SYSTEMS ENGINEERING DECISION PROCESS: OPTIONS ARE AVAILABLE

By

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To Alfred Loomis who said in 1939

"I appreciated then for the very first time the difference between the world of business, where a 20 percent decrease in cost is a major triumph, and the world of science, where nothing seems worth doing unless it promises an improvement by a factor of at least 10." (Conant 2002)

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CHAPTER I

BACKGROUND

The objective of my research is to investigate and develop tools and techniques such that systems engineering decision makers can make more effective financial and technology choice decisions. Using ideas from the financial domain, particularly that of Real Options Analysis (ROA), this research shows how this advanced cost evaluation tool can be applied in a systems engineering context. Next, the dissertation demonstrates how ROA ideas can be extended to technology alternatives leading to the development of a new advanced performance evaluation model called Technology Options Analysis (TOA). Subsequently, these two views (performance and cost) are merged into a unified algorithm to sequentially evaluate cost and performance alternatives so that a decision maker can gain an overall view of the value of delaying a technology decision until the most appropriate times. It is important to note that this research varies two (performance, cost) of the three (performance, cost, and schedule) primary objectives of a system; modifying the third is beyond the scope of this dissertation. These techniques provide system engineering decision makers with significantly more and, most importantly, new information upon which to base their decisions when making technology choices.

It is not uncommon for large complex systems to be expected to operate for decades and take years to design, develop and test before they enter the market place. Consider, for example, the Boeing 787, Airbus A380, or the Joint-Strike-Fighter (JSF), all of which are requiring billions of dollars and decades of development work (Steidle 1997; Struth 2000; 2001; Mecham 2005; Flight Level 350 2007). These are complicated systems that utilize and integrate sophisticated technologies in order to operate properly in complex environments. The potential design solutions for such complex systems typically consist of many different and competing technological alternatives. Evaluations of the competing technologies may continue until a final implementation decision is reached (Blanchard and Fabrycky 1998; Buede 2000; Rouse 2003). These decisions are important to the achievement of overall system objectives because they not only select the technologies to be utilized, they also determine and commit the majority of the total life cycle cost of the project (Buede 2000; Kerzner 2001; Blanchard 2004) as can be seen in Figure 1.1. With decisions of this type, Drucker (2006) and others have advocated that a decision should be made no later than necessary, but as late as possible as more time will normally provide a decision maker with additional information. For complex systems then the question is "How late in the decision process can the choice of technology effectively be made?" In order to succeed in such a complex problem space and environment, these programs have utilized the processes of Systems Engineering.

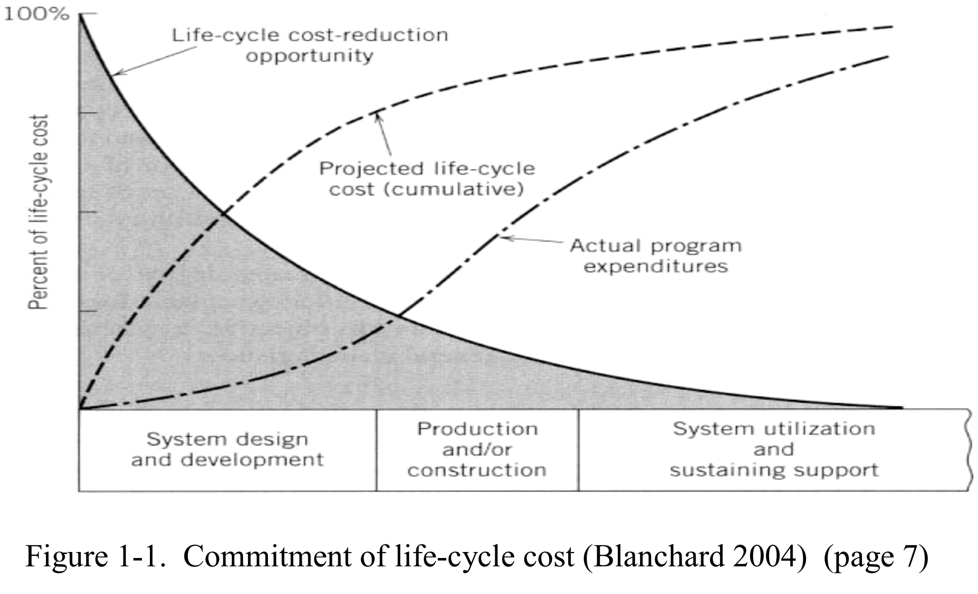

Figure 1-1. Commitment of life-cycle cost (Blanchard 2004) (page 7)

Systems Engineering (SE) is “An interdisciplinary approach and means to enable the realization of successful systems” (Sage and Rouse 1999). Systems Engineering is the integrative aspect of a system; it is the “glue” that assures that the diverse elements of a system can “bind” together to create a usable solution to the customer requirements over the total life of the project (Sabbagh 1996; Blanchard and Fabrycky 1998; Sage and Rouse 1999; Buede 2000; Fleeman 2001; Kerzner 2001; Blanchard 2004). Customer requirements, or originating specifications, are inputs to the Systems Engineering process and are normally categorized into three primary system objectives: schedule, performance, and cost.

1.1 Three Primary System Objectives

Schedule objectives are, simply put; the date the customer wants or needs the system to be operational. These objectives can be expressed in increments of capability or as the entire capability by a certain timeframe(s). For example, the Joint Strike Fighter has several schedule objectives. The 2001 System Design and Development contract required the delivery of 22 development evaluation aircraft in 2006 (Steidle 1997; Struth 2000). Other contract awards have required delivery of the first combat evaluation aircraft in 2008 and obtainment of initial operational capability (IOC) in 2011 (Steidle 1997; Struth 2000). For the Boeing 777, the schedule objective was the delivery of the first aircraft to United Airlines in accordance with their contract (Sabbagh 1996). This schedule objective was negotiated before the aircraft had been designed. Meeting schedule objectives can be critical to the acceptance of the system and customer satisfaction, as can be seen with the delays in delivering the Airbus A380 (Michaels 2006; Wall Street Journal 2006). The schedule objective gives the systems engineer the when. The next objective, the performance objective, defines what the system must do.

Performance objectives consist of desires about the operation of the system, such as; speed - how fast or slow, range - how far, payload - how large a load, accuracy - how close to a target, crew size - how many people to operate, reliability – how dependable, availability – how often, etc. Basically performance objectives are how the system is expected to operate in normal conditions. Such objectives are defined by a range of values usually with the minimum value designated as the threshold and the desired value described as the goal (Chambers 1986; Lake 1992; Sabbagh 1996; Steidle 1997; Blanchard and Fabrycky 1998; Parth 1998; Buede 2000; Struth 2000; Blanchard 2004). Nothing less than the threshold is acceptable and anything more than the goal is not necessary. For example, the Boeing 777 had several performance objectives: the range goal was 5,500 nm with a threshold of 3,500 nm when fully loaded and the empty plane weight had a goal of 293,000 lbs and a threshold - 298,000 lbs (Sabbagh 1996). Another example would be the FAA Wide-Area Augmentation System objective that during a precision landing it must warn a pilot of potentially hazardous misleading information within a goal of 5.2 seconds and a threshold of 6 seconds with a reliability of one error in ten million landings (GAO 2000). Example objectives for the Air Force's version of the Joint Strike Fighter are a combat threshold range of 450 miles with a goal of 600 miles and a maximum cruising speed threshold comparable to the F-16 (approximately mach 2 at altitude) (JIRD 1995). By having a range of values for the performance parameters of a system, various technology alternatives can be explored and evaluated relative to the cost objectives.

Cost objectives are the customer's acceptable range of expenditure on development, production, operations, and maintenance for the system. These objectives may include non-reoccurring and reoccurring expenses or simply a single cost broken out for each cost category (Chambers 1986; Frosberg and Mooz 1992; Lake 1992; Blanchard and Fabrycky 1998; Parth

1998; Sage and Rouse 1999; Struth 2000; Kerzner 2001; Rouse 2003; Blanchard 2004). They may be expressed on an annual basis, as a total life cycle cost or as a purchase cost (Chambers 1986; Frosberg and Mooz 1992; Lake 1992; Blanchard and Fabrycky 1998; Parth 1998; Sage and Rouse 1999; Struth 2000; Kerzner 2001; Rouse 2003; Blanchard 2004). Cost objectives are normally expressed in current year dollars and are articulated as thresholds and goals (Chambers 1986; Frosberg and Mooz 1992; Lake 1992; Blanchard and Fabrycky 1998; Parth 1998; Sage and Rouse 1999; Struth 2000; Kerzner 2001; Rouse 2003; Blanchard 2004). Unlike performance thresholds and goals, a cost threshold is the highest cost the customer is willing to pay and the goal is the desired cost. For example the threshold cost for a new Boeing 777 was approximately $210 million with a goal of $195 million (Sabbagh 1996). With the Joint Strike Fighter program, the cost focus of the program was affordability at all levels; development cost, production cost, and cost of ownership. In this program, all requirement trade-offs were evaluated against not only their operational value but also using cost as a parameter in design using a model referred to as “cost as an independent variable” (CAIV) (Steidle 1997; Struth 2000; Brady 2001). The goal for the Air Force’s version was a unit production cost of less than $28M with a threshold of $35M in government fiscal year 1994 dollars (Steidle 1997; Struth 2000).

To complete an overall evaluation of the system, a systems engineer needs to assess how well each technological alternative performs against the various objectives of schedule, performance, and cost. This is accomplished by assigning attributes to each alternative for each of the system objectives.

1.2 System Attributes

The method utilized to determine the schedule, performance, and cost attributes is critical to the analysis that leads to selection of the alternatives that best meets the systems engineer's goal of providing the customer with a system that performs the tasks desired, at a fair price, and delivered in the time frame requested (Blanchard and Fabrycky 1998; Sage and Rouse 1999; Buede 2000; Kerzner 2001; Blanchard 2004). To do this, System Engineering (SE) takes a customer's originating specifications and, by using a variety of processes, simulations, and models, conducts trade studies that output "optimal" system solutions (Buede 2000). This "optimal" solution or "best value" alternative is chosen from the group of potential alternatives. The selection is based on weighting the schedule, performance, and cost attributes associated with each technology, such that the solution is the choice that achieves the overall objectives in the most effective way.

The schedule attribute for an alternative is the probability that the technologies utilized by the alternative will be available when needed. Because a significant amount of research has been completed in schedule alternatives (Frosberg and Mooz 1992; Blanchard and Fabrycky 1998; Rochecouste 1999; Buede 2000; Kerzner 2001; Nicholas and Nicholas 2001; Rouse 2003; Allen and Sosa 2004; Cleland 2004; Roberts 2004; Rubenstein 2004), for the purpose of my dissertation, these will be considered as fixed or given.

To determine if the alternative performs the desired tasks, performance characteristics for an alternative are derived based on technical evaluations using one or more of the following techniques: similarity, testing, or modeling and simulation (Blanchard and Fabrycky 1998; Sage and Rouse 1999; Buede 2000; Kerzner 2001; Blanchard 2004). These technical evaluations provide information on potential solutions. Each alternative's performance characteristics are compared to the customer-weighted performance objectives to investigate how well the alternative performs the desired task. The output of this process is a single, aggregate performance value for each alternative. From a performance perspective, delaying the technology choice decision until the last possible point in time maximizes the information available and, as decision makers prefer more to less information (Eisenhardt 1989), such a delay can be valuable in making the best technology choice.

In order to compare the cost and financial impact of the various alternatives, a technique to summarize the monetary cost/value of each alternative is essential. The most common theory used in SE to evaluate cost is the time value of money (Blanchard and Fabrycky 1998; Sage and Rouse 1999; Buede 2000; Kerzner 2001; Barringer 2003; Blanchard 2004). Discounted cash flow (DCF) analysis is used to bring the total life cycle costs for each alternative to its current net present value (NPV). Each alternative's NPV is then evaluated, where alternatives with a negative NPV may be removed from the potential solution space, and the alternative with the highest NPV is selected.

This cost summarization approach works well if the costs drivers associated with each alternative are well understood and characterized (Hayes and Abernathy 1980). Consider the case when one alternative's cost is dependent on a highly variable process and a second alternative's cost is well known and stable. This might be the case when one technology solution is state of the art and the other is mature, for example, the use of composite technology in aircraft frames versus the use of aluminum. It is well known that normally highly variable cost driver stabilize once the new technology is more mature or a dominant design occurs (Christensen 1993; Christensen and Bower 1996; Christensen, Suarez et al. 1998; Srinivasan, Lilien et al. 2006), however, the NPV techniques do not have a mechanism for incorporating technology variability or for valuing delaying a decision until more information is available. Estimating when the variable technology will be available for use during the time frame of the project must be subjectively determined with little evidence. For example, while the technology experts believed that technology for the JSF would be available, the United States General Accounting Office (GAO) reported in 2001 that the Joint Strike Fighter cost objective would not be achieved because technologies critical to meeting this objective where not available at the proper maturity level (GAO 2001). There is also a difficulty on the opposite side of the technology curve: some alternatives may consist of technologies that are in the decline phase of the technology life cycle such that they will no longer be manufactured when the project starts production and the cost of reconstituting them is not captured. For example, a system designed around a Pentium IV processor with a 5 ¼" floppy drive has such legacy issues.

The NPV "optimal" solution process's basic theoretical premise is that it is possible and necessary to forecast the future because the complexity of the system requires technology choices be selected or "locked down" as soon as possible (Buede 2000). Such a premise holds if technology and cost forecasting is accurate and if no disruptive technologies (Christensen and Bower 1996) or radical innovation occur (Henderson and Clark 1990; Fine 1998). However, because different technologies mature at different rates (Fine 1998), if a technology choice is locked down too early, potential future opportunities will be missed; if it is locked down too late performance, cost, and schedule may be negatively impacted.

1.3 Assumptions of Systems Engineering

Systems Engineering is a well documented, well known discipline (Chambers 1986; Frosberg and Mooz 1992; Lake 1992; Blanchard and Fabrycky 1998; Parth 1998; Rochecouste

1999; Sage and Rouse 1999; Buede 2000; Gansler 2000; Kerzner 2001; Rouse 2003; Allen and Sosa 2004; Blanchard 2004; Roberts 2004; Rubenstein 2004). However, as with any discipline, it is based on certain assumptions. A few of these assumptions will be presented and how they may not hold true in today's environment will be discussed. First, the technologies utilized in a chosen design alternative must be available for the system's entire life. This requires a reasonably accurate technology forecast. However, it is not uncommon that the production phase of a system may occur 5 or more years after the initial technologies have been chosen. Such a system may be required to be maintained and repaired over the next 15 years or more (Lake 1992; Sabbagh 1996; Steidle 1997; Blanchard and Fabrycky 1998; Parth 1998; Rochecouste 1999; Buede 2000; Gansler 2000; Struth 2000; Rouse 2003; Blanchard 2004; Mecham 2005). For this to be true, technology forecasting assumes that the technologies will evolve in an incremental fashion and that performance improvements will be linear or at least known to an acceptable level of certainty and centered around a reasonable mean (Christensen and Bower 1996). This assumption is accurate for technologies that experience minor incremental innovations over many years and are therefore considered to have a slow clock speed (Henderson and Clark 1990; Fine 1998). However, some technology improvements demonstrate completely different trajectories (Christensen 1993; Christensen and Bower 1996).

Technology reality does not always fit neatly into such a well characterized box; exponential rates of improvement, disruptive technologies, and radical innovations shift markets or create new ones (Christensen 1993; Christensen and Bower 1996). In one of the most famous technology projections, in 1965 Gordon E. Moore made an observation that has become known as Moore's Law 'the number of transistors that can be placed in an integrated circuit has increased exponentially, doubling approximately every 2 years'. This defines an exponential growth curve that shows little sign of slowing. Just as technology improvements have grown at exponential rates, others have vanished altogether. Bubble memory was a very promising technology in the 1970s, but flopped commercially when hard disks proliferated in the 1980s (Christensen 1993; Christensen and Bower 1996). The mainframe computer which was once the only way to complete complicated mathematical problems has been replaced, first by the minicomputer and most recently by a grid of low cost personal computers. These are but two examples of how dynamic the technology environment can be. The problem becomes one of forecasting accuracy. For example, how likely is it that anyone will accurately forecast the availability and performance of all the technologies utilized in a newly designed aircraft, missile, or combine as it enters production in 2012?

In addition to dynamic technology growth, customer expectations may change during the development of the system. Explicitly the SE process requires a customer's requirements remain constant so alternatives can be compared and a solution found, yet is it known that customers continually reevaluate alternatives based on existing technologies, not on those technologies that existed at the time of their initial requirements specifications (Dilts and Pence 2004) .

Both of these situations demonstrate the need that technology upgrades or pre-planned project improvements be part of the customer's originating specifications so that they can be incorporated at the system design stage. The customer may want the option to insert a technology with a non-linear trajectory into the project sometime in the future. Hence a technique that allows for the option of delaying decisions is required. The financial community has developed and utilized a technique for valuing such an option for years.

1.4 Real Options Analysis and Structure of the Dissertation

When there is high uncertainty and decisions should be delayed until additional information is available, the finance community employs Real Options Analysis (ROA). The Black-Scholes-Merton method (Merton 1973) provides a methodology to value stock options. This research was extended into the realm of "real" items, i.e., physical, not financial, items, and became known as Real Options Analysis (ROA). ROA deals with uncertainty by determining values for delaying decisions until additional information is available. It is the ability of ROA to value the provision of more information at a later date than is typically available to the SE that makes extending ROA into the SE domain important. There are four reasons why ROA is important to SE: 1) when there is high uncertainty, the current technique of NPV fails to correctly estimate the value of alternatives; 2) the decision to build something may be irreversible but the decision to delay building it is always reversible; 3) delaying a decision can have a significant effect on the SE technology decision; and 4) ROA provides a method to calculate the value of delaying a decision, which is not available using NPV.

Chapter 2 extends this discussion of ROA and demonstrates that a Systems Engineering project is analogous to a real option such as a capital investment opportunity. It translates and maps the five basic ROA variables into the SE domain. Two ROA techniques, a deferral option and a compound option, are demonstrated for evaluating an example SE project. The example case is the design, development, and procurement of a new aircraft. A simple deferral option examines the value of waiting one year to make the decision to start the aircraft program until more information is available. The Net Present Value for the project is determined and then compared against the results of the deferral option analysis. Next, a more advanced ROA technique, compound option analysis, examines the situation where expenditures are required at various times to commence the next phase in the program. For this example, there are three expenditures, one each for design, development, and procurement. This type of situation is similar to what occurs in practice during the development of a new aircraft. The difference is in the amount and type of information that is available to the Systems Engineer from ROA versus NPV. I will demonstrate through example the value of ROA to the systems engineer. ROA only address value in the financial sense and cost is only one variable that concerns the systems engineer; performance is another and it is this variable that will be discusses in Chapter 3.

The premise of Chapter 3 is that technology selection occurs as the direct result of selecting a particular design alternative for a system and that when the technology selection is made, design dependent parameters are incorporated into the system's configuration (Blanchard and Fabrycky 1998; Kerzner 2001). It is these design dependent parameters that are either irreversible or reversible only with considerable penalty with regard to time, performance, or cost that make the technology selection especially sensitive to various the forms of risk, such as uncertainty over the technology's future performance, its ability to meet required performance parameters, uncertainty over its maturity, and uncertainty over its potential future availability. Such risk is not adequately addressed by existing SE tools. A new methodology, Technology Options Analysis (TOA), will be developed which will address this deficiency. Chapter 3 starts by demonstrating that technology options exist and are similar, but not 100% comparable, to financial real options. In the development of TOA, various system engineering and management of technology theories are utilized. Some examples are the effects of technology S-curves (Christensen 1993) and clock speed (Fine 1998) of an industry on making the decision. By understanding the current and future predicted performance of a currently available technology and a new technology Chapter 3 defines the five variables utilized in TOA and presents a methodology that provides the decision maker with information about the optimum time to make the technology decision, defined as the cross-over point and the value of waiting until the performance uncertainty is resolved or nearly resolved. One key finding from this chapter is the importance of this cross-over point because this aides the systems engineering decision maker in understanding that choices exists; 1) make the decision today or 2) to wait until the performance uncertainty with a new technology is resolved at the cross-over point.

Since performance and cost are not independent, Chapter 4 combines these two dimensions into a unified algorithm to sequentially evaluate cost and performance alternatives so that a decision maker can gain an overall view of the value of delaying a decision until the most appropriate time. Finally an example using actual data is presented that demonstrates the usefulness of this analysis technique.

Chapter 5 summarizes the findings of the dissertation, discusses the limitations of the research, and presents potential future areas for research.

1.5 Contribution of the Dissertation

By extending Real Options Analysis to include the Net Option Value technique so that the expenditures required for the implementation of multiple technologies during the development cycle are included, a useful value of waiting to make the technology selection is provided to the systems engineering decision maker. Based on this information the systems engineering decision maker should be able to make a more effective decision.

Through the development of Technology Options Analysis, the systems engineering decision maker now has the ability to determine when a new technology will overtake an existing one, how much performance gain might be achieved, and the risk of the new technology meeting its predicted future. The system engineering decision maker now understands the optimum time to make the technology selection and should be able to make a more successful technology decision.

The incorporation of Technology Options and Real Options Analysis into a unified algorithm provides simultaneous evaluation of performance and cost. By understanding the optimum time to make the technology selection based on resolving a new technology's future performance uncertainty and using that information to conduct the cost analysis a more insightful depiction of the value of waiting to make the technology selection is provided to the systems engineering decision maker. This can assist the decision maker in making more effective technical and financial choices.

CHAPTER II

REAL OPTIONS ANALYSIS

Technology selection is key to the success or failure of a systems engineering (SE) project (Sabbagh 1996; GAO 2000; GAO 2001; Rouse 2003; Dilts and Pence 2004; Editorial 2006; Michaels 2006; Flight Level 350 2007; Dilts and Pence IEEE-TEM under review). To arrive at the technology decision, systems engineers need to assess how well each technological alternative performs against various schedule, performance, and cost objectives (Sabbagh 1996; Blanchard and Fabrycky 1998; Sage and Rouse 1999; Buede 2000; Fleeman 2001; Kerzner 2001; Blanchard 2004). In order to compare the cost and financial impact of the various alternatives, a technique to summarize the monetary cost/value of each alternative is necessary and essential. The most common theory used in SE to evaluate cost is the time value of money (Blanchard and Fabrycky 1998; Sage and Rouse 1999; Buede 2000; Kerzner 2001; Barringer 2003; Blanchard 2004) where discounted cash flow (DCF) analysis is used to bring the total life cycle costs for each alternative to their current net present value (NPV).

Each alternative's NPV is then evaluated, where alternatives with a negative NPV may be removed from the potential solution space, and the alternatives with the highest NPV selected. It has been demonstrated that NPV works well when the cost drivers associated with each alternative are well understood and characterized (Hayes and Abernathy 1980). This is not normally the case, however, this early in a SE project's lifecycle (GAO 2001). Technology selection determines and commits the majority of the total life cycle cost of the project based on these technology decisions (Buede 2000; Kerzner 2001; Blanchard 2004). Drucker (2006) and others have advocated that a decision of this magnitude should be made no later than necessary, but as late as possible as more time will normally provide the decision maker with additional information.

NPV analysis has two primary difficulties. First, it has no mechanism to value delaying a decision. Second, it does not have the ability to differentiate between technologies whose associated costs have different volatilities. Real Options Analysis (ROA) on the other hand provides an analysis tool that uses the cost volatility as part of the evaluation process and provides a quantitative value that can be used to evaluate whether delaying a decision has a value greater than making the decision today. The use of ROA to evaluate SE projects, or alternatives within a project, has not been addressed in either the real options or SE literature and is the focus of this chapter. In order for systems engineers to appreciate the power of ROA, a more detailed description and examples of ROA are necessary then were given in Chapter 1. I begin by presenting the basic theories of ROA. Next, the five basic ROA variables are translated and mapped into the SE domain and the specifics of two ROA techniques that calculate the value of delaying a decision are presented. I then conduct sensitivity analysis on the ROA variables so the decision maker has an understanding of how variations in the variables could affect the decision and therefore the risk associated with those decisions. These results will have important implications to the systems engineer for two reasons: 1) the volatility of a technology's cost drivers are now part of the cost evaluation process and may impact the technology selection and 2) a quantitative evaluation can be conducted to determine if delaying a decision until more information is available is of greater value to the project than making the decision today.

2.1 Real Options

A real option is the right, but not the obligation, to take an action such as deferring, expanding, contracting, or abandoning a project at a predetermined cost called the exercise price, for a predetermined period of time or the life of the option (Quigg 1993; Kogut and Kulatilaka 1994; Dixit and Pindyck 1995; Trigeorgis 1995; Abel, Dixit et al. 1996; Trigeorgis 1996; Luehrman 1997; McGrath 1997; Luehrman 1998; Luehrman 1998; Merton 1998; Amram and Kulatilaka 1999; Benaroch and Kauffman 1999; Bollen 1999; Gardner and Rogers 1999; Jarrow 1999; Angelis 2000; Trigeorgis and Brennan 2000; Benaroch 2001; Copeland and Antikarov 2001; Schwartz and Trigeorgis 2001; Benaroch 2002; Razgaitis 2003; Copeland and Tufano 2004; van Putten 2004; Kauffman 2005). While the underlying financial aspects of options were completed by Merton (1973) and Black and Scholes (1973), Myers (1977) was the first to discuss the idea of a real option. Myers (1977) considered the idea that perhaps the cost of capital was being incorrectly specified in NPV and that the equilibrium capitalization rate used to calculate the hurdle rate was overestimated for firms that held valuable real options. Recall that a hurdle rate is the required return on an investment that a firm requires a project to exceed before accepting the project as financially viable. He introduced the concept that the value of a firm as a going concern depended on its future investment strategy and that it was useful to think of the firm as being composed of two distinct asset types: (1) 'real assets', which have market values independent of the firm's investment strategy, and (2) 'real options', which are opportunities to purchase real assets on possibly favorable terms.

Myers and Turnbull (1977) extended this work by noting that growth opportunities are affected by the observed systemic risk and therefore the correct discount rate could not be inferred. Myers acknowledged these shortcomings but continued to utilize the capital asset pricing model (CAPM), weighted average cost of capital, and net present value (NPV) for capital budgeting. However, the limitations of using traditional discounted cash flow (DCF) methods like NPV were starting to be recognized as they made it difficult to estimate the value of a firm's real options when there are many potential strategic options or high uncertainty.

McDonald and Siegel (1986) extended the work done by Myers in capital budgeting beyond the use of traditional DCF models. The question asked by McDonald and Siegel was "What is the appropriate way to decide whether or not to build a facility?" The decision to build a system was irreversible since once decided, the project would go forth until completion, but the decision to defer building was reversible, i.e., a decision maker could always decide to reverse the decision to wait by building the system. It was this asymmetry that their article explored. By assuming that the rate of growth in value for an irreversible project was similar to the growth rate of a stock they showed that the problem could be solved using geometric Brownian motion to determine the optimal time for a firm to invest (McDonald and Siegel 1986). Their research suggested that, under certain conditions, the decision to defer an irreversible investment was more valuable than traditional DCF indicated.

Pindyck (1991) followed this stream of research and he believed that capital investment behavior was poorly understood and that previous models ignored two important characteristics: irreversibility and investment delay. He and others (Quigg 1993; Dixit and Pindyck 1994; Dixit and Pindyck 1995; Trigeorgis 1995; Abel, Dixit et al. 1996; Schwartz and Trigeorgis 2001) showed that the ability to delay an irreversible decision could have a large effect on the decision to invest. This simple yet powerful fact undermined the theoretical foundation of the standard neoclassical investment models and invalidated the net present value rule utilized by the financial community. Pindyck (1991) further demonstrated that an irreversible investment opportunity was similar to a financial call option. He accomplished this by showing that the same methods used by Merton to determine the value of a financial option could be used to value a real option. This led to a wealth of research on applying Real Options Analysis in a wide variety of settings, including: research and development projects, capital investment projects, Information Technology projects, new product development and product life cycles, all in an attempt to more accurately value uncertainty and decision delay. This research resulted in a number of special journal issues (*Quarterly Review of Economics and Finance* (1998) and *Academy of Management Review* (2004) to name two) and ROA research has appeared in the *Harvard Business Review*, *IEEE Transactions on Engineering Management* and numerous other journals and books.

2.2 Real Options and Systems Engineering

In order to fully understand a real option and its potential value to SE, we need to initially focus on the components that make up a real option. In real options the action is about the making of a decision. The decision can be to defer a project, to increase or reduce the scope of the project, or to cancel the project. The cost (in financial terms, the exercise price) of a decision (action) is available to the systems engineer today and will remain valid for a set period of time (life of the option). Like a financial option, a real option depends on five basic variables:

1. S: the value of the underlying risky asset. This is the present value of the project, investment, or acquisition.
2. E: the exercise price. The amount of money needed to buy the asset (call option) or the money that will be received to sell the asset (put option).
3. τ : the time to expiration of the option. The length of time the decision may be deferred.

4. σ^2: the standard deviation of the value of the underlying risky asset. The riskiness of the project assets.
5. r_f: the risk-free rate of interest over the life of the option, i.e., time value of money.

There are four key points that systems engineers need to understand and appreciate. First, when options or alternatives coexist with uncertainty, it is well know that NPV fails to estimate the true value of the alternative (McDonald and Siegel 1986; Pindyck 1991; Dixit and Pindyck 1994; Kogut and Kulatilaka 1994; Dixit and Pindyck 1995; Trigeorgis 1995; Abel, Dixit et al. 1996; Kumar 1996; Trigeorgis 1996; Luehrman 1997; McGrath 1997; Luehrman 1998; Luehrman 1998; Merton 1998; Ahn, Boudoukh et al. 1999; Amram and Kulatilaka 1999; Benaroch and Kauffman 1999; Bollen 1999; Dong-Hyun, Boudoukh et al. 1999; Gardner and Rogers 1999; Angelis 2000; Benaroch and Kauffman 2000; Trigeorgis and Brennan 2000; Copeland and Antikarov 2001; Schwartz and Trigeorgis 2001; Benaroch 2002; Copeland and Tufano 2004; van Putten 2004). Second, the decision to build or initiate a system is irreversible but the decision to delay building it is not. Third, that delaying an irreversible decision can have a significant effect on the possible decision outcomes. And finally, and most importantly, it is possible to calculate the value of delaying a decision using the five basic real options variables presented above.

Within the SE process there exists a tension between desires and implementation. A true but often unstated reality is that systems engineers like to make all major decisions as soon as possible because doing so makes it much easier to manage the system's development. But the changing nature of some technologies makes this infeasible to implement. When Robert Merton (1998) looked back on the application of option-pricing theory he made the following observation:

> Many … option-pricing applications do not involve financial instruments. The family of such applications is called "real" options. … The common element for using option-pricing here is the same … : the future is uncertain (if it were not, there would be no need to create options because we know now what we will do later) and in an uncertain environment, having the flexibility to decide what to do after some of that uncertainty is resolved definitely has value. Option-pricing theory provides the means for assessing that value (1998:339).

As it does with financial instruments, ROA allows the Systems Engineer to have the best of both worlds: the ability to make decisions currently while preserving flexibility for future decisions.

2.3 Variable Mapping: From Financial to SE Domains

In order to demonstrate a SE project can be modeled using real options analysis three conditions must be satisfied: 1) demonstrate that a SE project is analogous to a type of real option; 2) translate the ROA variables into the SE domain; and 3) complete a realistic example problem to show the viability of the technique. With regard to the first condition, the analogy between financial options and corporate investments that create future opportunities has been well researched and documented (Pindyck 1991; Quigg 1993; Dixit and Pindyck 1994; Trigeorgis 1995; Abel, Dixit et al. 1996; Trigeorgis 1996; McGrath 1997; McGrath 1998; Ahn, Boudoukh et al. 1999; Amram and Kulatilaka 1999; Benaroch and Kauffman 1999; Bollen 1999; Dong-Hyun, Boudoukh et al. 1999; Gardner and Rogers 1999; McGrath 1999; Angelis 2000; Taudes, Feurstein et al. 2000; Trigeorgis and Brennan 2000; Copeland and Antikarov 2001; Schwartz and Trigeorgis 2001; Benaroch 2002; Razgaitis 2003; Kauffman 2005; Benaroch 2006). The previous chapter and earlier sections of this chapter, conceptually discussed how SE projects are analogous to real options but this chapter will further pursue this by use of examples.

With regard to the second condition, a mapping of the variables is shown in Table 2-1. This requirement is accomplished by showing how financial call option variables are mapped onto a capital investment opportunity, and the recognition that a capital investment is analogous to a SE project.

Table 2-1. Mapping Call Option Variables into a Capital Investment Opportunity and a Systems Engineering Project

Variable	*Call Option*	*Capital Investment Opportunity*	*SE Project*
S	Stock Price	Present value of a project's operating assets to be acquired	Present value of the SE project if it existed today
E	Exercise Price	Expenditure required to acquire the project asset	Investments required to design, develop, build, and test the SE project
τ	Time to expiration	Length of time the decision may be deferred	Length of time the decision may be deferred
r_f	Risk-free rate of return	Risk-free rate of return	Risk-free rate of return
σ^2	Variance of returns on stock	Riskiness of the project's assets	Riskiness of the project

A capital investment opportunity involves expending resources to design and build something that accomplishes a set of objectives (Pindyck 1991; Dixit and Pindyck 1995; Trigeorgis 1995; Gardner and Rogers 1999). One of the most common examples used in the real options literature for a capital investment is the building of a plant (Pindyck 1991; Gardner and Rogers 1999). Bringing a plant online involves expending resources to design, build, and furnish a structure that meets the desires and needs of the owners and investors. This is analogous to bringing a Boeing 777, or the Joint Strike Fighter, or the Federal Aviation Administration Wide-Area Augmentation System into existence.

The first variable of interest, S, for a capital investment opportunity, is the present value of the project's operating assets to be acquired. For an SE project this is similar to the present value of the SE project if it existed today. The expenditure (E) required to acquire the capital investment's asset is comparable to the investment necessary to design, develop, build, and test a SE project. "τ" defines the length of time the decision to start or abandon the opportunity or project can be delayed. The risk-free rate of return (r_f) defines what would be earned if "E" was placed in a risk-free investment like a savings account. Both "τ" and r_f have the same meaning in the SE and ROA domains. The uncertainty associated with a capital investment's asset is captured as σ^2. From an SE perspective, this is the uncertainty associated with the cost of the project. In order to more clearly demonstrate the porting of real options variables into the SE domain it is useful to utilize an example.

Dixit and Pindyck (1994) give a simple example of a capital investment opportunity. Consider a decision that must be made today to either expend \$1600K (E) in a capital investment such as a plant now, or to defer it until the end of the year (τ). Once made, the investment is irreversible (e.g., the plant has no salvage value) and the risk-free rate of return (r_f) over the life of the option is 10%. The present value of the plant's operating assets to be acquired (S) are calculated based on a first year output of $200K with an equal (50%) chance of either going up to $300K or down to $100K per year. This upward or downward change is the revenue volatility (σ^2) and is equal to 50% or .5. In either case, the change in output is assumed to be permanent.

Now, instead of a plant consider a SE project such as building an aircraft where the decision is whether to invest $1600K (E) to bring an aircraft to market or to defer the decision one year (τ). Once the decision is made it is irreversible and the risk-free rate of return (r_f) over the life of the option is 10%. The present value of the plane if it existed today (S) will be calculated based on its first year revenue of $200K. After the first year there is an equal (50%) chance or risk (σ^2) revenue will go up to $300K or decrease to $100K per year. This upward or downward change is the revenue volatility (σ^2) and is equal to 50% or .5. In either case the change in revenue is assumed to be permanent.

By way of these parallel examples, the two conditions considered necessary prior to modeling a SE project using real options analysis have been met: 1) demonstrate that a SE project is analogous to a type of real option, a capital investment opportunity and then; 2) translate the ROA variables into the SE domain.

By translating the ROA variables into the SE domain, the third condition can now be evaluated. Two ROA techniques will be used to demonstrate satisfying this condition. First an example of a simple deferral option will be presented. Next a compound sequential option problem will be developed that demonstrates how ROA can accommodate a phased SE development approach. In both cases the information provided will be compared and contrasted with that of their corresponding NPV analyses.

2.4 Net Present Value

Systems Engineers currently use traditional discounted cash flow (DCF) methods to assess a project by computing the project's net present value (NPV) (Newman 1980; Blanchard and Fabrycky 1998; Buede 2000; Kerzner 2001; Blanchard 2004). Simply stated, NPV is the difference between how much the assets (S) are worth (their present value) and how much they cost or the required expenditure (E). Table 2.2 defines the variables used in calculating Net Present Value.

Table 2-2. Net Present Value Variables and Meaning

Net Present Value Variables	*Meaning*
S	Present value of the SE project if it existed today
E	Investment required to design, develop, build, and test the SE project
r_f	Risk-free rate of return
P	Probability of the revenue increasing
(1-p)	Probability of the revenue decreasing
O_h	Value of the increased revenue (h=high or increasing)
O_l	Value of the decreased revenue (l=low or decreasing)

Let us revisit our earlier example: consider a decision whether to invest $1600K to design a new aircraft today or wait a year. The revenue generated by the aircraft today would be $200K for the first year and then either $300K or $100K for the second year on. Once the decision is made it is irreversible, that is, the airplane will be built.

The Net Present Value (NPV) formula is:

$$\text{NPV} = \text{S} - \text{E}$$

In this example E equals \$1600K and S needs to be determined.

S is the sum of the annual revenues after being discounted to the present. The revenue generated at the end of the first year is \$200K. The potential revenue stream for year two onward is either \$300K per year or \$100K per year with an equal probability therefore:

$$\begin{aligned}\text{RevenueForYearTwoOn} &= \text{p} * \text{O}_\text{h} + (1-\text{p}) * \text{O}_\text{l}\\ &= .5(\$300\text{K}) + .5(\$100\text{K})\\ &= 150\text{K} + 50\text{K} = 200\text{K per year}\end{aligned}$$

Using this information, S can be determined by discounting the future revenues using the risk free rate of return:

$$\text{S} = \sum_{t=0}^{\infty} \frac{\text{RevenueForYear}(t)}{(1+r_f)^t} = \sum_{t=0}^{\infty} \frac{\$200\text{K}}{(1+.1)^t} = \$2200\text{K}$$

Now that both S and E are know the NPV can be calculated.

$$\text{NPV} = \text{S} - \text{E}$$

$$\text{NPV}_{\text{E=\$1600}} = \text{S - E} = \$2{,}200\text{K} - \$1{,}600\text{K} = \$600\text{K}$$

When the NPV is positive as in this example, the decision is to invest and the investment occurs immediately.

In order to determine how sensitive this decision is to E, suppose the investment required for the aircraft increases by 50% to \$2,400K. The NPV in this example is:

$$\text{NPV}_{\text{E=\$2400}} = \text{S - E} = \$2{,}200\text{K} - \$2{,}400\text{K} = -\$200\text{K}$$

With the NPV negative, the decision is not to invest and the project would never be started.

These decisions are predicated on the assumption that no further information will become available. But what if more information will be available at the end of Year 1, and ROA can utilize this information? Let us examine this by first looking at the simplest form of a ROA, that of a deferral option.

2.5 Deferral Option

Within real options analysis a deferral option, involves making a decision to 1) expend resources today to acquire a system, 2) to defer the expenditure for a finite period of time, or 3) to never acquire the system. We can easily calculate the deferral option's value for the above example since Cox, Ross, and Rubinstein (1979) utilized probability theory to develop a binomial lattice approach to option pricing that employs discrete mathematics to achieve isomorphic results which are equivalent to the calculus used by Black-Scholes (1973). From a Systems Engineer's point of view, the advantage is that discrete mathematics is algebraic in nature and simpler to understand than are stochastic differential equations. A binomial decision tree or lattice approach is the most common way to solve these types of problems (Cox, Ross et al. 1979; Luehrman 1997; Copeland and Antikarov 2001; Copeland and Tufano 2004; van Putten 2004). Table 2.3 defines the variables used in calculating a Deferral Option.

Table 2-3. Deferral Option Variables

Deferral Option Variables	*Meaning*
$S_{i,j}$	Value of the SE project at point i,j i and j define the variable's position in the lattice
$PV_{i,j}$	Present Value of the SE project at point i,j
E	Investment required to design, develop, build, and test the SE project
τ	Length of time the decision may be deferred
r_f	Risk-free rate of return
p	Probability of the next outcome increasing
(1-p)	Probability of the next outcome decreasing
OV	Option Value
OC	Option Cost
NOV	Net Option Value

For this problem the lattice consists of three points defined by i and j such that: 1) i=j=0 is the initial point or in this example year one; 2) i=1and j=0 is the point associated with the revenue going up to $300K; and 3) i=0 and j=1 is the point associated with revenue going down to $100K.

The present value ($PV_{i,j}$), for any point i,j in the lattice, is determined using one of two formulas;

1) if an expenditure is required in order to realize the project at a point i,j then, the expenditures (E) are subtracted from the value of the project ($S_{i,j}$) associated with point i,j but in no case can $PV_{i,j}$ ever be less than zero. This leads to the following formula:

$PV_{i,j} = MAX[S_{i,j} – E, 0]$

2) if no expenditures are required then $PV_{i,j}$ is a function of $PV_{i+1,j}$ and $PV_{i,j+1}$ and depends on the probability of realizing either value such that:

$PV_{i,j} = (p * PV_{i+1,j}) + ((1\text{-}p)*PV_{i,j+1})$

Using the following information from the NPV example:

$S_{0,0}$ = \$200K, $S_{1,0}$ = \$300K, $S_{0,1}$ = \$100K
E = \$1,600K
p = 50% = .5
(1-p) = 1 - .5 = .5
r_f = 10% = .1
τ = 1 year

Then,

$$\begin{aligned}
\mathrm{PV}_{1,0} &= MAX\left[\sum_{\tau=1}^{\infty}\frac{\mathrm{S}_{1,0}}{(1+r_f)^{\tau}} - \frac{\mathrm{E}}{1+r_f}, 0\right]\\
\mathrm{PV}_{1,0} &= MAX\left[\sum_{\tau=1}^{\infty}\frac{\$300\mathrm{K}}{(1+.1)^{\tau}} - \frac{\$1{,}600\mathrm{K}}{1+.1}, 0\right]\\
\mathrm{PV}_{1,0} &= MAX\left[\frac{\$3300\mathrm{K}-\$1600\mathrm{K}}{1.1}, 0\right] = MAX\left[\frac{\$1700\mathrm{K}}{1.1}, 0\right] = MAX[\$1545\mathrm{K}, 0]\\
\mathrm{PV}_{1,0} &= \$1545\mathrm{K}
\end{aligned}$$

$$\mathrm{PV}_{0,1} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$100\mathrm{K}}{(1+.1)^{\tau}} - \frac{\$1{,}600\mathrm{K}}{1+.1}, 0\right] = MAX[-455\mathrm{K}, 0] = 0$$

$$\begin{aligned}
\mathrm{PV}_{0,0} &= \mathrm{p} * \mathrm{PV}_{1,0} + (1-\mathrm{p}) * \mathrm{PV}_{0,1}\\
\mathrm{PV}_{0,0} &= .5 * \$1545\mathrm{K} + (1-.5) * \$0 = \$773\mathrm{K}
\end{aligned}$$

The option value (OV) of deferring the decision is the difference between the present value at point 0,0 and the net present value calculation. For this example;

$OV_{E=1600} = PV_{0,0} – NPV_{E=1600}$ = \$773K - \$600K = \$173K

Systems engineers realize that there may be a cost associated with deferring a decision and this cost is captured in the variable OC. Assume the cost to wait for one year was determined to be \$75K then the net option value (NOV) can be calculated by subtracting the option cost (OC) from the option value (OV) or

$$NOV_{E=1600} = OV_{E=1600} - OC_{E=1600}$$
$$NOV_{E=1600} = \$173K - 75K = \$98K$$

For this example the decision is to defer the decision one year. The deferral option analysis provides the decision maker with five key pieces of information. First, if the revenue changes at the end the first year to the high return (\$300K per year), the present value at year one ($PV_{1,0}$) is \$1545K and, if the decision had been deferred, the decision would be to build the aircraft. Second, if the revenue at the end of the first year changes to the low return (\$100K per year), the present value at year one ($PV_{0,1}$) would be zero and, if the decision was deferred, the decision would be to abandon. This is important because it shows that if the decision is delayed until more information is available, e.g., the annual revenue after the first year is known, then two different outcomes should occur. The third piece of information is that the present value today ($PV_{0,0}$) of the deferral is \$773K, which leads to the fourth piece of information the value of the option or the option value ($OV_{E=1600}$) being equal to \$173K, which is the difference between the ROA PV and the NPV value. The fifth piece of information, the net option value (NOV), is perhaps the most important information available to the systems engineer because if the cost of the option (OC) is less than the value of the option (OV) the decision to defer is more valuable than starting the project today.

Now consider the second NPV example where E=\$2,400K,

$$\mathrm{PV}_{1,0} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$300\mathrm{K}}{(1+.1)^{\tau}} - \frac{\$2{,}400\mathrm{K}}{1+.1}, 0\right] = MAX[\$818\mathrm{K}, 0] = \$818\mathrm{K}$$

$$\mathrm{PV}_{0,1} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$100\mathrm{K}}{(1+.1)^{\tau}} - \frac{\$2{,}400\mathrm{K}}{1+.1}, 0\right] = MAX\left[-\$1{,}181\mathrm{K}, 0\right] = \$0$$

$$\mathrm{PV}_{0,0} = .5 * \$818\mathrm{K} + (1-.5) * \$0 = \$409\mathrm{K}$$

$$\mathrm{OV}_{E=2400} = \mathrm{PV}_{0,0} - \mathrm{NPV}_{E=2400} = \$409\mathrm{K} - \$0 = \$409\mathrm{K}$$

$$\mathrm{NOV} = \mathrm{OV} - \mathrm{OC} = \$409\mathrm{K} - \$75\mathrm{K} = \$334\mathrm{K}$$

In this example, the value of deferring is even more valuable than in the previous example as the deferral option analysis demonstrates. Because NOV > 0, the best decision is to defer the expenditure one year rather than to never start the system, as recommended by NPV analysis.

Next we will consider examples where the volatility of the revenue varies. In the third and fourth example the present value will be based on first year revenue of $200K that varies by ±75%, that is, it has an equal likelihood of going up to $350K or down to $50K per year from year two onward. The fifth and sixth examples will have the present value being based on first year revenue of $200K that varies by ±10% or has an equal chance of either going up to $220K or down to $180K per year from year two onward.

In all examples the change is assumed to be permanent. In order to calculate the NPV the future revenues must be determined:

$$\begin{aligned}\mathrm{RevenueForYearTwoOnAt75percent} &= \mathrm{p} * \mathrm{O_h} + (1-\mathrm{p}) * \mathrm{O_l} \\ &= .5(\$350\mathrm{K}) + .5(\$50\mathrm{K}) \\ &= 175\mathrm{K} + 25\mathrm{K} = 200\mathrm{K} \text{ per year}\end{aligned}$$

$$\begin{aligned}\mathrm{RevenueForYearTwoOnAt10percent} &= \mathrm{p} * \mathrm{O_h} + (1-\mathrm{p}) * \mathrm{O_l} \\ &= .5(\$220\mathrm{K}) + .5(\$180\mathrm{K}) \\ &= 110\mathrm{K} + 90\mathrm{K} = 200\mathrm{K} \text{ per year}\end{aligned}$$

Of interest to the systems engineer should be that the value of the revenue for year two on does not change no matter how large a difference between the high and low variance values. The values for revenue for year two on are identical to the previous example when year 2 on varied by 50%. This means that the value of S will not change and therefore the NPV remains the same. In the example where the investment (E) is \$1600K the NPV will still equal \$600K in spite of the changes to the volatility of the future revenue. However, with Real Options Analysis, the values for $S_{1,0}$ and $S_{0,1}$ are different and therefore the values of $PV_{1,0}$, $PV_{0,1}$, $PV_{0,0}$, OV, and NOV need to be calculated for both examples.

In the example where the volatility is +/- 75%:

$S_{0,0} = \$200K$, $S_{1,0} = \$350K$, $S_{0,1} = \$50K$
$E = \$1,600K$

Then,

$$PV_{1,0} = MAX\left[\sum_{\tau=1}^{\infty}\frac{S_{1,0}}{(1+r_f)^{\tau}} - \frac{E}{1+r_f},0\right]$$

$$PV_{1,0} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$350K}{(1+.1)^{\tau}} - \frac{\$1,600K}{1+.1},0\right]$$

$$PV_{1,0} = MAX\left[\frac{\$3850K - \$1600K}{1.1},0\right] = MAX\left[\frac{\$2250K}{1.1},0\right] = MAX[\$2045K,0]$$

$$PV_{1,0} = \$2045K$$

$$PV_{0,1} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$50K}{(1+.1)^{\tau}} - \frac{\$1,600K}{1+.1},0\right] = MAX[-\$954K,0] = 0$$

$$PV_{0,0} = p * PV_{1,0} + (1-p) * PV_{0,1}$$

$$PV_{0,0} = .5 * \$2045K + (1-.5) * \$0 = \$1022K$$

$$OV_{\sigma=+/-75,E=1600} = PV_{0,0} - NPV_{E=1600} = \$1022K - \$600K = \$422K$$

$$NOV_{\sigma=+/-75,E=1600} = OV_{\sigma=+/-75,E=1600} - OC_{E=1600}$$

$$NOV_{\sigma=+/-75,E=1600} = \$422K - 75K = \$347K$$

In summary, the decisions with E=1600 and high volatility (+/-75%) are:

- At Year 1, if "up" occurs, invest ($PV_{1,0} = \$2045K$)

- At Year 1, if "down" occurs, abandon the option ($PV_{0,1}$ = \$0) ,
- Today, Year 0, defer the decision one year ($NOV_{\sigma=+/-75,E=1600}$ = \$347K)

Based on this information, the decision maker would be better off deciding today to defer the expenditure one year rather than to start today because the NOV > 0.

In the example where the volatility is +/- 10%:

$S_{0,0}$ = \$200K, $S_{1,0}$ = \$220K, $S_{0,1}$ = \$180K

E = \$1,600K

Then,

$$PV_{1,0} = MAX\left[\sum_{\tau=1}^{\infty}\frac{S_{1,0}}{(1+r_f)^{\tau}} - \frac{E}{1+r_f}, 0\right]$$

$$PV_{1,0} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$220K}{(1+.1)^{\tau}} - \frac{\$1{,}600K}{1+.1}, 0\right]$$

$$PV_{1,0} = MAX\left[\frac{\$2420K - \$1600K}{1.1}, 0\right] = MAX\left[\frac{\$820K}{1.1}, 0\right] = MAX[\$745K, 0]$$

$$PV_{1,0} = \$745K$$

$$PV_{0,1} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$180K}{(1+.1)^{\tau}} - \frac{\$1{,}600K}{1+.1}, 0\right] = MAX[\$345K, 0] = \$345K$$

$$PV_{0,0} = p * PV_{1,0} + (1-p) * PV_{0,1}$$

$$PV_{0,0} = .5 * \$745K + (1-.5) * \$345 = \$545K$$

$OV_{\sigma=+/-10,E=1600} = PV_{0,0} - NPV_{E=1600}$ = \$545K - \$600K = -\$55K

$NOV_{\sigma=+/-10,E=1600} = OV_{\sigma=+/-10,E=1600} - OC_{E=1600}$

$NOV_{\sigma=+/-10,E=1600}$ = -\$55K – 75K = -\$130K

In summary, the decisions with E=1600 and low volatility (+/-10%) are:

- At Year 1, if "up" occurs, invest ($PV_{1,0}$ = \$745K)
- At Year 1, if "down" occurs, invest ($PV_{0,1}$ = \$345) ,

- Today, year 0, do not defer the decision (NOV $_{\sigma=+/-10,E=1600}$ = -\$130K) and build the system (NPV>0)

Based on this information, the decision maker would be better off deciding today not to defer the expenditure one year but rather starting it now because NOV < 0 and NPV > 0.

In the example where the investment (E) is \$2400K the NPV is still equal to -\$200K but the values for $S_{1,0}$ and $S_{0,1}$ are different and therefore the values of $PV_{1,0}$, $PV_{0,1}$, $PV_{0,0}$, OV, and NOV need to be calculated for both examples.

In the example where the volatility is +/- 75%:

$S_{0,0} = \$200K$, $S_{1,0} = \$350K$, $S_{0,1} = \$50K$

$E = \$2,400K$

Then,

$$PV_{1,0} = MAX\left[\sum_{\tau=1}^{\infty}\frac{S_{1,0}}{(1+r_f)^{\tau}} - \frac{E}{1+r_f}, 0\right]$$

$$PV_{1,0} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$350K}{(1+.1)^{\tau}} - \frac{\$2,400K}{1+.1}, 0\right]$$

$$PV_{1,0} = MAX\left[\frac{\$3850K - \$2400K}{1.1}, 0\right] = MAX\left[\frac{\$1450K}{1.1}, 0\right] = MAX[\$1318K, 0]$$

$$PV_{1,0} = \$1318K$$

$$PV_{0,1} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$50K}{(1+.1)^{\tau}} - \frac{\$2,400K}{1+.1}, 0\right] = MAX[-\$1850K, 0] = 0$$

$$PV_{0,0} = p * PV_{1,0} + (1-p) * PV_{0,1}$$

$$PV_{0,0} = .5 * \$1318K + (1-.5) * \$0 = \$659K$$

$$OV_{\sigma=+/-75,E=2400} = PV_{0,0} - NPV_{E=2400} = \$659K - \$0 = \$659K$$

$\text{NOV}_{\sigma=+/-75,E=2400} = \text{OV}_{\sigma=+/-75,E=2400} - \text{OC}_{E=2400}$

$\text{NOV}_{\sigma=+/-75,E=2400} = \$659\text{K} - 75\text{K} = \$584\text{K}$

In summary, the decisions with E=2400 and high volatility (+/-75%) are:

- At Year 1, if "up" occurs, invest ($\text{PV}_{1,0} = \$1318\text{K}$)
- At Year 1, if "down" occurs, abandon the option ($\text{PV}_{0,1} = \$0$) ,
- Today, year 0, defer the decision 1 year ($\text{NOV}_{\sigma=+/-75,E=2400} = \$584\text{K}$)

Based on this information, the decision maker would be better off deciding today to defer the expenditure one year rather than not to start it at all because the NOV > 0.

In the example where the volatility is +/- 10%:

$\text{S}_{0,0} = \$200\text{K}$, $\text{S}_{1,0} = \$220\text{K}$, $\text{S}_{0,1} = \$180\text{K}$

$\text{E} = \$2{,}400\text{K}$

Then,

$$\text{PV}_{1,0} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\text{S}_{1,0}}{(1+r_f)^\tau} - \frac{\text{E}}{1+r_f},0\right]$$

$$\text{PV}_{1,0} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$220\text{K}}{(1+.1)^\tau} - \frac{\$2{,}400\text{K}}{1+.1},0\right]$$

$$\text{PV}_{1,0} = MAX\left[\frac{\$2420\text{K} - \$2400\text{K}}{1.1},0\right] = MAX\left[\frac{\$20\text{K}}{1.1},0\right] = MAX[\$18\text{K},0]$$

$$\text{PV}_{1,0} = \$18\text{K}$$

$$\text{PV}_{0,1} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$180\text{K}}{(1+.1)^\tau} - \frac{\$2{,}400\text{K}}{1+.1},0\right] = MAX[-\$381\text{K},0] = \$0$$

$$\text{PV}_{0,0} = \text{p} * \text{PV}_{1,0} + (1-\text{p}) * \text{PV}_{0,1}$$

$$\text{PV}_{0,0} = .5 * \$18\text{K} + (1-.5) * \$0 = \$9\text{K}$$

$\text{OV}_{\sigma=+/-10,E=2400} = \text{PV}_{0,0} - \text{NPV}_{E=2400} = \$9\text{K} - \$0 = \$9\text{K}$

$$NOV_{\sigma=+/-10,E=2400} = OV_{\sigma=+/-10,E=2400} - OC_{E=2400}$$

$$NOV_{\sigma=+/-10,E=2400} = \$9K - 75K = -\$66K$$

In summary, the decisions with E=2400 and low volatility (+/-10%) are:

- At Year 1, if "up" occurs, invest ($PV_{1,0}$ = \$18K)
- At Year 1, if "down" occurs, abandon ($PV_{0,1}$ = \$0) ,
- Today, year 0, do not defer ($NOV_{\sigma=+/-10,E=2400}$ = -\$66K) and do not build the system because the original net present value was negative (NPV<0)

Based on this information, the decision maker would be better off deciding today not to defer the expenditure one year because NOV < 0 and never to start it because NPV <0. Table 2-4 presents the deferral option results based on the required expenditure (E) and the volatility (σ) of the revenue.

Table 2-4. Deferral Option Results

Example #	*E =* *σ =*	*NPV*	*NPV Decision*	*NOV*	*NOV Decision*
3	E=\$1600K σ=.75	\$600K	Build	\$347K	Defer
1	E=\$1600K σ=.5	\$600K	Build	\$98K	Defer
4	E=\$1600K σ=.1	\$600K	Build	-\$130K	Do not Defer – Decision based on NPV
5	E=\$2400K σ=.75	-\$200K	Abandon	\$584K	Defer
2	E=\$2400K σ=.5	-\$200K	Abandon	\$334K	Defer
6	E=\$2400K σ=.10	-\$200K	Abandon	-66K	Do not Defer – Decision based on NPV

These simple examples show the power of Real Options Analysis. The decision maker received five times the amount of information than was available using traditional NPV analysis and based on that information different decisions should be made. These, of course, are simple examples and do not reflect the more complex aspects of SE. There is an ROA technique that can be used for the more complex SE situations where the program employs a phased approach: compound options.

2.6 Compound Options – Sequential Options

In complex situations ROA makes use of compound options. Compound options occur often and were recognized as an important problem by Black/Scholes (1973) and first solved by Geske (1977). One of the most common compound option scenarios is when an option's value depends on another option, for example the first option (chronologically) is the right to buy the second option. In this phased investment situation where one option's value depends on another, sequential options analysis provides the decision maker with more information then is provided by NPV. Table 2-5 defines the variables used in calculating a Compound Option.

Table 2-5. Compound Option Variables

Compound Option Variable	*Meaning*	*Comments*
$S_{i,j}$	Value of the SE project at point i,j	i and j define the variable's position in the lattice
$E^N{}_y$	Investment required to design, develop, build, and test the SE project	N designates different sequential investments over time, i.e., 1st, 2nd, etc. y designates the year in which the investment actually takes place
τ	Length of time the decision may be deferred	
r_f	Risk-free rate of return	
σ	Volatility of project	Used to calculate the up and down factors
U	Up Factor	$u = e^{\sigma\sqrt{\tau}}$
D	Down Factor	d= 1/u
P	Probability of the $PV^N{}_{i,j,y}$ increasing	
(1-p)	Probability of $PV^N{}_{i,j,y}$ decreasing	
$PV^N{}_{i,j,y}$	Present Value of the SE project at point i,j evaluated using $E^N{}_y$ if appropriate	
OV	Option Value	
OC	Option Cost	
NOV	Net Option Value	

Consider the following example where an aircraft is constructed in phases. The first phase, an initial design phase cost ($E^1{}_0$) of \$250K must be started immediately. Following the design phase and at the end of the first year an expenditure ($E^2{}_1$) of \$750K is necessary to commence the detailed engineering phase. Once the detailed engineering phase is complete an expenditure (E^3_2 or E^3_3) of \$1,500K will be required within two years to build the first aircraft. By using the SE approach to designing an aircraft in multiple phases, a compound option results where the \$250K expenditure creates the right to expend \$750K at the end of the first year, and the exercise of that choice creates the right to expend \$1500K to purchase a new asset, an aircraft in either Year 2 or Year 3.

The first step is estimating what the value of the aircraft would be if it existed today. This can be done using traditional DCF methods. The second step is estimating how much this value is likely to move up or down during the period in question. In the deferral examples we assumed that the change in value from Year 2 on would be permanent, in this compound option example this is not assumed. Instead the distribution of possible aircraft values will be considered to be fairly standard and assumed to follow a log-normal distribution. This will mean that the factor to apply for an up movement is given by the formula *e* to the power of σ (volatility or variance) times the square root of τ (time interval) and the factor for a down movement is given by the inverse of the up factor. Other formulas can be used in cases where the distribution is not lognormal.

Assuming a lognormal distribution centered at \$200K with a variance of 50% the value of the aircraft if it existed today ($S_{0,0}$) can be determined using a standard DCF method:

$$S_{0,0} = \text{SalvageValue} + \text{AveReturn} = 0 + \sum_{t=0}^{\infty} \frac{\$200\text{K}}{(1+1)^t} = \$2200$$

The distribution of the possible aircraft values is then a multiplicative process that starts at $S_{0,0}$ and moves up or down based on σ with the up factor given by $e^{\sigma\sqrt{\tau}}$ and the down factor by 1/ $e^{\sigma\sqrt{\tau}}$. In this example the volatility (σ) is 50% or .5 and τ = 1 so the up factor u=$e^{\sigma\sqrt{\tau}} = e^{.5} =$ 1.65 and the down factor d=1/u=1/1.65=0.61.

$S_{1,0} = u * S_{0,0} = 1.65 * \$2200K = \$3627K$

$S_{0,1} = d * S_{0,0} = 0.61 * \$2200K = \$1334K$

Table 2-6 presents the $S_{i,j}$ values for the aircraft through Year 3. The detailed calculations are presented in Appendix B.

Table 2-6. Possible Aircraft Values (S)

	Today		*Year 1*		*Year 2*		*Year 3*
						$S_{3,0} =$	\$9860K
				$S_{2,0} =$	\$5980K		
		$S_{1,0} =$	\$3627K			$S_{2,1} =$	\$3627K
$S_{0,0} =$	\$2200K			$S_{1,1} =$	\$2200K		
		$S_{0,1} =$	\$1334K			$S_{1,2} =$	\$1334K
				$S_{0,2} =$	\$809K		
						$S_{0,3} =$	\$491K

The next step in sequential option analysis is to calculate and evaluate the present values ($PV^{N}_{i,j,y}$) at the point time that the last investment takes place, Year 3 in this example. The reason for starting the evaluation when the last investment takes place is because if all of the $PV^{N}_{i,j,y}$ values at those points are \$0 then the project should be abandoned and no other analysis is required. This is because the expenditures will always be more than the revenues the project can generate. The same techniques demonstrated in the deferral options section are applied for sequential options.

$PV^{N}_{i,j,y} = \mathit{MAX}[S_{i,j} - E^{N}_{y}, 0]$

$PV^{3}_{3,0,3} = \mathit{MAX}(S_{3,0} - E^{3}_{3}, 0) = \mathit{MAX}(\$9860K - \$1500K, \$0) = \mathit{MAX}(\$8360K, \$0)$

$PV^{3}_{3,0,3} = \$8860K$

$PV^{3}_{2,1,3} = \mathit{MAX}(\$3627K - \$1500K, \$0) = \mathit{MAX}(\$2127K, \$0) = \$2127K$

$PV^{3}_{1,2,3} = \mathit{MAX}(\$1334K - \$1500K, \$0) = \mathit{MAX}(- \$166K, \$0) = \$0$

$PV^{3}_{0,3,3} = \mathit{MAX}(\$491K - \$1500K, \$0) = \mathit{MAX}(- \$1009K, \$0) = \$0$

Two potential points are positive ($PV^3_{3,0,3}$, $PV^3_{2,1,3}$) therefore the sequential option analysis will continue.

To find the $PV^N_{i,j,y}$ at Year 2, two alternatives will need to be examined and then compared in order to determine the maximum $PV^N_{i,j,y}$. This is because in this example the production of the aircraft could start in Year 2 or be deferred until Year 3. The first set of calculations determines the project's value when the decision is to build the aircraft in Year 2.

$$PV^3_{2,0,2} = MAX(S_{2,0} - E_3, 0) = MAX(\$5980K - \$1500K, 0) = MAX(\$4480K, 0)$$
$$PV^3_{2,0,2} = \$4480K$$
$$PV^3_{1,1,2} = MAX(\$2200K - \$1500K, 0) = \$700K$$
$$PV^3_{0,2,2} = MAX(\$809K - \$1500K, 0) = \$0$$

The next set of calculations will determine the project's value in Year 2 if the decision is to defer building the aircraft until Year 3. In the deferral examples in the previous section the probability of high or low return was given. For this example the probability (p) of high will have to be calculated and the probabilistic method developed by Cox, Ross, and Rubinstein (1979) will be utilized (a detailed derivation is provided in Appendix B);

$$p = \frac{1 + r_f - d}{u - d} = \frac{1 + 0.1 - 0.61}{1.65 - 0.61} = \frac{.49}{1.04} = .47 \text{ and } 1\text{- } p = .53$$

The same techniques demonstrated in the deferral option section are applied to determine $PV^N_{i,j,y}$ but in the generalized case.

$$PV^N_{i,j,y} = MAX(\frac{(p * u * PV^N_{i,j,y}) + ((1-p) * d * PV^N_{i,j,y})}{(1 + r_f)}, 0)$$
$$PV^3_{2,0,3} = MAX(\frac{((0.47 * \$8360K) + (0.53 * \$2127K))}{(1 + 0.1)}, 0)$$
$$PV^3_{2,0,3} = MAX(\frac{(\$3958K + \$1120K)}{1.1}, 0)$$
$$PV^3_{2,0,3} = MAX(\frac{\$5078K}{1.1}, 0)$$

$$PV^3_{2,0,3} = MAX(\$4617K, 0) = \$4617K$$
$$PV^3_{1,1,3} = MAX(\$916K, 0) = \$916K$$
$$PV^3_{0,2,3} = MAX(\$0, \$0) = \$0$$

The final step is to compare the two sets of results and select the maximum value ($PV^N_{i,j,y}$) for Year 2.

$$PV^3_{2,0,y} = MAX(PV^3_{2,0,2}, PV^3_{2,0,3}) = MAX(\$4480K, \$4617K) = \$4617K = PV^3_{2,0,3}$$
$$PV^3_{1,1,y} = MAX(\$700K, \$916K) = \$916K = PV^3_{1,1,3}$$
$$PV^3_{0,2,y} = MAX(\$0, \$0) = \$0 = PV^3_{0,2,3}$$

In this example, the value of deferring the building of the aircraft until Year 3 was greater in all cases than the value of building it in Year 2. The remaining $PV^N_{i,j,y}$ values for Years 1 and 0 are now calculated.

$$PV^2_{1,0,1} = MAX(\frac{((0.47 * \$4616K) + (0.53 * \$916K))}{(1+0.1)} - \$750K, 0)$$
$$PV^2_{1,0,1} = MAX(\frac{(\$2186K + \$482K)}{1.1} - \$750K, 0)$$
$$PV^2_{1,0,1} = MAX(\frac{\$2668K}{1.1} - \$750K, 0)$$
$$PV^2_{1,0,1} = MAX(\$2425K - \$750K, 0) = MAX(\$1675K, 0) = \$1675K$$
$$PV^2_{0,1,1} = MAX(\$394K - \$750K, 0) = MAX(-\$356K, 0) = \$0$$
$$PV^1_{0,0,0} = MAX(\frac{((0.473 * \$1675K) + (0.527 * \$0))}{(1+0.1)} - \$250K, 0)$$
$$PV^1_{0,0,0} = MAX(\$721K - \$250K, 0) = MAX(\$471K, 0) = \$471K$$

Table 2-7 presents the $PV^N_{i,j,y}$ values for the aircraft and the decision associated with those values.

Table 2-7. Compound Option Present Values

	Today		*Year 1*		*Year 2*		*Year 3*
Expenditure	$E^1_0=\$250K$		$E^2_1=\$750K$				$E^3_3=\$1500K$
						$PV^3_{3,0,3}$ =	$8360K
				$PV^3_{2,0,3}$ =	$4617K		***Procure Aircraft***
		$PV^2_{1,0,1}$ =	$1675K		***Defer Procurement***	$PV^3_{2,1,3}$ =	$2127K
$PV^1_{0,0,0}$ =	$471K		***Start Detailed Design***	$PV^3_{1,1,3}$ =	$916K		***Procure Aircraft***
	Start Preliminary Design	$PV^2_{0,1,1}$ =	$0		***Defer Procurement***	$PV^3_{1,2,3}$ =	$0
			Abandon Project	$PV^3_{0,2,3}$ =	$0		***Abandon Project***
					Abandon Project	$PV_{0,3}$ =	$0
							Abandon Project

In order to understand how much the value of this flexibility is worth, the NPV for the aircraft must be calculated. In this example there are two potential NPVs because the expenditure to purchase the aircraft can occur in either Year 2 or Year 3 therefore both need to be calculated.

$$\mathrm{NPV} = \mathrm{S} - \mathrm{E}^1 - \mathrm{E}^2 - \mathrm{E}^3$$

$$\mathrm{NPV}_2^3 = \sum_{t=0}^{\infty} \frac{\$200\mathrm{K}}{(1.1)^t} - \$250\mathrm{K} - \frac{\$750\mathrm{K}}{(1.1)^1} - \frac{\$1500\mathrm{K}}{(1.1)^2} = \$28\mathrm{K}$$

$$\mathrm{NPV}_3^3 = \sum_{t=0}^{\infty} \frac{\$200\mathrm{K}}{(1.1)^t} - \$250\mathrm{K} - \frac{\$750\mathrm{K}}{(1.1)^1} - \frac{\$1500\mathrm{K}}{(1.1)^3} = \$141\mathrm{K}$$

In both cases the NPV is positive but the NPV if the aircraft is built at Year 3 is greater. So the decision reached by today's SE process would be to commence the project and build the aircraft at Year 3 based on a $\mathrm{NPV}^3{}_3$ of \$141K. The value of the option (OV) is the difference between the $\mathrm{NPV}^3{}_3$ and the project's value with flexibility or

$$\mathrm{OV} = \mathrm{PV}^1{}_{0,0,0} - \mathrm{NPV}^3{}_3 = \$471\mathrm{K} - \$141\mathrm{K} = \$330\mathrm{K}$$

For this example, deferring the decision to build the aircraft will have a cost of \$150K or a 20% increase during the detailed design phase.

$$\mathrm{OC} = \$150\mathrm{K}$$

The net option value is:

$$\mathrm{NOV} = \mathrm{OV} - \mathrm{OC} = \$330\mathrm{K} - \$150\mathrm{K} = \$180\mathrm{K}$$

The aircraft was designed around sequential reviews where the state of the project was determined at multiple decision points, 1) at the start of the initial design phase, 2) at the start of the engineering phase, and finally 3) when to purchase the aircraft. The NPV analysis could only differentiate between when to purchase the aircraft and therefore only provided two real pieces of information -- start the program and wait until Year 3 to build the aircraft. ROA provided additional information and that information was available now for every future decision point using sequential options analysis. There were 10 potential points that the program could arrive at over the three years and each point provided unique information to the decision maker about how to proceed. Table 2-7 displays the information and the recommended decisions. The value of this knowledge is that information is available today and key decision makers know what the decision will be at any point in time; this is the power of ROA. Understanding how sensitive this knowledge is to variations in the ROA variables provides the decision maker with an understanding of the riskiness associated with these decisions.

2.7 Sensitivity Analysis

The baseline for the sensitivity analysis was the first NPV/deferral example: the building of an aircraft where the decision is whether to invest \$1600K (E) to bring an aircraft to market or to defer the decision one year (τ); once the decision is made it is irreversible; the risk-free rate of return (r_f) over the life of the option is 10%; the value (S) of the plane, if it existed today, will be calculated based on its first year revenue of \$200K and after the first year there is an equal (50%) probability that the revenue will increase or decrease the second year, in either case the change in revenue is assumed to be permanent.

Sensitivity analysis was then performed on four of the five ROA variables and on the ratio of E to S. The fifth variable, time to expiration of the option (τ), was not varied since the decision could not be deferred beyond one year. For this analysis, the future revenue uncertainty σ was considered to be an independent variable and was varied between 0 and 1. The variables S, E, and r_f were considered to be dependent variables and each was varied independently while all the others were held constant. The first variable to be investigated is investment.

The investment (E) necessary to design, develop, build, and test the SE project was evaluated at $1600K ± 50%. The effect on the option's value for the various values of E was then plotted against the uncertainty in revenue cost and is shown in Figure 2-1. What the sensitivity analysis reveals is: 1) as the cost (E) approaches the value of the project (S), the uncertainty of the future revenue makes the value of the option greater or the sooner it diverges from the NPV solution, and 2) the slope of the line varies as a function of the average annual return and it starts to diverge at the same value for all three cases. This is what would be expected and it is the ability of ROA to account for the cost variance that is important to the decision maker. The next variable investigated was the risk free rate of return.

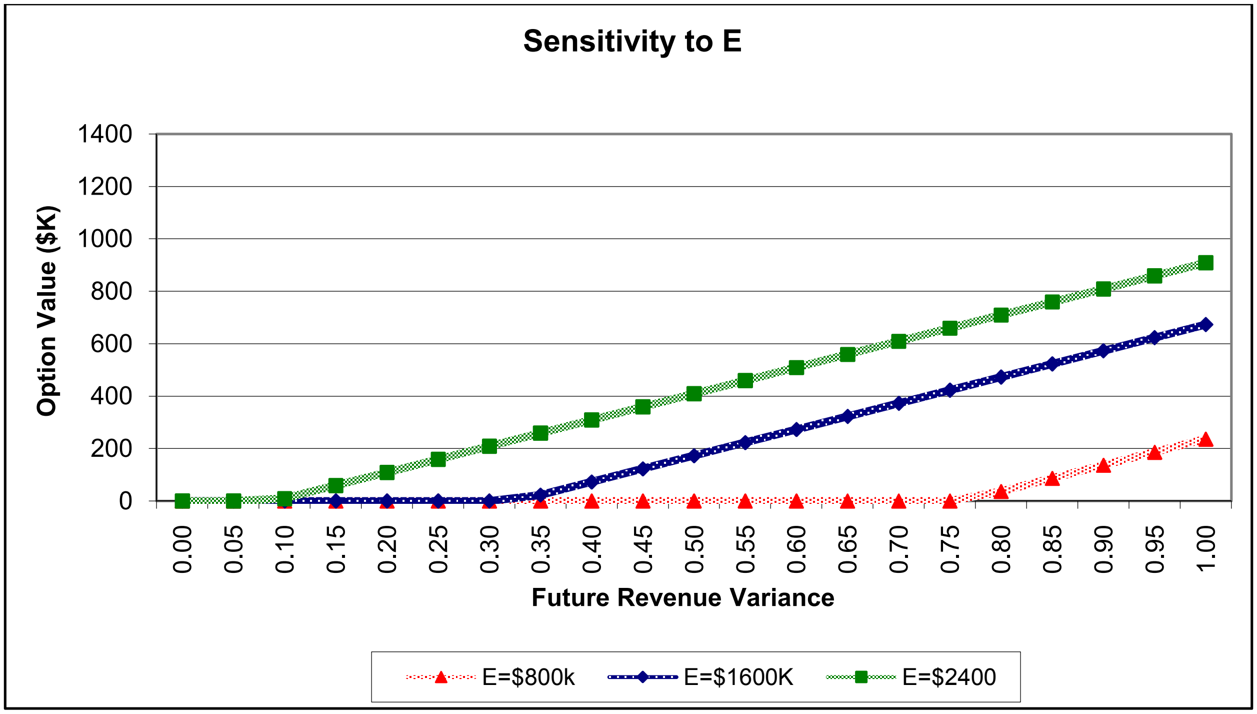

Figure 2-1. The Sensitivity of Option Value to Variations in E

Sensitivity of the option value as a function of the risk free rate of return (r_f) was evaluated at a rate of 5%, 10%, and 15%. Two significant events can be observed in Figure 2-2. First, when the interest rate was 5% the value of the option never exceeded the NPV solution so it never diverged from the NPV solution. This implies that for a low risk free rate of return the value of waiting, even when there is great uncertainty, has little to no value. The second event of interest was when the interest rate was 15%. The NPV decision changed from a build decision to an abandon decision. The impact of this decision reversal was the reason the option value never went to zero. This would suggest that when the risk free rate of return is high the value of waiting is large even with little uncertainty. The implication for the decision maker is that the risk free rate of return that is assumed can greatly impact the value of the option. Fortunately this value is objective since it is based on what could be earned by placing the investment in a savings account or some other type of riskless investment. Next the value of the revenue generated was varied.

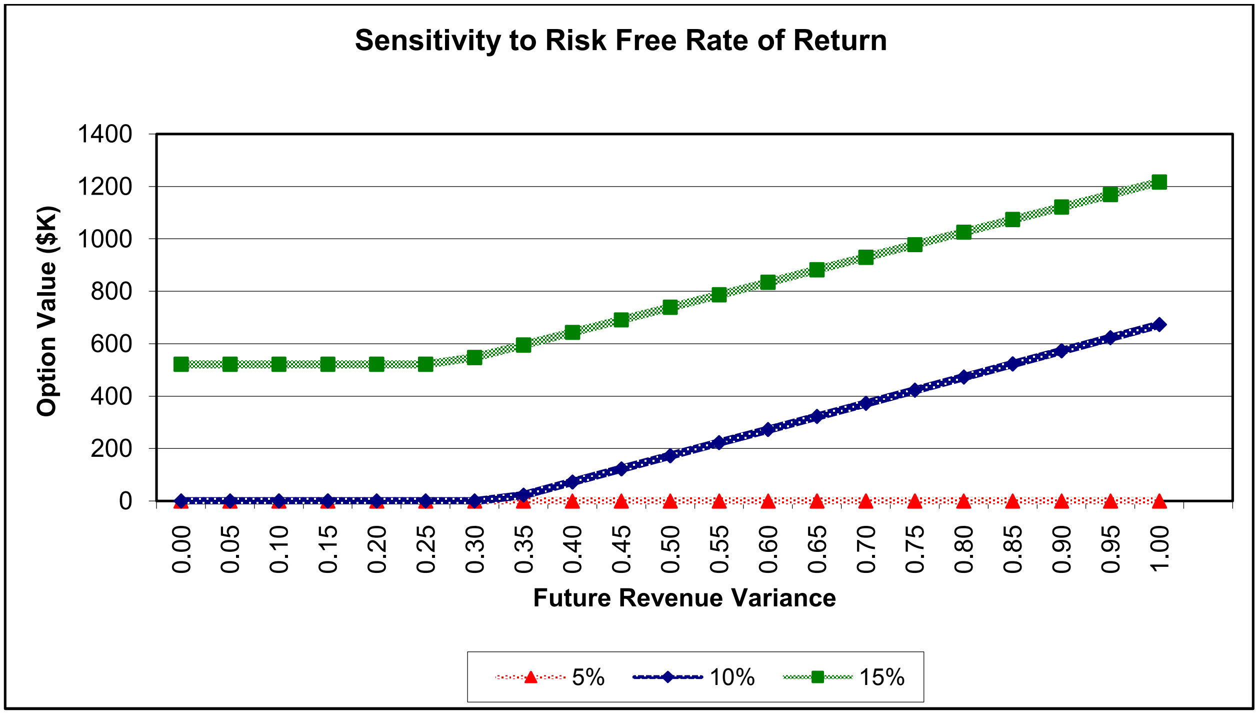

Figure 2-2. The Sensitivity of Option Value to Variations in the Risk Free Rate of Return

The sensitivity of the option value to changes in the project's value (S) if it existed today was accomplished by varying the annual return. Annual returns of $150K/year, $200K/year, and $300K/year where chosen. Utilizing these annual returns equated to S values of $1650K, $2200K, and $3300K, respectively. Figure 2-3 is similar to the findings seen in Figure 2-1; 1) as the difference between the cost (E) and the value of the project (S) decreases, the uncertainty of the future revenue makes the value of the option greater or the sooner it diverges from the NPV solution and 2) the slope of the line is 1/2 of the average annual return every 10% of variance once it starts to diverge. Further investigation into these suppositions is necessary before it can be generalized. If however it can be generalized it would provide another valuable piece of information to the decision maker as to when using ROA is appropriate. Based on the results of the sensitivity analysis for both S and E, it was believed that the ratio E/S might provide some insight into when the option value starts to diverge from the NPV solution and thereby offer additional insight into when the use of ROA would be useful.

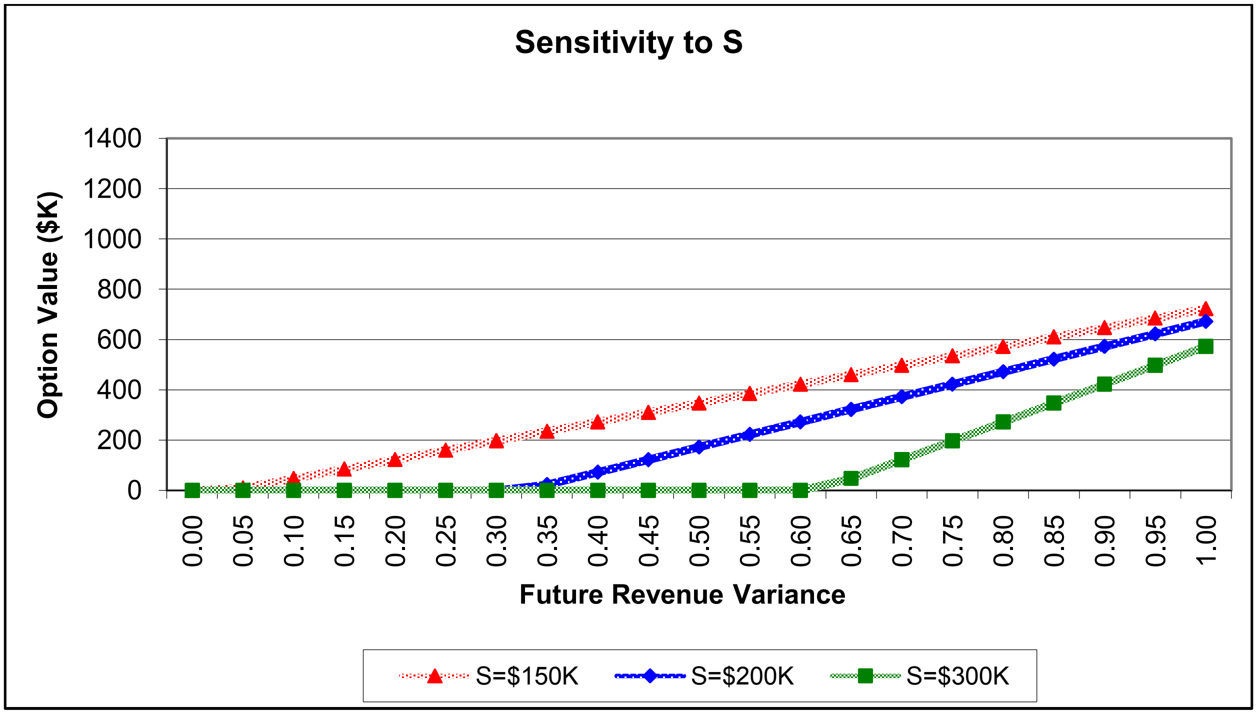

Figure 2-3. The Sensitivity of Option Value to Variations in S

An E/S ratio was derived using E=$1600K and S=$2200K such that E/S = $1600K/$2200K = 0.72. The annual revenues were varied as in the previous analysis, $150K/year, $200K/year, and $300K/year and produced S values of $1650K, $2200K, and $3300K, respectively. The E/S ratio of 0.72 was then applied to the S value and E was calculated.

$1650K * 0.72 = $450K, $2200K * 0.72 = $1600K

and

$3300K * 0.72 = $2400K.

Figure 2-4 clearly demonstrates that all three lines diverge from the NPV solution at the same point and follow the slope discussed above. This may imply that the boundary conditions as to when to use NPV or ROA techniques are functions of the system's cost to present value ratio (x intercept) and the systems annual revenue (slope). It must be understood that this are simple examples and a different result might occur for more complex annual revenue equations. Investigation into this area will be left for future research. Since a compound or sequential option is an extension of a deferral option, the variable sensitivity will be the same and therefore a specific example is not necessary. Based on an understanding of the sensitivity of ROA variables the decision maker now has all the information necessary to evaluate the riskiness associated with a ROA evaluation and the decisions it provides.

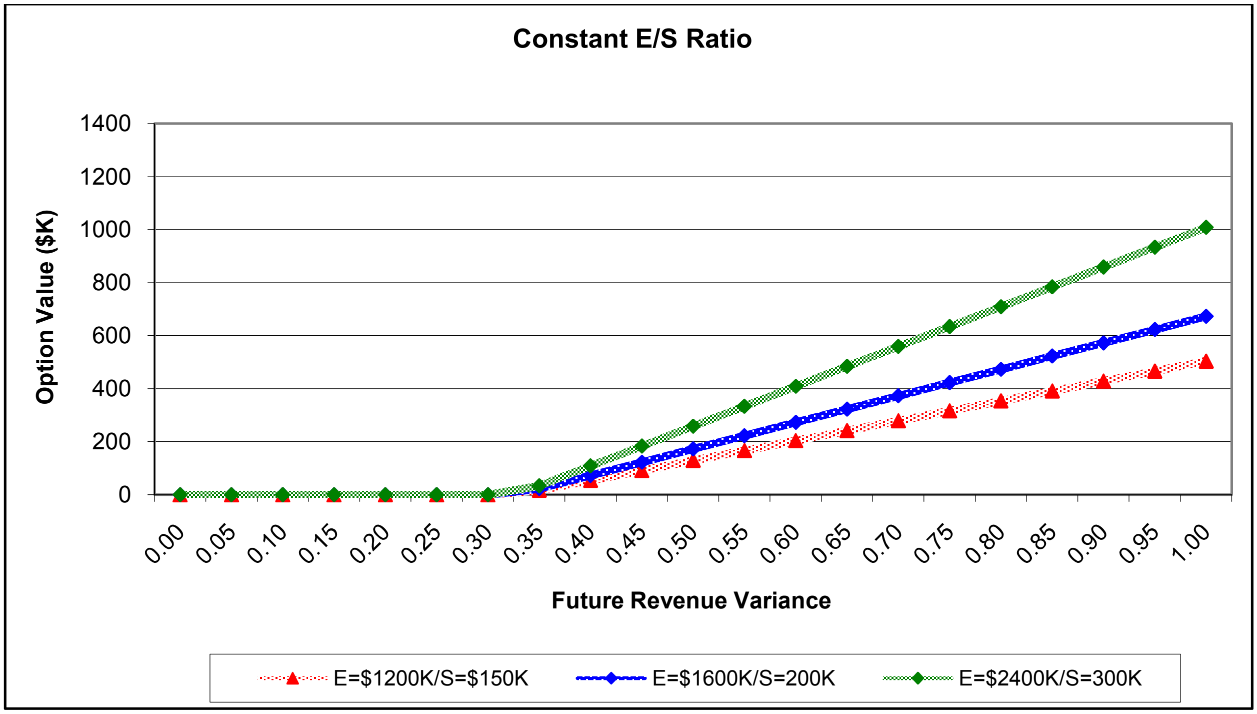

Figure 2-4. The Sensitivity of Option Value
to a Constant E/S Ratio

2.8 Summary

It order to utilize ROA in SE it was necessary to demonstrated that an SE project is analogous to a real option and that the ROA variables could be translated into the SE domain. If these objectives where meet it would allow ROA techniques to be utilized by SE. Both of the objectives were met and two ROA techniques, a deferral option and a compound option, were presented.

In the deferral option examples, the value of waiting to make a decision until more information was available was evaluated. In the examples presented, NPV analysis failed to either acknowledge a potentially negative risk by recommending the investment or to recognize the potential of a positive opportunity and recommending the project not start. ROA provided the decision maker with additional information and based on this information a different decision should have resulted, i.e., defer the decision.

In the first example the decision to expend $1600K today would have been the result given the NPV analysis and there would have been a 50% chance of it being the wrong decision, meaning the project had the opportunity to lose money. The opposite outcome would have been the result based on the NPV data in the second example since the decision maker would never have started the project and therefore would miss the opportunity to make money. In both of these examples ROA indicated that a decision to defer one year would ensure only positive opportunities would result, revenue goes up - build, revenue goes down – abandon. In the second set of examples it was demonstrated that NPV does not take into account the volatility associated with future revenue. ROA demonstrated that when the volatility was large it was of even greater value to wait one year. However, when the volatility was small, ROA reverted to NPV. This is as would be expected since one of the premises NPV is based on is that the cost drivers are well understood and characterized (Hayes and Abernathy 1980). These were simple examples and do not necessarily reflect the complexity of the SE process. In order to address this shortcoming, a compound option example was presented which would reflect a typical phased program used in SE.

The compound option example was based on a sequential program in which there were three stages to building an aircraft; initial design phase, detailed engineering phase, and procurement phase. Each one of the phases required an investment and occurred in different years. This type of situation was evaluated using sequential option analysis. A sequential option is one in which the first option chronologically is the right to buy the second option. For this example executing the initial design phase results in the right to invest in the engineering phase and that in turn results in the right to procure the aircraft. The only information NPV analysis provided for this example was whether to start the project or not and what year the aircraft should be procured. NPV analysis indicated that the program should go forward and that the aircraft should be procured in year 3. ROA showed that there were actually 10 decision points between the start of the program and the procurement of the aircraft. At each of these points, one today, two at Year 1, three at Year 2, and four at Year 3, information was available today on whether to invest, defer, or abandon. This information allows the Systems Engineer the ability, based on where the program is at any point in time, to know if the decision should be to execute, wait, or terminate a project now or in the future.

It is the ability of ROA to value uncertainty until additional information is available that will benefit SE remember: 1) when there is uncertainty, the current technique of NPV fails to correctly estimate the value of alternatives; 2) the decision to build something may be irreversible but the decision to delay building it is always reversible; 3) delaying a decision can have a significant effect on the SE technology decision; and 4) ROA provides a method to calculate the value of delaying a decision, which is not available using NPV. The value of this knowledge is that information is available today and key decision makers know what the decision will be at any point in time. Understanding how sensitive this knowledge is to variations in the ROA variables provides the decision maker with an understanding of the riskiness associated with these decisions.

Sensitivity analysis was performed on four of the five ROA variables and on the ratio of E to S. The fifth variable, time to expiration of the option (τ) was not varied since the decision could not be deferred beyond one year. For the analysis, the future revenue uncertainty σ was considered to be an independent variable and was varied between 0 and 1. The variables S, E, and r_f were considered to be dependent variables and each was varied independently while all the others were held constant. The results implied the following relationships between the variables and the value of the option: 1) when r_f is low there is little to no value in waiting, or when r_f is high the NPV solution may reverse and the value of waiting more valuable, 2) as the difference between the cost (E) and the value of the project (S) decreases, the uncertainty of future costs or revenue makes the value of the option greater, 3) the slope of the line once the value of the option starts to diverge from the NPV solution is a function of the annual revenue, the larger the annual revenue the greater the slope, and 4) the ratio of the system's cost to its present value defines the point where the option's value starts to diverge from the NPV solution (x intercept). These relationships suggest that it is possible for the system engineer to understand how dependent the ROA results are on any particular variable and therefore where the greatest risk to the project may exist.

The use of ROA will have important implications to the systems engineer for two reasons: 1) the volatility of a technology's cost drivers are now part of the cost evaluation process and may impact the technology selection, and 2) a quantitative evaluation can be conducted to determine if delaying a decision until more information is available is of greater value to the project in spite of the perceived higher cost than making the decision today. One should not forget the words of Robert Merton (1998) "the future is uncertain (if it were not, there would be no need to create options because we know now what we will do later)". ROA provides a quantitative answer to the question 'How late in the process can the systems engineering decision be made?' Real Options Analysis, however, only addresses value in a financial sense and cost is only one variable utilized by SE.

Performance is another important variable in the analysis of various alternatives being considered by the Systems Engineer. Chapter 3 will develop and present a new analytical method Technology Options Analysis that will model performance-using techniques similar to Real Options Analysis. Technology Options and Real Options Analysis should allow the Systems Engineer and decision maker to evaluate performance and cost using analysis methods that provide information not currently available. This information should help the Systems Engineer and decision maker answer this question: "Should the decision be made today or will more information become available that would make it worthwhile to wait?"

CHAPTER III

TECHNOLOGY OPTION ANALYSIS

Technology selection occurs as the direct result of selecting a particular design alternative for a system (Blanchard and Fabrycky 1998; Kerzner 2001). Of interest for my research is how that decision is made and can better techniques be developed to aid the systems engineer in making technology decisions? The basic steps in the design process are to define what the system must do, i.e., requirements (which can vary with time), and how well the system must do it, i.e., the system's technical performance metrics. Potential design solutions (alternatives) are then postulated, refined, and described based on various underlying technologies. As potential design alternatives become available they are synthesized, analyzed, and evaluated against the various technical performance metrics. Trade studies are conducted on the various design alternatives to determine which best meets the customer's requirements. With this process, the underlying technology becomes inherent in the system configuration (Blanchard and Fabrycky 1998; Buede 2000; Rouse 2003).

Despite the importance of technology selection in the Systems Engineering process, existing techniques have had limited success in predicting whether a technology's future performance will be achieved or not and there is a lack of a clear understanding as to why projects succeed or fail (Dilts and Pence 2004) . One problem with current methods is that they ignore two important characteristics of technology selection. First, when the technology selection is made, design dependent parameters are incorporated into the system's configuration and they become either irreversible or reversible with considerable penalty with regard to time, performance, or cost. Examples of such irreversible technology decisions include: fly-by-wire versus hydraulic control for aircraft, disc brakes versus drum brakes, or the differences in voltage and current draw between electronic technologies. Irreversibility makes technology selection especially sensitive to various forms of risk, such as uncertainty over the technology's future performance, its ability to meet required performance parameters, uncertainty over its maturity, and uncertainty over its potential future availability. It is for this reason System Engineering text books recommend that technology selection be locked down as soon as possible preferably at the projects preliminary design review (Blanchard and Fabrycky 1998; Buede 2000; Kerzner 2001; Rouse 2003).

Second, the current methods utilized to analyze potential design alternatives do not have a mechanism for quantifying the future performance uncertainty associated with a technology. Consider two alternative technology cases. In the first, technology *N* (new) is advanced state-of-the-art (e.g., it may only exist in the laboratory) but its projected future performance will dramatically exceed the customer's requirements *if it meets its expectations.* Unfortunately, technology *N's* future performance is uncertain. In the second case, technology *E* (existing), the technology is mature, stable, and meets the system's requirements now.

There are only two choices available to the decision maker: they can ignore technology *N's* potential, select technology *E*, and lose out if *N* reaches its future performance or they can select technology *N* and thereby accept the risk that it will not achieve its projected performance. The horizon for this decision is set by the project's timeline. What is needed is a third method, one that incorporates the future performance uncertainty associated with a technology into a decision support tool.

There are potential methods available that could be used to assist in understanding a technology's future performance uncertainty. One method would be for a knowledgeable and impartial individual to judge the uncertainty and assign a probability that the technology will reach its predicted future performance. The first problem with this method is where do you find an impartial expert on an emerging technology and will one always be available? The second issue would be the qualitative nature of this assessment? A second method would be to wait until more information about the new technology is available and make the decision based on the additional information. This would provide a quantitative assessment of the new technology's future performance.

In situations where there is significant uncertainty about the future and the ability to wait exists, an extensive amount of literature from other research areas have demonstrated that the ability to delay an irreversible decision can profoundly affect the decision. For example, with decisions of both high importance and high future uncertainty, Drucker (2006), Eisengardt (1989), and others advocate that a decision should be made no later than necessary, but as late as possible as more time will normally provide a decision maker with additional information to reduce uncertainty. This suggests that there is a utility for waiting. But for how long should a decision like this be deferred? The method that is developed in this chapter satisfies this need by providing to the decision maker the time the decision should be made.

3.1 Technology Options Analysis

Ashford uses the term Technology Options Analysis when he discusses ways to force innovation into regulated firms through government intervention (Ashford 1993; Finkel and Golding 1996; Ashford 2000). The methods and scenarios discussed by both Ashford and Finkel

(Finkel and Golding 1996) have to do with reduction of risk in highly polluting industries and not similar to the scenario or methods presented in this chapter.

The scenario of interest is one in which two technologies are being consider for the system. One is mature and meets all the requirements today and into the future. A new technology is being developed but it does not meet the requirements today but its future performance is projected to exceed the performance of the existing technology. This scenario is of interest because it means that at some time in the future the new technology's performance is predicted to exceed or cross-over the existing technology. The cross-over point (Figure 3-1) defines the optimum time to make the decision because prior to the cross-over point there is still uncertainty about the new technology's performance but after the cross-over point no additional information is required. The cross-over point (τ_{cop}) is determined by subtracting the new technology's currently demonstrated performance ($\mathrm{P_c}$) from the mature technology's performance (P_f) and dividing by the new technology's rate of performance change (r_p): $\tau_{cop} = [(P_f) - (\mathrm{P_c})] / r_p$. It is at that point the decision maker has all the information needed to make an informed decision.

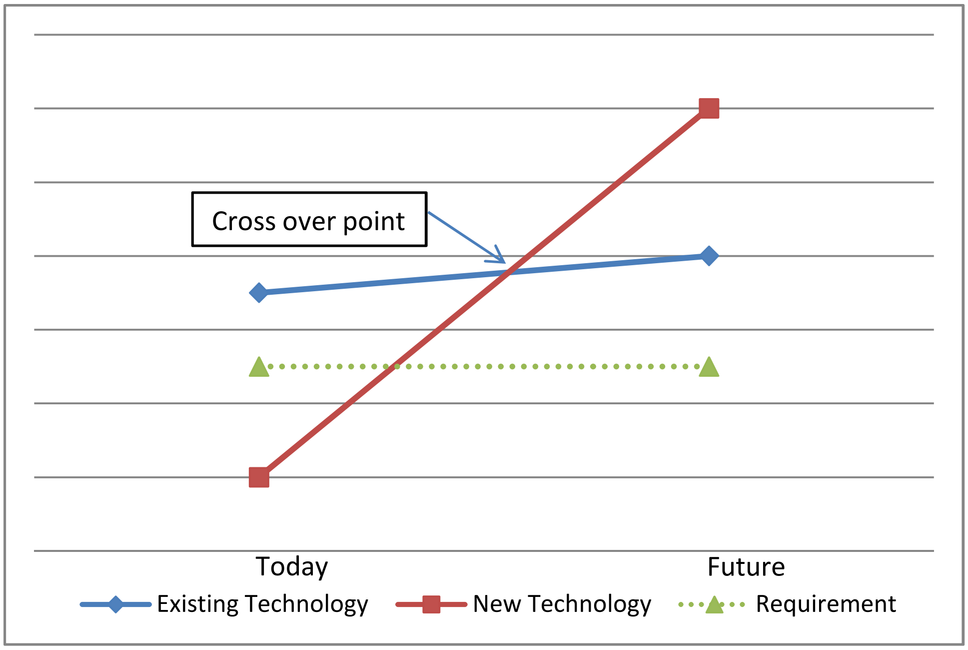

Figure 3-1. Cross-over Point that Defines Optimum Time to make the Decision

Based on the cross-over point information the decision maker has two choices, make the decision today or defer the decision until the cross-over point. The cross-over point is independent of the project and only depends on the technologies being considered. Suppose that the selection must take place by the system's critical design review. If the cross-over point does not occur prior to the critical design review the decision should be made today to use the existing technology. However, if the cross-over point occurs prior to the critical design review, the decision maker should consider deferring the decision until the uncertainty associated with the new technology's performance can be resolved. In addition to knowing whether to defer or not, the decision maker also knows what decision should be made at the cross-over point, use the existing technology if the crossing did not occur or use the new technology if it did. This method provides information to the decision maker today on when the decision should be made and what the outcomes should be. This is very similar to the information that is provided to a decision maker by a financial European call option or a simple real options deferral option. Determining the cross-over point does not require complicated mathematical techniques.

The cross-over point can be determined with only four pieces of information, the current demonstrated performance of the new and existing technologies and the predicted performance of the new and existing technologies sometime in the future. That information is available today from various sources; technology roadmaps, Delphi surveys, or the actual vendors. Using these four data points, two lines can be drawn, one from the existing technology's current performance to its future predicted performance and the second from the new technology's current performance to its future predicted performance. Each line represents a linear rate of change in performance, one for the existing technology and the other for the new technology. Where the lines intersect is the cross-over point (Figure 3-2). It is not necessary to know the actual shape of a technology's rate change of performance only that the underlying shape remains the same. If additional information about the predicted shape of the technology's rate of change of performance is known it can be used instead to generate the intersection point. The decision maker now knows when the decision can be made based on when the information will be available. Enhancements to this very simple technique could be made that would make it more informative by providing additional information.

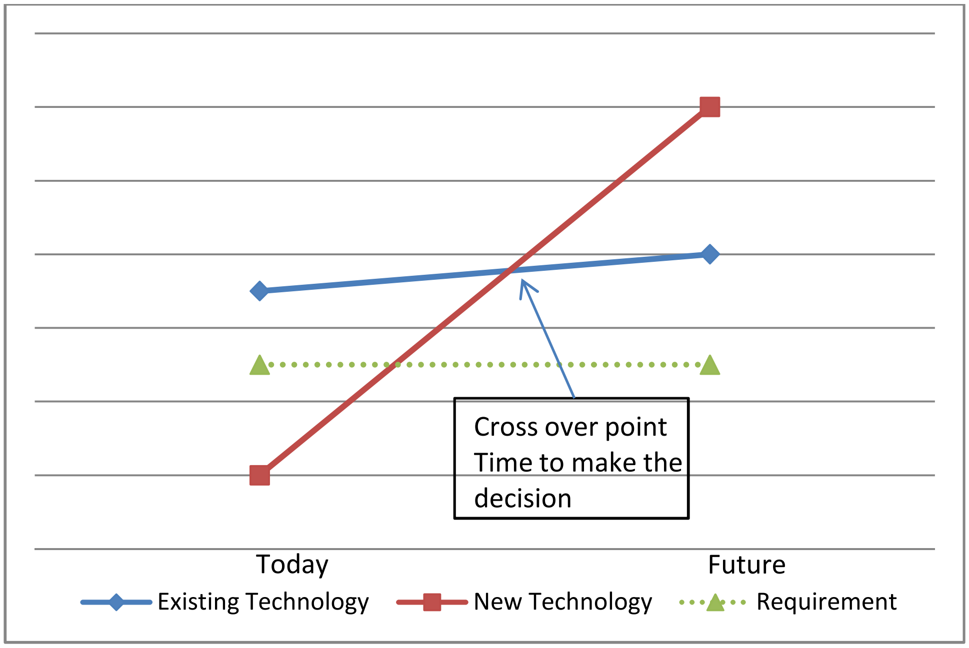

Figure 3-2. Identified Cross-over Point
for the Time to Make the Decision

The first enhancement to the technique will be to provide the decision maker with information on how much greater the performance of the new technology might be when compared to the existing technology. This performance difference (delta performance) between the new technology and the existing technology at a specified point in the future (Figure 3-3) is important because it provides the decision maker a quantitative appreciation of the value of deferring the decision. The performance delta cross-over point (τ_{cop}) is determined by subtracting the new technology's future predicted performance (P_f) from the mature technology's performance (P_f): $\Delta p = (P_f) - (P_f)$. The greater the delta in performance the more value in deferring the decision until more information is available. Gaining an insight into the risk of the new technology reaching its predicted performance will be the next enhancement.

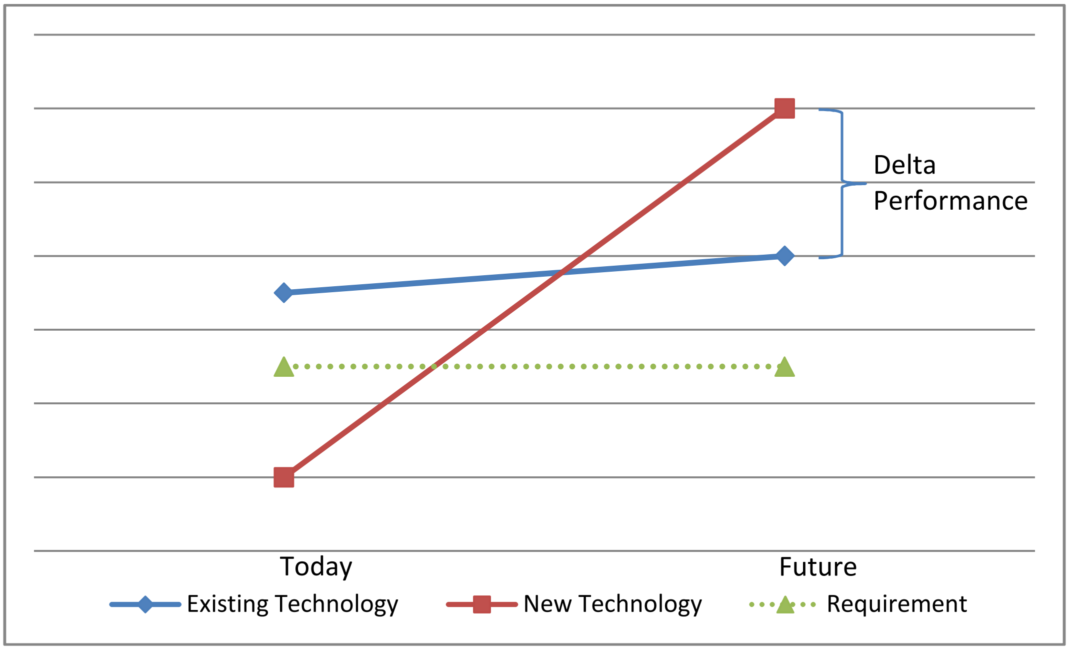

Figure 3-3. Performance Difference between New Technology and Existing Technology

The motivation behind the technique discussed above was to provide the decision maker with a riskless decision environment, i.e., wait until the information becomes available and then make the decision. But what is the likelihood or the risk that the cross-over point will occur at the time the technique predicts. Understanding that risk should be of benefit to the decision maker because if the risk is high a different decision may result even if the cross-over point and the performance delta indicate the decision should be deferred. Figure 3-4 uses the technique presented above to determine the cross-over point for three new technologies and one existing technology. It was constructed so that the cross-over point was the same for all three new technologies but with a different delta performance for each. The delta performance increases from new technology 1 to new technology 3. Suppose each new technology is evaluated one at a time against the existing technology using the techniques that have been presented so far. In each case the decision maker would decide to defer until the cross-over point. If the risk of reaching the cross-over point at the predicted time was equal for all three technologies then the decision maker should decide to abandon technology 1 and 2 because their value would be less than technology 3. Deciding to abandon technology 1 and 2 might be the right decision but it also may be the wrong decision if technology 3 fails to meet its predicted performance improvement. One more piece of information is needed to be provided to the decision maker, the projected rate of improvement for each new technology. The higher the projected rate of improvement the greater the risk the technology will not reach the cross-over point when predicted. The projected rate of improvement for each of the new technologies in the example above is the slope of their line. The risk of new technology 1 is less than new technology 2 which is less than new technology 3 of reaching the cross-over point when predicted. Having this additional information might result in a different decision being made. To this point we have assumed that information was only available today and at the cross-over point.

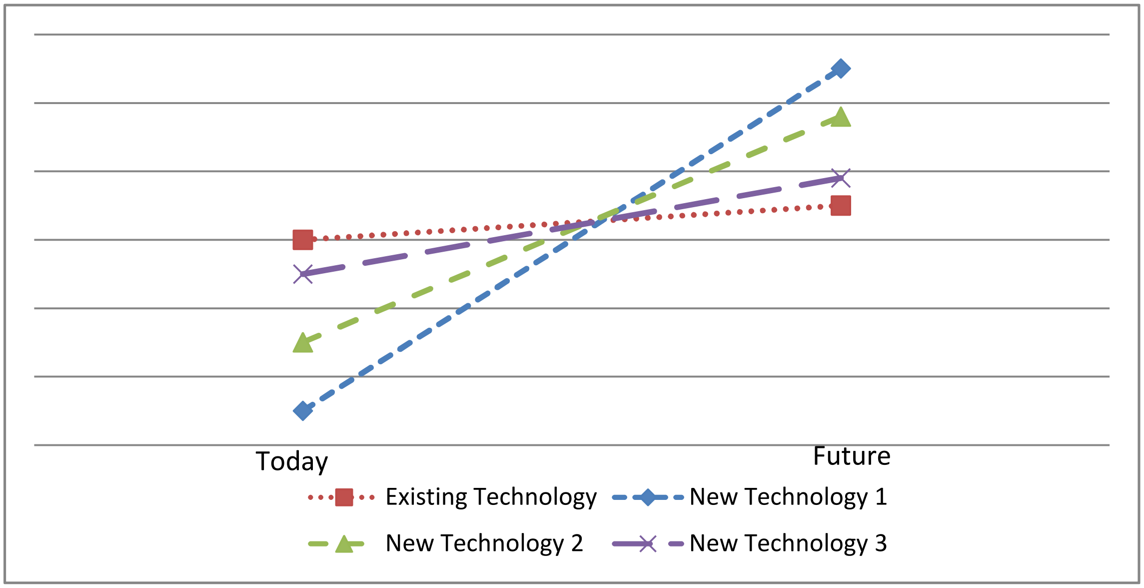

Figure 3-4. Comparison of Existing Technology and New Technology Alternatives

There is no reason to assume that information would not be forth coming at various points in time before the predicted cross-over point. When new performance information becomes available it can be used to refine and update the predicted cross-over point and performance difference calculations. This information may cause a change in the decision timing and could allow for a decision to be made earlier than originally predicted. In addition it should become clearer if the riskiness of the new technology in reaching the cross-over point is increasing or decreasing. These enhancements to the original technique provide additional information to the decision maker today on how much performance improvement the new technology may provide, the risk associated with the rate of performance improvement of the technology, and the ability to reassess the timing of the cross-over point and when the decision should be made. As a result of these improvements the method presented now makes available to the decision maker the same type of information provided by a financial American call option. The decision maker now knows when the decision can be made based on when the information will be available. Table 3-1 presents the Technology Option Analysis variables, their definition and meaning.

Table 3-1. Technology Option Analysis Variables
(with definitions and Meanings)

TOA Variable	*TOA Definition*	*Meaning*
P	Performance	Performance of the Technology
T	Requirement	Minimum performance required
τ_{cop}	Cross-over Point	When the new technology's performance intersects the existing technology's performance
Δp	Delta Performance	Difference between the existing technology's and the new technology's predicted performance
r_p	Projected Rate of Performance	Riskiness of the technology

Understanding how sensitive this knowledge is to variations in the TOA variables provides the decision maker with an understanding of the risk associated with these decisions.

3.2 Sensitivity Analysis

Sensitivity analysis was performed to see the effect new information about the performance of the emerging technology had on the cross-over point. For ease in demonstration, only one new technology will be evaluated. For this analysis, three situations were analyzed using high, medium, and low projected rates of performance. All of the calculations were accomplished by manipulating the standard equation for a line, y = mx + b. For Technology Options Analysis, y is the emerging technology's predicted future performance (P_f), m is the projected rate of performance (r_p), x is the time the predicted future performance will be achieved, and b is the emerging technology's current performance (P_c), thereby giving $P_f = r_p * x + P_c$. The projected rate of performance for the emerging technology was calculated by solving for the slope of a line given by $r_p = [(P_f) - (P_c)] / x$. The cross-over point ($\tau_{cop}$) is then calculated by setting the emerging technology's performance equal to existing technology's performance and calculating when that would occur, such that $\tau_{cop} = [(P_f) - (P_c)] / r_p$, where P_f is the existing technology's performance. For purposes of clarity the vertical axis scale was held constant so the difference in the slopes would be apparent.

For the first series, the emerging technology's delta performance (Δp) is predicted to be twice that of the existing technology in 8 time periods with an initial performance (P_c) of zero. The future predicted performance of the emerging technology (P_f) and the existing technology's future predicted performance (P_f) were held constant. A point half-way between today and the initial cross-over point (τ_{cop}) was chosen as the point when the new information would become available. The new information consisted of varying the emerging technology's performance between +87% and -87% from the predicted performance at the evaluation point. The reason +/-87% was chosen was because it would bound the potential future at the point being examined, i.e., the performance of the emerging technology had to be better than it is today but not as good as the existing technology. This is because if the emerging technology's performance was worse in the future the decision would be to use the existing technology and if the emerging technology was equal to the existing technology the decision would be to utilize the emerging technology.

Figure 3-5 displays the results for the situation where a high rate of performance change (slope of 65 degrees) is predicted for the emerging technology. Figures 3-6 displays the results for the situation where a medium rate of performance change (slope of 45 degrees) is expected out of the new technology. Figures 3-7 displays the results for the situation where a low rate of performance change (slope of 27 degrees) is expected out of the new technology. Table 3-2 presents the changes to τ_{cop} based on the new information. Two pieces of information can be extracted from the data. First is that the change to the cross-over point is independent of the projected rate of performance for the emerging technology. An increase or decrease in the demonstrated performance compared to the projected performance moves τ_{cop} the same amount. The second data point is that a change of at least 50% is required before the decision maker should consider changing the timing of the decision. These observations may be unique to this example and will be investigated further.

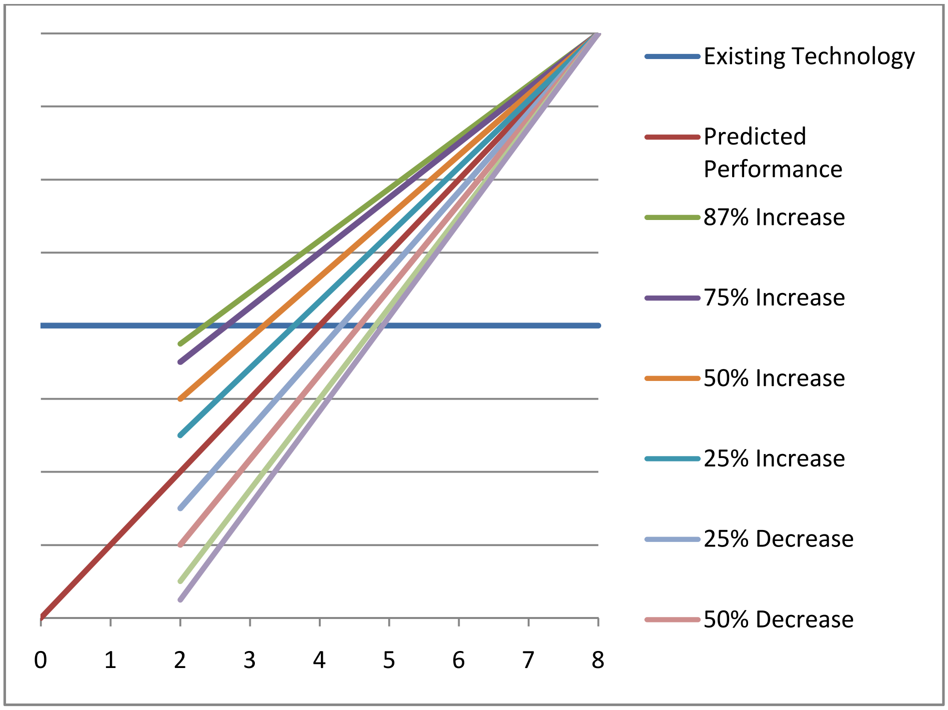

Figure 3-5. High Rate of Performance Change

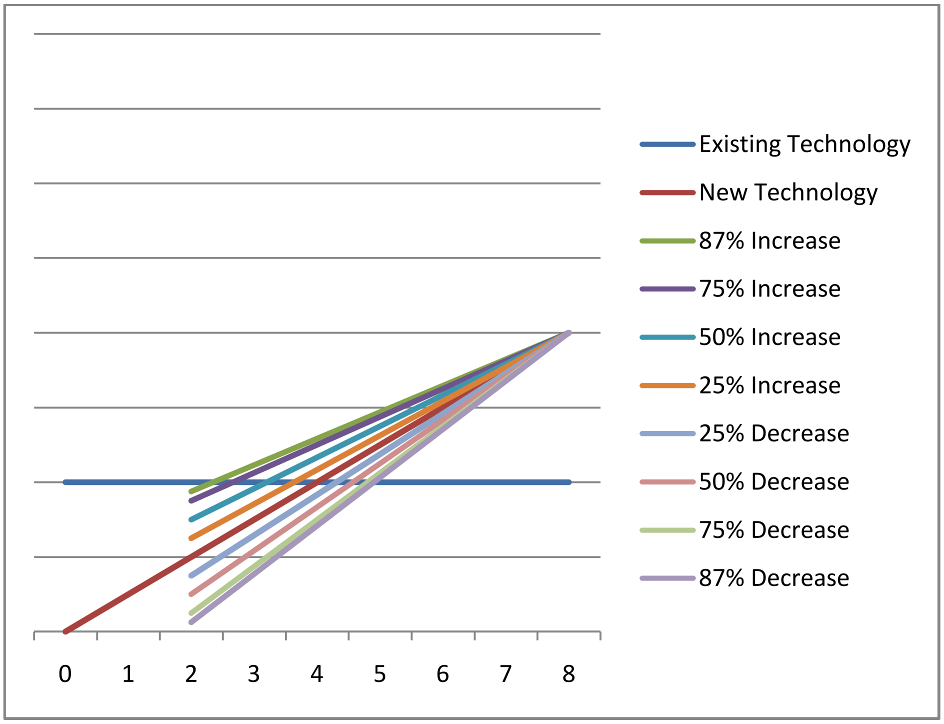

Figure 3-6. Medium Rate of Performance Change

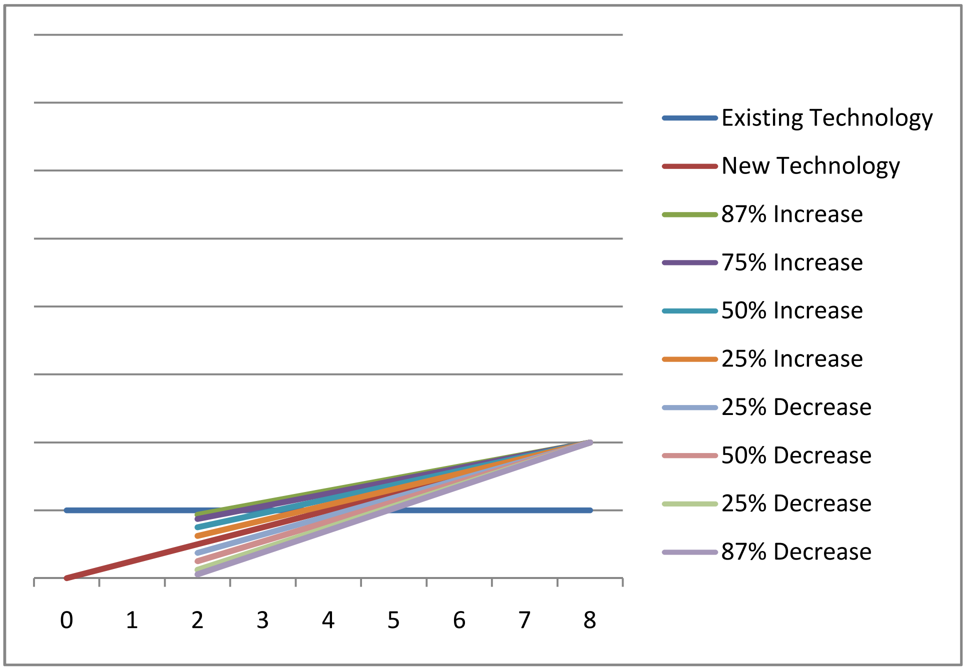

Figure 3-7. Low Rate of Performance Change

Table 3-2. New Cross-over Points based on Emerging Technology Performance Information

Change in Performance	Figure 3-5 *New Cross-over Point*	Figure 3-6 *New Cross-over Point*	Figure 3-7 *New Cross-over Point*	Delta to Original
87.5%	2.4	2.4	2.4	41%
75%	2.7	2.7	2.7	33%
50%	3.2	3.2	3.2	20%
25%	3.6	3.6	3.6	9%
0	4.0	4.0	4.0	0
-25%	4.3	4.3	4.3	-8%
-50%	4.6	4.6	4.6	-14%
-75%	4.8	4.8	4.8	-20%
-87.5%	4.9	4.9	4.9	-23%

Table 3-3 presents the changes to r_p based on the new information. Two pieces of information can be extracted from the data. First is that the percentage change to r_p as a function of the original performance trajectory compared to the new performance trajectory is constant for these examples. This might imply that as additional information about the emerging technology is received it is the variance to the predicted performance trajectory that describes the new risk to meeting the future projected performance. These observations may be unique to this example and will be investigated further.

Table 3-3. New Projected Rate of Performance on Emerging Technology Performance Information

Change in Performance	New r_p for High Performance Technology	New r_p for Medium Performance Technology	New r_p for Low Performance Technology	Percentage change from the original r_p
87.5%	1.4	0.7	0.4	71%
75%	1.5	0.8	0.4	75%
50%	1.7	0.8	0.4	83%
25%	1.8	0.9	0.5	92%
0	2.0	1.0	0.5	100%
-25%	2.2	1.1	0.5	108%
-50%	2.3	1.2	0.6	117%
-75%	2.5	1.3	0.6	125%
-87.5%	2.6	1.3	0.6	129%

For the second series, the emerging technology's delta performance (Δp) is predicted to be twice that of the existing technology in 8 time periods with an initial performance (P_c) of zero.

The existing technology's future predicted performance (P_f) and the emerging technology's projected rate of performance (r_p) were held constant. Holding r_p constant will cause the future predicted performance of the emerging technology ($\mathrm{P_f}$) to change as new information becomes available. A point half-way between today and the initial cross-over point (τ_{cop}) was chosen as the point when the new information would become available. The new information consisted of varying the emerging technology's performance between +87% and -87% from the predicted performance at that point ($\mathrm{P_{c2}}$).

Figure 3-8 displays the results for the situation where a high rate of performance change (slope of 65 degrees) is predicted for the emerging technology. Figures 3-9 displays the results for the situation where a medium rate of performance change (slope of 45 degrees) is expected out of the new technology. Figures 3-10 displays the results for the situation where a low rate of performance change (slope of 27 degrees) is expected out of the new technology. Table 3-7 presents the changes to τ_{cop} and Table 3-8 presents the % change between the emerging technology's originally predicted future performance and the updated predicted future performance based on the new information. Two pieces of information can be extracted from the data. First is that the change to the cross-over point and the percent change in predicted performance is independent of the projected rate of performance for the emerging technology. An increase or decrease in the demonstrated performance compared to the projected performance moves τ_{cop} and the future performance the same amount. The second data point is that a change of at least 50% is required before the decision maker should consider changing the timing of the decision. These observations may be unique to this example and will be investigated further.

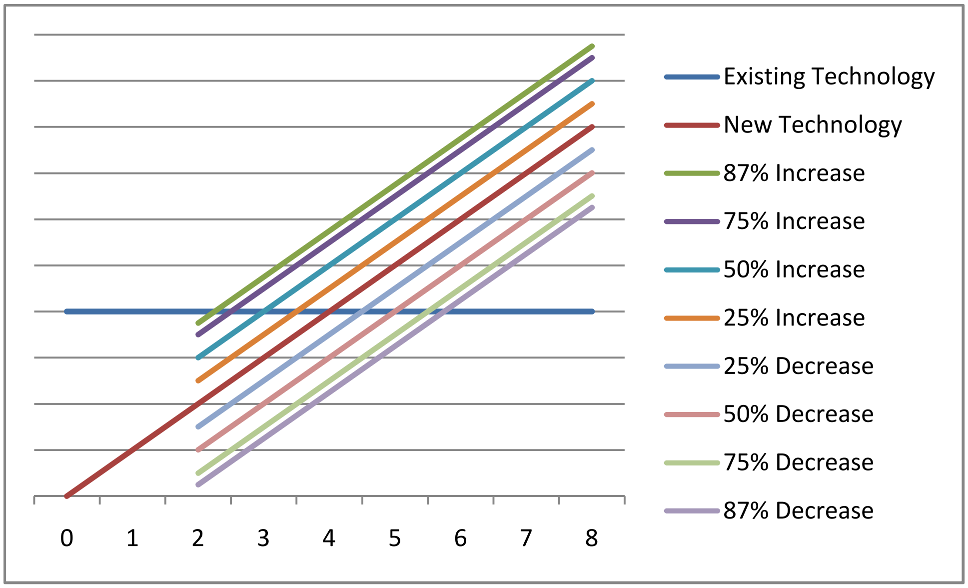

Figure 3-8. High Rate of Performance Change

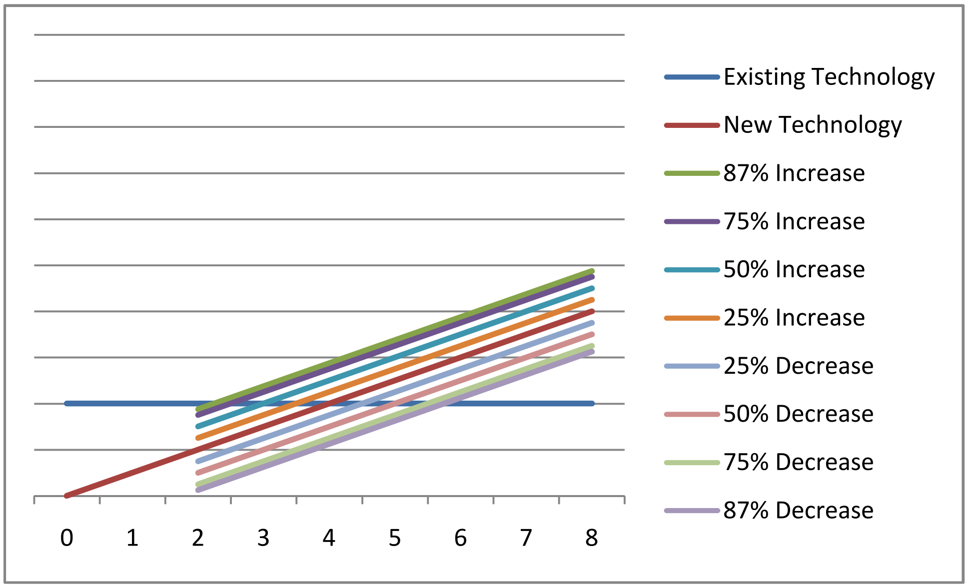

Figure 3-9. Medium Rate of Performance Change

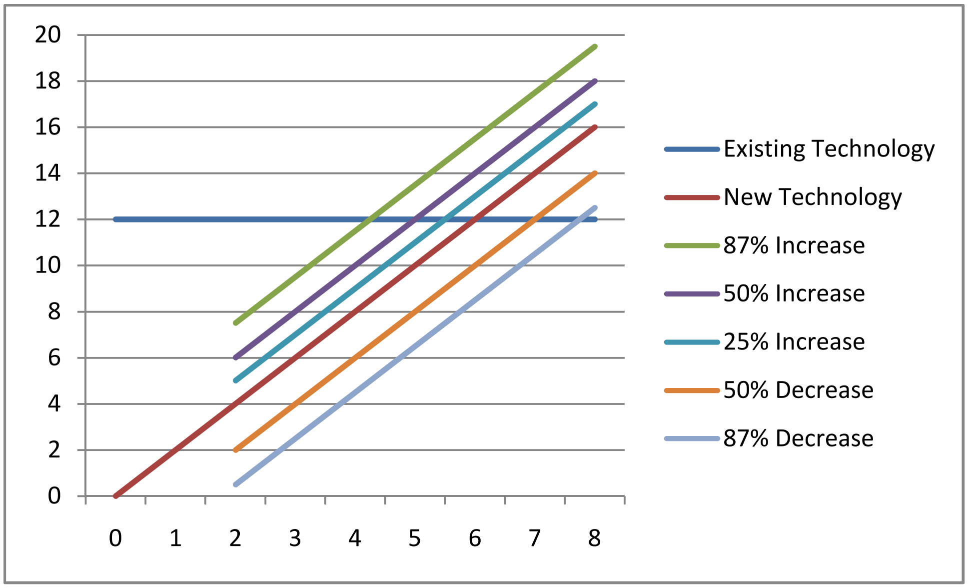

Figure 3-10. Low Rate of Performance Change

Table 3-4. New Cross-over Points based on Emerging Technology Performance Information

Change in Performance	Figure 3-8 New Cross-over Point	Figure 3-9 New Cross-over Point	Figure 3-10 New Cross-over Point	Delta to Original
87.5%	2.3	2.3	2.3	44%
75%	2.5	2.5	2.5	38%
50%	3.0	3.0	3.0	25%
25%	3.5	3.5	3.5	13%
0	4.0	4.0	4.0	0
-25%	4.5	4.5	4.5	-13%
-50%	5.0	5.0	5.0	-25%
-75%	5.5	5.5	5.5	-38%
-87.5%	5.8	5.8	5.8	-44%

Table 3-5. New Predicted Performance based on Emerging Technology Performance Information

Change in Performance	Figure 3-8 Improvement in Performance	Figure 3-9 Improvement in Performance	Figure 3-10 Improvement in Performance
87.5%	22%	22%	22%
75%	19%	19%	19%
50%	13%	13%	13%
25%	6%	6%	6%
0	0	0	0
-25%	-6%	-6%	-6%
-50%	-13%	-13%	-13%
-75%	-19%	-19%	-19%
-87.5%	-22%	-22%	-22%

For the third series, the emerging technology's performance will be held constant and the existing technology will be varied such that a 100%, 75%, and a 50% improvement will be projected over the existing technology in 8 time periods. The initial performance of the emerging technology will be equal to 20% of its future predicted performance. A point half-way between today and the initial cross-over point (τ_{cop}) was chosen as the point when the new information would become available. The new information consisted of varying the emerging technology's performance so it would not exceed the existing technology's performance or be below it initial performance at point (P_{c2}).

Figure 3-11 displays the results for the situation where the emerging technology doubles (100% increase) the performance of the existing technology. Figures 3-12 displays the results for the situation where the emerging technology exceeds the performance of the existing technology by 75%. Figures 3-13 displays the results for the situation where the emerging technology exceeds the performance of the existing technology by 50%. Table 3-9 presents the changes to τ_{cop} based on the new information. Two pieces of information can be extracted from the data. First is that the percentage change to the cross-over point is independent of the existing technology's performance when measured at a point halfway between the initial assessment point and the original cross-over point. The second data point is that a change of at least 50% in the demonstrated performance of the emerging technology is required before the decision maker should consider changing the timing of the decision. These observations may be unique to this example and will be investigated further.

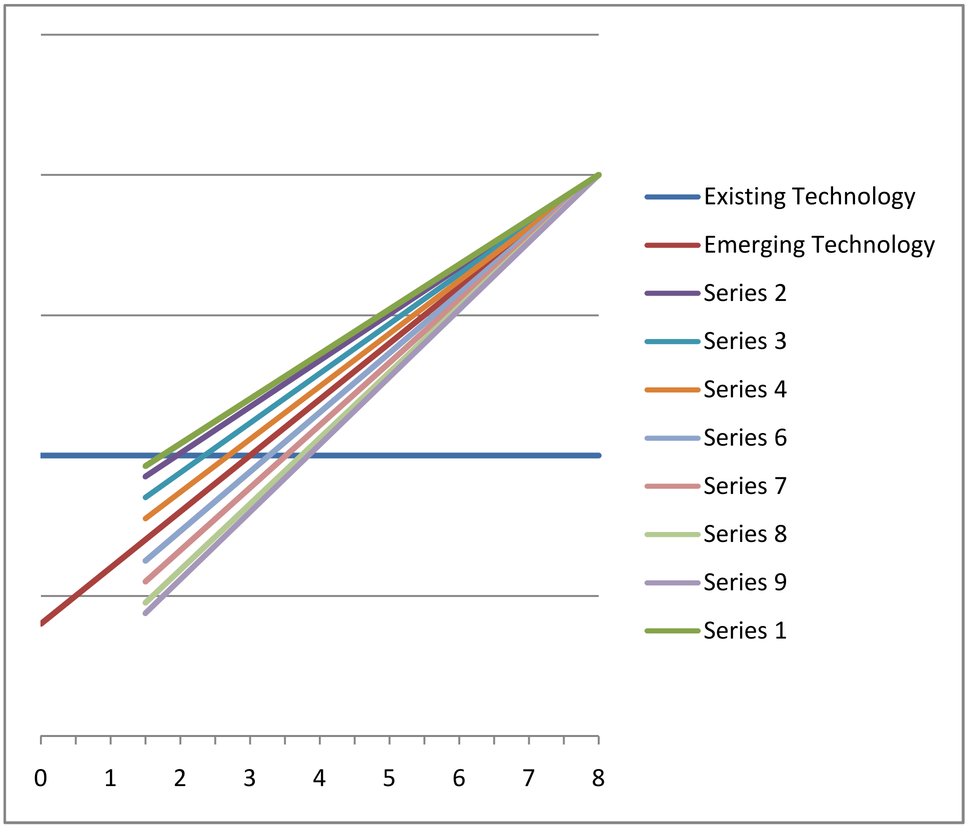

Figure 3-11. Performance gain of 100%

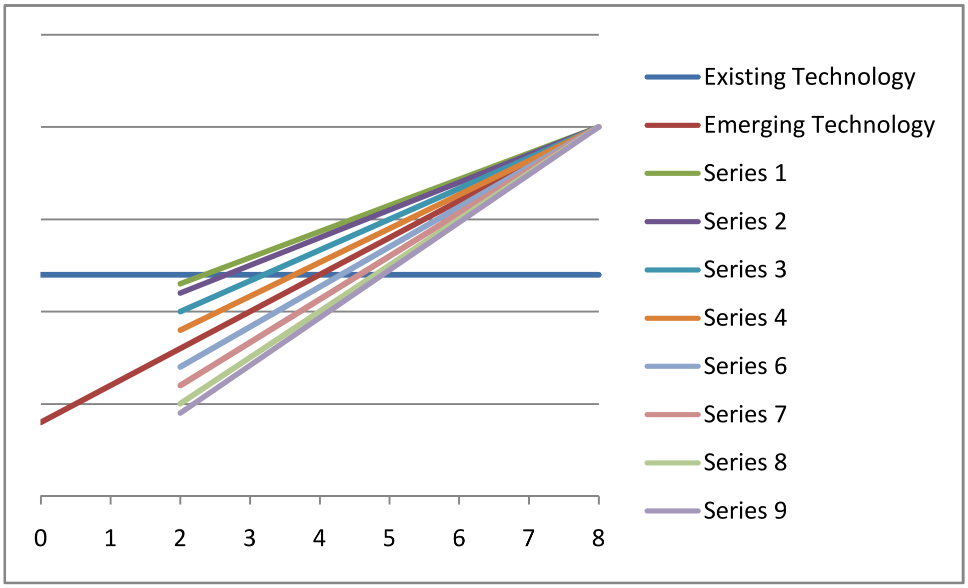

Figure 3-12. Performance gain of 75%

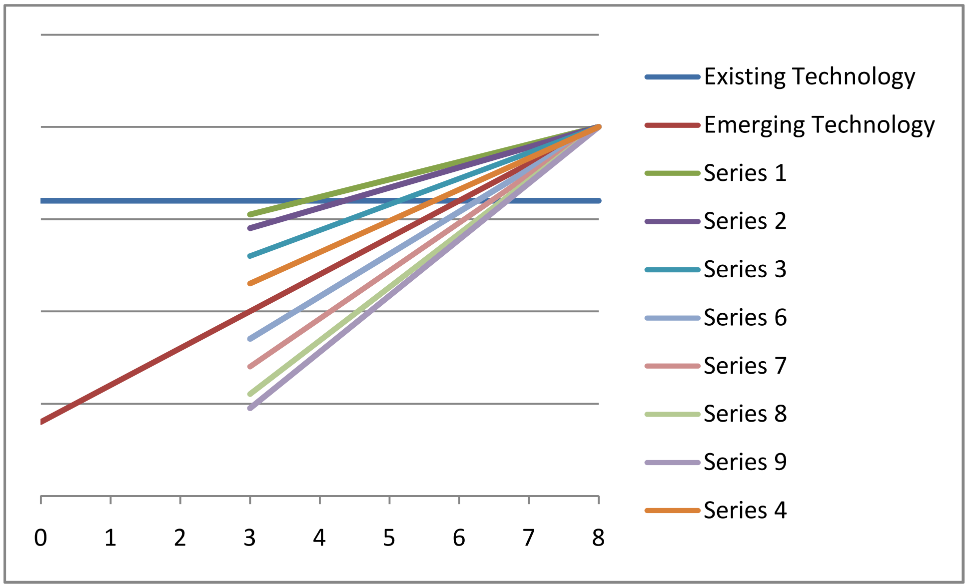

Figure 3-13. Performance gain of 50%

Table 3-6. New Cross-over Points based on Emerging Technology Performance Information

Movement of Cross-over Point – 100% increase in *Δp*	Figure 3-11 New Cross-over Point	Movement of Cross-over Point – 75% increase in *Δp*	Figure 3-12 New Cross-over Point	Movement of Cross-over Point – 50% increase in *Δp*	Figure 3-13 New Cross-over Point
42%	1.7	41%	2.4	37%	3.8
35%	2.0	33%	2.7	27%	4.4
22%	2.3	20%	3.2	14%	5.1
10%	2.7	9%	3.6	6%	5.6
0%	3.0	0%	4.0	0%	6.0
-9%	3.3	-8%	4.3	-4%	6.3
-17%	3.5	-14%	4.6	-8%	6.5
-25%	3.7	-20%	4.8	-10%	6.6
-28%	3.8	-23%	4.9	-11%	6.7

For the fourth and final series, the emerging technology's performance will be held constant and the existing technology will be varied such that a 100%, 75%, and a 50% improvement will be projected over the existing technology in 8 time periods. The initial performance of the emerging technology will be equal to 20% of its future predicted performance. A point 2 time periods into the future was chosen as the point when the new information would become available. The new information consisted of varying the emerging technology's performance so it would not exceed the existing technology's performance or be below its initial performance at the evaluation point.

Figure 3-14 displays the results for the situation where the emerging technology doubles (100% increase) the performance of the existing technology. Figures 3-15 displays the results for the situation where the emerging technology exceeds the performance of the existing technology by 75%. Figures 3-16 displays the results for the situation where the emerging technology exceeds the performance of the existing technology by 50%. Table 3-7 presents the changes to τ_{cop} based on the new information. Two pieces of information can be extracted from the data. First it appears that the change to the cross-over point is dependent on the ratio of the time the information is received to the time the cross-over is predicted to occur. In the case of improving performance the sooner the information is available the more effect the change has on moving the cross-over point. The second piece of information is the smaller the performance gain (Δp) for a given rate of performance (r_p) the further out in time the cross-over point (τ_{cop}) occurs. In order to further explore these ideas an emerging technology with a 50% Δp will be analyzed at a point 5 times period into the future. Figure 3-17 displays the results and Table 3-8 compares the results with Figure 3-16.

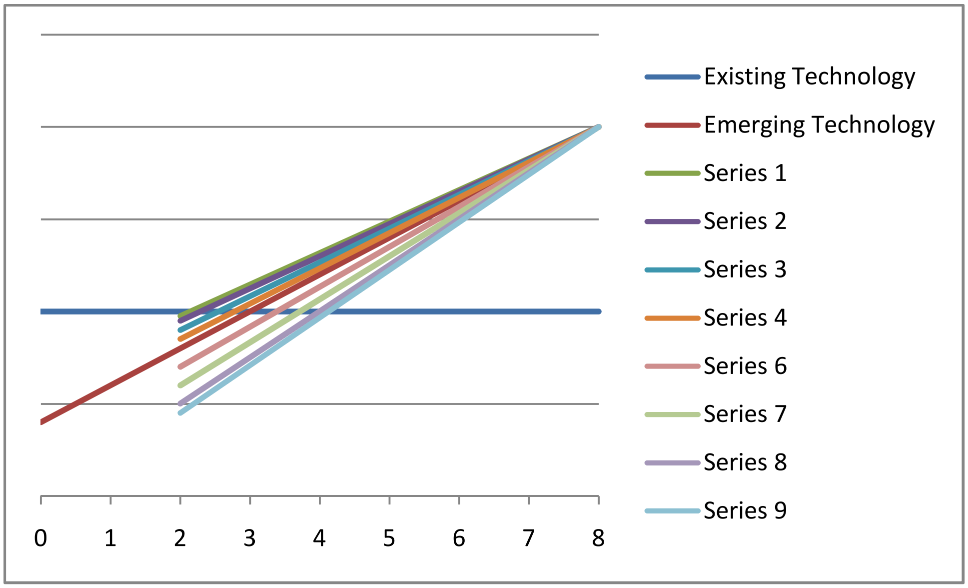

Figure 3-14. 100% Increase in performance
with new information at time 2

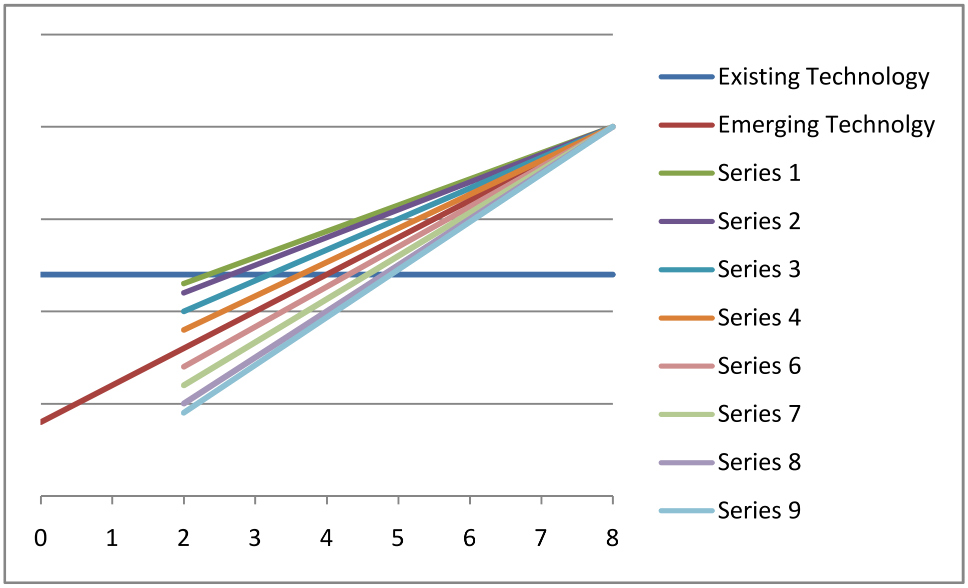

Figure 3-15. 75% Increase in Performance
with new information at time 2

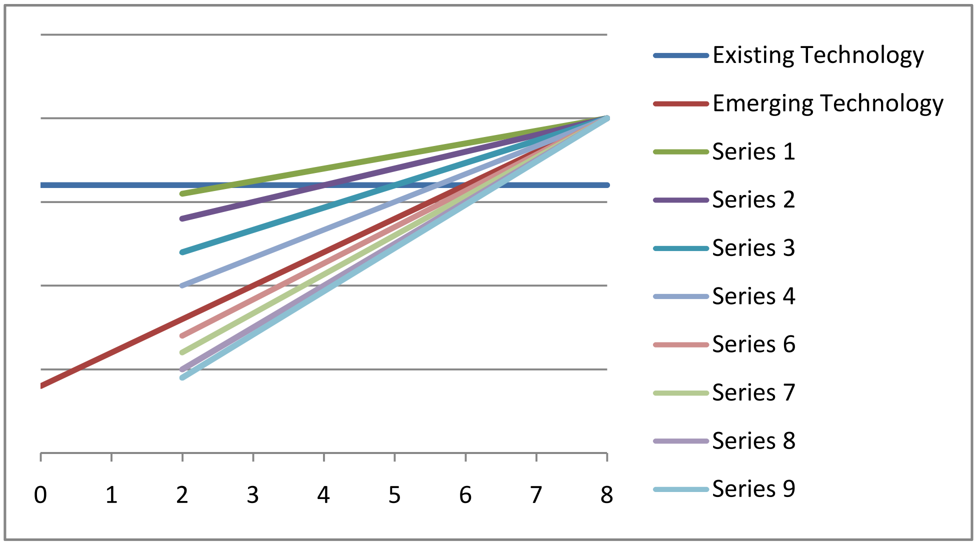

Figure 3-16. 50% Increase in Performance
with new information at Time 2

Table 3-7. New Cross-over Points Based on New Information at Time 2

Movement of Cross-over Point – *Δp* = 100%	Figure 3-14 New Cross-over Point	Movement of Cross-over Point – *Δp*=75%	Figure 3-15 New Cross-over Point	Movement of Cross-over Point – *Δp*=50%	Figure 3-16 New Cross-over Point
28%	2.1	41%	2.4	56%	2.7
24%	2.3	33%	2.7	33%	4.0
15%	2.5	20%	3.2	17%	5.0
7%	2.8	9%	3.6	7%	5.6
0%	3.0	0%	4.0	0%	6.0
-13%	3.4	-8%	4.3	-3%	6.2
-24%	3.7	-14%	4.6	-5%	6.3
-33%	4.0	-20%	4.8	-7%	6.4
-38%	4.1	-23%	4.9	-8%	6.5

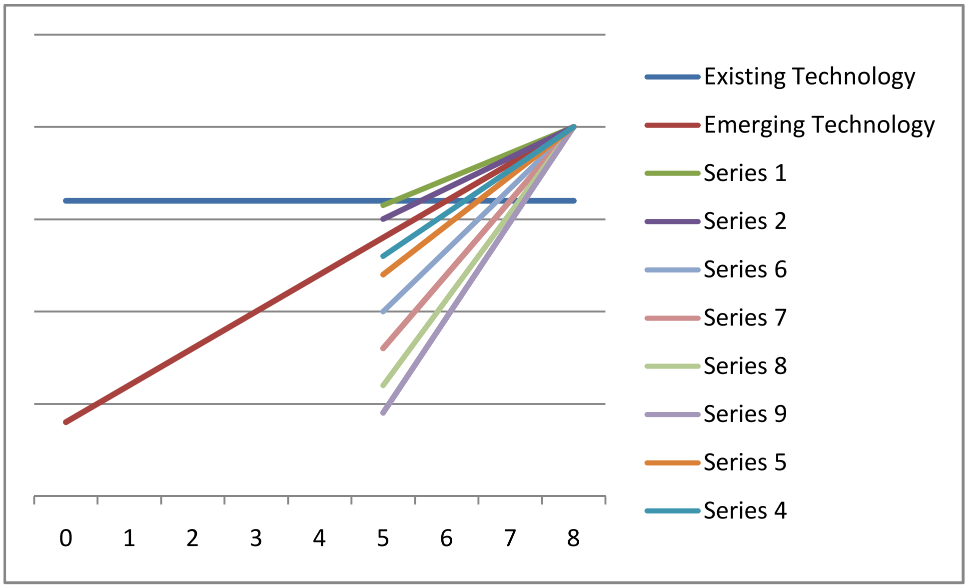

Figure 3-17. 50% Increase in performance with new information at time 5

Table 3-8. Updated r_p and τ_{cop} Based on New Information at Time 2 Compared to Time 5

Change in Performance	New r_p for Δp of 50% at time 2	New τ_{cop} for Δp of 50% at time 2	Change in Performance	New r_p for Δp of 50% at time 5	New τ_{cop} for Δp of 50% at time 5
94%	0.8	2.7	14%	1.4	5.2
75%	1.0	4.0	7%	1.7	5.6
50%	1.3	5.0	0%	2.0	6.0
25%	1.7	5.6	-5%	2.3	6.3
0%	2.0	6.0	-8%	2.7	6.5
-13%	2.2	6.2	-13%	3.3	6.8
-25%	2.3	6.3	-17%	4.0	7.0
-38%	2.5	6.4	-19%	4.7	7.1
-44%	2.6	6.5	-20%	5.2	7.2

The data in Table 3-8 appears to validate the observation that the change to the cross-over point is dependent on the ratio of the time the information is received to the time the cross-over is predicted to occur. In case 1, (Figure 3-16) the ratio is 2/6 and an improving performance would indicate the decision could be made sooner perhaps by as much as two to three time periods. In case 2, (Figure 3-17) the ratio is 5/6 and the amount the decision can move is limited to less than one time period. The other result of comparing figures 3-16 and 3-17 is that it clearly shows that the effect of not meeting the predicted performance has a larger effect and a greater rate of performance will be necessary to meet the predicted future performance. For example, missing the predicted performance by 13% at time 2 results in a change to r_p of 10% compared to a change of 65% at time 5. This sensitivity analysis was conducted based on changes between the future predicted performance and new demonstrated data arriving sometime in the futures. Additional sensitivity analysis should be conducted as part of future research to include forecasting variability, data uncertainty, and model uncertainty.

3.3 Summary

Technology selection can be a key to the success or failure of a project. Yet the existing techniques have had limited success in predicting whether a technology's future performance will be achieved or not. One of the key issues with the current methods is that they ignore two important characteristics of technology selection. First, when the selection is made design dependent parameters are incorporated into the system's configuration and they become either irreversible or reversible with considerable penalty with regard to time, performance, or cost. Second, the current methods utilized to analyze potential design alternatives do not have a mechanism for quantifying the future performance uncertainty associated with a technology.

The decision maker only has two choices when a mature, well established technology that meets the requirements today is competing against an advanced state-of-the-art but still immature technology that does not meet the requirements today but is projected to exceed the currently available technologies. They can ignore the new technology's potential and lose out if it achieves it or they can select the new technology and accept the risk that it will not achieve its projected performance. What should happen is the decision maker should delay the decision until more information about the new technology is available and the decision should be made based on that information. Therefore a new method is needed that incorporates the future performance uncertainty associated with a technology into a decision support tool that provides a quantitative assessment of the new technology's future performance, Technology Options Analysis.

The scenario described above is of interest because it describes a situation where at some time in a new technology's future it is predicted to exceed or cross-over the existing technology's performance. It is this cross-over point that defines the optimum time to make the decision because it is then when the uncertainty about the new technology's performance is answered. The cross-over point is independent of the project and only depends on the technologies being considered. Once it has been determined it can be compared to the project's time horizon and the decision to wait or not can be made. In addition to knowing whether to defer or not, the decision maker also knows what the outcomes will be when the cross-over point is reached, if the new technology has not achieved its predicted performance chose the mature technology or if the new technology has achieve its predicted performance chose the new technology. This sounds very similar to the results one would expect if they were considering investing in a financial call option or evaluating a real options deferral option. Enhancements were then considered that would provide additional information to the decision maker.

Several enhancements were discussed, the first provided the decision maker with information on how much greater the performance of the new technology might be compared to the existing technology. It was important because it provided the decision maker with a quantitative appreciation of the value of deferring the decision. The greater the delta in performance, the more value in deferring until more information was available. Understanding the risk of the new technology reaching its predicted performance was the next enhancement. This was represented as the projected rate of improvement for a technology. In addition to a high rate of change having a higher risk it was demonstrated during the sensitivity analysis that this risk is also present as the demonstrated performance varies from its predicted performance.

Sensitivity analysis was performed on the cross-over point, the emerging technology's delta performance, and the technology's projected rate of performance. There were several key observations from the sensitivity analysis. First was that the change to the cross-over point was independent of the projected rate of performance. Instead it was correlated to the relative change between the demonstrated performance and the projected performance. The magnitude was a function of the technology's delta performance and the time until the cross-over point. This was only true for cases where the delta performance was held constant, i.e., not allowed to change as new information is provided. When the technology's delta performance was allowed to change as new information was provided the change is correlated to the relative change between the demonstrated performance and the projected performance. The projected rate of performance was sensitive to both the relative change between the demonstrated performance and the projected performance and the time until the cross-over point.

Decision about technology selection however cannot be made strictly on how well it meets a set of performance requirements. Selection of a technology also involves the understanding of how much selecting that technology will cost. It is the next chapter that will address the benefit of waiting from both a performance and cost perspective by using Real Options Analysis and Technology Options Analysis.

CHAPTER IV

UNIFIED ALGORITHM ANALYSIS

As discussed throughout this work, technology selection is a key to the success or failure of a Systems Engineering project. Obviously no decision maker wants to make a decision that results in a project that is a failure; quite the opposite is true, but the decision maker can only make a decision based on the information available. Therefore, what decision aid can better utilize the information available to the decision maker to take into account the future uncertainties associated with cost and performance especially when the technology decision results in an irreversible or nearly irreversible architecture?

In the two previous chapters, the Systems Engineering decision tools were extended by developing new methods for evaluating systems engineering technology alternatives with respect to cost (Chapter 2) and performance (Chapter 3) using the framework of Real Options and Technology Options Analysis. This chapter combines those two dimensions into a unified algorithm to simultaneously evaluate cost and performance alternatives so that a decision maker can gain an overall view of the value of delaying a decision until the most appropriate time.
The algorithm is constructed based on a typical scenario faced by systems engineering decision makers. The scenario of interest is one in which two technologies are being consider for the new system. One is mature (Technology A) and meets all the requirements today and into the future. A new technology (Technology B) is being developed but it does not meet the requirements today but its future performance is projected to exceed the performance of the existing technology. This scenario is of interest because it means that at some time in the future the new

technology's performance is predicted to exceed or *cross-over* the existing technology. The dilemma for the systems engineer is this: if the mature technology is chosen it will be a good system if the new technology does not make it to market but if the new technology does mature sufficiently, the new system may be viewed as a mediocre product at best or a failure. This is a similar result to that of Pence and Dilts (Dilts and Pence 2004) . If the systems engineer gambles on the new technology it most likely will be a great product but if the new technology fails the new system fails. For this scenario then the question is "When in the decision process should the technology selection be made?"

4.1 Technology Options and Real Options Analysis Combined

The initial piece of data the decision maker requires is: should the emerging technology even be considered as a potential solution? Critical data for this decision is if the new technology's performance improves beyond that of existing technology; this will occur at what is defined as a cross-over point. Using TOA techniques the cross-over point (τ_{cop}) between the existing and the emerging technologies is calculated. Assume that Technology A is the existing technology and Technology B is the challenger. The timing and uncertainty associated with the cross-over point is evaluated relative to the project's time horizon. There are only two potential outcomes from this initial evaluation:

1) if the cross-over point occurs <u>outside</u> of the project's time horizon, then select the existing technology and **abandon** the emerging technology or
2) if the cross-over point occurs <u>within</u> the project's time horizon, then **continue** evaluating the option using TOA techniques to determine the performance change (delta) that could be expected between the new and existing technologies.

Of particular importance is the realization that, if the outcome is to abandon the emerging technology, valuable resources do not have to be spent conducting further analysis. If the outcome was to calculate the performance deltas, the results are provided to the decision maker. Based on this data, two outcomes can occur; 1) if the performance delta is insufficient, as determined by the decision maker, the emerging technology will be **abandoned**, 2) if the performance delta is sufficient, additional analysis using the techniques of NPV and ROA will be used to generate additional information that the decision maker will need to determine if the cost of performance flexibility is of sufficient value.

TOA data is utilized to determine the expenditure profiles during both NPV and ROA analysis by computing the cost associated with continuing the development with both technologies until τ_{cop}. This cost becomes the first expenditure (E_{AB}^{1}) and additional expenditures will be based on one of two cases:

1) Case 1: Technology B's performance does not exceed Technology A's, or
2) Case 2: Technology B's performance exceeds Technology A's performance at τ_{cop}.

The NPV for both cases is then calculated based on the new expenditure profile. In the situation where the NPV for both cases is positive, no further analysis is required; as the decision is to **carry both technologies** until τ_{cop} because, regardless of the state of technology maturity of either technology at τ_{cop} the technology will be sufficient for the needs of the new system. If however one or both of the NPV outcomes is 0, the analysis will continue using ROA techniques.

In complex situations where there exists a phased investment situation, ROA makes use of compound sequential options analysis. It utilizes the estimate of the value of the project if it existed today (S) obtained during the NPV analysis; future project values are calculated as a function of S. The next step is to calculate and evaluate the Present Value at each node starting at the nodes where the last expenditure takes place. If all of the Present Values are zero at the time the last expenditure takes place stop the analysis and **abandon** the technology. Otherwise, continue evaluating the remaining nodes working from the future to the present. Whenever all of the Present Values within an evaluation time period are 0, stop the analysis and **abandon** the technology. Once the Present Value analysis is completed, the final Present Value is used to determine the Net Option Value. The Net Option Value is determined by subtracting the NPV from the final Present Value. If in both cases the NOV is positive, **carry both technologies** until τ_{cop} because no matter what happens at τ_{cop} the technology will satisfy the system requirement. Chapter 2 described this compound sequential method in detail and it will be demonstrated as part of the example problem provided later in this chapter.

4.2 Real-life Example

In order to explain the method, a "real-life" example is utilized. There are two important initial notes: First, while the scenario is fictional, the cost and performance data used are actual information gathered from (BitMicro 2004; Kerekes 2005; Memtech 2006; TigerDirect 2006; Electonics 2007; Kerekes 2007; SanDisk 2007; SanDisk 2007; SuperTalent 2007; Tokar 2007; Kerekes 2008; Kerekes 2008; Mtron 2008; NewEgg 2008; NexTag 2008; RiData 2008; Samsung 2008; Technology 2008; Tokar 2008; Tokar 2008; BitMicro 2009; Curry 2009; Kerekes 2009; NexTag 2009; NexTag 2009; Shopping 2009; Tom's Hardware 2009; Tom's Hardware 2009; Tom 2009; Tom 2009; NexTag 2099). Second, while the decisions are specific to the example, the algorithm is generalizable to multiple systems engineering settings and decision making situations.

For this example, assume that it is 2006, just before the preliminary design for a proposed new system. The detailed design review will be conducted in 2008 and production will start in 2009. The technologies of interest are a hard disk drive (Technology A) and a solid state drive (Technology B). Technology A is mature, well-known and primarily subject to incremental innovation while Technology B is a new, less well-known, and still immature technology. Technology A meets all the requirements today and is expected to be available during production period for the system. Technology B does not meet the requirements today but is expected to exceed the requirements when the project goes into production in 2009. The financial risk free rate of return is 10%.

4.2.1 Technology Options Analysis

An important question to the success or failure of a project cannot be answered by the conduct of NPV or Real Options Analysis. That question is, when will Technology B's performance be equal to or better than Technology A's? The reason that question is important to the decision maker is because once the new technology is better than the current technology it tends to always be better. If the decision maker knew the answer to that question would a different decision result?

For this example the performance attribute of greatest importance is the number of megabytes (MB) that can be read per second from the drive, called the drive memory. This attribute is commonly benchmarked and referred to as a memory's sustained read capability measured in MB/s. For this project, assume the sustained read requirement is at least 70MB/s with a goal of 100MB/s.

Technology A is a hard disk drive (HDD) with a rotational speed of 10Krpm and an 8MB cache. Based on the rotational speed and cache size, its sustained read time performance trajectory is 72 MB/s. Since the read time for a hard disk drive is a function of the speed of the drive and the cache size one or both must be increased to improve performance. The technology roadmap provided by the company making this type of drive shows that its sustained read time is not expected to improve over the time frame of this development program (Curry 2009). Technology B is a solid state drive (SSD) and it has no moving parts. Its sustained read time is a function of transistor size and the number of bits per cell. In 2006 its predicted performance trajectory suggests that the performance of this SSD will double every 18 months (or so) (Kerekes 2005; Kerekes 2008) based on decreasing transistor size and the potential ability to store multiple bits in each cell. The SSD's demonstrated sustained read time performance in 2006 was 45 MB/s and this speed does not meet the project's requirement. Table 4-1 provides the projected performance based on its technology roadmap from 2007 until 2009 versus the HDD's performance. It is this predicted future performance that is the reason the systems engineer is interested in the solid state drive approach.

Table 4-1. Projected Read Times

Year	Technology B SSD Sustained Read Time *Predicted*	Technology A HDD Sustained Read Time *Known*
2007	75 MB/s	72 MB/s
2008	120 MB/s	72 MB/s
2009	180 MB/s	72 MB/s

The cross-over point for this example can be determined using the 2006 demonstrated sustained read performance and the predicted sustained read performance based Technology A and Technology B's performance trajectories presented in Table 4-1. Using these data points, lines are drawn, and where the lines intersect is the cross-over point τ_{cop}, see Figure 4-1.

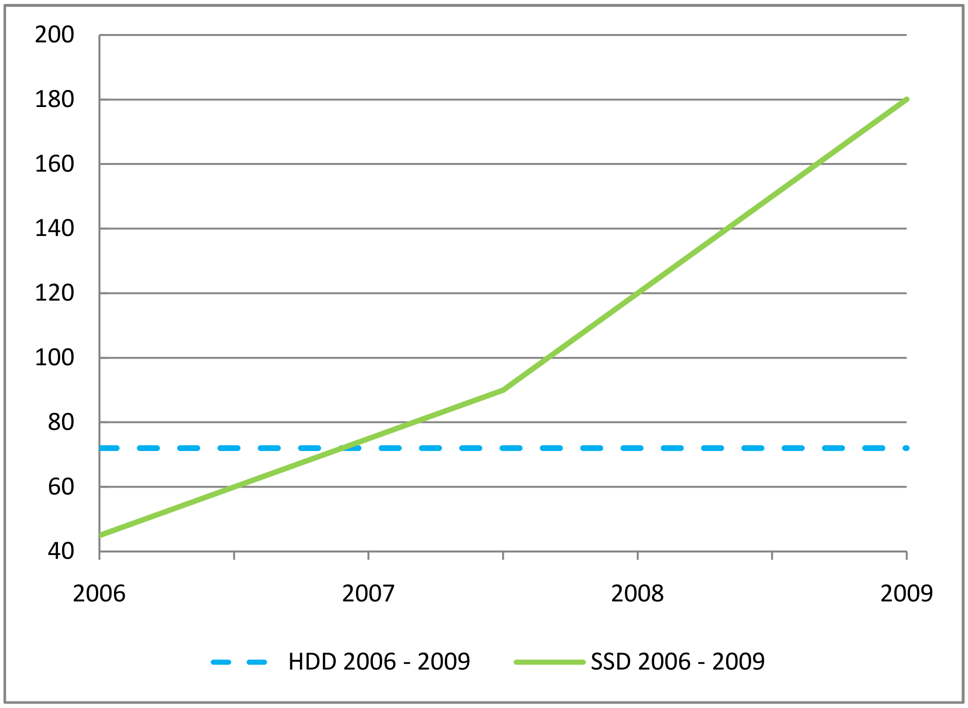

Figure 4-1. Cross-over Point for Projected Read Times

Because the initial doubling of the predicted performance for Technology B is better than Technology A (72 MB/s), see Figure 4-1 and Table 4-1, it meant that within the area of interest the performance trajectory was linear and the calculations can be accomplished by manipulating the standard equation for a line, y = mx + b[1]. Remember from Chapter 3 that for Technology Options Analysis, y is the emerging technology's predicted future performance (P_f), m is the projected rate of performance (r_p), x is the time the predicted future performance will be achieved, and b is the emerging technology's current performance (P_c), thereby giving $P_f = r_p * x + P_c$. The projected rate of performance for the emerging technology then can be calculated by solving for the slope of a line given by $r_p = [(P_f) - (P_c)] / x$. The cross-over point ($\tau_{cop}$) is then calculated by setting the emerging technology's performance equal to existing technology's performance and calculating when that would occur, such that $\tau_{cop} = [(P_f) - (P_c)] / r_p$, where P_f is the existing technology's performance.

The projected rate of performance is:

r_p = [90 MB/s – 45 MB/s]/18 months = 45 MB/s / 18 months = 2.5 MB/s /Month

The cross-over point (τ_{cop}) is:

τ_{cop} = [72 MB/s – 45 MB/s] / 2.500 MB/s / Month = 10.8 Months

The cross-over point for Technology B is within the time horizon of the project and based on that the next step is to determine its delta performance. The performance delta (Δp) between the new technology and the existing technology at a specified point in the future is calculated to provide the decision maker with a quantitative appreciation of the performance improvement based on deferring the decision. The greater the delta in performance the more value in deferring the decision until more information is available.

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[1] It should be noted that the technology curves can follow other shapes. For ease in demonstrating the algorithm, a linear trajectory is assumed.

The delta performance (Δp) values are presented in Table 4-2 and calculated for 2009 below:

Δp_s = 180 MB/s – 72 MB/s = 108 MB/s or a 150% improvement

Table 4-2. Projected Delta Performance

Year	Technology B SSD *Predicted* Sustained Read Time	Technology A HDD *Known* Sustained Read Time	Delta Performance	% Improvement
2007	75 MB/s	72 MB/s	3 MB/s	4%
2008	120 MB/s	72 MB/s	48 MB/s	67%
2009	180 MB/s	72 MB/s	108 MB/s	150%

The decision maker now knows that Technology B is expected to cross-over during the time horizon of the project. More specifically, the decision maker knows that delay the technology selection for approximately 11 months could produce a performance gain of 150%. Based on this information the systems engineer decides it would be of value to continue the analysis. The decision maker also decides that based on the uncertainty associated with Technology B's predicted performance that instead of using 10.8 as predicted by τ_{cop} the expenditure stream and the remaining analysis will be based on a value of 12 months or 1 year.[2]

Table 4-3 presents the hardware costs for each technology based on cost estimates from various sources (NewEgg 2008; NexTag 2008; NexTag 2009; NexTag 2009; Shopping 2009; NexTag 2099), the number of units to be procured during each phase of project, and the labor costs during each phase of the project. Labor costs are expected to be slightly different between Technology A and Technology B depending on whether Technology B's future predicted

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[2] This is used for convenience in the example. It does not materially impact the computations or the results.

performance is achieved. If Technology B reaches its full predicted performance a labor reduction of 20% will be realized and accounts for the difference between L_A^2 and L_B^2.

Table 4-3. Sample Problem Quantities and Costs

Variable	Meaning	Quantity	Cost
X	Number of Drives used in Preliminary Design	10	
$\$_A^1$	Cost of HDD during Preliminary Design		$190
$\$_B^1$	Cost of SSD during Preliminary Design		$1000
L_A^1	Labor Cost for HDD during Preliminary Design		$10,000
L_B^1	Labor Cost for SSD during Preliminary Design		$10,000
Y	Number of Drives used in Detailed Design	25	
$\$_A^2$	Cost of HDD during Detailed Design		$137.50
$\$_B^2$	Cost of SSD during Detailed Design		$250
L_A^2	Labor Cost for HDD during Detailed Design		$10,000
L_B^2	Labor Cost for SSD during Detailed Design		$8,000
Z	Number of Drives procured for production	2500	
$\$_A^3$	Cost of HDD during Detailed Design		$117.50
$\$_B^3$	Cost of SSD during Detailed Design		$125

The expenditures during each phase will vary between technologies and are further defined as follows and presented in Table 4-4:

E_A^1 is the expenditure required for incorporating the hard disk drive technology into the preliminary design and is equal to the cost ($\$_A^1$) of X hard disk drives plus a labor cost of L_A^1

E_B^1 is the expenditure required for incorporating the solid state drive technology into the preliminary design and is equal to the cost ($\$_B^1$) of X solid state drives plus a labor cost of L_B^1

E_A^2 is the expenditure required for incorporating the hard disk drive technology into the detailed design and is equal to the cost ($\$_A^2$) of Y hard disk drives plus a labor cost of L_A^2

E_B^2 is the expenditure required for incorporating the solid state drive technology into the detailed design and is equal to the cost ($\$_B^2$) of Y solid state drives plus a labor cost of L_B^2

E_A^3 is the expenditure required for procuring ($\$_A^3$) Z hard disk drives

E_B^3 is the expenditure required for procuring ($\$_B^3$) Z solid state drives

E_{AB}^1 is the expenditure required for incorporating both the hard disk drive and solid state drive technology into the preliminary design and is equal to the cost ($\$_A^1$) of X hard disk drives plus ($\$_B^1$) of X solid state drives plus ½ the labor cost of L_A^1 plus L_B^1

$E_A^{2'}$ is the expenditure required for incorporating only the hard disk drive into the preliminary design if the solid state drive fails to meet its performance trajectory and is equal to ½ the labor cost of L_A^1

$E_B^{2'}$ is the expenditure required for incorporating only the solid state drive into the preliminary design if the solid state drive meets its performance trajectory and is equal to ½ the labor cost of L_B^1

$E_A^{3'}$ is the expenditure required for incorporating the hard disk drive technology into the detailed design if the solid state drive fails to meet its performance trajectory and is equal to the cost ($\$_A^2$) of Y hard disk drives plus a labor cost of L_A^2

$E_B^{3'}$ is the expenditure required for incorporating the solid state drive technology into the detailed design if the solid state drive meets its performance trajectory and is equal to the cost ($\$_B^2$) of Y solid state drives plus a labor cost of L_B^2

$E_A^{4'}$ is the expenditure required for procuring ($\$_A^3$) Z hard disk drives if the solid state drive fails to meet its performance trajectory

$E_B^{4'}$ is the expenditure required for procuring ($\$_B^3$) Z solid state drives if the solid state drive meets its performance trajectory

Table 4-4. Computation of Variable Values

Variable	Equation	Value	Cost
E_A^1	$= (\$_A^1 * X) + L_A^1$	= ($190*10)+$10000	= $ 11,900
E_B^1	$= (\$_B^{1\,*} X) + L_B^1$	= ($1000*10)+$10000	= $ 20,000
E_A^2	$= (\$_A^2 * Y) + L_A^2$	= ($137.05*25)+$10000	= $ 13,437
E_B^2	$= (\$_B^2 * Y) + L_B^2$	= ($250*25)+$8000	= $ 14,250
E_A^3	$= \$_A^3 * Z$	= ($117.50*2500)	= $293,750
E_B^3	$= \$_B^3 * Z$	= ($125*2500)	= $312,500
E_{AB}^1	$= (\$_A^1 * X) + (\$_B^1 * X) + (½ *(L_A^1 + L_B^1))$	= ($190*10)+($1000*10)+ (½ * ($1000+$1000)	= $ 21,900
$E_A^{2'}$	$= ½ * L_A^1$	= ½ * $10000	= $ 5,000
$E_B^{2'}$	$= ½ * L_B^1$	= ½ * $10000	= $ 5,000
$E_A^{3'}$	$= (\$_A^2 * Y) + L_A^2$	= ($137.05*25)+$10000	= $ 13,437
$E_B^{3'}$	$=(\$_B^2 * Y) + L_B^2$	= ($250*25)+$8000	= $ 14,250
$E_A^{4'}$	$= \$_A^3 * Z$	= ($117.50*2500)	= $293,750
$E_B^{4'}$	$= \$_B^3 * Z$	= ($125*2500)	= $312,500

4.2.2 Net Present Value Analysis

NPV is the difference between how much the new system would be worth if it existed today (S) and how much the system will cost or all the required expenditures (E) to bring the project to reality.

$$\mathrm{NPV} = S - \Sigma E$$

The current value for the project if it existed today (S) was determined by computing the average of the production costs and development costs, and adding them together and was equal to \$245,000, the assumed risk free rate of return (r_f) is 10%, and the expenditures are listed in Table 4-4. This leads to the following equations:

When using Technology A exclusively

$$\mathrm{NPV}_A = S - E_A^1 - E_A^2 / (1+r_f)^2 - E_A^3 / (1+r_f)^3$$

$$\mathrm{NPV}_A = \$245{,}000 - \$11{,}900 - \$13{,}437/(1.1)^2 - \$293{,}750/(1.1)^3$$

$$\mathrm{NPV}_A = \$245{,}000 - \$11{,}900 - \$11{,}105 - \$220{,}699$$

$$\mathrm{NPV}_A = \$1{,}296$$

When using Technology B exclusively

$$\mathrm{NPV}_B = S - E_B^1 - E_B^2 / (1+r_f)^2 - E_B^3 / (1+r_f)^3$$

$$\mathrm{NPV}_B = \$245{,}000 - \$20{,}000 - \$14{,}250/(1.1)^2 - \$312{,}500/(1.1)^3$$

$$\mathrm{NPV}_B = \$245{,}000 - \$20{,}000 - \$11{,}777 - \$234{,}786$$

$$\mathrm{NPV}_B = -\ \$21{,}563 \longrightarrow 0$$

When starting the preliminary design with both technologies and Technology B fails to meet its performance estimates

$$\mathrm{NPV}_{AB\_A} = S - E_{AB}^1 - E_A^{2'} / (1+r_f) - E_A^{3'} / (1+r_f)^2 - E_A^{4'} / (1+r_f)^3$$

$$\mathrm{NPV}_{AB\_A} = \$245{,}000 - \$21{,}900 - \$5000/(1.1) - \$13{,}437/(1.1)^2 - \$293{,}750/(1.1)^3$$

$$\mathrm{NPV}_{AB\_A} = \$245{,}000 - \$21{,}900 - \$4545 - \$11{,}105 - \$220{,}699$$

$$\mathrm{NPV}_{AB\_A} = -\ \$13{,}249 \longrightarrow 0$$

When starting the preliminary design with both technologies and Technology B meets it performance estimates

$$NPV_{AB\_B} = S - E_{AB}^{1} - E_{B}^{2'} / (1+r_f) - E_{B}^{3'} / (1+r_f)^2 - E_{B}^{4'} / (1+r_f)^3$$

$$NPV_{AB\_B} = \$245{,}000 - \$21{,}900 - \$5000/(1.1) - \$14{,}250/(1.1)^2 - \$312{,}500/(1.1)^3$$

$$NPV_{AB\_B} = \$245{,}000 - \$21{,}900 - \$4545 - \$11{,}777 - \$234{,}786$$

$$NPV_{AB\_B} = - \$28{,}008 \longrightarrow 0$$

The NPV analysis provides the decision maker with the following information: the decision maker now knows that only the NPV_A is greater than zero. More specifically, the decision maker knows that $NPV_A = \$1{,}296$ and NPV_B NPV_{AB_A} and NPV_{AB_B} are all < 0. Based on this information the decision maker should decide it would be of value to continue the analysis.

4.2.3 Real Options Analysis

Table 4-5 defines the variables used in calculating a Compound Sequential Option Analysis.

The distribution of possible project values will be considered to be fairly standard and assumed to follow a lognormal distribution. As discussed in Chapter 2, this means that the factor to apply for an up movement is given by the formula *e* to the power of σ (volatility or variance) times the square root of τ (time interval) and the factor for a down movement is given by the inverse of the up factor.

Table 4-5. Compound Option Variables

Compound Option Variable	*Definition*	*Comments*
$S_{i,j}$	Value of the SE project at point i,j	i and j define the variable's position in the lattice
E_y^N	Investment required to design, develop, build, and test the SE project	y designates the technology N designates different sequential investments over time, i.e., 1st, 2nd, etc.
r_f	Risk-free rate of return	
σ	Volatility of project	Used to calculate the up and down factors
u	Up Factor	$u = e^{\sigma\sqrt{\tau}}$
d	Down Factor	d= 1/u
p	Probability of the $PV_y{}^N{}_{i,j}$ increasing	
(1-p)	Probability of $PV_y{}^N{}_{i,j}$ decreasing	
$PV_y{}^N{}_{i,j}$	Present Value of the SE project at point i,j evaluated using E_y^N if appropriate	
NOV	Net Option Value	

Assuming a lognormal distribution centered at the value for S ($245,000) utilized in the NPV analysis and based on the variance of the sums for the production and development costs a variance of 20%, the distribution of the possible project values is then a multiplicative process that starts at $S_{0,0}$ and moves up or down based on σ with a up factor (u) and a down factor (d). In this example the volatility (σ) is 20% or .2 and $\tau = 1$ so the up factor $u = e^{\sigma\sqrt{\tau}} = e^{.2} = 1.22$ and the down factor d=1/u=1/1.22=0.82. Therefore,

$S_{1,0} = u * S_{0,0} = 1.22 * \$245{,}000 = \$299{,}244$

$S_{0,1} = d * S_{0,0} = 0.82 * \$245,000 = \$200,589$

Table 4-6 presents the $S_{i,j}$ values for the project through 2009.

Table 4-6. Possible Project Values (S)

	2006		*2007*		*2008*		*2009*
						$S_{3,0}$ =	$446,419
				$S_{2,0}$ =	$365,497		
		$S_{1,0}$ =	$299,244			$S_{2,1}$ =	$299,244
$S_{0,0}$ =	$245,000			$S_{1,1}$ =	$245,000		
		$S_{0,1}$ =	$200,589			$S_{1,2}$ =	$200,589
				$S_{0,2}$ =	$164,228		
						$S_{0,3}$ =	$134,459

The next step in sequential option analysis is to calculate and evaluate the Present Values ($PV_y{}^N{}_{i,j}$) at the point in time that the last investment takes place, 2009 in this example. Remember that the reason for starting the evaluation when the last investment takes place is because if all of the $PV_y{}^N{}_{i,j}$ values at those points are $0 then the project should be abandoned and no other analysis is required. For the cases under consideration; case 1) is when starting the preliminary design with both technologies and Technology B fails to meet its performance estimates and case 2) is when starting the preliminary design with both technologies and Technology B meets it performance estimates, Table 4-4 lists all the potential expenditures for each year associated with the cases under consideration.

As discussed in Chapter 2, the $PV_y{}^N{}_{i,j}$ for any point i,j in the lattice, is determined using one of two formulas;

1) when expenditure is required in order to realize the project at a point i,j then, the expenditures (E) are subtracted from the value of the project ($S_{i,j}$) associated with point i,j but in no case can $PV_y{}^N{}_{i,j}$ ever be less than zero. This leads to the following formula:

$$PV_y{}^N{}_{i,j} = \mathrm{MAX}[S_{i,j} - E_y{}^N, 0]$$

2) when no expenditures are required then $PV_y{}^N{}_{i,j}$ is a function of $PV_y{}^N{}_{i+1,j}$ and $PV_y{}^N{}_{i,j+1}$ and depends on the probability of realizing either value such that:

$$PV_y{}^N{}_{i,j} = (p * PV_y{}^N{}_{i+1,j}) + ((1-p) * PV_y{}^N{}_{i,j+1})$$

In the two cases under consideration based on this example an expenditure takes place every year so equation 2 is not used. The equations for Case 1 for the nodes at 2009 are:

$$\begin{aligned} PV_{AB\_A}{}^4{}_{3,0} &= MAX(S_{3,0} - E_A{}^4, 0) \\ &= MAX(\$446419 – \$293750, \$0) \\ &= MAX(\$152669, \$0) = \$152669 \end{aligned}$$

$$\begin{aligned} PV_{AB\_A}{}^4{}_{2,1} &= MAX(S_{2,1} - E_A{}^4, 0) \\ &= MAX(\$299244 – \$293750, \$0) \\ &= MAX(\$5494, \$0) = \$5494 \end{aligned}$$

$$\begin{aligned} PV_{AB\_A}{}^4{}_{1,2} &= MAX(S_{1,2} - E_A{}^4, 0) \\ &= MAX(\$200589 – \$293750, \$0) \\ &= MAX(-\$93161, \$0) = \$0 \end{aligned}$$

$$\begin{aligned} PV_{AB\_A}{}^4{}_{0,3} &= MAX(S_{0,3} - E_A{}^4, 0) \\ &= MAX(\$134459 – \$293750, \$0) \\ &= MAX(-\$159291, \$0) = \$0 \end{aligned}$$

Using the same techniques the remaining $PV_y{}^N{}_{i,j}$ values were calculated and are presented in Table 4-7 for Case 1 and Table 4-8 for Case 2.

Table 4-7. Present Values for Case 1

Year	*2006*		*2007*		*2008*		*2009*
Expenditure	\$21,900		\$5,000		\$13,437		\$293,750
						$PV_{AB\_A}{}^{4}_{3,0}$	= \$152,669
				$PV_{AB\_A}{}^{3}_{2,0}$	= \$85,015		***Procure Technology A***
		$PV_{AB\_A}{}^{2}_{1,0}$	= \$48,985		***Use Technology A in Detailed Design***	$PV_{AB\_A}{}^{4}_{2,1}$	= \$5494
$PV_{AB\_A}{}^{1}_{0,0}$	= \$12,316		***Use Technology A***	$PV_{AB\_A}{}^{3}_{2,1}$	= \$0		***Procure Technology A***
	Use Both Technology A and B in Preliminary Design	$PV_{AB\_A}{}^{2}_{0,1}$	= \$0		***Abandon***	$PV_{AB\_A}{}^{4}_{1,2}$	= \$0
			Abandon	$PV_{AB\_A}{}^{3}_{0,2}$	= \$0		***Abandon***
					Abandon	$PV_{AB\_A}{}^{4}_{0,3}$	= \$0
							Abandon

Table 4-8. Present Values for Case 2

Year	*2006*		*2007*		*2008*		*2009*
Expenditure	$21,900		$5,000		$14,250		$312,500
						$PV_{AB\_B}{}^{4}{}_{3,0}$	= $133,919
				$PV_{AB\_B}{}^{3}{}_{2,0}$	= $70,789		***Procure Technology B***
		$PV_{AB\_B}{}^{2}{}_{1,0}$	= $39,952		***Use Technology B in Detailed Design***	$PV_{AB\_B}{}^{4}{}_{2,1}$	= $0
$PV_{AB\_B}{}^{1}{}_{0,0}$	= $6,007		***Use Technology B***	$PV_{AB\_B}{}^{3}{}_{2,1}$	= $0		***Abandon***
	Use Both Technology A and B in Preliminary Design	$PV_{AB\_B}{}^{2}{}_{0,1}$	= $0		***Abandon***	$PV_{AB\_B}{}^{4}{}_{1,2}$	= $0
			Abandon	$PV_{AB\_B}{}^{3}{}_{0,2}$	= $0		***Abandon***
					Abandon	$PV_{AB\_B}{}^{4}{}_{0,3}$	= $0
							Abandon

The value of the option to wait until the cross-over point to make the technology selection is the difference between the NPV and the project's Present Value. The Net Option Value (NOV) for Case1 and Case 2 are shown below:

$NOV_{AB\_A} = PV_{AB\_A}{}^{1}{}_{0,0} - NPV_{A}$

$NOV_{AB\_A} = \$12,316 - \$1,296$

$NOV_{AB\_A} = \$11,020$

$NOV_{AB\_B} = PV_{AB\_B}{}^{1}{}_{0,0} - NPV_{A}$

$NOV_{AB\_B} = \$6,007 - \$1,296$

$NOV_{AB\_B} = \$4,711$

For this example both of the Net Option Values are greater than zero, so the decision is to carry both technologies until the cross-over point and make the technology selection then. In both cases the options are sufficiently valuable to maintain.

4.2.4 The Technology Selection Decision at the Cross-over Point

It is now 2007 and the cross-over point has been reached, new performance data is available on both HDD (Technology A) and SSD (Technology B). Technology A is continuing to perform at 72MB/s for sustained reads and Technology B has crossed over Technology A's performance and is demonstrating sustained read times of 75MB/s. Cost information is also available for Technology A and Technology B, Technology A's cost has only decreased from \$190 to \$180 which places it above its cost projection and Technology B's cost is \$550 which places it on its cost projection. Based on this information the technology selection decision is to use the Technology B. The next section investigates the sensitive of the variables used in this example.

4.3 Sensitivity Analysis

The baseline for the sensitivity analysis was the example presented above. Sensitivity of the algorithm to variations in an emerging technology's performance trajectory are explored in this part of the chapter.

4.3.1 Technology Options Analysis

Technology B's performance trajectory was based on a forecast that the performance would double every 18 months. In order to understand how sensitive the algorithm is the baseline forecast was varied by +/- 6 months or approximately 33%. Table 4-9 lists the 2006 projected read times for performance trajectories of 12, 18, and 24 months for Technology B.

Table 4-9. Projected Read Times

Year	Technology B SSD Sustained Read Time *Predicted*			Technology A HDD Sustained Read Time *Known*
	12 Months	18Months	24 Months	
2006	45 MB/s	45 MB/s	45 MB/s	72 MB/s
2007	90 MB/s	75 MB/s	68 MB/s	72 MB/s
2008	180 MB/s	120 MB/s	90 MB/s	72 MB/s
2009	360 MB/s	180 MB/s	135 MB/S	72 MB/s

The cross-over points for the various performance trajectories from Table 4-9 are plotted in Figures 4-2 and 4-3.

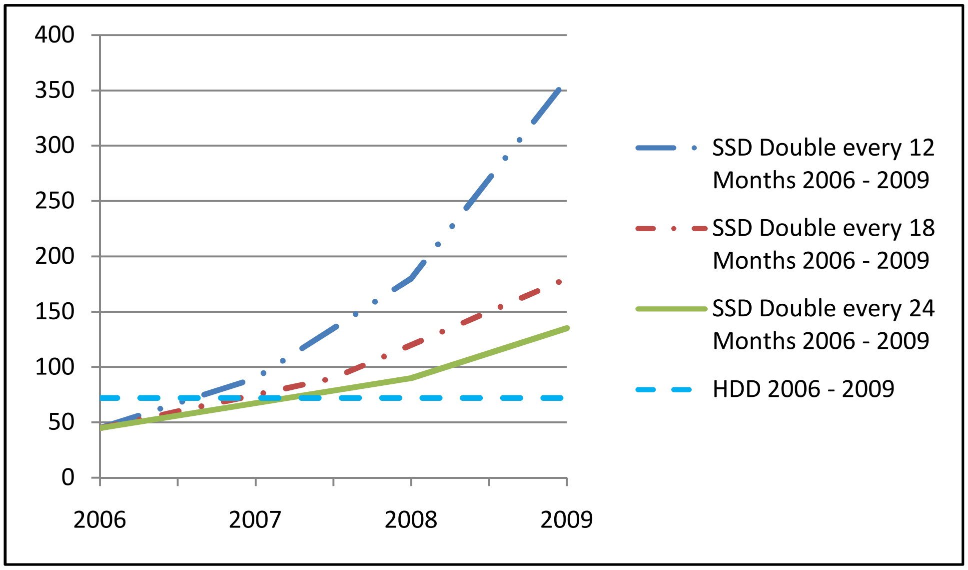

Figure 4-2. Cross-over Points for Various
Performance Trajectories (1/2)

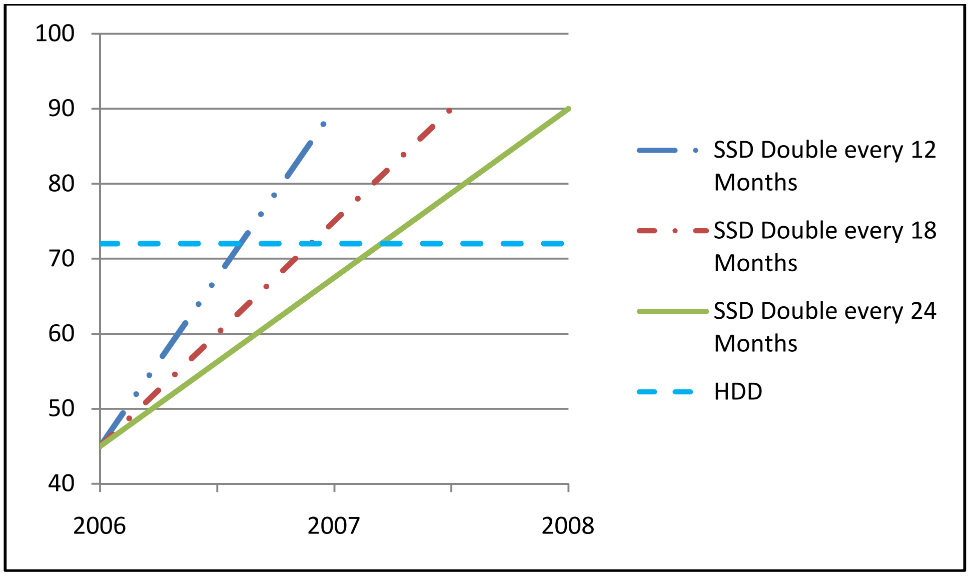

Figure 4-3. Cross-over Points for Various
Performance Trajectories (2/2)

Because the initial doubling of the predicted performance for Technology B is better than Technology A (72 MB/s) it is assumed that within the area of interest the performance trajectory is linear[3] and the calculations can be accomplished by manipulating the standard equation for a line as was done before.

The projected rate of performance for each performance trajectory:

$r_{p12months}$ = [90 MB/s – 45 MB/s]/12 months = 3.75 MB/s /Month

$r_{p18months}$ = [90 MB/s – 45 MB/s]/18 months = 2.5 MB/s /Month

$r_{p24months}$ = [90 MB/s – 45 MB/s]/24 months = 1.875 MB/s /Month

---

[3] As noted previously, the technology curves can follow other shapes. For ease in demonstrating the algorithm, a linear trajectory is assumed.

The cross-over point (τ_{cop}) for each performance trajectory:

$\tau cop_{12months} = [72\ MB/s - 45\ MB/s] / 3.75\ MB/s / Month = 7.2\ Months$

$\tau cop_{18months} = [72\ MB/s - 45\ MB/s] / 2.5\ MB/s / Month = 10.8\ Months$

$\tau cop_{24months} = [72\ MB/s - 45\ MB/s] / 1.875\ MB/s / Month = 14.4\ Months$

The cross-over points for all three performance trajectories are within the time horizon of the project and the next step is to determine their delta performance.

The delta performance (Δp) for each performance trajectory in 2009 based on Table 4-9 is:

$\Delta p_{12months} = 360\ MB/s - 72\ MB/s = 288\ MB/s$ *or a 400% improvement*

$\Delta p_{18months} = 180\ MB/s - 72\ MB/s = 108\ MB/s$ *or a 150% improvement*

$\Delta p_{24months} = 135\ MB/s - 72\ MB/s = 63\ MB/s$ *or a 88% improvement*

Table 4-10 presents the effect a 33% change on this particular performance trajectory had on the projected rate of performance, the cross-over point, and the delta performance. The data would indicate that for this example the cross-over point and the change in trajectory are perfectly correlated. This relationship is the same as was seen in the sensitivity analysis conducted in Chapter 3 and appears to apply when all potential trajectories start from the same demonstrated performance data point. Since the performance trajectories are exponential in this example and the delta performance calculations are not part of the linear region of interest the delta performance is correlated as a function of the time the evaluation takes place.

Table 4-10. Changes in Cross-Over and Delta Performance

	Double every 12 Months	Double every 18 Months	Double every 24 Months
r_p	3.75 MB/s/Month	2.5 MB/s/Month	1.875 MB/s/Month
% Change	50% faster	n/a	25% slower
τ_{cop}	7.2 Months	10.8 Months	14.4 Months
% Change	33% earlier	n/a	33% later
Δp	288 MB/s	108 MB/s	63 MB/s
% Change	166% faster	n/a	41% slower

Based on all the cross-over points being within the time horizon of the project and the amount of potential performance gain the sensitivity analysis will continue using the steps called out in the algorithm.

Using the data from Table 4-4 and the equations presented below, the new expenditure profiles based on each cross-over point was calculated and is presented in Table 4-11.

$E_{AB}{}^{1}{}_{12Months} = (X*(\$_A{}^1 + \$_B{}^1)) + [((L_A{}^1 + L_B{}^1)/24months)] * \tau_{cop12months}$

$E_{AB}{}^{1}{}_{18months} = (X*(\$_A{}^1 + \$_B{}^1)) + [((L_A{}^1 + L_B{}^1)/24)] * \tau_{cop18months}$

$E_{AB}{}^{1}{}_{24months} = (X*(\$_A{}^1 + \$_B{}^1)) + [((L_A{}^1 + L_B{}^1)/24)] * \tau_{cop24months}$

$E_A{}^{2'}{}_{12Months} = (L_A{}^1/24 \text{ months}) * (24 \text{ months} - \tau_{cop12months})$

$E_A{}^{2'}{}_{18months} = (L_A{}^1/24 \text{ months}) * (24 \text{ months} - \tau_{cop18months})$

$E_A{}^{2'}{}_{24months} = (L_A{}^1/24 \text{ months}) * (24 \text{ months} - \tau_{cop24months})$

$E_B{}^{2'}{}_{12Months} = (L_B{}^1/24months) * (24 \text{ months} - \tau_{cop12months})$

$E_B{}^{2'}{}_{18months} = (L_B{}^1/24 \text{ months}) * (24 \text{ months} - \tau_{cop18months})$

$E_B{}^{2'}{}_{24months} = (L_B{}^1/24 \text{ months}) * (24 \text{ months} - \tau_{cop24months})$

The detailed design and production cost remain the same both in their timing and their value.

Table 4-11. Calculated Cross-over Points

Variable	Cost
$E_{AB}^{1}{}_{12Months}$	= \$17,900
$E_{AB}^{1}{}_{18months}$	= \$20,900
$E_{AB}^{1}{}_{24months}$	= \$23,900
$E_{A}^{2'}{}_{12Months}$	= \$ 7,000
$E_{A}^{2'}{}_{18months}$	= \$ 5,500
$E_{A}^{2'}{}_{24months}$	= \$ 4,000
$E_{B}^{2'}{}_{12Months}$	= \$ 7,000
$E_{B}^{2'}{}_{18months}$	= \$ 5,500
$E_{B}^{2'}{}_{24months}$	= \$ 4,000
$E_{A}^{3'}$	= \$13,437
$E_{B}^{3'}$	= \$14,250
$E_{A}^{4'}$	= \$293,750
$E_{B}^{4'}$	= \$312,500

4.3.2 Net Present Value Analysis

Using the same values for S (\$245,000) and the risk free rate of return (10%) and the expenditure profiles as listed in Table 4-11, the NPV for all three performance trajectories were calculated and are presented below.

$NPV_{AB\_A\_12months} = -\$11,315 \longrightarrow 0$

$NPV_{AB\_A\_18months} = -\$12,752 \longrightarrow 0$

$NPV_{AB\_A\_24months} = -\$14,271 \longrightarrow 0$

$NPV_{AB\_B\_12months} = -\$26,074 \longrightarrow 0$

$NPV_{AB\_B\_18months} = -\ \$27{,}511 \longrightarrow 0$

$NPV_{AB\_B\_24months} = -\ \$29{,}030 \longrightarrow 0$

The NPV analysis information was not affected by the change to the performance trajectories.

4.3.3 Real Option Analysis

The present values ($PV_y{}^{N}{}_{i,j}$) were calculated using Table 4-6 and Table 4-11 and are presented in Table 4-12, Table 4-13, Table 4-14, and Table 4-15.

Table 4-12. Present Values for Doubling every 12 Month for Case 1

Year	*2006*		*2007*		*2008*		*2009*
Expenditure	$17,900		$7,000		$13,437		$293,750
						$PV_{AB\_A}{}^{4}{}_{3,0}$	= $152,699
				$PV_{AB\_A}{}^{3}{}_{2,0}$	= $85,015		***Procure Technology A***
		$PV_{AB\_A}{}^{2}{}_{1,0}$	= $44,965		***Use Technology A in Detailed Design***	$PV_{AB\_A}{}^{4}{}_{2,1}$	= $5494
$PV_{AB\_A}{}^{1}{}_{0,0}$	= $13,509		***Use Technology A***	$PV_{AB\_A}{}^{3}{}_{2,1}$	= $0		***Procure Technology A***
	Use Both Technology A and B in Preliminary Design	$PV_{AB\_A}{}^{2}{}_{0,1}$	= $0		***Abandon***	$PV_{AB\_A}{}^{4}{}_{1,2}$	= $0
			Abandon	$PV_{AB\_A}{}^{3}{}_{0,2}$	= $0		***Abandon***
					Abandon	$PV_{AB\_A}{}^{4}{}_{0,3}$	= $0
							Abandon

Table 4-13. Present Values for Doubling every 12 Month for Case 2

Year	2006		2007		2008		2009
Expenditure	$17,900		$7,000		$14,250		$312,500
						$PV_{AB\_B}{}^{4}_{3,0}$	= $133,919
				$PV_{AB\_B}{}^{3}_{2,0}$	= $70,789		***Procure Technology B***
		$PV_{AB\_B}{}^{2}_{1,0}$	= $36,270		***Use Technology B in Detailed Design***	$PV_{AB\_B}{}^{4}_{2,1}$	= $0
$PV_{AB\_B}{}^{1}_{0,0}$	= $7,435		***Use Technology B***	$PV_{AB\_B}{}^{3}_{2,1}$	= $0		***Abandon***
	Use Both Technology A and B in Preliminary Design	$PV_{AB\_B}{}^{2}_{0,1}$	= $0		***Abandon***	$PV_{AB\_B}{}^{4}_{1,2}$	= $0
			Abandon	$PV_{AB\_B}{}^{3}_{0,2}$	= $0		***Abandon***
					Abandon	$PV_{AB\_B}{}^{4}_{0,3}$	= $0
							Abandon

Table 4-14. Present Values for Doubling every 24 Month for Case 1

Year	*2006*		*2007*		*2008*		*2009*
Expenditure	$23,900		$4,000		$13,437		$293,750
						$PV_{AB\ A}{}^{4}_{3,0}$	= $152,699
				$PV_{AB\ A}{}^{3}_{2,0}$	= $85,015		***Procure Technology A***
		$PV_{AB\ A}{}^{2}_{1,0}$	= $51,024		***Use Technology A in Detailed Design***	$PV_{AB\ A}{}^{4}_{2,1}$	= $5494
$PV_{AB\_A}{}^{1}_{0,0}$	= $11,740		***Use Technology A***	$PV_{AB\_A}{}^{3}_{2,1}$	= $0		***Procure Technology A***
	Use Both Technology A and B in Preliminary Design	$PV_{AB\ A}{}^{2}_{0,1}$	= $0		***Abandon***	$PV_{AB\ A}{}^{4}_{1,2}$	= $0
			Abandon	$PV_{AB\ A}{}^{3}_{0,2}$	= $0		***Abandon***
					Abandon	$PV_{AB\ A}{}^{4}_{0,3}$	= $0
							Abandon

Table 4-15. Present Values for Doubling every 24 Month for Case 2

Year	*2006*		*2007*		*2008*		*2009*
Expenditure	\$23,900		\$4,000		\$14,250		\$312,500
						$PV_{AB\_B}{}^{4}{}_{3,0}$	= \$133,919
				$PV_{AB\_B}{}^{3}{}_{2,0}$	= \$70,789		***Procure Technology B***
		$PV_{AB\_B}{}^{2}{}_{1,0}$	= \$41,817		***Use Technology B in Detailed Design***	$PV_{AB\_B}{}^{4}{}_{2,1}$	= \$0
$PV_{AB\_B}{}^{1}{}_{0,0}$	= \$5,309		***Use Technology B***	$PV_{AB\_B}{}^{3}{}_{2,1}$	= \$0		***Abandon***
	Use Both Technology A and B in Preliminary Design	$PV_{AB\_B}{}^{2}{}_{0,1}$	= \$0		***Abandon***	$PV_{AB\_B}{}^{4}{}_{1,2}$	= \$0
			Abandon	$PV_{AB\_B}{}^{3}{}_{0,2}$	= \$0		***Abandon***
					Abandon	$PV_{AB\_B}{}^{4}{}_{0,3}$	= \$0
							Abandon

The Net Option Value (NOV) for the three trajectories are presented in Table 4-16.shown below:

Table 4-16. Net Option Value

	Double every 12 Months	Double every 18 Months	Double every 24 Months
NOV_{AB_A}	\$12,213	\$11,020	\$10,444
% Change	10%	n/a	5%
NOV_{AB_B}	\$6,139	\$4,711	\$4,013
% Change	30%	n/a	10%

For this example the Net Option Value's sensitivity to a 33% change in the performance trajectory did not change the recommended outcome. All of the Net Option Values are greater than zero, so the decision will still be to carry both technologies until the expected future cross-over point and make the technology selection then. In all cases the options are sufficiently valuable to maintain. This may be unique to this example and it is recommended that sensitivity analysis be conducted whenever this algorithm is used because each case will be slightly different. The sensitivity analysis conducted as part of Chapters 2 and 3 also hold when utilizing ROA and TOA techniques. The next section takes the steps presented and constructs an algorithm for the decision maker to utilize

4.4 Unified Algorithm

Algorithm Steps

1. Gather Data.
    a. Two Technologies with overlapping functionality

        i. One existing technology
            1. Meets requirements today and into the future
        ii. One emerging technology
            1. Does not meet requirements today but will surpass the performance of the mature technology in the future
    b. Obtain current performance data for both
    c. Obtain future predicted performance estimate for both
        i. Web/roadmaps/trade journals
    d. Obtain both technologies predicted rate of change of performance if available. If not available calculate using a linear approximation.
2. Using the rate of change of performance for the new technology
    a. Compute the τ_{cop} for the two technology options: A- existing technology, B- emerging technology.
        i. If τ_{cop} occurs outside the project's time horizon, then *abandon* the emerging technology B
        ii. If τ_{cop} occurs within the project time horizon, *continue*
3. Compute the - Δp in performance between Technology B and Technology A.
    a. If, the Δp is:
        i. Not sufficiently high, *abandon* the emerging technology B
        ii. Otherwise, *continue*
4. Compute the two expenditure profiles associated with continuing the development with both technologies until τ_{cop}

    a. Case 1: Technology B's performance does not exceed Technology A's at τ_{cop} so Technology A will be selected and
    b. Case 2: Technology B's performance does exceed Technology A's at τ_{cop} so Technology B will be selected, *continue*
5. Compute the NPV for Case 1, and Case2
    a. If the NPV is:
        i. Positive for Case 1 and Case 2, then start the development with *both* and proceed to step 8
        ii. Otherwise, *continue*
6. Calculate the Present Value for Case 1 and Case 2 using ROA compound sequential option techniques, *continue*
7. Calculate the Net Option Value (NOV) for Case 1 and Case 2
    a. If:
        i. Both NOVs are < 0, *abandon* that emerging technology B
        ii. Both OVs are >0, then start the development with *both* and *continue*

Only two of the four potential outcomes associated with NOV are addressed as part of this dissertation. For the cases where the NOV is <0 for one of the technology's and >0 for the other it is believed that that decision will be based on the decision maker's tolerance for risk. Table 4-16 provides a potential starting point for future research.

8. If new performance data on either technology arrives before the cross-over point then return to step 2, otherwise *continue,*
9. When τ_{cop} is reached evaluate performance and cost data and make the following decision:

a. If Technology B's performance is not greater than Technology A's (Case 1), select the existing technology A

If Technology B's performance is greater than Technology A's (Case 2), select the emerging technology B. Figure 4-4 presents the unified algorithm into a process flow chart.

Table 4-17. NOV Decision Table

Mature	New	Risk Seeking	Risk Avoiding
NOV>0	NOV>0	Wait	Wait
NOV<0	NOV<0	Abandon	Abandon
NOV<0	NOV>0	Wait	Wait
NOV>0	NOV<0	Wait	Abandon

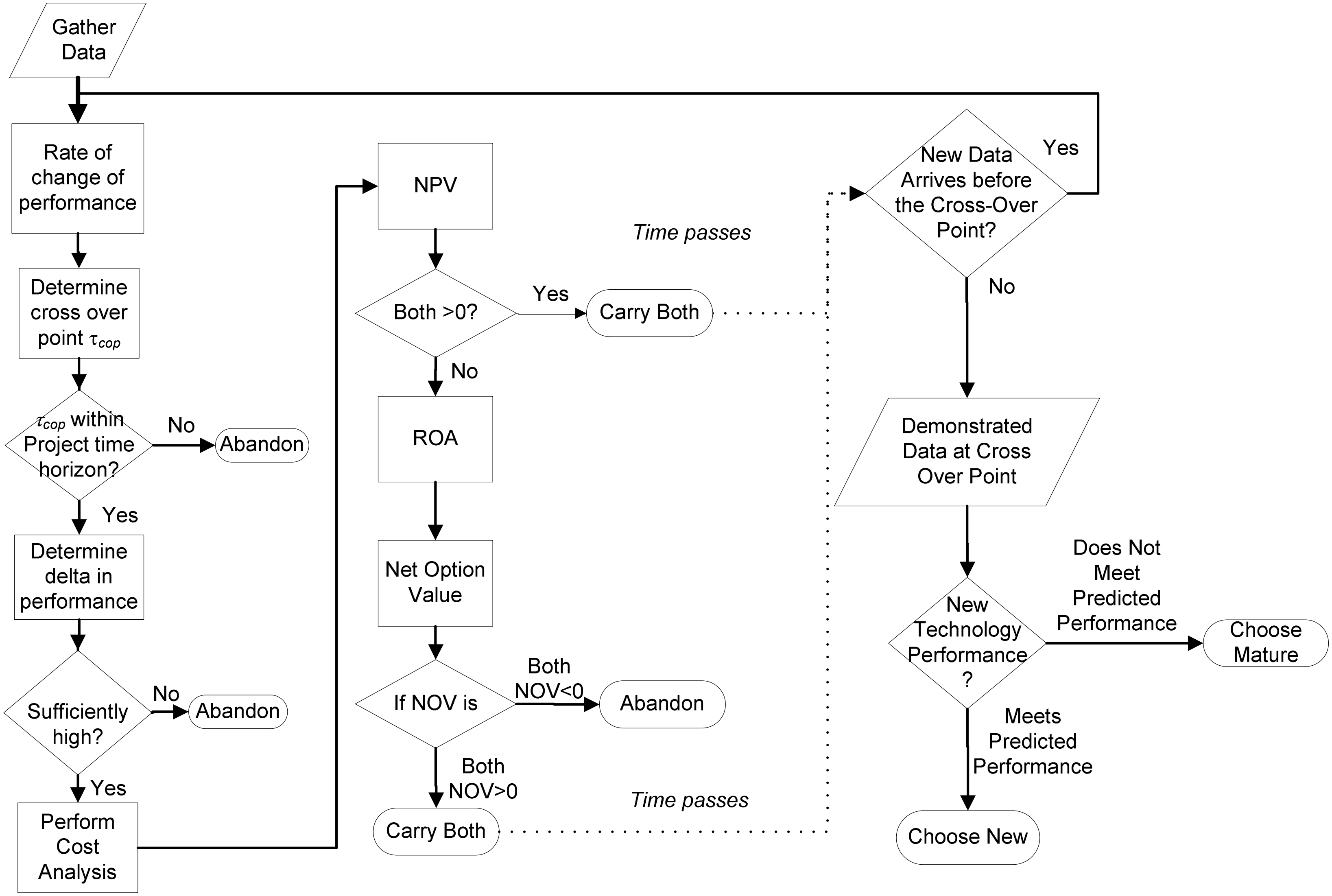

Figure 4-4. Process Flow Diagram of Unified Algorithm

4.5 Summary

The technology selection decision must be based on how well the technology meets a set of performance requirements and how much selecting that technology will cost. Such technology selection is a key to the success or failure of a Systems Engineering project. A decision aid therefore must be able to account for the uncertainties associated with cost and performance data, especially when the technology selection results in an irreversible or nearly irreversible architecture. Such an aid must present the decision space available to the decision maker in a useable and understandable output. In the two previous chapters, the Systems Engineering decision making process was extended by presenting and developing new methods for evaluating systems engineering technology alternatives with respect to cost (Chapter 2) and performance (Chapter 3) using the framework of Real Options and Technology Options Analysis. This chapter combined those two dimensions into a unified algorithm to simultaneously evaluate cost and performance alternatives so that a decision maker can gain an overall view of the value of delaying a decision until the most appropriate time.

The algorithm was constructed based on a common scenario faced by systems engineering decision makers. The scenario of interest was one in which two technologies are being consider for the system. One is mature and met all the requirements today and into the future, and a new technology is being developed but it did not meet the requirements today but its future performance is projected to exceed the performance of the existing technology. The algorithm starts by investigating wither a technology should even be considered. To do that it uses Technology Options Analysis and the techniques discussed in Chapter 3.

If, sometime in a new technology's future it is predicted to exceed or cross-over the existing technology's performance, this cross-over point defines the optimum time to make the decision because it is then when the uncertainty about the new technology's performance is answered. The cross-over point is independent of the project and only depends on the technologies being considered. Once it has been determined it can be compared to the project's time horizon and, if it is outside the time horizon, the decision it to abandon, or, if it is within the time horizon, continue with the analysis.

If the outcome is to continue the change (delta) in performance is determined next. Based on the magnitude of the gain in performance, the decision maker can either decide to start the cost analysis portion of the algorithm, or to abandon the new technology. If the decision is to continue the analysis, an expenditure profile should be calculated that takes into account carrying both of the competing technologies until the cross-over point. Once that is complete, the actual cost analysis begins using Net Present Value Analysis.

The NPV information answers the question as to whether the project with both technologies should be started immediately or not. If the NPV is positive for the cases where both technologies are carried until the cross-over point, no additional information is required. In the case where one or more of the NPVs are zero, Real Options Analysis is completed.

The compound sequential option from ROA is utilized because in most cases the SE project is based on a sequential program in which there multiple stages to building the project; initial design phase, detailed engineering phase, and procurement phase to name just a few. At each of the phases, a decision is required as to wither the project should continue or be abandoned. By using compound sequential option analysis the decision maker will be provided with an output that provides recommended decisions recommendation at all the key decision points in the future. The final Present Value of the compound sequential option is used to determine the value of waiting or the Net Option Value. It is the Net Option Value that provides the final piece of information the decision maker needs in order to understand the performance and cost impacts associated with the technology selection decision.

It is the ability of both TOA and ROA to value uncertainty until additional information is available that the algorithm takes advantage of and such information benefits the systems engineering decision maker when: 1) there is uncertainty in the future performance and cost of competing technologies, no current techniques are available to correctly estimate the cost/performance value of the alternatives; 2) the technology decision is or may be totally irreversible, but the decision to delay the technology decision is always reversible; and 3) delaying the technology selection can have a significant effect on the SE technology decision. The value of the algorithm developed in this chapter is that information is available today and it can be used to help key decision makers know what the decision should be at any point in time. Understanding how sensitive this knowledge was to variations a technology performance provides the decision maker with an understanding of the riskiness associated with these decisions.

CHAPTER V

SYNOPSIS AND CONCLUSIONS

The objective of my research was to investigate and develop tools and techniques such that systems engineering decision makers can make more effective financial and technology choice decisions. These decisions are important because the technology selection determines the final performance of the project and commits the majority of the total life cycle cost. This implies that technology selection is key to the success or failure of a systems engineering project. To arrive at the technology decision, a systems engineer needs to assess how well each technological alternative performs against various schedule, performance, and cost objectives. My research only investigated two (cost and performance) of the three (cost, performance, and schedule) primary objectives of a system; modifying the third (schedule) was beyond the scope of this dissertation. My research question was "When in the decision process should the technology selection be made?"

The most common method of evaluating the cost objective, Net Present Value, was presented and its inability to differentiate between technologies whose associated costs have different volatilities was demonstrated. Then Real Options Analysis was introduced as an analysis tool because of its ability to differentiate between technologies that have different cost volatilities. Specifically a compound sequential option was discussed in detail and an example of how this type of ROA analysis technique fit into the systems engineer process. ROA also introduced the concept to the SE decision maker that information was available now for every future decision point or milestone as part of the systems engineering process and how this could assist the decision maker in making more effective financial choices. The value of this information is that it is available today and the decision maker knows what the decision will be at any point in time (execute, wait, or terminate). Technology selection also impacts a project's final performance and its ultimate acceptance or rejection by the customer.

Despite the importance of technology selection the existing techniques for evaluating performance have had limited success in predicting whether a technology's future performance will be achieved or not. This is important because once a technology is chosen the architecture can become irreversible or nearly irreversible. The current methods available to analyze potential technology's performance uncertainties where unable to quantitatively answer the research question. A new analysis technique was needed that could take into account the uncertainty associated with a technology's future predicted performance. Technology Options Analysis (TOA) was developed using the idea that there can be value in waiting to make a decision until more performance information is available. For this reason it is similar to ROA but only in concept, not in execution. TOA methodology provides the decision maker with three key pieces of information; 1) the time a new emerging technology should cross-over the current technology, 2) the delta performance between the new and existing technology, and 3) an assessment of the risk associated with the new technology meeting its predicted performance. Based on this information the decision maker now can determine if there is value to delay the technology decision until the performance uncertainty with the new technology has been resolved and that should assist the decision maker in making more effective technical choices.

While the individual techniques developed are for evaluating future cost and performance, frequently these two variables cannot be looked at in isolation. The situation commonly faced by systems engineers is that the ROA and TOA analysis points may not occur concurrently. In order to solve this problem it was necessary to develop an algorithm that aligned the ROA analysis based on the TOA outputs. This would be necessary in order to fully answer my research question.

The algorithm is based on a scenario where at least two technologies are being considered for use in the system. One is mature and meets all the requirements today and into the future. A new technology is being developed but it does not meet all the requirements today but its future performance is projected to exceed the performance of the existing technology. However, there exists uncertainty relative to the new technology's performance and the new and existing technology's future cost. TOA analysis is performed first to determine if the new technology meets the project's time horizon and is of sufficient increase in performance to warrant further analysis. If the answer to both of these questions is yes than the analysis will continue utilizing the timing of the TOA cross-over point to perform NPV and ROA analysis. The final output of the algorithm is the value of waiting until the performance uncertainty is resolved or the option value. It is by using this algorithm that my research question, "When in the decision process should the technology selection be made?" is answered.

5.1 Sensitivity Analysis of the Unified Algorithm

When utilizing this algorithm the decision maker needs to be sensitive to the technology's predicted performance information. There are many potential sources for this information, technology or company roadmaps, surveys, or technical data surveys. Based on the sensitivity analysis if the new technology fails to meet its' near term performance predictions the risk of it meeting future performance predictions increase with the effect being more severe the later in the development time horizon that it is predicted to overtake the mature technology. The decision maker should also be sensitive to how the cost data is projected, like performance data it has many sources, some are more reliable than others. The sensitivity results for ROA demonstrated that the smaller the difference between the value of the project if it existed today (S) and the cost to acquire the project (E) the larger the option value. Variations in S and E will have an impact on the outcome of the algorithm. By understanding the source of the data the decision maker can adjust the uncertainty factors within TOA and ROA to see the impact on the outcome. For this reason it is recommended that when using this algorithm sensitivity analysis is performed by varying the performance trajectory and the cost data. This can be accomplished by selecting different values for the projected rate of performance (r_p) to generate a range of cross-over points (τ_{cop}) and various delta performance (Δp) values to understand the sensitivity of the performance data and by varying the volatility of the project (σ) and the investments to understand the sensitivity of the selection to the cost information. That way if there is sensitivity based on the data and the unique situation the decision maker will be aware of it

5.2 Research Contributions

By extending Real Options Analysis to include the Net Option Value technique so that the expenditures required for the implementation of multiple technologies during the development cycle are included, a more useful value of waiting to make the technology selection is provided to the systems engineering decision maker. Based on this information, the systems engineer should be able to make a more effective technology decisions.

Through the development of Technology Options Analysis, the systems engineering decision maker now has the ability to determine when a new technology will overtake an existing one (the cross-over point), how much performance gain might be achieved, and the risk of the new technology meeting its predicted future. The system engineer now understands the optimum time to make the technology selection and should be able to make a more informed technology decision.

The incorporation of Technology Options and Real Options Analysis into a unified algorithm provides simultaneous evaluation of performance and cost. By understanding the optimum time to make the technology selection based on resolving a new technology's future performance uncertainty and using that information to conduct the cost analysis a more insightful depiction of the value of waiting to make the technology selection is provided to the systems engineering decision maker. This should assist the decision maker in making more effective technical and financial choices and provides a rational and archiving of the decision process.

5.3 Limitations

As stated previously, this research only investigated two (cost and performance) of the three (cost, performance, and schedule) primary objectives of a system; modifying the third (schedule) was beyond its scope. This is a major limitation of the research that should be addressed by future research.

Additionally, only two technology choices were analyzed in the algorithms created. In many cases there are more than two choices, there this also is a natural extension of the research.

In order to utilize the algorithms developed, the following criteria must be meet; 1) at least two potential technologies should be under consideration for implementation in the SE project, 2) one technology must meet the minimum level of performance today while the other technology does not, 3) the technology that does not meet the minimum level of performance today is predicted to outperform the technology that does sometime in the future, 4) all the technologies must have a current demonstrated performance and as a minimum one predicted future performance value, and 5) all the technologies must have a future cost profile. In addition to the limitations associated with the algorithm each of the analysis techniques have limitations.

When using Real Options Analysis the choice of the value for the risk-free rate of return (*rf*) is often debated and argued. Based on the sensitivity analysis conducted on *rf* choosing a low value (<5%) will make the option value worthless and choosing a high value (>15%) will make the option value very valuable regardless in both cases of the cost uncertainty. The implication for the decision maker is that the *rf* that is assumed can greatly impact the value of the option.

How the technology curves are defined will influence the outputs when conducting TOA. For the examples in this dissertation, a linear approximation was considered appropriate. This will not always be the case since technology curves can follow other shapes. Based on the sensitivity analysis, when using a linear approximation instead of the actual technology curve, the error associated with the cross-over point increase as a function of the demonstrated performance delta. The implications for the decision maker is that the greater the performance delta today between the new and existing technologies the more important it is to understand the new technology's performance profile.

5.4 Future Research

From an application perspective, any systems engineering project considering the use of technologies that fit this scenario should consider using the algorithm before making a technology selection. For example, a project considering the use of organic light-emitting diodes (OLED) or photovoltaics might benefit from an understanding of when the new technology is predicted to surpass the current technology and what impact that might have on the success of their project.

From a research perspective, the algorithm should be further scrutinized and enhanced to: 1) accommodate multiple technologies to include the evaluation of multiple cross-over points and performance deltas,2) accommodate nested performance requirements, 3) determine if a performance-to-cost-of-waiting metric would be of value to the decision maker when the NOVs are not identical, and 4) incorporate error bands and probability curves around the sensitivity analysis to account for forecasting variability, data uncertainty, and model uncertainty.

5.5 Summary

My research has answered the question: what is the value to a system engineer for waiting to make a technology decision? This question has been answered three different ways: from a cost perspective, from a technology perspective, and from the union of these two viewpoints. While this research can and should be extended, the resulting algorithms extend the toolbox available to systems engineers in making technology decisions. These tools provide to the decision maker the value of waiting from both a performance and cost perspective and provide information on future decisions. The value of this information is that it is available today and the decision maker knows what the decision should be at any point in time (execute, wait, or terminate). Systems engineering decision makers no longer have to fear the future and worry about being second guessed by the development of new technologies, instead they can look forward to the future because they already understand the impact of technology on the outcome when the data does arrive, thus they are more likely to generate better solutions.

APPENDIX A

OPTION VALUATION OR DERIVATIVES

On 14 October 1997 the press release from the Royal Swedish Academy of Sciences said the Nobel Prize in economics was given “for a new method to determine the value of derivatives. Robert C. Merton and Myron S. Scholes have, in collaboration with the late Fischer Black, developed a pioneering formula for the valuation of stock options. Their methodology has paved the way for economic valuations in many areas. It has also generated new types of financial instruments and facilitated more efficient risk management in society.” Why a Nobel Prize? Scientific theories can be elegant and challenging but not very practical or they can be practical but not very elegant or challenging. As Robert Merton (1997) said, “Here we have both”.

Traders and investors all over the world use the Black-Scholes formula every day to value stock options. Their financial goal: maximize profits and minimize losses. The more generalized method devised by Robert Merton has turned out to be relevant to a larger audience and has created areas of research inside and outside the domain of financial economics. To appreciate the magnitude of their accomplishment however one has to understand what an option is, how an option works, and the state of the theory of option pricing or derivatives prior to their seminal work.

Black and Scholes (1973) defined an “option” as a financial instrument/document that is tradable and shows evidence of ownership. This instrument is commonly referred to as a security that gives the right, but not an obligation, to buy (call) or to sell (put) an asset, subject to certain conditions, within a specified period of time. Assets as used in this definition are considered to be a share of stock. Options are also defined based on when they can be exercised. An “American Option” can be exercised at any time up to the date the option expires and a “European Option” can only be exercised on a specific date in the future. The price paid for an asset when the option is exercised is called the “exercise price” or “striking price”. The day on which the option must be exercised is called the “expiration date” or “maturity date”. The simplest type of option, a “call option”, is one that gives the right to buy a single share of common stock at a preset price.

The French mathematician Louis Bachelier’s dissertation on the theory of speculation deduced an option pricing formula based on the assumption that stock prices followed Brownian motion with zero drift (Merton 1973). This work gave birth to continuous-time mathematics of stochastic processes and continuous-time economics of derivative security pricing and influenced both Ito’s (Black and Scholes 1973; Merton 1973)development of stochastic processes and Samuelson’s rational theory of warrant pricing (Black and Scholes 1973; Merton 1973; Merton, Simons et al. 1994; Jarrow 1999; Hull 2003). Both of these works would become important in the development of the Black-Merton-Scholes approach and led to the development of their option-pricing theory. Option pricing theory however sat dormant until the 1960s when subsequent researchers started to determine an option’s price using the maximizing conditions obtained from an investor’s optimal portfolio position. These valuation formulas depended on the expected return on the stock. But they all suffered from the same fundamental shortcoming in that risk premia was difficult to estimate and to use as risk shifts according to changing tastes and changing economic fundamentals.

Risk premia is the difference between the expected return on a security or portfolio and the "risk less rate of interest" or “risk-free rate of return” (r_f) (the certain return on a riskless security) and is often termed its risk premium. Underlying the terminology is the notion that there should be a premium (higher expected return) for bearing risk. This implies the value of an option to buy or sell a share depends on the uncertain development of the share price to the date of maturity. It is therefore natural to suppose that the valuation of an option requires taking a stance on which risk premium to use (Black and Scholes 1972; Black and Scholes 1973; Merton, Simons et al. 1994; Jarrow 1999; Hull 2003).

Prior to the Black-Scholes formula there were two difficulties that needed to be overcome. First, there were no generally accepted empirical models to determine an asset's risk premium that were consistent with the known regularities in the data. Secondly and most importantly, none of the valuation formulas offered a sense of how to hedge an option using a portfolio of the underlying stock and riskless borrowing or lending.

Black (1989) attempted to solve the problem by valuing the option in a continuous time setting using the capital asset pricing model (CAPM). He was able to obtain an implicit solution for an option's value that was described by a partial differential equation subject to boundary conditions but he was unable to find its general solution. Working with Myron Scholes they solved the equation using economic intuition and asset pricing formulas. In deriving their formula it was necessary to assume "ideal conditions" in the market for both the stock and the option. Specifically (Black and Scholes 1973) assumed:

a) The short term interest rate (r) is known and is constant through time
b) The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. Thus the distribution of possible stock prices at the end of any finite interval is log-normal. The variance rate of the return on the stock is constant.
c) The stock pays no dividends or other distributions.
d) The option is "European," that is, it can only be exercised at maturity.

e) There are no transaction costs in buying or selling the stock or the option
f) It is possible to borrow any fraction of the price of a security to buy it or to hold it, at the short-term interest rate.
g) There are no penalties to short selling. A seller who does not own a security will simply accept the price of the security from a buyer, and will agree to settle with the buyer on some future date by paying him an amount equal to the price of the security on that date.

Under these assumptions, the value of the option will depend only on the price of the stock and time and on variables that are taken to be known constants:

$$f(S, \tau: E) = S\Phi\,(d_1) - Ee^{-rr}\Phi\,(d_2),$$

$$d_1 \equiv [\log\,(S/E) + (r + \tfrac{1}{2}\,\sigma^2)\,t]/\sigma\,\sqrt{r},$$

$$d_2 \equiv d_1 - \sigma\,\sqrt{r_f}$$

where Φ is the cumulative normal distribution function, σ^2 is the instantaneous variance of the return on the common stock,

It was Merton (1973) who demonstrated a different way to derive their partial differential equation. His derivation was not based on CAPM but on a continuous-time construction of a perfectly hedged portfolio where the option price H(S, P, τ; E) is a function of the stock price (S), the riskless bond price (P), and the length of time to expiration (τ). A perfectly hedged portfolio consisted of common stock, the option, and riskless bonds with time to maturity equal to the expiration date of the option, such that the aggregate investment in the portfolio was zero. The price for any European option can be determined (Merton 1973) by,

$$\mathrm{H}(\mathrm{S},\mathrm{P},\tau;\mathrm{E}) = \mathrm{EP}(\tau)\,y\left[\mathrm{S}/\mathrm{EP}(\tau), \int_0^\tau \mathrm{V}^2(s)\,ds\right] \quad (38)$$

Consequently, for empirical testing or applications, one need only compute tables for the "standard" option price as a function of two variables, stock price and time to expiration, to be able to compute option prices in general (Merton 1973). In the special case of a non-stochastic and constant interest rate (i.e., $\delta = 0$, $P = e^{-rr}$, and $T \equiv \sigma^2\tau$) Merton's equation reduces to the Black-Scholes formula.

APPENDIX B

DEFERRAL OPTION VALUE
EXAMPLE CALCULATIONS

B.1 Example where E=$1,600K

$$\mathrm{PV}_{1,0} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\mathrm{S}_{1,0}}{(1+r_f)^{\tau}} - \frac{\mathrm{E}}{1+r_f},0\right]$$

$$\mathrm{PV}_{1,0} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$300\mathrm{K}}{(1+.1)^{\tau}} - \frac{\$1{,}600\mathrm{K}}{1+.1},0\right]$$

$$\mathrm{PV}_{1,0} = MAX\left[\frac{\$3300\mathrm{K}-\$1600\mathrm{K}}{1.1},0\right] = MAX\left[\frac{\$1700\mathrm{K}}{1.1},0\right] = MAX\left[\$1545\mathrm{K},0\right]$$

$$\mathrm{PV}_{1,0} = \$1545\mathrm{K}$$

$$\mathrm{PV}_{0,1} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\mathrm{S}_{0,1}}{(1+r_f)^{\tau}} - \frac{\mathrm{E}}{1+r_f},0\right]$$

$$\mathrm{PV}_{0,1} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$100\mathrm{K}}{(1+.1)^{\tau}} - \frac{\$1{,}600\mathrm{K}}{1+.1},0\right]$$

$$\mathrm{PV}_{0,1} = MAX\left[\frac{\$1100\mathrm{K}-\$1600\mathrm{K}}{1.1},0\right] = MAX\left[\frac{-\$500\mathrm{K}}{1.1},0\right] = MAX\left[-\$455\mathrm{K},0\right]$$

$$\mathrm{PV}_{0,1} = \$0$$

$$\mathrm{PV}_{0,0} = \mathrm{p} * \mathrm{PV}_{1,0} + (1-\mathrm{p}) * \mathrm{PV}_{0,1}$$

$$\mathrm{PV}_{0,0} = .5 * \$1545\mathrm{K} + (1-.5) * \$0 = \$773\mathrm{K}$$

$\mathrm{OV}_{E=1600} = \mathrm{PV}_{0,0} - \mathrm{NPV}_{E=1600}$ = $773K - $600K = $173K

$\mathrm{NOV}_{E=1600} = \mathrm{OV}_{E=1600} - \mathrm{OC}_{E=1600}$ = $173K - $75K = $98K

B.2 Example where E=$2400K,

$$\mathrm{PV}_{1,0} = MAX\left[\sum\nolimits_{\tau=1}^{\infty}\frac{\mathrm{S}_{1,0}}{(1+r)^{\tau}} - \frac{\mathrm{E}}{1+r},0\right]$$

$$\mathrm{PV}_{1,0} = MAX\left[\sum\nolimits_{\tau=1}^{\infty}\frac{\$300\mathrm{K}}{(1+1)^{\tau}} - \frac{\$2{,}400\mathrm{K}}{1+1},0\right]$$

$$\mathrm{PV}_{1,0} = MAX\left[\frac{\$3300\mathrm{K} - \$2{,}400\mathrm{K}}{1.1},0\right] = MAX\left[\frac{\$900\mathrm{K}}{1.1},0\right] = MAX\left[\$818\mathrm{K},0\right]$$

$$\mathrm{PV}_{1,0} = \$818\mathrm{K}$$

$$\mathrm{PV}_{0,1} = MAX\left[\sum\nolimits_{\tau=1}^{\infty}\frac{\mathrm{S}_{0,1}}{(1+r)^{\tau}} - \frac{\mathrm{E}}{1+r},0\right]$$

$$\mathrm{PV}_{0,1} = MAX\left[\sum\nolimits_{\tau=1}^{\infty}\frac{\$100\mathrm{K}}{(1+.1)^{\tau}} - \frac{\$2{,}400\mathrm{K}}{1+.1},0\right]$$

$$\mathrm{PV}_{0,1} = MAX\left[\frac{\$1{,}100\mathrm{K} - \$2{,}400\mathrm{K}}{1.1},0\right] = MAX\left[\frac{-\$1300\mathrm{K}}{1.1},0\right] = MAX\left[-\$1182\mathrm{K},0\right]$$

$$\mathrm{PV}_{0,1} = \$0$$

$$\mathrm{PV}_{0,0} = \mathrm{p} * \mathrm{PV}_{1,0} + (1-\mathrm{p}) * \mathrm{PV}_{0,1}$$

$$\mathrm{PV}_{0,0} = .5 * \$818\mathrm{K} + (1-.5) * \$0$$

$$\mathrm{PV}_{0,0} = \$409\mathrm{K} + 0 = \$409\mathrm{K}$$

$\mathrm{OV}_{E=2400} = \mathrm{PV}_{0,0} - \mathrm{NPV}_{E=2400}$ = \$409K - \$0K = \$409K

$\mathrm{NOV}_{E=2400} = \mathrm{OV}_{E=2400} - \mathrm{OC}_{E=2400}$ = \$409K - \$75K = \$334K

B.3 Example with High Volatility and E=$1600K,

$$\mathrm{PV}_{1,0} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\mathrm{S}_{1,0}}{(1+r)^{\tau}} - \frac{\mathrm{E}}{1+r},0\right]$$

$$\mathrm{PV}_{1,0} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$350\mathrm{K}}{(1+1)^{\tau}} - \frac{\$1{,}600\mathrm{K}}{1+1},0\right]$$

$$\mathrm{PV}_{1,0} = MAX\left[\frac{\$3850\mathrm{K} - \$1{,}600\mathrm{K}}{1.1},0\right] = MAX\left[\frac{\$2250\mathrm{K}}{1.1},0\right] = MAX[\$2045\mathrm{K},0]$$

$$\mathrm{PV}_{1,0} = \$2045\mathrm{K}$$

$$\mathrm{PV}_{0,1} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\mathrm{S}_{0,1}}{(1+r)^{\tau}} - \frac{\mathrm{E}}{1+r},0\right]$$

$$\mathrm{PV}_{0,1} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$50\mathrm{K}}{(1+.1)^{\tau}} - \frac{\$1{,}600\mathrm{K}}{1+.1},0\right]$$

$$\mathrm{PV}_{0,1} = MAX\left[\frac{\$550\mathrm{K} - \$1{,}600\mathrm{K}}{1.1},0\right] = MAX\left[\frac{-\$1050\mathrm{K}}{1.1},0\right] = MAX[-\$954\mathrm{K},0]$$

$$\mathrm{PV}_{0,1} = \$0$$

$$\mathrm{PV}_{0,0} = \mathrm{p} * \mathrm{PV}_{1,0} + (1-\mathrm{p}) * \mathrm{PV}_{0,1}$$

$$\mathrm{PV}_{0,0} = .5 * \$2045\mathrm{K} + (1-.5) * \$0$$

$$\mathrm{PV}_{0,0} = \$1022\mathrm{K} + 0 = \$1022\mathrm{K}$$

$\mathrm{OV}_{\sigma=+/-75,E=1600} = \mathrm{PV}_{0,0} - \mathrm{NPV}_{E=1600}$ = \$1022K - \$600K = \$422K

$\mathrm{NOV}_{\sigma=+/-75,E=1600} = \mathrm{OV}_{\sigma=+/-75,E=1600} - \mathrm{OC}_{E=1600}$ = \$422K - \$75K = \$347K

B.4 Example with Low Volatility and E=$1600K,

$$\mathrm{PV}_{1,0} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\mathrm{S}_{1,0}}{(1+r)^{\tau}} - \frac{\mathrm{E}}{1+r},0\right]$$

$$\mathrm{PV}_{1,0} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$220\mathrm{K}}{(1+1)^{\tau}} - \frac{\$1{,}600\mathrm{K}}{1+1},0\right]$$

$$\mathrm{PV}_{1,0} = MAX\left[\frac{\$2{,}420\mathrm{K} - \$1{,}600\mathrm{K}}{1.1},0\right] = MAX\left[\frac{\$820\mathrm{K}}{1.1},0\right] = MAX\left[\$745\mathrm{K},0\right]$$

$$\mathrm{PV}_{1,0} = \$745\mathrm{K}$$

$$\mathrm{PV}_{0,1} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\mathrm{S}_{0,1}}{(1+r)^{\tau}} - \frac{\mathrm{E}}{1+r},0\right]$$

$$\mathrm{PV}_{0,1} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$180\mathrm{K}}{(1+.1)^{\tau}} - \frac{\$1{,}600\mathrm{K}}{1+.1},0\right]$$

$$\mathrm{PV}_{0,1} = MAX\left[\frac{\$1{,}980\mathrm{K} - \$1{,}600\mathrm{K}}{1.1},0\right] = MAX\left[\frac{\$380\mathrm{K}}{1.1},0\right] = MAX\left[\$345\mathrm{K},0\right]$$

$$\mathrm{PV}_{0,1} = \$345\mathrm{K}$$

$$\mathrm{PV}_{0,0} = \mathrm{p} * \mathrm{PV}_{1,0} + (1-\mathrm{p}) * \mathrm{PV}_{0,1}$$

$$\mathrm{PV}_{0,0} = .5 * \$745\mathrm{K} + (1-.5) * \$345\mathrm{K}$$

$$\mathrm{PV}_{0,0} = \$372.5\mathrm{K} + \$172.5\mathrm{K} = \$545\mathrm{K}$$

$\mathrm{OV}_{\sigma=+/-10,E=1600} = \mathrm{PV}_{0,0} - \mathrm{NPV}_{E=1600}$ = \$545K - \$600K = -\$55K

$\mathrm{NOV}_{\sigma=+/-10,E=1600} = \mathrm{OV}_{\sigma=+/-10,E=1600} - \mathrm{OC}_{E=1600}$ = -\$55K - \$75K =- \$130K

B.5 Example with High Volatility and E=$2400K,

$$\mathrm{PV}_{1,0} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\mathrm{S}_{1,0}}{(1+r)^{\tau}} - \frac{\mathrm{E}}{1+r},0\right]$$

$$\mathrm{PV}_{1,0} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$350\mathrm{K}}{(1+1)^{\tau}} - \frac{\$2{,}400\mathrm{K}}{1+1},0\right]$$

$$\mathrm{PV}_{1,0} = MAX\left[\frac{\$3850\mathrm{K} - \$2{,}400\mathrm{K}}{1.1},0\right] = MAX\left[\frac{\$1450\mathrm{K}}{1.1},0\right] = MAX\left[\$1318\mathrm{K},0\right]$$

$$\mathrm{PV}_{1,0} = \$1318\mathrm{K}$$

$$\mathrm{PV}_{0,1} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\mathrm{S}_{0,1}}{(1+r)^{\tau}} - \frac{\mathrm{E}}{1+r},0\right]$$

$$\mathrm{PV}_{0,1} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$50\mathrm{K}}{(1+.1)^{\tau}} - \frac{\$2{,}400\mathrm{K}}{1+.1},0\right]$$

$$\mathrm{PV}_{0,1} = MAX\left[\frac{\$550\mathrm{K} - \$2{,}400\mathrm{K}}{1.1},0\right] = MAX\left[\frac{-\$1850\mathrm{K}}{1.1},0\right] = MAX\left[-\$1681\mathrm{K},0\right]$$

$$\mathrm{PV}_{0,1} = \$0$$

$$\mathrm{PV}_{0,0} = \mathrm{p} * \mathrm{PV}_{1,0} + (1-\mathrm{p}) * \mathrm{PV}_{0,1}$$

$$\mathrm{PV}_{0,0} = .5 * \$1318\mathrm{K} + (1-.5) * \$0$$

$$\mathrm{PV}_{0,0} = \$659\mathrm{K} + 0 = \$659\mathrm{K}$$

$$\mathrm{OV}_{\sigma=+/-75,E=2400} = \mathrm{PV}_{0,0} - \mathrm{NPV}_{E=2400} = \$659\mathrm{K} - \$0\mathrm{K} = \$659\mathrm{K}$$

$$\mathrm{NOV}_{\sigma=+/-75,E=2400} = \mathrm{OV}_{\sigma=+/-75,E=2400} - \mathrm{OC}_{E=2400} = \$659\mathrm{K} - \$75\mathrm{K} = \$584\mathrm{K}$$

B.6 Example with Low Volatility and E=$2400K,

$$\mathrm{PV}_{1,0} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\mathrm{S}_{1,0}}{(1+r)^{\tau}} - \frac{\mathrm{E}}{1+r},0\right]$$

$$\mathrm{PV}_{1,0} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$220\mathrm{K}}{(1+1)^{\tau}} - \frac{\$2{,}400\mathrm{K}}{1+1},0\right]$$

$$\mathrm{PV}_{1,0} = MAX\left[\frac{\$2420\mathrm{K} - \$2{,}400\mathrm{K}}{1.1},0\right] = MAX\left[\frac{\$20\mathrm{K}}{1.1},0\right] = MAX\left[\$18\mathrm{K},0\right]$$

$$\mathrm{PV}_{1,0} = \$18\mathrm{K}$$

$$\mathrm{PV}_{0,1} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\mathrm{S}_{0,1}}{(1+r)^{\tau}} - \frac{\mathrm{E}}{1+r},0\right]$$

$$\mathrm{PV}_{0,1} = MAX\left[\sum_{\tau=1}^{\infty}\frac{\$180\mathrm{K}}{(1+.1)^{\tau}} - \frac{\$2{,}400\mathrm{K}}{1+.1},0\right]$$

$$\mathrm{PV}_{0,1} = MAX\left[\frac{\$1980\mathrm{K} - \$2{,}400\mathrm{K}}{1.1},0\right] = MAX\left[\frac{-\$420\mathrm{K}}{1.1},0\right] = MAX\left[-\$381\mathrm{K},0\right]$$

$$\mathrm{PV}_{0,1} = \$0$$

$$\mathrm{PV}_{0,0} = \mathrm{p} * \mathrm{PV}_{1,0} + (1-\mathrm{p}) * \mathrm{PV}_{0,1}$$

$$\mathrm{PV}_{0,0} = .5 * \$18\mathrm{K} + (1-.5) * \$0$$

$$\mathrm{PV}_{0,0} = \$9\mathrm{K} + 0 = \$9\mathrm{K}$$

$\mathrm{OV}_{\sigma=+/-10,E=2400} = \mathrm{PV}_{0,0} - \mathrm{NPV}_{E=2400}$ = \$9K - \$0K = \$9K

$\mathrm{NOV}_{\sigma=+/-10,E=2400} = \mathrm{OV}_{\sigma=+/-10,E=2400} - \mathrm{OC}_{E=2400}$ = \$9K - \$75K = -\$66K

B.7 Compound Options – Sequential Options Example Calculations

Calculation of NPV:

$$NPV = \mathrm{S} - \mathrm{E}_1 - \mathrm{E}_2 - \mathrm{E}_3$$

Calculation of NPV when production takes place in Year

$$NPV_2 = \sum_{t=0}^{\infty} \frac{\$200\mathrm{K}}{(1.1)^t} - \$250\mathrm{K} - \frac{\$750\mathrm{K}}{(1.1)^1} - \frac{\$1500\mathrm{K}}{(1.1)^2} = \$2{,}200\mathrm{K} - \$250\mathrm{K} - \$682\mathrm{K} - \$1240\mathrm{K} = \$28\mathrm{K}$$

Calculation of NPV when production takes place in Year 3

$$NPV_3 = \sum_{t=0}^{\infty} \frac{\$200K}{(1.1)^t} - \$250\mathrm{K} - \frac{\$750\mathrm{K}}{(1.1)^1} - \frac{\$1500\mathrm{K}}{(1.1)^3} = \$2{,}200\mathrm{K} - \$250\mathrm{K} - \$682\mathrm{K} - \$1127\mathrm{K} = \$141\mathrm{K}$$

NPV used to evaluate ROA is the greater of NPV_2 and NPV_3.

$\mathrm{NPV}_2 < \mathrm{NPV}_3$ so NPV = \$141K

Calculation of potential values of the SE project

$S_{0,0}$ = \$2200K

$S_{1,0}$ = u * $S_{0,0}$ = 1.65 * \$2200K = \$3627K

$S_{0,1}$ = d * $S_{0,0}$ = 0.61 * \$2200K = \$1334K

$S_{2,0}$ = u *u* $S_{0,0}$ = 1.65 *1.65 * \$2200K = \$5980K

$S_{1,1}$ = u*d * $S_{0,0}$ = 1.65 * 0.61 * \$2200K = \$2200K

$S_{0,2}$ = d*d* $S_{0,0}$ = 0.61 * 0.61 * \$2200K = \$809K

$S_{3,0}$ = u*u*u * $S_{0,0}$ = 1.65 *1.65 *1.65 * \$2200K = \$9860K

$S_{2,1}$ = u *u*d* $S_{0,0}$ = 1.65 * 1.65 * 0.61 * \$2200K = \$3627K

$S_{1,2}$ = u*d*d * $S_{0,0}$ = 1.65 * 0.61 * 0.61 * \$2200K = \$1334K

$S_{0,3}$ = d*d*d * $S_{0,0}$ = 0.61 * 0.61 * 0.61 * \$2200K = \$491K

Table B-1. Possible Aircraft Values (S)

	Today		Year 1		Year 2		Year 3
						$S_{3,0}$ =	9860
				$S_{2,0}$ =	5980		
		$S_{1,0}$ =	3627			$S_{2,1}$ =	3627
$S_{0,0}$ =	2200			$S_{1,1}$ =	2200		
		$S_{0,1}$ =	1334			$S_{1,2}$ =	1334
				$S_{0,2}$ =	809		
						$S_{0,3}$ =	491

Calculation of Present Value (PV) at Year 3

$PV_{3,0} = MAX(S_{3,0} - E_3, 0) = MAX(\$9860K - \$1500K, 0) = MAX(\$8360K, 0) = \$8360K$

$PV_{2,1} = MAX(S_{2,1} - E_3, 0) = MAX(\$3627K - \$1500K, 0) = MAX(\$2127K, 0) = \$2627K$

$PV_{1,2} = MAX(S_{1,2} - E_3, 0) = MAX(\$1334K - \$1500K, 0) = MAX(-\$166K, 0) = \$0$

$PV_{0,3} = MAX(S_{0,3} - E_3, 0) = MAX(\$491K - \$1500K, 0) = MAX(-\$1009K,0) = \$0$

Calculation of Present Value in Year 2 when the decision is to not wait until Year 3 to exercise the option:

$PV^3_{2,0,2} = MAX(S_{2,0} - E_3, 0) = MAX(\$5980K - \$1500K, 0) = MAX(\$4480K, 0) = \$4480K$

$PV^3_{1,1,2} = MAX(S_{1,1} - E_3, 0) = MAX(\$2200K - \$1500K, 0) = MAX(\$700K, 0) = \$700K$

$PV^3_{0,2,2} = MAX(S_{0,2} - E_3, 0) = MAX(\$809K - \$1500K, 0) = MAX(-\$691K, 0) = \$0$

Calculation of Present Value in Year 2 when the decision is to wait until Year 3 to exercise the option:

$$\mathrm{PV_{i,j}} = \frac{(\mathrm{p} * \mathrm{u} * \mathrm{PV_{i,j}}) + ((1-\mathrm{p}) * \mathrm{d} * \mathrm{PV_{i,j}})}{(1 + r_f)}$$

$$\mathrm{PV_{i,j}}(1 + r_f) = \mathrm{p} * \mathrm{u} * \mathrm{PV_{i,j}} + (1-\mathrm{p}) * \mathrm{d} * \mathrm{PV_{i,j}}$$

$$1 + r_f = \mathrm{p} * \mathrm{u} + (1-\mathrm{p}) * \mathrm{d}$$

$$1 + r_f = \mathrm{p} * \mathrm{u} + \mathrm{d}\text{-}\ (\mathrm{p} * \mathrm{d})$$

$$1 + r_f = \mathrm{p} * (\mathrm{u}\ \text{-}\ \mathrm{d}) + \mathrm{d}$$

$$1 + r_f\text{-}\mathrm{d} = \mathrm{p} * (\mathrm{u}\ \text{-}\ \mathrm{d})$$

$$\frac{1 + r_f\ \text{-}\ \mathrm{d}}{\mathrm{u}\ \text{-}\ \mathrm{d}} = \mathrm{p}$$

$$\mathrm{p} = \frac{1 + r_f - \mathrm{d}}{\mathrm{u} - \mathrm{d}} = \frac{1 + 0.1 - 0.61}{1.65 - 0.61} = \frac{.49}{1.04} = .47 \text{ and } 1\text{- p} = .53$$

$$\mathrm{PV^3_{2,0,3}} = MAX\left(\frac{(\mathrm{p} * \mathrm{PV_{3,0,3}}) + ((1-\mathrm{p}) * \mathrm{PV_{2,1,3}})}{(1 + r_f)}, 0\right)$$

$$\mathrm{PV^3_{2,0,3}} = MAX\left(\frac{((0.47 * \$8360\mathrm{K}) + (0.53 * \$2127\mathrm{K}))}{(1 + 0.1)}, 0\right)$$

$$\mathrm{PV^3_{2,0,3}} = MAX\left(\frac{(\$3958\mathrm{K} + \$1120\mathrm{K})}{1.1}, 0\right)$$

$$\mathrm{PV^3_{2,0,3}} = MAX\left(\frac{\$5078\mathrm{K}}{1.1}, 0\right)$$

$$\mathrm{PV^3_{2,0,3}} = MAX(\$4617\mathrm{K}, 0) = \$4617\mathrm{K}$$

$$\mathrm{PV^3_{1,1,3}} = MAX\left(\frac{((\mathrm{p} * \mathrm{PV_{2,1,3}}) + ((1-\mathrm{p}) * \mathrm{PV_{1,2,3}}))}{(1 + r_f)}, 0\right)$$

$$\mathrm{PV^3_{1,1,3}} = MAX\left(\frac{((0.47 * \$2127\mathrm{K}) + (0.53 * \$0))}{(1 + 0.1)}, 0\right)$$

$$\mathrm{PV^3_{1,1,3}} = MAX\left(\frac{(\$1007\mathrm{K} + \$0)}{1.1}, 0\right)$$

$$\mathrm{PV}^3_{1,1,3} = MAX(\frac{\$1007\mathrm{K}}{1.1},0)$$

$$\mathrm{PV}^3_{1,1,3} = MAX(\$916\mathrm{K},0) = \$916\mathrm{K}$$

$$\mathrm{PV}^3_{0,2,3} = MAX(\frac{((\mathrm{p} * \mathrm{PV}_{1,2,3}) + ((1-\mathrm{p}) * \mathrm{PV}_{0,3,3}))}{(1+r_f)},0)$$

$$\mathrm{PV}^3_{0,2,3} = MAX(\frac{((0.47 * \$0) + (0.53 * \$0))}{(1+0.1)},0)$$

$$\mathrm{PV}^3_{0,2,3} = MAX(\$0,\$0) = \$0$$

The final Present Value for Year 2 is the maximum of $\mathrm{PV}^3_{\mathrm{i,j},3}$ and $\mathrm{PV}^3_{\mathrm{i,j},2}$

$\mathrm{PV}^3_{2,0,\mathrm{j}} = \mathrm{MAX}(\mathrm{PV}^3_{2,0,2}, \mathrm{PV}^3_{2,0,3}) = \mathrm{MAX}(\$4480\mathrm{K}, \$4617\mathrm{K}) = \$4617\mathrm{K} = \mathrm{PV}^3_{2,0,3}$

$\mathrm{PV}^3_{1,1,\mathrm{j}} = \mathrm{MAX}(\mathrm{PV}^3_{1,1,2}, \mathrm{PV}^3_{1,1,3}) = \mathrm{MAX}(\$700\mathrm{K}, \$916\mathrm{K}) = \$916\mathrm{K} = \mathrm{PV}^3_{1,1,3}$

$\mathrm{PV}^3_{0,2,\mathrm{j}} = \mathrm{MAX}(\mathrm{PV}^3_{0,2,2}, \mathrm{PV}^3_{0,2,3}) = \mathrm{MAX}(0, 0) = \$0 = \mathrm{PV}^3_{0,2,3}$

Calculation of Present Value (PV) at Year 1

$$\mathrm{PV}^2_{1,0,1} = MAX(\frac{((\mathrm{p} * \mathrm{PV}^3_{2,0,3}) + ((1-\mathrm{p}) * \mathrm{PV}^3_{1,1,3}))}{(1+r)} - \mathrm{E}^2_1,0)$$

$$\mathrm{PV}^2_{1,0,1} = MAX(\frac{((0.47 * \$4616\mathrm{K}) + (0.53 * \$916\mathrm{K}))}{(1+0.1)} - \$750\mathrm{K},0)$$

$$\mathrm{PV}^2_{1,0,1} = MAX(\frac{(\$2186\mathrm{K} + \$482\mathrm{K})}{1.1} - \$750\mathrm{K},0)$$

$$\mathrm{PV}^2_{1,0,1} = MAX(\frac{\$2668\mathrm{K}}{1.1} - \$750\mathrm{K},0)$$

$$\mathrm{PV}^2_{1,0,1} = MAX(\$2425\mathrm{K} - \$750\mathrm{K},0) = MAX(\$1675\mathrm{K},0) = \$1675\mathrm{K}$$

$$\mathrm{PV}_{0,1,1}^{2} = MAX(\frac{((\mathrm{p} * \mathrm{PV}_{1,1,3}^{3}) + ((1-\mathrm{p}) * \mathrm{PV}_{0,2,3}^{3}))}{(1+r_f)} - \mathrm{E}_{1}^{2},0)$$

$$\mathrm{PV}_{0,1,1}^{2} = MAX(\frac{((0.47 * \$916\mathrm{K}) + (0.53 * \$0))}{(1+0.1)} - \$750\mathrm{K},0)$$

$$\mathrm{PV}_{0,1,1}^{2} = MAX(\frac{\$434\mathrm{K}}{1.1} - \$750\mathrm{K},0)$$

$$\mathrm{PV}_{0,1,1}^{2} = MAX(\$394\mathrm{K} - \$750\mathrm{K},0) = MAX(-\$356\mathrm{K},0) = \$0$$

Calculation of Present Value (PV) at Year 0 with flexibility

$$\mathrm{PV}_{0,0,0}^{1} = MAX(\frac{((\mathrm{p} * \mathrm{PV}_{1,0,1}^{2}) + ((1-\mathrm{p}) * \mathrm{PV}_{0,1,1}^{2}))}{(1+r_f)} - \mathrm{E}_{0}^{1},0)$$

$$\mathrm{PV}_{0,0,0}^{1} = MAX(\frac{((0.473 * \$1675\mathrm{K}) + (0.527 * \$0))}{(1+0.1)} - \$250\mathrm{K},0)$$

$$\mathrm{PV}_{0,0,0}^{1} = MAX(\frac{(\$793\mathrm{K})}{1.1} - \$250\mathrm{K},0)$$

$$\mathrm{PV}_{0,0,0}^{1} = MAX(\$721\mathrm{K} - \$250\mathrm{K},0) = MAX(\$471\mathrm{K},0) = \$471\mathrm{K}$$

Table B-2. Systems Engineering Project's Value with flexibility using Compound Option Analysis

	Today		Year 1		Year 2		Year 3
Expenditures	E^1_0=\$250K		E^2_1=\$750K				E^3_{2OR3}=\$1500K
						$PV^3_{3,0,3}$=	8860
				$PV^3_{2,0,3}$=	\$4617K		
		$PV^2_{1,0,1}$=	\$1675K			$PV^3_{2,1,3}$=	\$2127K
$PV^1_{0,0,0}$ =	\$471K			$PV^3_{1,1,3}$=	\$916K		
		$PV^2_{0,1,1}$=	\$0			$PV^3_{1,2,3}$=	\$0
				$PV^3_{0,2,3}$=	\$0		
						$PV^3_{0,3,3}$=	\$0

The value of this flexibility or the option value (OV) is

$OV = PV^{1}_{0,0,0} – NPV_3 = \$471K - \$141K = \$330K$

The cost of waiting one year to exercise the production option has an OC of \$150K and the net option value (NOV) is

$NOV = \$330K - \$150K = \$180K$

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