MATHEMATICS

ON QUASICONVEX SUBSETS OF HYPERBOLIC GROUPS

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A geodesic metric space $\mathcal{X}$ is called hyperbolic if there exists $\delta \geq 0$ such that every geodesic triangle Δ in $\mathcal{X}$ is δ-slim, i.e., each side of Δ is contained in a closed δ-neighborhood of the two other sides. Let G be a group generated by a finite set $\mathcal{A}$ and let $\Gamma(G, \mathcal{A})$ be the corresponding Cayley graph. The group G is said to be word hyperbolic if $\Gamma(G, \mathcal{A})$ is a hyperbolic metric space. A subset Q of the group G is called quasiconvex if for any geodesic γ connecting two elements from Q in $\Gamma(G, \mathcal{A})$, γ is contained in a closed ε-neighborhood of Q (for some fixed $\varepsilon \geq 0$). Quasiconvex subgroups play an important role in the theory of hyperbolic groups and have been studied quite thoroughly.

We investigate properties of quasiconvex subsets in word hyperbolic groups and generalize a number of results previously known about quasiconvex subgroups. On the other hand, we establish and study a notion of quasiconvex subsets that are small relatively to subgroups. This allows to prove a theorem concerning residualizing homomorphisms preserving such subsets. As corollaries, we obtain several new embedding theorems for word hyperbolic groups.

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