Elements of Effective Tasks in Creating Positive Dispositions

A Capstone Project

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Abstract

Research has shown that while the discipline of mathematics focuses on negotiation, interpretation, and uncertainty, classroom mathematics often focus on rigidity, rules, and prescribed algorithms. This misalignment of mathematical practices in the classroom and mathematical practices in the discipline is negatively affecting both students' learning and students' dispositions (Boaler and Greeno 2000). Because students' dispositions develop in response to classroom practices, it is necessary for teachers to look at the ways in which they can better support the development of positive dispositions through task selection and modification. This paper explores a framework of the pivotal elements of task selection and modification, describing how explicit disposition goals can increase the effectiveness of instruction through discourse strategies, appropriate levels of difficulty, differentiation, goal alignment, and context. The paper then evaluates an example of a task from the *Interactive Mathematics Program: Year* 2 (IMP), showing how modifications to that task can help teachers better develop positive dispositions as well as implement effective instruction.

Keywords: task, disposition, discourse, received knowing, agency, intellectual authority, differentiation

Introduction

Teachers must make many decisions while planning and implementing a lesson. For instance, teachers must think about each student, the class as a whole, the task they will pose, what questions they will ask, how students might think, how they will assess the students' understanding. According to Lappan (1993):

No other decision that teachers make has a greater impact on students' opportunity to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages the students in studying mathematics. (p. 524)

Many other experts agree that choosing the task or activity for a lesson is one of the most important preparations that a teacher makes. Using Brophy and Alleman's (1991) definition, I will define the term "task" as "anything that students are expected to do, beyond getting input through reading or listening, in order to learn, practice, apply, evaluate, or in any other way respond to curricular content" (p. 9). These tasks affect the ways in which students develop understandings, but many argue that learning is more than that. Gresalfi (2009) says:

Learning is a process of developing *dispositions*, that is, ways of being in the world that involve ideas about, perspectives on, and engagement with information that can be seen both in moments of interaction and in more enduring patterns over time. (p. 329)

Based on the literature, I have created a framework that describes how to choose a task with two goals in mind: implementing effective instruction and developing positive dispositions. In order to communicate how this framework might help teachers to modify tasks, I have included a sample task and suggested possible modifications.

Dispositions

The dispositions created by narrow teaching practices in math classrooms can create many difficulties for students. Boaler and Greeno (2000) found that students in didactic classrooms had difficulty using their mathematical knowledge in new and varied situations. In contrast, students in discussion-based classrooms—those including negotiation and interpretation—were more able to use their knowledge in situations that required different applications (p. 172). In addition, Boaler and Greeno (2000) say, "The act of practicing procedures appears to become sufficiently dominant for many students that it obscures the meaning of the subject, and takes students' thoughts away from the concepts that are intended to be exemplified by the procedures" (p. 180). Boaler and Greeno (2000) argue further that students' knowledge is defined by the practices in which they learned the mathematics (p. 172). Because students develop dispositions in response to learning environments, students in classrooms with narrow procedural practices form dispositions towards mathematics centered on received knowing. Received knowing is a view of knowledge in which teachers and textbooks are the authority on mathematical correctness, and it is students' job to 'receive' that knowledge from the authorities (Boaler & Greeno, 2000, p. 174).

Many teachers might say that one of their goals is the continuation of students into further mathematical classes or careers. However, when classroom practices focus on rigidity and procedures, teachers are actually causing many students to leave the discipline. Boaler and Greeno (2000) argue that many able and successful mathematics students reject math in further studies or as a career because they want to pursue subjects that offer opportunities for creativity and agency (p. 187). Therefore, the students who continue with mathematics are often those who enjoyed the rigidity, rules, and lack of interpretive thought; they are then surprised to find

themselves unprepared for the expression and interpretation necessary for mathematicians at the college level and beyond. As Boaler and Greeno (2000) write, "The certainty they have come to enjoy . . . appears to be inconsistent with the mathematics with which they would engage at the highest mathematical levels" (p. 196). Teachers' rigid practices develop student dispositions centered on received knowing, therefore pushing able students out of the discipline and leaving unprepared those who do continue. The disconnect between the practices of mathematics in the classroom and mathematics as a discipline create inaccurate perceptions of mathematics that is harmful to students.

Framework

Discourse

Two important aspects of choosing or modifying a task is [a] what opportunities there are for discourse and [b] in what ways that discourse will be implemented. Discourse is defined as both the discussion between teacher and students and the discussion among students. Scholars in the field agree that teachers must think about discourse when choosing or planning implementation of a task. Not only does Gresalfi (2009) argue that choosing tasks that allow for students to discuss, question, and justify their answers allows students to engage with content more deeply, but she also says that these opportunities also have important consequences for disposition. Gresalfi (2009) writes, "These behaviors allow students to take on more responsibility for making meaning. When students are able to exercise authority for mathematical meaning-making, they are expected, obligated, and entitled to explain" (p. 362-363). Tasks that allow for discussions, questions, and justifications can shift the authority in the classroom for the better. These tasks show students that their ideas are valued and makes them, not the textbook, the deciders regarding mathematical "truths." Lampert (1990) emphasizes the

importance of intellectual authority. Having students justify, question, or revise their ideas helps them to develop positive dispositions around mathematics. Math, then, becomes no longer a class where the textbook and teacher decide what is right and where success means memorizing or following rules, but instead, "the practice of knowing mathematics in school is closer to what it means to know mathematics within the discipline by deliberately altering the roles and responsibilities of teacher and students in classroom discourse" (Lampert, 1990, p. 29). Therefore, teachers should choose or modify tasks to create opportunities for meaningful discourse in the classroom.

There is no perfect recipe for creating or choosing a task that allows for this kind of discourse. However, teachers can look for tasks that use words such as "justify," "explain," "decide," "why," or "create." While these words do not guarantee that a task will allow for valuable discussion, they may signify an aspect of open-endedness that often supports discourse in the classroom. Tasks that are repetitive in nature or have a prescribed algorithm to reach the solution will generally not support this kind of discourse. Teachers should ask themselves, "[In this task,] how are students expected, obligated, or entitled to talk to one another, to challenge one another's ideas, or to compare themselves with others?" (Gresalfi, 2009, p. 33). However, teachers need to think carefully about the way in which discourse is implemented once they have chosen a task. Gresalfi and Cobb (2006) emphasize the importance of positioning in the distribution of authority in the classroom (p. 51). Teachers can use a task that has opportunities for an authority shift, but without careful attention to their particular word choice, it could be rendered ineffective. For example, take a discussion where the teacher asks the students to justify but still responds with an indication of correctness; this scenario actually places the teacher as the holder of the authority and strips students of their agency. Instead, teachers should make an

effort to allow students to decide collectively on the correctness of ideas or to debate appropriately until they have modified an idea and deemed it correct. In this case, teachers serve more as moderators of discussions, rather than the holder of all mathematical knowledge.

Discourse centered on questioning, justifying, and revising ideas gives students responsibility for their own learning. Gresalfi and Cobb (2006) write, "Being responsible for others' understanding by having to 'convince' someone else requires higher standards of mathematical argumentation than simply being accountable to a teacher who gives confirmation of accuracy" (p. 52); therefore, this kind of discourse allows deeper understanding of concepts and material. In addition, literature on dispositions supports discourse as an aspect of tasks that shifts authority to increase agency in students. This creates further opportunity for students to develop positive dispositions in response to classroom practices. Finally, the result of this shift in authority more closely resembles mathematics as a discipline, setting students up for further success in the discipline by better preparing them for the kinds of thinking that mathematicians actually do. Lampert (1990) describes mathematics as a process of 'conscious guessing' about relationships, where ideas are constantly being vulnerable to revision (p. 30-31). When discourse in the classroom models the kind of mathematical "zig-zag" between proposed ideas and proposed revisions, it allows students to affiliate more closely and more importantly more accurately with math as a discipline (Lampert, 1990).

There are many different ways teachers can go about finding tasks; however, many teachers agree that creating effective tasks is much more difficult than it may sound; therefore, they rely on finding tasks within curricula or textbooks. This is a good strategy, but it is unlikely that a task taken from a curriculum set will be a perfect fit for a teacher's classroom, which means that they must be prepared to modify it to fit their classroom's needs. One curriculum

teachers might turn to for tasks is the *Interactive mathematics program: Year 2* (IMP). Figure 1, taken from the IMP curricula by Fendel, Resek, Alper and Fraser (2004), is just one example of a task teachers might find and decide to use in the classroom (p. 226).

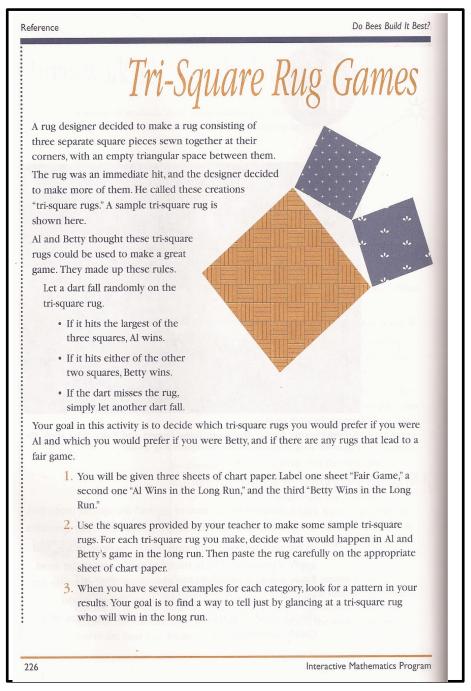


Figure 1
From Fendel, D., Resek, D., Alper, L., & Fraser, S. (2004). *Interactive mathematics program: Year 2*. Emeryville, CA: Key Curriculum Press.

One of the most important aspects of effective discourse is the way in which the roles and authorities are structured. As the literature recommends, tasks should allow students to construct meaning, rather than having the meaning structured for them with a reproducible algorithm.

Therefore, in implementing the task in Figure 1, teachers might want to consider removing the three numbered directions. By telling students how to label their paper and set up their discovery, the task is implying that the authority lies in the textbook rather than in students' construction of their own methods. Teachers can remove the numbered directions and make extra scratch paper, graph paper, and scissors available to students so that they may choose to work through it in a trial-by-example method; however, by removing the directions and allowing students to come up with this idea on their own, students create their own meaning and methods, structuring the role of the student as the one with intellectual authority. Instead of the numbered directions, teachers might modify these to say:

If you were playing the game as Al or Betty you might want to see if you could find an advantage in choosing a tri-square rug for the game. Your goal is to see if there is a way to tell just by glancing at a tri-square rug if someone will win in the long run. If so, who? (Adapted from Fendel et al. 2004)

Changing the phrasing to make it ambiguous as to whether or not there is a pattern gives an opportunity for differing opinions and creates a need for students to justify their answers.

Students now have to say whether or not there is a way to tell, and they have to support their idea with some kind of mathematical idea or pattern. This again opens the discourse to focus on student ideas, intellectual authority, and agency. Students can make communal revisions to student-derived ideas or decide what justifications are "enough" to convince them, giving the students the opportunity to participate in constructing their own mathematical meaning.

Level of Difficulty

While discourse is a large part of task selection, so is its level of difficulty: teachers must choose tasks that are at the optimal level. Driscoll (2007) focuses on the common problem of oversimplifying tasks and lowering cognitive demand in an effort to modify or personalize tasks for the classroom (p. 112). While teachers may want to modify a task in order to fit it best to their students and classrooms, they must be careful to maintain the cognitive demand of the task. Scholars such as Van de Walle (2006) say that a task must "take into consideration the students' current understanding. They should have the appropriate ideas to engage and solve the problem and yet still find it challenging and interesting" (p. 33). This idea, supported by other scholars such as Driscoll (2007) and Brophy and Alleman (1991), is not only important for teachers because it has shown to be effective in helping students engage at a deeper level with the material but is important because it develops positive dispositions. Mathematicians value the importance of uncertainty, an important part of mathematics as a discipline (Boaler and Greeno, 2000, p. 190). Too common in many math classrooms today is the belief by students that a problem should be solvable quickly and with one answer, and if you cannot solve it in just a few minutes, you should just give up (Corte, Eynde & Verschaffel, 2002). This attitude does not match up with the attitude of mathematicians, where uncertainty is a common aspect of problemsolving and where most problems are not solved within just a few minutes. Therefore, providing tasks to students that are at the appropriate level of difficulty—a place where the task meets with what students know but also challenge students—develops attitudes that support a positive disposition. Students may learn persistence and a value of uncertainty. If a task is at the appropriate level for students, they will often, "sense a difficulty that they would like to resolve and discuss" (Hiebert et al., 1997, p. 18). The difficulty may serve as a motivating factor for

students, allowing for deeper engagement with the content. Therefore, by choosing a task that is at the appropriate level of difficulty for the students completing it, teachers create both effective instruction and opportunities for students to develop positive dispositions.

While looking at Figure 1, it is again important to modify the three numbered directions. The modifications explained previously (see "Discourse") help better align this task to the students. Without the modifications, the task provides directions and a method, even telling students to look for a pattern. This does not model the kind of mathematics that mathematicians do in the discipline, where they value uncertainty. Therefore, modifying this section to open up the task for students to be uncertain where to begin might initiate the mathematical process of "zig-zag" between starting with an idea, trying it and evaluating it, and then modifying to a new idea.

Because every class is different, there is now way to know whether this task would be too easy or too difficult for certain classes; however, teachers can increase the difficulty of the task by creating extensions surrounding the idea of a fair game. Teachers can follow up the question surrounding whether or not there are rugs that lead to a fair game with questions such as:

If there are rugs that lead to a fair game, how do you know that it is fair? Justify your answer. If there are not, how do you know there are not? Can you create a new game similar to the tri-square rug game that does have opportunities for a fair game?

Teachers can ask many possible extension questions centered on students' creating new games with similar rule structures. These new games will change the way fairness is determined, increasing the difficulty of the task and bringing in new mathematical issues. If the task were too

easy for the class, teachers could include a short amount of time for students to play the game to increase their understanding of it or could include parts of the numbered directions as "hints."

Differentiation

In order for teachers to implement tasks at an appropriate level of difficulty for their students, it is imperative that the tasks are able to be differentiated for students who are at different levels of understanding. Finding a single task that is at an appropriate level for each student in the class is not only difficult, but, some teachers might argue, impossible! Therefore, in order to fit a task to an entire classroom, teachers must find tasks that are able to be scaffolded for a variety of learners and then modify them to include such differentiations.

One way supported by the literature is to choose or create a task that has many different correct methods. Fuson, Kalchman, and Bransford (2005) argue that students' believing that there are many methods supports their engagement, allowing students to maintain interest in persisting when a method does not work or when a method does not come immediately to them (p. 224). Instead of searching for one method, they have many chances for success. However, as Fuson et al. (2005) is careful to point out, "[Not] all strategies are equally good. But, students can learn to evaluate different strategies for their advantages and disadvantages" (p. 224). While some methods may take longer, may require a less complex understanding of the concepts, or may be inefficient, students can learn from the differences in methods between their own and other methods in the class. It is not only an important learning opportunity for students to learn from different strategies but also allows students at the lowest level to contribute a method as well as access the content. Hiebert et al. (1997) emphasizes the importance of meeting students where they are and linking content to what they already know, and tasks that have multiple methods allow teachers to do this (p. 22). In order for teachers to find tasks with multiple

methods, they should look for tasks that have multiple entry points to the task (Van de Walle, 2006). Teachers can ask:

Could students get started on this problem in multiple ways? Could they draw a picture, make a graph, or start with more advanced algebraic equations? Could they start with a specific example before generalizing? How might students at the lowest level start this? How about students at the highest level?

These kinds of questions support the teacher in choosing tasks or modifying tasks to include the multiple methods and entry points proposed by the literature.

Not only does a task with multiple entry points allow students at a variety of levels to access its content, it promotes the development of a positive disposition. Boaler and Greeno (2000) define figured worlds as, "places where agents come together and construct joint meanings and activities" (p. 173). By choosing tasks with multiple methods, teachers are creating such figured worlds supported by Boaler and Greeno, where everyone in the class creates meaning. A task with one prescribed algorithmic answer—where students' jobs are to reproduce the methods shown to them—teaches students more than just an algorithm: it teaches students that their ideas are not important or valued, that they don't have agency, and that math as a discipline is highly structured and rule-bound (Gresalfi & Cobb, 2006, p. 50). And, as discussed previously, this belief is detrimental to students' learning and to their pursuit of and future success in mathematics at higher levels. So, teachers' using a task with multiple methods shows students that they can evaluate, justify, and modify methods to create their own meaning and therefore gives students agency as mathematicians. However, it is not enough for teachers only to use such a task; they must implement it with the type of discourse discussed previously, showing students that their ideas are valued no matter the level of complexity in their thinking.

In some ways, the different elements of effective tasks for agency and disposition are so dependent on each other, that the modifications suggested for discourse and level of difficulty are modifications that are also appropriate for differentiation. The modification of the numbered directions in the task discussed previously (see "Discourse") expands the task and creates a need for justification. Instead of prescribing a method with the numbered directions, students are given the opportunity to approach the task from many methods. This allows students to access the task from different levels. Some students may start by playing the game or by creating paper examples of the game while others may access the task by creating drawings and immediately beginning with numerical area estimates. This allows students to use different methods as well as different entry points so that students can participate while having different previous levels of understanding. This can help students see that mathematics is not a prescribed set of rules, but rather is centered on problem-solving.

Goals

While differentiation and the level of difficulty for students is important, it most likely is not very worthwhile if the task is not aligned with the teacher's goals for learning outcomes. The task must be a means to achieving valuable goals that are linked with the curriculum (Brophy & Alleman, 1991). However, many scholars argue that goals must be more than procedures for the students to learn. As Gresalfi (2009) argues, "In the 21st century, being knowledgeable involves considering when to access facts, interrogate them, respond to them, and integrate them into daily activity" (p. 364). Being successful in today's world requires that students be able to make decisions about how to use mathematical knowledge and be able to transfer it to situations they have not encountered previously. This is especially true for students who pursue further math or for mathematicians in the field: they cannot be successful simply knowing a list of mathematical

skills without being able to use and apply those skills. Therefore, teachers should make sure to include a variety of goals that emphasize understanding of concepts and applicability as well as procedures and calculations. This allows students to develop dispositions surrounding mathematics that align with dispositions of mathematicians.

As Hiebert et al. (1997) writes, a task must leave behind important residue, which they define as "understandings as outcomes of solving problems" (p. 22). Teachers must first decide what residue they want students to take with them, and then choose a task that will allow students to achieve those understandings. However, in addition to accomplishing curricular goals, many scholars argue that pedagogy should include explicit goals focused on dispositions towards the discipline (Gresalfi & Cobb, 2006, 49). For example, Gresalfi and Cobb (2006) establish the difference between conceptual agency and disciplinary agency, where conceptual agency "involves choosing methods and developing meanings and relations between concepts and principles" and disciplinary agency focuses on applying pre-determined algorithms (p. 52). Gresalfi and Cobb's notion of conceptual agency aligns with literature by Hiebert et al. (1997) and Thompson, Philipp, Thompson, and Boyd (1994) that teachers need to include explicit goals around conceptual agency and positive dispositions. Thompson et al. (1994) writes:

It is important that students also appreciate that the most powerful approach to solving problems is to understand them deeply and proceed from the basis of understanding and that a weak approach is to search one's memory for the 'right' procedure. (p. 90)

Again, this is the strategy necessary for mathematicians and the kind of thinking valued by the discipline and therefore should be a goal for development by teachers when choosing or modifying a task for the classroom. By including conceptual agency as a goal for tasks, teachers

are not only supporting best practices for instruction, but they are supporting the development of positive mathematical dispositions.

In Figure 1, the task does not seem to have an explicit goal surrounding the development of positive dispositions. The prescribed directions for students to follow removes intellectual authority and also enforces the preconceived notion that mathematics is about rule following. However, when teachers consider the proposed modifications, one can see that with only a few modifications, the task has now been opened up to include disposition as an explicit goal. Teachers now have modified this task to include changes that will support the students' agency and views on mathematics as a disposition.

Context

Many teachers and experts agree that students are more successful when engaged in a task, and that one of the most engaging aspects of a task is its context. Best practice and culturally responsive pedagogy insist that students will learn more if the concepts are embedded in a context that is interesting and relevant to them. The literature says that students learn best through real contexts and situations because they are able to build meaning that is connected to their familiar experiences (Van de Walle, 2006). However, while most scholars agree that this is true, teachers must be careful. Scholars such as Hiebert et al. (1997), Lappan (1993), and Van de Walle (2006) warn teachers of choosing tasks based on context over choosing tasks based on the mathematics. Ideally, tasks will both have an intriguing or relevant context as well as mathematical significance. However, sometimes the context that makes tasks interesting can detract from or reduce the rigor of the mathematics (Van de Walle, 2006, p. 33). Hiebert et al. (1997) even emphasizes that "the intriguing or perplexing part of the situation should be the *mathematics*" (p. 19). Therefore, while many scholars support the value for engagement in

interesting and relevant contexts, teachers need to be selective when choosing or modifying tasks to make sure that the context of the problem highlights the important mathematical concepts.

This can be a difficult balance for teachers to create. However, by being cognizant of this balance, teachers can be careful in choosing tasks that keep the mathematics at the center of the activity while still maintaining context that is relevant or intriguing. Gresalfi & Cobb (2006) have shown that the dispositions developed can be influenced by context, and they advise teachers to choose tasks that "let students see mathematics as an activity that is reasonable to pursue for its own sake rather than as an activity that contradicts their 'peer' or 'social' identities" (p. 54). Also, in order for teachers to implement the kind of discourse and differentiation that can positively impact dispositions, they need to give the students a purpose for reasoning and making that reasoning public (Thompson et al., 1994, p. 87). Teachers can give students that occasion by choosing tasks with contexts that will be motivating.

Different teachers might argue whether or not the task in Figure 1 is relevant or interesting to students. For younger students, a task that takes the format of a game can be very fun and exciting in the classroom but older students might find it less interesting or relevant; however, with the modifications, it does fit one major component of the context element of effective tasks: in this task, the context does not detract from the mathematics. Students working on this task will be engaged in mathematics that has not been watered down for the sake of the context and allows students to be interested in the mathematics underlying the problem. Many students might be motivated by the modified task, as putting themselves in the position of Al and Betty as a player can be motivating in itself. If teachers wanted to modify the task in order to make it further motivating or engaging for students, they may rewrite the task to take place in the classroom. For example, Ms. Huebner might write the following:

Ms. Huebner's wacky sister invented a rug that is created with three square pieces sewn together at their corners, with an empty triangular space in between them. Unfortunately, her 'tri-square rugs' were not a hit, and she ended up with stacks and stacks of tri-square rugs in their house. Luckily, Ms. Huebner's inventive brother decided to make use out of the rugs by making a game for Ms. Huebner's students to play! Here are the rules: Let a dart fall randomly on the tri-square rug. If it hits the largest of the three squares, Timea (or another student in the classroom) wins. If it hits either of the other two squares, Cameron (or another student in the classroom) wins. If the dart misses the rug, simply let another dart fall. (Adapted from Fendel et al. 2004)

The rest of the previous modifications (see "Discourse") are then changed to use the names of students in the classroom. This modification has the potential to have a more relevant context while maintaining the mathematics as the interesting part of the task. However, instead of this being a task about two unknown people, it now has fun implications within the classroom and the students can feel more connected to it. This modification is not always necessary but is an option for teachers who think that their students may not connect with the current context.

Assessment

Throughout this paper, I have explored the literature on effective tasks and have focused on how these elements of effective tasks can be implemented in a way that contributes to the development of a positive mathematical disposition in students. The ways in which teachers use tasks, structure discourse, give intellectual authority and agency, and emphasize value in multiple methods all send students a message that can affect their mathematical disposition. It not only sends a message about what kind of thinking is valued in a teacher's specific classroom,

but it also sends a message about what math is like as a discipline, what students need in order to be successful in higher level courses or careers in mathematics, and what kind of characteristics mathematicians have or need. This is also true of assessment. The ways in which teachers assess the understanding of their students must align to the goals and values they have emphasized in the classroom. If the "bottom-line [of assessment] is error-free and mechanical performance, students come to believe that that is what mathematics is all about" (Schoenfeld, 1988, p. 15). This message contradicts the implications from instruction, and often the inferences taken from assessment will be the dominant ones. Perhaps if teachers have explicit goals about conceptual agency and disposition, this should be a part of the assessment itself. As presented earlier, there are necessary benefits for the education of our students in helping them develop positive dispositions. If this is such an important and necessary part of their future success, researchers and teachers should begin to think about not only how to develop positive dispositions but how to assess and track changes in disposition as a part of their assessment. This is an important area for future research in the field.

Conclusion

The literature emphasizes the importance of choosing or modifying tasks in order to implement the most effective instruction possible. Tasks should create opportunities for discourse, should be an appropriate level of difficulty, should allow for differentiation and multiple strategies, should be aligned with curricular goals, and should be embedded within contexts that highlight the mathematics. I have shown how each of these aspects of effective tasks also contributes to the development of positive dispositions in students and how these positive dispositions are necessary for student learning and success. Not only do dispositions affect the ways in which students learn and their understanding of material, but teachers can help

align dispositions more closely to the discipline of mathematics in order to prepare students for future success in the field. The overlap between the research on effective tasks and the research on dispositions shows that strategies for both of these areas support and further the success of the other. The awareness and explicit disposition goals will increase the effectiveness of discourse strategies, differentiation, and implementation; the effective use of differentiation, goal alignment, and discourse will add to the development of positive dispositions. Furthermore, I have shown that while these elements are difficult to find all within one task, teachers can modify existing tasks to address more completely all aspects of best practice. Using this framework as a guide, teachers can make dispositions a central component of choosing or modifying their classroom tasks.

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