AFFILIATION AND ENTRY IN FIRST-PRICE AUCTIONS

By

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To my wife Zhengfeng

and

To my parents Jianhua and Qixing

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CHAPTER I

INTRODUCTION

Introduction

Auctions have long been used as a means for price determination under a competitive setting and an environment where the information is incomplete. Various goods such as U.S. treasury bills, lands in China, pieces of art, mineral rights, timber rights are traded through different types of auctions all over the world. In addition to their practical importance, auctions have drawn much attention of economists for two reasons. First, auctions provide a perfect example to the game with incomplete information (Harsanyi (1967, 1968)) in that auctions have clearly stated rules and obvious features of incomplete information between the auctioneer and bidders, as well as among bidders. Therefore auctions have been used to verify the testable implications derived from theory. Second, plenty of field auction data are available, such as highway procurement auctions data and timber auctions data, which is one of the main driving forces that causes the rapid development of the empirical auction studies. Therefore since the seminal paper by Vickery(1961), auction theory developed within the game-theoretic framework has grown at a rapid rate, which not only helps us understand how auctions work, but also offers insight in analyzing many other economic problems. Among many studies, two polar cases, independent private value (IPV) and common value (CV) frameworks are intensively adopted in the literature. Within the IPV framework, the valuations of the object are private information of bidders, drawn from distributions independently, while within the CV framework, the object has the same ex post valuation to all bidders. Many celebrated theoretical results have been derived within these two paradigms. For example, within the symmetric IPV framework with risk-neutral bidders, all auction formats (first-price auction, second-price auction, English auction and Dutch auction) generate the same amount of expected revenue to the seller, which is called Revenue Equivalence Proposition (REP) derived by Vickery (1961). Within the CV framework, it is found that the number of bidders is not positively related with bids due to the winner's curse. Though quite a few applications have been made within these two paradigms, the main drawback of these two paradigms is being apart from the reality. In reality, bidders' values over the object might be neither independent from each other nor the same. Instead they might be dependent from each other. A good example of the source of such dependence is that values are affected by some unknown common factors.

In light of this, researchers have started to focus on affiliation, which is a more general concept in that it includes independent private value and common value frameworks as special cases. Affiliation is a concept borrowed from the multivariate statistics literature, in which it is called multivariate total positivity of order 2, and was first introduced into the auction literature by Milgrom and Weber (1982). Throughout the dissertation, the term affiliation follows the same definition as is given in Milgrom and Weber (1982) and will be stated in detail later. In addition to the fact that affiliation is a more general concept, several reasons make it important to the auction study. First, affiliation guarantees the existence and uniqueness of a monotone equilibrium in many scenarios (e.g. see Rodriguez (2000) for a two-person first-price auction with symmetric or asymmetric bidders). Second, under this general assumption, REP can be extended to a more general result derived in Milgrom and Weber (1982) that English auction has a higher expected revenue than the second-price auction, which in turn has a higher expected revenue than the first-price auction.

Third, several results within the IPV framework no longer hold. The effect of the number of potential bidders on bids is not always positive with affiliation among bidders due to the "affiliation effect" (Pinkse and Tan (2005)). The optimal reserve price converges to the true value of the seller as the number of bidders increases within the affiliated private value (APV) framework as is shown in Levin and Smith (1994). Therefore from both theoretical and empirical viewpoints, it is interesting and important to incorporate affiliation in the auction study.

Another important feature of the dissertation is taking bidders' entry decisions into account. Not only found in the auction literature, entry is also an important topic in other fields of industrial organization, especially in the studies of oligopoly markets. See, for example, Berry (1992) for a model of entry in the airline industry, and Seim (2006) for an entry model with endogenous product-type choices. In the auction framework, entry refers to the participations of bidders, which should be viewed as one part of decisions bidders make in an auction due to some nontrivial participation costs. That is, bidders make their entry decisions by comparing the expected profits and costs from participation. Ignoring the entry will cause the model to be misspecified and the estimation of the model to be biased. For example, one might overestimate the effect of the number of bidders on bids since there is a negative "entry effect" due to entry (Li and Zheng (2005)). To this end, a central theme in this dissertation is to take bidder's participation into account. In the theoretical framework, there are two prevailing ways of modeling the entry behavior of bidders in terms of the strategy adopted. One model is called pure strategy entry model, documented in Milgrom and Weber (1982) and Samuelson (1985), in which entry behavior is governed by a screening level of bidders' private signals. Within the IPV framework, it means that there exists a screening level of private values and only bidders whose private values are above the screening level enter the auction. The other model is called mixed strategy entry model proposed in Levin and Smith (1994), in which bidders who have the same entry cost randomize their entry decisions with a common entry probability. Note that by assuming that the entry costs follow a nondegenerated distribution, the mixed strategy entry model is purified to a pure strategy entry model where the entry behavior is governed by a cut-off point of entry costs. The bidders with entry costs smaller than the cut-off point participate in the auction and others do not. The key difference between the two types of models is the timing of acquiring private signals. In the former, bidders know their private signals prior to entry while they do not know their private signals until after they enter the auction in the latter and its purified variant. With these theoretical models and their variants, much empirical work has started to incorporate entry. Athey, Levin, and Seira (2004) compare the sealed-bid auctions and open auctions with entry by using data from U.S. Forest Service timber auctions. Krasnokutskaya and Seim (2007) analyze California Small Business program using highway procurement auction data with an entry and bidding two stage model within the IPV framework. Both papers adopt the entry model in which the entry behavior is controlled by the cut-off point of the entry costs. Li and Zheng (2005) use three different entry models including degenerated and nondegenerated distribution of entry costs to study highway mowing auction from the Texas Department of Transportation. Hortaçsu (2003) study the internet auctions with entry using data from eBay.com.

In view of the importance of affiliation and entry, this dissertation closely focuses on these two issues in first-price auctions, aiming at three goals. First, it develops a simple approach to test for affiliation among bidders' private information and applies the approach to the timber auction data in Oregon State. Second, it extends the APV model by taking entry into account with symmetric or asymmetric bidders. Lastly, it empirically analyzes the timber auction data in Oregon State by using the structural approach within the APV paradigm with entry and asymmetric bidders and study several policy related issues by counterfactual analyses. Although this dissertation includes three essays covering various topics, all incorporate affiliation and entry.

In the first essay, we propose a simple approach to test for affiliation in a more general framework, namely the affiliated value (AV) model, developed by Milgrom and Weber (1982). The test makes use of the information on bidders' entry behavior and is based on the insight that affiliation among bidders leads to affiliation among their entry behavior. The approach transforms the affiliation test problem to testing the positive correlation of a multivariate normal distribution. The test then is conducted through a simulation based method. We apply the test to the timber auctions organized by Oregon Department of Forestry (ODF) and find that bidders are affiliated with a small level. The proposed test is simple to implement and is widely applicable to various auction scenarios.

The second essay studies the APV model with entry and establishes an interesting relationship between the number of potential bidders and bids. Whether to encourage the competition is a question a seller should ask himself before designing an auction. Within the IPV framework without entry, competition is beneficial for the seller in that higher competition makes bidders bid more aggressively. It turns out, however, that it is not necessarily the case within the APV model with entry. Previously studied by Pinkse and Tan (2005) within the APV framework without entry and Li and Zheng (2005) within the IPV framework with entry, the relationship becomes more complicated in the model we studied. In this essay, we show that in the APV model with entry, there are three effects at work, namely "competition effect," "affiliation effect" and "entry effect," which have different effects on bids, therefore the total effect on bids really depends on the relative magnitudes of these three effects. We disentangle the total effect analytically and demonstrate it through an example.

The last essay extends the model in the second essay by considering asymmetric bidders. We develop a two stage entry and bidding model with asymmetric bidders within the APV framework and establish the existence and uniqueness of the equilibrium of the model, which is an extension of Lebrun (1999, 2006) who considers the IPV model without entry with N asymmetric bidders. Unlike the model in the second essay, there is no closed form for the equilibrium bid, which substantially complicates the estimation. In view of this we adopt a two step indirect inference method along with a numerical method solving equilibrium bids to estimate the parameters of the underlying distributions. We apply the model to the timber auctions in Oregon State, and identify the affiliation among private values and among entry costs. Using the structural estimates, we also quantify the effects of reserve prices, affiliation level and merger, to which the theory provides no guidance, through a set of counterfactual analyses.

The dissertation, especially the last essay, also contributes to the structural estimation of auction models, which was started with Paarsch (1992) who tests the IPV model against the CV model. Unlike the reduced form analysis, structural analysis of the auction data identifies the underlying distributions of private values and entry costs, which helps researchers to examine the effects of some exogenous variables such as reserve prices through counterfactual analyses without worrying about the "Lucas Critique." It assumes that the observed bids and entry behavior are the results of Bayesian-Nash equilibrium. One issue associated with that is the high dependence of the structural analysis on the information structure. Different information structures such as IPV, CV, and APV result in different equilibria and therefore different estimation results. While much work focuses on the

IPV framework, e.g. Donald and Paarsch (1993, 1996) for the maximum likelihood estimation; Laffont, Ossard and Vuong (1995) for the simulated nonlinear least square estimation (SNLLS); Guerre, Perrigne and Vuong for the two-step nonparametric estimation; Li and Zheng (2005) for the semiparametric estimation of an entry and bidding model; Athey, Levin and Seira (2004), and Krasnokutskaya and Seim (2007) for IPV model with entry and two types of asymmetric bidders, some researchers have studied structural estimation within the APV framework. For example, Li, Perrigne and Vuong (2000, 2002, 2003) and Campo, Perrigne and Vuong (2003) nonparametrically estimate an APV model in various settings. Li, Paarsch and Hubbard (2007) study an APV model of highway procurement auctions in Michigan Department of Transportation. Due to the complexity of the model in the third essay, we adopt a simulation-based approach called indirect inference to conduct the structural estimation. The approach avoids finding the likelihood function as in MLE and moment conditions as in GMM or SNLLS. It requires the calculation of the equilibrium bids, which do not have analytical forms and therefore are solved numerically and is computationally intensive.

The application of this dissertation has been focused on the timber auctions in Oregon. Due to the richness of timber auction data in the U.S. and elsewhere, timber auctions have been intensively studied from various perspectives. Baldwin, Marshall, and Richard (1997) study the possible bidder collusion at U.S. Forest Service timber sales. Brannman and Froeb (2000) study the merger, cartel and bidding preferences using the timber sales data in Oregon Department of Forestry. Haile (2001) study the bidding behavior in timber sales when bidders take the possible resale afterward into account. Li and Perrigne (2003) study the timber auctions with random reserve prices. Athey, Levin, and Seira (2004) utilize the U.S. Forest Service timber sales data to compare the sealed-bid and open auctions.

Lu and Perrigne (2008) propose a nonparametric estimation of risk aversion by using both open and sealed-bid timber sales. While the aforementioned work adopts an IPV framework without entry except Athey, Levin, and Seira (2004) and Li and Zheng (2007), who study entry and bidding in timber auctions, this dissertation studies timber sales in Oregon State within the APV framework with entry and asymmetric bidders.

Timber Auctions Data in Oregon State

This section describes the timber auctions data in Oregon State used in the first and third essays. Before an auction is advertised, the ODF "cruises" the selected tract of timber and obtains information of the tract, such as the composition of the species, the quality grade of the timber and so on. Based on the information it obtains, the ODF sets its appraised price for the tract, which serves also as the reserve price. After the "cruise," a detailed bid notice is usually released 4-6 weeks prior to the sale date, which provides information about the auction, including the date and location of the sale, species volume, quality grade of the timber, appraised price as well as other related information. Potential bidders acquire their own information or private values through different ways and decide whether and how much to bid. Bids are submitted in sealed envelopes that are opened at a bid opening session at the ODF district office offering the sale. The sale is awarded to the bidder with the highest bid. All the sales are therefore first price sealed bid scale auctions.

The original data contain 415 sales and 1501 observed bids in total from January 2002 to June 2007. To avoid the "skewed bidding" issue discussed in Athey and Levin (2001) we remove the sales with more than one bid species. In order to make the auctions as homogeneous as possible, we only make use of sales with Douglas-fir as the bid species, since it is the dominant bid species accounting for about 92%. Some sales are dropped due to the missing value. Therefore the final sample includes 282 sales and 1139 observed bids.

For each sale, we directly observe some sale-specific variables including the location and the region of the sale, appraised price, appraised volume, length of the contract, and diameter at breast height (DBH) as well. Noting that the bid species is often a combination of a mixture of several grades of quality, we use number 1, 2, $\cdots$, up to 18 to denote the letter-grades used by ODF so that the final grade of a sale is the weighted average of grades with volumes of grades as the weight. In addition to sale-specific variables, as shown in Brannman and Froeb (2000), hauling distance is an important bidder-specific variable that affects bidders' bidding decisions. However, hauling distance is not observed directly. We construct the hauling distance variable by transferring the location of a tract into latitude and longitude through the Oregon Latitude and Longitude Locator[1] and finding the distances between the tract and the mills of firms by using Google Map. To control the potential heterogeneity arising from the three locations and three types of sale, we construct four dummy variables, which are Region 1, Region 2&3, Clear and Recovery and Combo and Recovery.

The key information related to endogenous entry is the identities of potential bidders, which are not observed. Unlike some procurement auctions, where information on bidders who have requested bidding proposal is available and can be used as a proxy for potential bidders (Li and Zheng (2005)), we do not have such information in our case, as is usual for timber sale auctions. Therefore we follow Athey, Levin, and Seira (2004) and Li and Zheng (2007) to construct potential bidders. Specifically, we first divide all sales in the original data set into 146 groups, each of which contains all sales held in the same district in the same quarter of the same year. The potential bidders of a sale are then all

[1] It is available at http://salemgis.odf.state.or.us/scripts/esrimap.dll?name=locate&cmd=start

bidders who submit at least one bid in the sales of the group that the sale belongs to. In other words, all auctions in the same group have the same set of potential bidders. Note that in constructing the potential bidders we use the original data set including all auctions removed from the final sample. Summary statistics of the data are given in Table 1.

Table 1. Summary Statistics

	Observation	Mean	Std. Dev.
Entry Behavior	2055	0.5543	0.4972
Distance	2055	78.8476	47.1271
Bids	1139	386.8127	103.0914
Volume	282	3256.7520	2622.2350
Duration	282	781.5390	225.6230
Grade	282	10.2935	0.4549
DBH	282	16.5670	4.8455
# of Poten. Bidders	282	7.2872	2.9897
Region 1	282	0.8262	0.3796
Region 2&3	282	0.1596	0.3669
Clear, Recovery	282	0.4007	0.4909
Combo, Recovery	282	0.2731	0.4463

CHAPTER II

TESTING FOR AFFILIATION USING ENTRY BEHAVIOR

Introduction

First introduced by Milgrom and Weber (1982), affiliation has been widely used and has become an important concept in the development of auction theory. In the seminal paper by Milgrom and Weber (1982), affiliation is the crucial assumption of the main results in the paper. It implies the existence of a symmetric, increasing, pure strategy equilibrium for the first price auction and is the basis of the well known revenue ranking, that is under affiliation, the English auction has a higher expected revenue than the second price auction which in turn has a higher expected revenue than the first price auction. This revenue ranking result for affiliated bidders is in sharp contrast to the celebrated revenue equivalence result for independent bidders established by Vickrey (1961). Furthermore, in Rodriguez (2000) affiliation implies uniqueness of a monotone equilibrium in a two-person first-price auction with symmetric or asymmetric bidders. While recent work (e.g. Monteiro and Moreira (2006) and de Castro (2007)) has shown that for the existence and uniqueness of the monotone pure strategy equilibrium, affiliation is somewhat stronger and can be relaxed, the revenue ranking crucially relies on the affiliation assumption.[1] Furthermore, two standard implications derived under the assumption of the independent private signals do not hold any more when bidders' private signals are affiliated. First, the

[1] While de Castro (2007) derives a similar revenue ranking under a weaker assumption, it holds in a weaker sense in that it holds on average with respect to all functions of a set under consideration. As a result, for a specific density function in the set, the revenue ranking may break down. Under affiliation, however, for each affiliated density function, the revenue ranking always holds.

effect of the number of potential bidders on bids is not always positive with affiliation as in the independent private value (IPV) model due to the "affiliation effect" (Pinkse and Tan (2005)). Second, while the optimal reserve price under the IPV framework is always higher than the true value of the seller and independent of the number of bidders (Riley and Samuelson (1981), Myerson (1981)), it converges to the true value of the seller as the number of bidders increases within the affiliated private value (APV) paradigm as shown in Levin and Smith (1996).[2]

Affiliation has also played a key role in the structural analysis of auction data, which was started with Paarsch (1992) who tests the IPV model against the common value (CV) model, which are two polar cases of the affiliated value (AV) model studied in Milgrom and Weber (1982). Since the structural approach is to estimate an econometric model that is closely derived from theory, the assumption made on the underlying information structure is a key to the validity of the structural model. From an econometric point of view, failure in identifying affiliation among bidders may cause biased estimates of the structural parameters and result in misleading policy conclusions. For instance, one could overestimate bidders' private values when adopting the IPV paradigm for an actual APV model (Li, Perrigne, and Vuong (2002)). From a policy perspective, since one of the main advantages of the structural approach is to address policy related issues such as the optimal reserve price that should be used in an auction given the mechanism, it is important to assess whether bidders' private values are affiliated or independent given that the optimal reserve prices are quite different from each other under these two scenarios. In the empirical work using the structural approach, affiliation has been assumed by several studies. See, e.g. Li, Perrigne and Voung (2000) for the conditional independent private information (CIPI) model of

[2] Li, Perrigne and Vuong (2003) derive an explicit formula for the optimal reserve price in the symmetric APV model in terms of the joint distribution of the bidders' private values. The formula shows that the optimal reserve price implicitly depends on the number of bidders.

Offshore Continental Shelf (OCS) wildcat auctions, Li, Paarsch and Hubbard (2007) for the APV model of highway procurement auctions in Michigan Department of Transportation. Therefore, it is both interesting and important to test for the affiliation among bidders using field data.

In this chapter, we propose a simple approach to test for affiliation among the private information of bidders. Our test builds on the AV model of Wilson (1977) and Milgrom and Weber (1982) and take into account entry and endogenous participation. As is evidenced from the recent work (e.g., Bajari and Hortaçsu (2003), Athey, Levin and Seria (2004), Li and Zheng (2005, 2007), and Krasnokutskaya and Seim (2007)), entry is an important feature in auctions that should be taken into account in empirical analyses of auction data. We show that affiliation among potential bidders' private information (either private signals or entry costs) leads to affiliation among their entry decisions. Our test is then proposed based on this implication. From an econometric viewpoint, since our test is based on capturing affiliation through potential bidders' entry behavior, which can be modeled through a multivariate binary choice framework, we propose a simulated maximum likelihood estimation procedure extending the GHK methods developed by Geweke (1991), Börsch-Supan and Hajivassiliou (1993), and Keane (1994) to our setting. It is worth noting that our approach allows not only for testing for affiliation, but also for testing for asymmetry among potential bidders. It is thus general and flexible, and can be applied to various scenarios.

To the best of our knowledge, this chapter is the first one in the literature that proposes a test for affiliation among potential bidders' private information using potential bidders' entry behavior. The recent work by de Castro and Paarsch (2008) and Jun, Pinkse and Wan (2008) has proposed to use bids to test for affiliation in auctions without entry.

Notably, Jun, Prinkse and Wan (2008) apply their test to three data sets from the OCS, California Department of Transport, and Russian Federal Subsoil Resources Management Agency, and find that in most of the cases they cannot reject affiliation between bids and the number of bidders. They interpret this as most likely driven by endogenous entry, highlighting the importance of taking entry into account when testing for affiliation. It is worth noting that testing for affiliation using information from observed bids in a general framework with entry is complicated for several reasons. First, even for a symmetric AV model without entry, Laffont and Voung (1996) show that the private information cannot be identified without additional restrictions. Thus it is difficult to assess the validity of the affiliation assumption using a structural approach. Second, when entry is introduced to the analysis, using only observed bids to test for affiliation becomes challenging as observed bids are now from actual bidders who enter the auction and submit bids, while the affiliation is presumably an assumption made about the dependence structure across all potential bidders' private information that can affect both entry and bidding. In particular, without knowing enough about how bidders enter the auction, and whether the entry cost is dependent of the private signal, it is difficult for a researcher to decide whether to treat the observed bids as having a sample selection problem to start with when trying to use the observed bids to test for affiliation. Furthermore, there are some situations where using observed bids to test for affiliation could lead to misleading conclusions with entry process. For instance, in the IPV paradigm, if potential bidders' entry costs are affiliated, and they do not draw their private values until after the entry, and they decide whether to enter the auction based on comparing their entry costs with the expected profit from entering and winning, then it can be shown that observed bids are still independent, although the entry costs are affiliated. The literature on testing implications from auction models, which has been developed since Hendricks and Porter's pioneering work in 1980's (e.g. Porter (1995) for a survey of the work by Hendricks and Porter), on the other hand, has mainly focused on testing between private values and common values. For example, Paarsch (1992) estimates the structural models with a set of specifications within the IPV and CV paradigms and compares the goodness-of-fit of the specifications. Haile, Hong, and Shum (2003) test common values using an insight from the effect of the winner's curse on equilibrium bids. Hendricks, Pinkse and Porter (2003) propose a test for common values when there is a binding reserve price and using bids and the *ex post* realization of the auctioned object's value. The insight that is used in our testing procedure is that affiliation assumed in the joint distribution of private values or the joint distribution of entry costs implies affiliation among their entry decisions. Therefore an affiliation test can be proposed based on the potential bidders' entry behavior without using bidding information.

Since our approach is based on the testable implication of affiliation on entry behavior, the observed entry behavior is a key in our approach. While auctions with entry have been studied since 1980's (e.g., Levin and Smith (1994), Samuelson (1985), among others), the empirical analysis of auction data with entry has only drawn considerable attention recently. Several studies have attempted to analyze auction data taking endogenous participation into account. See, e.g. Bajari and Hortaçsu (2003) for internet auctions with entry using data from eBay.com, Athey, Levin and Seira (2004) for entry and bidding patterns in sealed bid and open auctions with heterogeneous bidders, Krasnokutskaya and Seim (2007) for the bid preference program and participation in California highway procurement, De Silva, Kosmopoulou and Lamarche (2007) for the effect of information release on entry and survival in procurement auctions, Li and Zheng (2005) for entry and competition effects in procurement auctions, and Li and Zheng (2007) for evaluating how bidders make entry and bidding decisions in Michigan timber auctions. Our test contributes to this part of the literature by exploiting the implication of affiliation of private signals or of entry costs among potential bidders to form an intuitive and easy-to-implement test for affiliation. On the other hand, a caveat of our test is that it is a reduced form in nature, and a confirmation of affiliation from the test can only indicate that there is an affiliation among potential bidders' private information. A further analysis such as a structural analysis should be used to disentangle the driving force for the affiliation of entry behavior, and to measure the extent to which private signals or/and entry costs are affiliated.

We demonstrate our test by applying the test to timber auctions organized by Oregon Department of Forestry (hereafter ODF). We find that bidders are affiliated. Also bidders are asymmetric in the sense that the hauling distance is highly significant in bidders' entry decisions. Our empirical application in timber auctions is interesting in its own right, in addition to serving as an illustration to our proposed testing procedure. Timber auctions have been studied extensively in the empirical literature, because the various auction formats have been used and provided researchers a ground for testing theory and comparing revenues from different auction formats, and also because of the richness of timber auction data in the U.S. and elsewhere.[3] Most of the (structural) empirical work in studying timber auctions, however, has used the IPV paradigm. We find through using our test in the ODF data that the affiliation is significant though at a relatively low level, which turns out to be the first result in the literature to assess affiliation in timber auctions.

This chapter is organized as follows. Section 2 describes two alternative entry models within the general AV framework and derive an implication of affiliation among bidders' private information on entry behavior, which is the theoretical foundation of our

[3]The work includes Paarsch (1997), Baldwin, Marshall and Richard (1997), Brannman and Froeb (2000), Haile (2001), Athey and Levin (2001), Li and Perrigne (2003), Haile, Hong and Shum (2003), Athey, Levin and Seira (2004), Li and Zheng (2007), among others.

test. The testing procedure is proposed in Section 3. In Section 4, we apply our test to timber sales organized by ODF. Section 5 concludes.

The Theoretical Framework

In this section we consider a first-price sealed-bid auction within the AV paradigm with a public reserve price r, where a single object is auctioned off to N risk-neutral potential bidders, extending the framework in Milgrom and Weber (1982) to accommodate asymmetric bidders. Affiliation is a terminology describing the dependence among random variables, which is equivalent to multivariate total positivity of order 2 (MTP_2) in the multivariate statistics literature (Karlin and Rinott (1980)). Here we use the same definition of affiliation as in Milgrom and Weber (1982).

Definition 1 *Let y and y' be any two values of a vector of random variables $Y \subseteq \mathbb{R}^N$ with a density $f(\cdot)$. It is said that all elements of Y are affiliated if $f(y \vee y') f(y \wedge y') \geq f(y) f(y')$, where $y \vee y' = (\max\{y_1, y_1'\}, \ldots, \max\{y_n, y_n'\})$ and $y \wedge y' = (\min\{y_1, y_1'\}, \ldots, \min\{y_n, y_n'\})$.*

Intuitively, affiliation means that large values for some of the components in Y make other components more likely to be large than small. Also, as shown in Milgrom and Weber (1982), for a twice continuously differentiable density $f(\cdot)$, it is affiliated if and only if for $i \neq j$, $\partial^2 \ln f / \partial y_i \partial y_j \geq 0$, that is, $\ln f$ is super-modular.

We denote the utility of bidder i from the object by $U_i = u_i(S, V)$, where V is a N-dimensional vector with support of $[\underline{v}, \overline{v}]^N$, whose i-th element V_i represents the private signal of bidder i, and S is an m-dimensional vector representing additional information.[4] Denote $F(s, v)$ and $f(s, v)$ the joint distribution and the joint probability density of S and

[4] Such a setup nests both IPV paradigm and CV paradigm. See Milgrom and Weber (1982) for detail.

V, respectively. Moreover, all potential bidders' entry costs $(k_1, ..., k_N)$ are drawn from a joint distribution $G(\cdot)$. $F(\cdot, \cdot)$ and $G(\cdot)$ are common knowledge to all potential bidders.

We consider two pure strategy entry models. The first model, referred to as Model 1, assumes that all potential bidders do not draw their private signals until after they decide to enter. Therefore, the entry cost in Model 1 can be interpreted as consisting of both information acquisition cost and bid preparation cost. The second model, referred to as Model 2, assumes that all potential bidders first draw their private signals, and then decide whether to enter the auction. In this case, the entry cost mainly consists of bid preparation cost. Specifically, in Model 1, denote potential bidder i's expected profit from entering the auction by Π_i, and the event that he enters the auction by $a_i = 1$. Then $\Pi_i = \sum_{a_{-i} \in A_{-i}} \int_{\underline{v}}^{\overline{v}} \pi_i \left(v_i | a_{-i}\right) dF_i \left(v_i\right) \Pr\left(a_{-i} | a_i = 1\right)$, where $a_{-i} \in A_{-i} = \{(a_1, \ldots, a_N) \,| a_j = 0 \text{ or } 1, j = 1, \ldots N, j \neq i\}$ is one possibility of the 2^{N-1} combinations of entry decisions of $N-1$ other potential bidders, $\pi_i \left(v_i | a_{-i}\right)$ is bidder i's profit at the bidding stage conditioning on his winning and on that his actual competitors' set is a_{-i}. Note that $\pi_i \left(v_i | a_{-i}\right) = \left(U_i - b_i\right) \Pr\left(B_{-i} < b_i | v_i; a_{-i}\right)$, where b_i is bidder i's bid at the bidding stage given a_{-i}, and B_{-i} denotes the maximum bid among other actual bidders.[5] For potential bidder i, he will enter the auction if $\Pi_i > k_i$, his ex ante expected profit from entering the auction exceeds the entry cost, otherwise, he will stay out.[6]

Model 2, on the other hand, assumes that for potential bidder i, $i = 1, ..., N$, he

[5] Since we are interested in using potential bidders' entry behavior to infer affiliation, we focus on the entry stage model, but not on the bidding stage. This can be viewed as an advantage of our approach, as what is required here is only the existence of a bidding equilibrium.

[6] Recently various simpler versions of Model 1 have been studied in the empirical auction literature. For example, Athey, Levin and Seira (2004) and Krasnokutskaya and Seim (2006) study a similar entry model within the IPV paradigm with two types of bidders, Li and Zheng (2005) study the symmetric version of the model within the IPV paradigm.

first draws a private signal v_i, and then compares v_i with the screening level defined as

$$v_i^* = \inf\left\{v_i | E\left[(U_i - r)\,1\left(V_j < v_j^*, j \neq i\right) - k_i | V_i = v_i^*\right] \geq 0\right\}. \tag{II.1}$$

As a result, the screening levels for all potential bidders $(v_1^*, ..., v_N^*)$ constitute solutions to a system of N equations given by (II.1) for i, $i = 1, ..., N$. For potential bidder i, he only becomes an actual bidder and submits a bid if his private signal exceeds the screening level, $v_i > v_i^*$.[7]

Note that in both models, the entry behavior of each potential bidder in an auction is determined by a cut-off point either for the entry cost (in Model 1), or for the private signal (in Model 2). It is then natural to think of affiliated bidders as more likely to have similar entry behavior. For instance, in Model 2, suppose that two bidders' private signals are affiliated and bidder 1 has a high private signal, which is above v_1^*. By the definition of affiliation, it is more likely that bidder 2 a has high private signal as well. Therefore the probability that both bidders 1 and 2 participate in the auction should be no less than the probability that bidder 1 participates and bidder 2 does not. The following proposition formalizes this intuition.

Proposition 1 *Let $D = (D_1, \ldots, D_N) \in \{0,1\}^N$ denote bidder $1, \cdots,$ bidder N's entry decisions.*

(1) In Model 1, if $k_1, ..., k_N$ are affiliated, then $D_1, \ldots, D_N$ are also affiliated.

(2) In Model 2, If $V_1, \ldots, V_N$ are affiliated, then $D_1, \ldots, D_N$ are also affiliated.

[7] This entry model, extending the symmetric entry model in Milgrom and Weber (1982) to the asymmetric case, is in a similar spirit to the one proposed by Samuelson (1985) within the IPV paradigm, in which potential potential bidders first draw their private values and adopt a pure strategy in their entry decision. An alternative entry model is developed by Levin and Smith (1994), and further studied in Li (2005) and Li and Zheng (2005), in which bidders make entry decision before learning their private information and therefore, their entry behavior is randomized, and under the symmetry assumption, each bidder has the same probability of participation.

Proof. (1) In Model 1, D_i is defined as $D_i = 1(k_i < \Pi_i)$ and thus is a non-increasing function of k_i. If $k_1, ..., k_N$ are affiliated, following Theorem 3 in Milgrom and Weber (1982), $D_1, \ldots, D_N$ are affiliated.

(2) In Model 2, D_i is defined as $D_i = 1(V_i > v^*)$ and thus is a non-decreasing function of V_i. If $V_1, \ldots, V_N$ are affiliated, following Theorem 3 in Milgrom and Weber (1982), $D_1, \ldots, D_N$ are affiliated. ■

Proposition 1 demonstrates that the theoretical framework we adopt in modeling potential bidders' entry decisions gives rises to affiliation among bidders' entry behavior as a result of affiliation in bidders' private information, either entry costs as in Model 1, or private signals as in Model 2. This is an intuitive result, and constitutes a basis upon which our test for affiliation is proposed.

The Affiliation Test

The insight we gain from the previous section is that affiliation among potential bidders' private information (either private values or entry costs) leads to affiliation among their entry decisions. Therefore we propose to test for affiliation among bidders' private information by testing for affiliation among their entry decisions. Since in practice we usually deal with a large number of (heterogeneous) auctions, we assume that we observe a $1 \times k_1$ auction-specific covariate vector, denoted by x_ℓ, $\ell = 1, ..., L$, where L is the number of auctions in the data set. For auction ℓ in the data, there are N_ℓ potential bidders. To control for the possible asymmetry among potential bidders, we include a $1 \times k_2$ bidder-specific covariate vector denoted by $z_{\ell i}$, $i = 1, ...N_\ell$. We define $D_{\ell i}$ to be 1 if potential bidder i enters the ℓ-th auction and 0 otherwise. We use a binary choice model to model the entry decision as follows,

$$D_{\ell i} = 1\left(x_\ell \beta + z_{\ell i}\gamma + \eta_\ell + \varepsilon_{\ell i} > 0\right) \tag{II.2}$$

where η_ℓ denotes the auction heterogeneity unobserved by the econometrician,[8] and $\varepsilon_{\ell i}$ denotes idiosyncratic error unobserved by the econometrician, and independent of x_ℓ, $z_{\ell i}$, and η_ℓ. Note here that we adopt a linear representation for the "reduced form" payoff function in modeling the entry decision, in which case, the payoff function can be arising either from a potential bidder's comparison of his entry cost with his cut-off level of expected profit as in Model 1, or from his comparison of his private signal with his cut-off level as in Model 2. Our approach can accommodate more general and flexible functional forms for the reduced form payoff function. The next proposition links the affiliation among $D_{\ell i}$ to the affiliation among $\varepsilon_{\ell i}$.

Proposition 2 *If $\varepsilon_{\ell 1}, \ldots, \varepsilon_{\ell N_\ell}$ are affiliated, then $D_{\ell 1}, \ldots, D_{\ell N_\ell}$ are affiliated.*

Proof. It follows the fact that $D_{\ell i}$ is a non-decreasing function of $\varepsilon_{\ell i}$. ■

Testing for affiliation among $D_{\ell i}$ now amounts to testing for affiliation among $\varepsilon_{\ell i}$ in our setting. To implement the test, we further assume that $\eta_\ell, \varepsilon_{\ell 1}, \ldots, \varepsilon_{\ell N_\ell}$ follow a joint normal distribution with mean $\mu = (0, \ldots, 0)'$ and covariance $\Sigma_{(N_\ell+1)\times(N_\ell+1)} = \begin{bmatrix} \sigma^2 & 0 & \cdots & 0 \\ 0 & 1 & & \rho \\ \vdots & & \ddots & \\ 0 & \rho & & 1 \end{bmatrix}$, implying that η_ℓ is independent of $\varepsilon_{\ell i}$, and $\varepsilon_{\ell i}$ and $\varepsilon_{\ell j}$ have a correlation denoted by ρ, the same across all the potential bidders/auctions. Under our setting, ρ represents the affiliation among $\varepsilon_{\ell 1}, \ldots, \varepsilon_{\ell N_\ell}$. This follows from Sarkar (1969) and Barlow

[8] As shown in Krasnokutskaya (2003), controlling for unobserved auction heterogeneity can be important in applications.

and Proschan (1975) who show that for $X = (X_1, \cdots, X_n) \sim N(0, \Omega)$, $X_1, \ldots, X_n$ are affiliated if and only if the off-diagonal elements of matrix $-\Omega^{-1}$ are nonnegative, which implies, in our case, that $\varepsilon_{\ell 1}, \ldots, \varepsilon_{\ell N_\ell}$ are affiliated if and only if $\rho \geq 0$. As a result, we can construct the following hypotheses to test for affiliation among the private signals of all potential bidders,

$$H_0 : \rho \geq 0,$$

$$H_1 : \rho < 0.$$

A by-product of our setting is that the symmetry assumption can be tested by testing $\gamma = 0$, since the possible asymmetry among potential bidders is accommodated by the bidder-specific variables $z_{\ell i}$.

A Smoothly Simulated Maximum Likelihood Estimator

The problem now becomes estimating the unknown parameters $\theta = (\beta, \gamma, \sigma, \rho)$ in the binary latent variable model. Under the specifications given above, we can write down the joint likelihood function for all N_ℓ potential bidders at the ℓ-th auction in terms of their entry decisions. For instance, the likelihood of all potential bidders participating in the ℓ-th auction is

$$\begin{aligned} p_\ell(\theta) &= \Pr\left(D_{\ell i} = 1, i = 1, \ldots, N_\ell\right) \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} 1\left(\varepsilon_\ell \in A_\ell\right) f\left(\varepsilon_{\ell 1}, \ldots, \varepsilon_{\ell N_\ell}, \eta_\ell\right) d\varepsilon_{\ell 1} \cdots d\varepsilon_{\ell N_\ell} d\eta_\ell \\ &= E_{\eta_\ell}\left[\int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} 1\left(\varepsilon_\ell \in A_\ell\right) f_\varepsilon\left(\varepsilon_{\ell 1}, \ldots, \varepsilon_{\ell N_\ell}\right) d\varepsilon_{\ell 1} \cdots d\varepsilon_{\ell N_\ell}\right], \end{aligned} \tag{II.3}$$

where A_ℓ denotes the set $\{\varepsilon_\ell | \varepsilon_{\ell i} > -(x_\ell \beta + z_{\ell i}\gamma + \eta), i = 1, \ldots, N_\ell\}$ and the last equality follows from the independence assumption of $\varepsilon_{\ell 1}, \ldots, \varepsilon_{\ell N_\ell}$ and η_ℓ. Similarly, we can express the likelihood of entry/non-entry of all potential bidders at all other auctions. In principle, θ can be estimated by MLE with the joint log-likelihood function defined as

$$\pounds(D, x, z; \theta) = \sum_{\ell=1}^{L} \ln p_\ell(\theta).$$

There are a couple of complications arising from implementing the MLE here. First, $p_\ell(\theta)$ involves multiple $(N_\ell + 1)$ integrations. Therefore when the number of potential bidders N_ℓ is large, the MLE becomes computationally intensive because of the large number of multiple integrals involved. This issue can be addressed through using the simulation based method to approximate the otherwise difficult-to-calculate integrals. Second, with the use of the simulation based method, the objective function may become non-smooth due to the index function $1(\varepsilon_\ell \in A_\ell)$. To address both issues, we adopt a smooth and unbiased simulator often called the GHK simulator after Geweke, Hajivassiliou, and Keane, which is found to be the most reliable method for simulating normal rectangle probability among others (e.g. Hajivassiliou, McFadden and Ruud (1996)). Specifically, in our case, the covariance matrix of $\varepsilon_{\ell i}$ is $\sum_{N_\ell} = \begin{bmatrix} 1 & & \rho \\ & \ddots & \\ \rho & & 1 \end{bmatrix}_{N_\ell \times N_\ell}$. There must exist a lower triangular matrix h_ℓ such that $h_\ell \cdot h'_\ell = \sum_{N_\ell}$. Then $\varepsilon_\ell = (\varepsilon_{\ell 1}, \ldots, \varepsilon_{\ell N_\ell})'$ can be written as $\varepsilon_\ell = h_\ell \xi_\ell$, where $\xi_\ell = (\xi_{\ell 1}, \ldots, \xi_{\ell N_\ell})'$ follows N_ℓ-variate standard normal distribution. Since h_ℓ is a lower triangular matrix, we have the following recursive formula for $\varepsilon_{\ell i}$.

$$\varepsilon_{\ell 1} = h_{\ell,11}\xi_{\ell 1} \tag{II.4a}$$

$$\varepsilon_{\ell 2} = h_{\ell,21}\xi_{\ell 1} + h_{\ell,22}\xi_{\ell 2} \tag{II.4b}$$

$$\vdots \tag{II.4c}$$

$$\varepsilon_{\ell N_\ell} = h_{\ell,N_\ell 1}\xi_{\ell 1} + \cdots + h_{\ell,N_\ell N_\ell}\xi_{\ell N_\ell}, \tag{II.4d}$$

where $h_{\ell,ij} = h_\ell(i,j)$. Then we can make simulation draws based on such a decomposition. For example, the probability that all bidders participate in the ℓ-th auction $p_\ell(\theta)$ in equation $(II.3)$ can be written as the following form

$$\begin{aligned} p_\ell(\theta) &= E_v\left[\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} 1(\xi_\ell \in B_\ell) f_\xi(\xi_{\ell 1},\ldots,\xi_{\ell N_\ell}) d\xi_{\ell 1}\cdots d\xi_{\ell N_\ell}\right] \\ &= E_v\left[\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty} g(x_\ell, z_\ell, \xi_l, v;\theta|v)\,\widetilde{g}(x_\ell, z_\ell, \xi_l, v;\theta|v)\, d\xi_{\ell 1}\cdots d\xi_{\ell N_\ell}\right] \\ &= E_v\left[E\left(g(x_\ell, z_\ell, \xi_l, v;\theta|v)\right)\right] \\ &= E\left(g(x_\ell, z_\ell, \xi_l, v;\theta)\right), \end{aligned}$$

where $\widetilde{g}(x_\ell, z_\ell, \xi_l, v;\theta|v) = 1(\xi_\ell \in B_\ell)\frac{\phi(\xi_1)\times\cdots\times\phi(\xi_{\ell N_\ell})}{g(x_\ell,z_\ell,\xi_l,v;\theta|v)}$, B_ℓ is the set derived from A_ℓ according to the transformation $(II.4)$, $v = \eta/\sigma$ and $g(x_\ell, z_\ell, \xi_l, v;\theta|v) = \left[1-\Phi\left(\frac{-x_\ell\beta - z_{\ell 1}\gamma - \sigma v}{h_{\ell,11}}\right)\right]\times \cdots\times\left[1-\Phi\left(\frac{-x_\ell\beta - z_{\ell N_\ell}\gamma - \sigma v - h_{\ell,N_\ell 1}\xi_1 - \cdots - h_{\ell,N_\ell N_\ell - 1}\xi_{\ell N_\ell - 1}}{h_{\ell,N_\ell N_\ell}}\right)\right]$. The second last equality follows the fact that $\frac{\phi(\xi_1)\times\cdots\times\phi(\xi_{\ell N_\ell})}{g(\xi_l|v)}1(\xi_\ell \in B_\ell)$ is actually a joint density function of ξ_ℓ conditional v. As a result, $p_\ell(\theta)$ can be approximated by $\widetilde{p}_\ell(\theta) = \frac{1}{T}\sum_{t=1}^{T} g_t(x_\ell, z_\ell, \xi_l, v;\theta)$ and $g_t(x_\ell, z_\ell, \xi_l, v;\theta)$ is simulated by the following procedure.

Step 1. First draw v from $N(0,1)$ and calculate $1-\Phi\left(\frac{-x_1\beta-\sigma v}{h_{11}}\right)$.

Step 2. Draw $\xi_{\ell 1}$ from $N(0,1)$ truncated at $\frac{-x_\ell\beta - z_{\ell 1}\gamma - \sigma v}{h_{\ell,11}}$ from below, and calculate

$1-\Phi\left(\frac{-x_\ell\beta-z_{\ell 2}\gamma-\sigma v-h_{\ell,21}\xi_{\ell 1}}{h_{\ell,22}}\middle|\,\xi_{\ell 1}\right).$

$\vdots$

Step N_ℓ. Draw $\xi_{\ell N_\ell-1}$ from $N(0,1)$ truncated at $\frac{-x_\ell\beta-z_{\ell N_\ell}\gamma-\sigma v-h_{\ell,N_\ell 1}\xi_1-\cdots-h_{\ell,N_\ell N_\ell-2}\xi_{\ell N_\ell-2}}{h_{\ell,N_\ell-1 N_\ell-1}}$ and calculate $1-\Phi\left(\frac{-x_\ell\beta-z_{\ell N_\ell}\gamma-\sigma v-h_{\ell,N_\ell 1}\xi_1-\cdots-h_{\ell,N_\ell N_\ell-1}\xi_{\ell N_\ell-1}}{h_{\ell,N_\ell N_\ell}}\middle|\,\xi_{\ell 1},\ldots,\xi_{\ell N_\ell-1}\right).$

Define the corresponding simulated likelihood function as follows

$$\widetilde{\mathcal{L}}(D,x,z;\theta)=\sum_{\ell=1}^{L}\ln\widetilde{p}_\ell(\theta).$$

Following Börsch-Supan and Hajivassiliou (1993), a smoothly simulated maximum likelihood estimator (SSMLE) can be proposed as

$$\widehat{\theta}_{SSMLE}=\arg\max_\theta\widetilde{\mathcal{L}}(D,x,z;\theta).$$

In practice, to guarantee that ρ is between -1 and 1 and σ is positive, we make two transformations, namely, $\rho=\frac{2}{1+\exp(\widetilde{\rho})}-1$ and $\sigma=\exp(\widetilde{\sigma})$.

As the SSMLE is essentially one type of the simulated maximum likelihood estimator, $\widehat{\theta}_{SSMLE}$ has the same asymptotic normal distribution as that of the usual MLE as the number of simulations, T, is large enough in the sense that $T/\sqrt{L}\to\infty$. Let $s_l(\theta)=\nabla_\theta(\ln p_l(\theta))'$, then $\sqrt{L}\left(\widehat{\theta}_{SSMLE}-\theta\right)\to N(0,\Lambda)$, where $\Lambda=E\left[s'_\ell(\theta)\,s_\ell(\theta)\right]^{-1}$(e.g. Train (2003)).

Note that here $(\varepsilon_{\ell 1},\ldots,\varepsilon_{\ell N_\ell})$ is assumed to follow a multivariate normal distribution, in which case, the GHK simulation method can be used to conduct the simulated maximum likelihood estimation. Our approach can be viewed as assuming a Gaussian copula for the joint distribution of $(\varepsilon_{\ell 1},\ldots,\varepsilon_{\ell N_\ell})$ with standard normal marginal of $\varepsilon_{\ell i}$, and can be extended to other specifications of the multivariate distribution for $(\varepsilon_{\ell 1}, \ldots, \varepsilon_{\ell N_\ell})$, such as non-normal distributions for the marginal distributions of $\varepsilon_{\ell i}$, and a Gaussian copula for the joint distribution.[9] In the same vein albeit a more general setting, a semiparametric approach can be developed extending the generalized bivariate probit model (Chen and Zhou (2007)), which leaves the marginal distributions of $\varepsilon_{\ell i}$ unspecified, to the multivariate case we consider here, though the computation intensity can be high.

Affiliation in Oregon Timber Auctions

This section applies our testing procedure to the timber sales organized by the ODF from January 2002 to June 2007. All auctions are held as first-price sealed-bid auctions and in a format of scale auctions.

Since we will focus on the entry behavior of the potential bidders to infer affiliation, we highlight the histogram of the entry proportion of the data in Figure 1, where the entry proportion is defined as the ratio between the number of the actual bidders and the number of potential bidders. Figure 1 presents some interesting features. As one can see from this figure, the auctions which have more than 90% entry proportion have the largest frequency. The number of auctions which have less than 30% or greater than 70% entry proportions is 160, which is a bit above a half of the total number of auctions in the data set. This can be viewed as an indication of a small level of affiliation among potential bidders.

Results

Table 2 presents the estimation results, which are obtained through the SSMLE with the number of simulations $T = 100$. Since our primary goal is to test for affiliation,

[9] For the concept of copula, which is to model joint distributions given marginal distributions, see Nelsen (1999). For using the copula approach to model joint distributions in auction models, see Li, Paarsch and Hubbard (2007). We also discuss it in detail in Chapter IV.

we are mainly interested in the estimate of ρ, which turns out to be 0.2281 with a standard error equal to 0.0493. This result suggests that ρ is significant and therefore bidders are affiliated. But the magnitude of ρ indicates that the level of affiliation is not high, though significant. This is consistent with the finding through the simple analysis of the entry proportion. The standard deviation of the unobserved heterogeneity is quite small and insignificant, which implies that the variables we use to control for auction heterogeneity have captured most of the auction heterogeneity that affects potential bidders' entry decision. Another interesting finding is that the hauling distance matters in the entry behavior and one percentage increase in the hauling distance will decrease the probability of entry by about 8.75% on average. As a result the hypothesis of symmetric bidders is rejected. This means that the hauling distance not only affects bidders' bidding behavior as found in Brannman and Froeb (2000), but also affects potential bidders' entry decisions. As far as auction-specific covariates are concerned, except that the duration of a contract and DBH are not significant, the estimates of all other covariates are intuitive. Potential bidders are more likely to enter the auction with higher volume or higher quality, as suggested by the positive coefficients of the log volume and log grade. On the other hand, the negative coefficient of the number of potential bidders implies that the competition may deter potential bidders' participation, which is consistent with the theoretical results in the literature (e.g. Li and Zheng (2005) for the relationship between the entry probability and the number of potential bidders).

To assess the robustness of our results, following a referee's suggestion, we reconduct the estimation using an alternative definition of potential bidders by defining potential bidders as all companies that participated in an auction in the same district in the previous 12 months, instead in the same quarter of the same year. We find that the estimate for ρ is about 0.1, and also significant, again supporting the affiliation hypothesis, though the estimated affiliation level is smaller than the one obtained using our original definition of potential bidders. Furthermore, we have also tried nonlinear specifications for the reduced form payoff function to check the sensitivity of the estimate for ρ to the specification of the payoff function. For instance, when the square of the number of potential bidders is included, the estimated ρ becomes 0.186 and also significant. These results demonstrate the robustness of the estimate of ρ, and thus of the proposed test.

Conclusions

Affiliation is an important assumption that has been used in both theoretical and empirical frameworks in studying auctions. It is thus an important econometric issue as to how to test for affiliation using field data. In this chapter, we propose a novel approach to test for affiliation. Using observed bids to test for affiliation is difficult when entry is present. We circumvent this problem by using only potential bidders' entry behavior based on the insight that affiliation among potential bidders' private information implies the affiliation among potential bidders' entry behavior.

Our testing procedure requires the estimation of a multivariate probit model. It is general as it can accommodate various scenarios such as asymmetric bidders and unobserved auction heterogeneity. We propose the SSMLE to estimate the model as the SSMLE overcomes computational complexity associated with the estimation and makes the estimation computationally tractable. It is worth noting that while our primary goal is to test for affiliation, our testing procedure can also be used to verify some other assumptions such as symmetry among bidders.

We apply our approach to the timber auctions held by ODF. Our results indicate that the affiliation among potential bidders' entry decisions is significant but of a small level. Furthermore, the potential bidders' entry decisions are affected by the hauling distance, meaning that the potential bidders are heterogeneous. These findings offer insight and guidance in terms of structural modeling, as in view of these results, a structural model that is used to study the timber auctions organized by ODF should take into account the affiliation among bidders' private values or/and bidders' entry costs, and asymmetry among potential bidders. In particular, from a policy viewpoint, how the optimal reserve price should be set according to the level of affiliation of private signals/entry costs can be addressed through the structural approach.

Table 2. Estimation Results

	Estimates	Std. Error
Log of Volume	0.1496*	0.0732
Log of Duration	0.014	0.1535
Log of Grade	5.3086*	2.3448
Log of DBH	−0.3006	0.3863
Potential Bidder	−0.0755*	0.0158
ρ^{**}	0.2281*	0.0493
σ^{**}	0.1560	3.6562
Log of Distance	−0.2064*	0.0223

∗ denotes 5% significance.

∗∗ The estimates and standard errors are the transformed ones.

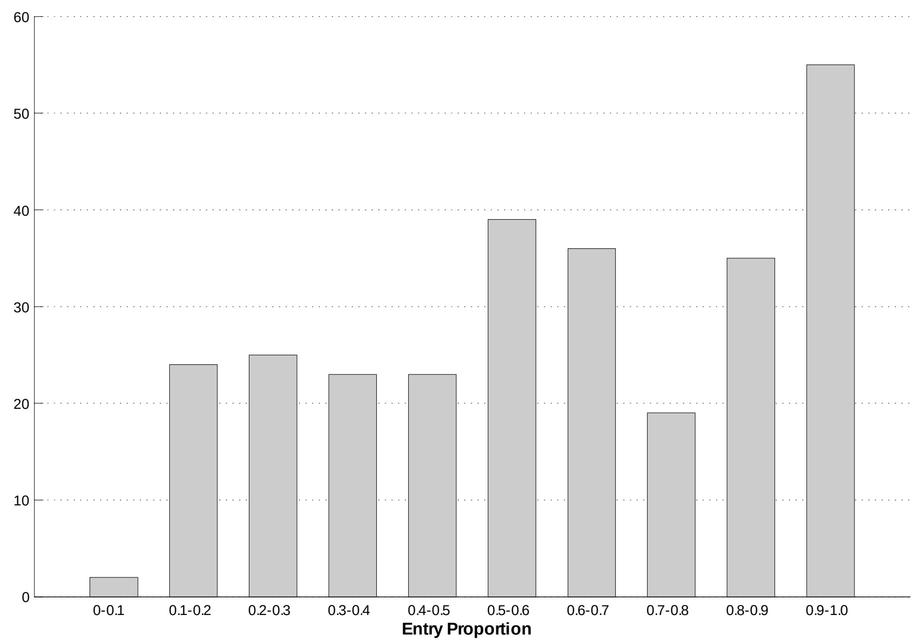

Figure 1. Histogram of Entry Proportion

CHAPTER III

AFFILIATION EFFECT, ENTRY EFFECT AND COMPETITION EFFECT

Introduction

This chapter studies the relationship between the number of potential bidders and bids when potential bidders' private values are affiliated and their entry decisions are endogenous. It decomposes the effect of the number of potential bidders on bids into three effects, namely "competition effect," "entry effect" and "affiliation effect," which can have different signs and interact altogether resulting in an ambiguous relationship between the number of potential bidders and bids.

While theoretical models of entry in auctions have been developed since 1980s, most of the work has focused on the IPV model. See, e.g., Samuelson (1985), McAfee and McMillan (1987), Levin and Smith (1994), Harstad (1990), and Kjerstada and Vagstad (2000) among others. This chapter studies theoretical implications of APV model with entry. In particular, it investigates the relationship between the number of potential bidders and bids, which has been an important issue in the auction literature. As established in Pinkse and Tan (2005), in the APV model without entry, in addition to the usual "competition effect," there is an additional effect specific to a class of APV models and opposite to the "competition effect," which they term as the "affiliation effect." For the IPV model with entry, Li and Zheng (2005) identify what they call "entry effect" that is opposite to the "competition effect" as well. This chapter shows that within the APV framework with entry, in addition to the "competition effect," both "affiliation effect" and "entry effect" are at work. As a result, the overall relationship between the number of potential bidders becomes ambiguous, and depends on the relative magnitudes among these three effects. Therefore, quantifying these effects and thus determining the overall relationship between the number of potential bidders and bids calls for a structural analysis of the auction data with the APV model and endogenous entry.

The remaining of this chapter is organized as follows. Section 2 presents the theoretical APV model with entry and public reserve prices. Section 3 studies the relationship between the number of potential bidders and bids by decomposing the effect of the number of potential bidders into the three effects analytically. Section 4 presents an example showing non-monotonic relationship between the number of potential bidders and bids and quantifies the total effect and three effects. Section 5 concludes.

Models

Based on the AV model developed by Milgrom and Weber (1982), we consider an APV model with entry. Consider a first-price sealed-bid auction, in which a single object is auctioned off to N homogenous and risk-neutral potential bidders with a public reserve price r.

Prior to the auction, bidders draw their private values v from the joint distribution of $F(v_{1,} \ldots, v_N)$ with the support of $[v^*, \overline{v}]^N$, and make their participation decisions accordingly. Bidder i incurs an entry cost k_i and submits a bid b_i if he decides to participate in the auction. The bidder with the highest bid wins the object and pays the amount he bids.

Due to symmetry, we look at bidder 1 in the following analysis without loss of generality.

Entry Decision

A non-trivial entry cost k, which mainly includes the preparation cost, may deter bidder 1 from entering the auction depending on his private value. Therefore before deciding how much to bid, bidder 1 has to make participation decision first. As is shown in Milgrom and Weber (1982) in the symmetric case the entry decision is governed by a screening level of the private values v^*. Only bidders who have private values greater than the screening level will participate in the auction.[1] The bidder c (denoting the cut-off bidder whose private signal is v^*) is indifferent from entering and not entering the auction, therefore v^* is determined by the zero profit condition:

$$(v^* - r)\, F_{V_{-1}|v_1}\left(v^*, \ldots, v^*|v^*\right) = k, \tag{III.1}$$

where $F_{V_{-1}|v_1}\left(v_2, \ldots, v_N|v\right)$ denotes the conditional joint distribution of $V_2, \ldots V_N$ conditioning on $V_1 = v$.

As one can see, the entry leads to the cut-off point that functions as a binding reserve price and thus changes the set of bidders who are willing to participate in the auction to compete.

Bidding Strategy

In the bidding stage, actual bidders submit their bids, which are determined by maximizing their expected profits conditional on their private signals. For bidder 1, his bid

[1] This entry model is a general model in that it includes the one proposed by Samuelson (1985) and further studied in Li and Zheng (2005, 2007) as a special case as the latter model considers the symmetric IPV paradigm. Another alternative entry model is developed by Levin and Smith (1994), in which bidders make entry decisions before learning their private information and their entry behaviors are randomized. Under the symmetry assumption, each bidder has the same probability of participation.

is determined by the following maximization problem

$$\max_{b_1} (v_1 - b_1) \Pr (B_j < b_1 | V_1 = v_1, j \neq 1), v_1 \in [v^*, \overline{v}].$$

As in the literature, we consider a strictly increasing Bayesian Nash bidding equilibrium $b_1 = s(v_1)$. In terms of the conditional joint distribution $F_{V_{-1}|v_1}$, the maximization problem can be written as follows,

$$\max_{b} (v - b) F_{V_{-1}|v_1} \left(s^{-1}(b), \ldots, s^{-1}(b) | v\right), v \in [v^*, \overline{v}]$$

where subscript 1 is suppressed due to symmetry and s^{-1} is the inverse function of bidding function.

The optimal bids are characterized by the first order condition of the maximization problem given in the following,

$$-F_{V_{-1}|v_1}(v, \ldots, v|v) + (v - s(v))(N-1) F_{2,V_{-1}|v_1}(v, \ldots, v|v) \frac{1}{\frac{\partial s(v)}{\partial v}} = 0$$

and a boundary condition $s(v^*) = r$, where $F_{2,V_{-1}|v_1}(v_2, \ldots, v_N|v) = \frac{\partial F_{V_{-1}|v_1}(v_2,\ldots,v_N|v)}{\partial v_2}$. The bidding strategy can be solved as

$$b = v - L(v|v) \int_{v^*}^{v} L(u|u)^{-1} du + L(v|v)(r - v^*), \quad \text{(III.2)}$$

where $L(v|v) = \exp\left(-\int_{v^*}^{v} \frac{(N-1)F_{2,V_{-1}|v_1}(t,\ldots,t|t)}{F_{V_{-1}|v_1}(t,\ldots,t|t)} dt\right)$.

The equilibrium of this game can be viewed as two parts, entry equilibrium and bidding equilibrium. In the entry equilibrium, there is a screening level of the private value for each bidder. Each bidder chooses participation as his strategy if his private value is above the screening level and stays out otherwise. The equilibrium bid is given by equation $(III.2)$. Although the model is illustrated through two parts, compared with the model with mixed entry strategy in Li and Zheng (2005), which extends Levin and Smith (1994), it is indeed a one stage game, since the private values are drawn prior to entry.

Affiliation Effect, Entry Effect and Competition Effect

This section is devoted to analyzing the relationship between bids and the number of potential bidders within the APV paradigm with entry. As is well known, due to the "competition effect" in the IPV model without entry, a higher level of competition makes bidders more aggressive, leading to a higher revenue to the seller. It is, however, not always the case when the entry is taken into account since there is an "entry effect" (Li and Zheng (2005)) representing the effect of the number of potential bidders on bids through the entry stage, which is opposite to the "competition effect." On the other hand, Pinkse and Tan (2005) find that in the APV model without entry, in addition to the "competition effect," another effect called "affiliation effect" has negative effect on bids. Similar to the "winner's curse effect" in the CV model, the winner in the APV model would think that he/she overestimates the common effect which affects his/her valuation. By taking this effect into account before they submit their bids and trying to alleviate this effect, bidders reduce their bids. It is shown in this section that in the model considered in the previous section all these three effects are at work. Before decomposing these three effects we first introduce two straightforward comparative statics.

Proposition 3 *Let C denote the corresponding copula of the joint distribution F and C_{11} denote the second derivative with respect to the first argument. Suppose C_{11} is nonnegative.*

As the entry cost k increases, the screening level v^* increases as well.

Proof. In terms of copula, $F_{N-1|1}(v^*, \ldots, v^*|v^*) = C_1(F_0(v^*), \ldots, F_0(v^*))$. We know that $C_1(F_0(v^*), \ldots, F_0(v^*))$ is increasing in v^* if

$$\sum_i^N C_{1i}(F_0(v^*), \ldots, F_0(v^*)) \geq 0.$$

Since $C_{1i} > 0$ for $i \neq 1$, the inequality follows from $C_{11} \geq 0$. ■

This condition is easily met by many copulas, such as Frank copula, Clayton copula, and Gumbel copula.

Proposition 4 *Suppose C_{11} is nonnegative. As the number of potential bidders increases, the screening level v^* increases as well.*

Proof. From zero profit equation $(III.1)$ in the entry stage, the entry cost can be viewed as a function of v^* and N, $k = k(v^*, N)$. Taking derivative with respect to N, we have $\frac{dk}{dN} = \frac{\partial k}{\partial v^*}\frac{\partial v^*}{\partial N} + \frac{\partial k}{\partial N}$. Since both k and N are exogenous, $\frac{dk}{dN} = 0$, leading to $\frac{\partial v^*}{\partial N} = -\frac{\partial k}{\partial N}/\frac{\partial k}{\partial v^*}$. $\frac{\partial k}{\partial v^*}$ is positive and $\frac{\partial k}{\partial N}$ is negative, which are clearly seen from equation $(III.1)$, therefore the result follows. ■

The intuition of this proposition is, as in Li and Zheng (2005), that since the entry cost remains the same, the average number of actual bidders should not change either. As a result, if the number of potential bidders increases, in order to keep the average number of actual bidders unchanged, v^* has to increase.

The next two propositions show that the total effect of the number of potential bidders on bids is a result of the "competition effect," the "entry effect" and the "affiliation effect" in the model. Before moving to the total effect of N, we first focus on the partial effect of N, that is the effect on bids holding v^* fixed.

Proposition 5 *Fixing entry strategy, that is fixing v^*, the effect of N on bids is ambiguous.*

Proof. From the optimal bidding equation $(III.2)$, $\frac{\partial b}{\partial N}$ is equal to the following,

$$
\begin{aligned}
\frac{\partial b}{\partial N} &= -\frac{\partial L(v|v)}{\partial N}\int_{v^*}^{v} L(u|u)^{-1}\,du - L(v|v)\int_{v^*}^{v}\frac{\partial L(u|u)^{-1}}{\partial N}du + \frac{\partial L(v|v)}{\partial N}(r-v^*) \\
&= \Big\{ L(v|v)\int_{v^*}^{v} R_2(t)\,dt \int_{v^*}^{v} L(u|u)^{-1}\,du - L(v|v)\int_{v^*}^{v}\left(L(u|u)^{-1}\int_{v^*}^{u} R_2(t)\,dt\right) du \\
&\quad - L(v|v)\int_{v^*}^{v} R_2(t)\,dt\,(r-v^*)\Big\} \\
&\quad + \Big\{ L(v|v)\int_{v^*}^{v} \partial\left[(N-1)(R_N(t)-R_2(t))\right]/\partial N dt \int_{v^*}^{v} L(u|u)^{-1}\,du \\
&\quad - L(v|v)\int_{v^*}^{v}\left(L(u|u)^{-1}\int_{v^*}^{u} \partial\left[(N-1)(R_N(t)-R_2(t))\right]/\partial N dt\right) du \\
&\quad - L(v|v)\int_{v^*}^{v} \partial\left[(N-1)(R_N(t)-R_2(t))\right]/\partial N dt\,(r-v^*)\Big\} \\
&\equiv b_{N1} + b_{N2},
\end{aligned}
$$

where $R_N(v) = \frac{F_{2,V_{-1}|v_1}(t,\ldots,t|t)}{F_{V_{-1}|v_1}(t,\ldots,t|t)}$, and $R_2(v) = \frac{F_{2,V_2|v_1}(t|t)}{F_{V_2|v_1}(t|t)}$. The terms in the first brackets, denoted by b_{N1} is positive since $R_2(t) > 0$ and $r < v^*$, while sign of the terms in the second brackets, denoted by b_{N2}, is ambiguous depending on the joint distribution of private values. We illustrate it using two different cases.

Case 1: If $F_{V_{-1}|v_1}(v|v)$ satisfies the strict monotone likelihood ratio property (MLRP) with respect to N, $b_{N2} > 0$. Then $\frac{\partial b}{\partial N} > 0$. (see Matthews (1987)).

Case 2: Like Pinkse and Tan (2005) we consider a conditional independent private values (CIPV) paradigm, a special case of APV model. Suppose bidders' private values $V_1, \ldots, V_N$ are affiliated through a random variable Z, and independent from each other conditional on z. Let $F_{CIPV}(v|z)$ and $f_{CIPV}(v|z)$ denote conditional distribution and density of V_i given $Z = z$, and $G_{CIPV}(z)$ and $g_{CIPV}(z)$ be the distribution and density of Z.

Under this setting, $\frac{F_{2,V_{-1}|v_1}(t,\ldots,t|t)}{F_{N-1|1}(t,\ldots,t|t)}$ and $\frac{F_{2,V_2|v_1}(t|t)}{F_{V_2|v_1}(t|t)}$ can be written as follows

$$\begin{aligned}\frac{F_{2,V_{-1}|v_1}(v,\ldots,v|v)}{F_{V_{-1}|v_1}(v,\ldots,v|v)} &= \frac{\int_{\underline{z}}^{\overline{z}} F_{CIPV}^{N-2}(v|z)\, f_{CIPV}^{2}(v|z)\, g_{CIPV}(z)\, dz}{\int_{\underline{z}}^{\overline{z}} F_{CIPV}^{N-1}(v|z)\, f_{CIPV}(v|z)\, g_{CIPV}(z)\, dz}, \\ \frac{F_{2,V_2|v_1}(v|v)}{F_{V_2|v_1}(v|v)} &= \frac{\int_{\underline{z}}^{\overline{z}} f_{CIPV}^{2}(v|z)\, g_{CIPV}(z)\, dz}{\int_{\underline{z}}^{\overline{z}} F_{CIPV}(v|z)\, f_{CIPV}(v|z)\, g_{CIPV}(z)\, dz}.\end{aligned}$$

Assume that F_{CIPV} satisfies MLRP, the proposition 1 in Pinkse and Tan (2005) shows that $\partial\left[(N-1)\left(R_N(v)-R_2(v)\right)\right]/\partial N < 0$, implying that $b_{N2} < 0$. The desired result follows. ■

Proposition 6 *There is no monotone relationship between bids and the number of potential bidders.*

Proof. From the optimal bidding equation $(III.2)$, the bid is a function of N and v^*, can be written as $b = b(N, v^*)$. Then $\frac{db}{dN} = \frac{\partial b}{\partial N} + \frac{\partial b}{\partial v^*}\frac{\partial v^*}{\partial N}$. Proposition 4 shows that $\frac{\partial v^*}{\partial N} > 0$, and

$$\frac{\partial b}{\partial v^*} = L(v|v)(N-1)R_N(v)\left(r - v^* - 2\int_{v^*}^{v} L(u|u)^{-1}\, du\right) - 2L(v|v) < 0,$$

which implies $\frac{\partial b}{\partial v^*}\frac{\partial v^*}{\partial N} < 0$. On the other hand, $\frac{\partial b}{\partial N}$ has an undetermined sign illustrated in proposition 3. Therefore the result follows. ■

As can be seen from the proofs of Proposition 5 and 6, we have actually decomposed the effect of the number of potential bidders into three parts, denoted by $\frac{db}{dN} = b_{N1} + b_{N2} + \frac{\partial b}{\partial v^*}\frac{\partial v^*}{\partial N}$. $\frac{\partial b}{\partial v^*}\frac{\partial v^*}{\partial N}$ is called the "entry effect" as in Li and Zheng (2005), since the number of potential bidders affects bids through v^*, which controls the equilibrium entry strategy. As the number of potential bidders increases, the "entry effect" makes bidders less aggressive. b_{N1} denotes the "competition effect" which is positive and b_{N2} represents the negative "affiliation effect." In the IPV framework, $b_{N2} = 0$, since $R_N(v) - R_2(v) = 0$.

An Example

In this section we illustrate the nonmonotonicity by a concrete example, which is mainly based on the example in Pinkse and Tan (2005), and quantify the total effect and the three effects. Suppose a common factor z has two possible values $z_1 = .01$ and $z_2 = 2$ with probabilities $g_1 = .15$ and $g_2 = .85$. Bidders' values $v \in [0,1]$ are affiliated through z but independent from each other conditional on z, with conditional distribution,

$$F_0(v|z) = \exp\left(z\left(1 - v^{-\beta}\right)\right), \text{where } \beta = 1,$$

which satisfies MLRP. In addition to this, there is a reserve price $r = .1$ and an entry cost $k = .05$. We set the entry cost small in order to make sure that the entry effect is not too large, which otherwise might dominate the "competition effect" together with the "affiliation effect" and make the bids monotonically decreasing in N. Two bidding functions corresponding to $N = 3$ and $N = 6$ are calculated and presented in Figure 2. It is clearly shown that the bids associated with 6 potential bidders is not always larger than the bids associated with 3 potential bidders. When the private value lies between $[0.4, 0.71]$, the bids with 3 potential bidders are larger. The difference of two bidding functions or the total effect of the change in the number of potential bidders is presented in Figure 3, represented by the solid line. The other three lines represent the "competition effect," the "entry effect," and the "affiliation effect." As is seen and expected, the "competition effect" is always positive, and the "entry effect" and the "affiliation effect" are negative. The "entry effect" is quite small, very close to zero, which is consistent the setup of the small entry cost in the example.

The dominance of the "affiliation effect" in the range of $[0.4, 0.71]$ is the reason which causes the nonmonotonicity of bids and the number of potential bidders.

Conclusion

Within the APV framework with entry, the relationship between with the number of bidders and the bids is not as clear as that within the IPV framework. The effect of the number of potential bidders depends on the relative magnitudes of three effects: "competition effect," "entry effect" and "affiliation effect." For a given data, quantifying these effects calls for the structural analysis to estimate the underlying distributions.

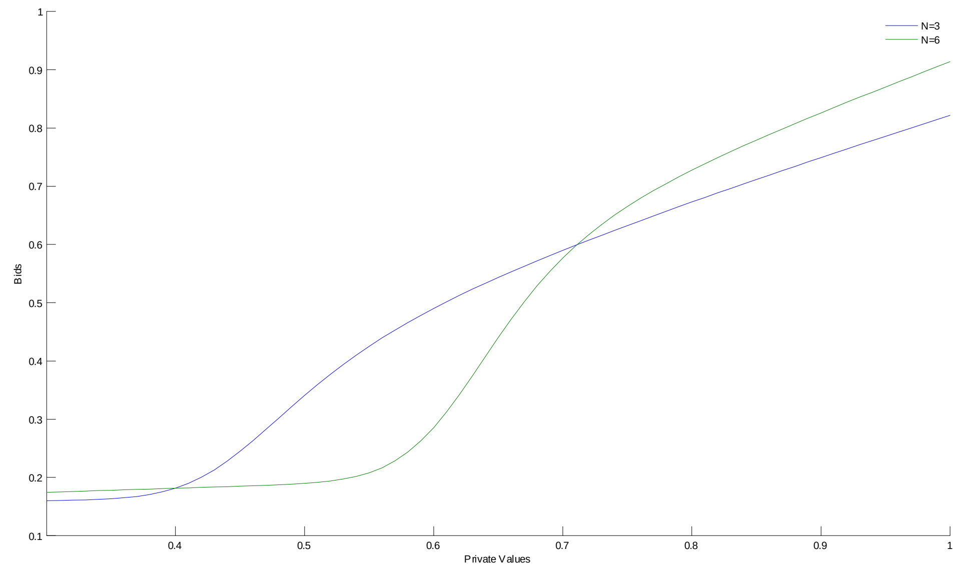

Figure 2. Two Bidding Functions with Different Numbers of Potential Bidders

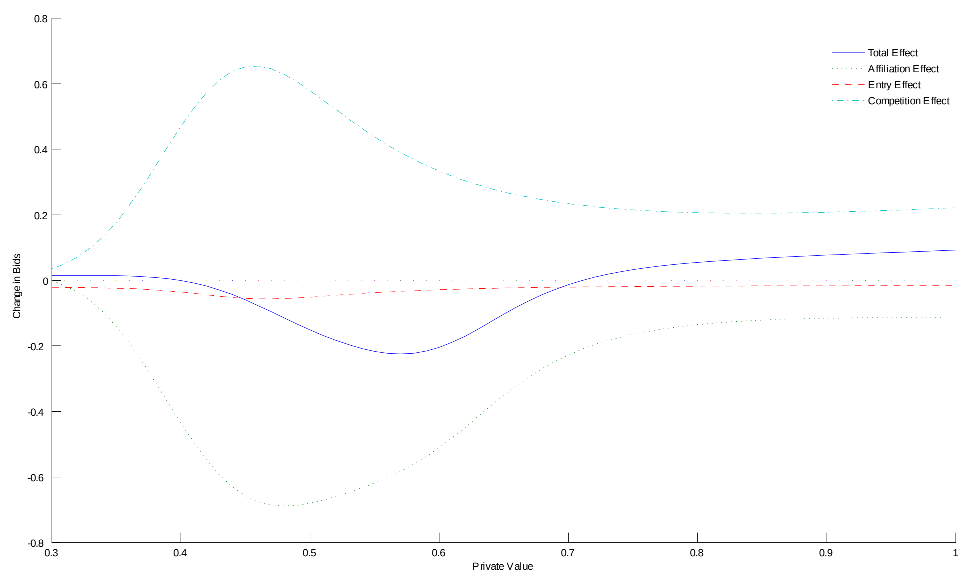

Figure 3. Total Effect and Three Effects of the Number of Potential Bidders

CHAPTER IV

AFFILIATION AND ENTRY WITH HETEROGENEOUS BIDDERS

Introduction

A celebrated result in auction theory is Vickrey's (1961) revenue equivalence theorem, which postulates that all the four auction formats (first-price sealed-bid, second-price sealed bid, English, and Dutch auctions) generate the same average revenue for the seller with symmetric, independent, and risk-neutral bidders. It is a powerful result that offers insight into how auction mechanisms work, and also raises important questions as to how this powerful result can be affected when the standard assumptions are relaxed. A large part of the auction theory has focused on answering these questions. Milgrom and Weber (1982) give revenue ranking with symmetric and affiliated bidders in which the English auction generates highest revenue among the four formats and the second-price auction ranks next; they also establish that with symmetric, affiliated, and risk-averse bidders who have constant absolute risk aversion, the English auction can generate at least as high revenue as the second-price auction. Myerson (1981) derives the optimal auctions with asymmetric bidders, and Maskin and Riley (1984) consider the case with risk-averse bidders. Levin and Smith (1994) extend the revenue equivalence and ranking results from Vickrey (1961) and Milgrom and Weber (1982) to the case with symmetric bidders (independent or affiliated) using mixed entry strategies.

Using timber sale auctions organized by the Oregon Department of Forestry (ODF), this chapter attempts to address a set of questions that include with heterogeneous bidders and when entry is taken into account, how the seller's revenue could change with the extent to which the bidders' private values are affiliated, and whether the reserve price currently set by the ODF is optimal with respect to maximizing the seller's revenue/profit. Moreover, merger and bidder coalition have been an important issue to economists interested in competition policy, yet no empirical work has studied this issue taking into account participation from potential bidders. That we consider heterogeneous bidders is motivated by the evidence from the previous work studying the timber auctions in Oregon (e.g. Brannman and Froeb (2000) using data consisting of oral auctions, and what we find in Chapter II using the same data used in this chapter), that hauling distance plays an important role in bidders' bidding (Brannman and Froeb (2000)) and entry (Chapter II) decisions. This means that bidders are asymmetric and heterogeneous. Furthermore, in chapter II we find small but strongly significant level of affiliation among potential bidders' private information (either private signals or entry costs) and Brannman and Froeb (2000) demonstrate the importance of affiliation to the merger application. Lastly, recent empirical work in auctions in general and in timber auctions in particular (e.g. Athey, Levin and Seira (2004), Bajari and Hortaçsu (2003), Kransnokutskaya and Seim (2007), Li and Zheng (2005, 2007)) has demonstrated that bidders' participation and entry decision is an integrated part of the decision making process that has to be taken into account when studying auctions. In view of these, in this chapter we attempt to study the timber auctions organized by the ODF within a general framework in which potential bidders are affiliated and heterogeneous, and they make entry decisions before submitting bids.

Auction theory offers little guidance in answering these questions for auctions with entry and asymmetric potential bidders with affiliated private values. On the other hand, to gain insight on these questions from an empirical perspective, one needs to observe two states of world, such as pre and post the change of the affiliation level, or pre and post merger. Usually in auction data, as is the case in our data, however, one cannot observe these two states of world. Therefore we adopt a structural approach in our empirical analysis. In particular, we estimate a game-theoretic auction model within the APV paradigm with asymmetric bidders and with entry. We then use the estimated structural parameters to conduct counterfactual analyses of our interest. We find that for a representative auction, the optimal reserve price should be much larger than the current one. In evaluating the merger effects we find that the merged bidder is very likely to participate in the auction regardless of the merging bidders' entry behavior and that the merger has little impact on other bidders' entry behavior. While the overall merger effect on the seller's revenue is not theoretically clear in our model, we find that at the current reserve prices and dependence levels, merger is beneficial to the seller, but it could mean a loss for the seller for some values of reserve price and dependence levels.

We develop an entry and bidding model for asymmetric bidders within the APV paradigm. Because of the general framework we adopt, the answers to the aforementioned questions of our interest depend on the interactions of affiliation, entry, and asymmetry, as well as competition. As is well known, the optimal reserve price in the symmetric IPV model without entry does not depend on the number of potential bidders. This result can change if entry is introduced (see, e.g., Levin and Smith (1994), Samuelson (1985), Li and Zheng (2007)), or if bidders have affiliated private values (Levin and Smith (1996), Li, Perrigne, and Vuong (2003)). In our case, on the other hand, assessing the optimal reserve price is complicated further by the APV framework with entry and asymmetric bidders. Therefore we can only address this issue through a counterfactual analysis using the structural estimates. Furthermore, while the effect of the number of potential bidders on winning bids and seller's revenue is clear in an IPV model with symmetric bidders and without entry, it becomes less clear in a more general setting, such as the IPV model with entry and symmetric bidders (Li and Zheng (2005, 2007)), and the APV model without entry (Pinkse and Tan (2005)). In Chapter III we find that in the symmetric APV model with entry, three effects, namely "affiliation effect," "entry effect" and "competition effect" interact together resulting in an ambiguous total effect of the number of potential bidders on bids. While we expect these three effects to remain in the APV framework with entry and asymmetric bidders, it becomes challenging to pinpoint them with asymmetric bidders. Since the effect of merger is closely related to how the seller's revenue changes with the set of potential bidders, i.e, not only the number of potential bidders, but also the identity of potential bidders when they are heterogeneous, and at the same time, theory does not yield good predictions, we rely on the structural analysis to gain insight on this issue.

Asymmetry is an indispensable element of the model given the asymmetric feature of the data. The analysis of the model, however, is complicated from both theoretical and econometric viewpoints due to the introduction of asymmetry. Because of the complexity of the model, and in particular, because that there is no closed form solution for the bidding function, we have to rely on some numerical approximation procedure. Moreover, while the structural analysis of auctions with asymmetric bidders has focused on the case with two types of bidders (Athey, Levin, and Seira (2004), Campo, Perrigne and Vuong (2003), and Kransnokutskaya and Seim (2007)), our model allows for all potential bidders to be different from each other, motivated by the fact that in our data, asymmetry is driven by the difference among bidders' hauling distances.

As mentioned in Chapter I, this chapter makes contribution to the growing literature of the structural analysis of auction data since Paarsch (1992). While the structural approach has been extended to the APV paradigm, this chapter is the first one in estimating a structural model within the APV paradigm and taking into account entry. On the other hand, while the recent work has started to pay attention to the problem of participation and entry, all the work has focused on the IPV framework with Bajari and Hortaçsu (2003) being an exception as they consider a common value (CV) model. In contrast, this chapter considers the entry problem within the APV paradigm, a more general framework.

Our empirical analysis of the timber auctions and the resulting findings offer new insight on timber sale auctions and policy related issues. While most of the empirical analysis of timber sale auctions is based on the IPV model without entry (e.g. Paarsch (1997), Baldwin, Marshall and Richard (1997), Haile (2001), Haile and Tamer (2003), Li and Perrigne (2003)) or the IPV model with entry (Athey, Levin and Seira (2004), Li and Zheng (2007)), ours is based on the APV model with entry and heterogeneous bidders. As a result, our findings can be more robust, and also can be more useful for addressing the policy-related issues as our analysis takes into account the affiliation effect, the entry effect, and the asymmetry effect. Moreover and probably more interestingly, we study the merger effect within the asymmetric APV framework with entry, and offer new insight into how merger as well as other issues related to competition policy can be affected by complications arising from affiliation, entry, and asymmetry, and how they can be addressed within a unified framework as adopted in this chapter.[1]

This chapter is organized as follows. In Section 2 we propose the asymmetric APV model with entry. Section 3 is devoted to the structural analysis of the data, and Section 4 conducts a set of counterfactual analyses studying the effects of reserve prices, affiliation

[1] It is worth noting that to the best of our knowledge, Brannman and Froeb (2000), considering oral timber auctions within an IPV paradigm without entry, is the only paper assessing the merger effect in auctions using the structural approach.

levels, and mergers. Section 5 concludes.

The Model

In this section we propose a theoretical two-stage model to characterize the timber sales, extending the models in Athey, Levin, and Seira (2004) and Krasnokutskaya and Seim (2007) with two groups of bidders within the IPV paradigm, and in Li and Zheng (2005) within the symmetric IPV paradigm, to the APV paradigm that allows potential bidders to be different from each other. Specifically, motivated by the finding of Brannman and Froeb (2000) that the hauling distance plays a significant role in bidders' bidding decision in oral timber auctions in Oregon, and the finding of Chapter II using the same data studied in this chapter that the hauling distance is important in potential bidders' entry decision and potential bidders are affiliated through their private information (either private values or entry costs), we consider a first-price sealed-bid auction within the APV paradigm with a public reserve price, entry, and asymmetric bidders.

In the model, a single object is auctioned off to N heterogenous and risk-neutral potential bidders, who are affiliated in their private information. Bidder i has a private entry cost k_i, including the cost of obtaining private information and bid preparation, and does not obtain his private value v_i until he participates in the auction. We allow both private values and entry costs to be affiliated across bidders, that is $V_1, \ldots, V_N$ and $K_1, \ldots, K_N$ jointly follow a distribution $F(\cdot, \ldots, \cdot)$ with the support of $[\underline{v} = r, \overline{v}]^N$, and a distribution $G(\cdot, \ldots, \cdot)$ with the support of $\left[\underline{k}, \overline{k}\right]^N$, respectively, where r is the public reserve price of the auction. Affiliation is a terminology describing the positive dependence among random variables, which was first introduced into the study of auctions by Milgrom and Weber (1982). In this chapter, it shares the same definition given in Chapter II.

We denote the marginal distribution and density of bidder i's private value by $F_i(\cdot)$ and $f_i(\cdot)$ and marginal distribution and density of bidder i's entry cost by $G_i(\cdot)$ and $g_i(\cdot)$, respectively, and assume that $f_i(\cdot)$ is continuously differentiable and bounded away from zero on $[\underline{v} = r, \overline{v}]$. The subscript of distribution function implies that all potential bidders are of different types. This assumption is motivated by the fact that heterogeneity among bidders arises from different hauling distances in our data.

The model differs the one considered in Chapter III in two ways. First it allows for asymmetric bidders. Second, the timing of acquiring private values is different.

Bidding Strategy

Because the entry decision is based on the pre-entry expected profit, which depends on the bidding strategy of bidder i, we first describe the bidding strategy of bidder i. We assume that bidder i knows the number of the actual competitors in the bidding stage,[2] and thus bidder i's bidding strategy is determined by the first order condition of the following maximization problem,

$$\max (v_i - b_i) \Pr (B_j < b_i | v_i; a_{-i}),$$

where B_j denotes the maximum bid among other actual bidders and

$$a_{-i} \in A_{-i} = \{(a_1, \ldots, a_N) | a_j = 0 \text{ or } 1, j = 1, \ldots N, j \neq i\}$$

is one possibility of the 2^{N-1} combinations of entry decisions of $N-1$ other potential bidders. Denote the number of actual bidders of the combination a_{-i} by $n_{a_{-i}}$. As usual we

[2]When the lower support of private value is below the reserve price, bidder i only knows the active bidders who participate in the auction but not actual bidders who submit bids. In our case, the number of active bidders is equal to the number of actual bidders, since the lower support of private value is assumed to be just the reserve price.

consider a continuously differentiable and strictly increasing bidding strategy, $b_i = s_i(v_i)$, therefore the first order condition is

$$-F_{V_{-i}|v_i}\left(s_j^{-1}(b_i), j \neq i|v_i\right) + (v_i - b_i)\sum_{j\neq i}^{n_{a_{-i}}} \frac{\partial F_{V_{-i}|v_i}\left(s_j^{-1}(b_i), j \neq i|v_i\right)}{\partial v_j}\frac{\partial s_j^{-1}(b_i)}{\partial b_i} = 0, \tag{IV.1}$$

where $F_{V_{-i}|v_i}$ denotes the joint distribution of $V_j, j \neq i$ conditional on $V_i = v_i$ and $s_i^{-1}(\cdot)$ is the inverse function of the bidding function of bidder i. A set of equation $(IV.1)$ for $i = 1, \ldots, n$ form a system of differential equations characterizing the equilibrium bids for all n actual bidders. We denote the post-entry profit of bidder i by $\pi_i(v_i|a_{-i})$.

Entry Decision

In the initial participation stage, each potential bidder i only knows his own entry cost, joint distributions of entry costs and private values. Therefore the entry decision of bidder i is determined by his pre-entry expected profit from participation, Π_i. Specifically, he participates in the auction only if his entry cost is less then Π_i. Let p_i denote the entry probability of bidder i, respectively. The ex ante expected profit Π_i is given by

$$\Pi_i = \sum_{a_{-i}\in A_{-i}} \int_{\underline{v}}^{\overline{v}} \pi_i(v_i|a_{-i})\, dF_i(v_i) \Pr(a_{-i}|a_i = 1), \tag{IV.2}$$

where $\Pr(a_{-i}|a_i = 1)$ is a function of $p = (p_1, \ldots, p_N)$. As a result, the pre-entry expected profit is the sum of 2^{N-1} products of the post-entry profits and corresponding probabilities with the unknown private value integrated out. On the other hand, the probability of entry is given by $p_i = \Pr(K_i < \Pi_i) = G_i(\Pi_i)$.

Note that although the number of potential bidders does not affect the bidding strategy in the bidding stage, it affects the number and the identities of actual bidders, which in turn have impact on the bidding strategy.

Characterization of the Equilibrium

Existence and uniqueness of the Bayesian Nash equilibrium with asymmetric bidders has been a challenging problem studied in the recent auction theory literature. See, e.g. Lebrun (1999, 2006) and Maskin and Riley (2000, 2003) within the IPV framework, Lizzeri and Persico (2000) within the APV framework and two types of bidders. The analysis of our model is further complicated by the introduction of affiliation and entry, as well as that we allow all potential bidders to be different from each other. To address the issue of existence and uniqueness in our case, we look at the case where the joint distribution of bidders' private values is characterized by the family of Archimedean copulas. For the copula concept and the characterization of the Archimedean copulas, see Nelsen (1999). Copula can provide a flexible way of modeling joint dependence of multivariate variables using the marginal distributions.

Specifically, by Sklar's theorem (Sklar (1973)), for a joint distribution $F(x_1, \ldots, x_N)$, there is a unique copula C, such that $C(F_1(x_1), \ldots, F_N(x_N)) = F(x_1, \ldots, x_N)$. For the Archimedean copulas, the copula C can be expressed as $C(u_1, \ldots, u_n) = \phi^{[-1]}(\phi(u_1) + \cdots + \phi(u_n))$, where ϕ is a generator of the copula and is a decreasing and convex function, and $\phi^{[-1]}$ denotes the pseudo-inverse of ϕ.[3] The family of Archimedean copulas include a wide range of copulas. For example, the generators $\phi(u) = \frac{1}{q}(u^{-q} - 1)$, $\phi(u) = (-\ln(u))^q$, and $\phi(u) = \ln\left(\frac{\exp(qu)-1}{\exp(q)-1}\right)$ correspond to the widely used Clayton copula,

[3] ϕ is a decreasing convex function from $[0,1]$ to $(0,\infty]$ with $\phi(1) = 0$. $\phi^{[-1]}$ is defined as

$$\phi^{[-1]}(u) = \begin{cases} \phi^{-1}(u), & 0 \le u \le \phi(0), \\ 0, & \phi(0) \le u \le \infty. \end{cases}$$

Gumbel copula, and Frank copula, respectively. Since we consider a differentiable bidding strategy, we have to confine ourself to the strict generator, that is $\phi^{[-1]} = \phi^{-1}$. Since $C_i(F_1(x_1), \ldots, F_N(x_N)) = F_{X_{-i}|x_i}(x_1, \ldots, x_N)$ (e.g. Li, Paarsch, and Hubbard (2007)), the first order condition $(IV.1)$ determining the equilibrium bids can be written as follows

$$\frac{ds_i^{-1}(b)}{db} = \frac{\phi^{-1\prime}\left(\sum_k \phi\left(F_k\left(s_k^{-1}(b)\right)\right)\right)}{\left(n_{a_{-i}}-1\right)\phi'\left(F_i\left(s_i^{-1}(b)\right)\right) f_i\left(s_i^{-1}(b)\right)\phi^{-1\prime\prime}\left(\sum_k \phi\left(F_k\left(s_k^{-1}(b)\right)\right)\right)} \left[\sum_{k\neq i}\frac{1}{s_k^{-1}(b)-b} - \frac{n_{a_{-i}}-2}{s_i^{-1}(b)-b}\right] \tag{IV.3}$$

and $\Pr(a_{-i}; p_1, \ldots, p_N | a_i = 1)$, for example, for the case that given the participation of bidder i, bidder 1 up to bidder $i-1$ participate in the auction while bidder $i+1$ up to bidder N do not, can be expressed as

$$\begin{aligned} & \Pr(a_1 = \cdots a_{i-1} = 1, a_{i+1} = \cdots a_N = 0 | a_i = 1) \\ = & \frac{\Pr(a_1 = \cdots a_i = 1, a_{i+1} = \cdots a_N = 0)}{\Pr(a_i = 1)} \end{aligned} \tag{IV.4}$$

where

$$\begin{aligned} & \Pr(a_1 = \cdots a_i = 1, a_{i+1} = \cdots a_N = 0) \\ = & C(p_1, \ldots, p_i, 1, \ldots, 1; q_k) - \sum_{i+1 \leq j \leq N} C(p_1, \ldots, p_i, p_j, 1, \ldots, 1; q_k) \\ & \cdots + (-1)^{N-i} C(p_1, \ldots, p_N; q_k), \end{aligned}$$

and $\Pr(a_i = 1) = C(1, \ldots, 1, p_i, 1, \ldots, 1; q_k)$.

Equilibrium of the model consists of two parts, entry equilibrium and bidding equilibrium. Based on the choice of Archimedean copulas for the joint distribution of private values, the existence of the equilibrium is guaranteed. Moreover, with some additional conditions, the bidding equilibrium is unique. The next paragraph describes the equilibrium formally.

Proposition 7 (Characterization of Equilibrium) *Assume (a) the marginal distribution of entry cost of bidder i, G_i is continuous over $\left[\underline{k}, \overline{k}\right]$ for all i; (b) marginal distribution of private value of bidder i is differentiable over $(\underline{v}, \overline{v}]$ with a derivative f_i locally bounded away from zero over this interval for all i; (c) joint distribution of private values follows an Archimedean Copula.*

i. Bidding Equilibrium *In the bidding equilibrium, bidder i adopts a continuously differentiable and strictly increasing bidding function $b_i = s_i(v)$ over $(\underline{v}, \overline{v}]$. The inverse functions of s_i for all i, $s_1^{-1}, \ldots, s_n^{-1}$ are the solution of the system of differential equations (IV.3) with boundary conditions (IV.5) and (IV.6) :*

$$s_i^{-1}(\underline{v}) = \underline{v} \tag{IV.5}$$

$$s_i^{-1}(\eta) = \overline{v}. \tag{IV.6}$$

for some η.

ii. Uniqueness of Bidding Equilibrium *Moreover, if $F_i(\underline{v}) > 0$ and $\frac{\phi^{-1\prime}(u)}{\phi^{-1\prime\prime}(u)}$ is decreasing in u, then the bidding equilibrium is unique.*

iii. Entry Equilibrium *In the entry equilibrium, bidder i chooses to participate in the auction if his entry cost is less than the threshold $\Pi_i(p)$ and stay out otherwise, where $p = (p_1, \ldots p_N)$ and p_i is the entry probability of bidder i and is determined by*

$$p_i = G_i(\Pi_i(p)). \tag{IV.7}$$

As is seen here, the existence of the entry equilibrium is equivalent to the existence of the entry probability p_i, given by the equation $(IV.7)$. Since Π_i is continuous in p_i and thus G_i is continuous over $[0,1]$, there exists a solution p_i of equation $(IV.7)$, according to Kakutani's fixed point theorem (Kakutani (1941)). To show the uniqueness of the bidding equilibrium is to show that there is a unique η such that $s_i^{-1}(\eta) = \overline{v}$. Then starting from η, according to Lipschitz uniqueness theorem, s_i^{-1} is unique over $(\underline{v}, \eta]$. Note that Clayton copula satisfies the condition for uniqueness that $\frac{\phi^{-1\prime}(u)}{\phi^{-1\prime\prime}(u)}$ is decreasing in u. The formal proofs are provided in Appendix A.

The Structural Analysis

We estimate the model proposed in the last section using the timber sales data. Our objective is to recover the underlying joint distributions of private values and entry costs using observed bids and the number of actual bidders. The structural inference in our case is complicated because of the generality of our model that accounts for affiliation, asymmetry, and entry. Our approach circumvents the complications arising from the estimation of our model and makes the structural inference tractable. First, to model the affiliation in a flexible way, we adopt the copula approach in modeling the joint distribution of private values and the joint distribution of entry costs.[4] Second, since we allow bidders to be asymmetric, the system of differential equations consisting of equation $(IV.3)$ that characterizes bidders' Bayesian Nash equilibrium strategies does not yield closed-form solutions. To address this problem we adopt a numerical method based on Marshall, Meurer,

[4]Li, Paarsch, and Hubbard (2007) use the copula approach to model affiliation within the symmetric APV framework without entry and propose a semiparametric estimation method.

Richard, and Stromquist (1994) and Gayle (2004). Third, because of the various covariates we try to control for and the relatively small size of the data set, the nonparametric method does not work well here. Therefore, we adopt a fully parametric approach.

Specifications

We adopt the Clayton copula to model the joint distributions of both private values and entry costs. With the generator of Clayton copula given above, the joint distribution of private value is specified as $F(v_1,\ldots v_n) = \left(\sum_i F_i(v_1)^{-q_v} - n + 1\right)^{-1/q_v}$, and the joint distribution of entry costs is specified as $G(k_1,\ldots,k_n) = \left(\sum_i G_i(v_1)^{-q_k} - n + 1\right)^{-1/q_k}$, where q_v and q_k are dependence parameters and F_i and G_i are the marginal distributions of private value and entry cost, which are specified as truncated exponential distributions given as follows, $F_{V_{\ell i}}(v|\mathbf{x}_{\ell i};\beta) = \frac{\frac{1}{\lambda_{v_{\ell i}}}\exp\left(-\frac{1}{\lambda_{v_{\ell i}}}\underline{v}\right) - \frac{1}{\lambda_{v_{\ell i}}}\exp\left(-\frac{1}{\lambda_{v_{\ell i}}}v\right)}{\frac{1}{\lambda_{v_{\ell i}}}\exp\left(-\frac{1}{\lambda_{v_{\ell i}}}\underline{v}\right) - \frac{1}{\lambda_{v_{\ell i}}}\exp\left(-\frac{1}{\lambda_{v_{\ell i}}}\overline{v}\right)}$, $G_{K_{\ell i}}(k|\mathbf{x}_{\ell i};\beta) = \frac{\frac{1}{\lambda_{k_{\ell i}}}\exp\left(-\frac{1}{\lambda_{k_{\ell i}}}\underline{k}\right) - \frac{1}{\lambda_{k_{\ell i}}}\exp\left(-\frac{1}{\lambda_{k_{\ell i}}}k\right)}{\frac{1}{\lambda_{k_{\ell i}}}\exp\left(-\frac{1}{\lambda_{k_{\ell i}}}\underline{k}\right) - \frac{1}{\lambda_{k_{\ell i}}}\exp\left(-\frac{1}{\lambda_{k_{\ell i}}}\overline{k}\right)}$ for bidder i of the ℓ-th auction, $\ell = 1,\ldots,L$, where L is the number of auctions, $\lambda_{v_{\ell i}}$ and $\lambda_{k_{\ell i}}$ are the private value and entry cost means and equal $\exp(\beta\mathbf{x}_{\ell i})$ and $\exp(\alpha\mathbf{x}_{\ell i})$, respectively, and $\mathbf{x}_{\ell i}$ is a vector of covariates that are auction specific or bidder specific, and in our case includes variables such as hauling distance, volume, duration, grade, and DBH.[5] In practice, $\underline{v}$ is equal to the reserve price of ℓ-th auction, $\overline{v}$ is equal to \$1500/MBF, the lower bound of entry cost is equal to zero and the upper bound $\overline{k}$ is \$940/MBF, an arbitrarily large number. We then model the joint distributions of private values and entry costs in auction ℓ as Clayton copula with different dependence parameters q_v and q_k. The use of the Clayton copula offers several advantages. First, it guarantees the existence and uniqueness of the equilibrium as discussed in Section 3.3. Second, it preserves the same dependence structure when the number of potential bidders changes. Third, it is

[5]Here we do not introduce unobserved auction heterogeneity into the model, as in chapter II we show that it does not have a significant effect in bidders' entry decisions.

relatively easy to draw dependent data from the Clayton copula, as it has a closed form that can be used to draw data recursively. Lastly, since q is the only parameter that measures the dependence, we can easily evaluate the impact of the dependence level on the end outcomes of an auction by changing the value of q.

Note that in these specifications, the asymmetry across potential bidders is captured by the inclusion of the hauling distance variable in $\mathbf{x}_{\ell i}$, while both α and β are kept constant across different bidders. This enables us to estimate a relatively parsimonious structural model and at the same time control for the asymmetry.

Estimation Method

Because of the complexity of our structural model, we employ the indirect inference method to estimate the model. Initially proposed in the nonlinear time series context by Smith (1993) and developed further by Gourieroux, Monfort, and Renault (1993) and Gallant and Tauchen (1996), the indirect inference method is simulation based and obtains the estimates of parameters by minimizing a measure of distance between the estimates for the auxiliary parameters of an auxiliary model using the original data and simulated data. More specifically, let θ denote the vector of parameters of interest, γ be the parameters of the auxiliary model, $\widehat{\gamma}_T$ and $\widehat{\gamma}_{ST}^{(p)}(\theta)$ be the estimates of the auxiliary model using the original data and the p-th simulated data out of P sets of simulated data from the model given a specific θ, respectively. Then the estimator of θ, denoted by $\widehat{\theta}_{ST}$, is defined as

$$\widehat{\theta}_{ST} = \arg\min_{\theta} \left[\widehat{\gamma}_T - \frac{1}{P} \sum_{p=1}^{P} \widehat{\gamma}_{ST}^{(p)}(\theta) \right]' \Omega \left[\widehat{\gamma}_T - \frac{1}{P} \sum_{P=1}^{P} \widehat{\gamma}_{ST}^{(p)}(\theta) \right], \tag{IV.8}$$

where Ω is a symmetric semi-positive definite matrix. Therefore to implement the indirect inference method, we have to draw data from the model for a given θ, which involves calculating the equilibrium bids and the thresholds of the entry costs. Basically we use numerical approximation method similar as the ones in Marshall, Meurer, Richard, and Stromquist (1994) and Gayle (2004) to find the equilibrium bids and iteration to find the equilibrium entry probabilities, both of which are illustrated in detail in Appendix B.

Though it avoids calculating the likelihood function or moment conditions, indirect inference requires to solve the equilibrium bids and equilibrium entry probabilities. When the number of potential bidders is large, it involves intensive computation. As is seen from equation $(IV.2)$, the pre-entry expected profit is a sum of 2^{N-1} terms, each of which requires to calculate equilibrium bids numerically. To address this issue, we only use a small portion of the data, that is we use the observations whose number of potential bidders are no more than 5, which makes the sample consist of 81 sales and 245 observed bids. The summary statistics are provided in Table (3). We propose a two step indirect inference method. In the first step, we apply the indirect inference method to the bidding stage only, which gives us the estimates of the distribution of private values. With these, we apply the indirect inference to the entry stage again and obtain the distribution of entry costs.

The auxiliary model, which is usually simpler than the original model and easier to estimate as well, plays an important role in the indirect inference method. In this chapter, following the idea in Li (2005) we employ a relatively simple and easy-to-estimate auxiliary model to make the implementation tractable and the inference feasible. Specifically, in the first step the auxiliary model is a linear regression of the observed bids and it is a Poisson regression of the number of actual bidders in the second step, which are described as follows

$$
\begin{aligned}
b_\ell &= \gamma_{10} + \sum_{h=1}^{H} X_{h\ell}\gamma_{11h} + \sum_{h=1}^{H} X_{h\ell}^2\gamma_{12h} + \cdots + \sum_{h=1}^{H} X_{h\ell}^m\gamma_{1mh} + \varepsilon_1, \\
\Pr(n_\ell = k) &= \frac{\exp(-\lambda_\ell)\,\lambda_\ell^k}{k!}, \lambda_\ell = \gamma_{20} + \sum_{h=1}^{H} X_{h\ell}\gamma_{21h} + \sum_{h=1}^{H} X_{h\ell}^2\gamma_{22h} + \cdots + \sum_{h=1}^{H} X_{h\ell}^m\gamma_{2mh},
\end{aligned}
$$

where b_ℓ is the average bid of auction ℓ, and $X_{h\ell}$, $h = 1, ...H$, denote the vector of auction-specific covariates of auction ℓ and the average of bidder-specific covariates, and H is the number of such covariates, which is 6 in our case. $m = 2$ makes our model over-identified.

An issue arising from the implementation of the indirect inference method is the discontinuity of the objective function of equation ($IV.8$) because of the discrete dependent variable (the number of actual bidders) in the auxiliary model that makes gradient-based optimization algorithm invalid. We address this issue by using simplex, a nongradient-based algorithm. Alternatively, one can follow Keane and Smith (2003) to smooth the objective function using a logistic kernel.

Estimation Results

Table 4 reports the estimation results. For the (marginal) private value distribution, all the estimated parameters are significant at the 5% level, and also have the expected signs.[6] Of particular interest is the parameter of the hauling distance variable, which is used to control for heterogeneity across bidders. The negative estimate means that the longer the hauling distance is, the less is the private value mean. Furthermore, the average marginal effect of the hauling distance variable is about -1.73, meaning that one mile increase in the distance would reduce the private value mean by \$1.73/MBF while everything else is fixed. In other words, one mile increase in the distance could reduce the private value mean by

[6]Standard errors are obtained through bootstrap.

0.21%. Another parameter of particular interest is the dependence parameter q_v in private values, which turns out to be relatively small ($q_v = 0.0983$) but significant. To get some idea of how large the dependence is with $q_v = 0.0983$, we use a measure called Kendall's τ (Nelsen (1999)), which is used to measure the concordance of random variables. Concordance is not really the same concept as affiliation, but measures the positive dependence in a similar way. Kendall's τ is defined as the probability of concordance minus the probability of discordance for the case with two random variables X and Y.

$$\tau_{X,Y} = \Pr\left[(X_1 - X_2)(Y_1 - Y_2) > 0\right] - \Pr\left[(X_1 - X_2)(Y_1 - Y_2) < 0\right].$$

For the Clayton copula, when the dimension of variables is equal to 2, the dependence parameter and Kendall's τ are bundled together by equation $\tau = q/(q+2)$. Therefore $q_v =$ 0..0983 implies that the event of any two bidders' private values being concordant is about 4.7% more likely than the event of being discordant.

Two points in the estimates in the distribution of entry costs are worth noting. First, the hauling distance variable is significant and positive in the entry cost distribution and its marginal effect is 1.6. Second, the dependence level among the entry costs is 0.5122, implying a Kendall's τ of 0.2. This indicates that the affiliation among the entry behavior is mainly driven by the affiliation among the entry costs.

Model Fit

In this subsection, we assess the model fit from the structural estimation. Because our model yields two main outcomes, namely, the number of actual bidders and bids submitted by the actual bidders, we assess the model fit through these two outcomes. Specifically, we use the structural estimates to conduct 1000 simulations, each of which contains 81 auctions. We pool all simulated bids and the simulated number of actual bidders and compare two histograms of the simulated data with those of our sample. The histograms are provided in Figure 4, from which we can see that the distributions of the simulated data match the distributions of the original data quite well. The means of the simulated bids and number of actual bidders are \$349.30/MBF and 3.37, respectively, which are quite close to \$376.69/MBF and 3.02 of the sample means.[7]

Counterfactual Analyses

With the estimated structural parameters we can now answer the questions put forward in the introduction section empirically. We focus on both end outcomes, namely, the number of actual bidders, and winning bids (or seller's revenue). We conduct counterfactual analyses on the 99th auction of our data, which is not included in the estimation sample. We use this auction as a representative auction, as the values of covariates of this auction are close to the average values of all auctions in our data set. In particular, the number of potential bidders in this auction is 7, about the same as the average number of potential bidders in the data. In doing so we assume that the estimates derived from a subset of the data fit the whole data.

The seller's expected unit revenue is given as follows

$$\begin{aligned} E(w) &= E(w|w>0)\Pr(w>0) + E(w|w<0)\Pr(w<0) \\ &= E(w|w>0)\Pr(w>0) + v_0 \Pr(w<0), \end{aligned}$$

[7] Alternatively we could compare the estimated private value mean and the true private value mean. The latter, however, is not observed. Through our private communications with a staff at the ODF, the average private value is within the range of \$800/MBF and \$1000/MBF. The estimated average private value from our estimation is about \$975.64/MBF, well falling inside this range.

where w denotes the winning bid and v_0 is the valuation of the timber to the seller, and the second equality follows the assumption that if the timber is not sold successfully then the seller gets his own value. In the following analyses we assume $v_0 = 0$, thus the expected revenue is equivalent to the expected profit.

Effects of Reserve Price and Dependence Level

Intuitively the effect of the reserve price can be seen from two aspects. On one hand, a higher reserve price is associated with a lower ex ante expected profit, i.e., a lower cut-off entry cost according to equation $(IV.2)$ as it narrows the integration range, and thus fewer participating bidders and lower probability of being sold, which may lower bids in our APV model with asymmetric bidders. On the other hand, a higher reserve price raises the lowest acceptable bids and of course makes bidders bid higher. Our counterfactuals shown in Figure 5 confirm such trade-off. The three panels in Figure 5 show how the reserve price affects the number of actual bidders, the probability of being sold and the seller's revenue. The number of actual bidders is decreasing in the reserve price as is shown in the first panel. The average number of participating bidders drops dramatically from 5.52 to 1.25 when the reserve price is raised from \$293.4/MBF to \$1320.4/MBF. The probability of being sold is negatively related to the reserve price when the reserve price is less than \$1000/MBF. The change in the winning bid is the final result of all effects associated with change in the reserve price. As is seen in the last panel, the optimal reserve price is around \$704/MBF, which is more than twice as large as the current reserve price. This implies that when the reserve price is below \$704/MBF the positive effect on the winning bid outweighs the negative effect associated with the lower probability of being sold.

The APV model we estimated also enables us to quantify the effects of the dependence level among bidders. To this end, we change the values of the dependence parameters of both private values and entry costs while keeping other parameters fixed. We are able to conduct such analysis as the change of the dependence parameter does not affect other parameters, which appear only in the marginal distribution of private values or entry costs. As in the analysis of the effects of the reserve price, we are interested in three effects of the dependence parameters. Results are provided in Figure 6 and Figure 7, in which the dependence levels are represented by Kendall's τ's, which are transformed from the dependence parameters q_v and q_k.[8] We use τ_v and τ_k to denote Kendall's τ's corresponding to q_v and q_k. The probability of being sold remains at 1 as dependence levels change which implies that the dependence levels do not affect the number of participating bidders as much as the reserve price does. The number of participating bidders is slightly negatively related with the dependence level of private values. The average number of participating bidders associated with a high dependence level, $\tau_v = 0.6$, is only about a half less than the one in the almost independence case. On the other hand, the number of participating bidders is not monotone with respect to the dependence level of entry costs, as is seen from the first panel of Figure 7. One thing worth noting is that a relatively large increase in the number of participating bidders is associated with the change of τ_k from almost 0 to 0.1. This is consistent with the intuition that affiliated bidders should have more similar entry behavior, which is also the idea of the affiliation test in Chapter II. Moreover, two dependence parameters have different effects on the winning bids. Unlike the fact that the dependence level of entry costs is strongly positively related with the winning bids, the dependence level of private values does not have a significant relationship with the winning bids.

[8]For the Clayton copula, $\tau = \frac{2^d}{2^d-1}\left[\prod_{i=1}^{d} \frac{1+(i-1)q}{2+(d-i)q} - \frac{1}{2^d}\right]$, where d is the dimension of the random variables, which is equal to 7 in our case.

Effect of Bidding Coalition or Merger

Our asymmetric model is ideal for evaluating the merger effects on the end outcomes of auctions, because asymmetry is intrinsically involved in the merger. The merged bidder will be different from other bidders even if they are symmetric pre-merger. For the purpose of measuring the effects of bidding coalition or merger, we conduct two hypothetical mergers, the "best" and "worst" mergers. In the "best" merger two least competitive bidders are merged, which means that two bidders with the longest hauling distances are merged in our case. On the contrary, in the "worst" merger two bidders with the shortest hauling distances are merged into one entity. On the other hand, according to the pre-merger entry behavior of the merging bidders, mergers can be divided into three groups: mergers between two participating bidders, mergers between two non-participating bidders, and mergers between one participating bidder and one non-participating bidder. It is obvious that most "worst" mergers belong to the first group while most "best" mergers belong to the second group, because strong bidders are more likely to participate in the auction than weak bidders do. We therefore focus on the merger effects of the first group "worst" mergers and the second group "best" mergers. These two polar cases should shed light on other mergers.

The private value V_m and the entry cost K_m of the merged bidder are defined as $V_m = \max(V_1, V_2)$ and $K_m = \min(K_1, K_2)$, respectively, assuming that bidder 1 and bidder 2 are merged without loss of generality where m denotes the merged bidder. Therefore the marginal distributions of private values and entry costs of the merged bidder are defined as $F_m(v_m) = C(F_1(v_m), F_2(v_m); q_v)$, and $G_m(k_m) = \widetilde{C}(1 - G_1(k_m), 1 - G_2(k_m); q_k)$ in terms of copula, where $\widetilde{C}$ is the survival copula associated with C. In practice we simulate 3000 auctions based on covariates of the representative auction and conduct "best" and "worst" mergers for these 3000 auctions and compare the pre-merger and post-merger end outcomes.

When it comes to the merge effects on entry, we are not only interested in the merged bidder's entry behavior, but also concerned with whether a merger induces non-participating bidders into the auction or crowds participating bidders out of the auction. The first panels of Figure 8 to Figure 13 demonstrate the interactions between entry behavior of bidders and the reserve price and two dependence levels in both "worst" and "best" mergers. The blue solid lines represent the entry probabilities of the merged bidders. The red dot lines are the probabilities of staying out of bidders who participated pre-merger and the probabilities of participation of bidders who did not participated pre-merger are represented by the green dash lines. As is seen from Figure 8, Figure 9, and Figure 10, in the "best" mergers the merged bidder always participates in the auction for any values of reserve prices and dependence levels, while in the "worst" mergers, the merged bidders participates in the auction with a large probability of more than 0.80 in most cases except for some high enough reserve prices (Figure 13) and some values of dependence levels of private values (Figure 12). This is not surprising because the merged bidder has a higher private value mean and lower entry cost mean. On the contrary the effects of both mergers on entry behavior of non-merging bidders are negligible, which means almost all non-merging bidders follow the same entry strategy post-merger. Considering all effects on entry, it is more likely that the auction loses one participating bidder due to the "worst" merger and gains one participating bidder in the "best" bidder.

The changes in the number and identities of participating bidders affect the final bids and thus the winning bids through several channels. The first channel is called the "competition effect." The increase or decrease in the number of participating bidders makes bidders more or less aggressive. To the merged bidder in “worst” merger, the competition between two merging bidders is removed due to the merger, which causes the merged bidder bid less. Second, the merger may affect the bids and winning bids through affiliation. Within the APV framework, a bidder would think that he may overestimate the common factor which affects all bidders' private values when he wins the auction. By taking this into account and trying to alleviate this effect, the bidder reduces his bid. This effect is called the “affiliation effect” in Pinkse and Tan (2005) and can make bidders bid more as the number of potential bidders decreases. Lastly, the merger yields a stronger bidder meaning a smaller marginal distribution, through which the merger affects bidders' bids and possibly the winning bids. Intuitively this should lower the winning bid in the “worst” merger and raise the winning bid in the “best” merger, because a stronger winner is undesired while a stronger competitor is desired in terms of the degree of competition. Note that the first two channels are essentially through the change in the number of participating bidders and the last one is effective through the change in identities of participating bidders. How merger affects the seller's revenue depends on the interactions of these effects. Because of the complexity of the model we consider, however, analytically we cannot quantity the extent to which each effect impacts on the seller's revenue. Therefore we can only rely on the counterfactual analysis to quantity the overall effect of merger on the seller's revenue as we do here.

The effects on the revenue are presented in the second panels of Figure 8 to Figure 13. The first thing we note is that at the current levels of reserve price and dependence levels, both types of merges are beneficial to the seller, yielding gains of \$113/MBF and \$73/MBF in “best” merger and “worst” merger respectively. This implies that the “competition effect” and the effect associated with a stronger competitor dominate the other effect in the “best” merger, while in the “worst” merger the affiliation effect might be the dominant one. The effects on the revenue vary as the reserve price varies. In the “worst” merger, the merger could do harm to the seller for some values of the reserve price, for example, \$700/MBF, as is shown in Figure 8. From Figure 9 and Figure 10, it can be seen that in the anti-merger, both dependence levels are negatively related with the change in the expected revenue which implies that high levels of dependence might alleviate the “affiliation effect” and strengthen the other two effects. On the other hand, in the “best” merger, the negative relationship between the merger effect on revenue change and the dependence levels is not that obvious. Like the interaction between the merger effect on the revenue and the reserve price, the change in revenue due to merger varies as the dependence levels vary as well. The difference between two types of mergers is that the “best” merger is always beneficial to the seller whatever the dependence levels are but the “worst” merger can result in some loss to the seller at some dependence levels such as $\tau_k > 0.28$ as shown in Figure 9.

To summarize, we find the following results regarding the merger effects.

i. The merged bidder almost always participates in the auction.

ii. The merger has little impact on non-merging bidders' entry behavior.

iii. Both “worst” and “best” mergers are beneficial to the seller at the current levels of reserve price and dependence levels.

iv. The “best” merger is better than “worst” merger in terms of the change in the seller's revenue at the current levels of reserve price and dependence levels.

Conclusion

In this chapter we study how affiliation and entry can affect bidders' bidding behavior and the seller's revenue using the timber sales auction data from the ODF. We develop an entry and bidding model with heterogeneous bidders within the APV framework, and establish the existence and uniqueness of the Bayesian-Nash equilibrium. We adopt a structural approach to obtain the estimates for the structural parameters in the bidders' private values distribution. We are able to quantify the extent to which the potential bidders' private values and entry costs are affiliated, respectively, and find that the affiliation among bidders' private information found in Chapter II is mainly driven by the affiliation among bidders' entry costs. We then use the structural estimates to conduct counterfactual analysis to address the policy-related issues. In particular, we quantify how the seller's revenue could change with the changes in the reserve price or the dependence level. Moreover, we quantify the merger effect and evaluate how it changes with the changes in the reserve price or the dependence level.

Since we allow bidders to be heterogeneous and have affiliated private values, and also take entry into account, our approach is general and closer to the real timber auction environment. On the other hand, the analysis of the end auction outcomes and welfare implications is complicated by the interactions of affiliation, asymmetry, and entry. The structural approach we propose offers a promising way to disentangle these effects through the counterfactual analysis in addressing policy-related issues such as the merger effect.

Table 3. Summary Statistics of the Sample

	Observation	Mean	Std. Dev.
Bid	245	376.6915	96.5321
# of Potential Bidders	81	4.2099	0.8619
# of Actual Bidders	81	3.0247	1.3036
Entry Proportion	81	.7251	.2852
Appraised Price	81	329.7940	95.5464
Distance	341	75.9155	45.3968
Volume	81	3644.333	3085.016
Duration	81	803.4355	212.5968
Grade	81	10.3236	.4570
DBH	81	16.7655	4.8213

Table 4. Estimation Results

	Private Value distribution		Entry Cost distribution	
	Coefficient	Std. Error	Coefficient	Std. Error
Distance	-.1414	.0098	1.3367	.0451
Volume	.0582	.0026	.0688	.0011
Duration	-.0785	.0071	.0296	.0004
Grade	.9770	.0762	1.0687	.0223
DBH	.1699	.0067	-.0770	.0011
Dependence	.0983	.0045	0.5122	.0062

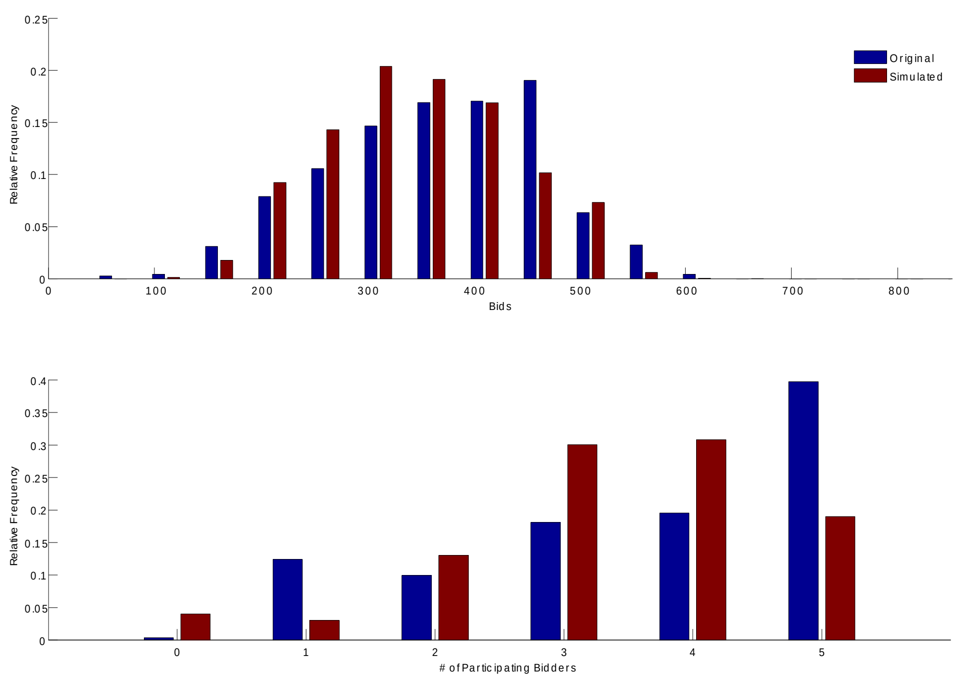

Figure 4. Histograms of Original and Simulated Data

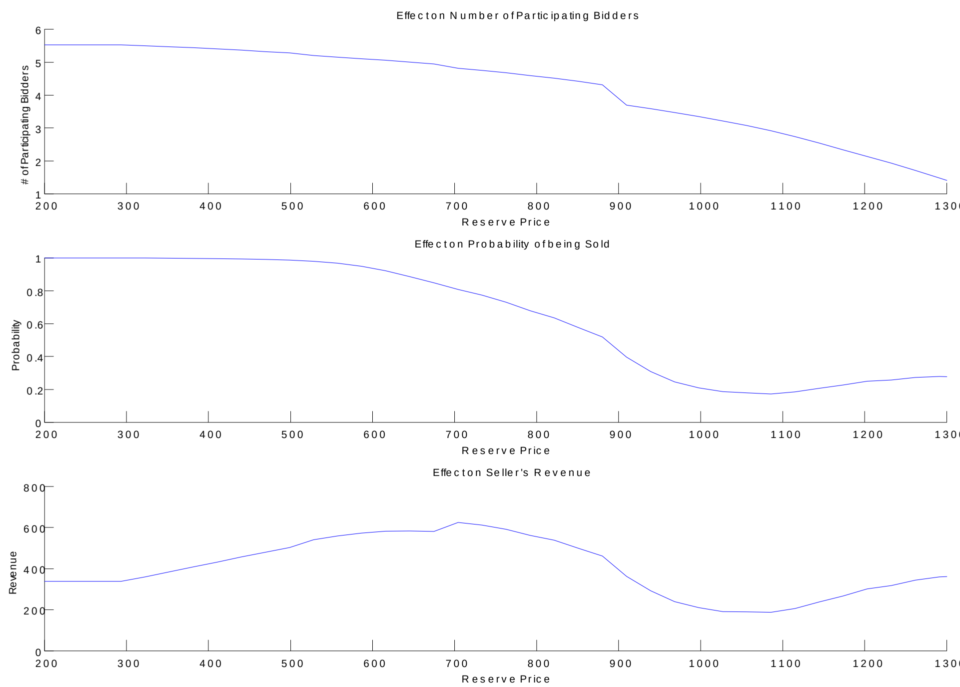

Figure 5. Effect of Reserve Price

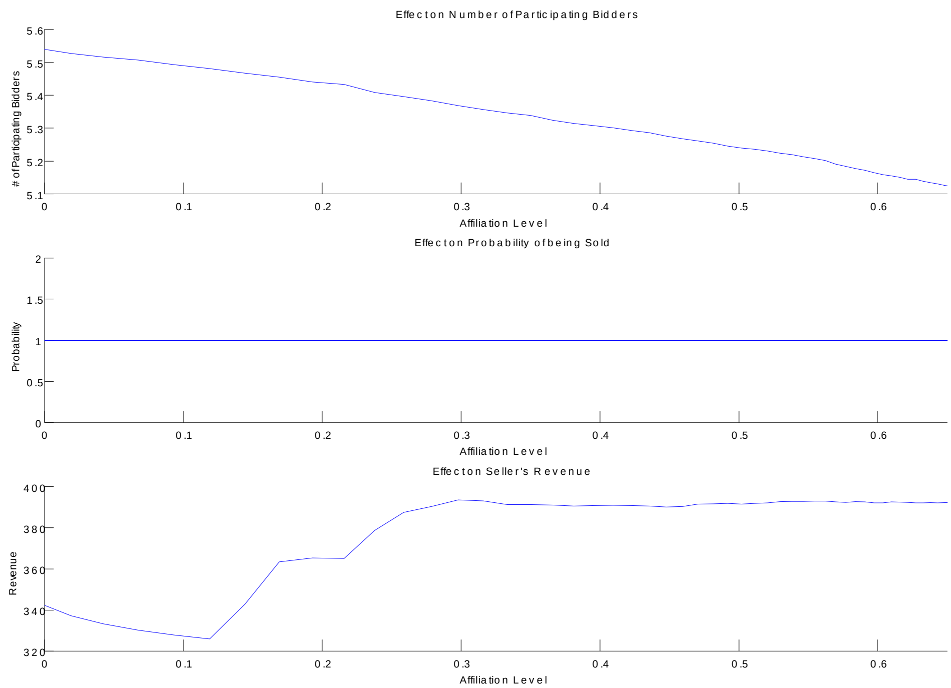

Figure 6. Effect of Dependence Level of Private Values

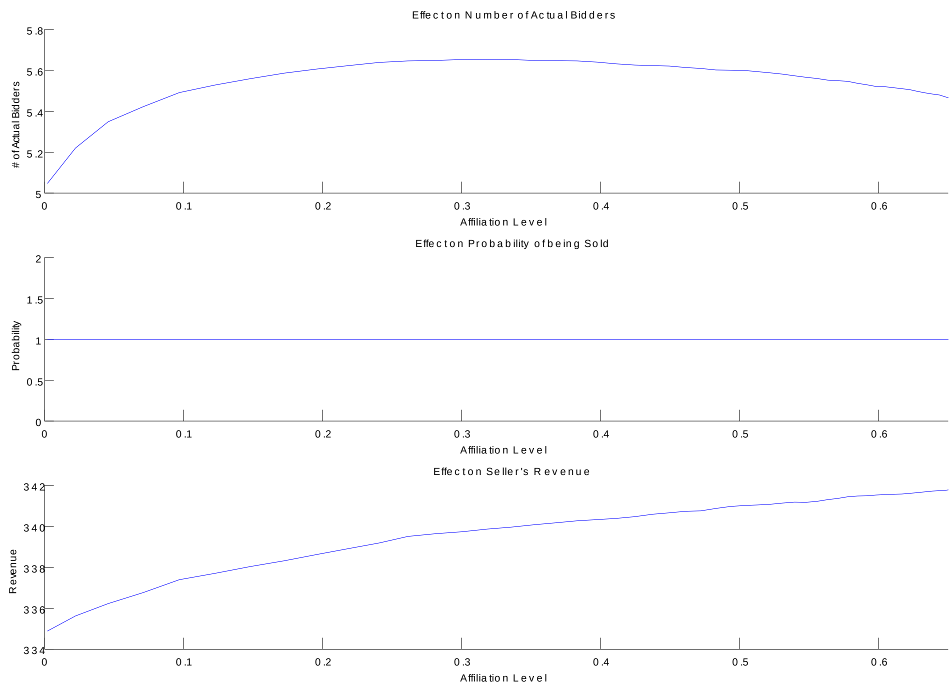

Figure 7. Effect of Dependence Level of Entry Costs

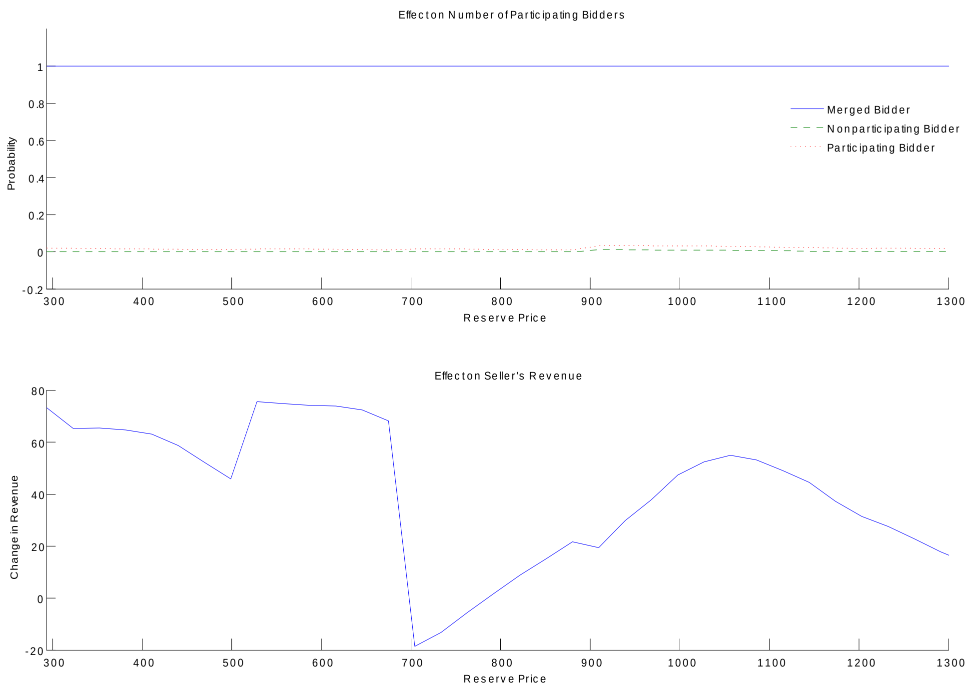

Figure 8. Interaction between "Worst" Merger and Reserve Price

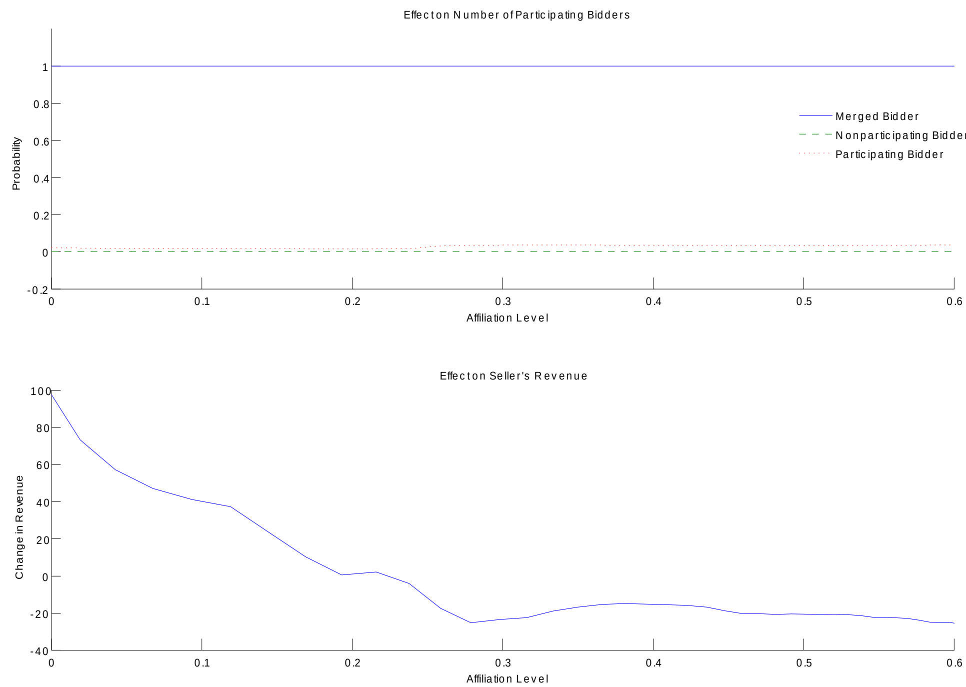

Figure 9. Interaction between "Worst" Merger and Dependence Level of Private Values

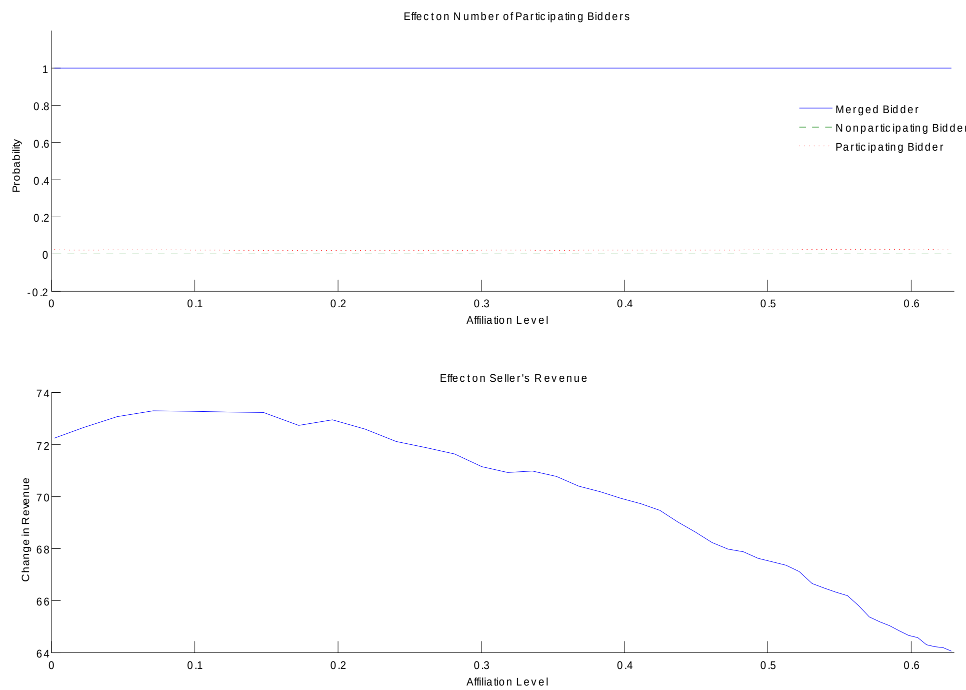

Figure 10. Interaction between "Worst" Merger and Dependence Level of Entry Costs

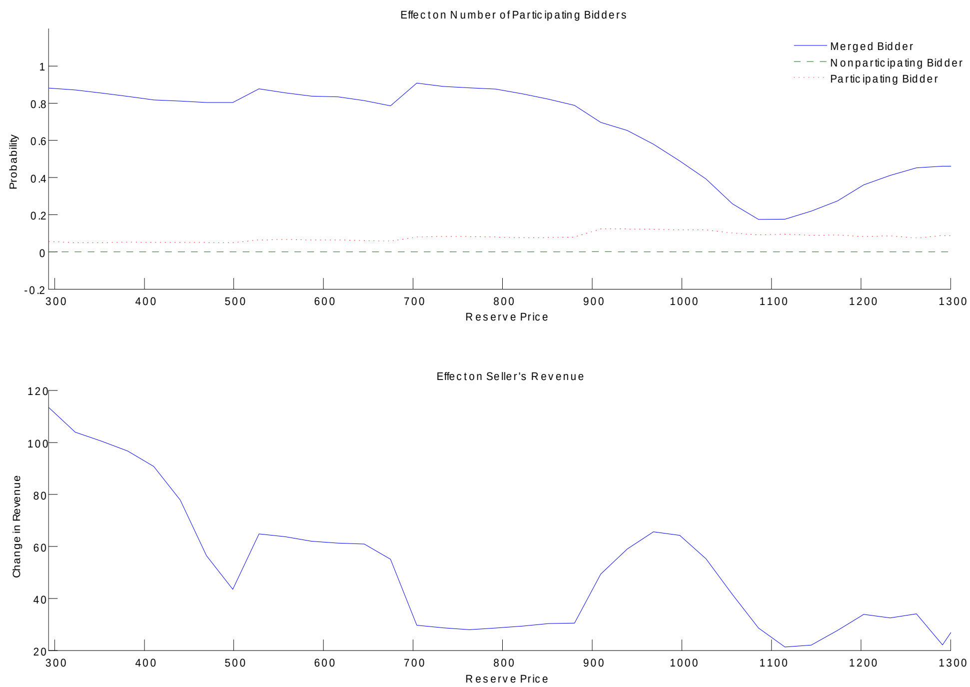

Figure 11. Interaction between "Best" Merger and Reserve Price

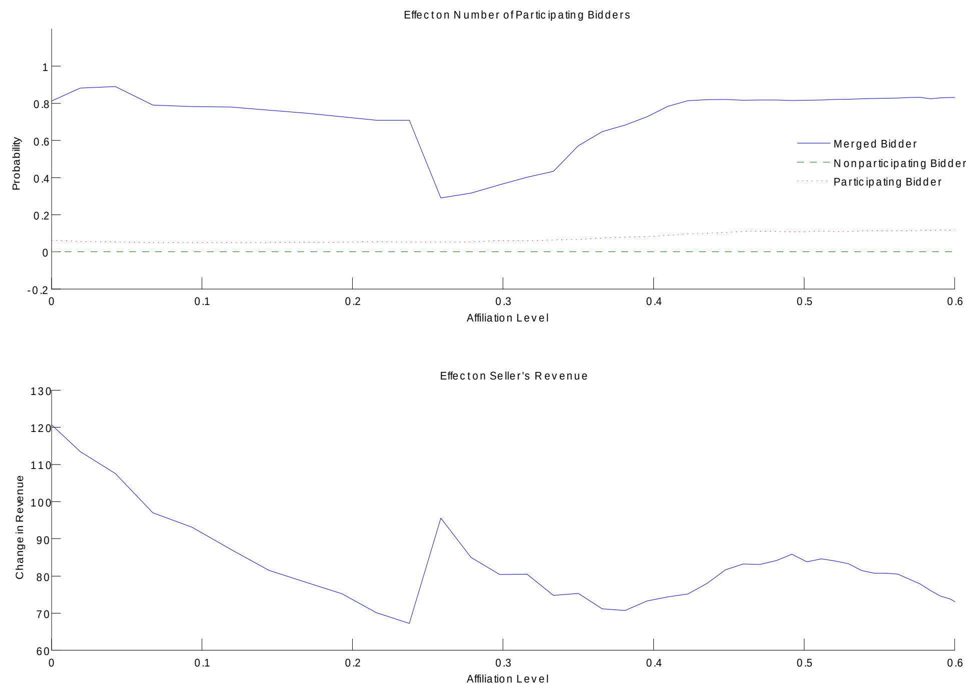

Figure 12. Interaction between “Best” Merger and Dependence Level of Private Values

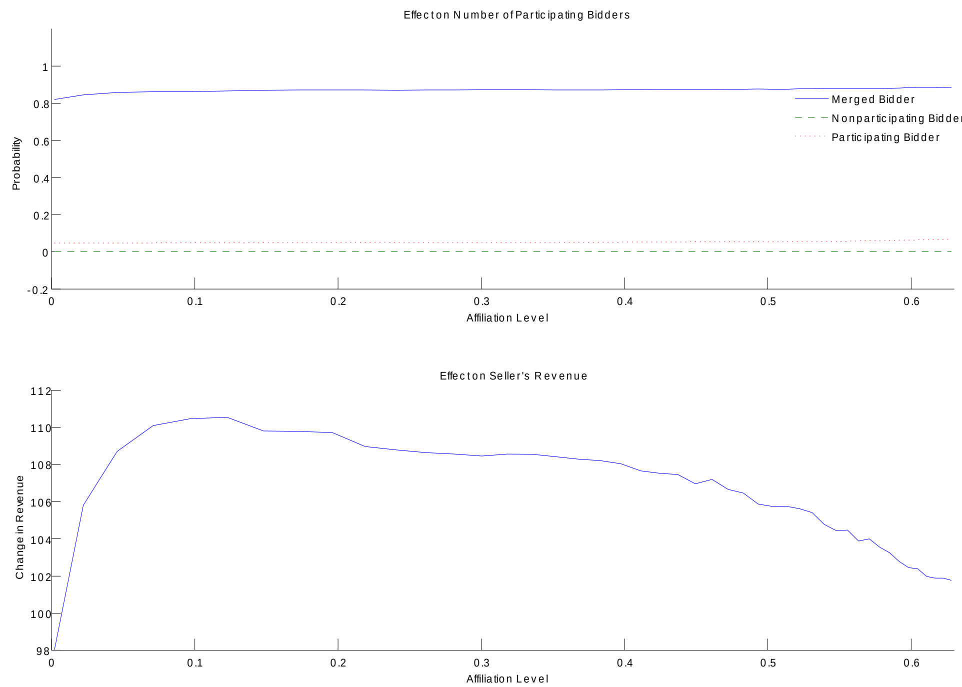

Figure 13. Interaction between "Best" Merger and Dependence Level of Entry Costs

APPENDIX A

Proof of Existence and Uniqueness of Equilibrium

Bidding Equilibrium

The proof of Proposition adapts Lebrun (1999, 2006). We first need the following lemma.

Lemma 1 *Consider a continuously differentiable and strictly increasing bidding strategy. Assume* $\frac{\phi^{-1\prime}(u)}{\phi^{-1\prime\prime}(u)}$ *is decreasing in* u. *If* $\widetilde{\eta} > \eta$ *and* $\widetilde{s}_i^{-1}(b)$ *and* $s_i^{-1}(b)$ *for all* i *are two solutions of the system of differential equations* $(IV.3)$ *with boundary condition* $(IV.6)$ *over* $(\widetilde{\gamma}, \widetilde{\eta}]$ *and* $(\gamma, \eta]$, *respectively, then the inverse bidding functions satisfy the following condition:* $\widetilde{s}_i^{-1}(b) < s_i^{-1}(b)$ *for all* b *in* $(\max(\gamma, \widetilde{\gamma}), \eta]$, *where* $\gamma > \underline{v}$.

Proof. Since we know that s_i^{-1} is strictly increasing over $(\gamma, \eta]$, we have $\widetilde{s}_i^{-1}(\eta) < s_i^{-1}(\eta) = \overline{v}$. Define g in $[\max(\gamma, \widetilde{\gamma}), \eta]$ as follows:

$$g = \inf\left\{b \in [\max(\gamma, \widetilde{\gamma}), \eta] \left| \widetilde{s}_i^{-1}(b') < s_i^{-1}(b'), \text{ for all } i \text{ and all } b' \in (b, \eta] \right.\right\}.$$

We want to prove that $g = \max(\gamma, \widetilde{\gamma})$. According to the definition of g, $\eta > g$. Suppose that $g > \max(\gamma, \widetilde{\gamma})$. By continuity, there exists i such that $\widetilde{s}_i^{-1}(g) = s_i^{-1}(g)$. From the definition of g, we also have $\widetilde{s}_j^{-1}(g) \leq s_j^{-1}(g)$ for all j. Moreover, there exists $j \neq i$ such that $\widetilde{s}_j^{-1}(g) < s_j^{-1}(g)$, because all the solutions coincide at the point g and therefore coincide in $(g, \eta]$ due to the fact that the right hand side of equation $(IV.3)$ is locally Lipschitz at $b = g$, which contradicts the fact that at point η $\widetilde{s}_i^{-1}(\eta) < s_i^{-1}(\eta)$.

From equation $(IV.3)$, we know $ds_i^{-1}(b)/db$ is a strictly decreasing function of $s_j^{-1}(b)$, for all $j \neq i$, since $\frac{\phi^{-1\prime}(u)}{\phi^{-1\prime\prime}(u)}$ is decreasing in u. Consequently, $d\widetilde{s}_i^{-1}(g)/db > ds_i^{-1}(g)/db$. Therefore there exists $\delta > 0$ such that $\widetilde{s}_i^{-1}(b) > s_i^{-1}(b)$, for all b in $(g, g+\delta)$. This contradicts the definition of g. ■

We then prove the first part of the proposition by showing that there exist an η, such that $s_i^{-1}(\eta) = \overline{v}$.

Let $i, 1 \leq i \leq n$ denote bidders who have the highest bids, denoted by η', at the upper bound of private value $\overline{v}$ and $j, 1 \leq j \leq n$ denote bidders who has the second highest bid, denoted by η, at the upper bound of private value $\overline{v}$. So $\eta' \geq \eta$.

For bidder i, we know that

$$\left(\overline{v} - \eta'\right) \Pr\left(B_{-i} < \eta' | \overline{v}\right) \geq \left(\overline{v} - \eta\right) \Pr\left(B_{-i} < \eta | \overline{v}\right).$$

It is obvious that $\Pr\left(B_{-i} < \eta' | \overline{v}\right) = 1$

$\Pr\left(B_{-i} < \eta | \overline{v}\right) = \Pr\left(b_j < \eta, b_k < \eta, k \neq i, j | v_i = \overline{v}\right) = \Pr\left(b_k < \eta, k \neq i, j | v_i = \overline{v}\right)$, since b_j is not larger than η.

$\Pr\left(B_{-j} < \eta | \overline{v}\right) = \Pr\left(b_i < \eta, b_k < \eta, k \neq i, j | v_j = \overline{v}\right)$.

Since the joint distribution of private values follows Archimedean copulas, we have

$$\begin{aligned}
\Pr\left(B_{-i} < \eta | \overline{v}\right) &= \phi^{-1\prime}\left(\sum_{k \neq i,j} \phi\left(F_k\left(s_k^{-1}(\eta)\right)\right) + \phi\left(F_j\left(s_j^{-1}(\eta)\right)\right) + \phi\left(F_i\left(\overline{v}\right)\right)\right) \phi'\left(F_i\left(\overline{v}\right)\right) \\
&= \phi^{-1\prime}\left(\sum_{k \neq i,j} \phi\left(F_k\left(s_k^{-1}(\eta)\right)\right) + \phi(1) + \phi(1)\right) \phi'(1)
\end{aligned}$$

and

$$\begin{aligned}
\Pr\left(B_{-j}<\eta|\overline{v}\right) &= \phi^{-1\prime}\left(\sum_{k\neq i,j}\phi\left(F_k\left(s_k^{-1}\left(\eta\right)\right)\right)+\phi\left(F_j\left(\overline{v}\right)\right)+\phi\left(F_i\left(s_i^{-1}\left(\eta\right)\right)\right)\right)\phi'\left(F_j\left(\overline{v}\right)\right) \\
&= \phi^{-1\prime}\left(\sum_{k\neq i,j}\phi\left(F_k\left(s_k^{-1}\left(\eta\right)\right)\right)+\phi\left(1\right)+\phi\left(F_i\left(s_i^{-1}\left(\eta\right)\right)\right)\right)\phi'\left(1\right)
\end{aligned}$$

If $F_i\left(s_i^{-1}\left(\eta\right)\right)<1$, then $\phi\left(F_i\left(s_i^{-1}\left(\eta\right)\right)\right)>\phi\left(1\right)$ and $\Pr\left(B_{-i}<\eta|\overline{v}\right)>\Pr\left(B_{-j}<\eta|\overline{v}\right)$ since $\phi'\left(1\right)<0$ and $\phi^{-1\prime}\left(x\right)$ is increasing in x. Therefore

$$\left(\overline{v}-\eta'\right)\Pr\left(B_{-j}<\eta'|\overline{v}\right)>\left(\overline{v}-\eta\right)\Pr\left(B_{-j}<\eta|\overline{v}\right)$$

since $\Pr\left(B_{-j}<\eta'|\overline{v}\right)=1$. But this is impossible because the optimal bid of bidder j at $\overline{v}$ is η, therefore we have $F_i\left(s_i^{-1}\left(\eta\right)\right)=1$ and $\eta'=\eta$.

Uniqueness of Bidding Equilibrium

Suppose that there exist two equilibria and thus two different values η and $\widetilde{\eta}$ such that the respective solutions $s_i^{-1}\left(b\right)$ and $\widetilde{s}_i^{-1}\left(b\right)$ are also solutions of the system of differential equations for all i. Without loss of generality, we can assume that $\eta<\widetilde{\eta}$. The value of $\ln\left(\Pr\left(v_j<s_j^{-1}\left(b_i\right),j\neq i|v_i\right)\right)$ at $b_i=\eta$ is strictly larger than the value of $\ln\left(\Pr\left(v_j<\widetilde{s}_j^{-1}\left(b_i\right),j\neq i|v_i\right)\right)$ at the same point. We have shown that $\widetilde{s}_i^{-1}\left(b\right)<s_i^{-1}\left(b\right)$ for all b in $(\underline{v},\eta]$. When b converges to $\underline{v}$, $s_i^{-1}\left(\underline{v}\right)=\underline{v}$.

On the other hand, the first order condition can be written as follows

$$\frac{d\ln\left(\Pr\left(v_j<s_j^{-1}\left(b_i\right),j\neq i|v_i\right)\right)}{db}=\frac{1}{s_i^{-1}\left(b_i\right)-b_i}.$$

Thus $\frac{d\ln\left(\Pr\left(v_j<s_j^{-1}(b),j\neq i|v_i\right)\right)}{db}<\frac{d\ln\left(\Pr\left(v_j<\widetilde{s}_j^{-1}(b),j\neq i|v_i\right)\right)}{db}$. Therefore, the difference between these two logarithms increases as b decreases towards $\underline{v}$. On the other hand, $\ln\left(\Pr\left(v_j < \underline{v}, j \neq i | v_i\right)\right)$ is a finite value since $F_j\left(\underline{v}\right) > 0$. Therefore for two solutions, $\ln\left(\Pr\left(v_j < s_j^{-1}\left(b_i\right), j \neq i | v_i\right)\right)$ cannot both converge to the same finite value as b decreases towards $\underline{v}$. Therefore η and $\widetilde{\eta}$ coincide and the equilibrium is unique.

Entry Equilibrium

The entry probability p_i is determined by equation $(IV.7)$. Let $p = (p_{1,} \ldots, p_n) \in [0,1]^n$ and $G_p = (G_1 \circ \Pi_1(p), \ldots, G_n \circ \Pi_n(p))$. Since $s_i(v)$ and G_i is continuous, the pre-entry expected profit Π_i and $G_i \circ \Pi_i$ is continuous in p. So $G_p : [0,1]^n \rightarrow [0,1]^n$ and is continuous in p. A fixed point of p follows Kakutani's fixed point theorem (Kakutani (1941)).

APPENDIX B

Solving Equilibrium Bids and Entry Probabilities

Equilibrium Bids

Note that with the choice of Clayton copula, the first order condition given in equation $(IV.3)$ can be written as follows,

$$(1+q)(v_i - b)\sum_{j\neq i}\frac{dF_j^{-q}\left(s_j^{-1}(b)\right)}{db} = -q\left(\sum_{k=1}^{n}F_k^{-q}\left(s_k^{-1}(b)\right) - n + 1\right).$$

Define $F_i^{-q}\left(s_i^{-1}(b)\right) = l_i(b)$, then $v_i = F_i^{-1}\left(l_i^{-\frac{1}{q}}(b)\right)$, and F.O.C. becomes

$$(1+q)\left(F_i^{-1}\left(l_i^{-\frac{1}{q}}(b)\right) - b\right)\sum_{j\neq i}l_j'(b) = -q\left(\sum_{k=1}^{n}l_k(b) - n + 1\right)$$

Rewriting all terms in the equation as polynomials

$$
\begin{aligned}
l_i(b) &= \sum_{j=0}^{\infty} a_{i,j}(b-b_0)^j, \\
l_i'(b) &= \sum_{j=0}^{\infty} (j+1)\,a_{i,j+1}(b-b_0)^j, \\
l_i^{-\frac{1}{q}}(b) &= \sum_{j=0}^{\infty} g_{i,j}(b-b_0)^j, \\
F_i^{-1}\left(l_i^{-\frac{1}{q}}(b)\right) &= \sum_{j=0}^{\infty} p_{i,j}(b-b_0)^j, \\
F_i^{-1}\left(l_i^{-\frac{1}{q}}(b)\right) - b &= \sum_{j=0}^{\infty} \widetilde{p}_{i,j}(b-b_0)^j, \\
F_i^{-1}(x) &= \sum_{j=0}^{\infty} d_{i,j}(x-x_0)^j, \\
x_i^{-\frac{1}{q}} &= \sum_{j=0}^{\infty} c_{i,j}(x-x_0)^j,
\end{aligned}
$$

where $\widetilde{p}_{i,0} = p_{i,0} - b_0$, $\widetilde{p}_{i,1} = p_{i,1} - 1$, and $\widetilde{p}_{i,j} = p_{i,j}$ for $j > 1$.

Computation of $p_{i,j}, g_{i,j}$: following the lemma in Appendix C in Marshall, Meurer, Richard, and Stromquist (1994), we have

$$p_{i,J} = \sum_{r=1}^{J} d_{i,r}\theta_{i,r,J} - z_J,\ p_{i,0} = F_i^{-1}\left(l_i^{-\frac{1}{q}}(b_0)\right) \tag{B.1a}$$

$$\theta_{i,r,J} = \sum_{s=1}^{J-r+1} g_{i,s}\theta_{i,r-1,J-s},\ \theta_{i,0,0} = 1, \tag{B.1b}$$

$$g_{i,J} = \sum_{r=1}^{J} c_{i,r}\varphi_{i,r,J}, \tag{B.1c}$$

$$\varphi_{i,r,J} = \sum_{s=1}^{J-r+1} a_{i,s}\varphi_{i,r-1,J-s}, \varphi_{i,0,0} = 1. \tag{B.1d}$$

Computation of $a_{i,j}$: from the F.O.C., we have

$$\begin{aligned}
& (1+q)\left(\sum_{j=0}^{\infty} \widetilde{p}_{i,j}\,(b-b_0)^j\right) \sum_{j\neq i}\sum_{s=0}^{\infty} (s+1)\, a_{j,s+1}\,(b-b_0)^s \\
= \;& -q\left(\sum_{k=1}^{n}\sum_{s=0}^{\infty} a_{k,s}\,(b-b_0)^s - n + 1\right) \\
& (1+q)\left(\sum_{j=0}^{\infty} \widetilde{p}_{i,j}\,(b-b_0)^j\right) \sum_{s=0}^{\infty} (s+1)\left(\sum_{j\neq i} a_{j,s+1}\right)(b-b_0)^s \\
= \;& -q\left(\sum_{s=0}^{\infty}\left(\sum_{k=1}^{n} a_{k,s}\right)(b-b_0)^s - n + 1\right) \\
& (1+q)\sum_{s=0}^{\infty} (s+1)\left(\sum_{r=0}^{s} \widetilde{p}_{i,s} \sum_{j\neq i} a_{j,s+1-r}\right)(b-b_0)^s \\
= \;& -q\left(\sum_{s=0}^{\infty}\left(\sum_{k=1}^{n} a_{k,s}\right)(b-b_0)^s - n + 1\right)
\end{aligned}$$

Comparing the coefficients of $(b-b_0)^s$ we have

$$(1+q)\,(s+1)\left(\sum_{r=0}^{s} \widetilde{p}_{i,s} \sum_{j\neq i} a_{j,s+1-r}\right) = -q\left(\sum_{k=1}^{n} a_{k,s}\right), \text{ for } s>0 \quad \text{(B.2a)}$$

$$(1+q)\, p_{i,0} \sum_{j\neq i} a_{j,1} = -q\left(\sum_{k=1}^{n} a_{k,0} - n + 1\right), \text{ for } s=0. \quad \text{(B.2b)}$$

Algorithm:

1. $d_{i,j}, c_{i,j}$ for $j = 1, \ldots, J$, can be computed by Taylor expansion. In practice, $J = 3$.
2. Decide $a_{i,0}, \widetilde{p}_{i,0}, \theta_{i,0,0}$,and $\varphi_{i,0,0}$ by the boundary conditions.
3. Calculate $\widetilde{p}_{i,1}$ from equations $(B.1)$ given $a_{i,0}, \widetilde{p}_{i,0}, \theta_{i,0,0}$,and $\varphi_{i,0,0}$.
4. Calculate $a_{i,1}$ from equations $(B.2)$ given $\widetilde{p}_{i,1}$.
5. Repeat step 3 and 4 until $a_{i,j}, j = 1, \ldots, J$ are calculated.

Now we have found the coefficients of the Taylor expansion of the inverse bidding function up to the J-th order, so we are able to find the equilibrium bid for a given private value for bidder i through the obtained Taylor expansion at a appropriate point. One issue regarding the algorithm is the boundary conditions. From the Proposition we know that there are two boundary conditions associated with the equilibrium. Note that it cannot be used here although the boundary condition at the lower bound of bids is known to us, since it causes the problem of singularity. Therefore we have to use the condition at the upper bound, which is unfortunately unknown to us. To address this problem we follow the method described in Marshall, Meurer, Richard, and Stromquist (1994) and Gayle (2004) to find the common η first. Roughly, it is to find an η which generates the best equilibrium bids at point $\underline{v}$ according to the algorithm described above.

Equilibrium Entry Probabilities

As is seen from the Proposition and equation $(IV.7)$ the equilibrium entry probabilities are determined by fixed point problem. We solve it through iteration, described in detail as follows,

1. Given an initial guess of $p_i^{old}, i = 1, \ldots, N$, we calculate the probabilities of all possible entry behavior occurring according to equation $(IV.4)$.

2. Given the calculated $\int_{\underline{v}}^{\overline{v}} \pi_i \left(v_i | a_{-i}\right) dF_i \left(v_i\right)$ for all possible entry behavior in equation $(IV.2)$ and associated probabilities given in step 1, we calculate the post-entry expected profit $\Pi_i, i = 1, \ldots, N$.

3. Calculate new entry probabilities according to $p_i^{new} = G_i \left(\Pi_i\right), i = 1, \ldots, N$.

4. If the difference of p_i^{new} and p_i^{old} is smaller than a given small positive number, ε, then $p_i^{new}, i = 1, \ldots, N$ are the equilibrium entry probabilities and the iteration stops;

otherwise, let $p_i^{old} = p_i^{new}$ and go to step 1.

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