ESSAYS ON THE STRUCTURAL ANALYSIS OF AUCTIONS

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Chapter I

INFORMATION COST IN TAKEOVER AUCTIONS

I.1 Introduction

In the sale of a company, the potential acquirors obtain various information on the target company to learn their synergy value. The confidential information revealed throughout the process not only plays a crucial role in the evaluation, but also has significant effect on future competition after the transaction. Specifically, a bidder would discount her synergy value whenever other bidders gain confidential information on the target because they are mostly industry rivals who may strategically exploit the competitive materials. Hence, bidders would lower their bids, and this causes a premium loss for the seller, which we call the *information cost*.

Hansen (2001) first introduces the notion of information cost in takeover auctions, which has been, since then, discussed in a number of articles in corporate finance, including Boone and Mulherin (2007); Rogo (2014); Schlingemann and Wu (2015). Legal studies of corporate takeovers also echo the *"legitimate proprietary concerns"* on sensitive information being disseminated to their industry rivals. Although the fiduciary duty requires that the boards of the target company act as auctioneers to maximize shareholders' benefits, the Delaware Court of Chancery indicated in many cases that a full-blown auction may not be desirable, as the cost could outweigh the benefits.[1] Among those cases, the potential loss of competitive information is one of the major concerns. For example, in the case of *Lear Corp. Shareholder Litigation*, the target company rejects a potential acquiror on the ground that there could be a risk of losing an initial bidder or the initial bidder would

[1] Delaware is the leading jurisdiction for publicly traded corporations listed on U.S. stock exchanges. More than half of such corporations have chosen to incorporate in Delaware. Delaware is also the leading jurisdiction for out-of-state incorporations, where a corporation headquartered in one state chooses to incorporate in another state. See for more information `https://corplaw.delaware.gov/facts-and-myths/#fn:1`.

pay less if the target engaged with additional bidders.[2]

In this chapter, we develop a takeover auction model, accounting for the synergy value discount, which we term the *information disclosure discount*. Specifically, we approximate the takeover transaction by the first-price sealed-bid auction with unknown number of bidders, reflecting a key institutional feature that the winning bidder makes her offer while being uninformed of the number of competitors and their bids.[3] Assuming that bidders are ex-ante identical yet privately learn their synergy values, we derive a symmetric Bayesian Nash equilibrium with a strictly increasing bidding strategy for the game induced by the auction.

This study contributes to theoretical studies on takeover auctions. The extant research largely resides on the idea that information acquisition is costly for the bidders; for example, Ye (2007), Quint and Hendricks (2015), and Lu and Ye (2016). Ye (2007) finds that bidders' entry cost cannot explain why the seller of a company uses an indicative bidding stage to shortlist bidders into the final bidding competition. Quint and Hendricks (2015) show that in a certain scenario, where the indicative bids sort bidders into a finite number of groups, the seller can use indicative bids to optimally shortlist bidders. Lu and Ye (2016) develop an optimal two-stage mechanism where bidders need to incur information acquisition cost to discover their values. These studies acknowledge the seller's active role of shortlisting final bidders in the takeover process, but they do not consider the costs incurred by the seller. Deviating from this strand of studies, we build a novel model, where information disclosure is costly to the seller, to provide a new insight on why the seller restricts bidders' entry in takeover auctions.

The paper proceeds as follows. Section I.2 provides the institutional background of a typical takeover process. Section I.3 develops a takeover auction model, derives an optimal

[2] See *In Re Lear Corp. Shareholder Litigation, 926 A.2d 94, 119*. More examples can be found in Sautter (2013).

[3] From our conversation with an experienced investment banker dealing with M&A and our reading of SEC filings, we learn that it is a common practice for the seller to keep bidders uniformed of the rivals, e.g., their identities and offers.

bidding strategy and establishes a symmetric BNE. Section I.4 constructs the information cost based on the developed model and illustrates how it affects sellers expected revenue. Section I.5 concludes by discussing policy implications.

I.2 Sale of Companies

Before modeling the takeover process, we describe a typical sale of a company in a private takeover. In general, a public takeover could follow by the announcement of a private takeover. For the purposes of this study, we describe only the private takeover process. See Boone and Mulherin (2007) and Hansen (2001) for a complete description.

I.2.1 Institutional Background

A corporate takeover is initiated either by a target company considering the sale of its enterprise as a strategic alternative or by an acquiring company launching an unsolicited inquiry. Once a takeover begins, the target hires an investment bank for financial advice and retains a law firm for legal counsel – we call the target and its advisors as the seller in this paper. The seller contacts prospective buyers with the delivery of a teaser and confidentiality and standstill agreements.[4] For public companies, Regulation Fair Disclosure concerns govern the content of the teaser; at this stage no material nonpublic information is revealed to the buyers.[5] Among the prospective buyers, those who are interested sign the confidentiality and standstill agreements and obtain the Confidential Information Memorandum (CIM), which contains some non-public information on the target company such as divisional data, order backlogs, proprietary contracts, and the R&D status. After the pre-

[4] The confidentiality agreement governs how buyers can use the information obtained and the standstill agreement precludes the prospective buyer from making unsolicited offers or purchasing the target's shares etc., in a specified period of time.

[5] The Regulation Fair Disclosure mandates that all publicly traded companies must disclose material information to all investors at the same time. According to this rule, the prospective buyers may not even know which company is on sale before signing the confidentiality and standstill agreements because it may constitute selective disclosure of material information, e.g., that the company is for sale, only to the contacted prospective buyers. See discussion in Hansen (2001).

liminary due diligence based on CIM, if further interested, a bidder submits a non-binding indication of interest that specifies a number of elements: (a) indicative purchase price (typically presented in a range) and form of consideration (cash vs. stock mix); (b) key assumptions to arrive at the stated indicative purchase price; (c) information on financing sources; (d) treatment of management and employees; (e) key conditions to signing and closing. For the research purpose of this paper, we consider the bidders indicating their interest as the *potential bidders* because only a few of them will be invited by the seller to the final round of comprehensive due diligence after reviewing their indications of interest.[6] These *final bidders*, who are invited, gain the most confidential materials through on-site visits, consultations with target management, etc. The seller's shortlisting decision depends on various considerations, such as speed to the transaction closing, fulfillment of fiduciary duties, disruption of business, and confidentiality concern. Note that the concerns on dissemination of confidential information is particular to takeover auctions and is the focus of this study. For example, in the sale of Spinnaker Exploration Company, the seller's financial advisor Randall & Dewey suggested that the company adopt a "*limited marketing approach as it minimized the exposure of Spinnaker's sensitive confidential information to a smaller group of competitors*" because "*the likely buyers for Spinnaker were all competitors.*"[7]

Following the due diligence, the final bidders submit their binding bids by the due date. The seller then analyzes the final bids and selects the winner to work on the final definitive agreement, which includes the purchase price, the method of payment, various deal protection devices, and fiduciary-out provisions. Upon signing the M&A agreement, the seller and buyer publicly announce the deal, which ends the takeover process.[8]

[6] The definition of *potential bidders* varies to serve specific research agendas. For example, Gentry and Stroup (2015) refer to the contacted prospective buyers as potential bidders in order to model their (entry) decisions on whether to sign the confidentiality agreement. Our definition of *potential bidders* serves our purpose to understand sellers' shortlisting behaviors.

[7] The *italic* texts are extracted directly from the *DEFM14A* document filed by Spinnaker to SEC on November 10, 2005.

[8] The announcement could be followed by a public takeover battle for the target. In practice, less than 4 percent of deals are subject to public competition; see Moeller, Schlingemann, and Stulz (2007). Moreover,

Throughout the entire process, the seller communicates with each final bidder to facilitate the bidders evaluation on the target and also to draw the best outcome from the deal. However, the seller keeps the final bidders uninformed of other bidders participation and their competing bids; see Gorbenko and Malenko (2014). Even if the seller might tell bidders about competition status, such information would not be verifiable because bidders do not observe the sellers interaction with other bidders, and it is not even reliable because all tentative offers are frequently modified and can even be withdrawn before the final definitive agreement. This is well supported by a number of SEC filings, e.g., some cases report that a bidder, the only invited one, kept modifying her bids and other cases show that bidders were revising their bids but all below the (tentative) highest standing offer.[9]

I.3 Takeover Auction Model

Based on the institutional background described above, we develop an auction model to approximate bidders' behavior in a takeover process. Given our research objective to understand bidders' value discount due to the release of confidential information, we can abstract away their entry decision, especially, the decision on whether to sign the confidentiality agreement, and only model their bidding behavior.[10]

I.3.1 Model Setup

As noted in I.2, the final bidders are uniformed of the number of competitors and their bids. To develop an auction model that captures this crucial informational structure, it is

when it happens, the seller compensates the original winner following the final agreement. Note also that Boone and Mulherin (2007), Gorbenko and Malenko (2014), and Gentry and Stroup (2015) study only the private takeover stage.

[9] From our conversation with an industry personnel dealing with M&A deals, moreover, we confirm that the seller does not release information on other bidders identities or tentative offers during the takeover process.

[10] Note also that Gentry and Stroup (2015) explicitly model this entry decision in their model where bidders who enter choose optimal bids using the value distribution conditional on their signal being larger than the cut-off point for entry. This conditional distribution amounts to the distribution of synergy value in this paper.

necessary to model bidders' perception on competition intensity when they are shortlisted for the final stage. As mentioned above, the seller's shortlisting decision depends on many factors, some of which can be inferred from the observed target characteristics, but others are private to the seller. Therefore, we assume that the final bidders regard the shortlisting rule as a random process and perceive it through learning the number of potential bidders N and the shortlisting probability

$$p_{n|N} := \Pr(n \text{ bidders are invited out of } N \text{ potential bidders })$$

with aids from their financial advisors. Since the financial advisors employ similar databases and analysis methods to learn $\{p_{n|N}, N\}$, we further assume that $\{p_{n|N}, N\}$ are common knowledge among the final bidders.[11] For theoretical developments in this section, we suppress the dependence on N for notational simplicity.

Moreover, since the bidders do not observe their opponents' bids, we employ the first price sealed-bid auction to specify bidders bidding behavior, where ex-ante identical bidders integrate out the uncertainty about the number of opponents by the shortlisting rule p_n and maximize their expected utility by choosing a final bid. In this paper, we regard bidders frequent modifications of their tentative bids and all the interactions with the seller outlined in I.2 as a complicated process, which is part of the final due diligence, where they collect data on the target to learn synergy values. We therefore consider only the final definitive offers as the submitted bids in the model.

To be specific, we focus on bidder i in the final bidding stage. After the comprehensive due diligence, she privately discovers her synergy value and discounts it to reflect the potential damage of competitive information revelation. To this end, we express bidder i's net

[11] A major method adopted by the investment bank is the precedent transactions analysis, where the information of the previous M&A transactions are collected from SEC filings and business databases such as SDC Platinum. See Chapter 2 of Rosenbaum and Pearl (2009) for more information. Note that we use the same data sources as illustrated in the previous section.

utility when she wins the auction with n final bidders as

$$u_i = v_i - D(n, v_i), \tag{I.1}$$

where v_i is her *synergy value* upon merging with the target and $D(n, v_i)$ denotes her *information disclosure discount*. We consider the synergy value as private information to the bidder because it reflects the bidder's opportunity costs of merging with the target and business compatibility that is particular to each bidder-target combination. We also assume that $(v_1, \ldots, v_n)$ are independent draws from the valuation distribution F_v with density f_v supported on $[\underline{v}, \overline{v}] \subset \mathbb{R}_+$, which is common knowledge among the final bidders. Note that the bidder symmetry can be justified because bidders do not know the identities of their opponents. Moreover, we maintain the following assumption regarding the structure of information disclosure discount.

ASSUMPTION I.1. *$D(n, v)$ satisfies*

i. $D(n, v)$ is increasing in n;

ii. $D(n, v)$ is increasing in v, but not faster than the identity;

iii. $D(n, \underline{v}) = 0$.

Assumption I.1 items (i) and (ii) reflect that a greater discount arises either as more competitors gain the confidential information or as a larger synergy value is at stake; item (ii) preserves the order of the bidders' valuations in the presence of the information disclosure discount, i.e., $v_i > v_j$ implies $u_i > u_j$; and item (iii) normalizes the discount on the lower boundary to be zero.

I.3.2 Equilibrium Bidding Strategy

In order to derive the Bayesian equilibrium for the game induced by the auction, we consider bidder i's optimal bidding. Suppose that all other bidders follow a strictly increasing bidding strategy $\beta(\cdot)$. Since $\{F_v, D, N, p_n\}$ is common knowledge in the bidding game, we write bidder i's expected payoff of bidding b as

$$\begin{aligned}\pi(b,v) &= \sum_{n=1}^{N} p_n \left[\prod_{j=1, j\neq i}^{n} Pr(\beta(v_j) \leq b)(v - D(n,v) - b) \right] \\ &= \sum_{n=1}^{N} p_n F_v(\beta^{-1}(b))^{n-1}(v - D(n,v) - b).\end{aligned}$$

By taking a derivative of this payoff function with respect to b, we obtain the first-order necessary condition for the bidder's optimization problem as

$$\sum_{n=1}^{N} p_n F_v(v)^{n-1} \left(\frac{\partial}{\partial v} \beta(v) \right) = \sum_{n=2}^{N} p_n (n-1) F_v(v)^{n-2} f_v(v) \left(v - D(n,v) - \beta(v) \right). \qquad \text{(I.2)}$$

Solving the differential equation (I.2), we obtain an analytical expression for $\beta(v)$. In order to simplify the expression, we denote bidder i's equilibrium winning probability by $H(v) := \sum_{m=1}^{N} p_m F_v(v)^{m-1}$ and its derivative by $h(v) := dH(v)/dv = \sum_{m=2}^{N} p_m (m-1) F_v(v)^{m-2} f_v(v)$. We then find the solution to (I.2) as

$$\begin{aligned}\beta(v) &= \frac{H(\underline{v})}{H(v)} \beta(\underline{v}) + \sum_{n=2}^{N} \frac{p_n}{H(v)} \int_{\underline{v}}^{v} (t - D(n,t)) dF_v(t)^{n-1} \\ &= \frac{p_1}{H(v)} \underline{v} + \sum_{n=2}^{N} \frac{p_n F_v(v)^{n-1}}{H(v)} \int_{\underline{v}}^{v} (t - D(n,t)) d \left[\frac{F_v(t)}{F_v(v)} \right]^{n-1},\end{aligned}$$

for which we use the boundary condition, $\beta(\underline{v}) = \underline{v}$: the bidder with $\underline{v}$ always bids $\underline{v}$, because any bid lower than $\underline{v}$ is not accepted and any bid higher than $\underline{v}$ exceeds her value. We may write $\beta(v)$ in a more intuitive form. To do so, we define the contingent bidding function

$$\widetilde{\beta}_n^I(v) := \begin{cases} \underline{v}, & \textbf{for } n = 1, \\ \int_{\underline{v}}^{v} (t - D(n,t)) d \left[\frac{F_v(t)}{F_v(v)} \right]^{n-1}, & \textbf{for } n = 2, \ldots, N. \end{cases}$$

Notice that when there is no information disclosure discount, i.e., $D(v,n) = 0$, $\widetilde{\beta}_n^I(v)$ for $n \geq 2$ is the optimal bidding strategy in the first price auction where the bidders know n. Hence, we consider $\widetilde{\beta}_n^I(v)$ as the bidding strategy that incorporates the value discount for each contingent n. Using this expression, then, we can re-write $\beta(v)$ as a weighted average of the contingent bidding function;

$$\beta(v) = \sum_{n=1}^{N} w_n(v)\widetilde{\beta}_n^I(v). \tag{I.3}$$

where $w_n(v) := p_n F_v(v)^{n-1}/H(v)$ is a conditional winning probability, i.e., $w_n(v) \geq 0$ for all n and v and $\sum_{n=1}^{N} w_n(v) = 1$ for all v by construction.

Now we establish the existence of a symmetric Bayesian Nash Equilibrium (BNE) characterized by the bidding strategy (I.3) by showing no bidder can profitably deviate from it when all rival bidders follow it.

PROPOSITION I.1. *Suppose that the bidding strategy (I.3) is strictly increasing in v, then* $\beta(v;N)$ *characterizes a symmetric monotone BNE.*

Proof. See Appendix A.1. □

I.3.3 Monotonicity of Equilibrium Bidding Strategy

The existence of BNE relies on the presumption of the strict monotonicity of $\beta(v)$, which does not follow the strict monotonicity of $\widetilde{\beta}_n^I(v)$ because the weights $w_n(v)$ may not be monotone and some of $w_n(v)$ even may decrease. We do not establish a sufficient and necessary condition for the monotonicity at the level of model primitives. Instead, we present a sufficient condition that only involves a reduced form parameter p_n to verify that there exists a set of structural parameters under which $\beta(v)$ is strictly increasing. This condition generally restricts p_n but it always holds for $N = 2$ and 3.

PROPOSITION I.2. *The bidding strategy (I.3) is strictly increasing if for any N and $m \leq N$,*

$$p_1 \geq \mathbb{1}(N \geq 3) \times \sum_{m=3}^{N} (m-2) p_m. \tag{I.4}$$

Proof. See Appendix A.2. □

I.4 Information Cost: An Illustration

Recall that the *information cost* is defined as the expected premium loss due to bidders' information disclosure discounts, which can be formally expressed as follows,

$$IC(v;N) = \sum_{n=1}^{N} w_n(v) D(n,v),$$

for a takeover auction with N potential bidders and a winner with a synergy value v.

Figure I.1 presents the equilibrium bidding strategies with and without considering the information disclosure discounts, and thus the information costs, for the following three hypothetical examples:

- **Example 1:** $N = 2$, $[p_1, p_2] = [0.7, 0.3]$ and $[D(1,v), D(2,v)] = [0, 0.10]$;
- **Example 2:** $N = 3$, $[p_1, p_2, p_3] = [0.5, 0.3, 0.2]$ and $[D(1,v), D(2,v), D(3,v)] = [0, 0.10, 0.15]$;
- **Example 3:** $N = 4$, $[p_1, p_2, p_3, p_4] = [0.4, 0.3, 0.2, 0.1]$ and $[D(1,v), D(2,v), D(3,v), D(4,v)] = [0, 0.10, 0.15, 0.20]$,

and the synergy value distribution F_v follows a mixture of Beta and Uniform distributions. Specifically, $v \sim \frac{3}{4}\mathscr{B}eta(2,4) + \frac{1}{4}\mathscr{U}(0,1)$.

Given the monotone equilibrium bidding strategy, the information cost is increasing with respect to the synergy value. The magnitude depends on the specification of the underlying primitives, i.e., p_n, F_v and $D(n,v)$. In Chapter II, we will identify and recover these underlying primitives empirically.

I.5 Conclusion

This chapter develops a game-theoretic model to approximate acquirors' bidding behavior in a takeover process, accounting for their valuation discount due to the dissemination of confidential information and their uncertainty about the number of actual opponents. We derive a symmetric Bayesian Nash equilibrium with a strictly increasing bidding strategy, and provide a sufficient condition for the monotonicity of the equilibrium bidding strategy. Finally, we construct the information cost resulting from acquirors' information disclosure discounts and illustrate how it affects the sellers expected revenue.

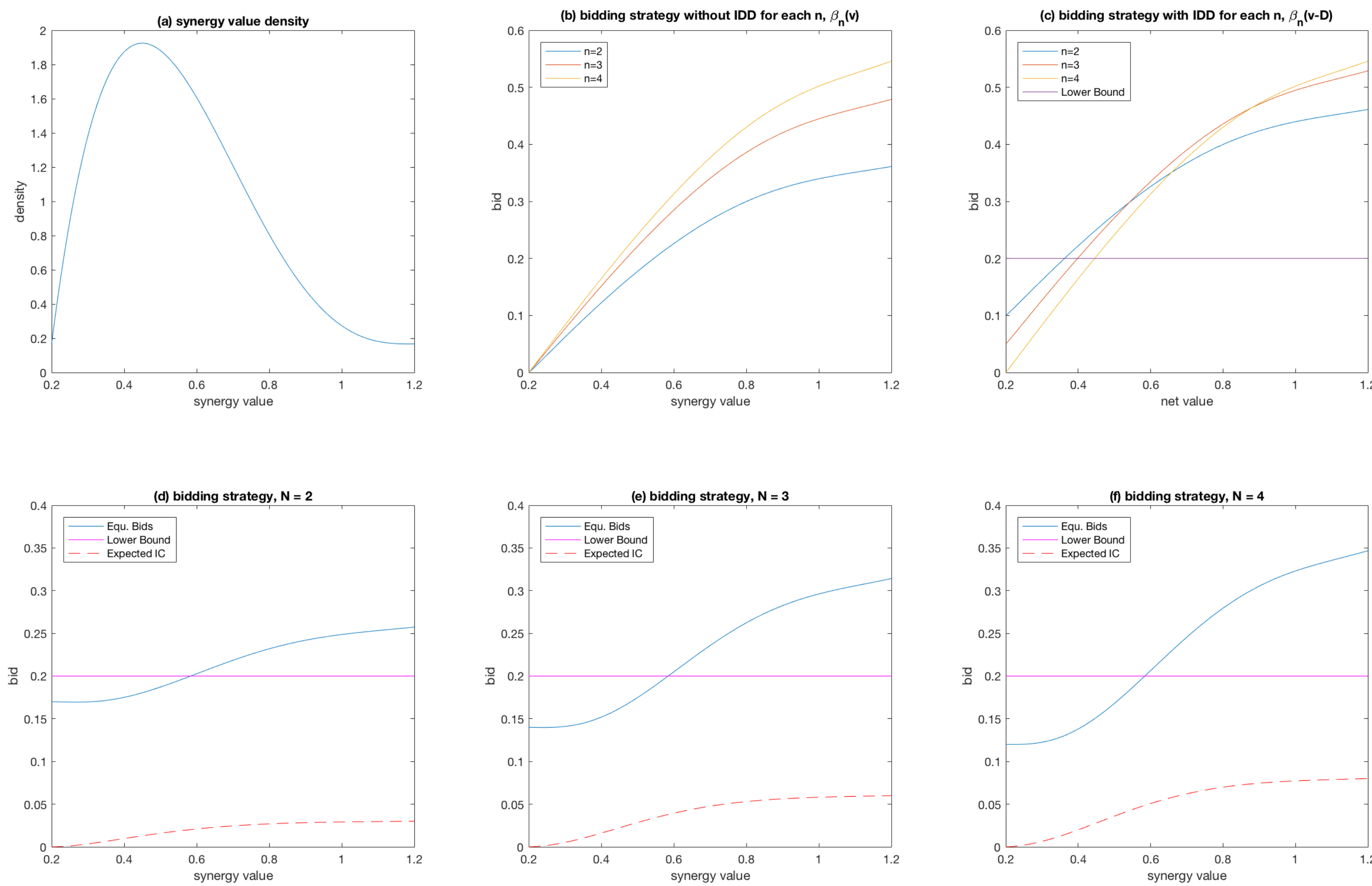

Figure I.1: Equilibrium Bidding Strategy $\beta(v)$ and Expected Information Cost.

Chapter II

HOW COSTLY TO SELL A COMPANY? A STRUCTURAL ANALYSIS OF TAKEOVER AUCTIONS

II.1 Introduction

The literature of the mergers and acquisitions (M&A) has reported that sellers of takeover auctions routinely restrict bidders' entry. In particular, Boone and Mulherin (2007) document that about a half of the company sales in their M&A sample invite one acquiror and even when they do more than one, they allow only a few. This observation seems to contradict the received theory: competition raises the seller's (expected) revenue.[1] We explain the entry regulation by the seller's costs to conduct a takeover auction.

We consider two well-documented costs: *operation cost* and *information cost*, that together affect sellers' shortlisting decisions. The operation cost includes advisory fees and other economic costs resulting from disruption of business, negative impacts on the employee morale, and foregoing of business opportunities, etc., see Rosenbaum and Pearl (2009) and DePamphilis (2014). While the operation cost reflects sellers private information on their own business, the information cost arises from bidders evaluation process and bidding behavior, which we is the focus of Chapter I. Specifically, a bidder would discount her synergy value whenever other bidders gain confidential information on the target because they are mostly industry rivals who may strategically exploit the competitive materials. Hence, bidders would lower their bids, and this causes a premium loss for the seller, which we call the information cost. Notice that the information cost is a unique feature of takeover auctions where the confidential information plays a crucial role in and even after the auction.

[1] Bulow and Klemperer (1996) show, for example, that no amount of bargaining power is as valuable to the seller as attracting one extra bona fide bidder. There is a range of situations where this claim can fail, e.g., auctions with almost common value in Klemperer (1998) and auctions with voluntary entry in Li and Zheng (2009). The takeover auction, however, does not fit into those frameworks.

Based on the game-theoretic model developed in Chapter I, we first establish the identification of the auction model allowing for the unobserved heterogeneity, which should be critical in empirical investigation of takeover auctions because the confidential information is latent by nature. In particular, we use the deconvolution method of Kotlarski (1966) to identify the distribution of unobserved heterogeneity by the within-auction bid variation.[2] After separating out the unobserved heterogeneity, we identify the distribution of synergy values and the information disclosure discount by the cross-auction bid variation and the exogenous variation of the number of potential bidders.

Integrating out the unobserved heterogeneity in the Bayesian framework, we analyze a sample of 287 takeover auctions of U.S. public companies that took place between 2000 and 2008. We find that the bidders lower their synergy value by 11.9 percent (posterior mean) for each additional opponent joining the bidding competition. In addition, after controlling for the observed characteristics, the unobserved auction heterogeneity accounts for 91.07 percent of the bid variation, corroborating the importance of the unobserved heterogeneity in empirical investigation of takeover auctions.

Our structural approach allows us to disentangle the information cost and competition effect via counterfactual analysis. The result indicates that the information cost is substantial; for example, the information cost lowers the predictive takeover revenue by \$6.4 million, for the average target company with market value of \$459 million in the consumer industry when the seller chooses 2 final bidders out of 4 potential bidders. Furthermore, we bound the seller-specific operation cost by contrasting the theoretical prediction on the seller's revenue against the observed shortlisting pattern. In the same example, we find that the operation cost is at least 4.1% of the transaction value, which is much more substantial than documented advisory fees: Hunter and Jagtiani (2003) report that advisory fees are on average 0.84% of the transaction value.

[2] Li and Vuong (1998) first introduce Kotlarski (1966) to identify models with measurement error. Using this technique, Li, Perrigne, and Vuong (2000) identify a class of auctions with conditionally independent private values and in a different context, Krasnokutskaya (2011) identifies auction models with unobserved heterogeneity.

The main contribution of this paper is to employ a structural method to measure the information cost and operation cost incurred by the seller in a sale of company. Many studies in corporate finance use the existence of the information cost to explain some stylized features in M&A transactions. For example, Boone and Mulherin (2007) find that negotiations and auctions accrue similar transaction premiums and suggest that the presence of information cost may explain the choice of sale mechanism. Schlingemann and Wu (2015) find that the likelihood of choosing auction over negotiation decreases in R&D intensity, which may proxy the cost of disclosing proprietary information. These studies, however, take a reduced form approach that cannot measure the magnitude of the information cost or disentangle the information cost from the competition effect on the revenue, as it is essentially structural.

There are a few articles that use a structural approach to study other aspects of takeover competitions. Taking the pool of acquirors as given, Gorbenko and Malenko (2014) estimate an auction model to study how strategic and financial bidders evaluate target companies and find that different targets appeal to different groups of bidders. Gentry and Stroup (2015) model potential acquirors' entry decision, i.e., a decision on whether to sign confidentiality agreements, and study how the pre-entry uncertainty on the target value affects the bidders' entry and bidding behavior. Unlike those papers, we focus on the seller's active role in determining the pool of final competitors and study how the seller's shortlisting affects bidding behavior, from which we can recover the costs incurred by the seller.

Moreover, none of the aforementioned empirical papers formally consider unobserved target heterogeneity. The confidential information is crucial in our analysis, as it is the source of the information cost, but it is latent by nature. We therefore take into account unobserved target heterogeneity in our econometric analysis and find that it is indeed economically significant in studying M&A transactions, as it explains 91.07% of the variation of the synergy value in our sample.

The paper proceeds as follows. Section II.2 introduces a sample of 287 M&A deals of U.S. public companies and conducts a preliminary analysis on the data. Assuming the observed bid data is generated from the BNE characterized in Chapter I, Section II.3 establishes the identification of the structural model with unobserved target heterogeneity. Section II.4 proposes a Bayesian method to estimate the structural parameters and Section II.5 reports the empirical results. Section II.6 conducts a range of counterfactual analyses to quantify the information cost and operation cost and to exam an alternative sale mechanism. Section II.7 concludes by discussing policy implications.

II.2 Takeover Auction Data

In this section, we first elaborate the construction of our dataset from various databases.[3] Then we conduct a reduced-formed investigation of the information cost hypothesis.

II.2.1 Data Description

The sample comes from the Mergers and Acquisitions database of the Securities Data Corporation (SDC) and consists of 287 takeover deals that were announced and completed between January 1, 2000 and September 6, 2008 and meet the following criteria:

(i) The target is a publicly traded non-financial (SIC codes 6000 - 6999 excluded) U.S. company.

(ii) The winning bid is made in cash only.

(iii) The winning bidder obtains 100% of target shares after transaction.

(iv) The deal is not a spin-off, recap, self-tender, exchange offer, repurchase, minority stake purchase, acquisition of remaining interest, or privatization.

[3] We thank Alexander Gorbenko and Andrey Malenko for sharing their data used in Gorbenko and Malenko (2014). Note that, following their instruction, we replicate the construction of the explanatory variables from the commercial databases of Compustat and CRSP. Moreover, in order to serve our research purpose, from the SEC filings, we collect information on the numbers of bidders at each stage of the takeover process and identify the cash-only bids for the final bidders.

(v) The deal is fulfilled by an auction, which we define as a deal with at least two buyers showing indications of interest.

(vi) The deal background is available in SDC, or EDGAR filings on the SEC, or MergerMetrics.

(vii) Quarterly financial data on the target is available on the Compustat database.

Conditions (i)-(iv) are imposed to make takeover deals and bids comparable. First, M&A deals from the financial sector are excluded because its valuation is fundamentally different from that for non-financial firms. For example, a high leverage normal to financial firms is an indicator of great distress to firms in other industries.[4] Second, the winning payment is cash-only, in which case the deal value is known with certainty. This excludes other payment arrangements whose values could depend on unobserved factors. For instance, in a "stock-for-stock" merger, the deal value depends on the unobserved winner's characteristics, and thus cannot be reliably compared to a cash-only deal or other bids involving payments in stocks. Third, the winner eventually owns 100% share of the target so that it is a full-scale merger rather than an equity investment. Finally, we exclude deals with motivations other than a business combination.[5]

Structural Variables from SEC Filings

For each takeover in the sample, we collect additional information from the background sections in filings of the US Securities and Exchange Commission (SEC).[6] In particular, we observe the bidder participation in each stage and the competing bids for each takeover

[4] Therefore, it is a common practice in corporate finance research to separate financial firms from others; e.g., Fama and French (1992).

[5] It is arguable that the true value of a bid depends on all terms of the final merger proposals, and thus is different from the cash value of the bid. We argue that, in the context of our structural model, this deviation reflects measurement error, which is captured as bidders' heterogeneity.

[6] Boone and Mulherin (2007) first use the hand-collected data from SEC filings to study the takeover competition. Deal backgrounds are contained in SC-TOT, 14D-9, PREM14C, DEFM14C, DEFS 14A, and S-4 filings, available at http://www.sec.gov/edgar/searchedgar/companysearch.html.

auction.[7]

First, we measure bidders' participation in two stages: (i) N: the number of potential bidders, i.e., the ones showing indications of interest; (ii) n: the number of final bidders, i.e., the ones shortlisted for the final due diligence. Note that our theoretical model is built upon N and n. Second, we complement the bid data by collecting all the reported losing cash-only bids from the SEC documents.[8] Following other empirical studies on M&A, we normalize each bid by the corresponding target stock price four weeks before the announcement date, and use the resulting (raw) bid premiums in our empirical study.[9]

Explanatory Variables from Compustat and CRSP

We use the same set of explanatory variables as in Gorbenko and Malenko (2014) to control for the observed target heterogeneity and the economic conditions. In particular, we include the target specific variables constructed from the Compustat database: (i) *Size* is the firm size defined as the book value of the total assets (in millions); (ii) *Leverage* is the market leverage defined as the ratio of the book value of debt to the sum of the market value of equity; (iii) *Q-ratio* is defined as the ratio of the sum of the market value of equity and the book value of debt to the book value of the target; (iv) *CashFlow* is the cash flow over the last four quarters; (v) *Cash* is the cash balance, i.e., the sum of cash, short-term investments, and marketable securities; (vi) *R&D* is the R&D expenses; and (vii) *Intangibles* represents accounting measure for the intangible assets; (viii) *Industry* is the industry category as defined by Fama and French (1997).[10] In addition, we scale the *CashFlow*, *Cash*, *R&D* and *Intangibles* by *Size*. Standard filters are applied to exclude unreasonable values of these covariates that are likely to be mistakes. Specifically, we exclude observations with

[7] SDC provides the final deal value, i.e., winning bid, for each auction; while SEC filings often document other (losing) bids.

[8] We exclude some anomalous losing cash-only bids from the sample because of its incomparability to other bids for various reasons: (1) bids are subject to conditions; (2) bidders cannot provide sufficient financing proofs; (3) bidders quit after winning the auction; etc. In some of these cases, the losing cash bids could be higher than the winning cash bids.

[9] See Eckbo and Thorburn (2009) for a discussion on the choice of base prices.

[10] The Fama-French five industry classification is used because the twelve industry classification results in few observations for some industries.

market leverage below zero and above 100%; Q-ratio in excess of 10; cash flow in excess of 10; and negative cash.

We also include the two economy-wide variables constructed from the Center for Research in Security Prices (CRSP): (i) *MarketReturn* is the market return defined as the cumulative return on the S&P 500 index over the 12 months prior to the announcement date; and (ii) *CreditSpread* is the credit spread defined as the rate on Moody's Baa bonds preceding the announcement date minus the rate on 10-year Treasury bonds on the announcement date.

Finally, our sample consists of 287 takeover auctions with a total of 372 observed bids and 193 auctions with winning bid only. Table II.1 reports some descriptive statistics of the auction and target characteristics for the full sample and across the Fama-French five industries. Panel A first confirms the common practice of entry restriction in all industries; the average invitation rate ranges from 62% to 74% across the five industries. In addition, we observe substantial takeover premiums across the industries, especially in the HighTech, Health, and Others.

Panel B investigates the accounting features of the target companies. The target firms in HighTech and Health industries appear to be growth companies, as indicated by their high Q-ratio, low leverage, and large cash balance. Companies in these two industries also stand out in their R&D spending and percentage of intangible assets.

II.2.2 Regulating Entry: An Information-Cost Hypothesis

As mentioned in Section I.2 of the institutional background, many factors affect sellers' shortlisting decisions. To better understand sellers' entry regulation, we consider a Probit regression (Model I) where the dependent variable is a dummy variable equal to 1 if a potential bidders is shortlisted by the seller and 0, otherwise. Model I includes a set of observed target characteristics and economic conditions described above. To see whether the result of Model I is driven by industry effects, Model II expands Model I to include

Table II.1
Descriptive Statistics of Auction and Target Characteristics

This table reports descriptive statistics (mean, standard deviation, and median) of the auction and target characteristics for the full sample across five industries as classified by Fama and French (1997). Within the five-industry classification, *Consumer* includes consumer durables, nondurables, wholesale, retail, and some services (laundries, repair shops); *Manufacturing* includes manufacturing and energy; *HighTech* includes business equipment, telephone, and television transmission; *Health* includes healthcare, medical equipment, and drugs; and *Others* includes mines, construction, construction materials, transportation, hotels, business services, and entertainment. Panel A reports the means and medians (in brackets) of N (the number of potential bidders), n (the number of final bidders), and n/N (the invitation rate), and the means and standard deviations (in parentheses) of the takeover premium (winning bid scaled by the target size). Panel B reports the means, medians (in brackets), and standard deviations (in parentheses) of the target characteristics. The sample contains 287 takeover auctions took place between January 1, 2000 to September 6, 2008.

	All	Consumer	Manufacturing	HighTech	Health	Others
Panel A						
N (Potential)	4.7	5.1	6.2	4.3	3.5	4.8
	[3]	[4]	[6]	[3]	[3]	[4]
n (Final)	2.7	2.8	3.1	2.6	2.5	2.7
	[2]	[2]	[2.5]	[2]	[2]	[2]
n/N (Invitation Rate)	0.68	0.62	0.62	0.70	0.74	0.70
	[0.67]	[0.62]	[0.65]	[0.71]	[0.88]	[0.75]
Premium	1.34	1.27	1.22	1.38	1.37	1.40
	(0.30)	(0.30)	(0.21)	(0.31)	(0.30)	(0.28)
Panel B						
Size (in millions)	659.0	719.6	731.6	569.5	435.8	975.5
	[183.72]	[287.48]	[161.58]	[101.39]	[238.71]	[380.55]
	(2345.0)	(1400.8)	(1898.6)	(3103.2)	(528.5)	(2449.3)
Leverage	0.15	0.23	0.19	0.07	0.13	0.27
	[0.04]	[0.13]	[0.09]	[0.00]	[0.02]	[0.21]
	(0.22)	(0.26)	(0.21)	(0.13)	(0.22)	(0.26)
Q-ratio	1.53	1.14	1.28	1.53	2.69	1.17
	[1.26]	[0.97]	[1.26]	[1.32]	[2.01]	[0.91]
	(1.17)	(0.62)	(0.50)	(0.99)	(1.99)	(0.81)
CashFlow	0.02	0.07	0.11	-0.03	-0.02	0.05
	[0.07]	[0.08]	[0.12]	[0.05]	[0.05]	[0.08]
	(0.26)	(0.10)	(0.08)	(0.35)	(0.24)	(0.21)
Cash	0.25	0.10	0.09	0.37	0.31	0.18
	[0.18]	[0.06]	[0.04]	[0.34]	[0.25]	[0.07]
	(0.23)	(0.09)	(0.13)	(0.22)	(0.25)	(0.24)
R&D	0.02	0.00	0.00	0.03	0.03	0.00
	[0.00]	[0.00]	[0.00]	[0.02]	[0.02]	[0.00]
	(0.03)	(0.00)	(0.01)	(0.04)	(0.04)	(0.00)
Intangibles	0.15	0.09	0.09	0.19	0.17	0.13
	[0.06]	[0.04]	[0.03]	[0.11]	[0.09]	[0.00]
	(0.19)	(0.15)	(0.14)	(0.19)	(0.22)	(0.20)
# of obs.	287	57	36	113	40	41

industry dummies following the five-industry classification of Fama and French (1997). Given the panel structure of our data, we further run a target-specific Random-Effect Probit regression (Model III) to control for the unobserved heterogeneity.[11]

The regression results in Table II.2 show that several target characteristics help to predict sellers' shortlisting decisions. Among those explanatory variables, we are particularly interested in the impact of intangible assets, representing accounting measure of many confidential items such as patents and trade secrets. A higher value of intangible assets suggests that the confidential information is more important to the target value.[12] All three regressions show significantly negative effects of intangible assets on the shortlisting probability, which supports our premise that the seller restricts bidders' participation concerning the dissemination of the confidential information.

II.3 Identification of the Structural Model

In this section, we study the identification of the takeover model, in which the observed bids are generated from the BNE described in Proposition I.1. As noted earlier, the final bidders observe the confidential information through the due diligence, but since it is confidential, the econometrician cannot observe it. In order to handle the problem of unobserved heterogeneity, we let (X, τ) be the vector of auction characteristics that the bidders observe in the final stage, where $X \in \mathscr{R}^k$ is observed by the econometrician but $\tau \in \mathscr{R}_+$ is not, i.e., unobserved heterogeneity. Since all the identification arguments are conditional on observed heterogeneity, we suppress the notational dependence on X in this section.

We consider a two stage valuation updating process: in the first stage, bidder i observes public information X and obtains an initial assessment ν_i on her synergy value v_i; upon

[11] For a given auction, a repeated sample of the shortlisting decision on each potential bidder is observed. The unobserved target-specific heterogeneity reflects the seller's private knowledge on the target and private consideration for the auction, e.g., the speed of sale and the disruption of the business. For a given auction, we treat the auction as an individual and regard the seller's shortlisting decision on each potential bidder as a response in different time.

[12] For example, it is well known that Coca-Cola has its own formula for their products, which is confidential. Its market value is recorded in intangible assets.

Table II.2
Determinants of Shortlisting Decisions

This table reports estimation results for three Probit regressions on shortlisting decisions, i.e., the dependent variable for all the regression models is the indicator for each potential bidder to be shortlisted. Model I includes, as regressors, all the observed target characteristics and market conditions, but not including industry dummies. Model II expands Model I by including industry dummies following the five-industry classification of Fama and French (1997). Model III runs a target-specific Random-Effect Probit regression. *t* statistics are in parentheses beneath the coefficient estimates and statistical significance is indicated by ***, **, and * for the 0.01, 0.05, and 0.10 levels. The sample size (total numbers of potential bidders from 287 takeover auctions) is $1,339$. The sample covers January 1, 2000 to September 6, 2008.

	Probit Model I	Probit Model II	Probit Model III (Random-Effect)
Industry-1 (*Consumer*)	-0.840***	-0.438	-0.246
	(-2.67)	(-1.29)	(-0.52)
Industry-2 (*Manufacturing*)	-0.874***	-0.273	-0.046
	(-2.79)	(-0.80)	(-0.10)
Industry-3 (*HighTech*)	-0.525*	-0.219	-0.045
	(-1.66)	(-0.64)	(-0.10)
Industry-4 (*Health*)	-0.530	-0.348	-0.185
	(-1.48)	(-0.92)	(-0.35)
Industry-5 (*Others*)	-0.645**	-0.258	-0.030
	(-2.01)	(-0.74)	(-0.06)
Log(Size)	0.119***	0.126***	0.126***
	(4.00)	(4.05)	(2.97)
Leverage	0.197	0.144	0.163
	(1.05)	(0.76)	(0.63)
Q-ratio	0.195***	0.122**	0.132**
	(3.80)	(2.48)	(2.35)
CashFlow	-0.412*	-0.405*	-0.546*
	(-1.78)	(-1.73)	(-1.76)
Cash	-0.235	-0.035	0.035
	(-1.05)	(-0.15)	(0.12)
R&D	2.057	0.988	0.712
	(1.12)	(0.67)	(0.57)
Intangibles	-0.883***	-0.643***	-0.579**
	(-4.16)	(-2.96)	(-2.07)
Market Return	0.110	0.734*	0.569
	(0.27)	(1.71)	(1.07)
Credit Spread	5.676	15.626	11.495
	(0.66)	(1.64)	(0.91)
N (# of potential bidders)		-0.092***	-0.105***
		(-9.75)	(-9.04)

being shortlisted to the final stage, she examines the confidential information that updates her value to $v_i = \tau \cdot \nu_i$. This structure then corresponds to the specification of the unobserved heterogeneity of Krasnokutskaya (2011). Since the synergy value is unique to each bidder-target pair, we consider ν_i as private information for bidder i. Moreover, since τ represents the confidential information, we assume that it is orthogonal to the first-stage estimates $\{\nu_1, \ldots, \nu_n\}$. Furthermore, we assume that the number of potential bidders N is exogenous to $(\tau, \{\nu_1, \ldots, \nu_n\})$ because an acquiring company's decision to make indication of interest, i.e., becoming a potential bidder, largely depends on her private opportunity costs, e.g., alternative investment opportunities. Formally, we make the following assumption,

ASSUMPTION II.1. *For each auction ,*

(i) *The joint distribution of* $(\tau, \nu_1, \ldots, \nu_n)$ *has the structure of*

$$F(\tau, \nu_1, \ldots, \nu_n | N) = F_\tau(\tau) \prod_{i=1}^{n} F_\nu(\nu_i),$$

where $F_\tau(\cdot)$ *is the marginal distribution of* τ *with* $\tau \in [\underline{\tau}, \overline{\tau}]$ *and* $\underline{\tau} > 0$*; and* $F_\nu(\cdot|N)$ *is the marginal distribution of* ν_i *with* $[\underline{\nu}, \overline{\nu}]$ *and* $\underline{\nu} > 0$*;*

(ii) $D(n, v_i) = D(n, \tau \cdot \nu_i) = \tau \cdot D(n, \nu_i)$ *for all* $n \in \{2, \ldots, N\}$*;*

(iii) $p_{n|N} = p_{n|N,\tau}$, *for all* $n = \{1, \cdots, N\}$.

Assumption II.1 item (i) combines the discussion above and our model assumptions in section I.3, item (ii) imposes the multiplicative structure on the information disclosure discount in the same way as the value, and item (iii) states that τ does not affect bidders' estimation of the shortlisting probability. As described in section I.3, bidders obtain the common knowledge of the shortlisting probability $p_{n|N}$ by studying precedent transactions using public information, i.e., X. Since the unobserved heterogeneities of these precedent takeovers are not publicly available, bidders do not know how τ affects the seller's shortlisting decision, and therefore, cannot update $p_{n|N}$ through τ.

Our objective is to examine if we can learn (F_ν, F_τ, D) from the data $(b_1, \ldots, b_n, N, n)$. Since the identification is a large sample statement, we assume that we have the bid distribution $G(b|N)$ with density $g(b|N)$ for all N in the sample. The identification consists of two steps: we first identify $F_\tau(\cdot)$ by exploring the within-auction bid variation and then identify $F_\nu(\cdot)$ and $\{D(n,\cdot)\}_{n=2}^N$ by the cross-auction bid variation and the exogenous variation of N.

First, we identify $F_\tau(\cdot)$ by deconvoluting the joint bid distribution following Krasnokutskaya (2011). Since a model with (τ, ν) and a model with $(\tau/2, 2\nu)$ generate the same bid distribution, a location normalization is introduced.

ASSUMPTION II.2. $\log\tau$ ***has a non-vanishing characteristic function with*** $E[\tau] = 1$**.**

The location normalization $E[\tau] = 1$ of Assumption II.2 is consistent with our interpretation of two-stage valuation updating process; that is, ν_i is an unbiased first-stage estimate of the true value v_i, i.e., $\nu_i = E(\tau) \cdot E(v_i) = E(v_i)$. Under Assumption II.1, we have $b_i = \beta(v_i; N) = \tau \cdot \beta_1(\nu_i; N)$, where $\beta_1(\nu_i; N)$ is a hypothetical bid that bidder i would have submitted if she observed $\tau = 1$. Let $\mathtt{b}_i := (b_i/\tau) = \beta_1(\nu_i; N)$ with an associated density $g_{\mathtt{b}}(\cdot|N)$. The following Proposition provides conditions for identifying $f_\tau(\cdot)$ and $g_{\mathtt{b}}(\cdot|N)$.

PROPOSITION II.1. *Under Assumptions II.1 and II.2, $F_\tau(\cdot)$ and $G_{\mathtt{b}}(\cdot|N)$ are nonparametrically identified.*

Proof. Under the hypothesis, the joint distribution of $(\log b_1, \log b_2)$ is directly identified. Since $\log \mathtt{b}_i$ has a compact support $[\log \underline{\nu}, \log \beta(\overline{\nu}; N)]$, its characteristic function is non-vanishing ; see Lemma $A1$ of Krasnokutskaya (2011). Under Assumption II.2, the characteristic function of $\log\tau$ is also non-vanishing. Since $\log b_i = \log\tau + \log \mathtt{b}_i$, the joint distribution of $(\log b_1, \log b_2)$ determines the joint distribution of $(\log\tau, \log \mathtt{b}_1, \log \mathtt{b}_2)$ up to a change of location; see Kotlarski (1966). We fix the location at $E[\tau] = 1$ (Assumption II.2), concluding the proof. □

Based on Proposition II.1 we regard $\{G_{\mathtt{b}}(\cdot|N)\}_N$ as known and we identify F_v and $D(n,\cdot)$. Since we use $G_{\mathtt{b}}(\cdot|N)$, we consider an auction with $\tau = 1$, for which we can rewrite the first order necessary condition (I.2) in terms of the bid distribution functions by the change of variable via the equilibrium bidding strategy, $\beta_1(v;N)$;

$$\sum_{n=2}^{N}(n-1)p_{n|N}G_{\mathtt{b}}(b|N)^{n-2}g_{\mathtt{b}}(b|N)(v-D(n,v)-b) = \sum_{n=1}^{N} p_{n|N}G_{\mathtt{b}}(b|N)^{n-1}. \tag{II.1}$$

Note that we express the dependence of the shortlisting probability $p_{n|N}$ on the number of potential bidders, N, which is useful for identification. In order to simplify the equilibrium relationship between the value and the bid in (II.1), we define

$$\lambda_N(b) := \frac{\sum_{n=1}^{N} p_{n|N}G_{\mathtt{b}}(b|N)^{n-1}}{\sum_{m=2}^{N}(n-1)p_{m|N}G_{\mathtt{b}}(b|N)^{m-2}g_{\mathtt{b}}(b|N)},$$

which is the ratio of the overall winning probability to its derivative, and

$$\eta_{n,N}(b) := \frac{(n-1)p_{n|N}G_{\mathtt{b}}(b|N)^{n-2}}{\sum_{m=2}^{N}(m-1)p_{m|N}G_{\mathtt{b}}(b|N)^{m-2}},$$

which forms a probability distribution, as it is positive and sum to 1 over n. Using the notations, we rewrite (II.1) as

$$v = b + \lambda_N(b) + \sum_{n=2}^{N} \eta_{n,N}(b)D(n,v), \tag{II.2}$$

where the second term on the right hand side is the markdown due to competition intensity and the third term is the weighted expected value deduction due to information disclosure discount. Note that $\lambda_N(\cdot)$ and $\eta_{n,N}(\cdot)$ are directly identified because they are determined by $G_{\mathtt{b}}(b|N)$. We can construct $\overline{N}-1$ equations from (II.2), one for each $N \in \{2,\ldots,\overline{N}\}$ with $\overline{N} \geq 2$. Using the equations, we hope to identify $F_v(\cdot)$ and $D(n,\cdot)$ for all $n \in \{2,\ldots,\overline{N}\}$, which gives $\overline{N}$ unknown functions. Since we have more unknowns than knowns, however, the model primitives would not be identified without a further structure. To this end, we consider the following specification.

ASSUMPTION II.3. *For any $v \in [\underline{v}, \overline{v}]$ and each $n = 2, \ldots, N$, the information disclosure discount has the form of $D(n,v) = (n-1) \cdot D(v)$ with $(n-1)dD(v)/dv < 1$.*

Assumption II.3 has an intuitive interpretation: since bidders do not know the identities of their competitors, they naturally treat each rival bidder as a representative competitor, and discount their synergy values by the same expected loss for each potential competitor. Accordingly, the discount factor $D(v)$ represents this expected loss per rival bidder. Besides, this specification alleviates the data requirement for identification and the empirical application later. The identification problem is now boiled down to identifying $D(v)$ and $F_{\mathsf{v}}(\cdot)$.

PROPOSITION II.2. *Under Assumptions II.1 and II.3, $D(\cdot)$ and $F_{\mathsf{v}}(\cdot)$ are identified by $G_{\mathsf{b}}(\cdot|N_1)$ and $G_{\mathsf{b}}(\cdot|N_2)$ with $N_1 \neq N_2$ when $p_{N_1|N_1} > 0$ and $p_{N_2|N_2} > 0$.*

Proof. See Appendix A.3. □

II.4 Estimation Method

II.4.1 Estimation Strategy

Now we bring the structural model and the data together to recover the model primitives. We observe a sample of T independent takeover auctions, $\{b_{1t}, \ldots, b_{n't}, N_t, n_t, X_t\}_{t=1}^{T}$ where $n'_t \leq n_t$ denotes the number of observed cash bids, i.e., winning cash bids and some losing cash bids.[13] We postulate that the omitted losing bids are lower than the winning bid because the target board of directors has the fiduciary duty to accept the superior offer; and

[13] Losing bids are omitted mainly for the following reasons: (a) a bidder is invited, but did not submit a final bid; (b) a submitted bid is not comparable to the winning bid, e.g, not a cash-only bid; and (c) a losing bid is simply not reported in the SEC filings.

therefore the cash values of the losing bids, reported or not, should be lower than the observed cash-only winning bid. Given the symmetric IPV paradigm, then, the joint density of the bids for auction t is given by

$$\prod_{i=1}^{n'_t} g(b_{it}|N_t, X_t; \tau_t, \theta)\,[G(b_t^w|N_t, X_t; \tau_t, \theta)]^{n_t - n'_t},$$

where b_t^w denotes the winning bid, τ_t is the unobserved heterogeneity, and θ collects all the model parameters, which will be specified later. Let $\boldsymbol{y} := \{b_{1t}, \ldots, b_{n't}\}_{t=1}^{T}$, $\boldsymbol{\tau} := \{\tau_t\}_{t=1}^{T}$, $\boldsymbol{n} := \{n_t\}_{t=1}^{T}$, and $\boldsymbol{X} := \{N_t, X_t\}_{t=1}^{T}$. Conditional on $(\boldsymbol{\tau}, \boldsymbol{X}, \theta)$, then, the joint density of the observed bids $\boldsymbol{y}$ and the number of final bidders $\boldsymbol{n}$ is given by

$$L(\boldsymbol{y}, \boldsymbol{n}|\boldsymbol{\tau}, \boldsymbol{X}, \theta) = \prod_{t=1}^{T} \left[p(n_t|N_t, X_t; \theta) \prod_{i=1}^{n'_t} g(b_{it}|N_t, X_t; \tau_t, \theta)\,[G(b_t^w|N_t, X_t; \tau_t, \theta)]^{n_t - n'_t} \right], \tag{II.3}$$

where $p(n_t|N_t, X_t; \theta)$ denotes the conditional shortlisting probability.

Now, we specify each element in (II.3) on the right hand side: the shortlisting probability and the bid distribution functions. We model the shortlisting probability by the binomial distribution, i.e.,

$$p(n = n_t|N_t, X_t; \delta) := \begin{pmatrix} N_t - 1 \\ n_t - 1 \end{pmatrix} \Phi(X_t'\delta)^{n_t - 1} \left[1 - \Phi(X_t'\delta)\right]^{N_t - n_t}, \tag{II.4}$$

where $\delta \in \mathscr{R}^k$. In order to construct the bid density, we need to specify several factors. First, we consider the valuation premium on target t in the form of

$$v_{it} := \frac{V_{it}}{M_t} = \tau_t \exp(X_t'\gamma)\varepsilon_{it} \tag{II.5}$$

where V_{it} is bidder i's absolute synergy value and M_t is the observed standalone value of the target.[14] Here, $\tau_t \exp(X_t'\gamma)$ with $\gamma \in \mathscr{R}^k$ refers to the common synergy premium and ε_{it} denotes the idiosyncratic synergy premium, specific to each bidder-target combination. We model the distribution of ε_{it} by the truncated log-normal distribution and the distribution of τ_t by the truncated normal distribution that is symmetric around its mean 1; i.e.,

$$\log \varepsilon_{it} \sim \mathscr{N}(0, h_\varepsilon^{-1}) \mathbb{1}\{\log \varepsilon_{it} \in [-d_1 {h_\varepsilon}^{-1/2}, d_1 {h_\varepsilon}^{-1/2}]\}, \tag{II.6}$$

$$\tau_t \sim \mathscr{N}(1, h_\tau^{-1}) \mathbb{1}\{\tau_t \in [1 - d_2 {h_\tau}^{-1/2}, 1 + d_2 {h_\tau}^{-1/2}]\}, \tag{II.7}$$

where h_ε and h_τ are the precision parameters of the normal distributions $\mathscr{N}(\cdot,\cdot)$, $d_1 = 2.58$ is a truncation constant which ensures the idiosyncratic synergy ε_{it} lies in a compact set, and $d_2 = 1.96$ is another truncation constant which ensures τ_t lies in a compact set.[15] We use $F_\varepsilon(\cdot|h_\varepsilon)$ and $f_\varepsilon(\cdot|h_\varepsilon)$ to denote the CDF and PDF of ε_{it} and $F_\tau(\cdot|h_\tau)$ and $f_\tau(\cdot|h_\tau)$ similarly for τ_t.

Second, we specify the discount factor $D(v_{it})$ as a linear function of v_{it} which is zero at the lower bound, i.e., $D(v_{it}|\eta) = \eta[v_{it} - \underline{v}_t]\mathbb{1}(\eta > 0)$, where the event that $\eta \leq 0$ which corresponds to no information discount. Then, together with Assumptions II.1 and II.3, the information disclosure discount is written as

$$D(n_t, v_{it}|X_t; \eta) = (n_t - 1)\tau_t \exp(X_t'\gamma)\eta[\varepsilon_{it} - \underline{\varepsilon}_{it}]\mathbb{1}(\eta > 0),$$

where $(n_t - 1)\eta < 1$ for any given n_t to be consistent with Assumption I.1 that valuation order is preserved in the presence of information disclosure discounts.

Now, we use $\theta := (\gamma, \delta, \eta, h_\varepsilon, h_\tau)$ to index the model primitives. We interpret observed bids as equilibrium outcomes, i.e., the latent synergy values are linked to the observed bids through the strictly increasing bidding function. Then, we derive the bid distribution

[14] Gorbenko and Malenko (2014) and Gentry and Stroup (2015) use a similar synergy value structure but without the unobserved heterogeneity τ_t.

[15] $d_1 := \Phi^{-1}(99.5\%) = 2.58$ and $d_2 = \Phi^{-1}(97.5\%) = 1.96$ with $\Phi(\cdot)$ being the CDF of $\mathscr{N}(0,1)$, so respectively the two truncated normal distribution cover 99% and 95% of the central areas of the Normal distributions symmetrically.

functions in (II.3) by change of variables under each parameter θ.[16] Finally, given the data $\boldsymbol{z} := (\boldsymbol{y}, \boldsymbol{n}, \boldsymbol{X})$ and a prior $\pi(\theta)$, the posterior density is

$$\pi(\boldsymbol{\tau}, \theta | \boldsymbol{z}) \propto \pi(\theta) \times \prod_{t=1}^{T} f_\tau(\tau_t | h_\tau) \times L(\boldsymbol{y}, \boldsymbol{n} | \boldsymbol{\tau}, \boldsymbol{X}, \theta),$$

and the posterior predictive mean is given as

$$E[c(\boldsymbol{\tau}, \theta) | \boldsymbol{z}] = \iint c(\boldsymbol{\tau}, \theta) \pi(\boldsymbol{\tau}, \theta | \boldsymbol{z}) d\boldsymbol{\tau} d\theta. \tag{II.8}$$

for a measurable function of interest, $c(\boldsymbol{\tau}, \theta)$.

II.4.2 Implementation

The posterior predictive mean (II.8) requires the integration of a high-dimensional function. To overcome the computational difficulty, we draw a sample $\{\boldsymbol{\tau}^{(s)}, \theta^{(s)}\}_{s=1}^{S}$ from the posterior distribution $\pi(\boldsymbol{\tau}, \theta | \boldsymbol{z})$ using Metropolis-within-Gibbs algorithm, then approximate the equation (II.8) by

$$\frac{1}{S} \sum_{s=1}^{S} c(\boldsymbol{\tau}^{(s)}, \theta^{(s)}) \xrightarrow{a.s} E[c(\boldsymbol{\tau}, \theta) | \boldsymbol{z}]. \tag{II.9}$$

The algorithm is summarized as follows with details given in Appendix B.2.

Prior We use the following prior distributions for $\theta = (\gamma, \delta, \eta, h_\varepsilon, h_\tau)$:

$$\begin{aligned} h_\tau &\sim \mathscr{G}a(\alpha_\tau, \lambda_\tau^{-1}) \mathbb{1}(d_2^2 \leq h_\tau), \\ h_\varepsilon &\sim \mathscr{G}a(\alpha_\varepsilon, \lambda_\varepsilon^{-1}), \\ \gamma &\sim \mathscr{N}(\mu_\gamma, \sigma_\gamma^2 I_k), \\ \delta &\sim \mathscr{N}(\mu_\delta, \sigma_\delta^2 I_k), \end{aligned}$$

[16] See Appendix B.1 for the detailed derivation of bid distribution functions.

and $\eta = \eta^* \mathbb{1}(\eta^* > 0)$ with $\pi_{\eta^*}(\eta^*) \propto \mathbb{1}(\eta^* > \underline{\eta}^*)$ for some $\underline{\eta}^* < 0$, where $\mathscr{G}a(\alpha, \lambda^{-1})$ is the CDF of Gamma distribution with shape parameter α and rate parameter λ, I_k is a k-dimensional identity matrix and α_τ, λ_τ, α_ε, λ_ε, μ_γ, σ_γ, μ_δ, σ_δ and $\underline{\eta}^*$ are known parameters. It is worth emphasizing that we set $\underline{\eta}^* < 0$ to allow the detection of no information disclosure discounts, and thus no information cost.

Gibbs Sampling At each iteration $s = 1, \ldots, S$, we split θ into scalar blocks and sample each scalar parameter separately, conditional on the most recent values of the other parameters.

Metropolis-Hastings Updating An adaptive Gaussian Metropolis-Hastings algorithm is used to update the parameters within each block sampling. We use Gaussian proposal distribution for all the parameters except $\gamma = (\gamma_1, \ldots, \gamma_k)$ and h_ε, for which truncated Gaussian proposal distributions are used by incorporating theoretical parameter bounds in order to improve computational efficiency. For each iteration, we tune the variance of the proposal distribution following Haario, Saksman, and Tamminen (2001, 2005). In particular, the variance is updated according to the variance of the parameter values sampled so far.

II.5 Estimation Results

In this section, we present the estimation results from our structural analysis. We run the sampling algorithm for $45,000$ iterations, then keep every 10th draw after an initial $15,000$ burn-in period. Eventually, this procedure gives a sample of $M = 3,000$ draws from the posterior distribution.

II.5.1 Parameter Estimates

The key model primitive of our interest is the information disclosure discount, which is summarized by the parameter η, the percentage synergy value discount for each additional competitor learning the confidential information. The posterior mean of η is 11.9% with the 95% credible interval of $[10.2\%, 13.2\%]$, which means that on average the synergy value will be offset by the information disclosure discount in an auction with 8 final bidders. Later in the counterfactual analyses, we will further demonstrate the economic significance of this discount in a series of specific takeover auctions.

Table II.3 summarizes the posterior distribution of the coefficients attached to auction covariates X_t by their posterior means and 95% credible intervals. The estimate of γ, reported in the column (A), represents the direct effects of the target characteristics and market conditions on the synergy values; see our specification (II.5). First, we find that the industry premiums are positive but not substantially different from each other, as their 95% credible intervals overlap. Second, among the observed firm-level characteristics, we find a negative effect of target size on the synergy value, which is likely due to decreasing returns to scale.[17] In addition, the positive predictive coefficients of *R&D* and *Intangibles* are intuitive: a main source of synergy value creation comes from technological enhancement in the combined entity. Finally, the finding that high market return lowers bidders' synergy valuation is intuitive since market return reflects bidders' opportunity costs of acquiring the target.

The estimate of δ, reported in the column (B) of Table II.3, demonstrates that a few observed target characteristics help to predict the shortlisting probability. This result supports our early assertion in section I.3 that bidders can estimate the shortlisting probability by studying the precedent transactions using their public information, i.e., observed target characteristics. Moreover, we find that the target-specific characteristics, i.e., *Log(Size)*, *CashFlow*, and *Intangibles*, rather than the general market conditions and industry factors, are more relevant to seller's shortlisting decisions. In particular, the negative association of intangible assets with the shortlisting probability suggests that bidder's entry is more

[17] An alternative explanation of the negative size coefficient could be due to our focus on takeovers with cash-only bids. Cash bidders usually finance the acquisition through investment banks and the cost of cash financing increases with the size.

Table II.3
Posterior Estimation of Parameters

This table reports the posterior means and 95% credible intervals of the coefficients on the set of target and auction characteristics and economic conditions for the specifications of the synergy value (II.5) and the specification of the shortlisting probability (II.4). The estimates are calculated based on $3,000$ draws from the posterior distribution.

	(A) γ's in valuation		(B) δ's in shortlisting	
Variables	Mean	95% Credible Interval	Mean	95% Credible Interval
Industry-1 (*Consumer*)	0.582	[0.407, 0.704]	-0.228	[-0.596, 0.108]
Industry-2 (*Manufacturing*)	0.522	[0.355, 0.661]	-0.048	[-0.413, 0.297]
Industry-3 (*HighTech*)	0.606	[0.431, 0.745]	0.012	[-0.391, 0.367]
Industry-4 (*Health*)	0.662	[0.443, 0.823]	-0.088	[-0.528, 0.385]
Industry-5 (*Others*)	0.651	[0.459, 0.806]	0.017	[-0.394, 0.464]
Log(Size)	-0.029	[-0.041, -0.013]	0.118	[0.075, 0.162]
Leverage	0.066	[-0.092, 0.243]	0.100	[-0.290, 0.521]
Q-ratio	-0.033	[-0.059, 0.003]	0.084	[-0.022, 0.206]
CashFlow	-0.014	[-0.255, 0.163]	-0.667	[-1.267, -0.090]
Cash	0.014	[-0.170, 0.202]	-0.119	[-0.587, 0.328]
R&D	0.861	[-0.258, 1.850]	0.272	[-1.398, 1.980]
Intangibles	0.082	[-0.105, 0.274]	-0.772	[-1.270, -0.312]
Credit Spread	0.083	[-1.530, 2.314]	0.136	[-1.858, 2.141]
Market Return	-0.536	[-0.788, -0.228]	0.261	[-0.372, 0.867]
N (# of potential bidders)			-0.083	[-0.102, -0.063]

tightly regulated for target companies with more intangible assets. This result is coherent with our premise of information costs.

II.5.2 Predictive Densities of τ_t and ε_{it}

Our specification (II.5) forms bidder i's synergy value by three components: the idiosyncratic bidder-target synergy ε_{it}, observed target heterogeneity $X_t'\gamma$, and unobserved target heterogeneity τ_t. Our specification along with the identification result allows us to measure the contribution of each of the three components on the variation of the bid data.

To this end, we define the (posterior) predictive density of τ as

$$f_\tau(\tau|\boldsymbol{z}) := \int f_\tau(\tau|h_\tau)\pi(h_\tau|\boldsymbol{z})dh_\tau. \tag{II.10}$$

where $f_\tau(\cdot|h_\tau)$ is the density of τ in (II.7) and $\pi(h_\tau|\boldsymbol{z})$ is the marginal posterior density of h_τ. We approximate (II.10) by an ergodic sample, $\{h_\tau^{(m)}\}_{m=1}^M$ drawn from the posterior, i.e., for each τ

$$\frac{1}{M}\sum_{m=1}^{M} f_\tau(\tau|h_\tau^{(m)}) \xrightarrow{a.s} f(\tau|\boldsymbol{z}),$$

and summarize its uncertainty by a pointwise 95% credible interval $[a,b]$ such that

$$95\% = \frac{1}{M}\sum_{m=1}^{M} \mathbb{1}\left\{f_\tau(\tau|h_\tau^{(m)}) \in [a,b]\right\}.$$

which forms a credible band as we vary τ. Specifically, we use the 2.5 and 97.5 percentiles for a and b. We similarly construct the predictive density of ε_{it} denoted by $f_\varepsilon(\cdot|\boldsymbol{z})$ and its 95% credible band.

Figure II.1 presents the predictive densities $f_\tau(\cdot|\boldsymbol{z})$ and $f_\varepsilon(\cdot|\boldsymbol{z})$ in solid lines with their 95% credible band in dashed lines. It shows that $f_\tau(\cdot|\boldsymbol{z})$ is more diffuse than $f_\varepsilon(\cdot|\boldsymbol{z})$, implying that a bidder can dramatically change her evaluation on the synergy value after learning non-public and non-quantifiable information such as confidential materials. We also consider the ratio of the variance of τ to the sum of variances of τ and ε,

$$\zeta(h_\tau, h_\varepsilon) := \frac{V(\tau|h_\tau)}{V(\tau|h_\tau) + V(\varepsilon|h_\varepsilon)},$$

which can be interpreted as the relative variation of the unobserved heterogeneity after controlling for the observed target heterogeneity X_t.[18] We find that the posterior mean of the

[18] Note that since τ is distributed as a truncated normal distribution, its variance is different from $1/h_\tau$. We estimate $V(\tau|h_\tau)$ by 10,000 simulation draws from $f_\tau(\cdot|h_\tau)$ for a given value of h_τ. We do similarly for $V(\varepsilon|h_\varepsilon)$.

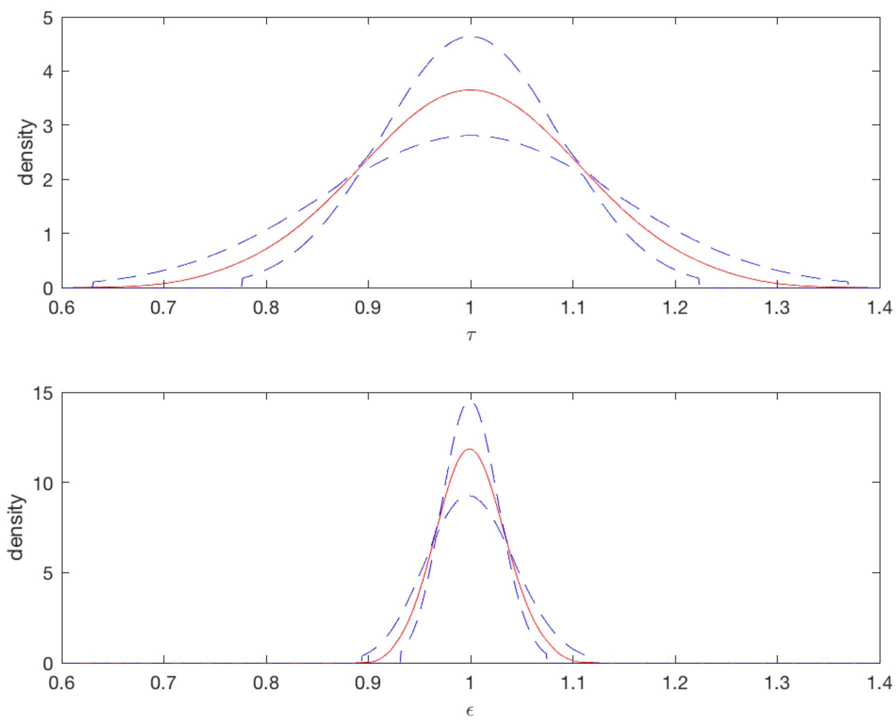

Figure 1. Predictive Densities of τ_t and ε_{it}. This Figure shows the predictive densities of the unobserved heterogeneity and the idiosyncratic synergy, based on $3,000$ draws from the posterior distribution.

ratio, $E[\zeta(h_\tau, h_\varepsilon)|z]$, is 91.07% with the 95% credible interval of $[85.52\%, 94.92\%]$, which suggests that the unobserved heterogeneity may play a critical role in bidders' evaluation of their synergy value in a takeover process.

II.6 Counterfactual Analyses

In this section, we conduct a range of counterfactual analyses. First, we disentangle the information cost and the competition gain. Then, we bound the operation cost by exploring the seller's shortlisting decision. Finally, we show that the English auction may reduce information cost and thus increase the seller's expected revenue.

Let $R(X, \tau, n|N, \theta)$ denote the expected revenue (i.e., takeover premium) when the seller of a target with (X,τ) invites n final bidders out of N potential bidders under the parameter θ. The multiplicatively separable valuation structure implies that the expected revenue is also multiplicatively separable, i.e., $R(X,\tau,n|N,\theta) = \tau \cdot R(X,\tau = 1,n|N,\theta)$. For the counterfactual analyses, we consider a range of auctions in the Consumer and HighTech industries with $\tau_t = 1$ and other covariates equal to the sample medians (see the first column of Panel B in Table II.1), and suppress the dependence of $R(X,\tau,n|N,\theta)$ on (X,τ). Formally,

$$R(n|N,\theta) := \int b_w g_w(b_w|N,n,\theta)db_w, \tag{II.11}$$

where b_w is the winning bid from the auction and $g_w(\cdot|N,n,\theta)$ is the corresponding probability density. Moreover, we define the posterior predictive revenue as

$$\begin{aligned} R(n|N,\boldsymbol{z}) &:= \int R(n|N,\theta)\pi(\theta|\boldsymbol{z})d\theta \\ &= \int b_w \left\{ \int \left[g_w(b_w|N,n,\theta)\right] \pi(\theta|\boldsymbol{z})d\theta \right\} db_w \\ &= \int b_w g_w(b_w|N,n,\boldsymbol{z})db_w \end{aligned}$$

where the second equality applies the Fubini's theorem and the last uses the definition of the posterior predictive density of b_w. That is, $R(n|N,\boldsymbol{z})$ is the posterior mean of the winning bid. Hence, we approximate it by $M^{-1}\sum_{m=1}^{M} b_w^{(m)}$ with $b_w^{(m)} \sim g_w(b_w|N,n,\theta^{(m)})$ for each draw $\theta^{(m)}$ from the posterior, and summarize its uncertainty by the 95% credible interval based on the 2.5 and 97.5 percentiles of $\{b_w^{(m)}\}_{m=1}^{M}$.

II.6.1 Information Cost and Competition Effect

Our structural model allows us to disentangle two opposing effects, i.e., competition effect and information cost, on the takeover premium when one more bidder is invited. Denote the revenue in the absence of information disclosure discount by $R_0(n|N,\theta)$, which is identical to $R(n|N,\theta)$ except that η is restricted to be zero. Then we define the total information cost (*TIC*) incurred when n final bidders are invited as the revenue loss induced by the information disclosure discount, i.e.,

$$\mathbf{TIC}(n;N) := R_0(n|N,\theta) - R(n|N,\theta).$$

Then the (marginal) information cost (*IC*) by inviting an additional bidder is

$$\mathbf{IC}(n,n+1;N) := \mathbf{TIC}(n+1;N) - \mathbf{TIC}(n;N). \tag{II.12}$$

We also define the competition effect (*CE*) as the revenue difference in the absence of the information disclosure discount, i.e.,

$$\mathbf{CE}(n,n+1;N) := R_0(n+1|N,\theta) - R_0(n|N,\theta). \tag{II.13}$$

Then, we can decompose the revenue change of inviting one more bidder, i.e., from n to $n+1$, into the competition effect (*CE*) and the information cost (*IC*) as follows,

$$\begin{aligned} R(n+1|N,\theta) - R(n|N,\theta) =& R_0(n+1|N,\theta) - R_0(n|N,\theta) \\ & - [R_0(n+1|N,\theta) - R(n+1|N,\theta)] \\ & + [R_0(n|N,\theta) - R(n|N,\theta)] \\ =& \mathbf{CE}(n,n+1;N) - \mathbf{IC}(n,n+1;N). \end{aligned}$$

Considering a series of auctions in the Consumer and HighTech industries with $N = (2,4,8)$, Table II.4 reports the predictive revenue, $R(n|N,\boldsymbol{z})$, along with its 95% credible intervals and the change of the predictive revenue along with its decomposition into the predictive competition effect, $E[\mathbf{CE}(n,n+1|N,\theta)|\boldsymbol{z}]$, and the predictive information cost, $E[\mathbf{IC}(n,n+1|N,\theta)|\boldsymbol{z}]$. Column (A) shows that acquirors generally value HighTech targets more than the Consumer targets ceteris paribus. This posterior prediction is consistent with the observed premium difference in Table II.1; for example, on average a HighTech target receives 11% higher premium than a Consumer target, i.e., 38% v.s. 27%. Column (C) reports the predictive revenue increments when the number of final bidders increases from $n-1$ to n, and columns (D) and (E) decompose the revenue increments into the information costs and competition effects. The result shows that the information cost reduces a substantial portion of the takeover premium; for example, for the median participation of $(N,n) = (4,2)$ in Consumer industry, the predictive premium decreases by 1.3%, which amounts to $6.4 million for a firm with an average market value in our sample, i.e., $495 million.[19]

II.6.2 Operation Cost

Suppose that the sellers choose the optimal number of final bidders to maximize their expect revenue. We can bound the operation cost by contrasting the theoretical prediction on the sellers' revenue against the observed number of shortlisted bidders.

Recall that n is unknown whereas N is common knowledge among the bidders, and the bidding strategy depends on N, but not on n. However, since $R(n|N,\theta)$ is the expectation of the highest bid among submitted n bids, we have $R(n|N,\theta) > R(n-1|N,\theta)$ for all $n \leq N$. Now let OC be the operation cost for inviting an additional bidder. Then, the seller would invite the n^{th} bidder if the expected revenue increment is larger than OC, i.e., $R(n|N,\theta) - R(n-1|N,\theta) \geq OC$. Similarly, he would not invite $(n+1)^{\text{th}}$ bidder if the marginal revenue is smaller than OC, i.e., $R(n+1|N,\theta) - R(n|N,\theta) < OC$. Therefore, when the seller chooses to invite n bidders, it must be that

$$R(n+1|N,\theta) - R(n|N,\theta) < OC \leq R(n|N,\theta) - R(n-1|N,\theta). \qquad \text{(II.14)}$$

[19] The market value is calculated as the target stock price four weeks before the announcement date times the outstanding common shares.

Table II.4
Predictive Revenue, Information Cost, and Competition Effect

This table reports the posterior predictive revenues, information costs, and competition effects for a range of auctions, each with a different participation configuration (N,n), but with the identical auction covariates – the median values of observed characteristics and the unobserved heterogeneity given as $\tau = 1$ for Consumer and HighTech industries. Columns (A) and (B) report the predictive revenues and 95% credible intervals, column (C) reports the predictive revenue increments when the number of final bidders increases from $n-1$ to n, and columns (D) and (E) decompose the revenue increments into the information costs and competition effects. The results are calculated based on $3,000$ draws from the posterior distribution.

(N,n)	(A) Revenue	(B) 95% CI for (A)	(C) Δ Revenue	(D) Info. Cost	(E) Comp. Effect
Consumer Industry					
(2,1)	1.018	[0.838, 1.222]	–	–	–
(2,2)	1.071	[0.886, 1.244]	0.053	0.003	0.056
(4,1)	1.116	[0.854, 1.412]	–	–	–
(4,2)	1.200	[0.932, 1.448]	0.083	0.013	0.097
(4,3)	1.240	[0.992, 1.462]	0.041	0.005	0.046
(4,4)	1.267	[1.043, 1.471]	0.026	0.003	0.029
(8,1)	1.141	[0.861, 1.439]	–	–	–
(8,2)	1.225	[0.951, 1.472]	0.084	0.032	0.116
(8,3)	1.265	[1.021, 1.493]	0.040	0.015	0.055
(8,4)	1.290	[1.076, 1.510]	0.026	0.009	0.035
(8,5)	1.308	[1.107, 1.523]	0.018	0.006	0.023
(8,6)	1.323	[1.123, 1.531]	0.015	0.004	0.018
(8,7)	1.332	[1.149, 1.534]	0.010	0.002	0.012
(8,8)	1.340	[1.161, 1.535]	0.008	0.002	0.009
HighTech Industry					
(2,1)	1.070	[0.876, 1.280]	–	–	–
(2,2)	1.134	[0.936, 1.307]	0.064	0.003	0.067
(4,1)	1.169	[0.892, 1.460]	–	–	–
(4,2)	1.261	[0.992, 1.504]	0.092	0.017	0.109
(4,3)	1.303	[1.055, 1.522]	0.042	0.006	0.049
(4,4)	1.331	[1.113, 1.531]	0.028	0.003	0.031
(8,1)	1.174	[0.898, 1.435]	–	–	–
(8,2)	1.257	[1.016, 1.484]	0.082	0.046	0.128
(8,3)	1.293	[1.081, 1.503]	0.036	0.021	0.057
(8,4)	1.316	[1.129, 1.514]	0.024	0.013	0.036
(8,5)	1.335	[1.159, 1.521]	0.018	0.009	0.028
(8,6)	1.347	[1.179, 1.528]	0.013	0.005	0.018
(8,7)	1.356	[1.188, 1.534]	0.009	0.003	0.012
(8,8)	1.365	[1.205, 1.539]	0.009	0.003	0.012

From the inequality (II.14), we can bound OC by contrasting the theoretical revenue and observed n. Formally, we have $OC \in [\underline{OC}, \overline{OC}]$, where for $n \in \{1, \ldots, N-1\}$,

$$\underline{OC} := R(n+1|N,\theta) - R(n|N,\theta) \tag{II.15}$$

and for $n \in \{2, \ldots, N\}$,

$$\overline{OC} := R(n|N,\theta) - R(n-1|N,\theta) \tag{II.16}$$

are the implied lower and upper bounds of OC, respectively. In addition, we can get a natural lower bound $\underline{OC} = 0$ for $n = N$; while for $n = 1$, we bound OC from above by setting $R(0|N,\theta) = 0$, i.e., the lowest possible value of a company.

Table II.5 reports the predictive lower and upper bounds of the marginal operation costs OC, defined in (II.15) and (II.16), for a series of hypothetical auctions with $N = (2,4,8)$ in Consumer and HighTech industries. The result indicates a substantial marginal operation cost to accommodate bidders in the final due diligence. For example, for the median participation of $(N,n) = (4,2)$ in Consumer industry, the marginal operation cost predictively consumes 4.1% to 8.3% of the takeover premium, which amounts to \$20.3 to \$41.1 millions for a firm with an average market value in our sample, i.e., \$495 millions. This estimate indicates a substantially higher economic cost, such as foregoing of business opportunities, than the recorded accounting cost in the sale of companies; for example, Hunter and Jagtiani (2003) report an average of 0.84% advisory fees. The operation cost and the information cost together explain the practice of entry regulation by the seller.

II.6.3 English Auction of Companies

The English auction reveals the number of final bidders to all competitors in the bidding game. By dissolving bidders' uncertainty on the number of bidders, the English auction can increase the seller's revenue when the actual pool of bidders is not large. For a series of

Table II.5
Estimated Bounds on Operation Cost

This table reports the predictive lower bounds ($\underline{OC}$) and upper bounds ($\overline{OC}$) of the operation costs for a range of auctions, each with a different participation configuration (N,n), but with the identical auction covariates – the median values of observed characteristics and the unobserved heterogeneity given as $\tau = 1$ for Consumer and HighTech industries. The results are calculated based on $3,000$ draws from the posterior distribution.

	(A) Consumer Industry		(B) HighTech Industry	
(N,n)	Predictive $\underline{OC}$	Predictive $\overline{OC}$	Predictive $\underline{OC}$	Predictive $\overline{OC}$
(2,1)	0.053	1.018	0.064	1.070
(2,2)	0.000	0.053	0.000	0.064
(4,1)	0.083	1.116	0.092	1.169
(4,2)	0.041	0.083	0.042	0.092
(4,3)	0.026	0.041	0.028	0.042
(4,4)	0.000	0.026	0.000	0.028
(8,1)	0.084	1.141	0.082	1.174
(8,2)	0.040	0.084	0.036	0.082
(8,3)	0.026	0.040	0.024	0.036
(8,4)	0.018	0.026	0.018	0.024
(8,5)	0.015	0.018	0.013	0.018
(8,6)	0.010	0.015	0.009	0.013
(8,7)	0.008	0.010	0.009	0.009
(8,8)	0.000	0.008	0.000	0.009

auctions with the number of final bidders n ranging from 2 to 8, we report the predictive revenues and their 95% credible intervals in Table II.6.[20]

In contrast to the predictive revenues in column (A) of Table II.4, where the first-price takeover auctions are conducted, we find that the English auction generate smaller seller's premium almost all the cases that we consider, excluding for the case with $(N,n) = (2,2)$. This may explain the prevalence of first-price auctions in takeover competitions; that is, the sellers strategically keep bidders uninformed of the number of opponents and their competing bids.

[20] We omit the counterfactual comparison of the English auction with only one bidder because it would never improve the takeover revenue than the current first-price auction, as the single bidder would bid the reserve price.

Table II.6
Predictive Revenue for English Auctions

This table reports the predictive revenues and 95% credible intervals for a series of hypothetical English auctions with the median values of explanatory variables and the unobserved heterogeneity $\tau = 1$ for Consumer and HighTech industries. The results are calculated based on $3,000$ draws from the posterior distribution.

	(A) **Consumer Industry**		(B) **HighTech Industry**	
n	Revenue	95% CI	Revenue	95% CI
2	1.138	[0.826, 1.535]	1.169	[0.855, 1.528]
3	1.087	[0.835, 1.405]	1.118	[0.867, 1.428]
4	0.966	[0.750, 1.235]	0.994	[0.782, 1.244]
5	0.816	[0.637, 1.046]	0.838	[0.666, 1.053]
6	0.648	[0.490, 0.861]	0.665	[0.510, 0.865]
7	0.467	[0.321, 0.670]	0.478	[0.329, 0.677]
8	0.277	[0.123, 0.486]	0.285	[0.127, 0.498]

II.7 Concluding Remark

This study finds that the information cost and operation cost incurred by the seller of a company can be economically significant, which explains the common practice that the seller limits bidders' participation. Our findings provide a few policy implications to the regulatory and judiciary authorities; and our structural method can be useful in resolving some legal disputes related to takeover transactions.

The large amount of the information cost, e.g., \$6.4 million for a firm with an average market value in the consumer industry with $(N,n) = (4,2)$, indicates a substantial loss of social welfare due to the informational externality. It advocates the reinforcement of regulation on the use of information acquired from the takeover process.[21] Moreover, our quantitative approach can be useful in settling some takeover lawsuits. In particular, the seller's shortlisting decision is often challenged in court by its shareholders, who may argue that the entry restriction discourages competition and hurts the shareholders' benefits. Given the observed target characteristics, our method provides a reasonable range of oper-

[21] It is beyond the scope of the paper to develop a formal decision method to choose a policy parameter because it requires additional elements outside the auction framework, e.g., the probability of detecting violation of the agreement and the socially desirable level of economic efficiency.

ation costs for similar transactions. These estimates may serve to guide the court judgment by checking whether the implied operation cost lies in a reasonable range.

In addition, the large seller's cost also explains why a target company is willing to sign an exclusivity agreement with a single bidder. The exclusivity agreement sends a credible signal to the bidder that no confidential information is exposed to other competitors and, thereby, preventing the bidder from discounting her evaluation on the target company. Finally, although we focus on successful takeovers in the analyses, the implication of information cost on failed takeovers is straightforward: when a deal fails to consummate, the market value of the target company is expected to fall because the target firm, as the eventual owner, should bear the information cost.

Chapter III

OPTIMAL AUCTION DESIGN WITH VOLUNTARY ENTRY

Entry costs are ubiquitous in an auction or bidding competition: bidders can at least have spent the time in other activities and enjoyed a positive payoff. In some auctions, this entry cost is low and negligible; while in some other cases, it can be formidably high. Moreover, entry costs may change over time as a result of technological development etc. For example, companies interested in bidding in an Outer Continental Shelf (OCS) auction usually spend several hundred thousand dollars per tract for a 3-D seismic survey of tracts for sale. A large literature documents the impact of this cost on the bidder's entry and bidding behavior; to name a few, Hendricks and Porter (1992) and Hausch and Li (1993). The survey cost decreased due to the advances in computing power; for example, before 1990, most of the oil companies conducted 2-D seismic surveys because the cost of 3-D analysis was dreadfully high.

The presence of entry costs deviates from the standard framework developed in Myerson (1981), in which every bidder has a positive instead of a zero reservation value. To induce entry, a feasible auction should provide each entrant with at least an expected entry payoff that covers her reservation value. Levin and Smith (1994) studies a class of two-stage auctions with entry costs just enough to admit some, but not all, potential bidders. *They show that a revenue-maximizing mechanism may involve entry fees, but should always have the reserve price at seller's reservation value.*

However, entry fees are either not applied or negligible in auction practice; meanwhile binding reserve prices are observed in many real-world examples, e.g., timber auctions and highway procurements. Taking this observation into consideration, we depart from Levin and Smith (1994) by studying an optimal auction design problem *without invoking entry fees*. In particular, we consider a two-stage auction with voluntary entry, where a potential bidder needs to incur an entry cost to learn her private value and participate in the second-stage auction and a reserve price is the only policy instrument available to the seller. Following Stegeman (1996) and Lu (2009), we focus on the semi-direct mechanisms, where the nonparticipants neither receive the object nor make any payments, and the entrants report their types (values) truthfully on the equilibrium path.

We first characterize bidders' equilibrium entry and bidding behaviors and then derive the optimal mechanism from a set of feasible mechanisms, i.e., satisfying the two-stage incentive-compatible conditions and the resource restriction. We find that the optimal mechanism should assign the asset to the bidder with the highest non-negative virtual value with an adjustment for the entry-stage incentivization. This optimal mechanism is implementable via a second-price auction with an optimal reserve price.

This paper contributes to the studies of optimal auction design problems with endogenous entry. Following the seminal work by Myerson (1981), the optimal auction design problem has been extended to various informational environment. Among them, there is a large literature considering the participation decision by the bidders when the participation cost is not negligible. For example, Samuelson (1985); Levin and Smith (1994); and more recently, Gentry, Li, and Lu (2017), who consider a two-stage model with selective entry, which encompasses, as two special cases, both Samuelson (1985) and Levin and Smith (1994). However, most of these studies allow the mechanism designer to use an entry fee/-subsidy as a policy instrument and their proposed optimal entry fees could be impractically high. Accounting for this practical restriction, this paper derives an optimal mechanism that embeds Myerson (1981) and Levin and Smith (1994) as two special cases.

The rest of this paper is organized as follows. Section III.1 sets up a symmetric auction model with voluntary entry. Section III.2 characterizes the symmetric equilibrium entry and bidding behavior within a set of feasible semi-direct mechanisms. Section III.3 derives the optimal mechanism for a wide range of entry costs and discusses its implication on the equilibrium entry, and Section III.4 provides an example to illustrate the optimal mechanism. Section III.5 concludes.

III.1 The Auction Model with Voluntary Entry

Our model setting is similar to Levin and Smith (1994). There is one seller selling one indivisible asset to $N(\geq 2)$ potential bidders. We use $\mathscr{N} = \{1,2,\ldots,N\}$ to denote the set of all potential bidders. The seller and bidders are assumed to be risk neutral and have additively separable utility functions for money and the asset being sold. The seller's reservation valuation for the asset v_0 is normalized to be 0.

We assume that bidders' value discovery is costly and covert. Considering a potential bidder i, she has to incur an entry cost $c \geq 0$ to discover her independent private value v_i, distributed according to a common distribution $F : [0,1] \to [0,1]$.[1] We adopt the regularity condition of "monotone non-decreasing hazard rate" on the valuation distribution $F(v)$, i.e., $h(v) := \frac{1-F(v)}{f(v)}$ is weakly decreasing in v. We further assume that the number of potential bidders N, the common entry cost c, and the seller's reservation value 0 are all revealed to bidders and the seller as common knowledge. The timing of the mechanism is as follows,

- **Time 0**: A seller announces and commits to a mechanism with anonymous rules.
- **Time 1**: Each potential bidder $i \in \mathscr{N}$ simultaneously and confidentially decides whether to incur an entry cost c to participate into the second-stage auction.
- **Time 2**: Upon paying the entry cost c, a subgroup of bidders $g \in 2^{\mathscr{N}}$ enter the second-stage auction. Each entrant $i \in g$ learns her valuation v_i, and sends a message to the seller.
- **Time 3**: Based on the messages from the entering bidders, allocations and payments are implemented according to the mechanism announced in **Time 0**.

[1] Without loss of generality, we assume the valuation support is $[\underline{v}, \overline{v}] = [0,1]$.

Mechanisms

A general mechanism is defined on a complete set of messages sent by the potential bidders, including the non-participating messages from nonparticipants. Let $\emptyset$ denote the null message sent by a nonparticipant, and restrict the message sent by an entrant in the unit space $[0,1]$. Then $\mathbf{M} := [0,1] \cup \{\emptyset\}$ is the message space for every potential bidder, and $\mathbf{m} = (m_1, m_2, \ldots, m_N) \in \mathbf{M}^N$ is the collection of messages received by the seller.

Following Stegeman (1996) and Lu (2009), we restrict attention to semi-direct mechanisms where the nonparticipants neither receive the object nor make any payments, and the entrants report their types (values) truthfully on the equilibrium path. A *semi-direct mechanism* $(\mathbf{p}, \mathbf{x})$ is thus characterized by the allocation rules, $p_i : \mathbf{M}^N \to [0,1]$, the probability that the object is awarded to bidder i, and the payment rules $x_i : \mathbf{M}^N \to \mathbb{R}$, the payment to the seller from bidder i, $\forall i \in \mathscr{N}$, satisfying,

(c1) $p_i(\mathbf{m}) = x_i(\mathbf{m}) = 0$ if $m_i = \emptyset$, $\forall i \in \mathscr{N}$, $\forall \mathbf{m} \in \mathbf{M}^N$;

(c2) $m_i = v_i$ if $m_i \neq \emptyset$, $\forall i \in \mathscr{N}$;

(c3) $x_i(\mathbf{m}) = 0$ if $p_i(\mathbf{m}) = 0$, $\forall i \in \mathscr{N}$;

(c4) $p_i(\mathbf{m}) \geq 0$ and $\sum_{i=1}^{N} p_i(\mathbf{m}) \leq 1$, $\forall i \in \mathscr{N}$, $\forall \mathbf{m} \in \mathbf{M}^N$.

Condition (c1) is the no passive reassignment (NPR) assumption adopted by Stegeman (1996), which reflects the fundamental relationship between the seller and the a nonparticipant: if a bidder does not participate in the bidding stage, both the seller and the bidder should maintain their status quo; condition (c2) restricts the second stage auctions in the space of direct mechanisms. Condition (c3) precludes the possibility of charging bidders an entry fee (or subsidizing them). Condition (c4) is the resource constraint, saying that the sum of all bidders' winning probabilities should not exceed 1.

Since the allocation and the payment rule for nonparticipants are fixed by the condition (c1), a *semi-direct mechanism* $(\mathbf{p}, \mathbf{x})$ is further reduced to a collection of stage-2 direct mechanisms, i.e.,

$$\{(p_i^g(\mathbf{v}), x_i^g(\mathbf{v})) | i \in g, g \in 2^{\mathscr{N}}\},$$

that satisfy the following *no-entry-fee* condition

$$x_i^g(\mathbf{v}) = 0, \text{ if } p_i^g(\mathbf{v}) = 0, \forall i \in g, \forall g \in 2^{\mathscr{N}} \tag{III.1}$$

and the reduced resource constraint,

$$p_i^g(\mathbf{v}) \geq 0, \text{and} \sum_{i \in g} p_i^g(\mathbf{v}) \leq 1, \forall i \in g, \forall g \in 2^{\mathscr{N}}, \tag{III.2}$$

where the dimensionality of $\mathbf{v}$ is determined by the cardinality of g, i.e., the number of entrants $n = |g|$.

III.2 Equilibrium Analysis

Focusing on the symmetric equilibrium, we now characterize bidders' equilibrium entry and bidding behaviors through backward induction. A semi-direct mechanism is *feasible* if and only if it is (Bayesian) incentive compatible for all bidders in both entry and bidding stages.

Let $\Pi_i(q_i; \overline{q})$ be the *ex ante* expected payoff of a potential bidder i when her entry strategy is summarized by an entry probability q_i while her opponents follow a common equilibrium entry probability $\overline{q}$. Then the (Bayesian) incentive compatible condition in the entry stage (**IC-1**) is given by,

$$\Pi_i(q_i; \overline{q}) \geq \Pi_i(q_i'; \overline{q}), \forall q_i', \overline{q} \in [0, 1], \forall i \in \mathscr{N}. \tag{III.3}$$

Suppose a group g of bidders decide to pay the entry cost c to learn their values. Denote

$\pi_i(v_i, v_i')$ the expected payoff of an entrant $i \in g$ with value v_i when she sends a message v_i' to the seller. Then the bidding stage (Bayesian) incentive compatible condition (**IC-2**) is given by

$$\pi_i(v_i, v_i) \geq \pi_i(v_i, v_i'), \forall v_i, v_i' \in [0,1], \forall i \in g, \forall g \in 2^{\mathscr{N}}. \tag{III.4}$$

III.2.1 Bidding Stage

Suppose that each potential bidder chooses to enter according to a given entry probability $\overline{q}$ and that eventually a group g of bidders with $n = |g|$ enter the bidding stage. Now considering the bidding decision made by an entrant $i \in g$, she does not observe the number of opponents $n-1$ and their reported values $\mathbf{v}_{-i}$, but knows that $n-1 \sim \mathscr{B}(N-1, \overline{q})$ and that $\mathbf{v}_{-i} \sim F(v)^{n-1}$. Recalling that the mechanism rules are *anonymous*, entrant i's expected winning probability from reporting value v is given by

$$P_i(v;\overline{q}) = \sum_{g \in 2^{\mathscr{N}}} \Pr(g|\overline{q}, N) E_{\mathbf{v}_{-i}} \left[p_i^g(v, \mathbf{v}_{-i}) \right] = \sum_{n=1}^{N} \Pr(n-1|\overline{q}, N-1) E_{\mathbf{v}_{-i}} \left[p_i^g(v, \mathbf{v}_{-i}) \right],$$

where $\Pr(n-1|\overline{q}, N-1) = C_{N-1}^{n-1} \overline{q}^{n-1}(1-\overline{q})^{N-n}$ is the probability that entrant i faces $n-1$ rivals. For a symmetric direct mechanism, IC-2 holds if and only if entrant i's expected payoff is given by the following envelope condition

$$\pi_i(v_i;\overline{q}) = \pi(0) + \int_0^{v_i} P_i(y;\overline{q}) dy = \int_0^{v_i} P_i(y;\overline{q}) dy, \forall v_i \in [0,1], \tag{III.5}$$

and her expected winning probability

$$P_i(v_i;q) \text{ is weakly increasing in } v_i, \forall i \in g, \tag{III.6}$$

where the minimum payment $\pi(0) = 0$ in equation (III.5) is required by the *no-entry-fee* condition (III.1). Note, from equation (III.5), that the mechanism outcome depends on the allocation rules $\{p_i^g(\mathbf{v})|i \in g, g \in 2^{\mathscr{N}}\}$ alone.

III.2.2 Entry Stage

Given the bidding stage expected payoff $\pi_i(v_i;\overline{q})$, now we characterize the symmetric equilibrium entry probability $\overline{q}$. Toward this end, consider the entry decision made by a potential bidder i facing $N-1$ potential rivals who make their entry decisions according to the given equilibrium entry probability $\overline{q}$. Then bidder i's *interim* expected payoff from entry is given by

$$\begin{aligned}
\pi(\overline{q}) &= E_{v_i}\left[\pi(v_i;\overline{q})\right] \\
&= \int_0^1 \left[\int_0^{v_i} P_i(y;\overline{q})dy\right] dF(v_i) \\
&= \int_0^1 \frac{1-F(v_i)}{f(v_i)} P_i(v_i;\overline{q}) dF(v_i) \\
&= E_{v_i}\left[\frac{1-F(v_i)}{f(v_i)} P_i(v_i;\overline{q})\right] \\
&= \sum_{n=1}^{N} \Pr(n-1|\overline{q},N-1) E_{\mathbf{v}}\left[\frac{1-F(v_i)}{f(v_i)} p_i^g(\mathbf{v})\right] \\
&= \sum_{n=1}^{N} \Pr(n-1|\overline{q},N-1) E_{\mathbf{v}}\left[p_i^g(\mathbf{v})h(v_i)\right],
\end{aligned}$$

where $h(v) = \frac{1-F(v)}{f(v)}$ measures the *"information rent"* paying to an entrant for her information advantage on her private value v, and the third equality is due to Fubini's Theorem. It is easy to show that $\frac{d\pi(\overline{q})}{d\overline{q}} < 0$, and its implication is intuitive: when no opponent presents in the bidding stage, i.e., $\overline{q} = 0$, an entrant would expect the highest entry payoff $\pi(0)$; and when all potential opponents participate, i.e., $\overline{q} = 1$, each entrant should expect the lowest entry payoff $\pi(1)$.

A potential bidder's *ex ante* expected payoff by playing an entry strategy q is then given by $\Pi(q;\overline{q}) = q\left[\pi(\overline{q}) - c\right]$. The following lemma characterizes the equilibrium entry probability $\overline{q}$,

Lemma III.1. *The entry-stage incentive compatible condition III.3 implies the following necessary conditions for an equilibrium entry probability* $\overline{q}$*,*

- *For* $\overline{q} \in (0,1)$*,* $\frac{d\Pi(q;\overline{q})}{dq}|_{q=\overline{q}} = \pi(\overline{q}) - c = 0.$
- *For* $\overline{q} = 0$*,* $\frac{d\Pi(q;\overline{q}=0)}{dq}|_{q=\overline{q}=0} = \pi(0) - c \leq 0.$ *When the entry cost strictly exceeds the highest expected entry payoff, i.e,* $c > \pi(0)$*, no bidder would participate.*
- *For* $\overline{q} = 1$*,* $\frac{d\Pi(q;\overline{q}=1)}{dq}|_{q=\overline{q}=1} = \pi(1) - c \geq 0.$ *When the entry cost is lower than the lowest expected entry payoff, i.e,* $c \leq \pi(1)$*, all potential bidders would participate.*

III.3 Optimal Mechanism

We are now ready to derive the seller's expected revenue from a feasible semi-direct mechanism. The total expected surplus (TS) from a two-stage mechanism with an equilibrium entry probability q is given by

$$\begin{aligned} TS &= E_{\mathbf{v}}\left\{\sum_{g}\Pr(g|q,N)\left[\sum_{i\in g}p_i^g(\mathbf{v})v_i - |g|\cdot c\right]\right\} \\ &= \sum_{n=1}^{N} n\Pr(n|q,N)E_{\mathbf{v}}\left[p_i^g(\mathbf{v})v_i\right] - \sum_{n=1}^{N} n\Pr(n|q,N)\cdot c \\ &= \sum_{n=1}^{N} n\Pr(n|q,N)E_{\mathbf{v}}\left[p_i^g(\mathbf{v})v_i\right] - Nqc, \end{aligned}$$

where the last equality uses a property of binomial distribution that $\sum_{n=1}^{N} n\Pr(n|q,N) = Nq$.

The total expected payoff accrued to N potential bidders is given by

$$\begin{aligned} N\Pi(q;q) &= Nq\left[\pi(q) - c\right] \\ &= Nq\sum_{n=1}^{N}\Pr(n-1|q,N-1)E_{\mathbf{v}}\left[p_i^g(\mathbf{v})h(v_i)\right] - Nqc \\ &= \sum_{n=1}^{N} n\Pr(n|q,N)E_{\mathbf{v}}\left[p_i^g(\mathbf{v})h(v_i)\right] - Nqc, \end{aligned}$$

where the last equality uses a property of binomial distribution that $Nq\Pr(n-1|\overline{q},N-1) = n\Pr(n|\overline{q},N)$. The seller's expected revenue ER is then given by

$$\begin{aligned} ER &= TS - N\Pi(q;q) \\ &= \sum_{n=1}^{N} n\Pr(n|q,N)E_{\mathbf{v}}\left[p_i^g(\mathbf{v})(v_i - h(v_i))\right] \\ &= \sum_{n=1}^{N} n\Pr(n|q,N)E_{\mathbf{v}}\left[p_i^g(\mathbf{v})J(v_i)\right], \end{aligned}$$

where $J(v) := v - h(v)$ is the traditional *"virtual value"* and is increasing in v as we adopt the regularity assumption of monotone hazard rate.

The seller's problem is to maximize ER with respect to the allocation rules $\mathbf{p} := \{p_i^g(\mathbf{v})|i \in g, g \in 2^{\mathscr{N}}\}$, satisfying the resource constraint (III.2), the monotonicity condition (III.6) and the necessary conditions in Lemma III.1. We consider this optimization problem in three scenarios categorized in Lemma III.1.

III.3.1 Full Participation, $q^* = 1$

First, we consider the scenario that all potential bidder participate in equilibrium, i.e., $q^* = 1$, in which case, **IC-1** requires that $c \leq \pi(1) = E_{\mathbf{v}}\left[p_i^{\mathscr{N}}(\mathbf{v})h(v_i)\right]$. We augment this inequality condition into the objective function through a non-negative Lagrange multiplier λ as follows,

$$\mathscr{L} = NE_{\mathbf{v}}\left[p_i^{\mathscr{N}}(\mathbf{v})J(v_i)\right] + N\lambda\pi(1) = NE_{\mathbf{v}}\left[p_i^{\mathscr{N}}(\mathbf{v})(v_i - (1-\lambda)h(v_i))\right].$$

Denote the solution to the new optimization problem by $p^{\mathscr{N}*}$ and λ^*, and define

$$w(v,\lambda^*) = v - (1-\lambda^*)h(v) = J(v) + \lambda^* h(v)$$

as the *"virtual value"* adjusted for the first-stage entry incentivization. This adjusted *"virtual value"* identifies two sources of surplus the seller can extract from a winning entrant: $J(v)$ measures the maximal surplus extracted from the bidding stage and $\lambda^* h(v)$ measures an additional surplus the seller can extract through inducing a desired competition intensity, i.e., q^*. Naturally, the Lagrange multiplier λ^* indicates the proportion of *"information rents"* $h(v)$ accrued to the seller.

From the Lagrangian objective function, the optimal allocation rule is straightforward:

$$p_i^{\mathcal{N}*}(\mathbf{v}) = \begin{cases} 1 & \text{if } i = argmax_{j\in\mathcal{N}}\{w(v_j,\lambda^*)\} \text{ and } w(v_i,\lambda^*) \geq 0, \\ 0 & \text{otherwise,} \end{cases} \qquad \forall i \in \mathcal{N}.$$

This optimal allocation rule states that the asset should be awarded to the bidder with the highest non-negative adjusted virtual value. Given the monotone hazard rate assumption, this allocation rule can be implemented via a second-price auction with an optimal reserve price r^* satisfying $r^* = (1-\lambda^*)h(r^*)$.

Let $\pi(q;r)$ denote the expected payoff from participating in a second-price auction with a reserve price r when all potential opponents participate with probability q. Next we examine the complementary slackness conditions that $\lambda^*[\pi(1;r^*)-c] = 0$ and $\lambda^* \geq 0$.

III.3.1.1 $\lambda^* = 0$

If $\lambda^* = 0$, the optimal reserve price r^* is determined by $r^* = h(r^*)$, which corresponds to Myerson's optimal reserve price, denoted by r^M; see Myerson (1981). IC-1 requires that the entry cost

$$c \leq \pi(1;r^M) = \int_{r^M}^{1} F^{N-1}(v)[1-F(v)]dv.$$

This result is intuitive: when the entry cost is so low that all potential bidders will participate, the seller cannot induce a higher competition intensity to extract additional surplus, and thus the IC-1 is not binding and $\lambda^* = 0$.

III.3.1.2 $\lambda^* > 0$

If $\lambda^* > 0$, then the slackness conditions suggests that $\pi(1; r^*) = c$, i.e.,

$$\int_{r^*}^{1} F^{N-1}(v)[1 - F(v)]dv = c.$$

Then λ^* is determined by $r^* = (1-\lambda^*)h(r^*)$. Since $h(\cdot)$ is a decreasing function and $\lambda^* > 0$, it is easy to show that $r^* < r^M$, which in turn implies that $c > \pi(1; r^M)$. Moreover, since $\pi(1; r^*)$ is decreasing in r^*, the largest entry cost satisfying $\pi(1; r^*) = c$ is arrived at seller's reservation value, i.e., $r^* = v_0 = 0$.

In summary, $\pi(1; r^M) < c \leq \pi(1; v_0)$: when the entry cost is high enough, the seller can use reserve price to induce a higher competition intensity and extracts additional surplus, and thus the IC-1 is binding and $\lambda^* > 0$.

III.3.2 Partial Participation: $q^* \in (0, 1)$

This is the case studied in Levin and Smith (1994), where the equilibrium entry probability q^* is uniquely determined by $\pi(q^*) = c$, and thus the *ex ante* expected equilibrium payoff $\Pi(q^*; q^*) = q^* [\pi(q^*) - c] = 0$. The expected revenue can be written as follows,

$$ER = TS = \sum_{n=1}^{N} n \Pr(n|q^*, N) E_{\mathbf{v}} \left[p_i^g(\mathbf{v}) v_i \right] - Nq^* c.$$

Then it is clear that the optimal allocation rule is, $\forall g$, $\forall i \in g$,

$$p_i^{g*}(\mathbf{v}) = \begin{cases} 1 & \text{if } i = argmax_{j \in g}\{v_j\} \text{ and } v_i \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

This optimal mechanism is the same as the one identified in Levin and Smith (1994), where the asset should be awarded to the bidder with the highest value as long as the value is not less than seller's reservation value, i.e., $v_0 = 0$. This mechanism can be implemented via a second-price auction with a reserve price equal to the seller's reserve value, i.e., $r^* = v_0 = 0$. The expected entry payoff is given by

$$\pi(q^*; v_0) = \sum_{n=1}^{N} \Pr(n-1|q, N-1) \int_0^1 F^{n-1}(v)[1-F(v)]dv.$$

It is easy to show that $d\pi(q; v_0)/dq < 0$, $\forall q \in (0,1)$: when the equilibrium entry probability increases, an entrant is more likely to face more rivals and should expect to pay a higher bid to win the auction, and thus her expected entry payoff $\pi(q^*; v_0)$ goes down. Therefore, in this scenario, $\pi(1; v_0) < c \leq \pi(0; v_0)$ since $\pi(q^*; v_0) = c$.

Even though the Lagrange multiplier λ^* does not appear in this case explicitly, we can infer that $\lambda^* = 1$ since it measures the proportion of *"information rents"* $h(v)$ accrued to the seller. When the entry cost is so high, the seller extracts all the *"information rent"* from the winning bidder by setting the reserve price $r^* = v_0$.

III.3.3 No Participation: $q^* = 0$

This scenario appears when the entry cost is so high that no entrant can recover her entry cost from any feasible semi-direct mechanism. Considering the most benevolent environment for an entrant in the bidding stage: no rivals and minimum reserve price, the entrant would guarantee to retain the asset and pay the minimum possible price, i.e., seller's reservation value, and her *ex ante* expected payoff is $\pi(0; v_0) = E(v)$. Therefore, if the entry cost is formidably high, i.e, $c > \pi(0; v_0)$, no rational bidder would participate, i.e., $q^* = 0$. In this case, any allocation rule satisfying the resource constraint (III.2) and the monotonicity condition (III.6) is trivially optimal.

Finally, we summarize the optimal mechanism and the induced equilibrium entry pattern in the following proposition.

PROPOSITION III.1. *Among all feasible mechanisms* $\{(p_i^g(\mathbf{v}), x_i^g(\mathbf{v})) | i \in g, g \in 2^{\mathcal{N}}\}$*, the following mechanism maximizes the seller's expected revenue:* $\forall g$, $\forall i \in g$,

$$p_i^{g*}(\mathbf{v}) = \begin{cases} 1 & \text{if } i = argmax_{j \in g}\{w(v_j, \lambda^*)\} \text{ and } w(v_i, \lambda^*) \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

and the payment rules are defined as

$$x_i^{g*}(\mathbf{v}) = \begin{cases} \max\{r^*, z_i^g(\mathbf{v}_{-i})\} & \text{if } i = argmax_{j \in g}\{w(v_j, \lambda^*)\} \text{ and } w(v_i, \lambda^*) \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

where $z_i^g(\mathbf{v}_{-i}) = \max_{j \in g, j \neq i}\{v_j\}$ *is the highest value among rival entrants in group g,* $\forall g$*. The optimal reserve price* r^**, the induced equilibrium entry* q^**, and the Lagrange multiplier* λ^* *are determined as follows,*

> ***Case I:*** *When* $0 \leq c \leq \pi(1; r^M)$, $r^* = r^M$, $q^* = 1$, *and* $\lambda^* = 0$*;*
>
> ***Case II:*** *When* $\pi(1; r^M) < c \leq \pi(1; v_0)$, r^* *is determined by* $\pi(1; r^*) = c$, $q^* = 1$, *and* λ^* *is determined by* $r^* = (1 - \lambda^*)h(r^*)$*;*
>
> ***Case III:*** *When* $\pi(1; v_0) < c \leq \pi(0; v_0)$, $r^* = v_0$, $q^* \in (0, 1)$ *is determined by* $\pi(q^*) = c$*, and* $\lambda^* = 1$*;*
>
> ***Case IV:*** *When* $c > \pi(0; v_0)$, $q^* = 0$, $r^* \geq v_0$.

III.4 An Example

Now we illustrate the optimal mechanism in a numerical example: a seller with reservation value $v_0 = 0$ sells one asset to $N = 2$ potential bidders, who draw values from $\mathcal{U}[0, 1]$ independently after paying a common entry cost c. We can compute the Myerson's optimal reserve price $r^M = 1/2$ and the three critical points of entry cost as follows,

$$
\begin{aligned}
c_1 &:= \pi(1; r^M) = 1/12; \\
c_2 &:= \pi(1; v_0) = \frac{1}{N(N+1)} = 1/6; \\
c_3 &:= \pi(0; v_0) = E(v) = 1/2.
\end{aligned}
$$

Figure III.1 presents the optimal reserve price r^*, the induced equilibrium entry probability q^* and the the proportion of *"information rents"* accrued to the seller λ^*.

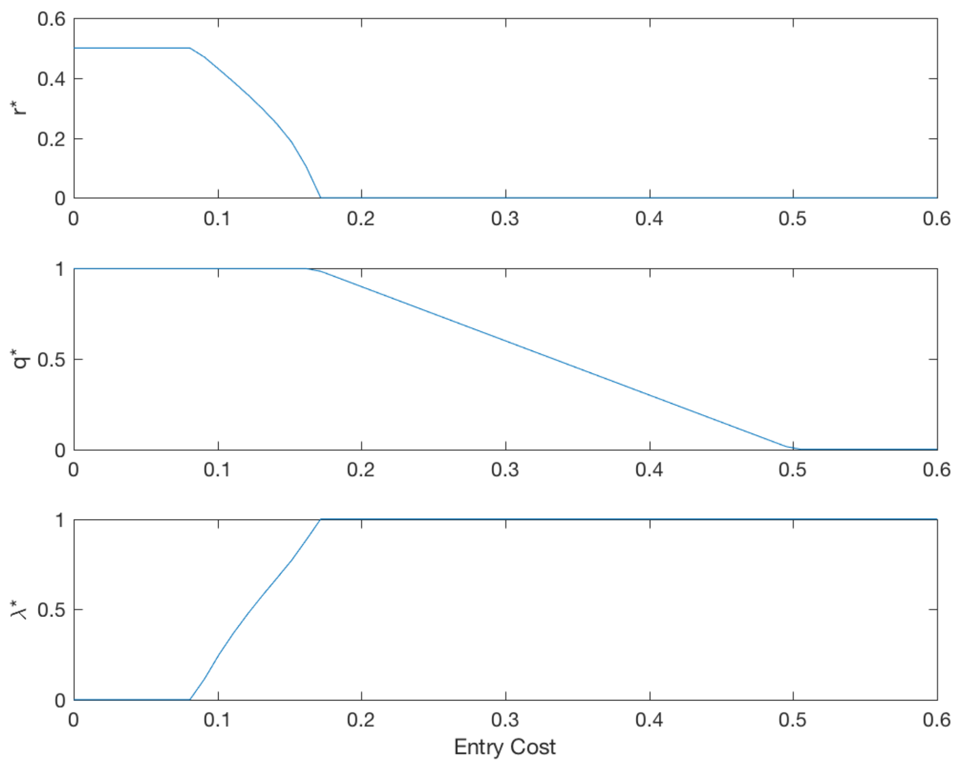

Figure III.1: Changes of r^*, q^* and λ^* along with Entry Cost.

III.5 Concluding Remarks

Departing from the extant studies on optimal auction design with endogenous entry, we explicitly assume away an entry fee/subsidy as a policy variable in order to mimic the real-world auction practices. We characterize bidders' equilibrium entry and bidding behaviors within the set of feasible semi-direct mechanisms, and find that the optimal mechanism should allocate the asset to the entrant with the highest non-negative virtual value adjusted for the entry-stage incentivization. The seller can extract additional information rent from the winner by using reserve price to induce a desired competition. The optimal reserve price is gradually decreasing with respect to the entry cost.

This chapter considers a symmetric IPV framework; while in practice, common value (CV) auction is also common, such as the OCS auction. It would be interesting to study the optimal auction design problem in the CV paradigm when an entry fee is not allowed. Another potential extension is to consider the selective entry framework studied in Gentry, Li, and Lu (2017) and see how the seller can use the reserve price to induce a desired signal threshold for entry .

Appendix A

TECHNICAL PROOFS

A.1 Proofs from Chapter I

Proof of Proposition I.1. This proof extends Proposition 2.2 in (Krishna, 2002) to our model with uncertain number of bidders and varying valuation.

Suppose that all but bidder i follow the bidding strategy (I.3). We argue that it is optimal for bidder i to follow (I.3) as well. Assuming that the bidding strategy (I.3) is an increasing and continuous function. Thus, in equilibrium the bidder with the highest value submits the highest bid and wins the auction. It is not optimal for any bidder to bid over $\beta(\overline{v};N)$.

First, with the rank preserving property assumed in Assumption I.1, we are able to rewrite the $\widetilde{\beta}_n^I(\cdot)$ as

$$\widetilde{\beta}_n^I(v) = \beta_n^I(\tilde{v}) := \begin{cases} \underline{\tilde{v}}, & \textbf{for } n = 1 \\ \frac{1}{\widetilde{F}_n(\tilde{v})} \int_{\underline{\tilde{v}}}^{\tilde{v}} \tilde{t} d\widetilde{F}_n(\tilde{t}), & \textbf{for } n \in \{2, \ldots, N\}, \end{cases}$$

where $\tilde{v} := v - D(n,v)$ with a CDF $\widetilde{F}_{\tilde{v}}(\cdot)$, and $\widetilde{F}_n(\cdot) := \widetilde{F}_{\tilde{v}}(\cdot)^{n-1}$.[1] Then, let $y := \beta^{-1}(b;N)$. The expected payoff of bidder i with value $v \in (\underline{v}, \overline{v})$ if he bids an amount $b = \beta(y;N)$ is

$$\begin{aligned} \pi(\beta(y;N), v; N) &= \sum_{n=1}^{N} p_{n|N} F_v(y)^{n-1} \left[v - D(n,v) - \beta(y;N) \right] \\ &= \sum_{n=1}^{N} p_{n|N} F_v(y)^{n-1} \tilde{v} - H(y|N) \beta(y;N) \end{aligned}$$

[1] To see this, note that the rank preserving assumption implies that

$$F_v(t) = Pr(v \leq t) = Pr(\tilde{v} \leq \tilde{t}) = \widetilde{F}_{\tilde{v}}(\tilde{t}).$$

$$
\begin{aligned}
&= \sum_{n=1}^{N} p_{n|N} F_v(y)^{n-1}\tilde{v} - \sum_{n=1}^{N} H(y|N) w_n(y) \beta_n^I(\tilde{y}) \\
&= \sum_{n=1}^{N} p_{n|N} F_v(y)^{n-1}\left[\tilde{v} - \beta_n^I(\tilde{y})\right] \\
&= p_{1|N}(v-\underline{v}) + \sum_{n=2}^{N} p_{n|N}\widetilde{F}_n(\tilde{y})\left[\tilde{v} - \frac{1}{\widetilde{F}_n(\tilde{y})}\int_{\underline{\tilde{v}}}^{\tilde{y}} \tilde{t}\, d\widetilde{F}_n(\tilde{t})\right] \\
&= p_{1|N}(v-\underline{v}) + \sum_{n=2}^{N} p_{n|N}\left[\widetilde{F}_n(\tilde{y})\tilde{v} - \int_{\underline{\tilde{v}}}^{\tilde{y}} \tilde{t}\, d\widetilde{F}_n(\tilde{y})\right] \\
&= p_{1|N}(v-\underline{v}) + \sum_{n=2}^{N} p_{n|N}\left[\widetilde{F}_n(\tilde{y})\,(\tilde{v}-\tilde{y}) + \int_{\underline{\tilde{v}}}^{\tilde{y}} \widetilde{F}_n(t)\, d\tilde{t}\right]
\end{aligned}
$$

Observe that $\pi(\beta(\underline{v};N);N),\underline{v};N) = 0$, so bidding $\beta(\underline{v};N) = \underline{v}$ is incentive compatible for the bidder with the lowest value. Next, we exam the change of profit by deviating from the bidding strategy (I.3),

$$
\pi(\beta(v;N),v;N) - \pi(\beta(y;N),v;N) = \sum_{n=2}^{N} p_{n|N}\left[\widetilde{F}_n(\tilde{y})(\tilde{y}-\tilde{v}) + \int_{\tilde{y}}^{\tilde{v}} \widetilde{F}_n(\tilde{t})\, d\tilde{t}\right] \geq 0,
$$

regardless of whether $\tilde{y} \leq \tilde{v}$ or $\tilde{z} \geq \tilde{v}$ because the term in the square brackets is non-negative for $n = 2, \ldots, N$.

We have thus shown that if all other bidders are following the strategy (I.3), an arbitrary bidder with a value of $v \in [\underline{v}, \overline{v}]$ cannot benefit by bidding anything other than $\beta(v;N)$. Therefore, $\beta(\cdot;N)$ given by (I.3) characterizes a symmetric equilibrium bidding strategy for the model. □

Proof of Proposition I.2. We rewrite (I.3) as follows,

$$
\begin{aligned}
\beta(v;N) &= \left(1 - \sum_{n=2}^{N} w_{n|N}\right)\underline{v} + \sum_{n=2}^{N} w_{n|N}(v)\widetilde{\beta}_n^I(v) \\
&= \underline{v} + \sum_{n=2}^{N} w_{n|N}(v)\left(\widetilde{\beta}_n^I(v) - \underline{v}\right) \qquad \text{(A.1)}
\end{aligned}
$$

Since $\widetilde{\beta}_n^I(v) - \underline{v}(\geq 0)$ is strictly increasing in v for all $n \geq 2$, then (I.3) is strictly increasing as long as $\omega_{n|N}(v)$ is nondecreasing in v for all $n \geq 2$. To simplify, we suppress dependence of v and N in $\omega_{n|N}(v)$.

$$\begin{aligned}
\frac{d}{dv}\omega_n &= \frac{1}{H^2}\left[(n-1)p_nF^{n-2}f\sum_{m=1}^{N}p_mF^{m-1} - p_nF^{n-2}F\sum_{m=2}^{N}p_m(m-1)F^{m-2}f\right] \\
&= \frac{1}{H^2}\left[(n-1)p_nF^{n-2}f\sum_{m=1}^{N}p_mF^{m-1} - p_nF^{n-2}f\sum_{m=2}^{N}p_m(m-1)F^{m-1}\right] \\
&= \frac{p_nF^{n-2}f}{H^2}\left[\sum_{m=1}^{N}p_m(n-1)F^{m-1} - \sum_{m=2}^{N}p_m(m-1)F^{m-1}\right] \\
&= \frac{p_nF^{n-2}f}{H^2}\left[\sum_{m=1}^{N}p_m(n-m)F^{m-1}\right].
\end{aligned}$$

Therefore, for $n = 2$, we obtain

$$\frac{d}{dv}\omega_2 = \frac{p_nF^{n-2}f}{H^2}\left[p_1 - \sum_{m=3}^{N}p_m(m-2)F^{m-1}\right] \geq \frac{p_nF^{n-2}f}{H^2}\left[p_1 - \sum_{m=3}^{N}p_m(m-2)\right] \geq 0,$$

where the last inequality is due to (I.4).

Note that $p_nF^{n-2}f/H^2 \geq 0$ and $\sum_{m=1}^{N}p_m(n-m)F^{m-1}$ is increasing with respect to n for any $n \geq 2$. So, $d\omega_2/dv \geq 0$ implies $d\omega_n/dv \geq 0$ for all $n > 2$, which in turn implies the strict monotonicity of equation (A.1). □

A.2 Proofs from Chapter II

Proof of Proposition II.2. Let $v(\alpha)$ be the α-quantile of $F_v(\cdot)$. Then $b_N(\alpha) = \beta(v(\alpha);N)$ because $b_N = \beta(v;N)$ and $\beta(v;N)$ is strictly increasing in v. Therefore, from equation (II.2), we have,

$$v(\alpha) = b_N(\alpha) + \lambda_N(b_N(\alpha)) + \sum_{n=2}^{N}\eta_{n,N}(b_N(\alpha))D(n, v(\alpha)) \qquad \text{(A.2)}$$

for any $\alpha \in (0,1]$ and an $N \geq 2$.

Now suppose we observe two distinct numbers, N_1 and N_2, of potential bidders and let

$N_2 > N_1 \geq 2$ without loss of generality. Under the Assumption (II.3), we can substitute $D(n, v(\alpha)) = (n-1)D(v(\alpha))$ into the equation (A.2). Then for $N = N_1$ and N_2, we have

$$v(\alpha) = b_N(\alpha) + \lambda_N(b_N(\alpha)) + \left[\sum_{n=2}^{N}(n-1)\eta_{n,N}(b_N(\alpha))\right] D(v(\alpha)). \qquad \text{(A.3)}$$

First note that $\sum_{n=2}^{N}(n-1)\eta_{n,N}(b_N(\alpha)) \neq 0$ given $p_{N|N} \neq 0$. Then by taking a difference of the two equations, we get

$$\begin{aligned}&(b_{N_2}(\alpha) - b_{N_1}(\alpha)) + (\lambda_{N_2}(b_{N_2}(\alpha)) - \lambda_{N_1}(b_{N_1}(\alpha)))\\ &+ \sum_{n=2}^{N_2}(n-1)\eta_{n,N_2}(b_{N_2}(\alpha))D(v(\alpha)) - \sum_{n=2}^{N_1}(n-1)\eta_{n,N_1}(b_{N_1}(\alpha))D(v(\alpha)) = 0,\end{aligned}$$

which leads to

$$\begin{aligned}D(v(\alpha)) &= \frac{(b_{N_1}(\alpha) - b_{N_2}(\alpha)) + (\lambda_{N_1}(b_{N_1}(\alpha)) - \lambda_{N_2}(b_{N_2}(\alpha)))}{\sum_{n=2}^{N_2}(n-1)\eta_{n,N_2}(b_{N_2}(\alpha)) - \sum_{n=2}^{N_1}(n-1)\eta_{n,N_1}(b_{N_1}(\alpha))}\\ &= \frac{(b_{N_1}(\alpha) - b_{N_2}(\alpha)) + (\lambda_{N_1}(b_{N_1}(\alpha)) - \lambda_{N_2}(b_{N_2}(\alpha)))}{\sum_{n=2}^{N_2} n \cdot \eta_{n,N_2}(b_2(\alpha)) - \sum_{n=2}^{N_1} n \cdot \eta_{n,N_1}(b_{N_1}(\alpha))}.\end{aligned}$$

The last equation is arrived by recognizing that $\sum_{n=2}^{N}\eta_{n,N}(b) = 1$ for any $b \in (\underline{b}, \overline{b}]$ and any $N \geq 2$ when $p_{1|N} < 1$. This concludes the identification of $D(v(\alpha))$ at α-quantile.[2]

With $D(v(\alpha))$ identified at each quantile $\alpha \in (0,1]$, we can identify $v(\alpha)$ for each $\alpha \in (0,1]$ by equation (A.3). As an immediately result, the valuation distribution $F_v(\cdot)$ is nonparametrically identified on $(\underline{v}, \overline{v}]$. And the information disclosure discount factor $D(v)$ is also nonparametrically identified on $(\underline{v}, \overline{v}]$ by equation (A.3). □

[2] $\sum_{n=2}^{N_2} n \cdot \eta_{n,N_2}(b_2(\alpha)) - \sum_{n=2}^{N_1} n \cdot \eta_{n,N_1}(b_{N_1}(\alpha)) \neq 0$. If it is zero, then the bidding strategy is not strictly monotone.

Appendix B

ESTIMATION DETAILS IN CHAPTER II

B.1. Derivation of Bid Distribution Functions

In order to construct the bid density, consider an auction with characteristics $(\tau, X) = (1,0)$ and N potential bidders. Notice that the success rate of the shortlisting probability (II.4) for this auction is $q := \Phi(X'\delta) = \Phi(0)$. We let (h_ε, η) index (F_ε, D), respectively. For this auction, we use $\beta(\cdot|q, N, h_\varepsilon, \eta)$ to denote the equilibrium bidding strategy. Then, interpreting observed bids in data as equilibrium outcomes, we may write

$$
\begin{aligned}
b_{it} &= \beta(v_{it}|\Phi(X'\delta), N, h_\varepsilon, \eta) \\
&= \beta(\tau \exp(X'\gamma)\varepsilon_{it}|\Phi(X'\delta), N, h_\varepsilon, \eta) \\
&= \tau \exp(X'\gamma)\beta(\varepsilon_{it}|\Phi(X'\delta), N, h_\varepsilon, \eta)
\end{aligned}
$$

for the auction with characteristics (X, τ). The bid density can then be written as The bid density can then be written as

$$
\begin{aligned}
g(b_{it}|N, X; \tau, \theta) =& \frac{f_\varepsilon\left[\beta^{-1}\left(\frac{b_{it}}{\tau \exp(X'\gamma)}|\Phi(X'\delta), N, h_\varepsilon, \eta\right)|h_\varepsilon\right]}{\tau \exp(X'\gamma)\beta'\left[\beta^{-1}\left(\frac{b_{it}}{\tau \exp(X'\gamma)}|\Phi(X'\delta), N, h_\varepsilon, \eta\right)|\Phi(X'\delta), N, h_\varepsilon, \eta\right]} \\
&\times \mathbb{1}\left\{\frac{b_{it}}{\tau \exp(X'\gamma)} \in \left[\underline{\varepsilon}, \beta(\overline{\varepsilon}|\Phi(X'\delta), N, h_\varepsilon, \eta)\right]\right\},
\end{aligned}
$$

where $\theta := (\gamma, \delta, \eta, h_\varepsilon, h_\tau)$. Accordingly, the distribution function of b_{it} is

$$
G(b_{it}|N, X; \tau, \theta) := F_\varepsilon\left[\beta^{-1}\left(\frac{b_{it}}{\tau \exp(X'\gamma)}|\Phi(X'\delta), N, h_\varepsilon, \eta\right)|h_\varepsilon\right].
$$

B.2. Posterior Sampling

Now we provide details on sampling $\{\boldsymbol{\tau}^{(s)}, \theta^{(s)}\}_{s=1}^{S}$ from the posterior distribution $\pi(\boldsymbol{\tau}, \theta|\mathbf{y})$ using Metropolis-within-Gibbs algorithm. Suppose the current parameter value is $\theta^{(s)} = (\gamma^{(s)}, \delta^{(s)}, \eta^{(s)}, h_{\varepsilon}^{(s)}, h_{\tau}^{(s)})$ and $\boldsymbol{\tau}^{(s)}$, we describe how we draw $\theta^{(s+1)}$ and $\boldsymbol{\tau}^{(s+1)}$ for $s = 1, \ldots, S$.

1. Draw $\gamma^{(s+1)} = (\gamma_1^{(s+1)}, \ldots, \gamma_k^{(s+1)})$: For each j starting from 1 to k, we update $\gamma_j^{(s+1)}$ using a slightly modified Gaussian Metropolis-Hastings algorithm. In order to improve computational efficiency, we compute the conditional support $[\underline{\gamma}_j^{(s)}, \overline{\gamma}_j^{(s)}]$ and draw a candidate $\tilde{\gamma}_j$ from the truncated normal distribution.

   Let $b^{\min}$ and $b^{\max}$ be the lowest and the highest (i.e. winning) observed bids in auction t respectively. Theoretically, any bid b_{it} is bounded by

$$\underline{\varepsilon}^{(s)} = \underline{b}^{(s)} \leq \frac{b_{it}}{\tau^{(s)} \exp(x\gamma^{(s)})} \leq \overline{b}^{(s)} := \beta(\overline{\varepsilon}^{(s)} | \Phi(x'\delta^{(s)}), N, h_{\varepsilon}^{(s)}, \eta^{(s)}),$$

   where $\underline{\varepsilon}^{(s)} = \exp(-dh_{\varepsilon}^{(s)-1/2})$ and $\overline{\varepsilon}^{(s)} = \exp(dh_{\varepsilon}^{(s)-1/2})$. Then if $x_{jt} > 0$, from the first inequality, we have

$$\overline{\gamma}_j^{(s)} := \min_{t \in \{1, \cdots, T\}} \left\{ \frac{1}{x_{jt}} \log \left[\frac{b^{\min}}{\underline{b}^{(s)} \tau^{(s)} \exp\left(\sum_{j' \neq j} x_{j't} \gamma_{j'}^{(s)}\right)} \right] \right\}$$

   and from the second inequality,

$$\underline{\gamma}_j^{(s)} := \max_{t \in \{1, \cdots, T\}} \left\{ \frac{1}{x_{jt}} \log \left[\frac{b^{\max}}{\overline{b}^{(s)} \tau^{(s)} \exp\left(\sum_{j' \neq j} x_{j't} \gamma_{j'}^{(s)}\right)} \right] \right\}.$$

   For $x_{jt} < 0$, we obtain similar support $[\underline{\gamma}_j^{(s)}, \overline{\gamma}_j^{(s)}]$, and do not repeat it here.

   We draw a candidate $\tilde{\gamma}_j \sim \mathcal{N}(\gamma_j^{(s)}, \sigma_{\gamma_j}^2) \mathbb{1}(\tilde{\gamma}_j \in [[\underline{\gamma}_j^{(s)}, \overline{\gamma}_j^{(s)}])$. In particular, we draw

$u \sim \mathscr{U}nif(0,1)$, then let

$$\tilde{\gamma}_j := \gamma_j^{(s)} + \sigma_{\gamma_j}\Phi^{-1}\left\{ u \cdot \left[\Phi\left(\frac{\overline{\gamma}_j^{(s)} - \gamma_j^{(s)}}{\sigma_{\gamma_j}} \right) - \Phi\left(\frac{\underline{\gamma}_j^{(s)} - \gamma_j^{(s)}}{\sigma_{\gamma_j}} \right) \right] + \left(\frac{\underline{\gamma}_j^{(s)} - \gamma_j^{(s)}}{\sigma_{\gamma_j}} \right) \right\}.$$

We then set $\gamma_j^{(s+1)} := \tilde{\gamma}_j$ with probability

$$\min\left\{ 1, \frac{\pi(\tilde{\theta})L(\boldsymbol{y},\boldsymbol{n}|\boldsymbol{\tau}^{(s)},\boldsymbol{X},\tilde{\theta})}{\pi(\hat{\theta}^{(s)})L(\boldsymbol{y},\boldsymbol{n}|\boldsymbol{\tau}^{(s)},\boldsymbol{X},\hat{\theta}^{(s)})} \times \frac{\Phi\left(\frac{\overline{\gamma}_j^{(s)}-\tilde{\gamma}_j}{\sigma_{\gamma_j}}\right) - \Phi\left(\frac{\underline{\gamma}_j^{(s)}-\tilde{\gamma}_j}{\sigma_{\gamma_j}}\right)}{\Phi\left(\frac{\overline{\gamma}_j^{(s)}-\gamma_j^{(s)}}{\sigma_{\gamma_j}}\right) - \Phi\left(\frac{\underline{\gamma}_j^{(s)}-\gamma_j^{(s)}}{\sigma_{\gamma_j}}\right)} \right\};$$

otherwise, $\gamma_j^{(s+1)} := \gamma_j^{(s)}$, where $\tilde{\theta} := (\gamma_1^{(s+1)}, \ldots, \gamma_{j-1}^{(s+1)}, \tilde{\gamma}_j, \gamma_{j+1}^{(s)} \ldots, \gamma_k^{(s)}; \delta^{(s)}, \eta^{(s)}, h_\varepsilon^{(s)}, h_\tau^{(s)})$ contains the most recent updated values for all other parameters but $\tilde{\gamma}_j$; and $\hat{\theta}^{(s)} := (\gamma_1^{(s+1)}, \ldots, \gamma_{j-1}^{(s+1)}, \gamma_j^{(s)}, \gamma_{j+1}^{(s)} \ldots, \gamma_k^{(s)}; \delta^{(s)}, \eta^{(s)}, h_\varepsilon^{(s)}, h_\tau^{(s)})$ contains the most recent updated values for all other parameters but $\gamma_j^{(s)}$.[1]

Finally, we tune σ_{γ_j} following Haario, Saksman, and Tamminen (2001, 2005). In particular, for the first 10 iteration $s = 1, \ldots, 10$, we use $\sigma_{\gamma_j} = 0.01$; then for $s = 11, \ldots, S$, we use $\sigma_{\gamma_j} = 1.54 \times \widehat{\text{Std}}(\gamma_j^{(1)}, \ldots, \gamma_j^{(s-1)})$ with probability 95% and $\sigma_{\gamma_j} = 0.0001$ with probability 5%.[2]

2. Draw $\delta^{(s+1)} = (\delta_1^{(s+1)}, \ldots, \delta_k^{(s+1)})$: For each j starting from 1 to k, we update $\delta_j^{(s+1)}$ by drawing a candidate $\tilde{\delta}_j \sim \mathscr{N}(\delta_j^{(s)}, \sigma_{\delta_j}^2)$ We then set $\delta_j^{(s+1)} := \tilde{\delta}_j$ with probability

$$\min\left\{ 1, \frac{\pi(\tilde{\theta})L(\boldsymbol{y},\boldsymbol{n}|\boldsymbol{\tau}^{(s)},\boldsymbol{X},\tilde{\theta})}{\pi(\hat{\theta}^{(s)})L(\boldsymbol{y},\boldsymbol{n}|\boldsymbol{\tau}^{(s)},\boldsymbol{X},\hat{\theta}^{(s)})} \right\};$$

otherwise, $\delta_j^{(s+1)} := \delta_j^{(s)}$, where $\tilde{\theta}$ and $\hat{\theta}^{(s)}$ are defined similarly for parameter δ_j. As before, we tune σ_{δ_j} by setting $\sigma_{\delta_j} = 0.01$ for first ten iterations, after which we

[1] Without confusion, we recycle the notations $\tilde{\theta}$ and $\tilde{\theta}^{(s)}$ for each parameter updating.

[2] The reference proves the convergence of the algorithm and provide a recursive formula to compute $\widehat{\text{Std}}(\gamma_j^{(1)}, \ldots, \gamma_j^{(s-1)})$ at each s.

use $\sigma_{\delta_j} = 1.54 \times \widehat{\text{Std}}(\delta_j^{(1)}, \ldots, \delta_j^{(s-1)})$ with probability 95% and $\sigma_{\delta_j} = 0.0001$ with probability 5%.

3. Draw $\eta^{(s+1)}$: Similarly, we draw a candidate $\tilde{\eta} \sim \mathcal{N}(\eta^{(s)}, \sigma_\eta^2)$ and update $\eta^{(s+1)} = \tilde{\eta}$ with probability

$$\min\left\{1, \frac{L(\boldsymbol{y}, \boldsymbol{n}|\boldsymbol{\tau}^{(s)}, \boldsymbol{X}, \tilde{\theta})}{L(\boldsymbol{y}, \boldsymbol{n}|\boldsymbol{\tau}^{(s)}, \boldsymbol{X}, \hat{\theta}^{(s)})}\right\};$$

where $\tilde{\theta}$ and $\hat{\theta}^{(s)}$ are defined similarly for parameter η. We tune the scale parameter σ_η as before.

4. Draw $h_\varepsilon^{(s+1)}$: Similar to step 1, we use the theoretical restrictions to truncate the proposal density in order to improve computational efficiency. From the following restriction,

$$\underline{\varepsilon}^{(s)} = \exp(-dh_\varepsilon^{(s)-1/2}) \leq \frac{b}{\tau \exp(x\gamma^{(s)})} \leq \overline{b}^{(s)} := \beta(\overline{\varepsilon}^{(s)}|\Phi(x'\delta^{(s)}), N, h_\varepsilon^{(s)}, \eta^{(s)}).$$

we obtain

$$\overline{h}_\varepsilon^{(s)} = \min_{t \in \{1,\ldots,T\}} d^2 \left\{\log\left[\frac{b^{\min}}{\tau^{(s)} \exp(x\gamma^{(s)})}\right]\right\}^{-2}.$$

Similarly, we could obtain the exact lower bound from the second inequality, but we do not because it requires us to compute the equilibrium bidding strategy for each $t \in \{1, \ldots, T\}$. Instead, we only use the fact that $h_\varepsilon > 0$. We draw the candidate $\tilde{h}_\varepsilon \sim \mathcal{N}(h_\varepsilon^{(s)}, \sigma_{h_\varepsilon}^2) \mathbb{1}(\tilde{h}_\varepsilon \in [0, \overline{h}_\varepsilon^{(s)}])$. In particular, we let

$$\tilde{h}_\varepsilon := h_\varepsilon^{(s)} + \sigma_{h_\varepsilon} \Phi^{-1}\left\{u \cdot \left[\Phi\left(\frac{\overline{h}_\varepsilon^{(s)} - h_\varepsilon^{(s)}}{\sigma_{h_\varepsilon}}\right) - \Phi\left(\frac{0 - h_\varepsilon^{(s)}}{\sigma_{h_\varepsilon}}\right)\right] + \left(\frac{0 - h_\varepsilon^{(s)}}{\sigma_{h_\varepsilon}}\right)\right\}$$

with $u \sim \mathscr{U}nif(0,1)$. We then update $h_\varepsilon^{(s+1)} = \tilde{h}_\varepsilon$ with probability

$$\min\left\{1, \frac{\pi(\tilde{\theta})L(\boldsymbol{y},\boldsymbol{n}|\boldsymbol{\tau}^{(s)},\boldsymbol{X},\tilde{\theta})}{\pi(\hat{\theta}^{(s)})L(\boldsymbol{y},\boldsymbol{n}|\boldsymbol{\tau}^{(s)},\boldsymbol{X},\hat{\theta}^{(s)})} \times \frac{\Phi\left(\frac{\bar{h}_\varepsilon^{(s)}-\tilde{h}_\varepsilon}{\sigma_{h_\varepsilon}}\right) - \Phi\left(\frac{0-\tilde{h}_\varepsilon}{\sigma_{h_\varepsilon}}\right)}{\Phi\left(\frac{\bar{h}_\varepsilon^{(s)}-h_\varepsilon^{(s)}}{\sigma_{h_\varepsilon}}\right) - \Phi\left(\frac{0-h_\varepsilon^{(s)}}{\sigma_{h_\varepsilon}}\right)}\right\}$$

where $\tilde{\theta}$ and $\hat{\theta}^{(s)}$ are defined similarly for parameter h_ε. We tune the scale parameter σ_{h_ε} as in the previous steps.

5. Draw $h_\tau^{(s+1)}$: h_τ is the hyperparameter that governs the distribution of the unobserved heterogeneity τ. Hence we draw $h_\tau^{(s+1)}$ from its conditional density given $\boldsymbol{\tau}^{(s)}$. Using the conjugacy of gamma prior for normal precision, the conditional density of h_τ is proportional to

$$\begin{aligned}\pi_\tau(h_\tau)\prod_{t=1}^{T} f_\tau(\tau|h_\tau) &\propto h_\tau^{\alpha_\tau-1}\exp(-\lambda_\tau h_\tau)\prod_{t=1}^{T} h_\tau^{\frac{1}{2}}\exp\left(-\frac{h_\tau}{2}(\log\tau)^2\right) \\ &\propto h_\tau^{\alpha_\tau+\frac{T}{2}-1}\exp\left[-\left(\lambda_\tau + \frac{1}{2}\sum_{t=1}^{T}(\log\tau)^2\right)h_\tau\right].\end{aligned}$$

Hence, we draw

$$h_\tau^{(s+1)} \sim \mathscr{G}a\left(\widehat{\alpha}_\tau, (\widehat{\lambda}_\tau^{(s)})^{-1}\right)$$

where $\widehat{\alpha}_\tau = \alpha_\tau + \frac{T}{2}$ and $\widehat{\lambda}_\tau^{(s)} := \lambda_\tau + \frac{1}{2}\sum_{t=1}^{T}(\log\tau^{(s)})^2$.

6. Draw $\boldsymbol{\tau}^{(s+1)}$: we draw $\tilde{\tau}$ simultaneously for each $t \in \{1,\ldots,T\}$. Since τ has to be positive, we draw $\tilde{\tau} \sim \mathscr{N}(\tau^{(s)}, \sigma_\tau^2)$ individually and collect $\tilde{\boldsymbol{\tau}} = \{\tilde{\tau}\}_{t=1}^{T}$. Then we update $\tau^{(s+1)} = \tilde{\tau}$ with probability

$$\min\left\{1, \frac{f_\tau(\tilde{\tau}|h_\tau^{(s+1)})L(\boldsymbol{y},\boldsymbol{n}|\tilde{\boldsymbol{\tau}},\boldsymbol{X},\theta^{(s+1)})}{f_\tau(\tau^{(s)}|h_\tau^{(s+1)})L(\boldsymbol{y},\boldsymbol{n}|\boldsymbol{\tau}^{(s)},\boldsymbol{X},\theta^{(s+1)})}\right\}.$$

We tune the scale parameter σ_τ as in the previous steps.

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