

AN EFFICIENT EQUATION GENERATION MECHANISM FOR A
COMPONENT-BASED MODELING SCHEME

By

Aparna Barve

Thesis

Submitted to the Faculty of the
Graduate School of Vanderbilt University
in partial fulfillment of the requirements
for the degree of

MASTER OF SCIENCE

in

Computer Science

May, 2008

Nashville, Tennessee

Approved:

Professor Gautam Biswas

Professor Xenofon Koutsoukos

This thesis is dedicated to my parents, Archana and Ashok Barve.

ACKNOWLEDGEMENTS

I would like to thank my advisor, Dr. Gautam Biswas, for providing valuable guidance during the course of my Masters research and for his insightful comments. I am grateful to my secondary advisor Dr. Xenofon Koutsoukos for his constructive feedback. I would also like to thank Mr. Nag Mahadevan for his timely suggestions and valuable inputs in working out the implementation details for my thesis. I appreciate all the help provided to me by my fellow graduate students at Vanderbilt University and at the Institute of Software Integrated Systems.

Last but not the least, I would like to acknowledge the support and encouragement provided by my family during my Masters studies.

TABLE OF CONTENTS

| | Page |
|---|------|
| LIST OF TABLES | vi |
| LIST OF FIGURES | vii |
| Chapter | |
| I. INTRODUCTION | 1 |
| I.1. Main Contributions | 2 |
| I.2. Organization of Thesis | 3 |
| II. BUILDING OBSERVERS FROM BOND GRAPH MODELS | 4 |
| II.1. Bond Graph Modeling Language | 4 |
| II.2. Causality in Bond Graphs | 5 |
| II.3. Generating Mathematical Models from BG | 6 |
| II.4. Building Kalman Filters | 7 |
| III. IMPLEMENTING KALMAN FILTERS FROM BOND GRAPH MODEL | 9 |
| III.1. Modeling Environment | 9 |
| III.1.1. Introduction to FACT | 9 |
| III.1.2. Extensions for n-port elements | 10 |
| III.2. Efficient methods for generating State Space Models | 13 |
| III.2.1. Data Structures | 14 |
| III.2.2. State Equation Generation Algorithm | 26 |
| III.2.3. Output Equation Generation Algorithm | 30 |
| III.2.4. Basic Equation Generation Algorithm | 30 |
| III.2.5. Extension for Transformers and Gytrators | 31 |
| III.2.6. Extensions to SCAP | 35 |
| III.3. Building the Kalman Filter | 36 |
| IV. EXPERIMENTAL RESULTS | 41 |
| IV.1. Hydraulic Actuator System | 41 |
| IV.1.1. Equation Generation for the Hydraulic Actuator Sys- tem | 42 |
| IV.1.2. Observer runs with measurement noise and modeling errors in the system model | 43 |
| IV.2. N-port system | 54 |

| | | |
|----|----------------------------|----|
| V. | CONCLUSIONS | 59 |
| | V.1. Future Work | 59 |
| | BIBLIOGRAPHY | 60 |

LIST OF TABLES

| Table | Page |
|--|------|
| IV.1. Description of the Actuator Parameters | 42 |
| IV.2. Mean Square Error Values | 51 |

LIST OF FIGURES

| Figure | Page |
|--|------|
| I.1. TRANSCEND System Architecture | 2 |
| II.1. Model Based Diagnosis | 7 |
| III.1. Bond Graph Elements Class Hierarchy | 11 |
| III.2. Class Hierarchy for N-port Elements | 12 |
| III.3. Constraints for N-port Elements | 13 |
| III.4. Schematic Diagram of a Two Tank System | 14 |
| III.5. Fixed and Multi-port Variable Causality Assignments and BD representation | 15 |
| III.6. 1- and 2-port Variable Causality Assignments and BD representation | 16 |
| III.7. A GME Bond Graph Model of a Two Tank System | 16 |
| III.8. A Block Diagram Model of a Two Tank System | 17 |
| III.9. Template of the Node Data Structure | 18 |
| III.10. Algorithm for assigning Bond numbers | 24 |
| III.11. Equation Generation trace for state variable e5 | 28 |
| III.12. Algorithm for generating State Space Equations | 29 |
| III.13. Basic Equation Generation Algorithm | 34 |
| III.14. Causal assignments and corresponding equations for a Transformer | 34 |
| III.15. Causal assignments and corresponding equations for a Gyrator . . . | 35 |
| III.16. Causality Assignment for nC and nI | 36 |
| III.17. Kalman Filter Equations for Process and Measurement Model . . . | 38 |
| III.18. Observer and System Output for a Two Tank System | 39 |

| | | |
|--------|---|----|
| IV.1. | A simplified model for the Hydraulic Actuator | 41 |
| IV.2. | A GME Bond Graph Model of Hydraulic Actuator System | 42 |
| IV.3. | Simulink Model of the Hydraulic Actuator System | 44 |
| IV.4. | Pressure Output of the Hydraulic Actuator System | 44 |
| IV.5. | A GME Bond Graph Model of a part of the Hydraulic Actuator System | 45 |
| IV.6. | Measured and Observed values for Actuator for a measurement noise of 2 percent | 48 |
| IV.7. | Measured and Observed values for Actuator for a measurement noise of 5 percent | 49 |
| IV.8. | Measured and Observed values for Actuator for a measurement noise of 8 percent | 50 |
| IV.9. | Measured and Observed values for Actuator for a measurement noise of 2 percent, S_e increased by 2 percent and R_{valve} decreased by 2 percent | 51 |
| IV.10. | Measured and Observed values for Actuator for a measurement noise of 2 percent, S_e decreased by 5 percent and R_{valve} increased by 5 percent | 52 |
| IV.11. | Measured and Observed values for Actuator for a measurement noise of 8 percent, S_e increased by 2 percent and R_{valve} decreased by 8 percent | 53 |
| IV.12. | Bond Graph model of the system with an n-port element | 54 |
| IV.13. | Observer and System Output for a Two Tank System implemented using nC | 57 |

CHAPTER I

INTRODUCTION

The thesis addresses the problem of building observers from component-based models of continuous dynamic systems. We build models using bond graphs, which provide a domain independent, energy-based, topological scheme for capturing the dynamic behavior of physical systems [9]. The observer plays an important part in online diagnosis schemes by providing reliable estimates of system state and system output in the presence of noise in the measurements and discrepancies in the system model. The estimated outputs are compared against the observed measurements to compute the system residuals. Non-zero residuals imply faults in the system.

The TRANSCEND (TRANSient based Continuous ENgineering Diagnosis) [12] provides such a model based system for monitoring and diagnosis of complex dynamic systems. As shown in Fig. I.1, the observer tracks the residuals

$$r = y - \hat{y},$$

i.e. the difference between the actual and the predicted observations. A non-zero residual triggers the fault diagnosis process.

We use the FACT [6] (Fault Adaptive Control Technology) system developed in the Generic Modeling Environment for diagnosis applications. It allows us to build component-oriented bond graph models of our systems and also provides a run time environment for carrying out fault diagnosis.

The steps involved in building the observer involve transforming the component-based bond graph model of the physical system to its corresponding block diagram

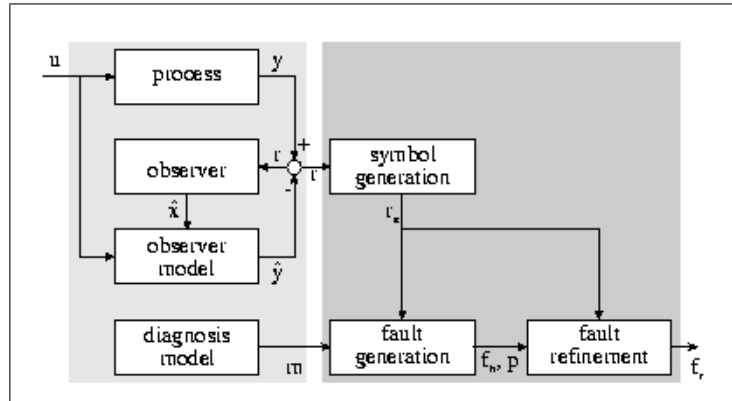


Figure I.1: TRANSCEND System Architecture

model and generating the state space equations and the output equations from this model. The observer is implemented as a Kalman filter.

I.1 Main Contributions

A previous implementation [6] of the observer in the FACT system did not explicitly use the causality information from the bond graph while generating equations for the system. Instead, it used a more brute force approach by generating equations for every component in the bond graph model and manipulating them over multiple iterations till all the equations could be represented in the state space form. This method of manipulating the multiple equations fragments simultaneously made the equation generation process inefficient and computationally complex. It was clear that this complexity could be significantly reduced by exploiting the computational causality of the bonds in the bond graphs [8].

The main contribution of the thesis is to implement an efficient equation generation mechanism for the component based modeling paradigm in FACT. Assigning causalities to the bonds captures the cause effect relationship between the bond graph

elements and considering these causalities in the equation generation algorithm significantly simplifies the process of deriving the state and output equations of the system. We use these models to construct a Kalman filter based observer to track continuous system behavior for linear systems. We also provide a mechanism for extending the modeling paradigm to include n-port elements that play an important part in the modeling on multi-domain physical systems.

I.2 Organization of Thesis

The thesis is organized as follows.

Chapter II of the thesis provides a background of the various components involved in designing state space observers, the Bond Graph Modeling Language, the concept of causality, equation generation, and Kalman filters. Chapter III provides a detailed overview of the implementation details, which includes the extensions made to the modeling paradigm, the algorithms used for equation generation, followed by the building of the Kalman filter from the generated equations. The simulation and the experimental results are discussed in Chapter IV. Chapter V elaborates on the future work and provides the conclusion for the thesis.

CHAPTER II

BUILDING OBSERVERS FROM BOND GRAPH MODELS

This chapter gives a brief overview of the process of building observers from bond graph models and presents a background for the problems addressed in the thesis. An overview of the bond graph modeling language that is used to construct the input models for our system is presented at the beginning of this chapter. In this chapter, we also explain the concept of causality in bond graphs, followed by the process of generating mathematical equations from the given bond graph models. Lastly, we discuss Kalman filters and show how they can be built based on the generated equations.

II.1 Bond Graph Modeling Language

The bond graph modeling language is a physics-based modeling language that provides a uniform, lumped-parameter, energy-based, topological framework for modeling across multiple physical domains. Bond graphs [3] are a domain independent language for modeling physical systems. The physical processes are directly represented as vertices in a directed graph, and the edges represent the ideal exchange of energy between the vertices. It is possible to model systems involving more than one physical domain like the thermal, electrical, fluid or mechanical domains using bond graphs. Bond graphs capture the exchange of energy involved in the physical process by making use of bonds. The various commonly used elements used in bond graphs are energy storage elements (C and I), energy dissipation elements (R), energy transformation elements (TF and GY), and input output elements (Se and Sf).

The connecting edges, called bonds, represent energy pathways between the elements. Each bond is associated with two variables: effort and flow. Every bond has a power direction and a computational causality direction [9]. Connections in the system are modeled by two idealized elements: 0- (or parallel) and 1- (or series) junctions. For a 0- (1-) junction, the efforts (flows) of all incident bonds are equal, and the sum of flows (efforts) is zero [2].

II.2 Causality in Bond Graphs

In bond graphs, the inputs and the outputs associated with the system components are characterized by effort causality and flow causality. Causality assignment is a process by which the bond variables effort and flow are partitioned into input-output sets. There exist four types of causal constraints in bond graphs [1].

1. *Mandatory causality* - The constitutive laws force the output port to take on a fixed causality that cannot be changed. Sources S_e and S_f have mandatory effort and flow causality, respectively.
2. *Preferred causality* - For the storage elements C and I, causality can be determined by the integral or derivative form of their behaviors. The preferred causality here refers to the integral causality form for these elements.
3. *Constrained causality* - For TF, GY, 0- and 1-junction elements, there are relations between the causality of the different ports of the elements defined by the algebraic relations between the variables associated with these ports.
4. *Indifferent causality* - Indifferent causality means that causal constraints on a port are not determined by the internal constraints of the component. The linear resistor element R exhibits indifferent causality since both power variables

- effort and flow - can be specified as input or output ports depending on the relations that hold for the adjacent junction.

Most traditional causality assignment procedures use a local constraint propagation scheme to label bond causality. From some starting point, usually one of the source elements, causality is assigned sequentially, until all element ports are labeled. These causality assignment procedures must also satisfy the four causality types and the fundamental causal constraints.

II.3 Generating Mathematical Models from BG

Topological bond graph models may be converted into a typical mathematical representation for system behavior by deriving the equations from the topological model. The flow variables associated with the inductive elements and the effort variables associated with the capacitive elements form the state variables of the system. Thus, our goal is to parse the bond graph I and C elements (one port or multi port) and generate the state equations individually for each state variable. This forms the state space model or the process model [4], as it models the transformation of the process state. Also, in order to link the output variables and the state variables, we need to derive the output equations.

Output equations need to be generated for all measured variables on the system. The equations for these variables are expressed in terms of the state variables and the input variables. These form the measurement model [4] that describes the relationship between the process state and the measurements.

II.4 Building Kalman Filters

Observers play an important part in model based diagnosis [10]. Residuals are computed as the difference between the observations and the predicted normal behavior. Whenever a non-zero residual is detected, the diagnosis algorithm is triggered. This is depicted in Fig. II.1 [10].

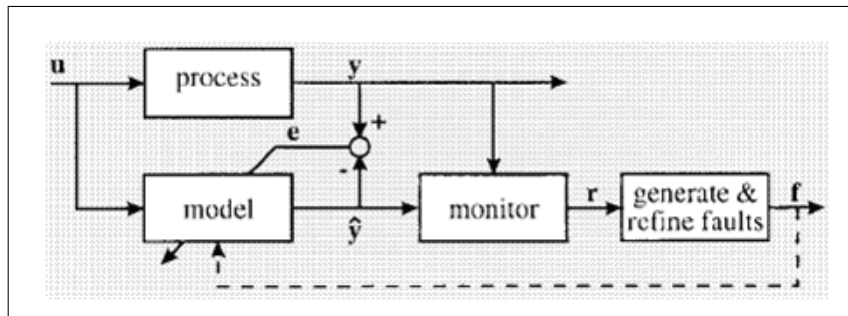


Figure II.1: Model Based Diagnosis

Here, u is the system input, y is the predicted normal behavior, \hat{y} is the estimated output, and r is the non-zero residual that triggers fault diagnosis.

In this thesis, we follow the FACT system and rebuild the Kalman filter based observer for diagnosis purposes. The Kalman filter [4] is a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of the process.

We adopt the form of Kalman filter implementation that addresses the general problem [4] of generating the best estimate of the state of a discrete-time controlled process that is governed by the linear stochastic difference equation

$$x_k = Ax_{k-1} + Bx_{k-1} + w_{k-1}$$

with a measurement $z \in \mathbb{R}^m$ that is

$$z_k = Hx_k + v_k$$

The random variables w_k and v_k represent the process and measurement noise respectively. The $n \times n$ matrix A in the difference equation relates the state variable at the previous time step $k-1$ to the state at the current step k . The $n \times l$ matrix B relates the optional control input u to the state x . The $m \times n$ matrix H in the measurement equation relates the state to the measurement z_k . These matrices are obtained from the coefficients of the state and the output equations that are generated from the bond graph model.

CHAPTER III

IMPLEMENTING KALMAN FILTERS FROM BOND GRAPH MODEL

This chapter begins with an overview of the modeling environment used to create component-based bond graph models. The chapter mainly discusses the algorithms for the efficient generation of state space equations, followed by the application of these equations in implementing the Kalman filter.

III.1 Modeling Environment

The Generic Modeling Environment (GME) provides a framework for defining custom modeling languages. Fault Adaptive Control Technology (FACT) [6] is a tool suite in GME which is used to build bond graph models graphically, and then use these models to construct model-based diagnosers. The following section discusses the aspects of FACT used in our implementation and the extensions made to FACT in order to provide modeling support for the n -port components.

III.1.1 Introduction to FACT

The FACT modeling paradigm defines the language for component-oriented modeling of physical systems using bond graphs [6]. FACT has two main components: (1) a design-time environment for modeling and model transformation, and (2) a run-time environment consisting of the model-based reasoner components [5]. The FACT modeling language provides a Bond Graph aspect to describe the hybrid bond graph model of the component or system, a Failure Propagation aspect to describe

the Temporal Fault Propagation model of the component or system, and an Interface aspect to describe the inputs and outputs of the system or plant [6]. The Bond Graph aspect allows the user to create bond graph models using various elements like signal ports, decision functions, energy ports, capacitors, inductances, resistors, transformers, gyrators, sources of flow/effort, zero and one junctions, etc. It allows the creation of hierarchical models.

III.1.2 Extensions for n -port elements

The R, C and I elements in the original FACT modeling paradigm are 1-port elements. However, to model physical systems involving multiple domains, and vector elements, the 1-port elements can be extended to n -port elements. For example, an n -port resistance can be used to depict convection in a thermo-fluid system, where the thermal resistance is modulated by the volume flow rate from the hydraulic domain. An n -port capacitance can be used to depict the temperatures of different bodies taking part in heat transfer. Similarly, an n -port inductance can be used to represent quantities like the mutual inductance in a motor or a generator system. There is an n -port transfer matrix associated with an n -port element that is used to represent the relation between its inputs and outputs.

In order to construct bond graph models for such systems that contain n -port elements, we have extended the current FACT paradigm to support multi-port elements as shown in Fig. III.1.

An additional element is added to the *BGElement* class i.e., *NPort*, along with the *OnePort* and *TwoPort* elements. The *NPort* element class is further subclassed as *nR* - the n -port resistance and the *NPortStorage* class, which represents nC and nI , as shown in Fig. III.2. Associated with all the n -port elements, we have an attribute n that gives the number of bonds that will be connected to this element.

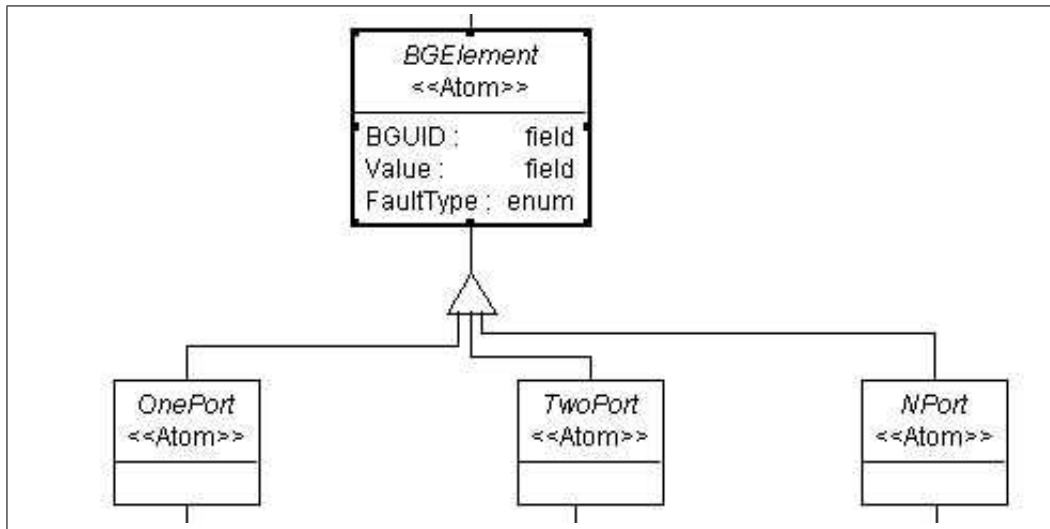


Figure III.1: Bond Graph Elements Class Hierarchy

Also, as described above we have an $n \times n$ transfer matrix associated with every n -port element.

Along with the transfer matrix we have a vector of initial values of length n for the n -port storage elements - nC and nI . These are the values that will be used as the initial values for the integration operation.

Also, as shown in Fig. III.3, we have a few additional constraints associated with the n -port elements. We need to ensure that an n -port element has been connected to exactly n number of bonds. Apart from that, we also need to check that the transfer matrix entered for the n -port element has a non-null value, and that the value has been specified in the right format.

If the initial value attribute is left null, a set of default values of all zeros is assumed. Also, for all bonds connecting to an n -port element its mandatory for the user to provide the ordering of those bonds from 1 to n with respect to the n -port element, such that every bond associated with the n -port element has a distinct number that lies between 1 and n .

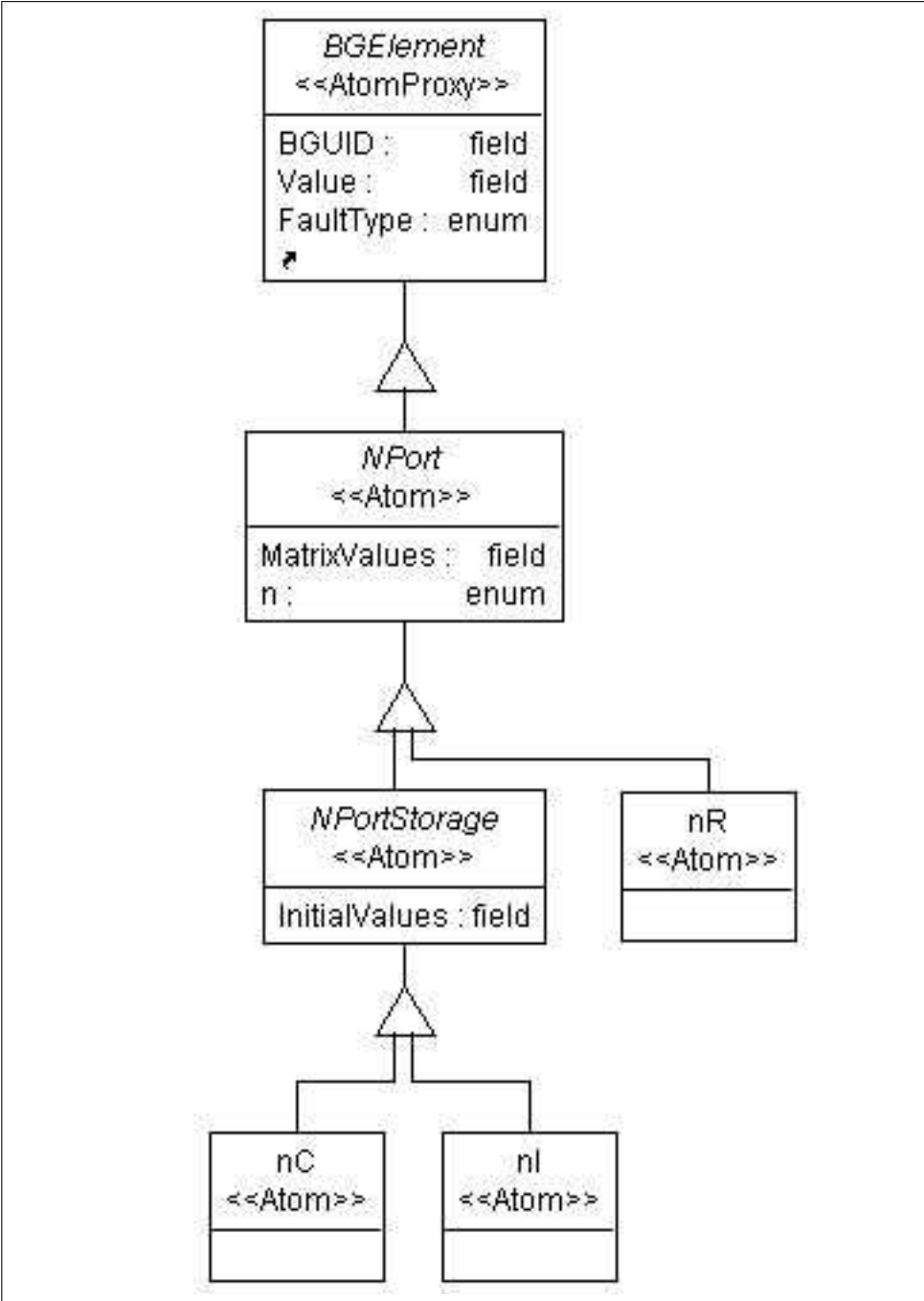


Figure III.2: Class Hierarchy for N-port Elements

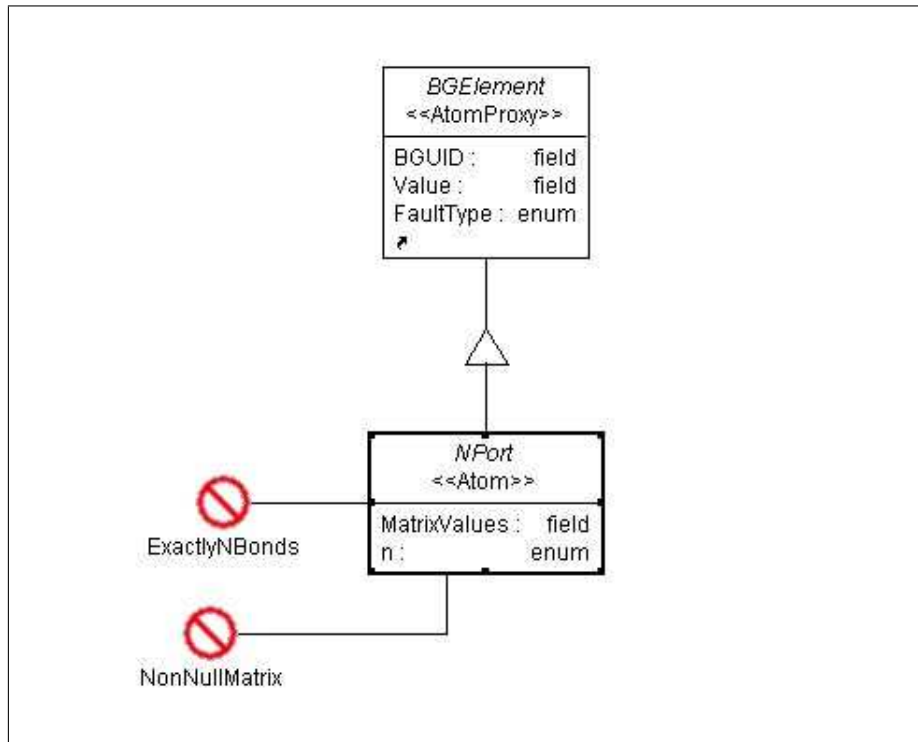


Figure III.3: Constraints for N-port Elements

III.2 Efficient methods for generating State Space Models

In order to derive the state space model, we need to represent the component-oriented bond graph model using a block diagram and other data structures that capture the details of all the nodes and bonds in the bond graph model. The equation generation algorithm identifies the state, input and the output variables for the given model and works on these data structures to generate the corresponding equations.

We consider the example of a two tank system for explaining the various parts in the following sections. Fig. III.4 shows the basic structure of a two tank system. It consists of a source of flow S_f , two tanks with capacities $C1$ and $C2$, and pipes with

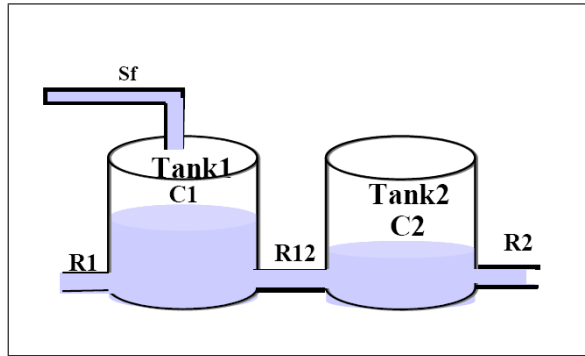


Figure III.4: Schematic Diagram of a Two Tank System

resistances $R1$, $R2$ and $R12$. Our goal is to measure the outflow rate from tanks 1 and 2 through pipes $R1$ and $R2$, respectively.

III.2.1 Data Structures

The basic data structures involved in the Equation Generation algorithm are the Block Diagram representation of a BG model, a mapping of bond numbers to the actual bond connections in the BG model, a mapping of junctions in the BG to their determining bonds and a flat data structure containing the information about all the nodes in a BG and the connections between them.

Block Diagram Representation

The block diagram (BD) formalism is a widely used graphical, computational scheme for describing simulation models of continuous and hybrid systems [7].

Fig. III.5 and Fig. III.6 show the causality assignment and the block diagram structure for every type of element in a bond graph [2]. The Sf, Se, C, and I elements have a single unique block diagram representation because their incident bonds have only one possible causal assignment, assuming integral causality for the C and the I elements. The R, TF and GY elements allow two causal representations each, and each one produces a different BD representation. A junction with m incident bonds

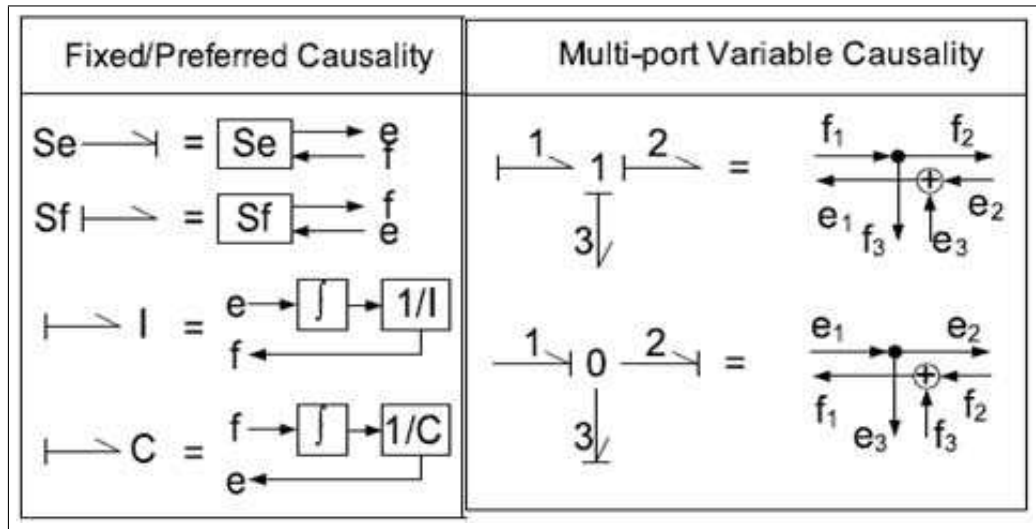


Figure III.5: Fixed and Multi-port Variable Causality Assignments and BD representation

can have m possible BD configurations. Mapping a junction structure to its BD is facilitated by the notion of the determining bond, which captures the causal structure for the junction.

Fig.III.7 shows the bond graph model of the two tank system and Fig. III.8 shows its corresponding block diagram model.

In the BD model of the two tank system, there is a block corresponding to every node of the BG model and there are Output blocks corresponding to every output port in the BG model. The *Flow1* and *Flow2* outputs represent the volume flow rates measured at pipes *R1* and *R2*, respectively.

Bond Graph Class - A Flat Data Structure

The input for the Equation Generation system is a GME Model constructed using the extended FACT paradigm. This model represents a simple or a hierarchical bond graph. To operate on this model, we first need to convert it to a flat model and then represent the flat model using an appropriate local data structure like an object so that it can be accessed by various parts of the algorithm without the overhead of

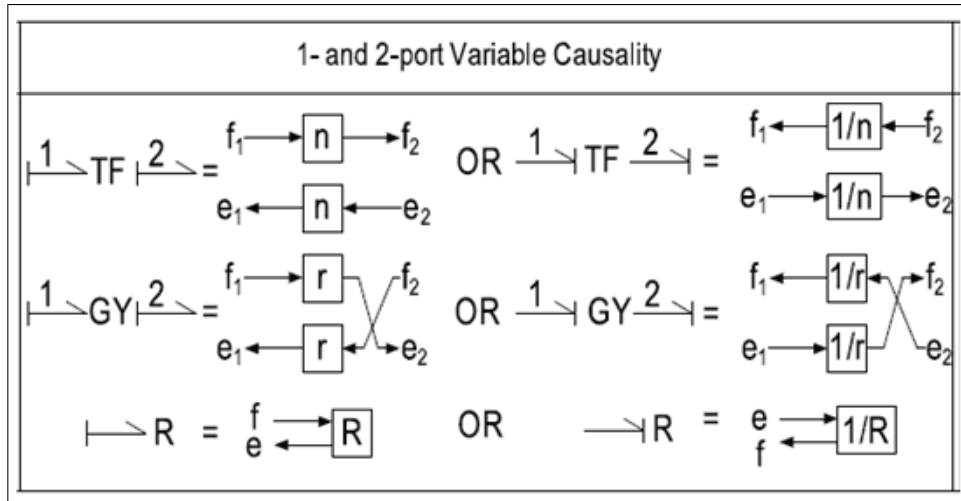


Figure III.6: 1- and 2-port Variable Causality Assignments and BD representation

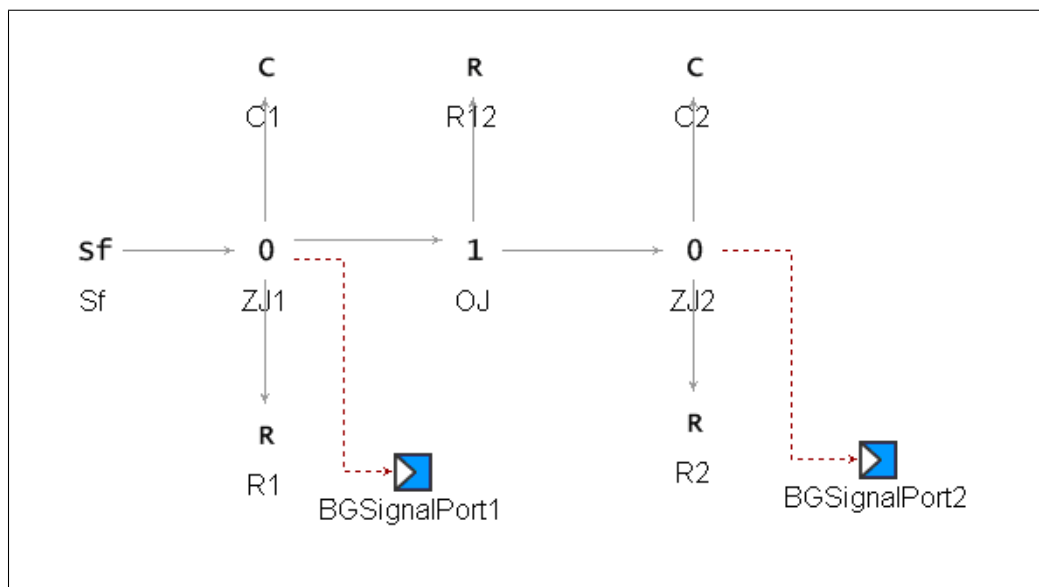


Figure III.7: A GME Bond Graph Model of a Two Tank System

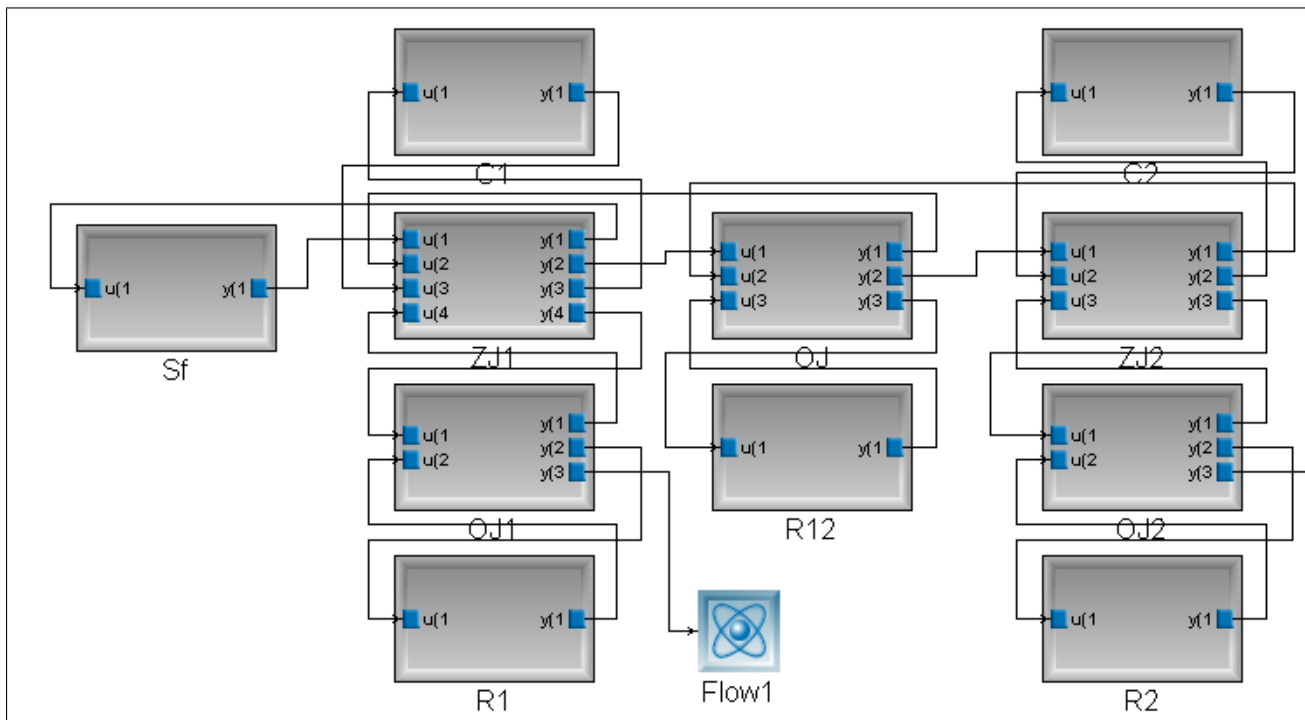


Figure III.8: A Block Diagram Model of a Two Tank System

opening or closing text files or the model files. This ensures fast access time making the algorithm more efficient.

The class *BondGraph* stores the various details about a given Bond Graph Model including the specifications for all the nodes in the bond graphs and the connections that exist between them.

The *BondGraph* class stores certain properties that are applicable to the entire bond graph. They can be enumerated as follows:

1. Total number of nodes in the bond graph
2. Total number of junctions in the bond graph
3. Total number of hybrid junctions in the bond graph
4. List of fixed junctions

```

For every node  $i$  in the BG
    node( $i$ ).name
    node( $i$ ).type
    node( $i$ ).number_of_bonds
    node( $i$ ).is_fault_candidate
    set of node( $i$ ).bonds
end

```

Figure III.9: Template of the Node Data Structure

5. List of determining bonds
6. Sample time

Fig. III.9 shows the data that is stored for every node in the bond graph.

The various data fields can be explained as below:

1. Node identifier
2. Name of the node
3. Type of the node
4. Number of bonds that it is connected to
5. Whether or not it is a potential fault candidate
6. The listing of bonds connected to this node
7. State of the node(On/Off - This parameter is applicable only when the node under consideration is a hybrid junction. In case of a junction in a continuous system, this parameter will default to an "on" value)
8. Whether or not the node (junction) is hybrid

9. Index of the node (junction) in the bond graph
10. The parameter n for an n -port element
11. A list of bonds that cannot act as the determining bond for this node (junction).

The bond graph stores the above data for every node in the form of a vector or an array of the size of the number of nodes in the flat bond graph model. Some of the properties stated above are populated while generating the data structure from the Bond Graph GME model, whereas certain others can only be filled in during the later parts of the algorithm. For instance, since this data structure has been specifically designed for its use in generating the causality assignment for the bond graph, some of its properties like the determining bonds list, etc. can only be populated during the process of running the causality update algorithm.

For storing the connectivity information for the bond graph, that is to capture the connections between different nodes, each node in the above data structure stores a set of nodes that it connects to. Thus, the size of this set will be equal to the number of bonds associated with this node. The order in which the various nodes appear in this list does not matter as long as we are not dealing with n -port elements. However, for n -port elements nR , nC and nI , the order is important because the user associates a distinct number between 1 to n with every bond connected to an n -port element, and thus the connecting nodes should be listed in the same order as specified by the user. This helps to associate the index of the connecting node in the list to the appropriate row in the $n \times n$ matrix supplied by the user, in the later parts of the algorithm.

The node data structure generated for the two tank system is as follows:

```
Node 1 - TwoTankSystem/TTS/R1
```

```
bg.node(1).name = '//TwoTankSystem/TTS/R1';
```

```
bg.node(1).type = 'R';
```

```
bg.node(1).numBonds = 1;  
bg.node(1).fault = 0;  
bg.node(1).bond(1)= 8;
```

Node 2 - //TwoTankSystem/TTS/R2

```
bg.node(2).name = '//TwoTankSystem/TTS/R2';  
bg.node(2).type = 'R';  
bg.node(2).numBonds = 1;  
bg.node(2).fault = 0;  
bg.node(2).bond(1) = 9;
```

Node 3 - //TwoTankSystem/TTS/R12

```
bg.node(3).name = '//TwoTankSystem/TTS/R12';  
bg.node(3).type = 'R';  
bg.node(3).numBonds = 1;  
bg.node(3).fault = 0;  
bg.node(3).bond(1) = 7;
```

Node 4 - //TwoTankSystem/TTS/C1

```
bg.node(4).name = '//TwoTankSystem/TTS/C1';  
bg.node(4).type = 'C';  
bg.node(4).numBonds = 1;  
bg.node(4).fault = 0;  
bg.node(4).bond(1) = 10;
```

```
Node 5 - //TwoTankSystem/TTS/C2
bg.node(5).name = '//TwoTankSystem/TTS/C2';
bg.node(5).type = 'C';
bg.node(5).numBonds = 1;
bg.node(5).fault = 0;
bg.node(5).bond(1) = 11;
```

```
Node 6 - //TwoTankSystem/TTS/Sf
bg.node(6).name = '//TwoTankSystem/TTS/Sf';
bg.node(6).type = 'Sf';
bg.node(6).numBonds = 1;
bg.node(6).fault = 0;
bg.node(6).bond(1) = 10;
```

```
Node 7 - //TwoTankSystem/TTS/OJ
bg.node(7).name = '//TwoTankSystem/TTS/OJ';
bg.node(7).type = 'OneJunction';
bg.node(7).numBonds = 3;
bg.node(7).state = 1;
bg.node(7).index = 1;
bg.node(7).hybrid = 0;
bg.node(7).bond(1) = 10;
bg.node(7).bond(2) = 11;
bg.node(7).bond(3) = 3;
```

```
Node 8 - //TwoTankSystem/TTS/OJ1
bg.node(8).name = '//TwoTankSystem/TTS/OJ1';
bg.node(8).type = 'OneJunction';
bg.node(8).numBonds = 2;
bg.node(8).state = 1;
bg.node(8).index = 2;
bg.node(8).hybrid = 0;
bg.node(8).bond(1) = 10;
bg.node(8).bond(2) = 1;
```

```
Node 9 - //TwoTankSystem/TTS/OJ2
bg.node(9).name = '//TwoTankSystem/TTS/OJ2';
bg.node(9).type = 'OneJunction';
bg.node(9).numBonds = 2;
bg.node(9).state = 1;
bg.node(9).index = 3;
bg.node(9).hybrid = 0;
bg.node(9).bond(1) = 11;
bg.node(9).bond(2) = 2;
```

```
Node 10 - //TwoTankSystem/TTS/ZJ1
bg.node(10).name = '//TwoTankSystem/TTS/ZJ1';
bg.node(10).type = 'ZeroJunction';
bg.node(10).numBonds = 4;
bg.node(10).state = 1;
bg.node(10).index = 4;
```

```

bg.node(10).hybrid = 0;
bg.node(10).bond(1) = 6;
bg.node(10).bond(2) = 7;
bg.node(10).bond(3) = 4;
bg.node(10).bond(4) = 8;

    Node 11 - //TwoTankSystem/TTS/ZJ2
bg.node(11).name = '//TwoTankSystem/TTS/ZJ2';
bg.node(11).type = 'ZeroJunction';
bg.node(11).numBonds = 3;
bg.node(11).state = 1;
bg.node(11).index = 5;
bg.node(11).hybrid = 0;
bg.node(11).bond(1) = 7;
bg.node(11).bond(2) = 5;
bg.node(11).bond(3) = 9;

    bg.numNodes = 11;
bg.numJunctions = 5;
bg.numHybridJunctions = 0;

```

Causality Assignment

Bond Graph models imply a causal structure and algorithms like the Sequential Causality Assignment Procedure (SCAP) [8] applied to well-formed BG models assign causal directions to all bonds in the model.

```

AssignBondNumbers
  For all nodes n in BondGraph bg do
    For all nodes j connected to node n
      If the pair<n, j> and the pair <j, n> are absent in the structure bonds
        Add pair<n, j> to bonds
      end
    end
  end
end

```

Figure III.10: Algorithm for assigning Bond numbers

After executing SCAP, every junction in the BG model is assigned a determining bond; this data is further used in the equation generation algorithm. The determining bond assignment for the two tank system model can be given as:

```

JunctionName ---> DeterminingBond
TwoTankSystem/TTS/OJ ---> TwoTankSystem/TTS/R12
TwoTankSystem/TTS/OJ1 ---> TwoTankSystem/TTS/R1
TwoTankSystem/TTS/OJ2 ---> TwoTankSystem/TTS/R2
TwoTankSystem/TTS/ZJ1 ---> TwoTankSystem/TTS/C1
TwoTankSystem/TTS/ZJ2 ---> TwoTankSystem/TTS/C2

```

Assigning Bond Numbers

The equations that are generated have effort and flow variables associated with different bonds. In order to generate these equations, we need to assign numbers to all the bonds in the bond graph model. The algorithm for assigning the bond numbers shown in Fig. III.10 operates on the the list of nodes that is a part of the *BondGraph* class described above.

The algorithm *AssignBondNumbers* iterates over all the nodes in the bond graph.

For every node, it iterates over all the connecting nodes associated with that particular node. If that pair of nodes has already been assigned a bond number (the order of nodes in the pair is immaterial), then it skips over to the next node, else it adds it to the bonds data structure; the index of the pair in the data structure automatically becomes the bond number associated with that pair.

Following is the bond number assignment made by the algorithm for the given two tank BG model.

Bond 1: R1 - 0J1
 Bond 2: R2 - 0J2
 Bond 3: R12 - 0J
 Bond 4: C1 - ZJ1
 Bond 5: C2 - ZJ2
 Bond 6: Sf - ZJ1
 Bond 7: 0J - ZJ1
 Bond 8: 0J - ZJ2
 Bond 9: 0J1 - ZJ1
 Bond 10: 0J2 - ZJ2

Storing variable coefficients

For a given input model, we need to extract the sets of input, state and output variables. Input variables are associated with the flows and efforts associated with Sf and Se blocks respectively. State variables are the efforts on the capacitances and the flows through the inertias. The flow through a one-junction or an effort at a zero-junction to which a sensor has been connected, form the output variables.

Thus, for the two tank system, we observe that $f6$, i.e., the flow associated with

Sf is the input variable, $e4$ and $e5$, i.e., the efforts on the two capacitances are the state variables, and $f1$ and $f2$, i.e., the flows measured at the two one junctions are the output variables.

Each equation that is generated is represented in terms of the state and the input variables. Thus, for every variable for which we need to generate an equation we need to associate it with a map; the key of this map is the state or the input variable index and the value is the coefficient associated with that variable for that particular equation. Initially, for each state or input variable in the map, the coefficient value is equal to zero. The value part keeps on getting updated during the equation generation process. When the equation generation algorithm has been executed for a given variable, we finally get a map which will have non-zero coefficient values for all the state and input variable taking part in the equation. If we need to generate the equations in symbolic form, this coefficient value is represented as a string instead of representing it in a numeric form.

III.2.2 State Equation Generation Algorithm

The *GenerateStateEquations* algorithm is used to generate the state equations given a Block Diagram bd . This block diagram is a computational representation of the given bond graph; it has blocks corresponding to every element in the bond graph. It also captures the connections between the different blocks in the form of input and output signals.

As seen in the previous section, for a given physical system represented by a bond graph, we have the flows through the inertias and the efforts on the capacitances as the state variables. The basic *EquationGeneration* algorithm described in Fig. III.13 is called for each of these state variables. The goal of the basic equation generation

algorithm is to represent a given flow or an effort variable associated with a particular bond in terms of state variables and input variables of the system.

For a capacitive element, with an associated effort e , flow f , and capacitance C , we have the state variable e represented as

$$\dot{e} = \frac{1}{C} \cdot f$$

Similarly, for an inductive element, with an associated effort e , flow f , and inductance I , we have the state variable f represented as

$$\dot{f} = \frac{1}{I} \cdot e$$

The basic principle in generating equations from a block diagram is to backtrack from the block under consideration till a point where we end up with a set of effort or flow signals associated with blocks that either represent the input variables of the system or the state variables.

We illustrate the working of this algorithm by stepping through the equation generation process for one of the state variables of the two tank system. Fig. III.11 shows the execution trace for the equation generation algorithm for $e5$. The parameters passed to the *GenerateFurther* function i.e., the Basic Equation Generation algorithm, are the coefficient, the bond variable (The variable is typically represented as a $\langle \text{flow}, \text{bond number} \rangle$ variable or an $\langle \text{effort}, \text{bond number} \rangle$ variable) and the name of the source block. We keep on backtracking till we reach a state variable or an input variable at which point we prune that branch of the tree. As we can see in Fig. III.11, the branches get pruned when we reach the state variables $e4$ and $e5$.

The state equation generation algorithm is as shown in Fig. III.12.

By running this algorithm, we are able to obtain the representation of every state

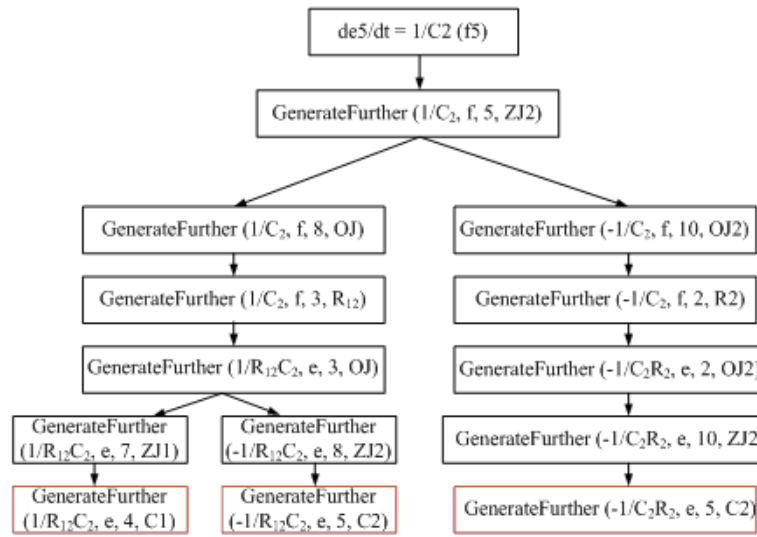


Figure III.11: Equation Generation trace for state variable e5

variable in the system in terms of the input and the state variables of the system, with the corresponding coefficients stored in a data structure. These coefficients are further used for calculating the A and the B matrices of the observer.

The execution of this algorithm for the two tank system model generates the following output:

IN NUMERIC FORM

$$de4/dt = -0.0240798 e4 + 0.00841728 e5 + 61.867 f6$$

$$de5/dt = 0.00841728 e4 - 0.012684 e5$$

IN SYMBOLIC FORM

$$de4/dt = ((((-1*(1/C1))/R12))+(-1*(1/C1))/R1)e4 + (-1*((-1*(1/C1))/R12))e5 + (1/C1)f6$$

$$de5/dt = ((1/C2)/R12)e4 + (((-1*((1/C2)/R12)))+(-1*(1/C2))/R2)e5$$

```

GenerateStateEquations (BlockDiagram bd)
  For all blocks blk in block diagram bd
    if blk.blockType = "C" or blk.blockType = "nC"
      For a C block, set n = 1
      For i = 1 to n          //n state variables
        Trace back to the n flow variable(s) that are used to calculate the effort
        across the capacitance and set is_effort = false
        For all flow variable(s) j = 1 to n
          Backtrack from the C or the nC block blk to get source_block.
          bond_number = getConnectingBond (blk, source_block)
          state_variable_row = Row in the data structure corresponding to
          state variable e<bond_number> (effort variable)
          coeff = get_matrix_value (i, j)          //1/C for C block
          GenerateFurther (coeff, is_effort, bond_number, source_block,
          state_variable_row)
        end
      end
    end
  else if blk.blockType = "I" or blk.blockType = "nI"
    For an I block, set n = 1
    For i = 1 to n          //n state variables
      Trace back to the n effort variable(s) that are used to calculate the flow
      through the inductance and set is_effort = true
      For all effort variable(s) j = 1 to n
        Backtrack from the I or nI block blk to get source_block
        bond_number = getConnectingBond (blk, source_block)
        state_variable_row = Row in the data structure corresponding to
        state variable f<bond_number> (flow variable)
        coeff = get_matrix_value (i, j)          //1/I for I block
        GenerateFurther (coeff, is_effort, bond_number, source_block,
        state_variable_row)
      end
    end
  end
end
end
end
end

```

Figure III.12: Algorithm for generating State Space Equations

III.2.3 Output Equation Generation Algorithm

The output equation generation algorithm invokes the basic equation generation algorithm for the output variables in exactly the same way as the state equation generation algorithm that invoked the basic equation generation algorithm for the state variables. In order to identify the output variable, we scan the Block Diagram for Output blocks and then trace back from them till we reach a block representing either a One Junction or a Zero Junction. A sensor connected to a one junction implies that the flow at that junction is measured, whereas a sensor connected to a zero junction implies that the effort at that junction is to be measured. To measure the flow or effort at a one or a zero junction respectively, we consider the determining bond for the junction, and then invoke the basic equation generation algorithm for these parameters.

For the two tank system model, f_1 and f_2 are the output variables. The equations generated for these variables are given as:

IN NUMERIC FORM

$$f_1 = 0.000253165 \text{ e}4$$

$$f_2 = 6.89655\text{e-}005 \text{ e}5$$

IN SYMBOLIC FORM

$$f_1 = ((1)/R_1)\text{e}4$$

$$f_2 = ((1)/R_2)\text{e}5$$

III.2.4 Basic Equation Generation Algorithm

The basic equation generation algorithm encapsulates the basic step of backtracking required to generate an equation for any variable. This algorithm will be called multiple times for generating an equation for a given state or an output variable. It

takes as its input, a source block blk in a Block Diagram that has been derived from a bond graph, the variable v for which we are interested in generating an equation for in the form of $\langle is_effort, bond_number \rangle$ (the Boolean value is_effort indicates whether we are determining an effort or a flow, and the $bond_number$ indicates the bond corresponding to the variable), the coefficient which is the multiplier of this variable in the entire equation of which the variable is a part, and the row_id . (The variable under consideration is a part of the equation that is being generated for another variable w . So, the row_id gives the row corresponding to the variable w .)

The algorithm is as shown in Fig. III.13

The equation generation procedure aims at expressing any given variable in terms of state variables and input variables. So, the algorithm checks for this condition in the very beginning. It terminates when it finds that v is either a state variable or an input variable, by storing the value of the coefficient for v . If the variable v is not a state or an input variable, the behavior of the algorithm varies depending on the type of block blk .

The working of the algorithm for the equation generation of a state variable in the two tank system can be seen in Fig. III.11.

III.2.5 Extension for Transformers and Gytrators

The following condition checks are added to the basic equation generation algorithm so that it can handle bond graphs having transformer and gyrator elements.

If the block type corresponds to a Transformer block TF, we get the node number corresponding to the $bond_number$ that was passed as an argument. Also, get the transformer ratio value n . Then we find out if the bond that is passed is an input to the TF or an output, and also find its corresponding input or output index. We also find the node corresponding to the other bond of the transformer as we need that to

```

GenerateFurther (coefficient, is_effort, bond_number, Block blk, row_id)
  if <is_effort, bond_number> is an input or a state variable
    Add coefficient to that variable's column for the row row_id in the
    state_variable_list
  return
end
if blk.blockType = "OneJunction"
  if we are dealing with a flow variable, (is_effort = false)
    detBond = GetDeterminingBond(One Junction corresponding to blk)
    Find the block src_blk corresponding to the other end of detBond
    GenerateFurther (coefficient, false, detBond, src_blk, row_id)
  else
    For all bonds bond connected to this One Junction
    where bond != bond_number
      Find the block src_blk corresponding to the other end of bond
      if bond.direction = bond_number.direction
        coefficient = coefficient *-1
      end
      GenerateFurther (coefficient, true, bond, src_blk, row_id)
    end
  end
end
end

```



```

else if blk.blockType = "ZeroJunction"

    if we are dealing with an effort variable, (is_effort = true)

        detBond = GetDeterminingBond(Zero Junction corresponding to blk)

        Find the block src_blk corresponding to the other end of detBond

        GenerateFurther (coefficient, true, detBond, src_blk, row_id)

    else

        For all bonds bond connected to this Zero Junction

        where bond != bond_number

            Find the block src_blk corresponding to the other end of bond

            if bond.direction = bond_number.direction

                coefficient = coefficient * -1

            end

            GenerateFurther (coefficient, false, bond, src_blk, row_id)

        end

    end

else if blk.blockType = "R"

    Find the block src_blk by backtracking from the R block

    bond_number = getConnectingBond(blk, src_blk)

    if input is an effort (is_effort = true)

        coefficient = coefficient * R_value as  $e = R * f$ 

        Backtrack to calculate the flow variable value

        GenerateFurther (coefficient, false, bond_number, src_blk, row_id)

    else

```

```

    coefficient = coefficient / R_value as f = e / R
    Backtrack to calculate the effort variable value
    GenerateFurther (coefficient, true, bond_number, src_blk, row_id)
end
end
end

```

Figure III.13: Basic Equation Generation Algorithm

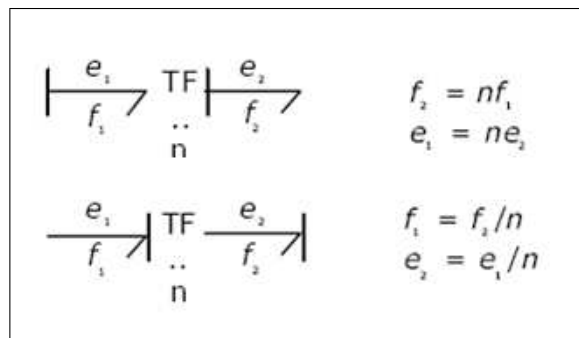


Figure III.14: Causal assignments and corresponding equations for a Transformer

continue the equation generation. For a transformer, we have the following two cases as shown in Fig. III.14.

From the given equations we see that, if the variable that we are considering for equation generation *var* is e1, then we need to call the basic equation generation algorithm for e2 with the coefficient value multiplied by n. If the variable *var* is e2, then we need to call the basic equation generation algorithm for e1 with the coefficient value divided by n. Similarly, if *var* is f1, then we need to call the basic equation generation algorithm for f2 with the coefficient value divided by n and if *var* is f2, then we need to call the basic equation generation algorithm for f1 with the coefficient value multiplied by n.

Now, if the block type corresponds to a Gyration block GY, we get the node number corresponding to the bond_number that was passed as an argument. Also, get the

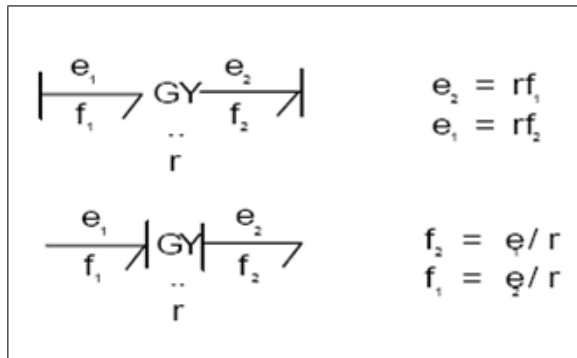


Figure III.15: Causal assignments and corresponding equations for a Gyrator

gyrator ratio value r . Then we find out if the bond that is passed is an input to the GY or an output, and also find its corresponding input or output index. We also find the node corresponding to the other bond of the transformer as we need that to continue the equation generation. For a gyrator, we have the following two cases as shown in Fig. III.15.

From the given equations we see that, if the variable that we are considering for equation generation var is e_1 , then we need to call the basic equation generation algorithm for f_2 with the coefficient value multiplied by r . If the variable var is e_2 , then we need to call the basic equation generation algorithm for f_1 with the coefficient value multiplied by r . Similarly, if var is f_1 , then we need to call the basic equation generation algorithm for e_2 with the coefficient value divided by r and if var is f_2 , then we need to call the basic equation generation algorithm for e_1 with the coefficient value divided by r .

III.2.6 Extensions to SCAP

In order to assign causalities to a BG model containing n -port elements, it is necessary to extend SCAP [8] to assign appropriate causalities to the bonds connected to the n -port elements. The n -port capacitance nC and the n -port inductance nI have a preferred integral causality, thus nC determining the effort on all n incident bonds

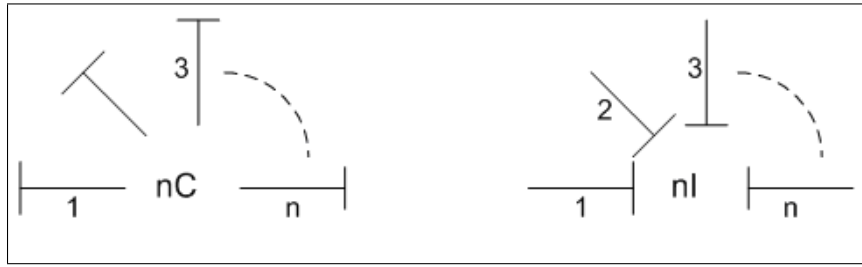


Figure III.16: Causality Assignment for nC and nI

and nI determining the flow for all n incident bonds. Fig. III.16 shows nC and nI in preferred integral causalities.

The n -port resistance nR exhibits indifferent causality just like its single port counterpart. The next chapter explains the handling of n -port elements in greater detail with the help of a sample system.

III.3 Building the Kalman Filter

The Kalman filter provides a mechanism for estimating the state of a discrete-time controlled process that is governed by a set of linear stochastic difference equations.

After identifying the state and the output variables of the system and after generating the corresponding equations for them using the state and the output equation generation algorithms, we obtain the matrices A, B, C and D such that they satisfy the following equations:

$$\dot{X}(t) = FX(t) + GU(t)$$

$$Y(t) = CX(t) + DU(t)$$

The second equation is in the form expected by the Kalman filter. However, we

need to transform the first equation from continuous to discrete form to get it in the following form:

$$X(t) = AX(t - 1) + BU(t - 1)$$

To convert from the continuous to discrete form, the method previously used [6] was the Zero Order Hold Discretization method. However, this is a crude approximate discretization and may not suffice to represent the dynamics of complex systems in a sufficiently accurate manner. First Order Hold, and Impulse trains are some of the other methods that can be used for the purpose of discretization. These methods are implemented by the `c2d` function in MATLAB [13] and will be used in the future to get more accurate results.

The values of the measurement noise covariance R , the process noise covariance Q , and the initial values for the set of state variables X , the initial values for the set of input variables U and the initial values for the estimation error covariance P are user-defined. Similarly, the period for which the simulation is run and the sampling frequency are all user-defined.

Fig. III.17 [4] shows the complete operation of the Kalman filter, including all the equations. Based on the values of the matrices A , B , C , D (after converting F , G to discrete time A , B) and the user inputs, the time update or the predictor equations [4] are executed for projecting forward (in time) the current state and error covariance estimates for the next time step.

These are followed by the execution of the measurement update or the corrector equations [4] that are responsible for the feedback i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate. This process is continued for period equal to the one entered by the user at the given sampling frequency.

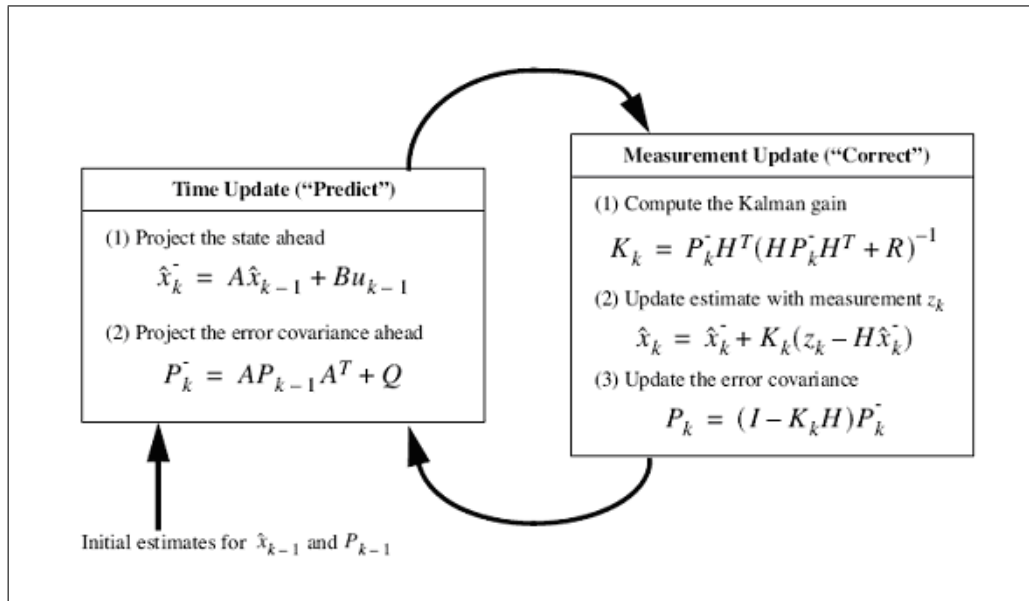


Figure III.17: Kalman Filter Equations for Process and Measurement Model

For the two tank system example that we have considered, the values for matrices A, B, C and D generated by our implementation are as follows:

The values of matrices A, B, C and D are given as:

A matrix:

$$[0.999759 \quad 8.41728e-005 \quad ; \quad 8.41728e-005 \quad 0.999873]$$

B matrix:

$$[0.61867 \quad 0]$$

C matrix:

$$[0 \quad 6.89655e-005; \quad 0.000253165 \quad 0]$$

D matrix:

$$[0 \quad 0]$$

The Kalman filter is built based on the above matrices and it takes the outputs from the Simulink model to produce a set of estimated output values. The set of estimated output values and the system generated output values for $f1$ and $f2$ are plotted against time in Fig. III.18

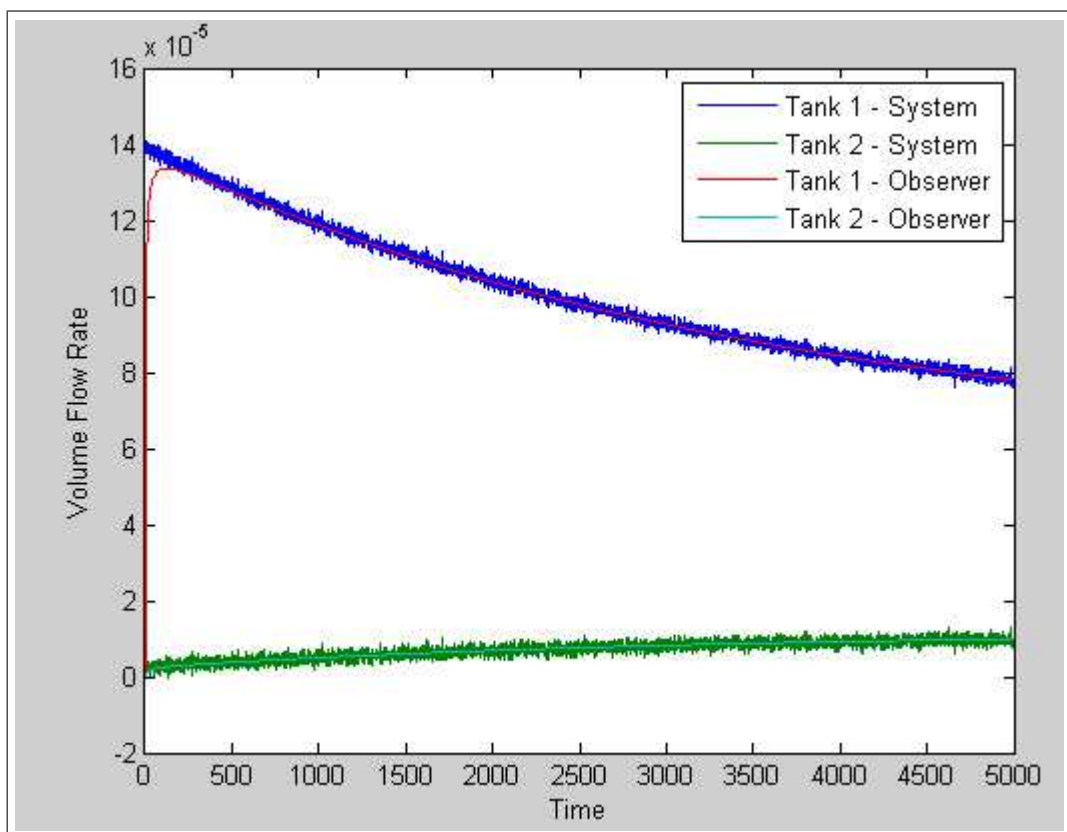


Figure III.18: Observer and System Output for a Two Tank System

By looking at the plot, it can be observed that our implementation of the observer closely tracks the system output. It does so in the presence of noise as well.

CHAPTER IV

EXPERIMENTAL RESULTS

In order to test our system for generating the state space and the output equations and the building of the Kalman filter, we consider two different systems.

IV.1 Hydraulic Actuator System

A simplified model of a hydraulic actuator system is shown in Fig. IV.1 [11].

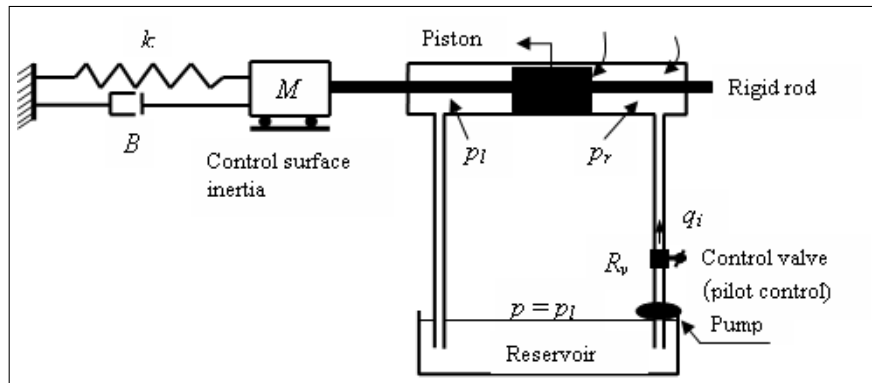


Figure IV.1: A simplified model for the Hydraulic Actuator

The actuator is composed of a single chamber, piston of cross sectional area A and volume of the chamber V_0 . The pressure difference between the right and the left sides of the chamber produces the force for piston displacement. The piston is connected to the load which is modeled as a simple mass spring damper system.

A centrifugal pump system is defined in terms of its input voltage and current, internal variables such as torque and angular velocity and output variables, such as fluid pressure and fluid flow rate. The energy transmitted to the pump is expended in turning the veins of the pump rotor. In the process, the fluid is pushed out with a certain pressure and flow rate. This flow of the liquid helps in moving the piston cylinder of the actuator system back and forth which in turn is connected to the load.

IN SYMBOLIC FORM

$$df5/dt = (-1*B_friction)*(1/Mass) f5 - ((1/Mass)*TF2)e6 - (1/Mass)e7 + ((1/Mass) e8$$

$$de6/dt = ((1/C2)*TF2) f5 - ((1/C2)/R_pipe) e6 + ((1/C2)/R_pipe) e10$$

$$de7/dt = (1/C_spring) f5$$

$$de8/dt = (-1*(1/C1)*TF2) f5 + (-1*((1/C1)/R_valve)) e8 + (((1/C1)/R_valve)*TF1) e9$$

In the above equations, $f5$ is the flow associated with the inertia $Mass$, $e6$ is the effort associated with the capacitance $C2$, $e7$ is the effort associated with the capacitance C_spring , and $e8$ is the effort associated with the capacitance $C1$. Effort $e9$ associated with the source of effort Se and the effort $e10$ associated with the source of effort Se_cap are the input variables of the system.

The following output equations are generated for the system:

$$f5 = f5$$

$$e6 = e6$$

$$e8 = e8$$

$$f13 = (TF2) f5,$$

where $f5$ is the Velocity sensor, $e6$ is the Cap_2 sensor, $e8$ is the Cap_1 sensor and $f13$ senses the pressure difference.

The Simulink model for this system and one of the outputs (pressure output) is shown in Fig. IV.3 and IV.4, respectively.

IV.1.2 Observer runs with measurement noise and modeling errors in the system model

For the purpose of analyzing the behavior of the observer under the presence of noise in the measurement and/or modeling errors, we consider a part of the actuator system described in the previous subsection. The current implementation of our

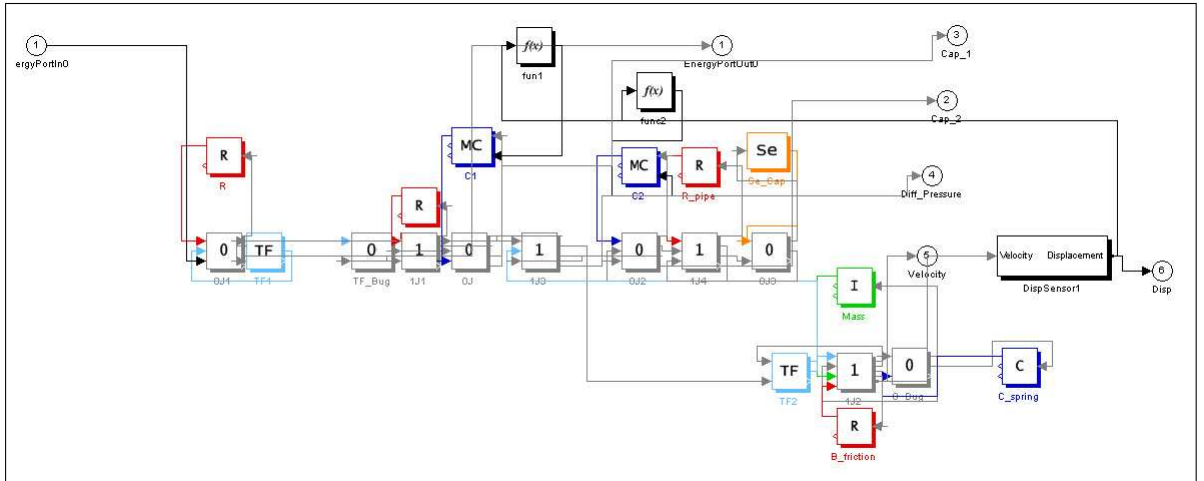


Figure IV.3: Simulink Model of the Hydraulic Actuator System

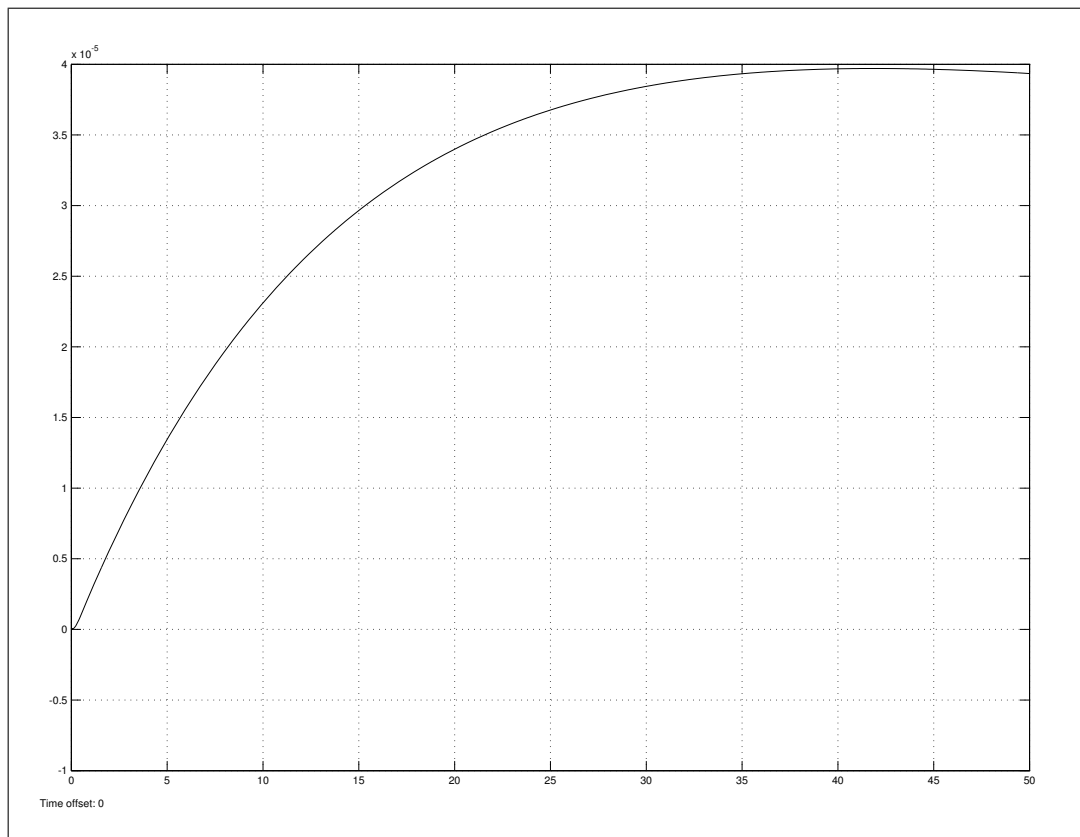


Figure IV.4: Pressure Output of the Hydraulic Actuator System

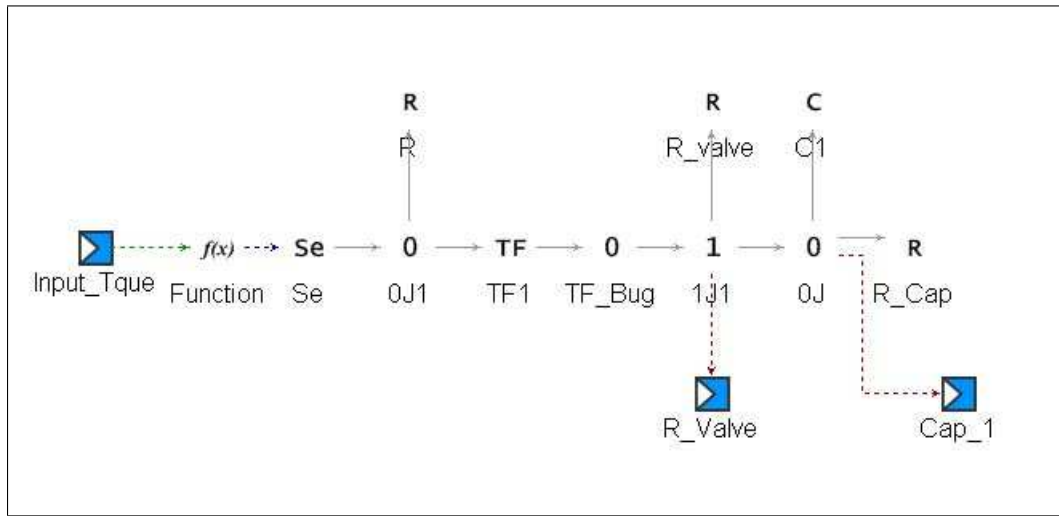


Figure IV.5: A GME Bond Graph Model of a part of the Hydraulic Actuator System

observer only supports linear systems; hence, we restrict ourselves to a part of the hydraulic actuator.

Fig. IV.5 shows the bond graph model of the part of the actuator system under consideration. The different components in the bond graph model are explained as below:

R is the resistance of the pipe connected from the pump to the cylinder of the actuator. Se represents the effort output of the centrifugal pump that drives the actuator. The transformer TF1 models the conversion of mechanical energy to fluid energy. The resistive element R_valve is used to control the flow of the liquid through the pipes. The capacitance C1 models the variable volume of the right hand side of the cylinder.

From the bond graph model, we observe that the system contains one state variable, i.e. the effort associated with the capacitance C1, one input variable i.e. the effort imposed by Se, and two measured variables - flow through R_valve and effort across the capacitance C1.

Our system generates the following causality assignment for the given bond graph:

```
JunctionName ---> DeterminingBond
Component/1J1 ---> Component/R_valve
Component/0J ---> Component/C1
Component/TF_Bug ---> Component/TF1
Component/0J1 ---> Component/Se
```

The bond numbers are assigned as follows: Bond 1: R_valve - 1J1

Bond 2: R - 0J1

Bond 3: R.Cap - 0J

Bond 4: C1 - 0J

Bond 5: Se - 0J1

Bond 6: TF1 - 0J1

Bond 7: TF1 - TF_Bug

Bond 8: 1J1 - TF_Bug

Bond 9: 1J1 - 0J

The following equations are generated by our system.

State space equations:IN NUMERIC FORM

$$de4/dt = -0.1001e4+0.0001e5$$

IN SYMBOLIC FORM

$$de4/dt = (((-1*((1/C1)/R_valve))) +(-1*(1/C1))/ R.Cap) e4 +(((1/C1)/ R_valve)/ TF1) e5$$

Output equations:

IN NUMERIC FORM

$$f1 = -1e-005 e4 + 1e-005 e5$$

$$e4 = 1e4$$

IN SYMBOLIC FORM

$$f1 = (-1*((1)/ R_valve)) e4 + (((1)/R_valve)/ TF1) e5$$

$$e4 = (1)e4$$

The A, B, C, D matrices generated for the building of Kalman filter from the above equations are shown below.

A matrix

$$[0.998999]$$

B matrix

$$[1e-006]$$

C matrix

$$[-1e-005 \ 1]$$

D matrix

$$[1e-005 \ 0]$$

The other parameters fed as an input to the Kalman filter are
Initial value of for X = [0.0]

Error covariance matrix P = Identity matrix of size 1 = [1]

Multiplier for the process noise covariance matrix Q = 1e-7

Multiplier for the measurement noise covariance matrix R = 1e-7

Sampling time T = 0.01 seconds

Period for which the model is run = 200 seconds

The output values for Cap_1 and R_valve generated by simulating the Simulink model are fed into the filter.

We analyze the performance of our observer by taking multiple runs of the model under different noise levels, and also, by taking multiple runs with small errors in

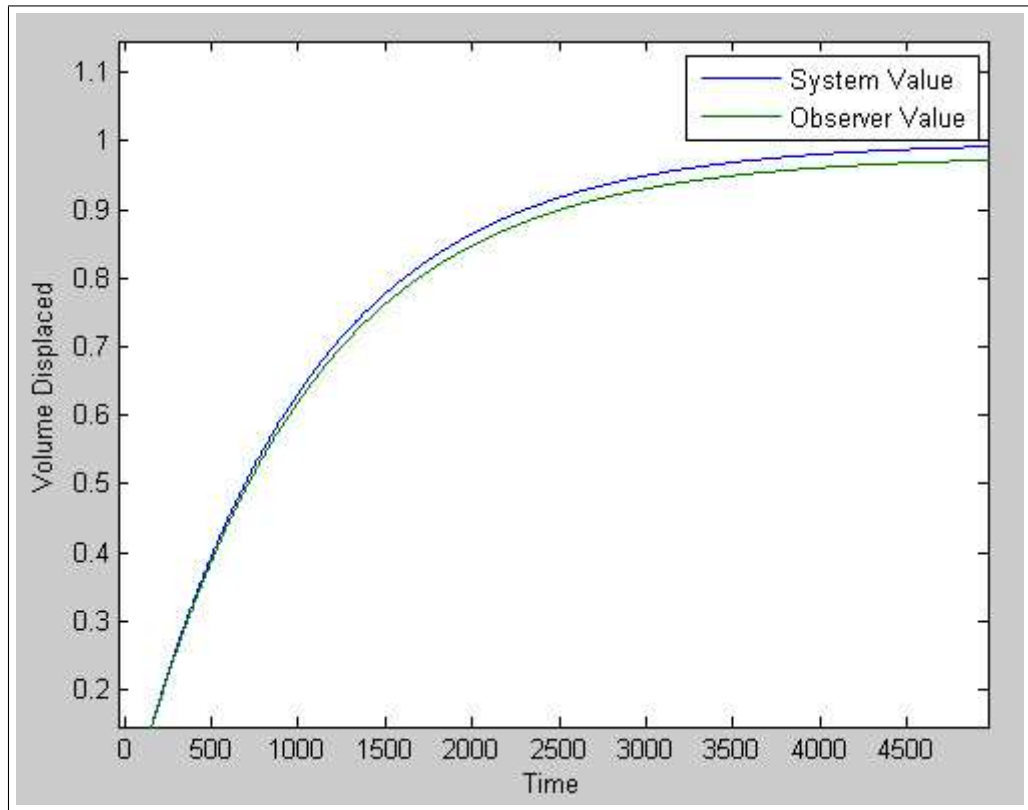


Figure IV.6: Measured and Observed values for Actuator for a measurement noise of 2 percent

model parameters along with some noise. Also, for each run, we calculate the mean squared value of the error in order to verify the working of the observer.

Figures IV.6, IV.7, IV.8 plot the measured and the observed outputs for noise values of 2, 5 and 8 percent in the measurement respectively. From Table IV.2, we can see that the mean square error increases as the measurement noise increases.

Next, we run the observer for a measurement noise of 2 percent and by introducing errors of 2, 5 and 8 percent in the parameter values of the input torque Se and the valve resistance R_{valve} simultaneously. When the torque is increased and the resistance decreased, the volume displaced increases whereas, when the torque is decreased and the valve resistance increased, we observe that the volume displaced decreases. This

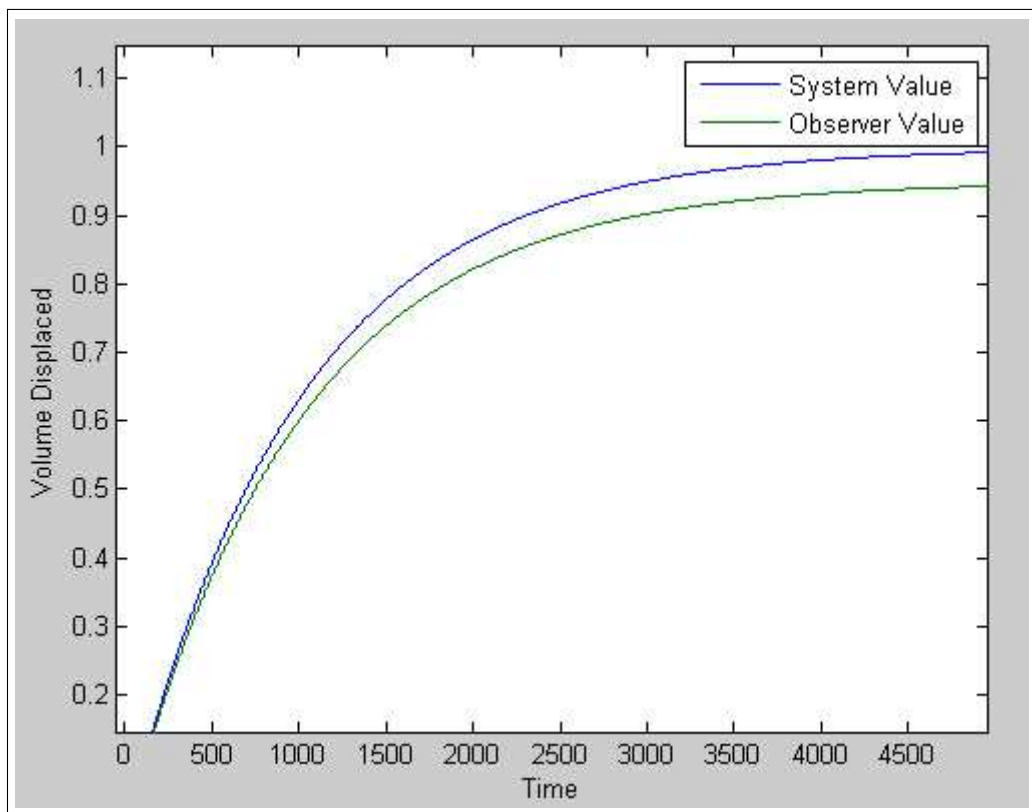


Figure IV.7: Measured and Observed values for Actuator for a measurement noise of 5 percent

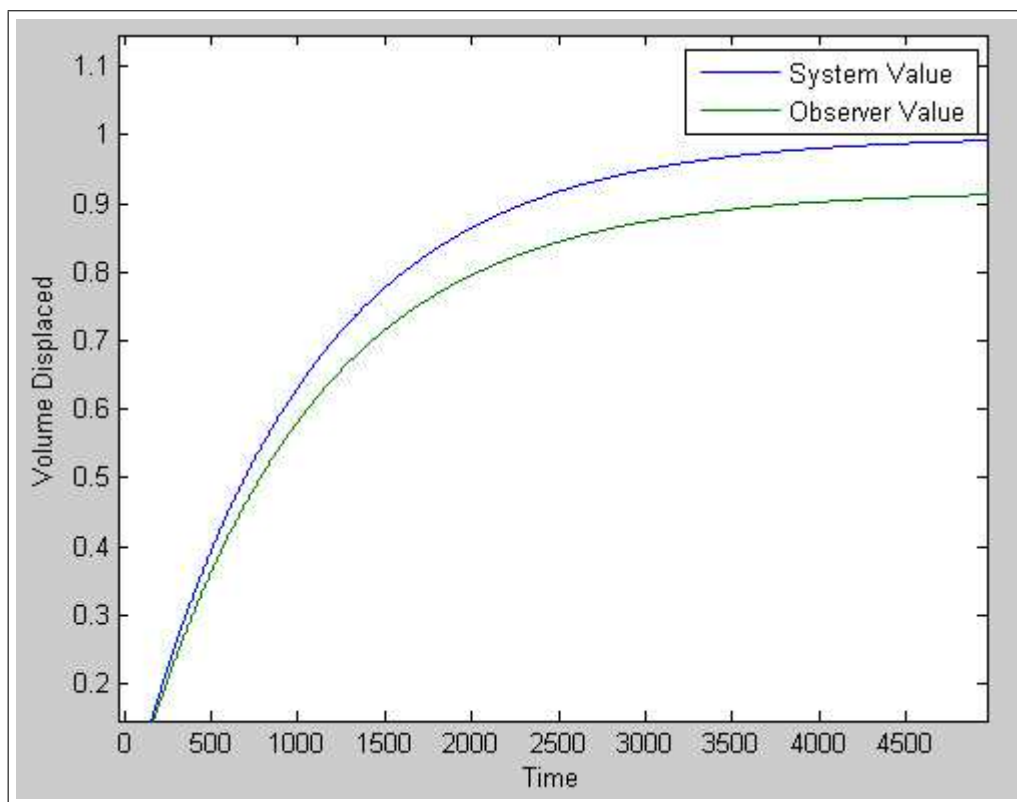


Figure IV.8: Measured and Observed values for Actuator for a measurement noise of 8 percent

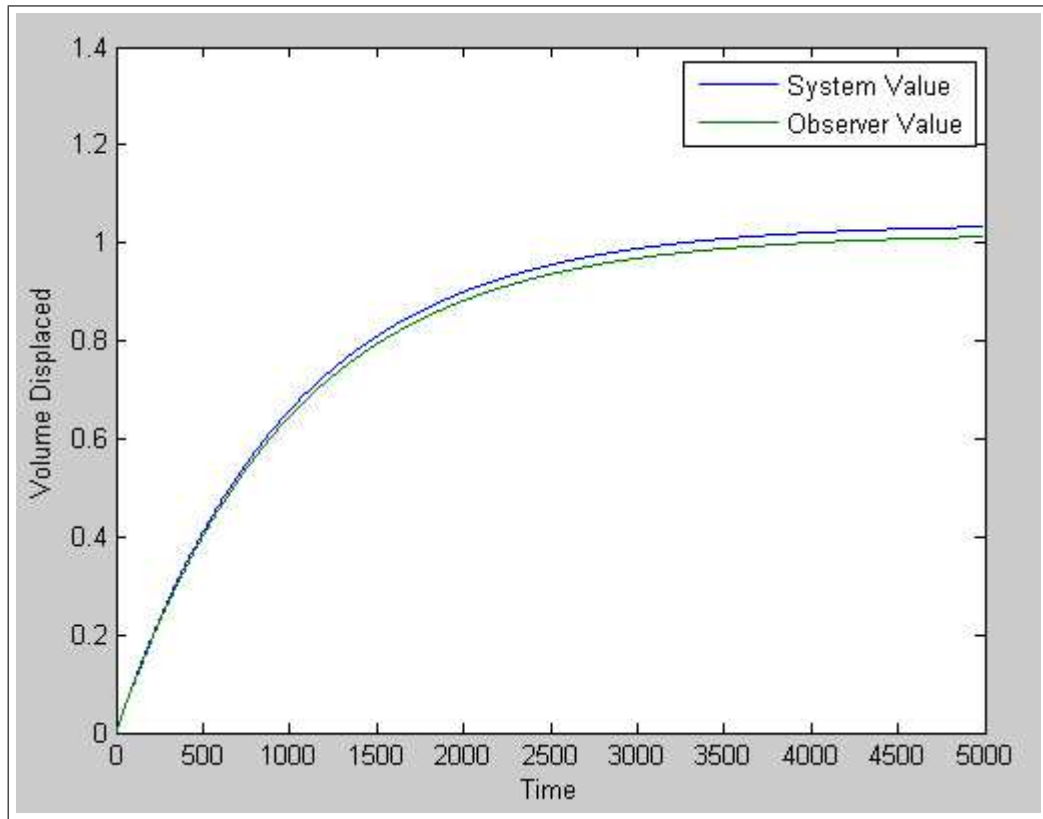


Figure IV.9: Measured and Observed values for Actuator for a measurement noise of 2 percent, S_e increased by 2 percent and R_{valve} decreased by 2 percent

concur with the expected behavior of the system. Figures IV.9, IV.10 and IV.11 show the plots for the same.

Table IV.2: Mean Square Error Values

| Noise % | Error % in S_e | Error % in R_{valve} | Mean squared error | Error % |
|---------|------------------|------------------------|--------------------|----------|
| 2 | 0 | 0 | 1.09712e-7 | 1.622e-7 |
| 5 | 0 | 0 | 1.27299e-7 | 1.953e-7 |
| 8 | 0 | 0 | 1.48611e-7 | 2.017e-7 |
| 2 | +2 | -2 | 1.18849e-7 | 1.455e-7 |
| 2 | -5 | +5 | 9.01745e-8 | 1.27e-7 |
| 2 | +8 | -8 | 1.51785e-7 | 1.647e-7 |

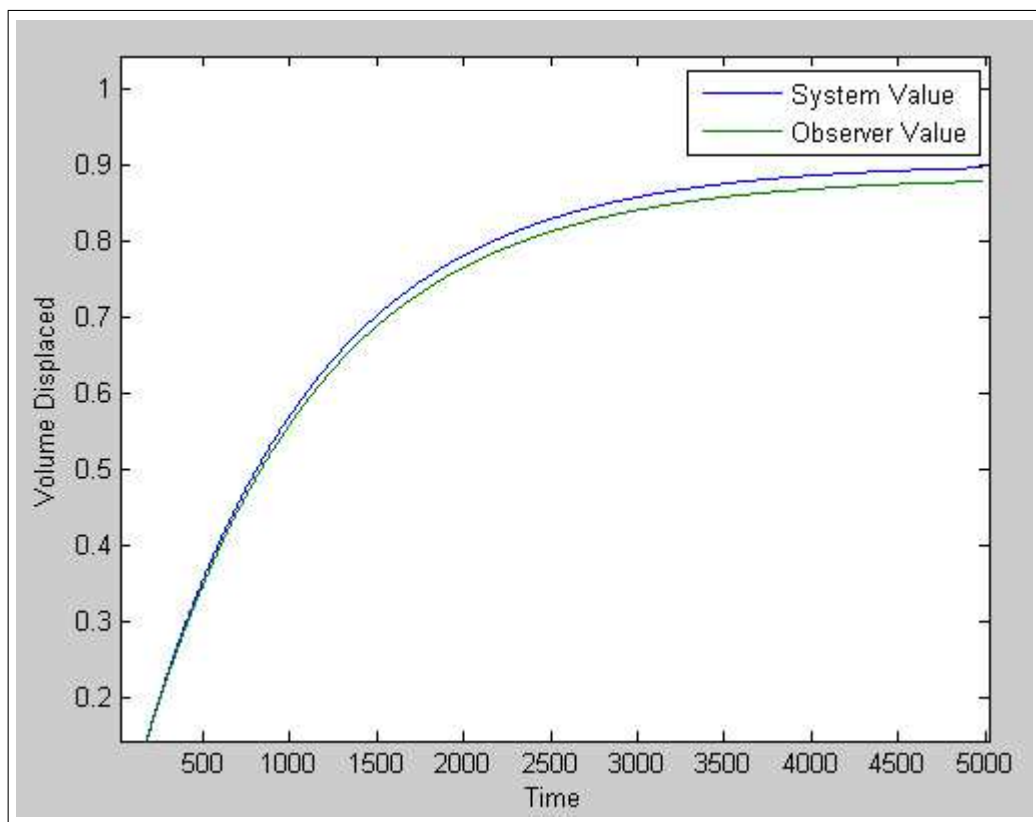


Figure IV.10: Measured and Observed values for Actuator for a measurement noise of 2 percent, S_e decreased by 5 percent and R_{valve} increased by 5 percent

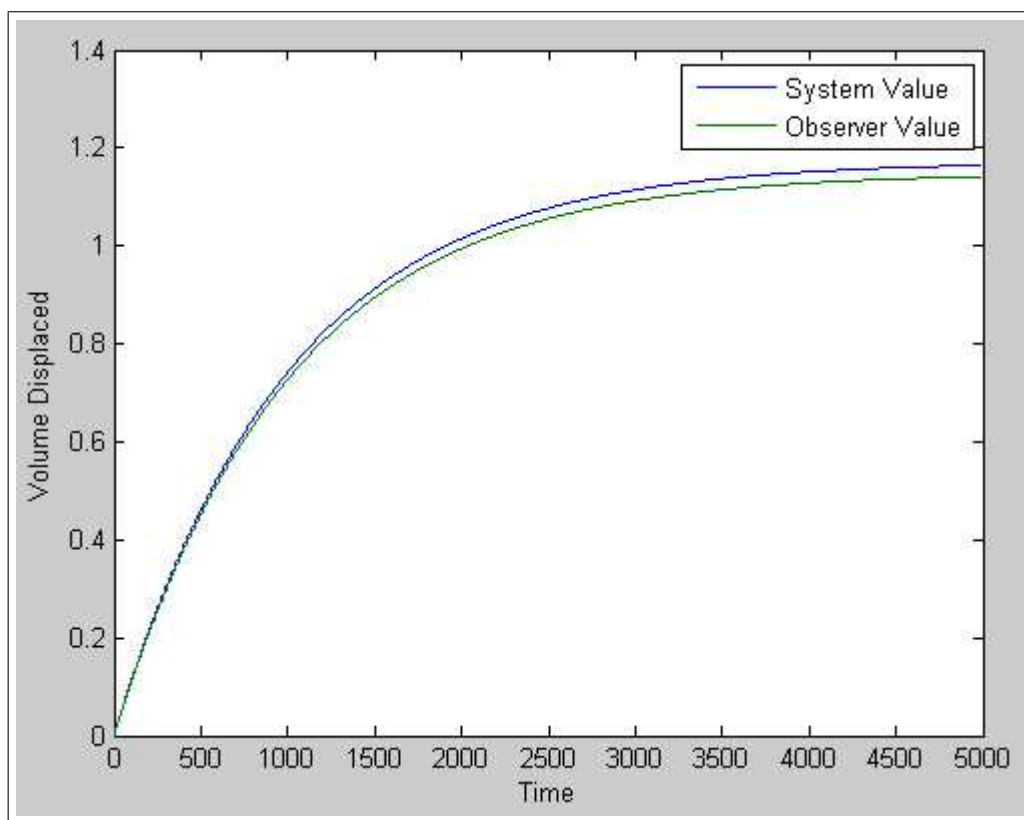


Figure IV.11: Measured and Observed values for Actuator for a measurement noise of 8 percent, S_e increased by 2 percent and R_{valve} decreased by 8 percent

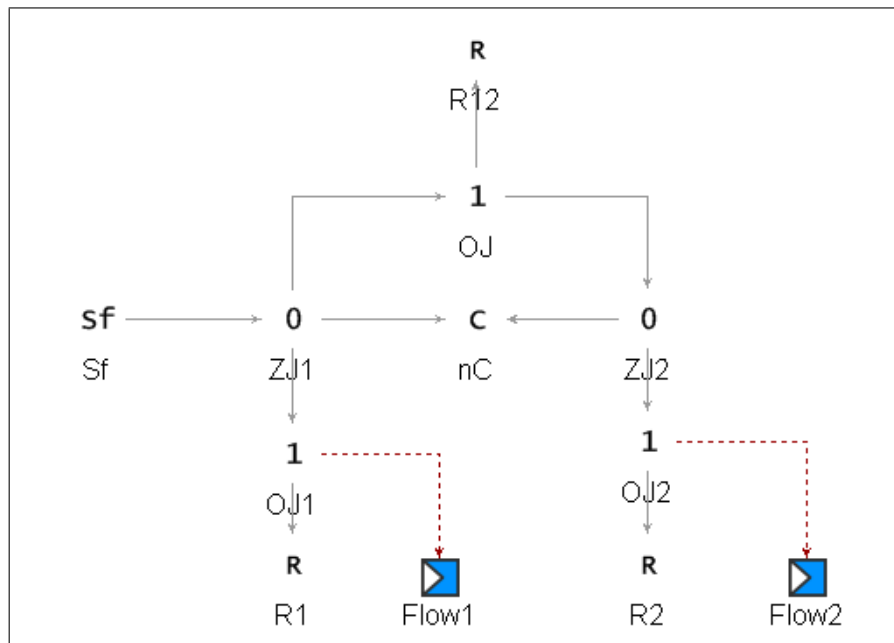


Figure IV.12: Bond Graph model of the system with an n-port element

IV.2 N-port system

This section discusses the causality assignment, equation generation and the generation of observer plots for a system containing an n-port element. We consider an equivalent of a two tank system that uses a 2-port capacitance, instead of 2 single port capacitances. The values of the parameters have been chosen to be equivalent to those used in the two tank system example in the last chapter. The purpose of this example is to explain and verify the working of our system for an n-port element, and not to suggest a possible application of n-port elements.

Fig. IV.12 shows the bond graph model of the system, where S_f represents the flow source, R_s represent the pipe resistances and nC is an n -port capacitance with $n=2$.

The matrix values entered for nC are

$$[1/C1 \ 0; \ 0 \ 1/C2]$$

where $C1$ and $C2$ correspond to the values of the two capacitances used in the equivalent system with 2 one-port capacities. By making use of the extended version of SCAP that supports n-port elements, the causalities get assigned in such a way that nC determines the effort at both the zero junctions connected to it. The bond assignment for the system is as shown below:

Bond 1: $R1 - 0J1$
 Bond 2: $R2 - 0J2$
 Bond 3: $R12 - 0J$
 Bond 4: $nC - ZJ1$
 Bond 5: $nC - ZJ2$
 Bond 6: $Sf - ZJ1$
 Bond 7: $0J - ZJ1$
 Bond 8: $0J - ZJ2$
 Bond 9: $0J1 - ZJ1$
 Bond 10: $0J2 - ZJ2$

The flow associated with Sf is the input variable $f6$. The efforts $e4$ and $e5$ associated with nC are the two state variables in the system and the volume flow rates measured at the two one junctions, namely $f1$ and $f2$ are the output variables of the system.

The state equations generated for the system are:

IN NUMERIC FORM

$$de4/dt = -0.0240798 e4 + 0.00841728 e5 + 61.867 f6$$

$$de5/dt = 0.00841728 e4 - 0.012684 e5$$

IN SYMBOLIC FORM

$$de4/dt = ((((-1*(1/nC(1,1)))/R12))+(-1*(1/nC(1,1)))/R1)e4 + (-1*((-1*(1/nC(1,1))$$

$$+(1/nC(1,1))f6$$

$$de5/dt = ((1/nC(2,2))/R12)e4 + (((-1*((1/nC(2,2))/R12)))+(-1*(1/nC(2,2)))/R2)e5$$

The output equations generated for the system are:

IN NUMERIC FORM

$$f1 = 0.000253165 e4$$

$$f2 = 6.89655e-005 e5$$

IN SYMBOLIC FORM

$$f1 = ((1)/R1)e4$$

$$f2 = ((1)/R2)e5$$

The values of the Kalman filter matrices A, B, C, D are: The values of matrices A, B, C and D are given as:

A matrix:

$$[0.999759 \ 8.41728e-005 \ ; \ 8.41728e-005 \ 0.999873]$$

B matrix:

$$[0.61867 \ 0]$$

C matrix:

$$[0 \ 6.89655e-005; \ 0.000253165 \ 0]$$

D matrix:

$$[0 \ 0]$$

We see that these values correspond to those generated for the equivalent two tank system model considered in the previous chapter, and are thus able to verify the working of our implementation for an n-port system.

The plot of the measured and the observed volume flow rate outputs against time is as shown in Fig. IV.13.

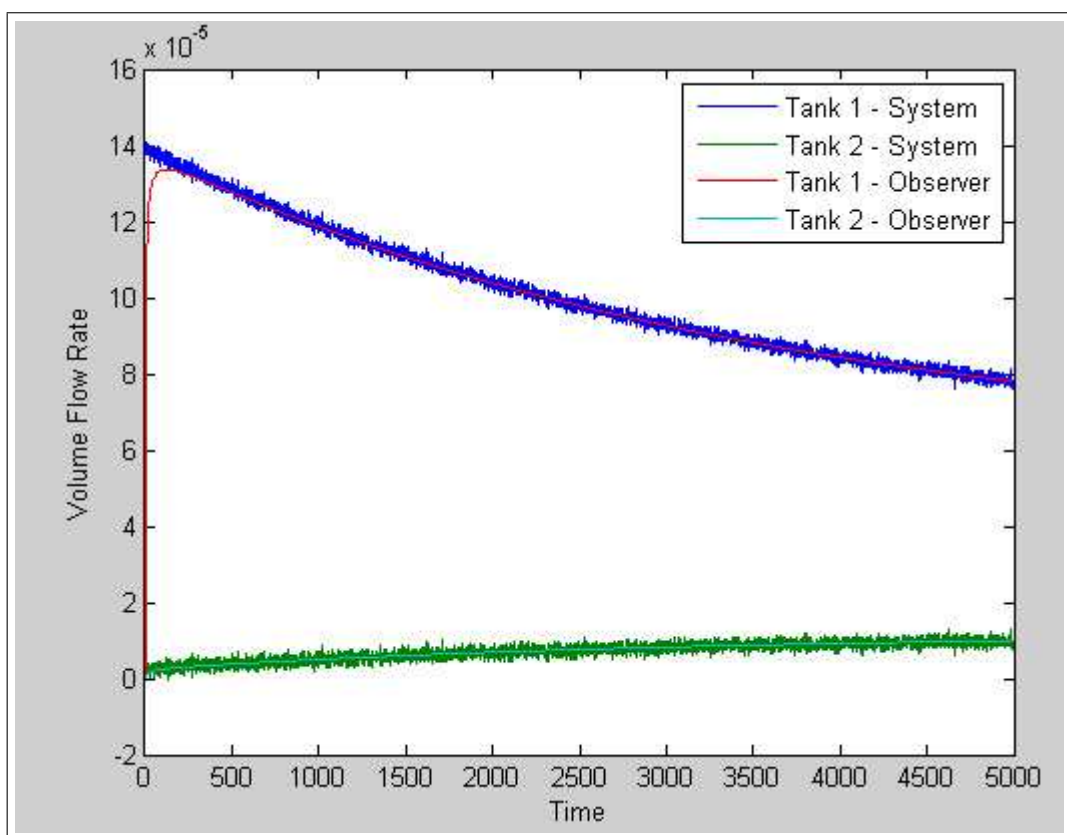


Figure IV.13: Observer and System Output for a Two Tank System implemented using nC

From the above two examples the correctness of our implementation can be verified.

CHAPTER V

CONCLUSIONS

After testing the implementation of our system for different component-based bond graph models of linear systems, we have verified the working of the our equation generation algorithm and our Kalman filter implementation. Also, by running the observer for multiple sets of data containing measurement noise and modeling errors, we observed, as expected, that the mean squared error increased with the increase in the measurement noise or with the increase in the deviation of model parameters from their nominal values, however, the observer still managed to track the system dynamics within acceptable error bounds.

V.1 Future Work

The current system works for models of linear systems. However, most complex systems contain a number of non linearities, and an important extension to the current algorithm will be to support such non-linearities by extending the equation generation algorithm to handle modulated elements and function blocks and changing the implementation of the observer from the Kalman filter to the Extended Kalman Filter. This will also allow us to build bond graph models containing n-port elements where the transfer matrix coefficients will not have to be constant values.

Another extension to our present implementation would be to support hybrid systems which are modeled as Hybrid Bond Graphs. The causality assignment and the equation generation algorithm will have to take the junction state into account, and make the effective changes to the system model.

BIBLIOGRAPHY

- [1] T. Wong, P. Bigras, K. Khayati, “Causality assignment using multi-objective evolutionary algorithms”, *Systems, Man and Cybernetics, 2002 IEEE International Conference on* Volume 4, 6-9 Oct. 2002, On page(s): 6 pp.
- [2] I. Roychoudhury, M. Daigle, G. Biswas, X. Koutsoukos, P. J. Mosterman, “A Method for Efficient Simulation of Hybrid Bond Graphs”, *International Conference on Bond Graph Modeling and Simulation (ICBGM 2007)* January, 2007, pp. 177-184
- [3] - J. Broenink, “Bond-Graph Modeling in Modelica”, *European Simulation Symposium 1997* Oct. 19-22
- [4] G. Welch, G. Bishop, “An Introduction to the Kalman Filter”, *Department of Computer Science, University of North Carolina at Chapel Hill*
- [5] G. Karsai, G. Biswas, S. Abdelwahed, N. Mahadevan, E. Manders, “Model-based software tools for integrated vehicle health management”, *Space Mission Challenges for Information Technology, 2006. SMC-IT 2006. Second IEEE International Conference on* 17-20 July 2006, On page(s): 8 pp.
- [6] FACT Documentation
- [7] M. Daigle, I. Roychoudhury, G. Biswas, X. Koutsoukos, “Efficient Simulation of Component-Based Hybrid Models Represented as Hybrid Bond Graphs”, *Technical Report ISIS-06-712, Institute for Software Integrated Systems, Vanderbilt University* Dec 2006
- [8] D. Karnopp, D. Margolis, R. Rosenberg, “Systems Dynamics: Modeling and Simulation of Mechatronic Systems”, *Third edn. John Wiley Sons, Inc., New York* 2000
- [9] J. Broenink, “Introduction to Physical Systems Modeling with Bond Graphs”, *University of Twente, Dept EE, Control Laboratory*
- [10] P. Mosterman, G. Biswas, “Diagnosis of continuous valued systems in transient operating regions”, *Systems, Man and Cybernetics, Part A, IEEE Transactions on*, Volume 29, Issue 6, Nov 1999, pp. 554 - 565
- [11] A. Moustafa, G. Biswas, S. Mahadevan, “Fault Diagnosis of Hydraulic Actuators using Bond Graphs”, *Progress Report for August 2007*
- [12] E. Manders, P. Mosterman, G. Biswas, L. Barford, “TRANSCEND: A System for Robust Monitoring and Diagnosis of Complex Engineering Systems”, Jan 14 1999

[13] MATLAB Documentation