

MONETARY REGIMES AND INFLATION

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Monetary Regimes and Inflation in the UK, 1976-2007

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Abstract

I offer a reappraisal of the British monetary policy reaction function based on evidence of structural change indicated by a break-point algorithm. Discussion is supplemented by an inflation pressure analysis within a New Keynesian general equilibrium model to determine the effectiveness of policy in the post-Bretton Woods era. I find that the timing of structural breaks is similar to but does not match precisely the breaks indicated by traditional narrative approaches and that policy has become more aggressive in combating inflation as implied by the Taylor Rule. The inflation pressure analysis complements this by showing that the underlying inflationary environment moderated significantly during the Bank of England's period of inflation targeting.

1 Introduction

British monetary policy has evolved substantially in the past quarter-century. Policymakers have moved through three distinct monetary regimes: broad money targets in the late 1970s and early 1980s, exchange-rate targets in the late 1980s, and inflation targets since the early 1990s. Each of these broad periods was conducted with varying levels of commitment by the monetary authorities to their targets, and within each regime there were frequent changes to the nominal target and stance of monetary policy.

With such an erratic history, it is of interest whether the effectiveness of British monetary policy has changed in a significant way since 1975. Taylor (1993) introduced a positive metric for

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determining the goals of monetary policymakers, expressed in simple interest-rate reaction functions as well as normative rules for judging the effectiveness of policy, particularly through an analysis of the magnitude of the coefficients on the reaction function. Since then, the literature on Taylor Rules has expanded considerably and various authors have estimated Taylor-type reaction functions for a wide cross-section of countries.

Prior approaches to estimating Taylor Rules, whether in the UK or more generally, either tend to estimate a single rule for long time periods, or exploit the narrative history to identify changes in announced central bank policy and estimate separately for each period. The first approach ignores the possibility of structural breaks in the monetary policy reaction function and risks biased coefficients on the parameters of interest. The second approach offers an improvement by acknowledging the possibility of structural change in the reaction function, but implicitly assumes that the change in actual policymaking occurs at the moment of the announcement of a change in policy. However, it is plausible that these two events may occur with a lag: an announced change in policy may reflect a change in de facto policymaking some quarters prior, or an announced change in policy may only be implemented with a lag. These possibilities pose serious challenges to the use of the narrative history in determining the timing of breaks in subperiods.

This paper will present two approaches to understanding UK monetary policy in the past quarter-century. First will be a discussion of the implied Taylor Rule implemented by the Bank of England across various subperiods from 1975-2005; these monetary policy regimes will be estimated empirically using the breakpoint method designed by Bai and Perron (1998, 2003). While it is true that the Taylor Rule approach has been applied to the UK in the past, a combination of the Taylor Rule with a statistical test of structural breaks is novel. The advantage of this is that by using breakpoint analysis, one may consider regime breaks without explicitly relying on the stylized regime changes present in the narrative history.

The second substantive section of this paper will develop a New Keynesian model for the United Kingdom and perform an inflation pressure analysis on the UK, following Weymark (2004, 2009). This will supplement the discussion of Taylor Rules by identifying the presence of inflation pressure in the British economy, providing information on how effective UK policymakers were in managing inflation and inflation expectations. The inflation pressure approach is new to the literature.

The rest of this paper is organized as follows. Section 2 reviews the relevant literature on

monetary regimes and structural breakpoints. Section 3 presents a short narrative history of central bank policy in England to provide context. Section 4 performs an analysis of the UK Taylor Rule based on the breaks indicated by the Bai-Perron breakpoint algorithm on the inflation time series. Section 5 describes the benchmark model to be used in the remainder of the paper. Section 6 discusses the results of the model specification and the inflation pressure analysis. Section 7 concludes.

2 Literature

The relevant literature may be partitioned into two strains: that on monetary policy rules in the UK and that on the econometric analysis of structural breaks. I take each up in turn.

2.1 Monetary Regime Changes

The modern analysis of monetary policy regimes began with Taylor (1993), which characterized the behavior of the Federal Reserve in terms of a reaction function that related the short-term nominal interest rate to inflation and the output gap. It has become typical to frame monetary policy in terms of such “simple” rules and to judge shifts in monetary regimes as shifts in the coefficients of the rule.

While the previous literature on the analysis of monetary policy rules is large, that concerning specifically the United Kingdom remains slim. The majority of studies either focus exclusively on the United States (for example, Clarida, Gali, and Gertler (1999)) or incorporate a variety of countries (for example, Clarida, Gali, and Gertler (1998)). There are two studies, however, that are close to this one in spirit and in application, Nelson (2001) and Adams, Cobham, and Girardin (2005). Both of these studies offer a careful treatment of the UK experience since the early 1970s and each allows for structural breaks in the monetary policy rule. As will be seen, both use the narrative history to inform their break points; the primary advantage of this work over the previous literature is the application of a statistical break point test.

Nelson (2001) provides the most thorough exploration of UK monetary policy reaction functions in the post-Bretton Woods era. That study estimates five Taylor Rules for the 1972-1997 period, chosen for the periods of monetary targets, exchange-rate targets, and inflation targeting. He

estimates domestic versions of the Taylor Rule, only incorporating external factors during the 1990-1992 ERM targeting period. The Taylor Rules estimated by Nelson show some heterogeneity across the five subperiods; however, no period before 1992 showed an inflation response coefficient above unity, while that post-1992 does show an inflation response coefficient above unity and also features and output gap response coefficient near one-half. For all periods, a lag term for the nominal rate is included and is highly significant.

Adams, Cobham, and Girardin (2005) (hereafter ACG) offer an analysis similar to Nelson (2001) but look at a slightly later period, 1985-2003. They use three subperiods: 1985-1990, the pre-ERM period; 1992-1997, the post-ERM period; and 1997-2003, the period in which the Monetary Policy Committee obtained independence from the Treasury (see the narrative history, below). The main innovation of this paper was to include a variety of external factors in the Taylor Rule estimation, in particular both the US and German interest rates. Previous studies either neglected external factors or only included the German interest rate as a possible source of external concern. The use of external factors allowed ACG to test both “domestic” and “international” versions of the Taylor Rule as well as “nested” rules which incorporate both domestic and international factors. A second innovation is the introduction of a distinction between regime shifts due to “monetary frameworks” and those due to shifts in “institutional constraints”, for example the shift towards central bank independence in 1998.

2.2 Structural Breaks

The analysis of structural breaks in both time-series and cross-sectional datasets has a long history in the econometrics literature. The method employed by Bai and Perron (1998, 2003) offers a variety of appealing characteristics. First, it is an explicitly time-series oriented test of structural change. Second, it includes procedures for the testing of a null hypothesis of, say, k breaks against $k + 1$, allowing for sequential estimation of breaks.

As previously mentioned, much of the prior work on applying structural change to Taylor Rule analysis is *ex post*: one first decides when to date the subperiods based on an analysis of the narrative history, then estimates separately for each period, and finally employs a test of structural change (for example, that of Chow (1960)). Our approach is to take an *ex ante* perspective: the break dates are estimated first using the data, and only then are Taylor Rules estimated from the

generated break dates.

3 A Monetary History

Following the breakdown of the Bretton Woods system in the early 1970s, formal monetary targets were first announced by the Bank of England in July 1976. From 1976 to 1981 the Bank experimented with targets for M3, with varying degrees of commitment to and discretion around the announced target. The early targets, from 1976 to 1979, were not particularly binding throughout; their main purpose seems to have been to signal a renewed focus by the Bank on inflation as opposed to unemployment. The targets became gradually more binding after 1979 and the introduction of the Thatcher government.

Again the use of announced targets in this early stage did not represent a wholesale movement towards rule-based policymaking. Rather, it reflected a shift of focus on the part of the central bank towards price stability, no doubt motivated by the spectacular inflation of the late 1970s. In addition, the announcement of targets provided a nominal anchor by which the public could form expectations about Bank policy, which would promote economic stability. Finally, it was believed that announced targets would allow the Bank to more quickly affect public inflation expectations.

The tighter targets of 1979-1981 were replaced in March of 1981 with a policy of monetary targets with far more discretion in the monetary regime. This was motivated by an internal shift in the Thatcher government towards containing the fiscal deficit. As such, the announced targets were often overshoot from 1981-1985 and it became clear that the targets were no longer serving their stated purpose as a method of providing a nominal anchor for private inflation expectations. In 1985 announced monetary targets were abandoned after a decade of use.

The abandonment of monetary targets in the mid-1980s was also influenced by external factors, most notably the emerging possibility of shadowing or outright entering the ERM. For a year, from March 1987 to March 1988, the Thatcher government entered an informal peg to the Deutschemerk; this was abandoned in 1988 but the Bank continued to monitor the exchange rate closely in its decision-making process. This was followed by the UK's formal entrance into the ERM in 1990, an experiment which lasted until 1992 and during which time the Bank surrendered domestic monetary independence in favor of a hard exchange-rate target.

The ERM collapsed in 1992 and this presaged another major shift in UK monetary policy. The Bank introduced an inflation target in late 1992 with the decision-making process to be shared by the Bank and the Treasury. Formally, interest-rate decisions were to be made by the Treasury under the advice of the Bank of England. This arrangement lasted until 1997. The newly-elected Labour government in 1997 gave the Bank of England operational independence in deciding how to set the interest rate; this arrangement has lasted until the present. The only major shift since 1997 was the switch, in 2003, from using the Retail Price Index to using the Consumer Price Index as the official price index that the Bank would monitor and respond to.

One might then summarize the history of UK monetary policy as one in which there were three broad regimes with considerable heterogeneity within those regimes. First, from 1975 to 1985 the Bank of England and Treasury followed monetary targets, though with varying degrees of enthusiasm. From 1985 to 1992 the UK followed an externally-focused monetary regime, with particular emphasis on exchange-rate targeting. Again the level of commitment to the exchange-rate target varied, though it was strongest in the explicit ERM period. Finally, since 1992 the Bank has targeted inflation using the short-term nominal rate as its instrument. This commitment was strengthened in 1997 when the Bank gained operational independence from the Treasury.

However the broad summary of three regimes should not obscure the considerable shifts that occurred within each regime. One result of this paper is to show which of these regime shifts and within-regime policy stance shifts actually influenced inflation and monetary policymaking as measured via a Taylor Rule.

4 Analysis of the UK Taylor Rule

The standard analysis of regime change tends to emphasize heavily the study of implied Taylor Rules across various time frames, usually chosen based on an historical analysis of central bank policy, and compare the implied coefficients on parameters of interest across subperiods. As mentioned above, both Nelson (2001) and ACG (2005) employ this method. Neither, however, uses a formal test of structural change to identify regime shifts in monetary policy. The present section remedies this gap in the literature and offers comment.

4.1 The Taylor Rule

As presented in Taylor's seminal 1993 paper, the Taylor Rule takes the form

$$i_t = \phi_0 + \phi_\pi \pi_{t-1} + \phi_y y_{t-1} \quad (1)$$

where i_t is the central bank policy instrument and π_{t-1} and y_{t-1} are the lagged values of inflation and the output gap, respectively; the output gap being measured as the deviation of output from its trend. The magnitude of the coefficients ϕ_π and ϕ_y are important; they determine the monetary authority's marginal reaction to changes in inflation and the output gap. Specifically, much importance has been attached to an inflation response coefficient above unity.

To see the importance of this, consider the usual interest-rate transmission mechanism of interest rates on aggregate demand. An increase in the inflation rate will be met with an increase in the nominal interest rate by the central bank. If the inflation response coefficient is less than unity, the net effect will be a decrease in the real interest rate, which will increase aggregate demand and fuel additional inflation. On the other hand, an inflation response coefficient above unity will lead the monetary authority to increase the nominal interest rate more than one-for-one in response to increases in inflation; this will increase the real interest rate, reduce the output gap and then reduce inflation (through the inflation-output gap relation present in the Phillips Curve).

In empirical work, such rules are often considered simple approximations of the much more complex decision-making procedure actually employed by central banks; they offer a guide to policy by measuring responsiveness to various indicators but are not usually posited as being the rule that the policymaker actually follows. The stylized rule presented above has received two major innovations in the subsequent decade of research. First, it is now common to acknowledge the interest-smoothing behavior of central banks by including a lag term of the nominal interest rate in the Taylor Rule; the second is to allow for the bank to consider not just past inflation but also expected future inflation in its decision-making process. Hence the empirical work on Taylor Rules generally focuses on equations of the form

$$i_t = \rho i_{t-1} + (1 - \rho)[\phi_0 + \phi_\pi E_t \pi_{t+k} + \phi_y E_t y_{t+l}] \quad (2)$$

where now the E_t terms denote expectations conditional on information known at time t and i_{t-1} is the one-period lag of the nominal interest rate; ρ provides an indicator of the degree of interest rate smoothing by central banks. Note that this incorporates the possibility of backward-looking behavior and also allows for different optimal leads for each of inflation and the output gap. Specifically, the monetary authority looks k periods in the future in response to inflation and l periods in the future in response to the output gap.

4.2 Estimation method

Prior studies have analyzed shifts in the coefficients of (2) by appealing to the historical record, identifying structural breaks in Bank policy with changes in the monetary target or changes in the policy target itself (monetary aggregate, exchange rate, or inflation). By contrast, I allow the data itself to guide the choice of subperiods by applying an econometric break-point analysis. There are several approaches by which one might identify the breaks in regime. First and most appealingly, one might run a breakpoint analysis on the Taylor Rule itself; however, since the Taylor Rule includes expectational variables, OLS estimation is inapplicable and it is OLS which underlies most breakpoint techniques, including the one which I employ. One might then estimate the breakpoints using a fully backward-looking Taylor Rule, as in (1), but this is also unsatisfactory; it ignores the fact that central banks are essentially forward-looking towards both inflation and the output gap. To avoid these problems, I instead perform the breakpoint analysis on the inflation time-series under the simple assumption that inflation follows an AR(1) process. A robustness check of performing the Bai-Perron breakpoint method on (1) showed similar results. However, the eventual estimation of the Taylor Rule will of course be based on (2).

Estimation is by OLS, using the Bai-Perron algorithm to identify point of structural change. It is assumed that there are m breakpoints, where the coefficients shift from one stable set to another. For m breaks, there are $m + 1$ *regimes*, and the structural change model takes the form

$$\pi_t = \alpha_{0,j} + \alpha_{1,j}\pi_{t-1} + \epsilon, \quad t = T_{j-1} + 1, \dots, T_j \quad (3)$$

for $j = 1, \dots, m$. The break dates are not imposed exogenously but are determined endogenously: the algorithm is to choose the m locations of breakpoints to minimize the sum of squared residuals

across all regimes. The algorithm optimizes for each choice of number of regimes, and each regime is evaluated based on the residual sum of squares, a Bayesian information criterion, and F-tests for the marginal effect of using $m + 1$ rather than m breaks. I use a combination of the residual sum of squares calculation (RSS) and the Bayesian Information Criterion (BIC) to determine the optimal number of breaks.

The results of the Bai-Perron technique are displayed in Table 1. The Bayesian Information Criterion, which tests for the optimality of breaks, strongly suggests one break. The residual sum of squares decreases with the number of breaks, by definition, but shows little significant improvement after three breaks. I choose to use three breaks, hence four regimes, for the remainder of the analysis.

The four regimes implied by the method are: 1976:I-1981:IV; 1982:I-1987:II; 1987:III-1992:II; and 1992:III-present. These subperiods correlate remarkably well with the changes in monetary policy discussed in the narrative history above. The first period covers the time frame of the introduction of monetary targets as well as the period of strict monetary targeting, ending with a two-quarter lag from the date at which the Bank of England relaxed its formal target. The second period covers the time in which monetary targets were lax and variable. The third period opens with the same quarter that the Bank of England began shadowing the Deutschemark and closes with the end of the ERM experiment. The fourth period begins with a one-quarter lead over the time in which the Bank introduced inflation targeting. Furthermore, note that the two additional breaks identified by the algorithm are in 1998 and 2004; these may be linked to the independence of the Monetary Policy Committee and the switch in inflation target from the Retail Price Index to the Consumer Price Index, respectively, though neither break is highly significant and I do not treat these two regimes separately.

4.3 The Taylor Rule under Estimated Subperiods

With the breakpoints established, it is possible to analyze the behavior of the UK Taylor Rule. This will be accomplished by estimating the Taylor Rule separately for each subperiod brought forth by the data.

Estimation of the Taylor Rule requires gathered data on interest rates, the inflation rate, and the output gap. I use quarterly data obtained from the UK National Statistics Office and Bank

of England. The nominal interest rate is chosen to be the three-month Treasury bill; alternative specifications using similar instruments (such as the three-month interbank rate) did not significantly affect results. Inflation is measured as year-over-year change in the core Retail Price Index. The output gap is measured by the deviation of log real GDP from its Hodrick-Prescott trend with a standard smoothing parameter. Alternative specifications using detrended output did not significantly affect results. Estimated coefficients for each subperiod appear in Table 2.

Throughout all four periods, the coefficient on the lagged interest rate is positive, statistically significant and less than 1, but higher than one-half. The range of values reported for this smoothing parameter are consistent with those found in other studies, though somewhat lower than typical in earlier periods and slightly higher in later periods. The coefficient on expected inflation is positive in all four periods and is significant in three of four periods. The coefficient on the expected output gap is significant in two of four periods and points in the correct direction for three of four periods; where it is negative, the coefficient is insignificant.

The first subperiod, from 1976:I-1981:IV, covers a time frame in which the Bank of England first adopted but only loosely adhered to monetary targets (1976-79) as well as the period in which monetary targets were held most strictly (1979-81). This subperiod has been found to be difficult to estimate in prior studies due to a unit root problem in the lagged interest rate (see especially Nelson 2001), but no such issues occur here. The coefficient on the lagged interest rate, 0.723, is typical of that seen in most Taylor Rule studies.

The coefficient on future inflation two periods ahead is statistically significant at the 5% level and points in the correct direction, however the response coefficient is below unity. This conforms broadly with prior studies of this period. The output gap response coefficient is insignificant but points in the correct direction. From a comparison of the two values, it appears that the Bank was roughly equally concerned with inflation and the output gap; but the main finding is that the response to inflation was tepid. Increases in inflation were not matched more than 1-for-1 with changes in the nominal interest rate, so that increases in inflation led to reductions in the real interest rate even after central bank intervention.

The second subperiod, from 1982:I to 1987:II, matches closely the period of loose monetary targets. Note that the degree of interest-rate smoothing declined to its lowest point in this period and that the response to inflation remains below unity, implying that monetary policy during this

period continued to only tepidly respond to changes in inflation. In addition, the response of policymakers to inflation became more erratic as evidenced by a lack of statistical significance in the inflation coefficient. Furthermore, note that the output gap response coefficient points in the wrong direction in this subsample, becoming negative; further, this variable remains statistically insignificant. All of these observations corroborate the narrative discussion that policy was relatively undisciplined in this period.

There is some anomaly in this second subperiod. It features largely falling inflation rates combined throughout as well as an estimated inflation response below unity. In most discussions, particularly Clarida, Gali, and Gertler (1999), it is noted that monetary policy can only be effective in reducing inflation if the response coefficient on inflation is above unity (so that changes in inflation are matched more than one-for-one with changes in the nominal interest rate, thus driving down the real interest rate when inflation is high to reduce aggregate demand); this is known as the Taylor Principle after Taylor (1993). However from 1982 to 1987 the inflation rate in the UK fell dramatically without an overly aggressive policy by the central bank. Why would this be the case?

In some sense it is an artifact of the endpoints. The inflation rate fell from 10.7% in 1982 to just 3.6% in 1987. The vast majority of these gains occurred in 1982; inflation declined slightly from 1983 to 1987 but largely remained in the 3% to 5% range from 1983 to 1987. A second explanation is that despite a relatively docile response coefficient, nevertheless the level of the (ex post) real interest rate was high in this period; indeed higher than the first period and rising throughout the period. Hence the level of the real interest rate was high enough to promote a disinflationary environment despite the Bank of England's relatively weak response to inflation as measured by the Taylor Rule.

The third period, from 1987:III to 1992:III, exactly parallels the period in which British monetary policy was most externally focused. It covers the DM-shadowing period of 1987-88 as well as the British involvement in the ERM experiment in 1992. The smoothing parameter returns to normal levels, and both the inflation and output gap response coefficients are significant and point in the correct direction.

The major point of note in the third period is that the inflation response coefficient rose above unity, in line with the Taylor Principle. This signals an aggressive central bank policy towards inflation, though in part this may be spurious: the Bank of England was highly externally fo-

cused during this period, shadowing the Deutschemark in 1987-88 and participating in the ERM experiment in 1991-92. However, despite this external focus and intense internal discussion within the Bank of the proper conduct of monetary policy, in terms of outcomes Bank policy was more aggressive towards inflation than it had been since the start of the 1970s. It is possible that the discipline imposed on the central bank by adopting loose exchange-rate targets fostered a degree of public confidence in the competence of the central bank and increased the Bank of England's effectiveness as measured by the Taylor Rule. In particular, it is likely that the Bank of England was able to take advantage of the credibility of the Bundesbank's reputation for inflation-fighting by tying British monetary policy to German monetary policy. The effect of this shows up in the Taylor Rule as an inflation response coefficient above unity.

The final period, 1992:IV to the end of 2007, encompasses the establishment of the Monetary Policy Committee, the introduction of inflation targeting, and the introduction of the independence of the MPC from political influence. All coefficients are statistically significant and point in the direction implied by theory. The smoothing parameter is unusually high, but from the confidence interval there is little risk of a unit root problem.

The inflation response coefficient is extremely high; well above unity. It is higher than the coefficient that Nelson (2000) reports for the 1994-97 period and higher than ACG report for either 1992-97 or 1998-2002, with their values clustering between 1.5 and 2.5. The output gap coefficient above unity is unique among prior studies (see the survey in ACG); however it is also the case that my estimation is for a far longer subperiod than any of those previously studied, and indeed combines several subperiods relative to earlier studies (which tend to split the data between pre-MPC independence and post-MPC independence where the time frame allows).

The overall results of this Taylor Rule exercise show that the Bank of England was relatively more aggressive towards inflation in the time since 1992; throughout 1976 to 1987 the policy response towards inflation was tepid, and the period 1987-1992 shows an inflation response coefficient not statistically significantly different from 1, though somewhat higher than earlier periods. Policymakers also increased their response to the output gap in the later two periods compared to those earlier.

5 Inflation Pressure: Definitions and Model

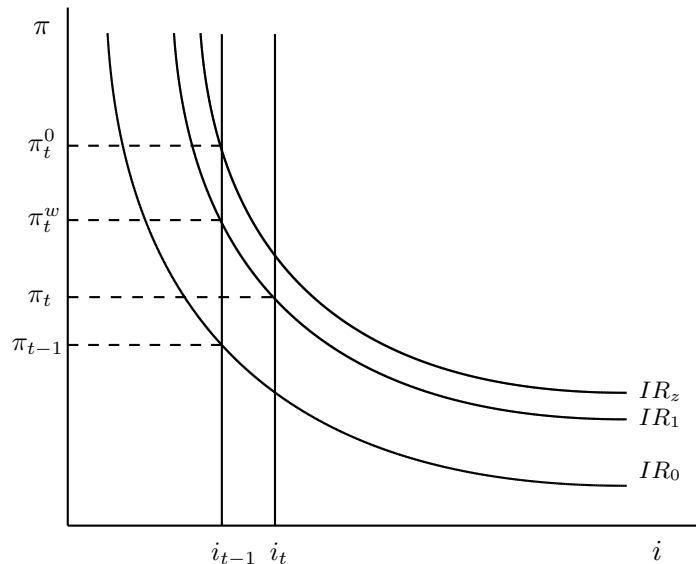
5.1 Inflation Pressure: Model-Independent Definitions

This section broadens the previous Taylor Rule discussion by analyzing inflation pressure in the British economy.¹ While the success of the central bank tends to be judged based on the level and variance of realized inflation, monetary policymakers have an additional goal beyond mere inflation moderation: the moderation of the underlying inflationary environment. Inflation pressure allows the analyst to be precise about what this “underlying environment” is and allows the analyst to compute quantitative estimates of the change in this environment. While the Bank of England was broadly successful in controlling realized inflation throughout the mid-1980s through late 2000s, the inflation pressure analysis will uncover the extent to which the Bank was also successful in moderating the inflationary environment it faced.

The economy is regularly affected by shocks, real and nominal, which affect output and inflation. The central bank reacts to these shocks by changing its policy interest rate, offsetting negative shocks with easier monetary policy and positive shocks with tighter monetary policy. Further, the reaction of the central bank to these shocks is to some degree known by the public, who then build likely monetary policy response into their expectations. The inflation pressure indicies provide a measure of what change in inflation would have materialized had the central bank not responded to shocks in the prior period (or more, if there is a significant lag between the setting of monetary policy and its effect on the economy) but instead had left its policy interest rate unchanged.

I first discuss how one should define the “underlying environment” that the central bank faces, and how to uncover it. The rate of inflation that would have been realized, had the central bank held policy constant and not attempted to mitigate those shocks, is a feasible way to define the environment that the central bank faces when it does choose to implement policy. What remains is to define carefully what it means to hold policy “fixed” so that one can uncover that environment. While a variety of measures may be proposed, the following paragraphs will make clear that the most natural measure of “holding policy fixed” is by defining “fixed policy” to mean the central bank holds its policy interest rate fixed for the duration that policy is held constant.

¹Both the term “inflation pressure” and the quantitative indicies are adapted from Professor Weymark’s research; they build on the concept of exchange-market pressure in Weymark (1995, 1998) and are first introduced in Weymark (2004) in the context of inflation in the United States.



To make these initial concepts clear, consider a stylized example in which the central bank's decision to change the interest rate affects the economy with no control lag; monetary policy affects the economy instantaneously. The central bank faces a tradeoff between inflation and the interest rate as shown in the figure. Begin at the end of $t - 1$, in which the central bank selected a combination of interest rate and inflation (i_{t-1}, π_{t-1}) on IR_0 . In the beginning of period t , shocks are realized; this pushes the central bank's tradeoff to IR_z . The measure *ex ante inflation pressure* is the vertical distance from π_{t-1} to π_t^0 ; that is, the inflation rate that would have been realized, had the central bank kept its interest rate at the level of i_{t-1} .

However, when agents form their expectations rationally, the inflation rate is a function of the expected change in the interest rate as a result of the central bank reaction to period t shocks. Hence once expectations have been adjusted, the actual tradeoff faced by the policy authority is IR_1 . The vertical distance from π_{t-1} to π_t^w measures *ex post inflation pressure*: the amount of inflation pressure remaining in the economy after the central bank's policy is implemented.

Formally, we define *ex ante inflation pressure* at time t ($EAI P_t$) and *ex post inflation pressure* at time t ($EPI P_t$) as:

$$EAI P_t = \pi_t^0 - \pi_{t-1} \quad (4)$$

$$EPI P_t = \pi_t^w - \pi_{t-1}. \quad (5)$$

I now go into detail as to what these two indicies are measuring.

Ex ante inflation pressure is the result of a policy experiment: it measures the change in inflation from $t - 1$ to t that would have been realized had the central bank chosen for one period not to implement policy to counteract shocks to the economy, and had announced this intention in $t - 1$ so that inflationary expectations reflected this change in policy. Hence it is a natural measure of the underlying inflationary environment of the economy. In particular, large values of EAIP relative to actual inflation indicate that the central bank must exert considerable stabilization policy to achieve its aims; relatively small values of EAIP relative to realized inflation indicate that the central bank is able to achieve its aims with relative ease.

Ex post inflation pressure measures the inflation pressure remaining in an economy once the central bank's policy has been realized. When the central bank is successful in moderating inflation pressure, only some of that moderation shows up in realized inflation; the remaining portion shows up in the change in the interest rate. Hence to obtain a quantitative value for the remaining inflation pressure left in an economy, one of the two (inflation or the interest rate) must be converted to equivalent units of the other. Ex post inflation pressure is a measurement exercise that converts the change in the interest rate into inflation-equivalent units, walking up the IR_1 curve and expressing the entirety of the inflation pressure in units of inflation alone.

Note in particular that since EPIP is calculated from the policy as implemented, it holds private sector expectations fixed. Ex ante inflation pressure, by contrast, is calculated by allowing agents to fully update their expectations of central bank policy given the newly announced one-period policy of keeping interest rates fixed. Hence while EPIP may be calculated without reference to the expectations-formation process, EAIP must take a stand on how expectations evolve. Throughout, following modern practice, I assume rational expectations. Actual calculation of these measures must take place in the context of a model, which I outline in the next section.

5.2 The Model

While the previous definitions are model-independent, actual estimation of inflation pressure must be computed by use of a macroeconomic model. Further, this model must contain two features: an interest-rate transmission mechanism that relates the interest rate to inflation, and an expectations transmission mechanism by which expected future inflation affects current inflation. The broad

class of New Keynesian general equilibrium models satisfies both of these conditions, in particular with an interest-rate transmission mechanism through the output gap and extensive use of forward-looking expectations. The model as implemented is based on the benchmark used in Clarida, Gali, and Gertler (1999).

The Clarida, Gali, and Gertler model consists of an IS schedule, Phillips Curve, and Taylor Rule. The Taylor Rule here is restated from above. The economy is characterized by the following equations:

$$\pi_t = \alpha_0 + \alpha_1 \pi_{t-1} + \alpha_2 E_t \pi_{t+r} + \alpha_3 y_{t-i} + \epsilon_t \quad (6)$$

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 E_t y_{t+s} - \beta_3 (i_{t-j} - E_{t-j} \pi_{t-j+1}) + \eta_t \quad (7)$$

$$i_t = \rho i_{t-1} + (1 - \rho) [\phi_0 + \phi_\pi E_t \pi_{t+k} + \phi_y E_t y_{t+l} + \sigma_t] \quad (8)$$

Equation (6) is a standard expectations-augmented New Keynesian Phillips Curve, derived from the Calvo price-setting assumption for monopolistically competitive firms. A priori, all coefficients are assumed to be positive; inflation depends positively both on lagged inflation and on expected future inflation; α_3 ties the inflation process to the (lagged) output gap. As written, the expectations term implies that the private sector anticipates inflation r periods ahead. There are a variety of justifications for including lagged inflation in the Phillips Curve; most pertinent here is that its inclusion is what drives the short-run tradeoff between inflation and the output gap; otherwise inflation would adjust instantly to changes in the output gap.

Equation (7) is the model's IS curve, with all variables as defined in the previous section: y_t denoting the output gap, π_t the inflation rate and i_t the nominal interest rate. This aggregate relationship may be derived from the microfoundations of a representative household's consumption Euler equation. The output gap y_t is dependent on the previous period's output gap y_{t-1} to reflect persistence, as well as the expected output gap $E_t y_{t+s}$ to capture the forward-looking behavior of agents in the model, who anticipate changes in the output gap s periods ahead. Finally, the output gap responds to the real interest rate in the usual manner. All coefficients are *a priori* expected to be positive; that is, there is an expected negative relationship between the output gap and the real interest rate, while both output gap persistence and output gap expectations are expected to factor positively in the current output gap.

Equation (8) is simply the Taylor Rule restated, which behaves as previously defined. The lagged interest rate coefficient ρ captures interest-rate smoothing and is expected to be less than unity; the remaining slope coefficients are expected to be positive, so that the central bank reacts positively to expected increases in both inflation and increases in the output gap (recalling that negative levels of the output gap represent recessionary periods, while positive levels of the output gap represent expansionary periods relative to trend).

The stylized equations above are written with variable leads in the expectations terms; in implementation these will be tied down in a manner that ensures strong model fit. The most important substantive element of the model is its lag structure. The interest-rate transmission mechanism evolves as follows: interest rate policy implemented at time t affects the output gap with a lag of j periods, which then affects the inflation rate with an additional lag of i periods; hence, the effect of monetary policy on inflation via the interest-rate transmission process occurs with a lag of $i + j$ periods. The effect of policy on current expectations of inflation, however, is immediate and affects current inflation at the rate α_2 .

6 Inflation Pressure: Results

6.1 Regression estimation

The entire model is estimated separately for each regime, equation-by-equation. Model results for the IS schedule and Phillips Curve are given in tables 3 and 4. All estimates are obtained using Hanson's (1982) GMM estimator for expectational variables: expectations of future variables were replaced with their actual future values and instrumented accordingly. Instruments used are lagged values of the output gap, inflation rate, and the 3-month T-bill rate as well as the 3-month interbank rate as an alternate measure of the interest rate. Since the validity of the inflation pressure analysis depends crucially on the strength of these model estimates, it is worthwhile to go through them in some detail.

The IS curve estimates are relatively weak compared to the Phillips Curve and Taylor Rule; this was true throughout an extensive specification exercise. The backward- and forward-looking components are significant in most samples; the forward-looking component (expectations of the future output gap) is insignificant only in the second period. While significant in all periods, the

magnitude of backward-looking behavior changes considerably through the subperiods. The rate of output gap persistence rises from one-third to two-thirds in the first two periods before stabilizing around one-half for the latter two periods. The degree of forward-looking behavior hovers around one-half in most periods, plunging to one-third in the second period. The real interest rate term is negative in all samples and is significant in three of four samples; its magnitude is lowest in the third period (in which it is also statistically insignificant) and the fourth period (in which it is significant in a one-sided test). The insignificance of the real interest rate term is a concern in the third subperiod; since the direct monetary transmission mechanism is through interest rates, inflation pressure estimates for the third subperiod must be treated with caution.

The Phillips Curve estimates are satisfactory. The forward- and backward-looking components are of the correct sign and have reasonable magnitudes; only one of the eight coefficients is not statistically significant at the 5% level (the forward-looking component for the fourth period). The degree of inflation persistence hovers around 0.75 to 0.80 in all samples; there is heterogeneity in the forward-looking component, with the magnitude of the forward-looking component declining in the third period before recovering in the fourth period. In some ways, this is significant: it implies that the degree of forward-looking behavior declined during the exchange-rate targeting phase, but recovered once the Bank of England implemented inflation targeting. In no case is the backward-looking component large enough to incite unit root concerns. The output gap coefficient is positive in all samples, as predicted by the theory, and is significant in all samples. Only the estimate for the final period is weak; this may be attributed to the fourth period capturing more long-run behavior than the other three (shorter) periods.

Taylor Rule results were reported in section 4. Again, the degree of interest-rate persistence is safely below unit-root levels in all samples. Coefficients behave as expected with the sole exception of the second period, in which the output gap response coefficient is negative and statistically insignificant. That the output gap coefficient points in the “wrong direction” has strong implications for the inflation pressure estimates; while the qualitative level of inflation pressure remains informative, the exact quantitative estimates are contaminated by this perverse behavior in the Taylor Rule. Again, this negative but insignificant coefficient matches the findings of ACG (2005) and Nelson (2001) for periods covering similar dates.

6.2 Ex Ante Inflation Pressure

With the benchmark model of (6) - (8) in hand and specification (timing of lagged and expectational variables) pinned down as in the previous section, we may return to inflation pressure within the context of the model. Inflation pressure is calculated as a counterfactual: how would inflation have evolved in the presence of a one-period deviation of the central bank from its Taylor Rule, instead leaving the policy rate unchanged? Since interest rates affect inflation only with a lag, to calculate inflation pressure it is necessary to iterate backwards through the Phillips Curve until all variables are expressed in terms of the $i + j$ lag. I demonstrate this using the IS and PC relationships derived in the empirical results: the control lag is three periods (one period for the output gap in the Phillips Curve, two more for the interest rate in the IS curve), the forward-looking component on inflation is two periods in the Phillips Curve, and the forward-looking component in the IS curve is one period. Then the policy experiment consists of holding i_{t-4} equal to i_{t-3} . Iterating once through the IS-PC relationship:

$$\begin{aligned}
\pi_t &= \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2 E_t \pi_{t+2} + \alpha_3 y_{t-1} + \epsilon_t \\
\pi_t &= \alpha_0 + \alpha_1 \{ \alpha_0 + \alpha_1 \pi_{t-2} + \alpha_2 E_{t-1} \pi_{t+1} + \alpha_3 y_{t-2} + \epsilon_{t-1} \} \\
&\quad + \alpha_2 E_t \pi_{t+2} + \alpha_3 \{ \beta_0 + \beta_1 y_{t-2} + \beta_2 E_{t-1} y_t - \beta_3 (i_{t-3} - E_{t-3} \pi_{t-2}) + \eta_t \} \\
&\quad + \epsilon_t
\end{aligned}$$

yields π_t as a function of π_{t-2} , i_{t-3} , y_{t-2} and five expectational variables. Iterating once more would yield π_t in terms of π_{t-3} , y_{t-3} , i_{t-3} and i_{t-4} along with several expectational variables.

Once this process is complete, current inflation will be a function of y_{t-4} , π_{t-4} , i_{t-3} , i_{t-4} and i_{t-5} as well as expectational terms and shocks:

$$\begin{aligned}
\pi_t &= A_0 + A_1 \pi_{t-4} + A_2 y_{t-4} + A_3 i_{t-3} + A_4 i_{t-4} + A_5 i_{t-5} \\
&\quad + B_1 \epsilon_{t-3} + B_2 \epsilon_{t-2} + B_3 \epsilon_{t-1} + \epsilon_t + B_4 \eta_{t-3} + B_5 \eta_{t-2} + B_6 \eta_{t-1} \\
&\quad + C_1 E_t \pi_{t+2} + C_2 E_{t-1} \pi_{t+1} + C_3 E_{t-2} \pi_t + C_4 E_{t-3} \pi_{t-1} \\
&\quad - C_5 E_{t-3} \pi_{t-2} - C_6 E_{t-4} \pi_{t-3} - C_7 E_{t-5} \pi_{t-4} \\
&\quad + C_8 E_{t-1} y_t + C_9 E_{t-2} y_{t-1} + C_{10} E_{t-3} y_{t-2}
\end{aligned}$$

where the A , B and C coefficients are nonlinear combinations of the α and β coefficients in the IS and Phillips curves. The various expectational terms are themselves iterated back to time $t - 4$ by the use of a rational-expectations-derived Minimal State Variable (MSV) solution. Ex ante inflation pressure is then calculated by holding i_{t-3} equal to i_{t-4} , quarter-by-quarter:

$$\begin{aligned} E A I P_t &= \pi_t^0 - \pi_{t-1} \\ &= \Delta\pi_t + \Omega x_t^e + \alpha_3 \beta_3 \Delta i_{t-3} \end{aligned} \quad (9)$$

where Ω is a vector of coefficients and x_t^e is the vector of expectational terms used in the above expression. A derivation of this expression is available in the Appendix.

6.3 Ex Post Inflation Pressure

Ex post inflation pressure denotes a measurement experiment given the policy that was actually implemented. Each of the expectational terms, then, is held constant. For the purposes of the measurement experiment, we substitute the IS curve into the PC equation and hold i_{t-4} equal to i_{t-3} , so that:

$$\pi_t^w = \alpha_0 + \alpha_1 \pi_{t-1} + \alpha_2 E_t \pi_{t+2} + \alpha_3 \{ \beta_0 + \beta_1 y_{t-2} + \beta_2 E_{t-1} y_t - \beta_3 (i_{t-4} - E_{t-3} \pi_{t-2}) + \eta_t \} + \epsilon_t$$

Ex-post inflation pressure is then

$$\begin{aligned} E P I P_t &= \pi_t^w - \pi_{t-1} \\ &= \Delta\pi_t + \alpha_3 \beta_3 \Delta i_{t-3} \end{aligned} \quad (10)$$

The previous expression is then applied to inflation in each quarter, using the values of α_3 and β_3 for their respective subperiod. A derivation of this expression is provided in the Appendix.

By contrast, ex ante inflation pressure considers the case where the one-period deviation from the Taylor Rule is fully anticipated by all agents. Hence, the effect on inflation is not only through the interest rate change (Δi_{t-3}) but also through the change in agents' expectations. Hence one cannot hold expectations constant; instead, we calculate the change in agents' expectations and

take these into account when defining ex ante inflation pressure.

6.4 Numerical Estimates of Inflation Pressure

Ex post inflation pressure is calculated and graphed on figure 2. Ex Post inflation pressure tends to be small: fewer than five basis points, and frequently below three basis points, throughout the entire sample period. This result is quite similar to that found in Weymark (2009), which finds that ex post inflation pressure tends to be on the order of a few basis points for Canada and Australia. Note however that since the actual change in the interest rate itself tends to be relatively small on a quarterly basis, it is indeed expected for EPIP to itself be small.

However, note that ex post inflation pressure moderates considerably after 1991; in some ways this is the result of changes in the interest rate being smaller in magnitude since the end of the ERM period. Ex Post inflation pressure was more variable during early policy regimes, and is especially erratic where there is a change between policy regimes. A fuller discussion of how the change in regimes impacted inflation must wait until calculations of ex ante inflation pressure are completed.

Ex Ante inflation pressure is plotted against the actual change in inflation in figures 3 and 4. For figure 3, note that in the first period, the monetary authority was successful in moderating the change in inflation that would have occurred if the policy authority had not changed its policy rate. indeed, it is most clear from period 1 what inflation pressure is measuring with respect to realized inflation: the central bank's actions are moderating what would have been large swings in inflation that would have occurred in the absence of central bank intervention. The second half of figure 3 shows dramatic swings in ex ante inflation pressure; while the numerical estimates here are of only weak reliability, the qualitative message is that the underlying inflationary environment faced by policymakers in the mid- to late-1980s was highly unstable. Given that much of the change in the inflationary environment is driven by changes in expectations, the substantive takeaway here is that the monetary policy authority had little to no control of private-sector expectations during this period, and the unraveling of expectations manifested itself in a wildly unpredictable inflationary environment. Returning to figure 1, which shows the macro aggregates for the UK, we see that rate of inflation proper declined during the mid- to late-1980s; what the inflation pressure analysis uncovers is that the substantial gains realized in inflation reduction were not accompanied

by a moderation in inflationary expectations by the public. These results corroborate the narrative history insofar as the Bank's policy during this period was erratic with respect to the instrument of policy and the path of the target variable.

In figure 4, note that the underlying inflationary environment, as measured by EAIP, is remarkably close to the actual difference in inflation. This implies a great deal of control by the central bank of private-sector expectations. This anchoring of expectations resulted in a mild inflationary environment, so that even a one-period deviation of the central bank from its inflation target would not drastically affect inflationary expectations. The anchoring of inflation expectations began in the third subperiod, during which time the Bank of England committed itself to a variety of constraints in the form of soft exchange-rate targets, while during the fourth period inflation pressure is almost indistinguishable from realized inflation, implying that an announced one-period deviation of the central bank from its Taylor Rule, with the bank returning to its Taylor Rule thereafter, would not significantly affect inflationary expectations or macroeconomic performance.

7 Conclusion

This paper reappraised the UK Taylor Rule under regime change using a time-series breakpoint technique. The statistical breaks in the inflation series were found to correlate closely with the announcement of changes in the stance of monetary policy, indicating that policymakers' announcements about the direction of policy affect inflation. Further, by estimating Taylor Rules separately for each subperiod it was found that the performance of monetary policy, measured by the size of the coefficients on the Taylor Rule, has improved since the mid-1970s.

The aggressiveness of policymakers towards inflation (as measured by the magnitude of the inflation response coefficient in the Taylor Rule) has increased markedly since the late 1970s. In addition, responsiveness to changes in the output gap has increased and become more stable; from an insignificant response of uncertain sign to a consistently positive level of response. This maturation of the decision-making process can be seen in the data: inflation has moved from high and variable levels in the 1970s through 1980s to low and stable levels since the beginning of inflation targeting in 1993.

The inflation pressure exercise has shown that British central bank policy has become more

effective over time in controlling the underlying inflationary environment that it faces; connecting this to the narrative history suggests strongly that adherence to some sort of policy rule, be it exchange-rate or inflation-targeting, aids tremendously in achieving this goal. Despite the erratic behavior of EAIP in the first and second periods, however, the Bank of England did manage to translate a highly unstable inflationary environment into important improvements in realized inflation. The Bank's stabilization efforts in the mid-1980s were successful in spite of the environment in which it worked; the Bank's successes in the 1990s and 2000s were matched by equally impressive improvements in underlying inflationary expectations.

8 Tables

Table 1: Bai-Perron RSS and BIC output

Number of breaks =	0	1	2	3	4	5
RSS	189.15	159.07	155.01	152.31	151.36	150.72
BIC	445.55	436.74	447.96	460.30	474.19	488.35

RSS: Residual Sum of Squares; BIC: Bayesian Information Criterion

BIC is minimized for one break; RSS shows little improvement after 3 breaks.

Table 2: Taylor Rule. Dependent variable: i_t

	1976:I-1981:IV	1982:I-1987:II	1987:III-1992:II	1992:III-2007:IV
i_{t-1}	0.723** (3.22)	0.504*** (3.58)	0.695*** (5.38)	0.909*** (21.23)
$E_t\pi_{t+2}$	0.217* (2.25)	0.201 (0.94)	0.327* (2.52)	0.623*** (3.85)
$E_t y_{t+2}$	0.171 (0.65)	-0.345 (-0.91)	0.252*** (4.00)	0.403*** (3.45)
Constant	0.067 (0.43)	0.334 (1.64)	-0.007 (-0.18)	-0.544*** (-3.56)
Response coefficients:				
ρ	0.723	0.504	0.695	0.909
ϕ_π	0.783	0.405	1.072	6.863
ϕ_y	0.615	-0.696	0.826	4.441
ϕ_0	0.241	0.673	-0.024	-5.992

Instruments: three lags each of t-bill, interbank rate, inflation, and the output gap.
t-statistics in parentheses.

Table 3: IS curve. Dependent: y_t

	1976:I-1981:IV	1982:I-1987:II	1987:III-1992:II	1992:III-2007:IV
y_{t-1}	0.370** (3.24)	0.637*** (3.48)	0.486*** (4.76)	.494*** (7.19)
$E_t y_{t+1}$	0.550*** (7.17)	0.324 (1.32)	0.544*** (6.17)	0.510*** (5.04)
$i_{t-2} - E_{t-2} \pi_{t-1}$	-0.066* (-2.22)	-0.111+ (-1.80)	-0.042 (-1.28)	-0.042+ (-1.93)
Constant	-0.001 (-1.00)	0.005 (1.19)	0.002 (1.26)	0.001+ (1.65)
N	16	22	20	61

Instruments: six lags each of interbank rate and the output gap.
 t -statistics in parentheses.

Table 4: Phillips Curve, periods 1-3. Dependent: π_t

	1976:I-1981:IV	1982:I-1987:II	1987:III-1992:II	1992:III-2007:IV
π_{t-1}	0.802*** (15.35)	0.727*** (14.25)	0.810** (21.24)	0.790*** (6.28)
$E_t \pi_{t+2}$	0.371*** (5.23)	0.428*** (3.61)	0.141* (2.13)	0.208 (0.89)
y_{t-1}	0.261** (3.21)	0.601** (2.87)	0.140* (2.46)	0.107+ (1.69)
Constant	-0.024*** (-4.75)	-0.003 (-0.80)	0.002 (1.05)	0.000 (-0.01)
N	18	22	20	60

Instruments: four lags each of interbank rate and inflation.
 t -statistics in parentheses

Table 5: Breaks in inflation time-series. Dependent: π_t

	1976:I-1981:IV	1982:I-1987:II	1987:III-1992:II	1992:III-
π_{t-1}	0.678*** (5.36)	0.856*** (16.28)	0.879*** (9.62)	0.752*** (15.10)
Constant	0.362* (2.51)	0.149* (2.68)	0.129 (1.33)	0.254*** (4.97)
N	22	22	20	63

t -statistics in parentheses

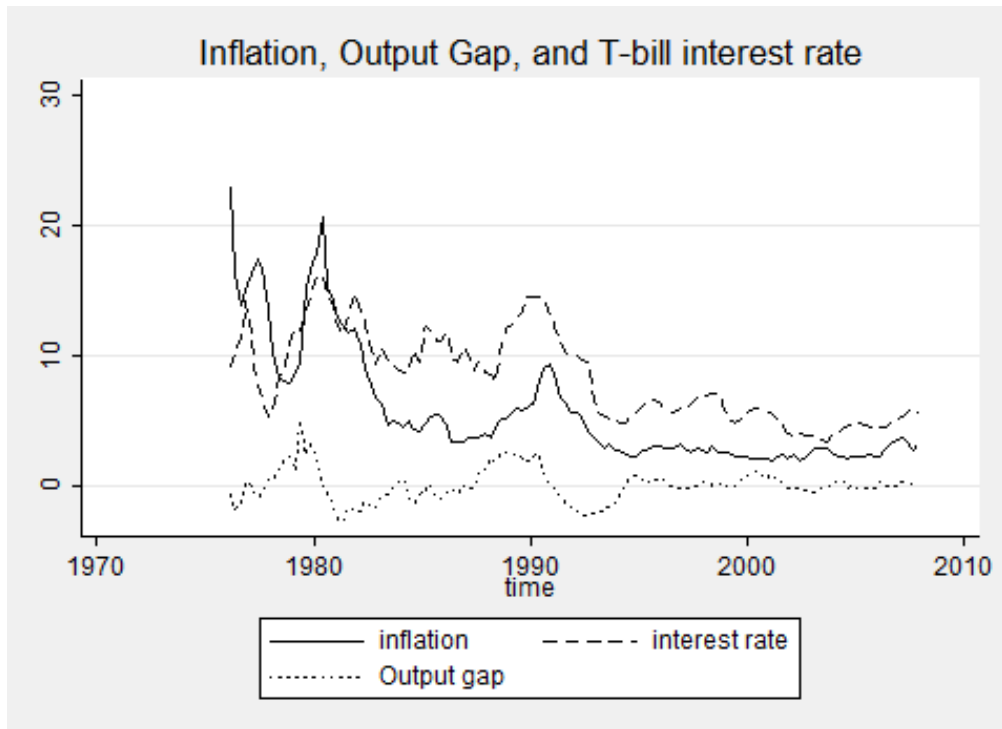


Figure 1: Macroeconomic time-series for the UK.

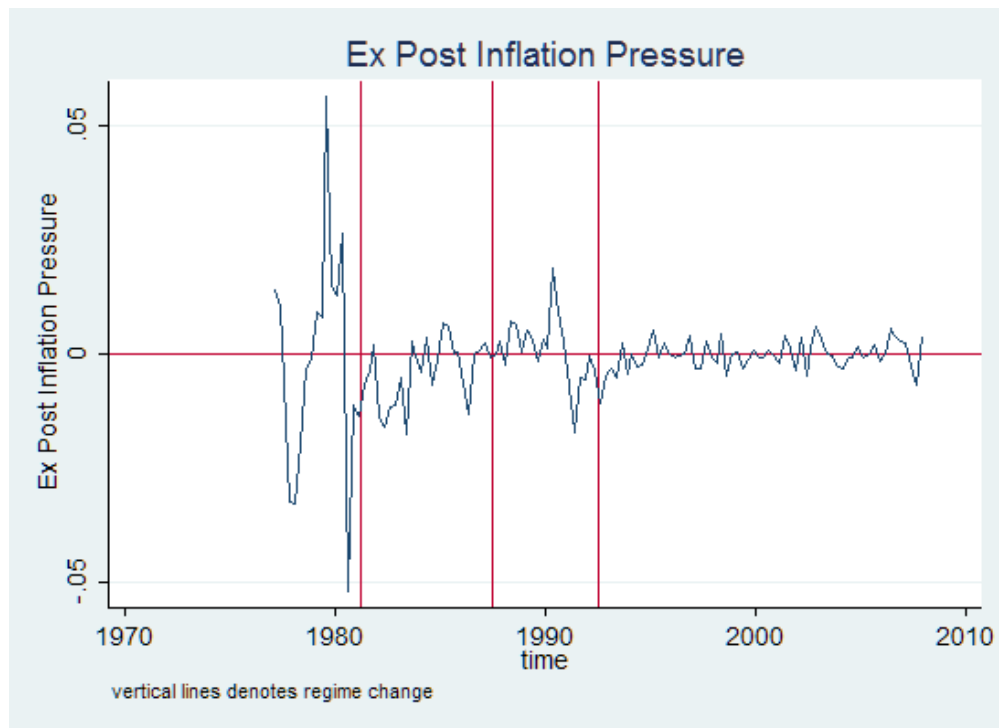


Figure 2: Ex Post Inflation Pressure.

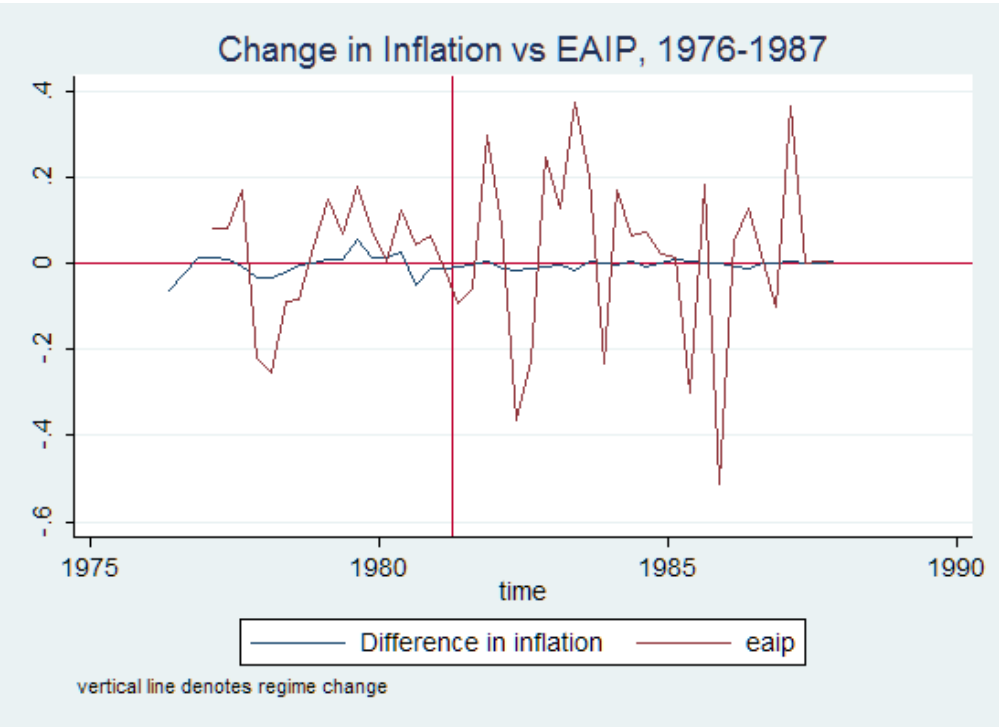


Figure 3: Ex Ante Inflation Pressure, First Two Periods

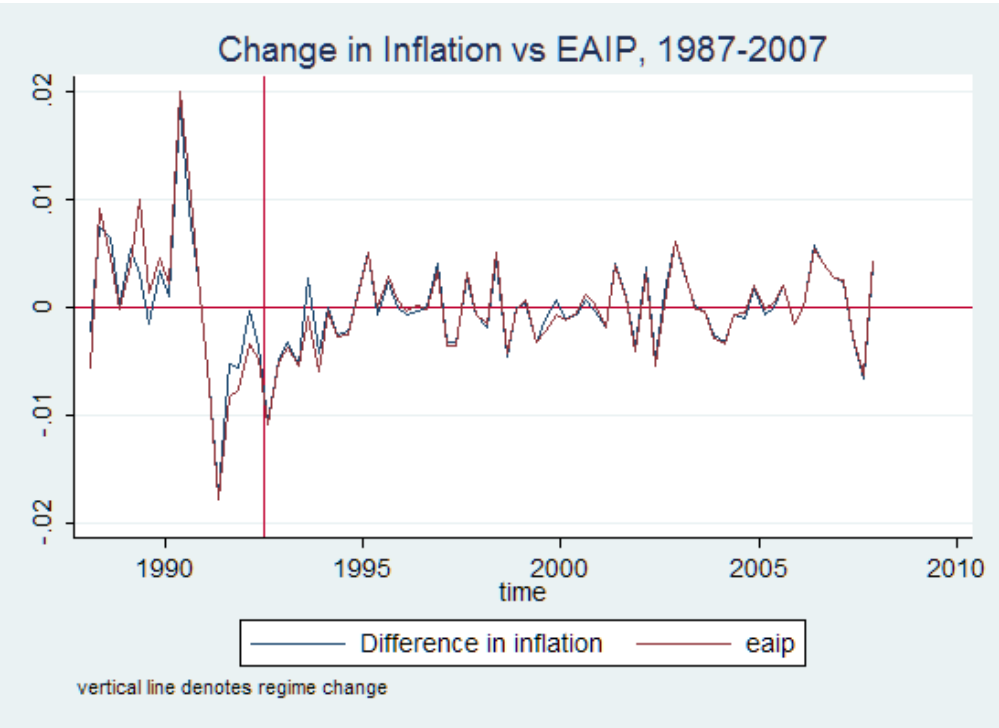


Figure 4: Ex Ante Inflation Pressure, Latter Two Periods

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A Derivation of Inflation Pressure indicies

Throughout this appendix, I use the following equations for IS, PC, and TR:

$$\begin{aligned}\pi_t &= \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2E_t\pi_{t+2} + \alpha_3y_{t-1} + \epsilon_t \\ y_t &= \beta_0 + \beta_1y_{t-1} + \beta_2E_t y_{t+1} + \beta_3(i_{t-2} - E_{t-2}\pi_{t-1}) + \eta_t \\ i_t &= \rho i_{t-1} + (1 - \rho)[\phi_0 + \phi_\pi E_t\pi_{t+2} + \phi_y E_t y_{t+2} + \sigma_t]\end{aligned}$$

In particular, note first the timing of the expectations; and second, that throughout the appendix β_3 is a negative number.

A.1 Ex Post Inflation Pressure

Begin with the IS-PC equation, substituting in one lag of the IS curve into the Phillips Curve:

$$\begin{aligned}\pi_t &= \alpha_0 + \alpha_3\beta_0 + \alpha_1\pi_{t-1} + \alpha_3\beta_1y_{t-2} + \alpha_3\beta_3i_{t-3} \\ &+ \alpha_2E_t\pi_{t+2} + \alpha_3\beta_2E_{t-1}y_t - \alpha_3\beta_3E_{t-3}\pi_{t-2} \\ &+ \alpha_3\eta_t + \epsilon_t\end{aligned}$$

Ex post inflation is a measurement experiment. Holding the policy environment (in this case, expectations) fixed, I calculate the level of inflation that would have prevailed if the policy authority had held its policy instrument fixed:

$$\begin{aligned}\pi_t^w &= \alpha_0 + \alpha_3\beta_0 + \alpha_1\pi_{t-1} + \alpha_3\beta_1y_{t-2} + \alpha_3\beta_3i_{t-4} \\ &+ \alpha_2E_t\pi_{t+2} + \alpha_3\beta_2E_{t-1}y_t - \alpha_3\beta_3E_{t-3}\pi_{t-2} \\ &+ \alpha_3\eta_t + \epsilon_t\end{aligned}$$

Notice that the coefficient on $\alpha_3\beta_3$ is now i_{t-4} and that all other terms are the same as in the actual inflation equation.

hence:

$$\begin{aligned}\pi_t - \pi_t^w &= \alpha_3\beta_3(i_{t-3} - i_{t-4}) \\ \pi_t^w &= \pi_t - \alpha_3\beta_3\Delta i_{t-3}\end{aligned}$$

By definition, ex post inflation pressure is

$$EPIP = \pi_t^w - \pi_{t-1} = \Delta\pi_t - \alpha_3\beta_3\Delta i_{t-3}$$

which completes the derivation.

A.2 Ex Ante Inflation Pressure

In this case, we allow agents to update their expectations about future variables. Ex Ante inflation pressure is a policy experiment, in which the inflation rate π_t^0 calculated is under a completely different policy regime than the observed π_t : in particular, π_t^0 is calculated under a one-period regime of $\rho = 1$ in the Taylor Rule.

Begin again with IS-PC:

$$\begin{aligned}\pi_t &= \alpha_0 + \alpha_3\beta_0 + \alpha_1\pi_{t-1} + \alpha_3\beta_1y_{t-2} + \alpha_3\beta_3i_{t-3} \\ &\quad + \alpha_2E_t\pi_{t+2} + \alpha_3\beta_2E_{t-1}y_t - \alpha_3\beta_3E_{t-3}\pi_{t-2} \\ &\quad + \alpha_3\eta_t + \epsilon_t\end{aligned}$$

Now, this is an equation which contains π_{t-1} and y_{t-2} . Both of these depend on i_{t-3} and i_{t-4} implicitly. Hence we iterate those variables backwards and continue to do so until π_t is an equation in terms of π_{t-4} , y_{t-4} , expectational variables, and errors.

Iterate back once:

$$\begin{aligned}\pi_t &= \alpha_0 + \alpha_3\beta_0 + \alpha_1\{\alpha_0 + \alpha_1\pi_{t-2} + \alpha_2E_{t-1}\pi_{t+1} + \alpha_3y_{t-2} + \epsilon_{t-1}\} + \alpha_3\beta_1y_{t-2} + \alpha_3\beta_3i_{t-3} \\ &\quad + \alpha_2E_t\pi_{t+2} + \alpha_3\beta_2E_{t-1}y_t - \alpha_3\beta_3E_{t-3}\pi_{t-2} \\ &\quad + \alpha_3\eta_t + \epsilon_t\end{aligned}$$

to obtain an equation for π_t entirely in terms of π_{t-2} and y_{t-2}

Once this process has been continued two more times (substituting in for $t-2$ variables in terms of $t-3$, then substituting in for $t-3$ variables in terms of $t-4$) one obtains the somewhat clumsy expression:

$$\begin{aligned}
\pi_t = & \alpha_0 + \alpha_0\alpha_1 + \alpha_3\beta_0 + \alpha_0\alpha_1^2 + \alpha_3\beta_0(\alpha_1 + \beta_1) + \alpha_0\alpha_1^3 + \alpha_3\beta_0(\alpha_1^2 + \beta_1^2 + \alpha_1\beta_1) \\
& + \alpha_2E_t\pi_{t+2} + \alpha_1\alpha_2E_{t-1}\pi_{t+1} + \alpha_1^2\alpha_2E_{t-2}\pi_t + \alpha_1^3\alpha_2E_{t-3}\pi_{t-1} \\
& - \alpha_3\beta_3E_{t-3}\pi_{t-2} - \alpha_3\beta_3(\alpha_1 + \beta_1)E_{t-4}\pi_{t-3} - \alpha_3\beta_3(\alpha_1^2 + \beta_1^2 + \alpha_1\beta_1)E_{t-5}\pi_{t-4} \\
& + \alpha_3\beta_2E_{t-1}y_t + \alpha_3\beta_2(\alpha_1 + \beta_1)E_{t-2}y_{t-1} + \alpha_3\beta_2(\alpha_1^2 + \beta_1^2 + \alpha_1\beta_1)E_{t-3}y_{t-2} \\
& + \alpha_1^4\pi_{t-4} + (\alpha_1^3\alpha_3 + \beta_1\alpha_3(\alpha_1^2 + \beta_1^2 + \alpha_1\beta_1))y_{t-4} \\
& + \alpha_3\beta_3i_{t-3} + \alpha_3\beta_3(\alpha_1 + \beta_1)i_{t-4} + \alpha_3\beta_3(\alpha_1^2 + \beta_1^2 + \alpha_1\beta_1)i_{t-5} \\
& + \alpha_1^3\epsilon_{t-3} + \alpha_1^2\epsilon_{t-2} + \alpha_1\epsilon_{t-1} + \epsilon_t + \alpha_3(\alpha_1^2 + \beta_1^2 + \alpha_1\beta_1)\eta_{t-3} + \alpha_3(\alpha_1 + \beta_1)\eta_{t-2} + \alpha_3\eta_{t-1}
\end{aligned}$$

Letting A_i , B_i and C_i 's replace the coefficients, the above simplifies to:

$$\begin{aligned}
\pi_t = & A_0 + A_1\pi_{t-4} + A_2y_{t-4} + A_3i_{t-3} + A_4i_{t-4} + A_5i_{t-5} \\
& + B_1\eta_{t-3} + B_2\eta_{t-2} + B_3\eta_{t-1} + B_4\epsilon_{t-3} + B_5\epsilon_{t-2} + B_6\epsilon_{t-1} + \epsilon_t \\
& + C_1E_t\pi_{t+2} + C_2E_{t-1}\pi_{t+1} + C_3E_{t-2}\pi_t + C_4E_{t-3}\pi_{t-1} \\
& - C_5E_{t-3}\pi_{t-2} - C_6E_{t-4}\pi_{t-3} - C_7E_{t-5}\pi_{t-4} \\
& + C_8E_{t-1}y_t + C_9E_{t-2}y_{t-1} + C_{10}E_{t-3}y_{t-2}
\end{aligned}$$

The first line groups the constant term with past variables. The second line groups all shock terms. The third line contains all two-period-ahead expectations from the Phillips Curve; the fourth and fifth lines, the one-period-ahead expectations from the IS curve.

For Ex Ante inflation, we (1) set the interest rate $i_{t-3} = i_{t-4}$ and additionally (2) allow agents to update their expectations given that $\Delta i_{t-3} = 0$; these updated expectations are denoted E^0 .

Hence ex ante inflation π_t^0 is:

$$\begin{aligned}
\pi_t^0 &= A_0 + A_1\pi_{t-4} + A_2y_{t-4} + (A_3 + A_4)i_{t-4} + A_5i_{t-5} \\
&+ B_1\epsilon_{t-3} + B_2\epsilon_{t-2} + B_3\epsilon_{t-1} + \epsilon_t + B_4\eta_{t-3} + B_5\eta_{t-2} + B_6\eta_{t-1} \\
&+ C_1E_t^0\pi_{t+2} + C_2E_{t-1}^0\pi_{t+1} + C_3E_{t-2}^0\pi_t + C_4E_{t-3}^0\pi_{t-1} \\
&+ C_5E_{t-1}^0y_t + C_6E_{t-2}^0y_{t-1} + C_7E_{t-3}^0y_{t-2} \\
&- C_8E_{t-3}^0\pi_{t-2} - C_9E_{t-4}^0\pi_{t-3} - C_{10}E_{t-5}^0\pi_{t-4}
\end{aligned}$$

Just as in EPIP, we calculate the difference in π_t and π_t^0 and rearrange to obtain a formula for π_t^0 in terms of π_t , Δi_{t-3} , and expectational terms. Notice, however, that two of these ex ante counterfactual expectations, $E_{t-4}^0\pi_{t-3}$ and $E_{t-5}^0\pi_{t-4}$, are predetermined in $t-3$ and hence are equal to their real-world counterparts. Hence, they will vanish and we must only keep track of eight expectational terms:

$$\begin{aligned}
\pi_t - \pi_t^0 &= A_3\Delta i_{t-3} + \sum_{j=1}^8 C_j(E x_j - E^0 x_j) \\
\pi_t^0 &= \pi_t - A_3\Delta i_{t-3} - \sum_{j=1}^8 C_j(E x_j - E^0 x_j)
\end{aligned}$$

where the x_j are the expectational terms associated with the C_j .

What remains is the determination of those expectations. A rational expectations equilibrium exists in which the variables π_t , y_t and i_t may be expressed in minimal-state variable (MSV) form as:

$$\begin{aligned}
\pi_t &= g_0 + g_1\pi_{t-1} + g_2y_{t-1} + g_3i_{t-1} + g_4i_{t-2} + g_5\epsilon_{t-1} + g_6\eta_{t-1} + g_7\sigma_{t-1} + g_8\epsilon_t + g_9\eta_t + g_{10}\sigma_t \\
y_t &= h_0 + h_1\pi_{t-1} + h_2y_{t-1} + h_3i_{t-1} + h_4i_{t-2} + h_5\epsilon_{t-1} + h_6\eta_{t-1} + h_7\sigma_{t-1} + h_8\epsilon_t + h_9\eta_t + h_{10}\sigma_t \\
i_t &= k_0 + k_1\pi_{t-1} + k_2y_{t-1} + k_3i_{t-1} + k_4i_{t-2} + k_5\epsilon_{t-1} + k_6\eta_{t-1} + k_7\sigma_{t-1} + k_8\epsilon_t + k_9\eta_t + k_{10}\sigma_t
\end{aligned}$$

which, again, is a recursive formula so that repeated back-substitution may be employed to express these in terms of i_{t-3} and i_{t-4} . The estimation of these g , h , and k coefficients is via Christopher

Sims' numerical GENSYS algorithm. ²

I demonstrate the process of computing ex ante expectations for a representative expectation, $E_{t-2}\pi_t$:

$$\begin{aligned}
E_{t-2}\pi_t &= g_0 + g_1\pi_{t-1} + g_2y_{t-1} + g_3i_{t-1} + g_4i_{t-4} \\
&= g_0 + g_1(g_0 + g_1\pi_{t-2} + g_2y_{t-2} + g_3i_{t-2} + g_4i_{t-3} + g_5\epsilon_{t-2} + g_6\eta_{t-2} + g_7\sigma_{t-2}) \\
&\quad + g_2(h_0 + h_1\pi_{t-2} + h_2y_{t-2} + h_3i_{t-2} + h_4i_{t-3} + h_5\epsilon_{t-2} + h_6\eta_{t-2} + h_7\sigma_{t-2}) \\
&\quad + g_3(k_0 + k_1\pi_{t-2} + k_2y_{t-2} + k_3i_{t-2} + k_4i_{t-3} + k_5\epsilon_{t-2} + k_6\eta_{t-2} + k_7\sigma_{t-2}) + g_4i_{t-2}
\end{aligned}$$

continue to back-substitute and collect terms, and the final expression is of the form:

$$\begin{aligned}
E_{t-2}\pi_t &= G_0 + G_1\pi_{t-3} + G_2y_{t-3} + G_3i_{t-3} + G_4i_{t-4} \\
&\quad + G_5\epsilon_{t-2} + G_6\eta_{t-2} + G_7\sigma_{t-2} + G_8\epsilon_{t-3} + G_9\eta_{t-3} + G_{10}\sigma_{t-3}
\end{aligned}$$

From here it is possible to hold $i_{t-3} = i_{t-4}$, so that:

$$\begin{aligned}
E_{t-2}^0\pi_t &= G_0 + G_1\pi_{t-3} + G_2y_{t-3} + (G_3 + G_4)i_{t-4} \\
&\quad + G_5\epsilon_{t-2} + G_6\eta_{t-2} + G_7\sigma_{t-2} + G_8\epsilon_{t-3} + G_9\eta_{t-3} + G_{10}\sigma_{t-3}
\end{aligned}$$

Subtracting $E_{t-2}^0\pi_t$ from $E_{t-2}\pi_t$ yields a simple expression:

$$E_{t-2}\pi_t - E_{t-2}^0\pi_t = G_3(i_{t-3} - i_{t-4})$$

Similarly, each $Ex_j - E^0x_j$ may simply be expressed as $R_j\Delta i_{t-3}$. Hence, ex ante inflation and

²MATLAB routines and documentation regarding the implementation of GENSYS and how to interpret GENSYS output may be found at <http://sims.princeton.edu/yftp/gensys/>

EAIP are expressed compactly by:

$$\pi_t^0 = \pi_t - \alpha_3 \beta_3 \Delta i_{t-3} - \sum_{j=1}^8 C_j R_j \Delta i_{t-3} \quad (11)$$

$$EAIP = \pi_t^0 - \pi_{t-1} = \Delta \pi_t - \alpha_3 \beta_3 \Delta i_{t-3} - \sum_{j=1}^8 C_j R_j \Delta i_{t-3} \quad (12)$$

This completes the derivation.