Geometer's Sketchpad®: Advancing Middle School Curriculum

Karleen Vogt

Vanderbilt University

Capstone
June 15, 2011

Abstract

This paper will investigate the implementation of a product of Key Curriculum Press®, Geometer's Sketchpad® (GSP), into a traditional middle school mathematics curriculum: Larson's *Middle School Math: Tennessee*. GSP, a dynamic and engaging software program, allows students to be able to explore geometric, algebraic, and statistical concepts through creation and manipulation of mathematical notions (Hinders, 1992). The students' ability to generate their own products and conjectures makes them agents in their own learning (Driscoll, 2007). GSP also leads to effective group work and whole class discussions. By looking at learners and learning, the learning environment, curriculum and instructional strategies, and assessment this paper will explore the benefits of implementing GSP in middle school classrooms. The introduction to this paper will also briefly examine the reform of traditional curricula and what implications this has for technology. The conclusion of the paper will provide educators with an understanding of the implementation of GSP in traditional type curricula and offer suggestions for execution of GSP in the middle school classroom. Four GSP tasks will be provided and discussed along with images of the completed task in the GSP software. These four tasks cover geometric, algebraic, and statistical understandings that are proposed by Tennessee and national standards.

Introduction

Curricula across the United States are progressively reforming in response to the focus on how people learn most effectively and how to improve the structure of acquiring knowledge. Specifically in mathematics, traditional approaches emphasize direct instruction where the teacher is the preeminent source of knowledge in the classroom. Additionally, discourse is not accentuated and concepts are often taught in isolation. Students are shown one standard technique and expected to reproduce the method or algorithm when necessary. The new developments in curricula have formed mathematics courses where discovery, discussion, and connectedness are at the center of instruction. Reform mathematics has focused on including conceptual understanding and application of mathematics to real life context, ideas often overlooked in traditional texts. Battista states, "For most students, mathematics is an endless sequence of memorizing and forgetting facts and procedures that make little sense to them (1999)." It is for these reasons, among others, that teachers have looked towards restructuring curricula and instruction to assist in bridging this gap between procedural and conceptual learning.

Regardless of the reform movement in mathematics, traditional curricula remain the norm across the United States. Frequently, teachers find themselves working from procedural based resources. Technology, however, can become a plausible avenue for teachers who still desire to incorporate engagement, discovery, and independent thinking, the hallmarks of reform mathematics, into their classroom. The mathematical investigations created through geometry software give students the opportunity to create and test conjectures, offering an active role in the learning process. The professor of curriculum and instruction in the College of Education at the University of Illinois at Urbana, Gloriana González, states that when her students "used dynamic

geometry software, they were more successful in discovering new mathematical ideas than when they used static, paper based diagrams ("Technology & Learning", 2010)." Furthermore, Keastan and Sinclair conducted a study that showed 71% of teachers incorporate geometry software to enhance student understanding of material (2009). Additionally, it was found that these computer based tasks helped to solve current and persistent difficulties teachers have in delivering the curriculum. These challenges include addressing misconceptions and providing visual aids, among other difficulties that teachers sometimes face (Keastan & Sinclair, 2009). Technology can be utilized as an instructional tool to enhance learning, transfer, and comprehension of mathematics concepts.

This project will investigate the implementation of a product of Key Curriculum Press®, Geometer's Sketchpad® (GSP), into a traditional middle school mathematics curriculum: Larson's *Middle School Math: Tennessee*. GSP, a dynamic and engaging software program, allows students to be able to explore by creation and manipulation of geometric, algebraic, and statistical concepts (Hinders, 1992). The students' ability to create their own products and conjectures makes them agents of their own learning (Driscoll, 2007). By looking at learners and learning, the learning environment, curriculum and instructional strategies, and assessment this project will explore the benefits of implementing GSP in middle school classrooms. The conclusion of this project will provide educators with an understanding of the implementation of GSP into traditional type curricula and offer suggestions for execution of GSP in the middle school classroom. Supplemental material for Larson's *Middle School Math* will be created using GSP, touching upon algebraic, geometric, and statistical concepts. The entire curriculum was reviewed and key concepts that could be enhanced by GSP were made into tasks that could be executed during instruction. Step-by-step task directions and images of the GSP task are

included. The four tasks that are discussed cover geometric, algebraic, and statistical concepts that are proposed by Tennessee and national standards.

Before discussing the specific GSP tasks, this paper will address learners and learning, the learning environment, curriculum and instructional strategies, and assessment through the general integration of GSP into the classroom.

Learners and learning

No two students reason, develop and learn in the same manner (Milner, 2010). Hence, there is not one correct way to present material to students. It is vital that in presenting mathematics to students it is done in multidimensional techniques that simultaneously engage and apply to students (Milner, 2010). Since technology is a necessary part of our everyday routine, Geometer's Sketchpad can act as a catalyst to foster algebraic, geometric, and statistical habits of mind. This program allows individuals or groups to display, construct, transform, and measure geometric shapes, algebraic functions, and statistical concepts (Exploring geometry with The Geometer's Sketchpad, 1992). Once a concept has been explored, one can adapt, drag, and manipulate images to develop generalizations (Driscoll, 2007). Through guidance, students can discover aspects of geometry, algebra, and statistics in a hands-on manner. The students' ability to create their own products and conjectures gives a sense of independence and accomplishment. Finally, GSP can address common student errors or naive conceptions (Kasten & Sinclair, 2009). Through manipulations and tests, students can see mistakes they may make in their reasoning.

It is through appropriately adapting traditional material to incorporate GSP that this paper will further touch upon the learner and learning. Examples will be given of suitable GSP activities/tasks that are dynamic, cognitively stimulating, and mathematically relevant.

Learning environment

The environment created surrounding the student often determines the motivation the student has for mathematics (Milner, 2010). A traditional curriculum involves individual work with minimal teacher-student interaction and limited use of manipulatives. However, in a computer lab, students are focused on using a tool to inspire learning and can communicate with both peers and the teacher (Battista, 2002). It should not be ignored that this type of environment involves creative monitoring. The computers must be set up in such a way that the teacher can monitor the student's actions. Putting the computers around the perimeter of the classroom may allow for proper observation but might be difficult for instruction. Rows, on the other hand, may be ideal for instruction but pose an issue for keeping students on task. Therefore, other lab management practices need to be applied appropriately. For example, when the teacher is speaking, he/she could ask the students to put their hands on the top of their heads to discourage working during instruction. Additionally, teachers should make sure certain websites are unavailable for students during lab hours.

Building upon the interactive classroom, GSP tasks are conducive to engaging in high level mathematical discussions (Battista, 2002). Since students are manipulating and transforming geometric, algebraic, and statistical figures, they can use their work to form conjectures and justify their findings (Driscoll, 2007). Not every property is proven or supported in the same fashion. Discussions of similarities and differences among approaches can also be highlighted (Battista, 2002).

It is through appropriately adapting traditional material to incorporate GSP that this paper will further touch upon the learning environment. Examples will be given of suitable GSP activities/tasks that create an engaging atmosphere centered on discourse and discussion.

Curriculum and instructional strategies

Given any middle school program, GSP can be incorporated as supplemental material. Additionally, GSP fits the widespread curriculum that is often found in 7th and 8th grade because it reaches to geometric, algebraic, and statistical concepts (KCP Technologies, 2009). GSP may work best in 60-90 minute class period so that the students get enough time to discover or practice the topic of study. Teachers can create interactive and dynamic activities which aid students in their own exploration of concepts, thus making the students' learning more notable (KCP Technologies, 2009). Additionally, it can be utilized as a demonstration tool or to review a process learned in geometry, algebra, or statistics. GSP tasks support small group or individual learning, along with reaching students at all levels of thinking.

GSP can complement effective curriculum designs such as those offered in *Understanding by Design* (Wiggins & McTighe, 2005). In the framework presented, essential questions are the central focus of a unit. According to Wiggins and McTighe an essential question is "deemed essential" if it is not easily answered, can "get to the heart" of a topic, and can be found outside of the classroom in everyday life (2005, p.107). GSP creates many open-ended opportunities through which everyday life experiences can be explored. Additionally, the software can be utilized for algebraic investigations, geometric problem solving, statistical inquiries, among other mathematical concepts.

It is through appropriately adapting traditional material to incorporate GSP that this paper will further touch upon the curriculum and instructional strategies. Examples will be given of suitable GSP activities/tasks that are interactive and support top curriculum designs.

Assessment

The hands-on interactive experience fostered by GSP provides a solid understanding of mathematical concepts and, thus, demonstrating a student's ability to better recall information during assessments. Not only does GSP prepare for summative assessments, but it can be used as a pre-assessment and formative assessment. Teachers may create GSP tasks that act in way to discover what the students understand before beginning a topic. Furthermore, as students work through tasks, they can self-reflect on the particular concepts being studied. While working on a task or after completion, students may insert a text box and include a summary about how they completed their construction and what they learned from this construction. Also during construction, teachers can gauge student comprehension of manipulations and justifications by asking appropriate pressing questions (Driscoll, 2007). The software allows for more indepth questioning because the tasks are complex, often with multiple solutions. GSP tasks can be printed out or saved on the computer for future reference. Finally, teachers may also choose to give a summative assessment in an engaging way using GSP.

It is through appropriately adapting traditional material to incorporate GSP that this paper will further touch upon assessment. Examples will be given of suitable GSP activities/tasks that can act as pre-assessments, formative assessments, and/or summative assessments.

Tasks

An overview of each task will be presented along with an explanation of its role in creating a more productive learning experience. Instructions for implementing the task and images of the GSP file will be included.

Task 1

This task could be completed in conjunction with Section 11.7: Slope-intercept form, page 577, in Larson's *Middle School Math: Tennessee.* In order to be successful with this activity, students should understand slope and recognize lines with positive, negative, undefined, and zero slopes (Tucker, 2010). Minimal experience with GSP would not reduce learning. The task is completed as follows:

Investigating Slopes lab   (Adapted from Tucker, 2010)

1. Start The Geometer's Sketchpad.

2. Draw a horizontal line segment across the center of the screen.

3. Find the midpoint of the line. Construct a circle from the midpoint that reaches the endpoints of the line segment.

4. Label the intersection of the line segment and the circle as points A and B.

5. Place six points at various places on one semicircular arc of the circle. Label the points C through H.

6. Connect each of these six points to the points A and B.

7. Measure the slopes, to the nearest hundredth, of each pair of line segments meeting at the six points on the semicircle. Record the slopes in the table below.

8. Measure the angles formed at each of the six points on the semicircle. Describe the angle measurements here: _____.

9. Use the GSP calculator to calculate the entries for the columns in the table.

Point Name	m_1, Slope to A	m_2, Slope to B	$m_1 + m_2$	$m_1 - m_2$	$m_1 \cdot m_2$	$m_1 \div m_2$	$1/m_1$	$1/m_2$
C								
D								
E								
F								
G								
H								

10. Look at the entries in the columns in the table. See if there are patterns among any of the values of the sum, difference, product, quotient, or reciprocals of the slopes of the line segments. Justify your answer.

11. What is the vocabulary word used to describe two lines that meet at the angle you measured in step 8?

12. Formulate a conjecture about the slopes of these lines. Justify your answer.

This particular task can prove to be a very powerful tool when investigating perpendicular lines. Using the built in calculator can ensure accuracy and save time performing tedious calculations. Students are able to quickly and precisely create perpendicular lines and 90° angles (Tucker, 2010). Tucker states, "The inscribed right triangle forces perpendicularity wherever the student places the point on the semicircle. In contrast with a paper exercise, here the accuracy of the angle measure is automatic (p. 606)." As a result, students are taking less time to complete the construction and more time analyzing patterns from the results (Tucker, 2010). Instead of memorizing facts about perpendicular lines, students are able to detect, observe and discover rules, forming a more solid understanding of why these theories exist.

In this task, students are introduced to more than concepts surrounding perpendicular lines. It can be proven that any angle inscribed in a semi-circle is a right angle. As students are determining correlations between perpendicular lines, they are utilizing this fact. This can be brought up during this discussion or can be revisited when angles and chords are investigated to connect mathematical concepts.

The main goal of this lesson is for the students to learn that the sum and difference vary but to see that the product is always -1 (Tucker, 2010). Additionally, students can determine that
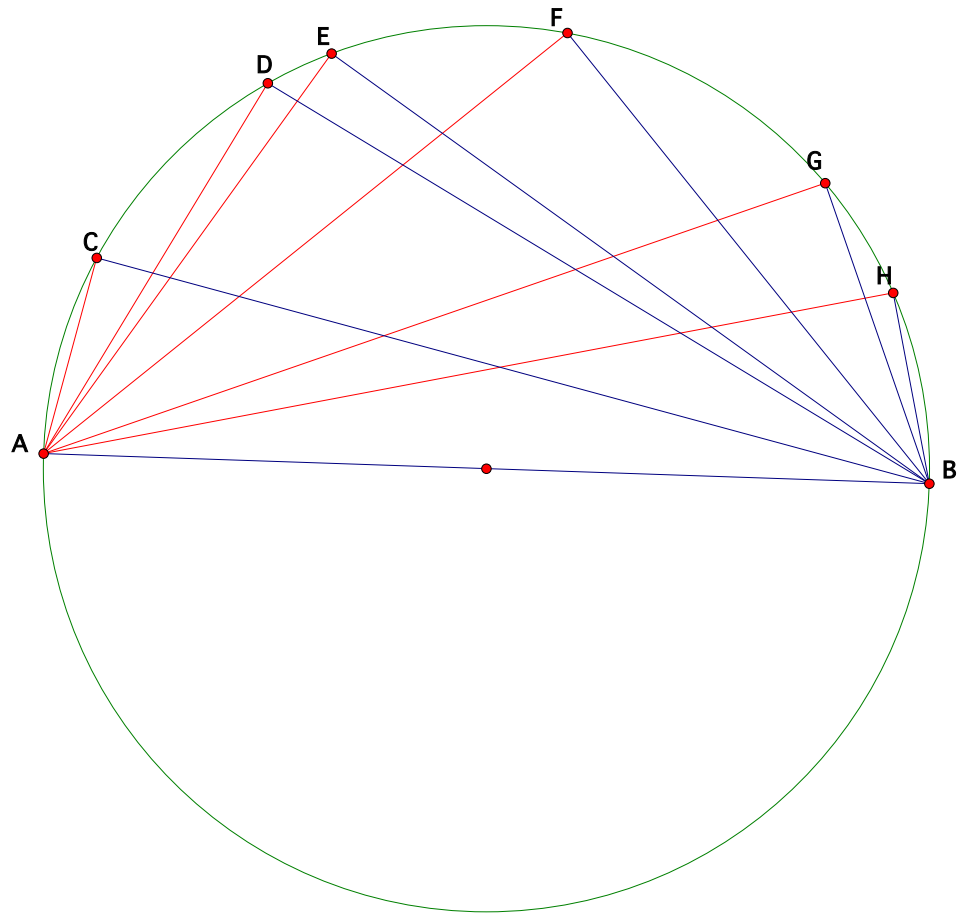
Investigating Slopes

Example Work:

Slope $\overline{AC}$ = 3.69   Slope $\overline{BC}$ = -0.27
Slope $\overline{AD}$ = 1.65   Slope $\overline{BD}$ = -0.61
Slope $\overline{AE}$ = 1.39   Slope $\overline{BE}$ = -0.72
Slope $\overline{AF}$ = 0.80   Slope $\overline{BF}$ = -1.25
Slope $\overline{AG}$ = 0.35   Slope $\overline{BG}$ = -2.89
Slope $\overline{AH}$ = 0.19   Slope $\overline{BH}$ = -5.29

$m\angle ACB = 90.00°$
$m\angle ADB = 90.00°$
$m\angle AEB = 90.00°$
$m\angle AFB = 90.00°$
$m\angle AGB = 90.00°$
$m\angle AHB = 90.00°$

$\left(\text{Slope } \overline{AC}\right)+\left(\text{Slope } \overline{BC}\right) = 3.41$

$\left(\text{Slope } \overline{AC}\right)-\left(\text{Slope } \overline{BC}\right) = 3.96$

$\left(\text{Slope } \overline{AC}\right)\cdot\left(\text{Slope } \overline{BC}\right) = -1.00$

$\dfrac{\left(\text{Slope } \overline{AC}\right)}{\left(\text{Slope } \overline{BC}\right)} = -13.58$

perpendicular lines have reciprocal slopes. Along with building conceptual knowledge, this task introduces many features of GSP and allows students to obtain more skills in using the software.

This particular task creates a powerful learning environment through discovery and discussion. In small groups or in a whole class discussion, the students can draw conclusions from data that they have collected. The students are prompted to investigate more measurements than needed to deduce the pattern (Driscoll, 1999). They must sort through the information, determining what is significant and what does not form a pattern. A student might argue for a pattern when adding m_1 and m_2, while another student might not be convinced of it. This allows the students to form informed opinions and support their findings.

Task 2

This task could also be completed in conjunction with Section 11.7: Slope-intercept form, page 577, in Larson's *Middle School Math: Tennessee.* In order to be successful with this activity, students should know what a function is, including input and output relationships. Additionally, they should be somewhat familiar with GSP.

Transforming Linear Equations in Slope-Intercept Form

1. Open Geometer's Sketchpad. Under **Graph** on the main tool bar, click **define coordinate system** (Make sure the grid form is in square grid or rectangular grid. Also, you can hide grid and just view the axes.).

2. Under **Graph**, click **plot new function** (Ctrl + G).

3. Under **Equation**, make sure **y-notation** is checked. To type in the function y = x, simply put an *x* in the white box. Notice the equation is shown above (some form of f(x) = x). Click okay.

4. **Positive coefficient:** Also graph $y = 2x$ and $y = \frac{2}{3}x$ on the coordinate system. What can you conclude about positive coefficients in respect to transforming linear equations in slope-intercept form? Justify your response.

   - **Note:** At any time you want to delete a function, highlight that function **only** and click the delete button.

5. **Negative coefficient:** Graph $y = -3x$ and $y = \frac{-5}{2}x$ on the coordinate system. What can you conclude about negative coefficients in respect to transforming linear equations in slope-intercept form? Justify your response.

6. **Positive Constant:** Graph $y = x + 3$ and $y = x + 1.5$ on the coordinate system. What can you conclude about positive constants in respect to transforming linear equations in slope-intercept form? Justify your response.

7. **Negative Constant:** Graph $y = x - 4$ and $y = x - \frac{1}{3}$ on the coordinate system. What can you conclude about negative constants in respect to transforming linear equations in slope-intercept form? Justify your response.

8. **Practice:** Anticipate the graph of the given function. Include a reasoning or justification for your graph. Use Geometer's Sketchpad to graph the equation and check your work.

Equation	Anticipated Graph	Justification/Reasoning	Actual Graph
$y = -4x + 7$			
$y = \frac{1}{4}x + 1$			
$y = 5x$			
$y = -x - 2$			
$y = 4$			

- What happens in the final graph? Why? Justify your answer.

Geometer's Sketchpad 15

$g(x) = 2 \cdot x$

$h(x) = \dfrac{2 \cdot x}{3}$

$q(x) = -2 \cdot x$

$r(x) = \dfrac{-5 \cdot x}{2}$

$s(x) = x+3$

$t(x) = x+1.5$

$u(x) = x-4$

$v(x) = x-\dfrac{1}{3}$

$w(x) = x$
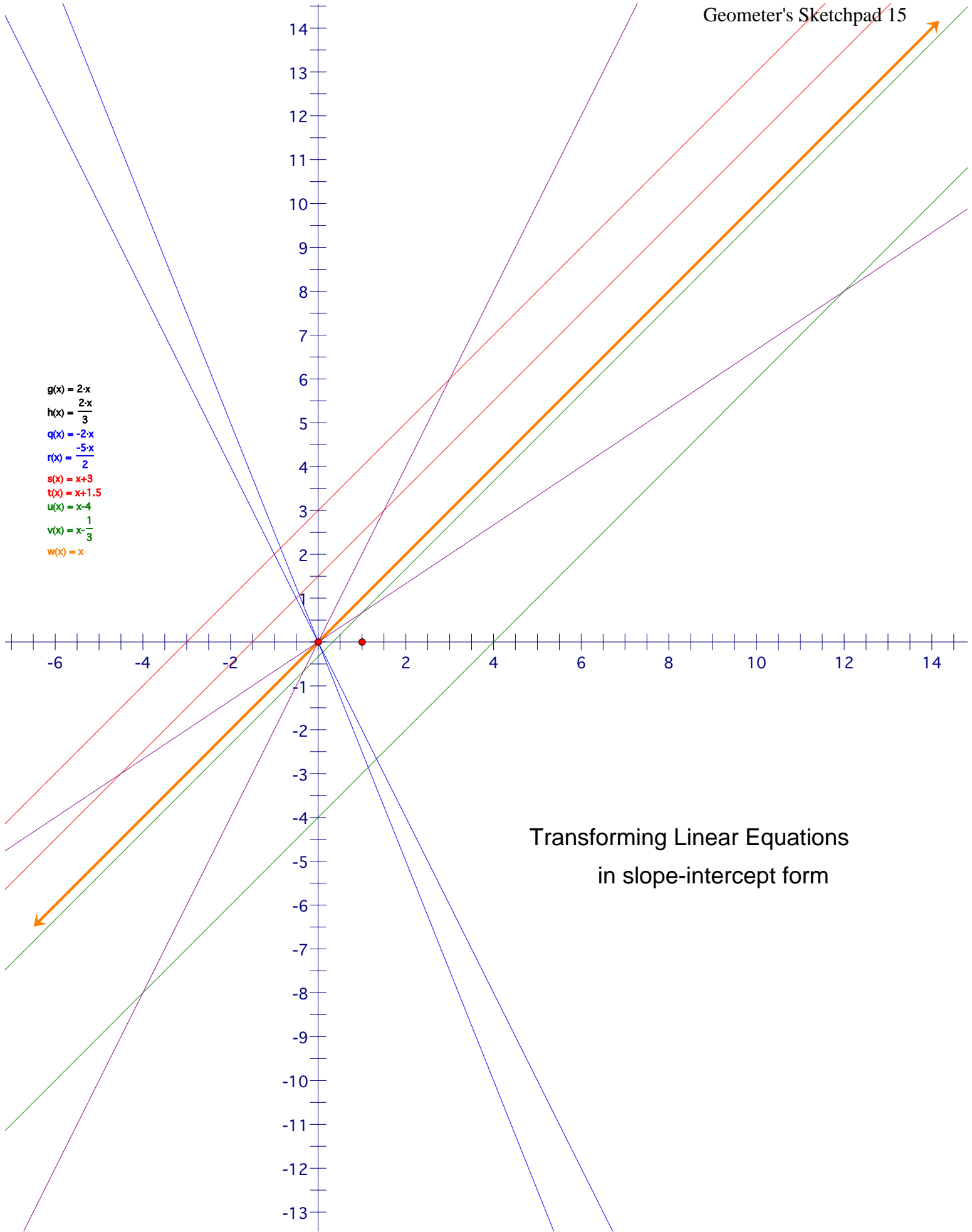
Transforming Linear Equations
in slope-intercept form

This task allows students to investigate algebraic expressions along with their graphical representation. In groups or individually, students can explore how negative and positive coefficients result in positive or negative slopes. The students will see how this changes the basic y=x graph. In addition, students will make generalization about the role of the y-intercept in the algebraic expression and graphical representation. Finally, after students identify patterns, they must anticipate other examples of algebraic expressions. They are even required to make a conjecture about the line y = 4.

In this task, the software can provide a way for the students to investigate algebraic expressions without focusing on the intricate particulars of graphing equations. The students are able to quickly graph the expression and gain immediate feedback through the visual. They do not have to spend excessive amounts of time on the graphing portion of the task and are instead able to concentrate on identifying patterns, developing explanations, creating conjectures, and forming generalizations (Driscoll, 199).

A strength of this GSP task is the visual support for transformations (Driscoll, 2007). The students read the algebraic expression and are asked to graph it using the function tool. They are then able to compare and contrast to a known function, creating a hypothesis of the patterns that they see. Additionally, students are required to support their reasoning for their anticipated graphs. This will hopefully allow the students to gauge their own comprehension and write about mathematical relationships. The students are making connections among multiple representations, reinforcement their understanding of the material.

Task 3

Tour 2: A Theorem About Quadrilaterals

Sometimes students will have trouble remembering or understanding a particular theorem. Discovering the theorem themselves or actively exploring its consequences can make a huge difference in their level of recall and understanding. This tour challenges you to discover a theorem. As you solve the challenge, you'll learn many new Sketchpad features.
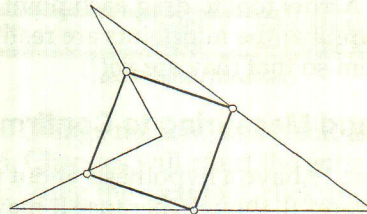
When the midpoints of the sides of a quadrilateral are connected, the resulting shape is always a _____.

What You Will Learn

- How to construct a polygon using the **Segment** tool
- How to show an object's label
- How to measure lengths and angles
- How to create a caption
- How to apply formatting options to text

The theorem stated above has a key word left blank. Our goal is to use Sketchpad to discover what word fills the blank. Don't worry if this theorem is unfamiliar or, on the other hand, if it's too simple. The point is to learn about how to use Sketchpad for making and testing conjectures.

Constructing a General Quadrilateral

A "general quadrilateral" is a quadrilateral with no special properties, such as congruent sides or right angles.

1. Start Sketchpad if it isn't already running. If it is running, choose **New Sketch** from the File menu.
2. Use the **Segment** tool to draw a segment.
3. Construct a second segment that shares one endpoint with the first.

After step 3

4. Construct two more segments to complete the quadrilateral.
5. Using the **Arrow** tool, drag some of the points and segments to make sure the figure holds together—does it pass the drag test?
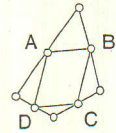

After step 5

Constructing the Midpoint Quadrilateral

Now that you have a quadrilateral, recall the theorem you're exploring. It begins "When the *midpoints* of the sides of a quadrilateral are *connected* . . . ." You will continue by constructing the four midpoints and then connecting them.

6. Deselect all objects by clicking in blank space. Select the four sides of the quadrilateral in clockwise or counter-clockwise order.

7. Choose **Midpoints** from the Construct menu.

You can also use the **Text** tool to show the labels. Click on an object with the **Text** tool to display (or later hide) its label. Double-click the label itself to change the label.

8. With the four midpoints still selected, choose **Show Labels** from the Display menu.

9. With the four midpoints *still* selected, choose **Segments** from the Construct menu.

10. Use the **Arrow** tool to drag each point in your figure. Especially make sure that the midpoints are really midpoints. (Can you drag them so that they are *not*?)

After step 9

Dragging and Measuring to Confirm Your Conjecture

You may already have a hypothesis about what kind of quadrilateral the inner figure is. Be careful, though, because it may be that shape only for a particular shape of the outer quadrilateral. You need to convince yourself that your conjecture holds *no matter what shape the outer quadrilateral is.* This requires dragging. To gain even more confidence in your answer, you'll take measurements as well.

11. Drag various parts of the original quadrilateral around, keeping an eye on the inscribed figure.

12. Now, select the four inner segments and choose **Length** from the Measure menu.

$m \overline{AB} = 1.15$ cm

When selecting three points to measure an angle in Sketchpad, the second selected point is the vertex.

13. Deselect all objects. Then select three consecutive midpoints and choose **Angle** from the Measure menu.

$m\angle ABC = 84.61°$

14. Select three consecutive midpoints again, with a different midpoint as the second selected point (vertex), and choose **Angle.** Repeat this step two more times, making sure a different midpoint is the second point selected each time.

15. Choose **Calculate** from the Measure menu. Click on the measure for $\angle ABC$, then click on the + sign (or type it on the keyboard), then click on the measure for $\angle BCD$, then click OK. What does this tell us?

$m\angle ABC + m\angle BCD = 180.00°$

16. Drag various parts of the original quadrilateral, making it wide, skinny, concave, and so on. What happens to the measurements? What changes, and what stays the same? Are you more confident of your conjecture now? As confident as you may feel, do you think you've *proven* the theorem?

Theorem about Quadrilaterals

m $\overline{AB}$ = 8.69 cm
m $\overline{BC}$ = 4.94 cm
m $\overline{CD}$ = 8.69 cm
m $\overline{DA}$ = 4.94 cm

m∠BAD = 63.62°
m∠ABC = 116.38°
m∠BCD = 63.62°
m∠CDA = 116.38°
m∠ABC+m∠BCD = 180.00°

When the midpoints of the sides of a quadrilateral are connected, the resulting shape is always a parallelogram.
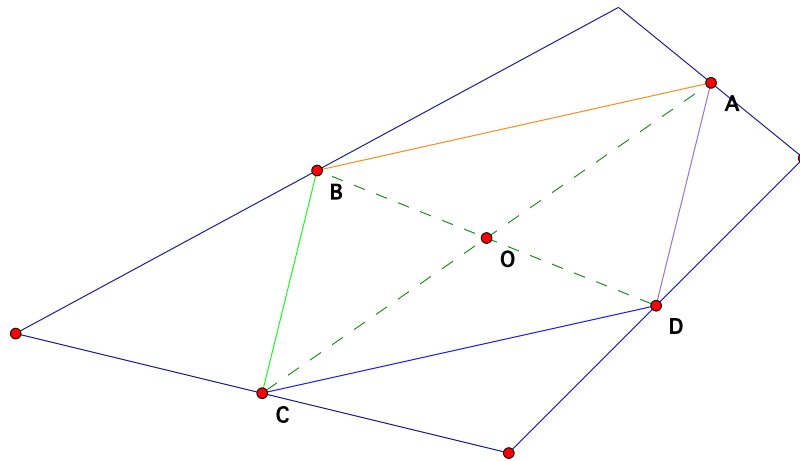
* Diagonals bisect each other.
* Opposite interior angles are congruent thus parallel sides
*Interior angles equal 360 degrees

A

B

O

D

C

This task could be completed in conjunction with Section 8.3: Quadrilaterals, page 386, in Larson's *Middle School Math: Tennessee.* In order to be successful with this activity, students should have some familiarity with properties of quadrilaterals and parallelograms, as well as with tools available in GSP. During the task, students will acquire additional techniques for effectively utilizing GSP.

Here the students discover that when the midpoint of the sides of a quadrilateral is connected, the resulting shape is always a parallelogram. In most classrooms, this would be among a list of properties concerning quadrilaterals that would have to be "committed to memory". Memorizing a theorem does not usually result in comprehension, nor allow for future recall. Discovering the theorem or actively exploring it can increase recall and understanding. ~~When students are involved in~~ "learning lists of properties is not nearly as important as being involved in the processes of developing and using a property-based conceptual system for reasoning about these shapes (Battista, 2002, p. 333)." In addition, it allows the students to take an abstract concept and make it more concrete (Battista, 2002). Similar to the task *Investigating Slopes*, students discover the answer to a theorem. In this task they explore the question, "When the midpoint of the sides of a quadrilateral is connected, the resulting shape is always a

_____? (Adapted from The Geometer's Sketchpad® Workshop Guide)"

Geometer's Sketchpad allows for differentiation among learners and acts as a support for higher order thinking. This particular task lends well to many differentiation strategies. The first is that the task supports learners at any level of geometric reasoning. Van de Walle proposed a five-level hierarchy of understanding spatial thought comprising level (0) Visualization, (1) Analysis, (2) Informal Deduction, (3) Deduction, and (4) Rigor (Van de Walle, 2007). Most middle school students would be at a level 1, 2, or 3. During this task, students form a

quadrilateral and are asked to investigate if it passes the drag test. With this investigation technique, students can begin to move from the level in which they are thinking to a higher level (Hinders, 1992). They can begin to explore the "all" case through the drag test or develop relationships around investigating properties. In the end, the students have to also prove and justify the results of their conjecture concerning the theorem. This allows each student to enter into the problem at their own level and utilize the parts of the software they see fit. In addition, there are 'Further Challenges' at the end of the section that could be utilized for higher level achieving students. These challenge problems both help students to organize their thoughts more effectively to create a stronger proof and allow students to extend their thinking to other shapes (The 'Further Challenge' are not included in this document but may be added for advanced students).

Specifically, this task also creates dynamic learning for the students. The students display, construct, transform, and measure geometric shapes (Exploring geometry with The Geometer's Sketchpad, 1992). As touched upon previously, once the quadrilateral or parallelogram is created, one can adapt, drag, and manipulate the image to develop generalizations (Driscoll, 2007). Through guidance, the students discover a theorem pertaining to all quadrilaterals in a hands-on manner. Finally, the students' ability to create their *own* products and conjectures gives a sense of independence and accomplishment. They are not simply listening to the teacher pour information into their brains. The learning environment is transformed into a place where the students hold the knowledge and can access it with through technology.

Task 4

Box and Whiskers

The *box-and-whiskers plot* (sometimes just called a *box plot*) is a recent development in statistical analysis. You cannot derive any detailed information from it, but it gives you a convenient, easily understood graphical representation of the data distribution.

SKETCH AND INVESTIGATE

1. Open **Box and Whiskers.gsp.**

The sketch contains ten data values represented as points on parallel lines. Above the points are a box and whiskers. You can change a value by sliding its corresponding point right or left. The data are ordered and displayed on the left, but the actual numerical values are not important for this activity.

2. Before answering any of the questions, take a minute to experiment with the sketch. Drag the data points and observe the effect.

Each of the following questions suggests a special shape for the box and whiskers. In each case, state whether it is possible. If it is not possible, explain why not. If it is possible, make a rough sketch of the data points that will create that configuration, and suggest a real data set that might make this happen. The first one is done as an example.

Q1 Can one whisker have zero length? Justify your answer.
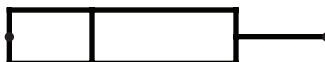
A1 This will occur if the lower one-fourth of the data points all have the same value. This might happen if there is a lower limit to the data range. Test a group of people to see how far they can throw a heavy weight. Those who cannot even lift the weight will all score zero.
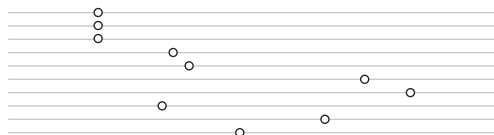
Q2 Can the median fall outside of the box? Justify your answer.

Q3 Can the box have zero width? Justify your answer.

Box and Whiskers
continued

Q4 Can both whiskers have zero length? Justify your answer.

3. Press the *Show Mean* button. The mean is represented by a green bar. Normally, the mean is not shown on a box-and-whisker plot. It appears here so that you can observe its relationship to the data distribution.

Q5 Can the mean fall outside of the box? Justify your answer.

Q6 Can the mean be greater than the maximum? Justify your answer.

Questions Q7–Q9 involve moving data points. In doing so, you are actually moving from one data set to another. In real life this could happen when a restaurant manager changes the prices of certain menu items, or when a sports team makes a player trade.

Q7 By moving one or more data points, can you move the mean without changing the box, the whiskers, or the median? Justify your answer.

Q8 Can you move a single data point without changing the mean? Justify your answer.

Q9 Can you move two data points without changing the mean? Justify your answer.

Q10 Can you change the median by moving a single data point? Justify your answer.
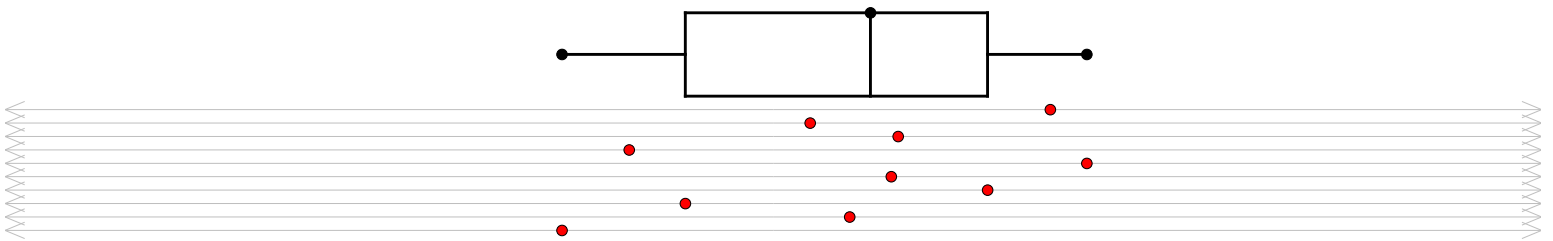
Box and Whiskers

$x_1 = 18.93$

$x_2 = 21.43$

$x_3 = 23.52$

$x_4 = 28.17$

$x_5 = 29.64$

$x_6 = 31.19$

$x_7 = 31.44$

$x_8 = 34.77$

$x_9 = 37.11$

$x_{10} = 38.46$

$Q_1 = 23.52$

$Q_3 = 34.77$

median = 30.42

mean = 29.47

Show Mean

Students can change the data points
to investigate different distributions.

Exploring Algebra 2 with Sketchpad
(C) 2007 by Key Curriculum Press

This task could be completed in conjunction with Section 12.2: Box-and-Whisker Plot, page 601, in Larson's *Middle School Math: Tennessee.* In order to be successful with this activity, students should have been taught quartiles and extreme values, in addition to the procedure for making a box-and-whisker plot. They should have done some work on interpreting the plot and using it to display data. This task was taken from *Exploring Algebra 2 with The Geometer's Sketchpad* written in 2007.

One way to foster statistical thinking is through balancing exploration and reflection of concepts. Mark Driscoll emphasizes this approach to nurture geometric thinking, arguing for the importance of trying various ways to approach a problem and regularly reflecting upon what is being learned (Driscoll, 2007). This approach for learning geometry may also be applied to this statistical task. Balancing exploration and reflection aids students in enhancing their problem solving techniques, as well as in evaluating the productivity of their investigation and the direction of the next moves. Using GSP to explore box-and-whisker plots emphasizes this trial and error approach. The students are able to quickly move around the data to explore things such as where the mean can fall in the data and on the box-and-whisker plot. They are able to compare the visual representation with the data.

This task can be used as either an exploratory tool or an assessment tool. As an exploratory tool, the students would follow the instructions as they have done with the other tasks. Through this type of implementation, teachers can gauge student understanding through their manipulations and conjectures (Driscoll, 2007). Additionally, the teacher can ask probing questions and engage in high level discussions with the students to assess their knowledge and gain an understanding of their nature of problem solving (Holyoak, 1995).

However, the students could also complete this individually or with a partner, as an end of the unit assessment. The unit may have talked about the sizes of the whiskers, where the mean can fall, and what affects the medium (for example). However, it might not have been the explicitly taught. If the students can reason through these small tasks, it would show that they understand the parts of a box-and-whisker plot. Since they can manipulate the data and view the resulting changes on the box-and-whisker plot, the students need to be able to justify their reasoning to each part. That is, they may draw the image or write down values of the data points. This immediate feedback, given through the GSP software, allows this task to be an effective pre- or post-assessment.

Conclusion

Given the effectiveness of GSP in fostering reasoning and sense-making in mathematics, the tasks can be time consuming to implement, and students may not be accustomed to investigating mathematics in this way. Additionally, the software can be expensive to purchase, difficult to install, and takes time to learn. Teachers need to be prepared to manage group work and discussion in their classroom. Despite these few limitations, GSP enhances the learning and understanding of mathematics.

Geometer's Sketchpad is effective first because of its influence on the learner. Students are motivated through the use of technology and being agents of their own learning. GSP reaches students at every level of mathematical thought and provides the necessary scaffold towards a more sophisticated understanding. Additionally, GSP demands higher order thinking and justification of conjectures. Finally, students are able to remember and recall what they learn

because GSP helps students to actively organize their thoughts, connect to previous learned knowledge, and balance exploration with reflection (Driscoll, 2007).

Also, GSP helps to create a more dynamic learning environment through highlighting big ideas and essential questions (Driscoll, 2007; Wiggins & McTighe, 2005). Its open-ended methods and justifications support overarching themes. Furthermore, instead of a traditional class where students sit in rows and the teacher is the source of knowledge, students are collaborating with each other and the teacher to draw their own conclusions and support their conjectures. Mathematical discourse is a direct result of GSP tasks, which strengthen comprehension of the concept at hand.

In addition to using GSP as a pre- or post-assessment tool, assessment of student learning occurs in day-to-day implementation of GSP. When students utilize GSP, teachers can gauge their understanding of their actions, manipulations, and justifications while problem solving (Driscoll, 2007). Through questioning, teachers can gain insight into student progress of mathematical connections.

As this country tends toward national standards and expectations, GSP is a powerful tool that helps all curricula align with the principals outlined in the NCTM standards. Geometer's Sketchpad:

(1) Emphasizes mathematical understanding, problem solving, and reasoning;

(2) Provides all students with opportunities to learn important mathematical ideas and skills;

(3) Provides opportunities for significant interactions of students and teachers, using a variety of communication methods (e.g., writing, explaining, presenting, defending);

(4) Incorporates the use of technology that enhances understanding; and

(5) Uses various forms of assessment to better gauge student understanding and to improve instructional practice (Rey et al, 1999).

These are deemed to be important goals by the NCTM and many educators. Geometer's sketchpad helps to obtain these goals. For the reasons presented in this paper, it should be a resource used by educators across the country.

In all future mathematics education preparation, teachers should be made aware of the conceptual framework of mathematical tasks and how GSP aligns with its goals and objectives. The mathematical tasks framework, thought to be an effective way to teach mathematics, has a *Task as Set-up, Implementation, and Discussion* phase (Stein, Grover, & Henningsen, 1996). The task as set-up phase is a time where the teacher and students discuss the context and mathematical relationships, without lowering the cognitive demand of the task. During the implementation stage, students are discovering or investigating the concept. Often this is accomplished through teacher questioning and group work. Finally, in a whole-class discussion, students show strategies and include justifications for work. Collectively, clear connections among ideas are established. Strong tasks "are characterized by features such as having more than one solution strategy, as being able to be represented in multiple ways, and as demanding that students communicate and justify their procedures and understandings in written and/or oral form" (Stein, Grover, & Henningsen, 1996, p. 456). The GSP tasks found in the paper support the above framework and the description of strong mathematical tasks. They provide an opportunity for discovery and lend well to whole class discussions. Knowing this framework would help educators acknowledge the importance of using GSP in their classroom.

Finally, future curriculum creators can use Geometer's Sketchpad to enhance their textbooks so that deeper mathematical relationships are made, students are able to recall more mathematics, and there is a connection between content and technology. GSP should not completely replace current mathematics curricula, but be used as a supplemental resource to develop strong mathematical discourse.

References

Battista, M. (1999). The Mathematical Miseducation of America's Youth Ignoring
Research and Scientific Study in Education. *Phi Delta Kappan*, 80(6).

Battista, M. (2002). Learning geometry in a dynamic computer environment. *Teaching Children Mathematics*, 8(6), 333-339. Education Module. (Document ID: 103719609).

Driscoll, M. (1999). *Fostering Algebraic Thinking: A Guide for Teachers, Grades 6-10.* Portsmouth, NH: Heinemann.

Driscoll, M. (2007). *Fostering Geometric Thinking*: *A Guide for Teachers, Grades 5-10*. Portsmouth, NH: Heinemann.

Exploring Algebra 2 with The Geometer's Sketchpad (2007). Key Curriculum Press Inc.

Exploring Geometry with The Geometer's Sketchpad. (1992). *The Arithmetic Teacher*, 40(3), 187. Education Module. (Document ID: 5218012).

Hinders, D. (1992). Technology Reviews: The Geometer's Sketchpad. *The Mathematics Teacher*, 85(5), 392. Research Library Core. (Document ID: 1867857).

Holyoak, K. J. (1995). Problem solving. In. E.E. Smith & D. N. Osherson (Eds.), *Thinking: An invitation to cognitive science*. (Vol. 3, pp. 267-296). Cambridge, MA: MIT Press.

Kasten, S., & Sinclair, N. (2009). Using Dynamic Geometry Software in the Mathematics Classroom: A Study of Teachers' Choices and Rationales. *The International Journal for Technology in Mathematics Education*, 16(4), 133-143. Education Module. (Document ID: 1929748201).

KCP Technologies (2009). The Geometer's Sketchpad® [On-line]. Available: http://www.dynamicgeometry.com/

Larson, Boswell, Kanold, & Stiff. (2005). *McDougal Littell Middle School Math: Tennessee.* Evanston, IL: McDougal Littell.

Milner, H.R. (2010). *Start where you are but don't stay there: Understanding diversity, opportunity gaps, and teaching in today's classrooms*. Cambridge: Harvard Education Press.

Reys, et al. (1999): Mathematics Curricula Based on Rigorous National Standards What, Why, and How? *Phi Delta Kappan* 80(6).

Stein, M., Grover, B., & Henningsen, M. (1996). Building Student Capacity for Mathematical Thinking and Reasoning: An Analysis of Mathematical Tasks Used in Reform Classrooms. *American Educational Research Journal*, 33(2), pp. 455-488.

Technology and Learning. (2010). *The Science Teacher*, 77(3).

The Geometer's Sketchpad® Workshop Guide. (2009). Key Curriculum Press Inc.

Tuckers, J. (2010). A Lesson on the Slopes of Perpendicular Lines. *Mathematics Teacher*, 103(8).

Van De Walle, J. A. (2007). Geometric thinking and geometric concepts. In J. A. de Walle (Ed.), *Elementary and middle school mathematics: Teaching developmentally* (pp. 407-451). Boston : Pearson.

Wiggins, G., & McTighe, J. (2005). *Understanding by design* (Expanded second edition). Alexandria, VA: Association for Supervision and Curriculum Development.