

Contribution of Nonsymbolic Representation Beyond Symbolic Equal Sign Instruction in
Second-Grade Classrooms

By

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CHAPTER 1

I. INTRODUCTION

When a child enters school, language forms the necessary foundation for academic and behavioral success (Tomblin et al., 2000). Language is required to understand academic instruction through listening and reading content as well as to demonstrate understanding by speaking and writing. As comprehension demands of classroom discourse become less related to the instructional content as students move through school (Nelson, 1985), an even more complex environment with varying levels of linguistic demands unfolds. Effective communication and comprehension skills are essential, as evaluation of understanding (or misunderstanding) happens via speaking and writing (Cazden, 2001).

Because teachers use oral language as the primary mode of academic instruction, and students use language to interact with teachers and peers, proficiency in language is essential to navigate the academic and social context of classrooms and schools. Communicative clarity and accuracy may be more valued in school than in any other context children encounter (Harmon & Watson, 2012; Peets, 2009). Although the expectation of linguistic accuracy and communicative ability may be higher in the classroom, it is likely the expectation of linguistic and communicative competency varies across different academic, social, and family contexts (Aulls, 1998; Cazden, 2001; Peets, 2009; Walshaw & Anthony, 2008).

Because language is used differently (and effectively) in different contexts, considering the idea of a “correct” English language and the potential consequences is important. Harmon and Wilson (2012) argue for a “broadening of what we mean by ‘grammatical’ in order to

recognize the fundamentally complex language structures that all speakers use” (p. 29). The *prescriptive* grammar movement began to describe many of the “wrongs” and “rights” of the English language (Parker & Riley, 2010). A prescriptive approach to grammar contains a set of rules based on how spoken and written language should correctly be used (i.e., there is a right way and a wrong way) and is the common standard in most classrooms in the United States. In contrast, *descriptive* grammar refers to how language is typically used. Because students may be communicating effectively with peers and family via grammar that would be described as descriptive, one supposition is that the traditional prescriptive approach to language and teaching in the classroom may serve as a catalyst for the increasing mismatch between the linguistic expectations (and subsequent feedback) a child experiences. For example, if a child uses the phrase “she went” with the intention of communicating the phrase “she said,” a prescriptive approach would consider “she went” to be incorrect, whereas a descriptive approach would consider it an effective way of using language to communicate. For students with co-occurring language and behavior problems, the higher linguistic demands of the classroom and the dichotomy between prescriptive and descriptive language may be particularly relevant because of the association between teacher instruction and student disruptive behavior (Gunter et al., 1994).

Problem Behavior and Classroom Interactions

Students with emotional and behavioral disorders (EBD) are often identified as having expressive language deficits that result in an inability to engage in and maintain fluent conversations (McDonough, 1989), and receptive language deficits that impair their ability to comprehend abstract concepts and decontextualized language (Warr-Leeper, Wright, & Mack, 1994). According to Patterson and Reid’s (1984) theoretical categorizations based on the study

of family member interaction, these problematic communicative interactions between teachers and their students fall under the umbrella of a coercive interaction model. Patterson (1982) describes this as a “negative reinforcement trap,” which is an interaction-based scenario created when both parties are reinforced by behaviors that ultimately have a negative impact on desired outcomes. In the classroom, this cyclic interaction pattern may have devastating consequences for students’ academic and social outcomes.

Harrison, Gunter, Reed, and Lee (1996) proposed a conceptual framework for understanding teacher instruction and negative reinforcement, hypothesizing that teacher instructional language may be aversive stimuli to students with EBD, and that these stimuli maintain and reinforce problem behavior. For example, researchers have suggested that linguistically complex instruction (e.g., sophisticated, abstract, vague, lengthy) may be misunderstood by or completely incomprehensible to students with language deficits (Harrison et al., 1996; Kevan, 2003). Because researchers have reported this negative cycle of interaction in classrooms with students at-risk for or with EBD (Gunter et al., 1994; Sutherland & Morgan, 2003; Wehby, Symons, & Shores, 1995), and many students with EBD have low language skills (Hollo, Wehby, & Oliver, 2014), it is possible that students engage in problem behavior to avoid instructional interactions with teachers, and that the problem behavior is reinforced by the delivery of a desired consequence (e.g., escape). This finding is consistent with the hypothesis that negative social behavior functions for some students as a communicative action.

Additional research has shown that aggressive behavior in classrooms makes the development of a negative teacher-student relationship more likely (Ladd & Burgess, 1999), and these negative relationships tend to stabilize over time (Henricsson & Rydell, 2004). Intuitively, the more negative interactions (and subsequent escape from academic tasks) that occur, the less

instruction that student receives. In one study, teachers of students with EBD in self-contained classrooms devote only 30% of their time to academic instruction (Wehby, Lane, & Falk, 2003), and these students consistently present low rates of academic engagement overall (Scott, Alter, & Hirn, 2011; Shinn et al., 1987) compared to their non-disabled peers (Briesch, Volpe, & Ferguson, 2014). Thus, the poor academic performance of students with EBD is no surprise.

Language and Behavioral Deficits

Language deficits and behavioral problems predict poor academic and social outcomes (Bradley, Doolittle, & Bartolotta, 2008; Hinshaw, 1992; Mogford-Bevan & Summersall, 1997; Nation, Clarke, Marshall, & Durand, 2004). These findings are not surprising, as language skills are needed to navigate the academic landscape, and problem behavior often interferes with social relationships and access to academic instruction. However, descriptive research suggests that these deficits are highly comorbid. Hollo and colleagues (2014) estimated that 81% of children and youth with emotional and behavioral disorders (EBD) had unidentified – thus untreated – language deficits. Yew and O’Kearney (2013) found that children with specific language impairment were twice as likely to develop externalizing, internalizing, and attentive behavioral deficits. These two meta-analyses, along with other descriptive research (e.g., Bornstein, Hahn, & Suwalsky, 2013; Botting & Conti-Ramsden, 2008; Bretherton et al., 2014; Camarata, Hughes, & Ruhl, 1988; Lindsay, Dockrell, & Strand, 2007; Nelson, Benner, Neill, & Stage, 2006; Peterson et al., 2013), suggest that the constructs of language and behavior are linked, and that the prevalence of co-occurring linguistic and behavioral deficits among children who struggle in school is probable. This relation has recently received federal attention as an area of need; this year, the National Center for Research in Special Education’s request for applications stated that “research is needed to better understand the relationship between language deficits and behavior

problems and to allow for the development of effective interventions for this population of students” (‘IES Request for Applications,’ 2016, p.28). However, much less is known about specifically how these deficits interact with school and classroom environments and how that interaction leads to poor social and academic outcomes.

Academic Consequences of Language and Behavioral Deficits

Oral language performance has been shown to predict achievement in reading (Dickinson, Golinkoff, & Hirsh-Pasek, 2010; Roth, Speece, & Cooper, 2002) and mathematics (Duncan et al., 2007; Jordan et al., 2013; Fuchs et al., 2005). Children who struggle behaviorally have severe academic (Coutinho, 1986; Hinshaw, 1992; Kent et al., 2011; Nelson, Benner, Lane, & Smith, 2004; Reid, Gonzalez, Nordness, Trout, & Epstein, 2004; Wagner, Kutash, Duchnowski, Epstein, & Sumi, 2005) and linguistic (Hollo et al., 2014; Benner et al., 2002) deficits, and children with language deficits demonstrate poor academic outcomes (Cantwell & Baker, 1980). Further, research continues to reveal low academic and classroom participation of both students with EBD (e.g., Scott et al., 2011) and language impairment (Fujiki, Brinton, Morgan, & Hart, 1999; Rice, Sell, & Hadley, 1991). Because (a) low language is associated with poor academic achievement, (b) children with problem behavior perform dismally on measures of academic achievement, (c) and low language is highly prevalent in children with problem behavior, deficits in achievement, behavior, and language are likely inextricable. As we continue to work toward improving the educational outcomes of these students, it may be important to consider if development of interventions aimed at improving outcomes in one area (e.g., language) may benefit outcomes in other deficit areas (e.g., problem behavior). Given the strong connection of language impairment to academic and behavior difficulties, it seems promising to understand how interventions targeting language problems might be used in in this

fashion.

Intervention for Children with Language Deficits

Current language intervention primarily focuses on language and literacy skill development. It is well known that learning to read helps improve oral language. However, broader language skills are necessary in developing reading comprehension skills (Catts, Fey, Zhang, & Tomblin, 1999; Muter, Hulme, Snowling, & Stevenson, 2004). Research on intensive language intervention for young children has an established literature base (e.g., Fricke, Bowyer-Crane, Haley, Hulme, & Snowling, 2013; Strong, Torgerson, Torgerson, & Hulme, 2011; Swanson, Fey, Mills, & Hood, 2005). This emphasis on young children, perhaps, is because oral language developmentally precedes phonological awareness (Burgess & Lonigan, 1998; Storch & Whitehurst, 2002), and phonological awareness plays a central role in reading fluency and comprehension (National Reading Panel, 2000). Although there are evidence-based interventions that have successfully remediated literacy deficits in children (e.g., Fletcher, Reid Lyon, Fuchs, & Barnes, 2007), phonological impairments, vocabulary deficits, and low oral language skills are associated with poor response to these types of interventions (Al Otaiba & Fuchs, 2006, Duff et al., 2008; Torgesen et al., 1999; Vadasy, Sanders, & Abbott, 2008; Vellutino et al., 1996).

These studies suggest that oral language intervention is needed in order for students to benefit from literacy interventions, but the extent to which the interventions are effective is not firmly established. Of further concern is that the majority of the language interventions reported in these studies were comprised of intensive, individual sessions over many weeks, and few studies have evaluated practices with school-age children. In a review by Cirrin and Gillam (2008), the authors' search of school-age language interventions yielded 21 studies. Of these

studies, more than 50% included participants in kindergarten and first grade, and no studies focused on students in middle or high school. Further, the range in study quality, design, and language outcome variables did not allow for any strong conclusions about the efficacy of the research base. As a result, school-based professionals have relatively little evidence in which to base their practice (Cirrin & Gillam). For example, it has been reported that 99.6% of speech-language pathologists rely on personal clinical experience to make clinical decisions, 78.7% use colleagues' opinions, whereas only 17.7% rely on research to make clinical decisions (Zipoli & Kennedy, 2005). Thus, this is an important area of research that is likely going to continue.

One approach for improving academic outcomes for children with language deficits can be pursued by repackaging pre-existing evidence-based interventions and strategies in a way that supports their language difficulties in a compensatory fashion. This concurrent focus may also provide an avenue for researchers to actively test strategies that have been shown to improve academic performance for students with language deficits, adding to the research literature while simultaneously providing practical strategies for practitioners.

Visual Support

For students with language difficulty, visual representations are a promising and potentially important instructional adaptation to consider. Visuals may be particularly important for children with language impairment, because they provide a cross-modality approach for encoding information (Washington & Warr-Leeper, 2013), which may be less impacted by language deficits (Ebbels, 2007; Wener & Archibald, 2011). Pictures provide a visual representation for children that can remain in view, unlike temporary and often rapid verbal information (Cohen et al., 2005; Tallal et al., 1996), allowing children to access other forms of processing without needing to simultaneously retain the stimulus in working memory. Studies

have demonstrated that computer- and paper-based visual aids improve outcomes for children with language deficits (e.g., Cohen et al., 2005; Ebbels & van der Lely, 2001; Ebbels, 2008; Gillam et al., 2008; Leonard et al., 2006; 2008). Further, visualization strategies have been shown to improve language outcomes beyond rehearsal strategies (Gill, Klecan-Aker, Roberts, & Fredenburg, 2003). In the context of instruction, visual cues support cognitive capacities, which refer to the amount of computational space or resources necessary for completing a mental task (Kail & Salthouse, 1994). Visual support may lighten cognitive load, promote encoding, reduce processing demands, and highlight important features (Ebbels, 2008; Ebbels & van der Lely, 2001). The support that visuals provide paired with explicit instruction and practice may lessen the burden placed on student's cognitive resources (Washington & Warr-Leeper, 2011), thus compensating for existing linguistic processing deficits. Incorporating visuals into academic instruction may implicitly support language skills while explicitly teaching important content and concepts.

The Present Study

Because there is robust evidence supporting the use of visuals to improve and support math outcomes (e.g., Gersten et al., 2009; Jayanthi, Gersten, & Baker, 2008; Jitendra et al., 2016; National Advisory Mathematics Panel, 2008), we posit that integrating visual support into class-wide instruction will profit a wide range of students, including those with language and/or behavior difficulties. Given our intent to support language deficits during instruction, it is encouraging that research in mathematics education suggests that visual, or nonsymbolic, modes of instructional practices may facilitate learning. Specifically, research that focuses on the teaching *mathematical equivalence* has generated promising findings in the use of visual instructional modes. These findings support using math contexts to explore the role of language

in instructional contexts. Designing and manipulating instructional delivery of equivalence is a promising vehicle to better understand the linkage between language deficits and classroom instruction.

Mathematical equivalence. Mathematical equivalence is the relation between two quantities that are equal and interchangeable (Jones, Inglis, Gilmore, & Dowens, 2012; Kieran, 1981) and is a fundamental concept in arithmetic and algebra. The equal sign should be understood as a relational symbol, indicating that a balanced relation exists between the amounts on either side of the equal sign. However, students often develop an operational understanding of the equal sign, which promotes the idea that the symbol means to *do something* (Saenz-Ludlow & Welgamuth, 1998) or to *find the total* (McNeil & Alibali, 2005). This operational conceptualization of the equal sign may be, in part, because of the overrepresentation of standard-form equations (e.g., $3 + 4 = _$) and underrepresentation of equations with operations on both sides of the equal sign. For example, Rittle-Johnson and colleagues (2011) analyzed all instances of the equal sign in a textbook series for grades 1–6 and found that only 4% of equations included operations on both sides of the equal sign. Because elementary school students generally perform poorly on mathematical equivalence problems (e.g., McNeil, 2008), explicit instruction that teaches the relational understanding of the equal sign (e.g., the equal sign means ‘the same amount’) is merited. Powell and Fuchs (2010) reported that third-grade students who received equal-sign tutoring form a better understanding of how equations represent word problem structure, suggesting that early equal-sign instruction may provide students with a foundation for more complex content (e.g., word-problem solving) and algebraic thinking. Given (1) the advent of the *Common Core State Standards* (National Governors Association for Best Practices & Council of Chief State School Officers, 2010), where algebraic

reasoning is a domain area in the elementary years, and (2) that many high school students are unprepared for algebra (NMAP, 2008), instruction and intervention that develops algebraic reasoning in elementary school is a priority.

Nonsymbolic representation. Beginning in elementary school, virtually all problems that students are presented and expected to solve are in symbolic form (i.e., numbers and operation signs). However, mathematics research has indicated that children may develop a better understanding of mathematics when presented in nonsymbolic form (i.e., dots, shapes, pictures) (Barth, Beckmann, & Spelke, 2008; Gilmore, McCarthy, & Spelke, 2007; Lipton & Spelke, 2005). This finding is consistent with the literature on equal-sign instruction, suggesting that nonsymbolic presentation facilitates improved reasoning and performance on equivalence problems. Sherman and Bisanz (2009) found that second grade students performed better on equivalence tasks that were presented using concrete manipulatives than they did on the same problems presented symbolically. In a second study, children received 16 symbolic problems and the same problems in a nonsymbolic format. Half the children received the symbolic problems first, and the other half received the nonsymbolic problems first. Children who received the nonsymbolic set of problems first performed better on the symbolic problems, suggesting that experience with nonsymbolic equivalence problems can improve performance on subsequent symbolic problems.

More recently, Driver and Powell (2015) demonstrated that students with and without mathematics difficulty who were presented with nonsymbolic of equivalence problems significantly outperformed those who solved the same problems presented in symbolic form. These studies suggest that students are capable of reasoning, and that comprehending symbolic representation is where students struggle. One noteworthy difference in these two studies is how

researchers presented nonsymbolic problems. While Sherman and Bisanz (2009) used manipulatives (wooden cylindrical blocks and plastic bins) as their nonsymbolic form of presentation, Driver and Powell used pictorial representations of equations (e.g., pictures of sheep and a fence). These presentation forms are aligned with frameworks like the concrete, representational, and abstract (CRA) sequence of instruction (e.g., Miller & Hudson, 2007). Sherman and Bisanz used concrete manipulatives, Driver and Powell used a representational form, and both studies measured performance on abstract problems (symbolic; numerals and signs). In each case, students performed significantly better on the nonsymbolic problems (concrete, representational) than the same problems presented in symbolic (abstract) form.

Based on the reviewed literature, it appears that mathematics instruction, specifically teaching equivalence, may be an appropriate method for testing active intervention components that we hypothesize will support students with low language skills. Manipulating visual instructional components for students with language deficits, identified mathematics difficulty, and typically developing students provides a mechanism to further understand the relation between language and mathematics instruction.

Rationale for the Proposed Study

The purpose of this study is to extend the literature in two ways. First, we aim to add to the language intervention literature by using high-quality equal sign instruction as a vehicle to better understand how language interacts with academic instruction. Further, due to the prevalence of language deficits in children with behavior problems, we explore whether problem behavior and inattention interact with a treatment designed to support language deficits. To do this, we (1) test the efficacy of whole-class symbolic equal sign instruction, (2) contrast those effects to nonsymbolic instruction, and (3) test for interactions between treatment and language,

and treatment and behavior.

Second, we aim to extend the mathematical equivalence literature by delivering equal-sign instruction to whole classes of second-grade students. Although prior research has successfully increased the relational understanding of the equal sign and equivalence problem performance, only one study has attempted to deliver intervention to classrooms of students. In this study (McNeil, 2008), researchers provided teachers daily lesson scripts of symbolic equal sign instruction, but the lessons were not monitored. Further, although students were randomly assigned to condition (arithmetic vs. non-arithmetic workbooks) within classrooms, the analysis did not report efforts to account for classroom-level differences. The current study controls instructional delivery in both conditions to better ensure that the only between-condition difference is problem representation (symbolic vs. nonsymbolic).

Finally, this intervention is a vehicle to test the relations between language ability, behavior, and the intervention conditions. Because (a) research suggests that visual support increases academic performance of students with language deficits, and (b) language deficits commonly accompany behavioral deficits, we explore for whom this active compensatory strategy is effective.

Research Questions and Hypotheses

1. Does class-wide equal sign instruction improve the relational understanding of the equal sign and problem-solving performance for second-grade students? We hypothesize that the nonsymbolic and symbolic intervention conditions will outperform control.
2. What is the added value of nonsymbolic equal-sign instruction? We hypothesize that nonsymbolic intervention will outperform the symbolic and control conditions.
3. Do language scores moderate either intervention condition? We hypothesize that the

effectiveness of symbolic intervention will be stronger for children with higher language, whereas language will not differentiate response to nonsymbolic intervention.

4. Do teacher ratings of behavior moderate either intervention condition? Given (a) the problem behavior and attentive behavior measures are teacher reports, and (b) the intervention was designed to compensate for *language* deficits, this is an exploratory research question.

CHAPTER 2

II. METHOD

Participants

After receiving approval from both the school district and the university institutional review board, we began recruiting for participation. All participants were 2nd graders in general education classrooms in schools within a large, urban school district in the Southeastern United States. We included all students who return signed parent consent forms, because this study was about testing the effects of whole-class intervention. Participants per classroom ranged from four to 19 students, with a full sample of 212 students from 21 classrooms in four schools. Although teachers were aware of their treatment assignment (control vs. intervention), they were not aware of the intervention variant or the study's hypotheses.

In this cluster-randomized design, we randomized 21 intact classrooms to one of three study conditions: symbolic intervention (SYM; $n = 64$ children), nonsymbolic intervention (NON; $n = 69$), and control (CON; $n = 79$). This study included no joiners. After randomization, seven SYM, five NON, and five CON children were absent on the day of post-test. They did not differ statistically from the remaining students on pretest measures and did not differ significantly on any pretest measure as a function of condition. We omitted these children from the analysis. The final analytic sample included 195 children: 57 in SYM, 64 in NON, and 74 in CON.

In SYM, NON, and CON respectively, 59.6, 46.9, and 51.4% of students were male. Only two students had been retained and they were in SYM; and four students in SYM and two students in CON received special education services; SYM had four students identified as

English language learners, and NON and CON each had two. In SYM, the percentages of Black, Hispanic, White, and other students was 26.3, 22.8, 47.5, and 3.4; in NON, 20.3, 14.1, 56.3 and 9.3, in CON, 33.8, 10.8, 52.7, and 2.7. Chi-square tests revealed no demographic group differences. See Table 1 for sample demographics.

Table 1
Student Demographics, Descriptive Data, and Moderators by Study Condition

Variable	Symbolic (<i>n</i> = 57)		Nonsymbolic (<i>n</i> = 64)		Control (<i>n</i> = 74)	
	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%
Males	34	59.6	30	46.9	38	51.4
Ethnicity/Race						
African American	15	26.3	13	20.3	25	33.8
Hispanic	13	22.8	9	14.1	8	10.8
White	27	47.5	36	56.3	39	52.7
Other	2	3.4	6	9.3	2	2.7
IEP	4	7.0	0	0.0	2	2.7
ELL	4	7.0	2	3.0	2	2.7
Retained	2	3.4	0	0.0	0	0.0
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Addition Fluency	13.9	6.5	14.4	6.0	13.1	5.4
Language Factor						
TACL-4 Vocabulary	8.9	2.3	9.4	2.3	9.6	2.2
TACL-4 Morphology	8.8	2.5	8.2	2.4	8.6	2.1
TACL-4 Syntax	8.5	2.5	8.2	2.4	8.2	2.6
CELF-5 Screener	14.1	3.9	13.8	3.5	14.4	3.6
Problem Behavior	37.5	9.8	37.0	14.1	48.6	26.7
Inattention	44.6	21.0	45.5	24.2	35.5	17.1

Note. IEP = individualized education plan; ELL = English language learner; TACL = *Test of Auditory Comprehension of Language*; CELF = *Clinical Evaluation of Language Fundamentals*.

Outcome Measures

We administered five outcome measures at pre- and posttest. We created *Symbolic Open Equations*, *Nonsymbolic Open Equations*, and *Closed Equations* by writing sets of addition problems with and without missing information where we varied the position of the equal sign as well as the number of addends on either side of the equal sign. We adapted a set of nonstandard open equations (Powell & Driver, 2012) to create *Complex Symbolic Equations*, and an equal sign task item (Matthews & Rittle-Johnson, 2009) to create our *Definition* item. All items were addition problems. See Table 2 for sample items.

Definition. The *Definition* item assessed understanding of the equal sign in a written format. The tester asked the students to define the equal sign. For this item, students were oriented to the page and given 30 seconds to write down the definition of the equal sign in a large box. Students were awarded 1 point for relational definitions (e.g., the same as, same amount, balance) and 0 points for operational or incorrect definitions (e.g., it means the answer, solve, add). The definition measure was not used for instruction or practice during intervention, but it indexes the relational understanding of the equal sign, which was emphasized in both intervention conditions.

Symbolic Open Equations. With *Symbolic Open Equations*, children were given 5 minutes to complete 16 problems. These problems were in symbolic form (i.e., number, not picture representation), and contained three-number problems (e.g., $4 = _ + 1$). Chronbach's alpha on this sample was 0.86.

Nonsymbolic Open Equations. *Nonsymbolic Open Equations* were identical to Symbolic Open Equations in value and placement of the equal sign. The only difference was in the representation of the problems (see Table 2 for comparison). Children had 5 minutes to

complete these 16 problems. Chronbach's alpha on the sample was 0.86.

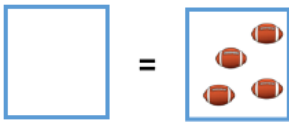
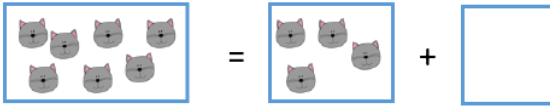
Complex Symbolic Equations. We defined *Complex Symbolic Equations* as four- or five-numbered nonstandard open equations (e.g., $4 + _ = 3 + 2$). Standard equations present the equal sign in the second to last position of a problem (e.g., $4 + 2 = 6$). These problems were nonstandard, meaning the equal sign was presented in a position other than second to last. Further, they contained at least four single-digit numbers, but no more than five. Children had 5 minutes to complete these 14 problems. Chronbach's alpha on this sample was 0.88.

Closed Equations. Finally, children had 5 min to complete *Closed Equations* problems. This set contained 18 closed equations. For these problems, children circled correct number sentences (e.g., $3 = 2 + 1$) and left incorrect number sentences uncircled (e.g., $2 = 4$). All problems were closed, in that they did not contain any missing information. Children were instructed to identify and circle the correct number sentences. Chronbach's alpha on this sample was 0.88.

Two research assistants independently scored all responses on all test protocols for each outcome measures. Any discrepancies were resolved to ensure 100% accurate scoring. Two research assistants independently entered responses on all of the scored outcome measures on an item-by-item basis into an electronic database, resulting in two separate databases. Formulae were written to highlight any discrepancies between the two independently entered databases. All discrepancies were resolved to ensure 100% accurate data entry.

Table 2
Sample Items From Outcome Measures

Nonsymbolic Open Equations



Symbolic Open Equations

$$\underline{\quad} + 6 = 9$$

$$7 = 4 + \underline{\quad}$$

$$4 = \underline{\quad}$$

Closed Equations

$$5 + 2 = 7$$

$$4 = 2 + 4$$

$$3 + 4 = 6 + 1$$

$$9 = 9$$

Complex Symbolic Equations

$$3 + 3 = 4 + \underline{\quad}$$

$$5 + 2 = \underline{\quad} + 3$$

$$4 + \underline{\quad} = 2 + 4$$

$$\underline{\quad} + 3 = 2 + 3$$

$$\underline{\quad} + 2 = 5 + 3 + 2$$

$$2 + 3 + 4 = 2 + \underline{\quad}$$

Definition

What does the equal sign (=) mean?

Note. Administered at Pre- and Posttest

Moderator Measures and Covariate

Language

To assess language, we used factor analysis to generate a composite language score using two direct assessment measures of language. We combined the three subtests (vocabulary, morphemes, syntax) of the Test for Auditory Comprehension of Language – 4th Edition (TACL-4; Carrow-Woolfolk, 2014) and the Clinical Evaluation of Language Fundamentals – 5th Edition (CELF-5; Wiig, Semel, & Secord, 2013). The TACL-4 is a verbally delivered language assessment that provides scores for vocabulary, morphology, and syntax. The examiner delivers a verbal stimulus, and the student is directed to point to the picture that best represents the meaning of the stimulus. No oral response is required from the student. Test-retest reliability ranges from .94 to .97. The CELF-5 screens students on morphology, syntax, semantics and pragmatics and is comprised of items from the CELF-5 diagnostic assessment. The purpose of this measure is to screen children to identify the potential need for further diagnostic testing to determine language disorder. No reliability estimates are provided for the screener.

Behavior

Teachers completed the *School Social Behavior Scales Rating Form – 2nd Edition* (SSBS-2; Merrell, 2008) to measure student problem behavior. This rating scale provides comprehensive ratings of social skills and problem behaviors. We used the Antisocial Behavior scale to index problem behavior. This scale includes 32 items that provides subscale scores for Hostility/Irritable, Antisocial/Aggressive, and Defiant/Disruptive. Teachers also completed the Strengths and Weaknesses of ADHD-Symptoms and Normal-Behavior (SWAN; J. M. Swanson, 2013) to index attentive behavior. This scale is an 18-item rating that corresponds to the criteria for ADHD from the Diagnostic and Statistical Manual of Mental Disorders (American

Psychiatric Association, 2000). Items are 7-point Likert scales with ratings; lower scores correspond with problematic behavior. The SWAN has been shown to significantly associate with other assessments of behavioral attention (Swanson et al., 2008).

Addition Calculation

We administered the addition subtest of the Second-Grade Calculations Battery (Fuchs, Hamlett, & Powell, 2003) to control for incoming addition fluency. Students had one minute to complete up to 25 addition problems. The score was the correct number of answers. The purpose of this measure was to control for student's incoming calculation skill, because all outcome measures in the current study were addition problems.

Equal Sign Intervention

The whole-class equal-sign intervention is referred to as *Equal Sign Exploration*. The intervention consisted of three 20-minute lessons. Interventionists were two doctoral students and three master's students majoring in special education. Interventionists participated in a 2-hour training to become familiar with and practice the lessons that were guided by lesson scripts. After independent rehearsal, each interventionist completed pseudo lessons and were required to meet or exceed 90% fidelity on the pretest and lesson scripts prior to live classroom lessons. Interventionists met with the PI at the end of their first intervention classroom to discuss the lessons and strategies around behavior management.

Intervention Implementation

Three 20-minute whole-class equal-sign lessons focused on teaching the relational understanding of the equal sign and two types of problem solving. Lesson 1 introduced *Equal Sign Exploration* and taught relational understanding of the equal sign (e.g., the same). In this lesson, we also taught students to be able to identify correct (e.g., $5 = 5$) and incorrect (e.g., $4 =$

2) equality statements. In Lesson 2, we reviewed the previous lesson, and introduced standard (e.g., $4 + _ = 7$) and nonstandard (e.g., $_ = 5 + 2$) open equation problem solving. In this lesson, we also taught students to identify correct and incorrect standard and nonstandard closed equations. In Lesson 3, we reviewed previous lessons and increased the difficulty of correct (e.g., $1 + 3 = 2 + 2$) and incorrect ($2 + 4 = 2 + 5$) closed equations, and the open equation problems (e.g., $5 + _ = 4 + 3$).

Contrast Condition Distinctions: SYM vs. NON

Scripts for intervention conditions were identical. All values and difficulty of example and practice problems were also identical between conditions. The only difference between the conditions was problem representation. In the SYM condition, all problems were represented in symbolic (i.e., numerical) form. In the NON, all problems were represented in nonsymbolic (i.e., visual/pictorial) form. The purpose of this distinction was isolate and compare the effect of nonsymbolic representation to symbolic representation and control. See Figure 1 for intervention representation contrast.

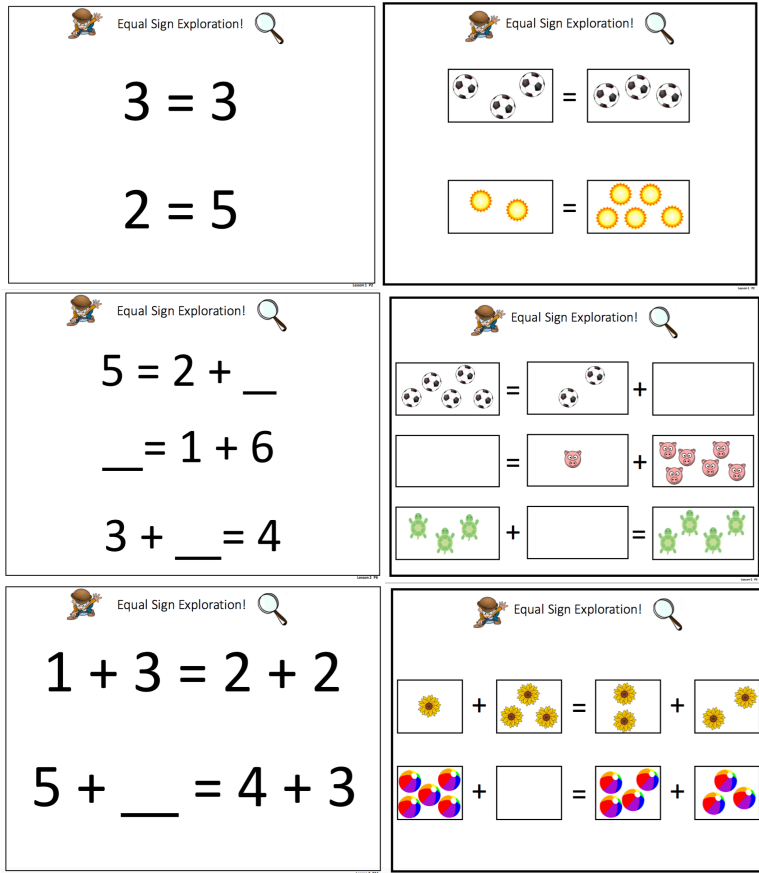


Figure 1. Examples of instructional posters used during intervention. Left = SYM. Right = NON.

Fidelity of Implementing Intervention

Every intervention session was audiotaped. Five research assistants listened to 100% of whole-class recordings ($n = 130$) while completing a checklist to identify core components implemented. The mean percentage of components addressed was 94.8 ($SD = 5.3$) in SYM and 95.3 ($SD = 6.2$) in NON. Another research assistant independently listened to 25% ($n = 26$) of intervention recordings to assess agreement. The mean difference in score was $< 2\%$.

Procedure

First, research assistants individually assessed consented students' language and working memory. Next, to assess pretreatment comparability amount conditions on addition fluency and equal-sign knowledge, research assistants administered our pre-test battery (outcome measures) and a measure of additional fluency (Fuchs, Hamlett, & Powell, 2003) to whole classrooms. In SYM and NON, three 20-min intervention sessions were delivered in the same week following pre-test. To assess intervention effects, interventionists readministered the outcome measures immediately following the final intervention lesson. All whole-class procedures (pre-test, Lesson 1, Lesson 2, and Lesson 3+posttest) were administered within the same week.

Individual assessment sessions (language measures) were audiotaped; 25% ($n = 105$) of tapes were randomly selected, stratifying by tester, for accuracy checks by an independent scorer. Agreement on test administration and scoring exceeded 98%. Testers were blind to study conditions when administering and scoring tests, and independent scorers were blind to condition of the participants on audiotapes. Two research assistants independently entered responses on 100% of all measures (rating scales, individual assessments) on an item-by-item basis into an electronic database, resulting in two separate databases. Formulae were written to highlight any discrepancies between the two independently entered databases. All discrepancies

were resolved to ensure 100% accurate data entry.

CHAPTER 3

III. RESULTS

Preliminary Analyses

We used multilevel regression models to test our research questions. We fit all models using maximum likelihood estimation in Stata 12 (StataCorp, 2011). We evaluated classroom- and school-level intraclass correlations (ICCs) on all outcomes due to the nested structure of the data. Classroom-level ICCs ranged from .14 to .27, and school-level ICCs ranged from .10 to .22. These results indicated a need for three-level models.

Chi-square tests confirmed that groups were comparable on pretest performance on addition skill, each equal sign measure, as well as relevant demographic variables. We analyzed the descriptive properties of all relevant variables, and log-transformed variables when needed. In all models, we interpret statistical significance after using Benjamini-Hochberg correction (Benjamini & Hochberg, 1995) as recommended by the What Works Clearinghouse (2014).

Does Equal-Sign Intervention Improve Problem-Solving and Understanding?

We fit multilevel regression models to test whether SYM and NON groups outperformed the CON group. For the *Definition* outcome (binary), we fit a multilevel logistic regression. In all models, we include a covariate for addition fluency, because all problems involved the addition property. Even though we established group equivalence at pretest on this measure, we elected to control for addition skill for theoretical purposes. Table 3 shows posttest means and standard deviations by condition. Table 4 shows model results. Both SYM and NON significantly outperformed CON on all equal-sign outcome measures. Effect sizes (Hedges' g)

and predicted probabilities are presented in Table 5.

The comparison between NON and CON consistently yielded the largest effect sizes. These effect sizes ranged from 0.53 to 0.78 across all outcome measures. The largest effect size represented the difference between NON and CON on *Nonsymbolic Open Equations*. All the contrasts between SYM and CON were statistically significant; however, the effect of the SYM condition on the nonsymbolic outcome was less than half ($ES = 0.31$) of the contrast between NON and CON ($ES = 0.78$).

Relative to the *Definition* outcome, we converted the odds ratios to predicted probabilities from the fixed part of the model. These estimate the predicted probability that children in each condition (NON, SYM, and CON) would produce a correct definition at posttest. The predicted probability for a correct definition was 77.2% in NON, 70.6% in SYM, and 14.0% in CON. The odds of a correct definition in NON and SYM were both greater and significantly different from CON.

Table 3
Pre- and Posttest Performance on Outcome Measures

Measure	Symbolic (<i>n</i> = 57)				Nonsymbolic (<i>n</i> = 64)				Control (<i>n</i> = 74)			
	Pretest		Posttest		Pretest		Posttest		Pretest		Posttest	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Symbolic Open Equations	9.7	4.8	11.9	5.1	10.2	4.5	12.2	4.3	9.1	4.2	9.7	5.0
Nonsymbolic Open Equations	8.1	5.1	10.3	5.0	8.9	4.6	12.1	3.7	7.8	4.1	8.8	4.7
Closed Equations	12.9	3.6	16.2	2.3	12.8	3.9	16.3	2.4	13.0	3.5	14.2	3.5
Complex Symbolic Equations	4.1	5.5	9.0	5.5	4.4	5.3	9.6	5.6	3.4	4.8	5.2	5.9
Definition (% correct)	10.5% (<i>n</i> = 6)		70.2% (<i>n</i> = 40)		6.2% (<i>n</i> = 4)		76.6% (<i>n</i> = 49)		6.8% (<i>n</i> = 5)		14.9% (<i>n</i> = 11)	

Note. Adjusted posttest means.

Table 4

Parameters for Multilevel Models Evaluating Effects of Equal-Sign Intervention

	Outcome				
	Symbolic Open	Nonsymbolic Open	Closed Equations	Complex Symbolic	Definition
Fixed <i>B</i> (SE)					
<i>B</i> [_cons]	2.96	3.23	8.47	2.90 (.76)	-2.34 (.50)
<i>p</i>	.000	.493	.000	.000	.000
CI	[1.70, 4.22]	[-2.35, 1.13]	[5.68, 11.25]	[1.40, 4.40]	[-3.33, -1.35]
<i>B</i> [SYM vs. CON]	1.67 (.52)	1.19 (.50)	1.88 (.37)	3.53 (.94)	3.16 (.69)
<i>p</i>	.001	.016	.000	.000	.000
CI	[.66, 2.69]	[.22, 2.16]	[1.15, 2.61]	[1.70, 5.36]	[1.80, 4.51]
<i>B</i> [NON vs. CON]	1.71 (.59)	2.49	2.06 (.70)	3.57 (.89)	3.45 (.65)
<i>p</i>	.003	.003	.003	.000	.000
CI	[.58, 2.85]	[.83, 4.15]	[.68, 3.44]	[1.83, 5.31]	[2.18, 4.71]
<i>B</i> [NON vs. SYM]	.04 (.86)	1.30 (.85)	.18 (.55)	.04 (.98)	.29 (.59)
<i>p</i>	.960	.126	.738	.968	.624
CI	[-1.63, 1.72]	[-.36, 2.96]	[-.90, 1.26]	[-1.89, 1.97]	[-.90, 1.45]
<i>B</i> [pretest]	.71 (.07)	3.23 (.43)	.34 (.05)	.65 (.13)	4.01 (1.23)
<i>p</i>	.000	.000	.000	.000	.001
CI	[.57, .85]	[2.38, 4.08]	[.25, .44]	[.39, .90]	[1.60, 6.42]
<i>B</i> [addition]	.01 (.04)	.07 (.04)	.09 (.02)	.21(.06)	---
<i>p</i>	.740	.054	.000	.000	---
CI	[-.07, .10]	[-.001, .14]	[.06, .12]	.10, .33]	---
Random					
school	.47 (.17)	.41 (.26)	.52 (.33)	.87 (.21)	.00 (.32)
classroom	.00 (.01)	.26 (.49)	.50 (.30)	1.03 (1.09)	.67 (.36)
residual	3.39 (.23)	3.01 (.22)	2.16 (.16)	4.71 (.22)	---
Log likelihood	-515.97	-483.99	-432.41	-584.12	-90.58

Note. Parameters are from two separate models with alternating reference categories. SYM = symbolic intervention condition. NON = nonsymbolic intervention condition. CON = control. We did not include addition fluency in the logistic regression, because the outcome of *Definition* was a written response item with no addition component.

Table 5
Main Effects, Between-Condition Effects, and Predicted Probabilities

	Symbolic Open	Nonsymbolic Open	Closed Equations	Complex Symbolic	Definition
Effect Size (<i>g</i>)					
SYM vs. CON	0.45*	0.31*	0.66*	0.67*	---
NON vs. CON	0.53*	0.78*	0.69*	0.76*	---
NON vs. SYM	0.06	0.41	0.04	0.11	---
Predicted Probability (%)					
SYM	---	---	---	---	70.6%
NON	---	---	---	---	77.2%
CON	---	---	---	---	14%

Note. Hedges' *g* effect sizes. SYM = symbolic intervention condition. NON = nonsymbolic intervention condition. CON = control. Definition outcome presented as the predicted probability of a correct answer.

Do Individual Differences Moderate the Treatment Effect?

Using a language factor score, we examined whether language ability moderated our overall intervention effects (i.e., total outcome score). That is, does treatment significantly interact with language using our total outcome score as an outcome? We used the same base multilevel model as we fit for the intervention main effects and entered interaction terms (language*NON, language*SYM). Results of this analysis yield a significant interaction (coefficient = 2.13, $SE = .93$, $z = 2.29$, CI [0.30, 3.96], $p = .02$) between language and the symbolic intervention condition relative to control, but no significant interaction (coefficient = 1.53, $SE = 1.13$, $z = 1.36$, CI [-.68, 3.75], $p = .18$) between language and the nonsymbolic intervention relative to control. See Figure 2 for the predicted relations between language and outcome scores by study condition.

Then, we fit a moderation model to determine whether there was a significant between-condition interaction. That is, we wanted to determine whether there was a significant language-by-treatment condition interaction when contrasting the treatment conditions against each other. Results of this analysis yielded a significant interaction (coefficient = 2.25, $SE = .89$, $z = 2.50$, CI [0.49, 4.01], $p = .01$) between language and SYM, but not between language and NON (coefficient = 1.64, $SE = 1.07$, $z = 1.53$, CI [-.45, 3.75], $p = .16$) relative to the other treatment group. This suggests that individual differences in language ability may matter in symbolic instructional contexts, where as they may be less meaningful for success during nonsymbolic instructional contexts.

We also explored the role of behavior. Moderator analyses did not yield any interactions between intervention conditions and problem behavior (SSBS-2) nor attentive behavior (SWAN). This suggests that teacher ratings of attentive behavior and problem behavior do not

significantly predict individual students' response to either *Equal Sign Exploration* intervention condition.

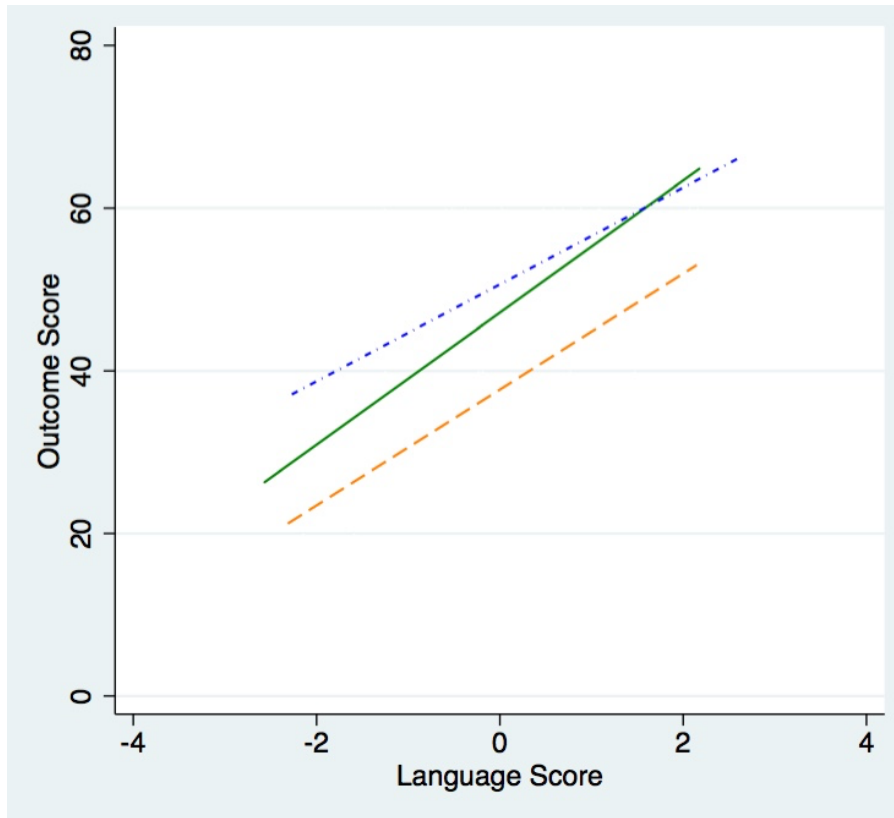


Figure 2. Visualization of the statistical interaction between SYM vs. CON and SYM vs. NON. Y-axis is the intervention outcome posttest; x-axis is the language factor score. Models control for pretest performance and addition fluency. SYM = solid line. CON = long dashes. NON = short dashes.

What is the Added Value of Nonsymbolic Intervention?

In addition to the main effect intervention conditions against control, we were also interested if there was added value to NON over and above SYM. That is, holding intervention content and verbal instruction constant, does nonsymbolic representation produce different effects as compared to symbolic instruction? Effect sizes of this contrast all favored NON and ranged from .06 to .41. However, none were statistically significant. The largest difference was on *Nonsymbolic Open Equations*, with an effect size of 0.41. There was no statistically significant difference between the intervention conditions in the predicted probability of a correct definition at posttest. However, relative to individual differences, child language moderated SYM but not NON. This finding suggests that children with higher language outperformed their peers with lower language skills in SYM. However, language ability did not predict response to NON. That is, language skills may be predictive of response to symbolic, numerical instruction, whereas language skills may not play as much of an influential role in individuals' response to nonsymbolic, visual instruction.

Social Validity

In an effort to gauge how practical our intervention was, we asked teachers to complete an exit survey that included 10 items on a five-point Likert-type scale to assess their current practice and opinions of the intervention content and potential. A response of 1 indicated strongly disagree, and a response of 5 indicated strongly agree. See Table 6 for average response from our intervention teachers. On average, the results suggest that the intervention was useful, pragmatic, and that students enjoyed the intervention. All intervention teachers in both conditions completed the exit survey, because they observed our research team deliver the intervention in their classrooms. We did not collect these data from control group teachers,

because they did not witness the intervention.

Table 6

Teacher Survey Responses (1 = Strongly Disagree and 5 = Strongly Agree)

Item	Mean	Item	Mean
The intervention focuses on an important mathematical concept.	4.64	I would recommend participation in a similar study to a colleague.	4.69
I have taught explicitly about the equal sign before.	3.11	I would attend/recommend professional development on this topic.	4.00
I am likely to explicitly teach about the equal sign in the future.	4.50	I am interested in including similar instruction into my regular practice.	3.93
The research team hindered my typical instruction.	1.07	My students enjoyed the intervention.	4.64
The research team met the needs of my schedule.	4.93	My students learned a lot from the intervention.	4.50

Note. Only intervention teachers ($n = 14$) completed this survey. Control group teachers did not have any exposure to the intervention.

CHAPTER 4

IV. DISCUSSION

The purpose of this study was to assess the efficacy of symbolic and nonsymbolic class-wide equal sign intervention and to examine language and behavior moderators of intervention. Intervention effects were measured using five equal-sign outcomes: *Symbolic Open Equations*, *Nonsymbolic Open Equations*, *Closed Equations*, *Complex Symbolic Equations*, and a *Definition* task. We wanted to assess whether providing brief, whole-class equal sign instruction improved equal-sign understanding and problem solving, extending previous research on class-wide equal-sign instruction (McNeil, 2008). In addition, we were interested in determining whether, in the context of the current study and sample, students with lower language skills would profit from nonsymbolic instruction (i.e., only visual representations presented during instruction and practice). That is, we hypothesized that visual instruction would alleviate students, particularly children with language deficits, from needing to process symbolic representations while learning equal-sign concepts.

We compared the outcomes of students who received SYM to control, students who received NON to control, and NON to SYM. Including SYM allowed us to isolate the effect of visual representation as compared to numerical representation during instruction. All verbal and instructional content was held constant; the only between-condition difference was the representation of the problems on lesson posters and practice problems.

Does Whole-Class Equal-Sign Intervention Improve Problem Solving and Understanding?

On all open equation problems (*Symbolic Open Equations*, *Nonsymbolic Open Equations*,

and *Complex Symbolic Equations*), students solved equations with missing information. Classrooms in both conditions received explicit instruction on how to solve these types of problems. In SYM, students learned how to complete symbolic problems by filling in the missing information so that both sides of the equal sign were the same amount. In NON, students received the same verbal instruction, but examples and practice problems were in nonsymbolic forms.

On *Symbolic Open Equations*, both NON and SYM conditions significantly outperformed control with respective effect sizes of 0.53 and 0.45. Even though students in the NON condition did not learn or practice with symbolic problems, they demonstrated significant gains on this outcome. Although not significantly different from each other, the effect size for NON when compared to control was larger than SYM compared to control. This result suggests that students benefited from NON instruction just as much as SYM on *Symbolic Open Equations*. On *Nonsymbolic Open Equations*, both intervention conditions also significantly outperformed control. These items were nonsymbolic representations of the same set of problems in *Symbolic Open Equations*. However, for these problems, the effect size for NON when compared to control was 0.78 and only 0.31 for SYM when compared to control – less than half of the NON vs. control effect size. These findings suggest that NON instruction may transfer to symbolic problems, but perhaps not as strongly the other way around. That is, nonsymbolic instruction and practice support symbolic problem solving more than symbolic instruction supports nonsymbolic problem solving. It may also be the case that students are, in general, more familiar with symbolic representation. Thus, we would expect children in NON to improve more than children in SYM on nonsymbolic problems, because they were exposed to and taught through the more unfamiliar, nonsymbolic representation.

In the case of *Complex Symbolic Equations*, the effect size for NON vs. CON was 0.76 and the effect size for SYM vs. CON was 0.67. This suggests that for symbolically-represented problems with missing information, both intervention conditions were effective on average. Further, these effect sizes were larger than the *Symbolic Open Equations*, suggesting that students in the intervention conditions profited more from the intervention when solving more complex symbolic problems.

For *Closed Equations*, students were instructed to identify and circle correct symbolic number sentences. For these problems, there was no missing information. The NON vs. control effect size was 0.69 and the SYM vs. control effect size was 0.66; both were significant. These problems were all presented in symbolic form. Thus, similar to *Symbolic Open Equations* and *Complex Symbolic Equations*, we can conclude that in this study, students who received NON performed at least as well as students who received SYM. This further suggests that nonsymbolic representation, completely absent of number symbols, can successfully transfer to symbolic problem solving after brief, class-wide instruction.

For our *Definition* task, students were asked to write down what the equal sign means. After intervention, the predicted probability of a correct answer was 77.2% in NON, 70.6% in SYM, and only 14% in control. Students who received intervention were much more likely to provide a correct definition of the equal sign at posttest than students in control. During intervention, we did not train this task. That is, in the three 20-min lessons, we did not ask students to practice writing out the definition. What we did provide was repetition of multiple correct meanings of the equal sign (e.g., the same as, both side of the equal sign are the same amount, balance). This item was the only item that required students to write something other than missing information in equations. These predicted probabilities indicate that exposure to

verbal iterations of equal-sign definitions can influence students' writing of the meaning of the equal sign independent from direct practice.

Overall, students who received *Equal Sign Exploration* in either nonsymbolic or symbolic form significantly outperformed students in control on all five outcomes. However, although the effect size between NON and SYM on *Nonsymbolic Open Equations* was .41, we did not detect any significant between-condition effects. Thus, we can conclude that NON and SYM were both superior to control, but not significantly different from each other based on performance on these five outcomes.

Does Nonsymbolic Intervention Compensate for Language Deficits?

One of the primary goals of this study was to support language deficits in the context of math instruction. We modeled our approach to answering this research aim after work on aptitude-by-treatment interactions, which demonstrate that specific learner characteristics can determine whether an instructional method will be effective or not (Chronbach & Snow, 1977, Snow 1978). We sought to test whether an instructional adaptation (NON) could render language ability (specifically, lower language skills) unimportant relative to success during intervention.

Results from our moderator analyses revealed a significant language by SYM interaction, whereas the interaction between language and NON was not significant. Figure 2 shows the significant interaction of SYM when compared to both CON and NON. These results suggest that child language ability is substantively meaningful for students who receive symbolic instruction, but not for students who receive nonsymbolic instruction. Child language skills may play an important role in successful response to SYM. However, children with lower language skills were, on average, just as successful during NON than their peers with higher language

skills. Thus, nonsymbolic instruction may compensate for language deficits in the context of a brief class-wide math intervention.

Does Behavior Interact with Treatment?

We explored whether teacher ratings of problem behavior or attentive behavior moderated either treatment condition. Results did not yield significant interactions for either measure. This suggests that both intervention conditions appear to be successful for students regardless of their behavior ratings. This may have implications for educating children and youth who present problem behavior and higher rates of inattention, because in the context of our intervention, individual differences in ratings did not predict response to either intervention condition. Our measures, however, were teacher rating scales of behavior, and they likely capture broader, less discrete aspects of behavior. It is possible that these more molar indices of behavior may not be as strongly related to the direct, observable behaviors that may be problematic in classrooms such as frequency of disruptive behavior or engagement during instructional time.

Were there Meaningful Differences Between Symbolic and Nonsymbolic Intervention?

We aimed to isolate the effect of nonsymbolic representation by delivering equal-sign instruction in two active conditions where representation of the problems was the only difference. This extends previous assessment work (Driver & Powell, 2015) by testing this representational difference in the context of intervention. Holding lesson content and verbal instruction constant, we did not detect any significant between-group differences on our outcomes. While the effect size between NON and SYM on *Nonsymbolic Open Equations* may be substantively meaningful ($g = 0.41$), the difference did not reach statistical significance. We did, however, find a meaningful difference when examining who may profit from each

intervention condition. We detected a significant interaction between language ability and SYM relative to NON, suggesting that language ability predicted a student's response to intervention when receiving symbolic instruction when compared to students receiving visual instruction. When comparing SYM to NON, our results posit that nonsymbolic instruction may be effective regardless of language ability. However, language ability may play a role in response to symbolic instruction over and above nonsymbolic instruction, suggesting that visuals may compensate for language deficits.

In summary, both intervention conditions significantly outperformed control on all outcomes. All the NON vs. CON effect sizes were larger than SYM vs. CON, but there were no significant differences between intervention conditions (i.e., NON vs. SYM). Language moderated SYM but not NON; neither measure of behavior moderated either intervention condition.

Limitations

Our results must be considered in conjunction with the study's limitations. First, we did not monitor what occurred in the control group classrooms during intervention. Even though our intervention was brief (total intervention including pre- and posttest was delivered in no more than five school days), we cannot describe exactly what content or lessons control group teachers delivered. We informed the control group teachers of their group assignment, and asked them to continue teaching as normal. We are confident that our intervention was highly focused on the equal sign, but it is possible that control group teachers covered similar concepts during the week between pre- and posttest.

Second, in order to accommodate the number of classrooms, we staggered the intervention over the course of three months. We controlled for potential learning across pre-

and post-test by delivering intervention in a small window – three to five days. We also, in most occasions, delivered intervention during the students’ math block, which reduced the chance that they were receiving additional and similar math instruction between pre- and posttest. However, in classrooms that we delivered intervention earlier in the school year may have had more room to grow relative to the classroom that we intervened in three months later (i.e., received more of the school math curriculum). This masks our ability to determine whether some growth was due to earlier intervention, or some smaller effects in classrooms were due to students having had more math instruction.

Although the sample is described as second grade classrooms, there are some features of our recruitment procedure that increased the homogeneity. Student participants came from classrooms of teachers who we met with and were highly motivated to participate. Teachers sent home consent forms with all their students, but response rate ranged from four to 18 ($M = 9.8$) across the 21 classrooms. Thus, our study examines the efficacy of intervention of second grade students for whom we have parental (or guardian) consent.

Relative to behavior, both moderators were rating scales that were completed by teachers, limiting the scope of interpretation. For example, we did not directly observe and index classroom behaviors such as disengagement or disruptive behavior. It is unclear whether direct behavioral observations or teacher ratings of behavior are the better choice for indexing the types of behaviors that low language may be more strongly associated with.

Overall, we tested if students learned the concept (relational definition of the equal sign) we taught and the types of problems we catered the instruction to. They demonstrated gains on all problem types in both intervention conditions compared to control. However, we did not have a measure of broader math performance. Although students made significant gains after

three brief lessons, they made gains on researcher-designed open and closed equations. We cannot infer that our intervention increased general math performance.

Future Directions

Our results lead us to the following recommendations for future research. First, we wonder about maintenance of intervention effects. We delivered a brief, three-session intervention program that targeted a specific skill, and our outcome measures were collected via immediate posttest. Future studies may examine whether gains from a brief math equivalence intervention maintained over a contextually meaningful amount of time. On a related note, future studies might design a similar, skill-focused intervention that is delivered over a longer period of time.

Our moderator analysis included a language factor score. Future studies should examine how different constructs of language may be more or less strongly related to math outcomes. For example, our language score was primarily comprised of receptive language tasks. Each of the three TACL subtests were solely receptive tasks. Only some of the CELF screener items included expressive language demands. Further, future research might examine the differential contribution of semantic, syntactic, and pragmatic language skill.

Neither of our measures of behavior significantly interacted with either intervention condition. Future studies should replicate these findings and compare them to more direct measures of classroom behavior, and actively recruit children with language and behavioral deficits to determine whether behavior is more strongly related to instruction format in children with more pronounced and/or comorbid deficits.

Based on our findings, a next logical step would be to explore whether nonsymbolic instruction lends itself to promoting response to instruction in other domains such as science and

language arts, particularly for children with language deficits. Future work should also extend this line of work to different aged children.

Relative to practice, we recommend that teachers incorporate nonsymbolic representation into their current math instructional practices and teach explicitly about the equal sign. The results of the present study suggest that children that receive math instruction using visuals only (NON) may outperform children who receive the same instruction using numbers only (SYM). Further, children with lower language skills may profit more from visually-presented problems. Thus, we recommend embedding nonsymbolic representation into lessons and practice.

Conclusion

Effective interventions are needed that explicitly mitigate language deficits in the classroom. Limitations notwithstanding, our results suggest that our overall intervention is effective, and that visually representing math problems during instruction and providing opportunities to practice solving visual problems is a promising avenue for children with and without low language skills.

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