

A Methodological Exploration of Financial and Economic Crises

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## INTRODUCTION

Following the 2008 Financial Crisis, the field of Macroeconomics entered a remarkable phase of methodological transformation. The reason for this is twofold. First, the standard approach of locally approximating equilibrium near a deterministic steady state is naturally ill-suited to incorporate occasionally binding constraints, multiple equilibria, or multiple steady states at least one of which is typically required to motivate occasional crisis episodes. Second, a consensus has emerged that in addition to the longitudinal evolution of aggregates such as output and inflation, macroeconomics further also ought to be able to address statistics that represent cross-sectional inequality. In effect, since providing insights into deep downturns requires a set of tools that are presently non-standard, my dissertation's topical focus of financial and economic crises has naturally lent itself to be complemented by an appraisal of contemporary macroeconomic methodology.

In Chapter One, I mimic the 2008 Financial Crisis by incorporating occasional financial fire sales into a macroeconomic framework. The financial sector's fragility arises from an occasionally binding maintenance margin imposed on investors by a broker. When binding, the margin constraint forces investors to liquidate assets in which case households lose a potentially substantial fraction of their accumulated retirement savings. In an effort to make up for lost savings following the fire sale, households curb consumer spending to which firms respond by scaling back production via employment and investment. Methodologically, the proposed framework serves as an illustration of the high degrees of state and parametric heterogeneity that can be accommodated when model primitives are chosen free of the constraints imposed by contemporary practice. Specifically, to mimic real-world decision-making, I depart from the literature by requiring that each agent's optimization problem be trivial to solve numerically which allows for a recursive simulation of the proposed economy by globally resolving each agent's problem each period.

In Chapter Two, I examine the gradual deterioration of the Greek state of sovereign finance after the country's yields switched from seemingly stationary to mildly explosive in early 2010. To rationalize the gradual nature of the ensuing crisis episode, I propose a sovereign debt model in which investors' reliance on external credit ratings causes debt crises to be slow moving. I find that the Greek state first became financially unsustainable in the fall of 2009, but the impending surge in yields remained latent until Greek credit ratings were downgraded by all major agencies in December of the same year. Given my model's multiple steady states, I further find that the Greek crisis may very well have been self-fulfilling, but even in the counterfactual event that perceived and actual credit risk had remained near zero beyond 2009, the resulting counterfactual Greek state would have been so fragile that eventual default was inevitable almost surely. Nevertheless, to the extent that time is a crucial factor in helping a country escape a financially fragile state, my framework's self-fulfilling, slow moving crises strongly support the notion that "breaking expectations" is an effective measure to combat a looming sovereign debt crisis.

In Chapter Three, I start with the observation that equilibrium selection in indeterminate



economies has for all intents and purposes remained ad-hoc. To add structure, I introduce the theoretical notion of resilience to formalize and quantitatively assess a stable equilibrium's capacity to absorb strategic perturbations while prevailing as a game's ultimately observed outcome. Adopting sunspots to represent such perturbations, I find that indeterminate economies typically contain pre-existing structures that lend themselves to be used for equilibrium selection. Specifically, these structures take the form of tipping points which 'physically' separate the empirically relevant, stable equilibria. Since unstable equilibria are tipping points, they thus serve a natural role in equilibrium selection. Using three canonical financial crises frameworks as examples, I show that to the extent that we care about the likelihood of observing bank runs and fire sales, we ought to care about the location such equilibria's unstable neighbors.

## CHAPTER I

# The Wealth Effect View of the Great Recession: A Behavioral Macroeconomic Model with Occasional Financial Fire Sales

### 1.1. Introduction

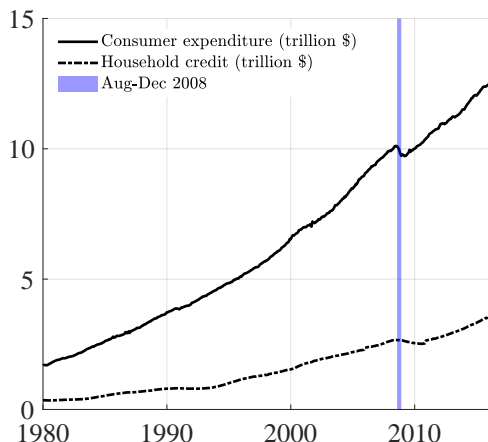
*How did the 2007-08 Financial Crisis morph into the Great Recession?* Over the course of the Great Recession, US employment fell by roughly 5% as seven million US workers were involuntarily separated from their employers and did not successfully find reemployment soon thereafter. The ensuing labor market recovery period spanned roughly a decade, a phenomenon that has become known as labor market *hysteresis*. The starting point of this paper is the assertion that the described, extraordinary labor market episode was *caused* by the preceding financial crisis.

Mian et al. (2013, 2014) document that the collapse in household net worth between 2007 and 2009 significantly and negatively affected consumer expenditure and, in turn, employment. To rationalize this effect, they propose that the observed aggregate demand slump may have either represented a voluntary response — the *wealth effect view of the Great Recession* — or it may have been mechanically forced by tightening consumer credit conditions — the *leverage view of the Great Recession*. While contemporary theory has predominantly emphasized the leverage view (Eggertsson and Krugman, 2012; Guerrieri and Lorenzoni, 2017), the latter has recently been challenged. Specifically, Jones, Midrigan, and Philippon (2018) argue that even though tightening borrowing constraints can potentially account for the slow recovery of the US economy following the Great Recession, they can in fact not — as also illustrated in Figure 1.1 — account for the abrupt and deep nature of the Great Recession itself.<sup>1</sup>

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<sup>1</sup>Similarly, Bernanke argues that the timing of the credit tightening does not align with

**Figure 1.1.** Consumer expenditure and consumer credit, 1980-2019



Notes: Figure 1.1 displays the evolution of US consumer expenditure and consumer credit between 1980 and 2019. If the observed slump in consumer expenditures in the fall of 2008 was indeed mechanical, we should expect to see — barring a major cross-sectional redistribution of credit — a one-for-one decrease of consumer credit for every dollar of lost consumer expenditure. Accordingly, the slow and gradual nature of the decline in consumer credit strongly suggests that the observed consumer response was in fact *not* mechanical, but voluntary.

In the spirit of Figure 1.1, the present paper thus argues that while the most recent financial crisis indeed did cause the Great Recession via aggregate demand, the corresponding, observed consumer response was driven by a voluntary wealth effect rather than by mechanical deleveraging. In turn, the paper’s main contribution lies in the theoretical formalization of the proposed wealth effect response to financial crises, namely by incorporating occasional financial fire sales into a macroeconomic general equilibrium model.

In terms of the particular variable of interest, my principal objective is to match the observed, extensive labor margin since 2008. I focus on the extensive margin because it serves as a natural first-order approximation of the degree to which financial crises affect households heterogeneously. For example, consider Figure 1.2 which depicts the evolution of decomposed US unemployment since 1987.

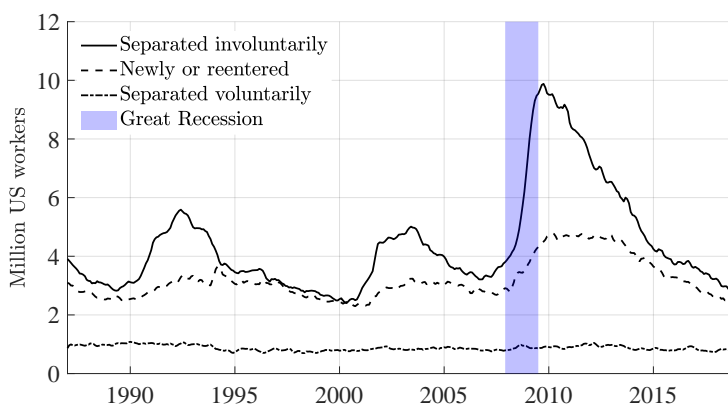
The fact that between 2008 and 2010, roughly seven million US workers were separated involuntarily and did not successfully find re-employment yields two main insights. First, the decline in employment observed during the Great Recession was driven by a broad slump in labor demand.<sup>2</sup>

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<sup>2</sup>Notice that a falling labor demand does not require an increase in corporate layoffs. In fact, “notwithstanding a spike in 2008”, separation only accounted for roughly one quarter of the observed variation in unemployment during the Great Recession (Shimer, 2012). To understand this, consider the following thought experiment. Suppose there is an exogenous, time-invariant separation rate. In this scenario, firms have to hire a certain number of workers each period just to retain a constant work force. To shrink their labor force during a downturn — when labor demand falls — firms then only need to reduce the number of new hires without ever having to lay off any workers.

Second, the crisis did not affect all households uniformly because some workers lost their employment involuntarily while others did not. Further notice that at any given time, only a small fraction of unemployed US workers report to choose unemployment voluntarily, but that the contrast between voluntary and involuntary unemployment was particularly stark during the Great Recession. Therefore, if financial crises cause extraordinary economic downturns and the corresponding rise in unemployment is overwhelmingly involuntary, the welfare implications of financial crises are particularly severe (see Shapiro and Stiglitz, 1985).

**Figure 1.2.** Decomposed stock of unemployed US workers, 1987-2019



Notes: Figure 1.2 displays the longitudinal evolution of decomposed US unemployment since 1987. The fact that roughly seven million US workers who were separated involuntarily did not successfully find re-employment between 2008 and 2010 yields two main insights. First, the observed contraction in employment reflects a broad slump in labor demand. Second, the crisis did not affect all households uniformly as some workers involuntarily lost their job while others did not. As stock variables, the displayed series effectively represent integrals over all corresponding past inflows (e.g. quits, layoffs, labor force entry) and outflows (e.g. hires, labor force exit). Figure 1.2 thus reveals the Great Recession’s *long-term* effects that remain hidden under the often cited quits, layoffs, and discharges measure. All data was taken from the Bureau of Labor Statistics (BLS) with job leavers and job losers (laid off + others) relabeled as voluntarily and involuntarily unemployed.

To formulate integrated policy recommendations for an economy that is subject to recurring crises, I embed both crisis origination and crisis transmission into a unified macroeconomic framework. Specifically, crisis origination is modeled explicitly because recognizing potential sources of financial fragility is instrumental in the process of *ex ante crisis prevention*. In turn, the fire sales are embedded in a macroeconomic model because we are not principally interested in the nominal crises themselves, but rather in their real effects. Understanding the relevant transmission mechanisms thus helps policy makers enact a sensible *crisis response* when a crisis is imminent.

Methodologically, I generate financial crises by way of an occasionally binding maintenance

margin which causes, if binding, a nominal collapse of the economy’s numéraire via fire sales.<sup>3</sup> In terms of the ensuing transmission, I propose the following wealth-effect-driven demand channel. First, during a financial crisis, households lose a potentially substantial fraction of their accumulated retirement savings. Second, to make up for the incurred losses in projected retirement wealth, households proceed by reducing consumer spending. Third, firms react to the upcoming slump in consumer demand by scaling back production via employment (and investment).<sup>4</sup>

To incorporate occasional financial crises into a macroeconomic framework, I propose the following key model elements. First, consumption goods producers must finance labor and capital rentals with commercial loans. Since production occurs prior to the realization of consumer demand, commercial loans are subject to credit risk. Banks follow an originate-to-distribute model with commercial loans being sold, securitized, and ultimately held by pension funds in the form of a collateralized loan obligation (CLO). In turn, an occasionally observed, fundamental demand signal yields a new secondary market CLO price with pension fund assets being marked-to-market accordingly. If a pension fund’s resulting equity position violates the prevailing maintenance margin, the broker issues a *margin call*, which induces, if liquidity is low, a *fire sale*.<sup>5</sup> Following the fire sale, households make up for lost retirement savings by curbing consumer spending.

To generate real effects from the nominal shock, I exploit the well established finding that wages are nominally downward rigid.<sup>6</sup> Specifically, I follow Solow (1979) and Akerlof and Yellen’s (1990) in assuming that exerted worker effort is sensitive to nominal wage cuts, in which case nominal demand slumps cause declines in demand for labor and investment. Recall, however, that beyond a sudden and sharp increase in unemployment, the Great Recession also featured a long and protracted labor market “hysteresis” (Yagan, 2019). To match the observed recovery period, I require an additional friction. For this, the labor market setup is borrowed from Weiss (1980) where

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<sup>3</sup>A maintenance margin is the minimum amount of equity required to maintain a margin account with a brokerage firm. If a declining market value of an investor’s portfolio causes equity to fall below the maintenance threshold, the broker issues a *margin call*. A margin call is a broker-issued liquidity demand with the aim of consolidating the corresponding investor’s equity position.

<sup>4</sup>Since production and sales are separated intratemporally in my model, firms rent capital and hire labor while demand is still uncertain.

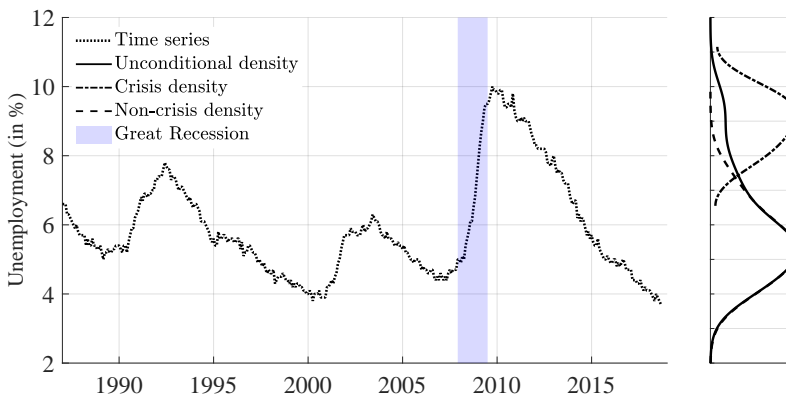
<sup>5</sup>Building on the extensive intuitive account provided by Shleifer and Vishny (2011), a fire sale is understood as a *forced* placement of sell orders irrespective of the corresponding asset’s fundamental value. Institutionally, fire sales thus constitute an example of a “portfolio-adjusting” trade, whereas orders based on new fundamental information may be described as “information-motivated” (Cuneo and Wagner, 1975). Margin calls serve as a natural way to induce investors to sell at a price at which they would not normally — in absence of the margin call — want to sell.

<sup>6</sup>See Fallick, Lettau, and Wascher (2016), Daly, Hobijn, and Lucking (2012), and Daly, Hobijn, and Wiles (2011) for recent empirical evidence that wages are nominally downward rigid.

information asymmetries between firms and workers give rise to a real downward wage rigidity which generates involuntary unemployment that persists. Specifically, since the lucrateness of each worker’s outside option is increasing in their skill, lowering wage offers always induces the highest skilled employees to quit first. Depending on the cross-sectional distribution of skill, adverse selection may then induce a threshold below which effective labor costs are *decreasing* in the wage. As a result, firms would rather face excess labor supply than paying wages below the threshold (Weiss, 1980).

The methodological contribution of the paper is twofold. First, the proposed framework rationalizes the statistics recorded during the Great Recession thereby elevating the corresponding ergodic tail out of the realm of statistical ‘outliers’. To illustrate this, consider Figure 1.3 which plots US unemployment against time and against its state space in the form of a set of estimated ergodic densities.

**Figure 1.3.** Decomposition of unemployment in crisis and non-crisis episodes



Notes: Figure 1.3 depicts three (normalized) estimated ergodic densities for US unemployment in the post-Volcker era. The unconditional density illustrates that unemployment exhibits a substantial upper tail with the two conditional densities confirming that said tail was recorded following the 2008 Financial Crisis. For purposes of partitioning the data, the “labor market crisis” episode is constructed by collecting all quarters that featured an unemployment rate higher than the previous peak in 1992. All density estimates were derived using the Gaussian kernel proposed in Botev et al. (2010) with mesh size  $2^{-7}$ . The data was recovered from BLS.

From the standpoint of economic intuition, it is unsurprising that unemployment reached its peak during the Great Recession. But if this is the case, we may conjecture that, as a general principle, if our economy of interest is subject to occasional crisis episodes, the data will be skewed, fat tailed, or, if downturns are particularly severe, even multimodal.<sup>7</sup> In the spirit of Stiglitz

<sup>7</sup>While Kocherlakota (2000) emphasizes the importance of exploring the business cycle’s cyclic asymmetry in general terms, Brunnermeier and Sannikov (2014) specifically argue that occasional crises episodes may give rise to ergodic multimodality. In either case, rationalizing the observed dynamics requires a strategy, theoretical and/or

(2018)<sup>8</sup>, one methodological contribution of this paper then lies in the *joint* rationalization of both the statistics recorded during crisis episodes as well as the statistics recorded during non-crisis episodes.

As a second methodological contribution, the proposed framework illustrates the high degrees of state and parametric heterogeneity that can be accommodated when model primitives are chosen subject to the cognitive constraint that agents are incapable of solving Euler equations in an internally consistent manner. In particular, I argue that even if our theory produces decision rules that appear appealing intuitively and match the data, but no real-world agent is realistically able to derive them, the modeler for all intents and purposes imposes behavior as the model primitive and thus invariably obfuscates the *actual tradeoffs* considered by those agents. In effect, internal consistency only serves as a useful benchmark if we can realistically assert that agents' behavior indeed derives from the proposed optimization problem. For example, a household considering the marginal cost of increasing consumption today realistically resorts to *quantifying* those costs in terms of lost (retirement) savings, not in terms of future consumption. This is because (i) an accurate probabilistic assessment of future consumption is prohibitively expensive, (ii) accumulated savings serve as a store of *value* in the sense that future consumption is strictly increasing in accumulated savings, and (iii) it is quantitatively convenient.

The remainder of the paper is organized as follows: Section 2 places the paper in the literature by examining the proposed framework's key elements. Section 3 presents the theoretical framework that formalizes the desired demand channel. Section 4 discusses the pursued parameterization strategy, illustrates quantitatively the macroeconomic transmission of a typical fire sale episode, and assesses various monetary policy options. Section 5 presents a summary of the gained topical and methodological insights. Section 6 concludes.<sup>9</sup>

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empirical, that extends beyond matching first and second data moments.

<sup>8</sup>“The most important challenge facing any macro-model is to provide insights into the deep downturns that have occurred repeatedly.”

<sup>9</sup>Appendices A through E contain a derivation of individual equilibrium strategies, a description of the employed data, a motivating discussion of the proposed consumption-savings problem as well as a technical note regarding the often cited notion of ergodicity, and an institutional dictionary.

## 1.2. Incorporating Fire Sales into a Macroeconomic Framework

The present work’s objective most closely resembles the recent article of Gertler, Kiyotaki, and Prestipino (2017) (GKP), but the pursued approaches differ substantially. First, the financial crises embedded in the macro model considered here manifest themselves in the form of a fire sale, whereas in GKP they take the form of a bank run. Second, macroeconomic transmission occurs via a slump in consumer demand in my model, whereas in GKP transmission occurs via aggregate supply as bank runs temporarily prevent banks from financing investment and less efficient intermediaries (households) taking their place. Methodologically, the proposed crises are similar in the sense that both types arise from an occasionally binding constraint, but equilibrium here is unique, whereas it is indeterminate in GKP.<sup>10</sup> I now turn to placing the paper in the literature more broadly by examining more closely the key elements — origination, transmission, and mitigation — of the simulated crises.

### *Crisis origination*

How do the simulated financial crises originate? The origination mechanism considered here resembles the canonical fire sale in Brunnermeier and Pedersen (2009). Since the CLO is purchased on margin, binding maintenance margin requirements induce demand to be non-monotonic (see Figure 1.4). Once the price of the CLO falls below the critical threshold, the Walrasian method of countering excess supply by lowering the price effectively *increases* excess supply because brokers are forcing investors to liquidate larger parts of their portfolio.<sup>11</sup> Maintenance margins thus gives rise to a price threshold below which there exists a “diabolic feedback loop” between falling asset prices and increasing margin calls (Brunnermeier and Pedersen, 2009). As such, the economic intuition underlying the proposed fire sale here is virtually equivalent to the one in Brunnermeier and Pedersen (2009) with the primary difference being that equilibrium here is unique because

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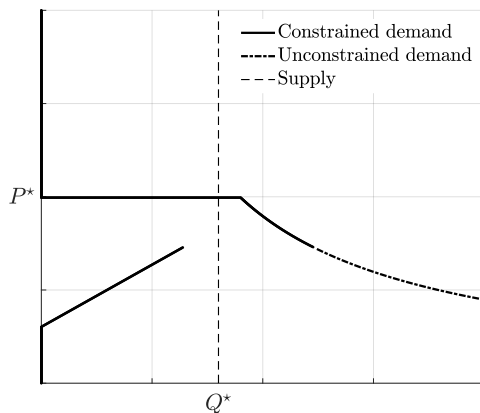
<sup>10</sup>The key point here is that while indeterminacy (and/or multistability) are conducive to generating extraordinary economic behavior, they are not necessary. Instead, my key methodological crisis component is that equilibrium is *fragile* in the sense that it is discontinuous (in the state space).

<sup>11</sup>In such a scenario, if liquidity shortage is symmetric, private market equilibrium ceases to exist such that financial markets collapse altogether unless a third party intervenes. I thus assume that, once a fire sale is imminent, the central bank supplies additional liquidity by deploying *unconventional* monetary policy.



maintenance margins are fixed.<sup>12,13</sup> In contrast, many articles in the literature model financial crises to manifest themselves in the form of equilibrium multiplicity.<sup>14</sup>

**Figure 1.4.** The secondary market for collateralized loan obligations (CLOs)



Notes: Figure 1.4 illustrates the effects of a prevailing maintenance margin. Since CLO demand is non-monotonic, an adverse price shock can spark a cascade of forced sell orders which causes excess supply to *increase* as the price falls.

### *Crisis transmission*

Following origination, how are nominal crises transmitted to the real sector? Since transmission via both aggregate demand and aggregate supply has been established empirically for the most recent financial crisis (Mian and Sufi, 2014; Chodorow-Reich, 2014), a natural next step is to ask which of the two channels was dominant in terms of its relative contribution to generating the Great Recession. The supply view suggests that financial crises cause contractions because financing production becomes more expensive for firms. Conversely, the demand view holds that recessions rather reflect firms reacting to shifts in aggregate demand as households reduce their consumption spending. While both views imply a contraction in output, the corresponding effects on the price level are opposite. Since the large declines in output observed during the Great Recession were not accompanied by a major price shift in either direction, anecdotal evidence supports

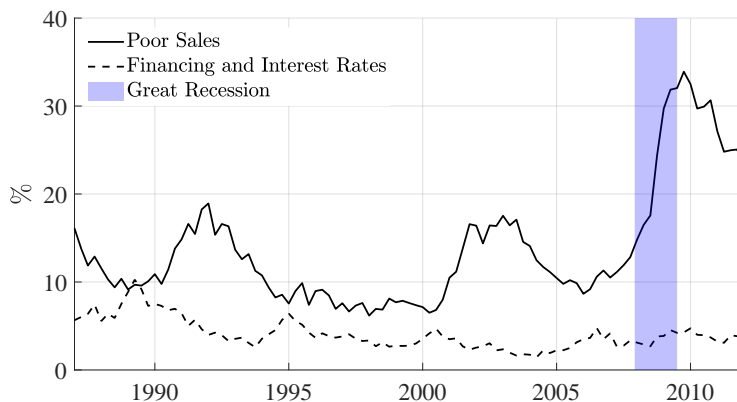
<sup>12</sup>While unique, equilibrium here is *fragile* in the sense that it is discontinuous in the state space. Since occasionally binding constraints typically induce such discontinuities, they require special attention when solving for equilibrium, but have been a subject of heightened interest since the 2008 financial crisis (see Cuba-Borda et al., 2019).

<sup>13</sup>Whether maintenance margins are set exogenously or endogenously is not paramount in terms of examining the macroeconomic transmission following a fire sale. Therefore, since I only require that fire sales in fact occur occasionally, the exogenous setting is preferred.

<sup>14</sup>Prominent examples other than Brunnermeier and Pedersen (2009) include the canonical bank run by Diamond and Dybvig (1983), the sovereign defaults by Calvo (1988). Lorenzoni and Werning (?) go even further in that they examine an economy featuring both multiple equilibria and multiple steady states. In Gertler, Kiyotaki, and Prestipino (2017), equilibrium is unique but discontinuous in the state space.

the aforementioned notion that supply and demand channels coexist. Focusing on housing net worth, Mian, Rao, and Sufi (2013) find that the recent financial crisis caused a severe slump in consumer demand. Weak aggregate demand in turn had a significant negative impact on employment successfully establishes the existence of a supply channel by exploiting bank-firm level data and estimates that at least one third of all employment losses at small to medium sized firms (SME) during the Great Recession are attributable to credit withdrawals by banks. However, Figure 1.5 provides some evidence that at least in terms of its relative contribution, aggregate demand was more important.

**Figure 1.5.** “What is the single most important problem facing your business today?”



Notes: Figure 1.5 reports business owners’ answer to the question “What is the single most important problem facing your business today?”. The fact that business owners were more worried about poor sales than securing finance during the Great Recession suggests that the relative contribution of aggregate demand in creating the recession was higher than the contribution of aggregate supply mechanisms. The data was retrieved from the online appendix in Mian and Sufi (2014).

Over the course of the 2008 Financial Crisis, US households lost close to 20% of their aggregate nominal wealth. There are two principal reasons why aggregate demand would suffer from such a shock. On one hand, households may curb their consumer spending because declining net worth causes borrowing constraints to tighten, which effectively induces a forced contraction. On the other hand, households may curb their spending voluntarily, namely to restore their lost savings. While the former mechanism has been coined the *household leverage view* of the Great Recession, the latter may be understood as the *wealth effect view* of the Great Recession. Even though the theoretical literature on aggregate demand following the 2008 Financial Crisis has predominantly emphasized leverage (Eggertsson and Krugman, 2012; Guerrieri and Lorenzoni, 2017), deleveraging over that time period was actually rather gradual. In effect, even though tightening borrowing constraints

may be able to account for the slow recovery following the Great Recession, they cannot account for the large and abrupt initial downturn that *was* the Great Recession (Jones, Midrigan, and Philippon, 2018). In contrast, as argued herein, households voluntarily substituting away from consumption towards savings can account for the Great Recession.

To elucidate the determinants of aggregate demand, the literature has extensively studied the heterogeneous nature of consumer response to wealth shocks (Mian, Rao, and Sufi, 2013) and income shocks (Parker et al., 2013). In either case, heterogeneity is important because a shock that is fixed in size will have a larger effect on aggregate demand if it is concentrated among households with higher marginal propensities to consume (MPC).<sup>15</sup> However, even though wealthy households have traditionally not been viewed as having high MPCs, the great majority of the Great Recession’s demand slump was driven by such households (Petev et al., 2011). Unsurprisingly, this is because rich households were the ones who incurred the highest wealth losses during the 2008 Financial Crisis, both in absolute and in relative terms (Heathcote and Perri, 2018). As a result, even though a hypothetical wealth collapse of the same size would have been more impactful if it had been distributed differently in the cross-section, the actually realized wealth collapse still generated a very significant response in consumer demand simply because it was so large in aggregate.<sup>16</sup>

The present approach departs from the aforementioned demand-side literature in two principal ways. First and foremost, my aggregate demand slump is generated via an endogenous, widespread loss in nominal household wealth, not via an exogenously tightening credit constraint or even the concavity of the policy function. In particular, because consumption is increasing in wealth and wealth collapses during financial crises, the consumption function’s second derivative is not of primary importance as long as it is monotonically increasing. Moreover, since my economy’s numéraire is given by money, fire sales only cause *nominal* wealth losses, which are by themselves insufficient to generate significant real downturns because prices and wages are principally capable of absorbing nominal shocks of an arbitrary size. I thus follow the New Keynesian tradition in

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<sup>15</sup>The proposition that households have heterogeneous MPCs goes back to Keynes (1936) who conjectured that the latter is decreasing in wealth. Zeldes (1989) and Carroll and Kimball (1996) show that the proposed concavity can be rationalized by pairing income uncertainty with a precautionary savings motive on part of the household. In turn, King (1994) emphasized that concavity can also arise if consumers face a binding borrowing constraint, an assertion reiterated by Carroll (2001) and exploited more recently by Kaplan and Violante (2014).

<sup>16</sup>Since MPCs are local objects, they almost surely yield unsatisfactory back-of-the-envelope approximation results if wealth collapses by 20% on average. In fact, if the consumption function is concave, such an approximation effectively serves as a lower bound.

assuming that wages are nominally downward rigid. Second, I follow the macroeconomic tradition of modeling firms explicitly, which allows for the desired investigation of the extensive labor margin.

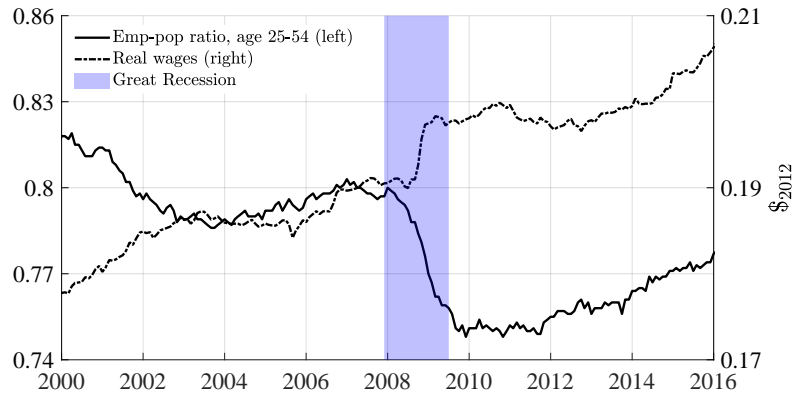
In terms of macroeconomic theory, early accounts of the nominal-real nexus had focused on aggregate demand (Keynes, 1936) and the accompanying phenomenon of deflation (Fisher, 1933), but the contemporary literature has effectively been dominated by supply-side shocks — most notably technology shocks — since Kydland and Prescott (1977, 1982). For example, the canonical “financial accelerator” literature, adverse technology shocks are exacerbated by deterioration of firm net worth which disincentivizes production via unfavorable external financing conditions (Bernanke, 1983; Bernanke and Gertler, 1989; Bernanke, Gertler, and Gilchrist, 1999). Conversely, Jermann and Quadrini (2012) introduce perturbations that originate in the financial sector, namely shocks that alter a firm’s borrowing capacity holding fixed the latter’s existing stock of collateral. In their model, the financial sector thus not only acts as an amplifier, but it constitutes a source of macroeconomic fluctuations in its own right. While output contractions are caused by binding quantity constraints in Jermann and Quadrini (2012), Gertler, Kiyotaki, and Prestipino (2017) appeal to lending and investment frictions that manifest themselves in the form of increasingly expensive capital loans following a bank run. Specifically, financing production is more expensive during a crisis because households — who temporarily own the entire capital stock — are less efficient “in handling investments”.<sup>17</sup>

In my framework, the key assumption that links financial crises to real downturns is that worker effort is sensitive to nominal wage cuts such that real wages inflate immediately and substantially following a fire sale. Figure 1.6 confirms that such an immediate and substantial real wage inflation took place in the United States during the Great Recession. Notice, however, that the protracted nature of the Great Recession is hard to reconcile with a rigidity that is purely nominal (Elbsy et al., 2016). And indeed, in my model, the nominal rigidity only serves as a transmission mechanism initially. In turn, the empirically observed hysteresis is generated via a collapsing capital stock which induces a protracted decline in labor productivity.

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<sup>17</sup>Since households incur a utility cost from holding capital beyond a certain threshold, they must be compensated to do so following a bank run. “The fire sale of assets from banks to inefficient households will lead to a *sharp rise in the cost of credit*, leading to an extreme contraction in investment and output.” Therefore, bank runs cause extreme recessions because financing production becomes more expensive for firms.

**Figure 1.6.** Average hourly real wage and working-age employment in the US since 1987



Notes: Figure 1.6 displays the evolution of the employment-to-working-age-population ratio and CPI-adjusted, average hourly wages in the post-Volcker era. During the Great Recession, real wages experienced an immediate and substantial spike while employment underwent a rapid and substantial decline. The proposed theory rationalizes this development as follows. Since worker effort is sensitive to nominal wage cuts, firms reacted to the slump in aggregate demand by curbing their demand for labor while leaving nominal wages largely unchanged. All data was retrieved from FRED.

The proposition that wages are nominally rigid is hardly new. Wage stickiness constitutes an integral part of the New Keynesian literature, but asymmetric downward rigidity has gained some additional momentum following the Great Recession.<sup>18</sup> A classic early reference in this regard is the *General Theory*, in which Keynes challenges the prevailing view that unemployment was voluntary during the Great Depression because wages were nominally rigid (Tobin, 1972). Unsurprisingly, in light of the theory proposed herein, the Great Depression was later found to have been amplified if not caused by nominally downward rigid wages (Akerlof et al., 1996; Bernanke and Carey, 1996).<sup>19</sup> More recently, Bakker (2015) replicates Bernanke and Carey’s (1996) cross-country exercise with Great Recession data and finds that rigid wages constitute a key driver of unemployment still.

From an empirical perspective, there is ample evidence that wages are downward rigid.<sup>20</sup> In fact, since “the existence of wage stickiness is not in doubt” (Kahneman et al., 1986), it is unsurprising that there exists a subliterate that has interviewed firms — as wage setters — to elicit exactly *why* wages are downward rigid. In a survey of 184 firms, Campbell and Kamlani (1997) find that the two mechanisms exploited herein — endogenous worker effort and adverse selection — are in

<sup>18</sup>Classic examples of New Keynesian DSGE models with symmetric wage rigidities include Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). Asymmetric downward rigidities are examined by Kim and Ruge-Murcia (2011) and, more recently, by Schmitt-Grohé and Uribe (2016) and Na et al. (2018).

<sup>19</sup>In this sense, moderate rates of inflation may serve as ‘grease’ for the labor market because it loosens the nominal wage constraints faced by firms (Kahneman et al., 1986).

<sup>20</sup>See McLaughlin (1994), Akerlof et al. (1996), Card and Hyslop (1997), Lebow et al. (2003), Fehr and Goette (2005), Dickens et al. (2007), Daly et al. (2011), Daly et al. (2012), Barattieri et al. (2014), Fallick et al. (2016).

fact the two principal reasons why firms are hesitant to cut wages. Bewley (1999) and Blinder and Choi (1990) also find strong evidence in support of the worker effort hypothesis, but the latter find no evidence that firms fear adverse selection in hiring. They hypothesize, however, that adverse selection may play a larger role in quits, a conjecture confirmed by Campbell and Kamlani (1997). On the worker side, Kahneman et al. (1986) find that nominal wage cuts are often perceived as unfair, although there may be mitigating circumstances in case the employer faces bankruptcy.

In line with the firm-side results discussed above, theory has traditionally attributed downward rigidity to the prevalence of ‘efficiency wages’, which formalize the notion that labor productivity is increasing in the wage: “you get what you pay for” (Solow, 1979). To generate the desired dependence, Weiss (1980) pairs heterogenous labor productivity with asymmetric information to induce adverse selection, whereas Shapiro and Stiglitz (1984) give workers the ability to shirk. Potential other reasons for the prevalence of efficiency wages are costly labor turnover (Stiglitz, 1974) and endogenous effort as captured by Akerlof’s positive “gift exchange” mechanism (1982) and its negative “fairness” counterpart (Akerlof and Yellen, 1990).

### *Crisis response*

This paper also relates to the literature that examines how policy makers should optimally address an ongoing crisis. For this, I proceed by first discussing a select set of works that address the Great Depression, before examining more recent insights gained during the Great Recession.

The classical reference for the policy response during the Great Depression is *A Monetary History of the United States* by Friedman and Schwartz (1963). The main thesis of their predominantly empirical exercise is to show that exogenous changes in monetary aggregates have real effects. In particular, they hypothesize that the Great Depression would have been less pronounced had the Federal Reserve acted less hawkishly.<sup>21</sup> Romer and Romer (2010) echo the formers’ hypothesis, but find the proposed link to be lacking in its theoretical foundation. They thus proceed by formulating a concrete transmission story, namely that money affects the real sector via increasing real interest rates as prices fall. Therefore, assuming the premise that the real downturn during the Great Depression was caused by deflation is accepted, Romer and Romer (2010) establish the desired causal link by empirically showing that monetary policy indeed fueled deflation at the time. In line

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<sup>21</sup>Recall that the Fed iteratively raised rates between 1928 and 1932.

with the above references, Christiano, Motto, and Rostagno (2003) construct a quantitative DSGE framework and find — by generating counterfactuals for a set of differing monetary policy rules — that the Federal Reserve could have muted the Great Depression had it acted more dovishly.

Before delving into the literature on the Great Recession, notice that the aforementioned analyses all consider conventional interest rate policy as the central bank’s primary if not only monetary policy lever. But, as we observed during the Great Recession, solely relying on conventional measures may be severely limiting insofar as nominal rates are subject to the zero lower bound. However, as the ultimate creator of the numéraire, the central bank can intervene in many ways that go beyond traditional interest rate policy if necessary. Examples of unconventional tools include emergency lending facilities as well as outright purchases and repurchase agreements of non-governmental assets.<sup>22</sup> In light of the widespread, recent deployment of such tools, it is unsurprising that researchers were quick to empirically examine and incorporate them in their models. Gertler and Karadi (2011) consider a DSGE framework, in which the central bank directly lends to the private sector when financial intermediaries are undercapitalized. They find that during crisis episodes, unconventional policy of this sort is beneficial irregardless of whether the zero lower bound binds, but that the benefits are particularly stark if it does bind. Similarly, Wu and Xia (2016) quantitatively assess the effects of unconventional policy on unemployment and find significant effects in the desired direction. In a more qualitative exercise, Kuttner (2018) concludes that the Federal Reserve’s actions were appropriate in the sense that the incurred costs are “dwarfed by the costs of the more protracted recession in the United States that likely would have occurred in the absence of the unconventional policies”.

### 1.3. A Recursive Sequential Game

The proposed economy is an infinitely repeated game with each period evolving sequentially and the main agents, households, maximizing multiple objectives per period. The latter do not boil down to a single objective because households *narrowly bracket* their decisions: Given the “mass of evidence, and the ineluctable logic of choice in a complicated world, [households] choose an option in each case without full regard to the other decisions” (Rabin and Weizsäcker, 2009). Specifically,

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<sup>22</sup>See Reis (Fall 2009) for a contemporaneous assessment of policy at the height of the Recession in late 2009.

I assume that households bracket their three main decisions as follows. First, they choose labor supply to maximize labor income. Taking the outcome of this first stage as given, they then choose how much of their income to consume and how much to invest in illiquid assets for retirement. Finally, they (re)partition the resulting stock of retirement savings into riskier equity claims and safer debt claims to maximize risk-adjusted returns.<sup>23</sup>

The private sector consists of consumption goods producers, capital goods producers, and financial intermediaries. Capital firms produce, own, and rent out capital to the consumption goods sector. In turn, consumption goods firms use capital and labor to produce and sell consumption goods to households. Since sectoral competition occurs via quantity in the consumption goods sector, sectoral size is a natural measure of competition. Since demand is uncertain at the time of production, higher competition leads to higher likelihoods of default. Banks scale up and down the economy’s artificial numéraire by issuing and subsequently selling commercial loans. Nonbank financial institutions (NBFIs: pension funds and brokerage firms) allow households to invest their nominal wealth in the form of equity and debt contracts respectively.

The public sector consists of a central bank and a government. The central bank enacts *conventional* monetary policy by periodically adjusting its interest rate target, which is perceived as ‘the current interest rate level’ by all agents. In addition, if a financial crisis is imminent, the central bank can, as the ultimate creator of the numéraire, deploy *unconventional* policy that extends beyond the periodic announcement and defense of the interest rate target. The government taxes labor income, disburses unemployment and retirement benefits, and issues a government bond.

### *Agents, Markets, and Prices*

Consider a dynamic closed economy populated by a set of infinitely many households  $J^{HH}$  of time-invariant measure  $\mu^{HH} \equiv \mu(J^{HH})$ .<sup>24</sup> Time is discrete with  $t \in \mathbb{N}$  denoting a period. Each household born in period  $t$  is member of generation  $t$  and lives for a finite, predetermined

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<sup>23</sup>Even though each household faces all three decisions in any given period, the described sequence of events effectively unfolds over the course of three periods. Intuitively, think of a household as prioritizing the decision of finding employment with an implied understanding that future consumption is increasing in income. After matching with an employer and working for a period, the household receives their paycheck and decides, over the course of the second period, what fraction of their earnings to spend on consumer goods and how much to save retirement. Finally, at the beginning of the third period, the household chooses how to (re)partition their accumulated retirement savings.

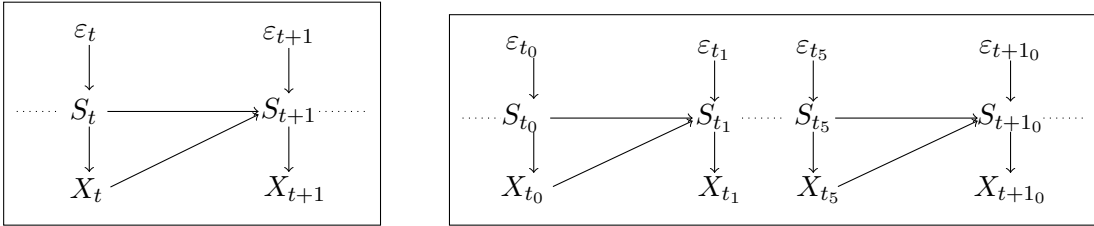
<sup>24</sup> $\mu^{HH}$  acts as a scaler of the economy and is not assumed to be unity. If  $x_{ht}$  denotes a household specific quantity, the corresponding economy-wide aggregate is given by  $x_t = \int_{J^{HH}} x_{ht} dh$ . Equivalently,  $x_t = \sum_{J^C} x_{ft}$  and  $x_t = \sum_{J^K} x_{ft}$  denote sectoral aggregates in the corporate sector.



number of periods  $T^L$ . At age  $T^R < T^L$ , households lose their productive labor endowment, exit the labor force, and finance their consumption via a government pension and accumulated retirement savings. After exiting the economy at age  $T^L$ , an old household's remaining property is, if applicable, bequeathed to a new household who takes its place. All initial generations are of equal size such that the labor force  $J^{LF} \subset J^{HH}$  is of time-invariant measure  $\mu^{LF} = \mu^{HH} \left( \frac{T^L - T^R}{T^L} \right)$ .<sup>25</sup>

The corporate landscape is made up of a consumption and a capital goods sector, each consisting of a time-invariant set of firms  $J^C$  and  $J^K$  respectively, as well as a set of banks  $J^B$ , pension funds  $J^{PF}$ , and brokerage firms  $J^{BR}$ . Each period unfolds sequentially with  $S_{t_j}$  denoting the system's state in subperiod  $j \in J^T = \{0, 1, 2, 3, 4, 5\}$  of period  $t$ .<sup>26</sup> Households are heterogenous such that  $S_{t_j}$  is infinite dimensional. State evolution is governed by equilibrium mappings from  $S_{t_j}$  to a vector of controls  $X_{t_j}$  and subject to exogenous innovations  $\varepsilon_{t_j}$ . The economy's stochastic environment, as captured by the joint density of  $\{\varepsilon_{t_j}\}_{j \in J^T, t \in \mathbb{N}}$ , is given by a probability space  $(\Omega, \mathcal{W}, \mu)$ . Figure 1.7 illustrates the described notion of intratemporal sequentiality.

**Figure 1.7.** State transition in a sequential setup



Notes: The above graphic illustrates the sequential nature of the recursive game. The endogenous state  $S_{t_j}$  and the exogenous innovation  $\varepsilon_{t_j}$  give rise to equilibrium as captured by the vector of controls  $X_{t_j}$ , which in turn yields the new endogenous state. Period  $t$  period ends after subperiod  $t_5$  at which point the new period  $t+1_0$  begins.

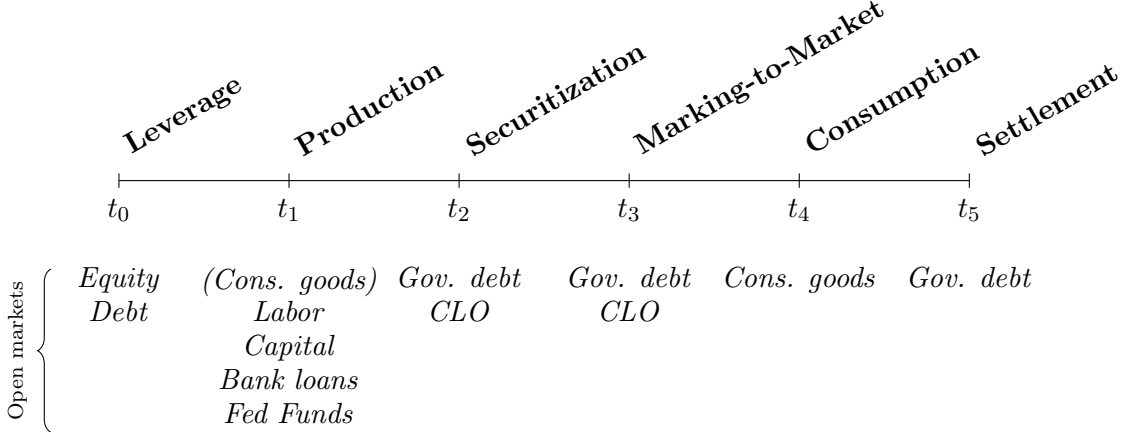
The six subperiods unfold as follows. At  $t_0$ , households partition their retirement portfolio into equity and debt while pension funds decide on their capital structure. During  $t_1$ , capital goods and consumption goods firms produce output while households supply labor across the two sectors. Consumption goods producers finance their production with a commercial loan originated and

<sup>25</sup>I assume that all of the working age population is in the labor force.

<sup>26</sup>I further proceed by highlighting random variables that are, at the contextually relevant point in time, non-predetermined by  $\tilde{X}$  with  $\tilde{X}_i$  denoting a specific agent's projection thereof. For example, knowing aggregate sectoral output is insufficient to infer the equilibrium consumption goods price at  $t_1$  because, at the time, the realization of the households' taste shock has not been observed.  $\tilde{P}_t$  thus denotes the corresponding random variable implied by the model with the accent simply highlighting the fact that its realization is unknown when firms are devising their optimal strategies. Moreover, since firms resort to approximating concurrent consumer demand with an isoelastic function,  $\tilde{P}_{it}$  denotes firm  $i$ 's price projection.

distributed by banks. At  $t_2$ , all commercial loans are pooled and securitized into a CLO that is held by pension funds. At  $t_3$ , the occasional observation of a noisy taste shock signal leads to an information-motivated repricing of the CLO. Pension fund equity is repriced as a residual of assets net of debt. If equity falls below the prevailing maintenance margin, a margin call is issued and debt must be repurchased. If pension fund liquidity is insufficient to satisfy the margin call, pension funds are forced to liquidate part of their portfolio in a fire sale. At  $t_4$ , the taste shock realization determines aggregate demand with market clearing determining the corresponding equilibrium price. Finally, during  $t_5$ , all claims are settled, the central bank announces a new risk free interest rate target, and the government auctions off a new issue of its bond. Figure 1.8 summarizes which markets are open in which subperiod.

**Figure 1.8.** The model’s intratemporal timeline



Notes: Figure 1.8 displays each period’s six subperiods including the respective markets that are open at each time.

Before describing in detail the proposed intratemporal sequence of events, I discuss the relevant market and equilibrium concepts. Markets are highly incomplete with trading exclusively taking place in recurrent spot markets, most of which only allow access to certain types of agents. For example, households cannot purchase commercial loans or securities directly, but they may hold an indirect claim on such assets via shares of NBFIs. The relevant equilibrium concept is the following.

**Recursive general equilibrium.** A recursive model economy is said to be of the general equilibrium type (RGE) if the price in each market is determined endogenously by pseudo-Nash equilibrium

in (each subperiod of) each period.<sup>27</sup>

Whenever prices are not set by market participants themselves, I entrust price discovery to a Walrasian auctioneer with the objective  $\max_{p \in \mathbb{R}^+} zp$ , where  $p$  is the price chosen by the auctioneer and  $z(p)$  denotes the corresponding market's excess demand in optimum. I further follow Arrow and Debreu (1954) in assuming that the auctioneer does not recognize the functional relationship  $z(p)$ . In effect, this implies that excess supply induces the auctioneer to lower the price, whereas excess demand induces the auctioneer to raise the price. General equilibrium thus requires  $z(p^*) = 0$  in all Walrasian markets.<sup>28</sup>

Table 1.1 summarizes all of my economy's intratemporal markets. In each market not labeled as Walrasian, pricing is determined by the designated agents. Notice that even in absence of a Walrasian auctioneer, competition may still lead to market clearing as exemplified by the markets for Fed Funds, commercial loans, capital, and the consumption good. Contrarily, in spite of competition, the market for labor may not clear, even in equilibrium.

**Table 1.1.** Markets and prices

	Price	Pricing	Clearing	Relative	Realized	Determined
$R_t^{FFR}$	Fed Funds Rate (FFR)	Banks	Yes	Yes	$t_1$	$t - 1$
$R_t^S$	Sovereign yield	Walrasian	Yes	No	$t_0 - t_4$	$t - 1$
$R_t^R$	CLO yield	Walrasian	Yes	Yes	$t_2$	$t_2$
$R_t^D$	Pension fund debt yield	Walrasian	Yes	Yes	$t_0$	$t_0$
$R_t^L$	Bank loan rate	Banks	Yes	Yes	$t_1$	$t_1$
$Q_t$	Capital	Firms	Yes	No	$t_1$	$t_1$
$W_t^C$	Labor	Firms	No	No	$t_1$	$t_1$
$P_t^C$	Consumption good	Firms	Yes	No	$t_3$	$t_3$

Notes: With the exception of the market for consumption goods, each non-Walrasian market features price competition. The market for consumption goods is special because firms must commit to producing a certain quantity before demand is known. Once production has occurred, the strategic environment switches to price competition. See Kreps and Scheinkman (1983) for a more general discussion of such a setup.

Notice that pricing of all financial claims directly or indirectly occurs *relative* to the return of investing at the predetermined risk free rate. The latter is determined as the central bank commits

<sup>27</sup>See Section 1.7 for a more extensive discussion of the employed equilibrium concept.

<sup>28</sup>Interestingly, Lucas and Sargent (1979) *define* market clearing in spirit of my proposed definition of general equilibrium: "One essential feature of equilibrium models is that all markets clear, or that observed prices and quantities are viewed as outcomes of decisions taken by individual firms and households". In contrast, market clearing is understood as a situation in which excess demand is zero,  $z(p^*) = 0$ , herein.

to intervening in bond markets such that  $R_t^{FFR}$  matches some previously announced target  $R_t^T$  in equilibrium.<sup>29</sup> I now turn to discussing in detail the proposed intratemporal sequence of events.

*Subperiod  $t_0$ : Leverage*

At the beginning of each period, households own an aggregate stock of liquid demand deposits  $w_{t_0}^L = d_{t_0}^{HH}$  in the amount of last period's income and accumulated retirement savings in the form of illiquid pension fund debt and equity  $w_{t_0}^I = w_{t_0}^D + w_{t_0}^E$  with corresponding uncertain returns  $\hat{R}_t^D$  and  $\hat{R}_t^E$ . Institutionally,  $w_{t_0}^E$  constitutes a direct claim on fund assets, whereas debt  $w_{t_0}^D$  is held via a broker. On the asset side, pension funds hold demand deposits  $d_{t_0}^{PF}$  and a fraction  $a_{t_0}^S$  of the government security  $S$  acquired in previous period's primary market. As illustrated in Figure 1.9, all demand deposits are fully backed by central bank credit  $v_{t_0}$  at this stage. Bank deposits are labeled  $M1$  because they are accepted as means of payment in exchange for goods and services in the economy. Conversely, pension fund IOUs are illiquid because they are not accepted as a means of payment and only pay off at retirement.

**Figure 1.9.** Aggregate balance sheet of the banking sector

Assets	Liabilities
Reserves $v_{t_0}$ ( $M0$ )	HH deposits $d_{t_0}^{HH}$ ( $M1$ )
	PF deposits $d_{t_0}^{PF}$ ( $M1$ )

Notes: At the beginning of each period, all bank deposits ( $M1$ ) are backed by central bank reserves ( $M0$ ). This is not always the case as banks can create  $M1$  by originating commercial loans.

At this time, households may opt to repartition their individual retirement portfolio. For this, they rely on a projected benchmark asset return of  $\tilde{R}_{ht}^A \equiv R_t^T + \bar{v}_t$  for each  $h \in J^{HH}$ , where  $\bar{v}_t$  is the geometric mean of the historical risk premium sequence  $\{\hat{R}_\tau^A - R_\tau^T\}_{\tau=0}^{t-1}$ . In deciding on their portfolio composition, households face a fundamental tradeoff between maximizing projected returns and limiting risk exposure with the latter increasing in  $w_{ht_0}^E/w_{ht_0}^I$  because equity is a residual claim. I then assume that households maximize risk-adjusted, *projected* returns as follows:

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<sup>29</sup>Since the only price that is nominally rigid in the current setup is consumption sector wage, conventional monetary policy is ineffective in the sense that it has real effects if and only if the nominal wage constraint binds. The nominal wage constraint binds during a crisis episode or if the central bank's target rate announcements vary substantially from period to period.

$$\begin{aligned}
V_0^{HH} (i \in J^{PF}, j \in J^B, w_{ht_0}^I) &= \max_{w_{ht_0}^E} w_{ht_0}^D \tilde{R}_{ht}^D + w_{ht_0}^E \tilde{R}_{ht}^E - \gamma_h^r \frac{w_{ht_0}^I}{2} \left[ \frac{w_{ht_0}^E}{w_{ht_0}^I} \right]^2 \\
\text{s.t. } w_{ht_0}^E &= w_{ht_0}^I - w_{ht_0}^D \\
\tilde{R}_{ht}^D &= R_{jt}^{D,b} \\
\tilde{R}_{ht}^E &= \tilde{R}_{ht}^A + \left( \tilde{R}_{ht}^A - R_{it}^{D,p} \right) L_{it}
\end{aligned}$$

where  $R_{jt}^{D,b}$  is the gross rate offered to each household by broker  $j$ ,  $R_{it}^{D,p}$  is the lowest gross rate charged to pension fund  $i$  across all brokers, and  $L_{it}$  pension fund  $i$ 's leverage. Idiosyncratic risk aversion is thus captured by and increasing in  $\gamma_h^L \in (0, \infty)$ . On the demand side, pension funds maximize projected return on equity subject to an *initial* margin requirement  $\delta_I$  to be relaxed by a *maintenance* margin  $\delta_M$  in subsequent subperiods<sup>30</sup>,

$$\begin{aligned}
V_0^{PF} \left( \tilde{R}_t^A \right) &= \max_{L_{it}} \tilde{R}_{it}^A + \left( \tilde{R}_{it}^A - R_t^D \right) L_{it} \\
\text{s.t. } L_{it} &\leq \delta_I
\end{aligned}$$

Since pension funds rely on the same asset return projection,  $\tilde{R}_{it}^A \equiv R_t^T + \bar{v}_t$  for each  $i \in J^{PF}$ , the household problem implies  $\tilde{R}_{it}^A > R_t^D$  such that funds strictly prefer debt finance in equilibrium. We have a corner solution with maximal leverage  $L_{it} = \delta_I$ ,  $w_{it_0}^E = \frac{1}{1+\delta_I} w_{it_0}^I$ , and  $w_{it_0}^D = \frac{\delta_I}{1+\delta_I} w_{it_0}^I$  for each  $i$ . Institutionally, equilibrium emerges as the auctioneer matches supply and demand for debt and equity across the two competitive brokers and pension funds.<sup>31</sup>

#### *Subperiod $t_1$ : Production*

The corporate sector consists of a set of consumption goods producers  $J^C$  and a set of capital producers  $J^K$ . In subperiod  $t_1$ , households choose to supply labor to a firm  $f \in J^F = J^C \cup J^K \cup J^U$ , where  $J^U = \{0\}$  represents voluntary unemployment. Letting  $n_{hft}$  and  $x_{hft}$  denote a household's firm-specific labor supply indicator and labor output, firm  $f$  produces the following sector-specific outputs,

<sup>30</sup>See Fabozzi (2015) for a discussion of the institutional details when buying assets on margin.

<sup>31</sup>See Section 1.7 for a discussion of the resulting equilibrium strategies.

$$y_{ft}^C = \begin{cases} z_t^C [k_{ft}^C]^\alpha [\mu^{LF} \int x_{hft} dh]^{1-\alpha} & \text{if } f \in J^C \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ft}^K = \begin{cases} z_t^K [\mu^{LF} \int x_{hft} dh] & \text{if } f \in J^K \\ 0 & \text{otherwise} \end{cases}$$

where  $x_{hft} = 1 \Rightarrow x_{hjt}$  for each  $j \neq f$  meaning that each worker can only be employed by one firm. As indicated, both goods are produced using technology  $z$  and labor  $x$ , but consumption goods production also requires capital  $k$ . The labor market setup is akin to Weiss (1980) in that households feature a time-invariant distribution of individual skill  $q_h \stackrel{\text{i.i.d.}}{\sim} G_q$ , but each household also chooses to exert a certain level of effort  $e_{hft}$ . A household's individual contribution to firm  $f$ 's labor output is then given by  $x_{hft} = n_{hft} q_h e_{hft}$ .

Prior to making their labor supply decision, each household receives a vector of firm-specific wage offers  $\{W_{hft}\}$ . While consumption sector offers are nominally fixed, offers in capital sector are tied to the individual contribution  $x_{hft}$  and thus take the form of a contract. This is because I assume that a worker's individual contribution  $x_{hft}$  is observed by the capital producers, but not the consumption goods producers.<sup>32</sup>

When choosing to supply labor to the consumption goods sector, households may fail to get matched, in which case they receive the same unemployment benefit as the voluntarily unemployed. The reason why involuntary unemployment exists is that my economy's labor market may exhibit excess labor supply *in equilibrium*.<sup>33</sup> The corresponding unemployment benefits are then calculated as a fraction  $\lambda^U \in (0, 1)$  of the lowest current consumption sector wage  $W_{hft}^U = \lambda^U \min_{f \in J^C} \{W_{ft}^C\}$ . Since  $W_{hft}^U$  is uniform across households and  $|J^U| = 1$ , it is notationally reduced to  $W_t^U$ .

Given the described wage offers and the uncertainty associated with applying for consumption sector jobs, I assume that labor supply is chosen to maximize the following maxmin or "worst case scenario" criterion.<sup>34</sup>

<sup>32</sup>This ensures that the lucrateness of each worker's outside option is increasing in their skill.

<sup>33</sup>If this is the case, each applicant is hired with equal probability with  $\hat{W}_{hft}(\omega)$  denoting the ex post realized wage given the event  $\omega$ .

<sup>34</sup>While extreme, this formulation is motivated by the premise that "a bird in the hand is worth two in the bush" as real-world households likely cannot infer true probability (see Knight, 1921; Savage, 1954). The setup, which may

$$V_1^{HH}(\{W_{hft}\}_{f \in J^F}) = \max_{f \in J^F} \left\{ \min_{\omega \in \Omega} \hat{W}_{hft}(\omega) - \zeta \mathbb{1}[f = J^U] \right\} \quad (1.1)$$

where  $\hat{W}_{hft}(\omega)$  denotes the ex post realized wage<sup>35</sup> and choosing voluntary unemployment carries a fixed utility cost of  $\zeta > 0$  for all  $h$ .

Once hired, workers are assumed to exert effort based on what they perceive as “fair” compensation (see Akerlof and Yellen, 1990). In the capital goods sector, fairness is not questioned because remuneration is directly linked to individual performance. In contrast, wage cuts in the consumption goods sector may be perceived as unfair because there is no transparent link between individual performance and effective wages. I specifically assume that workers retaliate nominal wage cuts as follows:

$$e_{hft} = \mathbb{1}[W_{ft}^C \geq \delta^W W_{ft-1}^C]$$

which effectively prevents firms from lowering wages below  $\delta^W W_{ft-1}^C$  and thus gives rise to a nominal downward rigidity.<sup>36</sup> Given the institutional setup described thus far, I now proceed by discussing in detail the objectives pursued by firms in subperiod  $t_1$ .

**Capital goods sector.**  $|J^K| > 1$  capital firms maximize contemporaneous monetary profits by choosing a capital rental price  $Q_{ft}$  and a wage function  $W_{ft}^K(x)$ . We have,

$$\begin{aligned} V_1^{FK}(z_t^K; k_{ft-1}^S, \{k_{jt-1}^S, P_{jt}^K, W_{jt}^K\}_{j \neq f}) &= \max_{Q_{ft}, W_{ft}^K(x)} Q_{ft} \min \{\bar{k}_{ft}^D(Q_{ft}), k_{ft}^S\} - \int_{J^{HH}} n_{hft} W_{ft}^K(x_{ht}) dh \\ \text{s.t. } k_{ft}^S &= (1 - \delta^D) k_{ft-1}^S + y_{ft}^K \\ y_{ft}^K &= \mu^{LF} \int z_t^K n_{hft} x_{ht} dh \end{aligned}$$

where  $\bar{k}_{ft}^D$  denotes a specific firm’s residual demand<sup>37</sup> given  $\{k_{jt}^S, P_{jt}^K\}_{j \neq f}$ . Since competition operates along the price margin, the quantities  $\{y_{jt}^K\}_{j \neq f}$  may not be taken as given. For example,

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alternatively also be thought of as a manifestation of “certainty effect” described in Tversky and Kahneman’s (1986), is extreme because households may choose to work for a capital producer even though working for a consumption goods producer entails a higher wage *almost surely*. In particular, recall that each household is of zero measure such that even if each consumption goods sector applicant is hired with probability one, a countable set of (measure zero) households may wind up unemployed involuntarily.

<sup>35</sup>As indicated,  $\hat{W}_{hft}(\omega) \in \{W_{ft}^C, W_t^U\}$  for  $f \in J^C$  and  $\hat{W}_{hft}(\omega) = W_{hft}$  for  $f \in J^K \cup J^U$ .

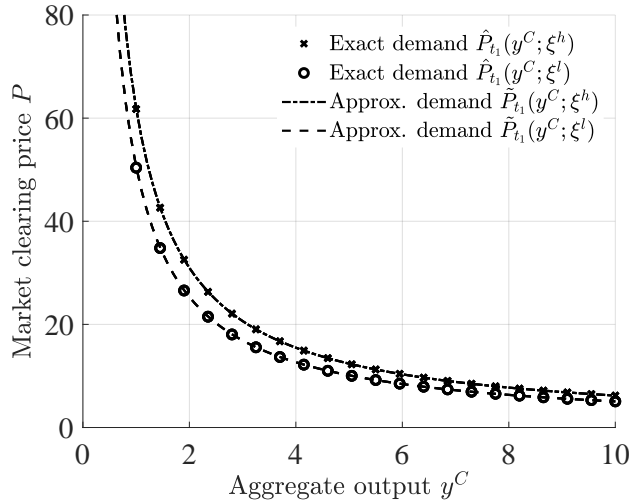
<sup>36</sup>As such, my specification behaviorally motivates the ad hoc rigidity proposed in Schmitt-Grohé and Uribe (2016).

<sup>37</sup>If multiple firms choose the same rental price of capital, residual demand is allocated proportionally.

a capital producer can always attract more labor by offering a very lucrative wage contract. Equilibrium in this market is characterized by a uniform, market clearing price strategy  $Q_{ft} = Q_t$  and a wage contract  $W_{ft}^K(x) = W_t^K(x) = z_t^K Q_t x$  for each  $f \in J^K$ .<sup>38</sup> Holding effort  $e_{ht}$  constant, notice then that  $W_{ht}^K = W_t^K(x_{ht})$  implies that a worker's sectoral wage offer is increasing in their skill  $q_h$  because individual labor output is given by  $x_{ht} = e_{ht} q_h$ .

**Consumption goods sector.**  $|J^C| > 1$  firms rent capital and hire labor to produce a homogenous, non-durable consumption good  $C$ . Since capital is rented, maximization and the implied equilibrium strategies are uniform across  $f \in J^C$  for each  $t$ . Since firms have no  $(M1)$  at the beginning of  $t_1$ , capital rentals and worker salaries are financed with a bank loan  $l_{ft}$  at the interest rate  $R_{ft}^L$ . Since consumption demand is a function of the uncertain taste shock  $\xi_t \sim G_{\xi_t|\xi_{t-1}}$  at the time of production,  $R_{ft}^L$  features a credit spread accounting for the case in which realized sales are insufficient to cover the face value of the loan. For purposes of realism and computation, it is further assumed that firms and banks locally approximate actual demand  $\hat{P}_{t_1}^C(y_t^C; \xi_t)$  with a function of the isoelastic type  $\tilde{P}_{t_1}^C(y_t^C; \xi_t) \equiv \tilde{\chi}_t^k(\xi_t)[y_t^C]^{-\tilde{\chi}_t^r(\xi_t)}$ .<sup>39</sup>

**Figure 1.10.** Actual vs. isoelastic approximation of aggregate demand



Notes: The message of the above figure is twofold. First, the proposed consumption-savings problem — to be introduced in subperiod  $t_4$  — generates aggregate demand that is approximately isoelastic, at least over the displayed domain. Second, the taste shock  $\xi_t$  acts as an exogenous shifter of the demand curve, which induces aggregate uncertainty at the time of production. Moreover, because demand is uncertain when firms hire workers and rent capital, bank loans can be subject to credit risk.

<sup>38</sup>For a derivation of the firms' equilibrium strategies, refer to Section 1.7.

<sup>39</sup>The approximation parameters  $\tilde{\chi}_{ft}^k(\xi_t)$ ,  $\tilde{\chi}_{ft}^r(\xi_t)$  are recovered numerically via a local symmetric difference quotient near last period's aggregate output.



Figure 1.10 illustrates that  $\tilde{P}_{ft_1}^C(y_t^C; \xi_t)$  serves as a very accurate approximation of  $\hat{P}_{t_1}^C(y_t^C; \xi_t)$  with the latter indeed being approximately isoelastic, at least over the displayed domain. After observing the realization of the taste shock in subperiod  $t_4$ , firms compete for customers via price which implies market clearing in equilibrium.<sup>40</sup> Ex ante, firms are then assumed to maximize expected profits as follows.

$$\begin{aligned}
V_1^{FC}(z_t^C, W_{ft-1}^C; R_{ft}^L, Q_t) &= \max_{l_{ft}, n_{ft}^C, W_{ft}^C} \mathbb{E}_{t_1} [\tilde{\Pi}_{ft}] \\
\text{s.t. } \tilde{\Pi}_{ft} &\equiv y_{ft}^C \tilde{\chi}_{ft}^k(\xi_t) \left[ \sum_{JC} y_{jt}^C \right]^{-\tilde{\chi}_{ft}^r(\xi_t)} - l_{ft} R_{ft}^L \\
y_{ft}^C &= z_t^C [k_{ft}^C]^\alpha \left[ \mu^{LF} \int n_{hft} q_h e_{ht} dh \right]^{1-\alpha} \\
k_{ft}^C &= (l_{ft} - \mu^{LF} n_{ft}^C W_{ft}^C - \mathcal{T}_t^F) / Q_t \\
n_{ft}^C &\leq \int n_{hft} dh \\
q_{ft}^C &= f(W_{ft}^C) \\
e_{ft}^C &= \mathbb{1}[W_{ft}^C \geq \delta^W W_{ft-1}^C]
\end{aligned}$$

where the expectation is taken with respect to  $G_{\xi_t|\xi_{t-1}}$  and the fact that the function  $f$ , to be derived shortly, is weakly increasing in  $W_{ft}^C$  may generate an additional downward rigidity.<sup>41</sup>

To understand the properties and the role of the function  $f$ , recall that each worker's capital sector wage offer  $W_{ht}^K = z_t^K Q_t q_h$  is increasing in their skill  $q_h$ , whereas — since optimization is symmetric across consumption goods producers —  $W_t^C$  is uniform across households. Given the utility specification in (1), a consumption sector wage  $W_t^C$  thus attracts all households satisfying  $q_h \leq \delta^U W_t^C / z_t^K Q_t$ . Holding fixed technology and prices, there then exists a marginal worker  $h^*$  with skill  $q_t^* \equiv \delta^U W_t^C / z_t^K Q_t$ . We then partition the set of households as follows: Since each worker can only apply for one position at a time, households with  $q_h > q_t^*$  choose to supply labor to the capital goods sector, whereas households with  $q_h \leq q_t^*$  apply for consumption sector jobs. Average sectoral productivity is then plotted in Figure 3.5a and is given by,

<sup>40</sup>The realization of  $\hat{P}_t^C(y_t^C, \xi_t)$  in turn determines the return of the bank's loan  $\hat{R}_{ft}^L = \min\{R_{ft}^L, y_{ft}^C \hat{P}_t^C / l_{ft}\}$ .

<sup>41</sup>Firms understand that markets will clear ex post, but take other firms' announced output as given in Cournot fashion ex ante. They thus implicitly assume that labor is not scarce in aggregate and thus do not recognize the possibility of affecting another firm's output by poaching its workers (by offering an infinitesimally higher wage). If this were allowed, the competitive nature of Bertrand competition would have to induce  $\mathbb{E}_{t_1}[\tilde{\Pi}_{ft}] = 0$ . While potentially more realistic and certainly very interesting, the corresponding analysis is, like price commitment, beyond the scope of this paper and thus left for future work. For a detailed discussion of the equilibrium strategies, refer to Section 1.7.

$$q_t^K = \mathbb{E}[q|q > q_t^*]$$

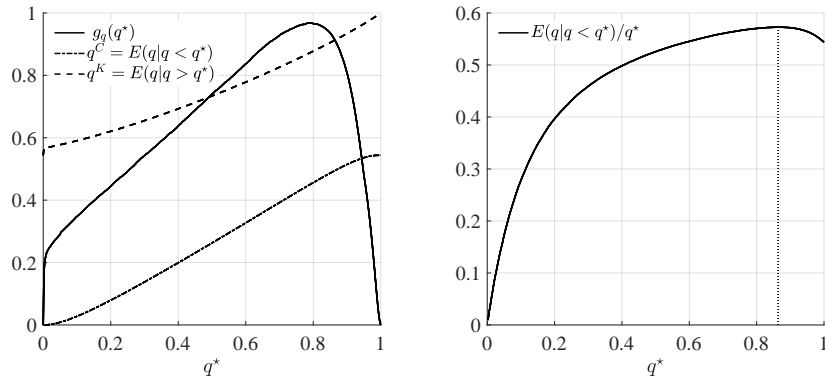
$$q_t^C = \mathbb{E}[q|q \leq q_t^*]$$

such that average labor productivity in the consumption goods sector  $q_t^C = \mathbb{E}[q|q \leq \delta^U W_t^C / z_t^K Q_t]$  is increasing in the wage offer  $W_t^C$ . This is because increasing the wage attracts more productive workers, which raises average labor productivity. Conversely, lowering wage offers always induces the highest skilled workers to quit first. Depending on the cross-sectional distribution of skill, adverse selection may then induce a lower wage threshold below which firms will never optimally make an offer. In particular, if the skill distribution  $G_q$  induces a unique maximizer  $\underline{W}_t^C$  of

$$\frac{\mathbb{E}\left[q|q < \frac{\delta^U W_t^C}{z_t^K Q_t}\right]}{W_t^C} \quad (1.2)$$

offering any wage  $W_{ft}^C < \underline{W}_t^C$  is strictly dominated by the strategy of offering the lower bound  $\underline{W}_t^C$ . This is because below the threshold, labor costs are effectively *decreasing* in the wage. Such a situation is shown in Figure 9, where the initial, local concavity of  $\mathbb{E}[q|q < q^*]$  induces a unique, interior maximum of  $\mathbb{E}[q|q < q^*]/q^*$ . In such a setting, optimality implies that firms will never offer wages below the threshold.<sup>42</sup>

**Figure 1.11.** Pairing heterogenous skill with asymmetric information



Notes: Figure 3.5a depicts average sectoral labor productivity for a rectified Gaussian distribution  $G_q$ , whereas Figure 3.5b illustrates the implied effective labor productivity per unit of wages paid (recall  $q_t^* = \delta^U W_t^C / z_t^K Q_t$ , where everything but  $W_t^C$  is taken as given by the firm). By definition, the conditional expectation  $\mathbb{E}[q|q < q^*]$  is increasing in  $q^*$ , which implies the following adverse selection mechanism from the point of view of firms. Lowering wage offers always induces the highest skilled workers to quit first, which depresses average labor productivity. This is not enough to generate the additional wage rigidity, for which I specifically require that (2) have an interior maximum such as the one highlighted in Figure 3.5 at  $x \approx 0.87$ . As illustrated, this desired property is satisfied by the chosen rectified Gaussian distribution because its mass at zero induces  $\mathbb{E}[q|q < q^*]$  to be locally concave.

<sup>42</sup>See Section 1.7 or Weiss (1980) for a proof.

**Banks.** Rather than disbursing commercial loans in reserves, banks artificially create previously nonexistent deposits (see Werner, 2014). This is acceptable to firms because deposits are accepted as means of payment irrespective of their backing.

**Figure 1.12.** *M1* creation by banks

Assets	Liabilities
Reserves ( <i>M0</i> )	Deposits ( <i>M1</i> )
+ Commercial loan	+ Firm deposit ( <i>M1</i> )

Notes: Commercial loans are not disbursed in reserves, but firms are credited with previously nonexistent deposits. As long as the reserve requirement is not binding, banks can thus instantly and artificially rescale the economy’s numéraire *M1*.

Assuming a reserve requirement of  $\lambda_{RR}$ , the banking sector is subject to the following aggregate constraint,

$$\lambda_{RR} \left[ \underbrace{d_{t_0}^{HH} + d_{t_0}^{PF} + d_{t_1}^F}_{v_{t_0}} \right] \leq v_{t_0} \quad (1.3)$$

which implies that the central bank can curb lending by reducing the amount of aggregate reserves (see Blinder and Stiglitz, 1983; Bernanke and Blinder, 1992). Conversely, as impressively illustrated by the fact that the money multiplier  $M_1/M_0$  was below 1 for virtually an entire decade between 2008 and 2018, boosting reserves in an effort to expand lending likely constitutes a doomed attempt to “push on a string”.<sup>43</sup> This is because banks need not use their reserves to extend loans.<sup>44</sup> In particular, rather than issuing the maximum possible amount of credit, banks can purchase risk free government bonds or lend to other banks instead,

$$d_{t_1}^F \leq \frac{1 - \lambda_{RR}}{\lambda_{RR}} \left[ v_{t_0} + \underbrace{\sum_{J^B} v_{bt_1}^B}_{=0} - \sum_{J^B} a_{bt}^S P_t^S \right] \quad (1.4)$$

where  $a_{bt}^S$  denote the quantity of *S* held by bank *b* and borrowed reserves must be zero in

<sup>43</sup>The pushing-on-a-string phrase dates back to the depression era during which it was used — by the incumbent Fed Chair Eccles (Wood, 2005) among others — to describe the impotence of contemporary monetary policy.

<sup>44</sup>While excess funds were lent overnight to other depository institutions prior to 2008, they now earn IOER when held with the Fed. Unsurprisingly, this has induced banks to hold vast amounts of excess reserves.

aggregate. Assuming that banks approximate future demand just like firms, or  $\tilde{P}_{bt_1}^C = \tilde{P}_{t_1}^C$  for each  $b \in J^B$ , I propose the following bank objective.

$$\begin{aligned}
V_1^B(v_{bt_0}) = & \max_{R_{bft}^L, a_{bft}^L, a_{bt}^S, v_{bt_1}^B} \min_{\xi_t \in \Xi} \left\{ \left( \sum_{JC} [l_{bft} - a_{bft}^L P_{bft}^L] \tilde{R}_{bft}^L(\xi_t) + a_{bt_1}^S V_t^S + v_{bt_1} \right) - d_{t_1}^T - v_{bt_1}^B R_t^{FFR} \right\} \\
\text{s.t. } & d_{bft_1}^F = l_{bft} \\
& d_{t_1}^T = d_{bt_0}^{HH} + d_{bt_0}^{PF} + \sum_{JC} d_{bft_1}^F - a_{bft}^L P_{bft}^L \\
& d_{t_1}^T \leq \frac{1}{\lambda_{RR}} v_{bt_1} \\
& v_{bt_1} = v_{bt_0} - a_{bt_1}^S P_t^S + v_{bt_1}^B \\
& P_{bft}^L = l_{bft} \mathbb{E}_{t_1} \left[ \tilde{R}_{bft}^L \right] / R_t^S \\
& \tilde{R}_{bft}^L(\xi_t) \equiv \min \left\{ R_{bft}^L, \frac{y_{ft}^C \tilde{P}_{bt_1}^C(\xi_t)}{\sum_{JB} l_{bft}} \right\}
\end{aligned}$$

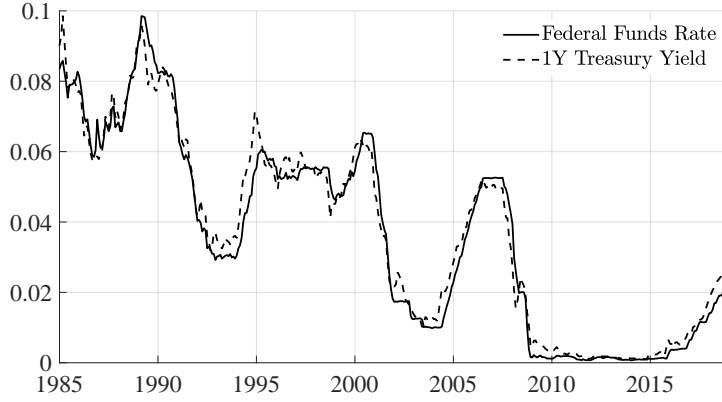
To understand the bank's problem, first notice that retaining a commercial loan on balance sheet entails the risk of generating a (marginal) profit short of the risk free rate. Given their maxmin risk preferences, banks will thus never opt to keep loans on balance sheet. Instead, banks follow an originate-to-distribute model in which each new loan is sold in its entirety ( $a_{bft}^L = 1$ ).<sup>45</sup> Loan sales are contractually set up by an auxiliary investment management company whose reservation price for the loan  $P_{bft}^L$  is taken as given.<sup>46</sup> Each loan's interest rate is thus not determined by the fundamental risk preference of the issuing bank, but by the prevailing secondary market price  $P_{bft}^L$ .

In effect, Bertrand competition among banks then leads to competitive loan pricing  $l_{bft} = P_{bft}^L$  and simultaneously ties the FFR to the exogenously set risk free interest rate  $R_t^{FFR} = R_t^S$ . The latter mechanism is known and exploited by the central bank in its setting of interest rates. As illustrated in Figure 1.13, it approximately holds in the data.

<sup>45</sup>See Gorton and Pennacchi (1995) for a historical account of the originate-to-distribute model and the typical moral hazard concerns associated with it. Brunnermeier (2009) argues that originate-to-distribute led to a significant deterioration of lending standards in the early 2000s and thus played a major role in the creation of the "housing bubble" prior to the Great Recession.

<sup>46</sup>It is assumed here that all financial market agents probabilistically assess the loans like a bank.

**Figure 1.13.** Federal funds rate vs. one-year treasury yield



Notes: Figure 1.13 illustrates the intimate empirical relationship between the Federal Funds Rate (FFR) and the one-year Treasury yield. In my theory, FFR and the one-period sovereign bond yield must be equivalent in equilibrium. This because investing in sovereign bonds yields the announced risk free rate such that banks are not willing to lend out their reserves returns at rates below it. At the same time, they are also not willing to borrow reserves at rates higher than the risk free rate because it would not be profitable. Both displayed series were taken from FRED.

**Pension funds.** Recall that pension funds began the period with an initial level of demand deposits  $d_{it_0}^{PF}$  and government securities  $a_{it_0}^S$ . Having chosen their capital structure in subperiod  $t_0$ , they now proceed by maximizing assets returns subject to a liquidity constraint imposed by the government regulator.

$$V_1^{PF}(d_{it_0}^{PF}, a_{it_0}^S, w_{it_0}^D) = \max_{d_{it_1}^{PF}, a_{it_1}^S} d_{it_1}^{PF} + a_{it_1}^S V_t^S$$

$$\text{s.t. } d_{it_1}^{PF} + a_{it_1}^S P_{t_1}^S \leq d_{it_0}^{PF} + a_{it_0}^S P_{t_1}^S \quad (1.5)$$

$$d_{it_1}^{PF} \geq \delta_L (d_{it_1}^{PF} + a_{it_1}^S P_{t_1}^S) \quad (1.6)$$

Since holding government bonds is more lucrative than holding money, because  $V_t^S > P_t^S$ , equation (1.5) binds in equilibrium.

#### *Subperiod $t_2$ : Securitization*

The financial sector allows households to indirectly hold claims in firms via pension funds and a collateralized loan obligation  $R$ . The latter is originated as the investment management company organizes all loan contracts in form a special purpose vehicle, whose shares are then sold to pension funds at  $t_2$ . Letting each commercial loan's face value, true ex post payoff, and projected payoff be denoted by  $V_{bft} \equiv l_{bft} R_{bft}^L$ ,  $\hat{V}_{bft} \equiv l_{bft} \hat{R}_{bft}^L$ , and  $\tilde{V}_{bft} \equiv l_{bft} \hat{R}_{bft}^L$ , the CLO's face value, ex post payoff, and projected payoff are given by,

$$\begin{aligned}
V_t^R &= \sum_{J^B} \sum_{J^C} V_{bft} \\
\hat{V}_t^R(\xi_t) &= \sum_{J^B} \sum_{J^C} \hat{V}_{bft}(\xi_t) \\
\tilde{V}_{it}^R(\xi_t) &= \tilde{V}_t^R(\xi_t)
\end{aligned}$$

for each  $i \in J^B \cup J^{PF}$ . Again, we have  $\tilde{V}_t^R(\xi_t) \neq \hat{V}_t^R(\xi_t)$  because of the isoelastic price approximation.<sup>47</sup> Institutionally, the sale of  $R$  is conducted by the Walrasian auctioneer as pension fund demand is determined by,

$$\begin{aligned}
V_2^{PF}(d_{it_1}^{PF}, a_{it_1}^S, w_{it_0}^D) &= \max_{d_{it_2}^{PF}, a_{it_2}^S, a_{it_2}^R} d_{it_2}^{PF} + a_{it_2}^S V_t^S + a_{it_2}^R \mathbb{E}_{t_2} [\tilde{V}_{it}^R] \\
\text{s.t.} \quad d_{it_2}^{PF} + a_{it_2}^S P_t^S + a_{it_2}^R P_{t_2}^R &\leq d_{it_1}^{PF} + a_{it_1}^S P_t^S \quad (1.7)
\end{aligned}$$

$$d_{it_2}^{PF} \geq \delta_L w_{it_0}^D \quad (1.8)$$

As long as funds are sufficiently liquid (including all  $S$  holdings), equilibrium must satisfy,

$$P_{t_2}^R = \frac{\mathbb{E}_{t_2}[\tilde{V}_t^R]}{R_t^S} \quad (1.9)$$

where  $R_t^S \equiv V_t^S/P_t^S$ . Equation (1.8) thus establishes (subjective) certainty equivalence between the lotteries associated with holding  $R$  and holding  $S$ . Since the investment management company appraise loans based on their value as part of the CLO, we have,

$$P_{bft}^L = \frac{l_{bft}}{\sum_{J^B} \sum_{J^C} l_{bft}} P_{t_2}^R \quad (1.10)$$

such that, by (9),  $P_{bft}^L$  is decreasing in  $R_t^S$  and thus  $R_t^T$ . This relationship simply captures that monetary policy operates along Bernanke's "cost-of-capital" channel, which postulates that firms' cost of finance is higher when monetary policy is tight (2007). Since all debt is rolled over each period, neither firm nor bank net worth play an amplifying role in the model.

### *Subperiod $t_3$ : Marking-to-market*

At the beginning of  $t_3$ , financial markets observe a signal  $\xi_t' \sim G_{\xi_t'|\xi_t}$  with probability  $\pi_s$ . If

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<sup>47</sup>We thus have  $\mathbb{E}_{t_2}[\tilde{V}_{it}^R] = \int_{\Xi} \tilde{V}_{it}^R(\xi_t) dG_{\xi_t|\xi_{t-1}}$  for each  $i \in J^{PF}$ .

observed, the signal is processed to generate the newly relevant conditional distribution  $G_{\xi_t|\xi'_t, \xi_{t-1}}$  using Bayes' theorem.<sup>48</sup> As projected demand is updated, funds revise their portfolios accordingly,

$$V_3^{PF}(d_{it_2}^{PF}, a_{it_2}^S, w_{it_0}^D) = \max_{a_{it_3}^S, a_{it_3}^R, d_{it_3}^{PF}} a_{it_3}^S V_t^S + a_{it_3}^R \mathbb{E}_{t_3}[\tilde{V}_t^R] + d_{it_3}^{PF}$$

$$\text{s.t. } d_{it_3}^{PF} \geq \delta_L w_{it_0}^D d_{it_3}^{PF} + a_{it_3}^S P_t^S + a_{it_3}^R P_{t_3}^R \geq \delta^M w_{it_0}^D$$

where  $\mathbb{E}_{t_3}[\tilde{V}_t^R] = \int_{\Xi} \tilde{V}_t^R(\xi_t) dG(\xi_t|\xi'_t, \xi_{t-1})$ . Because the liquidity constraint still binds across all funds, no purchases or sales of  $R$  take place. The new equilibrium satisfies,

$$P_{t_3}^R = \frac{\mathbb{E}_{t_3}[\tilde{V}_t^R]}{R_t^S}$$

such that, if a signal was observed, information-motivated trading has yielded a new risk-adjusted yield  $R_{t_3}^R \equiv V_t^R/P_{t_3}^R$  with the risk free rate remaining unchanged. Each fund's equity  $w_{it_3}^E$  is recalculated as a residual of assets net of debt  $w_{it_3}^I - w_{it_0}^D$ . If the maintenance margin requirement  $\delta_M w_{it_3}^E \geq w_{it_0}^D$  is violated, the broker issues a *margin call*  $MC_{it} \in \{0, 1\}$  demanding fund  $i$  bring its account up to the minimum maintenance level by repurchasing part of its debt using demand deposits.

$$MC_{it} = \mathbb{1}(\delta_M w_{it_3}^E < w_{it_0}^D)$$

$$= \mathbb{1}\left(w_{it_3}^I < \frac{\delta_I(1 + \delta_M)}{\delta_M(1 + \delta_I)} w_{it_0}^I\right)$$

$$= \mathbb{1}(w_{it_3}^I < \delta_{MC} w_{it_0}^I)$$

with a corresponding liquidity demand of  $\Delta_{it}^{MC} = w_{it_0}^D - \delta_M w_{it_3}^E$ . Facing a margin call, a fund's otherwise prevailing liquidity constraint  $d_{it_3}^{PF} \geq \delta_L w_{it_0}^D$  is lifted such that it may consolidate its position with the broker. However, since  $d_{it_3}^{PF}$  assumes non-negative values only, the fund may find itself in a position in which it must procure fresh liquidity either by selling part of its portfolio or the issuance of new debt.<sup>49</sup> If issuing new debt is impossible, brokers are assumed to proceed by *selling out* part of the fund's assets.

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<sup>48</sup>We have,

$$\Pr(\xi_t|\xi'_t, \xi_{t-1}) = \frac{\Pr(\xi'_t|\xi_t, \xi_{t-1}) \Pr(\xi_t|\xi_{t-1})}{\Pr(\xi'_t|\xi_{t-1})}$$

such that, as long as  $G_{\xi_t|\xi_{t-1}}$  and  $G_{\xi'_t|\xi_t}$  are known, it is straightforward to find  $G_{\xi_t|\xi'_t, \xi_{t-1}}$ .

<sup>49</sup>See Brunnermeier and Pedersen (2009) for an extensive account of funding vs. market liquidity.

$$\begin{aligned}
SO_{it} &= \mathbb{1}(d_{it_2}^{PF} < \Delta_{it}^{MC}) \\
&= \mathbb{1}\left(w_{it_3}^I < \frac{\delta_I(1 + \delta_M - \delta_L)}{\delta_M(1 + \delta_I)} w_{it_0}^I\right) \\
&= \mathbb{1}(w_{it_3}^I < \delta_{SO} w_{it_0}^I)
\end{aligned}$$

We must have  $\delta_{SO} < \delta_{MC}$ , which implies that all sellouts are preceded by margin calls, but not all margin calls are followed by a sellout. Letting  $\Delta_{it}^G = \Delta_{it}^{MC} - d_{it_2}^{PF}$  denote each fund's liquidity gap, notice that the sale of any convex combination  $[(\Delta_{it}^S, 0), (0, \Delta_{it}^R)]_\lambda$  with  $\Delta_{it}^S = \Delta_{it}^G / P_{t_3}^S$ ,  $\Delta_{it}^R = \Delta_{it}^G / P_{t_3}^R$  and  $\lambda \in [0, 1]$  would be sufficient to make the broker whole. However, since sellouts are symmetric across funds, no fund is capable of offering  $M1$  in exchange for either security. The only remaining option is to sell assets to the central bank. Absent any unconventional monetary policy, the central bank only purchases the risk free security at the previously announced target. Therefore, fund  $i$  can satisfy its broker's margin call if and only if the sale of  $a_{it_2}^S$  is sufficient to cover  $\Delta_{it}^G$ .<sup>50</sup> Otherwise, the broker proceeds to sell out  $R$  in a *fire sale*.

$$\begin{aligned}
FS_{it} &= \mathbb{1}(a_{it_2}^S P_{t_3}^S < \Delta_{it}^G) \\
&= \mathbb{1}(w_{it_3}^I < \delta_{FS} w_{it_0}^I)
\end{aligned}$$

Again, we must have  $\delta_{FS} < \delta_{SO}$  such that each fire sale is preceded by a sellout, but not each sellout is followed by a fire sale. In context of a fire sale, since there exists no private buyer for the risky security, the Walrasian auctioneer fails to locate an equilibrium price because the law of demand fails to hold: Lowering the security's price induces lower demand because margin calls increase. At this stage, I proceed by assuming that the monetary authority intervenes and provides liquidity in some fashion. In the benchmark case, policy makers agree to buy up the entire excess supply at a predetermined haircut  $\delta_{HC}$  below the fundamental price  $P_{t_3}^R$ . Alternatively, the central bank could also enter into repurchase agreements, install an emergency lending facility, and/or lower the interest rate target.

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<sup>50</sup>Since  $S$  remains liquid while  $R$  does not, asset composition of each fund's portfolio is crucial at this stage.



*Subperiod  $t_4$ : Consumption*

Recall that since households live for  $T^L$  periods, there are  $T^L$  overlapping generations at each time. Over the first  $T^R$  periods of life, households belong to the working-age population and supply labor to firms. While in the labor force, workers accumulate retirement savings by investing part of each period's income in illiquid financial claims. At retirement, the stock of previously accumulated financial wealth is liquidated and deposited in the corresponding household's bank account. During retirement, households receive a pension from the government and draw down their accumulated savings until the age of  $T^L$ , at which point they are replaced by a new household.

Since accumulated savings are illiquid until retirement, households cannot boost next period's consumption by saving more this period.<sup>51</sup> Instead, the relevant benefit associated with the cost of decreasing consumption today  $c_{ht}$  is given by a corresponding increase in projected retirement savings  $\tilde{w}_{ht}^P$ . The latter generate utility because households find retirement consumption too difficult to assess probabilistically, but understand that it is strictly increasing in accumulated savings.

As indicated by its designation, the *projection*  $\tilde{w}_{ht}^P$  serves as a household's best estimate of the effectively available funds at the time of retirement. Inspired by the Bureau of Labor Statistics' (BoL) guidelines on how to save for retirement,  $\tilde{w}_{ht}^P$  is derived via cumulative compounding with a benchmark return of  $\tilde{R}_t^A$ . Specifically, if  $\tau_{ht}^R$  denotes household  $h$ 's remaining number of periods in the labor force at time  $t$ , I assume,

$$\tilde{w}_{ht}^P = w_{ht_0}^I [\tilde{R}_t^A]^{\tau_{ht}^R} + s_{ht} w_{ht_0}^L \sum_{i=1}^{\tau_{ht}^R - 1} [\tilde{R}_t^A]^{i-1}$$

where  $w_{ht_0}^I$  is the beginning of period stock of illiquid retirement balances,  $w_{ht_0}^L$  are current liquid balances to be split between consumption and saving, and  $s_{ht}$  is the chosen savings fraction. In words, households base their retirement balance projection on the assumption that until retirement, they will save the exact same nominal amount each period and that all savings will generate a return of  $\tilde{R}_t^A$ .<sup>52</sup>

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<sup>51</sup>This assumption is by no means necessary, but it serves as a convenient way of emphasizing my main point, which is that the primary reason why households save is to accumulate wealth for retirement, not to shift consumption from today to tomorrow. An alternative specification would be to endogenize retirement with household utility being decreasing in the projected retirement age.

<sup>52</sup>For purposes of illustration, consider the following concrete example taken from the US DoL publication "Top 10 Ways to Prepare for Retirement" (see Section 1.7): Suppose you saved \$5,500 each year until retirement in 35 years and your money earned 7% annually. In that case, your projected retirement balance would be \$760'303.

For purposes of evaluation, households assess  $\tilde{w}_{ht}^P$  relative to a predetermined retirement goal  $w_{ht}^G$  that aims to capture the idea that accumulated savings substitute for labor income during retirement,

$$w_{ht}^G = (T^L - T^R)(1 - \lambda_R)\hat{W}_{ht-1}$$

where  $\lambda_U W_{t-1}$  is the projected per-period retirement benefit received from the government.<sup>53</sup> To generate the desired aggregate demand uncertainty, utility derived from retirement savings is subject to an aggregate shock  $\xi_t$ . Following the realization of  $\xi_t$ , firms compete to sell their existing inventories via price. Since production costs are sunk, equilibrium is given by a uniform price strategy  $P_t^C$  that clears the market.<sup>54</sup>

$$\int_{J^{HH}} c_{ht} = \sum_{J^C} y_{ft}^C$$

Finally, I assume that households only care about contemporaneous consumption and savings once retired. We have,

$$V_4^{HH}(w_{ht_0}^L, w_{ht_0}^I) = \max_{s_{ht}} \begin{cases} u(c_{ht}; \gamma^c) + \xi_t v\left(\frac{\tilde{w}_{ht}^P}{w_{ht}^G}; \gamma_h^w\right) & \text{if in labor force} \\ u(c_{ht}; \gamma^c) + v\left(s_{ht} - \frac{\tau_{ht}^L}{\tau_{ht}^L + 1}; \gamma^s\right) & \text{if retired} \end{cases}$$

$$\text{s.t. } c_{ht} = \frac{w_{ht_0}^L(1 - s_{ht})}{P_t^C}$$

$$\tilde{w}_{ht}^P = w_{ht_0}^I [\tilde{R}_t^A]^{\tau_{ht}^R} + s_{ht} w_{ht_0}^L \sum_{i=1}^{\tau_{ht}^R - 1} [\tilde{R}_t^A]^{i-1}$$

$$w_{ht}^G = (T^L - T^R)(1 - \lambda_R)\hat{W}_{ht-1}$$

where, for purposes of this paper,  $u$  and  $v$  are specified as follows,

$$u(x; \alpha) = x^\alpha \quad \alpha \in (0, 1)$$

$$v(x; \alpha) = \frac{1}{\alpha} [1 - \exp(-\alpha x)] \quad \alpha > 1$$

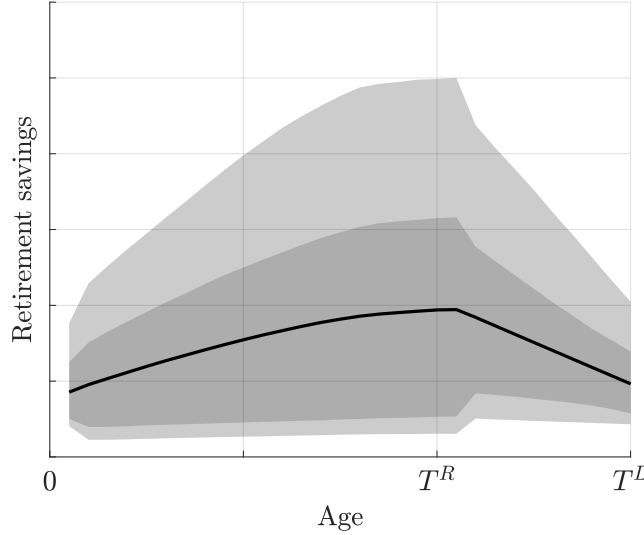
Since  $\lim_{c \rightarrow 0} u'(c; \alpha) \rightarrow \infty$ , we must have  $c_{ht} > 0 \quad \forall ht$  in equilibrium. In effect, the above

<sup>53</sup>As discussed shortly, the true retirement benefit is calculated as a fraction of the current going wage  $W_{t-1}$  on a period-to-period basis.

<sup>54</sup>Since firms need not commit to a price strategy prior to the observation of  $\xi_t$ , the entire taste shock is absorbed via price. This leads to unreasonably volatile rates of inflation. Introducing price commitment thus constitutes a natural next step in improving the model. See Section 1.7 for a discussion of equilibrium.

utility specification induces the aggregate demand function depicted in Figure 1.10 while inducing a bell-shaped profile over lifetime wealth. Figure 1.14 illustrates the evolution of lifetime wealth across households who live for thirty periods of which they are retired for the last ten.

**Figure 1.14.** The cross-sectional life cycle of household wealth



Notes: The above figure depicts the life cycle evolution of cross-sectional, beginning-of-period household wealth. Specifically, I report the mean as well as 68% and 95% confidence bands for a given age. During their working life, households accumulate wealth by saving their wage earnings and firm profits. In turn, following retirement, they proceed by financing part of their consumption by drawing down on accumulated savings. While labor income is zero during retirement, households still receive firm profits from their ownership of firms.

*Subperiod  $t_5$ : Settlement*

As firms repay their loans, banks credit pension funds with additional demand deposits in the amount of  $a_{it_3}^R \hat{V}_t^R$  such that fund  $i$ 's realized asset, debt, and equity returns are given by,

$$\begin{aligned}\hat{R}_{it}^A &= w_{it_4}^I / w_{it_0}^I \\ \hat{R}_{it}^D &= \min \{w_{it_4}^I, w_{it}^D R_t^D\} / w_{it}^D \\ \hat{R}_{it}^E &= (w_{it_4}^I - w_{it}^D \hat{R}_{it}^D) / w_{it}^E\end{aligned}$$

where  $w_{it_4}^I = d_{it_3}^{PF} + a_{it_3}^S V_t^S + a_{it_3}^R \hat{V}_t^R$ . Next period's funds, available for the purchase of the newly minted government bond, are then calculated as total assets net of expiring retirement balances,

$$w_{it+1_0}^I = w_{it_4}^I - \int_{J^{HH}} \mathbb{1}(\tau_{ht}^R = 1) \left[ w_{hit}^D \hat{R}_t^D + w_{hit}^E \hat{R}_t^E \right]$$

Institutionally, the sovereign bond  $S$  is rolled over as follows. First, in response to the prevailing

unemployment rate, the central bank issues next period's interest rate target<sup>55</sup>,

$$R_{t+1}^T = \max \{ \bar{R}^T (1 - \bar{n}_t^u)^{\kappa_n}, 1 \}$$

where  $\bar{n}_t^u = n_t^u - n^u$  is unemployment net of the natural rate  $n^u$ . Given  $R_{t+1}^T$ , households revise their asset return projections<sup>56</sup> and the treasury announces a new bond issue with face value  $V_{t+1}^S$  to be paid back at the end of next period. Third, the new issue is sold in an auction open to all pension funds and the central bank.<sup>57</sup> It is assumed that the central bank puts in a bid for the entire bond at  $R_{t+1}^T$  thereby rendering the security risk free.<sup>58</sup> The treasury thus knows it will at least generate revenues in the amount of  $V_{t+1}^S/R_{t+1}^T$  and pension funds know they must at least bid  $R_{t+1}^T$  to receive any portion of the issue. It is further assumed that the treasury guarantees the central bank, in return for making its securities risk free, to supply sufficient liquidity for the latter to be able to implement its policy  $V_{t+1}^S/R_{t+1}^T \geq \sum_{JPF} (1 - \delta^L) w_{it+10}^I$ , which implies that  $S$  is indeed priced by the central bank and not via Walrasian market clearing among funds. The final optimization problem of the period thus takes the following form,

$$\begin{aligned} V_5^{PF}(a_{t_3}^S, d_{it_3}^{PF}; R_{t+1}^T) &= \max_{R_{it+10}^S, a_{it+1}^{S,b}} d_{it+10}^{PF} + a_{it+10}^S V_{t+1}^S \\ \text{s.t.} \quad &a_{it+10}^{S,b} V_{t+1}^S / R_{it+1}^S + d_{it+10}^{PF} = w_{it+10}^I \\ &d_{it+10}^{PF} \geq \delta^L w_{it+10}^I \\ &a_{it+10}^S = a_{it+10}^{S,b} \mathbf{1}(R_{it+1}^S \leq R_{t+1}^T) \end{aligned}$$

where the equality of the last constraint neatly captures the central bank's ability to set interest rates exogenously. In particular, notice that, since the market is sufficiently liquid, it is not optimal to submit a bid below  $R_{t+1}^T$  such that each fund's optimal strategy  $(R^*, a^*)$  is given by  $(R_{t+1}^T, (1 - \delta^L) w_{it+10}^I R_{t+1}^T / V_{t+1}^S)$ . To avoid immediate default, the new bond must also at least cover the government's current liquidity gap,

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<sup>55</sup>Recall that the entire taste shock is absorbed along the price margin such that inflation targeting would not be contextually meaningful. Rather than camouflaging the fundamental pricing issue via a monetary targeting regime, the central bank is tasked to concern itself with unemployment only. Ex ante price commitment on the side of the firm constitutes a necessary next step because it would resolve this issue.

<sup>56</sup>Households interpret the monetary policy change as a permanent level-shift and revise their asset return projection as follows:  $\tilde{R}_{t+1}^A = R_{t+1}^T + \bar{v}_t$ , where  $\bar{v}_t$  denotes the average, additive risk premium as defined earlier.

<sup>57</sup>The bond is allotted proportionally among the highest bidders, but pension funds are prioritized over the central bank.

<sup>58</sup>While not in line with contemporary open market practice, the central bank engaging in the primary market is the only way to guarantee theoretical risklessness.

$$V_{t+1}^S = \min \left\{ \underbrace{V_t^S + \underbrace{(X_t - T_t)}_{\text{Primary deficit}} - \Pi_t^{CB}}_{\text{Deficit}}, \underbrace{\sum_{JPF} (1 - \delta^L) w_{it+10}^I}_{\text{Liquidity provision}} \right\} R_{t+1}^T$$

where  $X_t$  denote government expenditures,  $T_t$  is total tax revenue, and  $\Pi_t^{CB}$  are returned central bank profits. Letting  $\tau_{ht}^R$  denote household  $h$ 's number of year until retirement, the evolution of household monetary wealth  $w_{hit+10}^L$  and illiquid wealth  $w_{hit+10}^F$  is given by,

$$w_{ht+10}^F = \mathbb{1}(\tau_{ht}^R > 1) \left[ w_{ht0}^L (1 - s_{ht}) + \sum_{JPF} w_{hit}^D \hat{R}_t^D + w_{hit}^E \hat{R}_{it}^E \right]$$

$$w_{ht+10}^L = \underbrace{(1 - \tau_I) \hat{W}_{ht}}_{\text{Post-tax wage}} + \underbrace{w_{ht0}^L (1 - s_{ht})}_{\text{Current savings}} + \underbrace{\gamma_h^o \sum_{JF} \hat{\Pi}_{ft}}_{\text{Profits}} + \mathbb{1}(\tau_{ht}^R = 1) \underbrace{\sum_{JPF} w_{hit}^D \hat{R}_t^D + w_{hit}^E \hat{R}_{it}^E}_{\text{Liquidated retirement balances}}$$

where  $\gamma_h^o \stackrel{i.i.d.}{\sim} G_o$  denotes a household's time-invariant ownership share of the firms. Retired ( $n_{ht}^R = 1$ ) and unemployed ( $n_{ht}^U = 1$ ) households receive a pre-tax wage of  $\lambda_R W_t^C$  and  $\lambda_U W_t^C$  respectively. Finally, government tax income and expenditures are given by  $T_t = \int_{JHH} \tau_I \hat{W}_{ht} dh$  and  $X_t = \int_{JHH} [n_{ht}^R + n_{ht}^U] \hat{W}_{ht} dh$ . This concludes the period.

#### 1.4. A Typical Fire Sale Episode

To conduct a quantitative analysis, the proposed economy must be parameterized. For this, I partition  $\theta$  into two subsets  $(\theta_1, \theta_2)$ . First,  $\theta_1$  is calibrated<sup>59</sup> with reference to the literature, regulatory rules, or to specific data statistics. Second, because the computational requirements render a likelihood approach (e.g. particle filtering) impractical,  $\theta_2$  is estimated via simulated method of moments (SMM). In face of occasional, but rare crises, recovering relevant moments from the data is non-trivial. Primarily, this is because the number of observed crises in the employed dataset is so low that, even if observation horizons were sufficiently long to assert conditional convergence (crisis and non-crisis), the same is almost surely not true *unconditionally*. Instead of using recent US data to match the observed unconditional moments, via the observed crisis frequency, I thus proceed by borrowing the latter statistic from Barro’s (2006) treatment of “rare disasters” and subsequently targeting conditional moments only.

Now, before delving into the concrete moments to be matched, I would like to briefly pinpoint two principal issues arising from the Monte Carlo nature of SMM. First, since the duration of moment convergence crucially depends on the quality of state initialization, the system is initialized at the implied deterministic steady state for capital for each candidate  $\theta_2'$ . In an additional effort to improve finite sample performance, the selected sampling horizon  $\bar{\mathcal{T}}$  is augmented with a burn-in period  $\underline{\mathcal{T}}$ . The total simulation horizon is then given by  $\mathcal{T} = \underline{\mathcal{T}} + \bar{\mathcal{T}}$ , where the first  $\underline{\mathcal{T}}$  periods allow the economy to converge to its stationary distribution while time averages are only computed using data that is generated thereafter. Second, to ensure continuity of the loss function, the same sequence of random innovations must be recycled for all evaluations across  $\Theta_2$ . However, this implies that the resulting estimate  $\hat{\theta}_2$  is not only a function of the data and the model, but also of a random quantity that is the sequence of randomly drawn shocks. Traditionally, the estimation uncertainty resulting from this dependence is addressed via bootstrapping — by drawing a swarm of different shock sequences — but computational expense prohibits such a procedure in this case.

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<sup>59</sup>Calibration is understood as any procedure in which  $\theta_1$  is *chosen*, whereas estimation entails numerical (or analytic if possible) minimization of a predetermined, joint loss function of  $\theta_2$  given the data  $\hat{\theta}_2 = \operatorname{argmin}_{\theta_2} \mathcal{L}(\theta_2; Y)$ . Since identification is rarely verifiable across the entire space  $\Theta_2$ , even estimation requires careful ‘calibration’ of the initial guess  $\theta_{2_0}$ .

### Empirical targets

As indicated, rather than targeting unconditional statistics, I proceed by targeting moments that condition on whether or not the economy is in a state of crisis or not. The chosen statistics are summarized in Table 1.2 with  $\kappa^c$  and  $\kappa^{nc}$  denoting crisis and non-crisis statistics respectively,

**Table 1.2.** Empirical targets

Panel A: Statistics targeted via calibration		
$\pi_c$	Crisis frequency, annual	0.017
$\kappa_1^c$	Unemployment, recovery duration (in quarters)	40
$\kappa_2^c$	Nominal wealth lost during crisis	0.2
$\kappa_3^c$	Unemployment, crisis peak	0.1
$\kappa_4^c$	Output gap, crisis trough	-0.04
$\kappa_2^{nc}$	Unemployment, non-crisis mean	0.05
$\kappa_3^{nc}$	Labor share in capital goods sector	0.12
$\kappa_4^{nc}$	Gini coefficient (income), ergodic mean	0.4
$\kappa_5^{nc}$	Aggregate consumption/income, ergodic mean	0.66
$\kappa_6^{nc}$	Annualized equity risk premium $\tilde{R}_t^E - R_t^D$	0.1
Panel B: Statistics targeted via estimation		
$\kappa_7^{nc}$	Unemployment, ergodic standard deviation	0.007
$\kappa_8^{nc}$	Unemployment, quarterly persistence	0.86

As indicated, the targeted crisis frequency is taken from Barro’s treatment of “rare disasters” (2006) with a corresponding targeted employment recovery duration of ten years. Additional crisis targets include nominal wealth loss, calculated from the evolution of household net worth during the Great Recession, as well as the peak in output gap and unemployment recorded during the Great Recession. Data on unemployment and labor shares are obtained from the US Bureau of Labor Statistics, whereas all other data is taken from FRED. Finally, output is detrended via first differencing. Refer to Section 2.7 for a discussion of the data and the computation of each statistic.

Calibration

Table 1.3 summarizes the calibrated parameter values given the chosen time unit of one quarter.

**Table 1.3.** Calibration

Panel A: Literature		
$\alpha$	Capital share	0.4
Panel B: Specific targets <sup>†</sup>		
$\lambda_W$	Nominal wage rigidity	0.99
$\delta_D$	Capital depreciation	0.05
$\lambda_U$	Unemployment benefit	0.22
$\lambda_R$	Retirement benefit	0.5
$\gamma^c$	Consumption exponent	0.1
$\gamma^w$	Retirement savings parameter	3
$\gamma^r$	Risk aversion parameter	0.125
$\gamma^o$	Ownership parameter ( $G_o$ : log-normal)	$\mathcal{N}^L(-0.5, 1)$
$q$	Skill parameter ( $G_q$ : rectified normal)	$\mathcal{N}^R(1, 1.5)$
$\bar{R}^T$	Unconditional quarterly interest rate target	0.0125
Panel C: Regulatory parameters		
$\lambda_{RR}$	Required reserves ( $M0$ liquidity, bank)	0.1
$\lambda_M$	Liquidity constraint ( $M1$ liquidity, pension fund)	0.1
$\lambda_{IM}$	Initial margin (equity, pension fund)	0.2
$\lambda_{MM}$	Maintenance margin (equity, pension fund)	0.1
Panel D: Metaparameters		
$T^L$	Number of overlapping generations	30
$T^R$	Retirement age	20
$ J^C $	Size of consumption goods sector	100
$\mu^{HH}$	Measure of households	10
$N^{HH}$	Effective number of households	450

Notes: The set of calibrated parameters are partitioned into four subsets. First, the capital share of production in the consumption goods sector is set as is common in the literature. Second, nine parameters are set to target specific statistics from the data. The liquidity requirements are set based on observed practice, whereas the margin requirements deviate from observed practice in order to generate occasional fire sales. The metaparameters are chosen to scale the economy subject to the limitations imposed by computational constraints.

To match the initial labor market disruption, the nominal wage rigidity parameter is set to be  $\lambda_W = 0.99$ , which is in line with Schmitt-Grohé and Uribe (2016). Capital depreciation  $\delta_D$  is set equal to 5% quarterly to generate the slow employment recovery and a deterministic steady state of unemployment of 5%. While the liquidity requirements are set equal to values that constitute



common practice, the margin requirements are calibrated independently.<sup>60</sup> The unemployment benefit targets the prevailing level of unemployment, whereas retirement benefits are chosen in an ad hoc fashion. The distribution of skill  $G_q$ , risk preference  $G_s$ , and ownership  $G_o$  are set as follows: Targeting a full employment labor share in the capital goods sector of 12%,  $G_q$  was calibrated to induce a unique maximizer  $\underline{x}$  of  $\underline{f}_q(x) \equiv \mathbb{E}[q|q < x]/x$  at  $x = 0.88$ . In turn,  $G_s$  generates the desired consumption share of income while ownership  $G_o$  gives rise to the desired Gini coefficient for income. Finally, the number of overlapping generations was set to 30 to limit computational expense while the corresponding retirement age ensures a time-invariant working-age to total population ratio of  $\frac{2}{3}$ . The number of consumption sector firms is a natural metric for sectoral competition and was chosen to generate the desired credit spread on the commercial loans.<sup>61</sup> The effectively simulated number of households was finite for computational reasons, and set equal to 50 per generation.<sup>62</sup> The time-invariant measure of households  $\mu^{HH}$  and the initial level of  $M0$  only serve as tools to scale the economy, in real and nominal terms respectively, and were chosen to normalize initial output and the initial wage level to unity.

### *Exogenous drivers*

I proceed by parameterizing the technology and taste shock processes via SMM. For this, I assume that capital technology evolves according to an AR(1) process with ergodic mean  $\mu_k$ , persistence parameter  $\rho_k$ , and a shock standard deviation of  $\sigma_k$ . For purposes of simplicity, the taste shock is reduced to a Markov chain of state size two, good and bad. In the good state  $\xi^{\text{good}}$ , households care less about retirement savings and aggregate demand is high. In the bad state  $\xi^{\text{bad}}$ , households care more about retirement savings and aggregate demand is low. I now estimate the

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<sup>60</sup>The initial and maintenance margin are set to be lower than the actual thresholds of 50% and 25% required under the Federal Reserve Board’s Regulation T because the corporate sector setup does not generate enough loan repayment volatility to induce a margin call at those values. This is primarily due to the fact that loans are not only identical ex ante, but even ex post. Introducing price commitment and/or a spatial reallocation of households along a circular city may mitigate this problem, but both are left for future work.

<sup>61</sup>Since capital goods firms, pension funds and banks compete in Bertrand fashion, their corresponding number is irrelevant as long as it is greater than 1.

<sup>62</sup>Given the vast dimensionality of the model, I do not solve for policy or price functions across the entire state space. However, finding such functions is not necessary because even though each optimization problem takes the form of a functional equation, the latter are never self-referential as they are in the canonical Bellman setup. In consequence, this means that equilibrium at each point in the state space can be found irrespectively of equilibrium at any other point. The model is thus simulated by recursively solving for equilibrium anew each period. Exploiting parallelization, each period then takes roughly 30 seconds to simulate across 24 cores on Vanderbilt’s computing cluster. Notice that the combination of household heterogeneity and the life cycle nature of the model introduces an additional layer of complexity because the number of simulated households is finite. To prevent generational cycles arising from time-invariant sources, it is crucial that parameterization be equivalent across all generations.

size of the bad shock  $\xi^{\text{bad}}$  as well as the persistence  $\rho_k$  and volatility  $\sigma_k$  of the technology shock via SMM. To ensure continuity of the SMM loss function  $\mathcal{L}$ , the same sequence of random innovations is recycled for all evaluations across  $\Theta_2$ .<sup>63</sup> Table 3.1 reports the corresponding results.

**Table 1.4.** Simulated method of moments estimation

$\hat{\rho}_k$	Capital technology persistence	0.905
$\hat{\sigma}_k$	Capital technology shock volatility	0.005

Notes: The exogenous capital technology process is estimated via SMM. The parameters are just-identified as two moments are used to estimate two parameters. Since computation time is linear in the simulated horizon  $T$ , producing standard errors via Monte Carlo is infeasible.

The resulting capital technology process is then given by

$$z_t^K = (1 - \hat{\rho}_k)\mu_k + \hat{\rho}_k z_{t-1}^K + \varepsilon_t^K, \quad \varepsilon_t^K \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(0, \hat{\sigma}_k)$$

The employed Markov chain is calibrated to induce an ergodic demand slump frequency of  $\pi_{\text{recession}} = 0.3$ . The signal frequency  $\pi_{\xi'_t \in \Xi}$  and its accuracy — as given by the probability that the signal corresponds to the true state  $\pi_{\xi'_t = i | \xi_t = i}$  — were set to match the proposed yearly frequency of rare disasters  $\pi_c = 0.017$  in Barro (2006) while ensuring that a financial crisis emerges if and only if a bad state is followed by a bad signal.

$$\pi_c = \underbrace{\left( \pi_{\xi'_t = \xi^{\text{bad}} | \xi_t = \xi^{\text{good}}} \pi_{\xi_t = \xi^{\text{good}} | \xi_{t-1} = \xi^{\text{bad}}} + \pi_{\xi'_t = \xi^{\text{bad}} | \xi_t = \xi^{\text{bad}}} \pi_{\xi_t = \xi^{\text{bad}} | \xi_{t-1} = \xi^{\text{bad}}} \right)}_{\text{Likelihood that the observed signal is bad given a bad previous state}} \underbrace{\pi_{\xi'_t \in \Xi}}_{\text{Signal}} \underbrace{\pi_{\xi_{t-1} = \xi^{\text{bad}}}}_{\text{Bad state}}$$

The resulting process for the taste shock is summarized in Table 1.5.

**Table 1.5.** The taste shock

$\Xi = \{\xi^{\text{good}}, \xi^{\text{bad}}\}$	Taste shock space	{0.01, 0.5}
$\pi_{\xi_t = i   \xi_{t-1} = i}$	Markov chain	{0.86, 0.2}
$\pi_{\xi'_t \in \Xi}$	Signal frequency (i.i.d.)	0.05
$\pi_{\xi'_t = i   \xi'_t \in \Xi, \xi_t = i}$	Signal accuracy	{0.95, 0.997}
$\dagger i \in \Xi$		

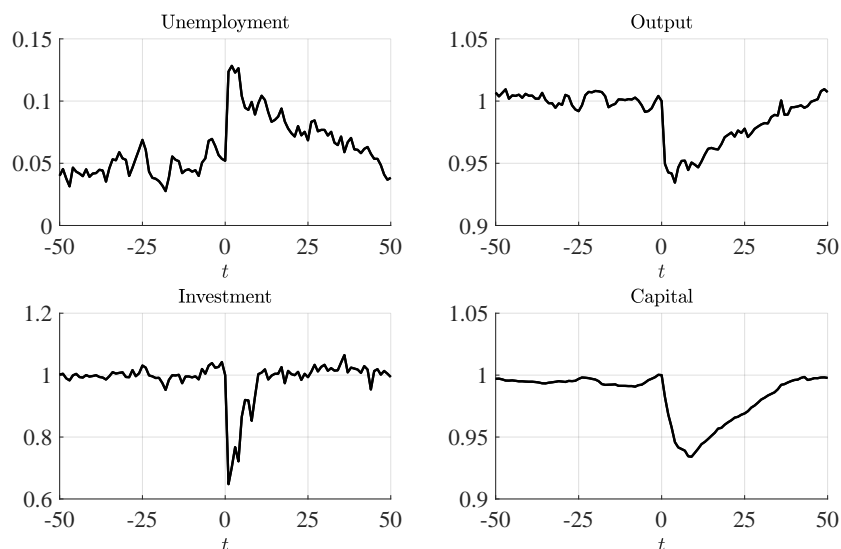
### *A typical fire sale episode*

Given the parameterized economy, I can now quantitatively examine the macroeconomic trans-

<sup>63</sup>Letting  $\hat{m}_i^{nc}$  denote the simulated moment corresponding to the target  $\kappa^{nc}$ , I am, for now, using the latter to weigh the corresponding moment conditions  $\mathcal{L} \equiv \sum_{i=7}^9 \left[ \frac{\kappa_i^{nc} - \hat{m}_i^{nc}}{\kappa_i^{nc}} \right]^2$ . The target is used in place of its sampled standard deviation for weighting because of the present computational constraints.

mission of a financial fire sale, an example of which is depicted in Figure 1.15.

**Figure 1.15.** An example fire sale



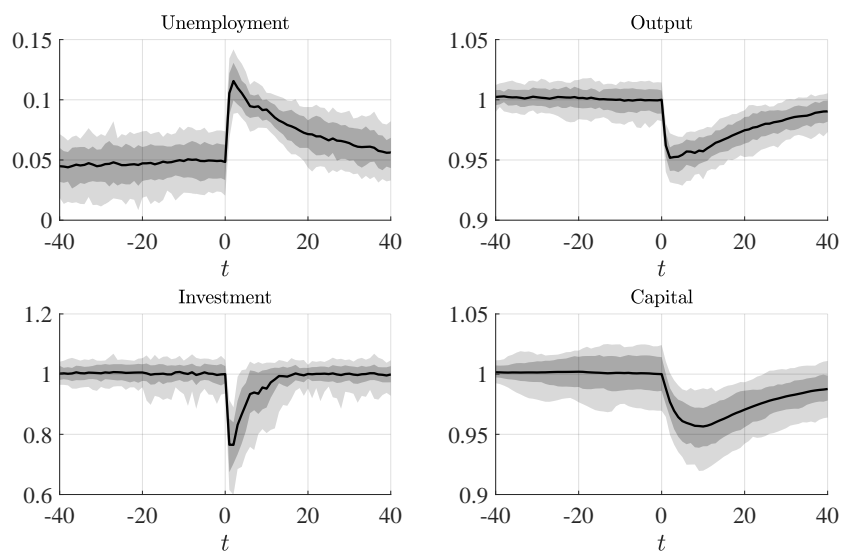
Notes: Figure 1.15 depicts unemployment, output, investment, and the capital stock for a given realization of technology in a 20 year window surrounding a fire sale at time  $t = 0$ . On average, workers lose roughly 20% of their nominal lifetime savings during the fire sale. In absolute terms, older workers who are nearing retirement are hit the hardest because they have accumulated more wealth than their younger counterparts. At time  $t = 1$ , households respond to the nominal shock by curbing consumption demand in nominal terms. Consumption goods producers anticipate this and react, because they are unable to freely adjust wages given the ‘fair wage’ constraint, by reducing investment and labor demand. By the time the nominal wage constraint no longer binds, around five periods after the initial shock, the reduced capital stock depresses labor productivity such that labor demand only slowly reverts back to the original steady state level.

First, notice that unemployment immediately more than doubles following the fire sale at  $t = 0$ . This initial bump is caused by the behavioral constraint imposed by worker effort, whereas persistence arises from the asymmetric information surrounding idiosyncratic worker productivity. In particular, facing the initial slump in household demand, firms do not lower wages by more than  $1 - \delta_W$  because otherwise workers would stop exerting any effort. This effect only lasts for a few quarters, after which excess unemployment persists because the lower level of capital depresses labor demand via labor productivity, an effect that is either mitigated or amplified by current technology. At that stage, firms do not lower wages to clear the market because of the selection issues arising from asymmetric information as discussed in Weiss (1980). Notice that the observed labor market recovery only takes thirty quarters and is thus quicker than the targeted window of ten years.

Importantly, notice that Figure 1.15 only depicts a single crisis for a given sequence of technology

shocks. We may then wonder what such a financial crisis looks like if the effects of technology are integrated out. For this, consider Figure 1.16 which depicts 100 simulated paths of my model economy for 100 paths of technology with the shaded areas representing 68% and 95% confidence bands.<sup>64</sup>

**Figure 1.16.** A typical fire sale



Notes: Figure 1.16 was created by simulating 100 fire sales given 100 different paths for technology. The resulting mean orbit, as depicted by the black line, represents a typical financial fire sale in the sense that the temporary effects of technology have been integrated out. The two shaded areas represent 68% and 90% confidence bands for the respective values.

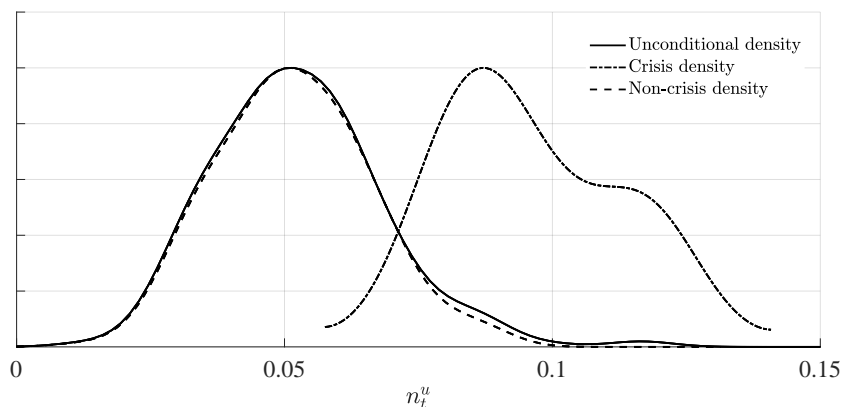
Figure 1.16 illustrates that the economy takes roughly ten years to revert back to its non-crisis ergodic benchmark. In particular, this implies that unemployment takes roughly ten years to recover *in expectation*. As indicated, the slow recovery is due to the loss in capital, which depresses labor productivity and takes roughly ten years to rebuild. Finally, consider Figure 1.17 in which I plot the estimated ergodic density for unemployment of my parameterized economy.

As desired, the resulting unemployment density features a substantial right tail including an additional mode with the corresponding statistics being recorded immediately following the fire sales. Crucially, further notice that the targeted crisis frequency borrowed from Barro (2006) is significantly lower than the observed frequency since the onset of the Great Moderation. The fact that the ‘crisis mode’ of the unconditional density depicted in Figure 1.17 is much less pronounced

<sup>64</sup>In the limit, as the number of Monte Carlo simulations approaches infinity, the economy converges to its conditional (crisis vs. no-crisis) ergodic distribution in which case the confidence bands are smooth.

than its empirical counterpart in Figure 1.3 is thus by construction. Whether or not the model is satisfactory in rationalizing the data should thus primarily be judged by way of the two conditional, crisis and non-crisis, densities.

**Figure 1.17.** Estimated ergodic density of unemployment for the parameterized model economy



Notes: The above figure depicts three estimated ergodic densities for simulated unemployment akin to Figure 1.3. Qualitatively, the densities look as desired. There is a substantial right tail that is recorded during the economic downturns following the fire sales. The circumstance that the ‘crisis mode’ of the unconditional density is much less pronounced than its empirical counterpart in Figure 1.3 simply reflects the fact that the targeted crisis frequency was taken from Barro (2006), not from the data since 1987. For purposes of contrasting the two graphs, the relevant densities for purposes of comparing the model with the data are thus the two conditional densities.

### *Implications for policy*

“How would behavior have differed had certain policies been different in specified ways?” (Lucas, 1977). The Great Depression and the Great Recession were both preceded by a financial crisis, but the monetary policy response varied greatly across the two episodes. In fact, Friedman and Schwartz (1963) view monetary policy as a primary root of the length and depth of the Great Depression, a perspective recently reevaluated by Romer and Romer (2013) and Christiano, Motto, and Rostagno (2003). Since my framework’s institutions were modeled to represent the US economy since the onset of the Great Moderation, the proposed theory does not provide a natural platform to quantitatively assess observed policies during the Great Depression. However, recall that deep downturns are caused by the widespread loss in retirement savings during the fire sales in my model.

**Insight.** Once a liquidity crisis is imminent, the success of crisis response may be measured by the degree to which it is able to contain nominal losses.

For example, to the extent that the Federal Reserve did not manage to prevent, or maybe even exacerbated, the nominal collapse during the Great Depression, my analysis suggests that policy

was not successful at the time. Extending beyond the qualitative insight discussed so far, I now turn to conducting a counterfactual, quantitative policy evaluation for my parameterized model economy. For this, notice that the central bank can, as the ultimate creator of the numéraire, at any time decide to provide additional *market and/or funding liquidity* (see Brunnermeier and Pedersen, 2009). In particular, recall that the simulated path depicted in Figure 1.16 was generated by targeting an aggregate nominal wealth loss of twenty percent, a quantity that is not invariant to policy. We may then wonder how the economy would have fared under a variety of alternative, unconventional monetary policy regimes.

**Table 1.6.** Monetary policy during a fire sale

Tool	Channel	Effect
No unconventional policy	-	Pension fund bankruptcy
Cut risk free target rate	Equity	Price increase of $S$ and $R$
Outright purchase of $R$	Liquidity	Reallocation of $R$
Repurchase agreement	Liquidity	Collateralized lending against $R$
Emergency lending	Liquidity	Collateralized lending against $R$

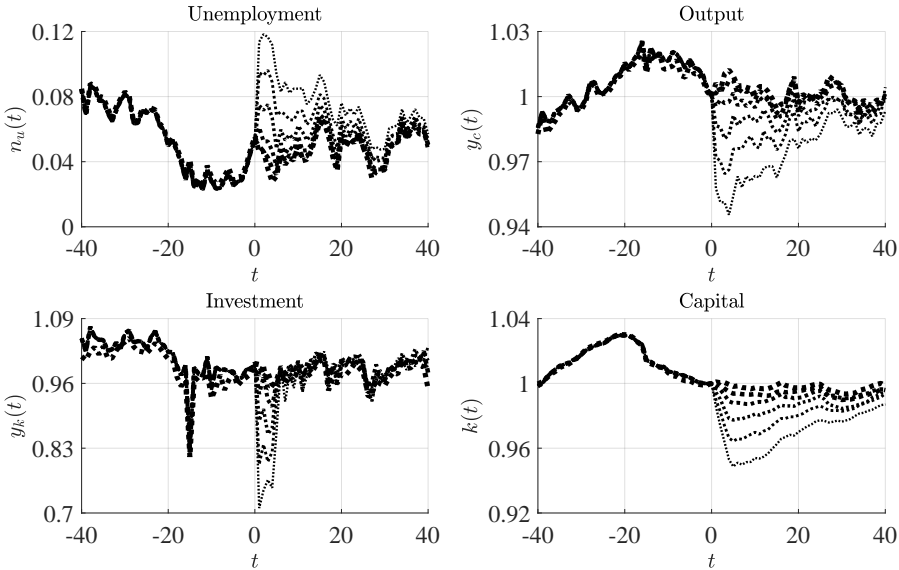
Notes: Table 1.6 lists a set of policy tools that the central bank has at its disposal during a financial fire sale. If policy makers decide to refrain from deploying unconventional policy, the Walrasian auctioneer drives the CLO price to zero and pension funds go bankrupt. Cutting the risk free interest immediately raises asset prices, which may already be enough to avert the crisis because it boosts pension fund equity. If not, the central bank may opt to purchase the CLO outright, to enter into a reverse repurchase agreement, or to lend to funds as a lender of last resort. Interestingly, any successful policy works by effectively circumventing the margin constraint imposed by the broker.

In face of a liquidity crisis, a natural first step to undertake for the central bank is to cut the previously announced interest rate target, thereby immediately boosting equity via asset prices. While this measure does not necessarily require any further action (e.g. asset purchases) on part of the central bank, it is likely insufficient to prevent a fire sale. If so, the central bank may provide additional liquidity via emergency lending or via outright purchases and repurchase agreements of non-governmental securities. In case of an outright purchase, the central bank agrees to take possession of the security *ex ante*, prior to the realization of its payout, whereas both repurchase agreements and emergency lending constitute a form of collateralized short-term lending, in which case the central bank assumes the risk of receiving the proceeds of the security *ex post*.

I start by assuming that, once a fire sale is imminent, the central bank proceeds by purchas-

ing the risky security outright in the open market. For this, consider Figure 1.18, which plots the macroeconomic transmission of a specific fire sale episode across different policy regimes. In particular, holding technology evolution constant, the different regimes are captured by a set of varying prices at which the central bank agrees to buy. Intuitively, the respective price thresholds metaphorically stand for the varying durations that the central bank waits as the Walrasian auctioneer iteratively lowers the resulting transaction price: The longer the central bank waits, the lower the price. As discussed previously, the severity of transmission uniquely depends on the degree to which nominal wealth collapses in my model. Therefore, as illustrated in Figure 1.18, more decisive central bank policy is mirrored by a less distressed real economy. In essence, once a crisis is imminent, more aggressive policies lead to less severe transmissions.

**Figure 1.18.** A fire sale across policy regimes



Notes: Figure 1.18 illustrates the macroeconomic transmission of the same financial crisis across a set of different policy regimes. In particular, each depicted series was generated using the same exact evolution of technology, but the price threshold at which the central bank starts absorbing the excess supply of the CLO during the fire sale at  $t = 0$  was varied. The more aggressive the central bank’s intervention, the smaller the nominal impact of the fire sale, and the more muted the macroeconomic transmission. In fact, if the central bank is willing to buy at the CLO’s fundamental value, the looming transmission to the real sector can virtually be contained in its entirety.

Instituting repurchasing agreements or emergency lending, on the other hand, effectively amounts to an exchange of the pension funds’ counterparty: A portion of leverage is transferred from the broker, who imposes the maintenance margin, to the central bank which acts as a ‘lender of last resort’. Importantly, I assume that the central bank does not impose a maintenance margin, but

instead insists on receiving preferred status in the chronology of payouts. As such, collateralized lending institutionally prevents fire sales by effectively circumventing the margin constraint imposed by the broker.

**Insight.** Fire sales are not symptomatic of fundamentally deteriorating assets, but of contemporary financial market institutions.

But if liquidity crises are an institutional problem, it is unsurprising that circumventing the respective institutions, as suggested above, serves as an effective mitigation mechanism. However, notice that all of the previously proposed policy only addresses scenarios in which a crisis is already imminent. We may thus wonder what can be done to strengthen systemic resilience *ex ante*. In light of the gained insights, an intuitive policy recommendation would be to disallow maintenance margins altogether. Banning margin calls would effectively eliminate the source of all fire sales, but it would inevitably also raise the costs associated with leverage.

## 1.5. Discussion

Empirical evidence suggests that financial crises affect real activity via both supply and demand (Chodorow-Reich, 2014; Mian, Rao, and Sufi, 2013), but contemporary macroeconomic theory overwhelmingly emphasizes channels of aggregate supply. The core contribution of this paper then lies in the formalization of a dynamic, wealth effect driven aggregate demand channel.<sup>65</sup> The proposed framework's financial sector is modeled to be fragile in the sense that it gives rise to occasional financial crises in the form of fire sales. Fire sales are followed by a reallocation of nominal claims from liquidity-strapped pension funds to the central bank. In an effort to make up for lost retirement wealth, the working-age population reacts by cutting consumption expenditures, in anticipation of which firms scale back production via employment and investment. To generate the targeted, steep and persistent real effects following a nominal collapse, I require a nominal and a real wage rigidity. In addition to the proposed transmission mechanism, the framework offers a variety of insights regarding policy, the nominal accounting of  $M1$ , and macroeconomic methodology.

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<sup>65</sup>I entirely abstract from tightening borrowing constraints faced by consumers, which likely also played a key role during the Great Recession (Mian, Rao, and Sufi, 2013).



### *On policy*

In my model, *conventional* monetary policy only directly affects commercial lending by way of the traditional cost-of-capital channel (Bernanke, 2007). While the central banks' interest rate announcements affect firms' cost of production — and thus potentially aggregate supply<sup>66</sup> — neither firm nor bank net worth play an amplifying role because all claims are rolled over each period. However, since households perceive monetary policy changes as permanent level-shifts, an increase in the policy rate also affects aggregate demand via intertemporal substitution. First, previously accumulated financial wealth is projected to generate more retirement wealth which depresses the incentives to save (wealth effect).<sup>67</sup> Second, each unit of additional savings is projected to generate more retirement wealth, which encourages saving (substitution effect).

The term *unconventional* monetary policy, which is exclusively enacted at  $t_3$  in my model, is used to describe any extraordinary action taken by the central bank to combat an imminent or ongoing financial crisis. For this, recall that when the maintenance margin constraint binds, the Walrasian auctioneer will drive the price of the CLO to zero unless the central bank intervenes. Naturally, the first option to consider is that the central bank continues to defend its interest rate target by purchasing as much of the sovereign bond from the pension funds until all brokers' margin calls are satisfied. When this is insufficient, the central bank has an array of additional, unconventional tools at its disposal as previously illustrated in Table 1.6. While lowering the interest rate target boosts asset prices directly, all other tools boost market or funding liquidity. As discussed, once a liquidity crisis is imminent, more aggressive policies lead to less severe transmissions. One might be tempted to argue that prescribing more aggressive policy ex post likely encourages moral hazard by inducing more risky behavior ex ante. However, notice that the crises examined herein do not arise from undesirably risky behavior on part of the investors. Instead, crises emerge from an institutional constraint, the maintenance margin, that is designed to shield debt holders from losing their investments. In fact, notice that all policies that successfully mitigate or prevent the real transmission effectively do so by circumventing the maintenance margin. A natural way to reduce systemic risk ex ante would then be to disallow maintenance margins altogether, a measure

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<sup>66</sup>Conventional monetary policy has no real effects unless the nominal wage constraint binds as is the case during a crisis episode.

<sup>67</sup>Since there exist no multi-period bonds, increasing the interest rate across periods does not depress the prices of any outstanding securities. The same is not true for unconventional policy enacted at  $t_3$ .

that would most likely increase the cost of leverage.

*On the numéraire, nominal wealth, and the role of banks*

My economy’s unit of account is given by noninterest-bearing demand deposits issued by banks. Agents exhibit demand for such deposits for a number of reasons. Consumption goods producers take out loans from banks because bank deposits are the only means of payment accepted by both capital suppliers and workers. Capital producers only accept deposits as means of payment because worker salaries must be paid in deposits. Households accept and retain  $M1$  throughout the period because consumption goods must be purchased in exchange for deposits. Pension funds hold  $M1$  because they are required do so by law.

Money creation is carried out by the central bank ( $M0$ ) and by commercial banks ( $M1$ ). The latter can create money because their deposits are, irrespective of their balance sheet counterpart, accepted as a means of payment within the private sector. Specifically, banks can credit firms with previously nonexistent deposits without having to expense any of their reserves. While such demand deposits that are not backed by central bank credit may be viewed as artificial, because they are fictitiously “invented” by banks (Werner, 2014), such a system allows banks to promptly respond to money demand by scaling up and down the numéraire. Conversely, a “full-reserve” alternative would almost certainly require the central bank to alter, if not abandon the contemporary practice of targeting interest rates (SNB, 2018).<sup>68</sup> Therefore, because banks are a key component of modern monetary systems in which the central bank tightly controls the nominal interest rates, but not monetary aggregates, the banking sector was modeled in the spirit of contemporary practice.

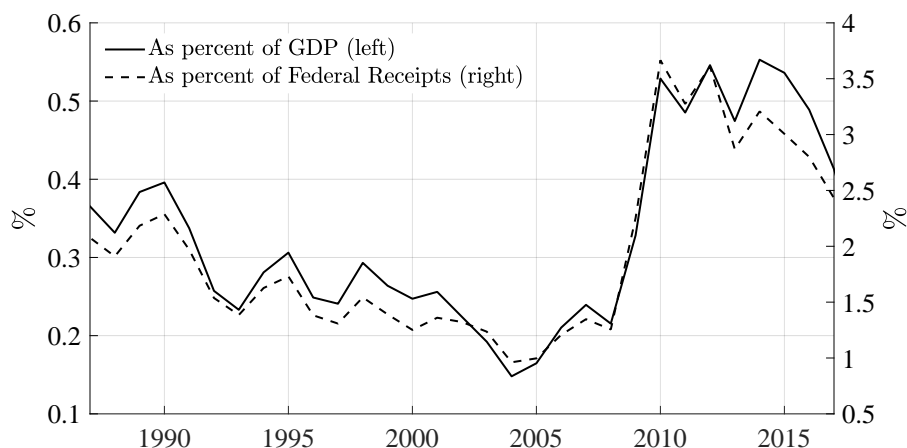
Since my framework does not feature long-term loans, it is impossible for a crisis to induce assets price dislocations across periods. This is because at the beginning of each period, when commercial loans have yet to be created, all household wealth is backed by  $M0$  and risk free bonds. Therefore, the only way for fire sales to affect real activity is via an idiosyncratic asset reallocation across households, an aggregate reallocation between liquid and illiquid asset holdings, or an aggregate reallocation of wealth between households and the government. In particular, given the zero-sum

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<sup>68</sup>If (1) were binding, the Walrasian constraints imposed by scarcity would lead banks to charge loan rates in excess of the otherwise prevailing competitive benchmark. In such a setting, the central bank’s task to choose appropriate amounts of high-powered money  $M0$  would become much more consequential and thus more difficult. If reserves were further required to be backed by an intrinsically valuable good (see Fama, 1980), the central bank would lose its institutional meaning entirely.

nature of nominal accounting, any loss incurred by the private sector as part of a fire sale must be offset by a corresponding profit by the central bank in my model.<sup>69</sup> While the same zero-sum logic does not apply in reality, because long-term assets do in fact exist, it is nevertheless worth noting that the Federal Reserve made significant profits following its recent, large-scale asset purchases as illustrated in Figure 1.19.

**Figure 1.19.** Federal Reserve transfers to the US Treasury



Notes: Figure 1.19 is a replication of a graph found in a Federal Reserve of St. Louis [blog post](#) (see Section 2.7). It principally serves as an illustration of the ballooning central bank profits following the Federal Reserve’s extraordinary asset purchases following the 2008 financial crisis. The data were taken from FRED and FRB.

*A final remark on the methodological approach*

What are adequate microfoundations? The macroeconomic community has for all intents and purposes reached a consensus that its models should be microfounded, but what constitute adequate microfoundations continues to be the subject of heated debate (see Stiglitz, 2018). If macro and micro had evolved congruently, the former would be a unifying field assembling the latest behavioral insights in the form of an overarching model of the macroeconomy (Blanchard, 2018). The reason that this is not currently the case is methodological: Whenever the state space is high dimensional, there exists a tradeoff between building rich model environments and retaining canonical conditional expectation objectives. For example, with the usual disclaimer that it serves as a good point of reference, Christiano et al. (2018) poignantly proclaim that “the assumption of rational expectations is *obviously* wrong”. Contrarily, the proposed theory in this paper is constructed based

<sup>69</sup>Since profits are returned to the treasury, government bonds purchased by the central bank effectively carry a zero percent interest rate.

on the view that the imposition of internal consistency is inadequate only if objectives are poorly chosen. In particular, since “expectations” are precisely defined mathematical objects implied by theory, instituting conditional expectation objectives requires a tremendous amount of institutional and statistical knowledge on part of the agent. Instead, I rely heavily on the literal definition of an expectation, namely “a belief that something will happen or is likely to happen” which yields the following methodological insight: There does not exist a tradeoff between insisting on internally consistent, individual optimization and building a rich macroeconomic model. In particular, I have shown the high degrees of state and parametric heterogeneity that can be accommodated when model primitives — the objectives — are chosen subject to the cognitive constraint that agents are incapable of solving Euler equations in an internally consistent manner.

But then, are the proposed microfoundations adequate? First, microfoundational adequacy does not hinge on the mathematical appeal of a model’s *prescribed* hypothetical behavior or the mathematical derivation thereof, but on how well the latter *describes* observed behavior (see Thaler and Shefrin, 1988). For example, “if Keynes was right that individuals saved a constant fraction of their income, an aggregate model based on that assumption *is* microfounded” (Stiglitz, 2018). However, as famously argued by Lucas (1976), even the ability to match observed behavior is insufficient to guarantee satisfactory model performance if policy is subject to change. As a result, individual behavior is now typically derived as a solution to mathematical problems of constrained optimization. Deriving behavior from optimization, rather than imposing it as a model primitive, is advantageous because it forces the theorist to disclose the fundamental economic tradeoffs that are claimed to govern the agent’s decision: “Theory helps keep track of benefits and costs” (Varian, 1993).

If the purpose of instituting objectives is to shed light on the fundamental tradeoffs considered by real-world agents, microfoundational validity not only hinges on how well prescribed behavior matches observed behavior, but also on *the former’s derivation by the agents*. In particular, even if a model produces decision rules that appear appealing intuitively and match the data, but no real-world agent is realistically capable to derive them, the corresponding microfoundations are unsatisfactory because the modeler for all intents and purposes imposes behavior as the model primitive and thus invariably obfuscates the *actual tradeoffs* considered by real-world agents. Internal consistency thus only serves as a useful benchmark if we can realistically assert that agents’

behavior indeed derives from the proposed optimization problem. In the real world, where agents are heterogenous, information is sparse, and uncertainty is epistemic, conditional-expectation objectives do not satisfy this requirement.<sup>70</sup>

In face of high degrees of high degrees of heterogeneity, information sparsity, and epistemic uncertainty, there are two principal ways to address the issue of internal consistency. We can either relax the required level of rationality on part of the agents — such as boundedly rational expectation formation schemes — or, alternatively, we can reduce institutional complexity by disposing of conditional-expectation objectives. While the contemporary literature has chosen to proceed in the former fashion, I argue that reducing institutional complexity is preferable for two reasons.<sup>71</sup> First, it preserves internal consistency as the effectively maximized objectives and the originally defined objectives are in fact congruent. Second, it improves credibility because real-world agents are almost surely incapable of deriving true conditional expectations anyway.<sup>72</sup> In this spirit, I have chosen objectives that — while still informed by the future — neither require knowledge of the entire state, nor of the full stochastic environment.<sup>73</sup> For example, rather than probabilistically assessing a stochastic sequence of future consumption, consumers assign utility to projected retirement wealth (via current liquid and illiquid savings) in my model. Instituting monetary wealth as a conceptual placeholder for future consumption makes intuitive sense because money serves, by definition, as a *store of value*. Of course, the store-of-value function of money derives from the fact that future consumption is strictly increasing in accumulated monetary wealth.

Reducing institutional complexity further entails the benefit that the costs of incorporating heterogeneity are *relatively* small. In particular, they are small *relative* to the corresponding benefit of gaining the ability to consider cross-sectional statistics beyond the mean as macroeconomic targets (e.g. Gini coefficients). At the same time, however, allowing for heterogeneity also raises

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<sup>70</sup>Neither the assumption that the infinite dimensional state is observed nor that its model implied transition is understood by real-world agents is tenable.

<sup>71</sup>See Krusell and Smith (2006) for a discussion of bounded rationality in the context of heterogenous agent setups.

<sup>72</sup>In ‘boundedly rational’ or ‘non-rational’ approaches, agents evaluate objectives that are incongruent with the objectives as originally defined. In this case, the proposed model primitives are ill-defined at best or agents internally inconsistent at worst. Either way, the corresponding framework inevitably violates Muth’s requirement that “expectations, since they are informed predictions of future events, are essentially the same as the predictions of the *relevant* economic theory” (1961).

<sup>73</sup>Selecting among such objectives naturally precludes the use of self-referential functional equations such as the ones that typically arise from infinite horizon setups. This is because solving the corresponding fixed point problem requires that agents, if they are to behave in an internally consistent manner, perfectly know and understand their economic and stochastic environment.

new questions, especially if the latter is parametric. For example, in my model, income inequality is chiefly driven by heterogeneous firm ownership with skill and time-varying factors only playing secondary and tertiary roles. This may, and likely should, be viewed as problematic because there does not exist a market for firm ownership such that income inequality is for all intents and purposes determined exogenously. But what if real-world income inequality were indeed primarily driven by time-invariant firm ownership? If this were so, irrespective of whether income inequality is viewed as desirable or undesirable, incorporating heterogeneous firm ownership would be key to understanding the effects of corresponding government policies.

Finally, recall that while parameterization is often viewed as successful if the resulting model generates a set of desired macro moments, credibility invariably hinges on whether the resulting parameter values are in fact consistent with microeconomic evidence (Chari, Kehoe, and McGrattan, 2009). Of course, allowing for parametric heterogeneity does not relieve the economic modeler of this credibility constraint. However, to the extent that real-world agents are in fact heterogeneous in their fundamental evaluation of costs and benefits, the incorporation of parametric heterogeneity constitutes a tremendous advancement in terms of matching microeconomic evidence. Moreover, if heterogeneity is deemed contextually inappropriate, the homogeneous case is, of course, still nested by the heterogeneous case.

## **1.6. Concluding Remarks and Outlook**

The key contribution of this paper lies in the formalization of a wealth effect driven aggregate demand channel in the transmission of financial crises. The proposed framework yields insights into optimal policy response while also providing a macroprudential recommendation regarding ex ante crisis prevention. In particular, since any crisis response that successfully prevents a deep downturn must do so by effectively circumventing the prevailing maintenance margin, policy makers may want to consider banning margin calls altogether. Naturally, we would expect that such a policy would increase the cost of leverage, which may or may not be viewed as desirable. Since the proposed framework does not generate any financial crises beyond liquidity induced fire sales, the logic here only applies to the prevention of crises of this type. To allow for high degrees of state and parametric heterogeneity, I have further proposed a non-standard methodological approach

that retains internal consistency and global solutions. Specifically, in an attempt to enhance the credibility of the employed microfoundations, each agent’s objective was chosen subject to the constraint that optimization must be trivial to achieve numerically.

In terms of future research, the proposed methodological approach opens up a natural avenue for analyzing cross-sections dynamically, a capability which the present paper has hardly even scratched the surface of. Going forward, I thus envision endowing households with a vector of sector-specific productivity and allowing firms to observe certain worker characteristics as in Weiss (1980). In addition to the taste shock, demand uncertainty could arise from a spatial reallocation of consumers along a circular city after production. Firms may further possess heterogeneous, potentially endogenous production technologies. Relating to technology, since modern likelihood methods are not subject to the traditional ergodicity constraints imposed by the method of moments, I further ultimately envision a growing economy that unifies trend and cycles akin to King, Plosser, and Rebelo (2002). Incorporating non-stationarity is intriguing because estimation would neither require nor allow for the atheoretical extraction of “trend” from the data.

## 1.7. Appendix

### *Equilibrium - Definition*

Since the economy analyzed herein features market clearing failures *in equilibrium*, I start by outlining the set of relevant definitions. For this, consider a pseudo-game, or an abstract economy, in which “an action by one agent affects both the payoff and the domain of actions of other agents” (Arrow and Debreu, 1954).<sup>74</sup> More formally, an abstract economy  $\Gamma_A$  emerges if some game  $\Gamma = [I, \{\Sigma_i\}_{i \in I}, \{u_i\}_{i \in I}]$  is augmented by a set of correspondences  $\{A_i\}_{i \in I} : \Sigma_{-i} \rightrightarrows \Sigma_i$ .

**Pseudo-Nash equilibrium.** A strategy profile  $\sigma = \{\sigma_i\}_{i \in I}$  is said to be a pseudo-Nash equilibrium (PNE) of an abstract economy  $\Gamma_A = [I, \{\Sigma_i\}_{i \in I}, \{u_i\}_{i \in I}, \{A_i\}_{i \in I}]$  if  $u_i(\sigma_i, \sigma_{-i}) \geq u_i(\sigma'_i, \sigma_{-i})$  for all  $i \in I, \sigma'_i \in A_i(\sigma_{-i})$ .

**Competitive equilibrium.** A specific market within an abstract economy is said to be competitive if pseudo-Nash equilibrium induces zero excess demand in said market. An abstract economy is said to be competitive if pseudo-Nash equilibrium induces zero excess demand in each market.

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<sup>74</sup>e.g. the price set by a Walrasian auctioneer restricts the set of permitted strategies by the market participants.

**Walrasian equilibrium.** A competitive market is said to be Walrasian if there exists one market participant who’s sole purpose is to act as the market’s *Walrasian* auctioneer. An abstract economy is said to be Walrasian if each market features a Walrasian auctioneer.

Crucially, notice that pseudo-Nash equilibrium may not be competitive such that the prevalence of market clearing failures need not imply disequilibrium in the pseudo-Nash sense. In fact, since not all markets in the recursive economy examined herein are competitive, the relevant equilibrium concept, recursive general equilibrium, must explicitly permit excess demand, positive or negative.

**General equilibrium.** An abstract economy is said to be of the general equilibrium type if the price in each market is determined endogenously by pseudo-Nash equilibrium.

**Recursive general equilibrium.** A recursive model economy is said to be of the general equilibrium type (RGE) if each price in each market is determined endogenously by pseudo-Nash equilibrium in (each subperiod of) each period.<sup>75</sup>

In addition to market clearing failures, notice that the proposed model economy features markets that are highly incomplete. Specifically, this is because each agent is restricted to engage in specific markets corresponding to her type.<sup>76</sup> Moreover, trading exclusively takes place in recurrent spot markets and real goods can only be acquired in exchange for  $M1$ .

#### *Equilibrium - Consumer goods*

Once production has occurred, all production costs are sunk. At  $t_4$ , the observation of the taste shock  $\xi_t$  pins down aggregate demand  $D(P)$  and thus each firm’s residual demand as a function of the vector  $\{P_{jt}^C, y_{jt}^C\}_{j \neq f}$ . It is assumed that, at this stage, firms compete in Bertrand fashion and

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<sup>75</sup>Notice that RGE encompasses both “dynamic stochastic” (DSGE) as well as “recursive-dynamic computable” (RDCGE) setups. It is well known that the combination of infinite horizons with rational expectations, a defining feature of DSGE, requires the model to be solved across a grid of possible states, a computational feat that is subject to the curse of dimensionality and thus particularly challenging in face of heterogeneity. Imposing that each component of each agent’s objective generates utility instantaneously, I can dispose of intertemporal conditional expectation functions and recursively solve for each period’s general equilibrium anew. This is viable because, when utility is exclusively instantaneous, the original value function is never self-referential (as it is in the canonical Bellman case). Then, since optimization does not require fixed point iteration, finding agents optimal decisions *given the current state and some price vector* ( $\neq$  decision rules) is computationally trivial. This property of the model is appealing intuitively because real-world agents do not spend hours, days, or even weeks searching for conditional strategies giving rise to fixed points across a grid of possible states. More realistically, we optimize intertemporally by projecting current decisions onto future states in a highly approximate manner (e.g. current savings onto retirement wealth). Rather than spending an overwhelming majority of computational resources on the derivation of decision rules, my computational expense chiefly derives from finding equilibrium prices across heterogenous households.

<sup>76</sup>No agent trades firm ownership or old capital because no such markets exist.



that, if multiple firms offer the same price, residual demand is allocated proportionally. We have,

$$\max_{P_{ft}^C} P_{ft}^C \min \left\{ y_{ft}^C, y_{ft}^{C,d} \right\}$$

where residual demand is given by  $y_{ft}^{C,d} = D(P_{ft}^C) - \sum_{j \in J^C} y_{jt}^C \mathbf{1}(P_{jt}^C < P_{ft}^C) - \sum_{j \in J^C} \frac{y_{jt}^C \mathbf{1}(P_{jt}^C = P_{ft}^C)}{y_{jt}^C + y_{ft}^C}$ . First, notice that no firm will optimally choose to set their price below the market clearing price  $P^* = \{P \in \mathbb{R} \mid D(P) = \sum_{j \in J^C} y_{jt}^C\}$  at which point  $y_{ft}^C$  is sold in its entirety irrespective of the competition's pricing. However, depending on other firms' offering prices, a firm may find it profitable to charge a price above  $P^*$ ,  $P_{ft}^C = D^{-1}(y_{ft}^C)$  if  $P_{jt}^C > D^{-1}(y_{ft}^C) \forall j \neq f$  for example. Before proceeding to discuss optimal pricing, I show that sales  $D(P)P$ , or household expenditures, are weakly decreasing in price.<sup>77</sup> For this, suppose the lowest available price  $P_0^C = \min_f \{P_{ft}^C\}$  induces a particular household to save  $s_0$  and consume  $c_0 = w^L(1 - s_0)/P_0^C$ . Assuming an interior solution, we must then have,

$$\begin{aligned} \gamma^c c_0^{\gamma^c - 1} \frac{w^L}{P_0^C} &= \xi_t \exp(-\gamma^w \bar{w}^G(s_0)) a \\ &\equiv m(s_0) \end{aligned}$$

where  $a > 0$  is a constant such that  $m' < 0$  and, crucially,  $s_0$  is the only non-predetermined argument of  $m$ . Now, suppose that the price increased to  $P_1^C = \lambda P_0^C$ ,  $\lambda > 1$ , but that the household responds by saving  $s_1 \leq s_0$ . We then have  $c_1 \geq c_0/\lambda$ , which in turn implies,

$$\begin{aligned} \gamma^c c_1^{\gamma^c - 1} \frac{w^L}{P_1^C} &\leq \gamma^c \left(\frac{c_0}{\lambda}\right)^{\gamma^c - 1} \frac{w^L}{\lambda P_0^C} \\ &= \lambda^{-\gamma^c} \gamma^c c_0^{\gamma^c - 1} \frac{w^L}{P_0^C} \\ &= \lambda^{-\gamma^c} m(s_0) \\ &< m(s_0) \\ &\leq m(s_1) \end{aligned}$$

where the last inequality follows by  $m' < 0$ . Therefore,  $P_1^C > P_0^C$  implies that the marginal benefit of saving strictly exceeds its marginal cost at any  $s_1 \leq s_0$ . Unless the respective household

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<sup>77</sup>See Kreps and Scheinkman (1983) for a more general treatment.

was already at the corner  $s_0 = 0$ , it will thus respond by increasing its savings or, equivalently, decreasing its expenditures such that we must have  $s_1 > s_0$ . Since the same logic applies equally across all households, we have  $\partial D(P^C)P^C / \partial P^C \leq 0$  over the entire domain of  $P^C$ . Moreover, on the subset of the domain where there exists at least one household not at the corner  $s_0 = 0$ , which must be satisfied for at least one realization of the taste shock in equilibrium<sup>78</sup>, the inequality is strict:  $\partial D(P^C)P^C / \partial P^C|_{P^*} < 0$  where  $P^*$  denotes the market clearing price in general equilibrium. Incorporating this insight in their estimation of market demand, firms rightfully impose  $\tilde{\chi}_t^r(\xi_t) \leq 1$  for each  $\xi_t$ . I now proceed to show that equilibrium at  $t_4$  is given by a uniform market clearing price strategy,

$$P_{ft}^C = P^* \text{ for each } f \in J^C$$

By contradiction: Suppose that, in equilibrium,  $\exists f$  such that  $P_{ft}^C > P^*$ . Then, if  $y_{ft}^C < y_{ft}^{C,d}$ , firm  $f$  reacts by raising its price. If  $y_{ft}^C = y_{ft}^{C,d}$ , some firm  $j$  has zero sales and thus optimally reacts by lowering its price (at least to  $P_{ft}^C - \varepsilon$ , maybe further). If  $y_{ft}^C > y_{ft}^{C,d} > 0$ , firm  $f$  is a marginal seller and thus has an incentive to lower prices because its projected sales are at least locally decreasing in price (sale increases are discrete if there are other marginal sellers). Similarly, if  $y_{ft}^C > y_{ft}^{C,d} = 0$ , firm  $f$  will lower its price to attain positive sales.

#### *Equilibrium - Production*

As indicated, firms do not understand that they can affect other firms' output by poaching workers. Taking as given the announced firm outputs, optimality requires,

$$\left( \frac{\partial}{\partial k_{ft}^C}, \frac{\partial}{\partial n_{ft}^C} \right) \left( \mathbb{E}_{t_1}[\tilde{S}_{ft}^C] - [k_{ft}^C Q_t + \mu^{LF} n_{ft}^C W_{ft}^C] R_{ft}^L \right) = (0, 0)$$

which implies

$$\begin{aligned} \frac{\partial \mathbb{E}_{t_1}[\tilde{S}_{ft}^C]}{\partial y_{ft}^C} \frac{\partial y_{ft}^C}{\partial k_{ft}^C} / \frac{\partial \mathbb{E}_{t_1}[\tilde{S}_{ft}^C]}{\partial y_{ft}^C} \frac{\partial y_{ft}^C}{\partial n_{ft}^C} &= \frac{Q_t R_{ft}^L}{\mu^{LF} W_{ft}^C R_{ft}^L} \\ \implies \frac{\partial y_{ft}^C}{\partial k_{ft}^C} / \frac{\partial y_{ft}^C}{\partial n_{ft}^C} &= \frac{Q_t}{\mu^{LF} W_{ft}^C} \end{aligned}$$

and therefore, given the Cobb-Douglas form of production, constant expenditure shares for

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<sup>78</sup>Otherwise, firms trivially have an incentive to produce less.

capital and labor,

$$k_{ft}^C Q_t = \frac{\alpha}{1-\alpha} \mu^{LF} n_{ft}^C W_{ft}^C \quad (1.11)$$

Since production is CRS and prices are taken as given, the marginal cost of producing additional units of output is equal to a constant  $\delta_t^C$ . To find  $\delta_t^C$ , I exploit the optimal capital-labor share and calculate the cost of producing a benchmark output with  $n_{ft}^C = 1$ ,

$$\begin{aligned} \delta_t^C &= \frac{\left( \left( \mu^{LF} \frac{\alpha W_t^C}{(1-\alpha) Q_t} \right) Q_t + \mu^{LF} W_t^C \right) R_t^L}{z_t^C \left( \mu^{LF} \frac{\alpha W_t^C}{(1-\alpha) Q_t} \right)^\alpha (\mu^{LF} q_t^C)^{1-\alpha}} \\ &= \frac{Q_t^\alpha [W_t^C]^{1-\alpha} R_t^L}{\alpha^\alpha (1-\alpha)^{1-\alpha} z_t^C [q_t^C]^{1-\alpha}} \end{aligned}$$

such that the marginal cost is linear in  $R_t^L$ .<sup>79</sup> Consumption sector firms thus implicitly optimize,

$$\max_{y_{ft}^C} \mathbb{E}_{t_1} \left[ y_{ft}^C \tilde{\chi}_t^k(\xi_t) \left[ \sum_{j^C} y_{jt}^C \right]^{-\tilde{\chi}_t^r(\xi_t)} \right] - \delta_t^C y_{ft}^C$$

Appealing to symmetry, it is then easy to show that in equilibrium, individual output solves the following equation,

$$\mathbb{E}_{t_1} \left[ \tilde{\chi}_t^k(\xi_t) [y_t^C]^{-\tilde{\chi}_t^r(\xi_t)} \left( 1 - \frac{\tilde{\chi}_t^r(\xi_t)}{N^C} \right) \right] = \delta_t^C \quad (1.12)$$

such that equilibrium output is, ceteris paribus, increasing in  $\tilde{\chi}_t^k$  and decreasing in  $\tilde{\chi}_t^r$ . Further notice that (10) can be written as,

$$\mathbb{E}_{t_1} \left[ \tilde{P}_t^C(\xi_t) \left( 1 - \frac{\tilde{\chi}_t^r(\xi_t)}{N^C} \right) \right] = \delta_t^C$$

which reduces to the familiar zero expected profits condition as  $N^C \rightarrow \infty$ .

### *Equilibrium - Capital*

I start by showing that consumption good sector equilibrium dictates that consumption producer capital expenditures  $k_t^C(Q_t)Q_t$ , and thus capital producer rental revenue, must be strictly decreasing in  $Q_t$ . From the firm's FOC for capital, we know,

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<sup>79</sup>As per usual, monetary policy does not have any real effects unless nominal frictions prevent markets from clearing. In particular, when prices are entirely flexible, all exogenous changes in  $R_t^T$  are absorbed by a corresponding joint level shift in  $W_t^C$  and  $Q_t$  in equilibrium. However, since fire sales are transmitted to the real sector via the nominal downward wage friction, the central bank can mitigate the real effects of a crisis by lowering the interest rate target and thereby absorbing part of the nominal shock. Conversely, the monetary authority could hypothetically also generate unemployment by sharply increasing the interest rate target at any given time.

$$\alpha \mathbb{E}_{t_1} \left[ \tilde{\chi}_t^k(\xi_t) [N^C y_{ft}^C]^{-\tilde{\chi}_t^r(\xi_t)} \left( 1 - \frac{\tilde{\chi}_t^r(\xi_t)}{N^C} \right) \right] y_{ft}^C = k_{ft}^C Q_t R_{ft}^L \quad (1.13)$$

In comparing different equilibria (taking as given two different values of  $Q_t$ ), it is useful to analyze the behavior of both sides of (10). Specifically, notice that  $\tilde{\chi}_t^k(\xi_t) \leq 1$  implies that the left hand side of (10) is (at least weakly) increasing in  $y_{ft}^C$ . Similarly, the left hand sides of (11) and (12) are strictly decreasing in  $k_{ft}^C$  and  $n_{ft}^C$  respectively,

$$\alpha \mathbb{E}_{t_1} \left[ \tilde{\chi}_t^k(\xi_t) [N^C y_{ft}^C]^{-\tilde{\chi}_t^r(\xi_t)} \left( 1 - \frac{\tilde{\chi}_t^r(\xi_t)}{N^C} \right) \right] \frac{y_{ft}^C}{k_{ft}^C} = Q_t R_{ft}^L \quad (1.14)$$

$$(1 - \alpha) \mathbb{E}_{t_1} \left[ \tilde{\chi}_t^k(\xi_t) [N^C y_{ft}^C]^{-\tilde{\chi}_t^r(\xi_t)} \left( 1 - \frac{\tilde{\chi}_t^r(\xi_t)}{N^C} \right) \right] \frac{y_{ft}^C}{n_{ft}^C} = \mu^{LF} W_{ft}^C R_{ft}^L \quad (1.15)$$

By contradiction: Suppose  $Q_0$  and  $Q_1 > Q_0$  give rise to two firm-bank equilibria as given by the individual strategies  $(W_0, n_0, k_0, l_0, y_0; R_0)$  and  $(W_1, n_1, k_1, l_1, y_1; R_1)$  satisfying  $k_1 Q_1 \geq k_0 Q_0$ . From (8), we then know that  $n_1 W_1 \geq n_0 W_0$  and thus  $l_1 \geq l_0$ . Without loss of generality, assume then that  $l_1 = a l_0$  with  $a \geq 1$ . Given  $Q_1 > Q_0$ , we must have  $k_1 < a k_0$ . Moreover, from the labor market setup we know that, in equilibrium,  $n_1 > n_0 \implies W_1 \geq W_0$  such that, because  $n_1 W_1 = a n_0 W_0$ , we must have  $n_1 \leq a n_0$ . Combining  $k_1 < a k_0$  and  $n_1 \leq a n_0$  yields  $y_1 < a y_0$ , which implies, by  $\delta_t^C > 0$ , that expected (projected) sales per unit of the loan have decreased,  $\mathbb{E}_{t_1}[\tilde{S}(y_1)]/l_1 < \mathbb{E}_{t_1}[\tilde{S}(y_0)]/l_0$ . Then, unless there exists no  $\xi_t \in \Xi$  such that sales fall short of  $l_0 R_0$ <sup>80</sup>, whether  $y_1 > y_0$  or  $y_1 \leq y_0$ , we must have  $R_1 > R_0$ . However, notice that  $y_1 > y_0$  implies  $k_1 > k_0$  or  $n_1 > n_0$  and thus, by (11) or (12),  $R_1 < R_0$ . Similarly,  $y_1 \leq y_0$  implies, by (10),  $R_1 \leq R_0$ . We have a contradiction. Therefore, since optimality requires  $\partial k_{ft}^C(Q_t) Q_t / \partial Q_t < 0$  in equilibrium at the individual firm level, we have  $\partial k_t^C(Q_t) Q_t / Q_t < 0$  in aggregate.

I proceed by showing that equilibrium in the capital goods sector is characterized by a uniform, market clearing capital rental price  $Q_{ft} = Q_t^*$  for each  $f$  and a sectoral wage schedule of  $W_t^K(y_{ht}^K) \equiv \max_f \{W_{ft}^K(y_{ht}^K)\} = y_{ht}^K Q_t$  for each  $h$ . Recall that the capital goods producing firms maximize contemporaneous monetary profits by choosing an individual price  $Q_{ft}$  and an individual wage offer  $W_{ft}^K(y_{ht}^K)$ ,

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<sup>80</sup>If there were no risk associated with  $l_0$  and  $l_1$ , we would have  $R_0 = R_1 = R^S$ , but this is contextually irrelevant as the primary purpose of the taste shock is to create such risk.

$$\begin{aligned}
V_1^{FK} (k_{ft-1}^S, \{k_{jt-1}^S, P_{jt}^K, W_{jt}^K\}_{j \neq f}) &= \max_{Q_{ft}, W_{ft}^K} Q_{ft} \min \{k_{ft}^{C,d}, k_{ft}^S\} - \int n_{hft}^K W_{ft}^K(y_{ht}^K) dh \\
\text{s.t. } k_{ft}^S &= (1 - \delta^D)k_{ft-1}^S + y_{ft}^K \\
y_{ft}^K &= \int n_{hft}^K y_{ht}^K dh
\end{aligned}$$

where  $k_{ft}^{C,d} = k_t^C(Q_{ft}) - \sum_{JK} k_{jt}^S \mathbb{1}(Q_{jt} < Q_{ft}) - \sum_{JC} \frac{k_{jt}^S \mathbb{1}(Q_{jt}=Q_{ft})}{k_{jt}^S + k_{ft}^S}$ . I start by discussing existence constructively: suppose that for each  $h$ , the highest available wage contract, offered by at least two firms, is given by  $W_{ft}^K(y_{ht}^K) = y_{ht}^K Q_t^*$ , where  $Q_{ft} = Q_t^*$  for each  $f$  and  $Q_t^*$  clears the market. Then, each employed worker generates zero marginal profits and has a competitive outside option. Therefore, lowering wage offers cannot be profitable because current employees will simply opt to work for another firm. Conversely, poaching a worker from another firm by offering a higher wage may increase a firm's output, but only at the cost of negative marginal profits: If  $Q_{ft}$  is left unchanged (or lowered), all product is still sold, but the marginal sale does not cover the marginal labor costs. If  $Q_{ft}$  is raised above  $Q_t^*$ , markets no longer clear. Increased total labor costs are then mirrored by shrinking sales because, as shown above, capital rentals  $k_t^C(Q_t)Q_t$  are strictly decreasing in  $Q_t$  if the consumption goods sector is in equilibrium.

Having shown existence, I proceed by proving uniqueness. For this, notice that, in equilibrium, all capital rentals must occur at a uniform price. More precisely,  $Q_{ft} = Q_t$  for all  $f$  satisfying  $k_{ft}^S > 0, \bar{k}_{ft}^D(Q_{ft}) > 0$ . By contradiction: Suppose that in equilibrium,  $\exists i, j$  such that  $Q_{it} > Q_{jt}$  and  $k_{jt}^S, k_{it}^S > 0, \bar{k}_{it}^D(Q_{it}) > 0$ . Then, if markets clear,  $j$  may, irrespective of wages paid, increase profits by raising  $Q_{jt}$  to  $Q_{it}$ . In case of excess demand, both firms find it profitable to raise their price to the market clearing price  $Q_t^*$ . Finally, in case of excess supply,  $j$  may increase profits by raising  $Q_{jt}$  to  $Q_{it} - \varepsilon$ . Since existence has been shown and price dispersion cannot support equilibrium, all capital rentals must occur at a uniform price  $Q_t$  in equilibrium. Given  $Q_t$ , a worker  $h$  must be offered  $W_t^K(y_{ht}^K) = y_{ht}^K Q_t$  in equilibrium. By contradiction: Suppose that, in equilibrium,  $\exists h$  such that  $W_t^K(y_{ht}^K) = \bar{W}_t^K(y_{ht}^K) \neq y_{ht}^K Q_t$ . If  $W_t^K(y_{ht}^K) < y_{ht}^K Q_t$ , at least one firm, to attract the profitable worker  $h$ , will deviate by offering  $W_{ft}^K(y_{ht}^K) = W_t^K(y_{ht}^K) + \varepsilon$ . Conversely, if  $W_t^K(y_{ht}^K) > y_{ht}^K Q_t$ , the hiring firm would incur a net loss by employing  $h$  such that it is more profitable to lower

its offer until  $W_{ft}^K(y_{ht}^K) \leq y_{ht}^K Q_t$ . Given  $Q_{ft} = Q_t$  for all  $f$  satisfying  $k_{ft}^S > 0, \bar{k}_{ft}^D(Q_{ft}) > 0$  and  $W_{ft}^K(y_{ht}^K) = y_{ht}^K Q_t$ , suppose that  $Q_t$  does not clear the market. Then, in case of excess demand, it is trivially profitable for each firm to raise prices to  $Q_t^*$ . In case of excess supply, since equilibrium capital expenditures  $k_t^C(Q_t)Q_t$  are strictly decreasing in  $Q_t$ , firms find it profitable to lower their price until the market clears at  $Q_t^*$ .

### *Equilibrium - Labor*

Workers supply labor to the consumption goods sector if the unemployment benefits exceed their respective outside option  $\delta^U W_t^C > q_h z_t^K Q_t$ . Aggregate labor supply is then given by,

$$\begin{aligned} n_t^{C,s} &= \Pr(q_h < \delta^U W_t^C / z_t^K Q_t) \\ &= G(\delta^U W_t^C / z_t^K Q_t) \end{aligned}$$

In other words, the wage necessary to attract any given worker is, ceteris paribus, increasing in  $z_t^K, Q_t$  and decreasing in  $\delta^U$ .<sup>81</sup> On the other side of the market, labor demand satisfies,

$$\begin{aligned} n_t^C &= \sum_{j^C} n_{jt}^C \\ &= \sum_{j^C} \frac{(1-\alpha)k_{jt}^C Q_t}{\mu^{LF} \alpha W_t^C} \\ &= \frac{(1-\alpha)k_t^C Q_t}{\mu^{LF} \alpha W_t^C} \end{aligned}$$

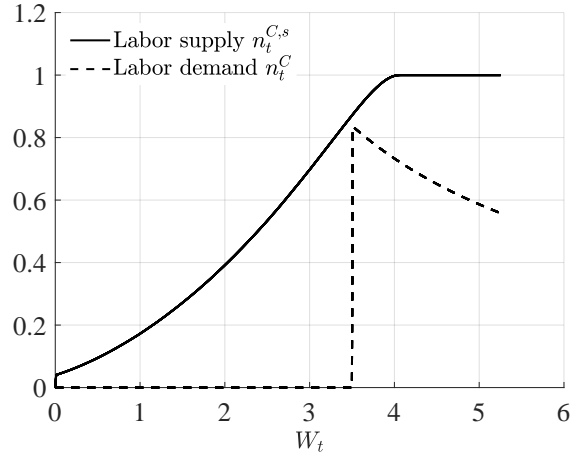
Since equilibrium requires  $W_t^C \geq \max\{\underline{W}_t^C, \delta^W W_{t-1}^C\}$ , the labor market may exhibit excess supply in equilibrium. For example, consider Figure 1.20 which depicts a situation where the labor market fails to clear. In particular, the market for labor may exhibit excess supply  $n_t^{C,s} > n_t^C$  for two reasons. First, firms never find it profitable to lower wages by more than  $(1 - \lambda^W)$  relative to last period's wage  $W_{t-1}^C$  because of concerns relating to worker effort. Second, the fact that offering higher wages increases average labor productivity induces the lower wage threshold  $\underline{W}_t^C$ .<sup>82</sup>

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<sup>81</sup>Since the equilibrium wage rate  $W_t^C$  is decreasing in  $\delta^U$ , both  $n_t^{C,s}$  and  $n_t^C$  are increasing in  $\delta^U$  in equilibrium. Therefore, if the demand shift dominates the supply shift, raising unemployment benefits induces lower equilibrium unemployment.

<sup>82</sup>Technically, the described adverse selection mechanism is not enough to generate the lower bound. To generate the lower bound, I require that (2) has an interior maximum, which requires that average productivity also be locally concave. The desired concavity property is provided by  $G_q$  being rectified Gaussian, a rather unrealistic assumption. For future work, I envision introducing scale efficiencies via nonlinear skill aggregation.

**Figure 1.20.** The labor market



Notes: Figure 1.20 shows the extensive margin of labor supply and demand for a given wage. Due to adverse selection across worker skill, there is a demand discontinuity because firms never find it optimal to offer a wage below  $\underline{W}_t$ .

Following the procedure of Weiss (1980), suppose there exists a wage offer  $W^o < \underline{W}_t^C = qz_t Q_t$ , a labor demand  $n^o$ , and a corresponding output  $y^o$  that maximize the firm's payoff. Holding labor costs  $C^o = W^o n^o$  fixed, the firm may alternatively employ  $n^* = C^o / \underline{W}_t^C < n^o$  workers at  $\underline{W}_t^C$ . From (2), we know that  $q^C(\underline{W}_t^C) / \underline{W}_t^C > q^C(W^o) / W^o$  and thus, since labor costs are fixed,  $q^C(\underline{W}_t^C) n^* > q^C(W^o) n^o$  in which case the firm produces  $y^* > y^o$  at the same cost  $C^o$ . By continuity of production and the strictly decreasing labor cost, the firm may alternatively also produce  $y^o$  at a reduced labor cost. Therefore, as long as the firm maximizes some measure of contemporaneous profit, we have induced a contradiction and thus shown that offering any  $W^o < \underline{W}_t^C$  is strictly dominated by the strategy of offering  $\underline{W}_t^C$  and employing  $n^*$ .

#### *Equilibrium - Demand deposits and reserves*

Banks are special in my model because they can artificially create previously non-existing units of the economy's numéraire by lending to firms. Since they are risk averse, banks never opt to keep a commercial loan on balance sheet, but they have the ability to infer what price  $P_{bft}^L$  can be obtained from selling a particular commercial loan  $(l_{bft}, R_{bft}^L)$  on the secondary market. Because competition must yield zero profits, we have  $l_{bft} = P_{bft}^L$ . Following creation, the newly minted demand deposits vanish as soon as the SPV contracts are executed. Finally, notice that banks also engage in the market for Fed Funds to satisfy the prevailing reserve requirement with the outside option of investing in the risk free bond pinning down  $R_t^{FFR} = R_t^S$ .

*Equilibrium - Debt and equity*

Given the auctioneer’s announcement  $R_t^D$ , households maximize a risk adjusted measure of projected asset returns, whereas funds maximize projected return on equity. The household’s optimality condition is given by,

$$\tilde{R}_t^E - \tilde{R}_t^D \geq \gamma_h^r \frac{w_{ht0}^E}{w_{ht0}^I}$$

which implies,

$$\frac{w_{ht0}^E}{w_{ht0}^I} = \min \left\{ \frac{1}{\gamma_h^r} (\tilde{R}_t^E - \tilde{R}_t^D), 1 \right\}$$

such that the optimal equity share is increasing in the projected risk premium  $\tilde{R}_t^E - \tilde{R}_t^D$  and decreasing in  $\gamma_h^r$ . Finally, the equilibrium interest rate  $R_t^D$  clears the market,

$$\int_{JHH} w_{ht0}^E = \frac{1}{1 + \delta_I} \int_{JHH} w_{ht0}^I$$

*Data sources*

The model was parameterized using quarterly US data from 1987 until 2017. The analysis is limited to this time period because the institutional monetary policy changes undertaken by former Fed chair Volcker are widely believed to have muted the business cycle (see Stock and Watson, 2002). Therefore, since the inclusion of structural changes invariably undermines the ubiquitous proposition of ergodicity, moment matching is conducted using data from the post-Volcker era, also known as the Great Moderation, only. All labor market data was obtained from the US Bureau of Labor Statistics, whereas the other series were sourced from the Federal Reserve of St. Louis database (FRED), the Federal Reserve Board (FRB), or Yahoo Finance. The following contains the origin of all data series displayed in the respective figures.

**Table 1.7.** Data sources<sup>†</sup>

Fig.	Series	Frequency (level), source
1.	Unemployment	Monthly (2008=0), LNS14000000 via <a href="#">BLS</a>
	Employment	Monthly (2008=0), LNS12300000 via <a href="#">BLS</a>
	Employment, 25-54	Monthly (2008=0), LNS12300060 via <a href="#">BLS</a>
2.	Job losers, on layoff	Monthly, LNS13023653 via <a href="#">BLS</a>
	Job losers, others	Monthly, LNS13025699 via <a href="#">BLS</a>
	Job leavers (quit)	Monthly, LNS13023705 via <a href="#">BLS</a>



	Reentrants	Monthly, LNS13023557 via <a href="#">BLS</a>
	New entrants	Monthly, LNS13023569 via <a href="#">BLS</a>
3.	Unemployment	Quarterly, LNS14000000 via <a href="#">BLS</a>
5.	Answers	Quarterly from Amir Sufi's <a href="#">Website</a>
6.	Employment	Monthly, LNS12300060 via <a href="#">FRED</a>
	Wages	Monthly, AHETPI via <a href="#">FRED</a>
	CPI	Monthly, PCEPILFE via <a href="#">FRED</a>
13.	FFR	Monthly, DFF via <a href="#">FRED</a>
	1Y Treasury	Monthly, GS1 via <a href="#">FRED</a>
19.	Transfers	Annual, B1190C1A027NBEA via <a href="#">FRED</a> and <a href="#">FRB</a>
	GDP	Annual, GDPA via <a href="#">FRED</a>
	Federal receipts	Annual, FYFR via <a href="#">FRED</a>

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<sup>†</sup>Notes: Figure 1.3 plots a Gaussian kernel density estimate of US unemployment since 1987. The estimate is constructed as in Botev et al. (2010) with a mesh granularity of  $2^{-7}$ . The employed input frequency is reduced to quarterly because the model is parameterized to match quarterly data. Figure 1.19 is a replication of a graph found in the Federal Reserve of St. Louis [blog post](#) “Fed Payments to Treasury and Rising Interest Rates” by Miguel Faria-e-Castro.

### *Top 10 Ways to Prepare for Retirement*

In my framework, households do not maximize expected lifetime utility over an infinite stream of consumption. First, this is because they only live a finite number of periods. More importantly, it's because deriving the distribution over future consumption implied by the model is extraordinarily challenging from a computational perspective. However, households understand that future consumption, retirement consumption in particular, is strictly increasing in accumulated lifetime savings. Rather than worrying about future consumption directly, my objective's relevant marginal benefit associated with the marginal cost of decreasing consumption today is thus given by the projected increase in accumulated retirement balances. This modeling choice entails the cost that future consumption only generates utility implicitly — because retirement consumption is increasing in retirement balances — such that the resulting decision rule is only implicitly governed by the tradeoff between consumption today and consumption in the future. Moreover, each household's future plan is reduced to the assumption that they will simply continue to save the exact same nominal amount each period until retirement. Admittedly, the proposed objective constitutes a stark departure from contemporary practice, but it is motivated by the real-life mathematical exercise depicted in Figure 1.21, which is frequently used, as illustrated by the fact that the US

Department of Labor advertises it, to illustrate the benefits associated with saving for retirement early.<sup>83</sup>

**Figure 1.21.** Retirement savings guidance from the US Department of Labor



Notes: The above illustration is taken from the US Department of Labor’s [publication](#) “Top 10 Ways to Prepare for Retirement”. It serves as an inspiration as to how households effectively assess the canonical tradeoff between consuming today and consuming in the future. In particular, I assume that households find probabilistic assessments of future consumption too complex and thus resort to setting themselves a nominal retirement savings goal. To achieve this goal, they go through the above cumulative compounding exercise while appealing to an ergodic benchmark interest rate.

As is the case in reality, pinning down a reasonable benchmark for the relevant interest rate used for compounding presents a challenging task. In the model, I assume that households view monetary policy shocks as permanent, namely by interpreting the risk free rate as ‘the current interest rate level’, and derive current asset return projections by adding the (ergodic) mean of all historic markups to the prevailing risk free rate.

### *SMM and ergodicity*

Consistency of SMM requires that the proclaimed data generating process  $X : \Omega \times T \mapsto S$ , defined on  $(\Omega, \mathcal{W}, \mu_{\mathcal{W}})$ , whose moments we want to match is ergodic.<sup>84</sup>

**Geometric ergodicity.** A process  $X : \Omega \times T \mapsto S$  is said to be geometrically ergodic if there exists  $\rho < 1$  and a time-invariant measure  $f_X : S \mapsto \mathbb{R}$  such that for any  $x_0 \in S$ ,

<sup>83</sup>In essence, the exercise illustrates the (nominal) marginal benefit of saving more today, again assuming that one saves the exact same amount each period until retirement.

<sup>84</sup>“In calculating asymptotic distributions, (geometric) ergodicity can substitute for stationarity since it means that the process converges (geometrically) to its stationary distribution” (Duffie and Singleton, 1993).

$$\lim_{T \rightarrow \infty} \rho^{-T} \|P_{x_0}^T - f_X\|_\nu = 0$$

where  $\|\cdot\|_\nu$  is the total variation norm and  $P_{x_0}^T : S \mapsto \mathbb{R}$  denotes the conditional distribution of  $X_T$  given  $X_0 = x_0$ . Assuming geometric ergodicity and some regularity conditions (see Duffie and Singleton, 1993), we have,

$$\begin{aligned} \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T g(X_t) &= \lim_{T \rightarrow \infty} \mathbb{E}[g(X_T)] \\ &= \int_S g(x) df_X(x) \end{aligned}$$

for any  $f_X$ -measurable function  $g$  and any initial condition  $X_0 = x_0 \in S$ . In particular, notice that aside from targeting canonical first and second moments — via the identity map and the squared error map —  $g$  may be chosen target any ergodic statistic including conditional expectations and quantiles. Letting  $X_\infty$  denote any random variable with the density  $f_X$ , consider for example  $g(x) = g_c(x) \equiv x\mathbb{I}(x \leq q)$ ,

$$\begin{aligned} \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^T X_t \mathbb{I}(X_t \leq q) &= \lim_{T \rightarrow \infty} \mathbb{E}[X_T | X_T \leq q] \\ &= \mathbb{E}[X_\infty | X_\infty \leq q] \end{aligned}$$

where  $\bar{T} \equiv \sum_{t=0}^T \mathbb{I}(X_t \leq q)$ . Alternatively, consider  $g(x) = g_q(x) \equiv \mathbb{I}(x \leq q)$ ,

$$\begin{aligned} \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \mathbb{I}(X_t \leq q) &= \lim_{T \rightarrow \infty} \mathbb{E}[\mathbb{I}(X_T \leq q)] \\ &= \lim_{T \rightarrow \infty} \Pr(X_T \leq q) \\ &= \Pr(X_\infty \leq q) \\ &= F_{X_\infty}(q) \end{aligned}$$

such that the LHS provides a consistent estimate of the  $q$ th quantile of the limiting cdf  $F_X$ .

**Table 1.8.** Agents

	Meaning
Banks	Intermediaries between households and firms which ‘artificially’ re-scale the economy’s numéraire by issuing and selling commercial loans.
Brokers	Intermediaries between households and pension fund.
Capital producers	Firms that own, produce, and lend capital to the consumption goods sector. Capital producers compete on price, but make positive profits if they have an inventory.
Consumption goods producers	Firms that take out commercial loans to borrow capital, hire workers, and produce goods. Production occurs while demand is uncertain such that commercial loans are subject to default.
Pension funds	Institution which manages households’ retirement savings by investing in risk free in risky securities.
Households	People who supply labor across sectors, choose to exert worker effort, and accumulate nominal balances to save for retirement.

**Table 1.9.** Institutional descriptions

	Meaning
Calibration	Practice of choosing a specific parameter value with reference to the literature or a specific moment of the data.
Commercial loan	Intraperiod loan issued and sold by bank. Ultimately held by pension funds in the form of a collateralized loan obligation (CLO).
Expectation	An agent-independent, well-defined mathematical object implied by theory.
Estimation	Econometric practice of fixating a parameter vector by way of minimizing a predetermined, joint loss function.
Ergodicity	System property that any initial condition yields a unique limiting distribution (see Duffie and Singleton, 1993). Since ergodicity implies eventual convergence to a unique distribution, an ergodic process is also stationary if initialized properly.
Fire Sale	Attempted forced sale of the risky security that causes a price implosion because excess supply is locally decreasing in price.
Fragility	Equilibrium discontinuity in the state space. Often induced by an occasionally binding constraint. Frequently present in indeterminate models (when a stable node collides and collapses with an unstable node), but not precluded by uniqueness as illustrated herein.
Initial margin	Minimum fraction of equity required to purchase assets using broker leverage.
Maintenance margin	Minimum fraction of equity required to retain an account with the broker.

Margin call	Debt repurchase demand issued by broker if equity falls below maintenance margin.
Monetary policy, conventional	Periodic announcement of new interest rate and defense thereof.
Monetary policy, unconventional	Any action taken by the central that extends beyond the implementation of conventional policy.
Projection	A hypothetic realization of a random variable that is not derived as a specific moment from the true conditional distribution. An agent-specific way of reducing stochastic complexity.
Pricing, absolute	Practice of valuing an asset by considering the fundamental benefits that it entails (see Cochrane and Culp, 2003).
Pricing, relative	Practice of valuing an asset by way of comparing it to an asset whose price is taken given (see Cochrane and Culp, 2003).
Sellout	Successful forced sale of the risk free security to the central bank.
Macroeconomic transmission	Proposition that there exists a causal link between nominal and real crises.

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## CHAPTER II

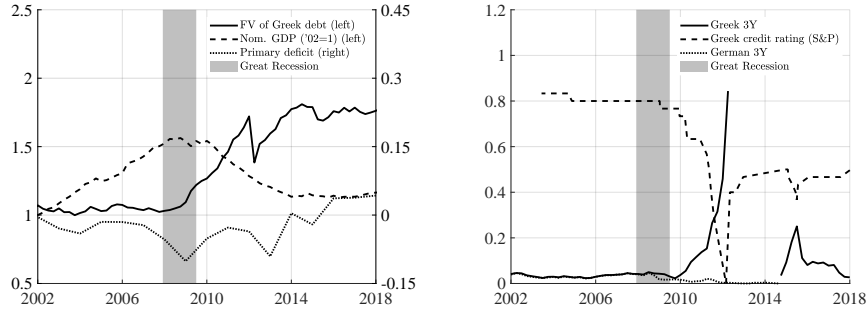
### Was the 2012 Greek Default Self-Fulfilling? A Sovereign Debt Model with Slow Moving Crises and Excusable Defaults

#### 2.1. Introduction

Following the 2008 Financial Crisis, the face value of Greek debt ballooned from roughly 100 percent of GDP in 2008 to about 170 percent of GDP in late 2011. This enormous rise can be decomposed into two phases. First, during the Great Recession, the Greek government temporarily engaged in extraordinary, expansionary fiscal policy to combat the local transmission of the global crisis such that debt-to-GDP rose via an increasing numerator. Interestingly, this initial phase of the crisis did not see a rise in Greek sovereign yields, which were falling right until the Great Recession ended in mid 2009. Unlike the Great Recession, the Greek recession did not end in mid 2009. Instead, the Greek economy fell into a long and severe contractionary, second phase and so debt-to-GDP continued to rise via a falling denominator.

As macroeconomic fundamentals continued to deteriorate following the Great Recession, Greek yields entered a dramatic, explosive regime in early 2010 which culminated in levels in excess of fifty percent two years later. Roughly three years after the initial fiscal deficits, the Greek debt crisis finally concluded in default when creditors voluntarily agreed to a haircut in March of 2012 after a ‘debt restructuring’ had long been deemed ‘inevitable’. The primary aim of this paper is then to construct a sovereign debt model that quantitatively accounts for the observed, extraordinary regime switch in Greek yields while also rationalizing two main features of the Greek crisis, namely that it unfolded gradually over the course of multiple years and that default was widely viewed as inevitable.

**Figure 2.1.** Greek macroeconomic fundamentals, sovereign bond yields, and credit ratings



Notes: Figure 2.1 depicts the evolution of select Greek macroeconomic fundamentals — debt-to-GDP (face value), nominal GDP, and primary deficits — as well as Greek secondary market yields and the corresponding Standard and Poor’s credit rating between 2002 and 2018. The subinterval of particular interest is the period between the 2008 Financial Crisis and the credit event in March 2012. Since there were no bonds with a remaining maturity of three years between March 2012 and March 2014, there are no corresponding yields displayed in Panel B during that period. The depicted numerical credit ratings were constructed by translating Standard&Poor’s alphabetical ratings into equidistant steps of  $\frac{1}{30}$  from zero to one.

To motivate the paper’s research question, I now provide a brief account of the dramatic rise in Greek yields between 2009 and 2012. As Greek macroeconomic fundamentals deteriorated during the Great Recession, all major credit rating agencies — Fitch, Moody’s, and Standard&Poor’s — downgraded Greek bonds for the first time in five years. In March of 2010, amid increased uncertainties associated with other sovereign borrowers throughout Europe’s southern periphery, the Greek government was the first to approach the European Union and the International Monetary Fund for financial assistance. After agreeing to structural reforms under the infamous ‘Memorandum of Understanding’ and being downgraded to ‘non-investment grade’, the country was plagued by political unrest, ever rising secondary market yields, and self-perpetuating credit risk assessments throughout 2010 and 2011. Finally, in March 2012 when a ‘debt restructuring’ — meaning default — had long become “inevitable” (Bank of Greece, 2014), creditors agreed to a nominal haircut of twenty percent, which in present value terms — as weighted average maturities more than doubled from seven to fifteen years — exceeded fifty percent (Zettelmeyer et al., 2013). But if default was inevitable at the time of the restructuring in March 2012, *when had the Greek financing scheme become financially unsustainable in the first place?*

An intuitive, but atheoretical way to assess when the Greek financing scheme became financially unsustainable would be to exclusively focus on the ‘local stationarity’ properties of the yield series depicted in Panel B of Figure 2.1. Specifically, notice that the evolution of Greek yields is visibly ‘explosive’ towards the end of the observed interval which, by conventional wisdom, serves as anec-

total evidence that the process has undergone a structural break.<sup>1</sup> And indeed, the implementation of a right-sided unit root test promptly rejects the null of a unit root against ‘mildly explosive’ behavior starting in March of 2010.<sup>2</sup> However, similar to Bohn’s (1998) argument regarding fiscal sustainability, the central point at the core of this paper is that even though the observation of explosive yields may anecdotally indicate a violation of financial sustainability, locally explosive behavior is not a necessary condition for a state to be financially unsustainable.<sup>3</sup> In fact, I find that even under the very conservative premise that the Greek government were willing to permanently spend a quarter of its tax income on unproductive interest outlays, the Greek financing scheme became financially unsustainable at least six months prior to exhibiting explosive yield dynamics, at a time when credit spreads had almost returned to zero. During the latency period when the Greek state was financially unsustainable but yields still appeared mean-reverting, the Greek treasury temporarily benefitted from a sequence of positive market perception shocks and a decrease in the risk free benchmark rate. Said decrease was, albeit effectively permanent, insufficient to make up for the ever-rising debt levels, which continued to advance the Greek state into deeper territories of the financially unsustainable region of the state space.

#### *Excusable default as optimal default*

Sovereign debt is unlike corporate debt in multiple ways. First, sovereign debt is not subject to external enforcement. Second, to avoid outright default, countries may be able to enter into an arrangement with organizations such as the International Monetary Fund or the European Stability Mechanism, or they could typically — albeit not Greece — also monetize the issuance of a new bond. Accordingly, sovereign defaults are unlike corporate defaults in that the latter are forced — via binding liquidity and solvency constraints — whereas the former, or at least their timing, are

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<sup>1</sup>While the series appears to have switched from a mean-reverting regime to an explosive regime, notice that a ‘local’ non-stationarity of the latter sort does not imply that the observed series as a whole is non-stationary. In fact, if it were indeed generated by a regime-switching process with a mean-reverting and an explosive regime, it may very well still be ergodic and thus — if initialized properly — strictly stationary.

<sup>2</sup>Interestingly, Greece entered into the infamous Memorandum of Understanding with the European Troika shortly thereafter: “The Memorandum averted a Greek default, which in April 2010 seemed inevitable” (Bank of Greece, 2014). See Section 2.7 for a discussion of the applied right-sided unit root test proposed by Phillips and Yu (2011).

<sup>3</sup>In his seminal treatment of fiscal sustainability, Bohn (1998) shows that a *fiscal rule* — a mapping from a country’s debt-to-GDP level to primary surpluses — that is at least linearly increasing is sufficient to guarantee fiscal sustainability in the sense that the government’s fundamental intertemporal budget constraint is satisfied. Importantly, it is further emphasized that the observation of a stationary debt-to-GDP ratio is neither necessary nor sufficient to establish fiscal sustainability such that testing the latter to conduct inference on the former does not constitute a rigorous approach.



typically voluntary such that they occur when it is perceived as optimal.<sup>4</sup>

To rationalize the specific types of contingencies that render default optimal, the literature typically appeals to a tradeoff between the benefits of lower debt and interest rate burdens versus the costs of a temporary fall in output. This modeling choice is seemingly supported by the observation that when default is imminent, debt-to-GDP is high, sovereign yields reach their peak, and — most importantly — output contracts (Mendoza and Yue 2012). However, using quarterly instead of annual data, Levy-Yeyati and Panizza (2011) document that while sovereign defaults typically do coincide with output declines, the latter actually precede the former and so defaults effectively “mark the beginning of economic recovery”. In fact, “growth rates in the post-default period are never significantly lower than in normal times” (Levy-Yeyati and Panizza, 2011).

The observation that defaults are typically preceded, but not followed by recessions raises the question in which direction the causality runs. In particular, if defaults in fact did cause the recessions that typically precede them without any corresponding costs *ex post*, then the contemporaneous decision to default is both self-fulfilling and trivial because the marginal costs of defaulting are then zero (Levy-Yeyati and Panizza, 2011). However, the default-as-a-cause proposition may very well suffer from reverse causality, namely if defaults do not cause, but are in fact caused by their preceding recessions (*post hoc et propter hoc*). As I argue herein, this would be the case if economic downturns boosted a government’s incentives to default, namely through higher levels of political excusability. In similar spirit, Grossman and van Huyck (1988) propose that sovereign debt be viewed as a contingent claim, where a default is *excusable* if and only if it is “justifiably associated with implicitly understood contingencies”. In this view, the marginal cost of default is decreasing in its excusability such that optimality and excusability are intimately interlinked.<sup>5</sup>

To motivate the government’s proposed tradeoff in my theory, I first acknowledge the existence of a principal-agent problem. While voters care to maximize their own utility, the incumbent government’s primary objective is to get re-elected. Accordingly, the primary costs of default *to the decision maker* are political as default typically lessens the government’s chance of getting re-elected. In turn, the primary benefit of default is an easing of the government’s budget constraint

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<sup>4</sup>Methodologically, this difference manifests itself in that firm default is typically modeled as being triggered exogenously, whereas sovereign defaults are typically modeled as occurring endogenously.

<sup>5</sup>The proposition that an excusable contingency had occurred in the case of Greece is supported by the fact that the ultimate debt exchange was “voluntary” and that virtually all negotiations were conducted by the creditors themselves (Zettelmeyer et al., 2013).

via falling a interest expense, which the government must weigh against the costs imposed by the political base. The political base generally prefers no default because they represent current creditors such that, to them, default effectively constitutes an expropriation. However, if eventual default in the foreseeable future is viewed as inevitable, then — in the spirit of Grossman and van Huyck (1988) — some creditors may be willing to *excuse* it. Specifically, my key assumption is that the proportion of the political base that views default as excusable (or even necessary) is increasing in the fraction of output that their government spends on ‘unproductive’ interest outlays. This is because increasing interest outlays must either be financed through more debt, higher taxation, or through much-despised reductions in public spending. In effect, since higher interest outlays translate to lower default costs while also increasing the benefits of default, likelihoods of default are increasing in the amount of output that a government spends on such outlays in my theory.

#### *Self-fulfilling vs. non-fundamental debt crises*

Dating back to the seminal contribution by Calvo (1988), sovereign default has been one of only few strands of the macroeconomic literature, in which the practical existence and empirical relevance of multiple equilibria appears to be widely accepted.<sup>6</sup> In fact, after proving that equilibrium in the canonical Eaton-Gersovitz (1981) model is unique, Auclert and Rognlie (2016) go so far as to interpret their result as a shortcoming of the model rather than as a vindication of the uniqueness proposition.<sup>7</sup>

In policy circles, the proposition that sovereigns may be subject to self-fulfilling debt crises has gained traction as well, even at the highest levels. After announcing that the European Central Bank was adding outright open market purchases of individual European sovereign titles to its monetary policy repertoire in September 2012, president Mario Draghi elaborated as follows,

“We are in a situation now where [...] large parts of the euro area in what we call a ‘bad equilibrium’, namely an equilibrium where you may have self-fulfilling expectations

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<sup>6</sup>Extending Calvo’s two-period model to the infinite horizon, Alesina et al. (1990) and Cole and Kehoe (2000) show a sovereign’s inability to commit to repayment can yield multiple, self-fulfilling equilibria. More recent examples of indeterminate models of sovereign finance include Conesa and Kehoe (2017), Lorenzoni and Werning (2019), and Bocola and Dovis (2019). The model by Lorenzoni and Werning (2019) features both multiple equilibria as well as multiple steady states.

<sup>7</sup>“Our objective is not to deny that sovereign debt markets can be prone to self-fulfilling crises, or that OMT may have ruled out a bad equilibrium. Instead, we hope that our results may help sharpen the literature’s understanding of the *assumptions that are needed for such multiple equilibria to exist.*” (2016)

that feed upon themselves and generate very adverse scenarios. So, there is a case for intervening, in a sense, to ‘break’ these expectations.”

To quantitatively assess the practical relevance of indeterminacy, Bocola and DAVIS (2019) decompose recent Italian credit spreads into *fundamental* and *non-fundamental* components. Specifically, their identifying assumption is that all default risk associated with the ‘default zone’ of their state space is fundamental, whereas all *rollover risk* associated with the ‘crisis zone’ — where equilibrium is indeterminate — is non-fundamental.<sup>8</sup> They find that rollover risk only played a modest role in driving the observed dynamics in Italian credit spreads, but concede that the model is unable to account for the large increase in Italian spreads during the fall of 2011. In particular, since measured fundamentals only fluctuated marginally at the time and the employed particle filter attributes only a small fraction of the observed variation in Italian spreads to the non-fundamental component, the latter’s extraordinary rise is largely attributed to measurement error of the credit spreads themselves. This bears further investigation for two reasons. First, measurement error in credit spreads is almost surely minimal, both in absolute and in relative terms. Second, the extraordinary rise in spreads in late 2011 is precisely what a sovereign default model ought to be able to explain. This paper addresses both of these concerns, namely by calling into question the premise that a self-fulfilling crises must be non-fundamental.

To understand how self-fulfilling crises can principally be fundamental, notice that the rollover crises described by Bocola and DAVIS (2019) are non-fundamental in the same way that expectations are non-fundamental in any rational expectations model. Specifically, whenever their state lies in the state space’s ‘crisis region’ where equilibrium is indeterminate, a *non-fundamental* sunspot is required to determine whether default occurs. As per usual, the underlying rational expectations assumption imposes that investors are capable of mapping the effects of their own behavior into the government’s behavior such that any additional information provided by an external agent would be entirely inconsequential. In stark contrast, Afonso et al. (2012) exploit recent, high frequency European data to show that sovereign credit rating updates do in fact significantly affect yield and CDS spreads.<sup>9</sup> I use this empirical fact to motivate my assumption that while investors successfully

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<sup>8</sup>Methodologically, this is carried out by constructing a counterfactual sequence of credit spreads by setting the conditional default probability in the crisis zone, captured by the sunspot variable  $\pi_t$ , to zero for all  $t$ . In effect, the proposed decomposition thus measures the fraction of Italian credit spreads that is accounted for by the crisis region of the state space relative to the fraction accounted for by the default region.

<sup>9</sup>Indeed, this is unsurprising as the mere existence of credit rating agencies implies that some investors do not

*derive* their optimization problem’s first order condition, they lack sufficient information to *evaluate* it. For lack of better information, they then replace the ‘rational’ conditional expectation with an analogous ‘behavioral’ object derived from the most recent credit risk assessment issued by the agency.<sup>10</sup> Since ratings are further only updated once per period in my theory, self-fulfilling crises occur gradually and are thus not “rollover crises” akin to Bocola and DAVIS (2019), but “slow moving crises” akin to Lorenzoni and Werning (2019). In the context of slow moving crises, since expectations feed back into fundamentals over the course of time, the two notions of a self-fulfilling crisis and a fundamental crisis are no longer mutually exclusive.

Aside from its empirical support, the proposition that expectation formation exhibits inertia has two main methodological advantages. First, since it turns indeterminacy into multistability, there is no need to conduct intra-temporal equilibrium selection. Second, the resulting slow moving nature of my crises is well-suited to explain the observed, gradual explosion in Greek yields. Specifically, notice that if macroeconomic fundamentals warranted an extraordinarily high rational-expectations-yield — say fifty percent as observed in Greece in early 2012— but previous yields were at more conventional levels, allowing for inertia in expectation formation naturally generates an explosive, but gradual rise *even if macroeconomic fundamentals remain unchanged*. As such, bounded rationality directly addresses a concern raised by Bocola and DAVIS (2019), whose model “has a hard time capturing the jump in spreads observed in the third quarter of 2011 with the fundamental shocks because [fundamentals] barely moved”.<sup>11</sup>

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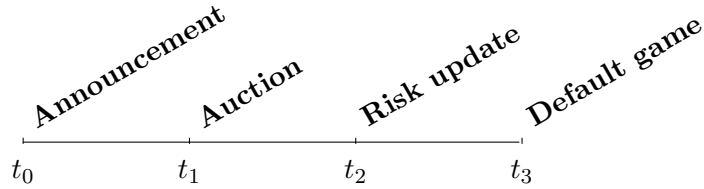
adhere to rational expectations. In contrast, the fact that different bidders submit different bids for the same bond is not sufficient for two reasons. First, since winning bidders can potentially affect the ultimate transaction price in real-world treasury auctions, submitting one’s fundamental valuation is not a dominant strategy. But even if it were, namely if treasury auctions were conducted in a multi-unit analogue of single-unit second-price auctions, different bidders may still have different valuations because of differing outside options and/or differing levels of risk aversion.

<sup>10</sup>Methodologically, this implies that expectation formation serves as an input of the model. In turn, this elevates expectations to the rank of a fundamental state, which serves as a necessary condition that “breaking expectations” constitutes a valid, dynamic strategy in the first place.

<sup>11</sup>Italian 3Y bond yields rose from 3.88% in late August to 8.21% in late November while German 3Y yields fell from 0.9% to 0.35% over the same time period. Although fundamentals may not have changed, this dramatic rise in spreads did coincide with a sequence of credit rating downgrades by all three major rating agencies. Precisely prior to the height of the surge, Moody’s downgraded Italian bonds by three levels (!) in the [first week of October](#).

## 2.2. A Model with Slow Moving Debt Crises and Excusable Defaults

Each period, my model evolves sequentially according to the following timeline.



First, following the realization of external financing needs and the risk free benchmark rate, the treasury announces the face value of a new bond to cover the government’s current liquidity gap. Second, primary market dealers derive their valuation of the new bond against a risk free outside option given a risk-neutral objective. When evaluating their first order condition, however, dealers lack precise information regarding the bond’s credit risk and so they resort to relying on a potentially outdated credit risk assessment issued by a rating agency. Third, following the auction, the rating agency updates its risk assessment and secondary market trading ensues. Fourth, after the Walrasian auctioneer has established equilibrium in secondary markets, the government enters into a strategic game with its political base to determine whether default occurs or not.

### *The auction*

At the beginning of each period, the treasury observes the exogenous realization of the primary deficit  $x$ , real economic growth  $g$ , and the risk free interest rate  $r$ . In contemporaneous per-GDP terms, present external financing needs (EFN) are then calculated as,

$$d \equiv \frac{V}{1 + g} + x$$

where  $V$  is the face value of an expiring bond that was issued in the previous period (in previous per-GDP terms).<sup>12</sup> In order to avoid immediate default, the proceeds  $l$  from the upcoming auction of a new bond with face value  $V'$  must satisfy  $l \geq d$ . Assuming that avoidable defaults carry severe political repercussions and that the treasury itself does *not* benefit from issuing any additional bonds, the treasury’s objective is simply given by,

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<sup>12</sup>Since real-world governments do not periodically refinance their entire stock of debt, recovering a sensible, analogous EFN measure from the data is non-trivial. See Section 2.7 for a discussion.

$$W^T(d) = \min_{V'} V' \quad \text{s.t.} \quad l \geq d$$

where the proceeds  $l$  are pinned down by principally unknown primary market yield  $y^P$  as follows:  $l = \frac{V'}{1+y^P}$ . However, since investors gauge credit risk by relying on a predetermined, external assessment  $\lambda^P$  that is also known to the treasury, the treasury can easily infer  $y^P = g_y(\lambda^P, r)$  via the known structural mapping  $g_y : \mathbb{R}^2 \mapsto \mathbb{R}$  and we have,

$$l = d$$

$$V' = d[1 + g_y(\lambda^P, r)]$$

To illuminate the origins of the spread  $y^P - r$ , I now turn to deriving  $g_y$ .

#### *Boundedly rational investors*

Unlike English auctions or markets that are presided over by a Walrasian auctioneer, treasury auctions are typically ‘blind’ in that participants never get to see anyone else’s actions. While participants may thus try to conduct inference as to how others value the good, such inference is unnecessary if the auction is second-price, in which case submitting one’s true valuation is a dominant strategy. To ensure that such unconditional bidding behavior is in fact optimal, I assume that our treasury securities are sold in the format of a multi-unit analogue of a single-unit second-price structure.<sup>13</sup>

In the auction,  $N$  investors submit a bid  $p$  to invest a fraction  $q$  of their wealth  $m$  into the new sovereign bond with face value  $V'$  and uncertain, binary payoff  $\hat{V}' \leq V'$  (default and no default). All wealth not allocated to the risky bond is diverted towards a risk neutral asset with a guaranteed return  $r$ . All investors have a risk neutral objective and thus *aim* to maximize,

$$W^I(m) = \max_{p,q} \frac{q^{pr} m}{p} \mathbb{E}[\hat{V}'] + (1 - q^{pr})m(1 + r)$$

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<sup>13</sup>Specifically, I assume that — contrary to contemporary practice — the uniform transaction price is set equal to the highest bid among all non-winning bidders. In contrast, actual treasury auctions typically set the transaction price equal to the lowest winning bid, which amounts to the multi-unit analogue of a single-unit first-price sealed-bid auction. Since “a bidder has a positive probability of influencing price in a situation where the bidder wins a positive quantity” in such auctions, bidders will not generally reveal their true valuation in optimum (Ausubel et al., 2014).

where  $q^{pr} = kq, k \in (0, 1]$  is allocated pro-rata across all winning bids.<sup>14</sup> As per usual, the above objective implies that each investor *fundamentally values* the security at  $\mathbb{V} = \frac{\mathbb{E}[\hat{V}']}{1+r}$  in the sense that we can write investor  $i$ 's best-response demand as,

$$\mathbb{B}(p^*) = \begin{cases} 0 & \text{if } p^* > \mathbb{V} \\ [0, 1] & \text{if } p^* = \mathbb{V} \\ 1 & \text{if } p^* < \mathbb{V} \end{cases} \quad (2.1)$$

where  $p^*$  denotes the transaction price implied by a particular strategy profile if the latter were observed.<sup>15</sup> In turn, assuming that  $p^* < \mathbb{V}$  is always perceived as a possible outcome (even if just for reasons of a trembling hand), bidding  $(\mathbb{V}, 1)$  is a dominant strategy for each investor.

To see why *implementing* the above policy may be non-trivial, I now consider the two principal factors that jointly determine the credit risk of lending to the government. Specifically, since  $\hat{V}'$  is binary — default and no default — we can multiplicatively decompose  $\mathbb{E}[\hat{V}']$  into the face value  $V'$  and the bond's credit risk  $\lambda$ ,

$$\begin{aligned} \mathbb{E}[\hat{V}'] &= (1 - \pi)V' + \pi\gamma V' \\ &= [1 - \pi(1 - \gamma)]V' \\ &\equiv (1 - \lambda)V' \end{aligned}$$

where  $\pi$  is the probability of default and  $1 - \gamma$  is the loss given default. The reason why determining  $\lambda$  is non-trivial is that it — along with its components  $\pi$  and  $\gamma$  — realistically depends on the outcome of the auction itself. In fact, determining the auction-implied  $\lambda$  is sufficiently difficult that real-world investors routinely rely on and react to external credit risk assessments (Afonso et al, 2012). I thus assume that investors are boundedly rational in the sense that they, by taking credit risk as given, do not recognize the effects of their own actions on the government's decision to default though the auction's outcome. For lack of better information, investors thus replace the fully 'rational' object  $\mathbb{E}[\hat{V}']$  in (1) with the readily available, behavioral object  $\mathcal{E}[\hat{V}']$ ,

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<sup>14</sup>I assume that investors collectively have 'deep pockets' in that net demand  $\sum_{i=1}^N q_i - q_j$  exceeds supply for every  $j \leq N$ .

<sup>15</sup>Recall that the second-price nature of the auction guarantees that  $p^*$  is independent of a bidder's bid if said bidder is allocated a positive quantity. Therefore, bidding  $\mathbb{V}$  is a dominant strategy and, as such, constitutes a best response across all strategy profiles *because the latter are in fact not observed*.

$$\mathcal{E}[\hat{V}'] \equiv (1 - \lambda^P)V' \quad (2.2)$$

by replacing the unavailable, auction-implied  $\lambda$  with the predetermined, readily available, most recent assessment  $\lambda^P$  issued by a credit rating agency. Clearly, this assumption has far reaching implications in that credit ratings effectively *cause* pricing, whereas they are trivially inconsequential in any rational expectations model. In effect, the *perceived valuation* of the bond is given by  $\mathcal{V} = \frac{\lambda^P V'}{1+r}$  and so we can rewrite (1) as,

$$\mathcal{B}(p^*) = \begin{cases} 0 & \text{if } p^* > \mathcal{V} \\ [0, 1] & \text{if } p^* = \mathcal{V} \\ 1 & \text{if } p^* < \mathcal{V} \end{cases} \quad (1')$$

Analogously to (1), since bidding  $\mathcal{V}$  constitutes a dominant strategy and  $p^* < \mathcal{V}$  is always perceived as a possible outcome, equilibrium is given by,

$$\Sigma^* = \left\{ (\mathbf{p}, \mathbf{q}) \in \mathbb{R}^{N \times 2} \mid (\mathbf{p}_i, \mathbf{q}_i) = (\mathcal{V}, 1) \forall i \in \mathbb{N}_{\leq N} \right\}$$

meaning that all investors submit the same bid  $(\mathcal{V}, 1)$ . The important point here is then that in equilibrium, we must have  $p^* = \frac{(1-\lambda^P)V'}{1+r}$ , which in turn — by  $y^P \equiv V'/p^* - 1$  — yields the desired mapping  $g_y$ ,

$$\begin{aligned} y^P &= \frac{1+r}{1-\lambda^P} - 1 \\ &\equiv g_y(\lambda^P, r) \end{aligned}$$

In turn, the auction result also determines the interest rate burden of debt  $e \equiv V' - l$ , or the fraction of the country's output that is spent on 'unproductive' interest expenses. Specifically, we have  $e = dg_y(\lambda^P, r)$ .

#### *Credit risk update and secondary markets*

Following the auction, the credit rating agency observes  $(V', y^P)$  and — exploiting its structural knowledge of the government's objectives and constraints — updates the credit risk assessment to



the auction-implied, true credit risk  $\lambda^{P'} = g_\lambda(V', y^P)$ .<sup>16</sup> In turn, a Walrasian auctioneer proceeds by equilibrating secondary market demand with the now fixed supply via the secondary market price  $p^S = \frac{V'}{1+y^S}$ . Secondary market trading then yields a new equilibrium price via a second update of (1),

$$\mathcal{B}(p^S) = \begin{cases} 0 & \text{if } p^S > \mathcal{V}' \\ [0, 1] & \text{if } p^S = \mathcal{V}' \\ 1 & \text{if } p^S < \mathcal{V}' \end{cases} \quad (1'')$$

where  $\mathcal{V}' = \frac{\lambda^{P'} V'}{1+r}$  and, in contrast to the primary market,  $p^S$  is observed by investors as the Walrasian auctioneer iteratively adjusts the price until we have,

$$\begin{aligned} y^S &= \frac{1+r}{1-\lambda^{P'}} - 1 \\ &= g_y(\lambda^{P'}, r) \end{aligned}$$

Finally, to illuminate the origins of credit risk  $g_\lambda$ , I now turn to discussing the political game which occasionally leads to default.

### *Default*

I model default as resulting from a political game between the incumbent government and the political base at the end of each period. The game's strategy space is given by the Cartesian product  $\Sigma^G \times \Sigma^B = \{\text{default, not default}\} \times \{\text{overthrow, not overthrow}\}$ . To incorporate the notion of default excusability (see Grossman and van Huyck (1988)), I assume that the payoffs of the political base are a function of current, per-GDP interest expenses,

		<b>Government</b>		
<b>Base</b>	$(b_3, g_4)$	$(b_2, g_3)$	Overthrow	
	$(b_4, g_k)$	$(b_1, g_l)$	Not Overthrow	
	Default	Not Default		

Notes: The above game depicts the political interaction between the incumbent government and the political base at the end of each period. In terms of the payoffs,  $i < j$  implies  $t_i > t_j, b_i > b_j$  and  $k, l \in \{1, 2\}$ .

<sup>16</sup>In combination with investors' bounded rationality, the assumption that the rating agency only updates its assessments after the auction is crucial in generating slow moving crises. Conversely, if credit ratings were accurately updated before the auction (by fully accounting for the ensuing bidding behavior), the resulting model would be observationally equivalent to its rational expectations counterpart.

While all payoffs are potentially endogenous, preference orderings are fixed with the exception of the government's assessment when they are not overthrown. Specifically, when the interest burden of debt is below a certain threshold  $\underline{e}$ , the government prefers not to default, whereas when the interest burden of debt is above  $\underline{e}$ , the government prefers to default. Moreover, I further assume that the political base's assessment of default depends on  $e$  such that we have  $b_3 - b_4 = g_e(e)$  with  $g_e > 0$  and  $g'_e < 0$ . Intuitively, this is because a larger fraction of the political base views default as excusable when the per-GDP interest burden of debt is high. For a detailed discussion of the chosen preference rankings, see Section 2.7.

To understand the depicted game's equilibria, we must naturally distinguish between the two cases  $e \leq \underline{e}$ . First, when  $e < \underline{e}$ , not defaulting is a dominant strategy for the government and equilibrium is given by (Not default, Not overthrow), which implies  $\pi = 0$ . Conversely, when  $e > \underline{e}$ , neither the government, nor the political base have a dominant strategy and we have a unique equilibrium in mixed strategies given by,<sup>17</sup>

$$\Pr(\sigma^G = \text{default}) = \frac{b_1 - b_2}{(b_1 - b_2) + (b_3 - b_4)}$$

$$\Pr(\sigma^B = \text{overthrow}) = \frac{t_1 - t_2}{(t_1 - t_2) + (t_3 - t_4)}$$

such that the probability of default  $\pi = \frac{b_1 - b_2}{(b_1 - b_2) + g_e(e)}$  is (at least weakly) increasing in  $e$  because default becomes more politically excusable when  $e$  rises. This mechanism will serve as the sole causal link that translates high sovereign bond yields into high probabilities of default.

In the event of default, I assume that the bond's face value is reduced to a stochastic fraction of GDP  $\xi$  with the resulting haircut being enforced uniformly across all investors.<sup>18</sup>

$$\hat{V}' = \begin{cases} V' & \text{if no default} \\ \xi & \text{in default} \end{cases}$$

such that  $\gamma = \min\{1, \xi/V'\}$ . Specifically, I assume that  $\xi$  is distributed logit-normally in  $(0, 2\bar{\xi})$

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<sup>17</sup>Clearly,  $i < j \Rightarrow t_i > t_j, b_i > b_j$  ensures  $\Pr(\sigma^G = \text{default}), \Pr(\sigma^B = \text{overthrow}) \in (0, 1)$ . See Section 2.7 for a discussion of the mixed-strategy case.

<sup>18</sup>The shock denoting a fraction of GDP and not the face value of the loan implies that credit risk is increasing in the outstanding level of debt. Specifically, holding  $\xi$  fixed, a higher debt level implies a higher loss given default.

with mean  $\bar{\xi}$ .<sup>19</sup> In effect, credit risk thus not only varies with the auction outcome  $(y^P, V')$  through the probability of default  $\pi$ , but also through the loss given default  $(1 - \gamma)$ .

$$\begin{aligned}\lambda &= \pi(1 - \gamma) \\ &= \frac{(1 - \xi/V')}{1 + g_e\left(\frac{y^P V'}{1 + y^P}\right)} \\ &\equiv g_\lambda(y^P, V')\end{aligned}$$

where I have assumed that  $b_1 - b_2$  is invariant to changes in  $(y^P, V')$  and, without loss of generality, set  $b_1 - b_2 = 1$ . Finally, I close the model by imposing a functional form for  $g_e(e)$ ,

$$g_e(e) = \begin{cases} \beta(e - \underline{e})^{-\alpha} & \text{if } e > \underline{e} \\ 0 & \text{otherwise} \end{cases}$$

where  $\alpha > 0, \beta > 0$  ensures that  $\pi < 1$  each period. Before examining my model's dynamical system representation, it should be noted that bounded rationality precludes indeterminacy in my theory. In turn, equilibrium uniqueness immediately implies that self-fulfilling crises cannot occur intra-temporally. Akin to Lorenzoni and Werning (2019), my self-fulfilling crises will thus be “slow moving” in that they unfold gradually, e.g. across periods.<sup>20</sup>

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<sup>19</sup>We have  $\xi = \frac{2\bar{\xi}}{1 + \exp(-\varepsilon\xi)}$ , where the shock  $\varepsilon\xi$  is Gaussian.

<sup>20</sup>While the “tipping point” (an unstable steady state) in their model is given by a certain level of debt, my model's tipping point will be certain level of yield as a function of the macroeconomic state (including the level of debt).

### 2.3. Dynamical System Representation

Unlike most macroeconomic models, the proposed economy herein features a closed-form representation of its implied dynamical system. In particular, the described economy is governed by the following set of equations,

$$y_t^S = \frac{1 + r_t}{1 - \pi_t(1 - \gamma_t)} - 1 \quad (2.3)$$

$$\pi_t = \frac{1}{1 + \max\{0, \beta(d_t y_t^P - \underline{e})^{-\alpha}\}} \quad (2.4)$$

$$\gamma_t = \min \left\{ 1, \frac{\xi_t}{d_t(1 + y_t^P)} \right\} \quad (2.5)$$

$$y_t^P = \left( \frac{1 + r_t}{1 + r_{t-1}} \right) (1 + y_{t-1}^S) - 1 \quad (2.6)$$

where the fact that last period's secondary market yield and risk free rate constitute relevant states is indicative of my investors' boundedly rational behavior. Further notice that our system is entirely parameterized by  $\theta = (\underline{e}, \alpha, \beta, \bar{\xi})$  and that — since the endogenous state is one-dimensional — we can principally further reduce the system's dimensionality by plugging (4)-(6) into (3),

$$y_t^S = \frac{1 + r_t}{1 - \frac{1 - \min \left\{ 1, \frac{\xi_t}{d_t \left( \frac{1+r_t}{1+r_{t-1}} \right) (1+y_{t-1}^S)} \right\}}{1 + \max \left\{ 0, \beta \left[ d_t \left[ \left( \frac{1+r_t}{1+r_{t-1}} \right) (1+y_{t-1}^S) - 1 \right] - \underline{e} \right]^{-\alpha} \right\}}} - 1 \quad (2.3')$$

given the state  $X_t = (y_{t-1}^S, r_t, d_t)$  and the shock  $\xi_t$ .<sup>21</sup> Equation (2.3') is 'simple' in the sense that it is one-dimensional and that it is available in closed-form, but it is visibly nonlinear and so even small changes in initial conditions can potentially cause large swings in asymptotic behavior. To gain a better understanding of the system's sensitivity to initial conditions, I shortly define *financial sustainability* to address the following *policy counterfactual*: how would yields evolve if the government were to indefinitely keep its monetary and fiscal states — the risk free rate and EFN — at a hypothetical, predetermined level? In addition, to assess whether the observed Greek crisis was self-fulfilling, I consider the a series of *empirical counterfactuals*. First, how would Greek yields have evolved in absence shock to the non-fundamental state  $\xi$ ? Second, how would Greek yields have evolved if spreads had not risen during the Great Recession?

<sup>21</sup>Technically,  $X_t$  also includes the lagged value of the risk free interest rate  $r_{t-1}$ .

*Financial sustainability: A policy counterfactual*

From the point of view of policy, the most relevant counterfactual is how we should expect yields to evolve if, going forward, macroeconomic fundamentals remained unchanged. The reason why this counterfactual is of particular interest is that policy makers effectively choose debt levels and interest rates while taking as given the inner workings of financial markets. To the extent that policy making tends to be local (should we reduce debt/lower interest rates?) rather than global (what is the optimal level of debt/interest rates?), policy markers naturally wonder how the government's financing conditions would evolve if it managed to stabilize macroeconomic fundamentals at the current or another predetermined level, e.g. if a particular macroeconomic state is *financially sustainable*.

Before defining financial sustainability, I first construct a new, auxiliary system for which I take as given some initial condition  $(y_0^S, d_0, r_0)$  and repeatedly iterate on (3') while setting  $d_t = d_0$ ,  $r_t = r_0$ , and  $\xi_t = \bar{\xi}$  for each  $t \geq 0$ . We get the following 'reduced' difference equation which fully describes the evolution of our counterfactual, deterministic system,

$$y_t^S = (1 + r_0) \left( \frac{1 + \max\{0, \beta[d_0 y_{t-1}^S - \underline{e}]\}^{-\alpha}}{\min\{1, \bar{\xi}/d_0(1 + y_{t-1}^S)\} + \max\{0, \beta[d_0 y_{t-1}^S - \underline{e}]\}^{-\alpha}} \right) - 1 \quad (2.7)$$

$$\equiv f(y_{t-1}^S | d_0, r_0; \theta)$$

with corresponding interest outlays in the amount of  $e_t = d_0 y_{t-1}^S$  for each  $t > 0$ .<sup>22,23</sup> Just like (3'), (7) is simple in the sense that it is one-dimensional and that it is available in closed-form, but it remains highly nonlinear such that small changes in initial conditions can potentially cause large swings in asymptotic behavior.<sup>24</sup> For example, consider Figure 2.2 which depicts  $f$  taking as given the macroeconomic state  $(d_0, r_0) = (1, 0.01)$  and the parameter vector  $(\alpha, \beta, \underline{e}, \bar{\xi}) = (2, 0.01, 0.05, 1)$ .

The primary lesson from Figure 2.2 is that the nonlinearities in  $f$  can give rise to multiple steady states. Specifically, for the given parameterization and the chosen macroeconomic state, (7) features a stable steady state at  $y = 0.03$  and an unstable steady state at  $y = 0.36$ . Therefore, any

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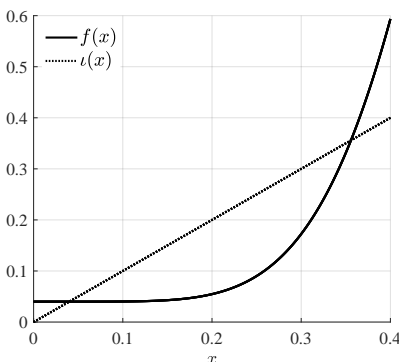
<sup>22</sup>This thought experiment naturally abstracts from default itself and thus should be viewed as describing the counterfactual evolution of yields prior to default only.

<sup>23</sup>Further notice that the numerator in the large fraction of (7) weakly exceeds the denominator and so we must have  $y_t^S \geq r_0 \forall t \geq 0$ .

<sup>24</sup>Of course, even though  $f$  is available in closed form, an explicit formula of  $y^S(t)$  as a function of  $(y_0^S, d_0, r_0)$  or of the difference equation's fixed points  $\mathcal{P}(d_0, r_0)$  cannot generally be found analytically.

initial condition  $y_0 \in [0, 0.36)$  asymptotically maps towards the good steady state  $\lim_{t \rightarrow \infty} y_t \approx 0.03$ , whereas any initial condition  $y_0 \in (0.36, \infty)$  induces an asymptotically diverging yield series. Accordingly, the unstable steady state  $y = 0.36$  constitutes a threshold beyond which yields diverge.<sup>25</sup> Accordingly, the distance between the favorable steady state and the unstable steady state in Figure 2.2 naturally serves as an intuitive measure of the former's resilience to exogenous perturbations.

**Figure 2.2.** A nonlinear transition function (cobweb)



Notes: Figure 2.2 depicts the difference equation (2.7) assuming a fundamental state of  $(d_0, r_0) = (1, 0.01)$  and illustrates two main points. First,  $f(y|d_0, r_0; \theta)$  is highly nonlinear. Second, the apparent nonlinearity of  $f$  gives rise to two separate steady states, namely at roughly  $y = 0.03$  and  $y = 0.36$ , only the former of which is stable. Once yields pass through the unstable threshold at  $y = 0.36$ , they diverge indefinitely (for the given parameterization).

To further illuminate the practical relevance of such fixed points, I now proceed by defining financial sustainability.<sup>26</sup>

**Financial sustainability.** A state  $X_0 = (y_0^S, d_0, r_0)$  is said to be  $\bar{e}$ -financially sustainable if and only if repetitively iterating on (7) given the initial condition  $X_0$  yields  $\lim_{t \rightarrow \infty} e_t \leq \bar{e}$ , where  $\bar{e}$  is a predetermined fraction of national output that a government is willing to permanently spend on ‘unproductive’ interest outlays.

It is evident that the proposed notion of financial sustainability is *conditional* in two ways. First, it conditions on what fraction of output a government deems acceptable to spend on interest outlays in the long run. Second, and more importantly, it also conditions on the premise that external financing needs and interest rates remain constant indefinitely. The practical implications

<sup>25</sup>Importantly, recall that this counterfactual does not describe the actual evolution of yields, which is determined by (3’), but it gives policy makers an idea how yields would evolve *if* they held macroeconomic fundamentals constant. As such, it reveals whether particular macroeconomic fundamentals are *financially sustainable*.

<sup>26</sup>Analogously, one could also define *fiscal sustainability* or *monetary sustainability* by instead focusing on the parallel fiscal and monetary subsystems. For example, abstracting from default, Bohn (1998) famously established that debt levels are strictly stationary if primary deficits fall more than linearly as debt levels rise.

of this second type of conditionality are most apparent when we examine what makes a state financially unsustainable: A state is financially unsustainable if the economic forces endogenous to financial markets will iteratively carry the state towards ‘unacceptable’ regions of the state space  $\mathcal{X}^E(\bar{e}|\theta) \equiv \{X_0 \in \mathbb{R}^3 \mid d_0 y_0 > \bar{e}\}$  unless the government manages to reduce its per-GDP level of debt — either through growth or primary surpluses — and/or the risk free rate falls. To the extent that policy making tends to be local (should we reduce debt/lower interest rates?) rather than global (what is the optimal level of debt/interest rates?), it is precisely this conditional nature of financial sustainability that makes it so practically relevant.

Given the above definition of financial sustainability, I can now partition the state space into the subset of financially sustainable states and financially unsustainable states.

$$\mathcal{X}^S(\bar{e}|\theta) \equiv \{X_0 \in \mathbb{R}^3 \mid \lim_{n \rightarrow \infty} d_0 f^n(y_0|d_0, r_0; \theta) \leq \bar{e}\} \quad (2.8)$$

$$\mathcal{X}^U(\bar{e}|\theta) \equiv \{X_0 \in \mathbb{R}^3 \mid \lim_{n \rightarrow \infty} d_0 f^n(y_0|d_0, r_0; \theta) > \bar{e}\} \quad (2.9)$$

where  $f^n$  is the  $n$ th iterate of  $f$ .<sup>27</sup> The sets captured by (8) and (9) effectively amount to a collection of *basins of attraction*, each mapping to either an ‘acceptable’ steady state, an ‘unacceptable’ steady state, or to no steady state at all (when the series diverges). To identify the set of financially sustainable states, I then solve (7) for a set fixed points, or steady states,  $\mathcal{P}(d_0, r_0)$ . By definition, we have,<sup>28</sup>

$$y^* = f(y^*|d_0, r_0; \theta) \quad (2.10)$$

for any  $y^* \in \mathcal{P}(d_0, r_0)$ . To understand the intuitive content of the fixed points in  $\mathcal{P}(d_0, r_0)$ ,  $d_0$  and  $r_0$  are then best thought of as *parameters* and so we can examine our dynamical system’s behavior by way of a bifurcation diagram.<sup>29</sup> For this, consider Figure 2.3 which depicts two sets of fixed points of  $f$ . In Panel A, I vary the debt level  $d_0$  while holding fixed  $r_0 = 0.03$ , whereas in Panel B, I vary  $r_0$  while holding fixed  $d_0 = 1$ .<sup>30</sup>

The key insight from Figure 2.3 is that higher debt levels and risk free interest rates are mirrored by a lower *tolerance* for yields as measured by the good steady state’s distance from its unstable

<sup>27</sup>Naturally, the financially sustainable region  $\mathcal{S}$  is increasing in  $\bar{e}$ . Further notice that  $d_0 r_0 > \bar{e}$  immediately implies that the initial condition is financially unsustainable irregardless of the value of the yield  $y_0$ .

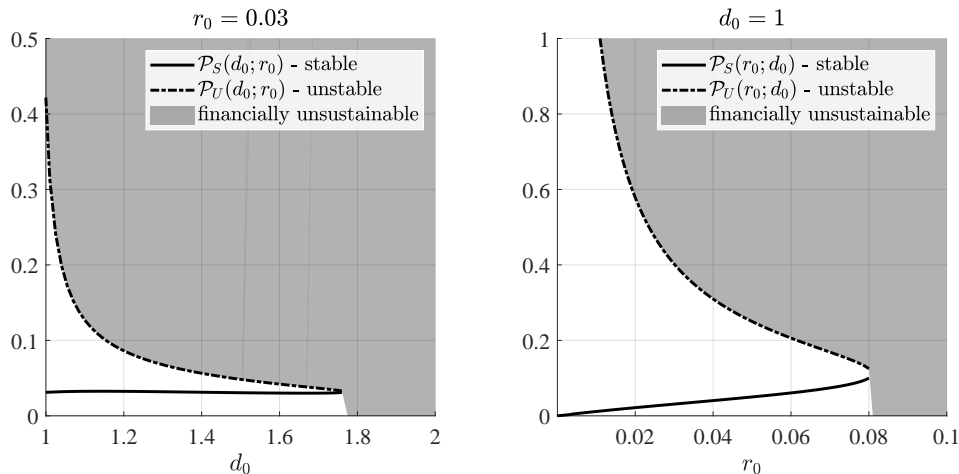
<sup>28</sup>Since the set of points satisfying (10) cannot be found analytically, I resort to using numerical techniques.

<sup>29</sup>Bifurcation diagrams illustrate the qualitative changes, or *bifurcations*, of a dynamical system’s asymptotic behavior with respect to its parameters such as  $d_0$  and  $r_0$  in our case.

<sup>30</sup>The economy continues to be parameterized by  $(\alpha, \beta, \underline{e}, \bar{\xi}) = (2, 0.01, 0.05, 1)$ .

counterpart  $\mathcal{P}_U(d_0, r_0) - \mathcal{P}_S(d_0, r_0)$ , an observation that is strongly supported by economic intuition.<sup>31</sup> Therefore, to the extent that a stable steady state’s *resilience* to stochastic perturbations is precisely given by  $\mathcal{P}_U(d_0, r_0) - \mathcal{P}_S(d_0, r_0)$ , Figure 2.3 impressively illustrates that a government’s financing scheme becomes less resilient as macroeconomic fundamentals deteriorate.<sup>32</sup> In the depicted *bifurcations* — at  $d_0 \approx 1.75$  and  $r_0 \approx 0.08$  in Figure 2.3 respectively — the ‘good’ stable node collides with the unstable threshold and, after briefly forming a saddle, ceases to exist altogether.<sup>33</sup>

**Figure 2.3.** Financially sustainable vs. financially unsustainable states (bifurcation diagram)



Notes: Figure 2.3 depicts the correspondence  $\mathcal{P} : \mathbb{R}^2 \rightrightarrows \mathbb{R}$  which maps the macroeconomic fundamentals  $(d, r)$  into fixed points of  $f$ . All states in the sustainable region  $\mathcal{S}$  (in white) asymptotically converge towards their corresponding stable fixed point, whereas all points in the unsustainable region  $\mathcal{U}$  (in gray) asymptotically diverge. In turn, first notice that over a subset of the state space, no bond yield is sufficiently low to stave off default asymptotically, even if the government were to be able to hold the debt level constant. For a risk free benchmark rate of 2%, the debt threshold beyond which this is the case is 2.15. For a debt level of 1, the risk free interest rate threshold is 6.5%. Secondly, even over the subset of the state space where low yields make for a financially sustainable state, some yields are still too high to avoid default asymptotically. Given the rapidly declining level of the corresponding threshold, the primary insight of Figure 2.3 is that worse macroeconomic fundamentals — higher debt levels and risk free interest rates — imply a lower *tolerance* for yields as measured by the unstable fixed points.

To get a more encompassing view of the government’s tolerance for yields, Figure 2.4 partitions the space of macroeconomic states into three regions: infinite tolerance, finite tolerance, and zero tolerance. In the infinite tolerance region, the initial condition  $y_0$  is irrelevant because we have a unique, stable steady state with low probability of default. In the finite tolerance region, the good steady state is complemented by an unstable steady state, beyond which yields diverge. In this region, the initial condition  $y_0$  is crucial which implies that tolerance is finite. Lastly, in the zero

<sup>31</sup>Abusing notation, we could analogously define *fiscal space* and *monetary space* as  $F(y_0, d_0, r_0) = \mathcal{P}_U^{-1}(y_0 | r_0) - d_0$  and  $M(y_0, d_0, r_0) = \mathcal{P}_U^{-1}(y_0 | d_0) - r_0$ .

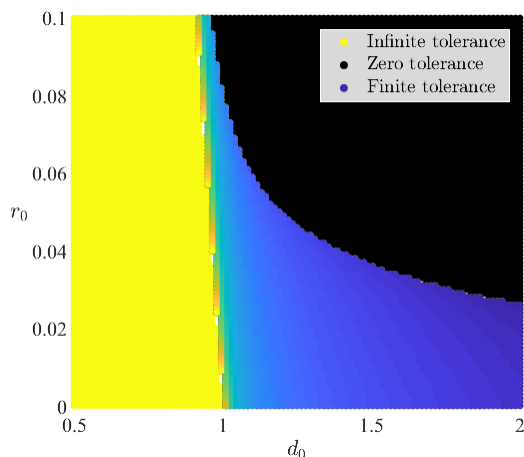
<sup>32</sup>See Holling (1973) for a discussion of different types of ‘resilience’ in the context of dynamical systems.

<sup>33</sup>This occurs as  $f$ , as displayed in Figure 2.2, shifts upward and loses its intersections with  $\iota$ .



tolerance region, initial conditions are once again inconsequential as yields diverge irregardless of  $y_0$ .

**Figure 2.4.** Exogenous state space



Notes: Figure 2.4 partitions the macroeconomic state space into separate regions of tolerance for yield. In the infinite tolerance region, the initial condition  $y_0$  is irrelevant because we have a unique, stable steady state with insignificant levels of credit risk. In the finite tolerance region, the good steady state is complemented by an unstable steady state, or threshold, beyond which yields asymptotically diverge. In this region, tolerance for yield is finite and infinitesimally small changes in initial conditions can dramatically alter asymptotic behavior. Lastly, in the zero tolerance region,  $y_0$  is once again inconsequential because yields will diverge irregardless of initial conditions.

Figure 2.4 illustrates that government’s tolerance for yields can be highly sensitive to small changes in macroeconomic fundamentals, especially along the debt dimension. For example, if the risk free interest rate is high — suppose  $r_0 = 0.08$  — a seemingly minuscule change in external financing needs from just below 100% of GDP to just above 100% of GDP is sufficient to carry the state from the infinite tolerance region through the finite tolerance region into the zero tolerance region. Moreover, notice that the finite tolerance subset of the state space may be thought of as a financially fragile region with a potential for self-fulfilling crises: If current yields are low, they will converge to the favorable steady state, whereas if they are high, they will diverge.

The reason why relatively small changes in initial conditions can lead to large, qualitative changes in the system’s asymptotic behavior is that the proposed data generating process is nonlinear. Now, while our nonlinearities manifest themselves most evident when they give rise to multiple conditional steady states and bifurcations such as the ones depicted in Figures 2-4, the main take-away from the policy counterfactual lies in the more general observation that even infinitesimally small changes in inputs can generate a wide range of asymptotic behavior (see Lorenz, 1963), an insight that will help us understand the observed explosion in Greek yields between 2010 and 2012.

## 2.4. The 2010-2012 Greek Crisis

To estimate  $\theta = (\underline{e}, \alpha, \beta, \bar{\xi})$ , I rewrite (3') in state space form as follows,

$$X_t = F(X_{t-1}, \varepsilon_t | \theta), \quad Y_t = X_t + \eta_t$$

where  $X_t = (y_{t-1}^S, r_t, d_t)$ ,  $\varepsilon_t = (\varepsilon_t^\xi, \varepsilon_t^r, \varepsilon_t^d)$ ,  $Y_t = (\hat{y}_{t-1}^S, \hat{r}_t, \hat{d}_t)$ , and  $\eta_t = (0, 0, \eta_t^d)$ .<sup>34,35</sup> I thus assume that Greek external financing needs are the only observable with measurement error, whereas Greek yields and the risk free rate are observed without such error,  $\hat{y}_t^S = y_t^S, \hat{r}_t = r_t$ . Since time series data is available for EFN and the risk free rate, notice that it is not necessary to specify the distribution of  $(\varepsilon_t^r, \varepsilon_t^d)$  for purposes of estimating our parameters of interest.<sup>36</sup> For now, I thus proceed by assuming that  $\varepsilon_t^\xi$  and  $\eta_t^d$  are uncorrelated Gaussian random variables and use their densities to derive a model-implied likelihood with a particle filter as described in Section 2.7. In turn, I choose a prior for each parameter and report the resulting posterior mode (MAP) and the posterior mean (BPM) in Table 2.1.

**Table 2.1.** Bayesian estimation

$\theta$	Interpretation	$\hat{\theta}_{MAP}$	$\hat{\theta}_{BPM}$	Prior
$\alpha$	Excusability elasticity	0.448	0.441	$\mathcal{U}(0, 5)$
$\beta$	Excusability level	0.870	0.879	$\mathcal{U}(0, 5)$
$\underline{e}$	Excusability threshold	0.015	0.015	$\mathcal{LN}(\log(1/60), 0.15)$
$\bar{\xi}$	Haircut central location	0.829	0.830	$\mathcal{LN}(\log(0.8), 0.1)$

Notes: Table 2.1 reports the Bayesian posterior mode  $\hat{\theta}_{MAP}$  and the Bayesian posterior mean  $\hat{\theta}_{BPM}$  recovered from the model-implied likelihood in conjunction with the depicted priors. The likelihood is constructed using the particle filter described in Section 2.7, which also contains a discussion of my priors as well as a graph that plots prior against posterior probability. In line with contemporary practice,  $\sigma = (\sigma_\xi, \sigma_\eta)$  is set externally. Specifically, I choose  $\sigma = (0.08, 0.03)$  such that a  $3\sigma_\xi$ -shock induces  $\xi_t = \bar{\xi} \pm 0.1$  and a  $3\sigma_\eta$ -shock induces  $\hat{d}_t = \bar{d}_t \pm 0.1$  respectively.

Given the similar values of  $\hat{\theta}_{MAP}$  and  $\hat{\theta}_{BPM}$ , all further analysis is conducted with  $\hat{\theta} \equiv \hat{\theta}_{MAP}$  only.<sup>37</sup> Intuitively, my parameter estimate produces three main objects of interest. First,  $\hat{e} = 0.015$  suggests that spending any amount below 1.5% of GDP on interest expenses is mirrored by zero credit risk because — in the language of Grossman and van Huyck (1988) — a corresponding

<sup>34</sup>See section 2.7 for a discussion of the employed data for  $Y_t$ .

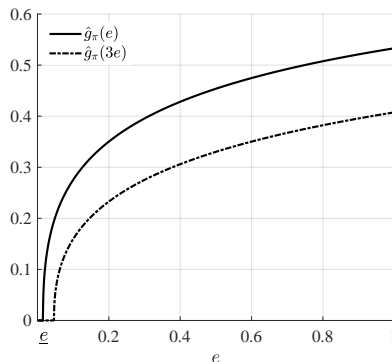
<sup>35</sup> $X_t$  technically also includes the lagged value of the risk free interest rate. Moreover, I did not derive the second and third dimensions of  $F$ , but they may be thought of — without loss of generality — as identity mappings in which case the distributions of  $\varepsilon_t^d$  and  $\varepsilon_t^r$  potentially/likely depend on the current state.

<sup>36</sup>Naturally, such a specification will be required for a counterfactual simulation of the parameterized economy.

<sup>37</sup>All counterfactuals are repeated with  $\hat{\theta}_{BPM}$ , which unsurprisingly yields qualitatively equivalent results.

credit event would be viewed as “unjustifiable repudiation”. Second, since credit risk is also zero if  $\xi_t \geq V_{t+1}$ , the median outcome  $\xi_t = \hat{\xi} = 0.83$  suggests that ‘on average’ external financing needs in the amount of 83% or below also entail no credit risk. Moreover, having set  $\sigma_\xi$  such that a three standard deviation shock induces  $\xi_t = \hat{\xi} \pm 0.1$ , EFNs in the amount of 73% entail zero credit risk at the 99.5% level.<sup>38</sup> The main point here is then that even without knowing the government’s objective, investors can — since  $e_t \leq \underline{e}$  and  $\xi_t \geq V_{t+1}$  each imply zero credit risk — obtain a rough credit risk assessment by examining current interest expenses  $e_t$  and the current per-GDP face value of debt  $V_{t+1}$ . Of course, exploiting our structural knowledge of the government’s decision to default allows for a more sophisticated, quantitative assessment. Specifically, augmenting  $\hat{e}$  with  $\hat{\alpha}$  and  $\hat{\beta}$  yields the model-implied probability of default.

**Figure 2.5.** Probability of default as a function of per-GDP interest outlays



Notes: Figure 2.5 illustrates model-implied, Greek probability of default  $\pi = \hat{g}_\pi(e) = [1 + \hat{\beta}(e - \hat{e})^{-\hat{\alpha}}]^{-1}$  as a function of the interest expense  $e$  determined in the auction. For ease of interpretability, I also plot  $\hat{g}_\pi(3e)$  to capture the probability of default as an approximate function of per-tax-income interest expense given that the Greek tax-to-GDP ratio historically roughly amounts to one third.

Figure 2.5 depicts the model-implied, Greek probability of default as a function of the interest expense determined in the auction. For example, if Greece spent all of its tax income on interest outlays in a given period,  $3e \approx 1$ , the probability that we observe a default *in said period* would roughly amount to 40%. Of course, if such a situation were to persist, say for  $n$  periods, the probability that we would observe default *in any period* would be much higher and equal  $1 - 0.6^n$ .

To illustrate the dynamic implications of my estimate  $\hat{\theta}_{MAP}$ , I now turn to re-examining the observed Greek series in light of the parameterized model. For this, I first assess model fit using a particle smoother, then discuss the previously introduced policy counterfactual with an application to Greece, and finally construct a sequence of three empirical counterfactuals to gauge how the

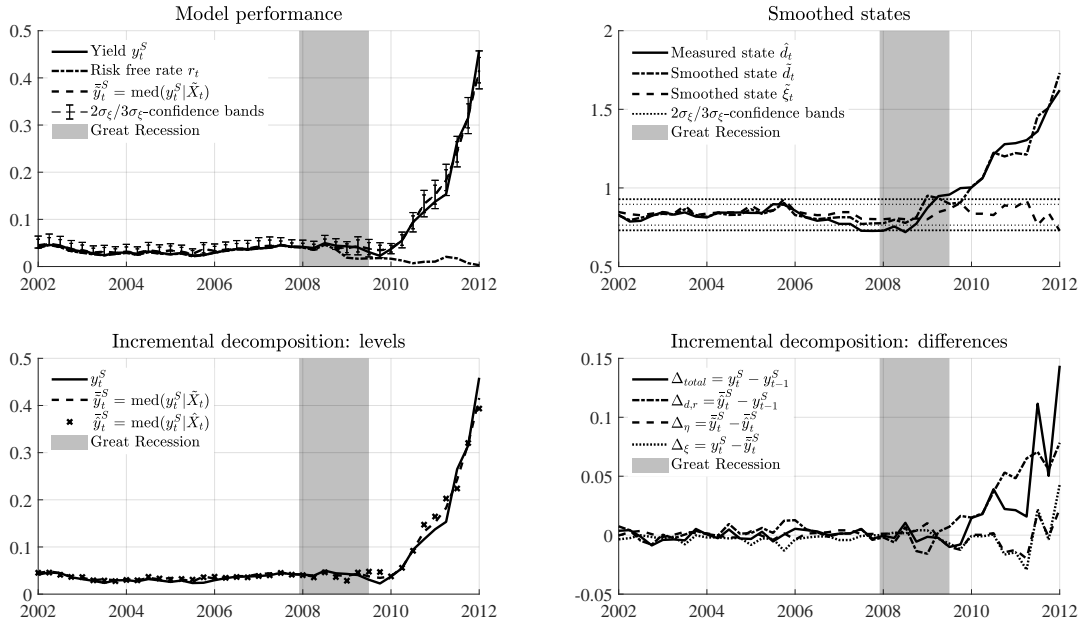
<sup>38</sup>Analogously, EFNs in the amount of 76% entail zero credit risk at the 97.5% level.

Greek economy would have fared under a variety of different counterfactual scenarios.

### Model fit

To assess overall model performance, I recover an estimate of each period’s state with a particle smoother. For this, recall that the ‘filtered’ state implied by the particle filter only accounts for past observations, whereas we would prefer a ‘smoothed’ estimate that exploits all available data.<sup>39</sup>

**Figure 2.6.** Model performance, smoothed states, and incremental factor decomposition



Notes: Panel A plots the realized state  $y_t^S$  against its conditional distribution implied by equation (2.3’), the smoothed state  $\tilde{X}_t = (y_{t-1}^S, r_t, \tilde{d}_t)$ , and the shock  $\varepsilon_t^\xi$ . Panel B displays the two smoothed states  $(\tilde{d}_t, \tilde{\xi}_t)$  recovered with the particle smoother. Given (3’), Panels C and D illustrate the *incremental* contribution of macroeconomic fundamentals, shocks, and measurement errors to the one-step evolution of Greek yields. Overall, the model performs well in that it simultaneously accounts for both the locally stationary phase as well as for the ‘mildly explosive’ phase starting in early 2010 while only requiring little measurement error.

Panel A plots the realized state  $y_t^S$  against its conditional distribution implied by equation (2.3’), the smoothed state  $\tilde{X}_t = (y_{t-1}^S, r_t, \tilde{d}_t)$ , and the shock  $\varepsilon_t^\xi$ . The reason why I choose to highlight the model-implied median (rather than the mean) is that the distribution is partially discrete as there is often a large point mass at  $y_t^S = r_t$  as any  $\xi_t \leq V_{t+1}$  implies a credit risk of zero. Overall, the model performs well in that it simultaneously accounts for both the locally stationary phase as well as for the ‘mildly explosive’ phase starting in early 2010.<sup>40</sup> The primary reason why the

<sup>39</sup>See Godsill, Doucet, and West (2004) for a discussion of the corresponding “forward-filtering, backward smoothing” logic. Using their notation, the particle filter yields  $p(X_t|Y_{1:t})$ , whereas we are interested in  $p(X_t|Y_{1:T})$ .

<sup>40</sup>Recall that Bocola and Dovis (2019) attribute a majority of the locally explosive rise in Italian credit spreads

model does well is twofold. First, the model’s nonlinearities are naturally well-equipped to turn small changes in inputs (macroeconomic fundamentals) into a large swings in outputs (yields).<sup>41</sup> Second, the inertia in expectation formation allows for latencies in the transmission of changes in macroeconomic fundamentals to observed yields. Specifically, while rational expectations may dictate drastic switches from one equilibrium to another, inertia in expectation formation translates indeterminacy into multistability, which turns discrete jumps between equilibria into a more gradual transmission between potentially distant steady states.

Panel B displays the two model-implied smoothed states  $(\tilde{d}_t, \tilde{\xi}_t)$ . For this, notice first that only moderate measurement error between the measured debt level  $\hat{d}_t$  and the smoothed debt level  $\tilde{d}_t$  is required to rationalize the observed explosion in yields. Second, to rationalize the minuscule credit spreads prior to the Great Recession, the smoothed shock series naturally follows  $\tilde{d}_t$ . After the Great Recession, it continues to follow a similar path as it had before and mostly stays within the  $2\sigma_\xi$  bands.

Finally, Panels C and D illustrate the *incremental* contribution of the observed macroeconomic fundamentals, the shock, and measurement error to the evolution of Greek yields. Both Panels tell the same story, namely that macroeconomic fundamentals account for the the majority of the evolution with the shock and measurement errors playing secondary and tertiary roles. However, since the displayed decomposition is only incremental, it does not reveal the *cumulative* effects of a specific factor. Specifically, the reader may wonder how Greek yields would have evolved in absence of shocks to the unobservable  $\xi_t$  altogether. This particular scenario serves as the first of three empirical counterfactuals which I discuss after the policy counterfactual.

I now turn to constructing four different types of counterfactual Greek states  $\{\check{X}_t\}$ . While all variables are computed for each counterfactual, I will primarily focus on counterfactual EFN  $\check{d}_t$  and counterfactual secondary market yields  $\check{y}_t^i$  in my discussion. For ease of exposition, I drop the secondary market yield’s  $S$  superscript and replace it with  $i \in \{a, b, c, d\}$  for each of the four counterfactuals.

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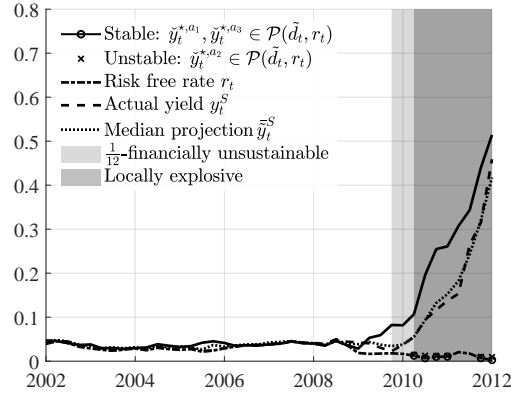
in late 2011 to measurement error.

<sup>41</sup>In this context, also notice that the varying size of the depicted confidence bands is not due to a change in the variance of the shock, but rather to the system’s nonlinearity in conjunction with changing macroeconomic fundamentals.

*Policy counterfactual: When did the Greek state become financially unsustainable?*

In my first counterfactual, I use the parameterized model to assess how Greek yields would have evolved asymptotically if the government, for each point in time, had managed to indefinitely stabilize its macroeconomic fundamentals — EFNs and the risk free interest rate — at their respective contemporaneous levels.<sup>42</sup> For this, I exploit the parameterized version of (7) to recover a *sequence of artificial dynamical systems*, one for each pair  $(\tilde{d}_t, r_t)$ . In turn, each system generates its own counterfactual evolution of future Greek yields  $\tilde{y}_{\tau \geq t}^a$  as well as a corresponding set of asymptotic fixed points  $\tilde{y}_t^{*,a_i} \in \mathcal{P}(\tilde{d}_t, r_t)$ ,  $i \in \{1, 2, 3\}$ . For this, I recover the set  $\mathcal{P}(\tilde{d}_t, r_t)$  as shown in Figure 2.7 by parameterizing and solving (10) with  $(\tilde{d}_t, r_t)$ .

**Figure 2.7.** Counterfactual A: Financial sustainability



Notes: Figure 2.7 plots evolution of Greek yields against the model-implied set of asymptotic fixed points  $\mathcal{P}(\tilde{d}_t, r_t)$  derived from equation (2.7). To recover the set  $\mathcal{P}(\tilde{d}_t, r_t)$ , I parameterize and solve (10) for fixed points. The main point here is to show why the Greek financing scheme became financially unsustainable precisely when spreads had nearly reached zero again, namely because the joint rise in  $\tilde{y}_t^{*,a1}$  and  $\tilde{d}_t$  caused a failure of the financial sustainability condition  $\tilde{d}_t \tilde{y}_t^{*,a1} \leq \bar{e}$ . The reason why  $\tilde{y}_t^{*,a1}$  is the relevant asymptotic fixed point in  $\mathcal{P}(\tilde{d}_t, r_t)$  is that actual yields consistently passed through its basin of attraction:  $\tilde{y}_{t-1}^S > \tilde{y}_t^{*,a2}$  whenever the latter existed. In turn, the fact that the existence of  $\tilde{y}_t^{*,a3}$  (and thus  $\tilde{y}_t^{*,a2}$ ) crucially depends on the risk free rate will prove consequential in the empirical counterfactuals later on.

Figure 2.7 illustrates that until early 2009, the recovered sequence of artificial fix points consists of a seemingly stationary sequence of unique, stable steady states  $\tilde{y}_t^{*,a1}$ . As discussed in section 2, uniqueness implies infinite tolerance in that yields converge to unique steady states irregardless of initial conditions. As macroeconomic fundamentals began deteriorating during the Great Recession, the permanent fixed point  $\tilde{y}_t^{*,a1}$  started rising rapidly and was occasionally complemented by two other steady states — a stable one  $\tilde{y}_t^{*,a3}$  and an unstable one  $\tilde{y}_t^{*,a2}$  — near the risk free

<sup>42</sup>This effectively amounts to a new counterfactual for each quarter.

rate.<sup>43</sup> Whenever multiple steady states did exist, Greek yields continued to diverge because they consistently passed through the basin of attraction of  $\check{y}_t^{*,a3}$  as evidenced by  $y_{t-1}^S > \check{y}_t^{*,a2}$  whenever the latter existed. The fact that  $\check{y}_t^{*,a2}$  briefly vanishes in 2011 will prove consequential later on.

To understand why observed Greek yields decreased while their asymptotic counterpart increased throughout 2009, it should be emphasized again that the fixed points in  $\mathcal{P}(\tilde{d}_t, r_t)$  do not represent an expectation or projection of the present state, which is captured by  $\bar{y}_t^S$ , but rather an asymptotic, central tendency of yields (potentially depending on the initial condition) in a counterfactual world in which the Greek government had managed to indefinitely stabilize its debt level and the risk free benchmark at their respective contemporaneous levels. While asymptotic yields rose in said world throughout 2009, the reason why actual Greek yields fell was twofold. First, during the first half of 2009, credit spreads increased substantially such that the slight fall in yields can only be accounted for by the observed decrease in the risk free interest rate. In turn, after credit spreads reached an inflection point in mid 2009, Greek yields continued to fall even though the risk free rate remained constant because perceived credit risk fell temporarily. In effect, Figure 2.7 thus suggests that Greek yields were bound to start increasing as early as December 2008, but the Greek government first benefitted from a fall in the risk free rate and then from a sequence of fortunate market perception shocks which allowed the impending surge to remain latent temporarily.

To answer the question when the Greek state became financially unsustainable and recover the financial sustainability indicator displayed in Figure 2.7, I first have to choose a maximum level of GDP that a government is willing to permanently spend on interest outlays  $\bar{e}$ . Since  $\bar{e}$  is not identified in the data, I conservatively assume  $\bar{e} = \frac{1}{12}$  because I suspect that no government is willing to permanently spend a quarter of its tax revenues on interest outlays (Greek tax-to-GDP ratio roughly amounts a third).<sup>44</sup> In effect, as shown in Figure 2.7, the Greek financing scheme started violating the financial sustainability condition in September of 2009, or two quarters prior to when Phillips and Yu's (2011) date stamping algorithm rejects a unit root vs. 'mildly explosive' behavior. I thus conclude that the Greek state became financially unsustainable at least half a

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<sup>43</sup>The reason why two steady states do not exist permanently is that changing fundamentals occasionally cause them to collide in a bifurcation as described in section 2. See Section 2.7 for an illustration of the evolution of the set  $\mathcal{P}(\tilde{d}_t, r_t)$  between 2008 and 2012.

<sup>44</sup>Said estimate is conservative in the sense that lower values of  $\bar{e}$  imply earlier crossings of the threshold. In effect, I have thus assumed that spending less than  $3\bar{e} = 0.05$  of tax income on interest outlays is generally accepted, whereas spending more than  $3\bar{e} = 0.25$  of tax income on interest outlays is entirely unacceptable.

year prior to exhibiting certifiably explosive dynamics, which immediately implies that seemingly stationary yield behavior is insufficient to guarantee financial sustainability.<sup>45</sup>

*Cumulative empirical counterfactuals: Was the Greek crisis self-fulfilling?*

To assess whether the Greek debt crisis was self-fulfilling, it is important to generate empirical counterfactuals that are — unlike the ones in Figure 2.6 — *cumulative* in that each period’s incremental counterfactual difference is carried over to the next. In this regard, a tempting, but naive approach would be to assume that the paths of external financing needs and risk free rates are invariant to changes in yields. Specifically, while I will argue that risk free rates evolve independently of Greek yields, the same is almost surely not true for the country’s own external financing needs. In fact, an exogenous increase in yields *ceteris paribus* implies a higher external financing need through the model-implied identity  $d_{t+1} = \frac{d_t(1+y_t^P)}{1+g_t} + x_t$  such that constructing empirical counterfactuals requires a specification of the evolution of primary deficits  $x_t$  and economic growth  $g_t$ .<sup>46</sup> Therefore, unless we are willing to assume that counterfactual primary deficits  $\check{x}_t^i$  and growth  $\check{g}_t^i$  would have precisely offset the effects of  $\check{y}_t^i$  on  $\check{d}_{t+1}^i$ , a rather grotesque assumption,  $\check{d}_{t+1}^i = d_{t+1}$  does not generally hold. For all empirical counterfactuals, I will then assume  $(r_t, g_t, x_t) \perp y_{t-1}^S, d_t$  which yields  $\check{r}_t^i = \hat{r}_t$ ,  $\check{g}_t^i = \hat{g}_t$ , and  $\check{x}_t^i = \hat{x}_t$  for each  $t$  and each  $i \in \{b, c, d\}$ .<sup>47</sup>

Similar to Bocola and DAVIS (2019), a natural approach to assess whether the Greek debt crisis was *non-fundamental* is to examine the cumulative effects of the market perception shock  $\varepsilon_t^\xi$ . For this, I construct a counterfactual evolution of the Greek state by setting  $\xi_t$  to  $\bar{\xi}$  for all  $t$  and plot the resulting series  $(\check{y}_t^b, \check{d}_t^b)$  against their observed/smoothed counterparts  $(y_t^S, \tilde{d}_t)$  in Figure 2.8.

Overall, Figure 2.8 shows that market perception was qualitatively inconsequential in the sense that the observed explosion in yields was going to occur with or without the shocks. Therefore, even though the shock remain important in terms of explaining the behavior of Greek yields quantitatively, Figure 2.8 lends strong support to the view that the Greek crisis was a *fundamental* crisis in that it arose from a deterioration in the macroeconomic fundamentals  $(\tilde{d}_t, r_t)$ . Figure 2.8 further illustrates that when the likelihood of default was low — between 2002 and 2010 — shocks

<sup>45</sup>See Section 2.5 for an assessment of the present Greek state.

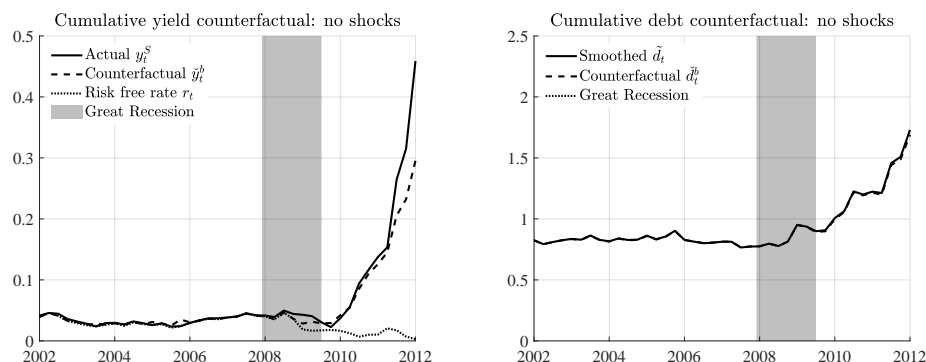
<sup>46</sup>The identity is model-implied because my hypothetical government refinances its entire stock of debt each period.

<sup>47</sup>This assumption may be viewed as unrealistic to the extent that higher interest rate expenses crowd out public spending and thus affect the primary deficit in the spirit of a *fiscal rule*. In turn, falling primary deficits may very well negatively affect growth via the fiscal multiplier.



to market perception have little to no impact on yields while debt is principally driven by economic growth and primary deficits. However, when the probability of default rose, as already seen in Figure 2.6, shocks to market perception can cause relatively large incremental swings in yields, which naturally also translates to a higher sensitivity of debt to such shocks.

**Figure 2.8.** Counterfactual B: Cumulative effect of the market perception shock



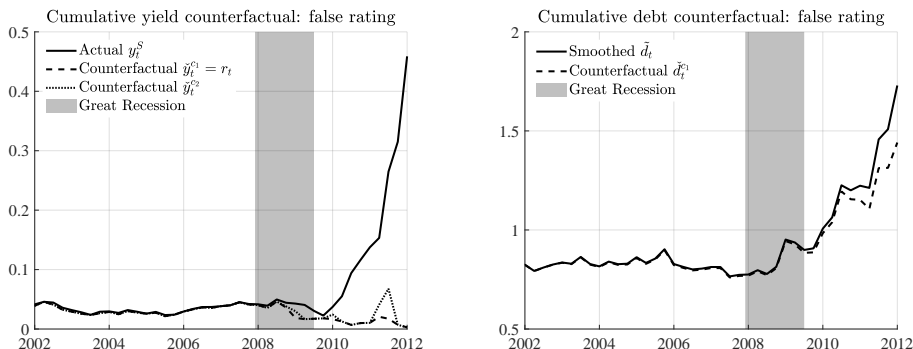
Notes: Figure 2.8 plots the observed Greek state against a counterfactual state in which  $\tilde{\xi}_t^b = \bar{\xi}$  for all  $t$ . While the shock is important in terms of explaining the behavior of Greek yields quantitatively, it is qualitatively inconsequential in the sense that the observed explosion would have occurred with or without the shocks.

To the extent that fundamental and self-fulfilling crisis are traditionally viewed as mutually exclusive, Figure 2.8 may be interpreted as evidence that the Greek crisis was not self-fulfilling because it was driven by a deterioration in macroeconomic fundamentals. However, recall that self-fulfilling crises are traditionally viewed as non-fundamental in the same way that expectations are viewed as non-fundamental. In my theory, however, since expectation formation exhibits inertia, expectations feed back into macroeconomic fundamentals over time and debt crises are slow moving. Specifically, deteriorating credit risk assessments induce higher interest outlays and increasing debt levels not *within*, but *across* periods. Moreover, since perceived credit risk constitutes a pre-determined state in the investors' decision making process, my credit rating agencies — because their ratings effectively *cause* pricing — potentially have the power to alter a country's ultimate fate as I will now illustrate in my third and fourth empirical counterfactuals.

To appreciate the implications of counterfactual credit ratings, recall my assumptions that investors rely on a rating agency to assess credit risk and that the agency updates its assessment to the model-implied credit risk after the auction. However, one could principally also imagine other updating schemes such as model-independent updates, updates prior to the auction, or even

multiple and real-time updates.<sup>48</sup> In this spirit, I now turn to examining the cumulative effects of a series of blanket, zero-credit-risk ratings  $\check{\lambda}_t^c = 0 \forall t$  and plot the resulting counterfactual evolution of the Greek state in Figure 2.9.

**Figure 2.9.** Counterfactual C: Cumulative effects of counterfactual credit ratings



Notes: Figure 2.9 depicts the counterfactual evolution of the Greek state assuming that perceived credit risk had remained at zero throughout the entire interval. Trivially, the corresponding yield counterfactual mirrors the risk free rate while the resulting counterfactual debt series lies below the smoothed series  $\tilde{d}_t$ . The main insight is then that the second counterfactual yield, one that does accurately account for credit risk but takes as given the counterfactual state, *also* closely mirrors — with the exception of mid 2011 — the risk free rate. This constitutes an impressive illustration of the self-fulfilling nature of debt crises as a more favorable credit risk perception in one period can evidently cause lower credit risk in the the following periods.

The results of this second counterfactual are quite striking. They are not striking because the counterfactual yield  $\check{y}_t^{c1}$  mirrors the risk free rate, which happens by construction, but rather because  $\check{y}_t^{c1}$  effectively coincides with  $\check{y}_t^{c2}$ , the latter of which accurately accounts for model-implied credit risk but *takes as given the counterfactual state*  $(\check{y}_{t-1}^{c1}, \check{d}_t^c, \check{r}_t)$ .<sup>49</sup> Therefore, if during and after the Great Recession Greek debt had continued to be treated as it had been before, namely as effectively carrying zero credit risk, it would have — with the exception of 2011 when a rise in the risk free rate briefly caused the good steady state to vanish — in fact continued to carry very little credit risk. Now, one may be tempted to argue that such a counterfactual is uninteresting because why should we care about a hypothetical sequence of ‘inaccurate’ credit ratings? The reason why we should care is that the employed particle filter has *fit the model* — including the measurement errors which rationalize the rise in credit spreads during the Great Recession — under the explicit

<sup>48</sup>Naturally, accurate assessments prior to the auction that fully account for bidding and government behavior would give rise to behavior that is observationally equivalent to the rational expectations case.

<sup>49</sup>Naturally, the big difference between  $y_t^S$  and  $\check{y}_t^{c2}$ , both of which accurately account for credit risk, reflects the fact that the effects of the alternative rating scheme cumulate over time via  $\check{d}_t^c$  and  $\check{y}_{t-1}^{c1}$ . This may thought of as *butterfly effect*, which captures the notion that in nonlinear systems, a minuscule change in initial conditions can lead to an enormous variation in outcomes asymptotically (see Lorenz, 1963).

assumption that the observed yield series accurately reflects the bonds' credit risk. But what if the true parameter  $\theta_0$  implied that the proposed counterfactual reflects credit risk more accurately?<sup>50</sup> Or in other words, *what if the initially observed widening in Greek spreads did not reflect an increase in fundamental credit risk?* If this were so, then Figure 2.9 lends support to the view that markets may have 'unnecessarily' driven Greece into default in which case we have a textbook example of a self-fulfilling crisis.

For the self-fulfilling narrative of the Greek crisis to be credible, one would ideally be able to pinpoint "the match that lit the fuse". Since investors look to credit risk assessments in my theory, the observation that all three major rating agencies downgraded Greek debt in December of 2009 right when spreads had returned to zero for the first time since the outbreak of the 2008 Financial Crisis serves as a natural candidate for said match.<sup>51</sup> In this spirit, one may be tempted to interpret Figure 2.9 as the counterfactual evolution of the Greek state in absence of this downgrade. As we will see in the fourth and final counterfactual, this is not so. In particular, this is not the case because even in the counterfactual event that perceived credit risk had remained at zero throughout the 2008 Financial Crisis, the entire Great Depression, and until the end of 2009, the resulting counterfactual Greek still would have been extremely fragile for two reasons. First, the existence of the favorable, low-credit-risk steady state was heavily dependent on the risk free interest rate remaining near zero. Second, even when the good steady state did exist, minimal perturbations would have been sufficient to carry the Greek state beyond the unstable node into the 'bad' steady state's basin of attraction. Therefore, even if the Greek default in 2012 had resulted from the credit rating downgrades in December of 2009, the complete lack in resilience of the counterfactual state shown in Figure 2.9 would almost surely have led to another credit event shortly thereafter. In this spirit, Figure 2.10 depicts the counterfactual scenario in which perceived credit risk is given by  $\tilde{\lambda}_t^{d_{\bar{t}}} = 0$  for all  $t < \bar{t} + 1$  (until the end of year  $\bar{t}$ ) while allowing the agency to issue model-implied credit risk assessments thereafter.

The main takeaway from Figure 2.10 is that the even the most favorable, counterfactual credit rating schemes generate counterfactual Greek states that would have been extremely fragile. Specif-

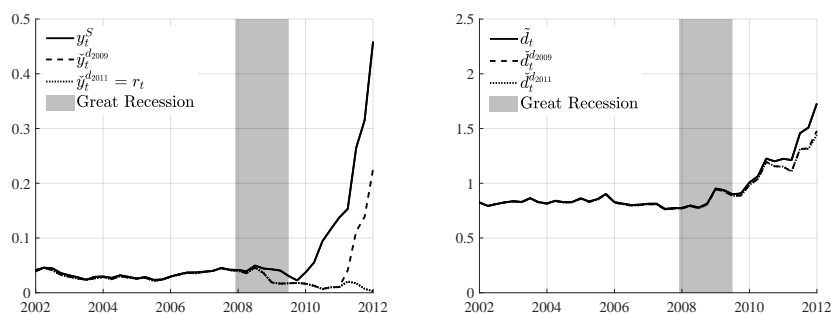
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<sup>50</sup>Notice that minuscule changes in  $\theta$ , especially  $\underline{\theta}$ , can have major implications regarding fundamental credit risk.

<sup>51</sup>Following an extraordinary election in October 2009, the newly elected Greek government released a new budget deficit estimate of 12.7%, which more than doubled the previous government's estimate. Unconvinced with the new government's plans to address the country's structural imbalances, the rating agencies reacted to this new information by lowering their credit ratings.

ically, even if credit risk assessments had remained at zero beyond the 2008 Financial Crisis and the Great Recession, only minor doubts regarding financial sustainability in conjunction with a small increase in the risk free rate would have been sufficient to trigger an asymptotic divergence in yields.

**Figure 2.10.** Counterfactual D: Cumulative effects of counterfactual credit ratings (reconsidered)



Notes: Figure 2.10 depicts the counterfactual evolution of the Greek state assuming two counterfactual credit ratings scenarios. In the first scenario, I force perceived credit risk to remain at zero until the end of 2009 while allowing risk pricing to occur as implied by the fitted model thereafter. The second scenario is equivalent to the previous counterfactual in which perceived credit risk remains at zero until the end of 2011. Notice that in the counterfactual with  $\tilde{\lambda}_t^{d_{2009}}$ , yields continue to mirror the risk free rate throughout all of 2010 during which they are assessed in an actuarially fair manner. This confirms the principal insight from Figure 2.9, namely that a more favorable credit risk assessment in one period causes actually lower credit risk in the following periods. However, the resulting counterfactual Greek state is extremely fragile such that combining actuarially fair risk assessments with the increase in the risk free rate in early 2011 is sufficient to induce an explosion in yields.

In summary, Figures 9 and 10 illustrate that we can only dismiss the proposition that the observed Greek crisis was self-fulfilling if we are willing to unconditionally defend the premise that credit spreads reflect an asset's actuarially fair value at all times.<sup>52</sup> In contrast, if we concede that spreads may have occasionally deviated from actuarially fair values in either direction, it is easy to construe a scenario — even holding fixed the model-implied sequence of shocks  $\tilde{\xi}_t$  — in which perceived *and* actual Greek credit risk would have remained low such that, in all likelihood, no credit event would have taken place in March of 2012. Even if the self-fulfilling narrative of the Greek debt crisis is accepted, however, there are two caveats associated with it. First, even if perceived credit risk had remained low and counterfactual Greek yields had continued to mirror the risk free rate, the Greek state would have been extremely fragile such that an eventual credit event would have been inevitable almost surely *unless*, in addition, counterfactual policy had substantially strengthened fundamentals. Second, self-fulfilling crises can only occur to sovereigns that have maneuvered into

<sup>52</sup>Notice that this follows immediately from equilibrium uniqueness.

the financially fragile subset of the state space in the first place. Nevertheless, to the extent that time may play a crucial role in helping a country escape a fragile state, my framework strongly supports Mario Draghi’s proposition that “breaking expectations” constitutes an effective measure to address a looming sovereign debt crisis.

Before concluding, I now assess whether the present Greek state is financially sustainable.

## 2.5. The Current Greek State

Since the credit event in 2012, Greece has been heavily reliant on the favorable terms provided to it by the Greek Loan Facility, the EFSF, the ESM, and the IMF. In concrete terms, the fraction of officially sourced Greek debt has consistently exceeded 80% since 2012, whereas it had stood at 16% and 26% in 2010 and 2011 respectively. In light of a future return to debt markets, investors naturally wonder whether the country’s current state would be financially sustainable given a predominantly private financing scheme

To assess the current state of affairs, first notice that the current face value of Greek debt — roughly 180% of GDP — is actually higher than it was in early 2012, but at the same time, the weighted average remaining maturity has increased from 6.3 years in 2011 to 20.5 years in 2019. Accordingly, we may naturally expect that our sustainability assessment will depend on the applied discount rate when deriving the present value of all current Greek obligations  $\hat{d}_{2020}$ . For example, if we were to apply current German yields as the discount rate, which are negative at virtually all horizons, the present value of all promised Greek payments would actually exceed their face value. To the extent that such an approach may be viewed as inadequate, I strengthen the robustness of my evaluation by parameterizing the function  $f$  from equation (2.7) with three separate values of  $\hat{d}_{2020}^i$ , each recovered using a different discount rate  $\delta_i \in \{0, 0.01, 0.02\}$  and a maturity of twenty years.

Figure 2.11 contains two main insights. First, the choice of any reasonable discount rate is qualitatively inconsequential as all parameterizations of  $f$  give rise to three steady states.<sup>53</sup> Specifically, in all three cases, there exists a ‘good’ stable steady state that precisely mirrors the risk free rate,

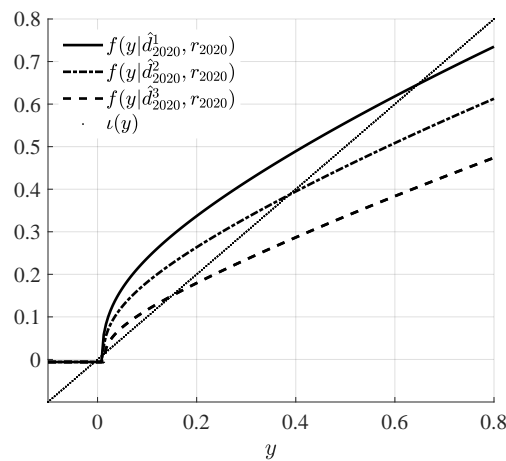
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<sup>53</sup>This is unsurprising insofar as only debt burden present values near  $\hat{\xi}$  or below generate a unique, favorable steady state. To reach such values given the present outstanding face value of 180%, the applicable (risk free) discount rate would need to be at least four percent. Recall that the relevant discount rate is given by the risk free rate because  $d_t$  measures the present value of having to service a given debt portfolio in absence of default.

an unstable steady state nearby, and a ‘bad’ stable steady state with yields in excess of ten percent. Also in all three cases, the ‘tipping point’ separating the two basins of attraction lies at roughly 1.5% such that initial conditions below 1.5% induce convergence towards the risk free rate, whereas initial conditions above 1.5% induce convergence towards the bad steady state. Accordingly, even conditioning on the historically low level of the risk free rate, returning to predominantly market-based financing scheme would create an extremely fragile situation, in which any event that would push yields above the 1.5% would likely map to default asymptotically.<sup>54</sup> The second insight, which lies even more heavily than the first one, is that the existence of the favorable steady state crucially hinges on the risk free rate remaining at historically low levels. If it were to rise only slightly above zero, the depicted transition function would lose two of its steady states through a bifurcation such that yields would converge towards the bad steady state irregardless of their initial condition.

In summary, while Figure 2.11 does allow for the interpretation that the current Greek state is financially sustainable, its fragility remains extremely high such that it would hardly be advisable to promote a quick return to a predominantly private financing scheme.

**Figure 2.11.** The current Greek state



Notes: Figure 2.11 parameterizes the transition function  $f$  from equation (2.7) with current macroeconomic fundamentals  $d_{2020} \in \{1.8, 1.48, 1.21\}$ ,  $r_{2020} = -0.005$  and  $\hat{\theta}$ . For all three values of  $d_{2020}$ , we have — as was often the case between 2009 and 2012 — a system with three steady states, one of which is unstable (at  $y \approx 0.015$ ) and thus serves as a ‘tipping point’.

<sup>54</sup>While Greek 3Ys are trading at 1.3% at the time of this writing, they briefly shot up to 4.3% amid the Coronavirus pandemic in mid March 2020.

## 2.6. Conclusion

As Greek macroeconomic fundamentals deteriorated following the 2008 Financial Crisis and the Great Recession, the country's bond yields entered a dramatic, explosive regime. In this paper, I have argued that the Greek state of sovereign finance became financially unsustainable at least half a year prior to exhibiting these explosive dynamics and so the seemingly stationary yield behavior throughout 2009 did not in fact reflect financial sustainability. The reason why the impending crisis remained latent for six months was that the Greek treasury temporarily benefitted from a decline in the risk free interest rate and from a sequence of positive market perception shocks. I further find that market perception shocks certainly play a role in explaining Greek yields quantitatively, but the Greek government was not driven to default by such shocks as the observed explosive yield behavior is qualitatively accounted for by the country's deteriorating macroeconomic fundamentals.

Since expectations are predetermined in my model, they feed back into fundamentals and debt crises are slow moving. Accordingly, the fact that the country was driven to default by fundamentals does not rule out the possibility that the crisis was self-fulfilling. Specifically, I have shown that if we concede that credit risk assessments may occasionally deviate from actuarially fair values, small perturbations of credit risk assessments during the Great Recession could have been sufficient to prevent a default in 2012. Intuitively, this is because the cumulative effects of a small change in initial conditions can cause large asymptotic swings in nonlinear systems — such as my key equation (2.3') — as was famously originally documented by Lorenz (1963). It is important to stress, however, that even if the Greek default in 2012 had indeed been caused by unnecessarily high credit spreads downgrades during 2009, the corresponding counterfactual — in absence of the downgrades — Greek state's complete lack in resilience to exogenous perturbations would almost surely have led to another credit event shortly thereafter unless counterfactual policy had strengthened fundamentals. This illustrates the broader point that self-fulfilling crises only occur once a sovereign has maneuvered into a financially fragile position in the first place. Nevertheless, granted that real-world investors indeed outsource credit risk assessments, my framework strongly supports Professor Draghi's proposition that “breaking expectations” constitutes an effective measure to delay, mitigate, or even prevent a looming sovereign debt crisis.

Finally, I find that given the current macroeconomic fundamentals, any sovereign yield below

1.5% renders the current Greek state financially sustainable. To this end, my theory permits the interpretation that the current Greek state is in fact financially sustainable, but fragility remains — much like in the counterfactual world of Figures 9 and 10 — extremely high such that it would hardly be advisable to promote a quick return to a predominantly private financing scheme.

## 2.7. Appendix

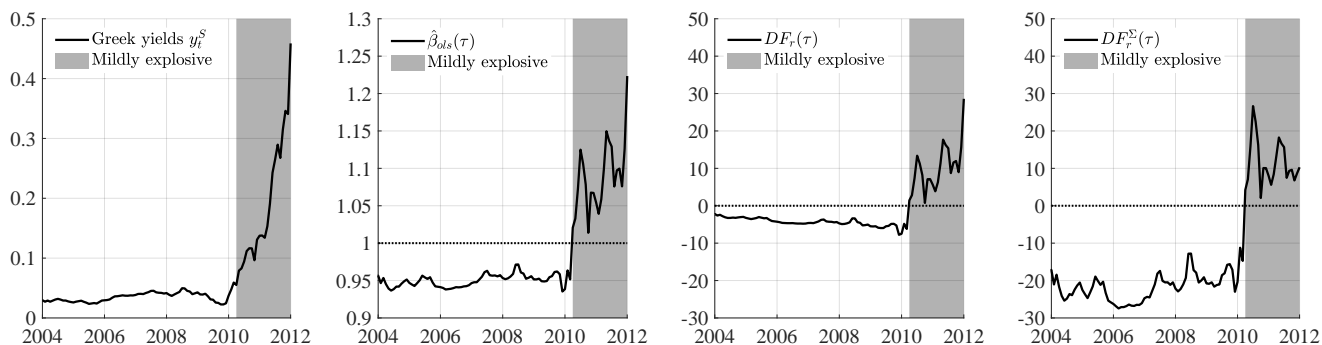
### *Locally explosive behavior in a linear setting*

To identify “mildly explosive” behavior, Phillips and Yu (2011) propose to carry out a set of right-sided unit root tests under the null that a time series is generated by a Gaussian AR(1), a linear model.<sup>55</sup> Specifically, taking the first  $\tau$  elements of a series with a total number of observations  $T$ , Phillips and Yu (2011) construct two sequences of estimators defined as,

$$DF_r(\tau) \equiv \tau[\hat{\beta}_{ols}(\tau) - 1], \quad DF_r^\Sigma(\tau) \equiv \hat{\Sigma}^{\frac{1}{2}}(\tau)[\hat{\beta}_{ols}(\tau) - 1]$$

where  $\hat{\beta}_{ols}(\tau)$  is the ordinary-least-squares slope estimate of the AR(1) and  $\hat{\Sigma}(\tau) \equiv \sum_{i=1}^{\tau} (y_{i-1}^S - \overline{y^S})^2 / \hat{\sigma}^2$ . Here, it is exploited that under  $H_0 : \beta = 1$ , both estimators weakly converge to a well defined distribution as  $\tau \rightarrow \infty$  (Phillips, 1987). Proceeding as such, as illustrated in Figure 2.12, both estimators reject  $H_0 : \beta = 1$  against  $H_1 : \beta > 1$  at a confidence level of 5% starting in March 2010.

**Figure 2.12.** A right-sided unit-root test



Notes: Figure 2.12 depicts the evolution of  $\hat{\beta}_{ols}(\tau)$ ,  $DF_r(\tau)$  and  $DF_r^\Sigma(\tau)$  as  $\tau$  more observation are included in the regression. As can be seen, the proposed procedure rejects  $H_0 : \beta = 1$  against  $H_1 : \beta > 1$  starting in March of 2010. This assessment is unsurprising insofar as the identified, explosive behavior is clearly visible in subfigure a).

Rejecting  $H_0 : \beta = 1$  against  $H_1 : \beta > 1$  need not imply that  $H_1 : \beta > 1$  is particularly sensible

<sup>55</sup>Specifically, we have  $y_{t+1}^S = \mu_y + \beta y_t^S + \varepsilon_{t+1}^y$  with  $\varepsilon_{t+1}^y \stackrel{i.i.d.}{\sim} \mathcal{N}(0, \sigma_y^2)$ .

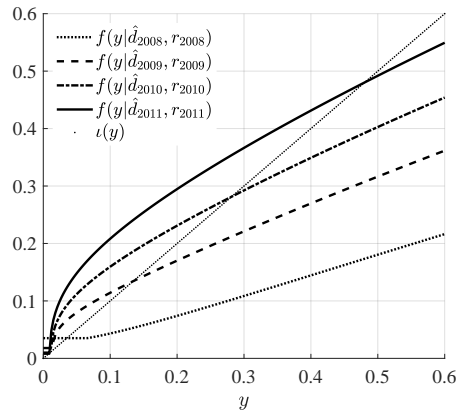


hypothesis. Specifically, recall that we have assumed that our process is AR(1). Examining the original yield series, it is evident that if the data is indeed generated by an AR(1), it is not credible that  $\beta$  exceeded unity over the entire observed interval.<sup>56</sup> Accordingly, the only remaining rationale that can save our *linear premise* is that the observed kink in Greek yields was in fact caused by a perturbation of the parameter  $\beta$ , or a *structural break*.

### *Uncovering the evolution of $\check{y}_t$*

Figure 2.13 explores the origins of the the rapid, explosive rise in the asymptotic fixed point  $\mathcal{P}(\tilde{d}_t, r_t)$  between 2008 and 2012 as shown in Figure 2.7. For this, I parameterize the difference equation (2.7) with the observed, fourth-quarter macroeconomic fundamentals — external financing need and risk free interest rate — from 2008, 2009, 2010, and 2011 respectively.

**Figure 2.13.** Transition function given for a set of observed Greek states



Notes: Figure 2.13 illuminates the origins of  $\check{y}_t$  by plotting the evolution of  $f$  against the identity function  $v$ . The four intersections represent the four fixed points towards which the parameterized system would have converged if Greek debt levels and the risk free interest rate had remained at their respective contemporaneous levels.

The main takeaway from Figure 2.13 is then that the deterioration in macroeconomic fundamentals following the 2008 Financial Crisis caused a continued upward tilt in  $f$ , which in turn gave rise to ever higher, or ‘worse’, asymptotic fixed points.

### *Strategic default*

The payoff table of the default game is motivated as follows. First, the government strictly prefers to remain in power, whereas the political base strictly prefers no default. Conditional on

<sup>56</sup>An analysis of the model-implied distribution of required shocks — virtually all negative between 2004 and 2010 — soundly rejects the zero-mean assumption in a local sense (as evidenced by  $\hat{\beta}_{ols} < 1$  for that time period).

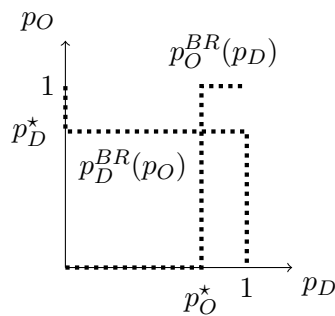
remaining in power, the government prefers to default if and only if  $e > \underline{e}$ . Conditional on being overthrown, the government would prefer no default such that the incoming delegation inherits the present fiscal imbalance, thus making the current administration look more favorably ex post. Conditional on default, the political base prefers punish the present government by overthrow it. Finally, conditional on no default, the political base prefers not to overthrow the government.<sup>57</sup>

The following table depicts the situation, in which we have  $e > \underline{e}$  such that the government prefers to default conditional on not being overthrown.

		<b>Government</b>		
<b>Base</b>	$(b_3, g_4)$	$(b_2, g_3)$	Overthrow	
	$(b_4, g_1)$	$(b_1, g_2)$	Not Overthrow	
		Default	Not Default	

Clearly, since  $b_i > b_j, g_i > g_j$  for each pair  $(i, j)$  with  $i < j$ , the above game does not feature a Nash equilibrium in pure strategies. Instead, letting  $p_D \equiv \Pr(\sigma^G = \text{default})$  and  $p_O = \Pr(\sigma^B = \text{overthrow})$ , unique equilibrium is given by the tuple of mixed-strategies depicted in Figure 2.14,

**Figure 2.14.** Mixed strategy Nash equilibrium



*A particle filter*

I estimate the parameter vector  $\theta$  by approximating the Bayesian posterior distribution  $\Pr(\theta|Y^T)$ ,

$$\theta_{\text{MAP}} \equiv \operatorname{argmax}_{\theta \in \Theta} \Pr(\theta|Y^T)$$

$$\theta_{\text{BPM}} \equiv \int_{\Theta} \theta \Pr(\theta|Y^T) d\theta$$

where  $\Pr(\theta|Y^T) \propto \mathcal{L}(Y^T|\theta) \Pr(\theta)$ ,  $\mathcal{L}(Y^T|\theta)$  is the true likelihood of the data  $Y^T$ , and  $\Pr(\theta)$  is the joint prior. The approximate sample analogues are then given by  $\hat{\Pr}(\theta|Y^T)$  and  $\hat{\mathcal{L}}(Y^T|\theta)$ .

<sup>57</sup>The simultaneous nature of the game is motivated by the idea that both overthrowing a government and orderly defaulting requires some preparation.

To construct the likelihood  $\hat{\mathcal{L}}$ , I use equation (2.3') to forward propagate a swarm of old state particle  $(y_{it-1}^S, r_{it}, \tilde{d}_{it})$  with random draws of the shock  $\varepsilon_t^\xi$  and measurement errors  $\eta_t^d$ . As discussed in the main text, we have,

$$X_t = F(X_{t-1}, \varepsilon_t | \theta), \quad Y_t = X_t + \eta_t$$

where  $X_t = (y_{it-1}^S, r_t, d_t)$ ,  $\varepsilon_t = (\varepsilon_t^\xi, \varepsilon_t^r, \varepsilon_t^d)$ ,  $Y_t = (\hat{y}_{it-1}^S, \hat{r}_t, \hat{d}_t)$ , and  $\eta_t = (0, 0, \eta_t^d)$  denote the unobserved state, the observables, a vector of fundamental shocks, and a vector of measurement errors. Since the first two elements of  $\eta_t$  are zero, I explicitly assume that the risk free interest rate and Greek yields are observed without measurement error.

In contrast to the classical particle filter advertised by Fernández-Villaverde and Rubio-Ramírez (2007), in which particles are generated by simulating shocks and weighting occurs via the joint density of measurement errors, I instead simulate measurement errors and ‘backward engineer’ the model-implied shock. Before discussing why this approach is preferred, I first show that it is valid. For this, notice that we can decompose the likelihood as follows,

$$\begin{aligned} P(Y^T | \theta) &= \Pr(Y_T | Y^{T-1}; \theta) \Pr(Y^{T-1} | \theta) \\ &= \prod_{t=1}^T \Pr(Y_t | Y^{t-1}) \\ &= \prod_{t=1}^T \iint \underbrace{\Pr(Y_t | \varepsilon_t, X_0, Y^{t-1}; \theta)}_{\text{Weigh particle}} \underbrace{\Pr(\varepsilon_t, X_0 | Y^{t-1}; \theta)}_{\text{Simulate particle}} d\varepsilon_t dX_0 \\ &= \prod_{t=1}^T \iint \underbrace{\Pr(Y_t | \eta_t, X_0, Y^{t-1}; \theta)}_{\text{Weigh particle}} \underbrace{\Pr(\eta_t, X_0 | Y^{t-1}; \theta)}_{\text{Simulate particle}} d\eta_t dX_0 \end{aligned}$$

where the last equality illustrates that we can principally simulate our particles by either drawing shocks or measurement errors (or even a mixture of the two) as long as our approach yields a distribution over the observation  $Y_t$  that we can evaluate with the remaining densities. This condition is satisfied in the canonical setup, in which all shocks are simulated and all observations allow for measurement error. In my case, however, the only endogenous state is observed without error. To generate the likelihood, I must thus either simulate a shock  $\tilde{\varepsilon}_{it}^\xi$  and backward engineer a measurement error  $\tilde{\eta}_{it}^d$  that rationalizes the new state  $y_t^S$  or I can alternatively simulate a measurement error  $\tilde{\eta}_{it}^d$  and backward engineer a corresponding shock  $\tilde{\varepsilon}_{it}^\xi$ . The reason why the latter approach is contextually more appropriate is that the model-implied error-to-shock mapping is analytically

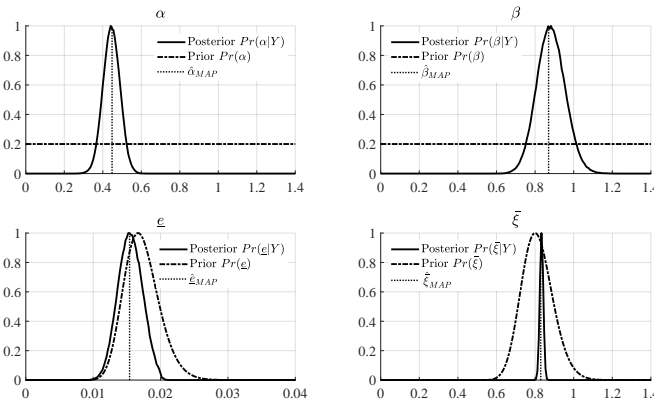
available, whereas the shock-to-error mapping is not. I thus choose to simulate measurement errors and backward engineer the model-implied shock because that way I can exploit the closed-form nature of the system (3') and do not need to resolve for equilibrium given each parameter candidate anew. Concretely, we thus have,

$$\begin{aligned}
\mathcal{L}(Y^T|\theta) &\equiv \Pr(Y^T|\theta) \\
&\simeq \prod_{t=1}^T \frac{1}{N} \sum_{i=1}^N \Pr(Y_t|\eta_{it}, x_{i0}, Y^{t-1}; \theta) \\
&\simeq \prod_{t=1}^T \frac{1}{N} \sum_{i=1}^N f_{\varepsilon}(\varepsilon_{it}|\eta_{it}, x_{i0}, Y^{t-1}; \theta) \\
&= \prod_{t=1}^T \frac{1}{N} \sum_{i=1}^N f_{\varepsilon\xi}(\tilde{\varepsilon}_{it}^{\xi})[1 - \tilde{\pi}_{it}] \equiv \hat{\mathcal{L}}(Y^T|\theta)
\end{aligned}$$

where  $\tilde{\varepsilon}_{it}^{\xi}$  and  $\tilde{\pi}_{it}$  are the shock — weighed by its density  $f_{\varepsilon\xi}$  — and the probability of default implied by the particle  $\tilde{\eta}_{it}^d$ . To ensure smoothness of the likelihood over the parameter space, I recycle the same measurement errors for each evaluation of  $\hat{\mathcal{L}}$ .

To recover  $\hat{\theta}_{MAP}$ , I choose priors  $\Pr(\theta)$  and maximize the resulting approximate posterior distribution  $\hat{\Pr}(\theta|Y^T) \propto \hat{\mathcal{L}}(Y^T|\theta) \Pr(\theta)$  with the `particleswarm` routine native to Matlab. I then iteratively sample one million draws from  $\hat{\Pr}(\theta|Y^T)$  using a Metropolis algorithm (initialized at  $\hat{\theta}_{MAP}$ ) and plot the resulting empirical posterior against its priors in Figure 2.15.

**Figure 2.15.** Estimation: prior vs. posterior probability



Notes: Figure 2.15 plots the posterior density of each parameter against its chosen prior. For  $\alpha$  and  $\beta$ , an uninformative prior is chosen so as to effectively only impose  $\alpha, \beta > 0$ . For  $\underline{\varepsilon}$ , I choose a lognormal with mode  $\frac{1}{60}$ , which corresponds — since Greece historically has a tax-to-GDP ratio of one third — to a target of roughly 5% of tax income. In turn, the standard deviation is set to target an effective lower bound of  $\frac{1}{100}$ , which corresponds to roughly 3% of tax income. Finally, the prior for  $\tilde{\xi}$  is chosen to target a mode of 0.8 with an effective lower bound of 0.6.<sup>58</sup>

<sup>58</sup>Here, the notion of an ‘effective lower bound’ captures the idea that posterior probability is low by construction

Finally, I calculate  $\hat{\beta}_{BPM}$  by taking the sample average along all dimensions of my empirical posterior. The posterior means  $\hat{\beta}_{BPM}$  and  $\hat{\xi}_{BPM}$  weakly exceed  $\hat{\beta}_{MAP}$  and  $\hat{\xi}_{MAP}$  which suggests that those distributions are right-skewed. Conversely,  $\hat{\alpha}_{MAP}$  weakly exceeds  $\hat{\beta}_{BPM}$  such that the posterior of  $\alpha$  is left-skewed. All counterfactuals are conducted with both candidate estimates to confirm that the results are indeed invariant to the choice in posterior mode versus posterior mean, which they are.

### Data

The data sources for all series displayed in Figure 2.1 are shown in Table 2.2

**Table 2.2.** Data sources

	Series	Source
Fig. 1A	Debt/GDP (Face value)	Quarterly, via <a href="#">Eurostat</a>
	Real GDP (2010 chained)	Quarterly, via <a href="#">Eurostat</a>
	Primary deficit/GDP	Annual, via <a href="#">ECB</a>
Fig. 1B	3Y Greek yields	Monthly, via <a href="#">Bank of Greece</a>
	3Y German yields	Quarterly, via <a href="#">Investing.com</a>
	Credit Rating	Aperiodic, via <a href="#">Trading Economics</a>
Other	Weighted av. maturity	Annual, via Greek <a href="#">Public Debt Management Agency</a>

The model is parameterized using quarterly data from 2001:I until December of 2011:IV. Before considering my employed observations —  $\hat{y}_{t-1}^S$ ,  $\hat{r}_t$ , and  $\hat{d}_t$  — in Table 2.3, it is important to note that real-world countries do not periodically refinance their entire stock of debt. This brings about two complications when taking my model to the data.

First, I must select a specific maturity to represent  $y_t^S$ . I choose a remaining maturity of three years (3Y) for the primary reason that it allows me to pick up credit risk fears well in advance while simultaneously also reflecting fears at much shorter horizons as evidenced by the fact that 3Y yields continued to rise when default was already imminent. This is because, as accurately anticipated by markets, holders of virtually all horizons were bailed in as part of the 2012 debt restructuring (as opposed to a sequential skipping of payments whenever they come due).<sup>59</sup>

unless the likelihood were to be extremely informative with a very high local mass.

<sup>59</sup>The main point here is that haircuts may be applied to bonds before they mature. Therefore, a bond with a remaining maturity of three years may still very well be subject to default within a quarter. Intuitively, the proposed framework is thus best interpreted as featuring a government which — rather than issuing additional bonds to finance its contemporaneous spending — refinances its entire portfolio with a new three year loan each quarter.

The second issue that arises from overlapping finance derives from the fact that measured, maturity-unadjusted debt-to-GDP values do not coincide with the modeled values such that recovering a real-world equivalent of the ‘external financing need’  $d_t$  is nontrivial. Specifically, recall that  $d_t$  quantifies the burden of all presently outstanding debt by computing the liquidity that would be needed to settle all outstanding claims now and thus with certainty. Therefore,  $d_t$  does *not* represent the canonical present value of the observed Greek debt — which accounts for default — but rather the present value of all outstanding claims *in absence of default*.<sup>60</sup> In effect, I calculate  $\hat{d}_t$  by discounting the observed per-GDP face value (FV) using weighted average maturities and the corresponding risk free rate. To mitigate concerns regarding the precision of  $\{\hat{d}_t\}$ , I allow for measurement error when constructing the particle filter.

**Table 2.3.** Observables used for estimation

Variable	In Table 2.2
$\hat{y}_t^S$	Secondary market yield
$\hat{r}_t$	Risk free rate
$\hat{d}_t$	External Financing Need

Notes: External financing needs  $\hat{d}_t$  are calculated by discounting the observed per-GDP face value of Greek debt  $\hat{FV}_t$  with the weighted average maturity of all outstanding debt  $\hat{m}_t$  and a correspondingly interpolated German Bund rate  $\hat{r}_t^{\hat{m}}$  (e.g. for a remaining maturity of six years, I linearly interpolate between the German 5Y and 7Y):  $\hat{d}_t = \hat{FV}_t / (1 + \hat{r}_t^{\hat{m}})^{\hat{m}_t}$ .

In constructing my empirical counterfactuals, I require additional data on nominal GDP growth and use it to recover the model-implied primary deficit using the identity  $x_t = d_t - \frac{V_t}{1+g_t}$  and the smoothed state estimate  $(\tilde{d}_t, \tilde{V}_t)$ .

**Table 2.4.** Observables used for counterfactuals

Theory	Data equivalent
$\hat{g}_t$	Nominal GDP growth
$\hat{x}_t$	Primary deficit

Notes: Nominal GDP growth  $\hat{g}_t$  is directly computed from the data while the primary deficit measure  $\hat{x}_t$  is constructed using the model-implied identity  $\hat{x}_t = \tilde{d}_t - \frac{\tilde{V}_t}{1+\hat{g}_t}$ .

<sup>60</sup>To the debtor, the burden of having to settle all outstanding claims in present value terms is invariant to current yields. Of course, current yields do affect the new bond’s face value that is required to refinance.

**Table 2.5.** Chronology of the 2010-2012 Greek sovereign debt crisis

Month	Event
October 2008	BoG: “The Greek economy exhibits serious structural weaknesses and chronic imbalances that have remained unaddressed for a protracted period.”
February 2009	BoG: “Greece must break with [...] a model of overconsumption, sizeable imports, and lasting twin deficits and debts.”
October 2009	Greek authorities announce that the 2009 budget deficit is more than double its projection while the 2008 deficit is also revised significantly.
April 2010	Greece officially requests financial support from the European Union (EU) and the International Monetary Fund (IMF).
May 2010	EU and IMF announce separate financial support programs totaling € 110 billion. The European Central Bank (ECB) continues to accept Greek bonds as collateral despite their below investment grade ratings.
October 2010	In addition to sluggish Greek tax revenues, concerns in Ireland and Portugal cause sovereign spreads to soar across all peripheral states.
June 2011	The Troika concludes that further reforms are inevitable. The country is plagued by public riots and political instability.
July 2011	Eurozone members agree to new ”measures designed to alleviate the Greek debt crisis and ensure the financial stability of the euro area as a whole”. Greek debt is downgraded to extremely speculative by all major rating agencies.
September 2011	The Troika abruptly leaves Athens after talks with the Greek government are unsuccessful. Media paint scenarios of a Greek default and Eurozone exit.
October 2011	Private investors are expected to agree to a haircut at a ”nominal discount” of 50 percent.
November 2011	Although George Papandreou wins a parliamentary confidence vote, the Prime Minister resigns shortly thereafter.
December 2011	The new Greek government releases its budget plan for 2012. While Greece’s fiscal state is predicted to improve, the expected recovery is largely due to the anticipated debt restructuring.
March 2012	After resolving technical and legal issues, the Greek government takes advantage of collective action clauses and successfully restructures its debt.

## CHAPTER III

### The Role of Unstable Equilibria in Equilibrium Selection: Introducing Structural Sunspots into the Diamond-Dybvig Model of Bank Runs

#### 3.1. Introduction

When equilibrium is indeterminate, we often appeal to the notion of stability to select among the set of equilibria that are empirically relevant. In this context, stable equilibria are viewed as empirically relevant because we may expect to observe such equilibria in the data, whereas unstable equilibria are empirically irrelevant because we would not expect to record such outcomes in the data. In this spirit, Brunnermeier and Pedersen (2009) discount the unstable equilibrium in their canonical fire sale model as “uninteresting”, whereas Diamond and Dybvig (1983) fail to acknowledge the existence of such solutions altogether. Extending the very same logic that renders unstable equilibria empirically irrelevant, this paper emphasizes the latter’s economic relevance by highlighting their natural role in *equilibrium selection*.

To select among the two canonical equilibria in their indeterminate bank run economy, Diamond and Dybvig (1983) speculate that households may coordinate their actions based on the observation of a commonly observed random variable. Postlewaite and Vives (1987) expand on this thought by recognizing that the introduction of such a coordination scheme effectively amounts to adding a state — even if just a sunspot — in which case equilibrium is in fact unique.<sup>1</sup> Following this logic, it has become widely accepted to render indeterminate equilibrium unique by appealing to sunspots that randomly select among the set of viable equilibria.<sup>2</sup> For example, in their canonical treatment of self-fulfilling debt crises, Cole and Kehoe (2000) introduce a sunspot that selects — conditional on the state lying in their ‘crisis zone’ — the default equilibrium with a time-invariant probability  $\pi$ . Time-invariance has recently been relaxed by Bocola and Dovis (2019), whose sovereign debt model explicitly allows the threshold  $\pi_t$  to exogenously vary with time. In turn, the fact that  $\pi_t$  is left to vary exogenously reflects the primary motivating observation of this paper, namely

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<sup>1</sup>Unlike in Diamond and Dybvig (1983), bank runs in Postlewaite and Vives (1987) are structural. Specifically, occasional bank runs occur because in certain states, one or both agents find it optimal to run *irrespective of the other’s action*.

<sup>2</sup>Here, viable is used synonymously with stable because only stable equilibria are deemed empirically relevant.



that contemporary equilibrium selection via sunspots has largely remained non-structural in that it omits information that is implied by the model’s own structure.

To understand the non-structural nature of contemporary sunspot equilibrium selection, recall that the term ‘sunspot’ was originally coined by Cass and Shell (1983) to capture uncertainty that is extrinsic in the sense that it does not affect an economy’s fundamentals. It is shown that even holding fundamentals fixed, sunspots can materially affect a game’s observed outcomes, namely when they select an outcome among multiple “certainty equilibria”.<sup>3</sup> As evidenced by its name, the concept of a sunspot remains vague in Cass and Shell (1983) so as to preserve generality. In contrast, the proposition that a subset of patient households react to negative news by queueing in front of the Diamond-Dybvig bank illustrates that sunspots are not vague by construction. I thus define a *structural sunspot* as an extrinsic variable which allows for a natural interpretation relative to the structure implied by a particular theory. In turn, I apply the notion of structural sunspots to three existing frameworks to assess their canonical equilibria’s resilience to such shocks.

Tailoring sunspots to a particular theory is beneficial because it forces the modeler to disclose two things. First, by definition, it forces the modeler to disclose the nature of the extrinsic forces that are presumed to be driving outcomes when equilibrium is indeterminate. Second, and more importantly for the purposes of this paper, it requires a disclosure of how strategic interactions are presumed to iteratively carry the system from an initial condition to the ultimately observed equilibrium outcome.<sup>4</sup> In this spirit, the idea at the core of this paper is that we can — rather than taking it as given — derive a model-implied distribution over observed outcomes by endowing a sunspot with a structural interpretation. In turn, the resulting distribution reflects a more transparent and internally consistent approach to conducting equilibrium selection.

To understand the intuition underlying my proposed equilibrium selection mechanism, first recall that we typically discard unstable equilibria as empirically irrelevant because we would not expect to observe such equilibria in the data. In turn, the reason why we would not expect to observe unstable equilibria in the data is that even infinitesimally small perturbations are sufficient

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<sup>3</sup>Equilibrium selection via sunspots differs from the ‘global game’ approach proposed by Carlsson and van Damme (1993) and famously implemented by Morris and Shin (1998, 2004). In the latter, uniqueness derives from uncertainty that is intrinsic because the indeterminate economy’s fundamentals are in fact perturbed. It further also differs from the ‘animal spirits’ described by Farmer (2011), where equilibrium is uncountably infinite because indeterminacy does not arise from nonlinearity or discontinuity, but rather from a missing equation in an underdetermined system. In such a setting, beliefs can interestingly be both primitive and rational.

<sup>4</sup>As we shall see, it is the latter requirement that will lend unstable equilibria their economic relevance.

to carry systems away from such equilibria. While such an approach to empirical relevance is perfectly reasonable, it begs the question why the same logic is not extended analogously to stable equilibria as well. Specifically, equilibria that are more resilient to exogenous perturbations are more likely to be observed in the data than equilibria that are less resilient to the same perturbations. I thus argue that it will be useful to characterize equilibria by the degree to which they are capable of absorbing exogenous perturbations while still emerging as a game's ultimately observed outcome, or their *resilience*.<sup>5</sup>

“There may be equilibrium which, though stable, is so delicately poised that, after departure from it beyond certain limits, instability ensues, just as [...] a stick may bend under strain [...] *until a certain point* is reached, when it breaks.”

*Irving Fisher (1933)*

As illustrated by the above quote, the primary factor which determines a stable equilibrium's resilience to exogenous perturbations is its location vis-à-vis the set of its neighboring 'tipping points'.<sup>6</sup> The primary insight underlying my proposed, structural equilibrium selection mechanism is that said tipping points coincide with the stable equilibrium's unstable neighbors, if they exist. This proposition is most easily illustrated via an analogy to the tipping point in a well-understood physical (dynamical) system. For this, consider a soccer player who randomly kicks a ball from a predetermined initial location in a two-dimensional space featuring two valleys separated by a hill. In this analogy, the floor of each valley represents a stable steady state, the top of the hill represents the tipping point, gravity represents the economic forces endogenous to the system, and the kick represents an exogenous perturbation. To assess the likelihood that the asymptotic location of the ball coincides with the floor of the kicker's own valley, we have to assess the likelihood that the ball's landing spot following the kick does not lie beyond the top of the hill.<sup>7</sup> In the special case that the kicker's location coincides with one of the three steady states, the likelihood that the ball ultimately returns to its origin corresponds to the latter's resilience. The key point here is then that the only information required to assess the origin's resilience is given by its distance from the top of the hill and the force of the kick. Therefore, the top of the hill serves a crucial role because

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<sup>5</sup>Globally stable equilibria are maximally resilient, whereas unstable equilibria are minimally resilient.

<sup>6</sup>If the latter are close, resilience is low. If they are far away, resilience is high.

<sup>7</sup>It is implicitly assumed here that all of the ball's kinetic energy is immediately absorbed upon its first impact such that the initial landing spot effectively determines its asymptotic location.

— in the face of gravity — knowing its relative location to the landing spot of the ball is sufficient to deduce to which floor the ball will ultimately converge.

The soccer ball analogy illustrates that to inform equilibrium selection using structural sunspots, we principally require three ingredients. First, we must know which initial conditions (the two valleys) endogenously map to which ultimately observed outcomes (the two floors). Second, we must specify the origin that is to be perturbed by the sunspot (kicker’s location). Finally, we require the sunspot itself (kick), which randomly transforms the origin into a perturbed initial condition (landing spot). In most indeterminate economies, pinning down the origin is not difficult. In absence of recent bank runs in the Diamond-Dybvig economy, for example, it is natural to think of the predetermined initial condition as being given by the ‘good’ no-run equilibrium. In turn, as suggested by the authors themselves, the sunspot may be best interpreted as a negative news shock that causes a random, initial fraction of fundamentally patient households to queue in front of the bank. It is only when the length of this initial queue crosses a certain tipping point that all patient households find it in their best interest to run.<sup>8</sup> Finally notice that at the tipping point itself, all patient households are precisely indifferent between running and not running and so the corresponding strategy must constitute an equilibrium in its own right. As per usual, the fact that the tipping point equilibrium is unstable renders it empirically irrelevant because we would never expect to observe it as an outcome. However, to the extent that it helps us determine the preferred, no-run equilibrium’s resilience to the sunspot and we care about the probability of a bank run, the tipping point is of utmost economic importance.

The remaining paper is organized as follows. Section 2 reconstructs the original Diamond-Dybvig economy and derives the set of all equilibria including the unstable ones. To obtain uniqueness, Section 3 augments the original economy with a structural sunspot that randomly perturbs the households’ strategy profile. The primary result is then that because the set of unstable equilibria effectively partitions the strategy space into disjoint basins of attraction, much like the top of a hill partitions a physical space into disjoint valleys, its properties yield an internally consistent, sunspot-implied probability of a bank run. Section 4 examines a repeated version of the Diamond-Dybvig game, in which the unstable equilibria’s location yields an analogous, sunspot-implied crisis

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<sup>8</sup>In all likelihood, long queues in front of the bank will be picked up by the local news such that everyone can update their priors based on the incoming information.

frequency. Section 5 collects the gained insights by outlining more generally the conditions under which an equilibrium may be described economically relevant. Section 6 briefly touches on the main implications for the fire sale models by Brunnermeier and Pedersen (2009) and Morris and Shin (2004).<sup>9</sup> Section 7 concludes.

### 3.2. The Diamond-Dybvig Model of Bank Runs

Diamond and Dybvig’s original setup features three time periods.

**Table 3.1.** The Diamond-Dybvig game

t	Game
0	Households receive and allocate/deposit resources
1	Patient and impatient types are realized, each household decides what fraction of deposits to withdraw
2	All remaining deposits are withdrawn

Notes: The Diamond-Dybvig economy unfolds over the course of three time periods  $t \in \{0, 1, 2\}$ . This paper only focuses on the indeterminate nature of equilibrium in the canonical subgame at  $t = 1$  assuming an ex ante optimal contract was negotiated at  $t = 0$ .

In period 0, since households don’t know their own type  $\theta_i \in \{\text{patient, impatient}\}$ , they insure against the risk of being impatient by entering into a demand deposit contract with a bank. In period 1, each household learns their type  $\theta_i$  and decides what fraction of deposits to withdraw given the terms of the contract. Famously, the latter subgame features multiple equilibria, one with only impatient types withdrawing everything and one with everyone attempting to withdraw everything, the *bank run*. In the final period, all remaining claims are settled.

The present paper focuses exclusively on the game played by households once they have observed their type at the beginning of  $t = 1$ . I thus assume that households have already entered into the ex ante optimal banking contract in period 0, which is discussed in Appendix A.

To illustrate the economic significance of the game’s unstable equilibria, I start by first collecting the set of all equilibria  $\mathcal{E}$ . I then proceed by augmenting the original game with a structural sunspot, imposing a specific process of intratemporal best-response dynamics, and deriving the sunspot-implied probability of a bank run. The main result is that the unstable equilibria are

<sup>9</sup>The benchmark model by Morris and Shin (2004) famously features equilibrium uniqueness in *threshold strategies*. In contrast, I will examine a simplified version of their economy that is indeterminate, but features no unstable equilibrium.

highly economically relevant because they serve as a tipping point that physically separates the two canonical equilibria and thus effectively pin down the likelihood of a run. If the game is repeated, the unstable equilibria further pin down the ergodic crisis probability, which captures how frequently bank runs occur over time.

### *Deriving the game $\Gamma$*

To collect the set of all equilibria, we must first define the game  $\Gamma = (I, \mathcal{W}, u)$  played by households at  $t = 1$ . First, the set of players  $I$  is given by a unit measure of households, each indexed by their respective individual location  $i \in I \equiv [0, 1]$ . After learning their type, each household  $i$  must decide what fraction  $w_i \in W \equiv [0, 1]$  of their deposits to withdraw early, meaning at  $t = 1$ . The strategy space  $\mathcal{W}$  is thus given by the following infinite Cartesian product,<sup>10</sup>

$$\begin{aligned} \mathcal{W} &\equiv \prod_{i \in I} W \\ &= \left\{ w : I \mapsto W \mid w(i) \in W \ \forall i \in I \right\} \end{aligned}$$

To complete the game, utility  $u$  is assumed to be given by,

$$u(c_{i1}, c_{i2} | \theta_i) = \begin{cases} v(c_{i1}) & \text{if } \theta_i = \text{impatient} \\ \rho v(c_{i2}) & \text{if } \theta_i = \text{patient} \end{cases}$$

where  $c_{i1} = w_i V_1$ ,  $c_{i2} = w_i V_1 + (1 - w_i) V_2$ , and  $(V_1, V_2)$  are realized bank deposit payoffs at  $t = 1, 2$ .<sup>11</sup>

### *Best responses*

Without loss of generality, assume that, following the realization of  $\Theta$ , all impatient types are contained in  $\underline{I} \equiv [0, \tau]$ . Then,  $w(i) = 1$  is a dominant strategy for each  $i \in \underline{I}$  irregardless of what strategies other households choose. Conversely, since  $V_1$  and  $V_2$  are perfect substitutes from the point of view of the patient types, the inequality  $V_1 \leq V_2$ , with  $V_1 = V_2$  as a special case, pins down the latter's best-response correspondence  $B$ . Specifically, we have,

<sup>10</sup>Like Diamond and Dybvig, I abstract from mixed strategies.

<sup>11</sup>Specifically,  $V_1(f_i, r_1) = r_1 \mathbb{1}[f_i < r_1^{-1}]$  and  $V_2(f, r_1) = \max\{R(1 - r_1 f)/(1 - f), 0\}$  as we shall see shortly.

$$B(w_{-i}^0) = \begin{cases} 0 & \text{(not run)} & \text{if } V_1 < V_2 \\ 1 & \text{(run)} & \text{if } V_1 > V_2 \\ W & \text{(indifferent)} & \text{if } V_1 = V_2 \end{cases}$$

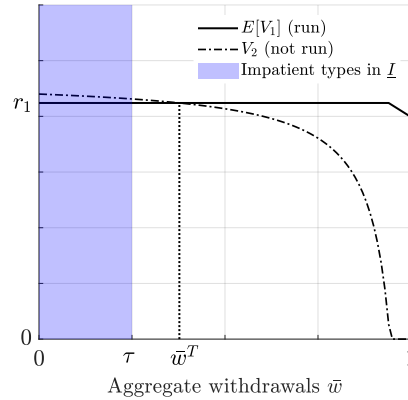
I follow Diamond and Dybvig in assuming that aggregate withdrawals  $\bar{w} \equiv \int_I w(i)di$  map into payoffs as follows,

$$V_1(\bar{w}_i, r_1) = r_1 \mathbb{1}[\bar{w}_i < r_1^{-1}] \quad (3.1)$$

$$V_2(\bar{w}, r_1) = \max\{R(1 - r_1\bar{w})/(1 - \bar{w}), 0\} \quad (3.2)$$

where  $\bar{w}_i \in [0, \bar{w}]$  is a randomly assigned place in line, and the returns  $(R, r_1)$  are taken as given.

**Figure 3.1.** The run and no-run payoffs as a function of aggregate withdrawals



Notes: Figure 3.1 depicts the certain payoff  $V_2$  against the expected payoff  $V_1$  as a function of  $\bar{w}$ . Importantly, notice that as long as  $V_2$  is positive,  $V_1$  is certain. Therefore, if  $V_1$  is uncertain, it must be preferred to  $V_2$  because  $V_2 = 0$  with certainty.

Importantly, patient households are only ever indifferent if the two payoffs are equivalent with certainty, which requires  $V_2 = V_1 = r_1$ . Therefore, indifference occurs if and only if,

$$V_2(\bar{w}, r_1) = r_1 \quad (3.3)$$

We can then solve (2) and (3) to recover the tipping point that renders all patient types indifferent between running and not running,<sup>12</sup>

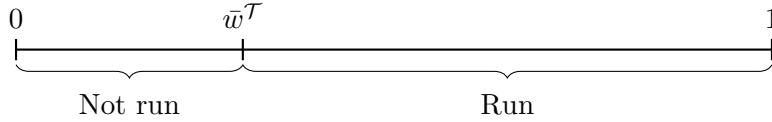
$$\bar{w}^T \equiv \frac{R - r_1}{r_1(R - 1)} \quad (3.4)$$

<sup>12</sup>Notice that  $\bar{w}^T$  is decreasing in  $r_1$  and converges to 0 as  $r_1 \rightarrow R$  and 1 as  $r_1 \rightarrow 1$ .

and rewrite  $B$  as follows,

$$B(w_{-i}^0) = \begin{cases} 0 & \text{(not run)} & \text{if } \bar{w}^0 < \bar{w}^T \\ 1 & \text{(run)} & \text{if } \bar{w}^0 > \bar{w}^T \\ W & \text{(indifferent)} & \text{if } \bar{w}^0 = \bar{w}^T \end{cases}$$

Each patient household thus strictly prefers to wait if  $\bar{w}^0 \in [0, \bar{w}^T)$  and to run if  $\bar{w}^0 \in (\bar{w}^T, 1]$ .



### Canonical equilibria

The two *canonical* equilibria of the described game are given by the ‘no-run profile’  $w^{NR}(i) = 0$  for each  $i \in \bar{I} \equiv (\tau, 1]$  and the ‘run profile’  $w^R(i) = 1$  for each  $i \in \bar{I}$ , but these equilibria only coexist if the model is parameterized ‘properly’.

**Proper parameterization (coexistence).** A parameter vector  $\theta_1 \equiv (\tau, \rho, R, r_1)$  is said to be *proper* if and only if — abusing notation —  $\tau < \bar{w}^T(\theta_1)$ .

In turn, the resulting set of canonical strategy profiles are,

$$\mathcal{E}^{NR} \equiv \left\{ w \in \mathcal{W} \mid w_i = \mathbb{1}[\theta_i = \text{impatient}] \right\}$$

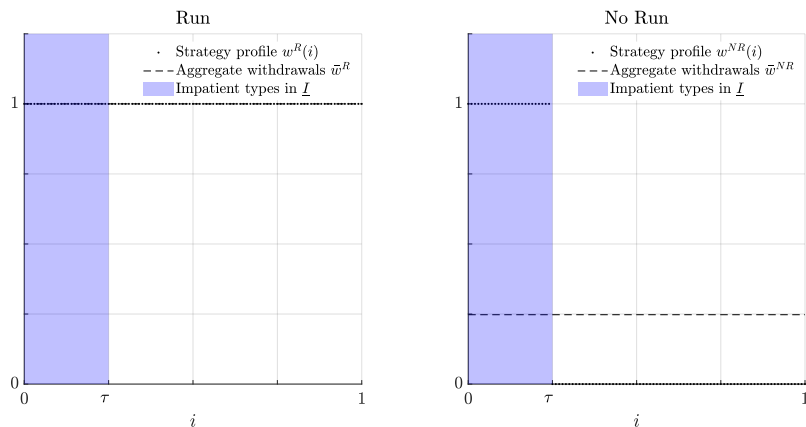
$$\mathcal{E}^R \equiv \left\{ w \in \mathcal{W} \mid w_i = \mathbb{1}[\theta_i \in \{\text{impatient}, \text{patient}\}] \right\}$$

Of course, if the location of the impatient types were not fixed, the cardinality of these sets would be infinite. However, since I have fixed  $\underline{I}$ , both sets are singletons:  $\mathcal{E}^{NR} = w^{NR}$ ,  $\mathcal{E}^R = w^R$ . Notes: Figure 3.2 illustrates the two canonical equilibria — bank run and no bank run — of the Diamond-Dybvig game holding fixed the location of all impatient households in  $\underline{I} \equiv [0, \tau]$ . In Panel A, each patient household chooses to withdraw all of their deposits,  $w_i = 1$  for each  $i \in \bar{I}$ , whereas in Panel B, each patient household chooses to withdraw zero deposits,  $w_i = 0$  for each  $i \in \bar{I} \equiv (t, \tau]$ .  $\bar{w}^j \equiv \int_I w^j(i) di$  denotes aggregate withdrawal amounts for  $j \in \{NR, R\}$ . In both equilibria  $w^{NR}$  and  $w^R$ , the patient households strictly prefer their chosen strategy  $w_i$  over any available alternative  $w'_i \in W$ .

### Tipping points

In addition to the canonical equilibrium strategy profiles  $w^{NR}$  and  $w^R$ , there further exists a third type of equilibrium, namely a *tipping point* at which each patient household is indifferent

**Figure 3.2.** The two canonical equilibria, run and no run, of the Diamond-Dybvig game



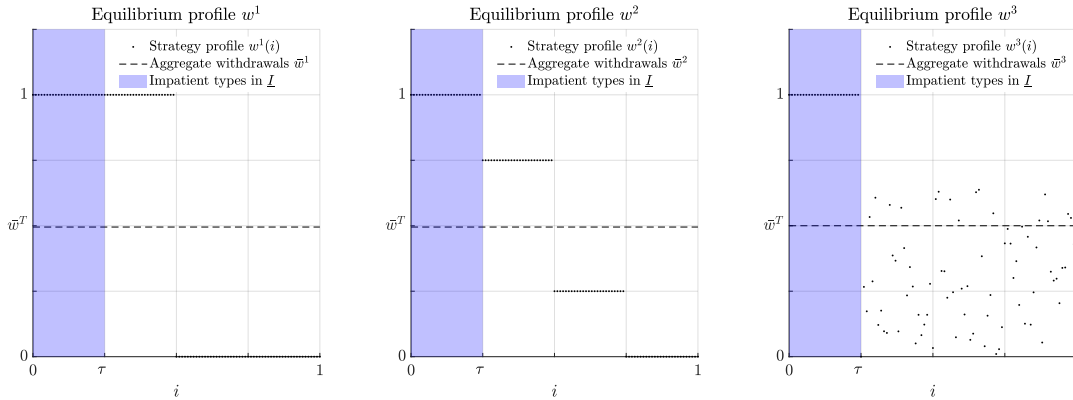
between withdrawing everything  $w_i = 1$ , withdrawing nothing  $w_i = 0$ , and any convex combination thereof  $w_i \in (0, 1)$ . As discussed above, since  $V_1$  and  $V_2$  are perfect substitutes from the point of view of the patient types, a necessary and sufficient condition for patient households to be indifferent is  $V_1 = V_2$ , which holds true if and only if  $\bar{w} = \bar{w}^{\mathcal{T}}$ . The corresponding third equilibrium set is thus given by,

$$\mathcal{E}^{\mathcal{T}} \equiv \left\{ w \in \mathcal{W} \mid w(i) = 1 \forall i \in [0, \tau], \bar{w} = \bar{w}^{\mathcal{T}} \right\}$$

where proper parameterization ensures that  $\mathcal{E}^{\mathcal{T}}$  is non-empty. Moreover, since  $\mathcal{W}$  features all functions  $g : [0, 1] \mapsto [0, 1]$ , it contains an infinite number of functions that satisfy  $\int_I w(i) di = z$  for any  $z \in [0, 1]$ . Therefore, as long as  $\mathcal{E}^{\mathcal{T}}$  is non-empty, its cardinality must be infinite. Figure 3.3 depicts three examples of such tipping points for an economy with  $0.25 = \tau < \bar{w}^{\mathcal{T}}(\theta_1) = 0.5$ .



**Figure 3.3.** Three unstable equilibria



Notes: Figure 3.3 illustrates three sample equilibria contained in  $\mathcal{E}^{\mathcal{T}}$  assuming  $0.25 = \tau < \bar{w}^{\mathcal{T}}(\theta_1) = 0.5$ . The point here is to show that many different types of strategy profiles, infinitely many in fact, satisfy the equilibrium condition  $\bar{w} = \bar{w}^{\mathcal{T}}$  as long as  $\tau < \bar{w}^{\mathcal{T}}$ .

Under proper parameterization, the equilibria depicted in Figure 3.2 — the ‘good’ no-run equilibrium and the ‘bad’ run equilibrium — may be described stable. Conversely, the three tipping points depicted in Figure 3.3 — as well as all other elements in  $\mathcal{E}^{\mathcal{T}}$  — may be described as unstable. The remainder and main contribution of this paper is to formalize in what sense the tipping points are unstable and to illustrate why they are still economically relevant, namely because they determine a neighboring stable equilibrium’s resilience to exogenous perturbations.

*Best-response dynamics and basins of attraction*

Before introducing the theoretical notions of stability and resilience, we must first define a dynamic strategy adjustment process to reach a game’s *ultimately observed outcome*  $w^*$ . For this, I specifically assume that given some initial strategy profile  $w^0$ , all households simultaneously iterate on their strategies by selecting an element from their best-response set,  $w_i^{j+1} \in B(w_{-i}^j)$ .<sup>13</sup>

Given the simultaneous nature of the strategic adjustment process, the initial condition  $\bar{w}^0 \leq \bar{w}^{\mathcal{T}}$  immediately determines whether  $w^*$  will be given by  $w^{NR}$ ,  $w^R$ , or some element in  $\mathcal{E}^{\mathcal{T}}$ .<sup>14</sup> We may then partition our strategy space into three subsets,

<sup>13</sup>If  $w^0 \in \mathcal{E}^{\mathcal{T}}$ , all patient households are indifferent how much to withdraw. In this case, I assume that each household will stick with their original strategy,  $w_i^{j+1} = w_i^j$  for each  $i \in I$ .

<sup>14</sup>Here, ‘immediately’ is meant to capture the fact that  $w^*$  is reached in a single iteration of the updating process.

$$\mathcal{B}^{NR} = \{w \in \mathcal{W} \mid \bar{w} < \bar{w}^T\}$$

$$\mathcal{B}^R = \{w \in \mathcal{W} \mid \bar{w} > \bar{w}^T\}$$

$$\mathcal{B}^T = \{w \in \mathcal{W} \mid \bar{w} = \bar{w}^T\}$$

Intuitively,  $\mathcal{B}^{NR}$  is the set of initial conditions that map to  $w^* = \mathcal{E}^{NR}$  as households update their strategies, whereas  $\mathcal{B}^R$  and  $\mathcal{B}^T$  map to  $\mathcal{E}^{NR}$  and  $\mathcal{E}^T$  respectively. In an analogy to dynamical systems, the three subsets  $\{\mathcal{B}^{NR}, \mathcal{B}^R, \mathcal{B}^T\}$  may be understood as *basins of attraction*<sup>15</sup>, each with a corresponding ‘uniform’ size of  $\{\bar{w}^T, 1 - \bar{w}^T, 0\}$ .<sup>16</sup>

In uniform terms, the set  $\mathcal{B}^T$  is thus infinitely smaller than either of the two basins  $\{\mathcal{B}^{NR}, \mathcal{B}^R\}$ . This serves as an indication as to why one may think of  $\mathcal{E}^T$  as empirically irrelevant, namely because any measure  $\mu^A : \mathbb{W} \mapsto [0, 1]$  (see footnote 8) that induces a continuous distribution over  $\bar{w}$  must yield  $\mu^A(\mathcal{B}^T) = \mu^A(\mathcal{E}^T) = 0$ . Similarly, the fact that each equilibrium in  $\mathcal{E}^T$  only contains a singleton in its basin of attraction, namely itself, serves as an indication as to why these equilibria are unstable. To gain a more precise understanding of stability, I now proceed by augmenting the original game  $\Gamma$  with a structural sunspot  $\xi$ . In turn, the perturbed game  $\Gamma^P = (\Omega, \Sigma, \mu, \xi)$  serves as a natural platform to formally introduce the notion of *resilience*.

### 3.3. A Structural Sunspot and Crisis Probability

Having equipped ourselves with the three basins  $\{\mathcal{B}^{NR}, \mathcal{B}^R, \mathcal{B}^T\}$ , we can now examine our equilibria’s stability and resilience properties. Specifically, we may wonder how *resilient* an equilibrium is to exogenous, strategic perturbations. For this, I proceed by augmenting the familiar economy  $\Gamma = (I, \mathcal{W}, u)$  with a structural sunspot  $\xi : \Omega \times \mathcal{W} \mapsto \mathcal{W}$  and a corresponding probability space  $(\Omega, \Sigma, \mu)$ . Given some initial condition  $w^0 \in \mathcal{W}$ , the sunspot  $\xi$  thus randomly maps  $w^0$  into some new profile  $w^1 \in \mathcal{W}$  based on the realization of the random event  $\omega \in \Omega$ ,

<sup>15</sup>Technically speaking,  $\mathcal{B}^T$  really contains an infinite number of basins, one for each element of  $\mathcal{E}^T$ .

<sup>16</sup>Given the infinite-dimensional nature of our strategy space, it is non-trivial to construct a ‘uniform’ measure on  $\mathcal{W}$  akin to the Lebesgue measure for finite-dimensional Euclidian spaces. However, for purposes of measuring the size of our partitions, constructing such a ‘fine’ measure is not necessary because a relatively sparse  $\sigma$ -algebra on  $\mathcal{W}$  is sufficient to ensure measurability of the random variable  $\bar{w} = m(w) \equiv \int_I w(i) di$ . In particular, we can construct a  $\sigma$ -algebra  $\mathbb{W} \equiv \cup_{A \in \sigma(I)} m^{-1}(A)$  and define  $\mu^W : \mathbb{W} \mapsto [0, 1]$  as  $\mu^W(A) \equiv \mathcal{L}(m(A))$  for each  $A \in \mathbb{W}$ , where  $\mathcal{L}$  is the Lebesgue measure on  $\mathbb{R}$ , which yields the desired, uniform measure space  $(\mathcal{W}, \mathbb{W}, \mu^W)$ . In particular, we have  $\mu^W(\mathcal{B}^{NR}) = \mathcal{L}(m(\mathcal{B}^{NR})) = \mathcal{L}([0, \bar{w}^T]) = \bar{w}^T$  and  $\mu^W(\mathcal{B}^R) = \mathcal{L}(m(\mathcal{B}^R)) = \mathcal{L}((\bar{w}^T, 1]) = 1 - \bar{w}^T$ .

$$w^1 = \xi(\omega, w^0)$$

Now, such a strategic perturbation is likely best interpreted in one of three ways. First, prior to learning any information regarding aggregate withdrawals, each household may have a prior that is subject to extrinsic news. Second, akin to the motivation of trembling hand equilibria via “slight mistakes” (Selten, 1975), a strategic perturbation could represent the fact that some households simply make mistakes when choosing their strategy. Finally, in the context of our Diamond-Dybvig economy, a perturbation could also arise from households misperceiving their own type.<sup>17</sup> In either case, assuming a default strategy to not run, the sunspot causes a subset of households to select a strategy that is irrational *ex ante*, but such initial mistakes can — as famously shown by Diamond and Dybvig (1983) — become self-fulfilling.

Following the sunspot, the perturbed strategy profile  $w^1 = \xi(\omega, w^0)$  yields a new, perturbed level of aggregate withdrawals  $\bar{w}^1 \equiv \int_I w^1(i) di$ , which may or may not trigger a bank run. For example, suppose that our economy starts out in the ‘good’ no-run equilibrium  $w^0 = w^{NR}$  and is randomly perturbed by  $\xi$ . We may then wonder how large of a perturbation is required for the bank run to emerge as the observed outcome  $w^* = w^R$ , and how likely it is that such an event occurs. To formalize and quantitatively assess this idea, I now proceed by defining the theoretical concepts of stability and resilience for the augmented game  $(I, \mathcal{W}, u, \xi)$ .

### *Stability and resilience*

**Stability.** An equilibrium  $w \in \mathcal{E}$  with corresponding basin of attraction  $\mathcal{B}$  is said to be (locally) *stable* if there exists  $\varepsilon > 0$  such that  $\{w' \in \mathcal{W} \mid \int_I w'(i) di \in [\bar{w} - \varepsilon, \bar{w} + \varepsilon]\} \subset \mathcal{B}$ .

Intuitively, an equilibrium in the Diamond-Dybvig setup is (locally) stable if small perturbations of  $\bar{w}$  are insufficient to push the economy away from it. Conversely, an equilibrium is said to be *unstable* if even an arbitrarily small perturbation of  $\bar{w}$  in either direction is sufficient to steer the economy towards another equilibrium in  $\mathcal{E}$ .<sup>18</sup> It is then evident why the tipping points in  $\mathcal{E}^T$  are all

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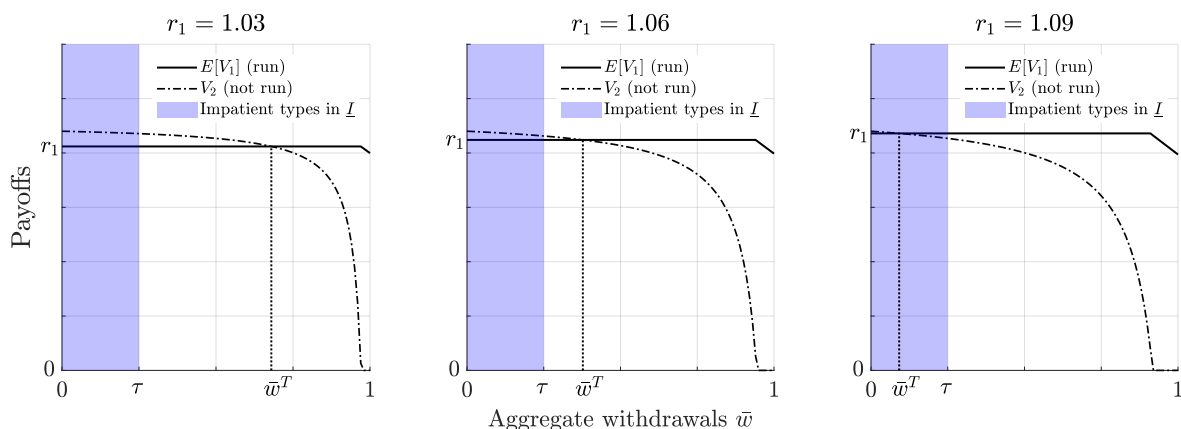
<sup>17</sup>The latter interpretation is less practical in the present context because an impatient household misperceiving their type only affects their preferred strategy when  $\bar{w}^0 < \bar{w}^T$ , but not when  $w^0 = w^R$ . In contrast, purely random strategic errors will alter outcomes irrespective of the initial condition  $w^0$ .

<sup>18</sup>See Frisch (1936) and Tinbergen (1941) for an early discussion of the importance of local stability properties.

unstable, namely because even an infinitesimally small, local perturbation of aggregate withdrawals  $\bar{w}$  near  $\bar{w}^T$  is sufficient to steer the economy towards one of the two canonical equilibria  $\{w^{NR}, w^R\}$ . Of course, this is because, all patient types strictly prefer to run if  $\bar{w} > \bar{w}^T$  and strictly prefer not to run if  $\bar{w} < \bar{w}^T$ .

Recall that while the property of stability serves as an indicator that an equilibrium can absorb local perturbations, it does not reveal how large of a perturbation the equilibrium is able to absorb. For example, consider Figure 3.4, which depicts the Diamond-Dybvig payoff structure economy for different values of the parameter  $r_1$ .

**Figure 3.4.** The tipping point nature of unstable equilibria



Notes: Figure 3.4 depicts three different payoff structures, each associated with a different level of the return  $r_1$ . Holding the technology  $R$  fixed, a higher return  $r_1$  implies a lower tipping point  $\bar{w}^T$  beyond which each patient household will withdraw all deposits. The key point is then that the distance  $\bar{w}^T - \tau$  serves as an indication of the shock size that the equilibrium  $w^{NR}$  is able to absorb. Notice that the third subfigure pictures an economy in which equilibrium is unique and given by  $w^R$  because no matter what the other patient types do, it is always in the best interest of each patient household to withdraw. Recall that such an economy is not permissible under proper initialization.

As indicated by Figure 3.4, the distance  $\bar{w}^T - \tau$  serves as a potentially useful measure to assess the no-run equilibrium's resilience to exogenous perturbations. In particular, we can already say that  $w^{NR}$  in Panel A is at least as resilient to exogenous perturbations as  $w^{NR}$  in Panel B if the structure of the sunspot  $\xi$  is held fixed.<sup>19</sup> In this spirit, the key insight of this paper is that unstable equilibria may provide pertinent information regarding the capacity of neighboring stable equilibria to absorb exogenous perturbations. To formalize this idea, I now define resilience.

**Resilience.** Given  $\xi$ , the *resilience*  $\lambda \in [0, 1]$  of an equilibrium  $w \in \mathcal{E}$  with corresponding basin of

<sup>19</sup>In subfigure (c), unique equilibrium is given by  $w^R$  as neither the no-run equilibrium, nor any tipping points exist. This case is precluded by proper parameterization.

attraction  $\mathcal{B}$  is defined as the likelihood that  $w$  still emerges as the observed equilibrium outcome  $w^*$  even after it has been perturbed by  $\xi$ ,  $\lambda \equiv \Pr(w^* = w | w^0 = w) = \Pr(\xi(\omega, w) \in \mathcal{B})$

As we shall see, stability has implications for resilience. Specifically, if  $\Pr(\bar{w}^1 | w^0)$  — and thus  $\xi$  — is continuous and unimodal, then the resilience of any unstable equilibrium is zero, whereas the resilience of any stable equilibrium is strictly positive. Concretely, we can now calculate the resilience of our two stable equilibria as follows,

$$\begin{aligned}\lambda^{NR} &= \mu(\xi^{-1}(\mathcal{B}^{NR}) \div w^{NR}) \\ \lambda^R &= \mu(\xi^{-1}(\mathcal{B}^R) \div w^R)\end{aligned}$$

where  $A \div B$  is defined as  $\{x \in C | x \times B \in A\}$  given  $A = C \times B$ . Equivalently, and maybe more intuitively, we can also write resilience in terms of the conditional cumulative density of perturbed aggregate withdrawals,<sup>20</sup>

$$\begin{aligned}\lambda^{NR} &\equiv \Pr(w^* = w^{NR} | w^0 = w^{NR}) \\ &= \Pr(\bar{w}^1 < \bar{w}^\mathcal{T}(\theta_1) | w^0 = w^{NR}, \theta_2) \\ &\equiv G_{\theta_2}^{NR}(\bar{w}^\mathcal{T}(\theta_1))\end{aligned}$$

where  $\theta_2$ , by parameterizing  $\xi$ , pins down the conditional distribution of  $\bar{w}^1$  given  $w^0$ . In turn, the perturbed game  $\Gamma^p$  is entirely parameterized by the vector  $\theta \equiv (\theta_1, \theta_2)$ . I then define,

**Proper parameterization (co-occurrence).** A parameter vector  $\theta$  is said to be *proper* if and only if — abusing notation —  $\Pr(\bar{w}^1 \leq \bar{w}^\mathcal{T}(\theta_1) | w^0, \theta_2) \in (0, 1)$  for any  $w^0 \in \mathcal{W}$ .

In words, a particular parameterization of the perturbed game is proper if and only if, irregardless of the initial condition  $w^0$ , both canonical equilibria  $\{w^{NR}, w^R\}$  occur as the game's ultimate outcome  $w^*$  with positive probability. Of course, this immediately implies  $\lambda^{NR}, \lambda^R \in (0, 1)$ .

#### *A concrete structural sunspot*

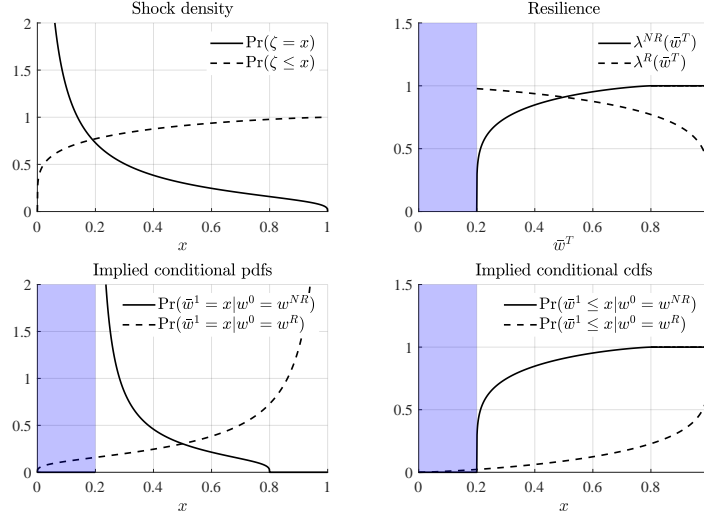
I continue to assume  $\theta_1 = (0.25, 0.98, 1.1, 1.03)$  while further imposing that the sunspot takes the form of an aggregate shock  $\zeta \sim G^\zeta$  by which some households randomly choose an inverse strategy  $w_i^1 = 1 - w_i^0$  and that the probability of such a mistake  $\zeta \in [0, 1]$  is uniform across all households. For example, suppose  $w^0 = w^{NR}$  and, as before,  $\tau = 0.2$  such that we have  $\bar{w}^0 = 0.2$ . In turn,

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<sup>20</sup>Analogously, we may define  $G^R(x) \equiv \Pr(x < \bar{w}^\mathcal{T} | w^0 = w^R)$  which yields  $\lambda^R = 1 - G^R(\bar{w}^\mathcal{T})$ .

an aggregate shock  $\zeta = 0.1$  perturbs  $w^0$ , meaning that 10% of all households choose an inverted strategy. In that case, perturbed aggregate withdrawals are  $\bar{w}^1 = 0.2(0.9) + 0.8(0.1) = 0.26$ , which may be sufficient to trigger a bank run depending on the  $\bar{w}^T \leq 0.26$ .

**Figure 3.5.** A structural sunspot example with its implied conditional densities and resilience



Notes: Figure 3.5 depicts resilience of  $w^0 \in \{w^{NR}, w^R\}$  and the corresponding implied conditional densities of  $\bar{w}^1$  for a given sunspot  $\xi$ . In particular, to concentrate the mass of the aggregate shock  $\zeta$  near zero while allowing for higher levels of irrationality occasionally,  $G$  is set equal to a beta distribution with shape parameters 0.2 and 1.3. Unsurprisingly, resilience of the two canonical equilibria are inversely related: When the tipping point  $\bar{w}^T$  is low, we have  $\lambda^{NR} < \lambda^R$ , whereas when  $\bar{w}^T$  is high, we have  $\lambda^{NR} > \lambda^R$ .

More generally, by the law of large numbers, we have  $\bar{w}^1 = \bar{w}^0 + \zeta(1 - 2\bar{w}^0)$ . Therefore, since a bank run is avoided if and only if  $\bar{w}^1 \leq \bar{w}^T$ , we can explicitly calculate resilience as follows,

$$\lambda^{NR} = G^\zeta \left( \frac{\bar{w}^T - \tau}{1 - 2\tau} \right) \quad (3.5)$$

$$\lambda^R = G^\zeta (1 - \bar{w}^T) \quad (3.6)$$

with the special case  $\lambda^{NR} = G^\zeta(\bar{w}^T)$  when  $\tau = 0$ . Unsurprisingly,  $\lambda^{NR}$  is increasing in  $\bar{w}^T$ , whereas  $\lambda^R$  is decreasing in  $\bar{w}^T$ . Figure 3.5 plots the resulting quantities of primary interest when  $\zeta$  follows a beta distribution with shape parameters 0.2 and 1.3.

### 3.4. The Repeated Game and Crisis Frequency

If the Diamond-Dybvig game were repeated, assuming that households' objectives remain unchanged and that a period's observed equilibrium outcome serves as the following period's initial

condition  $w_{t+1}^0 = w_t^*$ , the described perturbation  $\xi$  naturally induces a Markov chain for the observed state  $\{w^*\}$  in which case the tipping point  $\bar{w}^T$  not only pins down the conditional probability of a bank run, but also its ergodic probability, or its *frequency*. In particular, we have,

$$\begin{bmatrix} P_{NR} & P_T & P_R \\ 0 & 0 & 0 \\ 1 - P_{NR} & 1 - P_T & 1 - P_R \end{bmatrix} \begin{bmatrix} \pi_{NR} \\ \pi_T \\ \pi_R \end{bmatrix} = \begin{bmatrix} \pi_{NR} \\ \pi_T \\ \pi_R \end{bmatrix}$$

where  $P_i$  denotes the transition probability from state  $i \in \{NR, T, R\}$  to state  $NR$  and  $\pi_R = 1 - \pi_T - \pi_{NR}$  is the corresponding stationary, or ergodic, probability. Exploiting the fact that the above chain is reducible, since  $\pi_T = 0$ , and rewriting  $P_{NR} = \lambda_{NR}$  and  $P_R = 1 - \lambda_R$ , we have,

$$\begin{bmatrix} \lambda_{NR} & 1 - \lambda_R \\ 1 - \lambda_{NR} & \lambda_R \end{bmatrix} \begin{bmatrix} \pi_{NR} \\ \pi_R \end{bmatrix} = \begin{bmatrix} \pi_{NR} \\ \pi_R \end{bmatrix} \quad (3.7)$$

and can solve,

$$(1 - \lambda_R)\pi_R = (1 - \lambda_{NR})\pi_{NR}$$

$$\pi_R = 1 - \pi_{NR}$$

for  $\pi_R$  and  $\pi_{NR}$ . This yields,

$$\pi_R = \frac{1 - \lambda_{NR}}{2 - \lambda_{NR} - \lambda_R} \quad (3.8)$$

$$\pi_{NR} = \frac{1 - \lambda_R}{2 - \lambda_{NR} - \lambda_R} \quad (3.9)$$

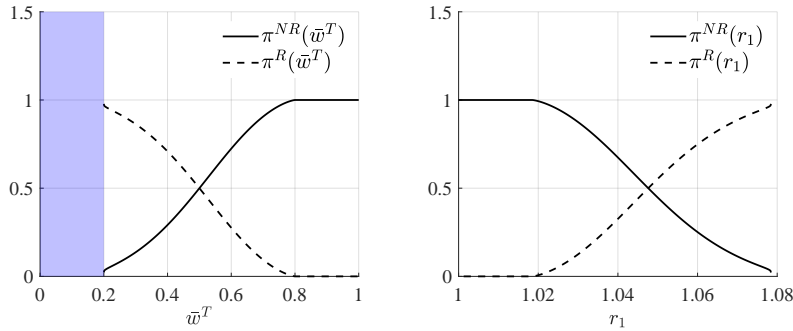
In turn, combining (8) and (9) with (5) and (6), we finally have,

$$\pi_R = G^{\pi^R}(\bar{w}^T)$$

$$\pi_{NR} = G^{\pi^{NR}}(\bar{w}^T)$$

which mathematically captures the notion that the unstable equilibria directly influence the frequency of observed crisis episodes. Figure 3.6 further illustrates this point by plotting the ergodic crisis probability against the tipping point  $\bar{w}^T$ ,

**Figure 3.6.** Crisis frequency as a function of the tipping point



Notes: Figure 3.6 depicts the ergodic probability of both canonical equilibria as functions of the tipping point  $\bar{w}^T$  and the promised return  $r_1$ . Of course, the two probabilities are inversely related because we must have  $\pi^{NR} = 1 - \pi^R$ . To the extent that the ergodic probabilities effectively capture the frequency of the corresponding equilibrium's occurrence over time, the tipping point  $\bar{w}^T$ , which is the key factor determining these probabilities, is highly economically relevant.

The primary insight from Figure 3.6 is that to the extent that we care about crisis frequencies, the tipping point  $\bar{w}^T$ , which effectively pins down  $\pi^R$ , is highly economically relevant. Section 5 discusses this idea in more detail by formally distinguishing between *empirical relevance* and *economic relevance*.

Panel B illustrates that we can further plot the ergodic probabilities as a function of the fundamental  $r_1$  by exploiting (4) to calculate  $\bar{w}^T$  (holding  $R$  and  $\rho$  fixed). Since higher returns  $r_1$  imply a lower tipping point  $\bar{w}^T$ ,  $\lambda^{NR}$  is decreasing in  $r_1$ , whereas  $\lambda^R$  is increasing in  $r_1$ . This simply means that smaller spreads between the physical return  $R$  and the promised return  $r_1$  cause more frequent crises over time, a proposition that is strongly supported by economic intuition.

### 3.5. The Economic Relevance of Unstable Equilibria

I now proceed by defining and applying the notions of empirical and economic relevance to both the single-shot and the repeated Diamond-Dybvig game.

**Empirical relevance (general).** A set of equilibria  $\mathcal{E}^*$  is said to be *empirically relevant* if and only if the probability that an element in  $\mathcal{E}^*$  emerges as the ultimately observed outcome  $w^*$  is strictly positive.

**Economic relevance (general).** A set of equilibria  $\mathcal{E}^*$  is said to be *economically relevant* if and only if it is empirically relevant or if the properties of its elements positively affect the distribution over the ultimately observed outcome  $w^*$ .



Since the single-shot game requires initialization, relevance derives from a conditional distribution and is thus only a local statement. In the repeated game, empirical and economic relevance derive from the ergodic density instead and initialization is irrelevant unless both equilibria are absorbing. I thus continue to assume proper parameterization to ensure that both canonical equilibria exist and that they are recurring.

*Single-shot game*

**Empirical relevance (single-shot).** For a given parameterization  $\theta$  and initialization  $w^0$  of the single-shot game, an equilibrium set  $\mathcal{E}^* \subseteq \mathcal{E}$  is said to be *empirically relevant* if and only if  $\Pr(w^* \in \mathcal{E}^* | w^0) > 0$ .

For each  $j \in \{NR, R, T\}$ , we can then rewrite the empirical relevance criterion as  $\Pr(w^* = \mathcal{E}^j | w^0, \theta) = \Pr(w^1 \in \mathcal{B}^j | w^0, \theta) > 0$ . Given the regularity condition that  $\Pr(\bar{w}^1 = x | w^0, \theta)$  be continuous in  $x$  for each  $w^0 \in \mathcal{W}$ , it follows immediately that the set of all unstable equilibria  $\mathcal{E}^T$  is empirically irrelevant because the conditional probability that any such equilibrium emerges as the game's ultimate outcome  $w^*$  is zero:  $\Pr(w^* \in \mathcal{E}^T | w^0, \theta) = \Pr(w^1 \in \mathcal{B}^T | w^0, \theta) = \Pr_{\theta_2}(\bar{w}^1 = \bar{w}^T(\theta_1) | w^0) = 0$  for any  $w^0 \in \mathcal{W}$ .

As far as the canonical equilibria are concerned, there are principally three cases. One in which both are empirically relevant, and two in which only one of the two is empirically relevant. Which of the three cases is applicable for a particular combination  $(\theta, w^0)$  depends on the distribution of  $\bar{w}^1 | w^0$  relative to the tipping point  $\bar{w}^T$ . Specifically,  $w^{NR}$  is uniquely empirically relevant if  $\Pr_{\theta_2}(\bar{w}^1 \leq \bar{w}^T | w^0, \theta_1) = 1$ ,  $w^R$  is uniquely empirically relevant if  $\Pr_{\theta_2}(\bar{w}^1 \leq \bar{w}^T | w^0, \theta_1) = 0$ , whereas both are empirically relevant if  $\Pr_{\theta_2}(\bar{w}^1 \leq \bar{w}^T | w^0, \theta_1) \in (0, 1)$ , as guaranteed by proper parameterization.

**Lemma 1.** Under proper parameterization, the set of empirically relevant equilibria  $\mathcal{R}^{EM} \subset \mathcal{E}$  is given by the set of canonical equilibria  $\mathcal{E}^C$  for any initial condition  $w^0 \in \mathcal{W}$ .

*Proof.* First, notice that  $\mathcal{E}^C \subset \mathcal{R}^{EM}$  follows immediately from the properness requirement that  $\Pr(\bar{w}^1 \leq \bar{w}^T(\theta_1) | w^0, \theta_2) \in (0, 1)$  for any  $w^0 \in \mathcal{W}$ . In contrast,  $\mathcal{R}^{EM} \cap \mathcal{E}^T = \emptyset$  follows immediately from the regularity condition that  $\Pr(\bar{w}^1 = x | w^0)$  be continuous in  $x$ . We thus have  $\mathcal{R}^{EM} = \mathcal{E}^C$ .

**Economic relevance (single-shot).** For a given parameterization  $\theta$  and initialization  $w^0$  of

the single-shot game, an equilibrium  $w \in \mathcal{E}$  is said to be *economically relevant* if and only if it is empirically relevant or if every parametric perturbation from  $\theta_1$  to  $\theta'_1$  that induces a shift in equilibrium from  $w$  to  $w'$  with  $\bar{w}' \notin [\bar{w} - \varepsilon, \bar{w} + \varepsilon]$  also induces  $\Pr(w^* = x|w^0, \theta') \leq \Pr(w^* = x|w^0, \theta)$  for some  $x \in \mathcal{E}$  and  $\forall \varepsilon > 0$ .

By definition, an empirically relevant equilibrium is also economically relevant. Conversely, empirical irrelevance can (under improper parameterization), but need not imply economic irrelevance, which constitutes the paper's main proposition.

**Proposition 1.** Under proper parameterization, the set of economically relevant equilibria  $\mathcal{R}^{EC} \subseteq \mathcal{E}$  in the single-shot game is given by the set of all equilibria  $\mathcal{E}$  for any initial condition  $w^0 \in \mathcal{W}$ .

*Proof.* By Lemma 1, both canonical equilibria are economically relevant. We are thus left to show that the set of unstable equilibria  $\mathcal{E}^T$  is economically relevant as well. For this, we only need to show that one arbitrary equilibrium's likelihood of being the game's ultimately observed outcome  $w^*$  is sensitive to the described parametric shift. For example, recall  $\Pr(w^* = w^{NR}|w^0, \theta) = \Pr(\bar{w}^1 \leq \bar{w}^T(\theta_1)|w^0, \theta_2)$ . Therefore, since a shift of  $\mathcal{E}^T$  as described implies  $\bar{w}^T(\theta'_1) \leq \bar{w}^T(\theta_1)$ , we have  $\Pr(w^* = w^{NR}|w^0, \theta') \leq \Pr(w^* = w^{NR}|w^0, \theta)$  because  $\Pr(\bar{w}^1 = x|w^0)$  was assumed to be unimodal and continuous in  $x$ .<sup>21</sup>

### *Repeated game*

**Empirical relevance (repeated).** For a given parameterization  $\theta$  of the repeated game, an equilibrium  $w \in \mathcal{E}$  is said to be *empirically relevant* if and only if its ergodic probability is strictly positive.

**Lemma 2.** Under proper parameterization, the set of empirically relevant equilibria  $\mathcal{R}^{EM} \subseteq \mathcal{E}$  in the repeated game is given by the set of canonical equilibria  $\mathcal{E}^C$ .

*Proof.* Since the conditional probability of reaching the set  $\mathcal{E}^T$  is zero for any initial condition, the only equilibria to consider are the two canonical equilibria in  $\mathcal{E}^C$ , which form the states of the Markov chain in section 5. As illustrated by (8) and (9), the only case in which either may be

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<sup>21</sup>Further notice that it is irrelevant whether  $w^{NR}(\theta'_1) = w^{NR}(\theta_1)$  is true or not as the distribution over  $w^*$  changes either way.

empirically irrelevant is if the other were absorbing,  $\lambda^{NR} = 1$  or  $\lambda^R = 1$ . These cases are ruled out by proper parameterization.

**Economic relevance (repeated).** For a given parameterization  $\theta$  of the repeated game, an equilibrium  $w \in \mathcal{E}$  is said to be *economically relevant* if and only if it is empirically relevant or if every parametric perturbation from  $\theta_1$  to  $\theta'_1$  that induces a shift in equilibrium from  $w$  to  $w'$  with  $\bar{w}' \notin [\bar{w} - \varepsilon, \bar{w} + \varepsilon]$  inevitably also alters the ergodic distribution  $\pi$  for every  $\varepsilon > 0$ .

**Proposition 2.** Under proper parameterization, the set of economically relevant equilibria  $\mathcal{E}^{EC} \subseteq \mathcal{E}$  in the repeated game is given by the set of all equilibria  $\mathcal{E}$ .

*Proof.* By Lemma 2, both canonical equilibria are economically relevant under proper parameterization. We are thus left to show that the set of unstable equilibria  $\mathcal{E}^{\mathcal{T}}$  is economically relevant as well. For this, we only need to show that one arbitrary equilibrium's ergodic probability is sensitive to the described parametric shift. In Proposition 1, we have already shown that the resilience of  $w^N R$  is affected by such a shift. Since the same is true for  $w^R$  (but in the opposite direction) under proper parameterization, (8) and (9) immediately imply that the ergodic probabilities of  $w^{NR}$  and  $w^R$  are not invariant to parametric shifts of this sort.

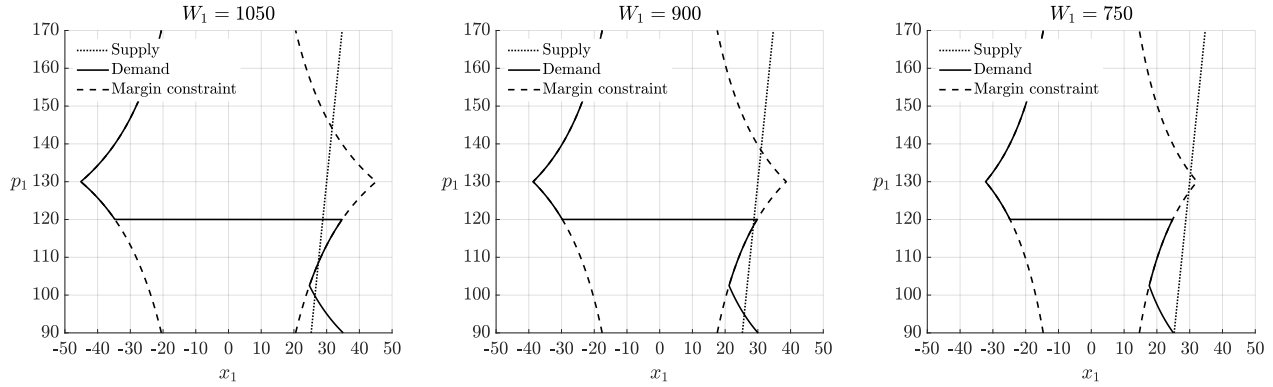
In this section, I have shown that the unstable equilibria are economically relevant in that they can and will materially affect observed outcomes if households are subject to structural sunspots. Before concluding, I now turn to examining two additional financial crises frameworks in light of the insights gained thus far. For this, I choose the canonical fire sale models by Brunnermeier and Pedersen (2009) and Morris and Shin (2004).

### 3.6. Further Applications: Fire sales and Liquidity Black Holes

For my second application of structural sunspots, I choose the canonical fire sale model by Brunnermeier in Pedersen (2009). The primary reason why I choose this framework as a second example is that the authors discard their unstable equilibrium as “uninteresting”. To illustrate why the latter may in fact be viewed as very interesting, I now reproduce their two key graphs in Figures 7 and 8.

Notes: Panel B and C of Figure 3.7 reproduce Brunnermeier and Pedersen's Figure 3, whereas Panel A is used to illustrate more visibly the unstable equilibrium that separates the two stable equilibria. As in Diamond-Dybvig, the unstable equilibrium is important because it tells us how resilient the preferred, ‘liquid’ equilibrium  $p_1 = 120$  is to

**Figure 3.7.** Supply and demand of the financial asset (BP: Fig. 3)



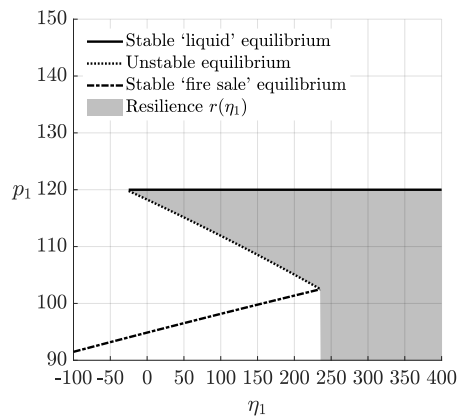
exogenous perturbations. Assuming  $W_1 = 900$  as in Panel B, for example, a minuscule (vertical) tremble by the Walrasian auctioneer is already sufficient to throw our economy into a fire sale equilibrium with  $p_1 \approx 94$ . Conversely, the preferred equilibrium is much more resilient to such perturbations if  $W_1 = 1050$  as in Panel A, whereas it does not even exist in Panel C.

Figure 3.7 depicts the supply and demand for a financial asset. The reason why demand is non-monotonic is that buyers purchase the asset on margin, which is calculated based on the asset's price. Given the depicted margin pricing scheme, equilibrium is indeterminate in Panels A and B, whereas it is unique in Panel C. In the benchmark case, as pictured in Panel B, the 'liquid' equilibrium ( $p_{\text{liq}} = 120$ ) is complemented by the 'fire sale' equilibrium ( $p_{\text{fs}} \approx 94$ ) and an unstable tipping point in between ( $p_{\text{tp}} \approx 118$ ).

Assuming that pricing is conducted by a Walrasian auctioneer, a natural way to think of structural sunspots would be to perturb the auctioneer's initialization of the tâtonnement process. Specifically, rather than successfully locating the price at which excess demand is precisely zero on her first try, the auctioneer may announce an initial guess of  $p_0 = 115$ . This will cause no harm in the economy depicted in Panel A as the auctioneer will ultimately converge back to the 'liquid' equilibrium, but the same is not true for the economy in Panel B. After the auctioneer registers excess supply at  $p_0$ , she reacts the only way she knows how, namely by lowering the price. In effect, a relatively small perturbation of the 'liquid' equilibrium is sufficient to throw the economy into a fire sale. In turn, we can derive the resilience of  $p_{\text{liq}}$  by defining a structural sunspot as given by the conditional distribution of the Walrasian guess  $p_0$  given the predetermined origin  $p_{\text{liq}}$ . Specifically, resilience is then given by  $r_{\text{liq}} \equiv \Pr(p_0 \geq p_{\text{tp}} | p_{\text{liq}})$ , where the unstable tipping point plays a crucial role once again.

To illustrate resilience across a range of scenarios, Figure 3.8 reproduces BP’s Fig.2, and plots resilience as a function of the shock  $\eta_1$  by augmenting the original graph with the missing unstable equilibrium. As before, we can see that at  $W_1 = 900$  ( $\eta_1 = 0$ ), the preferred equilibrium can only withstand perturbations up to -2.5, beyond which the auctioneer drives the economy towards a fire sale. In summary, augmenting the economy with a structural sunspot reveals why the unstable equilibrium is important, namely because it effectively pins down the preferred equilibrium’s resilience to exogenous perturbations.

**Figure 3.8.** Resilience of the ‘liquid’ equilibrium (BP: Fig. 4)



Notes: Figure 3.8 is a reproduction of Brunnermeier and Pedersen’s Figure 4. To illustrate the economic significance of the unstable equilibrium, I use the function  $r(\eta_1)$  to represent the ‘liquid’ equilibrium’s resilience to exogenous perturbations. For  $\eta_1 < -25$ , for example, the ‘liquid’ equilibrium is lost through a bifurcation with the unstable equilibrium (see Panel C of Figure 3.8).

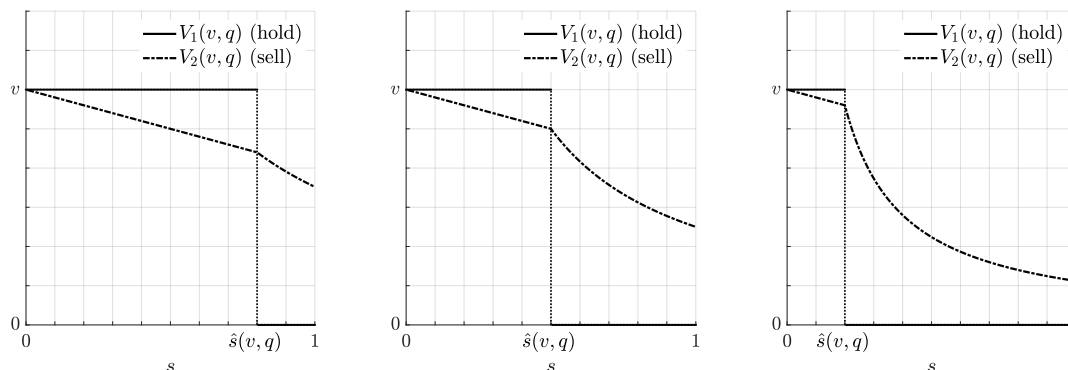
*What if there is no unstable equilibrium?*

For my third and final application, I examine an indeterminate economy that does *not* feature an unstable equilibrium. For this, I will consider a simplified version of the liquidity black hole model by Morris and Shin (2004) in which the loss limits of all traders are homogenous. Even though the resulting economy does not exactly correspond to the one examined by the authors, it is highly interesting nonetheless because it gives rise to two stable equilibria that are not separated by an unstable equilibrium in between.<sup>22</sup> To see this, consider Figure 3.9, which reproduces Fig.1

<sup>22</sup>In the face of heterogenous loss limits that are private information, Morris and Shin (2004) consider the set of all *threshold strategies*  $\bar{v}_i : \mathbb{R} \mapsto \mathbb{R}$  in which agent  $i$  sells if and only if their loss limit exceeds a critical threshold  $q_i > [\bar{v}_i]^{-1}(v)$ . In turn, their main result lies in the proof that the set of *equilibrium threshold strategies* is a singleton and given by  $\bar{v}_i(q) = v^*(q) \equiv \{x \in \mathbb{R} | x - q - c \exp(\frac{q-x}{2(q+x)}) = 0\}$  for each  $i$ . To show uniqueness, it is exploited that, holding  $v$  fixed, observing one’s own loss limit  $q_i$  implies a posterior distribution over the state  $\theta$ , which in turn implies a posterior distribution over aggregate sales  $s$ . Further exploiting that, by definition, a trader on the desired threshold is precisely indifferent between holding and selling, the posterior distribution over  $s$  is then used to derive

from Figure 3.9 and compares a trader’s expected payoff across the two possible strategies as a function of aggregate sales.

**Figure 3.9.** The payoffs to holding and selling as a function of aggregate sales (MS: Fig. 1)



Notes: Panel A of Figure 3.9 is a reproduction of Fig. 1 in Morris and Shin (2004), which plots a trader’s expected payoff across the two strategies  $\sigma_i \in \{\text{hold, sell}\}$  conditional on aggregate sales  $s$ . Selling is strictly preferred iff  $s > \hat{s}$ , whereas holding is strictly preferred iff  $s \leq \hat{s}$ . Importantly, notice that if  $s = \hat{s}$ , holding is strictly preferred and so the two payoffs never intersect. In effect, the two canonical, stable equilibria are *not* separated by an unstable one in between. Nevertheless,  $\hat{s}$  still represents a tipping point and thus has important implications regarding resilience. As can be seen in Panels B and C, the ‘liquid’ equilibrium’s resilience to sunspots is decreasing in  $q$  via  $\hat{s}$ .

First, notice that Figure 3.9 bears a strong resemblance to Figure 3.1, which captures the Diamond-Dybvig households’ payoffs to running and not running. For the chosen parameterization in Panel A — the loss limit  $q$ , the asset value expectation  $v$ , and an additional parameter  $c$  — traders *strictly* prefer to hold iff aggregate sales  $s$  satisfy  $s \leq \hat{s}(v, q) = 0.8$ , whereas they strictly prefer to sell if  $s > 0.8$ . Assuming that implied aggregate sales are observed for any chosen strategy profile  $\sigma_0$ , it is then easy to see that in the displayed economy, the two strategy profiles in which each agent sells and each agent holds both constitute a stable equilibrium. However, since utility is discontinuous at  $s = \hat{s}$ , the existence of two stable equilibria does not generally imply that there must exist a third, unstable one in between.<sup>23</sup>

Even though  $s = \hat{s}$  does not represent an unstable equilibrium, it still constitutes a ‘tipping point’. Therefore, augmenting our economy with a structural sunspot will still produce an internally consistent equilibrium selection mechanism. Specifically, consider a Diamond-Dybvig type sunspot

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the above equation that  $q$  and  $v$  must satisfy if the point  $(q, v)$  is to lie on the threshold. Methodologically, it is important to notice that a successful derivation of the model-implied distribution over aggregate sales crucially hinges on the assumption that each agent knows that everyone commits to the same threshold strategy  $(q_i, v) \mapsto \{\text{hold, sell}\}$  as described. If this is not so, then agents will have to rely on other sources of information to conduct inference on  $s$ , which is the situation I examine here.

<sup>23</sup>By Kakutani’s Fixed point theorem, an unstable equilibrium would exist if  $\hat{s}_i$  were continuously distributed across  $i$  with cdf  $G$  satisfying  $G(0) = 0, G(1) = 1$ . If  $G = u(0, 1)$ , equilibrium would be uncountably infinite.

$\xi$ , where a news shock causes an initial fraction of traders  $s_1 = \int \sigma_1 di = \int \xi(\sigma_0) di$  to submit a limit sell order. Upon learning  $s_1$ , each trader re-evaluates their strategy  $\sigma_{1i}$  by picking a corresponding best-response (which we know is a singleton), and iteratively submits/cancels a limit sell order if necessary. This structure of best-response dynamics implies that the economy enters a liquidity black hole iff  $s_1 > \hat{s}$  such that recovering the sunspot-implied density over  $s_1$  is sufficient to derive the resilience of the ‘liquid’ equilibrium  $\Pr(s_1 \leq \hat{s} | \sigma_0)$ .<sup>24</sup>

The liquidity black hole model by Morris and Shin (2004) illustrates that while unstable equilibria are a convenient structures to exploit for equilibrium selection with sunspots, they are not required. Instead, all that is required is a ‘tipping point’, which may or may not take the form of an unstable equilibrium.<sup>25</sup> In fact, as long as the sunspot induces a distribution over the tipping point variable that is continuous, the specific nature of the tipping point is perfectly inconsequential.

### 3.7. Conclusion

Indeterminate economies typically offer structures that lend themselves to be used for equilibrium selection. Specifically, such structures take the form of ‘tipping points’, which can be exploited — since they ‘physically’ separate the empirically relevant, stable equilibria — to add structure to the process of equilibrium selection. To obtain uniqueness using the model-implied tipping points, I thus propose to augment indeterminate economies with sunspots that allow for a natural interpretation relative to the model’s own structure. In turn, since unstable equilibria are tipping points, they serve — as illustrated by my first two applications — a natural role in equilibrium selection. Conversely, even tipping points that are not unstable equilibria, such as the one in my third application, can still be used. In fact, as long as the sunspot induces a continuous distribution over the tipping point variable, the latter’s specific nature is perfectly inconsequential.

### 3.8. Appendix

#### *Deriving the optimal allocation*

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<sup>24</sup>Notice that Morris and Shin (2004) also a density over  $s$ , but this density is a posterior, model-implied density. In effect, they examine a one-shot game in which traders must select a strategy without the possibility to change it in light of new information.

<sup>25</sup>Rather than flat hilltops, tipping points that are not unstable equilibria may be envisioned as sharp, asymmetric mountain peaks.

Each household is impatient with probability  $\tau$  and maximizes expected utility across the two possible states. Since the solution of the decentralized problem and the planner's problem are equivalent, I solve,

$$\begin{aligned} \max_{c_1, c_2} \quad & \tau u(c_1) + (1 - \tau)\rho u(c_2) \\ \text{s.t.} \quad & c_2 = \frac{(1 - \tau c_1)R}{(1 - \tau)} \end{aligned} \quad (3.10)$$

which may be rewritten as,

$$\max_{c_1} \quad \tau u(c_1) + (1 - \tau)\rho u\left(\frac{R}{(1 - \tau)} - \frac{\tau R c_1}{(1 - \tau)}\right)$$

The resulting FOC is then given by,

$$\begin{aligned} u'(c_1) &= \frac{1 - \tau}{\tau} \rho u'(c_2) \frac{\tau}{1 - \tau} R \\ &= \rho R u'(c_2) \end{aligned} \quad (3.11)$$

where (10) and (11) jointly pin down the *ex ante* optimal allocation across the two possible states.<sup>26</sup> Assuming CRRA utility with risk aversion parameter  $\gamma > 1$ , we can then calculate the optimal ex ante allocation as,

$$c_1^* = \frac{1}{(1 - \tau)(\rho R)^{\frac{1}{\gamma}}/R + \tau} \quad (3.12)$$

$$c_2^* = \frac{(\rho R)^{\frac{1}{\gamma}}}{(1 - \tau)(\rho R)^{\frac{1}{\gamma}}/R + \tau} \quad (3.13)$$

where  $\rho < 1$  implies  $(\rho R)^{\frac{1}{\gamma}}/R < 1$  for CRRA utility such that we must have  $c_2^* > (\rho R)^{\frac{1}{\gamma}}$  and  $c_1^* > 1$ , as pointed out by Diamond-Dybvig.<sup>27</sup> It is further also evident that  $c_2^* \rightarrow c_1^*$  as  $\gamma \rightarrow \infty$ .

### *Decentralizing the optimal allocation*

When types are not observable, the optimal allocation is not achievable in a market for contingent claims. However, as described by Diamond and Dybvig, the optimal allocation described by (12) and (13) can be achieved via a bank deposit contract that offers the following  $t = 1$  and  $t = 2$  payoffs per unit of not-withdrawn deposits,

<sup>26</sup>Notice that marginal utility only indirectly depends on  $\tau$ , namely through the market clearing condition (10).

<sup>27</sup>Interestingly,  $c_1^* > 1$  may not hold if utility were given by  $u(c) = c^\alpha$ ,  $\alpha \in (0, 1)$  because  $(\rho R)^{\frac{1}{1-\alpha}} \leq R$ . In fact, we can calculate the return threshold for which the optimal contract precisely yields  $c^* = 1$ :  $R = \rho^{1/\alpha}$ . In turn, this implies that  $c_1^* > 1$  is **not** generally a necessary condition for an improvement over the competitive allocation  $(1, R)$ .



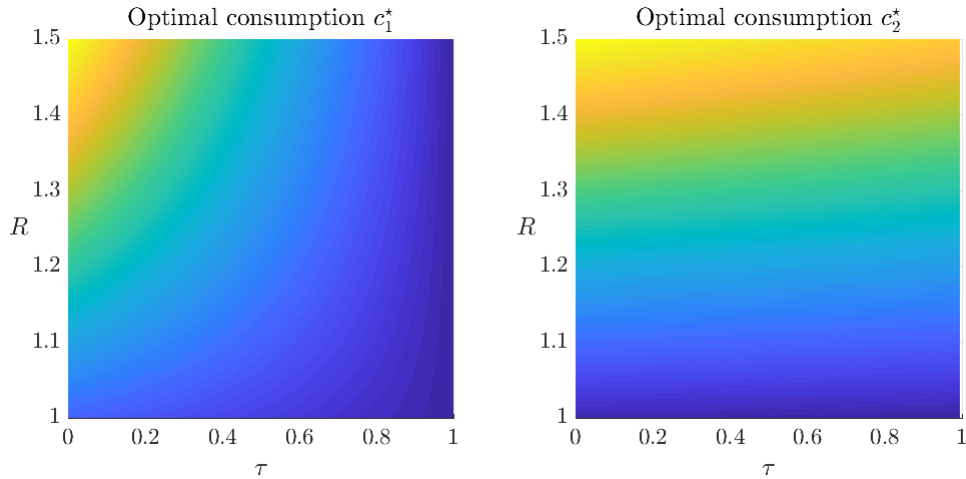
$$V_1(\bar{w}_j, r_1) = r_1 \mathbb{I}[\bar{w}_j < 1/r_1]$$

$$V_2(\bar{w}, r_1) = \max\{R(1 - r_1\bar{w})/(1 - \bar{w}), 0\}$$

In particular, the described contract decentralizes the optimal allocation as an equilibrium outcome if  $r_1$  is set equal to  $c_1^*$ . In this case, the game induced by the realization of the distribution of states features as a decentralized Nash equilibrium the strategy profile in which only the impatient withdraw and all patient types wait.<sup>28</sup> The corresponding total amount of deposits withdrawn at  $t = 1$  is  $\bar{w}r_1 = \tau c_1^*$ , whereas the total amount of deposits withdrawn at  $t = 2$  is  $(1 - \tau)c_2^* = (1 - \tau c_1^*)R$ . The described strategy profile is Nash because the contract satisfies the self-selection criterion, meaning that neither type would prefer the other's allocation (or a convex combination of the two).

Of course, the described game famously also features a Nash equilibrium in which all agents choose to withdraw their deposits early. The latter strategy profile is Nash because the resources at  $t = 1$  are insufficient to satisfy all claims such that, if everyone runs, waiting yields certain consumption of zero, which constitutes the worst possible outcome and is thus clearly not optimal.<sup>29</sup>

**Figure 3.10.** Optimal consumption in the Diamond-Dybvig model



Notes: Figure 3.10 depicts a heat map of the optimal consumption bundle  $(c_1^*, c_2^*)$  as functions of the two parameters  $\tau$  and  $R$ . Notice that  $c_1^*$  is more sensitive to  $\tau$  than is  $c_2^*$  with both being similarly sensitive to changes in  $R$ .

<sup>28</sup>Analogously to the original  $n$ -person game examined by Nash (1950), equilibrium here is given by a ‘self-countering’ point  $w \in \mathcal{W}$  satisfying  $u_i(w_i|w_{-i}) \geq u_i(w'_i|w_{-i})$  for each  $w'_i \in W$  and each  $i \in I$ .

<sup>29</sup>However, in addition to the canonical Inada conditions,  $\lim_{x \rightarrow 0} u'(x) \rightarrow \infty$ ,  $\lim_{x \rightarrow \infty} u'(x) \rightarrow 0$ ,  $u''(x) < 0 \forall x \in (0, \infty)$ , one may also want to choose  $u$  such that  $u(0) = M > -\infty$  because (expected) utility is not well-defined otherwise.

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