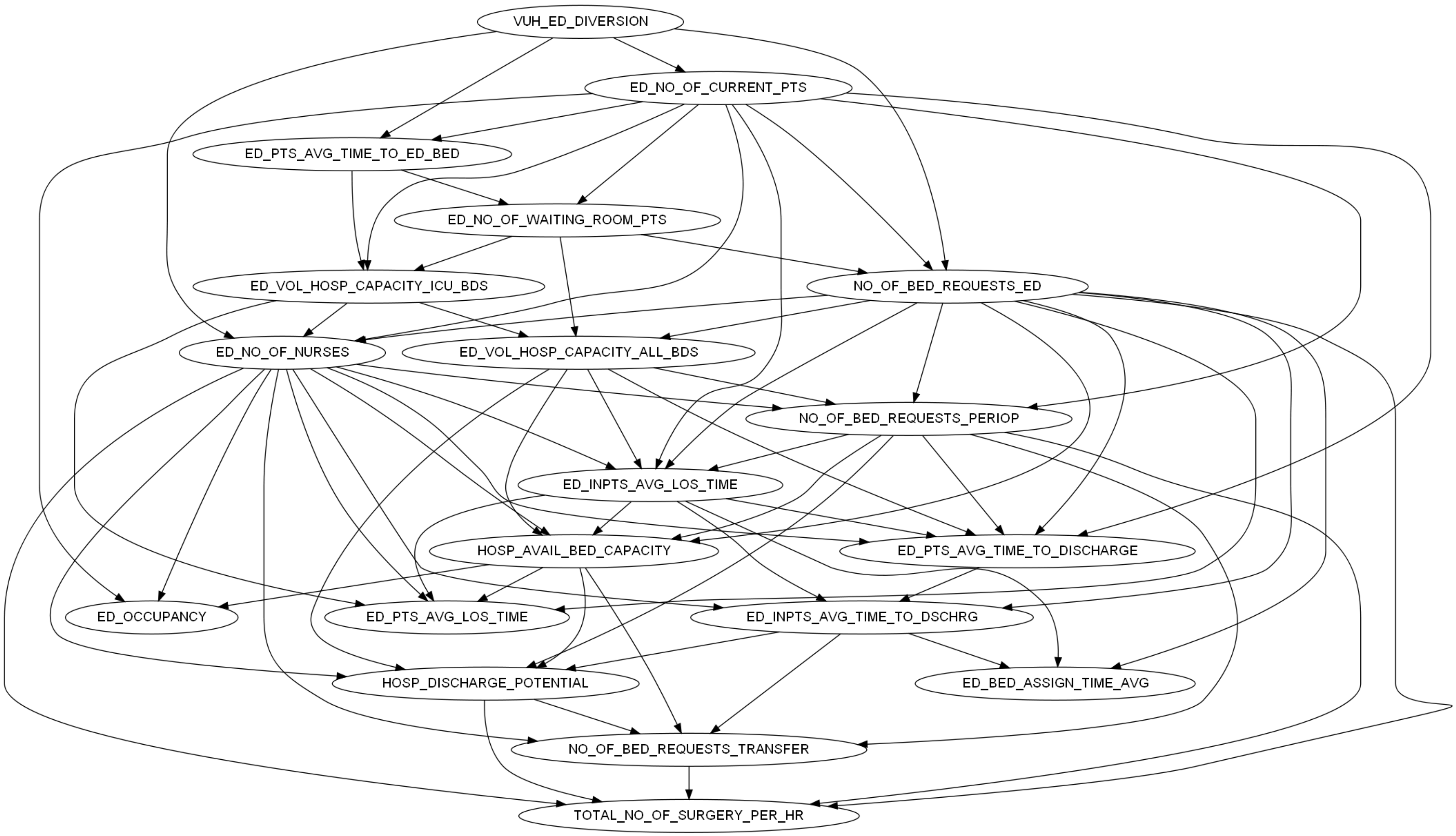


Figure 26 Semi-Supervised

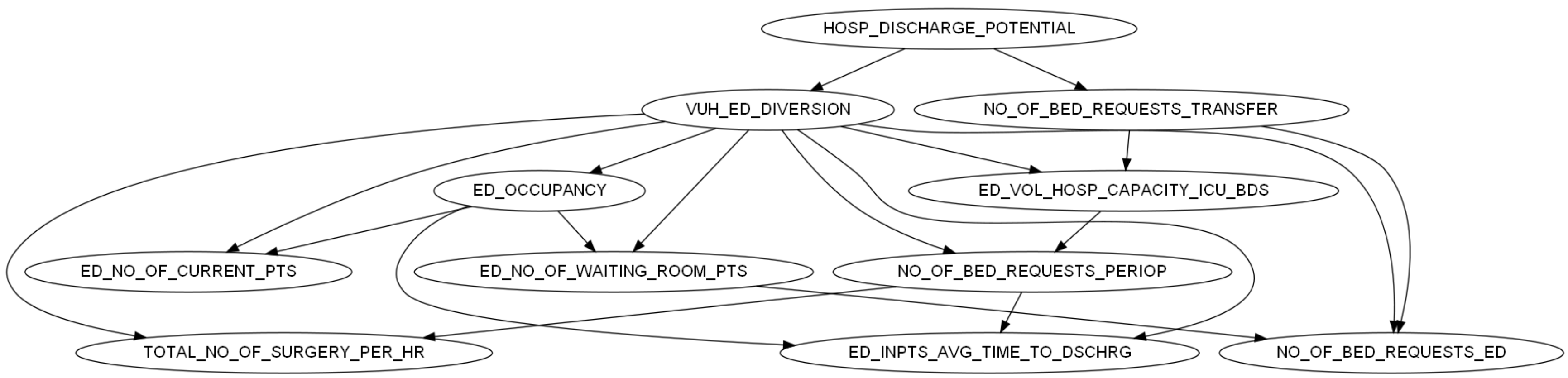


Figure 27 Expert Developed

APPENDIX C

EQUAL WIDTH NETWORK STRUCTURES

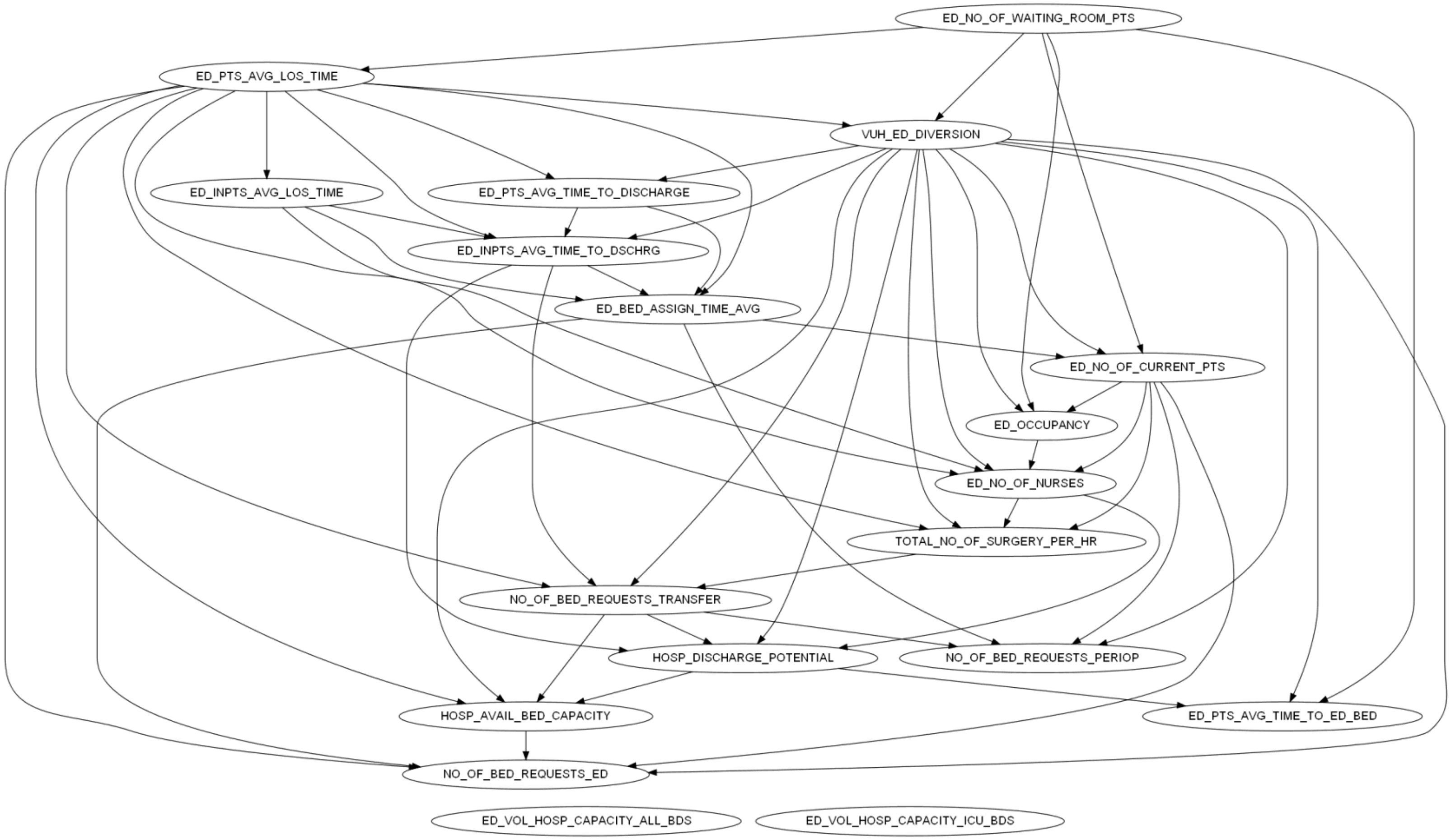


Figure 28 Markov Blanket

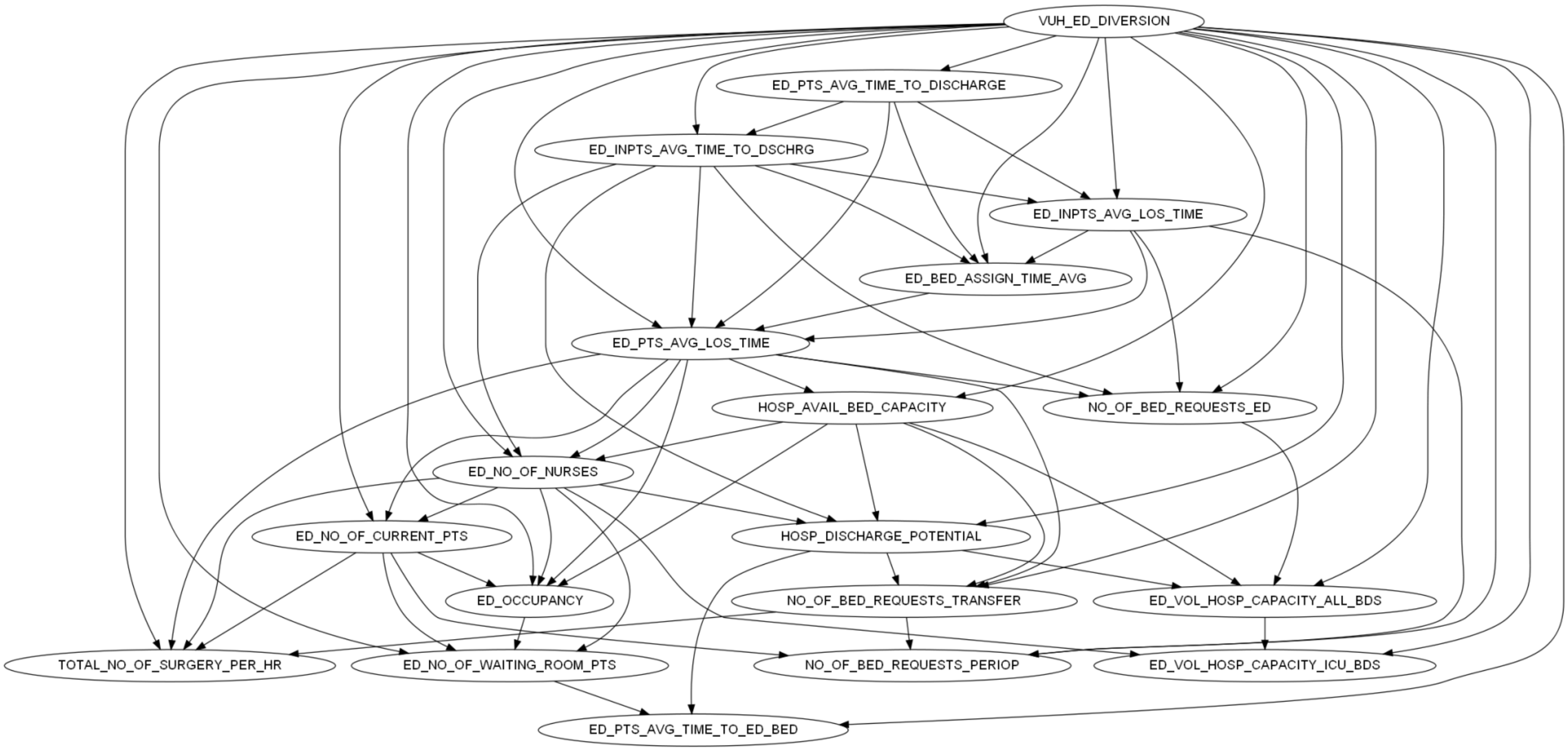


Figure 29Augmented Naive Bayesian Network

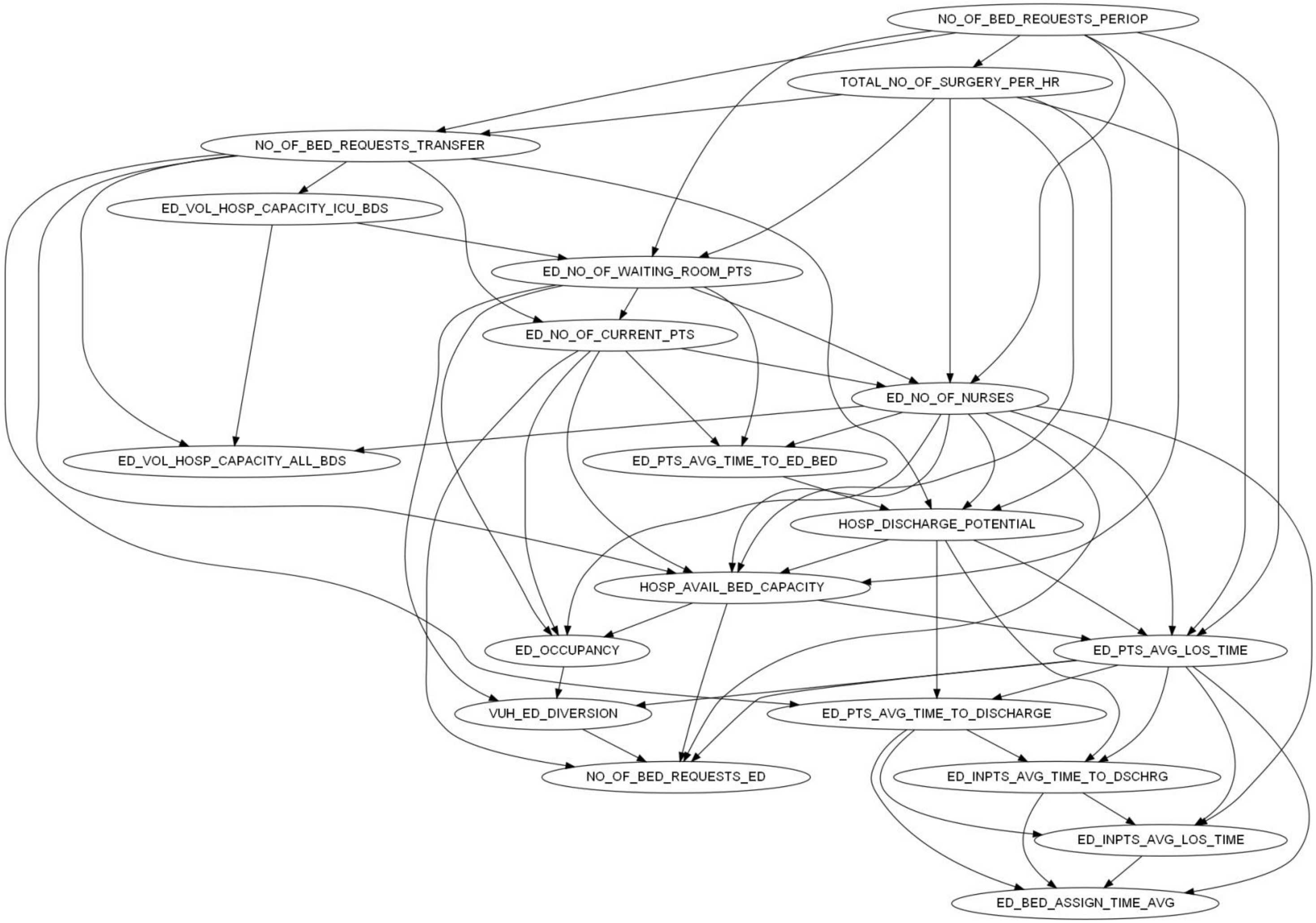


Figure 30 MMHC

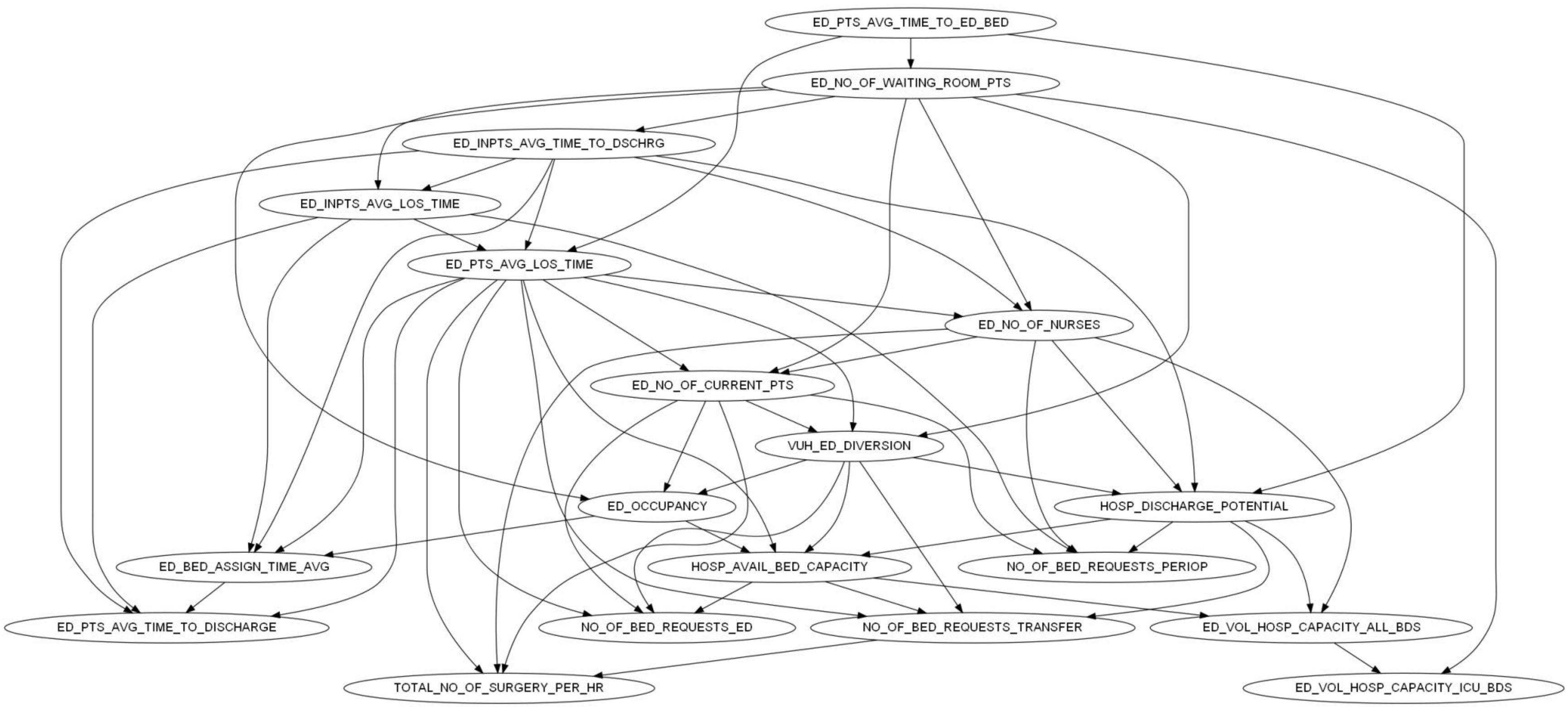


Figure 31 Taboo Search

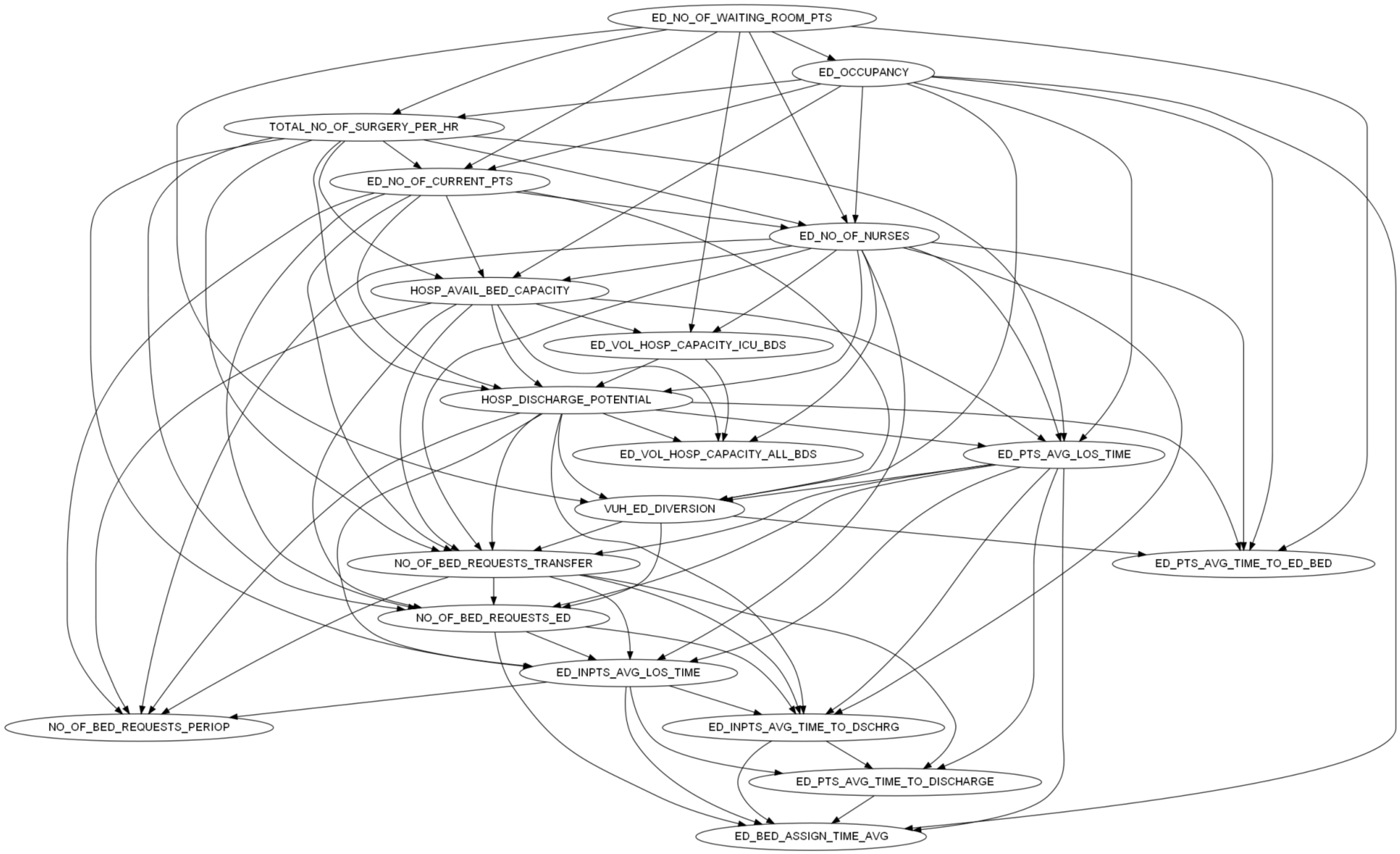


Figure 32 Greedy Search

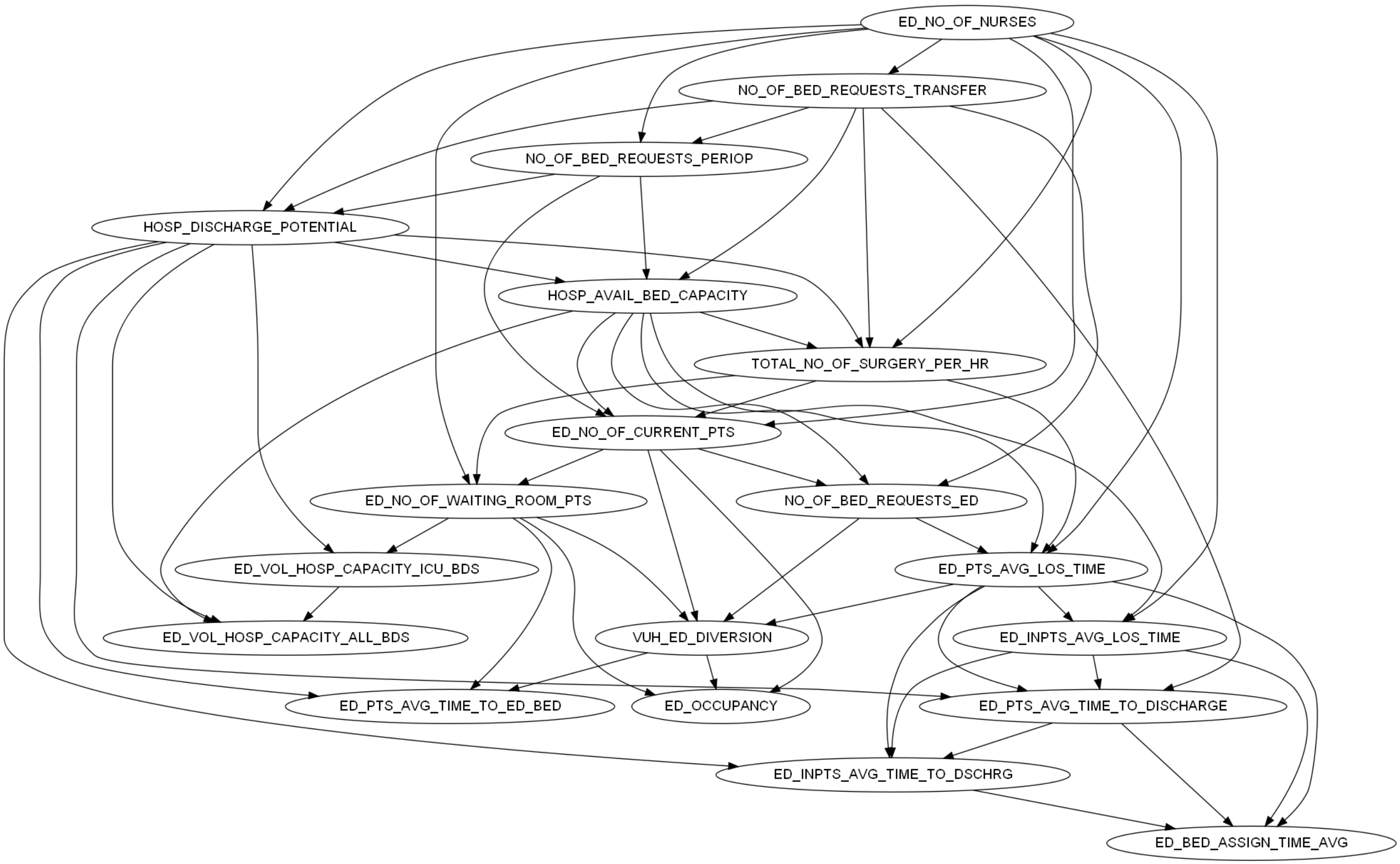


Figure 33 Semi-Supervised

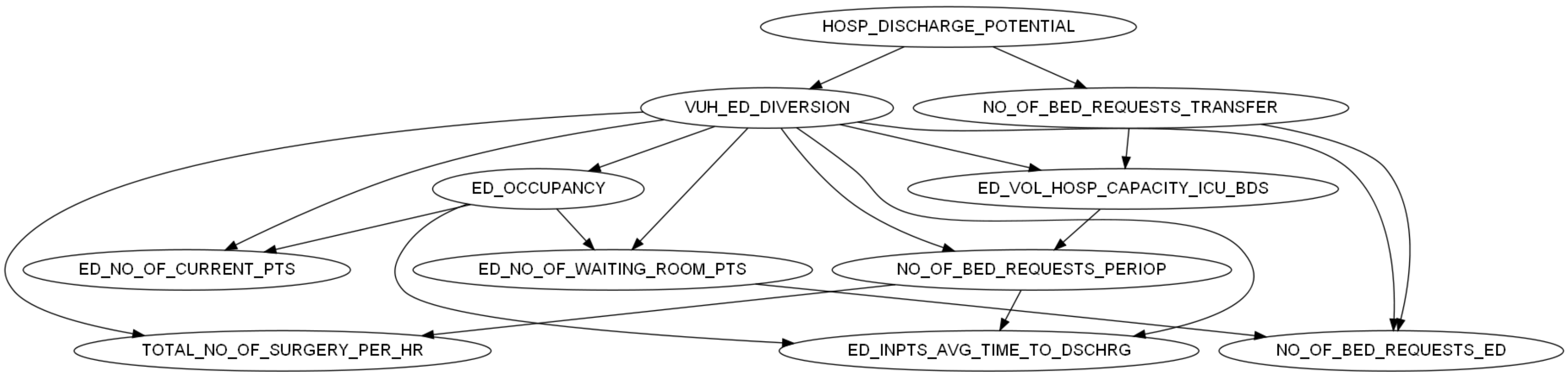


Figure 34 Expert-developed

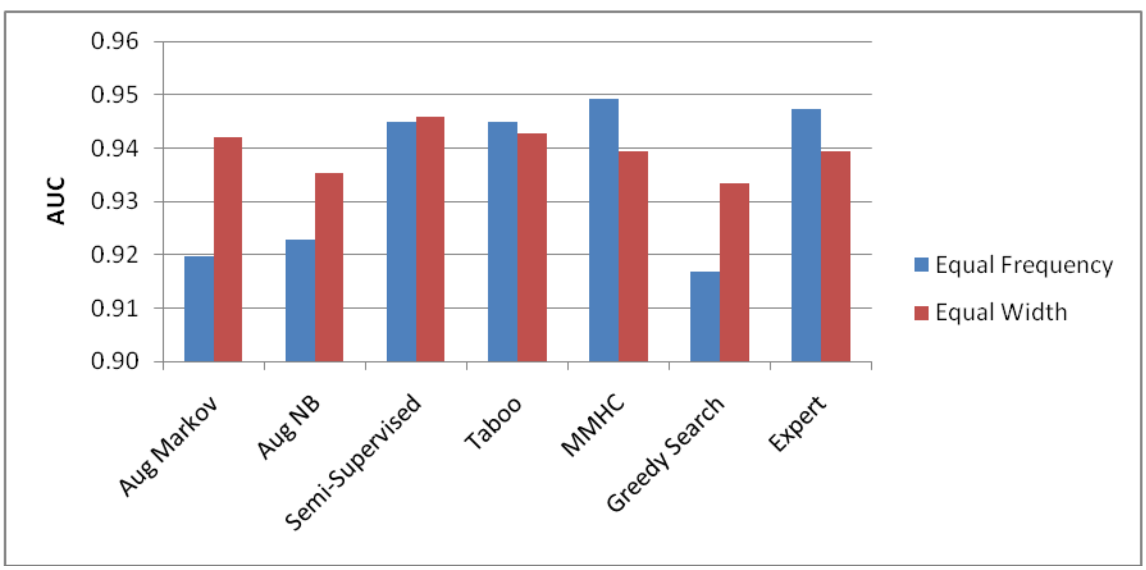
APPENDIX D

NETWORK EVALUATION RESULTS

Predicting 1 hour in Advance

	Equal Frequency Discretized Data Set			
Algorithms	**AUC**	**SE of AUC**	**NLL**	**AIC**
Aug Markov	0.9198	8.88E-04	35757.05	87652.09
Aug NB	0.9229	8.82E-04	35521.55	89031.10
Semi-Supervised	0.9449	7.21E-04	26650.50	80761.01
Taboo	0.9449	7.10E-04	26904.52	67515.04
MMHC	0.9491	6.89E-04	25531.81	61551.63
Greedy Search	0.9168	8.95E-04	34756.18	501444.37
Expert	0.9473	6.96E-04	32474.70	65679.40

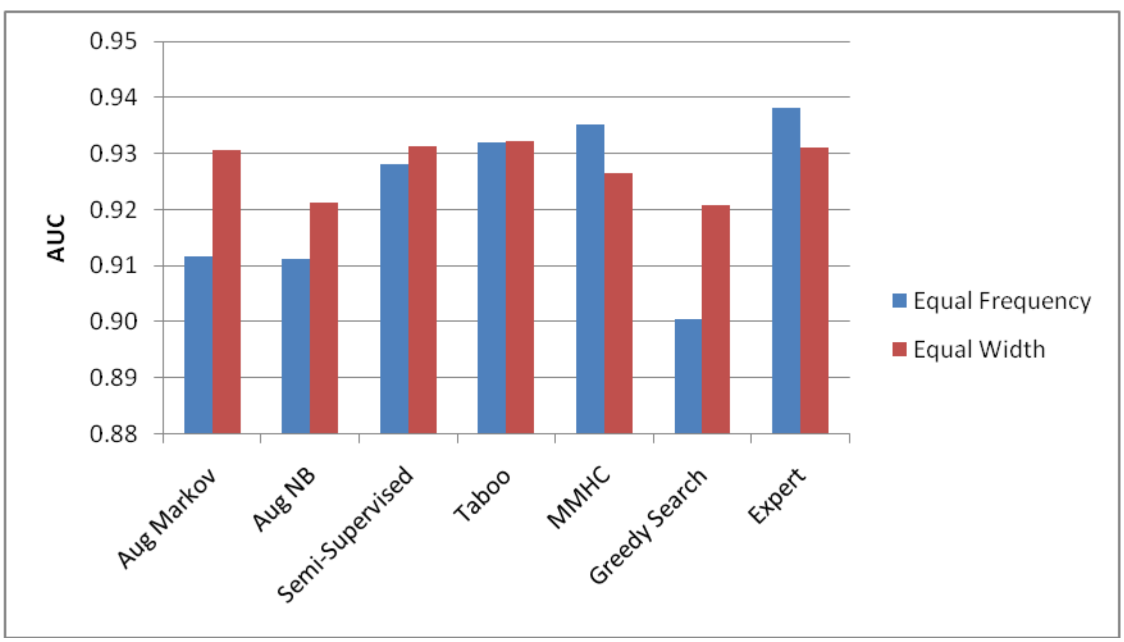
	Equal Width Discretized Data Set			
Algorithms	**AUC**	**SE of AUC**	**NLL**	**AIC**
Aug Markov	0.9421	7.59E-04	28166.23	62498.47
Aug NB	0.9352	8.28E-04	30595.78	69955.57
Semi-Supervised	0.9460	7.15E-04	26202.74	59067.48
Taboo	0.9427	7.49E-04	26575.39	59944.77
MMHC	0.9395	7.55E-04	27603.22	72066.44
Greedy Search	0.9334	8.14E-04	29669.45	125326.90
Expert	0.9395	8.04E-04	26916.09	55138.18



Predicting 2 hours in Advance

	Equal Frequency Discretized Data Set			
Algorithms	**AUC**	**SE of AUC**	**NLL**	**AIC**
Aug Markov	0.9117	9.38E-04	36900.42	89938.84
Aug NB	0.9112	9.54E-04	38739.68	95467.36
Semi-Supervised	0.9282	8.52E-04	29954.08	87368.16
Taboo	0.9320	8.11E-04	29371.32	72448.65
MMHC	0.9352	8.10E-04	28711.69	67911.39
Greedy Search	0.9005	9.98E-04	38010.68	507953.37
Expert	0.9381	7.74E-04	34691.88	70113.77

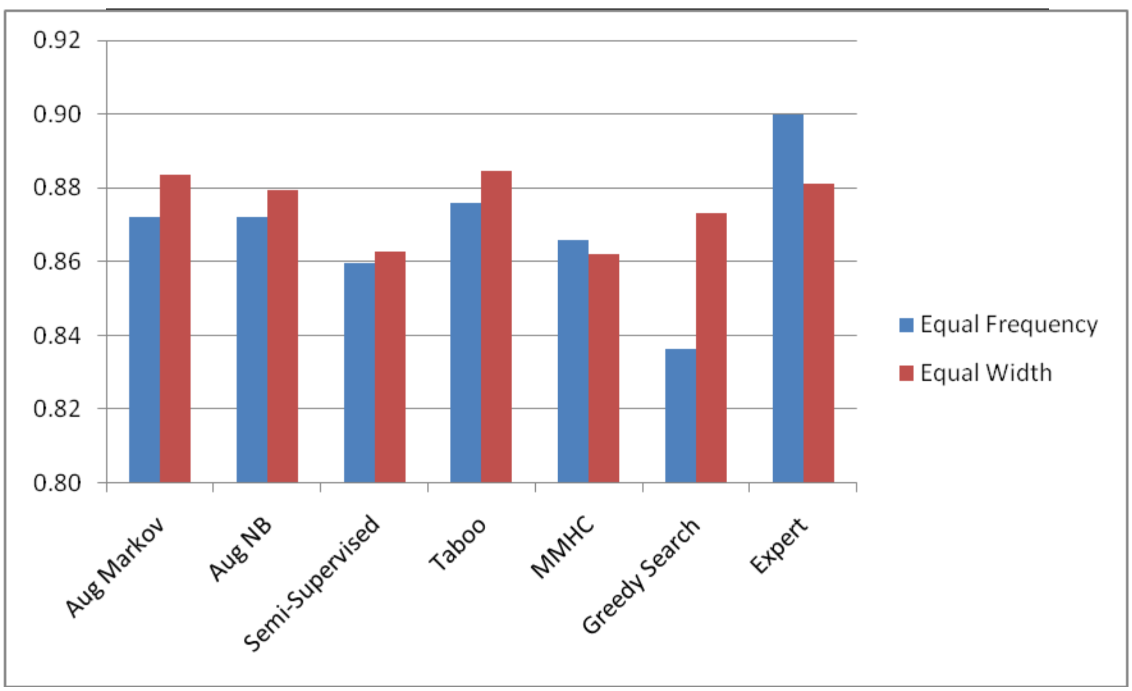
	Equal Width Discretized Data Set			
Algorithms	**AUC**	**SE of AUC**	**NLL**	**AIC**
Aug Markov	0.9305	8.57E-01	30612.86	67391.73
Aug NB	0.9213	9.44E-04	33692.40	76148.80
Semi-Supervised	0.9313	8.40E-04	29127.56	64917.11
Taboo	0.9322	8.49E-04	28739.30	64272.59
MMHC	0.9264	8.72E-04	29873.50	76607.01
Greedy Search	0.9207	9.12E-01	32201.49	130390.98
Expert	0.9311	8.65E-04	28697.59	58701.17



Predicting 4 hours in Advance

	Equal Frequency Discretized Data Set			
Algorithms	**AUC**	**SE of AUC**	**NLL**	**AIC**
Aug Markov	0.8720	1.17E-03	43546.21	103230.42
Aug NB	0.8720	1.18E-03	47463.51	112915.01
Semi-Supervised	0.8597	1.27E-03	39633.81	106727.61
Taboo	0.8758	1.18E-03	37674.12	89054.24
MMHC	0.8659	1.26E-03	38967.56	88423.12
Greedy Search	0.8362	1.36E-03	49039.18	530010.36
Expert	0.8998	1.05E-03	41403.5	83537.00

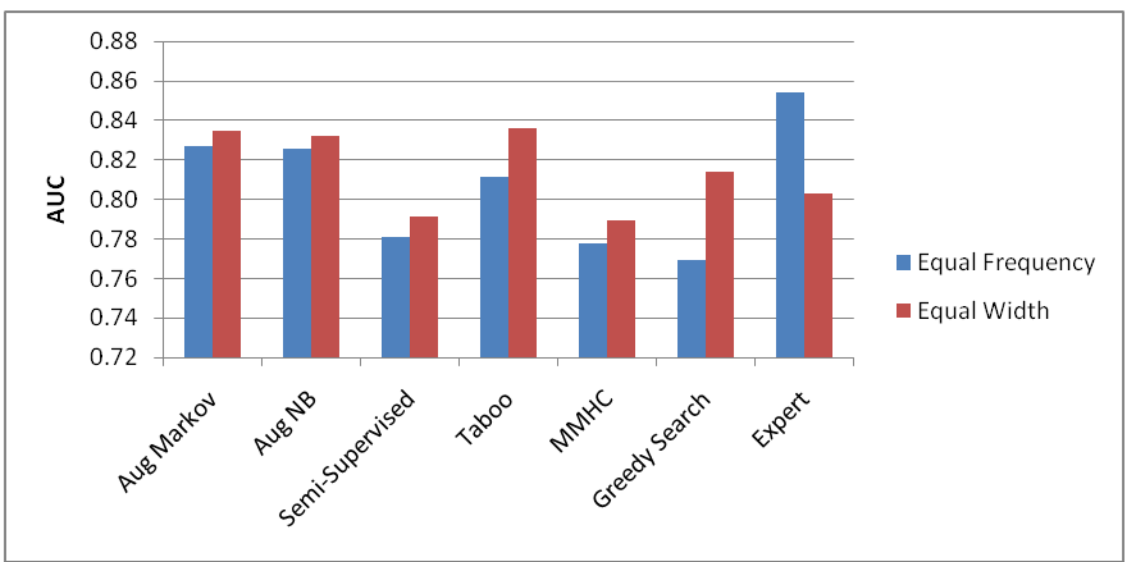
	Equal Width Discretized Data Set			
Algorithms	**AUC**	**SE of AUC**	**NLL**	**AIC**
Aug Markov	0.8836	1.16E-03	38510.03	83186.05
Aug NB	0.8794	1.21E-03	40649.64	90063.28
Semi-Supervised	0.8627	1.29E-03	39243.43	85148.87
Taboo	0.8848	1.15E-03	36867.49	80528.98
MMHC	0.8620	1.31E-03	38925.58	94711.16
Greedy Search	0.8732	1.20E-03	40799.77	147587.53
Expert	0.8810	1.21E-03	37280.02	75866.04



Predicting 6 hours in Advance

	Equal Frequency Discretized Data Set			
Algorithms	AUC	SE of AUC	NLL	AIC
Aug Markov	0.8269	1.37E-03	51039.79	118217.58
Aug NB	0.8257	1.39E-03	54594.66	127177.32
Semi-Supervised	0.7813	1.59E-01	46777.38	121014.77
Taboo	0.8115	1.49E-03	44149.72	102005.45
MMHC	0.7777	1.65E-03	47005.46	104498.92
Greedy Search	0.7693	1.61E-03	58588.38	549108.76
Expert	0.8542	1.28E-03	47258.07	95246.14

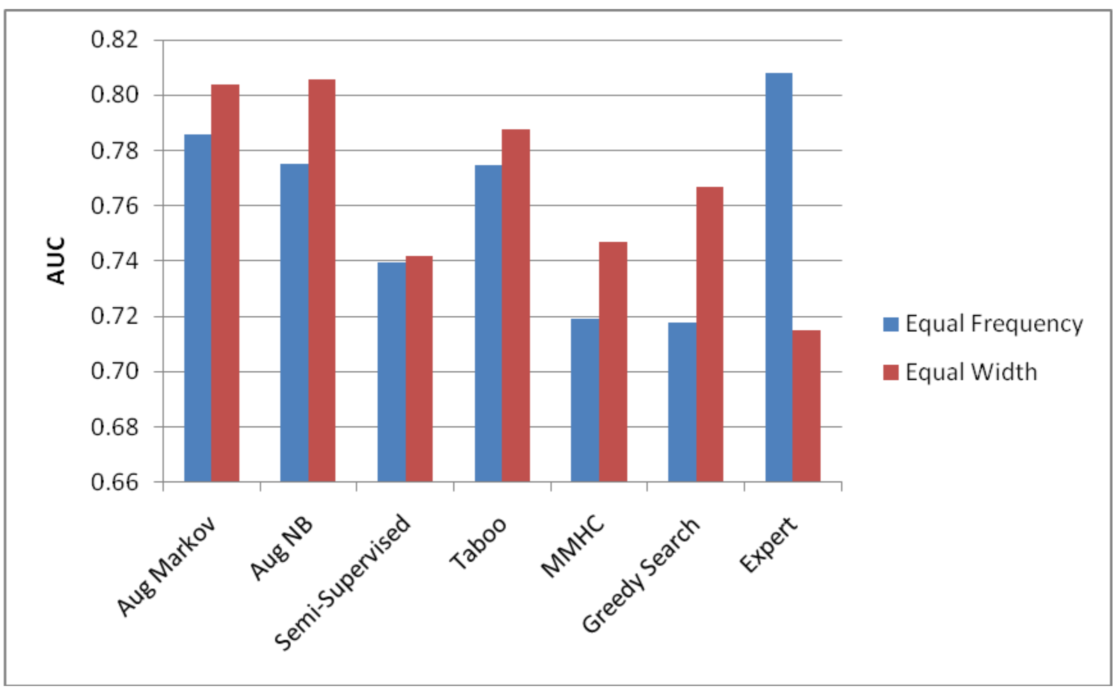
	Equal Width Discretized Data Set			
Algorithms	AUC	SE of AUC	NLL	AIC
Aug Markov	0.8349	1.43E-03	44396.08	94958.16
Aug NB	0.8323	1.44E-03	46362.11	101488.22
Semi-Supervised	0.7914	1.61E-03	45826.09	98314.18
Taboo	0.8360	1.40E-03	42687.69	92169.38
MMHC	0.7896	1.63E-03	45761.95	108383.90
Greedy Search	0.8142	1.49E-03	47682.42	161352.84
Expert	0.8030	1.62E-03	45708.11	92722.21



Predicting 8 hours in Advance

	Equal Frequency Discretized Data Set			
Algorithms	**AUC**	**SE of AUC**	**NLL**	**AIC**
Aug Markov	0.7857	1.53E-03	56778.54	129695.08
Aug NB	0.7751	1.60E-03	60699.49	139386.98
Semi-Supervised	0.7394	1.73E-03	49451.81	126363.63
Taboo	0.7745	1.60E-03	47082.70	107871.40
MMHC	0.7193	1.78E-03	50154.43	110796.86
Greedy Search	0.7179	1.74E-03	63152.91	558237.82
Expert	0.8081	1.47E-03	50623.57	101977.15

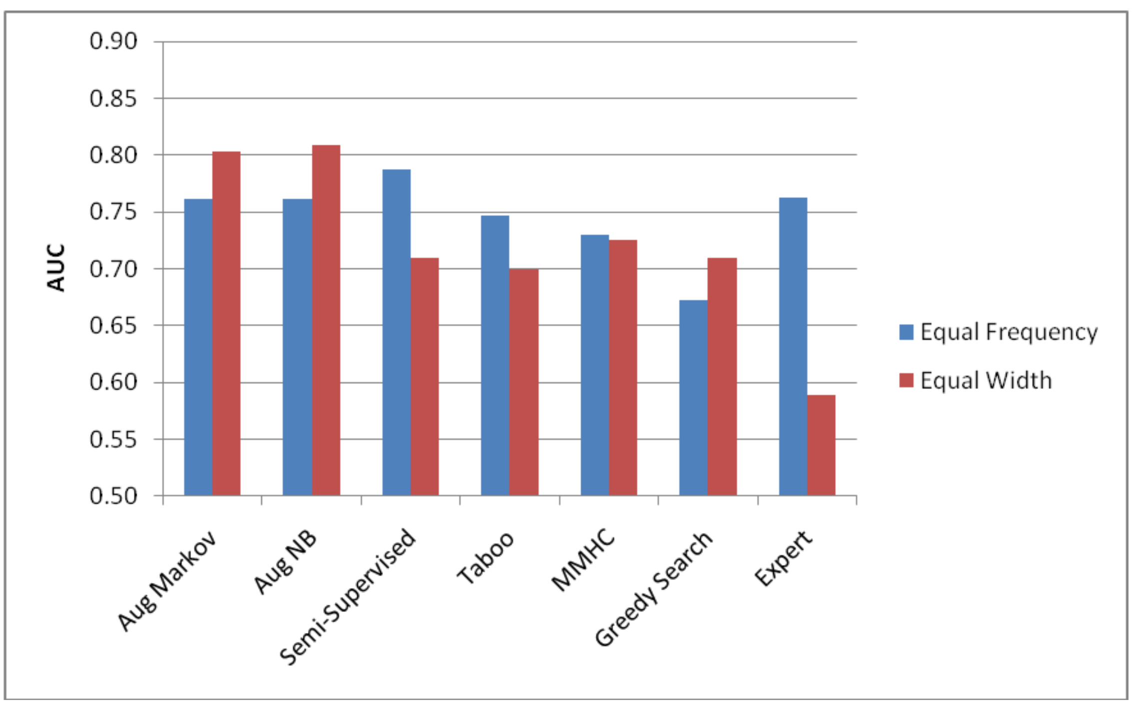
	Equal Width Discretized Data Set			
Algorithms	**AUC**	**SE of AUC**	**NLL**	**AIC**
Aug Markov	0.8037	1.54E-03	46981.12	100128.23
Aug NB	0.8058	1.55E-03	48414.40	105592.80
Semi-Supervised	0.7420	1.80E-03	48437.01	103536.01
Taboo	0.7877	1.59E-03	47551.95	101897.90
MMHC	0.7470	1.76E-03	48454.30	113768.60
Greedy Search	0.7666	1.66E-03	50943.09	167874.18
Expert	0.7150	1.90E-03	51791.67	104889.33



Predicting 12 hours in Advance

	Equal Frequency Discretized Data Set			
Algorithms	**AUC**	**SE of AUC**	**NLL**	**AIC**
Aug Markov	0.7619	1.62E-03	61996.33	140130.67
Aug NB	0.7613	1.65E-03	62770.93	143529.85
Semi-Supervised	0.7879	1.59E-03	46186.32	119832.64
Taboo	0.7469	1.70E-03	49496.43	112698.85
MMHC	0.7298	1.77E-03	50276.56	111041.13
Greedy Search	0.6725	1.85E-03	68207.27	568346.54
Expert	0.7623	1.65E-03	55102.35	110934.70

	Equal Width Discretized Data Set			
Algorithms	**AUC**	**SE of AUC**	**NLL**	**AIC**
Aug Markov	0.8032	1.51E-03	46904.20	99974.40
Aug NB	0.8091	1.45E-03	47550.60	103865.20
Semi-Supervised	0.7093	1.90E-03	50823.81	108309.62
Taboo	0.6997	1.86E-03	53700.80	114195.60
MMHC	0.7255	1.80E-03	50167.88	117195.77
Greedy Search	0.7095	1.81E-03	55723.21	177434.43
Expert	0.5891	2.05E-01	55782.44	112870.89



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