RECONCEPTUALIZING SUPPORTING TEACHERS' LEARNING ACROSS THE SETTINGS OF PROFESSIONAL DEVELOPMENT AND THE CLASSROOM

By

Qing Zhao

Dissertation

Submitted to the Faculty of the

Graduate School of Vanderbilt University

in partial fulfillment of the requirements

for the degree of

DOCTOR OF PHILOSOPHY

In

Teaching and Learning

May, 2011

Nashville, Tennessee

Approved:

Professor Paul Cobb

Professor Rogers Hall

Professor Ilana Horn

Professor Thomas Smith

Professor Martin Simon

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

The overall purpose of this dissertation is to contribute to a conceptualization of the relations between teachers' learning in the setting of professional development and their instructional practices in the classroom. Specifically, this dissertation involves two distinctive yet related goals, which are (1) articulating a theoretical framework for conceptualizing teachers' activities across the two settings and (2) illustrating the proposed conceptualization using data collected during a five-year teacher professional development project. The purpose of the first goal is conceptual, laying out an interpretive framework for understanding such relations by drawing on existing theoretical tools. The second goal involves an empirical analysis grounded in a five-year teacher development project[1]. Although each has a different emphasis, the two goals are interrelated. On the one hand, the conceptual framework constitutes an analytic lens through which teachers' learning in professional development is examined and interpreted. Consequently, it contributes to delineating the critical aspects involved in the empirical analysis. On the other hand, the empirical analysis serves to operationalize the conceptualization by contextualizing it in a longitudinal teacher professional development program.

The data for this dissertation study were collected from the third year of a five-year collaboration with a group of middle-school mathematics teachers who worked in different schools in the Jackson Heights Public School District. This urban

---

[1] Members of the research team included Paul Cobb, Kay McClain, Teruni Lamberg, Jose Luis Cortina, Chrystal Dean, Jana Visnovska, Melissa Gresalfi and myself.

school district served a 60% minority student population and was located in a state with a high-stakes accountability program. The district had received an external grant to support its reform efforts prior to the research team's collaboration with the teachers. In particular, the district had adopted a new mathematics curriculum, but a significant proportion of the middle-school teachers continued to use the traditional textbook series as the primary basis for their instruction. During the five-year collaboration, the research team conducted three one-day work sessions during the first school year and six one-day work sessions during each of the subsequent four school years as well as a three-day work session each summer. Our long-term goal in working with the teachers was to support their development of instructional practices in which student reasoning would be placed at the center of instructional planning and decision making.

In the following chapters, I first propose a theoretical framework for conceptualizing the relations between teachers' learning in the setting of professional development and their activities in the classroom. To do so, I start by unpacking the significance of understanding such relations both from the theoretical and the pragmatic point of views. I then present an overview of the current literature on mathematics teacher professional development. From this brief examination of literature emerge critical issues that build a case for developing a new conceptualization. I then propose a bi-directional conceptualization for accounting for teachers' learning across different settings by drawing on an array of conceptual tools from the literature. Following Chapter II, Chapter III presents an empirical analysis examining teachers' learning as they participated in activities across both settings. The purpose of Chapter III is to document the emergence of the bi-directional conceptualization in our work with the collaborating middle-school mathematics teachers. Chapter IV concludes the dissertation by discussing the design implications of the bi-directional conceptualization.

CHAPTER II

ARTICULATING A THEORETICAL FRAMEWORK FOR CONCEPTUALIZING TEACHERS' ACTIVITIES ACROSS THE TWO SETTINGS

Introduction

The ultimate goal of mathematics education reform as envisioned in the NCTM standards is to support all students' learning of important mathematical concepts and procedures with understanding (NCTM, 2000; Stigler & Hiebert, 1999). Implicit in this reform agenda is the requisite that mathematics teachers engage in substantial professional learning which will enable them to develop qualitatively different ways of teaching that place students' mathematical reasoning at the center of instructional decision making (Carpenter, Blanton, Cobb, Franke, Kaput & McClain, 2004; Franke & Kazemi, 2001; Lampert & Ball, 1998; Little, 1993; Wood, 1999). This realization has placed extraordinary demands on teacher professional development, which has long been recognized as the primary means to support teachers' professional learning and consequently, improvements in their classroom practice (Borko, 2004; Loucks-Horsley, Hewson, Love & Stiles, 1998, Tirosh & Graeber, 2003).

Inherent in the goal of teacher professional development is the expectation that the type of learning that is supported in the setting of professional development should lead to significant reorganization of teachers' activities in another setting, the classroom. This expectation has become so widely accepted that it often requires little justification in teacher education literature. Meeting it, however, has proved to be far from trivial. The field has witnessed too many instances where innovative professional development efforts resulted in little or no impact on teachers' classroom instruction (Cobb, 1996; Dawson, 1999; Lanier & Little, 1986; Tirosh & Graeber, 2003). Regardless of researchers' continuous efforts to design and support teachers' professional development, changes in classroom mathematics instruction do not always occur as intended. Thus, an immediate and pragmatic challenge posed to teacher educators necessarily involves how to design professional development activities so that teachers can relate what they learn to their classroom practices and, as a result, reorganize their current ways of teaching. To accomplish this, Mousley and Sullivan (1997) noted a general orientation that they considered critical in guiding the design of teacher professional development activities.

> Teacher education programs need to find ways to perturb [teachers'] existing conceptions of…teaching and learning, as well as the wider contexts of school and society, to create a milieu in which change is a desired state. This needs to be done, however, in ways that retain [teachers'] control over the content, direction and pace of change. (p.32)

To tackle this immediate and pragmatic challenge, numerous studies have been conducted to answer questions such as how professional development activities should be organized so that they are relevant to teachers' classroom instruction, what kind of classroom artifacts should be used in professional development, how professional development tasks can target at teachers' instructional practices specifically and effectively.

Although such studies have generated valuable resources for the field to draw on, there yet remains a conceptual challenge: How can we, as teacher educators, justify and account for the effectiveness of various approaches to support teachers' learning across the setting of professional development and their classroom? In their thorough examination of teacher education research over the last 20 years, Cochran-Smith and Lytle (1999) looked beyond the specificity of various professional development practices and call attention to the underlying assumptions and suppositions held by researchers in various professional development efforts. As they pointed out,

> there are radically different conceptions of teacher learning, including varying images of knowledge; of professional practice; of the necessary and/or potential relationships that exist between the two…Different conceptions of teacher learning—although not always made explicit—lead to very different ideas about how to improve teacher education and professional development…(p.249)

Following Cochran-Smith and Lytle's (1999) distinction between concrete approaches and underlying conceptions, any teacher professional development design can be considered as comprised of two levels (see Figure 1). The level of action, which is typically more observable, usually consists of concrete ideas about how teacher professional development should be carried out. In other words, this level mainly involves an action plan that delineates the overall organization of the professional development activities, the selection of specific artifacts, and/or the choice of topics for discussion with participating teachers. In contrast, the level of justification appears to be much less visible and often remains "unexamined and tacit" (Cochran-Smith & Lytle, 1999). It is typically composed of suppositions and assumptions held by researchers—for instance, how teachers learn, how teachers' learning in professional development relates to their classroom practices, what supports or hinders such learning, and what are the critical aspects in motivating changes in teachers' practices. This underlying level involves justifications that undergird the action plan and, ideally, gives rationality and consistency to the seemingly discrete activities on the level of action.

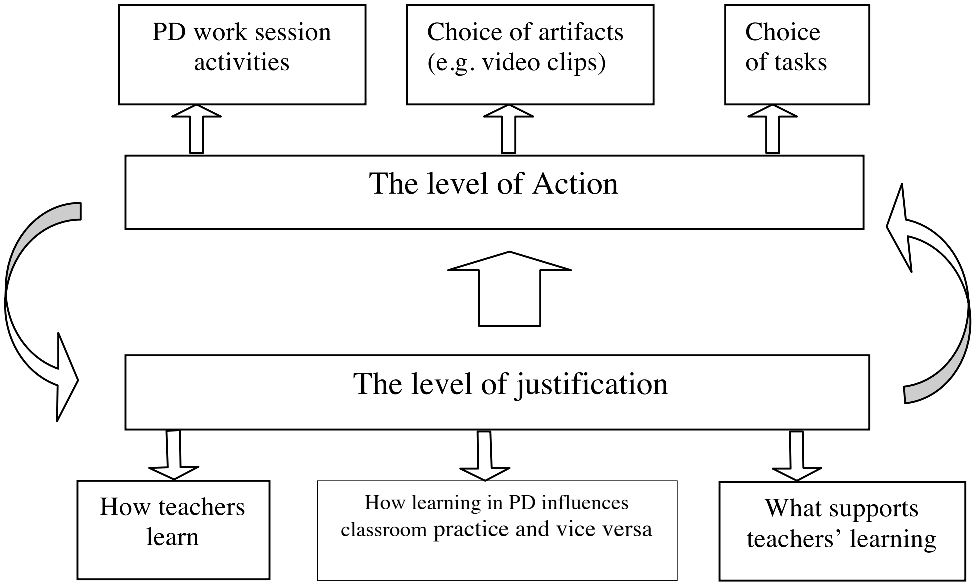

Figure 1. The Two Levels in the Design of Professional Development

The distinction between the level of action and the level of justification in teacher professional development design implies a two-part process when it comes to accounting for the effectiveness of teacher professional development programs. When such account focuses primarily on the level of action, the goal is typically to search for alternative strategies to bring about connections between the designed professional development activity and teachers' classroom practice whereas researchers' underlying conceptions remains to a large extent tacit. This is what Argyris and Schön (1978) described as *single-loop learning*, in which the emphasis is on "techniques and making techniques more efficient" (Smith, 2001, p.10). In contrast, an account of a different nature occurs when we make it an explicit goal to conceptualize the use as well as consequences of specific designs and articulate the underlying assumptions that we hold as researchers when we strive to support teachers' learning across the setting of the professional development and the classroom. The resulting account is what Argyris and Schön called double-loop learning (Argyris and Schön, 1978), in which researchers carefully scrutinize their underlying conceptions and examine how these conceptions shape the way in which various strategies and consequences are framed (Smith, 2001).

Cochran-Smith and Lytle (1999) focused explicitly on developing an account of the second type in their review of recent teacher education literature. They contended that an articulation of researchers' conceptions that underlie professional development initiatives can be both theoretically and pragmatically significant. Theoretically, this articulation has the potential of contributing to the generation of conceptual frameworks that are currently lacking in mathematics teacher education (Cohen, 1998). Pragmatically, by explicitly grounding the action plan in researchers' underlying conceptions and by making the latter subject to public scrutiny, we expand our purview of what is analyzable and workable in the process of supporting teacher learning. Significantly, Cochran-Smith and Lytle's work indicates a second, equally critical layer in the challenge that is inherent in supporting changes in teachers' instructional practice through professional development—the need to articulate, scrutinize and, when necessary, reconceptualize the relations between teachers' learning in professional development and their instructional practices in the classroom.

Current Conceptualizations of the Relations

To this point, I have discussed the significance of conceptualizing the relations between teachers' learning in the setting of professional development and their instructional practices in the classroom. The purpose of this section is to examine how such relations are currently conceptualized in teacher education. In doing so, I argue that, as a critical issue often escapes serious consideration, a reconceptualization is needed.

My review of teacher education literature is guided by the distinction between the level of action and the level of justification. A professional development plan that encompasses the envisioned learning goals and the instructional starting point necessarily involves conceptions—whether implicitly assumed or explicitly articulated—about the specific ways in which teacher learning in professional development is going to influence their classroom practices (and vise versa). In addition to examining the concrete professional development practices that have been developed and tested by various researchers, it is also important to explicate the underlying conceptions that researchers hold for supporting teachers' learning even when the underlying conceptions are implicit.

In the analysis presented below, three differing conceptualizations can be identified from the literature, each embracing a distinct set of assumptions about how changes in teachers' classroom practices occur as a result of their participation in professional development. In presenting key characteristics of each conceptualization, I attend to researchers' envisioned plan for change, the status of teachers' classroom practices in the design of professional development, and researchers' assumptions about what motivates teachers' to change their classroom practices.

A Uni-directional Conceptualization

In the last 20 years, teacher professional development has undergone significant changes that reflect evolving views of the relations between teachers' learning in PD and their classroom practices (Little, 1993; Sullivan, 2003). Prior to mid '90s, teacher professional development was dominated by a deficit model, in which the emphasis was frequently placed on technical training (Little, 2004; McIntyre & Hagger,1992). The implicit assumption that underlies this type of professional development is that teachers' current classroom practices are inadequate and need to be fixed. Dawson described (1999) this approach as "based on judging what is right and wrong, paying little attention to what mathematics teachers are actually doing in their classrooms, and looking outside for the 'right' way, the newest 'fix' " (p.148).

In the deficit model of development, teacher learning across the setting of professional development and the classroom is frequently conceptualized in terms of uni-directional relations (Clarke & Hollingsworth, 2002; Dawson, 1999; Jaworski, 1999; Little, 1993): Teachers are offered opportunities to develop new insights into instructional practices in professional development in the expectation that they will then apply them in their classrooms in a relatively straightforward manner. Teaching, therefore, is understood as "a process of applying received knowledge to a practical situation" (Cochran-Smith & Lytle, 1999, p.257).

This uni-directional conceptualization is reflected in the "knowledge-for-practice" conception, described by Cochran-Smith and Lytle (1999) as a significant strand of research in the teacher education literature. They pointed out that this conception hinges on the ideas that there exists a formal knowledge base developed and refined by researchers, and that developing knowledge of this type will eventually lead to effective instructional practices in the classroom. Consequently, professional development plans that are premised on a uni-directional conceptualization typically attempt to equip teachers with forms of expertise that researchers believe are important for the development of effective instructional practices. Such designs often focus on (a) additional skills or insights that researchers think are crucial for teachers to develop, (b) new tools or technologies that will support teachers' development of the desired forms of expertise, and (c) the specific manner in which teachers should engage in professional development activities in order to develop the desired expertise (Little, 1993).

A uni-directional conceptualization to a large extent circumscribes the types of interventions that are perceived feasible in teacher professional development. In this case, the design focuses almost exclusively on what can be accomplished in the setting of the professional development, and teachers' classroom is considered the location where the outcome of the professional development design is tested and evaluated. It is worth clarifying that a uni-directional conceptualization does not necessarily imply that researchers devalue knowledge of teachers' classroom practices although this might often be the case with traditional one-shot professional development workshops (Willson & Berne, 1999). Sometimes, the planned changes in professional development may very well be a consequence of careful studies of teachers' classroom practices. However, such studies tend to be evaluative and focus almost exclusively on what needs to be fixed in teachers' current ways of teaching.

In the plethora of professional development efforts that are guided by uni-directional conceptualization, the overall effectiveness is largely dependent on teachers coming to realize the relevance of the professional development and as a result, attempting to apply what they have learned to their classroom instruction. In other words, teachers' motivation to change their mathematics instruction is typically an unarticulated pre-requisite in the uni-directional conceptualization. On the other hand, if teachers' motivation to change is to be enhanced, it is often done through external administrative methods (e.g., teachers being pressured to attend professional development) or the use of a combination of rewards and sanctions (Spillane, 2005).

"Resistance and inertia" (Spillane, 2005, p.390) are therefore considered the primary causes when a professional development fails to engage teacher learners as expected.

Criticisms of this model for professional development have focused mainly on the limited role of teachers' current instructional practices in the design of professional development (Brown & McIntyre, 1993; McIntyre & Hagger, 1992). McIntyre and Hagger (1992) contended that teachers are unlikely to develop a genuine motivation to engage in professional development unless it is focused on problems grounded in their everyday work. This view explicitly challenges the deficit model in which teachers' classroom practices are made visible almost exclusively for evaluative purposes. Similar criticism was made by Castle and Aichele (1994), who posited that uni-directionally conceptualized professional development programs, "although well intentioned, are doomed to failure" (p.3).

> Professional knowledge cannot be transferred. Rather, it is constructed by each individual teacher bringing his or her "live experiences" as a learner and teacher to an education setting and interacting with the environment in a way that relates new knowledge to previously constructed knowledge in an attempt to make the best sense of the new knowledge. (p.4)

A second type of criticism questions the locus of knowledge in this development model. As Cochran-Smith and Lytle (1999) pointed out, in this model, researchers are assumed to be the possessors and teachers the consumers of knowledge. Teachers' knowledge of their classrooms is therefore not recognized as a potential resource on which to build but rather as something to be replaced (McIntyre & Hagger, 1992). As a result, teachers' primary obligation, when participating in professional development, is to adopt and implement faithfully the instructional practices envisioned by researchers, and consequently, to act as mediators between researchers and the classroom.

A Dichotomized Conceptualization

Lampert and Ball (1998) noted a different type of professional development approach in which what teachers learn in professional development and what they do in the classroom are divided along the lines of theory and practice. Although relatively common in pre-service teacher education, this conceptualization is also apparent in some in-service professional development programs. As observed by Lampert and Ball (1998),

> [in the setting of professional development, teachers] encounter generalized theoretical knowledge and methods based on synoptic views of learning and teaching …Their work in classrooms is presumed to provide them with practical aspects of the work best learned in the field, such as sunning a class discussion listening to student, reviewing homework, and writing on the board. (p.25)

In the dichotomized conceptualization, "theory and practice are divided both physically and conceptually" (Lampert & Ball, 1998, p.24). Physically, teachers are expected to learn a distinct body of knowledge in each setting, either theoretical (formal) or practical (informal). Conceptually, what teachers need to know in order to be effective in the classroom is believed to be composed of theories of teaching that transcend everyday practice and practical knowledge that deals with the technical aspects of teaching but often lacks an orienting framework.

The distinction between the theoretical and practical knowledge in a dichotomized conceptualization mirrors a long-standing division in conceptualizing the relations between teachers' beliefs and practice. The theoretical knowledge addressed in the professional development is presumed to support the emergence of more productive beliefs about teaching and learning, which are then expected to lead to fundamental changes in classroom practices. Professional development that follows a dichotomized conceptualization is often based on this linear, casual chain of teacher development (Clarke & Hollingsworth, 2002). The practical knowledge, on the other hand, is often seen as a summation of technical aspects of teaching. It is considered to come from learning from experienced teachers in the field rather than from professional development directly. As a result, learning from practice here means learning from *good* practice rather than treating teachers' current classroom practices as a site for their learning. As summarized by Lampert and Ball (1998), in a dichotomized conceptualization, "rarely is theory examined in practice" and "little attention is paid to what it means to learn in and from one's practice" (p.25).

In the dichotomized conceptualization, the relations between knowing and doing across the setting of professional development and the classroom is generally seen as unproblematic. In this type of professional development, it is assumed that what is worth learning would be evident to participating teachers (Cohen, 1998). Teachers are implicitly expected to engage in professional development activities with a pre-existing interest in improving their classroom teaching and a willingness to apply the learned theories in the context of their classroom instruction. It is rarely a priority for researchers to purposefully support teachers in applying the theories learned in the professional development to their classroom instruction, nor in reflecting on their instructional practices to verify these theories.

An Interrelated Conceptualization

More recently, a number of researchers have argued that professional development should be situated in the context of teachers' classroom practices. Ball and Cohen (1999) have proposed a two-step model for professional development design that involves first "identifying central activities of teaching practices" and then "creating or selecting materials that can usefully depict that work" (p.11). Ball and Cohen's (1999) proposal that teachers should learn "in and from practice" (p.10) has spawned a number of practice-based approaches, among which the use of classroom artifacts as well as accounts of classroom practices have become particularly prominent (Hatch & Grossman, 2009; Katims & Tolbert, 1998; Little, 2004; Little, Gearhart, Curry, & Kafka, 2003). These researchers contended that only in this way would teachers have the opportunities to engage in activities central to their daily practices and develop insights directly relevant to improving their work (Smith, 2003).

Central to this proposal is an increasing awareness that teachers' practices in their classrooms can and should serve as an resource on which researchers can draw to inform professional development designs. Teachers' learning in professional development is no longer considered uni-directional, merely traveling from the setting of professional development to the classroom. Instead, an important criterion for determining the effectiveness of a professional development involves how closely it is tied to teachers' experiences, needs and practices that are grounded in the setting of the classroom (Ball & Cohen, 1999; Franke, Carpenter, Fennema, Ansell, & Behrend, 1998; Nelson, 1997; Putnam & Borko, 2000; Schifter, 1998). In other words, the relations between teachers' activities across the two settings are considered *interrelated*. In contrast to the uni-directional view, the classroom in the interrelated conceptualization is viewed both as a site for teacher learning and a potential resource on which researchers can draw to inform the design of professional development.

In an interrelated conceptualization, the primary purpose for attending to teachers' classroom practice is to ground professional development design in the context of teaching. In doing so, researchers acknowledge the centrality of cultivating teachers' motivation to change by purposefully identifying professional development opportunities within the context of classroom teaching so that teachers may perceive it as relevant and useful to their work. This explicit attention to teachers' practice in turn indicates an elevated status attributed to teachers' classroom knowledge. Teaching, in the interrelated conceptualization, is acknowledged to be a complex enterprise, involving constant decision making in a highly uncertain environment (Wilson & Berne, 1999). Therefore, the quality of teaching cannot be simply improved by linearly "transferring" what teachers learn in professional development to their classroom. Instead, improvement in teachers' instructional practices requires a collaborative effort between teachers and researchers in which the professional expertise is believed to come in great part from inside classroom teaching.

Teacher professional development that is based upon interrelated conceptualization is often characterized as practice-based. However, it is critical to clarify that being practice-based does not necessarily imply that professional development is premised on the interrelated conceptualization. For example, professional development that reflects a deficit model can sometimes be grounded in teachers' classroom practice, with a focus on identifying potential fixes in teachers' current ways of teaching. In this latter type of "practice-based" professional development, teachers' learning is not conceptualized in a qualitatively different way than in the uni-directional conceptualization.

The idea of situating professional development design in the context of teachers' classroom practice has inspired a number of promising approaches for supporting teachers' learning. Two broad types of approaches can be distinguished in terms of the specific manner through which the alignment is brought about between teachers' learning in the setting of professional development and their instructional practices in the classroom. The two approaches can be described as (a) bringing the classroom into professional development and (b) carrying professional development into the classroom.

Bringing the classroom into professional development. The first approach, which I call "bringing the classroom into the professional development," centers on scrutinizing practices or, more frequently, records of practices (Lampert & Ball, 1998) in professional development settings. Three most commonly used types of records of practice include classroom originating artifacts such as student work (Chamberlin, 2005; Katims & Tolbert, 1998; Kazemi & Franke, 2004; Saxe, Gearhart, & Nasir, 2001), video-recorded classroom episodes (Richardson & Kile, 1999; Sherin & van Es, 2005) and records of integral aspects of instructional practices such as lesson plans (Lewis, Perry, & Murata, 2006). These various records of practices are considered promising for connecting the professional development activities to teachers' classroom practices as they aim to "equip teacher with the intellectual resources likely to be helpful in navigating the uncertainties of interpreting student thinking" (Ball, 1997, p. 808). The Cognitively Guided Instruction (CGI) professional development program conducted by Franke, Fennema, and Carpenter (Fennema, Franke, Carpenter, & Carey, 1993; Franke et al, 1998; Franke & Kazemi, 2001; Kazemi & Franke, 2004) is emblematic of this approach, capitalizing on classroom artifacts such as student work.

The CGI research is based on the hypothesis that when teachers have well-structured knowledge about how students' thinking in specific mathematical domains develops and can use this knowledge to interpret the reasoning of their students in the classrooms, their instructional practice will improve. To this end, the CGI researchers involved teachers in conversations and activities to examine student work that was generated in the course of classroom activities. This approach satisfied what Little

(2004) described as "triangle studies" (p.105) that investigate the relations between professional development, classroom practice and student learning. The use of student work in the CGI research was pivotal and served dual purposes. First, from the researchers' perspective, it constituted records of students' mathematical reasoning. Engaging teachers in conversations about student work served to help teachers focus on how students might think or reason mathematically. Second, student work constituted a mundane but indispensable aspect of teachers' everyday practice in the classroom (Franke & Kazemi, 2001; Kazemi & Franke, 2004). Artifacts that are deeply rooted in classroom practices were viewed by the CGI researchers as a medium through which the classroom could be represented as a focus of activity in professional development.

The CGI researchers conjectured that in the course of analyzing students' mathematical solutions, teachers would be surprised by the diversity as well as level of sophistication in students' mathematical reasoning. This surprise would in turn prompt teachers to continue eliciting their students' solutions and to try to understand their mathematical significance—practices that were previously unfamiliar to many teachers. Rather than attempting to "fix" the way in which teachers typically examined student work or thought of student reasoning, CGI researchers encouraged teachers to share their own perspectives in professional development and attempted to build on them (Lampert & Ball, 1998). This orientation to teachers' classroom instruction is an important characteristic of the interrelated conceptualization.

Carrying professional development into the classroom. The second approach involves what I call "carrying the professional development into the classroom." In contrast to "bringing the classroom into the professional development," this approach takes artifacts or practices that are purposefully introduced by the researchers as the starting point in working with teachers. Examining and discussing these artifacts or practices in relation to teachers' classroom instruction are conjectured to help teachers develop insights into teaching and eventually motive changes in their instructional practices.

The Developing Mathematical Ideas (DMI) curriculum designed by Schifter, Bastable and Russell (1999) is primarily aligned with the second approach. The central tenet of DMI was that deepened knowledge of mathematical content would prompt teachers to rethink what it meant to teach mathematics. Instead of focusing on records of practices from the classroom, the central design of DMI was composed of researcher-selected video clips exemplifying students' thinking (Remillard, 2002; Schifter, 2001, 2004). These classroom episodes were designed to simulate a typical classroom environment with which teachers could identify, while downplaying details that would differentiate one teacher's classroom from another. In DMI, both the selection of classroom episodes and the modification of the activity structure reflected the researchers' ideas of what teachers needed to learn in order to improve their classroom practices and how such learning should be organized and facilitated. The DMI developers conjectured that working with materials designed by the researchers constituted an important means of motivating teachers to reflect on their own classroom practices and to inquire further into mathematics teaching and learning. In an effort to relate teachers' learning in professional development sessions to their classroom, the DMI curriculum offered opportunities for teachers to share their student work, conduct student interviews and videotape their classroom teaching. The discussions organized around these artifacts (e.g., student work or teaching video) were expected to serve as a catalyst that would enhance teachers' engagement in the researcher-designed learning materials.

The two approaches that I have delineated are not mutually exclusive. In both approaches, teachers are expected to bring certain aspects of their classroom practice into professional development and, at the same time, to apply what they learn back to their classroom teaching. In doing so, they are supported in constantly testing, modifying, and regenerating their knowledge of mathematics teaching and learning. The key distinction between the two approaches is that, in the first approach (e.g., CGI), the goal is oriented towards scrutinizing the classroom internally, whereas the second approach (e.g., DMI) aims at providing teachers with external resources to examine their classroom instruction.

Developing a Reconceptualization

The recent proposal for situating professional development in the context of teachers' classroom practice has generated important implications for conceptualizing the relation between teachers' activities across the two settings. The interrelated conceptualization, in particular, reflects a significant shift in the field. Various research efforts, such as CGI and DMI, have significantly contributed to operationalizing Ball and Cohen's two-step proposal for teacher education which involves identifying central aspect(s) of teaching together with critical records of practice that can be usefully incorporated into the design of professional development. Such research efforts are, however, motivated primarily by a pragmatic concern of teacher educators; that is, what professional development strategies can best support teachers in connecting their learning to their classroom instruction. To this end, researchers have explored a number of important questions that include:

- What are the central aspects of teaching that need to be addressed in professional development?

- What are the critical records of practice that have the potential to lead to meaningful learning opportunities?
- How should the discussion in professional development be structured in order to effectively address teaching?

Answers to these questions, although undoubtedly significant, do not lend themselves to the development of a fully elaborated theoretical framework through which a deeper explanation of teacher development can be generated (Stein, Silver, & Smith, 1998).

Goldsmith and Schifter (1997) reminded us of the complexity involved in "learning in and from practice." They pointed out that teachers who engaged in professional development activities might not have "useful images from their personal experiences" (p.25) to guide their participation in these activities, thus creating a barrier of experience. This concern was echoed in the questions that Little (2004) raised with regard to the use of classroom artifact (e.g., student work) in professional development settings.

> One recurrent issue lies in the representation of classroom practices of learning and teaching in out-of-classroom contexts in which teachers come together for purposes of professional development. How much of students' thinking or learning is made evident by the student work available for consideration, and what additional resources enable teachers to make the most from looking at student work? (p. 110)

In my view, these voiced concerns are highly relevant, indicating the possibility that teachers may interpret student work differently and not view it as a record of student reasoning. Building on these concerns, a series of questions begin to emerge: What do classroom artifacts such as student work come to represent for teachers? What pedagogical value do teachers attribute to students' work in their daily instruction? What does it mean when teachers interpret classroom events differently than expected by researchers? And eventually, the question that Lampert and Ball (1998) asked:

"Teaching and teacher education…How do they connect? How might they connect?" (p.23).

Answers to these questions are critical. They orient us to think about what might be "meaningful variations in 'opportunity to learn' at the practice level" (Little, 2004, p. 111) and what can be feasibly accomplished when records of classroom practice are incorporated into the professional development as a means to support teacher learning. To answer these questions, it then becomes necessary for teacher professional developers to look beyond concrete professional development strategies on the level of action (see Figure 1). More importantly, we should scrutinize how we justify these strategies and whether and to what extent such justifications are legitimate, in the process engaging in what Argyris and Schön called double-loop learning (Argyris and Schön, 1978). Unfortunately, however, these issues have rarely been made a focus of investigation in their own right even in practice-based professional development programs, thus limiting their contribution mostly to innovating design strategies on the level of action. To this point, little conceptual work has been done to explicate the complexity involved in supporting teacher learning across different settings although this remains a central concern in the field.

In the remainder of this chapter, I draw on existing theoretical constructs in the literature to develop a foundation for conceptualizing the relations between teacher learning in professional development and their practices in the classroom. I find the notion of *consequential transition* (Beach, 1999, 2003) particularly relevant in that it explicitly addresses the challenge of conceptualizing people's activities across different settings. Importantly, it offers a range of theoretical tools that are relevant in developing a conceptual framework for articulating such relations on the level of justification. I then go on to introduce another critical component in the proposed conceptualization, which involves documenting teachers' instructional practice as seen from their own perspective.

Understanding the Relations through Consequential Transition

Beach (1999, 2003) used the notion of consequential transition to characterize the process through which learning would evolve as individuals participated in activities across different settings. Although he focused mainly on how students' learning in school related to their learning in other social settings, this construct carries significant implications for understanding and examining teachers' learning as they engage both in professional development activities and classroom teaching.

In the mathematics classrooms, teachers develop intuitions, rationales and beliefs towards what they do as a teacher (Borko, 2004); they plan, assess and interact with their students in particular ways that make sense to themselves given their particular institutional niche (Cobb, McClain, Lamberg, & Dean, 2003, Elmore, 2000; Spillane, Halverson, & Diamond, 2001). A transition takes place as teachers move from the mathematics classroom to the setting of professional development, in which researchers aim to achieve a specific agenda for supporting teacher learning. Teachers' prior experiences from classroom teaching inevitably shape their interpretations of and participation in these professional development activities, sometimes in ways that may not be anticipated by researchers[2] (Simon & Tzur, 1999; Heinz, Kinzel, Simon, & Tzur 2000). From Beach's point of view, this transition can be consequential in terms of teachers' learning if their participation in professional

---

[2] This is the case if and only if the purpose of teachers' participation in professional development is understood as to improve their classroom practices. In cases where teachers participate in professional development only to earn credentials or satisfy administrative requirements, it is unlikely that they will make an effort to relate what they learn to their classroom teaching.

development is oriented towards reorganizing their classroom practices, and if their classroom teaching constitutes the context for them to make sense of, reflect on and apply what they learn in professional development.

Developmental coupling. One of the methodological tools that Beach proposed for analyzing consequential transition was that of a developmental coupling, which "encompasses aspects of both changing individuals and changing social activity” (Beach, 1999, p.120). Beach clarified that developmental coupling would necessarily involve artifacts—student work for example—that reified practices and transcended different social activities in which people participated.

To illustrate the notion of a developmental coupling, Beach drew on the research he conducted in Nepal that involved two groups of individuals both making transitions between school and work. The first group was shopkeepers who attended adult education classes to learn arithmetic in order to improve their shopkeeping skills (see Figure 2a). A transition took place as the shopkeepers moved back and forth between work and school. The second group consisted of high-school graduates who were learning to become shopkeepers, making a one-way transition from school to work (see Figure 2b).

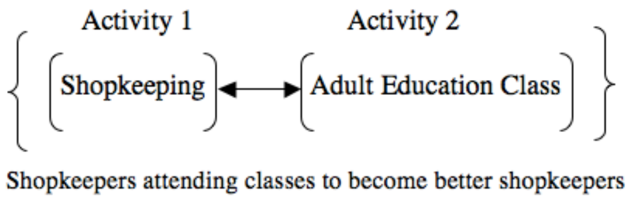

Figure 2a. Developmental Coupling as a Unit of Analysis—
The Case of the Shopkeepers

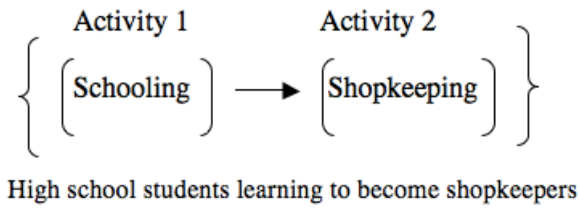

Figure 2b. Developmental Coupling as a Unit of Analysis—
The Case of the Students

The findings revealed that upon completing the adult education classes, the shopkeepers chose to drop the operational signs whereas the high-school students adhered to the written notations. Beach argued that the contrasting forms of arithmetic reasoning emerged not as a result of individuals' participation in a single activity nor the properties of the individuals. Rather, it was in the transitions between the activities of schooling and shopkeeping that these new forms of reasoning came into being. Although the arithmetic notations were emphasized in school mathematics that both the shopkeepers and the students had learned, the significance of the signs were different to these two groups of people. From the shopkeepers' perspective, these notations were not directly useful to their shopkeeping practice, a practice that had been long established before they entered the adult education classes. Therefore, no matter how much the signs were emphasized in school mathematics, the shopkeepers chose to drop these signs as they failed to see any relevance to the routine practice of recording transactions in the shop. The students, however, preferred to stick to the signs not because they found them useful to shopkeeping but because they wanted to reflect their status of having received formal education. Unlike the shopkeepers, the students' activities across the two settings were not oriented towards the reorganization of shopkeeping practices. Therefore, methodologically, Beach (1999) argued that the two activities directly involved in a transition should constitute a single unit of analysis and needed to be examined in juxtaposition.

Developmental coupling provides a useful tool for conceptualizing the process of supporting teacher learning across different settings. It extends the unit of analysis to include multiple activities that are involved in the transition. Specifically, it implies that teacher learning in professional development needs to be interpreted against the background of their classroom practices; likewise, changes in teachers' classroom practices cannot be sufficiently accounted for without an explicit reference to their learning in the setting of professional development. For example, in order to understand how teachers reason with records of classroom practice (e.g., student work, classroom tasks and assignments, video recorded episodes of student problem solving) in professional development, it then becomes imperative to look at how similar artifacts or activities are constituted in the context of classroom teaching. In this sense, developmental coupling provides useful guidance when conducting retrospective analysis, the purpose of which is to understand how changes in teachers' classroom practice occurred as a result of their participation in professional development.

In addition to guiding retrospective analysis, developmental coupling can also be adapted to help anticipate the extent to which the conjectured means to support learning is viable. For example, in the shopkeepers' case, had the researchers known beforehand how the activity of using arithmetic signs was constituted in both settings, they would have been able to anticipate that it would be unlikely for the shopkeepers to continue to use the signs once they returned to their shop. Figure 3 illustrates the two different roles of arithmetic signs, as they became constituted in the adult education class and the shop respectively.

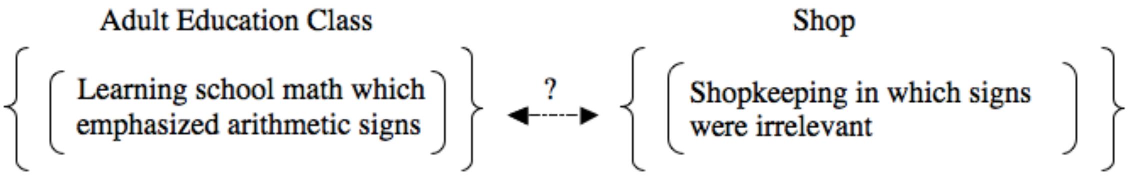

Figure 3. Developmental Coupling—the Shopkeepers' Use of Arithmetic Signs

In contrast to the actual developmental coupling that involves activities that have already occurred, *the anticipated developmental coupling* is by nature hypothetical. It juxtaposes the *intended activity* researchers have designed to support teachers' learning in professional development with the *existing activity* that teachers are familiar with in the context of their current instructional practices.

To illustrate the anticipated developmental coupling, I take as an example the use of student work in the early CGI research (Kazami & Franke, 2004) that focused on basic number concepts. The CGI researchers intended to use student work to support teachers in understanding the diversity in students' mathematical reasoning and eventually capitalizing on it in their instructional planning and decision making. This chain of design conjectures is outlined in the *intended activities* in Figure 4, from which it can be seen that student work was expected to constitute a reification (Wenger, 1998) of student reasoning within the context of professional development activities.

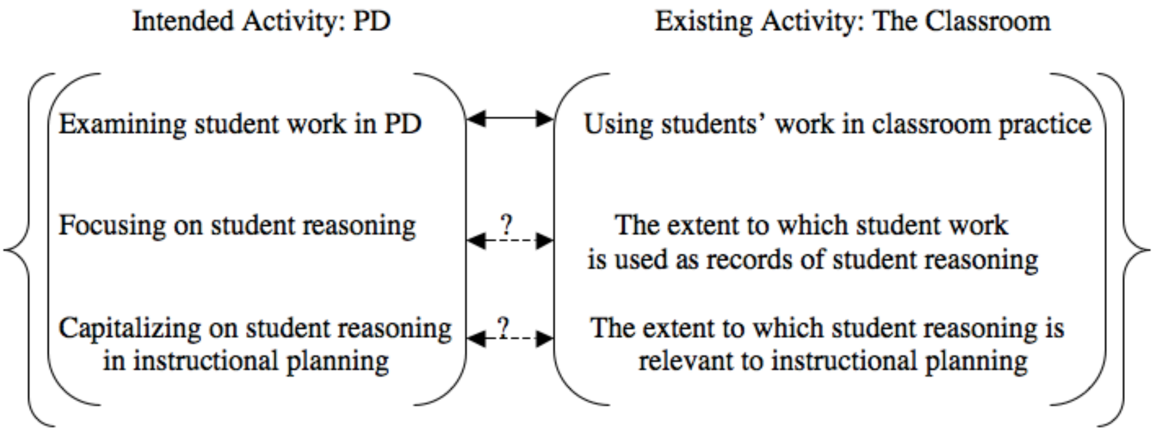

Figure 4. Anticipated Developmental Coupling in Professional Development Design

In the *existing activities* at the classroom level, the teachers did use student work regularly in their instructional practice, thus a solid arrow is drawn to indicate the two-way movement of the physical artifact. However, based on the CGI researchers' account, it is less obvious how the teachers used student work in their own classrooms, or whether their use of student work was compatible with the instructional practice of focusing on student reasoning. For example, it would be potentially problematic if, from the teachers' perspective, student work only reified the outcome of instruction and should mainly be used as an assessment tool. Another important issue that remains tacit in the CGI researchers' account is whether student reasoning had any currency or visibility within the context of classroom teaching. If the answer is negative, we can imagine that a direct focus on student reasoning in professional development may be thought of as irrelevant, impractical, or intimidating by the teachers.

Just as the shopkeepers ignored the arithmetic signs because they found them irrelevant to recording store transactions, teachers may have vastly different ideas about how useful student work or students' reasoning is with regard to their teaching at the classroom level. Without a solid and useful image of the *existing activities* (in this case, the mathematics instructional practices) in which teachers have engaged in their own mathematics classroom, researchers are to a large extent operating in dark when it comes to designing professional development activities.

Although having generated valuable outcomes in terms of supporting the learning of the collaborating mathematics teachers, in a later project that focused on algebraic thinking, the CGI researchers began to realize the importance of the missing account of the existing activity and its relevance to designing professional development (Franke, Carpenter, & Battey, 2007). This realization was spurred by the difficulty that the researchers experienced in getting teachers to engage in conversations around student reasoning even when student work was used in professional development. Part of the reason, as pointed out by Franke and her colleagues, was because the participating teachers' current instructional practices did not involve a focus on students reasoning but instead emphasized getting correct answers from the students. Reflecting on this research experience and comparing it to their previous work on basic number concepts, Franke and her colleagues contended that this new content focus on algebraic reasoning required different mathematical as well as pedagogical learning from the teachers that were largely unavailable in their current ways of teaching (Franke et al., 2007).

By examining the anticipated developmental coupling, professional development designers are able to predict whether an intended consequential transition is likely to occur, how it is going to occur, and/or whether the intended means of supporting teachers' learning is likely to be valid. In addition to developmental coupling, Beach introduced a second conceptual tool—leading activity—for understanding the participants' interpretations and experience as they moved from one setting to another. He argued that it was important to look beyond the settings or activities directly involved in developmental coupling and consider broader institutional, societal and cultural contexts on the macro-level, which, he believed, would have a profound impact on learning.

Leading activity. A leading activity, as the term suggests, is an activity that overshadows other activities and influences one's participation in multiple settings (Beach, 1999; 2003). Construed broadly, a leading activity reflects the perceived continuity of activities from the participant's perspective. To illustrate, I return to the Nepali example in Beach's work (1999).

The two groups of Nepali villagers developed different relations to the transitions between school and work. By trying to understand the leading activity from the participants' perspective, we are able to add another dimension to developmental coupling (see Figure 5).

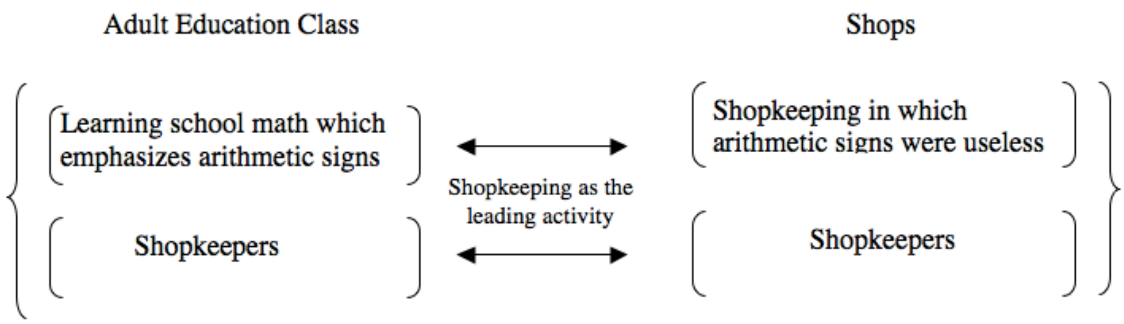

Figure 5: The Leading Activity in the Shopkeeper's Case

For both shopkeepers and high school students, the activities across settings essentially involved schooling and shopkeeping. However, the transition that the shopkeepers made between work and school involved a leading activity of shopkeeping. They attended adult education classes for the purpose of becoming more proficient at their job and as a result, they developed a repertoire of arithmetic strategies that were directly relevant to running their business. In both settings, they saw themselves as shopkeepers. In contrast, the high school students saw the two activities as disconnected, with one focusing on learning for credentials and the other on selling goods to make a profit. Most of the high school students had little understanding of what shopkeeping was about. This disconnect led the students to see school mathematics as unrelated to but above its counterpart used in shopkeeping practice. As a result, they adhered to the arithmetic notations to indicate their educational status even though these signs were not pragmatically useful for recording transaction in shops.

When participating in professional development, teachers may choose to engage for a variety of reasons: to gain credentials, to maintain professionals status within school or school district, to comply with administrative requirement, and/or to learn to teach more effectively. The different rationales significantly shape what and how teachers would learn as they participate in professional development activities. For a professional development to effectively influence teachers' classroom instruction, it is critical that the leading activity from the teachers' perspective is to improve their classroom teaching. This characteristic of teacher education is analogous to that of the shopkeepers' experience: in both cases, the leading activity is oriented towards improving one's own professional practice, be it teaching or shopkeeping.

Understanding what the leading activity is or is becoming from the teachers' perspective opens up the possibility that teachers' learning experiences in professional development might be significantly different than those anticipated by researchers. It therefore becomes critical for researchers to make sure that the potential leading activity is in alignment with the envisioned goals for professional development. This, however, does not imply that the leading activity is static or there is nothing researchers can do to influence teachers' perception of it. As Beach clarified, what counted as the leading activity might change as a result of individuals' participation in various activities. In terms of supporting teachers learning, this implies that professional development should include a negotiation of the leading activity. For example, with proper means of support, teachers may shift from viewing their participation in professional development as compulsory--mainly to satisfy administrative requirement--to seeing it as useful to improving their classroom instruction. In this sense, the leading activity can become an entity of development in its own right. In teacher education literature, the process of such negotiation is often kept in a black box. Documenting and unpacking this process can be of great value to teacher educators.

The notion of consequential transition illuminates a reconceptualization that centers on a *bi-directional interplay* between teachers' activities in the professional development and in the classroom. The implication of the bi-directional conceptualization is two-fold. On the one hand, it offers a conceptual framework for *retrospectively* analyzing and interpreting teachers' activity across the two settings. The bi-directionality characterized in this reconceptualization extends the unit of analysis to include *practices* that become constituted across both settings. Therefore, accounts of the use of classroom artifacts in professional development sessions have to consider how teachers use the same set of artifacts in their own instructional practices. On the other hand, a bi-directional conceptualization can guide the *prospective* design of professional development by enabling researchers to anticipate potential problems that may arise as teachers engage in the planned activities even when classroom artifacts or practices are incorporated in the professional development design. This reconceptualization centered on the *bi-directional interplay* elaborates what is often missing from the level of justification that underlies the interrelated conceptualization (see Figure 1).

Documenting Teacher's Classroom Practice through Accounts of Instructional Reality

In a bi-directional account of teachers' learning in professional development, an in-depth knowledge of their instructional practices in the classroom becomes imperative. Without it, researchers would have little resource to draw inferences about teachers' *existing activities* as shown in Figure 4. An understanding of teachers' current instructional practices is therefore crucial because it constitutes an essential component in the bi-directional construct.

Many approaches for documenting teachers' classroom practices that have been developed over the years are, by nature, observer-centered. They often characterize a researcher-developed rubric against which teachers' classroom practices are compared (Ball & Rowan, 2004). Typically, researchers create such rubrics by first identifying a constellation of core instructional practice that, from their perspective, determines the overall quality of teaching. A range of teaching practices with different levels of sophistication are then described under each identified core instructional practice so that the actual instructional practices observed in teachers' classrooms can be located somewhere on this scale. The resulting rubrics are presumed to allow a relatively objective assessment of the quality of teachers' classroom practices so that researchers with proper training can generate similar accounts of the observed teaching. Although this approach is adequate for evaluative purposes (especially for comparative studies on a large scale), it is less suitable for the purpose of informing a design of professional development that takes seriously the bi-directional conceptualization. The pre-conceived rubrics may help expose the gap between teachers' current observed practices and the desired teaching practices. However, little resource is provided to understand *why* teachers teach in a specific matter and/or how teachers might experience the transition as they move from the classroom to the professional development.

How then should teachers' classroom practices be analyzed? Central to the bi-directional conceptualization is the relation between teachers' activities across different settings *as teachers experience them*. Consequently, for a characterization of teachers' classroom practices to be useful for the purpose of understanding the bi-directional interplay across the two settings, it needs to encompass the perspectives that teachers hold towards teaching and learning. The term *accounts of instructional reality*, as I use it, encompasses researchers' accounts (1) of the perspectives that teachers hold towards teaching and learning mathematics, (2) of the teachers' perceptions of their role in the classroom and of their obligations as mathematics teachers, (3) of the instructional challenges and frustrations that they encounter and their explanations of them, and (4) of the valuations and expectations they hold towards various aspects of their instructional world (e.g., towards instructional resources that are made available to them) (Zhao, Visnovska, Cobb, & McClain, 2006). In other words, the accounts of instructional reality depict the world of teaching as teachers see and experience it. This characterization requires that explicit efforts should be made to capture not only the interrelated nature of various aspects of teachers' classroom practices but more importantly, their rationales for teaching in certain ways but not the others.

A key aspect in developing accounts of instructional reality is to assume teachers' instructional decisions as rational from their perspective (Simon & Tzur, 1999). Viewing teachers as rational decision-makers may seem counterintuitive especially in cases when teachers' current instructional practices differ significantly from those advocated by the reform. However, if we fail to take teachers' perspectives seriously, we may be at the risk of positioning teachers as deficient, having little to bring to the professional development; or we may overlook opportunities for supporting teachers' learning that may arise as their perspectives become explicit topics of conversation in professional development activities. PD efforts that fail to take teachers' perspective seriously often fall back to the uni-directional conceptualization and focus on filling the gaps between teachers' current instructional practices and the envisioned ones. The problematic nature of this approach is well indicated by the frustrations of both teachers who end up thinking professional development as unhelpful (Putnam & Borko, 2000) and professional developers who struggle to solicit teachers' interest and engagement (Franke, Kazemi, Carpenter, Battey, & Deneroff, 2002).

In documenting teachers' instructional reality, it is therefore crucial that researchers make an explicit commitment to take the perspectives and instructional practices that teachers develop as reasonable and coherent from their own viewpoints (Leatham, 2006; Simon & Tzur, 1999). Operating on the basis of this assumption enables researchers to avoid characterizing teachers and their practices solely as deficient. Instead, researchers are obliged to look for explanations of teachers' instructional practices (even when they may seem insensible or ineffective from an expert's perspective) until a reasonably coherent account is developed. As a result, an observation of a seemingly insensible or ineffective instructional decision in the classroom does not merely conclude with a negative assessment of the teacher's competence. It becomes a focal point for which the researchers need to account so that it can be seen as a reasonable and coherent component that fits within the landscape of the teacher's instructional reality. It is this kind of explanation of what teachers do and why they do it that provides valuable guidance for researchers in designing to support teachers' learning.

Simon and his colleagues pursued an agenda of this type as they attempted to capture the interrelated aspects of teachers' practice (Heinz et al., 2000; Simon & Tzur, 1999; Tzur, Simon, Heinz, & Kinzel, 2001). In doing so, they proposed the construct of *accounts of practice*. The construct of accounts of practice shares a number of commonalities with accounts of instructional reality. First, both define practice broadly to include not merely what teachers do but also their interpretations, construals, and rationales that underlie their action of teaching. Importantly, instead of dichotomizing teachers' practice and their beliefs, both notions view them as complementary aspects in teachers' world of teaching.

Second, both make a strong commitment to understand the coherence and rationality of teachers' instructional practices *from the teachers' perspective*. As Simon and Tzur (1999) noted, researchers do not have direct access to teachers' perspectives on their practices. Therefore, both types of accounts are reconstitutions of how teaching looks from the teachers' perspective, but as the *researchers* understand it. Heinz et al. (2000) made an important clarification about this approach of accounting for teachers' practices. They argued that this interpretive account of teachers' practices is particularly useful in that it "allows us to consider teachers' practices in light of ideas that are currently important to the mathematics education community, but which may not be accessible or important to the teacher in question" (Heinz et al., 2000, p. 88). This orientation also guides the generation of accounts of instructional reality in which the goal is not to merely synthesize what teachers say or do on the surface level but to understand the, often implicit, rationales that give coherence to what is observed.

Finally, the third commonality between the two constructs for documenting teachers' instructional practices is that the primary purpose for developing both accounts is to inform teacher development designs. Simon and Tzur (1999) stressed this interventionist stance.

> Our commitment in using the accounts of teaching practices methodology is to arrive at an appropriate (given the data) articulation of the teacher's current practice in a way that portrays the reasonableness of all the teacher's observed actions. Thus, whereas the accounts of practice serve to identify areas for teacher development, they also provide a foundation for the researchers to hypothesize on the basis of an understanding of the current practice, how that development might proceed. (p. 255)

Although both accounts of practice and accounts of instructional reality emphasize researchers' interpretation of how teaching mathematics looks from the teachers' perspective, the latter distinguishes itself from the former in two major ways in that (1) it takes a broader landscape of teaching into consideration, and (2) it explicitly accounts for the "problematic" aspects of teachers' instructional practices. To elaborate on the first distinction, accounts of teachers' instructional reality not only involve accounting for teachers' conceptions of mathematics teaching and learning—what is mathematics, how students learn, and what supports their learning (Heinz et al., 2000)—but also other aspects that significantly affect teaching from the teachers' point of view. Examples may include issues of student motivation, classroom management, or parental involvement. Additionally, instructional resources, especially how they are used in the practice of teaching is also considered as an important source of explanation in accounting for teachers' instructional reality (Bowen & McClain, 2005). This is because what teachers do and how they justify their practices are often bound up with the particular instructional resources (e.g., textbooks, state-mandated curriculum, or copies of students' work) that they use in the classroom (cf. Cobb et al, 2003; Bowen & McClain, 2005). For example, Cobb et al (2003) illustrated how the use of certain curriculum materials oriented teachers towards covering the mathematical content to a considerably greater extent than focusing on their students' mathematical reasoning. Other researchers who investigated the role of textbooks in classroom instruction have explicitly linked teachers' instructional practices to the instructional materials that were made available to them (Remillard, 1999; Stein & Kim, 2006).

Moreover, in developing an account of teachers' instructional reality, it is important that we consider the institutional context in which teachers develop and refine their practices (Cobb et al., 2003; Elmore, 2000; Lieberman & Miller, 1992; Spillane et al., 2001). As mentioned earlier, teachers' instructional experiences, the development of their instructional practices in particular, are not merely confined to the four walls of the classroom. The institutional context in which teachers work has a profound impact on how they approach teaching and learning. What are the obligations of being a teacher in a particular school or school district? What are the support or assessment structures available to facilitate classroom teaching? How is teaching made visible in a particular school or school district? By grappling with these aspects of institutional context, teachers constantly adapt their instructional practices to satisfy the institutional criteria for what it means to teach effectively. Answers to these and related questions generate critical insights for researchers as they seek to understand teachers' classroom practices that go far beyond the walls of the classrooms.

The second distinction between the accounts of practice and accounts of instructional reality is that the latter requires an articulation and understanding of the "problematic" aspects of teachers' instructional practices. The term "problematic aspects" here has dual meanings. First, it refers to the aspects of teaching and learning that are highly problematic from the teachers' perspective. This orients researchers to the struggles and challenges that teachers experience in their classroom as well as the repertoire of strategies—no matter how inappropriate they may seem—that they employ to resolve such tensions. The purpose of identifying these problematic aspects in accounting for teachers' instructional reality is to inform the design of professional development activities, specifically by delineating possible path for collaboration that might carry pragmatic value to the participating teaches (Walen & Williams, 2000). McIntyre & Hagger (1992) described this type of professional development as "genuine professional development" in which teachers are "motivated to solve the difficult problems which they experienced in their work and to fulfill their educational aspirations for their students more effectively" (McIntyre & Hagger, 1992, p.280).

The second type of problematic aspects that is emphasized in accounts of instructional reality concerns aspects of teachers' practices that do not fit with the researchers' current understanding of what might be reasonable from the teachers' perspective. These aspects in teachers' instructional reality are particularly significant in that they indicate the potential incongruities between researchers' anticipations and teachers' actual experiences in professional development, and are often the causes of ineffective professional development efforts. The goal here, however, is not to identify gaps or potential fixes in teachers' current practices. Rather, it further orients researchers to account for the noted differences in perspectives. In a study conducted by Leatham (2006), the researcher strove to understand teachers' beliefs as sensible system and argued cogently for its methodological significance.

> As observers, we may not find [certain beliefs of the teachers] sensible. It may not seem logical, rational, justifiable, or credible…But our incredulity does not

> diminish another's coherence… Observations of seeming contradictions are…perturbations, and thus an opportunity to learn… When a teacher acts in a way that seems inconsistent with the beliefs we have inferred, we look deeper, for we must have either misunderstood the implications of that belief, or some other belief took precedence in that particular situation. (p.5)

This orientation towards understanding the teachers' belief systems as described by Leatham (2006) is largely compatible with the basic tenet in understanding teachers' instructional reality. In the latter, such incongruities indicate potential issues that may become the source of misunderstandings if they are not negotiated explicitly in the course of collaboration. By alerting researchers to issues of this nature, the account of instructional reality can potentially shape the prospective goals in a professional development design.

Putting Together a Reconceptualization

I have so far described a bi-directional conceptualization of the relations between teachers' learning in the setting of the professional development and their practices in the classroom by drawing upon the notion of consequential transition (Beach, 1999) and accounts of instructional reality (Zhao et al., 2006). Three aspects are critical in the resulting conceptualization that centers on the bi-directional interplay between teachers' learning in professional development and their teaching in the classroom.

The first critical aspect concerns how teachers perceive the nature of their activity as they are simultaneously involved in professional development activities and in classroom teaching. On a general level, it is important for professional developers to understand (and renegotiate if necessary) what teachers see as the overall purpose for participating in professional development sessions. Whether the goal, as teachers see it, is to improve their classroom teaching or to simply fulfill administrative obligations would significantly influence their participation and learning in the professional development sessions.

The second aspect concerns the unit of analysis. Rather than examining teachers' activities separately in each setting, it becomes critical to juxtapose teachers' activities across both settings in order to (1) develop a more complete and accurate account of teachers' learning or lack thereof and (2) guide the prospective design of professional development by anticipating any potential problems that may arise as teachers participate in activities across these different settings. When the unit of analysis is set as such, it becomes imperative for practice-based professional development approaches to be grounded in a detailed analysis of teachers' instructional practices. As a result, a designed professional development activity (e.g., looking at student work) can be examined against its counterpart (e.g., how teachers actually use student work in their classroom instruction) at the classroom level. Only then can teacher educators be able to anticipate the extent to which teachers would perceive the designed activities as relevant to their classroom instruction and whether the conjectured means to support teachers' learning is valid.

The third aspect in the bi-directional conceptualization highlights a means for documenting teachers' instructional practices. An indispensable aspect of this conceptualization involves a documentation of teachers' current practices so that the dual unit of analysis can be used. Accounts of teachers' instructional reality, unlike many observer-centered accounts of teaching, focus not only on what teachers do on the observable level but more importantly, on the underlying rationales that teachers hold towards teaching and learning. These underlying rationales become the backbone of the bi-directional interplay through which teachers' classroom practices and their learning in professional development can be interpreted and examined. The inquiry into teachers' instructional reality is also motivated by a deep-rooted concern for design; that is, the need to search for potential issues of interest that teachers might find meaningful to engage in the setting of professional development. This approach of trying to understand teachers' classroom practices by placing their sense-making at the center is consistent with the overall approach of building on teachers' current classroom practices. It reflects researchers' efforts to de-center and adopt teachers' perspectives when interpreting the relations between teachers' classroom experiences and their experiences in the professional development.

The bi-directional reconceptualization is significant for both theoretical and pragmatic reasons. Theoretically, it makes an initial contribution to the development of an elaborated framework for conceptualizing the relations between teachers' learning in the setting of professional development and their instructional practices in the classroom. It is widely acknowledged among teacher educators and researchers that the ultimate goal for teacher professional development is to bring about improvements in classroom instructional practice and thus in students' learning of mathematics. The primary contributions of the prior research have been to develop concrete professional development strategies for "connecting" teachers' activities across the two settings. Little conceptual work has been done to understand what is involved in supporting teachers' learning across these settings, although many have called for a theoretical framework of this type (Borko, 2004; Cohen, 1998). Building on the prior research, the bi-directional conceptualization is consistent with diSessa and Cobb's (2004) characterization of an ontological innovation that involves "hypothesizing and developing explanatory constructs, new categories of things in the world that help explain how it works" (p. 177).

Pragmatically, the bi-directional reconceptualization offers a conceptual tool for interpreting and analyzing teachers' activities across different settings. At the same time, it invites professional development designers to consider their underlying design assumptions that can potentially shape what is perceived as viable means of supporting teachers' learning. As a result, the reconceptualization contributes to expand what researchers may view as workable and analyzable in the process of supporting teachers' learning.

CHAPTER III

UNDERSTANDING THE ROLE OF STUDENT WORK IN SUPPORTING TEACHERS' LEARNING ACROSS DIFFERENT SETTINGS: THE EMERGENCE OF THE BI-DIRECTIONAL CONCEPTUALIZATION

Introduction

An effective mathematics teacher professional development (PD) is expected to bring about changes in classroom instructional practices and ultimately, improvements in students' learning of mathematics (Borko, 2004; Hiebert, Morris, Berk, & Jansen, 2007; Whitcomb & Borko, 2009). Inherent in this characterization is the expectation that teachers' learning in PD should lead to significant changes in their mathematics instructional practices in the classroom. How to support teachers' learning across the setting of PD and the classroom therefore stays at the very core of every PD effort (Ball & Forzani, 2009; van Es & Sherin, 2010).

As simple and straightforward as this goal may sound, it places tremendous challenges upon mathematics teacher educators. Regardless of researchers' efforts to design and support mathematics teachers' learning, teachers' participation in carefully designed PD activities does not necessarily lead to intended changes in their classroom mathematics instruction (Franke et al. 2007; Forgasz & Leder 2008; Kazemi & Hubbard, 2008; Ma, 1999). Thus, an immediate and pragmatic challenge for teacher educators necessarily involves how to design PD activities so that teachers can relate what they are learning to their classroom practices and, as a result, reorganize their current ways of teaching.

In recent years, the field addresses this challenge by beginning to shift towards a "new paradigm for professional development" (Stein, Smith, & Silver, 1999), in which teachers' classroom practices have become the subject of inquiry (Ball &

Cohen, 1999; Jaworski, 1994; Steinberg, Empson, & Carpenter, 2004). This new practice-based approach is in sharp contrast with the traditional teacher workshops in which knowledge is often expected to travel uni-directionally from PD to the classroom. In the latter form of PD, it is often assumed that teachers will develop insights into teaching and students' learning in PD sessions and then apply them in the classroom in a uni-directional fashion. Designs for supporting teachers' learning typically focus on equipping teachers with forms of expertise that researchers believe are important for the development of effective instructional practices in the classroom (Borko, 2004; Clarke & Hollingsworth, 2002).

The practice-based approach directly challenges the uni-directional view. It acknowledges the complicated nature of teachers' learning across settings and emphasizes that PD should be situated in the context of teachers' instructional practices. In doing so, researchers begin to view teachers' learning in PD and their instructional practices as interrelated. For example, Ball and Cohen (1999) call for teacher PD activities to be centered on the records of practices. These records of practice, as they point out, should be directly relevant to teachers' classroom instruction and at the same time can generate leverage to "equip teacher with the intellectual resources likely to be helpful in navigating the uncertainties of interpreting student thinking" (Ball, 1997, p. 808). Practice-based approach reflects a growing consensus among teacher educators that an effective PD design should be centered on the core practices of mathematics teaching and closely tied to teachers' classroom experiences (Ball & Cohen, 1999; Franke et al., 1998; Grossman & Mcdonald, 2008; Nelson, 1997). As Wei, Darling-Hammond, and their colleagues point out, "the content of professional development is most useful when it focuses on concrete tasks of teaching, assessment, observation, and reflection" (Wei, Darling-Hammond, Andree, Richardson, & Orphanos, 2009, p. 3).

In striving to support teachers' learning across different settings, research following practice-based approach constantly seeks to answer one question: how can teacher educators support teacher learning across the two distinct settings in a coherent manner so that teachers perceive what they learn in PD as meaningful and relevant to their classroom practices while at the same time, reorganize instructional practices as envisioned by the teacher educators? Two types of inquiries are often emphasized in the practice-based approach to this end: Which aspects of mathematics teachers' classroom practice should be represented in PD (Grossman & Mcdonald, 2008; Kazemi, Lampert, & Ghousseini, 2007) and how they should be represented (Hatch & Grossman, 2009; Little, 2003, 2004; van Es & Sherin, 2010).

Research focusing on these issues has accumulated important insights that are pragmatically valuable to inform the design of PD. For example, evidence has shown that a few types of records of practice can be potentially useful in supporting teachers' learning in PD, including classroom originating artifacts such as student work (Chamberlin, 2005; Katims & Tolbert, 1998; Kazemi & Franke, 2004; Saxe et al., 2001), video-recorded classroom episodes (Hatch & Grossman, 2009; Richardson & Kile, 1999; Sherin, 2007; Sherin & van Es, 2005) and records of integral aspects of instructional practices such as lesson plans (Lewis et al., 2006). These records of practice are identified because they represent critical aspects of teachers' classroom instructional practices and therefore would be more likely to enhance the relevance between PD and teachers' classroom instruction. More importantly, researchers contend that these records of practice can give rise to opportunities for teachers to examine critical aspects of classroom instruction in a way that is consequential for improving the quality of their mathematics instruction. It goes without saying that for the records of practice to be used productively in PD, researchers have to not only choose the critical representation of teachers' classroom instructional practice but carefully design and guide these PD activities (Chamberlin, 2004).

Undoubtedly, these lines of research have greatly enriched what may be perceived as an action plan for supporting teachers' learning across different settings. However, records of teachers' instructional practice are not silver bullets and do not always result in desired learning outcomes in spite of researchers' careful planning and implementation of PD activities. In a more recent project that focused on algebraic thinking, the CGI researchers unexpectedly encountered difficulties in engaging teachers in PD activity around student work as they intended in spite of the fact that these activities were carefully planned (Franke et al., 2007). The activity of looking at student work did not seem to constitute an opportunity for the teachers to examine students' mathematics thinking as it did in the earlier PD programs designed by the same researchers with a focus on number sense. Franke et al. reflected on this incident and conjectured that the content focus on algebraic reasoning (as compared to number sense) required significantly different mathematical as well as pedagogical knowledge from the teachers. They then went one step further to examine what it was about student work that enabled it to support teachers' learning across the settings of PD and the classroom and more importantly, what led it failed to do so (Kazemi & Hubbard, 2008).

The CGI researchers' case of algebraic reasoning is revealing in that it indicates there is yet something about the use of records of practices that has not been fully understood. The variation of learning opportunities associated with records of practice clearly warrants further investigations. Indeed, if we do not understand why sometimes things do not work, we do not truly understand why sometimes they do.

The CGI researchers' efforts to unpack the use of records of practices in PD bring to the fore an issue of greater significance; how do we as a field conceptualize why (or why not) records of practice afford opportunities to support teachers' learning? To this point, considerable effort has been expended to find out *what* records of practice can be useful and *how* they can be useful in PD, but this issue of *why* they can be potentially useful has received far less attention. A conceptualization of such nature is called for by Cochran-Smith and Lytle (1999) in their review of teacher education research over the last two decades.

> There are radically different conceptions of teacher learning, including varying images of knowledge; of professional practice; of the necessary and/or potential relationships that exist between the two…Different conceptions of teacher learning—although not always made explicit—lead to very different ideas about how to improve teacher education and professional development…(p.249)

A teacher PD design, as Cochran-smith and Lytle (1999) see it, involves two distinct yet related levels. Take practice-based approach for example. The level of action, which is typically more observable, usually articulates what records of practices should be used in PD and how they should be represented and used in PD activities. It offers concrete ideas to the make the PD design more effective in terms of supporting teachers' learning. In contrast, the level of justification appears to be much less visible and often remains "unexamined and tacit" (Cochran-smith and Lytle, 1999). It is typically composed of suppositions and assumptions held by researchers while specific actions of design are being carried out. In practice-based approaches, the level of justification may involve researchers' conceptualizations of the records of practice—why these records of practice are useful and what exactly is useful about them. The level of justification constitutes a source of explanation that gives rationality and unity to the seemingly discrete activities on the level of action.

The distinction between the level of action and justification implies a two-part process in accounting for the effectiveness of a teacher PD design. When an account focuses primarily on the level of action, its goal is to seek alternative strategies for bringing about the connection between teachers' classroom practice and PD activity while researchers' underlying conceptions remain unexamined. This is what Argyris and Schön (1978) describe as single-loop learning, in which the emphasis is on "techniques and making techniques more efficient" (Smith, 2001, p.10). In contrast, an account of different nature occurs when researchers carefully scrutinize their underlying conceptions and examine how they may shape the way in which various strategies and consequences are framed (Smith, 2001). Argyris and Schön name an account of the second type double-loop learning (Argyris and Schön, 1978).

Cochran-Smith and Lytle (1999) explicitly call for an account of the second type, arguing that an articulation of researchers' conceptions that underlie PD initiatives can be both theoretically and pragmatically significant. Theoretically, it has the potential of contributing to the generation of conceptual frameworks that are currently lacking in mathematics teacher education (Cohen, 1998). Pragmatically, by explicitly grounding the action plan in researchers' underlying conceptions and by making the latter subject to public scrutiny, we expand our purview of what is analyzable and workable in the process of supporting teacher learning.

The purpose of this chapter is to contribute to the knowledge of the second type by framing as a paradigm case our research team's efforts to use student work in the context of collaborating with a group of middle-school mathematics teachers. This research experience made it necessary for us to articulate, examine, and eventually substantially reconceptualize the issue of supporting teachers' learning across the setting of PD and the classroom. Specifically, this chapter examines a chain of events that took place in the third year of our collaboration when we used student work as a means of supporting teachers' learning whereas the teachers' actual learning experiences unexpectedly deviated from our conjectures. By presenting an analysis of how the activity around student work became constituted in the PD sessions and the researchers' reflection upon it, I frame our research experience with the collaborating teachers as a case of broader significance, that of supporting teachers' learning across the setting of PD and the classroom.

Using Student Work in Practice-Based Approach to Support Teachers' Learning

The use of records of teachers' instructional practice has increased in popularity in mathematics teacher PD as a promising means to support learning (Ball & Cohen, 1999; Little et al., 2004; Smith, 2003). This trend is undergirded by a strong consensus among researchers and teacher educators that teachers should learn in, from and for practice (Lampert, 2010). A practiced-based approach to teacher PD entails a shift from focusing on what teachers know and believe to what they actually do in the classroom (Ball & Forzani, 2009; Grossman & McDonald, 2008). The actual tasks and activities involved in teachers' work should therefore stay at the center of PD design. In order to recapture the complexity as well as richness of teaching practices in the out-of-classroom PD, researchers and teachers educators often draw on records of teachers' practice and use them to create opportunities for teachers to collaboratively inquire into various aspects of teaching but to do so in a supported and facilitated setting of PD. Grossman and McDonald (2008) describe this approach as decomposition of practice that breaks down the instructional practices that can be complex and messy in the classroom and introduce them into PD for the purpose of teaching and learning.

Student work is particularly favored as a record of practice not only because it constitutes a critical aspect of mathematics teaching but because it affords opportunities for teachers to explore and analyze students' mathematics reasoning. This latter focus on student mathematical reasoning is considered critical to the overall improvement of mathematics instruction (Ball, 1997; Hiebert et al., 2007; Rodgers, 2002; Sowder, 2007; van Es & Sherin, 2010;). As noted by Kazemi and Franke (2004),

> student work, as a tool for professional development, has the potential to influence professional discourse about teaching and learning, to engage teachers in a cycle of experimentation and reflection, and to shift teachers' focus from one of general pedagogy to one that is particularly connected to their own students. (p.3)

However, as Lampert and Ball (1998) have cautioned us, student work does not by itself constitute a curriculum for teachers' learning. It is merely a record of practice, with a promising potential to become an integral component within a thoughtfully laid PD design. It is therefore important for us, researchers and teacher educators, to further articulate and specify the link between student work and the potential PD design for supporting teachers' learning. To fully understand how student work is currently used in teacher PD, I contend that it is necessary to examine three related issues:

- Why student work is used,
- How student work is used,
- Underlying conceptualizations and unanswered questions that are inherent in current ways of using student work to support teachers' learning.

Why Student Work is Used

Student written work is a common classroom artifact originating from the daily routine of instructional practice. Teachers use student work almost everyday to evaluate students' performances, make decisions about instruction, gather and examine school-level achievement data and accomplish many other goals. No matter how it is used, it is without a doubt a necessary constituent in the practice of teaching. The value of looking at student work in PD is two-fold.

First, when used in PD, student work acts as a medium through which aspects of real classroom instruction is represented in the out-of-classroom setting. Because of the realness embedded in student work, there is a good chance that teachers will find the PD activities directly relevant and useful once student work is brought into the focus. In other words, student work has the potential to allow researchers and teachers educators to break the boundaries of PD and the classroom so that teachers experience PD activities as focusing squarely on their instructional practices in the classroom. Additionally, when teaching back at the classroom level, teachers will have the chance to reflect on their learning in PD as they interact with students and student work.

Second, the value of looking at student work also resides in its potential for bringing student mathematical thinking more coherently and explicitly into teachers' instructional practices (Ball, 1997; Carpenter, Fennema & Franke, 1996; Goldenberg, Saunders, & Gallimore, 2004; Schifter, 2001; Steinberg et al., 2004). Research has shown that teaching for understanding requires teachers to place student mathematical thinking at the center of instructional planning and decision making (Ball, 1997; NCTM, 2000; Sowder, 2007). From the researchers' perspective, student work constitutes records of students' mathematical reasoning. It reveals students' interpretation of the problem, and their grasp on the key mathematical ideas as well as their misconceptions. By closely analyzing student work, teachers are invited to the opportunities to expand their understanding of students' mathematical thinking and to develop an appreciation of the diversity of students' methods that may appear surprisingly different from those on which instruction has focused. More importantly, this knowledge is believed to have the potential to inform teachers' instructional planning so that they can anticipate how their students would solve specific tasks, decide which instructional tasks will be appropriate to use, and predict the common misconceptions that students are likely to develop (Chamberline, 2005; Kazemi & Franke, 2004; Katims & Tolbert, 1998; Little, 2004; Saxe et al., 2001).

How Student Work is Used

Although student work can be a valuable resource for supporting teacher learning in PD, designing and orchestrating these PD activities is intricate work. Teachers educators have come to the consensus that merely by bringing teachers together to look at student work does not necessarily lead to learning opportunities for teachers (Kazemi & Franke, 2004; Little, 2004). Meaningful conversations need to take place in order for teachers to learn about students thinking (Ball & Cohen, 1999; Chamberline, 2005).

The nature of the activity. First and foremost, for the activity of looking at student work to be productive, it should move beyond an evaluative stance in which the goal is to assess teachers' instructional competence (Little, 2003). Instead, the activity should provide a supportive platform for teachers to open up integral aspects of their instructional practice and engage in collegial conversations that focus on student learning. Little (2003) addresses the significance of this orientation by drawing on Hutchins' notion of "horizon of observation" (Hutchins, 1996, p.52).

> Hutchins (1996, p. 52) employs the term ''horizon of observation'' to define the extent to which elements of a work environment are available as a learning context. This horizon of observation structures how completely novices or newcomers are able to see, hear, and participate in the work in question: its central tasks, tools and instruments, relevant categories and terms, and lines of communication. (p.918)

When teachers use student work in their own teaching, they often do so in isolation with few opportunities to discuss and share with fellow teachers how they look at or what they think of student work. As a result, their horizon of observation can be extremely limited. To truly make the practice of teaching a focus of inquiry, teachers should be equipped with the opportunity to compare multiple perspectives and explore with certain degree of openness when looking at student work. Only in doing so can they expand their horizon of observation and get to the key dimensions as well as significant nuances that are critical for doing their job effectively. This is, however, much less likely to happen if the activity is merely organized to evaluate rather than inquire.

The implementation of the activity. Allowing teachers to explore student work with certain degree of openness does not mean that it is completely up to teachers to decide how student work should be analyzed and investigated. As a matter of fact, the teacher educators' role in implementing PD activities should orient and support teachers' learning as they examine student work (Chamberlin, 2005; Little, 2003, 2004).

It is generally agreed that for the activity of looking at student work to be productive, teacher educators should actively involve themselves in the discussion and analysis. Their role should be co-participants rather than merely observers. When cross examining three PD programs that involved the use of student work, Little, Gearhart, Curry and Kafka (2003) identified a number of common practices that the teacher educators in these programs employed to shape the conversation about student work.

First, the teacher educators carefully directed the conversation to make sure it was focused on student thinking. When necessary, they reoriented the conversation so that it would not be about the background of the assignment or the student, or about the right and wrong evaluations that teachers would normally give to a student work. Instead, teachers were encouraged to interpret students' solutions even if it appeared illogical (see also Chamberlin, 2005).

Second, the PD facilitators shaped the participation structure as well as the topic of the conversation. They did so by building a series of mini-inquiries into the activity of looking at student work. The teachers would be asked to engage with a specific issue at a time and the facilitators would decide when the teachers should engage in individual analysis or whole-group discussions. If an issue of significance emerged, it would also be the teacher educators' job to direct the group's attention and make sure it was sufficiently addressed. Third, the teacher educators deliberately created opportunities for the teachers to bring to the table their questions and concerns triggered by the activity of looking at student work. In doing so, they needed to guide the development of certain group norms so that the teachers would feel safe to ask questions or challenge each other.

In addition to these three common practices observed by Little et al (2003), Chamberlin (2005) also noted another key dimension in the teacher educators' role when it comes to the use of student work. She argued that it would be critical for teacher educators to question teachers' interpretations of student work as a way to create opportunities for reflection as well as to bring researchers' perspectives to the conversation. As a result, she noted that the teachers developed more insights into their student thinking.

The structure of the activity. When guiding PD activities featuring student work, teacher educators often break down the activity to multiple phases, each having distinct yet related objectives (Kazemi & Franke, 2004). These phases may include:

- Inviting teachers to describe with detail the strategies that they observe from student work,
- Guiding teachers to compare, contrast and categorize the strategies in terms of their level of mathematical sophistication,
- Introducing terminology to label the strategies that reveals the development of key mathematical ideas within the content focus,
- Leading teachers to discuss how they would build on students' strategies in their instructional planning and why.

Although the exact sequence and details may vary, teacher educators often choose to start the series of activities by first inviting teachers to describe and share students' strategies without making judgment. They do so mainly for two reasons. First, this activity is most closely related to what teachers may do everyday in their own classrooms although they may do so from entirely evaluative point of view. Starting with this activity helps to ease teachers in without making them feel threatened. Second, this arrangement gives teacher educators an opportunity to negotiate what it means to look at student work in the context of PD if teachers appear to approach student work in a significantly different way, such as checking the correctness of calculations. For the activity to be productive, it is important for teachers to move beyond the evaluative perspective and look for evidence of students' thinking—what students did to solve the problem, why they chose to do so, and what it revealed about their understanding of the key mathematical ideas at work. When necessary, teacher educators can build on various observations made by the teachers to clarify how student work needs to be interpreted as well as the level of detail that would be helpful.

Underlying Conceptualizations and Unanswered Questions

Various practice-based research projects involving the use of student work have been motivated primarily by a pragmatic concern—how to design PD activities so that teachers' learning in PD and their teaching in the classroom can be productively connected. To this end, a number of important research questions have been explored including:

- What is the potential usefulness of student work in supporting teachers' learning?
- What should be the central focus of looking at student work in PD?
- What should be the role of teacher educators in orchestrating the activity of looking at student work?
- How should the discussions around student work be structured and what may be the leading questions to orient teachers?

In addition to these concrete concerns of what and how to design PD, there yet remains a conceptual challenge of why. As discussed earlier in this chapter, Cochran-Smith and Lytle (1999) have explicitly called for an articulation of researchers' conceptions that underlie PD initiatives. To address this concern, it becomes necessary to examine how the use of student work is justified within the context of supporting teachers' learning, and whether and to what extent such justifications are legitimate even when such conceptions are often implicated rather than explicated in many PD efforts. In this section, I do so by focusing on two conceptual aspects, both of which are critical to the design of PD—the role of student work in supporting teachers' learning across settings, and the assumed relations between looking at student work and changes in teachers' classroom instructional practices. In doing so, I also discuss the unanswered questions involved in these conceptions.

The role of student work. As I have discussed earlier, from the teacher educators' perspective, student work has duel roles. First, it is a critical and original artifact in the classroom. By looking at student work across settings, certain aspects of classroom instruction can be represented in PD in a realistic and lively matter. In other words, the presence of the artifact across both settings is considered to help connect PD to teachers' classroom instruction. A second, equally important aspect in the role of student work concerns its affordances to reify students' mathematical thinking. Again, from the teacher educators' perspective, student work is not merely the end product of a thinking process. Instead, it contains valuable information about the mathematical thinking that students are developing as they engage with these problems. As a result, by closely examining student work, teachers would have the opportunity to focus on students' mathematical thinking.

Although these conceptualizations of the role of student work are undeniably insightful, the process through which these roles can be fully realized may require further investigation. Two concerns are particularly relevant. First, when student work is used to represent aspects of teachers' instructional practice in PD, it is rarely questioned what exact kind of instructional practices are being represented or whether this kind of instructional practice can generate potential conflict with the one that teacher educators have in mind for teachers to develop. Goldsmith and Schifter (1997) have reminded us of the complexity involved in "learning in and from practice."

They point out that teachers who engage in PD activities may not have “useful images from their personal experiences” (p.25) to guide their participation in these activities. Similar concerns are also shared by Sherin (2007). She argues that teacher educators need to be aware of the perspectives that teachers bring to PD, especially veteran teachers who have already developed a habitual way of looking at classroom artifacts and events from years of experience. Hatch and Grossman (2009) further elaborate on this challenge by contending that there are many things that records of practice may not capture directly—things such as “larger contexts in which work may be situated, overarching purposes, histories, and long-term relationships invisible in daily interactions” (Hatch & Grossman, 2009, p.70)—yet will remain salient in influencing how theses records eventually get constituted in PD activities.

In the case of student work, it is not surprising that, given the reality of mathematics teaching and learning in the United States, the routine use of student work in teachers’ instructional practices does not commonly involves a focus on students’ underlying mathematical ideas (Elmore, 2000). Instead, it is more frequently used for evaluative and administrative purposes, thus creating a potential conflict with the way in which student work is intended to be used in the setting of PD. These issues raised by various researchers allude to the need for a deeper and more accurate understanding of what aspects of teachers’ instructional practices are wrapped up in student work and how they may afford or restrain the learning opportunities as teachers engage in looking at student work in PD.

A second, related concern is the extent to which teachers would come to share teacher educators’ perspective which sees student work as records of students’ mathematical thinking. Recall the Algebraic case analyzed by the CGI researchers (Franke et al., 2007). The teachers’ struggles indicated that issues involving mathematical content focus might have been at play. Additionally, in a classroom where teachers' primary use of student work is typically to evaluate student performance, it can be a critical yet difficult shift for teachers to begin to focus on student thinking. Two questions therefore need to be further clarified: How can we be sure as teacher educators that teachers will see what we see? Or if they do not, what can we do to support a shift in their perspectives? Unfortunately, neither question has been addressed sufficiently in current research on student work.

The relations between looking at student work and changes in classroom instructional practices. A typical chain of change that is often assumed in current research efforts can be illustrated by Figure 6.

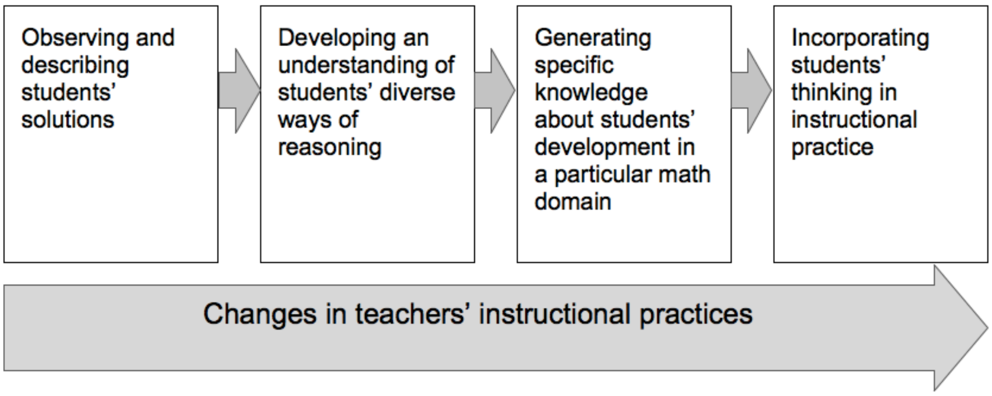

Figure 6. Chain of Change in Teachers' Instructional Practices

In this model of change, it is anticipated that the shifts depicted in Figure 6 will occur as teachers participate in a series of mini-inquiries designed and orchestrated by teacher educators. A critical yet implicit assumption is that the need to place student reasoning at the center of instructional decision making will become self-evident to teachers as they go through these activities. In this model, it is necessary that teachers, at certain point in time, will concur with teacher educators that a focus on student thinking is critical to teaching and learning at the classroom level, and the primary question of concern is merely how to realize this in action. The relations between focusing on student thinking and quality of mathematics instruction therefore are assumed to become a shared understanding among teachers and teacher educators.

However, viewing student thinking in this way may not come naturally for most teachers, especially when we consider the institutional contexts in which they work. Gamoran, Anderson, Quiroz, Secada, Williams, and Ashmann (2003) argue that the institutionalized nature of teaching as a private activity constrains the generation of intellectual and social resources for instructional improvement, as well as the extent to which teaching could become a subject of inquiry at the classroom level. This finding is in accordance with Elmore's (2000) analysis in which he contends that teachers in public schools are often left to their own devices to improve instructional practices. As a result, teaching mathematics at the classroom level often involves idiosyncratic practices, aimed at getting the correct answers and endorsing memorization of procedures with little exploration of significant mathematical ideas (Elmore, 2000; Stigler & Hiebert 1999).

These findings suggest that, prior to their participation in PD, teachers are unlikely to have developed a perspective to value student thinking in their daily contexts of teaching. As a result, an important aspect of teacher educators' job will have to involve supporting a critical shift in perspective so that teachers will come to share the view that student thinking can and should be used as a resource to inform classroom teaching. This shift in perspective is by no means trivial and often requires explicit and conscious support from teacher educators.

Understanding how this shift can be realized is of great value to teacher educators as it allows us to unpack, adapt and refine various means of supporting teachers' learning when working in new PD settings. However, in the current model of change, the significance of this shift in teachers' perspective does not seem to be fully recognized. Even when it does, the specific means of supporting this shift often remains underspecified. From the literature, little can be inferred about the rationales for why certain means of support were chosen and envisioned to be effective in supporting this shift.

In summary, the brief review of literature reveals that the use of student work is currently conceptualized mainly in terms of the two-way movements of the artifact between PD and the classroom. Students' classroom work is usually introduced in PD with the expectation that it will help establish a common ground between the two settings and make classroom teaching a focus of inquiry. Once student work is analyzed and discussed in the context of PD, teachers are expected to use it back in the classroom as a resource for instruction that orients them towards student thinking. In other words, the presence of the same artifact across both settings is considered critical in connecting PD to teachers' classroom instruction. However, there is little explicit discussion in the literature about how researchers view the relations between the perspectives and experiences that teachers bring to the PD via student work and the learning they take back to the classroom (Kazemi & Hubbard, 2008). It seems to be important that researchers examine what student work actually reifies from the teachers' perspective, especially when the record of practice is often chosen by researchers with a pre-assumed understanding of its meaning.

For the activity of looking at student work to be effective, two critical shifts in perspective have to take place. The first shift involves teachers coming to see student work as records of students' reasoning and second seeing students reasoning as a resource to plan for instruction. In my view, to fully understand of how these shifts can be supported in the context of PD, teachers educators need to reconceptualize the use of student work in a way that goes beyond the two-way movements of artifact but instead focus on the practices that teachers are developing as they engage in activities across different settings.

A common thread can be found between the unanswered questions that I have noted and Little's research on the use of student work (Little, 2003; Little, 2004; Little et al., 2003). When considering the significance of these issues, she wrote:

> In some respects, these dilemmas may be resolved with the simple passage of time, as groups gain familiarity and facility with particular procedures. Persistence matters, and some tradeoffs remain inevitable. However, we argue that groups would also benefit from tackling the dilemmas head on, reserving time to reflect on the assumptions underlying a given protocol or process and the degree to which it provides a fit with the participants' own purposes and resources. (p. 191-192)

The ensuing analysis in this chapter offers an initial effort to examine in detail a dilemma that a group of middle-school mathematics teachers encountered as they engaged in looking at student work in PD. Their dilemma led us, the researchers, to examine our assumptions that underlay our design decisions around student work. It was through this research experience that the need to reconceptualize the relations between teachers' learning in PD and their instructional practices in the classroom began to emerge.

Background of the Analysis

As I have discussed earlier, the analysis documented in this chapter draws on a paradigmatic case in which student work was used in the context of working with a group of middle-school mathematics teachers. In this section, my goal is to situate the analysis by discussing in detail various aspects that are relevant to understanding our work with the teachers.

Participants and Research Setting[3]

The data in this analysis were collected during the third year of a five-year PD design experiment (cf. Cobb, Confrey, diSessa, Lehrer & Schauble, 2003) with a group of middle-school mathematics teachers who worked in a large urban district in the southeast United States that served a 60% minority student population. The district was located in a state with a high-stakes accountability program in which students were tested in mathematics at each grade level. School and district administration responded to the accountability pressures of state standardized tests by attempting to monitor and regulate teachers' instructional practices. The research team was invited by the district's mathematics coordinator and provided teacher development in statistical data analysis. At the time when we started to work in the district, the teachers' informal professional networks were extremely limited and their mathematics instruction was highly privatized (Cobb et al, 2003; Dean, 2004, 2005). In addition, the teachers' instructional practices were rather homogeneous, focusing primarily on covering instructional materials rather than attending to students' mathematical reasoning.

In the first two years of our collaboration, the group was consisted of nine teachers, all selected by the district's mathematics coordinator. At the beginning of the third year, three teachers left the group[4] and six teachers from the same school district were invited to join. Throughout the third year, the group consisted of a total of 12 teachers working in five different schools in the same district. Additionally, the

---

[3] Pseudonyms are used in this analysis to protect the identities of the participating teachers as well as the school district.
[4] The three teachers left the group for reasons including job relocation, pregnancy and change of career.

district's mathematics coordinator participated regularly as a member of the group. In the first year of our five-year collaboration, the research team conducted a two-day initial summer session and three one-day work sessions during the school year, followed by a three-day summer session at the end of the school year. The team then conducted six one-day work-sessions and a three-day summer session during each school year for the remainder of our collaboration.

The overarching goal in working with the teachers was to support their development of instructional practices that place student reasoning at the center of their instructional decision making. To this end, we engaged the teachers in activities from a statistical instructional sequence that was developed during prior NSF-funded classroom design experiments (Cobb, 1999; McClain & Cobb, 2001). The instructional sequence was justified in terms of both the emergence of successive forms of statistical reasoning that became normative in the design experiment classrooms and the specific means that supported their emergence (Cobb, 1999; Cobb, McClain, & Gravemeijer, 2003). It was specifically designed to support the kind of instructional practices envisioned by the current mathematics education reform.

In our work with the teachers, the statistical instructional sequence was critical in supporting the teachers' as well as their students' learning. We conjectured that if we could support the teachers' reconstruction of this instructional sequence including its underlying rationales in the context of PD, they would be able to adapt, test, and modify the sequence in their classrooms and thus effectively support their students' statistical learning. It is worth stressing that in order for teachers to use the statistical instructional sequence effectively in their own classrooms, it was essential that they develop relatively sophisticated understandings of statistical ideas. Supporting teachers' learning of statistics was central to our work with the teachers throughout the entire collaboration.

The Statistical Instructional Sequence

In the classroom design experiments in which the statistical instructional sequence was originally designed, tested, and refined (Cobb & McClain, 2004; McClain & Cobb, 2001), the researchers have identified five aspects of the classroom environment that proved critical in supporting the students' statistical learning. They are:

1. The focus on central statistical ideas
2. The computer-based tools the students used
3. The instructional activities
4. The organization of classroom activities
5. The whole-class discussion of students' analyses

From our perspective as researchers, it was important that the collaborating teachers became aware of the importance of attending to these five means of support while planning and conducting statistics lessons. As a result, these five aspects oriented our work with the teachers and were apparent in the organization of the PD activities.

The focus on central statistical ideas. The central mathematical idea that guided the design of the instructional sequence as well as our work with the teachers was that of distribution (Cobb, et al., 2003; Dean, 2005). We considered it critical that students would come to reason about data in terms of distributions. Notions such as mean, mode, median, skewness, spread-outness, and relative density would then be viewed as ways of describing how specific data sets are distributed. As a result, various statistical representations or inscriptions would emerge as different ways of structuring data.

The computer-based tools. Three computer-based tools were developed as a primary means of supporting students' learning while simultaneously providing them with tools for data analysis. As noted by Cobb and McClain (2001), each tool offered students several ways of structuring data. Students could order, partition, and otherwise organize data points in a relatively immediate way. Importantly, as Cobb (1999) noted, "these options do not correspond to a variety of conventional inscriptions. Instead, we drew on the research literature to identify the various ways in which students structure data when given the opportunity to conduct genuine analyses" (Cobb, 1999, p.12). Two computer tools were primarily used in our work with the teachers during year 3.

The first computer tool was designed to facilitate students' initial explorations of univariate data sets. It provides a means of ordering data values, partitioning, and otherwise organizing small sets of data in a relatively immediate way. Each individual data point is inscribed as a horizontal bar, the length of which signifies the numeral value of the data point. The color of each bar could be either pink or green, thus enabling two data sets to be contrasted and compared. In the case of the Blood Drive activity, data were generated to compare the number of people who donated blood at two types of locations, supermarkets parking lots and community centers, so that the Red Cross could decide which type of locations attracted more blood donors.

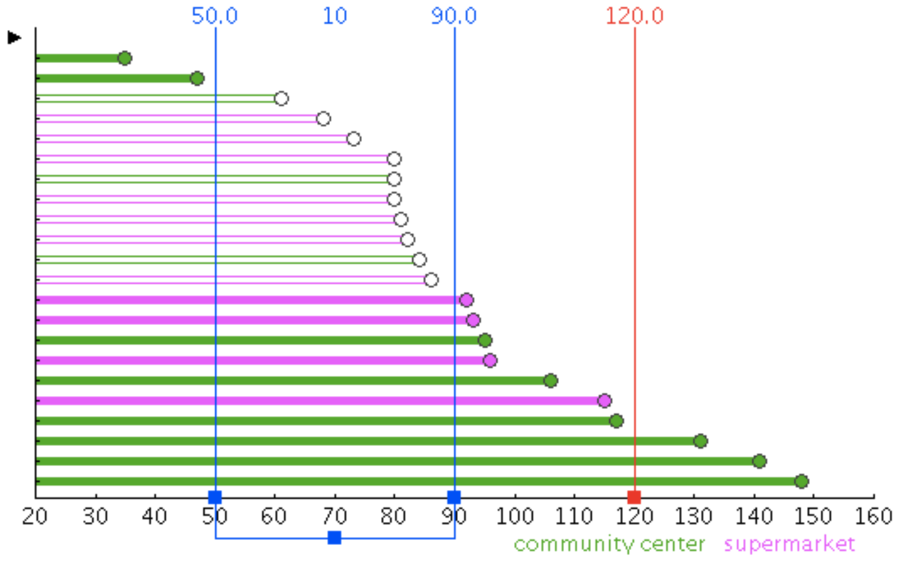

Figure 7. The Blood Drive Dataset in the First Computer Tool as Sorted by Size

Each bar in Figure 7 shows a single data value, and in the case of the Blood Drive activity, the number of people who donated blood at one blood drive. The teachers could sort the data by size and by color. In addition, they could hide either data set, and could also use the value bar (shown as the red vertical line in Figure 7) to partition the data sets and to find the value of any data point by dragging it along the horizontal axis. Further, they could find the number of data points in any interval by using the range feature (shown as the blue lines in Figure 7). The 11 green bars in Figure 7 represent the number of people who donated blood when 11 blood drives were conducted at community centers. Similarly, the 11 pink bars show the results when blood drives were conducted at supermarkets.

The purpose in grounding tasks in specific problem scenarios (e.g., deciding whether to hold future blood drives at supermarkets or community centers) was to support students in viewing the data as measures of a relevant attribute of a real phenomenon (e.g., the effectiveness of a blood drive) rather than merely as numbers without context. A second goal was to support students in organizing the data in a manner that would give insight into the question under investigation. The data were specifically design so that two types of arguments would likely to be brought up, one that compared the total of each dataset (i.e., the total number of people who donated blood at the community centers and at the supermarkets respectively) and a second that focused on the variability of each dataset (e.g., the number of donors at the supermarket locations was more consistent than the number of donors at the community center locations). It would then be important for students to discuss which argument is more insightful by situating the numbers within the problem context in which they are examined.

The second computer tool can be viewed as an immediate successor of the first in that the endpoints of the bars that each signifies a single data point in the first tool have, in effect, been collapsed down onto the axis so that data are now inscribed as an axis plot as pictured in Figure 8.

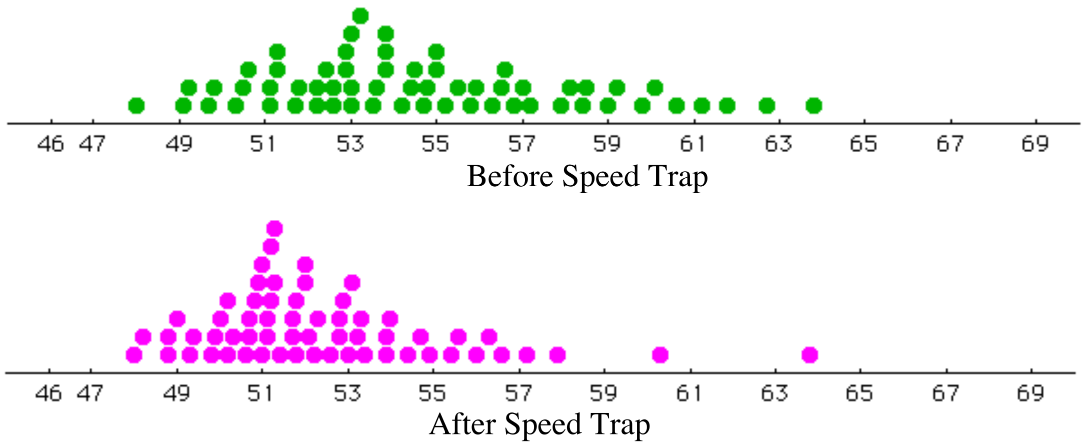

Figure 8. The Speed Trap Dataset in the Second Computer Tool

The instructional intent when designing the second computer tool was to support the emergence of more sophisticated ways of comparing and analyzing datasets with larger or sometimes unequal numbers of data values. The tool offers a range of ways to structure data. The mostly frequently used ones include structuring the data by (1) making your own groups by dragging one or more value bars to chosen locations on the axis in order to partition the data set, (2) partitioning the data into four equal groups so that each group contained one-fourth of the data (precursor to the box-and-whiskers plot) and (3) organizing data into groups of a fixed interval width along the axis (precursor to the histogram) (Cobb, 1999; McClain & Cobb, 2001). The number of points in each partition is shown on the screen and adjusts automatically as the bars are dragged along the axis.

One type of instructional activity designed for the second computer tool involved analyzing data sets with equal data values. For example, the Speed Trap activity involved comparing speeds of cars before and after a speed trap was put in place (see Figure 8) in order to decide whether the speed trap was effective. The data consisted of speeds of 60 cars measured before the speed trap was put in place, and speeds of another 60 cars two months later after the police implemented a speed trap on the same section of the highway. Each dot represents the speed of a car in miles per hour.

A second type of instructional activity involved analyzing data sets with an unequal number of data values. For example, the Migraine task involved analyzing data on the amount of time it took patients to get relief from a migraine headache (see Figure 9).

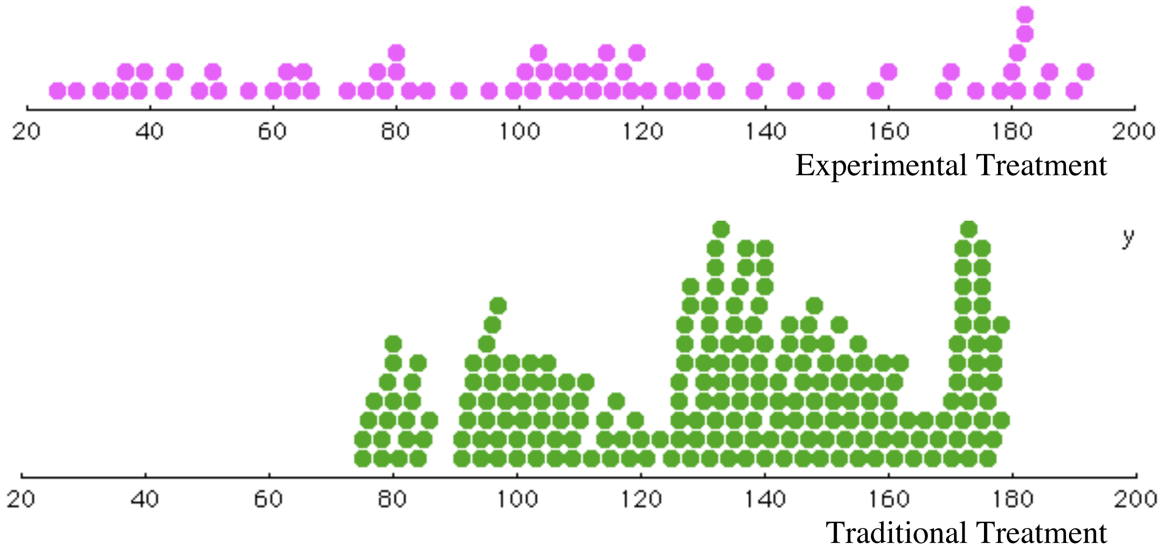

Figure 9. The Migraine Dataset in the Second Computer Tool

Each dot represents a patient's relief time measured in minutes. Two hundred and twenty people were treated with a traditional drug represented by the green dots. Sixty-eight people were treated with an experimental drug represented by the pink dots. Students were asked to make a recommendation to a hospital director about which drug should be used to treat migraines. The intent of this instructional activity was to support students in reasoning multiplicatively in terms of relative frequency rather than additively in terms of absolute frequency when they compared data sets with unequal number of data values.

The instructional activities. The instructional activities designed for the statistical sequence involved the investigative spirit of genuine data analysis from the outset. Similarly, in our work with the teachers in which the statistical instructional sequence played a central role, it was pivotal that the teachers came to view the nature of the activity as genuine data analysis rather than as manipulating numbers to produce the correct answer. As shown by the analysis of students' learning during the statistical design experiment (Cobb, 1999; Cobb, McClain, & Gravemeijer, 2003), this orientation in turn ensured that students would come to view these activities as realistic for a purpose that they consider legitimate. The instructional activities that were designed to this end involved either (a) analyzing a single data set in order to understand a phenomenon, or (b) comparing two data sets in order to make a decision or judgment (e.g., analyzing the T-cell counts of AIDS patients who had enrolled in two different treatment protocols to determine which treatment is more successful). The end product of these instructional activities typically involved students writing a report of their analyses to present to a specific audience to help make a decision based on their analyses (e.g., the chief medical officer of a hospital who will use the reports to make a decision about which treatment the hospital will adopt).

In addition, it proved critical during the statistical design experiment that the instructional activities should focus on significant statistical ideas. In approaching this challenge, Cobb and his colleagues viewed the various data-based arguments that the students produced as a primary resource on which the teacher could draw to orchestrate whole-class conversations that focused on significant statistical ideas.

The organization of classroom activities. The organization of classroom activities needed to support the investigative spirit of genuine data analysis while providing opportunities for significant statistical ideas to emerge (Cobb & McClain, 2004). An activity during the statistical design experiment typically involved (a) a whole-class discussion of the data generation process, (b) an individual or small-group activity in which the students usually worked at computers to analyze data, and (c) a whole-class discussion of the students' analyses.

In the first phase, the teacher introduced the instructional activity by talking through the data generation process with the students. These conversations often involved discussions during which the teacher and students together framed the particular phenomenon under investigation (e.g., migraine), clarified its significance (e.g., the importance of developing more effective treatments), delineated relevant aspects of the situation that should be measured (e.g., quickness of relief), and considered how they might be measured (e.g., recording patients' relief time). The teacher then introduced the data the students were to analyze as being generated by this process. The purpose of the data generation discussion is to ensure that students would come to view the data as measures of an aspect of a situation and their job to analyze data in order to answer questions that they agreed were significant. This orientation helped steer students away from seeing data as merely numbers handed down to them.

During the second phase of an instructional activity, the students then analyzed the data either individually or in small groups using a computer tool. The final product of their investigation involved written solutions in which data would be presented in a particular way to support their decision or judgment and a letter written to a specific audience explaining how the conclusion was derived. The third and final phase of an instructional activity consisted of a whole-class discussion of the students' analyses during which the various ways in which the students had structured the data were compared and contrasted so that significant ideas would emerge and become a topic of conversation for the entire class.

The findings from the statistical design experiment guided the structure of the PD activities to a considerable extent. While working with the teachers, the research team frequently engaged the teachers first as learners in a selected statistical instructional activity before asking them to teach the same activity in their own classrooms. The researchers organized the activity into the above three phases when supporting the development of the teachers' understanding of the key statistical ideas.

The group would then collectively talk about why it was important to organize the activity as such and the significance of each of the three phases by drawing on their own experiences as learners. We conjecture that teachers' own learning experiences would help them reflect on the specific aspects of the learning environment that were conducive to their learning of statistics and, as a result, begin to attend to these aspects when they later taught statistical activities in their own classrooms. It is worth noting that our goal was not for the teachers to imitate the moves of an expert teacher. Rather, the focus was to enable them to examine the underlying pedagogical justifications so that they would be in a position to begin to make informed decisions and judgments about how they might adapt the instructional sequence to their own classrooms.

Whole-class discussion of students' analyses. The final key aspect of a classroom environment that Cobb and colleagues identified involved a whole-class discussion of students' analyses. It is as students explain and justify their analyses during a whole-class discussion that a teacher has the opportunity to discuss what count as acceptable data-based arguments. In the statistical design experiment, the teacher and students established relatively early in the experiment that an acceptable argument had to justify why certain method of structuring the data was relevant to the question under investigation. This establishment of norms delegitimized analyses in which students simply produced a collection of calculations (e.g., mean, median, range) rather than attempted to identify trends and patterns in the data that were relevant to the issue under investigation.

Additionally, it is critical that the whole-class discussion is organized so that mathematically significant issues that can advance the instructional agenda become explicit topics of conversation. During the statistical design experiment, the teacher's interventions were critical in ensuring that this occurred. She carefully planned for the whole-class discussion to capitalize on the students' reasoning and did so by identifying data analyses that, when compared and contrasted, might give rise to substantive statistical conversations (McClain, 2002).

In our work with the teachers in PD, whole-class discussion also constituted a means for supporting the teachers' learning in two important ways. First, as we engaged the teachers as learners in analyzing data, we used whole-class discussions as an opportunity to negotiate with them norms or standards for what counted as an acceptable data-based argument, ensure that significant statistical ideas emerged as topics of conversation, and keep the teachers engaged as their own analyses became the topic of the discussion. Second, because the teachers were expected to teach with their students the statistical activities that they had themselves analyzed, the research team conjectured that it would be critical to discuss with the teachers what kind of whole-class discussion should be organized and how to organized it so that their students would find the activity of analyzing data worthwhile. These topics of conversation were frequently addressed during the work sessions and became especially explicit when the PD activities focused on student work. The teachers were then asked to think about how to build on students' solutions as they planned for the whole-class conversation.

In summary, the five aspects that proved effective in supporting the students' learning in the statistical sequence guided the research team's view of what counted as a productive statistical activity in working with the teachers. More importantly, they shaped what we considered to be viable means to support the emergence of a productive statistical activity when we designed the PD activities to engage the teachers.

Documented Developments during the First Two Years

In her dissertation, Dean (2005) summarized three important developments that took place in the first two years of our collaboration with the teachers. These developments constituted the background against which student work was used in year 3 as a means of supporting teachers' focus on students' reasoning in their instructional planning.

The first development occurred nineteen months into the collaboration when the group evolved into a genuine professional teaching community (Grossman, Wineburg, & Woolworth, 2001). Dean made a distinction between a group of teachers who merely meet to discuss issues of mutual interest and a professional teaching community based on the three criteria that Wenger (1999) proposed to define a community of practice. The three criteria are joint enterprise, mutual engagement, and a shared repertoire. Towards the end of the second year, the teacher group was characterized by

- a joint enterprise of improving mathematics instruction and student learning in which the goal was to ensure students understand the central mathematical ideas and be able to perform more than adequately on standardized tests and to acquire and utilize resources to make that possible,
- identifiable norms of members' mutual engagement which involved the teachers freely challenging and critiquing each other's reasoning as well as an increasingly normative obligation to justify their pedagogical arguments in terms of supporting students' mathematical learning rather than the need to cover the content for the standardized tests,

- a shared repertoire of tools and practices for achieving the common goals such as normative ways of reasoning with the computer-based tools and classroom artifacts that were introduced to the PD activities (e.g., student work).

The second noteworthy development involved a gradual but salient change in the way that the teachers perceived institutional constraints in relation to their teaching as well as the feasibility of changing such conditions. At the beginning of the collaboration, the teachers considered themselves to have little control over the instructional decisions that were made in their schools, and the pressure to cope with the standardized tests with very limited resources available was a source of frustration for the teachers. During the PD work sessions, through lengthy conversations about the affordances and constraints of the teachers' institutional context, the group started to view themselves no longer as merely passively constrained by their institutional context. Instead, they worked towards the goal of involving school administrators in conversations about how to improve mathematics teaching and learning. This way of reasoning with the institutional context continued to develop during year (Visnovska, 2009).

Last but not the least, the teachers became increasingly proficient in statistical data analysis as they engaged as learners with the statistical instructional sequence. This development was significant given the fact that the teachers initially had limited experience in analyzing data with a focus on variability and distribution (Dean, 2005). By the end of year 2, the teachers had become relatively proficient in developing data-based arguments that focused on the big idea of distribution. In doing so, they could use the computer-based tools effectively to organize and analyze data, reasoned about data multiplicatively, and developed a variety of strategies to analyze data based on trends and patterns that gave insight into the questions they were addressing.

As the teachers developed sophistication in analyzing data, the research team started to support them in adapting the statistical instructional activities to their classrooms. At about the same time, the teachers voiced a desire to engage in joint planning and to observe each other's teaching. Towards the end of the second year, the teachers were comfortable with videotaping their teaching of the statistical instructional activities and discussing the videotaped lessons within the professional teaching community.

Given these prior developments that occurred during the first two years, the research team conjectured it would be feasible to use student work in the PD sessions. In doing so, our primary goal was to support the teachers in focusing on students' statistical reasoning and in using student reasoning as a resource when they planned for instruction.

The Researchers' Initial Conceptualization of the Use of Student Work

In year 3, the research team built on the previous developments within the professional teaching community as we continued to support the teachers' mathematical as well as pedagogical reasoning. The specific goals while working with the teachers involved supporting teachers in: (1) deepening their own statistical understanding, (2) making sense of the pedagogical intent of the instructional sequence (by means such as identifying key statistical ideas), (3) developing a perspective to examine students' statistical solutions, and (4) using students' current statistical reasoning as an instructional resource to support the forms of statistical reasoning toward which the instructional sequence is built (Visnovska, 2009).

Typically, during each one-day work session, we engaged the participating teachers in a series of activity cycles that centered on instructional tasks selected from the statistical instructional sequence. The teachers would first participate in these instructional tasks as learners before they used them with students in their classrooms. As discussed earlier, we conjectured that the teachers' own learning in these statistical activities would constitute an experiential referent when they later engaged in discussions about the pedagogical intent of the instructional sequence as well as means for supporting their students' learning of statistics. During the work sessions, a complete cycle of activity typically involved: (1) the teachers solving a selected task from the statistical sequence during the work session, (2) the researchers leading teachers to reflect on their own learning of statistics, (3) the teachers teaching the same task with their students after the session, and (4) the teachers bringing students' written work to the following work session for group discussion.

Based on the prior developments within the professional teaching community (mainly in terms of teachers' increasingly sophisticated statistical reasoning and deprivatized classroom practices), we conjectured that it would be sensible to use student work as a means to support the teachers in focusing on students' statistical reasoning. It is worth clarifying that from the outset of our collaboration with the teachers, we attempted to move beyond the uni-directional conceptualization of the relation between teachers' learning in the PD sessions and their instructional practices in the classroom. Our conceptualization at that time was consistent with an interrelated conceptualization that emphasizes a focus on artifacts or practices originating from the classroom. The focus on student work in the third year of our work was aligned with Ball and Cohen's (1999) vision for PD that centers on classroom-related materials and teacher experiences "immediate enough to be compelling and vivid" (p.12). Specifically, our decision to focus on student work was based on three rationales. First, because student work was an integral aspect of teaching, we conjectured that making it a focus of activity would enhance the pragmatic value of the PD sessions in relation to the teachers' classroom practices. In addition, we conjectured that the teachers would openly critique and challenge each other's interpretations of student work because teaching was now deprivatized. Finally, we conjectured that open discussions of this type would give rise to opportunities for the teachers to gain insight into the diversity of their student reasoning that would be useful when they attempted to build on their students' solutions in their instruction, especially when they conducted whole-class discussions. These interrelated rationales reflected both our conscious effort to build on the teachers' classroom practices in PD and our conceptualization of the relations between the teachers' activities in PD sessions and their classrooms at that time.

An important goal we had planned for the activity of looking at student work was to support the teachers' ability to orchestrate effective whole-class discussions in which students' various statistical strategies would become the focus of conversation. As I have clarified earlier, in our view as researchers, the role of whole-class discussion is crucial in supporting students' learning of mathematics. On the one hand, it makes mathematically significant issues topics of discussion, creating opportunities for students to challenge and justify each other's solutions; on the other hand, an effectively orchestrated whole-class discussion draws on students' solutions, thus making them feel valued for their mathematical contributions, and as a result, gives a reason for student to engage in discussing mathematically significant ideas.

Importantly for the researchers, a whole-class statistical discussion involves more than students standing up and taking turns to present their solutions. Instead, it should involve teachers carefully selecting the mathematical solutions worthy of discussion, eliciting significant mathematical explanations and justifications from individual students to share with the entire class, and sequencing students' solutions so that important mathematical ideas can be compared and contrasted. In order for teachers to orchestrate productive whole-class discussions, the research team considered it important for them to develop a perspective on interpreting and assessing various statistical strategies that their students developed. Student work, in our opinion, would be a valuable resource to this end.

In summary, a critical aspect in our collaboration with the teachers in year 3 involved the use student written work. We anticipated that discussions of various students' statistical solutions would serve as an important instructional resource for the participating teachers, especially when they were to plan and orchestrate a whole-class conversation. However, as will be documented in the ensuing analysis, the activity of looking at student work did not go as we intended.

Data

The data in this analysis were collected as part of the five-year collaboration with a group of middle-school teachers. For the purposes of this analysis, two main bodies of data are examined, one documenting the PD sessions and the other teachers' classroom practices. Both bodies of data were collected during the third year of our collaboration with the teachers, which occurred during the 2002-2003 school year. The PD data involved data collected from the six one-day work sessions throughout the entire school year during which student written work was used as a means to support the teachers' learning. The classroom data that are examined in this analysis involved modified teachings sets that were collected at the end of the school year after the completion of all six work sessions.

Data Collected from the PD Sessions

The six one-day work sessions conducted during the third year of the collaboration with the teachers were held either at the district staff development center or at one of the middle schools in the district where the participating teachers worked. We video- and audio-recorded all work sessions using two video cameras and three audio-recorders. Additional audio recordings were also made to document the individual group discussions that were often held between several teachers. A set of field notes were created by a member of the research team and shared with other members after the work session. Sometimes when significant developments were noted during a work session, a written summary was generated and shared among the research team. Additional data also included copies of all material artifacts produced by the teachers during the work sessions such as copies of student work that the teachers analyzed, their written analyses of statistical problems, their written responses to questions posed by researchers, and chart paper that recorded significant ideas or issues raised during both whole-group and small-group discussions. The final data source that is relevant to my analysis is a log of the research team's ongoing design conjectures. This log included audio-recordings of all debriefing meetings and written records of design conjectures as we planned for PD activities, significant developments that took place in teachers' learning, and revised conjectures for supporting the collaborating teachers' learning.

Data Collected to Document Teachers' Classroom Practice

A modified teaching set (Simon & Tzur, 1999) was generated twice a year to document the instructional practices of the participating teachers. A modified teaching set entailed videotaping each teacher's lesson and then conducting follow-up audio-recorded teacher interviews on issues that emerged in the course of instruction. The observed classroom session served as a context within which the teachers could be oriented to address issues that were of research interest to the interviewer. The purpose of collecting a teaching set (as opposed to classroom observations or teacher interviews alone) was to understand teachers' practices, including the rationales behind their instructional decision making as well as their perceptions of instructional successes and challenges.

The modified teaching sets that are examined in this analysis were collected at the end of year 3. However, they were originally unscheduled but then conducted specifically for the purpose of accounting for the unexpected ways through which the teachers examined student work during the work sessions previously held during year 3. To this end, the interview questions were purposefully grounded in concrete episodes from the teachers' classroom instruction, instructional planning and decision making. The data that were generated this way allowed us to interpret teachers' interview responses within the context of their classroom instruction, while at the same time gaining access to the key instructional goals and struggles that these teachers were facing in their classrooms. During the interviews, we typically asked the teachers to

- describe the class they were teaching,
- describe the instructional goals they had planned for this lesson, especially the instructional challenges that they thought they might

encounter and the means through which they planned to overcome these challenges,

- evaluate the extent to which their instructional goals were fulfilled in the lesson and whether future adjustments might be needed and why,
- explain how the lesson relate to the ones that were before and after it,
- explain why or why not the teacher chose to organize the instruction in a particular way (e.g., whole-class discussion vs. small group activity or students' individual work),
- articulate what they think the students were learning from the lesson and why they were learning.

A more detailed account of the interview protocols is included in Appendix A. It is important to stress that the interview protocol was not meant to be followed strictly by the interviewers. On the contrary, it was critical that the interviewer could adjust the protocol spontaneously to address issues of potential significance that emerged during the classroom observation or the interview.

At the end of the third year of our collaboration, I and another member of the research team worked together to collect the teaching sets on all except one participating teachers over a period of a week[5]. During the interviews, we were cautious about the social context that was co-constructed between the interviewee and the interviewer and particularly made sure that the teachers did not perceive themselves to be evaluated or feel their professional status threatened. A debriefing meeting was conducted each day between the members to formulate tentative interpretations of the collected data thus far. When needed, the semi-structured

---

[5] The teacher did not participate in this data collection due to personal scheduling problems.

interview protocol was adjusted to include the emerging issues of significance. Due to technical problems, among the 11 collected teaching sets, one teacher interview was inaudible and therefore excluded from the analysis.

As a member of the research team, I was involved in all aspects of data collection. This included contributing to the development of the semi-structured interview protocols, collecting modified teaching sets, contributing to the generation of the research log, and planning as well as implementing the PD sessions. I was also responsible for developing field notes, making video and audio recordings, and collecting and organizing all materials that the teachers produced in our collaboration.

Method of Analysis

The overarching goal of the analysis is to understand the relations between teachers' activities across different settings. It then becomes imperative that data collected in both the PD and the classroom be analyzed and examined in a coordinated manner. Two sub-analyses can be further identified. The first sub-analysis is to document how the activity of looking at student work became established during the work sessions in year 3, and to illustrate how and to what extent this way of reasoning with student work was different to or compatible with our intent as researchers. Switching the locus from PD to the teachers' classrooms, the second sub-analysis is to examine teachers' classroom instructional practices and thereby generate insights into why the teachers came to use and reason with student work in that particular way. Taken together, these two analyses provide a basis for reconceptualizing teachers' activities across different settings. In the following discussion, I clarify the method of analysis for the two bodies of data respectively.

Analysis of the Activity of Looking at Student Work in the PD Sessions

The goal for analyzing teachers' activities in the PD sessions is to understand how the activity of looking at student work became established during the work sessions in year 3. In doing so, I document the normative ways in which the teachers came to use and reason with student work as they participated in the PD activities.

Before I discuss the specific approach that I used to analyze the normative practices around student work, it is important that I first clarify the notion of a norm. A norm can be identified by discerning patterns and regularities in the ongoing interactions of the members of a group. It is seen not as an individualistic property but instead a joint or collective accomplishment of the members of a group (Voigt, 1995). A primary consideration when conducting analyses of this type is to be explicit about the types of evidence used to determine that a norm has been established so that other researchers can scrutinize the rigor of the analysis (Dean, 2005).

A first, relatively robust type of evidence occurs when a particular way of reasoning or acting that initially has to be justified is itself later used to justify other ways of reasoning or acting (Stephan & Rasmussen, 2002). A second, more robust type of evidence is indicated by Sfard's (2000) observation that normative ways of acting are not mere arbitrary conventions for members of a group that can be modified at will. Instead, these ways of acting are value-laden in that they are constituted within the group as legitimate or acceptable ways of acting. This observation indicates the importance of searching for instances where a teacher appears to violate a conjectured group norm in order to check whether his or her activity is constituted as legitimate or illegitimate. In the former case, it would be necessary to revise the conjecture whereas, in the latter case, the observation that the teacher's activity was constituted as a breach of a norm provides evidence in support of the conjecture (cf. Cobb, Stephan, McClain, & Gravemeijer, 2001). Finally, a third and even more direct type of evidence occurs when the members of a professional teaching community talk explicitly about their respective obligations and expectations. Such exchanges typically occur when one or more of the members perceive that a norm has been violated.

In documenting the normative practices that became established when the teachers looked at student work in the PD sessions, I focused on three types of practices, including

- the normative way in which the teachers described students' solutions,
- the normative way in which the teachers categorized students' solutions and ranked them according to their level of mathematical sophistication,
- the normative views on how these students' solutions could be used to inform instructional planning and decision making.

These three types of practices each constituted a specific focus of activity when we engaged the teachers in looking at student work. It therefore makes sense to track the successive patterns of interactions as the teachers engaged with student work with each specific focus in mind. Given that the primary goal for the use of student work in the PD sessions was to support a focused attention on students' reasoning, it is important that the analysis of the normative practices around the use of student work explicate whether and to what extent student reasoning was made visible in the PD activities.

The specific approach that I used to analyze data collected from the work sessions involves a method described by Cobb and Whitenack (1996). Adapted from Glaser and Strauss' (1967) constant comparative method, this approach was developed for analyzing longitudinal data sets that are generated during design experiments. Initial tentative and eminently revisable conjectures are continually tested and revised while working through the data chronologically during the retrospective analysis. As new episodes are analyzed, they are constantly compared with current conjectured themes or categories, resulting in a formulation of claims or assertions that span across the entire data set but yet remain empirically grounded. As noted by Glaser and Strauss (1967), negative cases that appear to contradict a current category are of particular interest and are used to further refine the emerging categories. This process involves four phases that I describe below.

The first phase of analyzing the normative practices around the use of student work entailed reading through the field notes of the work sessions in chronological order and documenting the episodes when student work was used as a focus of PD activities. In this phase of analysis, my purpose was to create a preliminary log of PD activities involving student work. Using the preliminary log as a guide, the second phase involved working through the data corpus generated during the first phase to create a content index of the specific foci of activities. As discussed earlier, the analysis focused on three types of normative practices that became established as the teachers examined student work in PD. I therefore classified the identified episodes as either describing solutions, categorizing and ranking solutions, or planning instruction with student solutions based on the specific focus of the activity. Episodes that do not fall into these categories were classified as others. In this phase of analysis, my purpose was to create a content index without differentiating between the episodes in terms of how well they might reveal normative practices of the group.

In the third phase of the analysis, I relied on the previously created content index while working chronologically through the data for a second time. My goal in this phase was to identify episodes that contained direct evidence of the three types of normative practices established within the group. I supplemented the content index created in the second phase with detailed descriptions of the identified episodes. The focus of analysis of the three normative practices around student work is illustrated in the Table 1.

Normative practices	Focus of analysis
The normative way in which the teachers described students' solution types	The analysis documents the extent to which the teachers became obliged to interpret students' solutions in terms of students' mathematical reasoning
The normative way in which the teachers categorized and ranked students' solutions according to their level of sophistication	The analysis documents the extent to which the teachers became obliged to categorize students' solutions in terms of whether the solutions reflected the big idea of distribution and how effectively the data were structured to reflect it.
The normative views on how these students' solutions could be used to inform instructional planning and decision making	The analysis documents the extent to which the teachers became obliged to justify their instructional decisions in terms of the diverse ways students' reasoning identified in student work.

Table 1. Focus of Analysis of Normative Practices around Student Work

While working through the data in the third phase, I formulated, tested, and refined conjectures about each normative practice and documented the evidence for my claims. Each claim was substantiated or modified while analyzing subsequent episodes. In other words, the conjectured normative practice of the group was subject to further testing and, if necessary, revision until the last episode in the data corpus was analyzed. The completion of this process resulted in an empirically grounded chains of conjectures, refutations, and revisions that focus on the normative ways that the teachers came to use and reason with student work during the PD sessions.

In the last phase of the analysis, I examined the resulting chain of conjectures and refutations from the third phase against the conjectures that guided the design of the PD session activities involving the use of student work. My purpose in involving our design conjectures in the analysis was to make explicit the intended ways of reasoning with student work that the research team aimed to support as well as the extent to which the teachers' learning in the work sessions either fit with or deviated from these intentions. In this way, I juxtaposed the chain of conjectures and refutations of the teachers' actual ways of reasoning with student work with the analysis of the collected teaching sets in order to arrive at a rich explanatory account of the teachers' learning in the work sessions. The final product of the last phase is an empirically grounded account of the teachers' learning in the PD supplemented with explicit reference to their instructional practices in the classroom. The resulting account is bi-directional in nature, consisting of a network of mutually reinforcing assertions that span across both the data collected from the PD work sessions and from the collaborating teachers' classrooms.

Analysis of the Modified Teaching Sets Collected from the Classroom

My purpose in analyzing the modified teaching sets in this chapter was to develop an account of the collaborating teachers' instructional practices as well as their conceptions towards teaching and learning. In doing so, I specifically look for explanations of why the teachers looked at student work in a way that deviated from our intention[6]. In analyzing the modified teaching sets, I took the position that the perspectives and instructional practices the teachers had developed were reasonable and coherent from their viewpoints (Leatham, 2006; Simon & Tzur, 1999). Operating on the basis of this assumption enables researchers to avoid characterizing teachers and their practices solely in deficient terms. Instead, researchers are obliged to look for explanations of teachers' instructional practices—even when they may seem insensible or ineffective from an expert's perspective—until they develop a coherent account. As a result, an observation of a seemingly insensible or ineffective instructional decision in the classroom does not merely conclude with a negative assessment of the teacher's competence. Instead, it becomes a focal point for which the researchers need to account so that it can be seen as a reasonable and coherent component that fits within the landscape of the teacher's instructional reality. Accounts of this type that explain what teachers do and why they do it are directly relevant to the analysis. They generated insights for us to understand the teachers' perceptions of the PD activities in relation to their classroom instruction.

The analysis of the modified teaching sets encompassed both the videotapes of the teachers' classroom instruction and subsequent audio-recorded interviews. I examined the classroom videos to identify salient aspects of the teachers' classroom instruction. Specifically, I focused on (a) the structure of the lesson, (b) the nature of

[6] In our actual work with the teachers, the analysis of the modified teaching sets was also driven by a second purpose, which was to inform the design of the PD activities by identifying potentially promising means of support. This second purpose, although crucial to our work with the teachers, is beyond the scope of this chapter as the primary focus here is to understand and account for the teachers' activities around student work in the PD sessions. It will, however, be discussed in Chapter IV as I discuss the design implications of the proposed conceptualization for supporting teachers learning via the means of PD.

the instructional tasks, and (c) students' mathematical obligations during the lesson (Cobb, Stephan, McClain, & Gravemeijer, 2001). In doing so, I looked for regularities and patterns in the ways that the teacher and students acted and interacted as they completed instructional activities and discussed solutions (Cobb & Yackel, 1996). These three aspects of the teachers' instruction oriented me to tease out the primary means that the teachers used to support students' learning and the extent to which student reasoning was made visible during the classroom instruction. Because the follow-up interviews provided access to the teachers' perspectives on classroom events, I was able to complement the analysis of their instruction with their interpretations of what happened and their justifications of why they chose to organize the classroom instruction in a particular way.

I formulated and refined categories as I coded the audio-recorded interviews (Strauss & Corbin, 1990) in order to identify critical aspects of the teachers' conceptions of teaching and learning. Four themes emerged that oriented the analysis, including (1) accounting for students' learning, (2) strategies to support student learning, (3) perceptions of students' mathematical obligations, and (4) instructional challenges encountered. I consider these four aspects of instruction critical because each of them offers insights into the role that students' reasoning played in the teachers' instruction. This in turn generated insights into whether our goal of focusing on students' reasoning via the use of student work was viable at the time. Technically, these themes constituted the major topics of conversation in the interview protocol (see Appendix A) and therefore can be fairly easily teased out.

The process of data analysis took several rounds. First, I went through the entire interview data and segmented each interview into episodes in which the conversation focused on a single theme or topic. This was relatively easy to accomplish given that the conversation was driven by a semi-structured protocol that was organized by topics. I then decided to which category each of the identified topics of conversation should be classified. Topics that did not fall into any of the four categories discussed above were classified as others. The major themes together with various teachers' responses that fit into that theme category emerged in the course of this process.

Once the entire interview data were classified, I further examined the nature of the teachers' interview responses under each theme. In doing so, I focused on the extent to which a particular viewpoint or viewpoints was shared among the teachers within each theme category. The criterion for deciding whether a particular viewpoint was shared was that it was voiced by at least half of the teachers. In other words, for a response to be considered *shared*, it needed to be expressed explicitly by a majority of the teachers during the interviews. The resulting product from this round of analysis was the identification of the most widely shared perspectives that the teachers held towards issues highlighted in the four theme categories.

It is important to note that I did not examine the four theme categories in isolation. Rather, I analyzed each in light of the others to note any possible interrelations. These tentative interpretations served as the basis for generating hypotheses about the teachers' instruction as seen and understood from their perspectives and were subject to continual revision until three conditions were satisfied, which were (1) all data were completely analyzed, (2) no contradicting evidence could be further identified and (3) it was possible to account for the teachers' unexpected participation in the PD sessions. These hypotheses were then summarized to generate an account of teaching as seen and understood from the teachers' perspective that bore a high level of sharedness among the teachers.

Two important clarifications need to be made concerning this approach of data analysis. First, to create a rich account of what teaching might look like from the teachers' perspective, I looked for both commonalities and differences across the group. I was, however, more concerned with the commonalities given that my purpose for analyzing the modified teaching sets was to identify general patterns across the group that would explain the normative ways that the teachers came to use and reason with student work in PD sessions, especially when they deviated from our original intent. Additionally, the focus on the commonalities across the group was made possible by the characteristics of the particular group of teachers with whom we worked. At the outset of our collaboration, the teachers' instructional practices were rather homogeneous which, to a large extent, was a result of their lack of informal professional networks and the highly private nature of their mathematics instruction (Cobb et al, 2003). As they participated in the PD activities, their learning was accompanied by the identifiable patterns of various normative practices that were constituted within this group as legitimate ways of participation (Dean, 2005). These two conditions combined gave rise to the significant commonalities across the teachers and thus the feasibility to focus on them in the analysis of the modified teaching sets.

Last but not the least, when analyzing teaching, my focus was not to micro-examine the specific instructional moves or decisions occurred in each classroom, which may appear to be various and diverse from teacher to teacher. Instead, my goal was to explicate salient aspects of the underlying conceptions that the teachers often held towards teaching and learning, and that gave coherence and rationality to what was observed on the surface level. Compared to specific instructional moves or decisions, these conceptions often remained relatively stable across various teachers, and thus made it feasible for the analysis to focus on commonalties across the teachers. The final result of the analysis therefore comprised an account in which teaching was characterized in a way that was compatible with most of the teachers' conceptions yet allowed for the possibility of individual differences.

The second clarification that I would like to make is that the analysis of the modified teaching sets does not give an objective profile of each individual teacher. Instead, I actively drew on my own perspective as a researcher when attempting to tease out the common yet significant conceptions that the teachers had towards teaching and learning. In their work to document teachers' classroom practices, Heinz et al. (2001) made a similar clarification about this approach of accounting for teachers' perspective. They argued that this interpretive perspective on teachers' practices is particularly useful in that it "allows us to consider teachers' practices in light of ideas that are currently important to the mathematics education community, but which may not be accessible or important to the teachers in questions" (p. 88). Therefore, even though I was committed to take the teachers' perspectives and instructional practices as reasonable and coherent from their viewpoints, the analysis inevitably reflects my perceptions of key aspects of teaching as I formulated conjectures about what teaching might look like from the participating teachers' point of view.

The soundness of the analysis was strengthened both by my commitment to reconstructing teaching as coherent from the teachers' perspective and by the attention I gave to the institutional context in which the teachers worked. On the one hand, I continuously checked for incoherence within the conjectured account until I considered all seemingly conflicting pieces of evidence. On the other hand, the analysis was oriented to answer the question "how was the teachers' view reasonable given their school and district context?" Asking this question proved to be relevant to the analysis in that it allowed me to rule out interpretations that were not sensitive to the social aspects of the teachers' work or that portrayed them as making unconstrained decisions in an institutional vacuum.

Looking at Student Work in Year 3

The reader will recall that our overarching goal in working with the teachers was to support the emergence of productive statistical instructional practices that place students' statistical reasoning at the center of instructional planning and decision making. In year 3 of our collaboration with the teachers, the research team decided to engage the teachers in looking at student written work produced in their own classrooms as they taught the selected activities from the statistical instructional sequence with their students. The decision was based on prior developments mainly in terms of the teachers' increasingly sophisticated statistical reasoning as well as deprivatized classroom practices. We designed a series of activities in which the teachers were asked to (1) describe what types of statistical solutions were apparent in student work and how students might be thinking about the problem statistically, (2) categorize students' solutions and rank them in terms of their level of mathematical sophistication, and (3) propose ways to build on these various way of reasoning in the ensuing whole-class discussion.

As discussed earlier, from our perspectives as researchers, student work had the potential to connect the PD activities to teachers' classroom practices (Kazemi & Franke, 2004). Additionally, we conjectured that the activity of looking at student work would provide opportunities for the teachers to understand and anticipate the diversity of students' mathematical reasoning. We also intended to support the teachers to later use this knowledge as an important resource to plan for a whole-class discussion in which mathematically significant issues could emerge as topics of conversations. These rationales reflected our conscious efforts to build on the teachers' classroom practices. It also reflected our interrelated conceptualization of the relations between the teachers' activity in PD sessions and their classrooms at that time.

In the following discussion, I analyze how the teachers engaged in the student work activity to illustrate the aspects in our original conjectures that proved to be unviable. I then discuss how this mismatch oriented our further collaboration with the teachers and eventually led us to reconceptualize the relations between teachers' learning in PD and their instructional practices in the classroom.

Describing Students' Solutions

When the teachers looked at student work during the work sessions in year 3, they were first asked to describe what types of solutions that they saw in their student work, and to determine how the students might have been thinking about the problem statistically. From the researchers' perspective, the purpose of addressing these issues was three fold. First, it gave the researchers an opportunity to orient the teachers towards students' mathematical thinking processes that could be inferred from the student work, and away from an exclusively evaluative stance that focused on the correctness of answers. Second, we hoped the discussion of student solution types would help the teachers become aware of the diversity of students' reasoning and later be able to anticipate the range of solutions that students might produce when solving specific statistical instructional activities. Third, we intended that this focus on various types of students' solutions would serve as a starting point so that the group would then be able to move on to differentiating between these identified types of solutions in terms of their level of mathematical sophistication. Eventually, we hoped that the knowledge that the teachers gained by looking at student work would support them in collectively thinking about the issue of building on the diversity of students' reasoning in a whole-class discussion.

A genuine interest to understand students' solutions. The teachers appeared to be genuinely interested in figuring out what specific methods the students were using. During the work sessions in year 3, the teachers made various comments that helped us understand why this was the case. For example, in the first work session in year 3 (September 2002), the researcher who led the PD activities talked about the importance of focusing on students' mathematical understanding when examining student work. Two teachers then made a comment indicating that the PD activities allowed them to begin to see more than the right answer in student work.

Muriel: [Our study group[7] will be] looking at the kids' work… how can we look at the work and even if the kids have the right answers…how can we go deeper than that and say "ok, this one has some real understanding?"… That's probably how we can get into this conversation [about students' understanding]. I don't feel like I am expert enough to help them look at this [student work] and then say "yes, my kids fully understood this", or "no, they didn't yet"…but that's where I see our study group can go.

Wesley: I think how we assessed student work is very different than when they have EOG [end of grade assessment] and assess student work. And what I have been able to do is to start at more math knowledge in my students' work and build on that rather than just look at it and go "alright, another paper". I think [this is] the biggest growth I've seen in myself…

[7] The study group was organized by the mathematics leaders in the district during the school year that focused on the use of the reform textbook series.

The opinion expressed by the two teachers seemed to be shared within the group as two other teachers were visibly nodding with agreement and the rest of the listening teachers did not voice a contrary view. Later, in the third session of the year (November 2002), the teachers again indicated that they valued looking at student in the work sessions. The following conversation took place as one of the researchers asked the group what it meant for them if their students came up with non-standard ways of solving a problem.

Wesley: I think every time we do [student work] and every time I've got the stack of student work in front of me and I go through it, I go "This is shit."

Researcher: Yeah…

Wesley: When I go through it again, and we sit down and talk about it [in the work sessions], I find so much to honor about that child. And I learned to respect what that child can do with what that child's got.

Erin: And [when we looked at student work in the work sessions], the focus pulled off the grade… 'Cause a lot of times when you ask people [in my school] how they did on that quiz or whatever, first thing is always "well, I had everybody made an A" ... and it goes immediately to the grade, not what they do of it.

Wesley: Yeah. You know, taking the time to really look at what students are doing and thinking [is important]. It takes time and training.

Looking beyond the right or wrong answer seemed to be a shared understanding among the teachers when it came to examining student work in the work sessions. The teachers valued this orientation because it enabled them to share student work without worrying about being evaluated negatively as teachers if their students had produced incorrect answers. At the same time, they seemed to appreciate the opportunity to develop some understanding of whether their students were actually learning. As Wesley indicated in his comments, looking at student work took time and training, but it helped him to respect what his students did with what they knew.

When describing students' solutions, the teachers enthusiastically discussed what the students did at a relatively detailed level and sometimes making conjectures about why students produced certain types of solutions. For example, in Session 4 (January 2003), the teachers examined student work produced in the Blood Drive task. As the reader will recall, the goal of this instructional activity was to decide whether supermarkets or community centers are better locations for blood drives based on the numbers of people who donated blood (Figure 10) at each type of locations.

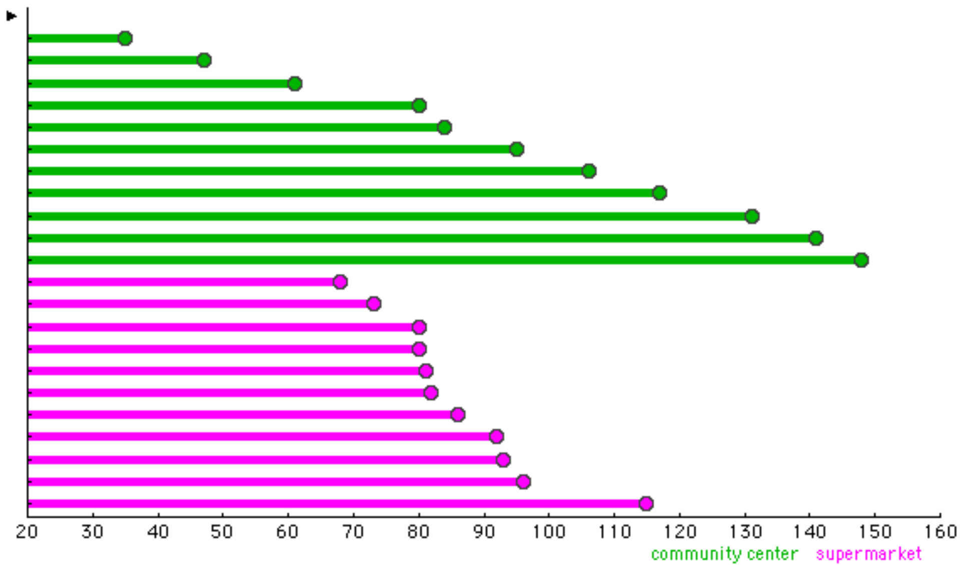

Figure 10. The Blood Drive Dataset as Sorted Both by Color and by Size

One group of students argued[8] that the supermarkets were better locations because the data were "sort of in the middle." To support this argument, the group created a graph to represent the data collected from the supermarkets (Figure 11).

---

[8] This argument made by the students was documented on the teaching video, which was recorded from the teachers' classrooms and later shared during the work session.

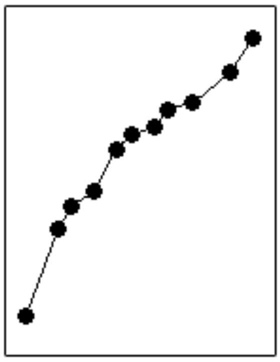

Figure 11. Recreated Student Work from the Blood Drive Task

In the following excerpt, the teachers discussed the students' representation. In doing so, they focused on making sense of what the students did rather than on evaluating what the students did not do but should have done. The short conversation cited below lasted about 50 seconds during which 7 teachers participated and, at times, spoke simultaneously. It appeared that the teachers were deeply engaged in this type of conversations. Conversations of this nature and with this level of detail occurred frequently during year 3 when the teachers described and categorized students' solutions.

Kate: What was she drawing? What was the line that she was drawing?

Muriel: She only drew, she took the points for just the supermarket… and did a line graph because she wanted to show that it was more consistent.

Helen: Yeah…

Muriel: But she did not compare it to anything.

Researcher: So, her argument was consistency…

Lisa: Yeah, she never said the word.

Muriel: No. But that was her argument.

Kate: That was her argument.

Researcher: …And then she is trying to show that this is more consistent by drawing that graph…

Muriel: Yeah, and assuming that they [the other students] got the other information, "This is the one [graph] I want."

Wesley: That's rather sophisticated argument then. Lots of assumptions there.

Muriel: Yes.

Erin: You said she did a line… she did a line graph showing?

Muriel: She just put the dots up there and connected them in a line to show they were close together.

Lisa: …But it didn't show that.

Erin: [Agreeing with Lisa] it [the line graph] showing them going up. Yeah. I guess I'm..

Wesley: I think she did x [axis] and y [axis] and they are equal to each other.

Muriel: I don't know.

Kate: This [line created by the student] is rather straight. It shouldn't be straight.

Jane: It also looked like she was doing a little bit of [the] total [argument].

"Consistency" and "total" were two types of solutions methods most frequently seen in the instructional activities designed for the first computer tool. The consistency type of solutions involved comparing the variability of each dataset and choosing one dataset because of its greater consistency. In contrast, the total type of solutions involved determining which dataset had greater sum of all data values. From our perspective as researchers, the spirit of genuine data analysis would require students to justify why either greater consistency or a larger sum was relevant in a specific situation by relating to the phenomenon under investigation. Only in this way could we be sure that the numbers were not merely numbers to the students but "numbers with a context" (Cobb & Moore, 1997, p.801).

Linking students' solutions to the classroom instruction. In trying to understand what the students had done, the teachers also attempted to figure out why they produced certain types of solutions in all but one work session in which student work was examined. For example, shortly after the above excerpt occurred, the teachers questioned why the students created the graph by focusing on what the teacher might have done instructionally in the classroom.

Kate: Did you just give them the dots or did you give them the bars?

Muriel: They had the bars with the dots at the end…

Wesley: In order?

Muriel: Yeah, in order.

Kate: I am just thinking…if you just give them the dots, that will be their tendency to connect the dots. Turn it to the side, connect the dots. That's where the line graph came from. …

Wesley: Did she turn it to the side?

Muriel: She did.

Kate: I am just looking at her presentation… she turned it this way [motioning a 90 degree rotation to the left] and connected the dots.

In this exchange, the teachers tried to figure out how the students connected the dots as shown in Figure 11. One of the teachers conjectured that if the data were given to the students with only the end points (see Figure 12), this might explain the "line graph" representation.

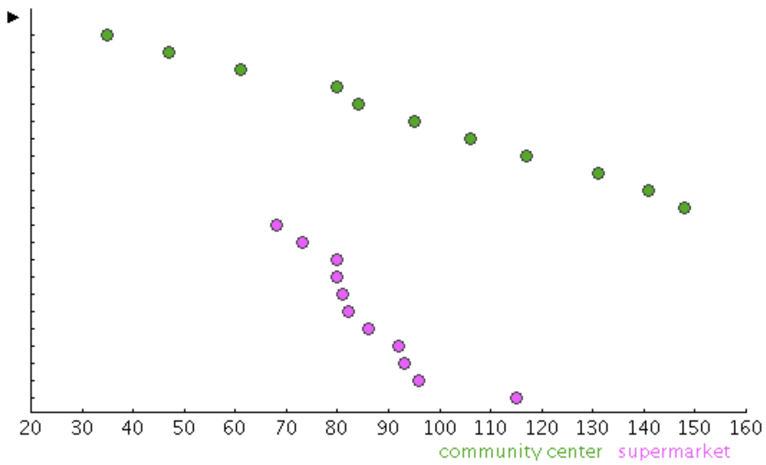

Figure 12. The Blood Drive Dataset Shown with End Points Only

This tendency to connect the type of students' solutions to aspects of classroom instruction became stronger in the latter sessions of year 3. It was clear that the teachers did so in order to understand students' solutions rather than to evaluate how well the teacher taught the instructional activity. For example, as the above discussion continued, another teacher described how some of her students only compared the maximum values in the two data sets (see Figure 10) because they thought the data were ordered chronologically by days. She then conjectured that this was because she did not clarify this issue sufficiently when introducing the data. Two other teachers agreed with her conjecture and reported that they had similar problems in their classrooms. Throughout year 3, it became normative that if something unexpected occurred in student work, the teachers would look for explanations in the classroom instruction that the students had received.

It is worth noting that when trying to establish a link between classroom instruction and student work, the teachers focused primarily on what the students did on the surface level, such as the types of graphs they created, specific cuts points they used to partition data or certain data values they chose to compare. As the teachers did so, they thought of the relationship mostly in a linear and causal fashion. As illustrated by the above example, the teachers conjectured that the graph (Figure 11) created by the students might have been a result of the data representation that they had been given by the teacher. For another example, in looking at student work from the Speed Trap task (Session 5, February 2003), the group agreed that the students' choice of a cut point was due to the teachers' mentioning of a specific speed limit. Importantly, in establishing such links, the teachers were not explicit about the extent to which the classroom instruction might have oriented the students to engage in genuine data analysis in which their various ways of structuring the data would have to be justified in terms of the question under investigation.

Focusing on mathematical procedures and processes. Although the teachers routinely elaborated on the details of students' solutions, their focus was often descriptive of what the students had produced in the written work. In doing so, they tended to key in on the mathematical procedures or processes used by the students rather than the justifications that the students provided for structuring data in certain ways. One such example can be found when the teachers looked at student work generated in the Speed Trap task (Figure 13), the goal of which was to decide whether a speed trap was effective by comparing two sets of 60 cars' speeds before and after the speed trap was put in place.

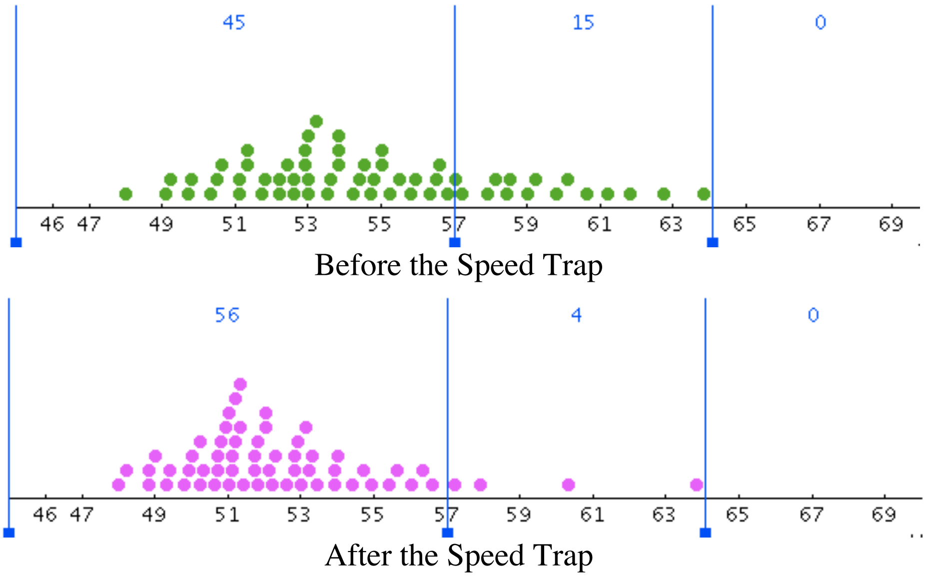

Figure 13. Recreated Student Work in Speed Trap Task

During the entire activity, there was only one instance in which the teachers considered how the students justified the ways in which they had structured the data, or asked other teachers to give such justifications when they described their students' solutions. The researchers made a number of attempts to press for a focus on students' statistical justifications, but once the teachers had answered the question, they immediately returned to students' mathematical procedures or processes. The following excerpt is illustrative of such conversations. Erin and Kate below shared the student work collected from the Speed Trap activity that they taught together.

Erin:	These [work sheets] were split into groups and a lot of these just look at comparing it with the same sections, the sub-grouping on both graphs. And especially this one…it looked at from 57 up to 64. They compared the same parts of the graph on both.
Researcher:	How did they choose the parts? Did they use the speed limit or…?
Kate:	We didn't define the speed limits. To them, that is where they saw the biggest break…
Researcher:	Hold on, I don't fully understand. So where are the breaks…?

Kate: They see it at 57. They see the big change up here and after 57. So they drew their lines down to see the difference…

Researcher: So they were comparing how many [people] were in each of the three sections?

Kate: To them, they are more concerned with the higher speeders. …So they are either looking at the upper end or the peak.

Wesley: So [choosing to focus on] upper-end will be creating partitions…

Erin: Yeah. And this one, it's like they did different subgroups. They decided to go between 53 and 61 [on the before data] and they shifted it to 50 and 54 [on the after data]…

In the above excerpt, the teachers did not share the students' statistical justifications unless were specifically asked to do so by the researchers. Instead, they focused on what the students had done. This tendency remained relatively stable throughout the third year of our collaboration.

Summary. From our perspective as researchers, it was significant that the teachers had developed a genuine interest in describing and attempting to understand student work in a non-evaluative way. This, to us, was an accomplishment in its own right given the fact that the teachers' instructional practices had been highly privatized and the only times they ever looked at student work in their teaching was to evaluate the correctness of students' answers[9]. The new orientation towards student work is a key characteristic of a productive PD activity involving the use of student work as depicted in the teacher education literature (Chamberline, 2005; Kazemi & Franke, 2004; Little, 2003). However, a closer examination revealed that the teachers' primarily focus when they described students' solutions was to articulate students'

---

[9] The teachers were able to adopt this orientation towards student work because of significant changes that took places within the group during the first two years of our collaboration. A detailed analysis was documented in Dean's (2005) dissertation that focused exclusively on developments that occurred in the first two years.

mathematical procedures and processes rather than students' statistical reasoning. As will become clear in the ensuing discussion, even with such genuine interest in looking at student work, the teachers still struggled with the idea of building on students' solutions when the focus of the activity turned to instructional planning.

Categorizing and Ranking Students' Solutions

After the teachers had described their students' solutions, they were asked to categorize them and order them in terms of their level of mathematical sophistication. Most of the teachers became increasingly proficient in discriminating between students' solutions and ranking them by their level of sophistication. However, as they did so, they focused mainly on the mathematical methods that the students used rather than ways in which they reasoned about statistics (e.g., how students interpreted the problem, how they justified their methods and why they thought one way of structuring data would be more convincing than others).

Categorizing solutions based on types of mathematical methods. During the third session of year 3 (November 2002), the teachers discussed how to categorize students' solution from the Blood Drive task in groups. One of the groups, composed of Lisa, Wesley, Kate and Jane, came up with five categories as shown in Figure 14.

1. NO math. Not based on the data. (e.g., Parking lot was not clean).
2. No math reasoning but some math vocabulary (a statement of "most stores"):
3. Comparing the biggest and the smallest values (but not in terms of range).
4. Totals and averages—the decision was community centers were better
5. Totals, averages, ranges and consistency—the decision was supermarkets were better

Figure 14. Categories of Students' Solutions from the Blood Drive Task Created by the Teachers

Each category in Figure 14 represented a specific mathematical method that the students used to arrive at their decision. This way of categorizing solutions was consistent across all teacher groups as they all identified total (or average), range, and comparing individual data values as different categories. Significantly, none of these categories examined how students reasoned statistically about the blood drive situation when they chose to use a specific mathematical method.

During the last session in year 3 (March 2003), the teachers examined student work from the Migraine activity. As the reader will recall, this activity involved analyzing data on the amount of time that it took patients to get relief from a migraine headache (Figure 15). Each dot therefore represented a patient's relief time measured in minutes.

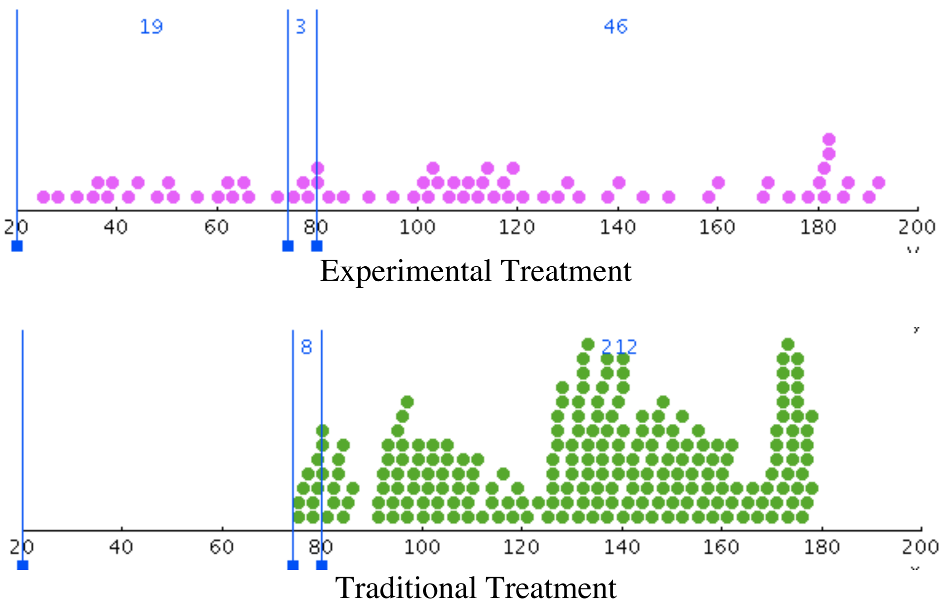

Figure 15. The Migraine Dataset with Cut Points Created at 75 and 80 minutes

One category of solutions that the teachers identified was cut point, in which the students would partition the data at a certain value. During the PD activity, the teachers discussed whether a specific solution should fall into the cut point category.

Erin: The majority of my group said…well, between 75 and the lower on the experimental [treatment], there were x number of people there who got relief. And if you go to traditional, 75 and lower, there was nobody who got relief. So that was the reason they were thinking…they said faster relief is better.

Wesley: Well, that's a lot like mine…

Erin: Was this [75] your cut point?

Wesley: No… I was thinking…well, I don't know…my last category was looking at a drop in the minimum value, a drop in the low value. Is that what you got?

Kate: That's a cut point though…

Wesley: You are talking about things below the cut point, below 80?

Lisa: Yeah, below 80.

Erin: …And this one picked 75…

Wesley: To me, that was a cut point.
Erin: [Together with Lisa at the same time] Yes…

Wesley: And that's like cut point with counts and but not converted into percentages.

Muriel: Right.

Lisa: [Same time] Right, definitely.

During this exchange, the teachers seemed to agree that three solutions—partitioning the data at the value of 75, partitioning at the value of 80, and looking at data on the lower end of the distribution— all belonged to the cut point category because they all involved partitioning the data. The fact that no one questioned choices of different cut points or discussed which one might result in a more convincing statistical argument indicated that these solutions appeared essentially the same to the teachers. As indicated again by this example, the teachers seemed to focus primarily on mathematical methods as they tried to categorize types of solutions from students work.

The teachers' approach to categorize students' solutions was in contrast with our view as researchers. Even though we concurred that it was important to identify the specific methods the students used, a key question for us involved to what extent the students were truly analyzing data. The choice to focus on total or consistency (range) therefore is not an acceptable statistical argument in itself but has to be justified in terms of how well this way of structuring data reveals trends or patterns in the data that give insight into the question under investigation. In other words, even when two methods both involve partitioning data, they might still be significantly different from our perspective: One but not the other way of partitioning may lead to the identification of relevant patterns in the data. During the work sessions, we tried to clarify this orientation each time the group examined categories in students' solutions. However, the teachers continued to focus on types of mathematical methods as they categorized students' solutions.

Ranking students' solutions based on the sophistication of arithmetical calculations. In addition to creating categories of students' solutions, the teachers also ranked them in terms of their level of mathematical sophistication. For example, the categories that the teachers identified in Figure 13 were ranked from the least to the most sophisticated, an order that was almost identical to the other groups' rankings. As researchers, we agreed with the teachers that solutions 1 and 2 were the least sophisticated among all identified types of solutions because they were not based on the data. We also concurred that solution 3, although data-based, focused on individual data values rather than the entire dataset and was therefore less sophisticated. However, our ranking of the two most sophisticated solutions differed from that of the teachers. From the teachers' perspective, both solution 4 and 5 were data-based and their decision to rank solution 5 as the most sophisticated was mainly because more mathematics was involved in the latter. As one of the teachers commented, “we ranked this one higher because it used more tools to look at it.” In contrast, to us, it was critical that the students justified their methods based on the question under investigation. For example, in the Blood Drive activity, students would have to articulate why looking either at total of data values or at the consistency of the data would allow them to decide whether supermarkets or community centers were better locations for conducting future blood drives.

In Session 6 (March 2003) when the teachers examined student work from the Migraine task, one teacher described a student solution involving percentages. Another teacher immediately followed up by making a positive comment about this type of solution.

> Helen: I had a child who looked at 100 minutes. And they found the percentages for each one. And 41% had relief in experimental while 19% had [relief] in traditional. So they basically chose experimental.
>
> Wesley: Percentage with cut point…I wanted percentages [in my class]. I had a lot of cut points [type of solutions in my class], but I didn’t have percentages to go with them. So that is a much higher level thing to me.

Calculating percentages in this task is clearly a sensible thing to do because of the unequal number of data values in the two data sets. To be more specific, a cut point created at the value of 100 would show that a greater number of people got relief within 100 minutes with the traditional drug (43 as compared to 29 with the experimental drug) (see Figure 16). It is therefore necessary to take account of the differing sample sizes by comparing percentages of the data sets.

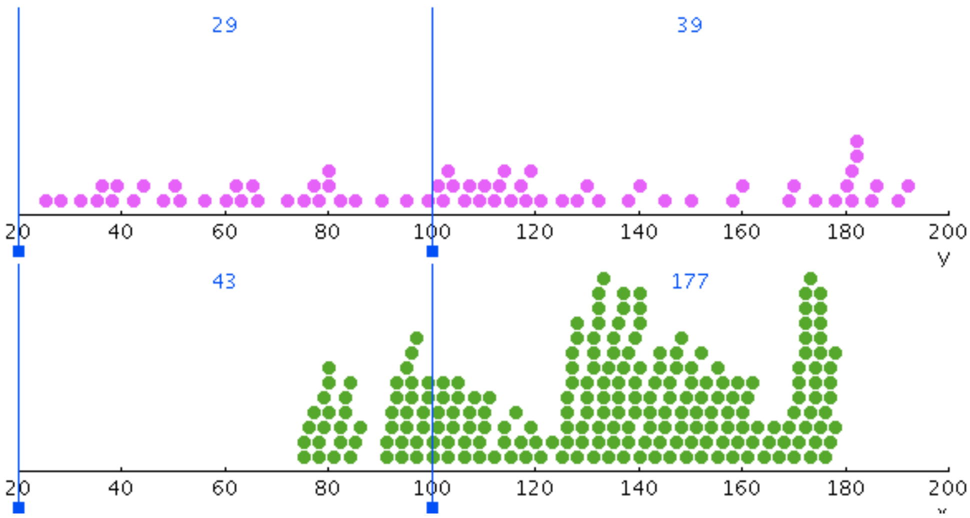

Figure 16. The Migraine Dataset with Cut Points Created at 100 minutes

However, the teachers did not specify why the students chose to use percentages—whether they were reasoning multiplicatively with the data or simply performing the calculations because they thought they were expected to do so. It was likely that Wesley appreciated this type of solutions because it involved more complicated mathematical procedures. Wesley's valuation of this solution seemed to be shared within the group. Another teacher said later in the work session that she ensured that the solution of "numbers and percentages" was presented last in her class because she thought it was the most sophisticated one and wanted the rest of class to pay attention to it.

Summary. As the teacher created categories of students' solutions, they focused on identifying solution methods rather than students' statistical reasoning. For example, all solutions involving the creation of a cut point would be considered essentially the same no matter what the justification was and how well each helped to show trends in the data. The rankings created by the teachers indicated that their understanding of what counted as a sophisticated statistical solution was different from ours. The teachers judged solutions that were unrelated to the data or the question under investigation to be less sophisticated. Examples included personal opinions (e.g., the supermarket parking lots were not clean) or calculations of mean, median and mode only. However, once the solutions did involve the data and made reference to the question, the teachers assessed them primarily in terms of the sophistication of arithmetical calculations involved in each solution without paying attention to whether such ways of organizing data helped to show relevant trends or patterns in the data.

The teachers' way of ranking students' solutions were intriguing to the researchers, especially given that they had become increasingly sophisticated in analyzing data during the third year of our collaboration. As learners in the statistical activities, the teachers found it important to focus on the distribution of the entire dataset, justify their solutions in terms of how well it helped detect trends and patterns within the data and make decisions based on the question under investigation (Dean, 2005; Visnovska, 2009). However, when it came to student work, their focus was on the mathematical methods that the students used as well as the sophistication of arithmetical calculations involved in each method.

Building on Students' Solution in the Whole-class Discussion

As the reader will recall, the purpose in looking at students' work was to identify student solutions that could be used as a resource for planning whole-class discussions. Therefore, after the teachers categorized and ranked students' solutions, the researchers always asked the question, "How would you as a teacher build on these various solutions in a whole-class discussion?" The purpose of this question was to engage the teachers in discussing their instructional decisions based on their knowledge of the diversity of students' statistical reasoning and in sharing their justifications for making those decisions. In this way, we conjectured that the teachers would be oriented towards thinking of student work as a resource for instructional planning.

However, these efforts remained to a large extent unsuccessful throughout year 3. The teachers did not seem to fully understand our question as we intended, and students' statistical reasoning was mostly invisible in their responses. To examine how the teachers interpreted this question in detail, I focus on three topics that emerged during our conversations—the purpose of a whole-class discussion, the instructional strategies to make whole-class discussions effective, and the meaning of building on students' reasoning.

The purpose of a whole-class discussion. There were clear indications that the teachers seemed to think that having a whole-class discussion was useful. As one of the teachers commented during session 4, a whole-class discussion gave students the "summary and closure" and "wrapped it up" (Wesley, Session 4 Year 3). When explicitly asked to share what they thought the general goals were for a whole-class discussion, the teachers brought up issues that were mostly unrelated to student reasoning.

Brian: Share in turn their viewpoints.

Jane: You want students to feel successful.

Hazel: They need to be comfortable when presenting.

Wesley: Self-esteem

Hazel: You keep the environment in your classroom where they feel secure.

Jane: We want the students to disagree but when they do, we want them to disagree with the idea, not with the person.

Lisa: I had to stop my class at one time and told my students their job was to understand what the presenter…what the argument was…and not to fight. Chill [laughing].

Brian: I agree with this.

Erin: We talked about using it as a time for reflection…I'd be worried about some of the kids being put on the spot and not able to defend it and you got the whole mess going. It is time for reflect. Ok, so is this [the solution] you are gonna do? Maybe write down some thoughts.

The various responses that the teachers gave had little to do with students' learning of statistics. Instead, six teachers explicitly addressed issues such as supporting students' self esteem, creating a safe classroom environment, and showing respect to each other during the work sessions. When the teachers were pressed to talk specifically about the instructional goals for a whole-class discussion, they focused on a general purpose of having students to learn from each other.

Researcher: What are you trying to get at in the whole-class discussion?

Christine: There is more than one answer. There is more than one-way of seeing it.

Muriel: [Students] learning from each other…

Erin: … Students are still looking for one correct answer…They are waiting for you to say " you are right " and "you are wrong".

Jane: I think it is exciting for them to be able to hear, you know, you did something that was completely different from the teachers' saying " we are gonna focus on median today"… This [task from the statistical instructional sequence] could have gone in any directions. Take the data and look at it and talk about all these things.

Wesley: It is a teaching moment… they talked about median and you guys didn't. You brought up range. So you guys are gonna do median and range and they are gonna do standard deviation or whatever. So the students could hear examples. Kid to kid example is a lot more resilient in their brains than teachers.

In the excerpt cited above, it seemed that the instructional significance of a whole-class discussion as seen by the teachers resided in the opportunities that it might provide for students to present and share solutions within the class and ultimately learn from each other. During session 5 of year 3 (February, 2003), the researchers asked the teachers about students' obligations during a whole-class discussion.

> Researcher: What do students have to do to be perceived as a good student in that discussion? What do they have to do?
>
> Muriel: Listen
>
> Researcher: Did they have to understand?
>
> Erin & Kate: [Shaking head]
>
> Kate: I don't think that they understand necessarily other presentations. There's a big rush at the end of the presentation period. You know you're gonna run out of time…to put that big space between the next day… And most of the time in presentations they are so concerned about their own. They won't pay attention to others.
>
> Wesley: They're still talking about who is gonna stand up and present.
>
> Kate: Yeah, they fight about who goes first.

The teachers' responses indicated again that their primarily intention for doing a whole-class discussion was for students to listen to each other. A whole-class discussion from the teachers' perspective therefore seemed to involve a series of students' presentations.

The concern of time constraints was brought up multiple times during year 3. For example, during session 4 (January 2003), one teacher announced that he was planning to conduct whole-class discussions twice a week because he realized that his students respected each other more than they would respect him during a discussion and therefore he wanted to create more opportunities for them to present their solutions to the class (Wesley, Session 4, Year 3). This decision surprised the rest of the group as they currently only conducted a whole-class discussion on a bi-weekly basis on the average and did not believe Wesley would be able to find time to do it more frequently.

Part of the reason for the time pressure that the teachers felt was that they attempted to conduct an entire statistics instructional activity including a whole-class discussion during a single class period. Even though the researchers requested that the teachers include a whole-class discussion in their teaching of the statistical activities, video-recording of their classrooms indicated that their discussions usually only lasted 7 to 10 minutes with three to four groups of students presenting or students each presenting individually. In contrast, during the classroom design experiment in which the statistical instructional sequence was originally developed, the researchers collected student work at the end of one class period and examined it to plan a whole-class discussion that usually lasted the entire next class period. Furthermore, rather than a series of separate student presentations, these discussions involved comparing and contrasting selected solutions that brought significant statistical issues to the fore. An instructional activity therefore typically spanned two class periods. However, for the teachers, a whole-class discussion seemed to mark the conclusion of an instructional activity in which student work became the final product of (as opposed to a resource) for instruction.

The instructional strategies to make whole-class discussions effective. As I have discussed, the primary purpose of a whole-class discussion as seen by the teachers was for students to present and listen to different solutions. Therefore, it would be problematic from the teachers' perspective if all students came up with similar solutions or if they would not listen to each other during the whole-class discussion. To tackle these problems, the teachers proposed two types of strategies.

The first type of strategies was to ensure that significantly different (if not all) solutions were presented. As a result, the teachers hoped that the students would have the opportunity to hear and eventually master them. For example, "total" and "consistency" can be viewed as two different methods in the Blood Drive activity. It was then problematic from the teachers' perspective that the students did not produce both methods, as indicated by the following comment made by a teacher in Session 3 (November 2002).

> Marci: Just to try to get them to think beyond totals as being the answer to everything that may seem as though total is the best answer…Try to get them to explore some of the other measures of central tendency maybe, other things that they may not have tried, like we talked about the range. And there is probably some kid, and their reason for choosing that large range versus the smaller range was probably based on what they were picturing as total. Because all of the papers that we looked at, it didn't matter what they chose as their reasoning, they all went back to being the total.

To solve this potential problem, the teachers proposed a variety of instructional strategies as shown in the following excerpts.

Excerpt 1 (Session 3, September 2002):

> Lisa: Kate had a good point as divide the class and say: OK, you [first half of the class] have to argue for supermarket, and you [second half] have to argue for community center. Have them do their argument and then say, ok, who would you go for now? Or where would you go now? And see if any of them would move [to a different conclusion].

> Kate: Because once my kids stopped at community centers, they never ever gave even a look to see, because I told them that you could [choose] either answer. But they've stopped as soon as they found the community center. I only had one group who actually looked to see if they could find the reason to justify the supermarkets.

Excerpt 2 (Session 3, September 2002):

> Muriel: But what do you do when everybody has the same argument? Everyone of them said community center…

Marci: I would change the question. I would say, look, the American Red Cross is going to switch all blood drives to supermarket. I want to look at the data and tell me why they made this decision. That's going to help them to see beyond the totals.

Erin & Jane: That's a good question.

Excerpt 3 (Session 5, February 2002):

Jane: I think after I did the Blood Drive data, they were so split down the middle and argumentative that their way was, you know, the right way, and they couldn't see the other way. I really wanted to almost assign them which one, which way they had to go and let them debate it out.

In all three excerpts, the teachers were trying to find ways to make sure that both "total" and "consistency" solutions would be presented in class. To this end, they proposed to either assign the students to a specific stance or create a scenario in which the students would have to use the other method. It seemed important to the teachers that the students would be able to master both methods.

The teachers' understanding of the purpose of whole-class discussions was clearly different from our view as researchers. Although we agree that it would be important for students to consider data from different perspectives, the critical issue in our view is that whole-class discussions focus on what counts as a good argument or an adequate way to justify an argument no matter which way students choose to organize data.

The second type of strategies that the teachers proposed to make a whole-class discussion effective was to ensure that the students paid attention to others' presentations. The teachers expressed their concern in this regard on four different occasions during the work sessions. In the following excerpt, the teachers talked about various strategies to this end (Session 4, January 2003).

Brian: We could make it a little bit of a contest and give reward for the single best presentation.

Wesley:	Or maybe let them vote but they can't vote on their own.
Erin:	Exchange the graphs [created by the students] and label each one—graph 1, graph 2 not telling them whose it is [and ask them to rank the graphs].

Interestingly, the proposed strategies had little to do with making mathematics instruction more relevant or meaningful for the students. As a matter of fact, when researchers asked the teachers what motivated students to listen to each other, over half of the teachers said they did not know

The meaning of building on students' solutions. During the first four sessions throughout year 3, the research team's repeated efforts to clarify what it might mean to build on students' solution in whole-class discussions were not successful. The teachers continued to focus on getting the right types of solutions for presentation and making sure that students listened to each other—the two types of instructional strategies that I have discussed above. The teachers' responses seemed to indicate that getting students to present was an end in itself. However, to us, it was merely the first step in a whole-class discussion. More importantly, the teacher him or herself would have to play a critical role to ensure that the discussion focused on key statistical ideas.

During Session 5 (February, 2003), we tried again to clarify this orientation with the teachers. By this time, the teachers had realized that we wanted a different response from them and as a result, became increasingly uncertain about how to respond. One of the researchers then clarified once again what we meant by the question of "how to build on students' solutions" as shown in the following excerpt.

Researcher:	These are the types of things you've got from the kids to work with. Now we want you to revisit the issue—what is it in here that you can build on? We purposefully asked the question this way. So rather than ask the question "did they get it?" or "how good was it?" we want you to say "ok, I've got several kids reasoning this way, I've got this group reason that way and I've

got this group reason that way." If we have a discussion of these solutions, what can we build on? If I compare these two types of solutions what can I build on? In other words, we are saying this is your staring point what your kids have done.

The researcher's comments were received with a silence that lasted for seven seconds before one of the teachers said that she "need[ed] to get a better mix" of good methods for the presentation. The intent of the question of how to build on students' solutions seemed puzzling to the teachers. The teachers then responded by reiterating the types of solutions that they saw in their student work. After about four minutes, the researcher attempted to further clarify the intent of the question.

Researcher: Did the question of what to build on make any sense?

All: [Collectively and hesitantly] Yes.

Researcher: What the kids have done was what you get for free. That's the material you get to work with. What can you build on what the kids have done to go somewhere.

[Silence, 6 seconds, followed by a joke unrelated to the topic of discussion]

Lisa: Well, it seems like you can build in different directions right?

Researcher: That's right. What are some of those directions based on what the kids have done? It not like there is necessarily a right direction. What are some possibility and how would you justify those?

Lisa: I know what I am gonna do with mine. I think you always know with your kids. But mine is totally different than Jane's.

Researcher: Why don't you all focus on this one set [Lisa's student work]?

Wesley: To me that the upper end, the higher level cognition is to partition on their own, irrespective of the speed limit. That whole component of mine is missing.

[Silence, 22 seconds]

Researcher: Maybe we should start talking about this 'cause I am not sure…

Muriel: When I was arranging how the groups went [during the presentation], I want them to learn from the other group. So that is kind of taking them in the direction of where they can go, right?

It seemed that the teachers were unclear about the role that they might play when attempting to "build on students' solutions." While three teachers attempted to answer the researcher's question, the rest of the teachers remained silent. Lisa's comment seemed to indicate that she was uncertain how the ensuing conversation was going to unfold given the fact that the students' solutions might vary significantly from one class to another. Wesley, on the other hand, indicated that what he considered to be the most sophisticated solution was missing from his student work and therefore it would be difficult for him to talk about what to build on. Both Lisa and Wesley's comments were accompanied by a certain degree of uncertainty. Muriel's comment, however, seemed to indicate again that, to her, the teacher's role would involve organizing a whole-class presentation of students' solutions and making sure that students listen to each other.

To further clarify what we meant by building on students' solutions, the researcher then went on to give an example of how to focus the discussion on mathematically significant issues by capitalizing on students' solutions.

Researcher: Can I give you an example? When we did this actual task [of Speed Trap], this was the first when we did [activities from the second computer tool], we got really excited over these types of solutions which had no calculation and where kids just talked about shape. We got one solution and her name was Jamie. Because what statistics is about…it is about detecting trends in whole groups of data…What you are looking for is trends over the long term. So you are not looking for what is the speed of the next car if it comes onto the road—we don't know. The speeds are all over the place. Why we got excited over this [Jamie's focus on shape], the most elementary solution possible? These kids were pointing to the general differences in the data that were actually relevant to the problem. The shift downward [in the after dataset] actually means something in terms of what they are trying to understand.

However, the intent of this clarification seemed to be misinterpreted by some teachers. One teacher thought it was about helping students master a certain mathematical method (i.e., shape) rather than about using a solution to engage the students in a discussion of why it would be adequate to focus on the shape of the data. This teacher then proposed to explicitly ask his students to create a graph that would outline the shape of the data.

Wesley: So when I get my kids to present…I might have got them all up and said I want you to visually show me…

Researcher: These are different ways of thinking about discussion. The goal for us is not just to have kids present. The goal for us is to use the kids' solutions to have discussions about issues that we think are important. It is not just to have kids present. Kids present is like step one. You are gonna choose which kids to present because there are some issues you want to get out.

Wesley: [Drawing triangles to represent the shape of the Speed Trap data, as shown in Figure 17] to ask [the students] as teams to come with a visual representation of what these two [students who focused on shape] were saying ... [long pause]. The majority of my students did not go towards the general trend in before and after so I want them all to come up… these two did not give a picture, just numbers. So I want them to move them to thinking about summarizing as a picture.

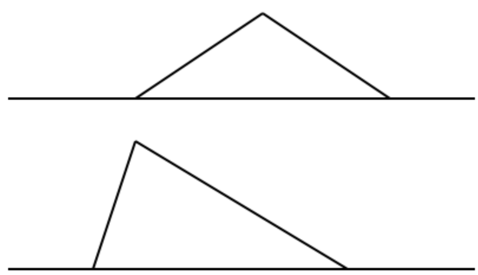

Figure 17. Recreated Graphs of Wesley's Drawing of the Speed Trap Data

Wesley seemed to suggest that his role as a teacher involved finding ways to get students to produce desired types of solutions. Realizing once again that the question did not make sense to the teachers, the researcher tried to stress that the issue for us was why it would be sensible to create certain types of solutions given the context of the problem. During the researcher's relatively lengthy explanation (which lasted about 5 minutes), two teachers asked a short clarifying question while the rest remained silent.

A few minutes later in the same session(Session 5, February 2003), Wesley returned to the issue of how he would build on students' solution.

> Wesley: The original question what is there that I personally would build on… and based on what I see right there [in my student work], I will be praising the hell of the ratios [type of solutions] and say "consider a visual representation of the data, and next time rather than numbers, give me numbers and a picture to go along with it and summarize what is going on, and consider other reasons for splitting up data than the way you did."

The researchers then explained that simply by praising solutions that focused on ratio would not be very helpful because it did not allow the rest of the class to understand why it was sensible to examine ratio given the context of the problem. It was likely that the other students might interpret this as a cue to produce ratio type of solutions while remaining oblivious to the reason for doing so. Frustrated, Wesley then proposed telling students directly.

> Wesley: I don't know. I am off.
>
> Erin: I'd like to hear what you think…
>
> Wesley: In practice, in the reality of the classroom, if this is the first time I was going through this [activity] and the first time the kids have ever done an activity like this, I think I just have to throw it in their face. I really do. I don't want to be subtle about it.
>
> Researcher: You can throw it in their face but our experience is you throw and they don't catch it.
>
> Wesley: They did. That group got it. So I want everyone to hear and draw attention. And two more groups may get it at that time…But I don't see the damage or negative as they are getting ready to leave [the class], saying "look at what these people did, this is incredible and we will talk about it"...

The above excerpts reveal how the teachers might have understood "building on students' reasoning" during the discussion of student work. While the teachers remained mostly silent during these conversations, the responses that they did make indicate that they would choose between (1) requesting students to produce the desire solutions, (2) praising the desired solutions in the hope that others would mimic and (3) telling students directly. In our view, none of these proposals used student work as a starting point for orchestrating whole-class discussions that focused on central mathematical ideas.

After a lengthy discussion of about an hour during which the researchers repeatedly attempted to clarify what we meant by the question, one of the teachers finally admitted with frustration that he had little image of what building on students' solutions might look like in practice. His comments provided strong evidence that we, the teachers and the researchers, did not have a common ground for talking about how to use student work for instructional planning.

Wesley:	I am very uncomfortable…
Researcher:	With what?
Wesley:	I am very uncomfortable with what I think with the lack of - with my understanding of the answer. I am very uncomfortable with my lack of understanding of my answer to the question what is there that I can build on. And I am not trying to continue the conversation, I am just trying to make a statement that I am not sure I've got it. And there is another agenda item or something like that I am equally and encouragingly moving on, or whatever. But I am juts not sure, I feel..
Researcher:	And I think you're not alone from what I can make out. Is that fair?
Muriel:	[Together with Erin Kate, nodding]
Lisa:	The kids were good and they've done it. They are getting it.
Wesley:	I mean I feel good about what my kids did and I would be confidently walking to the classroom the next day and not feel like I was creating educational malpractice.

Researcher: No, let me put this in perspective. This is, long discussion that we had before about how to approach that issue. ... We are talking about something that is WAY out there.

Muriel: And we aren't there yet either [teachers laughing]…

Summary. It became increasingly apparent during year 3 that the teachers were struggling to see the difference between having a show-and-tell presentation of student work and purposefully building on students' solutions to highlight mathematically significant issues (e.g., whether it would be adequate to look at the shape in the Speed Trap data). A whole-class discussion, as seen by the teachers, primarily involved students making presentations of their solutions, which helped explain why the teachers kept referring to it as a "whole-class presentation."

The normative justifications for devoting instructional time to whole-class discussions included that (a) students were more likely to learn from their peers than from the teacher and (b) presentations would help improve students' social skills, communicational skills and self-esteem. Critically, these justifications did not include a focus on students' reasoning. Instead, the goal for the teachers was that students would listen to the desired solutions strategies in the hope that they would produce similar solutions later. For the teachers, building on students' solutions seemed to involve either directly telling students to solve the problem in a particular way or modeling the more sophisticated solution for the students.

Accounting for the Collaborating Teachers' Instructional Practices

The research team intended that student work would come to constitute a record of students' reasoning for the teachers and orient them to focus on it in their instructional planning and decision making. However, the teachers' orientation towards the use of students' work was at odds with our conjectures. First, our conjecture that student work would come to represent records of students' reasoning for the teachers proved to be unviable. The teachers did seemed to value the opportunity to investigate what the students were doing and developed insights about whether they were learning. However, at the same time, their focus was mainly on describing students' mathematical procedures and processes, and on categorizing students' solutions based on types of mathematical methods rather than on students' underlying statistical reasoning.

Second, the teachers' appreciation for looking at student work did not necessarily lead them to think about it as instructionally relevant. Our efforts during year 3 to support the teachers in viewing student work as a resource for instructional planning and decision making were clearly not successful. More specifically, the teachers seemed to have different images of (1) the purpose of a whole-class discussion, (2) how to conduct an effective whole-class discussion and (3) what it meant to build on students' solutions shown in student work. As the analysis I have presented indicates, the teachers approached student work as the final product of an instructional activity, which usually ended with students taking turns to present their solutions.

These deviations from our original conjectures led the research team to question the interrelated conceptualization that had underpinned our design decisions about the use of student work. It was apparent that what we had intended to accomplish in the work sessions—for student work to become an instructional resource—was alien to the teachers' instructional experiences and not grounded in the context of their classroom teaching. Despite the teachers' genuine efforts to participate in and contribute to the work session activities, they saw little relation between the work session discussion and what they did as teachers in their classrooms. In attempting to understand this impasse, the research team concluded that there was something about these teachers' classroom practices that we were yet to understand. We therefore conducted a series of modified teaching sets (Simon & Tzur, 1999) with all participating teachers in order to investigate the teachers' classroom practices further.

As discussed earlier, a central principle that guided our analysis of the teaching sets was that the teachers' instructional practices were reasonable and coherent within their landscape of teaching and learning. It was therefore critical to develop accounts of their instructional practices that portrayed their decisions as rationale (Leatham, 2006). I therefore ground the analysis in the teachers' (inferred) instructional experiences in order to understand how the teachers taught and why they taught in a particular manner.

The analysis drew directly on the teaching sets that were collected at the end of year 3. The reader will recall that a teaching set was composed of a video-taped lesson accompanied by a follow-up interview with the same teacher. A researcher from the research team and myself worked together to conduct both the classroom observations and the follow-up interviews.

The analysis of the modified teaching sets is organized to address two salient issues that indicated differences in perspective between the researchers and the teachers. These two issues are (1) the role of students' reasoning in the teachers' classroom instruction and (2) the role of whole-class discussions in supporting students' learning of mathematics. An understanding of these two issues generated insights about why our goal of focusing on students' reasoning via the use of student work was unviable at the time.

The Role of Students' Reasoning in the Teachers' Classroom Instruction

The role of student reasoning in the teachers' classroom instruction constituted a focal point of investigation as we set out to understand the teachers' instructional practices. This was primarily because our efforts to orient the teachers towards students' reasoning via the use of student work had been unsuccessful in the PD. Analysis of the modified teaching sets revealed that student reasoning was to a large extent invisible in the teachers' classroom instruction.

The goal of learning mathematics. Learning mathematics, as the teachers saw it, mainly involved learning about mathematical procedures. During the observed lessons, all teachers (except one[10]) repeatedly reminded the students of the relevant rules or formulas before letting them work on problems. At the same time, six of them also tried to explain these rules and formulas by using visual representations. The teachers considered such explanations useful because the students would then have a better chance to remember what they were supposed to do once they understood it. However, such understanding did not seem to be considered absolutely vital to students' learning of mathematics. According to all four teachers who were teaching algebra during the observed lessons, some mathematical topics (such as algebra) were intrinsically difficult for students to understand and therefore needed to be learned through practice until students could eventually carry out the procedures. The type of mathematical learning that the teachers aimed to support seemed to mainly involve

[10] The only teacher from whom the researchers did not observe any instances of an emphasis on rules was teaching an activity from the statistical instructional sequence during the observed lesson. Through the entire lesson, the students worked on the problem in groups as the teacher walked around the classroom and listened to their arguments.

remembering the rules and formulas and correctly applying them in different problem situations.

In 10 out of the 11 observed lessons[11], the teachers' instruction was primarily focused on mathematical procedures. They would either give a direct instruction by demonstrating the correct solutions step-by-step, ask the students to report their answers and sometimes describe the calculational steps to the rest of the class, or do both. During either process, the conversation in the classroom largely centered on what was the correct answer and how to perform the correct mathematical procedures rather than issues such as how students interpreted the problem or why they decided to solve the problem in a specific way.

When students asked questions about problems, the teachers attempted to answer them with respect to the calculational steps, often by restating the procedures they had demonstrated earlier in the lesson. Based on the observations, alternative methods of solving problems were acceptable in all classrooms as long as they produced correct answers. Students' failure to produce the correct answers was typically treated as either a miscalculation, unfamiliarity with the procedure, or insufficient amount of practice rather than indications of their lack of access to the significant mathematical ideas. As a result, the teachers would reiterate the rules, review the calculational procedure (often by breaking it down to small steps) or simply ask the students to double-check the calculation. In cases like these, students' solution methods were not revisited to identify the source of the difficulty. Instances of this type were observed in all the teachers' classrooms. This account of students'

---

[11] The lesson in which an activity from the statistical instructional sequence was taught did not involve any whole-class instruction and therefore was excluded from the count.

non-understanding was further corroborated by seven teachers during the follow-up interviews. Jane’s comment below is representative in this regard.

> Jane: I can never explain why they don't get it cause sometimes it is something so easy to me. They may need to hear the comments again or see one more example. Trying to reiterate even though the class is learning a little bit new stuff

The fact that students’ mathematical reasoning was not implicated in the teachers’ accounts of students’ non-understanding indicated the peripheral role that students’ reasoning played in these teachers’ instructional practices.

Students’ obligations in the mathematics classrooms. The students in the observed lessons were obliged to follow the teacher’s directions and perform specified instructions. Examples included copying down answers or procedures into the notebook, answering questions when called upon, solving the problem using required methods, placing instructional materials in the appropriate place, and staying on task during the class. Students who failed to do so were reprimanded by the teacher.

Another important obligation of the students involved paying attention. From the observations, 10 teachers explicitly reminded the students to pay attention to the teacher and to other students who were sharing their answers with the class. Three teachers commented during the interview that they had developed a classroom routine in which they would randomly call on students and ask them to answer a quick question or simply repeat what had just been said in class as a way to make sure they were indeed listening attentively. Listening as directed was considered important in all classrooms.

When solving problems in class, students were obliged to provide the correct answer and, if required by the teacher, explain their solutions to specify the steps in the procedures that they used in arriving at their answer. Students were expected to ask questions if they did not understand the instruction. However, these questions were mainly calculational and focused on the specifics of the procedures rather than different ways of interpreting and reasoning about the problems. In most classrooms (8 out of 11), these questions were directed exclusively to the teacher and not the rest of the students. Only in three classes (all of which were advanced classes) did we observe instances of students being held accountable to each other as well as the teacher. In these classes, the students were expected to question and evaluate each other's solutions even though the goal was still to check the correctness of both the answer and the steps in the mathematical procedure.

During the interviews, the teachers were asked to state what they considered to be the students' obligations in the mathematics classrooms. Only three teachers included students' understanding the problem in their responses while the rest focused on obligations that were consistent with the two obligations discussed above.

Instructional challenges for the teachers. The teachers considered students' unwillingness to comply and failure to pay attention highly problematic in their teaching. During the interviews, six teachers indicated that these two issues were often the underlying causes of student "not getting it". Brian and Muriel's comments are representative in this regard.

Researcher: Sometimes there are mathematical concepts that students just don't get. They're having difficulty with them. When this happens, what do you think usually causes the problem?

Brian: Honestly, I would say it's not that the concept that is difficult. It's that they chose to tune out and then didn't hear the end. Didn't care. I spend all the time talking about (inaudible). And then all of the sudden in the last action all they've got to say is "I didn't get it." The biggest problem is motivational, and paying attention in class.

Researcher: I know sometimes students just don't get it. So what do you think from your experiences? What usually causes these problems?

Muriel: [When students don't get it, it is either that] they really don't get it or they are saying "I don't get it" because they haven't paid attention to what I said three times. We are after the [state-wide tests] and they want to just kick back and relax and not have to do anything. So sometimes, they will look at their homework and [say] "I didn't get it at all," "I don't get this," but [it is] because they didn't try it.

The teachers seemed to view students paying attention as a critical precondition for and sometimes synonymous with student mathematical learning. It seemed that the most prevalent instructional challenge for the teachers involved how to motivate students so that they would be engaged in instruction, as indicated by nine out of the 10 teachers interviewed. For most of the teachers, this was a daunting challenge, as Brian clarified during his interview.

Brian: The biggest problem is motivational. And paying attention in class… So really part of being an 8th grade math teacher, the biggest part of it is finding ways to motivate the kids and get a good classroom environment going…The differences between [various instructional] methods in terms of the outcomes of the kids' understanding are not big compared to the difference between a kid that's unmotivated and a kid who is motivated. …It's very frustrating and difficult. It's incredibly hard to motivate the kids. All the standard motivators for a big chunk of my kids don't matter. I mean grades, disciplinary stuff.

In these comments, Brian voiced many other teachers' frustrations related to engaging the students. From the teachers' perspective, students' motivation seemed to be a determining factor but was largely out of the teacher's control. During the interviews, three teachers explicitly talked about students' motivation being an inherent quality that had little to do with the teacher. Additionally, seven teachers indicated that the major difference between their advanced and regular mathematics classes was that students in the advanced classes were "intrinsically good students" and "naturally this way." As a result, the teachers treated the classes differently, as revealed by all the teachers except one who only taught advanced classes. For the regular classes, the teachers placed more emphasize on disciplinary issues and drill and practice. The problems were usually broken down more and taught at a slower pace. In contrast, the teachers felt that they had more flexibility with advanced classes and the activities they did were more investigative in nature.

During the interviews, the teachers brought up two broad types of strategies that they used for engaging the students. The first type of strategies was most prevalent among the teachers but had little to do with mathematics instruction itself. Instead, it involved using grades, parents, and disciplinary measures (e.g., giving warning or reprimanding students) as motivators to keep students on task but often lacked effectiveness, as indicated by Brian in his comments. In contrast, the second type of strategies focused more on adapting mathematics instruction to elicit students' engagement. Specifically, the teachers adopted a sugar-coating approach (Dewey, 1913/1975) that involved including something in the lessons that students might enjoy, such as computer games or free time to surf the internet. As a result, they expected the students to buy in to the mathematics activities that often came later.

It is revealing from the researchers' perspective that the instructional challenges the teachers described remained to large extent mathematics free. Understanding students' mathematical reasoning did not seem to be within the purview of their instructional challenges for the teachers. The strategies that the teachers used to address these challenges were external to the mathematical activity at hand. To us, this indicated that the teachers did not view the mathematics itself as a primary source of engagement for students.

Accounting for students' learning. During the follow-up interviews, we sought explanations to help explain why students' reasoning was largely invisible in the observed classrooms. The teachers' comments indicated that they seemed to view students' development of mathematical understanding as an elusive and unpredictable process. From their perspective, the connection between their instruction and students' learning outcomes was a black box. For example, the first few teachers interviewed spoke of mysterious "a-ha" moments. We viewed this as a revealing issue and asked the rest of the teachers to comment on "a-ha" moments as well. Overall, six teachers were asked about this topic and all of them considered it impossible to plan for these learning moments in advance. The following four excerpts are representative of the teachers' views in this regard.

> Researcher: How would you account for "a-ha" moments? Do you as a teacher have some control over those "a-ha" moments?
>
> Kate: I don't know [how to explain "a-ha" moments]. I like it when [my students] finally get it… It's fun to watch them have those moments… Sometimes it is a struggle. Lot of the units I find they struggle in the first three lessons, maybe four, and by the time they get to the fifth they finally understand all the other ones. Sometimes it's hard to convince them that they're gonna have to struggle through the first ones to put it all together. … I wish I had that control [over "a-ha" moments]. I don't.
>
> Researcher: When sometimes kids really learn something, how would you account for that learning? Why do you think that learning happened?
>
> Erin: I think a lot of it – comes from just them sitting there and doing it themselves instead of waiting for me to tell them…[I] assigned [students] to do [a homework] investigation that barely got started. They came the next day with all of the questions [answered]. The cycle got started… So they did not need me after all. They just needed more of interaction, of talking and stuff. Lot of it comes from them talking about their answers… It just amazes me that me talking just did not get to them. But then I said: let's just do it again, pretend you've never done it, and then just read your answers and make sure all of you just have the same answers. And all of a sudden I just had more kids to which it made sense.
>
> Researcher: Some students may have the "a-ha" moments. Do you think as a teacher you have some control over those moments? In your lesson plans, are you designing for them?

Naomi: It ["a-ha" moment] just happens … I can never know where they will have [it]… I can kind of guess. But something I think it is really easy and they [struggled a lot]. Something I thought really difficult and they get it. So it depends on the situation. I am not planning for it at all. … I don't know, sometimes, [I] just write it and they see it.

Researcher: You know, there are the special moments, "Ah, I get it!" What do you think brings about those special moments? Well, it doesn't always have to be those "a-ha!" moments but I'm very much interested in how you'd account for why and how learning happens.

Brian: It ["a-ha" moment] happens different times, sometimes it's just gonna be a student finally sort of… they just got to calm themselves, get out of their mood of talking and playing, calm themselves and engage with the problem and then it's like a totally internal thing. Doesn't involve any input from me.

From the teachers' perspective, it seemed that students' learning was to some extent characterized by these epiphanies that had little to do with what the teacher did. The teachers' experiences with "a-ha" moments seemed to indicate that it was difficult (if not impossible) to deliberately plan for and guide students' learning. During the interviews, most (9 out of 10) teachers believed that the role of their instruction in supporting students' learning was somewhat secondary. Without an access to more productive way of thinking about and working with the diversity in students' mathematical reasoning, it seemed reasonable for these teachers to attribute learning to inherent, personal qualities of individual students (e.g., intelligence or willingness to learn).

Revisiting the use of student work in PD. The observation that student reasoning was largely invisible in the teachers' classroom instruction helped the research team understand the ways in which the teachers looked at student work in the work sessions.

First, the general focus on student reasoning was largely incompatible with the teachers' classroom experiences. To the teachers, the overall goal of mathematics instruction appeared to involve mastery of procedures and the ability to solve problems correctly. This, rather than student various ways of reasoning, constituted the primary basis for making instructional decisions. In other words, if students were able to produce the correct answer, there was no reason from the teacher's perspective to further the discussion. In cases where they failed to do so, the discussion would often involve iteration of the procedures, breaking down the steps and additional demonstrations until students could produce the correct answer following similar procedures. This finding helped explain why the teachers focused on the details of students' procedures when they described and ranked students' solutions, but glossed over issues such as how students might have interpreted the statistics problem and what their justification was when organizing data in a particular way.

Second, even if the teachers had understood what it meant to focus on student reasoning, the instructional relevance of doing so might have been tenuous for most of them. The teachers seemed interested in knowing what was going on in students' minds as an end in itself. However, knowledge as such did not seem to be helpful for instruction. The most prevalent instructional challenge identified by the teachers involved how to motivate students so that they would engaged in the instruction. The strategies that they used to tackle this challenge were largely extraneous to mathematics instruction, such as using awards and sanctions or capitalizing on students' non-academic interests that had little to do with the instructional activity at hand.

As the analysis has shown, the teachers' views on student learning seemed to involve paradoxical perspectives. On the one hand, based on their observations that students often came to understand the problem in mysterious ways on their own, the teachers seemed to think of learning as a process that was elusive, unpredictable and sometimes out of their control. On the other hand, based on the notion that learning mathematics was mainly about mastery of the procedures and processes, the teachers emphasized students' paying attention and following directions and seemed to think learning would happen naturally as a result. Either way, student reasoning was not within the purview of their instructional practices. Consequently, students' failure to understand would be attributed to either their intrinsic personal qualities (e.g., intelligence) or their lack of motivation to pay attention.

To the teachers, the value of looking at student work was more than simply evaluating the correctness of students' answers. From their perspective, it involved looking for evidence of students' understanding, which involved the ability to perform the correct procedures in different problem situations. The analysis of the teaching sets convinced us that the teachers treated student work primarily as records of mathematical procedures that students produced rather than records of their reasoning.

The Role of Whole-Class Discussions in Supporting Students' Learning of Mathematics

As discussed earlier, the PD activity around student work in year 3 was largely ineffective. A large part of this ineffectiveness was due to the fact that the teachers did not consider student work to be a resource for planning a whole-class discussion that built on the various ways that students had reasoned about the problem. Therefore, a second important goal in accounting for the teachers' classroom instruction was to understand the nature of whole-class discussions in the teachers' classroom instruction and its role in supporting students' learning of mathematics.

The nature of whole-class discussions in the teachers' classroom. In the lessons that we observed, the nature of whole-class discussions primarily involved students taking turns to report their answer and sometimes specify the calculational steps in their method (usually upon the request of the teacher) while the rest of the class listened. During the observations, a whole-class discussion of students' solutions was only observed in seven (out of 11) lessons (see Table 2), all of which followed the same general pattern with only one exception (which I will discuss later).

During the whole-class discussion, the teacher would call for an answer from the students. Once the teacher heard a correct answer, he or she would often record it on the board or the overhead projector for other students to see and ask how many students had also got the same result. Sometimes the teacher would also ask students to articulate the calculational steps that they used to arrive at the answer. If the answer given by the student was incorrect, the teacher would often skip to the next student until a correct one was provided. If it appeared that most students failed to get the correct answer, the teachers would reiterate the rules or demonstrate the steps again. In five out of seven classes where a whole-class discussion was observed, there was no discussion of why an answer was incorrect. For their part, the students did not ask questions to find out why their solution was wrong although they might have informal discussions with other students about it. In the only two classes where students' mistakes were addressed, the discussion focused on identifying the misstep in the calculational procedure and correcting it. Additionally, in six out of the seven classes in which a whole-class discussion was observed, the students did not ask each other questions during the discussion and the speaking student was therefore only held accountable by the teacher.

The only whole-class discussion that did not follow the pattern discussed above was in Naomi's class, in which she asked various students to come to the board and solve problems in front of the class. The class would then collectively decide whether a solution (including the calculational steps) was correct and ask the student who wrote on the board to clarify or explain his or her solution. Only in this class did students' solutions truly become a topic of conversation. However, the questions asked by the students were still calculational in nature and were not oriented towards students' interpretations of the problem situation. The clarifications and explanations that students provided were considered acceptable as long as they made reference to the rules and the correct procedures. In all classes, different solutions were not discussed to compare how they were different or which one was a better way of solving the problem. In fact, during one lesson, a student explicitly asked if he could do the problem his own way and the teacher replied, "yes, as long as you did the entire problem."

The purpose of whole-class discussions. In the observed lessons, a whole-class discussion occurred either after student working individually or in groups, or at the beginning of a lesson when the teacher checked students' homework (see Table 2).

	WCD observed	Placement in lesson	Length of WCD (minutes)	Length of lesson (minutes)	Structure of the lesson (minutes)	
Brian	Yes	After IW	7	45	Introduction Individual Work WCD Individual Work Direct Instruction Individual Work	5 2 7 6 13 12
Kate	Yes	After GW	1	40	Review Group Work WCD DI-Practicing problems Group Work	5 11 1 3 20
Lisa	Yes	After GW	8	45	Introduction Group Work WCD Introducing a Problem Group Work WCD Introducing a Problem Group Work WCD Introducing a Problem Individual Work	2 7 4.5 1.5 3.5 2.5 2 2 1 4 15
Wesley	Yes	After IW	12	40	Introduction Direct Instruction Individual Work WCD Direct Instruction	8 4 4.5 12 21.5
Naomi	Yes	After IW	15 24	85	Individual Work WCD WCD-homework Direct Instruction Individual Work	13 15 24 17 16
Muriel	Yes	Beginning of the lesson	38	50	WCD-homework Individual Work	38 12
Amy	Yes	Beginning of the lesson	25	50	WCD-homework Direct Instruction Individual Work	25 8 17
Erin	No	N/A	N/A	45	Review-Introduction Group Work	8 37
Marci	No	N/A	N/A	40	DI-Practice problems	40
Helen	No	N/A	N/A	45	Group Work	45
Jane	No	N/A	N/A	50	DI-Review DI-Practicing problems	12 28

Table 2. The Structure of the Lessons Observed
(DI-Direct Instruction; WCD-Whole-class Discussion)

The primary purpose of a whole-class discussion of students' solution seemed to be evaluative, as indicated by both classroom observations and the teachers' responses during the interviews. The teachers conducted whole-class discussions to check whether the students had finished the problems and whether they were able to use the correct procedure to arrive at the correct answer. As I indicated earlier, a whole-class discussion primarily involved students responding individually to the teacher's questions. Student solutions did not constitute a topic of discussion per se. Rather, they were used as an assessment tool by the teachers, as Erin indicated in her comment below.

> Researcher: What did you do with student solutions during the whole-class discussion?
>
> Erin: Put on an overhead graph…Let [the students] see it up there. The checkpoint is the whole-class discussion, just review of the lesson. Ideally, you'd like to have it at the end of the class but it just does not happen. So we will have the review and the set up for the next.

This orientation was in sharp contrast with our view on whole-class discussions as researchers. As the reader will recall, we view whole-class discussions as an important means of supporting students' learning by building on the diverse ways in which students have reasoned about the problem. In this process, the teacher's role is critical, as he or she will have to carefully select solutions to be contrasted or compared so that significant mathematical issues can emerge as topics of conversation. However, it seemed that from the teachers' perspective, the primary role of a whole-class discussion in supporting student learning was to create opportunities for students to listen to the correct solutions, which hopefully would help them to produce similar ones in their own work. As seven teachers explained during the interviews, this was because students were more likely to listen to their peers rather than the teacher. As a result, the teachers' interventions during whole-class discussions were often minimal, involving things as such recording students' solutions, deciding the student to be called upon, asking clarifying questions about answers or steps, and indicating the correctness of the answer. This was the case even in Naomi's classroom where the quality of the whole-class discussion was ahead of the rest of the classrooms.

For the teachers, a whole-class discussion was no more significant than other phases of lessons. As a matter of fact, five teachers clarified during the interviews that they preferred whole-class discussions over group work (or vise versa) mainly in terms of classroom management issues rather than the quality of the learning opportunities that would arise for the students.

> Kate: I think the small groups is definitely the best way to go… [When I put them to small groups] I only have 5 or 6 groups to listen to. I don't have 24 individuals raising their hand and saying they didn't understand. Versus I've 6 groups, I can deal with one and other 5 are almost taking care of themselves. So then I can rotate… So that makes it actually easier for me. They are not getting yelled at for talking. It makes your discipline better.

> Muriel: Sometimes I do a whole-class discussion because I don't have time for them to do [small group work] in their class… Sometimes [they] are not being focused in their groups so I have to do it a whole-class discussion.

> Helen: [I prefer whole-class over small group activities because] I have more control of [students'] work. That's just me. I know when they are talking about things they are not supposed to be talking about. I feel like if I have more control we can get more done.

From the teachers' perspective, it seemed that whole-class discussions as such were dispensable and could be substituted by students working in groups. Whole-class discussions were essentially the same as the latter, only conducted on a large scale. According to six teachers during the interviews, it was during the group work or individual activity that the teachers expected to talk with students about their solutions in depth as they walked around the classroom. Whole-class discussions, on the other hand, were mainly useful for teachers to evaluate students' learning and for students to hear each other's solutions.

Revisiting the use of student work in PD. By comparing and contrasting the whole-class discussions as they were constituted in the teachers' classroom to the type of whole-class discussions that the research team was aiming for, we started to gain insight into why the question of how to build on student solutions did not make sense to the teachers.

First, the teachers viewed whole-class discussions as "checkpoints" for instruction in which student work (and the solutions that they produced) served primarily as a tool for retrospective assessment rather than a resource for prospective planning. A Whole-class discussion therefore involved presentations of the results of students' learning rather than a significant opportunity to support learning in itself.

Second, for most of the teachers, the only image of building on student solutions that they had involved creating opportunities for students to hear other's solutions. This perspective was actually compatible with their view that learning mathematics mainly involved mastery of procedures and processes. If mathematics is essentially procedural, then having opportunities to see or hear the calculational steps in order to learn to reproduce them would be reasonable and sufficient for the purpose of learning of mathematics. This finding helped explain why the teachers only got as far as discussing the sequence of the presentations during the work sessions when thinking about how to build on student solutions.

One might conclude that the collaborating teachers' instructional practices were problematic as they did not seem to focus on teaching mathematics in meaningful ways. To see these teachers' practices as reasonable, it is important to consider that the teachers worked in a world where neither decisions about what mathematics to teach nor their students' motivation to learn mathematics seemed to be under their control. The teachers thus attempted to support their students' learning by focusing on what they considered instructionally feasible from their perspective. The prior analyses on the institutional context (especially analyses on the accountability and support structures within the institutional context) in which the collaborating teachers worked also helped the research team to understand the regularities in teachers' practices (Cobb et al, 2003).

As indicated earlier, in the institutional context in which the participating teachers worked, the administrations responded to the accountability pressures of state testing by attempting to monitor and assess teachers' classroom instruction. Principals' drop-in visits to teachers' classrooms and faculty meetings were conducted on a regular basis. During these routine (at times weekly) visits, principals checked for teachers' coverage of the district prescribed objectives as well as for students' engagement. For them, good mathematics teaching seemed to involve students' good behavior and appearance of paying attention to the mathematics instruction (Dean, 2005; Visnovska, 2009). During the school faculty meetings, the teachers regularly discussed means to discipline misbehaving students and strategies for enhancing state test performance of borderline-failing students. As a result, the teachers had limited opportunities to reflect upon issues of student mathematical reasoning, as it was not a part of the discussion in any school-wide activities. Nor was it included in the vision for mathematics instruction in the district (Dean, 2005). On the other hand, students paying attention was a persistent concern for the administrators as well as the teachers, as it was viewed to prevent student from misbehaving and thus constituted as a legitimate indicator of good teaching and student learning.

The prior analyses of the institutional context provided a backdrop against which the teachers' conceptions of teaching and learning in their schools gained rationality, coherence, and legitimacy. A consideration of the institutional context in which the teachers' practices were situated served as a basis for evaluating the validity of conjectures that were formulated in the course of analyzing the collected teaching sets. For example, it led us to refute the conjecture that the teachers deliberately chose to ignore students' mathematical reasoning in their instructional planning. Instead, we were obliged to understand why student reasoning was not on the teachers' horizon while they engaged in teaching mathematics and attempted to satisfy institutional requirements at the same time.

Towards a Bi-directional Reconceptualization

Our analysis of the teaching sets resulted in two important insights. First, it enabled us to understand why the teachers took an evaluative orientation towards the use of students' work in the sessions. Second, we came to realize that the ways in which we assumed student work would be used in the sessions did not fit with how the teachers used student work in their classrooms. In Wenger's (1998) terms, student work was a reification of students' reasoning within the context of our practices as researchers and teacher educators. In contrast, student work was a reification of the outcome of instruction for the teachers within the context of their classroom practices (Cobb, Zhao, & Dean, 2009). These insights led us to question our assumption that teachers' activities in PD and in their classrooms could be related by focusing PD activities on artifacts that originate in the classrooms.

Beach's (1999) notion of consequential transitions proved to be particularly useful in rethinking the relations between teachers' activity in PD sessions and their classroom practices. In Beach's terms, transitions between settings occur when teachers shift from engaging in classroom teaching to participating in PD activities, and vice versa. For Beach, these transitions are consequential if and only if teachers' participation in PD sessions is oriented towards reworking their classroom practices. This perspective gives rise to two implications for PD.

The first implication is that PD activities should be designed so that teachers can relate their participation in work sessions to their classroom practices. In the case of the teachers with whom we worked, our design conjectures implicitly assumed that the teachers used student work as a reification of student reasoning in their classrooms. As we have illustrated, this assumption was unviable.

The second related implication of Beach's perspective on people's activity in different settings is made explicit in his proposal to use the construct of "developmental coupling" as a methodological tools for analyzing consequential transitions. As Beach defines it, a development coupling "encompasses aspects of both changing individuals and changing social activity" (Beach, 1999, p.120). Beach clarifies that a developmental coupling necessarily involves artifacts—student work for example—that reify practices and transcend different social activities in which people participate. Developmental coupling extends the unit of analysis to include multiple activities that are involved in the transition, thereby providing conceptual guidance for retrospective analyses of teachers' learning as they participate in activities across different settings. Concretely, it implies that teachers' learning in PD needs to be interpreted against the background of their classroom practices; likewise, changes in teachers' classroom practices cannot be accounted for sufficiently without an explicit reference to their learning in the setting of PD. This implication clarifies that when the same artifact is used in activities in different settings (e.g., student work is used both in PD activities and the classroom), its constitution in one setting needs to be understood in relation to how it is used in the other setting. In the case of student work, the questions that might be addressed when conducting an analysis of this type include: How do the participating teachers typically use students' work in their classroom practices? What pedagogical value do they attribute to students' work in the context of those practices? Are there significant differences between the teachers' use of student work in their classrooms and the ways in which the researchers envision it being used in PD sessions?

In addition to guiding retrospective analyses of teachers' learning, the construct of developmental coupling can also be adapted to help anticipate the extent to which prospective PD design conjectures are likely to yield the expected learning experiences for teachers. In contrast to a realized developmental coupling that involves activities that teachers have already experienced, the anticipated developmental coupling is by nature hypothetical. It juxtaposes the anticipated coupling envisioned by researchers with the actual coupling that teachers are likely to experience based on what they know and do about mathematics teaching and learning in their classrooms.

To illustrate, I take as an example our research experience around the use of student work. As discussed earlier, we intended to use student work to make visible the diversity in students' mathematical reasoning and eventually to help the teachers build on this reasoning in their instructional planning. This chain of design conjectures is outlined in the anticipated coupling (see Figure 18), from which it can be seen that student work is expected to constitute a reification (Wenger, 1998) of student reasoning within the context of PD activities. In the actual coupling, as student work is indeed regularly used in teachers' classroom practice, a solid arrow is drawn to indicate the two-way movement of the physical artifact. However, had we better understood of the teachers' instructional practices involving student work, we would have considered it highly unlikely that the teachers used student work in a way that was compatible with the instructional practice of focusing on students' reasoning. Additionally, for this PD activity to unfold as we expected, it is necessary that student reasoning had some currency or visibility within the context of classroom teaching. As this was not the case, a direct focus on student reasoning in PD was likely to be viewed as irrelevant, impractical, or intimidating from the teachers' perspective.

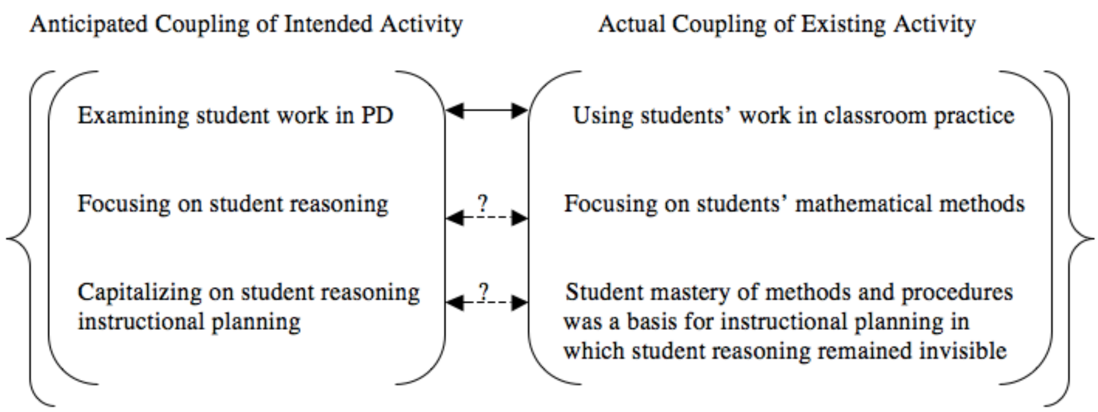

Figure 18. Anticipated Developmental Coupling in PD Design

By articulating the prospective developmental coupling, PD designers can anticipate whether an intended consequential transition is likely to occur, how it is going to occur, and whether the intended means of supporting teachers' learning is likely to be valid.

The notion of consequential transition grounds a reconceptualization that centers on a bi-directional interplay between teachers' activity in the PD and in the classroom. The implication of the bi-directional conceptualization is two-fold. On the one hand, it offers a conceptual framework for retrospectively analyzing and interpreting teachers' activity across the two settings. The bi-directionality characterized in this reconceptualization extends the unit of analysis to involve practices that become constituted across both settings. Therefore, when examining the use of classroom artifacts in PD sessions, their constitution has to be contextualized in teachers' classroom practices that involve the use of the same set of artifacts. On the other hand, a bi-directional conceptualization can guide the prospective design of PD by enabling researchers to anticipate potential conflicts that may arise as teachers engage in the planned activities even when classroom artifacts or practices are incorporated in the PD activities. This explication of the bi-directional interplay contributes to elaborate the missing dimension in the level of justification that underlies PD designs.

Conclusion

In this chapter, my goal has been to make an initial contribution to the development of an elaborated framework for conceptualizing the relations between teachers' learning in the setting of PD and their instructional practices in the classroom. The bi-directional conceptualization that I propose orients us to think about what might be "meaningful variations in opportunity to learn" (Little, 2004, p. 110) and what can be feasibly accomplished when records of classroom practice are incorporated into PD as a means to support teachers' learning. Most importantly, it enables researchers and teacher educators to examine PD design from a more conceptual perspective, one that goes beyond concrete empirical PD strategies.

The bi-directional conceptualization also gives rise to the need to characterize teachers' classroom practices from their own perspective. An analysis of teachers' instructional reality, unlike many observer-centered accounts of teaching, focuses not only on what teachers do on the observable level but more importantly, on the underlying rationales for their instructional practices. These underlying rationales constitute the backbone of the bi-directional interplay through which teachers' classroom practices and their learning in PD can be interpreted and examined. The inquiry into teachers' instructional reality is also motivated by a deep-rooted concern for teacher professional design; that is, the need to search for potential issues of interest that teachers might find meaningful and relevant to address in the setting of PD. This approach of trying to understand teachers' classroom practice by placing their sense-making at the center is fundamentally consistent with the approach of building on teachers' current classroom practices. It reflects researchers' efforts to de-center and adopt teachers' perspective when interpreting the relations between teachers' classroom experiences and their experiences in the PD.

The bi-directional conceptualization is consequential for designing PD to support changes in teachers' classroom practices. It indicates that, prior to working with teachers, it is important that researchers develop relatively detailed accounts of the collaborating teachers' instructional practices and thus of the ways in which they use key artifacts. Researchers will then be able to make informed conjectures about the extent to which the design PD activities will become constituted as expected and thus to anticipate potential problems that are likely to arise as well as means to overcome them. Two aspects of teachers' classroom practices are particularly worthy of attention. The first concerns the extent to which students' reasoning is made visible in teachers' classroom practices whereas the second involves identifying issues that are pragmatically relevant to the teachers in the context of their instructional practices and that can be leveraged to achieve the PD agenda of supporting their learning across the two settings.

Additionally, the bi-directional conceptualization implies that during the course of collaboration (longitudinal PD programs in particular), it is important that the researchers are informed by analyses of the collaborating teachers' developing classroom practices as well as by analyses of their activity in the PD sessions. Recall again that our intended use of student work proved to be unviable in the PD sessions. We would not have understood why the teachers did not view the activity as relevant to their classroom practices had we not conducted an additional round of data collection in order to analyze those practices.

Finally, the bi-directional conceptualization shapes the explanation of the teachers' learning and also results in credible accounts for why particular design decisions did not work as expected. For example, to account for why student work did not support the learning of the teachers as intended, we focused on the lack of alignment between how we envisioned student work might be used in the work sessions and how it was constituted in the teachers' classroom practices. This type of explanation is potentially generalizable to other cases in which there is a similar lack of alignments between the use of artifacts in professional development sessions and the classroom.

CHAPTER IV

FURTHER DISCUSSION OF THE DESIGN IMPLICATIONS

As illustrated by the analysis in Chapter III, it was apparent that a direct focus on student reasoning in the professional development (PD) activities was not viable because it lacked relevance to the teachers' instructional experiences at the classroom level. At the same time, the research team considered it equally invalid to abandon our original goal for working with the teachers; that is, to support them in developing instructional practices that would place student reasoning at the center. We therefore were in search of a new common ground between the envisioned focus on student reasoning in PD and the teachers' current instructional practices at the classroom level. Specifically, we were in need of an alternative focus of activity that would appear immediate and meaningful enough for the teachers to engage yet would allow us to reestablish the instructional relevance of student reasoning.

To this end, we found the accounts of the teachers' instructional reality a useful resource to draw on. As we developed these accounts, the issue of students' motivation emerged as a potentially viable option that would allow us to further our collaboration with the teachers without loosing sight of the ultimate goal of focusing on student reasoning. Our decision was primarily based on two reasons.

First, the accounts of the teachers' instructional reality indicated that the teachers viewed students' motivation as essentially inherent, determined mainly by societal or economical factors beyond the classroom. For this reason, seven teachers explained during the interviews that one of the major differences between the regular and advanced classes was that the latter were good students who were "naturally this way." Additionally, the issue of how to motivate students to engage in mathematics instructional activities was identified by six (out of 10) teachers during the interviews as the most prevalent instructional challenge in their teaching of mathematics. To the teachers, student motivation was highly problematic yet it largely determined students' engagement in instructional activities and eventually their learning of mathematics. We therefore conjectured that most teachers would find student motivation a sensible and relevant issue to address in the context of PD. Our conjecture was also based on the analyses of the institutional context in which these teachers worked (Cobb et al., 2003). As indicated earlier, the school and district administrations emphasized students paying attention to instruction and considered it one of the most important indicators of good mathematics instruction. Given the prevalent concern of engaging students in instructional activities, it seemed reasonable to expect that the teachers would find conversations about student motivation worthwhile.

Second, the focus on student motivation provided a potential link to student reasoning. By purposefully challenging the teachers' current views on student motivation, we planned to create a perturbation that would lead to an alternative perspective on the same issue. We expected this alternative perspective to be substantially more situational, relating students' classroom mathematical experiences to the extent to which they felt motivated to learn or participate in classroom activities. Pedagogically, it would then become meaningful for the teachers to discuss and examine viable means of supporting their students' development of positive mathematical experiences. From our view as researchers, a crucial aspect of such positive mathematical experiences would involve student developing meaningful understanding of significant mathematical ideas. In this way, we conjectured that the focus on students' motivation would eventually support the teachers in seeing student reasoning as instructionally relevant. In other words, we expected the readjusted focus on issues of motivation to constitute an alternative context in which we would be able to construct collaboratively with the teachers the relevance between student reasoning and instructional planning.

A series of PD activities were designed and facilitated to support changes in teachers' perspectives on students' motivation and later the relations between student reasoning and their classroom instruction. Our first goal when designing these activities was to challenge the widely accepted notion among the teachers that motivation is inherent. To us, this notion of motivation is problematic in that it assumes the "unmotivated" students cannot be effectively taught and thus deprives these students of opportunities to learn meaningful mathematics. We therefore intended to create a perturbation for the teachers that would challenge their current views on motivation.

To this end, we used interview data of a group of eighth-grade students who participated in a statistics design experiment while at the same time attending their regular algebra class[12] (Cobb & Hodge, 2007). The statistics class was organized to focus on significant mathematical ideas and the students were expected to explain and justify their solutions with regard to the problem under investigation rather than simply produce the correct answers. In contrast, the algebra class can be generally described as a traditional mathematics classroom, in which the focus was to produce the correct answers. Students were obliged to follow the teacher's direction at all

---

[12] The interview data were collected as part of the classroom design experiment during which the statistical instructional sequence was designed and tested.

times but were not held accountable to explain how they interpreted the problem mathematically.

We purposefully focused on these interviews because the same group of students talked about their mathematical obligations as well as their level of engagement in the two classrooms in vastly contrasting ways. We asked the teachers to analyze the interview excerpts combined with a description of the mathematics instruction in each classroom. The teachers were surprised to note that the same students who appeared to be highly motivated in the statistics class seemed rather unmotivated in the algebra one. They therefore had to look for explanations that were situated within the mathematics instruction as it was constituted in each classroom. As a result of this activity, the teachers began to develop an interest in understanding what it was about the statistics instruction that motivated the students to learn.

The research team capitalized on the teachers' interest by introducing the next activity in which they were asked to analyze a series of video clips from the statistics classroom. The goal of this activity was to identify key aspects of the instruction that motivated the students' engagement. In analyzing the video clips, we attempted to orient the teachers to examine the statistics instruction from the perspective of participating students. For example, one of the questions that we posed to the teachers was what makes an instructional activity worth engaging from the students' perspective. As a result of participating in these activities, the teachers gradually came to see that it was possible for a teacher to influence student motivation in learning mathematics through the means of classroom instruction. More specifically, the teachers came to the realization that one of the important reasons that the students were engaged in the statistics lessons was because their solutions were made a topic of conversation that led to significant mathematical ideas.

The research team capitalized on the readjusted focus of student motivation in the three-days summer sessions at the end of year 3 and throughout year 4. The issue of motivation was considered a promising focus not merely because it was an issue of relevance from the teachers' perspective but because it enabled us to attend to what was instructionally important for the teachers without overthrowing our overarching agenda for supporting the teachers' learning. The validity of this readjusted focus for PD activities was substantiated by the analysis of data collected in the last year (fifth year) of our collaboration with the teachers. The analysis reveals that, by the end of the fifth year, the teachers routinely considered various ways in which students might reason about data when they adapted the statistics instructional materials to their own classrooms (cf. Visnovska, 2009).

My intent in outlining the modified PD activities is to illustrate how the account of the teachers' instructional reality significantly enabled us to work more effectively in supporting their learning. Our decision to build on the issue of motivation was based on careful consideration of both the conceptions that the teachers had developed in their own classroom and the views that we hoped to support in the context of PD activities. As a matter of fact, by juxtaposing these perspectives across settings, we intended to capitalize on the conceptual conflict that the teachers were likely to experience. More importantly, as I have illustrated, we conjectured that this conflict would lead us to reexamine student reasoning in the light of motivating students to engage in instructional activities.

The bi-directional conceptualization that emerged in our work with the teachers involved dual purposes. It generated insights that would explain why the use of student work in PD did not yield the learning opportunities as expected and at the same time, illuminated alternative routes of collaborating with the teachers that would eventually build towards student reasoning. The readjusted focus on issue of motivation was made possible because the research team took seriously teachers' instructional practices and build on them to inform the design of PD activities. Research on student learning of mathematics has long recognized the importance of understanding student various ways of reasoning and using them as a resource to build towards the envisioned instructional goals. Analogously, a parallel focus on teachers' instructional practices should be placed at the center of PD designs to inform teacher educators' planning and decision making. Only then can teachers perceive PD activities as truly relevant and meaningful to the improvement of their mathematics instruction at the classroom level while at the same time participate in worthwhile PD activities.

APPENDIX A

TEACHER INTERVIEW PROTOCOL

1) About the class
    - What kind of class are you teaching? How would you like to characterize your class?
    - If a new student is coming to join your class and he is asking other students about what he should do in order to be successful in your math class, what do you think your students would tell the new comer?
    - What is the difference between the regular and the advance class from your perspective?
2) About today's lesson
    - What concerns you most when you were planning for today's lesson?
    - What do you think are the most difficult concept for students in today's lesson?
    - What did you do to address the difficulty?
    - What is your expectation of a successful lesson?
    - How did you decide your pacing for today's class? How did you decide to go faster or to slow down?
    - What is the previous lesson and what is the next lesson?
    - What are some changes that you made today in your instruction?
    - How did you decide the problems you used in today's lesson? What is your rationale for choosing them?
3) Orientation to students' reasoning
    - How do you cope with the diversity present in your classroom? What do you do specifically in your instruction to account for the diversity? Do you plan differently for different classes that you are teaching?
    - Sometimes when students just don't get it, what do you think the problems might be? What do you usually do to deal with this kind of situation/to help students understand?
    - Why do you think the Aha moments happen? Do you think the teachers have any control over it?
    - What do you think as the usefulness of the whole-class discussion that you had at the end of your class? Both for you as a teacher and for the students? What about group work?
4) Means of support learning/instructional challenges
    - How did you manage to engage the students in today's lesson? Why do you think the students were engaged?
    - How do you keep students on task?

REFERENCES

Argyris, C. and Schön, D. (1978), *Organizational learning: A theory of action perspective*. New York: McGraw-Hill.

Ball, D. L. (1997). What do students know? Facing challenges of distance, context, and desire in trying to hear children. In B. Biddle, T. Good, & I. Goodson (Eds.), *International handbook on teachers and teaching* (pp. 679-718). Dordrecht, Netherlands: Kluwer Press.

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In G. Sykes & L. Darling-Hammond (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3-32). San Francisco: Jossey Bass.

Ball, D. L., & Forzani, F. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education, 60*(5), 497-511.

Ball, D. L., & Rowan, B. (2004). Introduction: Measuring instruction. *The Elementary School Journal*, *105* (1), 3-10.

Beach, K.D. (1999). Consequential transitions: A sociocultural expedition beyond transfer in education. *Review of Research in Education, 24*, 101- 139.

Beach, K.D. (2003). Consequential transitions: A developmental view of knowledge propagation through social organization. In T. Tuomi-Gröhn & Y. Engeström (Eds.), *Between school and work: New perspectives on transfer and boundary-crossing* (pp. 39-62). NY: Pergamon

Borko, H.(2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher, 33(8)*, 3-15.

Bowen, E., & McClain, K. (2005). *Accounting for agency in teaching mathematics*. Paper presented at the research presession of the National Council of Teachers of Mathematics, Anaheim, CA.

Brown, S. & McIntyre, D. (1993). *Making sense of Teaching*. Buckingham: Open University.

Carpenter, T., Blanton, M., Cobb, P., Franke, M., Kaput, J., & McClain, K. (2004). *Scaling Up Innovative Practices in Mathematics and Science*. Research Report of the National Center for Improving Student Learning and Achievement in Mathematics and Science.

Carpenter, T.P., Fennema, E. & Franke, M. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *Elementary School Journal*, 97(1), 3–20.

Castle, K., & Aichele, D. B. (1994). Professional development and teacher autonomy. In D. B. Aichele & A. A. Coxford (Eds.), *Professional development for teachers of mathematics* (pp. 1-8). Reston, VA: National Council of Teachers of Mathematics.

Chamberlin, M. T. (2004). Design principles for teacher investigations of student work. *Mathematics Teacher Education and Development, 6*, 61-72.

Chamberlin, M. (2005). Teachers' discussions of students' thinking: Meeting the challenge of attending to students' thinking. *Journal of Mathematics Teacher Education, 8*, 141–170.

Clarke, D.J. & Hollingsworth, H. (2002) Elaborating a model of teacher professional growth. *Teaching and Teacher Education*, *18*(8), 947-967.

Cochran-Smith, M., & Lytle, S. L. (1999). Relationships of knowledge and practice: Teacher learning in communities. In A. Iran-Nejad & C. D. Pearson (Eds.), *Review of research in education* (Vol. 24; pp. 249–305). Washington, DC: American Educational Research Association.

Cobb, P. (1996). Justification and reform. *Journal for Research in Mathematics Education*. *27 (5)*, 516-520

Cobb, P. (1999). Individual and collective mathematical learning: The case of statistical data analysis. *Mathematical Thinking and Learning, 1*, 5-44.

Cobb, P., Confrey, J., diSessa, A. A., Lehrer, R., & Schauble, L. (2003). Design experiments in education research. *Educational Researcher, 32*(1), 9-13.

Cobb, P., & Hodge, L. L. (2007). Culture, identity, and equity in the mathematics classroom. In N. S. Nasir & P. Cobb (Eds.), *Diversity, equity, and access to mathematical ideas* (pp. 159-171). New York: Teachers College Press.

Cobb, P., & McClain, K. (2004). Principles of instructional design for supporting the development of students′ statistical reasoning. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning, and thinking* (pp. 375-396). Dordrecht, Netherlands: Kluwer.

Cobb, P., McClain, K., & Gravemeijer, K. (2003). Learning about statistical covariation. *Cognition and Instruction, 21*(1), 1-78.

Cobb, P., McClain, K., Lamberg, T. d. S., & Dean, C. (2003). Situating teachers' instructional practices in the institutional setting of the school and school district. *Educational Researcher, 32*(6), 13-24.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *Journal of the Learning Sciences*, *10*, 113–164.

Cobb, P., & Whitenack, J. W. (1996). A method for conducting longitudinal analyses of classroom videorecordings and transcript. *Educational Studies in Mathematics, 30*, 213-228.

Cobb, P., & Yackel, E. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.

Cobb, P., Zhao, Q., & Dean, C. (2009). Conducting Design Experiments to Support Teachers' Learning: A Reflection From the Field. *Journal of the Learning Sciences, 18*(2),165-199.

Cohen, D. (1998). Experience and education: learning to teach. In D. L. Ball, & M. Lampert (Eds.), *Teaching, multimedia, and mathematics* (pp.167-187). New York: Teachers College Press.

Dawson, S. (1999). The enactive perspective on teacher development: 'A path laid while walking'. In B. Jaworski, T. Wood, & A. J. Dawson (Eds.), *Mathematics teacher education*: *Critical international perspectives* (pp. 148-163). London: Falmer Press.

Dean, C. (2004). Investigating the development of a professional mathematics teaching community. In D. E. McDougall & J. A. Ross (Eds.), *Proceedings of the XXVI-the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 1057-1063). Toronto, Ontario, Canada.

Dean, C. (2005). *Supporting the learning and development of a professional mathematics teaching community*. Unpublished Dissertation, Vanderbilt University, Nashville, TN.

Dewey, J. (1913/1975). *Interest and effort in education*. Carbondale, IL: Southern Illinois University.

diSessa, A. A., & Cobb, P. (2004). Ontological innovation and the role of theory in design experiments. *Journal of the Learning Sciences, 13*, 77-103.

Elmore, R. (2000). *Building a new structure for school leadership*. Washington, DC: The Albert Shanker Institute.

Fennema, E., Franke, M. L., Carpenter, T. P. & Carey, D.A. (1993). Using children' knowledge in instruction. *American Educational Research Journal*, 30, 555-583.

Forgasz, H. J., & Leder, G. C. (2008). Beliefs about mathematics and mathematics teaching. In P. Sullivan & T. Wood (Eds.), *International handbook of mathematics teacher education, Vol. I: Knowledge and beliefs in mathematics teaching and teaching development* (pp. 173–192). Rotterdam, The Netherlands: Sense Publishers.

Franke, M. L., Carpenter, T. P., & Battey, D. (2007). Content matters: The case of algebraic reasoning in teacher professional development. In J. Kaput, D. Carraher, & M. Blanton, (Eds.) *Algebra in the Early Grades* (pp. 333-359). Hillside, NJ: Lawrence Erlbaum.

Franke, M.L., & Kazemi, E. (2001). Learning to teach mathematics: Developing a focus on students' mathematical thinking. *Theory into Practice, 40,* 102-109.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Understanding teaching and classroom practice in mathematics. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225–256). Reston, VA: National Council of Teachers of Mathematics.

Franke, M. L., Carpenter, T., Fennema, E., Ansell, E., & Behrend, J. (1998). Understanding teachers' self-sustaining, generative change in the context of professional development. *Teaching and Teacher Education, 14,* 67-80.

Franke, M. L., & Kazemi, E. (2001). Teaching as learning within a community of practice: Characterizing generative growth. In T. Wood & B. Nelson & J. Warfield (Eds.), *Beyond classical pedagogy in elementary mathematics: The nature of facilitative teaching* (pp. 47-74). Mahwah, NJ: Lawrence Erlbaum Associates.

Franke, M. L., Kazemi, E., Carpenter, T. P., Battey, D., & Deneroff, V. (2002, April). *Articulat- ing and capturing generative growth: Implications for professional development*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.

Gamoran, A., Anderson, C. W., Quiroz, P. A., Secada, W. G., Williams, T., & Ashman, S. (2003). *Transforming teaching in math and science: how schools and districts can support change*. New York: Teachers College Press.

Glaser, B. & Strauss, A. (1967). *The discovery of grounded theory: Strategies for qualitative research*. Chicago: Aldine.

Goldenberg, C., Saunders, B., & Gallimore, R. (2004). Settings for change: A practical model for linking rhetoric and action to improve achievement of diverse students. Final report to the Spencer Foundation: Grant #199800042). Long Beach: California State University at Long Beach.

Goldsmith, L., & Schifter, D. (1997). Understanding teachers in transition: Characteristics of a model for developing teachers. In E. Fennema & B. S. Nelson (Eds.), *Mathematics teachers in transition*. Mahwah, NJ: Lawrence Erlbaum Associates.

Grossman, P., & McDonald, M. (2008). Back to the future: Directions for research in teaching and teacher education. *American Educational Research Journal*, *45*(1), 184-205.

Grossman, P., Wineburg, S., & Woolworth, S. (2001). Toward a theory of teacher community. *The Teachers College Record*, *103*(6), 942-1012.

Hatch, T. & Grossman, P. (2009). Learning to look beyond the boundaries of representation. *Journal of Teacher Education. 60, (1):* 70-85.

Heinz, K., Kinzel, M., Simon, M. A., & Tzur, R. (2000). Moving students through steps of mathematical knowing: An account of the practice of an elementary mathematics teacher in transition. *Journal of Mathematical Behavior*, 19, 83-107.

Hiebert, J., Morris, A. K., Berk, D., & Jansen, A. (2007). Preparing teachers to learn from teaching. *Journal of Teacher Education, 58*, 47-61.

Hutchins, E. (1993). Learning to navigate. In S. Chaiklin & J. Lave (Eds.), *Understanding practice* (pp. 35-63). New York: Cambridge University Press.

Jaworski, B. (1988). Mathematics teacher research: Process, practice, and the development of teaching. *Journal of Mathematics Teacher Education*, 1, 3–31.

Jaworski, B. (1999). *The plurality of knowledge growth in mathematics teaching..* (1999). In B. Jaworski, T. Wood, & S. Dawson (Eds.), *Mathematics teacher education*: *Critical international perspectives* (pp. 180-211). London: Falmer Press.

Katims, N. & Tolbert, C.F. (1998). Accomplishing new goals for instruction and assessment through classroom-embedded professional development. In L. Leutzinger (Ed.), *Mathematics in the middle* (pp. 55–64). Reston, VA: National Council of Teachers of Mathematics.

Kazemi, E., & Franke, M.L. (2004). Teacher learning in mathematics: Using student work to promote collective inquiry. *Journal of Mathematics Teacher Education, 7,* 203-235.

Kazemi, E., & Hubbard, A. (2008). New directions for the design and study of professional development: Attending to the coevolution of teachers' participation across contexts. *Journal of Teacher Education*, 59, 428-441.

Kazemi, E., Lampert, M., & Ghousseini, H. (2007). *Conceptualizing and using routines of practice in mathematics teaching to advance professional education*. Report to the Spencer Foundation, Chicago.

Lampert, M. (2010). Learning teaching in, from, and for practice: What do we mean? *Journal of Teacher Education, 61*(1-2), 21-34.

Lampert, M. & Ball, D. L. (1998). *Teaching, multimedia, and mathematics*. New York: Teachers College Press.

Lanier, J., & Little, J. (1986). Research on teacher education. In M. Wittrock (Ed.), *Handbook of research on teaching* (3rd. ed., pp. 527-569). New York: Macmillan.

Leatham, K. (2006). Viewing mathematics teachers' beliefs as sensible systems. *Journal of Mathematics Teacher Education, 9*(2), 91–102.

Lewis, C., Perry, R., & Murata, A. (2006). How should research contribute to instructional improvement? The case of lesson study. *Educational Researcher, 35,* 3-14.

Lieberman, A., & L. Miller. (1992). *Teachers, their world and their work: Implications for school improvement*. New York: Teachers College Press.

Little, J. W. (1993). Teachers' professional development in a climate of reform. *Educational Evaluation and Policy Analysis*, 15(2), 129–151.

Little, J. W. (2003). Inside teacher community: Representations of classroom practice. *Teachers College Record 105*(6): 913-945.

Little, J. W. (2004). "Looking at student work" in the United States: Countervailing impulses in professional development. In C. Day & J. Sachs (Eds.), *International handbook on the continuing professional development of teachers*. Buckingham, UK: Open University.

Little, J. W., Gearhart, M., Curry, M. and Kafka, J. (2003). Looking at student work for teacher learning, teacher community and school reform. *Phi Delta Kappan, 85* (3), 185-192.

Loucks-Horsley, S., Hewson, P.W., Love, N., & Stiles, K.E. (1998). *Designing professional development for teachers of science and mathematics*. Thousand Oaks, CA: Corwin Press.

McIntyre, D., & Hagger, H. (1992). Professional-development through the Oxford internship model. *British Journal of Educational Studies*, *40*(3), 264-283.

Ma, L. (1999). *Knowing and teaching elementary mathematics*. Mahwah, NJ: Erlbaum.

McClain, K. (2002). Teacher's and students' understanding: The role of tools and inscriptions in supporting effective communication. *Journal of the Learning Sciences, 11*(2&3), 217- 249.

McClain, K., & Cobb, P. (2001). Supporting students' ability to reason about data. *Educational Studies in Mathematics, 45*, 103-129.

Mousley, J. & Sullivan, P. (1997). Plenary paper: Dilemmas in the professional education of mathematics teachers. In E. Pehkonnen (Ed.), *Proceedings of the International group for the Psychology of Mathematics Education* (pp. 31-46). PME: Lahti, Finland.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of

Mathematics.

Nelson, B. (1997) Learning about teacher change in the context of mathematics education reform: where have we come from. In E. Fennema & B. Scott Nelson (Eds.), *Mathematics teachers in transition* (pp. 3 –19). Mahwah, NJ: Lawrence Erlbaum Associates.

Putnam, R.T., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher, 29* (1), 4-15.

Remillard, J. T. (1999). Cirriculum materisals in mathematics education reform: A framework for examining teachers' curriculum development. *Curriculum Inquiry, 29 (3), 315-342*.

Remillard, J. T. (2002). Supporting teachers' professional learning by navigating openings in the curriculum. *Journal of Mathematics Teacher Education, 5, 7-34*.

Richardson, V., & Kile, R. S. (1999). Learning from videocases. In M. A. Lundeberg, B. B. Levin & H. L. Harrington (Eds.), *Who learns what from cases and how?* (pp. 121-136). Mahwah, NJ: Lawrence Erlbaum.

Rodgers, C. R. (2002). Seeing student learning: Teacher change and the role of reflection. Harvard Educational Review, 72(2), 230–253.

Saxe, G. B., Gearhart, M., & Nasir, N. S. (2001). Enhancing students' understanding of mathematics: A study of three contrasting approaches to professional support. *Journal of Mathematics Teacher Education, 4*(1), 55-79.

Schifter, D. (2004). Developing mathematical ideas: A resource for teaching mathematics. Retrieved December 1, 2004, from http://www.cbmsweb.or/NationalSummit/WG_Speakers/schifter.htm

Schifter, D. (1998). Learning mathematics for teaching: From a teachers' seminar to the classroom. *Journal of Mathematics Teacher Education 1,* 55–87.

Schifter, D. (2001). Learning to see the invisible: What skills and knowledge are needed to engage with students' mathematical ideas? In T. Wood, B.S. Nelson & J. Warfield (Eds.), *Beyond classical pedagogy: Teaching elementary school mathematics* (pp. 109–134). Mahwah, NJ: Lawrence Erlbaum Associates.

Schifter, D., Bastable, V., and Russell S. J. (1999). *Number and Operations: Making Meaning for Operations, Casebook and Facilitator's Guide*. White Plains, NY: Dale Seymour Publications.

Sfard, A. (2000). On reform movement and the limits of mathematical discourse. *Mathematical Thinking and Learning, 2*, 157-189.

Sherin, M. G. (2007). The development of teachers' professional vision in video clubs. In R. Goldman, R. Pea, B. Barron, & S. Derry (Eds.), *Video research in the learning sciences* (pp. 383–395). Mahwah, NJ: Erlbaum.

Sherin, M. G., & van Es, E. A. (2005). Using Video to Support Teachers' Ability to Notice Classroom Interactions. *Journal of Technology and Teacher Education,13*(3), 475-491.

Simon, M. A., & Tzur, R. (1999). Explicating the teacher's perspective from the researchers' perspective: Generating accounts of mathematics teachers' practice. *Journal for Research in Mathematics Education, 30*, 252-264.

Smith, M. K. (2001). *'Donald Schon: learning, reflection and change', the encyclopedia of informal education*. Retrieved December 11, 2006 from http://www.infed.org/thinkers/et-schon.htm

Smith, M.S. (2003). *Practice-based professional development for teachers of mathematics*. Reston: NCTM.

Sowder, J. T. (2007). The mathematical education and development of teachers. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 157–224). Reston, VA: National Council of Teachers of Mathematics.

Spillane, J. (2005). Local theories of teacher change: the pedagogy of district policies and programs. *Teachers College Record*, *104*(3), 377-420.

Spillane, J. P., Halverson, R., & Diamond, J. B. (2001). Towards a theory of leadership practice: Implications of a distributed perspective. *Educational Researcher*, *30*(3), 23–28.

Stein, M.K., & Kim, G. (2006). *The role of mathematics curriculum in large-scale urban reform: An analysis of demands and opportunities for teacher learning*. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Stein, M. K., Silver, E. A., & Smith, M. S. (1998). Mathematics reform and teacher development: A community of practice perspective. In J. G. Greeno & S. V. Goldman (Eds.), *Thinking practices in mathematics and science learning* (pp. 17- 52). Mahwah, NJ: Lawrence Erlbaum Associates.

Stein, M. K., Smith, M. S. and Silver, E. A. (1999). The development of professional developers: Learning to assist teachers in new settings in new ways. *Harvard Educational Review 69*(3), 237-269.

Steinberg, R., Empson, S. B. & Carpenter, T. P. (2004). Inquiry into children's mathematical thinking as a means to teacher change. *Journal of Mathematics Teacher Education, 7*(3), 237-267.

Stephan, M., & Rasmussen, C. (2002, April). *Classroom mathematical practices in differential equations*. Paper presented at the Research Pre-session of the

Annual Meeting of the National Council of Teachers of Mathematics, Las Vegas, NV.

Stigler, J.W., & Hiebert, J. (1999). *The teaching gap*. New York: Free Press.

Strauss, A. L., & Corbin, J. (1990). *Basics of qualitative research: Grounded theory, procedures and techniques*. Newbury Park: Sage.

Sullivan, P. (2003). Incorporating knowledge of, and beliefs about, mathematics into teacher education. *Journal of Mathematics Teacher Education 6,* 293–296.

Tirosh, D., & Graeber, A. O. (2003) Challenging and changing mathematics teaching classroom practices. In A.J. Bishop, M. A. Clements, C. Keitel, J. & Leung, F. K. Leung (Eds.), *Second international handbook of mathematics education (pp. 643-687)*. Dordrecht: Kluwer.

Tzur, R., Simon, M. A., Heinz, K., & Kinzel, M. (2001). An account of a teacher's perspective on learning and teaching mathematics: Implications for teacher development. *Journal of Mathematics Teacher Education*, *4*, 227-254.

van Es, E.A. & Sherin, M.G. (2010). The influence of video clubs on teachers' thinking and practice. *Journal of Mathematics Teacher Education, 13*, 155-176.

Visnovska, J. (2009). *Supporting mathematics teachers' learning: Building on current instructional practices to achieve a professional development agenda*. Unpublished Dissertation, Vanderbilt University, Nashville, TN.

Voigt, J. (1995). Thematic patterns of interaction and sociomathematical norms. In P. Cobb & H. Bauersfeld (Eds.), *Emergence of mathematical meaning: Interaction in classroom cultures* (pp. 163-201). Hillsdale, NJ: Lawrence Erlbaum Associates.

Walen, S., & Williams, S. (2000). Validating classroom issues: Case method in support of teacher change. *Journal of Mathematics Teacher Education, 3*, 3–26.

Wei, R.C., Darling-Hammond, L., Andree, a., Richardson, N., & Orphanos, S. (2009). *Professional learning in the learning profession: A status report on teacher development in the U.S. and abroad*. Dallas, TX: National Staff Development Council.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and Identity*. New York: Cambridge University Press.

Whitcomb, J., Borko, H., & Liston, D. (2009). Growing talent: Promising professional development models and practices. *Journal of Teacher Education 60* (3), 207-212.

Wilson, S. M., & Berne, J. (1999). Teacher learning and the acquisition of professional knowledge: An examination of research on contemporary professional development. In A. Iran-Nejad & P. D. Pearson (Eds.), *Review of research in education* (Vol. 24, pp. 173-209). Washington, DC: American Educational Research Association.

Wood, T. (1999). Approaching teacher development: Practice into theory. In B. Jaworski, T. Wood, & A. J. Dawson (Eds.), *Mathematics teacher education*: *Critical international perspectives* (pp.163-179). London: Falmer Press.

Zhao, Q., Visnovska, J., Cobb, P., & McClain, K. (2006, April). *Supporting the mathematics learning of a professional teaching community: Focusing on teachers' instructional reality*. Paper presented at the annual meeting of the American Educational Research Association Conference, San Francisco, CA.