The Lucas Paradox Revisited:
Tracking the Returns to Capital across the Development Spectrum

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0:: Abstract

Developing markets exhibit a tendency to deliver higher returns on capital than developed markets – but also exhibit more dispersion in these returns. The persistence of this inequality in returns across countries is known as the Lucas Paradox, after Lucas (1990) raised the concept to prominence, by exploring limiting influences on the mobility of capital. Rather than focus on such aspects as the different structural and institutional conditions of the world (i.e. *why* this effect might occur), this paper investigates the realized riskiness of the different markets in depth (i.e. *how* this supposed paradox is actually playing out), decomposing the opportunity space to some representative investor along many dimensions. Then a framework is advanced to penalize properly the two distinct layers of implicit costs associated with the variance of returns – and so reassess the residual return differential across the development spectrum. This framework largely amounts to focusing on how geometric mean (arithmetic mean net of variance) returns are optimized – and then measuring risk aversion after this "convexity correction." More specifically, the representative investor is conceived of as a large institution capable of conducting arbitrage on a global scale and of borrowing capital to exploit any such major market imbalances – an agent that seeks to maximize the realizable long run growth rate of his capital as he defers all of his consumption until some indefinite final period.

1:: Introduction

The question at hand is straightforward: does capital flow from rich regions to poor ones until returns are equalized? This notion of return parity across countries reflects an intuitive and well-established neoclassical assumption, forming a cornerstone in the framework of arbitrage-free asset pricing. In other words, if the markets offer some nontrivial persistent return premium to investors for supplying capital to less developed countries, then surely rational agents will recognize this large-scale imbalance and act accordingly, by shifting their allocation to the developing countries where incomes are low and capital is relatively scarce – or even by hedging their portfolios by betting against capital returns in the developed market space, thereby allowing for the acceleration of the flow of capital to where the returns are highest. But the opportunity space of realizable gains is complex. The true optimal leverage, corresponding to the capacity for capital mobility with all the agents considered together, for the representative investor, can be established using multiple derivations; this paper demonstrates with confidence that the optimal leverage is at least one and very likely more than one; however, the precise threshold of optimal leverage is very difficult to estimate exactly, even with stationary parameters in the return distribution. Yet it is safe to say that the investor can safely apply a leverage of two, meaning that for every dollar he has in capital, he borrows one dollar. The prospect of safely borrowing money to exploit imbalances in the global return distribution will lead to informative and realistic results for how capital might flow across the development spectrum. In other words, a micro-foundational approach is utilized to determine exactly how the representative agent (an large institutional investor) might exploit the supposed global return gap.

The rest of this paper will be organized into four sections: (2) gauging the limits to capital mobility from an *a priori* analysis, (3) applying that analysis to determine a reduced-form baseline estimate of risk aversion without the added context of consumption data, (4) implementing this estimation process in an *a posteriori* investigation of the data on international capital returns drawn primarily from Version 8.0 of the Penn World Table, and (5) then drawing conclusions to assess the strength of the Lucas Paradox. Also, an appendix and a list of references will be included at the end.

2:: Two Layers to the Implicit Cost of Risk: Sensitivity and Aversion

Before the impact of risk can be assessed, the definition of returns on capital must be established. There are two principal types of capital: physical and financial. This paper will explore the returns to aggregate physical capital by country, with financial capital in the background as a separate sphere. Accordingly, the returns to physical capital can be considered as an aggregate yield of output (Y) of the stock of capital (K) net of the replacement cost required by depreciation (δ) in the next period. The output to capital ratio is rescaled by the elasticity exponent (α) in the Cobb-Douglas Production Function. The elasticity is largely assumed to be invariant across time and countries, with a reasonable range of 0.3 to 0.4. Returns are in USD and carried out for at least both endpoints of this reasonable range for α.

$$R_{j,t}= \alpha(Y_{j,t}/K_{j,t})-\delta_{j,t+1}$$

[[0.1]]

The utility of intermediate consumption (C) is restricted to zero, reflecting the institutional nature of the representative investor who simply wants to maximize capital in some indefinite final period, which is identical to maximizing the geometric mean return. Imposing this restriction facilitates a baseline on risk aversion estimation in the multiple-period context; any residual return differential, adjusted for the higher variance in poorer countries, can subsequently be explained by the covariance of consumption and returns (Henriksen 2014), a second stage of analysis outside the scope of this paper.

$$C_T=K_T, \text{ where } C_{t\neq T}=0$$

[[0.2]]

The general objective function will be to maximize the geometric mean return over many periods, which is identical to maximizing expected capital in the indefinitely distant final period, where r_t is the return on the risky asset in time t and L is the leverage (or exposure) level. If L=1, which many authors implicitly assume, then the investor is putting all of his capital into an asset; if L=0.5, which the investor might choose if there is either (a) excess volatility or (b) he is risk averse or (c) both; if L=2, which, as demonstrated in the next section, constitutes a solid decision for the investor to make, then the investor is pursuing some opportunity akin to statistical arbitrage by investing all of his capital plus the same amount in borrowed capital. On the aggregate level, with N total agents, M=N*L, where M is the measure of capital mobility.

$$max\{\Pi_{1:T}(1+Lr_t)\}$$
$$max_L\{\Sigma_{1:T}(ln(1+Lr_t))\}=>$$
$$0=\Sigma_{1:T}(L/(1+Lr_t))$$

[[0.3]]

The implications of [[0.3]] will be thoroughly considered in the rest of Section 2, under various functional distributions, discrete and continuous; in Section 4 this equation will be applied to the data. This question of how much the investor should expose himself to overall is often taken for granted, with the focus jumping first to how much of each group should be held in the portfolio with an implied L=1. Here, in the short run at least, L is allowed to drift above 1 (arbitrage feasible and exploited) or below 1 (excess risk requires capital be held in reserve), in order to maximize final capital by maximizing the geometric mean. In the long run, there is some constraint on the ability of an agent to borrow capital from the rest of the world indefinitely.

Sensitivity

True arbitrage implies that the capacity for capital mobility can be indefinitely high, that the optimal leverage/exposure of the representative agent can approach infinity if the investor can capture a major imbalance. However, there are certain significant constraints to infinite capital mobility, both from an optimization standpoint and in the credit markets. The core idea here is that the representative agent aims to optimize his returns over many periods, not just one, and accordingly this aim corresponds to maximizing the geometric mean of returns, not the arithmetic mean. Moreover, it can be established that, for all risky markets with nonzero variance, the arithmetic mean is definitively greater than the geometric mean. Indeed, the difference between the two measures of central tendency increases as the effective variance of returns increases. Starting with first principles, consider the most basic case of a risky investment: there is a fixed probability p that the investor wins some fixed multiple b of his exposure x, and a complementary fixed probability q=1-p that he loses some fixed multiple a of his exposure x. There is no uncertainty to these binomial payoffs; the investor knows these fixed parameters. We further define the exposure x as the percentage of the investor's capital placed at risk and y as his capital, which can be assumed to start at 1 for the sake of simplicity. The investor is allowed to repeatedly bet at his desired frequency, which could be every year or whenever.

$$y=(1+bx)^p(1-ax)^q$$

Let v be the monotonically increasing and concave natural logarithmic function, strictly for computational convenience; here it is implied that maximizing one-period log utility is identical to maximizing multiple-period linear utility:

$$v=\ln(y)$$

$$v=p*\ln(1+bx)+q*\ln(1-ax)$$

$$dv/dx=bp/(1+bx)-aq/(1-ax)$$

Set the derivative to zero:

$$v'=dv/dx:=0=bp/(1+bx)-aq/(1-ax)$$

$$0=bp(1-ax)-aq(1+bx)$$

$$bp-aq=abpx+abqx=abx(p+q)$$

$$x=(bp-aq)/(ab(p+q))$$

$$x_{optimum}=(bp-aq)/(ab)$$

[[1]]

$$x_{optimum}=p/a-q/b$$

Or, more generally,

$$x_{optimum}=E[\Delta y/y]/(E[\Delta y/y|\Delta y/y<0]E[\Delta y/y|\Delta y/y>0])$$

[[2]]

Check the second order condition.

$$v''=b^2p(1+bx)^{-2}-a^2q(1-ax)^{-2}$$

$$0>b^2p(1+bx_{optimum})^{-2}-a^2q(1-ax_{optimum})^{-2}$$

$$0>b^2p(1+(bp-aq)/a)^{-2}-a^2q(1-(bp-aq)/b)^{-2}$$

$$0>b^2p(p(b+a)/a)^{-2}-a^2q(q(b-a)/b)^{-2}$$

$$0>(ab)^2(p^{-1}(b+a)^{-2}-q^{-1}(b-a)^{-2})$$

$$q^{-1}(b-a)^{-2}>p^{-1}(b+a)^{-2}$$

$$p(b+a)^2>q(b-a)^2$$

$$pb^2+2pab+pa^2>(1-p)b^2-2(1-p)ab+(1-p)a^2$$

$$(2p-1)b^2+(2p-1)a^2+2ab>0$$
$$2p(a^2+b^2)>(a-b)^2$$
$$p(a^2+b^2)(a-b)^{-2}>0$$

The product here must be greater than zero since all components are positive, so $x_{optimum}$ is a true maximum.

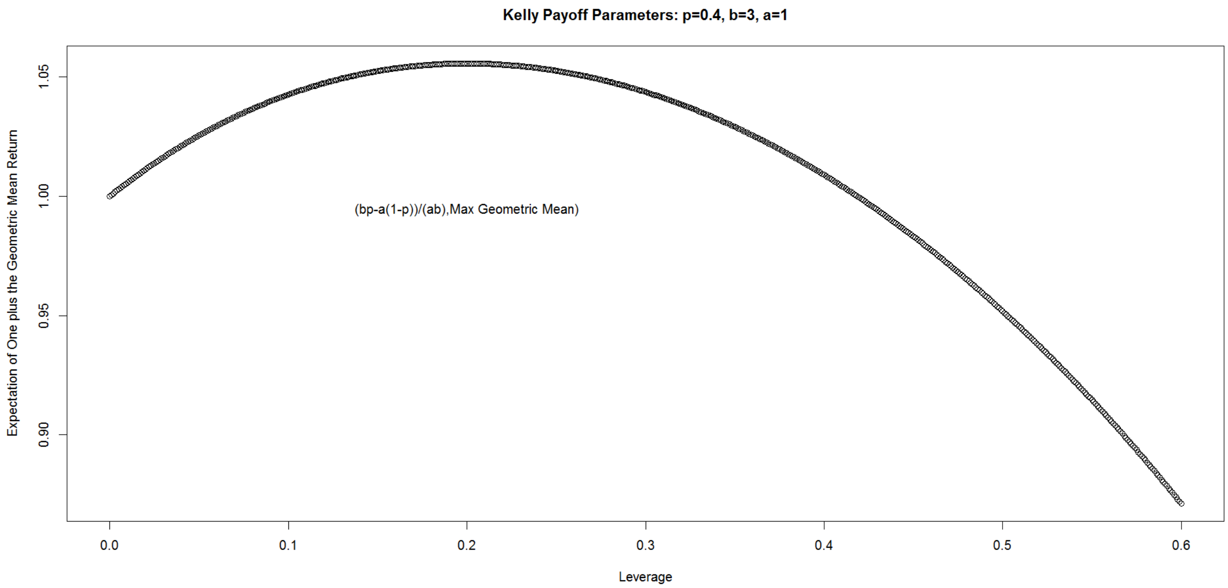
$$p>0,\ a^2+b^2>0,\ (a-b)^{-2}>0$$

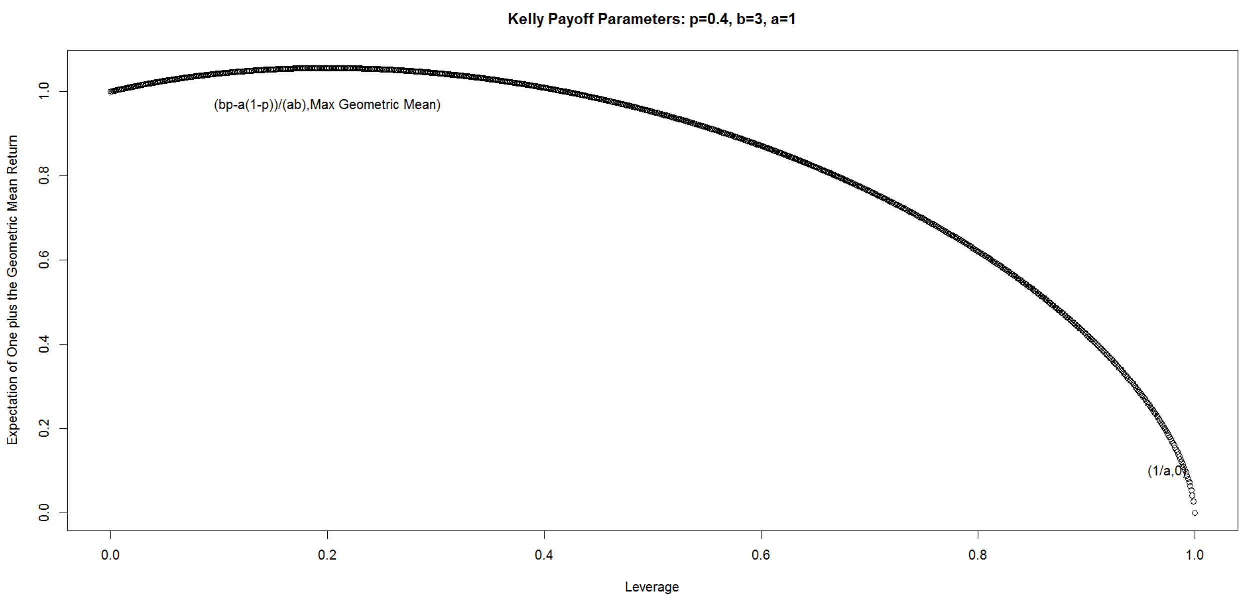
The optimal exposure $x_{optimum}$ in this restricted setting is thus the ratio of the arithmetic mean to some measure of its risk, which corresponds roughly to the variance; this concept of applying a risk-deflator to the arithmetic mean in this simplified setting is known as the Kelly Criterion. The true variance for this non-Normal distribution ($pq(b+a)^2$) does not capture the 3rd and 4th moments, which are not held at zero as is the case with the Gaussian probability density function; the true variance is close to the true optimal deflator in the case where the upside and downside are similar to typical asset returns. It turns out that this mean/variance ratio, known as the Merton Rule for the normal distribution, is very powerful and extends to other distributions. Effectively, even for the risk-neutral investor, there exists some measure of risk that functions as the deflator on the arithmetic mean of returns. In the case of normal and lognormal return distributions, simulations have been conducted to confirm the efficacy of mean/variance as an optimizing rule. If the investor only placed one bet – as in he did not have the option to reinvest – then the optimization for the risk-neutral agent is insensitive to the variance since there is only one outcome and it does not vary. In this single-bet case, the investor would be willing to run the risk of ruin, which is definitively distinct from the vast majority of real-world multi-stage risk-taking scenarios. It only applies when there is a singular or terminal allocation decision. Next, insert the optimal exposure $x_{optimum}$ into the utility function:

$$v_{optimum}=p*\ln(p(1+b/a))+q*\ln(q(1-a/b))$$
$$=\ln(q(1-a/b))+p*\ln(p/q*b/a*(b+a)/(b-a))$$
$$y_{optimum}=(q(1-a/b))*(p/q*b/a*(b+a)/(b-a))^p$$

To demonstrate the power of this optimization, consider two examples, where the payoffs reflect a positive skew and then a negative skew, respectively. First, the investor is offered a bet with p=0.4, b=3, and a=1. Thus, $x_{optimum}$=0.4/1-0.6/3=0.2. The investor loses most of the time, but wins much more when he does win. So he maximizes his utility (and capital) by only risking 20% every round of the betting. In the long run, by risking more, the investor will gain less and even lose in the extreme. If the investor sets x=0.41862, then he will break even in the long run, and so his capital will eventually approach zero beyond this threshold. If instead the investor faces parameters with p=0.8, b=0.2, and a=0.4, then $x_{optimum}$=0.8/0.4-0.2/0.2=1. If he sets x=1.745172, he will just breakeven.

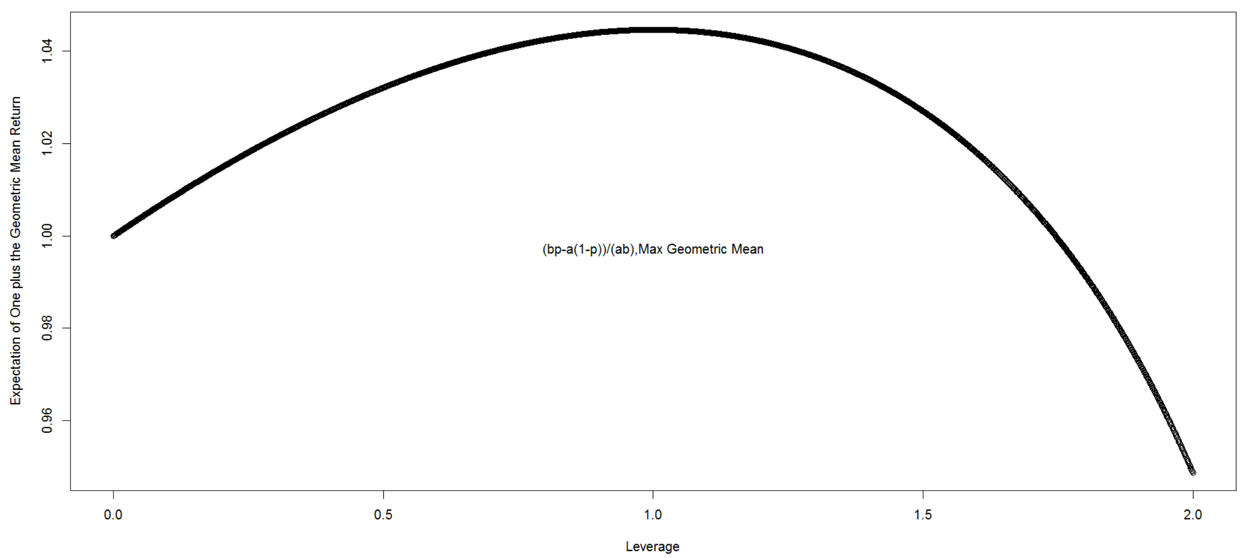
For the first case, the realizable gains are displayed, first with leverage somewhat near the optimum and then over the full range, assuming zero opportunity cost.



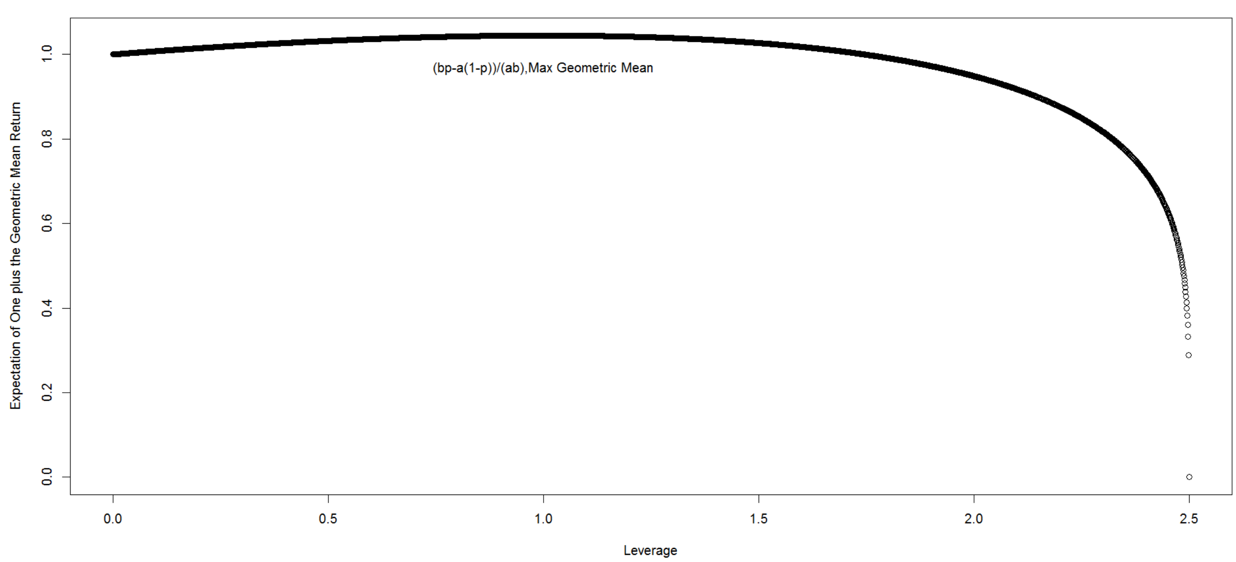


For the second case, the realizable gains are displayed, first with leverage somewhat near the optimum and then over the full range, assuming zero opportunity cost.

Kelly Payoff Parameters: p=0.8, b=0.2, a=0.4



Kelly Payoff Parameters: p=0.8, b=0.2, a=0.4



This rule – deflating the arithmetic mean by risk in order to determine the expose level that maximizes the geometric mean in the long run – is powerful. It carries over to other distributions – but requires separate derivations. Specifically, two other return distributions are considered: the normal and lognormal. Even though the leverage optimization, corresponding to the reasonable limit on the degree of capital mobility, is very dependent on the distribution, some indication of excess risk relative to reward can be established. In other words, for a given arithmetic mean return, the variance can be so high that the risk neutral investor can generate a higher geometric mean by withholding investment and keeping reserved in the risk-free safe alternative, even if this risk-free opportunity cost is zero. By extension, equation [[1]] can be changed to include nonzero opportunity cost.

Optimizing by the same process that led to [[1]],

$$y=(1+bx-r(x-1))^p(1-ax-r(x-1))^{1-p}$$

results in

$$x_{optimum}=(1+r)[(p(b-r)-(1-p)(a+r))/((b-r)(a+r))]$$

[[1b]]

$$x_{optimum}=(1+r)[p/(a+r)-(1-p)/(b-r)]=p[(1+r)/(a+r)]-(1-p)[(1+r)/(b-r)]$$

$$x_{optimum}=[(pb-(1-p)a)-r]/[(b-r)(a+r)/(1+r)]$$

It can be deduced that having a high arithmetic mean is more important when opportunity cost is higher, although the correspondence is not direct, since $x_{optimum}$ shrinks rapidly as r approaches b. However, the relationship between the evolution of opportunity cost and the evolution of mean returns will not be focused on. Rather, for many purposes, a realistic opportunity cost around 5% (E[r]=0.05) will be considered as some long run steady state level, matching up closely to the median one year Treasury rate since the 1950s.

Next the focus shifts from the simple discrete distribution to the continuous case. The log-normal distribution has the same two parameters for central tendency and dispersion, but μ denotes the geometric mean, not the arithmetic mean; the variance is also different than its regular counterpart. Importantly, lognormal distributions are positively skewed, since returns are bounded by -1 on the downside but can theoretically approach infinity on the upside. Formally, Y is lognormal if

$$\ln Y \sim N(\mu,\sigma^2)$$

In the case of returns,

$$r=-1+Y=-1+e^{N(\mu,\sigma 2)}$$

The moments for the returns are transformed.

$$\textit{ArithmeticMean}[r]=e^{\mu+0.5*\sigma\text{^}2}-1$$

$$\textit{Median}[r]=\textit{GeometricMean}[r]=e^{\mu}-1$$

$$\textit{Var}[r]=(e^{2\mu+\sigma\text{^}2})*(e^{\sigma\text{^}2}-1)=(\textit{ArithmeticMean}[r])^2(e^{\sigma\text{^}2}-1)$$

Thus, the mean/variance ratio after the log transform yields a different relationship.

$$L=\textit{ArithmeticMean}[r]/\textit{Var}[r]=(e^{\mu+0.5*\sigma\text{^}2}-1)/[(e^{2\mu+\sigma\text{^}2})*(e^{\sigma\text{^}2}-1)]$$

$$=1/[\textit{ArithmeticMean}[r](e^{\sigma\text{^}2}-1)]$$

[[3]]

Since L varies inversely with μ in this lognormal case, the Merton Rule does not carry over well, even though the positive skew inherent in the lognormal distribution should boost the true L; rather, the positive skew complicates the estimation process. It turns out that having this positive asymmetry inherent in the log-normal return distribution makes a meaningful difference in the mean/variance ratio – and consequently the estimation of optimal leverage. Specifically, the lognormal variance overstates the true downside risk relative to the upside. The distortion in the mean/variance ratio switching from the normal to the log-normal for a host of selected parameters can best be handled by simulation. After calculating the actual geometric mean using the code in the appendix, under various leverage levels, with a large number of random lognormal returns resampled a very large number of times, it seems at first that it is possible to generate a higher geometric mean than at L=1 when L>1, but eventually the risk of ruin is realized for a nontrivial variance. By the definition of the lognormal support, r_{min}=0+ ε, where ε is some trivially small positive number, so a hypothetical return of -0.9999 with L=1.2 would result in an effective return of (1.2)*(-0.9999) = -1.1988 < -1. Any investment generating a return less than or equal to -1 is ruined and ends up with a final level of capital of zero. To follow through on this principle more rigidly, if we apply a linear transformation to the lognormal, we get an interesting result.

$$\ln Y \sim N(\mu,\sigma^2)$$
$$b*Y+c=b*e^{N(\mu,\sigma2)}+c$$
$$b*Y+c=e^{\ln(b)+N(\mu,\sigma2)}+c$$
$$b*Y+c=e^{N(\mu+\ln(b),\sigma2)}+c$$

The scaling (b) and shifting (c) actually both result in shifting. For the effect of leverage, since (for the basic case of L=2) some arithmetic yields (1+r)*2-(2-1)=1+2r,

$$b=L$$
$$c=-(L-1)$$
$$L*Y-(L-1)=L*(Y-1)+1=e^{N(\mu+\ln(L),\sigma2)}-(L-1)$$

Thus, this leveraged distribution has an arithmetic mean shifted by

$$\Delta\mu=\ln(L)-L+1$$

[[4]]

$$d(\Delta\mu)/dL=1/L-1$$

If L=1, then

$$\Delta\mu=0,\ d(\Delta\mu)/dL=0$$

However, we are interested in optimizing the geometric mean, not the arithmetic mean. We know that the former is less than the latter in every instance with positive variance. But it is difficult to estimate the geometric mean here, since the leveraged distribution is no longer lognormal. That being said, as L increases so does the volatility drag (the gap between the arithmetic and geometric means), so the optimal L would logically be below 1, since above that there would be further drag on a lower arithmetic mean.

$$\mu>g$$

The normal approximation of the geometric mean ($g=\mu-0.5*\sigma^2$) holds up loosely in the lognormal case, so if L>1

$$\Delta\mu>\Delta g, \text{ where } \Delta\mu<0$$

Since the adjustment costs, which are indefinite generally and conceivably higher in poorer and less open regions, are assumed to be nontrivial, the pure-form Ito calculus, over extremely short time periods, utilized to obtain the Merton Rule does not hold. Next, a derivation is performed to estimate $L_{optimum}$ with minimal constraints on the functional form of the return distribution. Ultimately, even after the fact, it is not clear what the optimal leverage is exactly; the true return distribution does not have to match perfectly with any parametric probability density function.

Regardless of the parametric form of the distribution, the rule of maximizing expected utility holds in all cases.

$$U \neq E[1+r]$$
$$U = E[\ln(1+r)]$$

[[5]]

So, moderate risk aversion (logarithmic utility) in the one period case corresponds exactly to risk neutrality (linear utility) in the multi-period case. For any distribution of returns, maximizing the geometric mean is tantamount to maximizing final wealth,

$$max\{\Pi_{1:T}(1+r_t)\}$$
$$=max\{\Sigma_{1:T}(ln(1+r_t))\}$$
$$=max\{E[\ln(1+r)]\}$$

[[6]]

For $r^2<1$,

$$max\{E[U]\}$$
$$=max\{E[\ln(1+r)]\}$$

The Mercator series expansion converges for $r^2<1$.

$$=max\{E[r-r^2/2+r^3/3-r^4/4+r^5/5+\ldots]\}$$
$$\approx max\{E[r-r^2/2]\}$$
$$=max\{E[r(1-r/2)]\}$$

The expectation of the product minus the product of the expectation defines the covariance.

$$=max\{E[r]E[1-r/2]+Cov[r,1-r/2]$$
$$=max[E(r)E[1-r/2]-Var[r]/2\}$$

Allocation decision: buy X of one security with zero risk-free rate ($r_f=0$) opportunity cost, so L=X/K

$$E[r]=L\mu$$
$$Var[r]=(L\sigma)^2$$

$$max\{E[r]E[1-r/2]-Var[r]/2\}=>$$

Set the derivative to zero.

$$0=d[L\mu(1-L\mu/2)-L^2\sigma^2/2]/dL$$
$$0=\mu-L\mu^2-L\sigma^2$$
$$L= \mu/(\mu^2+\sigma^2)$$

For sufficiently small μ, where the Mercator series truncation is most representative,

$$L_{optimum}= \mu/\sigma^2$$

[[7]]

Borrowing from earlier work done by Merton, a simpler derivation can be examined for the normal distribution. In this normal setting, a special scaling property results, derived via Ito's Lemma as the parameters approach zero.

$$\ln(K_T)-\ln(K_0)=L[(\mu-L\sigma^2/2)T+\sigma t^{1/2}Z]$$

Here Z represents a standard normal shock.

$$Z\sim N(0,1)$$

For one (infinitesimal) step forward (T=1):

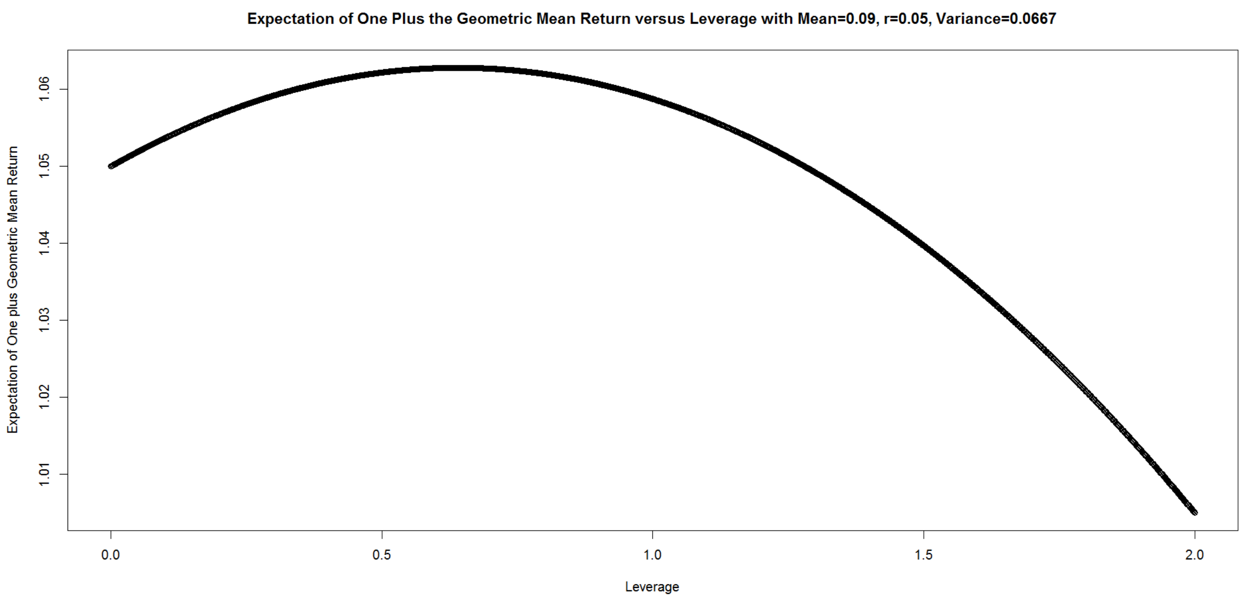
$$g=L\mu-(L\sigma)^2/2$$

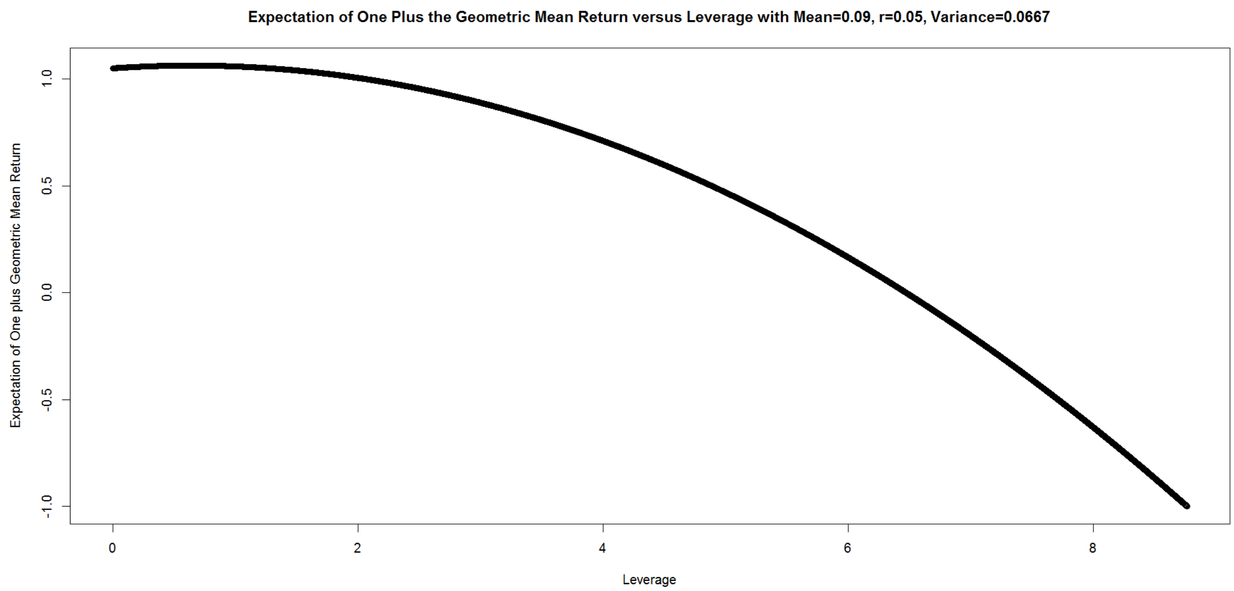
$$g'=0=\mu-L\sigma^2$$

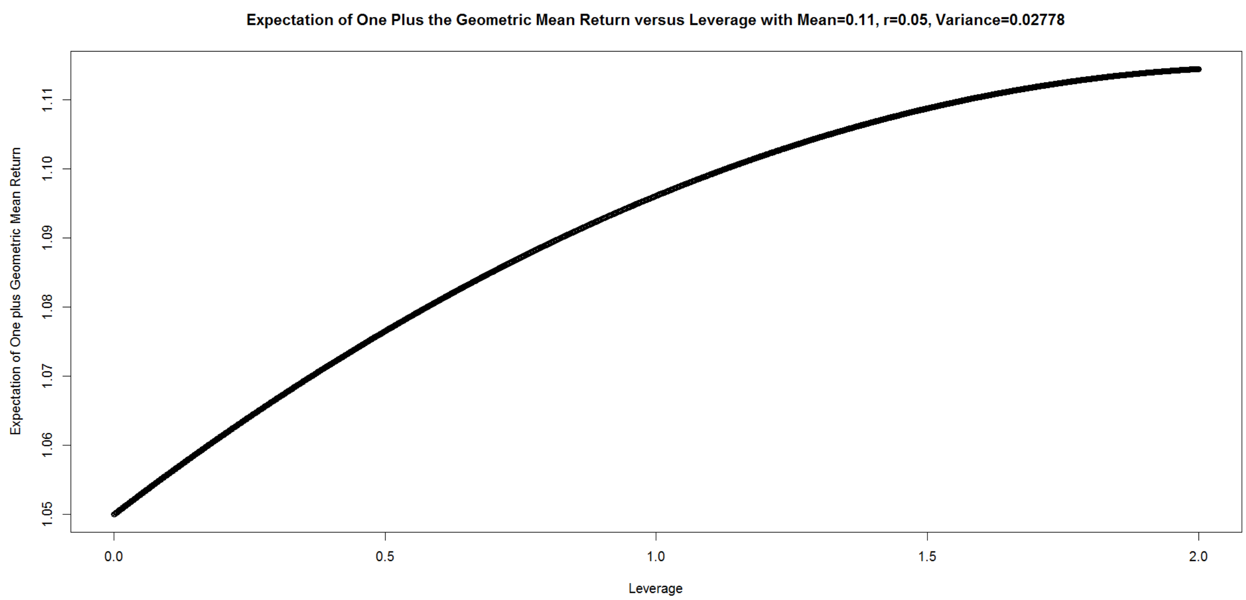
$$L_{optimum}=\mu/\sigma^2$$

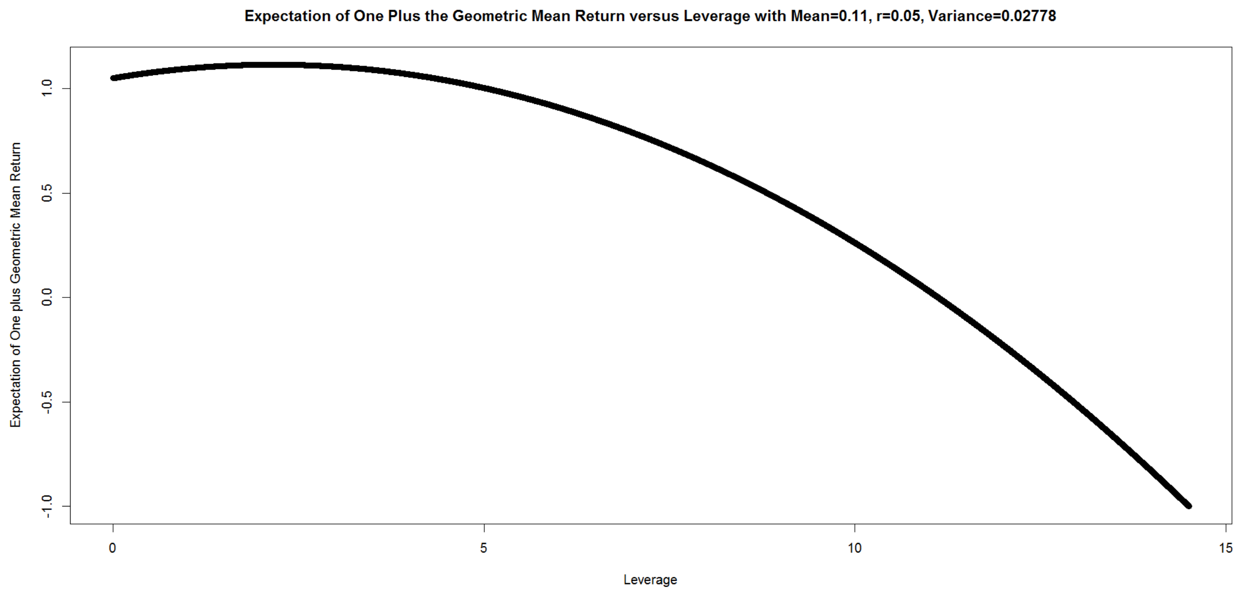
[[8]]

By the first order condition, $L_{optimal}=\mu/\sigma^2$ corresponds to exactly the mean/variance ratio; in the above equation, it must be noted than the growth rate, ignoring random shocks, scales linearly with the arithmetic mean reduced by half the variance, which is itself scaled by the exposure level L. The graphs below depict the curves of some leveraged normal distribution with hypothetical parameters that led to equation [[8]].




Expectation of One Plus the Geometric Mean Return versus Leverage with Mean=0.09, r=0.05, Variance=0.0667
Expectation of One plus Geometric Mean Return
1.0
0.5
0.0
-0.5
-1.0
0
2
4
6
8
Leverage


Expectation of One Plus the Geometric Mean Return versus Leverage with Mean=0.11, r=0.05, Variance=0.02778
Expectation of One plus Geometric Mean Return
1.11
1.10
1.09
1.08
1.07
1.06
1.05
0.0
0.5
1.0
1.5
2.0
Leverage



All of the above extensions require a sufficiently large amount of data in order for each one to be accounted for appropriately with precision. Centuries of data would yield more sound estimations. However, these extensions nonetheless serve to buttress the key insight here: that the arithmetic mean returns must be deflated by some measure of risk (roughly corresponding to the variance), in order to determine the optimal allocation for maximizing the geometric mean of returns, thereby implying the representative agent is truly a rational (yet possibly risk-averse) utility-maximizer. The specific method of penalizing instability and framing the risk-deflator will lead us to assess more appropriate estimates of genuine risk aversion preventing capital from flowing to poor countries, thereby retarding development. After calibrating these allocations based on the arithmetic mean reduced by the risk deflator, then the condition of parity of the rescaled geometric returns will be robustly tested. Specifically, these rescaled geometric mean returns will be regressed on lnGDPpc, with the null hypothesis being a slope of zero and the plausible alternative being a slope less than zero, reflecting a certain degree of genuine risk aversion. More generally, for any distribution of returns, equation [[6]] can be extended

$$max_L\{\Sigma_{1:T}(ln(1+Lr_t))\}=>$$
$$0=\Sigma_{1:T}(L/(1+Lr_t))$$

[[0.3]]

This condition can be handled by simulation for any distribution, parametric or not. For each country – and for all countries together – a resampling procedure can be conducted to determine the optimal leverage (L_0). This estimation process allows for nonparametric optimization, given that the true distribution of returns is most likely not exactly lognormal since negative returns are so uncommon; however, the empirical distribution can also be matched to the lognormal, for the sake of conformity.

So before risk aversion can be accounted for, the primary layer of the cost of risk, which can be called sensitivity, must be captured. Everyone is sensitive to risk because, ceteris paribus, it reduces the geometric mean return, which the risk neutral agent attempts to maximize. Since all risky markets exhibit nonzero variance, it has been established that $L_{optimum}$ is finite – and so capital mobility of all agents collectively is finite. But what is the true limit on capital mobility? There are two important thresholds: the soft limit, which reflects $L_{optimum}$ under various assumptions about the distribution of returns and the opportunity cost, and the hard limit, which reflects the maximum leverage L_{max} that can be applied without going bankrupt and being ruined. The hard limit can be more explicitly determined, although it is statistically less stable since it depends on one outlier, the minimum return.

$$Lr_{min}-(L-1)r_f=-1$$

$$L_{max}=-(1+r_f)/(r_{min}-r_f)-\varepsilon$$

[[10]]

Here, ε is some trivially small positive number approaching zero. In the entire span of all observations across countries and years, the minimum returns were rare and not severe: for alpha=0.3, r_{min}= -0.04591, corresponding to L_{max}=10.947999 with given opportunity cost r_f=0.05, and L_{max}=21.781999 with r_f=0; for alpha=0.4, r_{min}= -0.039431, L_{max}=11.741 with r_f=0.05, and L_{max}=25.360999 with r_f=0. Indeed, this hard boundary, while explicitly calculable, is very unstable and dependent on the assumptions of the distribution of returns and opportunity cost.

Aversion

Behavioral economists have contended frequently that the pain of losing one dollar is much greater in magnitude than the joy of winning one dollar. And there absolutely exist psychological costs of losing – the deadweight cost of feeling like a loser and other such emotional forces that might drag on future productivity. These complications are part of why the representative agent considered here should be thought of as a large institution with a completely negligible current consumption requirement and is capable of allocating efficiently and investing anywhere in the world when an opportunity is recognized. Moreover, the investor is not very constrained by the credit markets unless his effective leverage greatly exceeds one (if L*>>1, then r_{credit}>>r_f). The fundamental condition for risk aversion follows.

$$U(E[K])>E[U(K)]$$

This essential tenet of risk aversion holds in all contexts, where capital or consumption is considered. On a concrete level, it is obvious that utility should exhibit diminishing marginal returns to wealth. If the representative agent scales his consumption positively with his current wealth level, then – since everyone must consume some set of necessities to survive – if wealth drops below some critical level corresponding to the poverty line, then the pain of losing would become overwhelming. If instead the agent's wealth level is sufficiently high, then there is a negative externality to losing money – namely, that this agent would either get fired from his job as a portfolio manager or would gradually lose his client base. In order to deal with these nuances in a consistent fashion, Constant Relative Risk Aversion

(CRRA) – also known as iso-elastic – preferences will be adopted; this functional form possesses the advantage of being indifferent to the frame of reference that is initial wealth, although it was argued above that approaching ruin or poverty might escalate the aversion. A pair of CRRA utility functions is proposed.

Constant Relative Risk Aversion (CRRA):

$$U_1=(K^{1-\gamma}-1)/(1-\gamma), \text{ where } \gamma=-K*(d^2U_1/dK^2)/(dU_1/dK) \text{ and } \gamma \text{ is constant}$$

Constant Absolute Risk Aversion (CARA):

$$U_2=(1-\Gamma)/\Gamma*(K/(1-\Gamma))^{\Gamma}, \text{ where } \Gamma=-(d^2U_1/dK^2)/(dU_1/dK) \text{ and } \Gamma \text{ is constant}$$

Intermediate consumption is restricted to zero, reflecting the institutional nature of the representative investor who simply wants to maximizes capital in some indefinite final period.

$$C_T=K_T, \text{ where } C_{t\neq T}=0$$

[[0.2]]

Risk neutrality for the first equation corresponds to zero ($\gamma=0$), whereas for the second equation it corresponds to one ($\Gamma=1$). Both equations are special cases of the hyperbolic form. More generally, to frame equation [[11]] properly, a baseline on recursive preferences is sought, where zero utility is derived from intermediate consumption, with Epstein-Zin parameters restricted to the limits of their domains.

$$U_t=[(1-\beta)C_t^{\rho}+\beta(E[U_t^{\alpha}])^{\rho/\alpha}]^{1/\rho}$$

Set $\beta=1,\alpha=1,\rho=0$.

$$U_t=E[U_t]$$

The key insight here is that there are two distinct layers to risk that absolutely must be accounted for separately. There are observable degrees of variance (as well as skew and kurtosis) that drag on long run compounded growth, and function as deflating factors on the arithmetic mean when the agent is venturing to optimize his geometric mean. If the investor simply compares the arithmetic means of various countries against their respective logarithmic GDP per capita, then the Lucas Paradox appears to be dominant. It is here where this paper departs from that of Henriksen (2014). If we simply regressed arithmetic mean returns on log GDPpc, there would appear to be massive risk aversion in the global capital markets, which might only be justified by the structural incongruity of poor countries versus rich ones, such as monopolies that actively retard foreign competition or government-supported capital controls that both reflect a lack of openness – factors that critically might result in sufficiently high adjustment costs to capital. Only looking at the arithmetic mean returns for each countries would be valid if and only if the variance of returns were held perpetually at zero, an obviously false condition which only applies to short-duration government bonds of mature developed countries. These prohibitive adjustment costs would then deprive the representative agent of the opportunity to reallocate periodically, forcing him to make a major fixed capital commitment in just one period and basically stick with it over a long duration; in other words, lack of openness would generate a new and analytically problematic idiosyncratic risk factor reflecting a lack of liquidity.

3:: Methods

The true degree of risk aversion is difficult to capture, still more difficult to capture without examining consumption patterns. Essentially, the goal is in framing how much money is being left on the table, under two conditions – fully credit constrained investors (L=1) and partially credit constrained (L=2). Both conditions limit the investor to being less exposed than the estimation of optimal exposure presented in the results section, but a leverage of two is not excessive and would hold up in the future if the worst case loss increases significantly. Two derivations are proposed in this section. The first ends up being a nonlinear function of the mean/variance relationship, the second a linear one. Even though the focus of this paper is on long run geometric mean return maximization, for convenience only one step forward will be analyzed – a practice valid only under serial independence of the returns, a property also known as "partial myopia." So, while developing countries might be converging to developed ones, thereby reducing their variance and mean returns over time, the historically observable data will be assumed to hold going forward, representing a sample from some static governing distribution. For a reduced-form baseline version of the relative risk aversion parameter independent of inter-temporal consumption requirements, with a derivation based in part on (Jacquier 2011), let

$$E[K_t]=K_0(1+g)^t$$

$$1+g=e^{L\mu-0.5*(L*\sigma)\wedge 2-(L-1)r}$$

For simplicity, set

$$K_0=1,\ t=1$$

Then solve the objective function.

$$max_L\{E[U(K(L))]\}=>$$

$$0=E[U'(K)*K'(L)]$$

The covariance is the expectation of the product minus the product of the expectations.

$$0=Cov[U'(K),K'(L)]+E[U'(K)]*E[K'(L)]$$

Stein's Lemma is used to decompose the covariance further.

$$0=E[U''(K)]*E[K'(L)]*Cov[K(L),K'(L)]+E[U'(K)]*E[K'(L)]$$

$$\gamma\equiv-E[U''(K)]/E[U'(K)]=1/Cov[K(L),K'(L)]$$

[[12]]

$$Cov[K(L),K'(L)]=\beta_{K',K}*\sigma^2{}_K$$

$$\beta_{K',K}=\Delta K'/\Delta K=K''/K'$$

$$=[(e^{L\mu-0.5*(L\sigma)\wedge 2-(L-1)r})*((\mu-r-L\sigma^2)^2-\sigma^2]/[(e^{L\mu-0.5*(L\sigma)\wedge 2-(L-1)r})*(\mu-r-L\sigma^2)]$$

$$=((\mu-r-L\sigma^2)^2-\sigma^2)/(\mu-r-L\sigma^2)$$

$$=(\mu-r-L\sigma^2)-\sigma^2(\mu-r-L\sigma^2)^{-1}$$

$$=>Cov[K(L),K'(L)]=\sigma^2((\mu-r-L\sigma^2)-\sigma^2(\mu-r-L\sigma^2)^{-1})$$

$$=>\gamma=1/[\sigma^2((\mu-r-L\sigma^2)-\sigma^2(\mu-r-L\sigma^2)^{-1})]$$

$$=((\mu-r)/\sigma^2-L)/((\mu-r-L\sigma^2)^2-\sigma^2)$$

[[13]]

$$\gamma_{L=(\mu-r)/\sigma^2}=\gamma_{(\mu-r)/\sigma^2}=0$$
$$\gamma_1=((\mu-r)/\sigma^2-1)/((\mu-r-\sigma^2)^2-\sigma^2)$$

[[14]]

Then, to assess the absolute risk aversion level, a derivation based in part on Sandmo (1970) is presented. Here, eta is the risk premium (the difference between the expected final consumption and the certainty-equivalent level) and h is the approximation for standard deviation of the risky outcome.

$$\Gamma=2\eta/h^2$$
$$h^2=(L\sigma)^2$$
$$\eta=[L\mu-0.5*(L\sigma)^2-(L-1)r]-r$$
$$\Gamma=2((\mu-r)/\sigma^2)/L-1$$

[[15]]

$$\Gamma_{L=1,r=0}=2\mu/\sigma^2-1$$

[[16]]

$$\Gamma_{L=(\mu-r)/\sigma^2}=1$$

Risk aversion metrics can be computed for each country or each portfolio – and then measured across the factor that is used to construct the portfolio: GDPpc, GDPpw, and ECI. Based on the various derivations of optimal leverage in the risk sensitivity segment of section two, the mean/variance ratio functions as a good rule of thumb but is not perfect, other than for cases with very small variance. Consequently, for practical purposes, L should be set somewhere between one and the optimal ($1<L^*<L_{optimal}$), depending on the confidence that the investor has in the estimation of the true return distribution. Given the overall distribution of all country-year observations, this modification corresponds to setting the effective leverage at one and then gearing it up to two ($L^*=1,2$). However, this minimalist estimation of risk aversion is ascertainable without any consumption data; this estimation is very important to establish before consumption is considered. Consumption is the next dimension to investigate, for the explicit purpose of establishing some arbitrage-free baseline on the degree of risk aversion. In other words, this procedure captures the first layer of the implicit costs of risk, and the covariance of consumption with returns forms the second layer, firmly on top of the first. The evaluation of this second layer of implicit costs, conducted via the framework of Epstein-Zin recursive preference, will be left to other research projects. But it is sufficient to note that, if the representative investor prefers to consume anything in the time in between the first and final periods (valid for virtually all agents yet trivial for the largest ones), then his risk aversion will be higher than this reduced-form base metric. However, since the effective leverage (L^*) applied to each country is unobservable, different key values are tested from 1 to 2. However, all of the effort to configure different leverage levels should be constrained by the accounting identity that the net borrowing and lending must balance out, satisfying the universal clearing-condition that aggregate leverage is pinned to unity ($E[L^*]\equiv1$). In other words, it is inconceivable that the aggregated effective leverage for any group of countries should deviate indefinitely from unity. Moreover, maintaining a yearly rebalanced constant leverage different from one ($L^*\neq1$) is only feasible under trivial

adjustment costs, which are indefinitely high when aggregate physical capital is considered, especially for poor countries that might lack open capital markets (as Lucas points out in detail in his original paper). It should be noted that the relative risk aversion metric is nonlinear transform of the mean/variance ratio, whereas the absolute risk aversion metric is a linear transform.

In spite of all the mathematical complications, there is sufficient evidence that $L_{optimum}>1$, even with nontrivial adjustment costs and transaction costs – mainly because the left tail of the return distribution is so limited.

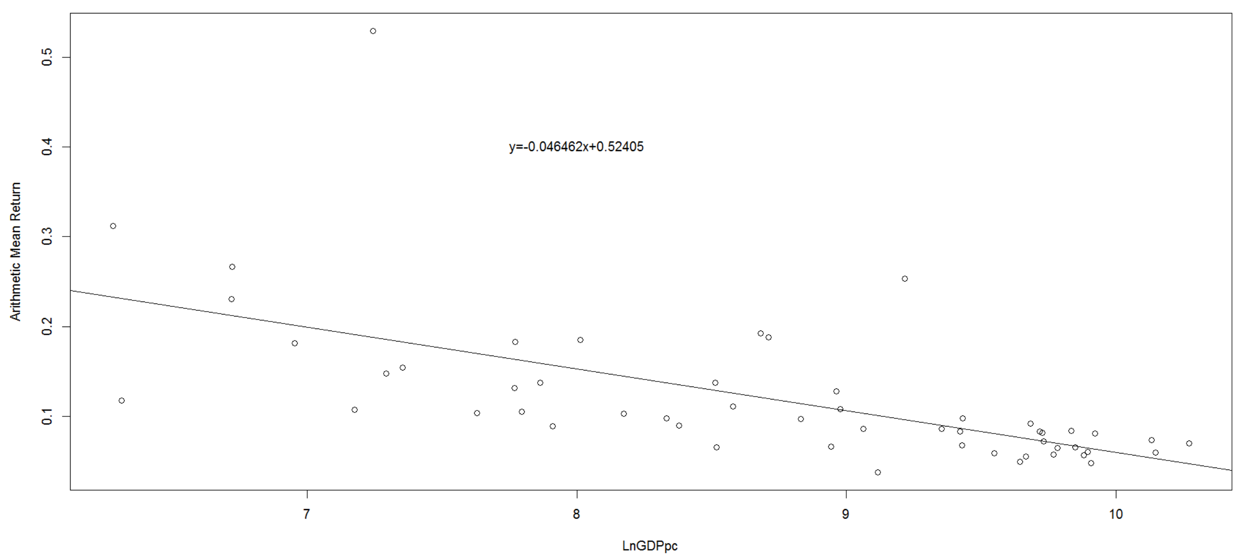
4:: Results

It can clearly be seen that the variance of the returns to capital in poor countries is higher than in rich countries. Some countries, mainly those associated with the CIS and formerly under the Soviet system, lack sufficient data to be analyzed with the appropriate precision. Since only 25 years have elapsed since their respective time series began, there is the potential for small sample bias, which is problematic in general and crucially so in this risk related context. It is commonly accepted that the number of observations exceed T>30 for this small sample bias to become adequately reduced in significance.

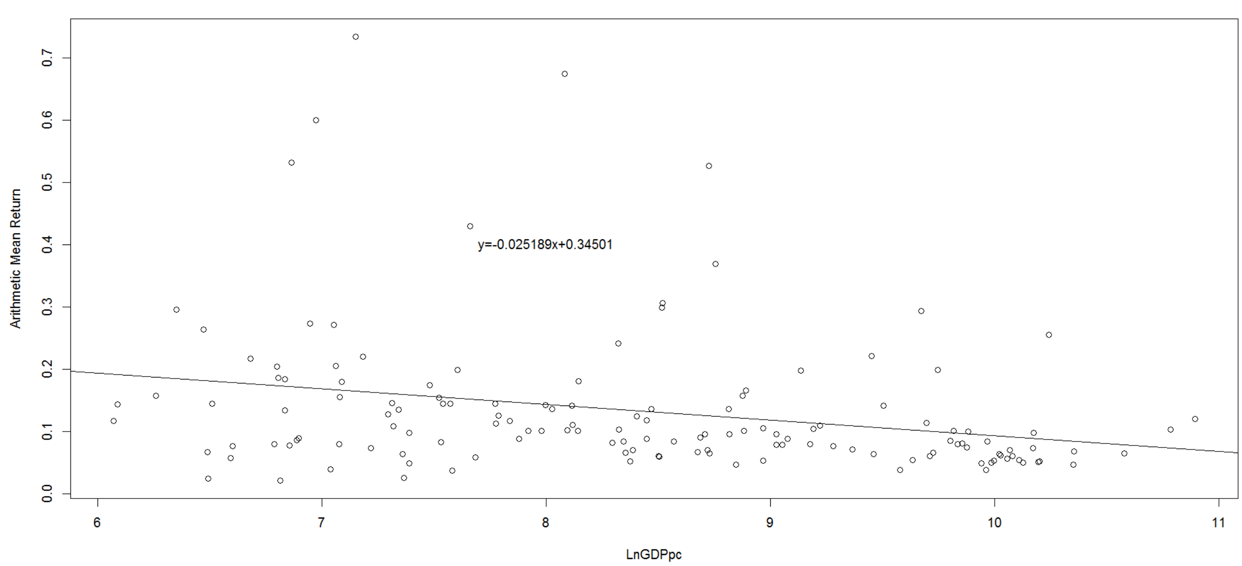
If the estimate for the genuine degree of risk aversion is sufficiently greater than zero or one, corresponding to risk neutrality, then the Lucas Paradox survives. Moreover, if risk aversion is high in this context, then the implications are very interesting. For one thing, high risk aversion might indicate an arbitrage opportunity is not being exploited, but this conclusion might be wishful thinking. For one thing, the whole estimation process implies than the brief (in statistical terms) historical sample yields the exact true parameters governing the future distribution of returns, which is a weak assumption; further work might endeavor to incorporate Bayesian updating to the estimation process for recovering these true parameters for the marginal distributions, but this complexity is outside the scope of this paper. Before the actual degree of risk aversion can be properly assessed, the mean/variance ratio must be evaluated for every country over different time periods, under varied assumptions. If the ratio is below one ($L_{optimum}\approx\mu/\sigma^2<1$), which is not the case except for a few countries, then there is some degree of excess variance – since the risk-neutral geometric mean return maximizer will withhold investment and keep some portion of his capital ($1-L_{optimum}$) in the debt market.

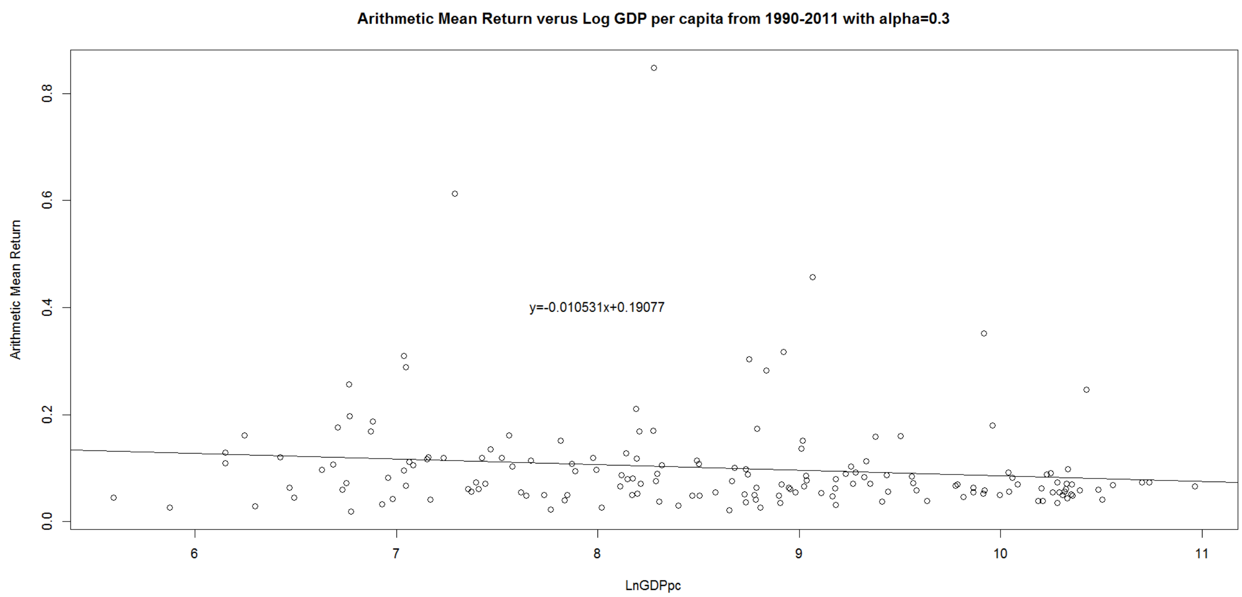
Here the primary relationship is displayed with the arithmetic mean returns for each country unadjusted for variance.

Arithmetic Mean Return verus Log GDP per capita from 1950-2011 with alpha=0.3

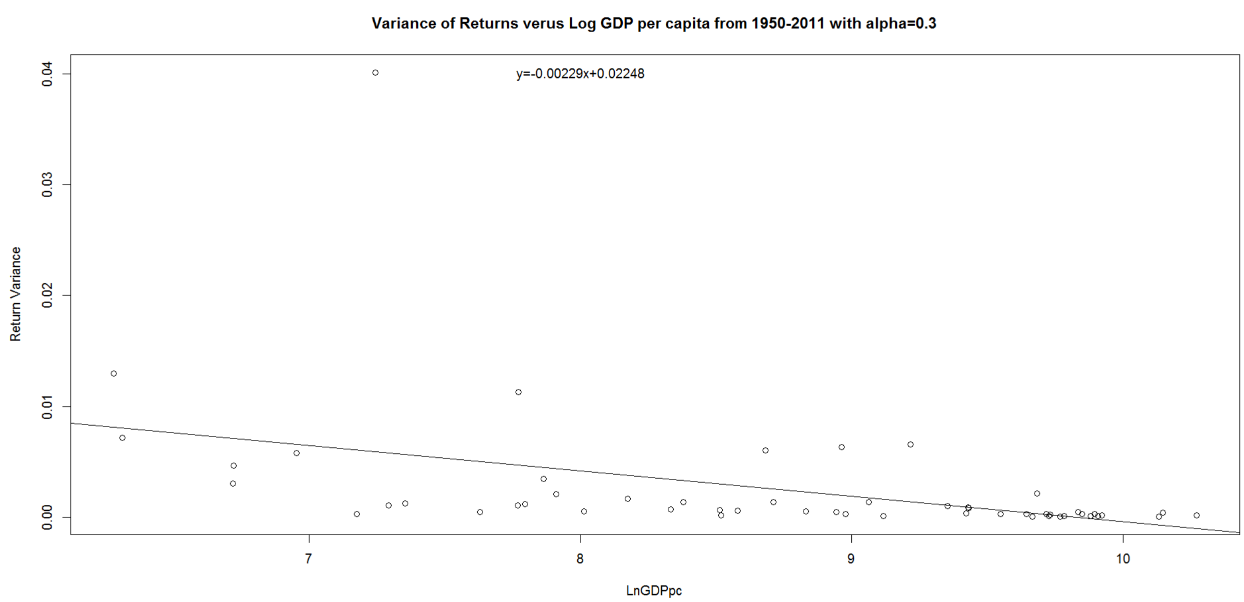


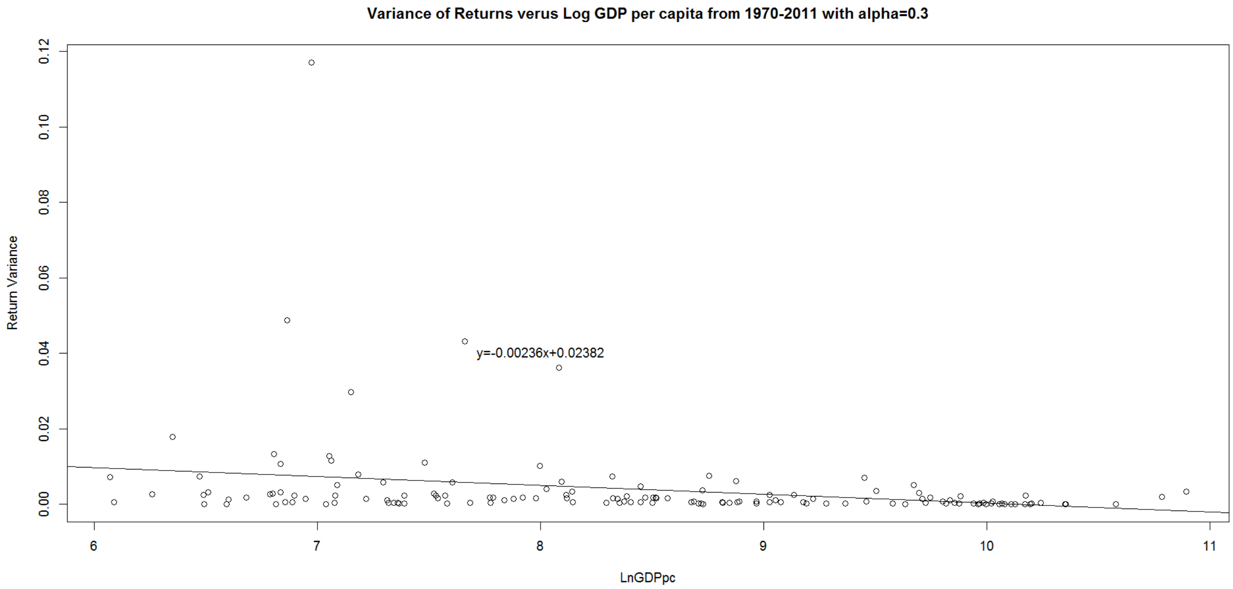
Arithmetic Mean Return verus Log GDP per capita from 1970-2011 with alpha=0.3

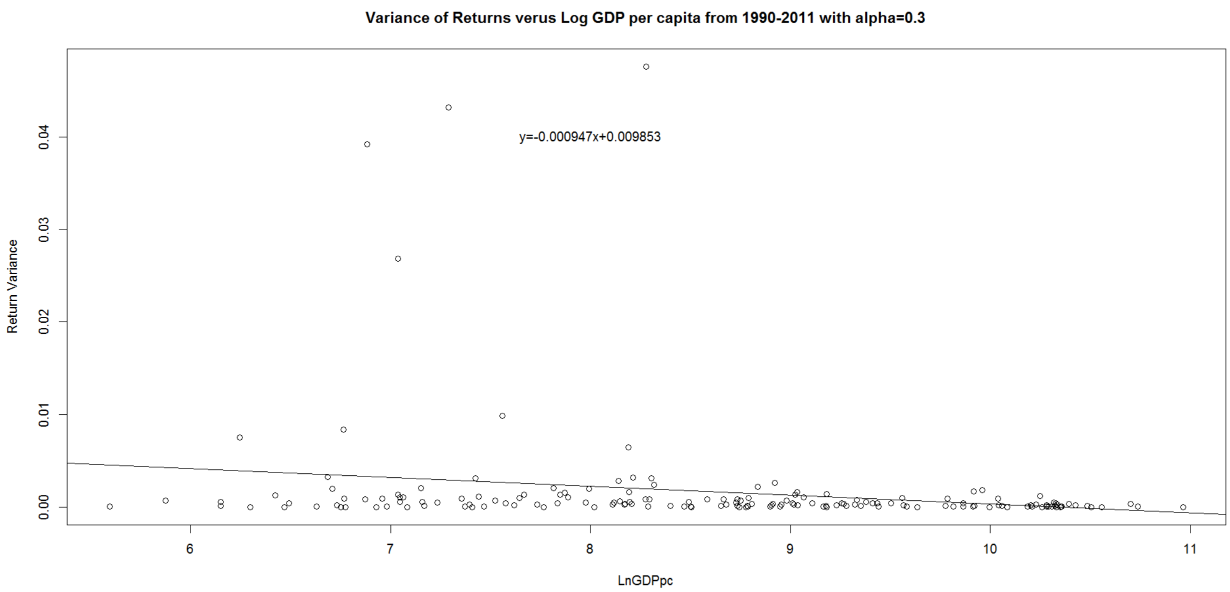




However, while the arithmetic mean return decreases with logarithmic income per capita, the variance decreases too.







This result is intuitive and consistent with the most fundamental principle of efficiency in capital markets – generating a higher reward requires taking on more risk. But what is the relative rate of change in mean against variance? In other words, as the arithmetic mean return decreases as income increases, does the variance decrease faster? In order to assess this question, the individual slopes will be rescaled. The process of dividing the slopes by the intercepts, all of which having a p-value less than 0.05, results in the following relative values.

Slope/Intercept	Regression of Return Mean on Log Income Per Capita	Regression of Return Variance on Log Income Per Capita
1950-2011	-0.08866	-0.10172
1970-2011	-0.073011	-0.098875

1990-2011	-0.055206	-0.096154

Since the magnitudes of the values in the first column are all smaller than the magnitude of value in the second column, the variance is decreasing faster than the mean as income increases, on a relative basis. However, the magnitudes of the values in the first column are decreasing faster than the magnitudes of the values of the second column over time. The implication of this observation, tentatively, is that, even if there is some convergence in the mean returns of poor and rich countries over time, the risk in poor countries remains persistently higher in the poor countries. Moreover, there is a limit to the extent to which the means can converge if the variance does not also converge. This limit is important, albeit hard to measure since the optimal leverage (and corresponding capacity for capital mobility) is somewhat ambiguous dependent on the true return distribution, which does not have to be stationary. Now the same relationship is reassessed using the geometric mean return. In the following tables, B denotes the slope estimate. The slopes are more statistically significant than in the prior regressions with the arithmetic means, with most having p-values less than 0.01.

B::μ_G:lnGDPpc	*1950-2011*	*1970-2011*	*1990-2011*
alpha=0.2	-0.03134	-0.01731	-0.00733
alpha=0.3	-0.04646	-0.02518	-0.01053
alpha=0.4	-0.06140	-0.03289	-0.01366
alpha=0.5	-0.07620	-0.04046	-0.01673

The elasticity of returns with respect to GDP per capita, under four different assumptions for the fixed alpha (the invariant elasticity of GDP with respect to capital), over three different time frames

B::μ_G:lnGDPpw	*1950-2011*	*1970-2011*	*1990-2011*
alpha=0.2	-0.02032	-0.02088	-0.00815
alpha=0.3	-0.02994	-0.03079	-0.01173
alpha=0.4	-0.03952	-0.04058	-0.01525
alpha=0.5	-0.04907	-0.05027	-0.01873

The elasticity of returns with respect to GDP per worker, under four different assumptions for the fixed alpha (the invariant elasticity of GDP with respect to capital), over three different time frames

B::ln(1+μ_G):lnGDPpc	*1950-2011*	*1970-2011*	*1990-2011*
alpha=0.2	-0.02842	-0.01525	-0.00664
alpha=0.3	-0.03968	-0.02071	-0.00897
alpha=0.4	-0.04961	-0.02541	-0.01101
alpha=0.5	-0.05848	-0.02952	-0.01282

B::ln(1+μ_G):lnGDPpw	*1950-2011*	*1970-2011*	*1990-2011*
alpha=0.2	-0.01926	-0.01872	-0.00737
alpha=0.3	-0.02713	-0.02592	-0.00999
alpha=0.4	-0.03431	-0.03226	-0.01228
alpha=0.5	-0.04089	-0.03789	-0.01432

The correlation drops from an R^2~0.3 to ~0.05, moving from the 1950-2011 period to the periods starting in 1970 and 1990. Moreover, using geometric means in the regression instead of arithmetic means as the dependent variable results in a lower p-value for the slope estimate B; with arithmetic means it is significant at the 5% level, and with geometric means it is significant at the 1% level.

The effect of poorer countries generating higher geometric mean returns is now tested for robustness by substituting a completely distinct metric, the Economic Complexity Index (ECI), for logarithmic income. The ECI is a structural measure that averages together all of a country's revealed comparative advantages, using the Balassa definition, in the export market for goods.

B::μ_G:ECI	*1970-2011*	*1990-2011*
alpha=0.2	-0.02230	-0.01680
alpha=0.3	-0.03290	-0.02500
alpha=0.4	-0.04340	-0.03320
alpha=0.5	-0.05380	-0.04130

B::ln(1+μ_G):ECI	*1970-2011*	*1990-2011*
alpha=0.2	-0.01990	-0.01510
alpha=0.3	-0.02760	-0.02110
alpha=0.4	-0.03430	-0.02650
alpha=0.5	-0.04030	-0.03120

The ECI, representing the standard score for a structural model of the diversity and exclusivity of each country's product space, is the distinct substitute for lnGDPpc and lnGDPpw. The consistency between the two types of explanatory variables confirms that there is a genuine *development* effect on capital returns.

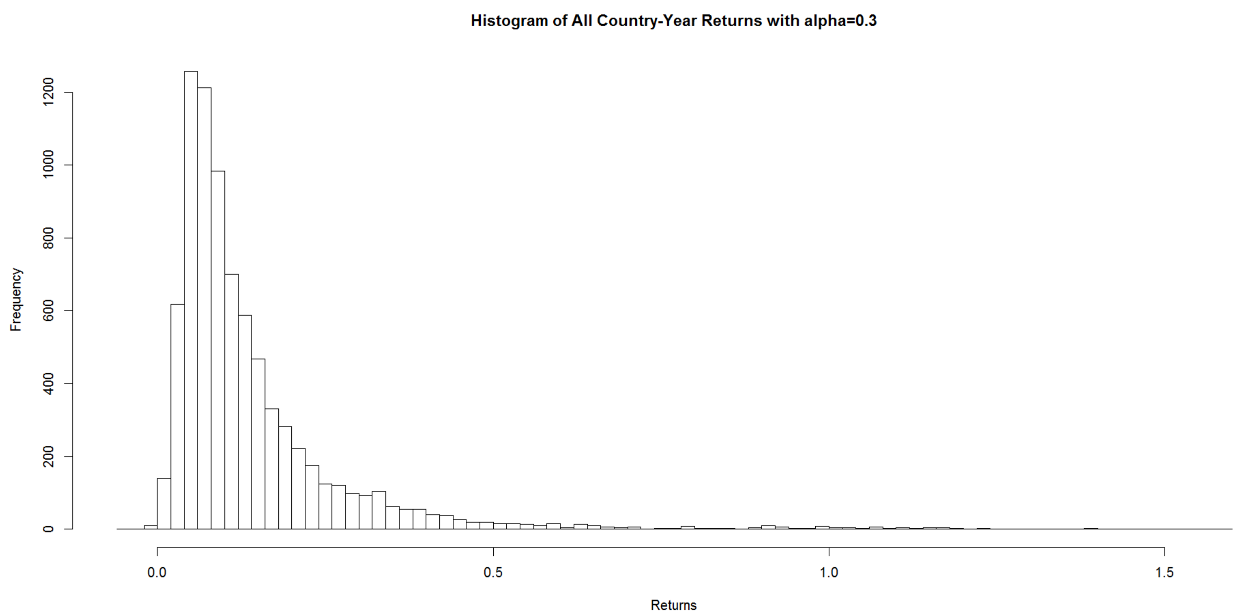
Next, the direct reward-risk association is displayed, with B0 being the intercept and B1 the slope.

***B0::μ_G:ln(**1+σ^2)*	*1950-2011*	*1970-2011*	*1990-2011*
alpha=0.2	0.04670	0.05720	0.04100
alpha=0.3	0.08790	0.10400	0.08060
alpha=0.4	0.12800	0.15000	0.12000
alpha=0.5	0.16700	0.19600	0.15900

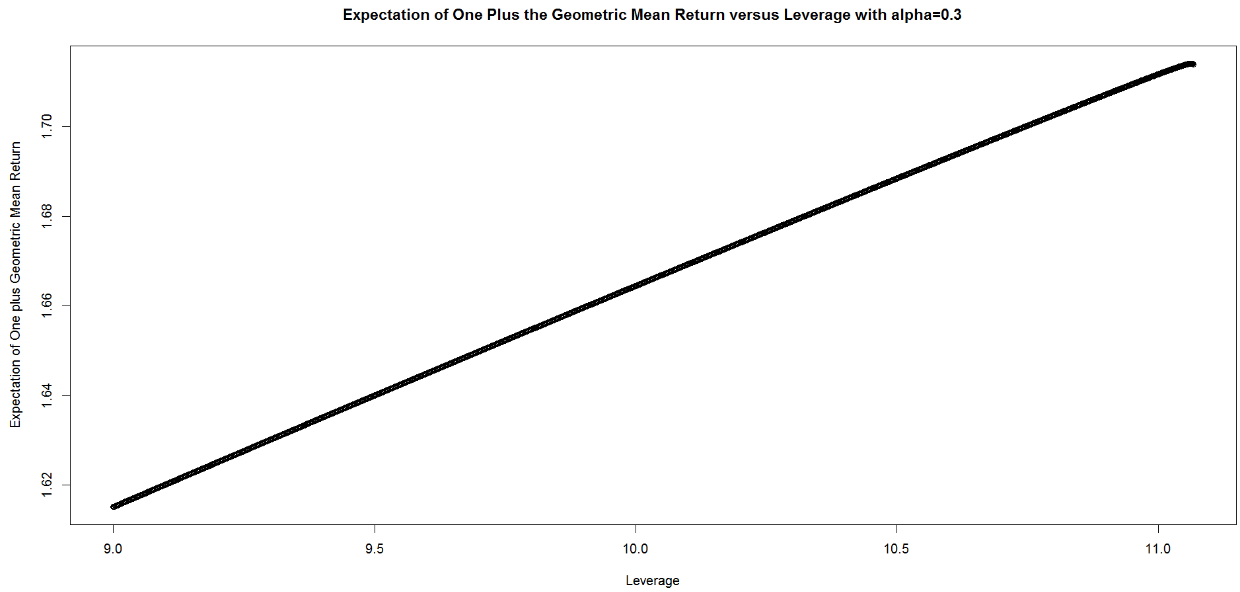
B1::μ_G:ln(1+σ^2)	*1950-2011*	*1970-2011*	*1990-2011*
alpha=0.2	14.26270	8.52410	13.55900
alpha=0.3	12.23260	7.34500	11.59700
alpha=0.4	11.32500	6.87000	10.68000
alpha=0.5	10.87000	6.66200	10.18000

Geometric mean returns at L=1 versus the logarithmic transform of variance, to allow for preferences to reveal diminishing marginal gains to taking on more risk

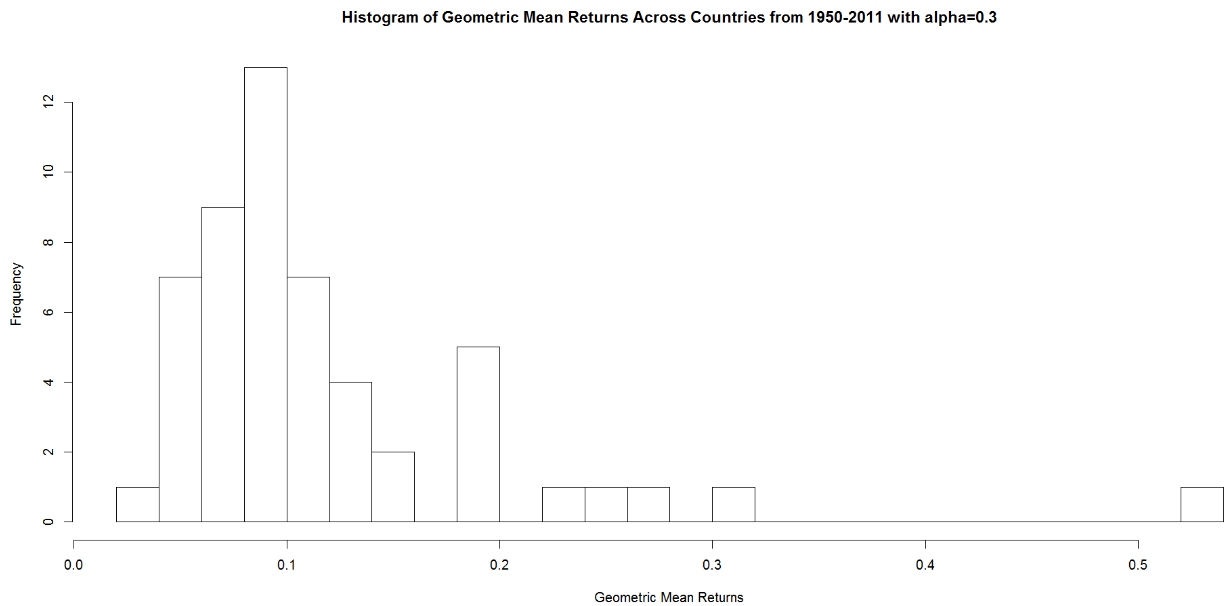
Taking a step back, the distributions of returns are displayed without the income dimension.

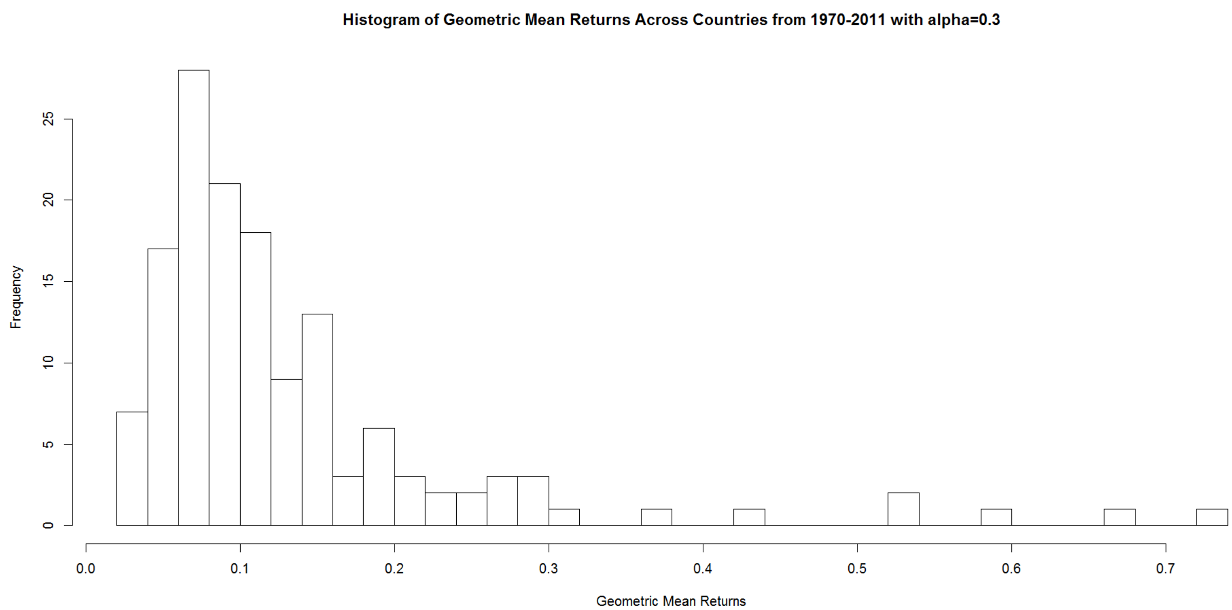


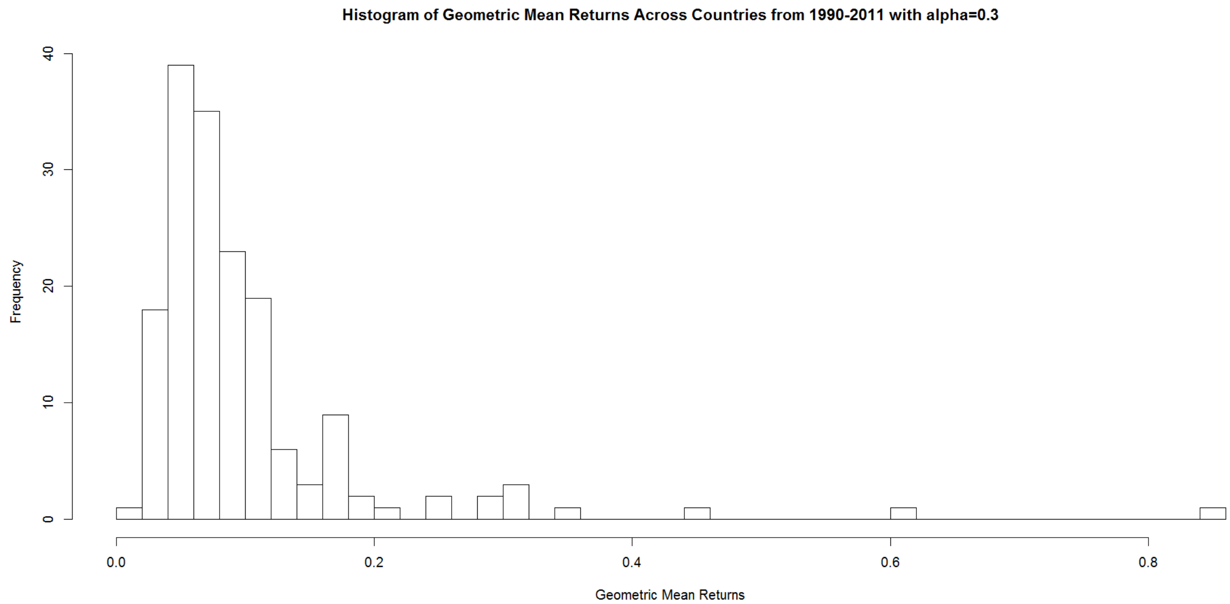
Next the geometric mean of returns plus one, with a random draw from the above histogram of all country-year returns, is displayed. Since the distribution is so favorable, with a small left tail and large right tail, the optimal leverage is very high, but the maximum leverage, at which bankruptcy is reached, is only one extremely small increment higher; this extremely rapid turn from optimal to failure can barely be seen on the graph below, with the apparent right hand endpoint turning down. So, perhaps ironically, optimizing the exposure to this highly favorable distribution of returns is dangerous. If this aggregate empirical distribution is discretized to the Kelly case reflected in equation [[1]], then p=0.99864,b=0.12526, and a=0.01746. (The R script for this curve is straightforward and listed in the appendix.)



The key limit here is right below L_{max}=10.948; due to the favorability of the distribution, perhaps paradoxically, this curve dangerously falls off a cliff, meaning that the soft limit ([[0.3]], [[1b]]) of the optimal leverage is smaller than the hard limit ([[10]]). In other words, the exception is the rule here; if the rarest outlier, the minimum return, dictates the effective limit on the capacity of capital mobility. Even though the minimum observed return is less than 5%, there is nothing preventing the minimum from being more extreme in the future; philosophically, especially given the limited time period of the observations, the biggest loss is always on the horizon. Some extreme shock like a natural disaster could destroy a huge portion of the capital stock in a small poor country, thereby increasing the depreciation rate close to, say, 50%, which is a critical level for the L=2 case. Or a political regime shift to a command economy might compromise the investor's capital commitment to one country. These natural disaster and political revolution type of shocks can and do happen; on a portfolio level, these events will have a muted impact, but the risk is not eliminated – the probability is just reduced, but not to zero (*Pr*[catastrophe]>0). The optimal leverage is clearly greater than one, but pushing it to the apparent limit is precarious; overshooting is deadly. So all countries together permit some level of borrowing, but it is not clear that the representative investor would borrow more to invest in the poor countries, given the drag on leveraged returns from the high variance. In fact, it is the other way around, as long as interest rates are not too high; with low interest rates, rich countries can be levered somewhat more highly than poor countries.

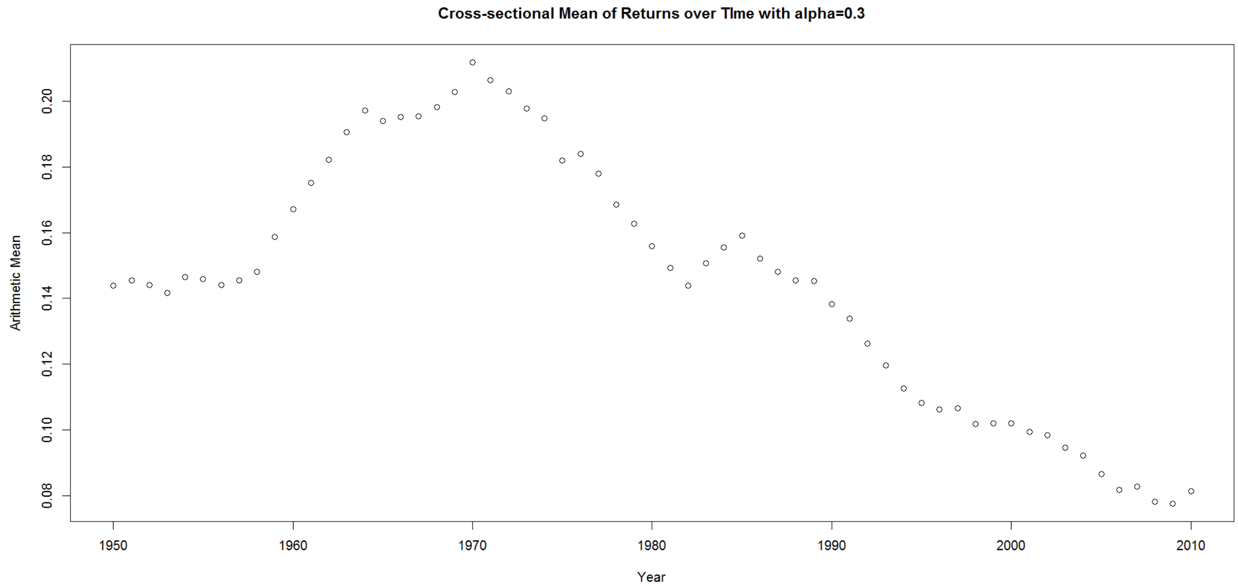

Histogram of Geometric Mean Returns Across Countries from 1950-2011 with alpha=0.3
Frequency
0
2
4
6
8
10
12
0.0
0.1
0.2
0.3
0.4
0.5
Geometric Mean Returns


Histogram of Geometric Mean Returns Across Countries from 1970-2011 with alpha=0.3
Frequency
0
5
10
15
20
25
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Geometric Mean Returns

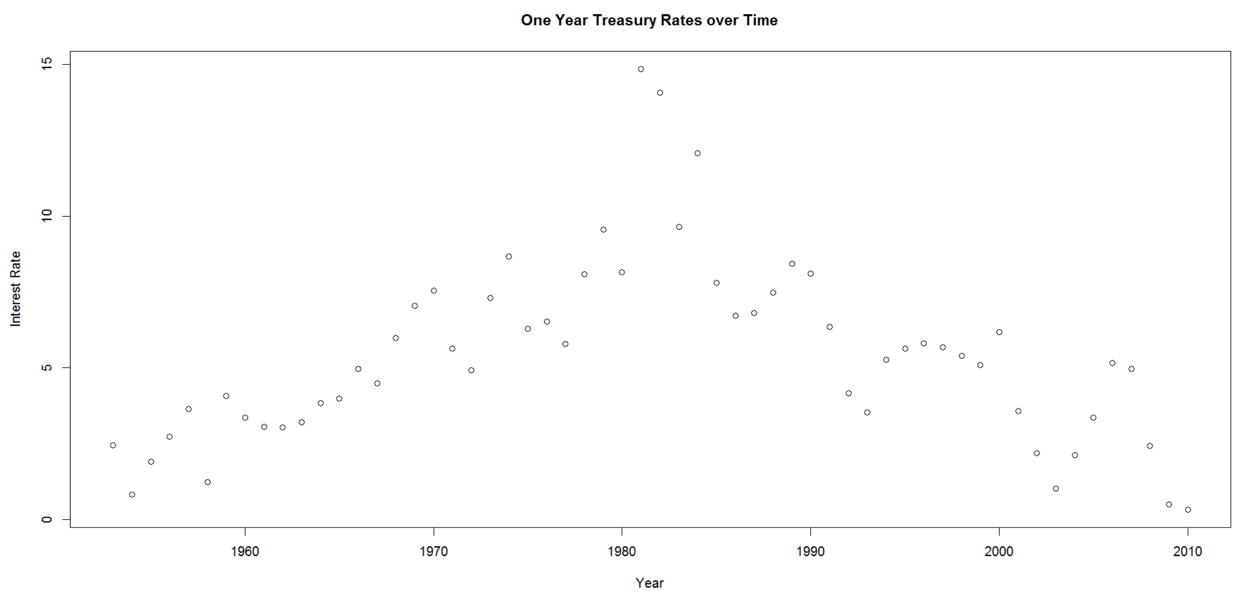


It is obvious that the left tail is not "fat." Negative returns are rare and small. The minimum return was less than 5%. If an investor borrowed 8.999 dollars with interest rate of 5%, close to the median opportunity cost over the entire 1950-2011 period based on the one-year US Treasury rate, he would have survived the 5% loss just barely without going bankrupt. Moreover, the distribution is positively skewed, so all the approximations relying on manipulating the normal distribution and restricting the variance to be small should be taken with a grain of salt.

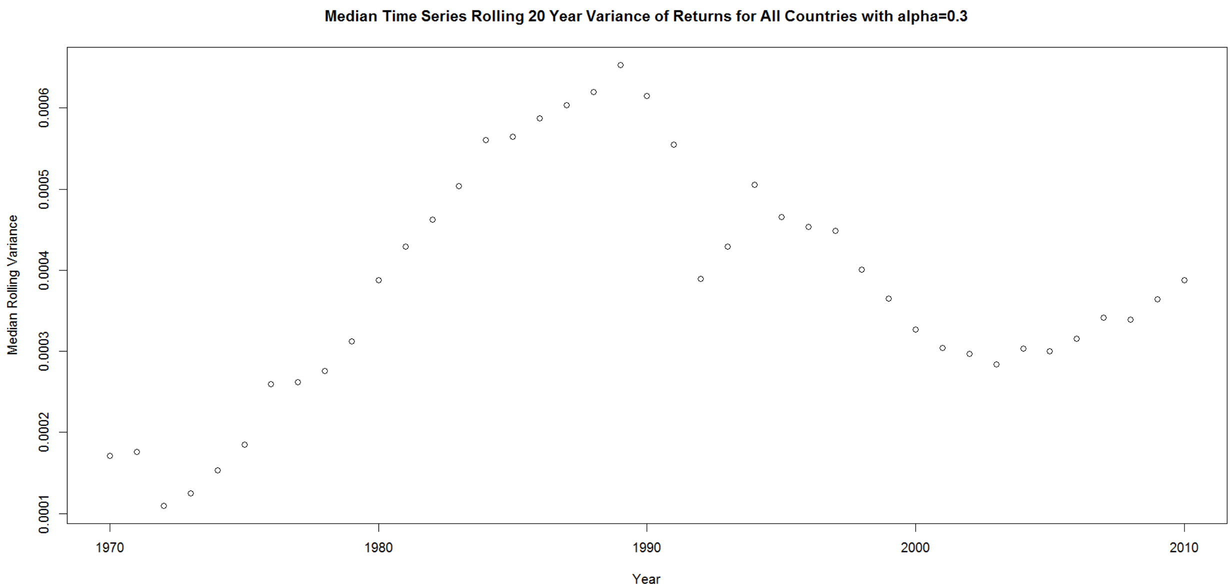
Next, the cross-sectional mean and the cross-sectional variance are displayed. The cross-sectional variance has decreased over time, indicating some degree of mean return convergence; this converge has taken place as the cross-sectional mean of returns has decreased over time as well, which weakens the case for convergence. It should be noted that the paths of both cross-sectional metrics has not been smooth, and there is no guarantee that the distribution governing returns is stationary.



The cross-sectional means seem to rise and fall with interest rates, but the correlation between interest rate levels and the levels of the cross-sectional means is low (R^2= 0.087556).



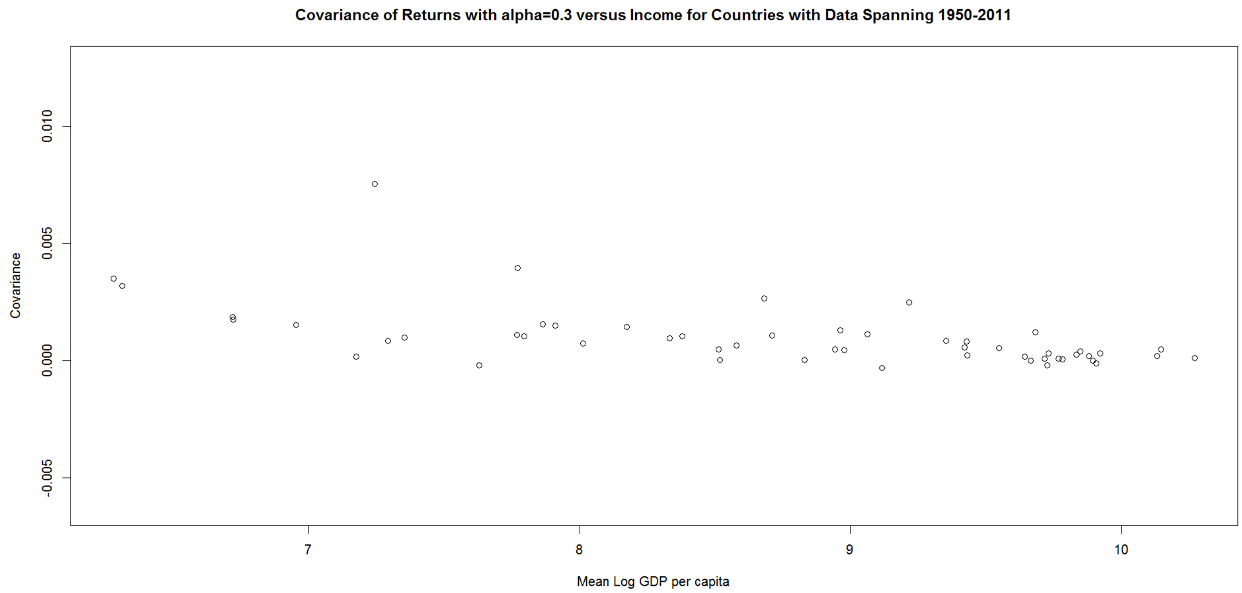
Also, mean returns are declining over time as mainly poor countries are added to the PWT, so the case for opportunity in poor countries is further weakened. Next the rolling time series variance of all countries over the past 20 years is displayed. Mathematically, if the period of calculation is 20 years, then the corresponding lag is 10 (20/2), meaning the latest observation reflects the rolling variance 10 years ago. It should be noted that this rolling variance is not decreasing. This is somewhat surprising considering that the cross-sectional mean of returns has declined, but it is not inconsistent with the cross-sectional variance decreasing over time. The decrease in cross-sectional variance combined with increase in the time series variance indicates that the risk of the middle income countries has increased even as the risk of the poor countries has somewhat lessened; that being said, the variance of poor countries is still much higher than the variance of middle income countries, just not as much as it used to be.

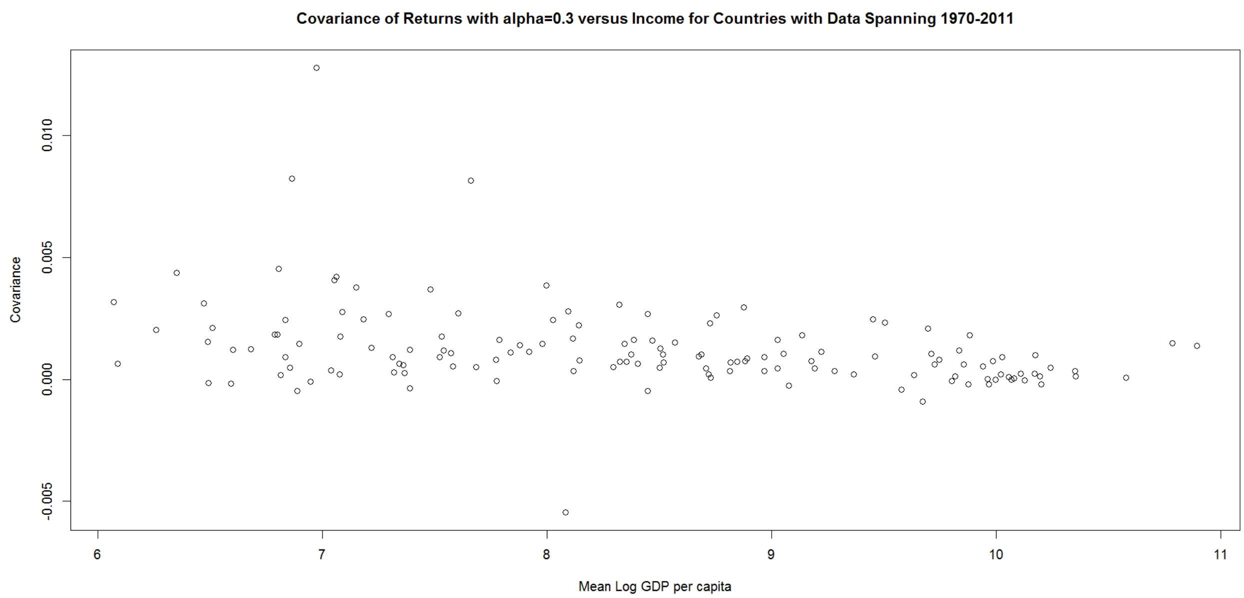


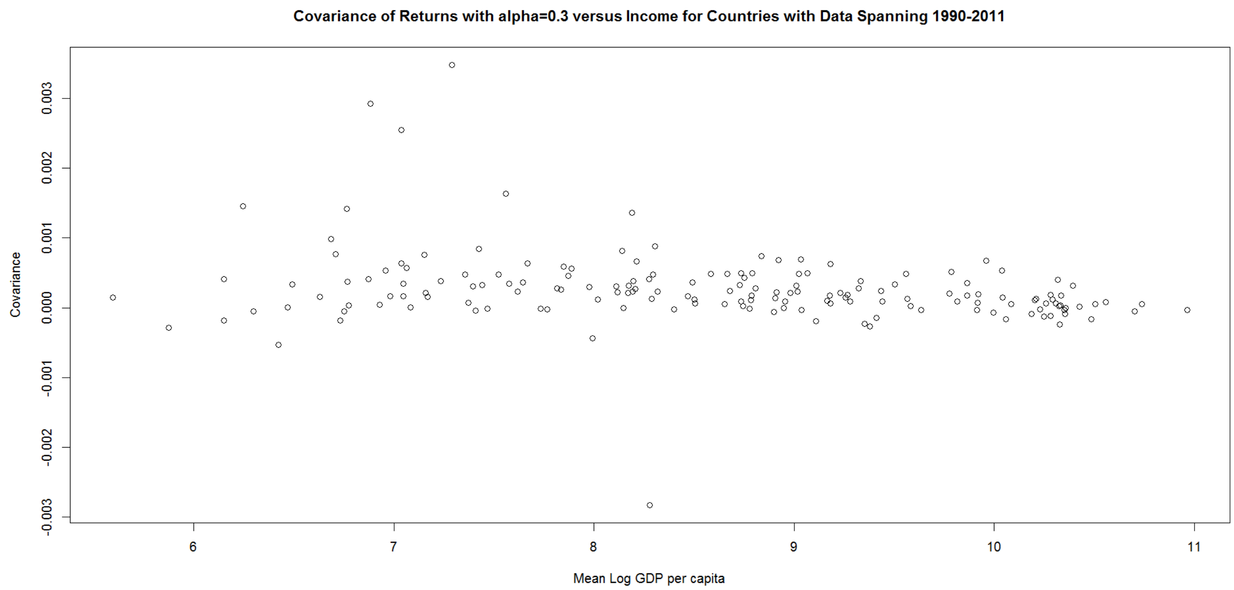
The null hypothesis of the neoclassical return parity across the world implies a slope of geometric means against countries' log GDP per capita to be zero; the Lucas Paradox stipulates that the slope is less than zero, although Lucas and others failed to make the crucial distinction between arithmetic and geometric means. This zero-slope null hypothesis implies some type of equilibrium; this equilibrium can take on either the strong-form or the weak-form.

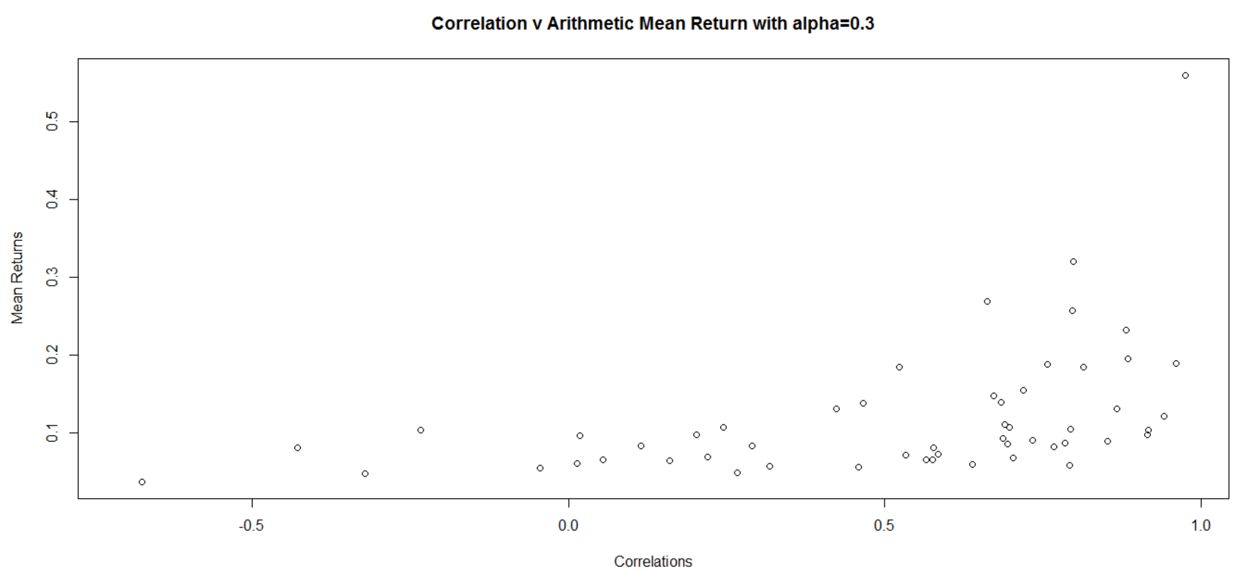
The strong-form equilibrium implies that inefficiency is quickly exploited as the capacity for capital mobility is infinite; this condition is simply false. The weak-form equilibrium implies that the inefficiency of high returns to capital in poor countries due to relative capital scarcity is exploited but not necessarily quickly, as the capacity for capital mobility is finite and the individual investor's leverage can only temporarily rise above one – and as the information about the opportunity space is incomplete and delayed. The weak-form condition reflects reality but is more difficult to explore, as the precise capacity for capital mobility is uncertain even if it is definitely finite. In fact, infinite capital mobility (L_{max}->∞) is only possible if it is impossible for any one country-year return observation to be below the opportunity cost threshold ($r_{min}>r_f$); this inequality implies that the investor does not go bankrupt but could still generate a negative geometric mean at a sufficiently high leverage. However, as long as variance is nonzero, the soft limit is below the hard limit ($L_{optimum}<L_{max}$), so infinite capital mobility is bogus.

The returns in poor countries are not less correlated to the global opportunity space as a whole, as the following graphs indicate. So the variances of poor countries are higher without less covariance to compensate.

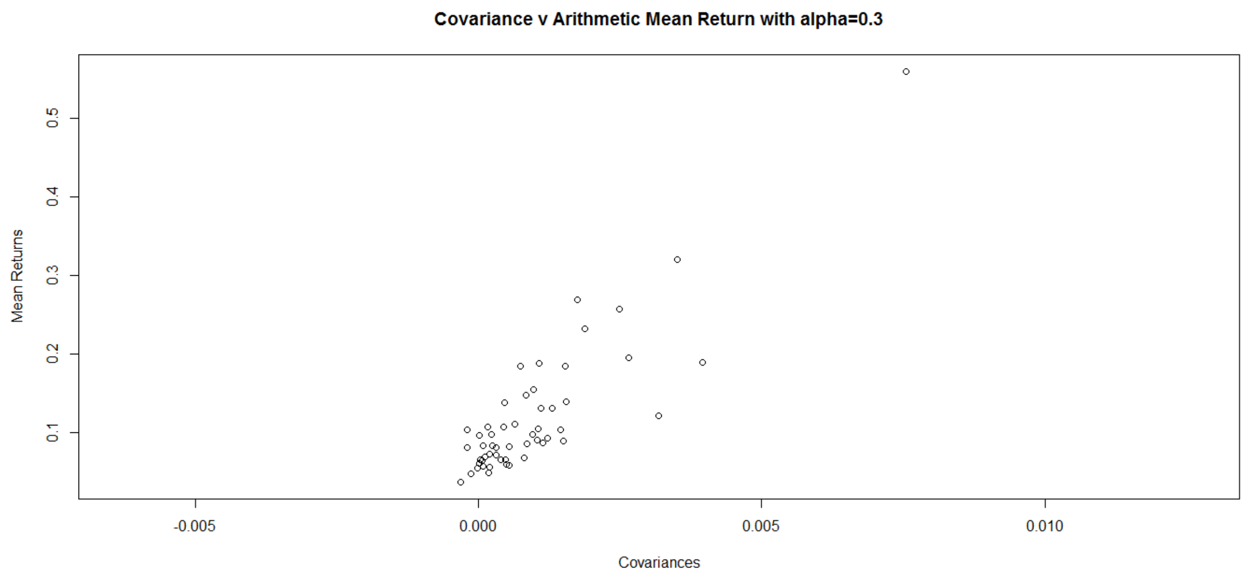

Covariance of Returns with alpha=0.3 versus Income for Countries with Data Spanning 1950-2011
Covariance
0.010
0.005
0.000
-0.005
7
8
9
10
Mean Log GDP per capita


Covariance of Returns with alpha=0.3 versus Income for Countries with Data Spanning 1970-2011
Covariance
0.010
0.005
0.000
-0.005
6
7
8
9
10
11
Mean Log GDP per capita

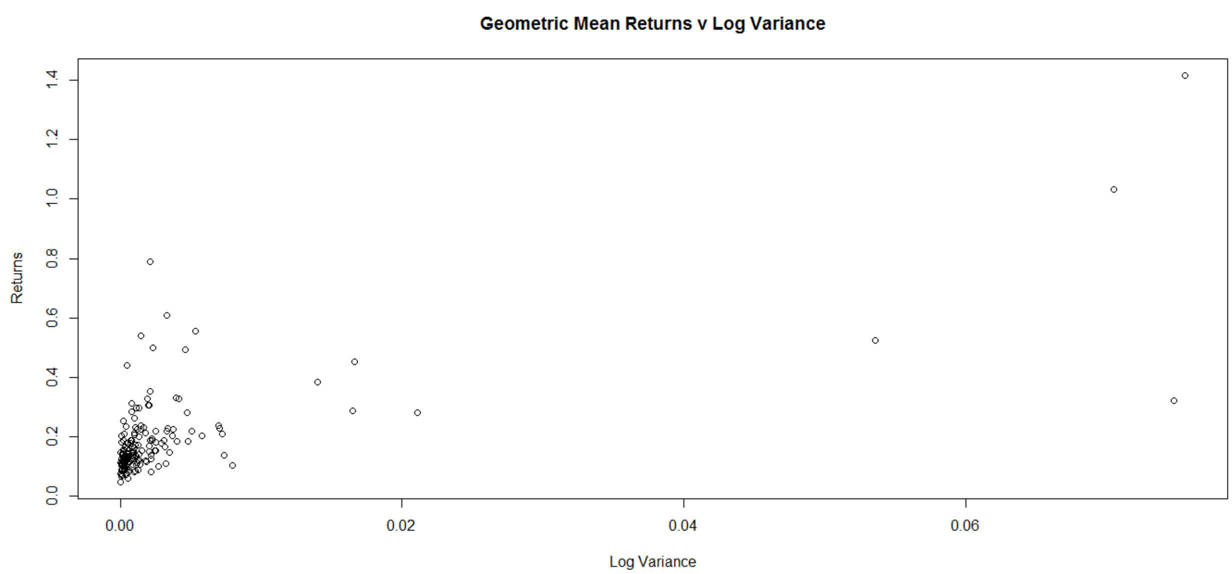




For countries with data spanning the full range of years, there is no real compensation in terms of diversification for investing in high mean countries. In fact, the effect seems to be the opposite.



The slope coefficient of mean return on covariance, from 1950-2011 is positive with a corresponding p-value of less than 0.001.



Here is a depiction of the core reward to risk tradeoff from 1990-2011.

Next, the performance metrics of each group are displayed in detail.

Setting L=1, for alpha=0.3, sorted into equally sized and weighted tiers by GDPpc, the expectations for separate countries in each group:

1950-2011	*Arithmetic Mean*	*Geometric Mean*	*Variance*	*Covariance*
Tier 1 (Poor)	0.18022	0.17663	0.0054961	0.0018625

Tier 2 (Middle)	0.1123	0.11132	0.0017048	0.00081964
Tier 3 (Rich)	0.067418	0.067248	0.00031898	0.000229
1970-2011	*Arithmetic Mean*	*Geometric Mean*	*Variance*	*Covariance*
Tier 1	0.17492	0.16916	0.0083312	0.0020928
Tier 2	0.1415	0.13972	0.0027107	0.0010729
Tier 3	0.091978	0.091398	0.0010288	0.00057804
1990-2011	*Arithmetic Mean*	*Geometric Mean*	*Variance*	*Covariance*
Tier 1	0.11182	0.10985	0.0031074	0.000461
Tier 2	0.1132	0.11199	0.0017115	0.00024305
Tier 3	0.079593	0.079413	0.00033749	0.000097695

Setting L=1, for alpha=0.3, sorted into equally sized and weighted portfolios by GDPpc, the expectations for combined countries in each group:

1950-2011	*Arithmetic Mean*	*Geometric Mean*	*Variance*	*Covariance*
Portfolio 1	0.18218	0.1806	0.0027311	0.002321
Portfolio 2	0.1123	0.11191	0.00070232	0.00093918
Portfolio 3	0.067418	0.067335	0.00015729	0.00024534
1970-2011	*Arithmetic Mean*	*Geometric Mean*	*Variance*	*Covariance*
Portfolio 1	0.17492	0.17292	0.003506	0.0027377
Portfolio 2	0.14247	0.14207	0.00072194	0.0012145
Portfolio 3	0.091978	0.091826	0.00028322	0.00065147
1990-2011	*Arithmetic Mean*	*Geometric Mean*	*Variance*	*Covariance*
Portfolio 1	0.11182	0.11135	0.00088354	0.000581
Portfolio 2	0.1132	0.11312	0.00015635	0.00024044
Portfolio 3	0.079593	0.079559	0.000065694	0.00010673

For a few developing countries, the GDP data starts one year before data on capital stock is available; these countries get sorted but the return data point is ignored for that year. Without such countries, portfolio grouping should indicate identical arithmetic mean returns against the tiered individual country returns. Then, as long as the countries within each portfolio are not perfectly correlated, the geometric mean return of the portfolio will exceed that of the tier. The covariance indicates the measure of each group's returns relative to the all-country equal-weight average.

Setting L=2, for alpha=0.3, sorted into equally sized and weighted portfolios by GDPpc, the expectations for combined countries in each group, assuming r_f=0.05:

1950-2011	*Arithmetic Mean*	*Geometric Mean*	*Variance*	*Covariance*
Portfolio 1	0.31437	0.30863	0.0090098	0.004642
Portfolio 2	0.1746	0.17314	0.0025195	0.0018784
Portfolio 3	0.084835	0.08451	0.00060877	0.00049067
1970-2011	*Arithmetic Mean*	*Geometric Mean*	*Variance*	*Covariance*
Portfolio 1	0.29985	0.29256	0.011574	0.0054754
Portfolio 2	0.23495	0.23347	0.002441	0.0024289
Portfolio 3	0.13396	0.13338	0.0010389	0.0013029
1990-2011	*Arithmetic Mean*	*Geometric Mean*	*Variance*	*Covariance*
Portfolio 1	0.17363	0.17187	0.003125	0.001162
Portfolio 2	0.17640	0.17609	0.00055547	0.00048088
Portfolio 3	0.10919	0.10905	0.00025131	0.00021346

+ Allowing the investor to increase his leverage beyond one results in the middle income countries outperforming the poor countries by even more than in the base case of L=1. On top of that, the variance is much lower.

Relative and absolute risk aversion metrics, given L=1 and L=2 for each country by income tier, are displayed. The true optimal is difficult to determine exactly, but there is sufficient evidence that $L_{optimum}$>1. Beyond L=2, the effective borrowing rate might be significantly higher than the risk-free rate, however. Moreover, limiting L to 2 should allow the investor to avoid ruin, since the minimum return was not close to -50%.

L=1 ::

1950-2011	**gamma1, alpha=0.3**	**Gamma1, alpha=0.3**	**gamma1, alpha=0.4**	**Gamma1, alpha=0.4**
Tier 1	8924.505	119.8001	3115.896	110.0881
Tier 2	21430.59	174.17	6850.627	181.3378
Tier 3	129539.1	426.2797	39289.61	498.2888
1970-2011	**gamma1, alpha=0.3**	**Gamma1, alpha=0.3**	**gamma1, alpha=0.4**	**Gamma1, alpha=0.4**
Tier 1	3943.468	67.48073	1795.474	85.29355
Tier 2	11016.54	133.7567	5440.103	139.673

Tier 3	73892.1	233.216	20169.08	283.0688
1990-2011	**gamma1, alpha=0.3**	**Gamma1, alpha=0.3**	**gamma1, alpha=0.4**	**Gamma1, alpha=0.4**
Tier 1	14737.48	145.8038	8796.597	217.6642
Tier 2	35629.64	225.1729	18418.42	238.9565
Tier 3	129513.6	430.4599	42351.88	470.3793

Median risk aversion metrics over the three time frames for each income tier under the two assumptions for alpha, given a constant opportunity cost of r_f=0.025.

1950-2011	**gamma1, alpha=0.3**	**Gamma1, alpha=0.3**	**gamma1, alpha=0.4**	**Gamma1, alpha=0.4**
Tier 1	8288.391	109.233	3332.34	105.8149
Tier 2	25688.47	126.3991	7980.597	160.6794
Tier 3	227468.1	317.4188	45876.02	443.7641
1970-2011	**gamma1, alpha=0.3**	**Gamma1, alpha=0.3**	**gamma1, alpha=0.4**	**Gamma1, alpha=0.4**
Tier 1	3089.196	56.89398	1671.103	71.52058
Tier 2	14758.78	102.3849	6471.433	121.1934
Tier 3	58787.38	168.908	26149.39	239.89
1990-2011	**gamma1, alpha=0.3**	**Gamma1, alpha=0.3**	**gamma1, alpha=0.4**	**Gamma1, alpha=0.4**
Tier 1	14023.97	109.897	9175.091	149.7107
Tier 2	25142.62	163.366	13958.8	205.9218
Tier 3	144681.4	317.5506	48187.27	413.9055

+ Median risk aversion metrics over the three time frames for each income tier under the two assumptions for alpha, given a constant opportunity cost of r_f=0.035.

1950-2011	**gamma1, alpha=0.3**	**Gamma1, alpha=0.3**	**gamma1, alpha=0.4**	**Gamma1, alpha=0.4**
Tier 1	6866.626	98.56431	4319.132	99.51715
Tier 2	45047.3	85.62517	8592.861	132.4122
Tier 3	186161.6	205.4902	55884.97	373.9449
1970-2011	**gamma1, alpha=0.3**	**Gamma1, alpha=0.3**	**gamma1, alpha=0.4**	**Gamma1, alpha=0.4**
Tier 1	2015.168	46.97914	1736.584	59.58264
Tier 2	6964.468	81.22681	5702.588	107.1724

Tier 3	60329.91	127.695	27797.69	199.1595
1990-2011	**gamma1, alpha=0.3**	**Gamma1, alpha=0.3**	**gamma1, alpha=0.4**	**Gamma1, alpha=0.4**
Tier 1	13904.51	76.5379	7474.852	125.0805
Tier 2	9545.191	101.5592	16913.03	183.9746
Tier 3	108871.1	169.1294	58126.14	359.1286

Median risk aversion metrics over the three time frames for each income tier under the two assumptions for alpha, given a constant opportunity cost of r_f=0.045.

1950-2011	**gamma1, alpha=0.3**	**Gamma1, alpha=0.3**	**gamma1, alpha=0.4**	**Gamma1, alpha=0.4**
Tier 1	8447.668	87.89565	3790.751	93.21943
Tier 2	935.6598	60.94747	10634.16	107.9495
Tier 3	33224.11	71.84697	82517.71	304.1256
1970-2011	**gamma1, alpha=0.3**	**Gamma1, alpha=0.3**	**gamma1, alpha=0.4**	**Gamma1, alpha=0.4**
Tier 1	960.1161	39.1983	1562.401	53.1508
Tier 2	8179.7	68.95626	5754.88	88.6428
Tier 3	32174.83	48.08273	27300.74	152.8522
1990-2011	**gamma1, alpha=0.3**	**Gamma1, alpha=0.3**	**gamma1, alpha=0.4**	**Gamma1, alpha=0.4**
Tier 1	7289.072	48.65406	5081.822	99.02071
Tier 2	5429.68	49.27445	10291.23	150.3442
Tier 3	22183.95	73.93416	58419.08	296.4674

Median risk aversion metrics over the three time frames for each income tier under the two assumptions for alpha, given a constant opportunity cost of r_f=0.055.

L=2 ::

1950-2011	**gamma2, alpha=0.3**	**Gamma2, alpha=0.3**	**gamma2, alpha=0.4**	**Gamma2, alpha=0.4**
Tier 1	7594.523	59.40007	3409.085	54.54403
Tier 2	21607.51	86.58498	7021.209	90.16888
Tier 3	131381.1	212.6399	39496.17	248.6444
1970-2011	**gamma2, alpha=0.3**	**Gamma2, alpha=0.3**	**gamma2, alpha=0.4**	**Gamma2, alpha=0.4**
Tier 1	3800.572	33.24037	2037.682	42.14677

Tier 2	11775.08	66.37835	5650.186	69.33648
Tier 3	76834.16	116.108	20370.9	141.0344
1990-2011	**gamma2, alpha=0.3**	**Gamma2, alpha=0.3**	**gamma2, alpha=0.4**	**Gamma2, alpha=0.4**
Tier 1	15073.94	72.40192	8857.499	108.3321
Tier 2	35884.51	112.0864	20330.45	118.9782
Tier 3	134313.7	214.7299	42597.52	234.6896

Median risk aversion metrics over the three time frames for each income tier under the two assumptions for alpha, given a constant opportunity cost of r_f=0.025.

1950-2011	**gamma2, alpha=0.3**	**Gamma2, alpha=0.3**	**gamma2, alpha=0.4**	**Gamma2, alpha=0.4**
Tier 1	8388.3	54.11649	3964.199	52.40743
Tier 2	27580.44	62.69955	8221.303	79.83972
Tier 3	231890.1	158.2094	46180.83	221.3821
1970-2011	**gamma2, alpha=0.3**	**Gamma2, alpha=0.3**	**gamma2, alpha=0.4**	**Gamma2, alpha=0.4**
Tier 1	2772.497	27.94699	1734.586	35.26029
Tier 2	15715.48	50.69245	6406.963	60.09672
Tier 3	50960.47	83.954	26405.76	119.445
1990-2011	**gamma2, alpha=0.3**	**Gamma2, alpha=0.3**	**gamma2, alpha=0.4**	**Gamma2, alpha=0.4**
Tier 1	16422.76	54.44849	9236.037	74.35535
Tier 2	25222.28	81.18302	14216.59	102.4609
Tier 3	145602.1	158.2753	48457	206.4528

Median risk aversion metrics over the three time frames for each income tier under the two assumptions for alpha, given a constant opportunity cost of r_f=0.035.

1950-2011	**gamma2, alpha=0.3**	**Gamma2, alpha=0.3**	**gamma2, alpha=0.4**	**Gamma2, alpha=0.4**
Tier 1	7206.076	48.78216	3577.149	49.25857
Tier 2	33758.57	42.31259	8752.897	65.70612
Tier 3	187336.1	102.2451	56795.8	186.4724
1970-2011	**gamma2, alpha=0.3**	**Gamma2, alpha=0.3**	**gamma2, alpha=0.4**	**Gamma2, alpha=0.4**
Tier 1	2362.536	22.98957	1667.108	29.29132

Tier 2	6597.654	40.1134	5792.043	53.08621
Tier 3	52736.95	63.34748	28308.73	99.07977
1990-2011	**gamma2, alpha=0.3**	**Gamma2, alpha=0.3**	**gamma2, alpha=0.4**	**Gamma2, alpha=0.4**
Tier 1	14381.25	37.76895	7545.874	62.04026
Tier 2	9622.128	50.2796	17309.63	91.48729
Tier 3	109972.3	84.06471	58555.88	179.0643

Median risk aversion metrics over the three time frames for each income tier under the two assumptions for alpha, given a constant opportunity cost of r_f=0.045.

1950-2011	**gamma2, alpha=0.3**	**Gamma2, alpha=0.3**	**gamma2, alpha=0.4**	**Gamma2, alpha=0.4**
Tier 1	9194.359	43.44782	3840.956	46.10971
Tier 2	982.5012	29.97373	11345.32	53.47475
Tier 3	37814.32	35.42349	76239.05	151.5628
1970-2011	**gamma2, alpha=0.3**	**Gamma2, alpha=0.3**	**gamma2, alpha=0.4**	**Gamma2, alpha=0.4**
Tier 1	500.5452	19.09915	1605.292	26.0754
Tier 2	8447.225	33.97813	6244.364	43.8214
Tier 3	33804.55	23.54137	27741.78	75.92611
1990-2011	**gamma2, alpha=0.3**	**Gamma2, alpha=0.3**	**gamma2, alpha=0.4**	**Gamma2, alpha=0.4**
Tier 1	7277.802	23.82703	5280.939	49.01036
Tier 2	2685.543	24.13723	9440.653	74.67209
Tier 3	19919.04	36.46708	58824.18	147.7337

Median risk aversion metrics over the three time frames for each income tier under the two assumptions for alpha, given a constant opportunity cost of r_f=0.055.

Since both the relative risk aversion (gamma) and the absolute risk aversion (Gamma) are derived based on an approximation of small variance, even though the actual variance is large, these metrics are best interpreted relative to each other – more precisely, the risk aversion of some tier relative to another over a single time frame, and of one tier relative to the same tier over different time frames. Any error or distortion arising from increasing the variance beyond small values should thus be negated, or at least mitigated for one of the absolute or relative metrics. Overall, these metrics further undermine the case for the validity of the Lucas Paradox

5:: Conclusion

The Lucas Paradox, predicated on the return on capital differential, appears to persist through time – but at a much reduced level compared to what it once was. In other words, capital does flow from rich countries to poor ones – but at an appropriately slow rate given how high the variance of returns in poor countries is relative to rich ones. Here, "slow" reflects the finding that it took until the 1990-2011 period before the geometric mean return of the poor countries converged to that of the middle income countries; however, both the rich countries still generate the lowest geometric mean return, albeit with remarkably much less variance (all of this evidence implicitly being at L=1). So, while there is evidence for some convergence of returns, the convergence is far from complete and nuanced. The returns of poor countries have converged down to the middle income countries but the variance of the middle income portfolio is only around one sixth of the poor country portfolio. So, even if the poor countries have transitioned out of high capital scarcity by achieving lower returns as of 2011, the representative investor should strongly prefer middle income countries going forward, given the persistently high variance of returns in the poor countries; the agent faces two groups of countries offering roughly the same reward, yet the poor group is remarkably less stable. So, if it is acknowledged that the poor group has converged to the middle group, the middle group must now converge with the rich group before the poor group can meaningfully converge further. All of these considerations on convergence have been through the lens of L=1, corresponding to a representative investor being credit-constrained when it comes to exploiting global return imbalances. If it is acknowledged that the agent can borrow money in an attempt to arbitrage these imbalances, then the estimation of the true degree of risk aversion is much more nuanced. Almost all of the various risk aversion metrics, both relative and absolute under both assumptions for alpha over the three time periods, did not steadily increase over time. However, the decreases came from the 1950 to 1970 period, in contrast to the apparent convergence in geometric mean return coming from the 1990 to 2011 period. Since the relative risk aversion is a nonlinear transformation of the mean/variance ratio and the absolute metric is a linear one, if the variance shifts quickly down to the zero lower bound, then the implied optimal leverage can increase rapidly, leaving the investor at L=1 and L=2 definitively under-leveraged. However, if the investor is truly credit-constrained (L=1), then he might as well tolerate the higher variance and not allocate to the rich countries.

Ultimately, unless interest rates are high (r_f>>0.05), the representative investor might as well set L to 2 and invest in the middle income rich countries; allocating to the poor countries requires taking on much more risk with little added diversification. In other words, the Lucas Paradox is highly overstated, now more than ever.

6:: References

[1] Lucas jr., R. (1990). Why Doesn't Capital Flow from Rich to Poor Countries? *The American Economic Review, 80*(2), 92-96. Retrieved on 2015-03-30: <http://www.jstor.org/stable/2006549>

[2] Henriksen, E., David, J., & Simonovska, I. (2014, December). The Risky Capital of Emerging Markets. Retrieved on 2015-03-30: <http://www.nber.org/papers/w20769>

[3] Merton, R. (1969). Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case. *The Review of Economics and Statistics, 51*(3), 247-257. Retrieved on 2015-03-30: <http://www.jstor.org/stable/1926560>

[4] Sandmo, A. (1970). The Effect of Uncertainty on Saving Decisions. *The Review of Economic Studies, 37*(3), 353-360. Retrieved on 2015-03-30: <http://www.jstor.org/stable/2296725>

[5] Kelly jr., J. (1956). A New Interpretation of Information Rate. *The Bell System Technical Journal*. Retrieved on 2015-03-30: <http://www.princeton.edu/~wbialek/rome/refs/kelly_56.pdf>

[6] Jacquier, E., & Polson, N. (2011, August). Asset Allocation in Finance: A Bayesian Perspective. Retrieved on 2015-03-30: <http://faculty.chicagobooth.edu/nicholas.polson/research/papers/Asset.pdf>

[7] Hidalgo, C., & Hausmann, R. (2009). The Building Blocks of Economic Complexity. *Proceedings of the National Academy of Sciences, 106*(26), 10570-10575. Retrieved on 2015-03-30: <http://www.ncbi.nlm.nih.gov/pmc/articles/PMC2705545/?tool=pmcentrez>

[8] Crucini, M., & Chen, K. (2014). Trends and Cycles in Small Open Economies: Making The Case For A General Equilibrium Approach. CAMA Working paper 76/2014. Retrieved on 2015-03-30: <http://dx.doi.org/10.2139/ssrn.2536634>

7:: Appendix

For three different interest rates, the mean/variance ratios of the returns on capital for each country are listed, first with alpha=0.3 and then 0.4, from the three main initial points for the data until 2011.

Setting r_f=0.025:

Country	0.3MeanVar 1950	0.3MeanVar 1970	0.3MeanVar 1990	0.4MeanVar 1950	0.4MeanVar 1970	0.4MeanVar 1990
Angola	NA	72.93011	72.19775	NA	83.81506	92.78574

Albania	NA	32.90215	29.31635	NA	39.65297	92.24804
Argentina	48.48897	67.37835	233.1918	46.40955	70.33648	323.5862
Armenia	NA	NA	84.18751	NA	NA	87.00504
Antigua and Barbuda	NA	71.66767	306.5131	NA	69.03391	261.7942
Australia	227.6992	1940.371	1459.439	453.7026	2718.273	2136.857
Austria	31.87581	49.48567	534.6035	32.77142	57.85408	756.5798
Azerbaijan	NA	NA	33.97125	NA	NA	32.61283
Burundi	NA	49.28908	141.2325	NA	45.67731	139.2962
Belgium	622.2331	425.0069	232.2866	764.6082	523.6107	291.9675
Benin	NA	16.2027	51.10111	NA	15.37282	53.68859
Burkina Faso	NA	38.74117	806.3543	NA	37.01172	805.6516
Bangladesh	NA	30.93644	44.85454	NA	28.58339	41.6151
Bulgaria	NA	147.166	232.7863	NA	156.9377	261.576
Bahrain	NA	29.87374	48.6793	NA	28.41263	52.39466
Bahamas	NA	100.1348	83.76805	NA	94.13746	78.4047
Bosnia and Herzegovina	NA	NA	55.54189	NA	NA	55.17366
Belarus	NA	NA	58.91299	NA	NA	71.02174
Belize	NA	168.6874	373.1296	NA	163.4743	359.8292
Bermuda	NA	501.5506	931.8954	NA	493.2129	867.5078
Bolivia	170.6437	214.4134	166.3759	173.6027	230.8872	167.534
Brazil	98.4731	82.33884	116.6301	96.11184	81.52584	124.0064
Barbados	NA	53.16353	189.7405	NA	48.99272	180.9203
Brunei	NA	28.16996	126.7257	NA	27.29575	139.4667
Bhutan	NA	57.0646	22.42241	NA	104.8474	231.5552
Botswana	NA	47.8939	36.56144	NA	50.16268	40.17062
Central African Republic	NA	324.312	1901.421	NA	406.5057	1983.841
Canada	270.5548	352.1416	575.6884	286.4921	379.5993	601.2458
Switzerland	87.50251	159.4113	313.9471	113.1325	235.368	542.721
Chile	NA	98.73473	291.3203	NA	100.4383	291.9244
China	NA	55.19418	74.84966	NA	51.88148	73.89059
Cote d`Ivoire	NA	30.64352	922.0747	NA	29.53941	957.9226

Cameroon	NA	45.25149	126.2539	NA	44.00651	125.8784
Congo, Dem. Rep.	13.46321	13.46321	197.6771	12.44369	12.44369	237.4269
Congo, Republic of	NA	14.08954	39.65875	NA	13.744	48.92471
Colombia	249.9317	281.1908	312.1161	269.9298	289.8198	313.3322
Comoros	NA	145.6225	508.8505	NA	197.9849	1154.892
Cape Verde	NA	2.877907	-133.196	NA	82.38268	464.6905
Costa Rica	120.9821	188.1355	362.9507	118.5138	181.9104	342.1929
Cyprus	88.88455	76.98374	451.086	149.2004	120.0207	557.8986
Czech Republic	NA	NA	242.8952	NA	NA	310.6538
Germany	717.0934	801.4413	669.2859	874.0594	984.5024	779.5325
Djibouti	NA	46.35142	127.9059	NA	44.60505	153.2558
Dominica	NA	137.3354	401.1473	NA	134.8688	375.0134
Denmark	129.3431	305.277	267.8244	147.7629	384.8173	354.9405
Dominican Republic	NA	65.01525	246.3	NA	59.8259	230.3873
Ecuador	NA	90.53979	229.4159	NA	101.0158	295.0632
Egypt	13.31797	10.10638	29.09744	12.96513	9.882041	27.24355
Spain	60.72484	97.33079	124.1614	59.16656	101.4001	143.987
Estonia	NA	NA	131.7532	NA	NA	138.9925
Ethiopia	22.71435	15.76227	18.74725	20.79051	14.4581	17.40638
Finland	82.80663	125.0419	104.1169	110.6749	217.5933	176.3376
Fiji	NA	185.4792	143.1945	NA	183.3534	141.6513
France	194.1369	242.0406	165.6638	208.2195	267.3403	182.5914
Gabon	NA	29.61894	30.7858	NA	31.13735	35.37364
United Kingdom	389.8522	297.4556	197.5746	402.4312	309.9586	206.189
Georgia	NA	NA	14.94197	NA	NA	15.91817
Ghana	NA	127.2932	630.7172	NA	155.0847	727.4261
Guinea	NA	223.869	169.2954	NA	210.2767	155.8176
Gambia, The	NA	57.14003	78.80187	NA	56.08709	79.6408
Guinea-Bissau	NA	-69.0918	-338.195	NA	116.0912	243.253
Equatorial	NA	19.80917	61.09712	NA	18.7554	57.51241

Guinea						
Greece	NA	203.4234	249.5951	NA	252.9781	317.1365
Grenada	NA	29.71457	146.5999	NA	29.81305	154.0376
Guatemala	284.4767	274.6632	381.8698	273.1182	263.7042	359.6338
Hong Kong	NA	34.97353	57.72041	NA	36.23143	72.28115
Honduras	97.38173	86.32635	63.68145	102.8056	93.17615	74.33271
Croatia	NA	NA	239.2758	NA	NA	283.8525
Hungary	NA	241.2137	197.353	NA	254.8568	208.0927
Indonesia	NA	67.97495	36.56505	NA	60.76834	33.36653
India	287.4967	205.9544	169.1724	287.7167	218.3619	186.5325
Ireland	87.58498	73.64679	53.73917	88.16325	77.60535	54.84571
Iran	NA	82.51945	918.7695	NA	93.87264	989.517
Iraq	NA	53.38395	36.77336	NA	50.45028	35.71002
Iceland	253.0694	415.6427	342.2739	301.7866	514.291	436.9368
Israel	168.216	156.244	2706.193	182.2147	172.826	3278.063
Italy	110.671	123.0313	131.4768	133.6674	164.0482	229.3743
Jamaica	NA	120.1949	612.8083	NA	119.4053	607.4725
Jordan	NA	23.62776	45.25691	NA	22.91455	55.02381
Japan	48.17864	63.56022	86.76859	53.18724	80.70608	185.0654
Kazakhstan	NA	NA	40.40887	NA	NA	47.81475
Kenya	114.7956	104.8858	185.0136	112.9213	104.2692	190.3839
Kyrgyzstan	NA	NA	53.2818	NA	NA	51.23514
Cambodia	NA	100.934	2564.38	NA	92.88001	2440.961
St. Kitts & Nevis	NA	146.1474	175.9195	NA	142.0408	170.0494
Korea, Republic of	NA	102.7613	88.34185	NA	107.6853	98.38969
Kuwait	NA	32.84571	106.0482	NA	31.81037	109.3042
Laos	NA	125.4702	136.846	NA	132.0003	139.5311
Lebanon	NA	19.3385	56.28304	NA	22.99029	94.63017
Liberia	NA	18.01557	3.104609	NA	21.16684	22.07918
St. Lucia	NA	46.34921	111.5933	NA	44.62514	104.3378
Sri Lanka	33.23544	43.22205	120.5151	32.18858	44.13657	119.9782
Lesotho	NA	21.63571	153.7642	NA	21.3021	201.9506
Lithuania	NA	NA	113.2345	NA	NA	109.6726
Luxembourg	273.2702	292.1853	589.829	300.633	317.1666	649.7476

Latvia	NA	NA	123.2963	NA	NA	133.1842
Macao	NA	361.6297	215.1055	NA	351.8255	211.3988
Morocco	14.50232	12.1665	80.49774	13.87214	11.94366	117.5213
Moldova	NA	NA	30.71218	NA	NA	45.44844
Madagascar	NA	15.47694	25.04771	NA	15.02624	25.9641
Maldives	NA	47.37609	80.03537	NA	45.63038	78.40671
Mexico	293.0045	248.1375	331.5124	276.7683	235.9252	327.6581
Macedonia	NA	NA	60.23694	NA	NA	63.14134
Mali	NA	33.06022	74.2691	NA	32.39877	76.9134
Malta	NA	285.6961	225.788	NA	320.3941	254.2068
Montenegro	NA	NA	113.0864	NA	NA	125.1107
Mongolia	NA	26.22642	18.76981	NA	27.16023	24.60514
Mozambique	NA	192.4652	591.5578	NA	178.6899	541.8755
Mauritania	NA	90.6817	385.0873	NA	100.4746	566.9044
Mauritius	28.39755	22.31954	38.89386	27.37361	22.09398	43.89535
Malawi	NA	41.88193	45.17937	NA	44.78389	65.75179
Malaysia	NA	52.51596	140.4785	NA	90.31537	576.5563
Namibia	NA	157.1165	687.3042	NA	166.5281	732.4259
Niger	NA	-2.4663	135.3136	NA	222.6899	512.3646
Nigeria	27.61294	35.42051	72.61932	26.00249	34.07287	66.36522
Netherlands	407.6908	448.0162	249.7372	439.1864	508.145	282.1985
Norway	124.3416	128.6366	187.4733	146.2168	166.3428	217.9383
Nepal	NA	12.73087	59.1038	NA	11.73559	56.61137
New Zealand	199.5806	235.1756	382.3024	212.7967	252.9968	400.3452
Oman	NA	57.5326	59.70308	NA	60.9525	63.82349
Pakistan	104.162	73.07828	64.87921	98.74607	69.31783	62.70516
Panama	166.1649	217.4755	247.0768	159.2674	217.9239	249.1057
Peru	47.18601	39.86369	584.7862	45.27078	39.55859	741.8394
Philippines	68.74543	45.1976	124.424	64.59245	42.87443	129.9707
Poland	NA	105.1232	237.7594	NA	107.1253	235.9577
Portugal	44.18734	51.933	192.3862	47.31232	61.12747	257.9728
Paraguay	NA	28.00231	148.2764	NA	26.64438	143.6489
Qatar	NA	39.09156	369.3544	NA	40.68243	421.2069
Romania	NA	352.7971	499.4353	NA	389.549	550.6046
Russia	NA	NA	29.34558	NA	NA	49.51534

Rwanda	NA	11.14471	28.23826	NA	10.84542	26.97247
Saudi Arabia	NA	24.44819	286.3916	NA	28.09259	461.734
Sudan	NA	24.62353	14.39993	NA	24.5509	14.44106
Senegal	NA	32.01526	39.65389	NA	32.05058	46.16646
Singapore	NA	52.40314	82.05616	NA	59.67595	102.918
Sierra Leone	NA	5.673694	4.762485	NA	5.447713	4.626135
El Salvador	68.64393	64.68585	165.9635	62.56793	58.71679	148.906
Serbia	NA	NA	87.14675	NA	NA	128.5802
Sao Tome and Principe	NA	35.67134	80.70799	NA	42.28704	161.2472
Suriname	NA	36.74173	35.1257	NA	39.39092	47.78101
Slovak Republic	NA	NA	393.6406	NA	NA	465.6976
Slovenia	NA	NA	216.3544	NA	NA	266.1331
Sweden	126.1319	219.5235	210.4972	140.1223	248.3684	235.4216
Swaziland	NA	13.50789	4.500217	NA	14.21483	9.017794
Syria	NA	52.83489	30.64392	NA	50.54472	30.28632
Chad	NA	166.8059	234.0824	NA	152.9863	217.9536
Togo	NA	101.2978	820.2228	NA	106.9795	896.9012
Thailand	30.9827	20.97699	3.151309	34.70853	25.23885	42.46441
Tajikistan	NA	NA	23.32556	NA	NA	28.23396
Turkmenistan	NA	NA	43.62851	NA	NA	81.29808
Trinidad & Tobago	35.35312	28.71391	232.0001	33.11724	27.29764	217.0915
Tunisia	NA	19.7607	49.33316	NA	20.05091	69.94925
Turkey	132.7592	111.1847	183.96	141.7099	117.5722	189.7953
Taiwan	NA	33.00931	68.89183	NA	32.22757	73.04834
Tanzania	NA	27.36069	161.7034	NA	29.58642	207.2386
Uganda	52.15621	109.1329	185.3463	48.52013	98.19835	166.4189
Ukraine	NA	NA	-14.5496	NA	NA	43.63114
Uruguay	16.85419	50.46237	194.0906	16.14339	52.23876	222.666
United States	575.6729	803.4952	943.5247	649.2681	883.5065	989.8239
Uzbekistan	NA	NA	163.7395	NA	NA	150.8004
St.Vincent &	NA	148.2701	114.4936	NA	137.8855	106.3751

Grenadines						
Venezuela	84.0865	106.1145	205.94	91.16888	130.3364	267.4538
Vietnam	NA	25.43629	14.26062	NA	23.07542	13.35594
Yemen	NA	NA	11.21805	NA	NA	10.33135
South Africa	144.8349	260.8169	1872.698	153.3434	281.7735	2172.987
Zambia	NA	18.10164	74.18451	NA	17.89676	86.28061
Zimbabwe	NA	18.81833	18.17095	NA	19.0253	18.30986

Setting r_f=0.035:

Country	0.3MeanVar 1950	0.3MeanVar 1970	0.3MeanVar 1990	0.4MeanVar 1950	0.4MeanVar 1970	0.4MeanVar 1990
Angola	NA	51.35007	43.2652	NA	70.81717	75.3609
Albania	NA	21.10347	-29.4205	NA	32.66119	56.62992
Argentina	41.0568	51.69245	128.003	42.05513	61.09672	256.3747
Armenia	NA	NA	68.98026	NA	NA	77.54036
Antigua and Barbuda	NA	67.55964	283.91	NA	66.33404	248.976
Australia	126.7708	1185.363	900.9908	369.9197	2240.159	1757.041
Austria	27.16751	36.12569	257.3196	29.85275	49.79977	599.3117
Azerbaijan	NA	NA	29.84948	NA	NA	30.12418
Burundi	NA	45.6212	124.5114	NA	43.3517	128.6038
Belgium	427.6763	288.9624	156.248	646.2634	441.7358	245.7671
Benin	NA	15.33611	43.98197	NA	14.80302	49.14063
Burkina Faso	NA	35.54261	694.5236	NA	34.97724	736.5887
Bangladesh	NA	28.97107	40.05107	NA	27.34662	38.75422
Bulgaria	NA	126.3443	188.7433	NA	143.5317	233.6511
Bahrain	NA	26.58701	37.83672	NA	26.37999	45.74132
Bahamas	NA	94.42702	78.37841	NA	90.46907	75.00847
Bosnia and Herzegovina	NA	NA	49.66146	NA	NA	51.45767
Belarus	NA	NA	38.48195	NA	NA	58.89211
Belize	NA	162.7274	359.783	NA	159.3978	350.7674
Bermuda	NA	479.8201	889.9656	NA	478.2939	840.3549
Bolivia	149.0792	190.155	148.6571	159.9941	214.429	156.1146
Brazil	84.95648	69.80771	90.53362	87.85315	73.88968	108.2468

Barbados	NA	51.21062	183.9559	NA	47.71151	176.9639
Brunei	NA	25.26733	100.8454	NA	25.46774	123.1514
Bhutan	NA	9.779249	-168.624	NA	76.33252	128.3923
Botswana	NA	41.85856	30.65215	NA	46.36628	36.57836
Central African Republic	NA	223.7062	1403.302	NA	341.1307	1697.318
Canada	222.3388	279.0759	445.7283	256.5394	335.1038	526.4777
Switzerland	62.30228	86.97386	121.3888	96.98223	191.5736	425.9859
Chile	NA	86.51823	246.4314	NA	92.67031	264.9928
China	NA	49.77523	63.30951	NA	48.51307	66.85582
Cote d`Ivoire	NA	28.92292	838.2114	NA	28.40265	901.4429
Cameroon	NA	41.78597	112.9855	NA	41.77447	117.5108
Congo, Dem. Rep.	12.0689	12.0689	100.5999	11.58414	11.58414	181.8164
Congo, Republic of	NA	13.18497	31.18055	NA	13.15151	43.3971
Colombia	188.201	219.5126	232.5547	234.3905	254.8511	270.9994
Comoros	NA	47.06811	-184.256	NA	148.2242	753.2989
Cape Verde	NA	-59.3908	-602.156	NA	42.84284	207.1635
Costa Rica	113.5883	174.8195	334.3738	113.6022	173.3523	324.5364
Cyprus	16.8185	21.91538	267.9119	106.4314	87.58107	448.8912
Czech Republic	NA	NA	160.2388	NA	NA	259.1221
Germany	479.0918	517.4836	449.4536	731.442	816.6856	652.5655
Djibouti	NA	39.10642	84.66858	NA	40.31715	126.6422
Dominica	NA	134.6138	391.8751	NA	132.9289	368.759
Denmark	97.65679	200.6171	162.3547	128.2325	321.0284	291.1383
Dominican Republic	NA	59.23931	213.8037	NA	56.29986	211.1685
Ecuador	NA	68.86376	131.5849	NA	87.88208	239.0858
Egypt	13.06872	9.874948	27.56198	12.78939	9.719806	26.26397
Spain	50.80781	73.63112	84.63709	53.41553	87.96331	120.6748
Estonia	NA	NA	110.4526	NA	NA	125.6424
Ethiopia	21.94497	15.20391	17.41801	20.28628	14.09212	16.56483
Finland	48.34563	34.7312	32.19544	89.64389	162.0825	132.0486

Fiji	NA	166.8723	127.1965	NA	171.7281	131.8661
France	153.0521	179.4306	121.3127	182.911	229.7189	156.3839
Gabon	NA	25.50566	23.37402	NA	28.63823	31.10761
United Kingdom	320.953	247.0024	166.5159	361.1777	279.3063	187.0527
Georgia	NA	NA	11.82037	NA	NA	13.99288
Ghana	NA	75.65997	454.6525	NA	128.016	628.6399
Guinea	NA	203.6087	151.6549	NA	198.1919	145.7452
Gambia, The	NA	52.81657	69.74029	NA	53.27544	73.97844
Guinea-Bissau	NA	-252.831	-873.966	NA	10.44885	-72.5484
Equatorial Guinea	NA	19.02911	56.2869	NA	18.23567	54.53338
Greece	NA	134.3334	158.3118	NA	210.7797	261.6481
Grenada	NA	28.36945	136.7461	NA	28.87923	147.2459
Guatemala	266.7009	257.14	355.2722	261.5369	252.3579	342.9031
Hong Kong	NA	30.37426	39.61498	NA	33.27577	61.20248
Honduras	88.26103	77.03542	54.61008	96.50856	86.79601	67.99713
Croatia	NA	NA	162.3337	NA	NA	237.7316
Hungary	NA	194.6609	155.4241	NA	226.771	183.1029
Indonesia	NA	62.34015	33.05908	NA	57.42728	31.27724
India	252.6182	181.547	147.6498	266.411	202.7167	172.5996
Ireland	75.63399	61.50811	45.71339	80.83972	70.10243	49.89609
Iran	NA	59.64843	556.6826	NA	80.38864	810.7134
Iraq	NA	47.24868	31.73547	NA	46.75394	32.68343
Iceland	173.7961	256.0954	200.9516	253.2531	419.9875	353.265
Israel	139.2169	128.2527	2108.518	163.8593	155.0406	2885.376
Italy	78.25426	72.82306	32.25798	113.7411	133.9536	171.4755
Jamaica	NA	101.4072	539.3075	NA	108.229	561.8032
Jordan	NA	20.59252	28.91792	NA	21.01237	44.90521
Japan	36.99278	38.16289	-2.8031	46.65842	66.5315	133.5967
Kazakhstan	NA	NA	27.3053	NA	NA	39.95675
Kenya	105.4467	96.2501	165.3578	106.863	98.64293	177.6786
Kyrgyzstan	NA	NA	46.9718	NA	NA	47.3441
Cambodia	NA	84.51499	2246.553	NA	83.77085	2251.82
St. Kitts &	NA	126.9774	145.6111	NA	130.5757	152.9688

Nevis						
Korea, Republic of	NA	84.15304	65.45134	NA	96.56366	85.24334
Kuwait	NA	28.40468	82.88042	NA	29.08668	95.43101
Laos	NA	102.9602	108.5134	NA	118.1312	123.4615
Lebanon	NA	13.85032	6.658947	NA	19.62127	68.74999
Liberia	NA	13.89205	-10.3495	NA	18.70899	14.83603
St. Lucia	NA	45.02123	107.799	NA	43.70864	101.8187
Sri Lanka	30.33012	37.60455	101.0246	30.3337	40.58497	108.5069
Lesotho	NA	17.79404	67.25935	NA	18.98984	151.4505
Lithuania	NA	NA	100.5097	NA	NA	101.7564
Luxembourg	211.8734	218.9547	445.8773	263.5863	274.9381	565.5967
Latvia	NA	NA	96.69801	NA	NA	117.0361
Macao	NA	314.3597	185.9184	NA	322.7411	193.4294
Morocco	13.61845	11.18538	48.76609	13.29655	11.31016	97.39036
Moldova	NA	NA	9.929153	NA	NA	33.54799
Madagascar	NA	14.54116	22.04668	NA	14.40507	24.04275
Maldives	NA	43.35912	69.17084	NA	43.09015	71.93704
Mexico	257.6309	217.0767	282.2785	255.6632	217.3648	297.8155
Macedonia	NA	NA	48.53704	NA	NA	56.06843
Mali	NA	31.70119	69.39398	NA	31.47133	73.56881
Malta	NA	224.115	171.7479	NA	280.7809	220.8843
Montenegro	NA	NA	82.18302	NA	NA	106.7139
Mongolia	NA	21.82478	11.34898	NA	24.45263	20.12171
Mozambique	NA	176.2605	534.5823	NA	168.5858	507.3473
Mauritania	NA	67.466	259.48	NA	86.287	472.1486
Mauritius	26.7358	20.6805	31.94473	26.27075	21.00583	39.48027
Malawi	NA	33.82985	22.84716	NA	40.00687	52.8337
Malaysia	NA	28.36676	-99.6458	NA	73.5774	409.5949
Namibia	NA	129.8482	552.7148	NA	149.8752	652.1111
Niger	NA	-172.458	-169.405	NA	109.0509	308.2717
Nigeria	25.8831	32.23643	65.10117	24.8845	32.05581	61.97849
Netherlands	305.1619	323.4865	180.9681	380.3887	434.545	241.6192
Norway	89.48308	81.82333	133.5527	124.8246	137.4058	185.1946
Nepal	NA	11.98109	48.73274	NA	11.25439	50.52366
New	165.3666	188.0367	315.6785	191.511	224.5921	360.0064

Zealand						
Oman	NA	50.83952	49.73146	NA	56.66169	57.96921
Pakistan	96.13873	67.03055	57.69372	93.66836	65.53553	58.32692
Panama	151.45	198.0661	225.02	150.0999	205.3881	234.8206
Peru	41.20431	33.19698	341.4202	41.67481	35.63002	599.0487
Philippines	60.2104	39.3556	93.82669	59.68089	39.50008	112.9837
Poland	NA	85.75496	185.5712	NA	95.58944	206.8676
Portugal	37.03425	38.7437	101.9586	42.8088	53.08237	208.038
Paraguay	NA	25.52657	121.8135	NA	25.11292	128.8487
Qatar	NA	34.18015	293.6484	NA	37.52215	373.7083
Romania	NA	265.8492	369.7526	NA	337.0684	473.6171
Russia	NA	NA	6.60997	NA	NA	35.93073
Rwanda	NA	10.93971	27.04522	NA	10.70043	26.17615
Saudi Arabia	NA	17.73052	78.19863	NA	23.99704	341.4836
Sudan	NA	24.28795	14.16829	NA	24.30515	14.27115
Senegal	NA	27.73141	28.88245	NA	29.35933	39.53382
Singapore	NA	43.0574	58.20864	NA	53.62326	88.39966
Sierra Leone	NA	5.588336	4.507455	NA	5.38759	4.454592
El Salvador	65.33468	61.11278	154.4678	60.4748	56.49268	141.9053
Serbia	NA	NA	33.55132	NA	NA	97.03572
Sao Tome and Principe	NA	28.49703	29.85561	NA	37.54334	126.3482
Suriname	NA	30.60554	23.62647	NA	35.46534	40.10166
Slovak Republic	NA	NA	277.5865	NA	NA	396.5784
Slovenia	NA	NA	151.5218	NA	NA	225.9128
Sweden	104.8028	171.6276	166.8222	126.4697	219.3075	208.8864
Swaziland	NA	11.8247	1.30519	NA	13.16203	7.149903
Syria	NA	48.4689	27.44209	NA	47.79979	28.2839
Chad	NA	160.1215	225.2227	NA	148.6254	212.0741
Togo	NA	82.2743	648.7544	NA	95.17111	788.9818
Thailand	26.18911	16.50478	-20.6077	31.54593	22.37306	27.93781
Tajikistan	NA	NA	13.87373	NA	NA	22.71486
Turkmenistan	NA	NA	2.166084	NA	NA	56.93229
Trinidad &	33.83316	27.28688	214.6768	32.13225	26.37019	206.4339

Tobago						
Tunisia	NA	17.69372	30.37936	NA	18.72603	58.29834
Turkey	114.3226	93.84142	160.4566	129.8746	106.9364	175.1946
Taiwan	NA	30.22284	58.72434	NA	30.47714	66.74665
Tanzania	NA	23.19698	116.1339	NA	26.94257	178.6829
Uganda	50.02257	103.4811	174.5821	47.13397	94.70793	159.7868
Ukraine	NA	NA	-64.6643	NA	NA	15.00434
Uruguay	15.27076	41.24807	143.0145	15.15221	46.62435	191.029
United States	456.1062	618.1208	725.1521	572.9429	771.7229	864.92
Uzbekistan	NA	NA	152.5038	NA	NA	143.7241
St.Vincent & Grenadines	NA	142.8782	110.0714	NA	134.3158	103.4609
Venezuela	63.69955	68.76731	118.8078	78.9375	107.6081	215.0833
Vietnam	NA	24.16347	13.25092	NA	22.26171	12.70575
Yemen	NA	NA	10.84511	NA	NA	10.08543
South Africa	128.0071	223.9281	1577.246	142.2417	257.9835	1973.682
Zambia	NA	16.40193	56.68037	NA	16.7987	75.3498
Zimbabwe	NA	18.54193	17.96076	NA	18.82007	18.15372

Setting r_f=0.045

Country	0.3MeanVar 1950	0.3MeanVar 1970	0.3MeanVar 1990	0.4MeanVar 1950	0.4MeanVar 1970	0.4MeanVar 1990
Angola	NA	29.77003	14.33265	NA	57.81928	57.93605
Albania	NA	9.304802	-88.1573	NA	25.66941	21.01181
Argentina	33.62463	36.00655	22.81408	37.70071	51.85697	189.1632
Armenia	NA	NA	53.773	NA	NA	68.07568
Antigua and Barbuda	NA	63.45161	261.3069	NA	63.63417	236.1577
Australia	25.84234	430.3552	342.5427	286.1368	1762.045	1377.225
Austria	22.45921	22.76571	-19.9644	26.93408	41.74545	442.0436
Azerbaijan	NA	NA	25.72772	NA	NA	27.63553
Burundi	NA	41.95332	107.7903	NA	41.02609	117.9114
Belgium	233.1195	152.918	80.20946	527.9186	359.8609	199.5666
Benin	NA	14.46952	36.86283	NA	14.23322	44.59268
Burkina	NA	32.34406	582.6929	NA	32.94276	667.5257

Faso						
Bangladesh	NA	27.00569	35.24761	NA	26.10984	35.89334
Bulgaria	NA	105.5227	144.7004	NA	130.1257	205.7262
Bahrain	NA	23.30027	26.99415	NA	24.34734	39.08798
Bahamas	NA	88.71923	72.98877	NA	86.80067	71.61223
Bosnia and Herzegovina	NA	NA	43.78102	NA	NA	47.74168
Belarus	NA	NA	18.05092	NA	NA	46.76249
Belize	NA	156.7673	346.4364	NA	155.3213	341.7057
Bermuda	NA	458.0896	848.0358	NA	463.375	813.202
Bolivia	127.5146	165.8966	130.9382	146.3855	197.9708	144.6951
Brazil	71.43985	57.27658	64.43718	79.59446	66.25352	92.48729
Barbados	NA	49.25772	178.1714	NA	46.4303	173.0075
Brunei	NA	22.3647	74.9651	NA	23.63974	106.836
Bhutan	NA	-37.5061	-359.67	NA	47.81769	25.22937
Botswana	NA	35.82322	24.74286	NA	42.56987	32.9861
Central African Republic	NA	123.1003	905.1824	NA	275.7557	1410.796
Canada	174.1228	206.0102	315.7682	226.5868	290.6084	451.7097
Switzerland	37.10204	14.53645	-71.1695	80.83197	147.7791	309.2507
Chile	NA	74.30174	201.5425	NA	84.90237	238.0612
China	NA	44.35627	51.76935	NA	45.14465	59.82105
Cote d`Ivoire	NA	27.20231	754.3481	NA	27.26588	844.9631
Cameroon	NA	38.32045	99.7171	NA	39.54242	109.1432
Congo, Dem. Rep.	10.67459	10.67459	3.522743	10.7246	10.7246	126.2059
Congo, Republic of	NA	12.2804	22.70234	NA	12.55901	37.86949
Colombia	126.4702	157.8345	152.9932	198.8513	219.8825	228.6665
Comoros	NA	-51.4863	-877.363	NA	98.4636	351.7057
Cape Verde	NA	-121.66	-1071.12	NA	3.302996	-50.3635
Costa Rica	106.1945	161.5036	305.797	108.6906	164.7942	306.8799
Cyprus	-55.2476	-33.153	84.7378	63.66251	55.14148	339.8838
Czech Republic	NA	NA	77.58241	NA	NA	207.5903

Germany	241.0901	233.526	229.6213	588.8246	648.8688	525.5985
Djibouti	NA	31.86143	41.43128	NA	36.02924	100.0286
Dominica	NA	131.8923	382.6029	NA	130.9889	362.5045
Denmark	65.97053	95.95725	56.88496	108.702	257.2395	227.3362
Dominican Republic	NA	53.46337	181.3074	NA	52.77383	191.9497
Ecuador	NA	47.18774	33.75385	NA	74.7484	183.1085
Egypt	12.81948	9.643515	26.02652	12.61366	9.557571	25.28439
Spain	40.89077	49.93145	45.11281	47.66449	74.52647	97.36263
Estonia	NA	NA	89.15205	NA	NA	112.2923
Ethiopia	21.17558	14.64556	16.08878	19.78205	13.72613	15.72328
Finland	13.88463	-55.5795	-39.726	68.61292	106.5717	87.7595
Fiji	NA	148.2653	111.1985	NA	160.1027	122.0809
France	111.9674	116.8205	76.96157	157.6024	192.0975	130.1765
Gabon	NA	21.39239	15.96223	NA	26.1391	26.84158
United Kingdom	252.0539	196.5493	135.4573	319.9243	248.6541	167.9164
Georgia	NA	NA	8.698757	NA	NA	12.06759
Ghana	NA	24.02678	278.5878	NA	100.9473	529.8538
Guinea	NA	183.3485	134.0143	NA	186.107	135.6728
Gambia, The	NA	48.49311	60.6787	NA	50.46379	68.31608
Guinea-Bissau	NA	-436.57	-1409.74	NA	-95.1935	-388.35
Equatorial Guinea	NA	18.24905	51.47669	NA	17.71595	51.55434
Greece	NA	65.24336	67.02844	NA	168.5812	206.1596
Grenada	NA	27.02433	126.8924	NA	27.94541	140.4541
Guatemala	248.9251	239.6167	328.6747	249.9556	241.0116	326.1724
Hong Kong	NA	25.77499	21.50955	NA	30.32011	50.1238
Honduras	79.14034	67.74449	45.53872	90.21147	80.41586	61.66154
Croatia	NA	NA	85.39162	NA	NA	191.6106
Hungary	NA	148.1081	113.4953	NA	198.6852	158.1131
Indonesia	NA	56.70535	29.55312	NA	54.08621	29.18796
India	217.7396	157.1397	126.1273	245.1054	187.0715	158.6668
Ireland	63.683	49.36944	37.68761	73.51619	62.59952	44.94647
Iran	NA	36.77741	194.5958	NA	66.90465	631.9097

Iraq	NA	41.1134	26.69758	NA	43.0576	29.65684
Iceland	94.52283	96.5481	59.62936	204.7196	325.684	269.5932
Israel	110.2178	100.2614	1510.843	145.5039	137.2552	2492.689
Italy	45.83758	22.61484	-66.9609	93.81473	103.859	113.5767
Jamaica	NA	82.61942	465.8067	NA	97.0527	516.134
Jordan	NA	17.55728	12.57892	NA	19.11019	34.7866
Japan	25.80693	12.76556	-92.3748	40.1296	52.35691	82.12813
Kazakhstan	NA	NA	14.20173	NA	NA	32.09874
Kenya	96.09774	87.61443	145.7019	100.8048	93.01664	164.9732
Kyrgyzstan	NA	NA	40.66181	NA	NA	43.45305
Cambodia	NA	68.09599	1928.726	NA	74.66169	2062.678
St. Kitts & Nevis	NA	107.8074	115.3027	NA	119.1106	135.8882
Korea, Republic of	NA	65.5448	42.56082	NA	85.442	72.09698
Kuwait	NA	23.96365	59.7126	NA	26.36298	81.55786
Laos	NA	80.45025	80.18072	NA	104.2621	107.3919
Lebanon	NA	8.36213	-42.9651	NA	16.25225	42.86981
Liberia	NA	9.76853	-23.8037	NA	16.25113	7.592867
St. Lucia	NA	43.69326	104.0048	NA	42.79215	99.29968
Sri Lanka	27.4248	31.98704	81.53415	28.47881	37.03337	97.03552
Lesotho	NA	13.95236	-19.2455	NA	16.67757	100.9504
Lithuania	NA	NA	87.7849	NA	NA	93.8401
Luxembourg	150.4765	145.7241	301.9257	226.5397	232.7095	481.4458
Latvia	NA	NA	70.0997	NA	NA	100.888
Macao	NA	267.0897	156.7312	NA	293.6566	175.4599
Morocco	12.73458	10.20426	17.03444	12.72096	10.67665	77.25946
Moldova	NA	NA	-10.8539	NA	NA	21.64753
Madagascar	NA	13.60538	19.04564	NA	13.7839	22.12141
Maldives	NA	39.34215	58.30632	NA	40.54991	65.46737
Mexico	222.2574	186.0159	233.0447	234.5581	198.8044	267.9729
Macedonia	NA	NA	36.83713	NA	NA	48.99552
Mali	NA	30.34216	64.51887	NA	30.54388	70.22421
Malta	NA	162.5339	117.7079	NA	241.1677	187.5619
Montenegro	NA	NA	51.2796	NA	NA	88.31718
Mongolia	NA	17.42314	3.928146	NA	21.74503	15.63828

Mozambique	NA	160.0557	477.6069	NA	158.4816	472.819
Mauritania	NA	44.2503	133.8726	NA	72.09943	377.3927
Mauritius	25.07404	19.04146	24.99561	25.1679	19.91769	35.0652
Malawi	NA	25.77776	0.514956	NA	35.22984	39.91561
Malaysia	NA	4.217555	-339.77	NA	56.83942	242.6335
Namibia	NA	102.5799	418.1254	NA	133.2224	571.7963
Niger	NA	-342.449	-474.124	NA	-4.5881	104.1789
Nigeria	24.15325	29.05234	57.58302	23.76652	30.03875	57.59176
Netherlands	202.6329	198.9569	112.199	321.5911	360.9449	201.0399
Norway	54.62458	35.01003	79.63209	103.4325	108.4689	152.451
Nepal	NA	11.23131	38.36168	NA	10.7732	44.43594
New Zealand	131.1527	140.8977	249.0546	170.2252	196.1873	319.6675
Oman	NA	44.14643	39.75984	NA	52.37087	52.11492
Pakistan	88.11549	60.98281	50.50823	88.59065	61.75322	53.94868
Panama	136.7351	178.6568	202.9632	140.9324	192.8523	220.5354
Peru	35.2226	26.53026	98.05407	38.07883	31.70146	456.2579
Philippines	51.67537	33.51361	63.2294	54.76933	36.12573	95.99673
Poland	NA	66.38675	133.383	NA	84.05357	177.7774
Portugal	29.88116	25.5544	11.53098	38.30527	45.03726	158.1032
Paraguay	NA	23.05084	95.35066	NA	23.58145	114.0485
Qatar	NA	29.26874	217.9423	NA	34.36186	326.2098
Romania	NA	178.9013	240.07	NA	284.5878	396.6295
Russia	NA	NA	-16.1256	NA	NA	22.34611
Rwanda	NA	10.7347	25.85218	NA	10.55544	25.37983
Saudi Arabia	NA	11.01284	-129.994	NA	19.90148	221.2332
Sudan	NA	23.95237	13.93666	NA	24.05941	14.10125
Senegal	NA	23.44755	18.111	NA	26.66808	32.90117
Singapore	NA	33.71165	34.36113	NA	47.57057	73.88127
Sierra Leone	NA	5.502978	4.252424	NA	5.327468	4.283048
El Salvador	62.02543	57.5397	142.972	58.38168	54.26857	134.9046
Serbia	NA	NA	-20.0441	NA	NA	65.49125
Sao Tome and Principe	NA	21.32272	-20.9968	NA	32.79965	91.44912
Suriname	NA	24.46936	12.12724	NA	31.53975	32.42231
Slovak	NA	NA	161.5324	NA	NA	327.4592

Republic						
Slovenia	NA	NA	86.68929	NA	NA	185.6924
Sweden	83.47358	123.7317	123.1472	112.8171	190.2466	182.3512
Swaziland	NA	10.14151	-1.88984	NA	12.10922	5.282011
Syria	NA	44.10291	24.24027	NA	45.05486	26.28148
Chad	NA	153.437	216.3631	NA	144.2645	206.1946
Togo	NA	63.25076	477.286	NA	83.36268	681.0624
Thailand	21.39553	12.03256	-44.3667	28.38333	19.50727	13.41121
Tajikistan	NA	NA	4.421906	NA	NA	17.19576
Turkmenistan	NA	NA	-39.2963	NA	NA	32.5665
Trinidad & Tobago	32.31319	25.85985	197.3534	31.14727	25.44274	195.7764
Tunisia	NA	15.62675	11.42556	NA	17.40116	46.64743
Turkey	95.88603	76.49811	136.9532	118.0392	96.30054	160.5939
Taiwan	NA	27.43638	48.55685	NA	28.7267	60.44495
Tanzania	NA	19.03326	70.56443	NA	24.29872	150.1272
Uganda	47.88894	97.82929	163.8179	45.74782	91.21751	153.1547
Ukraine	NA	NA	-114.779	NA	NA	-13.6224
Uruguay	13.68732	32.03378	91.93843	14.16103	41.00994	159.392
United States	336.5394	432.7465	506.7795	496.6176	659.9393	740.0161
Uzbekistan	NA	NA	141.268	NA	NA	136.6478
St.Vincent & Grenadines	NA	137.4863	105.6493	NA	130.7461	100.5467
Venezuela	43.31259	31.42013	31.67555	66.70612	84.87969	162.7127
Vietnam	NA	22.89065	12.24122	NA	21.44799	12.05556
Yemen	NA	NA	10.47217	NA	NA	9.839506
South Africa	111.1793	187.0394	1281.793	131.14	234.1936	1774.378
Zambia	NA	14.70222	39.17623	NA	15.70063	64.41899
Zimbabwe	NA	18.26553	17.75056	NA	18.61484	17.99757

```
######################################
# R Appendix

## curve of one plus geometric mean against leverage
## for the empirical distribution of all return observations given alpha=0.3
r=0.05

g=NULL
for(k in seq(9,-(1+r)/(exp(min(na.omit(alpha0.3returns)))-1-r),0.001))
{
rescaled_returns=k*exp(na.omit(alpha0.3returns))-(k-1)-r*(k-1)-1
y=exp(mean(log(1+rescaled_returns)))
g=rbind(g,c(k,y))
}
```