Partisanship in Legislative Bargaining

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1 Introduction

Shared interests and interests in others' well-being powerfully influence decision-making. While individual persons decide their own actions, decisions are not often made without consideration for their effects on friends, associates, or allies. In legislative contexts, shared interests are demonstrated in partisan connections among individuals and groups. Whether these connections manifest as membership in a formal political party or as looser associations, this behavior suggests that participants in legislative processes are not purely self-interested. However, the mere presence of a shared interest does not indicate identical interests or equal concern. An interest may be more highly valued for one person who holds it than another. Further, if these interests are for the well-being of different persons, then we may reasonably assume a person values his or her own well-being over that of others. As legislative bargaining plays an important role in the allocation of resources at many levels of society, modelling the effects of partisanship on its outcomes is a significant step in better understanding these processes. While many previous legislative bargaining models have varied the structure of the legislative system and some have varied legislator preferences, the present study introduces and investigates the equilibrium effects of treating legislators as partisan rather than purely self-interested .

The present study follows previous work in the field of bargaining theory, particularly building on the seminal work on legislative bargaining of Baron and Ferejohn (1989). Baron and Ferejohn propose a model of an infinite-session majority-rule system of n legislators, where the individual to propose a distribution of resources in each session is chosen at random. In the simple model, amendments to proposals are disallowed, and the probability of a legislator being chosen as a proposer is $\frac{1}{n}$. The n legislators vote on a proposal, which succeeds if it receives more than $\frac{n}{2}$ votes in favor. To obtain a unique equilibrium, the Baron-Ferejohn model relevant to the present study assumes stationarity. Stationarity requires that each player's continuation value remain the same across all subgames. Thus, when faced with the same subgame, each player will always choose the same strategy. The previous history

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of the game will not affect player's strategies, and strategic play across subgames is not possible.

Where the number of legislators n is odd, Baron-Ferejohn predicts an equilibrium outcome with the formation of a coalition of $\frac{n-1}{2}$ legislators, not including the proposer, in favor of the first proposal. This passable proposal grants a fractional share of $\frac{\delta}{n}$ of the resource to be divided to each of the non-proposer members of the coalition and the remaining $1-\frac{\delta(n-1)}{2n}$ to the proposer, where δ is the discount factor for waiting to the next session for a winning proposal. $\frac{\delta}{n}$ is the continuation payoff for a non-proposing player, which is his or her expected payoff should the proposal fail and the next session begin. A limitation of the Baron-Ferejohn model is its assumption that every player's utility corresponds directly to the share of the resource that he or she receives, i.e. players are assumed purely self-interested and risk-neutral.

The present study attempts in part to develop a model that can explain the outcomes of concerns for the absolute well-being of one's partisans. To this end, we solve a variant of the Baron-Ferejohn model, limited to three players and modified by the assumption of linear utility functions reflecting partisan preferences. The utility functions employed to represent partisanship allow for varying degrees of interest for the self versus interest for one's partisan, though they presume zero concern for payoff to players outside of the partisan alliance. For illustrative and comparative purposes, the model of unanimous bargaining with these preferences is examined along the way, and the results of Baron-Ferejohn's model are considered.

Additional results of interest pertain to the relationship between intentions and outcomes in complex systems. Several previous studies that have diverged from modelling participants in the majority-rule bargaining process as purely interested in a particularistic share of the resource indicate that the equilibrium effects of these variations in preference can be counter-intuitive. Montero (2007) shows that players with a certain class of inequity-averse utility functions produce greater inequity of resource shares in stationary majority-rules bargaining equilibrium than the purely self-interested actors of Baron-Ferejohn. Under the common inequity-averse utility function of Fehr and Schmidt (1999), a player's inequity-aversion equally or more severely affects his utility when the inequity arises from his share of the resource being less than others' shares. When applied to the Baron-Ferejohn model, inequity aversion of this sort acts similarly to risk aversion by increasing the incentives to accept an offered proposal immediately for fear of being excluded from a future winning proposal. The formateur can leverage this aversion in order to offer lower shares to his coalition partners, and his power to do so increases with the degree of inequity aversion. Therefore, the equilibrium effect of players who are more inequity-averse is greater inequity.

Volden and Wiseman (2007) demonstrate that for certain valuations of particularistic and collective goods, when valuation by each legislator of the collective good increases, a proposal formateur may extract an increased particularistic share for himself with a corresponding decrease in the share of the resource devoted to the collective good. These results are extremely counter-intuitive as we would expect an increased aversion to inequity by all bargaining participants to produce greater equality in shares of the resource and an increased valuation of a collective good by all participants to produce more of that collective good. The present study will answer also whether the equilibrium outcomes of partisan preferences divide resources among partisans in a manner that conforms with a presumed intent of a partisan alliance to improve the material welfare of participants. We find certain degrees of partisanship do produce worse material outcomes in equilibrium for some partisans than would exist if all legislators were purely self-interested.

Additionally, by introducing models of partisanship in both the unanimity and majority-rule games, we are able to compare equilibrium outcomes of these bargaining structures. Particularly, we can make intrapersonal comparisons of the payoffs and *ex ante* expected payoffs of both partisan and non-partisan players. These results may be informative with regards to which of these different bargaining structures is preferable to partisan and non-partisan players, respectively. We produce the intuitive result that majority parties prefer a lower required number of votes for passage and minority parties or non-partisans prefer a higher requirement. However, we show that this result holds even when the majority party legislators expect to have a chance of being excluded from the winning proposal and when the minority party expects to have a chance of being the formateur, by virtue of the random proposer rule.

2 The Model

For the three players, we assume linear utility functions reflecting partisan preferences of the form

$$u_1(x_1, x_2, x_3) = x_1 + \beta x_2$$

$$u_2(x_1, x_2, x_3) = \beta x_1 + x_2$$

$$u_3(x_1, x_2, x_3) = x_3$$

where x_1 is the payoff as a fractional share of the resources to be divided to the first partisan player, x_2 is the payoff to the other partisan, and x_3 is the payoff to the player outside of the partisan relationship. $\beta \in (0, 1)$ relates a player's preference for payoff to his or her partisan as weighted against his preference for his or her own payoff.

The proposer seeks to maximize his utility, given the constraint on resources such that the proposed distribution does not exceed the total resources available, or

$$x_1 + x_2 + x_3 = 1$$

The proposer is similarly constrained by non-negativity constraints, which prevent any person receiving a negative payoff.

$$x_1 \geq 0$$

$$x_2 \geq 0$$

$$x_3 \geq 0$$

3 Unanimous Bargaining

A proposer, or formateur, is selected randomly from among the players with probability $\frac{1}{3}$. In unanimous bargaining, we require that all three players accept a proposed resource division of the form (x_1, x_2, x_3) for the proposal to pass. As the proposer supports his own

proposal, he must offer a continuation payoff to the other two players such that the utility of each player supporting the bargain meets or exceeds the expected present value of his or her future utility should the proposal fail. (As the utility functions are linear, we can directly discount them rather than discounting the resource inputs without meaningful difference.) The common discount factor is $\delta \in [0, 1]$, and each player has a $\frac{1}{3}$ probability of being the proposer in the next round should the proposal fail. The horizon is infinite with rounds of bargaining continuing until a proposal is passed.
Each player upon becoming the formateur faces a unique subgame described below. However, before outlining those, we should define further notation.

x_i^j : the payoff x_i to player i when player j is the formateur

3.1 Player 1's Formateur Subgame

$$\max u_1 = x_1^1 + \beta x_2^1$$

$$\text{subject to } x_1^1 + x_2^1 + x_3^1 = 1$$

$$u_2 = \beta x_1^1 + x_2^1 \geq \frac{\delta}{3}[\left(\beta x_1^1 + x_2^1\right) + \left(\beta x_1^2 + x_2^2\right) + \left(\beta x_1^3 + x_2^3\right)]$$

$$u_3 = x_3^1 \geq \frac{\delta}{3}[x_3^1 + x_3^2 + x_3^3]$$

As the formateur has greater gain from allotting more of the resources to himself than to others, he will not offer more than is necessary to pass the proposal in continuation payoffs.

3.2 Player 2's Formateur Subgame

$$\max u_2 = \beta x_1^2 + x_2^2$$

$$\text{subject to } x_1^2 + x_2^2 + x_3^2 = 1$$

$$u_1 = x_1^2 + \beta x_2^2 \geq \frac{\delta}{3}[\left(x_1^1 + \beta x_2^1\right) + \left(x_1^2 + \beta x_2^2\right) + \left(x_1^3 + \beta x_2^3\right)]$$

$$u_3 = x_3^2 \geq \frac{\delta}{3}[x_3^1 + x_3^2 + x_3^3]$$

Notably, this problem is symmetric to player 1's subgame.

3.3 Player 3's Formateur Subgame

$$\max u_3 = x_3^3$$

$$\text{subject to } x_1^3 + x_2^3 + x_3^3 = 1$$

$$u_1 = x_1^3 + \beta x_2^3 \geq \frac{\delta}{3}[\left(x_1^1 + \beta x_2^1\right) + \left(x_1^2 + \beta x_2^2\right) + \left(x_1^3 + \beta x_2^3\right)]$$

$$u_2 = \beta x_1^3 + x_2^3 \geq \frac{\delta}{3}[\left(\beta x_1^1 + x_2^1\right) + \left(\beta x_1^2 + x_2^2\right) + \left(\beta x_1^3 + x_2^3\right)]$$

3.4 Stationary Equilibria in Unanimous Bargaining

We then obtain the stationary equilibrium payoffs given a certain player is chosen as the initial formateur.

Proposition 3.1 *For $\delta \in [0, \frac{3\beta}{1+2\beta})$, a set of pure strategies is a stationary subgame perfect equilibrium when player 1 is the formateur if and only if it has the following form: (1) player 1 proposes to receive $1-\frac{\delta}{3}$, offers 0 to player 2, and offers $\frac{\delta}{3}$ to player 3; (2) player 2 votes for any proposal in which he receives $x_2 \geq \frac{\delta}{3}(1+\beta) - \beta x_1$; (3) player 3 votes for any proposal in which he receives at least $\frac{\delta}{3}$. The first proposal receives a unanimous vote, so it passes.*

Proposition 3.2 *For $\delta \in [\frac{3\beta}{1+2\beta}, 1]$, a set of pure strategies is a stationary subgame perfect equilibrium when player 1 is the formateur if and only if it has the following form: (1) player 1 proposes to receive $\frac{3-2\delta-\beta\delta}{3(1-\beta)}$, offers $\frac{\delta-3\beta+2\beta\delta}{3(1-\beta)}$ to player 2, and offers $\frac{\delta}{3}$ to player 3; (2) player 2 votes for any proposal in which he receives $x_2 \geq \frac{\delta}{3}(1+\beta) - \beta x_1$; (3) player 3 votes for any proposal in which he receives at least $\frac{\delta}{3}$. The first proposal receives a unanimous vote, so it passes.*

Proposition 3.3 *For $\delta \in [0, 1]$, a set of pure strategies is a stationary subgame perfect equilibrium when player 3 is the formateur if and only if it has the following form: (1) player 3 proposes to receive $1-\frac{2\delta}{3}$ and offers $\frac{\delta}{3}$ each to players 1 and 2; (2) player 1 votes for any proposal in which he receives $x_1 \geq \frac{\delta}{3}(1+\beta) - \beta x_2$; (3) player 2 votes for any proposal in which he receives $x_2 \geq \frac{\delta}{3}(1+\beta) - \beta x_1$. The first proposal receives a unanimous vote, so it passes.*

As demonstrated in appendix A, the cases of player 1 and 2 as formateur are symmetric. For any formateur, the formateur's share of the resource decreases and the other players' shares increase with increases in δ. Patient players have greater bargaining power against the formateur as they perceive a less severe value decline from delaying their payoffs. With increases in β, the partisan formateur's share increases and the partisan partner's share decreases. Proposition 3.1 shows that the partisan partner's share can decrease even to zero for sufficiently-high values of β in relation to δ as the range of δ for that proposition increases in β. We can potentially explain this as a result of the shared pride of party success diminishing the perceived need for a material share to oneself.

Note that, for high partisan preference or a low discount factor, player 3 will receive a positive offer from a partisan formateur while the formateur's fellow partisan will receive an offer of zero. This could be explained as strongly-committed partisans, particularly those with high time preference, having diminished bargaining power amongst their own party members.

The equilibrium passing proposal for the same unanimity game with self-interested players is $\left(1-\frac{2\delta}{3}, \frac{\delta}{3}, \frac{\delta}{3}\right)$. Regardless of β, for $\delta \in [0, 1]$, $x_1^1 \geq \frac{3-2\delta}{3}$ and $x_2^1 \leq \frac{\delta}{3}$. Thus, the partisan formateur has a higher equilibrium share than in the unanimous game among self-interested players and the formateur's partner has a lower equilibrium share. (See appendix D.) The non-partisan player 3 receives the same share of $\frac{\delta}{3}$ as in the self-interested unanimous game. Perhaps unsurprisingly, the self-interested player 3 as the formateur replicates the equilibrium outcome of bargaining among three self-interested players. An innate partisan alliance of any strength between player 1 and player 2 does not give them additional bargaining power against player 3 under the unanimity requirement. Here, partisan preferences merely redistribute the resource to the partisan with proposal control from the partisan without it.

As β approaches 0, the payoffs when a partisan is selected as formateur converge to the payoffs predicted for a similar trilateral bargaining process among purely self-interested players. Indeed, setting $\beta = 0$ removes the partisan preference and replicates those preferences in the player's utility functions.

4 Majority Rule Bargaining

We now move to a case similar to that of Baron-Ferejohn, where a majority voting rule is sufficient to pass a proposal. Thus, the selected formateur need only incentivize one other player to support the proposal and form a minimum winning coalition. Therefore, he will offer the continuation payoff to one player and zero to the other. We again need to define some new notation.

$x_i^{3,k}$: the payoff x_i to player i when player 3 is the formateur and offers continuation payoff to player $k = 1, 2$

That player 3 chooses randomly (i.e. with probability $\frac{1}{2}$) between offering continuation payoff to player 1 and player 2 is somewhat intuitive but is shown in appendix C. As a result, player 3 has two formateur subgames that are symmetric.

4.1 Player 1's Formateur Subgame

$$\max\ u_1 = x_1^1 + \beta x_2^1$$

$$\text{subject to } x_1^1 + x_2^1 + x_3^1 = 1$$

and

$$u_2 = \beta x_1^1 + x_2^1 \geq \frac{\delta}{3}[\left(\beta x_1^1 + x_2^1\right) + \left(\beta x_1^2 + x_2^2\right) + \frac{1}{2}\left(\beta x_1^{3,1} + x_2^{3,1}\right) + \frac{1}{2}\left(\beta x_1^{3,2} + x_2^{3,2}\right)]$$

or

$$u_3 = x_3^1 \geq \frac{\delta}{3}[x_3^1 + x_3^2 + \frac{1}{2}x_3^{3,1} + \frac{1}{2}x_3^{3,2}]$$

Differing from the unanimous case, player 1 will only propose positive payoff to one other player to form a minimum winning coalition. As player 2 is player 1's partisan, player 1 will prefer to offer continuation payoff to player 2 over player 3 for all $\beta \in (0, 1)$ (as shown in appendix E). Thus, we can substitute a constraint setting player 3's payoff to zero in place of the last constraint, $x_3^1 = 0$.

4.2 Player 2's Formateur Subgame

$$\max u_2 = \beta x_1^2 + x_2^2$$
$$\text{subject to } x_1^2 + x_2^2 + x_3^2 = 1$$

and

$$u_1 = x_1^2 + \beta x_2^2 \geq \frac{\delta}{3}[\left(x_1^1 + \beta x_2^1\right) + \left(x_1^2 + \beta x_2^2\right) + \frac{1}{2}\left(x_1^{3,1} + \beta x_2^{3,1}\right) + \frac{1}{2}\left(x_1^{3,2} + \beta x_2^{3,2}\right)]$$

or

$$u_3 = x_3^2 \geq \frac{\delta}{3}[x_3^1 + x_3^2 + \frac{1}{2}x_3^{3,1} + \frac{1}{2}x_3^{3,2}]$$

As in the unanimity case, this problem is symmetric to player 1's subgame. Thus, we can similarly substitute a constraint setting player 3's payoff to zero in place of the last inequality, x_3^2.

4.3 Player 3's Formateur Subgame 1

In this subgame, which occurs with probability $\frac{1}{2}$ given the player 3 is selected as the formateur, player 3 offers the continuation payoff to player 1 and a zero share to player 2.

$$\max u_3 = x_3^{3,1}$$
$$\text{subject to } x_1^{3,1} + x_2^{3,1} + x_3^{3,1} = 1$$
$$u_1 = x_1^{3,1} + \beta x_2^{3,1} \geq \frac{\delta}{3}[\left(x_1^1 + \beta x_2^1\right) + \left(x_1^2 + \beta x_2^2\right) + \frac{1}{2}\left(x_1^{3,1} + \beta x_2^{3,1}\right) + \frac{1}{2}\left(x_1^{3,2} + \beta x_2^{3,2}\right)]$$
$$x_2^{3,1} = 0$$

4.4 Player 3's Formateur Subgame 2

In this subgame, which is symmetric to the one above and occurs with probability $\frac{1}{2}$ given the player 3 is selected as the formateur, player 3 offers the continuation payoff to player 2 and a zero share to player 1.

$$\max u_3 = x_3^{3,2}$$
$$\text{subject to } x_1^{3,2} + x_2^{3,2} + x_3^{3,2} = 1$$
$$x_1^{3,2} = 0$$
$$u_2 = \beta x_1^{3,2} + x_2^{3,2} \geq \frac{\delta}{3}[\left(\beta x_1^1 + x_2^1\right) + \left(\beta x_1^2 + x_2^2\right) + \frac{1}{2}\left(\beta x_1^{3,1} + x_2^{3,1}\right) + \frac{1}{2}\left(\beta x_1^{3,2} + x_2^{3,2}\right)]$$

4.5 Stationary Equilibria in Majority Rule Bargaining

We then obtain the stationary equilibrium payoffs given a certain player is chosen as the initial formateur.

Proposition 4.1 *For $\delta \in [0, \frac{6\beta}{(1+\beta)(2+\beta)})$, a set of pure strategies is a stationary subgame perfect equilibrium when player 1 is the formateur if and only if it has the following form: (1) player 1 proposes to receive 1, offers 0 to player 2, and offers 0 to player 3; (2) player 2 votes for any proposal in which he receives $x_2 \geq \frac{2\delta(1+\beta)}{6-\delta-\beta\delta} - \beta x_1$; (3) player 3 votes for any proposal in which he receives at least $\frac{2-\delta-\beta\delta}{6-\delta-\beta\delta}$. The first proposal receives a majority vote by players 1 and 2, so it passes.*

Proposition 4.2 *For $\delta \in [\frac{6\beta}{(1+\beta)(2+\beta)}, 1]$, a set of pure strategies is a stationary subgame perfect equilibrium when player 1 is the formateur if and only if it has the following form:(1) player 1 proposes to receive $\frac{6-3\delta-3\beta\delta}{(6-\delta-\beta\delta)(1-\beta)}$, offers $\frac{2\delta+3\beta\delta+\beta^2\delta-6\beta}{(6-\delta-\beta\delta)(1-\beta)}$ to player 2, and offers 0 to player 3; (2) player 2 votes for any proposal in which he receives $x_2 \geq \frac{2\delta(1+\beta)}{6-\delta-\beta\delta} - \beta x_1$; (3) player 3 votes for any proposal in which he receives at least $\frac{2-\delta-\beta\delta}{6-\delta-\beta\delta}$. The first proposal receives a majority vote by players 1 and 2, so it passes.*

Proposition 4.3 *For $\delta \in [0, 1]$, a set of pure strategies is a stationary subgame perfect equilibrium when player 3 is the formateur if and only if it has the following form: (1) player 3 proposes to receive $\frac{6-3\delta-3\beta\delta}{6-\delta-\beta\delta}$, offers $\frac{2\delta(1+\beta)}{6-\delta-\beta\delta}$ to either player 1 or 2 at random, and offers 0 to the other player; (2) player 1 votes for any proposal in which he receives $x_1 \geq \frac{2\delta(1+\beta)}{6-\delta-\beta\delta} - \beta x_2$; (3) player 2 votes for any proposal in which he receives $x_2 \geq \frac{2\delta(1+\beta)}{6-\delta-\beta\delta} - \beta x_1$. The first proposal receives a majority vote by player 3 and whomever is offered positive payoff, so it passes.*

The cases of player 1 and 2 as formateur are symmetric. Likewise, the two subcases of player 3 as formateur are symmetric. For all formateurs, the formateur's share decreases and the coalition partner's share increases with increases in δ. As in the unanimity case, patient players have greater bargaining power against the formateur. With increases in β, the partisan formateur's share increases and the partisan coalition partner's share decreases. We can again explain this as a result of the shared pride of party success diminishing the perceived need for a material share to oneself.

In the extreme case of Proposition 4.1, the partisan formateur is able to obtain all of the resource in equilibrium. Again, strongly-committed partisans, particularly those with high time preference, have diminished bargaining power amongst their own party. However, the threshold $\delta < \frac{6\beta}{(1+\beta)(2+\beta)}$ for this extreme case is higher for any β than the threshold in unanimous bargaining of $\delta < \frac{3\beta}{2+\beta}$. That is, the non-proposing partisan's bargaining power decreases to zero more rapidly with δ under majority rule than unanimity. This feature could be explained by the threat of the partisan formateur forming a minimum winning coalition with the nonpartisan player and the partisan coalition partner's fear of being left out of a future bargain led by player 3 as formateur should the current proposal fail.

When player 3 is the formateur, the continuation payoff he offers to player 1 or 2 increases and player 3's payoff to himself decreases with increases in β. Unlike in the unanimity game, stronger partisan preference enables the partisans to extract higher continuation payoffs even when the nonpartisan player is the formateur. Under majority rule, partisan preference also of course denies payoff to a non-partisan minority when a partisan player is the formateur.

As shown by Baron and Ferejohn, the equilibrium passing proposal for the majority-rule game with self-interested players is $\left(1-\frac{\delta}{3}, \frac{\delta}{3}, 0\right)$, where the player to receive his continuation payoff is chosen randomly by the formateur. For "very low" ratios of β to δ, $x_1^1 \leq 1-\frac{\delta}{3}$ and $x_2^1 \geq \frac{\delta}{3}$. However, for larger ratios, $x_1^1 > 1-\frac{\delta}{3}$ and $x_2^1 < \frac{\delta}{3}$. That is, when partisanship is very weak and players are patient, the partisan coalition partner can extract a larger share from the partisan formateur than he could were players self-interested. However, when partisanship is moderate or strong, the partisan formateur can offer a smaller share to his partner.

As β approaches 0, the payoffs when a partisan is selected as formateur **do not** converge to the payoffs predicted for a similar bargaining process among purely self-interested players. Since any $\beta > 0$ encourages the partisan formateur to offer payoff solely to his partisan, this partisan preference has serious effects on the equilibrium outcome even for near-zero β. We can consider the limits of the shares given in Proposition 4.2 here, because small values of β ensure that δ is above the necessary threshold for that case.

For the partisan proposer's share,

$$\lim_{\beta \to 0} x_1^1 = \frac{6-3\delta}{6-\delta} \leq 1-\frac{\delta}{3}.$$

For the partisan coalition partner's share,

$$\lim_{\beta \to 0} x_2^1 = \frac{2\delta}{6-\delta} \geq \frac{\delta}{3}.$$

For the nonpartisan proposer's share,

$$\lim_{\beta \to 0} x_{3,1}^3 = \frac{6-3\delta}{6-\delta} \leq 1-\frac{\delta}{3}.$$

For the nonpartisan proposer's coalition partner's share,

$$\lim_{\beta \to 0} x_{3,1}^3 = \frac{2\delta}{6-\delta} \geq \frac{\delta}{3}.$$

A small degree of partisan preference drastically affects the equilibrium shares when a partisan is the formateur, giving the formateur's partisan a guarantee of being the coalition partner and, thus, a higher continuation payoff This increases the partisans' continuation payoffs as compared to game among the self-interested players when the nonpartisan is the formateur as well.

5 Comparison of the Unanimous and Majority Rule Equilibrium Outcomes

5.1 Shares

For the relevant range of δ and β, the partisan formateur's share in the majority-rule model is greater than or equal to that of the unanimity model. This reflects the long-established point in the multilateral bargaining literature that requiring a higher approval rate for passage of a proposal reduces the bargaining power of the proposer and increases that of each other player. The partisan coalition partner's share under majority rule is greater than under unanimity if $\delta > \frac{6\beta}{(1+\beta)(1+2\beta)}$. For impatient or strongly partisan players, the partisan formateur can obtain the share that went to player 3 in unanimity and additional concessions from his partisan coalition partner. Of course, the non-partisan player 3 receives 0 under majority rule instead of $\frac{\delta}{3}$ as in unanimity when a partisan is the formateur.

When the nonpartisan player is the formateur, he receives a smaller share under majority rule than unanimity if $\delta > \frac{3(1-\beta)}{1+\beta}$. His coalition partner always receives a greater share than one of his coalition partners under unanimity, i.e. $\frac{\delta}{3}$. For $\delta > \frac{3(1-\beta)}{1+\beta}$, the coalition partner to the nonpartisan formateur receives a share greater than $\frac{2\delta}{3}$, the sum of the shares of both partisans under unanimity. Of course, the partisan outside of the coalition receives a share of 0 instead of $\frac{\delta}{3}$ as under unanimity.

5.2 Payoffs

As throughout the rest of this paper, we continue to define payoff in terms of a player's utility rather than solely his share of the resource. Under unanimity, a partisan proposer's equilibrium payoff is $\left(\frac{3-2\delta}{3}\right)(1+\beta)$. Under majority, this payoff is $\left(\frac{6-3\delta+3\beta\delta}{6-\delta-\beta\delta}\right)(1+\beta)$. The partisan proposer's equilibrium payoff under majority rule is greater than under unanimity.

For all cases, player 3's payoff is equivalent to his share. Therefore, the relationships between player 3's payoffs in the unanimity and majority-rule models are the same as those of his shares discussed above.

5.3 Expected Shares

Since the formateur is randomly selected with probability $\frac{1}{3}$, we can find the expected share of each player under unanimous rules by multiplying that fraction by the sum of his shares for each case of a different player as proposer.

$$E[x_j^U] = \sum_{i=1}^{3} \frac{1}{3} x_j^i$$

$$E[x_j^U] = \frac{1}{3}$$

For majority rule, the additional subcases when player 3 is the formateur cause the computation to be slightly different.

$$E[x_j^M] = \frac{1}{3}x_j^1 + \frac{1}{3}x_j^2 + \frac{1}{6}x_j^{3,1} + \frac{1}{6}x_j^{3,2}$$

$$E[x_1^M] = E[x_2^M] = \frac{2}{6 - \delta - \beta\delta}$$

$$E[x_3^M] = \frac{2 - \delta - \beta\delta}{6 - \delta - \beta\delta}$$

The expected shares of partisan players are higher under majority rule than unanimity, consistent with most of the results of the above section 5.1. Likewise, the expected share of the nonpartisan player is lower under majority rule. We have discussed above that the partisan amicability between player 1 and player 2 allows them to often extract larger shares from the nonpartisan player when his consent is not required for a proposal to pass. We see here that this results in greater expected inequity in shares of the resource than under majority rule.

5.4 Expected Payoffs

We can similarly find the expected payoffs for each player. For the partisans, the expected payoff is $\frac{1}{3}(1+\beta)\left(\frac{6-\beta\delta}{6-\delta-\beta\delta}\right)$ under majority rule and $\frac{1}{3}(1+\beta)$ under unanimity. Thus, the partisans expect to be better off under majority rule and will prefer this bargaining structure *ex ante*.

Since player 3's payoff is equivalent to his share, his expected payoff equals his expected share under both unanimity and majority rule. Thus, his expected payoff under majority rule is less than under unanimity. The nonpartisan expects to be worse off under majority rule and will prefer the unanimous bargaining structure *ex ante*.

6 Conclusion

Partisan preferences have significant effects on stationary equilibrium shares under both unanimous and majority rule three-player bargaining when a partisan player is randomly chosen as the formateur. These effects are muted under unanimity when a nonpartisan is the formateur, but arise under majority rule even in that case.

Under both voting rules, partisan preferences rarely produce an equitable distribution of resources between partisans that might be intuitively expected as an end of the alliance. Rather, strong partisan preferences in combination with high impatience (i.e. a discount factor closer to 0) concentrates payoff to the proposer to a greater degree than the model with purely self-interested players. Yet, in the majority rule game, weak partisan preference with a high degree of patience does result in a more equitable distribution of the resource between the partisan formateur and his coalition partner. We may attribute this to strongly-partisan players having lower credibility of refusing a proposed payoff that benefits their partisans. Ultimately, the advantages of the partisan alliance appear to lie in greater stability in being selected as the coalition partner and the ability to obtain higher continuation payoffs from the non-partisan formateur. Thus, we can predict that a partisan facing a formateur outside of his party will be able to secure a greater share of the resource that a player who lacks partisans.

Nonpartisans or members of the minority party will prefer *ex ante* a unanimous voting rule over a majority rule in dividing a resource. Those partisans that are confident that they will be in the majority will prefer a majority voting rule. With odd n purely self-interested players, each player has a $\frac{n+1}{2}$ chance of being in a winning coalition; in the three player case, each player thus has a $\frac{2}{3}$ chance of being in a winning coalition. With a partisan connection between two players of three total, each partisan player has a $\frac{5}{6}$ chance of being in a winning coalition, while a nonpartisan has only a $\frac{1}{3}$ chance. Presumably, a similar result holds for larger legislatures. A unanimous voting rule requires that the winning coalition always contain all members of the legislature. Therefore, we may expect a stable majority party to attempt to impose a majority voting rule, and a stable minority party to support a higher requirement for passage, even when that party expects to have a chance to make the budgetary proposal and be included in the winning coalition.

The zero share of the resource that is distributed to the non-proposing partisan in the extreme cases of Propositions 3.1 and 4.1 may provide insight into the budgetary priorities of a majority party. Even if a substantial subset of party members support some allocation of the resource, the resource may instead be allocated to the purposes of the formateur, if party members are sufficiently committed to each other goals. Strong partisan preference may be conceived of as pure friendship and interest in one's partisan's particularistic share of the resource. However, we can also conceive of each partisan's share being directed toward the funding of a certain set of policies. Then, partisan preference reflects one's support for the proposed policies of one's partisan in comparison to one's own. As the strength of partisan preference increases, the diversity of majority-party policies that are funded declines and the amount of funding for the remaining policy set increases (assuming a fixed budget). Likewise, a decrease in partisan preference will result in a broader range of funded majority-party policies.

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A Proof of Propositions 3.1-3.3 (Unanimity Cases)

A.1 Proposition 3.2

From the continuation payoff inequalities across the three proposer subgames, we have the following.

$$\beta x_1^1 + x_2^1 \geq \frac{\delta}{3-\delta}[\left(\beta x_1^2 + x_2^2\right) + \left(\beta x_1^3 + x_2^3\right)] \tag{1}$$

$$x_3^1 \geq \frac{\delta}{3}[x_3^1 + x_3^2 + x_3^3] \tag{2}$$

$$x_1^2 + \beta x_2^2 \geq \frac{\delta}{3-\delta}[\left(x_1^1 + \beta x_2^1\right) + \left(x_1^3 + \beta x_2^3\right)] \tag{3}$$

$$x_3^2 \geq \frac{\delta}{3}[x_3^1 + x_3^2 + x_3^3] \tag{4}$$

$$x_1^3 + \beta x_2^3 \geq \frac{\delta}{3-\delta}[\left(x_1^1 + \beta x_2^1\right) + \left(x_1^2 + \beta x_2^2\right)] \tag{5}$$

$$\beta x_1^3 + x_2^3 \geq \frac{\delta}{3-\delta}[\left(\beta x_1^1 + x_2^1\right) + \left(\beta x_1^2 + x_2^2\right)] \tag{6}$$

Since player 1 will maximize his payoff as proposer, $x_1^1+\beta x_2^1$, he will minimize x_3^1. If constraint (2) is not tight, then the proposer will transfer some of the resource from player 3 to himself, regardless of whether constraint (1) is tight. Since the terms on the right hand side of the inequality are non-negative, constraint (2) will be a stronger constraint than non-negativity for x_3^1.

$$x_3^1 \geq \frac{\delta}{3}[x_3^1 + x_3^2 + x_3^3] \tag{2}$$

$$\Leftrightarrow x_3^1 \geq \frac{\delta}{3-\delta}[x_3^2 + x_3^3]$$

By an analogous operation on constraint (4),

$$x_3^1 \geq \frac{\delta}{3-\delta}\left[\frac{\delta}{3-\delta}\left(x_3^1 + x_3^3\right) + x_3^3\right]$$

$$\Leftrightarrow \left(1 - \frac{\delta^2}{\left(3-\delta\right)^2}\right) x_3^1 \geq \frac{\delta}{3-\delta}\left[\left(1 + \frac{\delta}{3-\delta}\right) x_3^3\right]$$

$$\Leftrightarrow \left[\frac{9-6\delta}{\left(3-\delta\right)^2}\right] x_3^1 \geq \left[\frac{3\delta}{\left(3-\delta\right)^2}\right] x_3^3$$

$$\Leftrightarrow x_3^1 \geq \frac{\delta}{3-2\delta} x_3^3.$$

By the non-negativity constraint on x_3^3,

$$x_3^1 \geq \frac{\delta}{3-2\delta} x_3^3 \geq 0.$$

Constraint (2) is tight.

$$x_3^1 = \frac{\delta}{3}[x_3^1 + x_3^2 + x_3^3] \tag{2.1}$$

Similarly, since player 2 will maximize his payoff as proposer $\beta x_1^2 + x_2^2$, he will minimize x_3^2. Since the terms on the right hand side of the inequality are all non-negative, constraint (4) will be a stronger constraint than non-negativity for x_3^2. Constraint (4) is tight.

$$x_3^2 = \frac{\delta}{3}[x_3^1 + x_3^2 + x_3^3] \tag{4.1}$$

From (2.1) and (4.1), we find player 3's equilibrium share is equal for player 1 and player 2 as proposer.

$$x_3^1 = x_3^2 \tag{7}$$

Substituting this equation into (2.1), we obtain

$$\begin{aligned} x_3^1 &= \frac{\delta}{3}[x_3^1 + x_3^1 + x_3^3] \\ \Leftrightarrow \left(1 - \frac{2\delta}{3}\right) x_3^1 &= \frac{\delta}{3} x_3^3 \\ \Leftrightarrow \left(\frac{3 - 2\delta}{3}\right) x_3^1 &= \frac{\delta}{3} x_3^3 \\ \Leftrightarrow x_3^1 &= \left(\frac{\delta}{3 - 2\delta}\right) x_3^3. \end{aligned}$$

By (7),

$$x_3^1 = x_3^2 = \left(\frac{\delta}{3 - 2\delta}\right) x_3^3 \tag{8}$$

Since player 3 will maximize his payoff as proposer, x_3^3, he will minimize both x_1^3 and x_2^3. Constraints (5) and (6) are tight.

$$x_1^3 + \beta x_2^3 = \frac{\delta}{3 - \delta}[\left(x_1^1 + \beta x_2^1\right) + \left(x_1^2 + \beta x_2^2\right)] \tag{5.1}$$

$$\beta x_1^3 + x_2^3 = \frac{\delta}{3 - \delta}[\left(\beta x_1^1 + x_2^1\right) + \left(\beta x_1^2 + x_2^2\right)] \tag{6.1}$$

Summing equations (5.1) and (6.1), we obtain

$$(1 + \beta)\left(x_1^3 + x_2^3\right) = \frac{\delta}{3 - \delta}(1 + \beta)\,[x_1^1 + x_2^1 + x_1^2 + x_2^2].$$

Dividing by $(1+\beta)$ and substituting the resource constraints, we find

$$1 - x_3^3 = \frac{\delta}{3-\delta}[1 - x_3^1 + 1 - x_3^2].$$

From (8), we obtain

$$\begin{aligned}
1 - x_3^3 &= \frac{2\delta}{3-\delta}\left[1 - \left(\frac{\delta}{3-2\delta}\right)x_3^3\right] \\
\Leftrightarrow (3-\delta)\left(1 - x_3^3\right) &= 2\delta\left[1 - \left(\frac{\delta}{3-2\delta}\right)x_3^3\right] \\
\Leftrightarrow \left[\delta - 3 + \left(\frac{2\delta^2}{3-2\delta}\right)\right]x_3^3 &= 2\delta - 3 + \delta \\
\Leftrightarrow \left(\frac{3\delta - 2\delta^2 - 9 + 6\delta + 2\delta^2}{3-2\delta}\right)x_3^3 &= 2\delta - 3 + \delta \\
\Leftrightarrow \left(\frac{9\delta - 9}{3-2\delta}\right)x_3^3 &= 3\delta - 3 \\
\Leftrightarrow \left[\frac{9(\delta-1)}{3-2\delta}\right]x_3^3 &= 3(\delta - 1) \\
\Leftrightarrow \left(\frac{9}{3-2\delta}\right)x_3^3 &= 3 \\
\Leftrightarrow x_3^3 &= \frac{3-2\delta}{3}. \qquad \text{(P3.3)}
\end{aligned}$$

Thus, player 3's equilibrium share as proposer is $1 - \frac{2\delta}{3}$. By (8),

$$\begin{aligned}
x_3^1 = x_3^2 &= \left(\frac{\delta}{3-2\delta}\right)\left(\frac{3-2\delta}{3}\right) \\
\Leftrightarrow x_3^1 = x_3^2 &= \frac{\delta}{3}. \qquad \text{(P1.3)}
\end{aligned}$$

Player 3's equilibrium share when player 1 or player 2 is the proposer is $\frac{\delta}{3}$.
Since player 1 will maximize his payoff as proposer , $x_1^1 + \beta x_2^1$, he will minimize x_2^1 in favor of x_1^1. From constraint (1),

$$x_2^1 \geq \frac{\delta}{3-\delta}[\left(\beta x_1^2 + x_2^2\right) + \left(\beta x_1^3 + x_2^3\right)] - \beta x_1^1.$$

With the restriction $\delta \in [\frac{3\beta}{1+2\beta}, 1]$, we do not risk violating the non-negativity constraint by minimizing x_2^1 as far as possible in the continuation inequality. This inequality is tight.

$$x_2^1 = \frac{\delta}{3-\delta}[\left(\beta x_1^2 + x_2^2\right) + \left(\beta x_1^3 + x_2^3\right)] - \beta x_1^1$$

Constraint (1) is tight.

$$\beta x_1^1 + x_2^1 = \frac{\delta}{3-\delta}[(\beta x_1^2 + x_2^2) + (\beta x_1^3 + x_2^3)] \tag{1.1}$$

Similarly, since player 2 will maximize his payoff as proposer , $\beta x_1^2 + x_2^2$, he will minimize x_1^2 in favor of x_2^2. From constraint (3),

$$x_1^2 \geq \frac{\delta}{3-\delta}[(x_1^1 + \beta x_2^1) + (x_1^3 + \beta x_2^3)] - \beta x_2^2.$$

This constraint is tight again with the restriction $\delta \in [\frac{3\beta}{1+2\beta}, 1]$.

$$x_1^2 = \frac{\delta}{3-\delta}[(x_1^1 + \beta x_2^1) + (x_1^3 + \beta x_2^3)] - \beta x_2^2$$

Constraint (3) is tight.

$$x_1^2 + \beta x_2^2 = \frac{\delta}{3-\delta}[(x_1^1 + \beta x_2^1) + (x_1^3 + \beta x_2^3)] \tag{3.1}$$

Substituting (6.1) into the right side of equation (1.1), we find

$$\begin{aligned}
\beta x_1^1 + x_2^1 &= \frac{\delta}{3-\delta}\left[(\beta x_1^2 + x_2^2) + \frac{\delta}{3-\delta}[(\beta x_1^1 + x_2^1) + (\beta x_1^2 + x_2^2)]\right] \\
\Leftrightarrow \left(1 - \frac{\delta^2}{(3-\delta)^2}\right)(\beta x_1^1 + x_2^1) &= \frac{\delta}{3-\delta}\left(1 + \frac{\delta}{3-\delta}\right)(\beta x_1^2 + x_2^2) \\
\Leftrightarrow \left(\frac{9 - 6\delta + \delta^2 - \delta^2}{(3-\delta)^2}\right)(\beta x_1^1 + x_2^1) &= \frac{\delta}{3-\delta}\left(\frac{3-\delta+\delta}{3-\delta}\right)(\beta x_1^2 + x_2^2) \\
\Leftrightarrow \left(\frac{9-6\delta}{(3-\delta)^2}\right)(\beta x_1^1 + x_2^1) &= \frac{\delta}{3-\delta}\left(\frac{3}{3-\delta}\right)(\beta x_1^2 + x_2^2) \\
\Leftrightarrow \left[\frac{3(3-2\delta)}{(3-\delta)^2}\right](\beta x_1^1 + x_2^1) &= \frac{3\delta}{(3-\delta)^2}(\beta x_1^2 + x_2^2) \\
\Leftrightarrow \beta x_1^1 + x_2^1 &= \frac{\delta}{3-2\delta}(\beta x_1^2 + x_2^2) \\
\Leftrightarrow \beta x_1^2 + x_2^2 &= \frac{3-2\delta}{\delta}(\beta x_1^1 + x_2^1). \qquad (9)
\end{aligned}$$

Similarly, substituting (5.1) into the right side of equation (3.1), we find

$$\begin{aligned}
x_1^2 + \beta x_2^2 &= \frac{\delta}{3-\delta}\left[(x_1^1 + \beta x_2^1) + \frac{\delta}{3-\delta}[(x_1^1 + \beta x_2^1) + (x_1^2 + \beta x_2^2)]\right] \\
\Leftrightarrow x_1^2 + \beta x_2^2 &= \frac{\delta}{3-2\delta}(x_1^1 + \beta x_2^1). \qquad (10)
\end{aligned}$$

Summing (9) and (10), we obtain

$$(1+\beta)\left(x_1^2+x_2^2\right)=\left(\frac{3-2\delta}{\delta}\right)\left(\beta x_1^1+x_2^1\right)+\left(\frac{\delta}{3-2\delta}\right)\left(x_1^1+\beta x_2^1\right)$$

$$\Leftrightarrow (3-2\delta)(\delta)(1+\beta)\left(x_1^2+x_2^2\right)=(3-2\delta)^2\left(\beta x_1^1+x_2^1\right)+\left(\delta^2\right)\left(x_1^1+\beta x_2^1\right).$$

By (7),

$$x_3^1=x_3^2 \tag{7}$$

$$\Leftrightarrow 1-x_1^1-x_2^1=1-x_1^2-x_2^2$$

$$\Leftrightarrow x_1^1+x_2^1=x_1^2+x_2^2. \tag{7.1}$$

Substituting (7.1) into the above, we find

$$(3-2\delta)(\delta)(1+\beta)\left(x_1^1+x_2^1\right)=(3-2\delta)^2\left(\beta x_1^1+x_2^1\right)+\left(\delta^2\right)\left(x_1^1+\beta x_2^1\right)$$

$$\Leftrightarrow \left(\delta-\delta^2-3\beta+5\beta\delta-2\beta\delta^2\right)x_1^1=\left(3-5\delta+2\delta^2-\beta\delta+\beta\delta^2\right)x_2^1$$

$$\Leftrightarrow \left(\frac{\delta-\delta^2-3\beta+5\beta\delta-2\beta\delta^2}{3-5\delta+2\delta^2-\beta\delta+\beta\delta^2}\right)x_1^1=x_2^1. \tag{11}$$

By (7) and (P1.3),

$$1-x_1^1-x_2^1=\frac{\delta}{3}$$

$$\Leftrightarrow x_1^1+x_2^1=1-\frac{\delta}{3}.$$

By (11),

$$\Leftrightarrow x_1^1+\left(\frac{\delta-\delta^2-3\beta+5\beta\delta-2\beta\delta^2}{3-5\delta+2\delta^2-\beta\delta+\beta\delta^2}\right)x_1^1=\frac{3-\delta}{3}$$

$$\Leftrightarrow \left(\frac{3-4\delta+\delta^2-3\beta+4\beta\delta-\beta\delta^2}{3-5\delta+2\delta^2-\beta\delta+\beta\delta^2}\right)x_1^1=\frac{3-\delta}{3}$$

$$\Leftrightarrow \left[\frac{(1-\beta)(3-\delta)}{3-\beta\delta-2\delta}\right]x_1^1=\frac{3-\delta}{3}$$

$$\Leftrightarrow x_1^1=\frac{3-\beta\delta-2\delta}{3(1-\beta)}. \tag{P1.1}$$

Player 1's equilibrium share as proposer is $\frac{3-\beta\delta-2\delta}{3(1-\beta)}$.

By (7) and (P1.1),

$$\left[\frac{3-\beta\delta-2\delta}{3(1-\beta)}\right]+x_2^1=\frac{3-\delta}{3}$$

$$\Leftrightarrow x_2^1=\frac{(3-\delta)(1-\beta)-3+\beta\delta+2\delta}{3(1-\beta)}$$

$$\Leftrightarrow x_2^1=\frac{\delta-3\beta+2\beta\delta}{3(1-\beta)}. \tag{P1.2}$$

Player 2's equilibrium share with player 1 as proposer is $\frac{\delta-3\beta+2\beta\delta}{3(1-\beta)}$.

By a parallel argument from (9) and (10), we find that in equilibrium $x_2^2 = \frac{3-\beta\delta-2\delta}{3(1-\beta)}$ and $x_1^2 = \frac{\delta-3\beta+2\beta\delta}{3(1-\beta)}$. This agrees with the immediately apparent symmetry of the partisan proposers' subgames.

Player 1's equilibrium share as proposer is $\frac{3-\beta\delta-2\delta}{3(1-\beta)}$, player 2's equilibrium share with player 1 as proposer is $\frac{\delta-3\beta+2\beta\delta}{3(1-\beta)}$, and player 3's equilibrium share with player 1 as proposer is $\frac{\delta}{3}$. Therefore, since we assumed above player 1's payoff maximization, player 1 will offer these shares. Since we derived these shares from the continuation inequalities, the shares satisfy those constraints, and the other players will accept the proposal. The case for player 2 as proposer is symmetrical. This completes the proof of Proposition 3.2.

A.2 Proposition 3.1

Note that we can derive the equilibrium continuation payoff for a partisan as follows.

$$u_p = \beta x_1^1 + x_2^1 = \beta x_2^2 + x_1^2$$

$$\Leftrightarrow u_p = \beta\left[\frac{3-\beta\delta-2\delta}{3(1-\beta)}\right] + \left[\frac{\delta-3\beta+2\beta\delta}{3(1-\beta)}\right]$$

$$\Leftrightarrow u_p = \frac{\delta-\beta^2\delta}{3(1-\beta)}$$

$$\Leftrightarrow u_p = \frac{\delta(1-\beta^2)}{3(1-\beta)}$$

$$\Leftrightarrow u_p = \frac{\delta}{3}(1+\beta)$$

Thus, a player i with partisan j will accept any offer where $x_i + \beta x_j \geq \frac{\delta}{3}(1+\beta)$.

We can also derive the equilibrium continuation payoff (and share) for player 3.

$$u_p = x_3^1 = x_3^2$$

$$\Leftrightarrow u_p = \frac{\delta}{3}$$

Suppose $x_1^1 = 1 - \frac{\delta}{3}$, i.e. the proposer player 1 takes all of the remainder and offers 0 to player 2. Then, the following continuation inequality must hold for player 2.

$$0 + \beta\left(1-\frac{\delta}{3}\right) \geq \frac{\delta}{3}(1+\beta)$$

$$\beta\left(1-\frac{\delta}{3}\right) \geq \frac{\delta}{3}(1+\beta)$$
$$\beta \geq \frac{\delta}{3}(1+2\beta)$$
$$\delta \leq \frac{3\beta}{1+2\beta}$$

Therefore, for $\delta \in \left[0, \frac{3\beta}{1+2\beta}\right)$, the utility value $\beta\left(1-\frac{\delta}{3}\right)$ of the proposer's partisan exceeds his continuation utility when he is offered 0 of the resource. Thus, an offer by a partisan proposer to keep $\frac{3-\delta}{3}$, give 0 to his partisan, and give $\frac{\delta}{3}$ to player 3 is a stationary subgame perfect strategy for $\delta \in \left[0, \frac{3\beta}{1+2\beta}\right)$. The proposer aims to maximize his utility, $x_p + \beta\left(1-\frac{\delta}{3}-x_p\right)$, which he can accomplish by maximizing his payoff x_p as $\beta < 1$. Since $1-\frac{\delta}{3}$ is the highest possible payoff for the proposer compatible with player 3 receiving his continuation payoff without violating the resource and non-negativity constraints, this is the only stationary subgame perfect outcome for the partisan proposer with $\delta \in \left[0, \frac{3\beta}{1+2\beta}\right)$. This completes the proof of Proposition 3.1.

A.3 Proposition 3.3

Player 1's continuation payoff inequality when player 3 is the proposer is:

$$x_1^3 + \beta x_2^3 \geq u_p$$
$$\Leftrightarrow x_1^3 + \beta x_2^3 \geq \frac{\delta}{3}(1+\beta).$$

Since player 3 as proposer will minimize both x_1^3 and x_2^3 to maximize his payoff, this constraint is tight.

$$x_1^3 + \beta x_2^3 = \frac{\delta}{3}(1+\beta) \tag{5.2}$$

Similarly, from player 2's continuation payoff inequality, we obtain

$$\beta x_1^3 + x_2^3 = \frac{\delta}{3}(1+\beta) \tag{6.2}$$

Equations (5.2) and (6.2) give

$$x_1^3 + \beta x_2^3 = \beta x_1^3 + x_2^3$$
$$\Leftrightarrow (1-\beta)\, x_1^3 = (1-\beta)\, x_2^3$$
$$\Leftrightarrow x_1^3 = x_2^3. \tag{12}$$

By (12), (P3.3) and the resource constraint, we find

$$x_1^3 + x_2^3 + x_3^3 = 1$$

$$\Leftrightarrow 2x_1^3 + \left(1 - \frac{2\delta}{3}\right) = 1$$

$$\Leftrightarrow x_1^3 = \frac{\delta}{3}. \tag{P3.1}$$

By (12),

$$x_2^3 = \frac{\delta}{3} \tag{P3.2}$$

The equilibrium shares to the partisans when player 3 is proposer are $\frac{\delta}{3}$ each. Player 3's equilibrium share as proposer is $1 - \frac{2\delta}{3}$. Since we assumed player 3's payoff maximization, player 3 will offer these shares. Since we derived these shares from the continuation inequalities, the shares satisfy those constraints, and the other players will accept the proposal. This completes the proof of Proposition 3.3.

B Proofs of Propositions 4.1-4.3

As shown in appendix C, $x_1^{3,1} = x_2^{3,2}$ and $x_1^2 + \beta x_2^2 = \beta x_1^1 + x_2^1$. Applying the resource constraints to the first equation, we have

$$1 - x_3^{3,1} = 1 - x_3^{3,2}$$

$$\Leftrightarrow x_3^{3,1} = x_3^{3,2}.$$

Thus, player 3's two proposer subgames are symmetric. Applying the resource constraints to the latter equation, we have

$$(1 - x_2^2) + \beta x_2^2 = \beta x_1^1 + (1 - x_1^1)$$

$$\Leftrightarrow 1 + (\beta - 1)x_2^2 = (\beta - 1)x_1^1 + 1$$

$$\Leftrightarrow x_2^2 = x_1^1.$$

Again, applying the resource constraints gives

$$1 - x_1^2 = 1 - x_2^1$$

$$\Leftrightarrow x_1^2 = x_2^1.$$

B.1 Proposition 4.3

Thus, the proposer subgames of player 1 and player 2 are symmetric. From (5.4) in appendix C, we have the following constraint.

$$x_1^{3,1} = \frac{\delta}{3}[(1+\beta) + \frac{1}{2}x_1^{3,1} + \frac{1}{2}\beta x_2^{3,2}]$$

$$\Leftrightarrow x_1^{3,1} = \frac{\delta}{3}[(1+\beta) + \frac{1}{2}x_1^{3,1} + \frac{1}{2}\beta x_1^{3,1}]$$

$$\Leftrightarrow x_1^{3,1} = \frac{\delta}{3}[(1+\beta) + \frac{1}{2}(1+\beta)\, x_1^{3,1}]$$

$$\Leftrightarrow \left(\frac{3 - \delta(1+\beta)}{6}\right) x_1^{3,1} = \frac{\delta}{3}(1+\beta)$$

$$\Leftrightarrow x_1^{3,1} = \frac{2\delta(1+\beta)}{3 - \delta(1+\beta)}$$

$$\Leftrightarrow x_1^{3,1} = \frac{2\delta(1+\beta)}{6 - \delta - \beta\delta} \tag{P3.3}$$

$$\Leftrightarrow x_2^{3,2} = \frac{2\delta(1+\beta)}{6 - \delta - \beta\delta}$$

By the resource constraint,

$$\begin{aligned}1-x_3^{3,1} &= \frac{2\delta\left(1+\beta\right)}{6-\delta-\beta\delta}\\ \Leftrightarrow x_3^{3,1} &= \frac{6-\delta-\beta\delta-2\delta\left(1+\beta\right)}{6-\delta-\beta\delta}\\ \Leftrightarrow x_3^{3,1} &= \frac{6-3\delta-3\beta\delta}{6-\delta-\beta\delta} \qquad \text{(P3.1)}\\ \Leftrightarrow x_3^{3,2} &= \frac{6-3\delta-3\beta\delta}{6-\delta-\beta\delta}.\end{aligned}$$

The equilibrium shares to the a single partisan when player 3 is proposer is $\frac{2\delta(1+\beta)}{6-\delta-\beta\delta}$. The other partisan receives a zero share. Player 3's equilibrium share as proposer is $\frac{6-3\delta-3\beta\delta}{6-\delta-\beta\delta}$. Since we assumed player 3's payoff maximization, player 3 will offer these shares. Since we derived these shares from the continuation inequality of a partisan player, the shares satisfy that constraint, and the partisan player offered a positive share will accept the proposal. This completes the proof of Proposition 4.3. Note also that $x_3^{3,1}$ is equal to the partisan continuation payoff, so that payoff is $\frac{2\delta(1+\beta)}{6-\delta-\beta\delta}$.

B.2 Proposition 4.2

We can set the continuation payoff for player 2 when player 1 is proposer equal to the partisan continuation payoff, as the constraint is tight by player 1's payoff maximization With the restriction $\delta \in [\frac{6\beta}{(1+\beta)(2+\beta)}, 1]$. Otherwise, assuming tightness will violate the non-negativity condition on x_2^1.

$$\begin{aligned}\beta x_1^1 + x_2^1 &= \frac{2\delta\left(1+\beta\right)}{6-\delta-\beta\delta}\\ \Leftrightarrow \beta x_1^1 + \left(1-x_1^1\right) &= \frac{2\delta\left(1+\beta\right)}{6-\delta-\beta\delta}\\ \Leftrightarrow \left(\beta-1\right)x_1^1+1 &= \frac{2\delta\left(1+\beta\right)}{6-\delta-\beta\delta}\\ \Leftrightarrow \left(1-\beta\right)x_1^1 &= \frac{6-3\delta-3\beta\delta}{6-\delta-\beta\delta}\\ \Leftrightarrow x_1^1 &= \frac{6-3\delta-3\beta\delta}{\left(6-\delta-\beta\delta\right)\left(1-\beta\right)} \qquad \text{(P2.1)}\end{aligned}$$

By the resource constraint,

$$\Leftrightarrow 1-x_2^1 = \frac{6-3\delta-3\beta\delta}{\left(6-\delta-\beta\delta\right)\left(1-\beta\right)}$$

$$\Leftrightarrow x_2^1 = \frac{\left(6-\delta-\beta\delta\right)\left(1-\beta\right)-\left(6-3\delta-3\beta\delta\right)}{\left(6-\delta-\beta\delta\right)\left(1-\beta\right)}$$

$$\Leftrightarrow x_2^1 = \frac{2\delta + 3\beta\delta + \beta^2\delta - 6\beta}{(6-\delta-\beta\delta)(1-\beta)}. \tag{P2.2}$$

Player 1's equilibrium share as proposer is $\frac{6-3\delta-3\beta\delta}{(6-\delta-\beta\delta)(1-\beta)}$, player 2's equilibrium share with player 1 as proposer is $\frac{2\delta+3\beta\delta+\beta^2\delta-6\beta}{(6-\delta-\beta\delta)(1-\beta)}$, and player 3's equilibrium share with player 1 as proposer is 0. Therefore, since we assumed above player 1's payoff maximization, player 1 will offer these shares. Since we derived these shares from the player 2's continuation inequality, the shares satisfy that constraint, and player 2 will accept the proposal. The case for player 2 as proposer is symmetrical. This completes the proof of Proposition 4.2.

B.3 Proposition 4.1

Suppose $x_1^1 = 1$, i.e. the proposer player 1 takes all of the resource and offers 0 to player 2 (and, of course, 0 to player 3). Then, the following continuation inequality must hold for player 2.

$$0 + \beta(1) \geq \frac{2\delta(1+\beta)}{6-\delta-\beta\delta}$$

$$\Leftrightarrow \beta \geq \frac{2\delta + 2\beta\delta}{6-\delta-\beta\delta}$$

$$\Leftrightarrow 6\beta - \beta\delta - \beta^2\delta \geq 2\delta + 2\beta\delta$$

$$\Leftrightarrow 6\beta \geq 2\delta + 2\beta\delta + \beta\delta + \beta^2\delta$$

$$\Leftrightarrow 6\beta \geq \left(2 + 3\beta + \beta^2\right)\delta$$

$$\Leftrightarrow \delta \leq \frac{6\beta}{2+3\beta+\beta^2}$$

$$\Leftrightarrow \delta \leq \frac{6\beta}{(1+\beta)(2+\beta)}$$

Therefore, for $\delta \in \left[0, \frac{6\beta}{(1+\beta)(2+\beta)}\right)$, the utility value β of the proposer's partisan exceeds his continuation utility when he is offered 0 of the resource. Thus, an offer by a partisan proposer to keep 1, give 0 to his partisan, and give 0 to player 3 is a stationary subgame perfect strategy for $\delta \in \left[0, \frac{6\beta}{(1+\beta)(2+\beta)}\right)$. The proposer aims to maximize his utility, $x_p + \beta(1-x_p)$, which he can accomplish by maximizing his share x_p as $\beta < 1$. Since 1 is the highest possible payoff for the proposer without violating the resource and non-negativity constraints, this is the only stationary subgame perfect outcome for the partisan proposer with $\delta \in \left[0, \frac{6\beta}{(1+\beta)(2+\beta)}\right)$. This completes the proof of Proposition 4.1.

C Proof that Player 3 as Formateur Proposes a Share to 1 and 2 with Equal Probability

If the equilibrium continuation values of the shares $x_1^{3,1}$ and $x_2^{3,2}$ are not equal, then player 3 will have incentive to only offer a positive share to the player with the lower continuation value. However, if player 3 does this, the player offered a positive share will have a higher continuation payoff, so player 3 will want to offer a positive share to the other player instead. Thus, $x_1^{3,1}$ and $x_2^{3,2}$ are equal in equilibrium. Let $Q \in [0,1]$ be the probability that player 3 offers a positive share to player 1, and let $1-Q$ be the probability he offers a positive share to player 2.

$$\beta x_1^1 + x_2^1 \geq \frac{\delta}{3}[\left(\beta x_1^1 + x_2^1\right) + \left(\beta x_1^2 + x_2^2\right) + Q\left(\beta x_1^{3,1} + x_2^{3,1}\right) + (1-Q)\left(\beta x_1^{3,2} + x_2^{3,2}\right)] \tag{1}$$

$$x_3^1 = 0 \tag{2}$$

$$x_1^2 + \beta x_2^2 \geq \frac{\delta}{3}[\left(x_1^1 + \beta x_2^1\right) + \left(x_1^2 + \beta x_2^2\right) + Q\left(x_1^{3,1} + \beta x_2^{3,1}\right) + (1-Q)\left(x_1^{3,2} + \beta x_2^{3,2}\right)] \tag{3}$$

$$x_3^2 = 0 \tag{4}$$

$$x_1^{3,1} + \beta x_2^{3,1} \geq \frac{\delta}{3}[\left(x_1^1 + \beta x_2^1\right) + \left(x_1^2 + \beta x_2^2\right) + Q\left(x_1^{3,1} + \beta x_2^{3,1}\right) + (1-Q)\left(x_1^{3,2} + \beta x_2^{3,2}\right)] \tag{5}$$

$$x_2^{3,1} = 0 \tag{6}$$

$$\beta x_1^{3,2} + x_2^{3,2} \geq \frac{\delta}{3}[\left(\beta x_1^1 + x_2^1\right) + \left(\beta x_1^2 + x_2^2\right) + Q\left(\beta x_1^{3,1} + x_2^{3,1}\right) + (1-Q)\left(\beta x_1^{3,2} + x_2^{3,2}\right)] \tag{7}$$

$$x_1^{3,2} = 0 \tag{8}$$

$$x_1^{3,1} = x_2^{3,2} \tag{9}$$

Since the proposer in any subgame will seek to maximize his payoff, all the above continuation constraints are tight. Now, we substitute the zero shares of (6) and (8) into (5) and (7).

$$x_1^{3,1} = \frac{\delta}{3}[\left(x_1^1 + \beta x_2^1\right) + \left(x_1^2 + \beta x_2^2\right) + Q x_1^{3,1} + (1-Q)\beta x_2^{3,2}] \tag{5.1}$$

$$x_2^{3,2} = \frac{\delta}{3}[\left(\beta x_1^1 + x_2^1\right) + \left(\beta x_1^2 + x_2^2\right) + Q\beta x_1^{3,1} + (1-Q) x_2^{3,2}] \tag{7.1}$$

Since the continuation values for player 1 and 2 are the same when player 3 is the proposer, they are otherwise the same.

$$x_1^2 + \beta x_2^2 = x_1^{3,1} = x_2^{3,2} = \beta x_1^1 + x_2^1 \tag{10}$$

By the above equation (10), we can modify (5.1) and (7.1) as follows

$$x_1^{3,1} = \frac{\delta}{3}[\left(x_1^1 + \beta x_2^1\right) + \left(\beta x_1^1 + x_2^1\right) + Q x_1^{3,1} + (1-Q)\beta x_2^{3,2}] \tag{5.2}$$

$$x_2^{3,2} = \frac{\delta}{3}[\left(x_1^2 + \beta x_2^2\right) + \left(\beta x_1^2 + x_2^2\right) + Q\beta x_1^{3,1} + (1-Q)x_2^{3,2}] \tag{7.2}$$

$$x_1^{3,1} = \frac{\delta}{3}[(1+\beta)\left(x_1^1 + x_2^1\right) + Qx_1^{3,1} + (1-Q)\beta x_2^{3,2}] \tag{5.3}$$

$$x_2^{3,2} = \frac{\delta}{3}[(1+\beta)\left(x_1^2 + x_2^2\right) + Q\beta x_1^{3,1} + (1-Q)x_2^{3,2}] \tag{7.3}$$

By the resource constraints,

$$x_1^{3,1} = \frac{\delta}{3}[(1+\beta) + Qx_1^{3,1} + (1-Q)\beta x_2^{3,2}] \tag{5.4}$$

$$x_2^{3,2} = \frac{\delta}{3}[(1+\beta) + Q\beta x_1^{3,1} + (1-Q)x_2^{3,2}] \tag{7.4}$$

By equation (9),

$$\frac{\delta}{3}[(1+\beta) + Qx_1^{3,1} + (1-Q)\beta x_2^{3,2}] = \frac{\delta}{3}[(1+\beta) + Q\beta x_1^{3,1} + (1-Q)x_2^{3,2}]$$

$$\Leftrightarrow Qx_1^{3,1} + (1-Q)\beta x_2^{3,2} = Q\beta x_1^{3,1} + (1-Q)x_2^{3,2}$$

$$\Leftrightarrow Q(1-\beta)x_1^{3,1} = (1-Q)(1-\beta)x_2^{3,2}$$

$$\Leftrightarrow Qx_1^{3,1} = (1-Q)x_1^{3,1}$$

$$\Leftrightarrow Q = (1-Q)$$

$$\Leftrightarrow 2Q = 1$$

$$\Leftrightarrow Q = \frac{1}{2}$$

Thus, player 3 as proposer chooses to offer a positive share to player 1 or player 2 with probability $\frac{1}{2}$ each.

D Comparative Statics

D.1 Unanimous Bargaining

$$\frac{\partial x_1^1}{\partial \beta} = \begin{cases} \frac{1-\delta}{(1-\beta)^2} \geq 0 & \beta \in \left(0, \frac{\delta}{3-2\delta}\right] \\ 0 & \beta \in \left(\frac{\delta}{3-2\delta}, 1\right) \end{cases}$$

$$\frac{\partial x_2^1}{\partial \beta} = \begin{cases} \frac{\delta-1}{(1-\beta)^2} \leq 0 & \beta \in \left(0, \frac{\delta}{3-2\delta}\right] \\ 0 & \beta \in \left(\frac{\delta}{3-2\delta}, 1\right) \end{cases}$$

$$\frac{\partial x_3^1}{\partial \beta} = 0$$

$$\frac{\partial x_1^1}{\partial \delta} = \begin{cases} \frac{1}{3} - \frac{1}{1-\beta} < 0 & \delta \in \left[\frac{3\beta}{1+2\beta}, 1\right] \\ 0 & \delta \in \left[0, \frac{3\beta}{1+2\beta}\right) \end{cases}$$

$$\frac{\partial x_2^1}{\partial \delta} = \begin{cases} \frac{1}{3} + \frac{\beta}{1-\beta} > 0 & \delta \in \left[\frac{3\beta}{1+2\beta}, 1\right] \\ 0 & \delta \in \left[0, \frac{3\beta}{1+2\beta}\right) \end{cases}$$

$$\frac{\partial x_3^1}{\partial \delta} = \frac{1}{3} > 0$$

$$\frac{\partial x_1^3}{\partial \beta} = 0$$

$$\frac{\partial x_2^3}{\partial \beta} = 0$$

$$\frac{\partial x_3^3}{\partial \beta} = 0$$

$$\frac{\partial x_1^3}{\partial \delta} = \frac{1}{3} > 0$$

$$\frac{\partial x_2^3}{\partial \delta} = \frac{1}{3} > 0$$

$$\frac{\partial x_3^3}{\partial \delta} = -\frac{2}{3} > 0$$

D.2 Majority Rule Bargaining

$$\frac{\partial x_1^1}{\partial \beta} = \begin{cases} \frac{3\left(12-12\delta-4\beta\delta+(1+\beta)^2\delta^2\right)}{(1-\beta)^2(6-\delta-\beta\delta)^2} > 0 & \delta \in \left[\frac{6\beta}{(1+\beta)(2+\beta)}, 1\right] \\ 0 & \delta \in \left[0, \frac{6\beta}{(1+\beta)(2+\beta)}\right) \end{cases}$$

$$\frac{\partial x_2^1}{\partial \beta} = \begin{cases} -\frac{3\left(12-12\delta-4\beta\delta+(1+\beta)^2\delta^2\right)}{(1-\beta)^2(6-\delta-\beta\delta)^2} < 0 & \delta \in \left[\frac{6\beta}{(1+\beta)(2+\beta)}, 1\right] \\ 0 & \delta \in \left[0, \frac{6\beta}{(1+\beta)(2+\beta)}\right) \end{cases}$$

$$\frac{\partial x_3^1}{\partial \beta} = 0$$

$$\frac{\partial x_1^1}{\partial \delta} = \begin{cases} -\frac{12(1+\beta)}{(1-\beta)(6-\delta-\beta\delta)^2} < 0 & \delta \in \left[\frac{6\beta}{(1+\beta)(2+\beta)}, 1\right] \\ 0 & \delta \in \left[0, \frac{6\beta}{(1+\beta)(2+\beta)}\right) \end{cases}$$

$$\frac{\partial x_2^1}{\partial \delta} = \begin{cases} \frac{12(1+\beta)}{(1-\beta)(6-\delta-\beta\delta)^2} > 0 & \delta \in \left[\frac{6\beta}{(1+\beta)(2+\beta)}, 1\right] \\ 0 & \delta \in \left[0, \frac{6\beta}{(1+\beta)(2+\beta)}\right) \end{cases}$$

$$\frac{\partial x_3^1}{\partial \delta} = 0$$

$$\frac{\partial x_1^{3,1}}{\partial \beta} = \frac{12\delta}{(6-\delta-\beta\delta)^2} \geq 0$$

$$\frac{\partial x_2^{3,1}}{\partial \beta} = 0$$

$$\frac{\partial x_3^{3,1}}{\partial \beta} = -\frac{12\delta}{(6-\delta-\beta\delta)^2} \leq 0$$

$$\frac{\partial x_1^{3,1}}{\partial \delta} = \frac{12(1+\beta)}{(6-\delta-\beta\delta)^2} \geq 0$$

$$\frac{\partial x_2^{3,1}}{\partial \delta} = 0$$

$$\frac{\partial x_3^{3,1}}{\partial \delta} = -\frac{12(1+\beta)}{(6-\delta-\beta\delta)^2} \leq 0$$