PARENTAL EARLY MATH SUPPORT: THE ROLE OF PARENTAL KNOWLEDGE ABOUT EARLY MATH DEVELOPMENT

By

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For my mother who has been the epitome of persistence

For my nieces, nephew, and the generations to come, may you dream courageously and run tenaciously towards your dreams.

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First, I honor God for providing all that I have needed throughout this journey including courageous faith, groundedness, tenacity, community, and a passion for learning and growth. I am beyond grateful for my mother who has been my biggest inspiration and champion. Thank you for always finding a way to help me pursue my seemingly unthinkable dreams. I am grateful for all the big and small acts of service, the daily prayers, and the encouraging words. I also thank my father who sparked my love for math and for learning in my early years and who has been my loudest cheerleader over the years. I thank my brother for being there in countless ways including believing and investing in me. I thank my sister, the rest of my family, and my amazing friends for every kind word and every warm hug.

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Finally, I celebrate myself for doing *it* courageously in the midst of….

PREFACE

If Yuh Waa Good

(Original Poem Written in Jamaican Creole)

Naa sah!
A wha mek my nose a run so?
Likkle mose mi piece a house ketch a fia
No, tru ting, me a no liad!
A bay midnight oil mi a bun
An pon top a dat, from both sides of the candle, bay wax a run

Yuh see,
Di real reason,
….an mind you, sometimes mi figet,
Is me no jus waa gud, me waa di best
Me deh pon di grind 24/7,
me go di extra mile
Di ting is….
Me just believe inna hard work-
plain an simple-
in giving my best,
pursuing my dreams
even wen every ting a try put me to di tes.

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CHAPTER 1

INTRODUCTION

Children's mathematics knowledge prior to formal schooling is predictive of their later math and broader academic achievement as well as other life factors including their socioeconomic status (SES) in adulthood (Claessens & Engel, 2013; Duncan et al., 2007; Fyfe et al., 2019; National Research Council, 2009; Ritchie & Bates, 2013; Watts et al., 2014). Parent-child experiences around numeracy are positively associated with children's early math knowledge (see the metanalysis and systematic review conducted by Daucourt et al., 2021 and Mutaf-Yıldız et al., 2020 respectively). Much less is known about parent-child experiences around patterns, predictable relations ranging from an alternating sequence of shapes to functional relationships between two variables, despite evidence that knowledge about patterns is an important component of children's mathematics development (e.g., Fyfe et al., 2019; Sarama & Clements, 2004; Zippert et al., 2020). Thus, it is important to understand why and how parents provide early numeracy and patterning support. The current study aimed to understand the nature and role of parents' knowledge about early numeracy and patterning development in their numeracy and patterning support. Before I describe the current study, I briefly review previous research on the role of parents in supporting both types of knowledge including sources of variability in parents' support. Finally, I propose an updated parent socialization model for early math development and discuss how the current study aimed to provide evidence for the updated model.

The Nature and Role of Parents' Early Math Support

Parent-child Numeracy Experiences

Young children whose parents expose them to more frequent and more complex number-related experiences at home have more advanced concurrent and later number knowledge (see Mutaf-Yıldız et al., 2020 for a review). For example, Mutaf-Yıldız and colleagues (2020) found that there is a positive association between reported parent-child engagement in numeracy activities at home and children's mathematical skills across 37 studies. Longitudinal links also exist between parent-child numeracy experiences and children's number knowledge. For example, parents' numeracy engagement with their children who were about to begin kindergarten predicted children's numerical knowledge at the end of the school year (Skwarchuk et al., 2014).

The complexity of parents' numeracy input, the extent to which they focus on more advanced early numeracy skills that are within preschoolers' zone of proximal development, seems particularly important for supporting children's numeracy development. Several researchers identify simple arithmetic and symbolic magnitude comparison as the most advanced early numeracy skills since these focus on numerical operations unlike other early numeracy skills like rote counting (e.g., Skwarchuk et al., 2014). A systematic review revealed that the frequency of parents' numeracy support focused on the most advanced numeracy concepts, but not the frequency of their numeracy support focused on more foundational concepts, was positively associated with children's early mathematical skills (Mutaf-Yıldız et al., 2020). Notably, parents report providing input about the most advanced numeracy concepts significantly less often than input about foundational number concepts according to several studies (Blevins-Knabe & Musun-Miller, 1996; Ramani et al., 2015; Saxe et al., 1987; M. Susperreguy et al., 2020; Zippert & Ramani, 2017). Specifically, parents reported engaging their children in foundational number activities such as counting objects multiple times per week while only engaging in advanced number activities like simple arithmetic about once per month on average (e.g., Blevins-Knabe & Musun-Miller, 1996; Missall et al., 2015; Thompson et al., 2017; Zippert & Rittle-Johnson, 2020). Interestingly, Zippert and Ramani (2017) found that while every parent reported some engagement in foundational numeracy activities, several parents reported that they never engaged their preschoolers in advanced numeracy activities. Additionally, parents' observed number talk (without researcher input) has rarely been about advanced number concepts (i.e., arithmetic and magnitude comparison, e.g., Ramani et al., 2015; Vandermaas-Peeler et al., 2012). Thus, past research suggests that parents of preschoolers tend to miss opportunities to provide input about more advanced ways of understanding and using numbers that can help push their children's numeracy development. This might be an indication that parents have little knowledge about early numeracy development and may benefit from information about the numeracy concepts that preschoolers can typically develop, and which may be in their child's zone of proximal development.

Parent-child Patterning Experiences

The nature and role of parents' early patterning support are less understood than their numeracy support. There is growing evidence that parents' efforts to support their children's patterning development relate to their children's patterning and broader math skills (Rittle-Johnson et al., 2015; Zippert et al., 2020; Zippert & Rittle-Johnson, 2020). Additionally, there seems to be wide variability in parents' support of more foundational patterning skills like identifying patterns, and very little support for more advanced patterning skills (i.e., extending patterns, linking or abstracting patterns, and identifying pattern units; Zippert et al., 2020). Further, variability across small-scale studies in which parents were asked directly about the frequency of their patterning support suggests that there is wide variability among parents and that additional research with larger samples is needed.

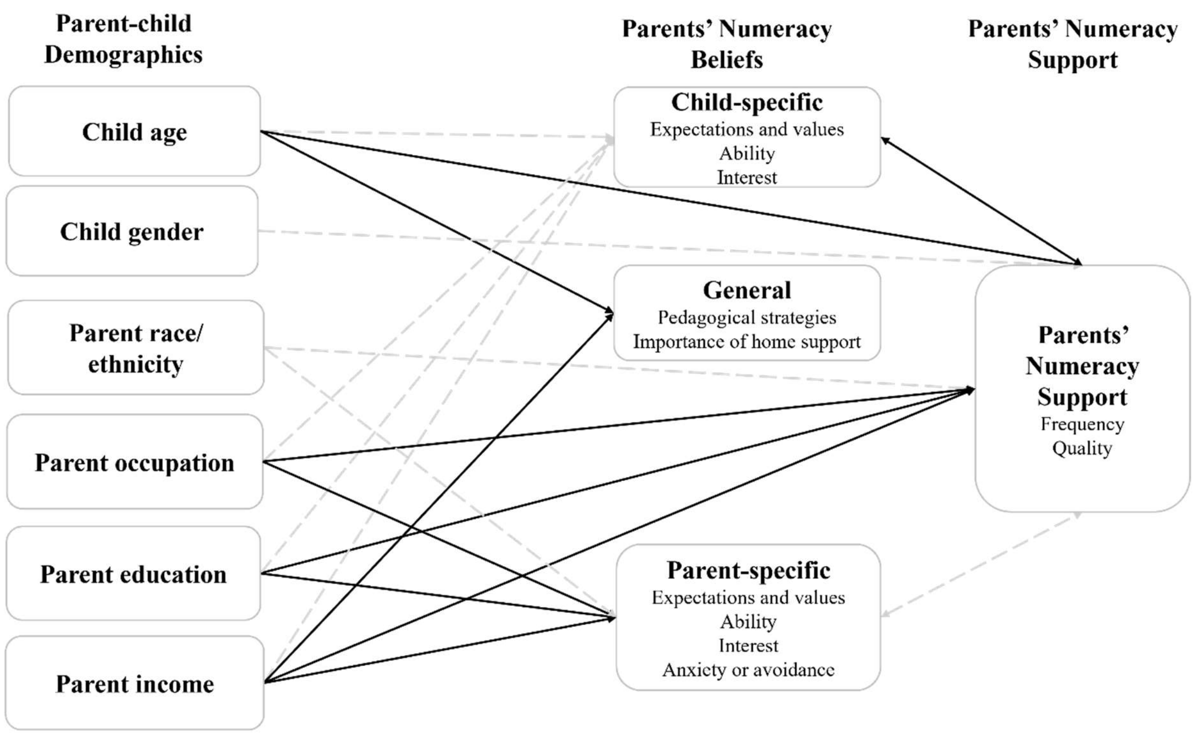
In *summary*, parents vary substantially in the frequency and complexity of the early numeracy and patterning support that they provide to their children. Further, variability in the frequency and complexity of their support is related to their children's early math knowledge. Thus, the factors that influence parents' early math support must be understood especially given the importance of early math skills for later academic and life outcomes.

Parents' Math Beliefs About Their Children

The parent socialization model posits that parents' beliefs affect their academic socialization of their children which in turn influences their children's academic knowledge (Eccles et al., 1983; Jacobs et al., 2004). This theory has been extended to include preschool children and the math-related beliefs and parent-child experiences which are relevant for this age group (e.g., Skwarchuk et al., 2014). A recent review summarized in the Parent Early Numeracy Socialization (PENS) model revealed that parents hold several types of numeracy-related beliefs (Douglas et al., 2021; see Fig. 1). Parents' child-specific numeracy beliefs were found to be more consistently related to their numeracy support than their numeracy beliefs about themselves and their more general numeracy beliefs. Further, parents hold several types of child-specific numeracy beliefs which are uniquely and differentially related to their early numeracy support. Next, I briefly describe the types of child-specific numeracy beliefs that have been studied and how they relate to parents' early numeracy support and review the literature on parents' child-specific patterning beliefs.

Figure 1

Parent Early Numeracy Socialization (PENS) Model



Note. Broken lines indicate relations for which there is mixed evidence.

Expectations and Values

Eccles and colleagues discuss "parents' expectations for their child's probable success at [a] task" (Eccles, 1993, p. 154) and "[parents] expectations for their child's performance in math" (Eccles1993, p. 156). This is similar to Zippert & Rittle-Johnson's (2020) question "How well do you think your child will do in each of these areas in Kindergarten?" and Kleemans and colleagues' (2012) question “To what extent do you expect your child to have mastered the following skills at the end of kindergarten?”). Zippert and Rittle-Johnson (2020) did not report whether this belief was related to parents’ support or children’s skills, however, Kleemans and colleagues' (2012) found that it was a unique predictor of children’s math skills, but was unrelated to parents’ math support.

I argue that other studies have reported on measures of parents’ *value* of their child’s abilities rather than *expectations* for their children’s numeracy, patterning, or math development given the operationalization of the variable (Elliott & Bachman, 2018; LeFevre et al., 2009; Skwarchuk et al., 2014; Susperreguy et al., 2020; Susperreguy et al., 2022). Specifically, parents have been asked ‘‘How important is it for your child to achieve each of the following benchmarks before starting Grade 1?’’ and “How important is it to you that your child does well in each of these activities?”. These questions seem to measure parents’ beliefs about *attainment task value* for their children which Eccles and colleagues defined “as the relative personal/identity-based *importance* attached by individuals to engage in various tasks or activities” (Eccles & Wigfield, 2020, p. 5) and discussed as “parents’ view of the value of the task for each particular child” (Eccles et al., 1993, p. 154). I also argue that Zippert & Rittle-Johnson (2020) measured a second type of parental value belief with their question “How useful do you think each of these kinds of skills will be to your child in the future?” as this seems like an operationalization of *utility value* which is described as relating to “how well a particular task fits into an individual's present or future plans” (Eccles & Wigfield, 2020, p., 5)

Parents rate their values about their preschoolers’ numeracy development (described as *expectations* in the home math environment literature) as neutral to high on average (e.g., Skwarchuk et al., 2014; Zippert & Rittle-Johnson, 2020). Notably, parents’ values about their children's numeracy development are positively related to the frequency and complexity of their numeracy support (Napoli et al., 2021; Skwarchuk et al., 2014; M. Susperreguy et al., 2020). Specifically, parents who reported higher values about their children's numeracy development also reported more frequent numeracy support, including support focused on more complex numeracy concepts.

To date, only one study has examined parents' patterning beliefs. Parents reported high values of their preschoolers' patterning development on average (Zippert & Rittle-Johnson, 2020). However, their values about their preschoolers' patterning development were not significantly related to the frequency of their reported patterning support. Notably, the study only had sufficient power to detect a large effect while studies that have reported significant relations between parents' values for their preschoolers' numeracy development and their numeracy support yielded small and medium effects. Thus, a study with a larger sample might detect a significant relationship between parents' values about their preschoolers' patterning development and their patterning support. Research examining how parents' values for their preschoolers' patterning development are related to the complexity of their patterning support is also needed. Taken together, more research is needed on both parents' *expectations* for and *value* of their children's early math development (as operationalized in alignment with the situated expectancy-value theory).

Ability

Parents tend to rate their 3- to 5-year-olds' early numeracy ability as average to very good (Huntsinger et al., 2016; Zippert & Rittle-Johnson, 2020) and to be fairly accurate at rating their children's specific numeracy abilities in comparison to direct assessments of children's skills by experimenters (Huntsinger et al., 2016; LeFevre et al., 2002; Lin et al., 2021; Zippert & Ramani, 2017). When parents were inaccurate, they tended to overestimate rather than underestimate their children's numeracy abilities (Zippert & Ramani, 2017). Further, parents were less accurate about their children's knowledge of more advanced rather than foundational numeracy concepts. Parents who rated their child's numeracy ability higher tended to also report supporting numeracy at home more frequently (Uscianowski et al., 2020; Zippert & Rittle-Johnson, 2020). Their beliefs were also positively related to the complexity of their reported numeracy support (Uscianowski et al., 2020; Zippert & Ramani, 2017). Thus, there is consistent evidence of positive relations between parents' beliefs about their children's numeracy ability and the frequency and complexity of their numeracy support.

Parents believed their preschoolers were fairly good at patterning activities on average (Zippert & Rittle-Johnson, 2020). Additionally, parents' beliefs about their preschoolers' patterning ability were positively related to the frequency of their reported patterning and broader math support in the same study. Notably, no study has examined how parents' beliefs about their preschoolers' patterning ability relate to the complexity of their patterning support.

Interest

Parents report believing that their 3- to 5-year-olds like numeracy activities "fairly well" to "very much" according to four studies (Fluck et al., 2005; Huntsinger et al., 2016; Saxe et al., 1987; Zippert & Rittle-Johnson, 2020). However, previous research only provides anecdotal evidence that parents' beliefs about their children's numeracy interests are related to the frequency of their numeracy support. Specifically, during an interview, some parents spontaneously reported that they have observed that their children were interested in numeracy and that they provided more frequent numeracy support in response to that interest (Cannon & Ginsburg, 2008). However, in a different study, parents' beliefs about their children's numeracy interests were not significantly correlated with their self-reported frequency of numeracy support, but this study was only powered to detect a large effect (Zippert & Rittle-Johnson, 2020). As noted previously, a study with a larger sample might reveal smaller significant relations. Additionally, no study has examined whether parents' beliefs about their children's numeracy interests relate to the complexity of their numeracy support.

As with parents' other patterning beliefs, little is known about how they view their preschoolers' interest in or liking of patterning activities. Parents reported that they believed their preschoolers were fairly interested in patterning activities on average (Zippert & Rittle-Johnson, 2020). However, parents' beliefs about their preschoolers' interest in patterning were unrelated to the frequency of their patterning support in the same, fairly small study.

In sum, parents' child-specific numeracy beliefs are often related to the frequency and complexity of their numeracy support. However, little is known about how parents' child-specific patterning beliefs relate to their patterning support. Thus, additional research is needed on the extent to which parents' child-specific patterning beliefs influence the patterning support that they provide their children. Such research will clarify how the PENS model can be broadened to include parents' early patterning support.

Parents' Knowledge of Math Development

Parents' knowledge about early math development might help explain variability in their math beliefs and support. Indeed, a recent study suggests that more positive parental numeracy beliefs can be fostered by providing parents with information about a numeracy skill (Douglas &

Rittle-Johnson, in prep). Further, previous research on parents' broader academic support and parenting suggests that parents' knowledge about development is important (Rowe et al., 2016; Sonnenschein & Sun, 2017). Specifically, parents' knowledge about child development, including their awareness of developmental norms and milestones, and of strategies for promoting children's growth, is theorized to shape their parenting beliefs and their efforts to support their children's development (e.g., how they interact with their children including the parenting strategies they use; (Bornstein et al., 2010). Indeed, parents' knowledge about child development is predictive of their academic support and their children's academic skills (Rowe et al., 2016; Sonnenschein & Sun, 2017). For example, parents' knowledge of child development while their children were 9 months old predicted the frequency of their literacy support to their children when they were 4 years old which in turn predicted children's reading and math skills at kindergarten entry (Sonnenschein & Sun, 2017). Additionally, parents' knowledge about child development helped explain differences in the frequency of their academic support and their children's academic skills that were associated with SES (Rowe et al., 2016; Sonnenschein & Sun, 2017).

Importantly, experimental research indicates that parents' knowledge about child development is malleable and is causally related to their beliefs and academic support as well as their children's outcomes. For instance, providing mothers with information about child development via baby books during their child's first year improved their maternal self-efficacy (belief about their ability to parent) and their children's language skills which were measured when their children were 1.5-years-olds (Albarran & Reich, 2014). Similarly, providing mothers with information about child development via baby books during their child's first year improved their beliefs about the importance of reading and the value of having resources to support reading (Auger et al., 2014). Thus, existing evidence indicates that parents' knowledge about development is an important and malleable predictor of their efforts to support their children's development.

However, very little is known about how parents' knowledge about early math development relates to their early math beliefs and support. A few studies suggest that parents vary in their knowledge about early numeracy development. Notably, there is some evidence that parents have limited accurate knowledge of early numeracy development (DeFlorio & Beliakoff, 2015; Fluck et al., 2005; Skwarchuk, 2009; Zippert & Ramani, 2017), but little is known about how this relates to their numeracy beliefs or support. Further, no study has examined parents' knowledge about early patterning development. Thus, research is needed on the nature and role of parents' knowledge about early patterning and numeracy development. Past research on parents' broader academic support and parenting highlights the potentially important role this knowledge may play.

Parents' Socioeconomic Status (SES)

Parents' knowledge about early math development may be an important pathway through which their socioeconomic status affects their math beliefs and support and their children's math skills. Several studies have shown that indicators of parents' financial resources are positively related to their numeracy beliefs and support (e.g., Casey et al., 2018; DeFlorio & Beliakoff, 2015; Vandermaas-Peeler et al., 2009) and their patterning support (Zippert & Rittle-Johnson, 2020). Further, one study suggests that parents' knowledge about early math development varies with their financial resources (Deflorio & Beliakoff, 2015). Specifically, parents with more financial resources (those who were not eligible for state or federally subsidized preschool and could pay tuition for private school) demonstrated a more accurate understanding of which math skills are within most 5-year-olds developmental range compared to parents with less financial resources. It is unknown whether parents' knowledge about early math development also varies with their highest educational attainment, another aspect of their SES which is often positively related to their numeracy beliefs and support (Gaylord et al., 2020; Thompson et al., 2017). Further, no study has examined how parents' knowledge about early math development relates to their math beliefs and efforts to support their children's math development or whether it helps explain SES differences in their beliefs or support. Notably, parents' knowledge about child development helped explain differences in the frequency of their academic support and their children's academic skills that were associated with parents' highest educational attainment (Rowe et al., 2016). Additionally, improving parents' knowledge of child development among a predominantly low-income sample led to parents having higher maternal self-efficacy and their children having better language skills (Albarran & Reich, 2014). Thus, parent knowledge of math development might be an important and malleable source of SES-related variability in their math beliefs and support.

An Updated Parent Socialization Model for Early Math Development

The literature on the role of parents in children's early math development aligns with the proposed pathways theorized in socialization models including the Parent Early Numeracy Support (PENS) model (Douglas et al., 2021). However, the PENS model needs to be expanded to capture additional factors that influence parents' efforts to support their children's early academic development. In addition, there are several gaps in the literature, especially pertaining to the role of parents in supporting children's early patterning development. The current study helps expand theory about and provide the necessary evidence for the role of parents' knowledge about early math development, child-specific math beliefs, and socioeconomic status in parents' efforts to support their children's early math development.

I propose that parents' knowledge about early math development is an important factor to include in an updated parent socialization model for math development. Evidence from parents' broader academic support suggests that their knowledge about development influences their academic support and their children's academic skills (Rowe et al., 2015; Sonnenschein & Sun, 2017). Thus, I theorize that parents' knowledge about early math development (i.e., about patterning and numeracy) is related to their beliefs about their children's math ability, their expectations and values for their preschoolers' math development, and their math support. I also predict that parents' knowledge about early math development is positively related to their highest educational attainment and that it may help explain the relationship between parents' SES and math beliefs as well as between parents' SES and math support.

In *summary*, the literature on the role of parents in children's early math development aligns with the proposed pathways theorized in existing parent socialization models (Douglas et al., 2021; Eccles et al., 1983). However, additional research is needed to expand the models to better understand the nature and role of parents' knowledge about early patterning and numeracy development in their efforts to support their children's patterning and numeracy development. Importantly, this will allow for the development of more informed strategies for helping parents to better support their children's math development.

Current Study

The current study aimed to understand the nature and role of parents' knowledge about early patterning and numeracy development in their efforts to support their children's patterning and numeracy development. More specifically, I aimed to develop and validate a measure of parents' knowledge about early math development and to answer two main research questions. First, how does parents' knowledge about math development relate to their child-specific math beliefs and support, and their socioeconomic status? I hypothesize that parents' knowledge about numeracy development will be positively related to their expectations for their child's numeracy development, their value of their child's numeracy abilities, the frequency and complexity of their numeracy support, and their highest educational attainment. I will explore how parents' knowledge about numeracy development relates to their other child-specific numeracy beliefs (i.e., their beliefs about their child's numeracy ability and interest) and other parent-child demographic characteristics including their income level. I make the same hypotheses for patterning.

Second, to what extent does parents' knowledge about numeracy and patterning development uniquely predict their numeracy and patterning support? I hypothesize that parents' knowledge about early numeracy development will be a positive predictor of the frequency and complexity of their numeracy support at home, above and beyond their child-specific numeracy beliefs (i.e., expectations for their child's numeracy development, values regarding their child's numeracy abilities, their beliefs about their child's numeracy ability, and their beliefs about their child's numeracy interest), socioeconomic status (i.e., their income level and highest educational attainment), and child's age. Likewise, I make the same hypotheses for patterning.

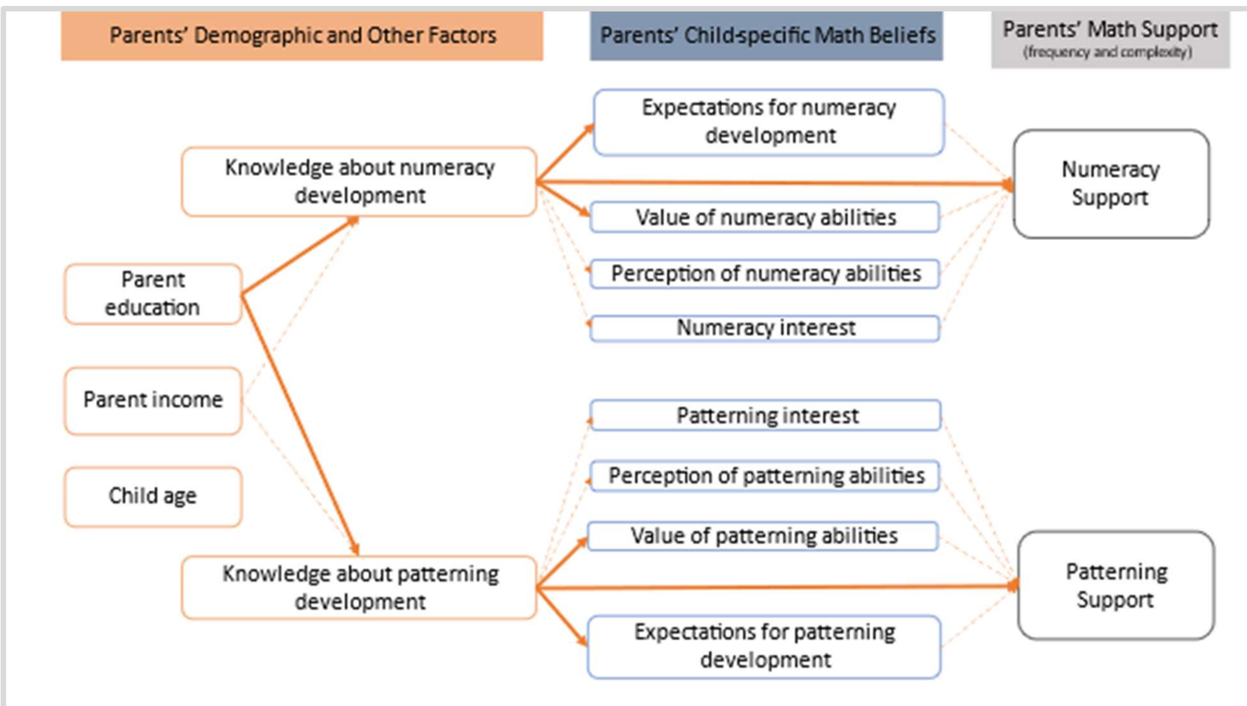
Researcher Positionality

I identify as a lifelong learner and a teacher who values education and believes that all individuals deserve adequate educational opportunities, access, and preparedness. I also identify as a first-generation scholar and a Black, Jamaican woman who engaged in P-12 education in formal and informal spaces in Jamaica and higher education in the US including at Fisk

University. I grew up knowing that my family and friends had high expectations for my academic and life success. I enjoyed academic success and was otherwise involved at school. Additionally, math was my favorite subject throughout my preschool to high school education and one of my earliest memories is doing math with my dad. I acknowledge that my beliefs and positionalities have shaped my research interests and may otherwise be reflected in my research processes as is true for all researchers (Holmes, 2020).

Figure 2

Model showing main and exploratory hypotheses tested in the current study.



Note. Solid lines indicate the main hypotheses.

METHOD

Participants

Three hundred and forty-four parents of 3- and 4-year-olds participated in the study. According to analyses conducted by Gatsonis and Sampson (1989), the study would be sufficiently powered for 5 bivariate correlational analyses as needed to test the first hypothesis with a sample of 317 (R = .20, p = .05). Additionally, an a priori power analysis indicated that a sample of 343 participants would have 80% power to detect a small to medium effect (f^2 = .0572) of seven predictors on parents' math support in a linear multiple regression. The effect size used in the power analysis was estimated from previous research on the relationship between parents' value of their child's numeracy development and their numeracy support; Skwarchuk et al., 2014; Susperreguy et al., 2020). Notably, the power analysis was conducted with a Bonferroni-corrected alpha to account for the number of linear multiple regressions needed to test the second hypothesis. Specifically, an alpha of .0125 was used (alpha divided by the number of analyses i.e., .05/4) given that one regression would be conducted for each of four dependent variables (i.e., the complexity of numeracy support, the frequency of numeracy support, the complexity of patterning support, and the frequency of patterning support).

A second a priori power analysis conducted with the Bonferroni-corrected alpha described above indicated that a sample of 405 participants would yield sufficient power to detect a small to medium effect (f^2 = .0572) of three additional predictors. As such, I aimed to recruit a larger sample than the number needed for the planned test of the second hypothesis (n = 405) so that I could include up to three additional variables identified during preliminary analyses as predictors. However, I was not able to recruit a larger sample in a reasonable amount of time so only seven previously identified variables will be included as predictors of parents' math support.

Demographics

Similar percentages of parents reported about their 3-year-old (52%) or 4-year-old (48%) while more parents reported about their sons (61%). About half of their children had attended preschool during the previous school year (2020-2021; 58%) and did not receive special education services at school (62%). Almost all parents reported that their 3- or 4-year-old heard English at home (99%). About half of the parents had more than one child (55%), including 20% with children who were 5 years or older.

Most parents identified as the child's primary caregiver (94%) and about half were mothers (57% including 1 grandmother). I aimed to recruit a sample that is representative of the national population in terms of educational attainment (US Census Bureau, 2020); however, the current sample is more highly educated than the US population. Specifically, over half of the parents had a bachelor's degree (55%), while 21% had some college education, a 2-year degree or less education, and 24% had some graduate education or a graduate degree. Parents also reported the highest educational attainment of the child's other parent or legal guardian if applicable, with over half having a bachelor's degree (see Table 1). About two-fifths of the parents reported a household income of \$45,000-\$89,000 (41%), while 28% reported \$44,999 or less, and 31% reported \$90,000 or more (see Table 2). A chi-square test indicated that participating parents' highest level of educational attainment is related to their household income, $X2$ (4, $N = 344$) = 49.01, $p < .001$ (see Table 3). Most parents reported receiving financial assistance for their child's preschool attendance (65%).

Table 1

Highest Educational Attainment of Current Sample and US Population

Highest Educational Attainment	Percentage of Participants		
	Participating Parent	Child's Other Parent	US Population
Elementary (or middle school)	0.6	0.4	3.0
Some high school	0.3	1.5	6.5
High school diploma or GED	5.2	13.1	27.8
Some college or 2-year degree	15.1	15.0	27.6
Bachelor's degree	54.9	53.1	22.1
Some graduate work	2.6	0.8	-
Master's, professional or doctoral degree	21.2	16.2	12.7

Table 2

Distribution of Household Income for Current Sample

Household Income	Percentage of Participants
Less than $27,000	7.3
$27,000 to $44,999	20.3
$45,000 to $89,999	40.7
$90,000 to $134,999	25.3
$135,000 or more	6.4

Note. Income bins were determined using the PEW research formula for low, middle, and high income and the US median income of $67,521 according to the 2020 US census.

Table 3

Distribution of Parents' Educational Attainment by Their Household Income

Highest Educational Attainment	n	Income		
		Less than $45,000	$45,000 - $89,000	$90,000 or more
Less than a bachelor's degree	73	46%	41%	12%
Bachelor's degree	189	28%	45%	28%
More than a bachelor's degree	82	11%	31%	59%

Most parents were White (77%) while 8% were Black or African American, 5% were Asian or Pacific Islander, 4% were Biracial or Multiracial, and 3% were American Indian or Alaska Native. A few parents indicated that they did not identify as any of the previously described races or ethnicities (2%) or were unsure about or preferred not to share their race (1%). Additionally, 20% of parents identified as Hispanic or Latine. Most were employed either full-time (79%) or part-time (11%) and some were pre-K or elementary school (36%) teachers. About half of the parents reported that they had previously or were currently participating in a program where they receive information about family engagement. Parents were from 46 states across the United States (see Table 4).

Measures

Knowledge about Early Math Development Survey

A previously used measure was adapted to assess parents' knowledge about early numeracy and patterning development (Deflorio & Beliakoff, 2015). Specifically, the measure was adapted to measure a wider variety of numeracy and patterning skills in a series of five rounds of pilot data collection, analysis, and revision with 288 parents not included in the current sample. See Appendix A for a more detailed description of how the current measure differed from the previous measure as well as each round of analysis done to revise the measure. Parents in the current study were presented with a list of 10 numeracy and 12 patterning abilities or skills that typically emerge between ages 3 and 8 years among typically developing children in the United States (e.g., ''Name the written numbers from 1 to 10…'', "Count a row of 15 objects…''), as listed in Table 5 and 6. They were asked whether they believe most children in the United States have developed each skill by their fifth birthday. Parents also reported on 10 spatial skills which served as distractors. Parents' affirmative responses to the 7 numeracy and 9 patterning items about skills that a majority of children have by the time they are 5 years old (Claessens & Engel, 2013; Clements & Sarama, 2014; Litkowski et al., 2020) were scored as 1. Parents' negative responses to 3 numeracy and 3 patterning skills that are not typically present until after age 5 were also scored as 1. All other responses were scored as 0. I report on descriptive statistics for, and reliability and validity of the revised measure in the results section.

Child-Specific Math Beliefs Survey

The parental beliefs survey was composed of items adapted from previous instruments (Eccles et al., 1983; LeFevre et al., 2009; Zippert & Rittle-Johnson, 2020). Parents reported on survey items about their child-specific numeracy and patterning beliefs. They also reported on distractor topics such as early literacy and language skills. They rated their beliefs on 7-point Likert scales.

Values of Numeracy and Patterning Abilities. Parents were asked "How important is it for your child to achieve each of the following benchmarks before first grade?" and "How useful do you think each of these kinds of skills will be to your child in the future?". Their ratings of the five numeracy items (e.g., "Counting and naming numbers" and "Know simple sums (for example, 2 + 2)") were averaged as a measure of their value of their child's numeracy abilities (α = .65). Likewise, their ratings of the five patterning items (e.g., "Noticing and making patterns" and "Identify the part that repeats in a pattern") were averaged as a measure of their value of their child's patterning abilities (α = .79).

Expectations for Numeracy and Patterning Development. Parents were asked, "How well do you think your child will do in each of these areas in Kindergarten?". Their ratings of the two numeracy items (i.e., “Counting and naming numbers” and “Comparing the magnitudes (size) of numbers”) were averaged as a measure of their expectations for their child’s numeracy development (α = .71). Likewise, their ratings of the two patterning items (i.e., “Noticing and making patterns” and “Figuring out what should come next in patterns”) were averaged as a measure of their expectations for their child’s patterning development (α = .87).

Abilities. Parents were asked, “How good is your child currently in each area listed below?”. Their ratings of the two numeracy items (i.e., “Counting and naming numbers” and “Comparing the magnitudes (size) of numbers”) were averaged as a measure of their perception of the level of their child’s numeracy abilities (α = .70). Likewise, their ratings of the two patterning items (i.e., “Noticing and making patterns” and “Figuring out what should come next in patterns”) were averaged as a measure of their perception of the level of their child’s patterning abilities (α = .85).

Interest. Parents were asked, “How much does your child like each of the following activities?”. Their ratings of the two numeracy items (i.e., “Counting and naming numbers” and “Comparing the magnitudes (size) of numbers”) were averaged as a measure of their perception of their child’s numeracy interests (α = .67). Likewise, their ratings of the two patterning items (i.e., “Noticing and making patterns” and “Figuring out what should come next in patterns”) were averaged as a measure of their perception of the level of their child’s patterning interests (α = .90).

Table 4

Distribution of Participating Parents' State of Residence

States	Percentage of Participants
California	13.4
Texas	9.6
Florida	5.8
Indiana	5.2
New York	5.2
Alabama	4.9
Pennsylvania	4.4
Washington	4.1
Ohio	3.8
Colorado	3.5
Louisiana	3.5
Illinois	3.2
New Jersey	2.9
North Carolina	2.9
Tennessee	2.6
Georgia	2.0
Kentucky	2.0
Massachusetts	2.0
Michigan	2.0
South Carolina	1.7
Maryland	1.5
Utah	1.5
Wisconsin	1.2
Minnesota, Nevada, Oklahoma, Oregon	0.9[a]
Arizona, Iowa, Missouri, Montana, Nebraska, Virginia, West Virginia	0.6[b]
Arkansas, Connecticut, Delaware, Hawaii, Idaho, Kansas, Mississippi, New Mexico, North Dakota, South Dakota, Vermont, Wyoming	0.3[c]

Note.

[a] Each state had 3 participating parents (0.9%). [b] Each state had 2 participating parents (0.6%). [c] Each state had 1 participating parent (0.3%).

Table 5

Descriptive Statistics for Measure of Parents' Knowledge about Early Numeracy Development

Knowledge measure item	*M (SD)* Overall	Less than Bachelor's	Bachelor's	More than Bachelor's	Cronbach's Alpha if item deleted	Item-total correlation	Standardized Factor Loading	Order of Items
Within Developmental Range								
Count a row of 15 objects (for example, count 15 plastic worms)	.94 (.23)	.96(.20)	.94(.23)	.94(.24)	0.59	0.16	-	1
Counts out the correct number of things when asked for a specific number of things up to 10 (for example gives 6 cookies when asked for 6 cookies)	.89(.32)	.95(.23)	.88(.32)	.84(.37)	0.56	0.3	0.43	13
Name the written numbers from 1 to 10 (for example, points to the 9 when asked "where is the number nine?"	.86(.35)	.95(.23)[a]	.87(.33)	.74(.44)	0.54	0.36	0.53	14
Solve small addition or subtraction problems presented with objects (for example, 3 blocks and 2 blocks is ___ blocks)	.82(.39)	.78(.42)[b]	.86(.35)	.74(.44)	0.56	0.29	0.27	16
Tell which of two spoken numbers between one and ten is bigger (for example, says "five" in response to "Which is bigger, five or two?"	.76(.43)	.84(.37)	.76(.43)	.67(.47)	0.53	0.36	0.51	23
Tell which of two written numbers between one and ten is bigger (for example, points to the written number 9 when shown the written numbers 2 and 9 and asked "Which is bigger")	.76(.43)	.77(.43)	.77(.42)	.72(.45)	0.53	0.38	0.49	25
Answer questions by adding or subtracting small numbers (for example, says "three" in response to "If you have four stickers and then you give me one of your stickers, how many stickers would you have left?")	.68(.47)	.66(.48)	.70(.46)	.67(.47)	0.56	0.31	0.34	29
Beyond Developmental Range								
Solve single-digit addition or subtraction problems presented on flashcards (for example, 5 + 3= ___)	.44(.5)	.62(.49)[c]	.35(.48)	.51(.50)	0.69	0.37	0.46	8
Recite number words from 1 to 100	.31(.46)	.51(.50)[a]	.24(.43)	.30(.46)	0.49	0.52	0.71	11
Name the written numbers from 1 to 100 (for example, says the word "ninety-three" when shown the written number 93)	.37(.48)	.56(.50)[a]	.31(.46)	.35(.48)	0.46	0.52	0.73	27

Note. Statistics for items within each subscale.

[a] Parents without a bachelor's degree had higher knowledge than other parents, *p*'s < .05.

[b] Parents with a bachelor's degree had higher knowledge than parents with a graduate degree, *p*'s < .05.

[c] Parents without a bachelor's degree had higher knowledge than parents with a bachelor's degree who in turn had higher knowledge than parents with a graduate degree, *p*'s < .05

Table 6

Descriptive Statistics for Measure of Parents' Knowledge about Early Patterning Development

Knowledge measure item	*M(SD)* Overall	Less than Bachelor's	Bachelor's	More than Bachelor's	Cronbach's Alpha if item deleted	Item-total correlation	Standardized Factor Loading	Order of Items
Within Developmental Range								
Copy a pattern someone else makes in the same way (for example, your child beats a drum in a loud-soft pattern just like do)	.81(.39)	.92(.28)[a]	.81(39)	.73(.45)	0.63	0.30	0.29	32
Use colored beads to make a simple pattern, such as a "blue-purple" pattern	.80(.40)	.81(.40)	.81(.39)	.76(.43)	0.63	0.29	0.3	3
Continue a pattern of cubes (for example, blue, blue, red, red, blue, blue, red, red, _______, _______, _______, _______)	.78(.42)	.84(.37)	.77(.42)	.74(44)	0.62	0.33	0.43	2
Makes a repeating pattern (for example, makes a clap, spin, snap, clap, spin, snap pattern)	.76(.43)	.78(.42)	.79(.41)	.66(.48)	0.61	0.39	0.47	31
Figure out what should come next in a simple pattern (for example: clap, stomp, clap, stomp, _______, _______)	.74(.44)	.88(.33)[a]	.69(.46)	.72(.45)	0.62	0.35	0.41	4
Make the same kind of simple pattern in their bracelet as their friends' bracelet, but using different colors (for example, your child makes a yellow-green pattern to match a friend's red-blue pattern)	.72(.45)	.55(.50)[b]	.75(.44)	.82(.39)	0.63	0.30	0.44	22
Fill in the missing part of a pattern made of repeating objects (for example: circle, square, square, circle, square, ____, circle, square, square)	.65(.48)	.70(.46)	.68(.47)	.56(.50)	0.59	0.46	0.58	18
Identify two patterns that follow the same rule made with different materials (for example, a block-block-ball pattern and a sun-sun-moon pattern are similar)	.63(.48)	.53(.50)	.67(.47)	.62(.49)	0.62	0.35	0.51	10
Beyond Developmental Range								
Use number patterns to solve problems (for example, fills in the blanks when presented with 26, 22, 18, __, 10, __)	.58(.50)	.84(.37)[a]	.49(.50)	.56(.50)	0.63	0.62	0.77	12
Figure out what comes next in a growing pattern that subtracts two cubes for each stair in a staircase (for example, 9 cubes, 7 cubes, 5 cubes, 3 cubes, __)	.55(.50)	.81(.40)[a]	.47(.50)	.51(.50)	0.65	0.6	0.76	20
Describe even numbers (for example, 2, 4, 6, 8, 10) as "skipping" every other number on a 100s chart	.49(.50)	.74(.44)[a]	.40(.49)	.49(.50)	0.74	0.53	0.61	30

Note. Statistics for items within each subscale.
[a] Parents without a bachelor's degree had higher knowledge than other parents, p's <.05.
[b] Parents without a bachelor's degree had lower knowledge than other parents, p's <.05.

Parent-Reported Math Support Survey

An adapted version of a survey used in previous research (Zippert & Rittle-Johnson, 2020; see Tables 7 and 8) served as a measure of parents' support of their preschoolers' numeracy and patterning development at home. Parents reported how frequently they engaged their preschoolers in numeracy activities (n = 15) and patterning activities (n =15) in the past month. Specifically, they rated their engagement in each activity with their preschooler using a 6-point Likert-type scale, where 0 = never, 1 = once a month or less, 2 = few times a month, 3 = about once a week, 4 = few times a week, 5 = almost daily or daily. They also reported how frequently they engage their preschooler in spatial activities (n = 7) using the same scale as distractors.

Most items were adapted from a previously used, reliable instrument (Cronbach alpha of numeracy and patterning subscales ranged from .75 to .90; Zippert & Rittle-Johnson, 2020). Zippert & Rittle-Johnson's (2020) measure was based on previous reliable instruments (Dearing et al., 2012; LeFevre et al., 2009; Rittle-Johnson et al., 2015). We added examples to several items to clarify the items and highlight activities that may be culturally salient for minoritized families within and outside of the US. Relatedly, we also added four additional game-related numeracy and patterning items including one focused on outdoor games which may be more popular in communities and countries with less access to manufactured play spaces and toys (e.g., Kinkead-Clark, 2021). We further adapted the survey to measure parent-child engagement in additional activities that correspond with early numeracy and patterning skills that are included in assessments of and standards around preschoolers' math development (e.g., (Clements & Sarama, 2014; National Research Council, 2009; Rittle-Johnson et al., 2015, 2020). Specifically, we added eight items that relate to recognizing, copying, and completing patterns, identifying the pattern unit, symbolic magnitude comparison, and arithmetic. Finally, we added six items (primarily open-ended) to allow parents to share activities not captured by the existing survey and to share additional details about how they support their children's numeracy and patterning development.

To measure the frequency of parents' numeracy support at home, parents' ratings of their engagement in all numeracy activities were averaged (n = 15; α = 91). Likewise, to measure the frequency of parents' patterning support at home, parents' ratings of their engagement in all patterning activities were averaged (n = 15; α = .95). To measure the complexity of parents' numeracy support at home, the frequency of parents' reported engagement in activities focused on symbolic magnitude comparison and simple arithmetic were averaged (n = 4; α = .84). To measure the complexity of parents' patterning support at home, the frequency of parents' reported engagement in activities focused on extending patterns, abstracting patterns, and identifying pattern units were averaged (n = 4; α = .91).

Procedure

Parents of 3- and 4-year-olds were recruited to participate in the study using CloudResearch. CloudResearch, formerly known as TurkPrime Data Acquisition Platform for the Social Sciences (TurkPrime), is an internet-based research platform that integrates with Amazon's crowdsourcing platform Mechanical Turk (MTurk; (Litman et al., 2017). After providing informed consent, parents completed surveys on their child-specific math beliefs, knowledge about early math development, math support at home, and their demographics. They also completed several attention checks that were embedded in the survey such as "To show that you are paying attention, please select the 'none of the above' option as your answer".

Participants who failed at least one attention check were not included in the study (i.e., the final sample includes 344 participants). Participants were paid $10 for completing the survey.

Table 7

Descriptive Statistics for Numeracy Home Activities

Item	*M (SD)*	Frequency Scale		Complexity Scale	
		Cronbach's alpha if item deleted	Item-Total Correlation	Cronbach's alpha if item deleted	Item-Total Correlation
Count items	2.23(.89)	0.90	0.49	-	-
Count out loud without objects	2.05(.94)	0.91	0.46	-	-
Talk about written numbers (for example, "That's a 7")	1.95(1.02)	0.90	0.52	-	-
Read books that show and talk about numbers (for example, "One Fish, Two Fish", "The Very Hungry Caterpillar")	1.88(1.01)	0.90	0.55	-	-
Watch TV shows or videos that show and talk about numbers (for example, "Peg + Cat", "Monster Math Squad")	1.85(1.03)	0.91	0.45	-	-
Compare quantities (for example, when playing card games or serving food for dinner, or sharing toys)	1.72(1.01)	0.90	0.58	-	-
Add simple sums or talk about number facts (for example, 2+2=4)	1.55(1.11)	0.90	0.65	0.79	0.69
Play computer games or use apps or interactive websites that include number games (for example, "Elmo Loves 123s", "PBS Kids math games")	1.52(1.13)	0.90	0.54	-	-
Compare written numbers (for example, "5 is bigger than 4")	1.49(1.11)	0.90	0.69	0.81	0.64
Practice subtracting items (for example, when playing with 2 toy cars, asking "How many cars will you have if I take away one of your cars?")	1.40(1.10)	0.90	0.68	0.78	0.71
Play board games that involve numbers (for example, "Chutes & Ladders", "Drafts/Checkers", "Ludi/Ludo", "Dominos")	1.28(1.13)	0.90	0.66	-	-
Create art that involves numbers	1.24(1.13)	0.90	0.66	-	-
Play hand or movement games that involve numbers (for example, "Slide/Back Front", "Down by the River", "Hide and Seek", "Chinese Skip/Chinese Jump Rope")	1.23(1.13)	0.90	0.62	-	-
Compare the cost of items when shopping (for example, "This milk costs less because it costs $3 and the other milk costs $4.")	1.22(1.17)	0.90	0.69	0.81	0.65
Play card games that involve numbers	1.21(1.12)	0.90	0.66	-	-

Note. 0 = Never, 1 = Once a Month, 2 = A few times a month, 3 = Once a week, 4 = A few times a week, 5 = daily

Table 8

Descriptive Statistics for Patterning Home Activities

Item	*M (SD)*	Frequency Scale		Complexity Scale	
		Cronbach's alpha if item deleted	Item-Total Correlation	Cronbach's alpha if item deleted	Item-Total Correlation
Read books that show or talk about patterns (for example, "Beep, Beep, Vroom, Vroom" or "Brown Bear Brown Bear What Do You See?")	1.62(1.02)	0.95	0.58	-	-
Recognize a repeating pattern (for example, stripes on a shirt)	1.47(1.07)	0.95	0.77	-	-
Watch TV shows or videos that show and talk about patterns	1.46(1.10)	0.95	0.64	-	-
Make patterns with objects or sounds (for example, putting blocks in a red-green-red-green pattern)	1.44(1.07)	0.95	0.76	-	-
Play computer games or use apps or interactive websites that include pattern games	1.43(1.12)	0.95	0.60	-	-
Figure out what comes next in a pattern	1.35(1.10)	0.95	0.79	0.89	0.75
Describe patterns in words	1.34(1.11	0.95	0.8		
Copy a pattern by making the same pattern with the same materials	1.30(1.04)	0.95	0.77	-	-
Figure out what part is missing in a pattern	1.32(1.11	0.95	0.82	-	-
Create art that involves patterns (for example, stripes or checkers)	1.32(1.07)	0.95	0.75	-	-
Figure out which part of a repeating pattern repeats over and over	1.27(1.10)	0.95	0.81	0.87	0.8
Discuss patterns in days of the week, months of the year, or seasons	1.26(1.10)	0.95	0.72	-	-
Talk about what makes two patterns similar (for example, say "Both patterns are an every-other-one pattern")	1.17(1.09)	0.95	0.84	0.87	0.82
Copy a pattern by making the same kind of pattern but with different materials (for example, use circles and squares to make the same kind of pattern as in a red-blue pattern)	1.17(1.07)	0.95	0.82	0.88	0.79

Note. 0 = Never, 1 = Once a Month, 2 = A few times a month, 3 = Once a week, 4 = A few times a week, 5 = daily

Analytic Plan

The analytic plan was preregistered (https://aspredicted.org/2YG_39S). The primary analyses were bivariate correlational analyses and regression analyses. First, bivariate correlations and one-way ANOVAs were conducted to answer the first research question (how does parents' knowledge about early math development relate to their child-specific math beliefs and support, and their socioeconomic status?). Relatedly, I explored how parents' knowledge about early math development relates to other parent-child demographic characteristics such as their child's age. Second, four linear multiple regressions were conducted to answer the second research question (to what extent does parents' knowledge about numeracy and patterning development uniquely predict their numeracy and patterning support). A linear multiple regression model was conducted for each of our dependent measures from the parent-reported math support survey, based on two dimensions: (a) numeracy or patterning and (b) frequency or complexity. Predictor variables in the model were parents' knowledge and child-specific beliefs about the particular topic (patterning or numeracy) and their socioeconomic status. Note, I mistakenly listed eight predictors while stating that I would include seven predictors based on the previously described power analysis in the preregistration. Preliminary analyses were conducted to identify which of the eight predictors is least related to parents' math support and therefore would be excluded from the main analysis. Analyses with all eight predictors are reported in Appendix B.

RESULTS

The primary aim of the current study was to understand the nature and role of parents' knowledge about early patterning and numeracy development in their efforts to support their children's patterning and numeracy development. More specifically, the current study aimed to develop and validate a measure of parents' knowledge about early numeracy and patterning development and to test two main hypotheses about the role of their knowledge.

Validating the Measure of Parents' Knowledge about Early Numeracy and Patterning Development

Reliability

Cronbach alpha was computed for all items about numeracy skills and revealed that they did not function as a reliable scale ($\alpha = .33$). Likewise, Cronbach alpha revealed that patterning items did not function as a reliable scale ($\alpha = .41$). Confirmatory Factor Analyses (CFA) and bivariate correlation analyses suggested that items corresponding with skills or abilities that are *within* the developmental range for a majority of 5-year-olds functioned differently than items corresponding with skills or abilities that are *beyond* the developmental range. Specifically, a model with one factor for numeracy items that are within the typical developmental range and a separate factor for numeracy items that are beyond the typical developmental range had a significantly better model-data fit than a model with all numeracy items on a single factor, $\chi2(3) = 157.51$, $p < .001$. In addition, the numeracy within subscale and the numeracy beyond subscale were negatively correlated, $r(342) = -.27$, $p < .001$. Similarly, a model with one factor for patterning skills that are *within* the typical developmental range for children 5 years old and younger and a separate factor for patterning skills that are *beyond* the typical developmental range had a significantly better fit than a model with all patterning items on a single factor, $\chi2(4) =119.19$, $p < .001$. Model fit statistics are reported in Table A2. The patterning subscales were also negatively correlated, $r(342) = -.32$, $p < .001$. Given that the current study focuses on parents of children less than 5-year-olds, the main analyses focus on parents' knowledge about items that are *within* the typical developmental range for children who are 5 years old or younger. Next, I discuss the reliability and validity of the numeracy and patterning subscales with items measuring skills within the developmental range. The reliability and validity of the numeracy and patterning subscales with items measuring skills beyond the developmental range are reported in Appendix C.

Cronbach alpha was computed for items focused on numeracy skills that are within the developmental range and revealed higher reliability ($\alpha = .59$) than the larger scale with all items, suggesting that a scale with these items would be a more appropriate measure of parents' knowledge about early numeracy development than the larger scale. The Kuder-Richardson Formula 20 (KR20), often considered a better measure of reliability for tests with dichotomous variables, yielded a similar estimate (.60). One item ("Count a row of 15 objects") had a low item-total correlation. As such, it was excluded from further analyses. Likewise, Cronbach alpha and KR20 were computed for patterning items that are within the developmental range and revealed higher reliability ($\alpha = .65$ and KR20 = .66) than the larger scale with all items, suggesting that a scale with these items would be a more appropriate measure of parents' knowledge about early patterning development than the larger scale. As with the numeracy-within subscale, one item ("Sort a set of objects into 3 groups based on color such as red, blue, and green") had a low item-total correlation and was excluded from further analyses.

Validity

Construct Validity. The previously described CFA models indicated construct validity. Specifically, all numeracy items loaded significantly onto a 2-factor model (with a factor for the within subscale and a factor for the beyond subscale) suggesting that the items measure the same theoretical construct (see Table 5). Importantly, the model fits the data well according to several indices such as a nonsignificant chi-square, Comparative Fit Index > .9, Root Mean Square Error of Approximation < .08, and Adjusted Goodness of Fit > .9. Likewise, all patterning items loaded significantly onto a 2-factor model (with a factor for the within subscale and a factor for the beyond subscale) suggesting that they measure the same theoretical construct (see Table 6). The 2-factor patterning model also fits the data well according to several indices such as a nonsignificant chi-square, Comparative Fit Index > .9, Root Mean Square Error of Approximation < .08, and Adjusted Goodness of Fit > .9.

Convergent Validity. As a measure of convergent or concurrent validity, I examined whether each subscale was significantly related to parents' beliefs about their child's corresponding ability. I found evidence of convergent validity for both parents' knowledge about early patterning and about early numeracy. Specifically, parents' knowledge about early patterning development (average of items corresponding to patterning skills within the developmental range except for the previously identified item) was significantly correlated with their perception of their child's patterning abilities, $r(342) = .24$, $p < .001$. Parents' knowledge about early numeracy development (average of items corresponding to numeracy skills within the developmental range except for the previously identified item) was also significantly correlated with their perception of their child's numeracy abilities, $r(342) = .11$, $p = .036$.

Discriminant Validity. I found evidence of discriminant validity for parents' knowledge about early numeracy development and early patterning development. Specifically, parents' knowledge about early numeracy development was not correlated with their perception of their child's literacy abilities, $r(343) = .05$, $p = .342$. Additionally, parents' knowledge about early patterning development was not correlated with their perception of their child's literacy abilities, $r(343) = .09$, $p = .112$.

Descriptive Analyses

Next, I examined the descriptive statistics for parents' knowledge about early numeracy and patterning development. Parents accurately classified 79% of items about numeracy skills that are within the developmental range for most five-year-old children ($M = .79$, $SD = .23$). Their knowledge about early numeracy development (average accuracy on skills within the developmental range) was substantially left-skewed as indicated by a skewness statistic that is less than -1 and a ratio of skewness to standard error of skewness that is less than 2 (skew = -1.09, SE = .13; ratio = -8.35, kurtosis = .94). Notably, the Item Response Theory (IRT) score of parents' knowledge about early numeracy development did not show substantial skew or kurtosis (skew = -.75, SE = .13, kurtosis = -.29), suggesting that it is a more statistically sound measure of parents' knowledge of numeracy development.

Parents accurately classified 74% of items about patterning skills that are within the developmental range for most five-year-old children ($M = .74$, $SD = .24$). Neither measure of parents' knowledge of patterning development showed substantial skew or kurtosis (total score: skew = -.63, SE = .13, kurtosis = -.31; IRT score: skew = -.31, SE = .13, kurtosis = -.89). For consistency, parents' IRT scored knowledge of numeracy and patterning development are used in future analyses.

How Does Parents' Knowledge About Early Math Development Relate to Their Child-Specific Math Beliefs and Support and Their Socioeconomic Status?

I hypothesized that parents' knowledge about math development would be positively related to their expectations for their child's math development, their value of their child's math abilities, the frequency and complexity of their math support, and their highest educational attainment, for both numeracy and patterning. I also explored how parents' knowledge about math development relates to their perception of their child's math abilities and interests. Findings are reported below and are summarized in Tables 9-11.

Parents' Knowledge About Early Numeracy Development

In partial support of the hypothesis, parents' knowledge about early numeracy was positively related to their beliefs about the value of their children's numeracy abilities, but not their expectations for their children's numeracy development. Parents' knowledge about early numeracy was positively related to their perception of their children's numeracy abilities and interests. Again, in partial support of the hypothesis, parents' knowledge about early numeracy was positively related to the frequency but not the complexity of their numeracy input. Contrary to my hypothesis, parents' knowledge about early numeracy development was negatively rather than positively related to their highest level of educational attainment. Specifically, post hoc comparisons revealed that parents with more education than a bachelor's degree had lower knowledge about children's early numeracy development than parents with a bachelor's degree and parents with less education than a bachelor's degree. Parents with a bachelor's degree did not have significantly different knowledge about children's numeracy development from parents with less education. Follow-up analyses on the individual knowledge measure items revealed that only parents' knowledge about preschoolers' ability to identify written numerals up to 10 and to solve simple arithmetic problems using manipulatives varied with their education.

To further explore the relationship between parents' knowledge about early numeracy development and their educational attainment, I examined whether parents' educational attainment and knowledge about early numeracy development covaried with their employment status. Parents' highest educational attainment was significantly related to their employment status, $X2$ (2, N = 344) = 75.02, $p < .001$, with most parents with graduate education and a bachelor's degree being employed full time (89%) and most parents with less education than a bachelor's degree being unemployed or employed part-time (58%). Additionally, parents' knowledge about early numeracy development was related to their employment status, with parents who were employed full-time having lower knowledge than parents who were employed part-time or who were unemployed (see Table 11). Notably, parents' knowledge was unrelated to their educational attainment after controlling for their employment status, $F(2, 340) = 2.69$, p = .069.

Parents' Knowledge About Early Patterning Development

As hypothesized, parents' knowledge about early patterning development was positively related to their beliefs about the value of their children's patterning abilities and their expectations for their children's patterning development. It was also positively related to their perception of their children's patterning abilities and interests. Additionally, as hypothesized parents' knowledge was positively related to both the frequency and complexity of their patterning input. However, contrary to my hypothesis, parents' knowledge about early patterning development did not vary significantly with their highest level of educational attainment.

Overall, the findings provided partial support for my hypotheses. Parents' knowledge about early math development was consistently related to their value of their child's math abilities and the frequency of their math support for both numeracy and patterning. However, parents' knowledge about early math development was only sometimes related to their expectations for their child's math development, the complexity of their math support, and their highest educational attainment.

Table 9

Correlations among parents' numeracy support, knowledge about early development, and their child-specific beliefs

Variable	1	2	3	4	5	6	*M(SD)*
1. Knowledge about early numeracy development	-						0.79(0.23)
2. Expectation for child's numeracy development	.08	-					5.61(1.23)
3. Value of child's numeracy abilities	.22***	.49***	-				4.75(0.66)
4. Perception of child's numeracy abilities	.11*	.51***	.49***	-			5.27(1.29)
5. Perception of child's numeracy interest	.19***	.51***	.44***	.76***	-		5.48(1.22)
6. Frequency of numeracy support	.12*	.30***	.42***	.62***	.54***	-	3.39(0.93)
7. Complexity of numeracy support	.07	.22***	.35***	.61***	.49***	.90***	3.15(1.30)

Notes. Parents' knowledge about early numeracy development is the average proportion correct while their beliefs and support are the average ratings on 7-point scales.
*** p < .001. * p < .05

Table 10

Correlations among parents' patterning support, knowledge about early development, and their child-specific beliefs

Variable	1	2	3	4	5	6	*M(SD)*
1. Knowledge about early patterning development	-						0.74(0.24)
2. Expectation for child's patterning development	.11*	-					5.74(1.06)
3. Value of child patterning abilities	.32***	.44***	-				4.56(0.82)
4. Perception of child's patterning abilities	.24***	.53***	.59***	-			5.14(1.38)
5. Perception of child's patterning interest	.23***	.52***	.58***	.80***	-		5.31(1.39)
6. Frequency of parents' patterning support	.30***	.20***	.48***	.59***	.57***	-	3.12(1.15)
7. Complexity of parents' patterning support	.30***	.17**	.46***	.59***	.57***	0.96***	2.95(1.36)

Notes. Parents' knowledge about early patterning development is the average proportion correct while their beliefs and support are the average ratings on 7-point scales.
*** $p < .001$. ** $p < .01$. * $p < .05$.

Table 11

Parents' Knowledge about Early Numeracy and Patterning Development by Their Socioeconomic Status

Variables	Numeracy			Patterning	
	N	M (SD)	F value	M (SD)	F value
Education					
Less than a bachelor's degree	73	0.84 (0.20)	3.93*	0.75 (0.24)	1.96
Bachelor's degree	189	0.83 (0.18)		0.75 (0.24)	
More than a bachelor's degree	82	0.76[a] (0.25)		0.70 (0.22)	
Income					
Less than $45,000	95	0.81 (0.21)	0.13	0.71 (0.24)	0.69
$45,000-$89,999	140	0.82 (0.18)		0.75 (0.21)	
$90,000 or more	109	0.81 (0.23)		0.74 (0.26)	
Child Receives Financial Aid [b]					
Yes	221	0.80 (0.23)	0.87	0.72 (0.24)	0.09
No	106	0.78 (0.24)		0.76 (0.23)	
Employment Status					
Full time	272	0.78(0.24)	7.23**	0.73(0.24)	1.48
Part time or unemployed	72	0.85(0.18)		0.76(0.22)	

Notes. ** $p < .01$. * $p < .05$

[a] Parents with more than a bachelor’s degree had significantly lower knowledge than parents with less education, $p < .05$.

[b] This excludes 17 parents who reported that they were unsure whether they receive financial assistance for their child to attend preschool.

To What Extent Does Parents’ Knowledge About Numeracy and Patterning Development Uniquely Predict Their Numeracy and Patterning Support

Preliminary Analyses

First, I examined the descriptive statistics for parents’ reported numeracy and patterning support at home. The average frequency of parents’ overall numeracy support at home (average rating of their engagement in 15 numeracy activities with their child) was about once per week (M = 3.39, SD = .93). The average frequency of their numeracy support *focused on more* *complex numeracy skills* (average rating of their engagement in four activities focused on symbolic magnitude comparison and simple arithmetic) was also about once per week (M = 3.15, SD = 1.30). There was not substantial skewness or kurtosis for the frequency (skewness = -.33, kurtosis = -.38) nor complexity (skewness = -.63, kurtosis = -.40) of parents' numeracy support given that values were between -1.00 and 1.00.

The average frequency of parents' patterning support at home (average rating of 15 patterning activities) was also about once per week (M = 3.12, SD = 1.15). The average frequency of their patterning support *focused on more complex patterning skills* i.e., four activities focused on extending patterns, abstracting patterns, and identifying pattern units was also about once per week (M = 2.95, SD = 1.36). Neither the frequency (skewness = -.44, kurtosis = -.43) nor complexity (skewness = -.46, kurtosis = -.63) of parents' patterning support had substantial skewness or kurtosis.

Next, bivariate correlations, one-way ANOVAs, and independent samples t-tests were used to determine which of the preregistered predictors would be included as covariates in the main analyses testing the effect of parents' knowledge about early math development on their math support. The frequency and complexity of parents' math support were significantly correlated with all four child-specific math belief variables (see Tables 9 and 10). Additionally, the frequency and complexity of parents' math support varied significantly with their highest level of educational attainment and household income (see Table 12). Finally, the frequency and complexity of math support did not vary significantly among parents of 3-year-olds and parents of 4-year-olds (see Table 12), so child age was excluded from the main analyses.

Table 12

Parents' math support by their educational attainment, income, and child age.

Variable	Numeracy				Patterning			
	Frequency		Complexity		Frequency		Complexity	
	M(SD)	F value	M(SD)	F value	M(SD)	F value	M(SD)	F value
Educational Attainment	-	21.10***		28.49***	-	18.89***	-	23.92***
Less than bachelor's	2.81(.76)[a]		2.22(1.26)[a]		2.44(1.05)[a]		2.05(1.26)[a]	
Bachelor's degree	3.50(.92)		3.31(1.23)		3.25(1.09)		3.13(1.28)	
More than a bachelor's	3.65(.98)		3.58(1.10)		3.44(1.14)		3.34(1.29)	
Household Income	-	6.06**		3.80*	-	3.59*	-	4.57*
Less than $45,000	3.17(.92)		2.95(1.21)		2.93(1.06) [c]		2.72(1.23)	
$45,000 - $89,000	3.37(.83)		3.06(1.28)		3.08(1.03)		2.87(1.30)	
$90,000 or more	3.61(1.01)[b]		3.42(1.37)[b]		3.35(1.32)		3.26(1.49)[b]	
Child Age	-	.99		3.33	-	.02	-	.02
3-year-old	3.34(.97)		3.02(1.37)		3.12(1.22)		2.94(1.42)	
4-year-old	3.44(.87)		3.28(1.21)		3.13(1.07)		2.96(1.29)	

Notes. *** $p < .001$. ** $p < .01$. * $p < .05$

[a] Both categories were significantly different from less than a bachelor's degree, $p < .05$. [b] Both categories were significantly different from $90,000 or more, $p < .05$. [c] Less than $45,000 significantly different from $90,000 or more, $p < .05$

Main Analyses

I hypothesized that parents' knowledge about early math development would be a positive predictor of the frequency and complexity of their math support at home, above and beyond their child-specific math beliefs and socioeconomic status. To determine the extent to which parents' knowledge about early numeracy and patterning development predicts their numeracy and patterning support, four linear regression analyses were performed with one of the four math support measures as the dependent variable in each analysis. The first regression block included parents' highest level of educational attainment and household income. Next, their beliefs about the value of their child's math (numeracy or patterning) abilities, their expectations for their child's math development (numeracy or patterning), and their perception of their child's current math ability and interests (numeracy or patterning) were entered into the second regression block. Finally, parents' knowledge about early math development (numeracy or patterning) was entered in the third block to examine whether parents' knowledge about early math development (numeracy or patterning) was a unique predictor of their math support after controlling for other parent-child factors. I tested for multicollinearity by estimating variance inflation factor (VIF) scores for all predictor variables, and all VIF scores for predictors were less than 4, indicating that multicollinearity was not biasing the results (Forthofer et al., 2007).

Parents' Numeracy Support

Contrary to my hypothesis, parents' knowledge about early numeracy development did not uniquely predict their reported numeracy support. However, parents' highest educational attainment was a unique, positive predictor of both the frequency and complexity of their numeracy support. Additionally, their beliefs about the value of their child's numeracy abilities and their perception of their child’s current numeracy abilities were also unique, positive predictors of both the frequency and complexity of their numeracy support. Parental expectation for their child’s numeracy development was a unique, negative predictor of the complexity of their numeracy support, but was only marginally predictive of the frequency of their numeracy support (i.e., not at the Bonferroni adjusted alpha level, $p = .0125$). Interestingly, parents’ expectations positively predicted their support in a model without their other child-specific beliefs (see Appendix D) although VIF scores and correlational analyses did not reveal issues of multicollinearity. Notably, the final regression models which included parents’ knowledge about early numeracy development, child-specific numeracy beliefs, and socioeconomic status explained over 40% of the variance in the frequency and complexity of their numeracy support (see Table 13).

Parents’ Patterning Support

As hypothesized, parents’ knowledge about early patterning development was a unique, positive predictor of their reported patterning support. In particular, for each standard deviation increase in knowledge, the frequency and complexity of parents’ patterning support increased by .15. In addition, parents’ highest educational attainment was a unique, positive predictor of both the frequency and complexity of their patterning support. Additionally, all measured child-specific patterning beliefs were unique predictors of both the frequency and complexity of parents’ patterning support, except that their value of their child’s patterning abilities did not predict the complexity of their patterning support (at the Bonferroni adjusted alpha level, $p = .0125$). The child-specific patterning beliefs were positive predictors except for parents’ expectation for their child’s patterning development which was a negative predictor. As with numeracy, parents’ expectations positively predicted their support in a model without their other child-specific beliefs (see Appendix D) although VIF scores and correlational analyses did not reveal issues of multicollinearity. Notably, the final regression models which included parents' knowledge about early patterning development, child-specific patterning beliefs, and socioeconomic status explained over 45% of the variance in the frequency and complexity of their patterning support (see Table 13).

Table 13

Linear regression predicting parents' numeracy and patterning support.

Variables (Final Block)	Numeracy Support				Patterning Support			
	Frequency [a]		Complexity [b]		Frequency [c]		Complexity [d]	
	β	t	β	t	β	t	β	t
Educational attainment	0.13	2.81**	0.19	4.11***	0.14	3.12**	0.17	3.81***
Household income	0.07	1.69	0.02	0.44	0.03	0.73	0.04	0.97
Value of child math ability	0.2	3.95***	0.15	3.03**	0.16	2.94**	0.13	2.55†
Expectation for child's math development	-0.11	-2.19†	0.17	-3.30**	-0.19	-3.88***	-.23	-4.78***
Perception of child current math ability	0.44	6.5***	0.49	7.15***	0.31	4.20***	0.31	4.31***
Perception of child math interest	0.14	2.13†	0.1	1.45	0.26	3.81***	0.29	4.26***
Knowledge about early math development	0.02	0.5	0.0	0.1	0.15	3.35***	0.15	3.65***

Notes. Parents' beliefs and knowledge about numeracy and patterning were used in numeracy and patterning models respectively.

[a] $R^2 = .45$, F(7, 336) = 39.52, $p < .001$. [b] $R^2 = .43$, F(7, 336) = 36.53, $p < .001$. [c] $R^2 = .46$, $F(7, 336) = 40.64$, $p < .001$. [d] $R^2 = .48$, $F(7, 336) = 44.33$, $p < .001$.

**** p < .001. ** p < .0125. †p < .05.*

Table 14

Parents' Knowledge About Early Math Development by Parent-child Demographic Variables of Interest

Variables	N	Numeracy		Patterning	
		M (SD)	t value	M (SD)	t value
Child Age					
3 years old	179	0.82 (0.19)	2.26*	0.77 (0.22)	3.09***
4 years old	165	0.76 (0.26)		0.70 (0.24)	
Preschool Attendance					
Yes	198	0.78 (0.23)	1.13	0.75 (0.22)	-1.29
No	146	0.80 (0.23)		0.72 (0.25)	
Child Gender					
Boy	210	0.80 (0.23)	-1.01	0.74 (0.24)	-0.56
Girl	134	0.78 (0.24)		0.73 (0.24)	
Special Education Services					
Yes	131	0.77 (0.25)	1.94*	0.74 (0.24)	-0.33
No	213	0.81 (0.22)		0.74 (0.24)	
Relation to Child					
Father	149	0.79 (0.24)	0.15	0.74 (0.24)	0.14
Mother or Grandmother	195	0.80 (0.22)		0.74 (0.24)	
Number of Children					
One Child	156	0.79 (0.24)	0.27	0.74 (0.23)	-0.01
More than One Child	188	0.80 (0.22)		0.73 (0.24)	
Parent employment					
Full time	272	0.78(0.24)	2.69**	0.73(0.24)	1.22
Part time or unemployed	72	0.85(0.18)		0.76(0.22)	
Parent is a Preschool Teacher					
Yes	84	0.83 (0.19)	1.29	0.79 (0.22)	2.94***
No	260	0.78 (0.24)		0.72 (0.24)	
Parent is a Preschool or Elementary Teacher					
Yes	123	0.79 (0.24)	0.75	0.76 (0.22)	-1.69
No	221	0.80 (0.23)		0.72 (0.24)	

*** $p < .001$ ** $p < .01$ * $p < .05$

Exploratory Analyses

Parents' Knowledge about Early Math Development and Parent-child Demographics

I explored whether parents' knowledge about early math development relates to demographic characteristics of interest that were not explored in the main hypotheses and whether they previously or were currently participating in a program in which they received information about early academic development (see Table 14). Specifically, I examined whether parents' knowledge about early numeracy and patterning development was related to their household income, parental role, child's age, or child's gender given that parent's math beliefs and early math support are sometimes related to these demographic factors (e.g., DeFlorio & Beliakoff, 2015; LeFevre et al., 2002; Thompson et al., 2017; Vandermaas-Peeler et al., 2009). I also examined whether parents' knowledge was related to whether their child receives special education services at school given the large percentage of parents who reported that their 3- or 4-year-old child receives special education services at school (38%). I also examined whether parents' knowledge about early numeracy and patterning development was related to whether their child attended preschool the previous year, whether they were pre-K teachers, and the number of children they have since these could be sources of their knowledge.

Parents' knowledge about early numeracy development was not significantly related to any of the identified demographic characteristics of interest except for their child's age and whether their child receives Special Education services at school (see Table 14). Specifically, parents of 3-year-old children had significantly more accurate knowledge about early numeracy skills than parents of 4-year-old children. Parents whose children did not receive Special Education services at school had significantly more accurate knowledge about early numeracy skills than parents whose children receive Special Education. Parents' knowledge about early patterning development was not significantly related to any of the identified demographic characteristics of interest except for their child's age and whether they were preschool teachers. As with numeracy, parents of 3-year-old children had significantly more accurate knowledge about early patterning development than parents of 4-year-old children. Parents who identified as preschool teachers had higher knowledge about early patterning development than parents who did not identify as preschool teachers.

Next, consider whether they previously or were "currently receiving text messages about skills young children can learn and how parents can support them at home (ready4k program)". Parents who were currently participating in a program in which they received information about early academic development had less accurate knowledge about early numeracy development (M = .77, SD = .24) than parents who were not participants (M = .81, SD = .22), $t(342) = 2.14$, p = .033. Similarly, parents who had previously participated in the program had less accurate knowledge about early numeracy development (M = .77, SD = .24) than parents who were not (M = .81, SD = .22), $t(342) = 1.98$, p = .049. However, parents' knowledge about early patterning development did not vary based on whether they were currently or had previously participated in a program in which they received information about academic development.

Parents' Knowledge as a Predictor of Their Beliefs

To further explore the relationship between parents' knowledge about early math development and their child-specific numeracy beliefs, eight linear regressions were performed with each child-specific belief as the dependent variable in one analysis (see Table 15). Parents' other child-specific beliefs were controlled for in all models. Numeracy models also included parents' highest educational attainment, employment status, child age, whether the child receives

Special Education services at school, and whether parents participated in the Ready4k program the previous year as covariates since these were related to parents' knowledge about early numeracy development in previous analyses. Patterning models also included child age and whether parents were preschool teachers as covariates since these were related to parents' knowledge about early patterning development in previous analyses. There were no issues with multicollinearity as estimated by variance inflation factor (VIF) scores which were less than 4 for all predictors (Forthofer et al., 2007). Overall, parents' knowledge was a unique, positive predictor of their perception of their child's current abilities and interest and their value of their child's development for both numeracy and patterning even after controlling for their other child-specific math beliefs. However, parents' knowledge about early numeracy and patterning development was not a unique predictor of their expectations for their child's numeracy or patterning development.

Table 15

Linear regressions predicting parents' child-specific math beliefs by their knowledge about early math development.

Child-specific Belief (Outcome Variables)	Numeracy Knowledge		Patterning Knowledge	
	β	t	B	t
Value of child ability	.25	4.55^{***}	.20	3.95^{***}
Expectation for child's development	.08	1.54	.09	1.64
Perception of child current abilities	.18	3.78^{***}	.29	5.75^{***}
Perception of child interests	.24	4.68^{***}	.20	3.89^{***}

Note. Each model included parents' child-specific math beliefs (besides the outcome belief) and parent-child demographic variables that were related to their knowledge about early math development in previous analyses as covariates.

Parent-reported Sources of Their Knowledge about Early Math Development

Next, I report on parents' responses to the question "Overall, how did you decide which answers to choose when deciding which academic skills most children in the United States develop by age five?". They typed their response to this question immediately after answering the knowledge measure questions. We coded parents' responses using a coding scheme that was developed based on parents' responses to the same question during a pilot round of data collection (see coding scheme and frequency of codes in Table 16). Three individuals coded the data. The coders double-coded 40% of the data in three rounds and had good reliability in the final round of coding (kappa = .96) and across all three rounds (kappa = .83). Forty-three percent of parents' responses did not fit the coding scheme (e.g., some parents responded to the question with a single word). Of the 196 parents whose responses were coded, most mentioned that they thought about their experience with other children (54%) and/ or their participating child's current abilities and their expectations for their child's growth (45%). Very few parents mentioned their knowledge of benchmarks or other information about children's early math development (16%).

Summary of Results

The study findings are summarized in Figure 3. Specifically, in partial support of my hypothesis, parents' knowledge about early numeracy development was significantly and positively related to all of the measured child-specific numeracy beliefs, except for parents' expectations for their child's numeracy development. Additionally, it was positively correlated with the frequency of their numeracy support; however, it was not a unique predictor. As hypothesized, parents' knowledge about early patterning development was positively related to all of the measured child-specific patterning beliefs and was positively correlated with and predictive of both the frequency and complexity of their patterning support. Contrary to my hypothesis, parents' highest educational attainment was not consistently related to their knowledge about early math development.

Table 16

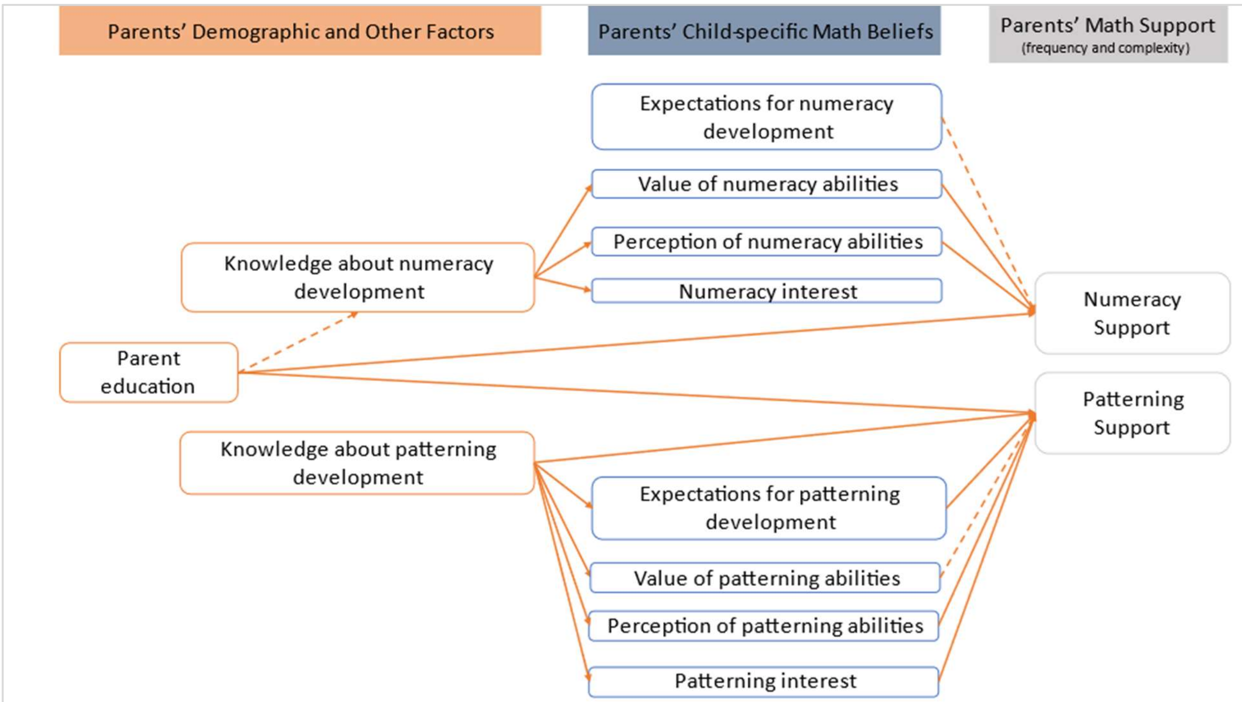
Parents' Responses to how they decided on their answers about early math development

Code	Definition	Example	%
Child's current ability and/or expectations for growth	Parents consider their 3- or 4-year-old child's current ability and potential development within a year or two.	"I just thought of what my daughter knows now and what I thought would be realistic for her to know in a year"	45
Experience with other children	Parents consider their experience or interaction with their own older children, other children, or their memory of their knowledge while they were children.	"I thought about what my older kids were able to do by the age of 5" "...what I see kids in his class do…"	54
Benchmark or other resources	Parents consider the information they have received from schools, research articles, or other resources.	"I've thought about some articles I've read and some information that teacher friends and her pediatrician have mentioned." "... what they teach at the classroom."	16
Average expectations	Parents consider what they think an average child can do by age 5 or what is realistic for 5-year-olds without specifying their source of knowledge.	"I thought about where I see my child in five years and compared that to the average child."	31

Note. Percentage of parents whose responses fit the coding scheme (n = 196). The responses of 43% of the total sample (n = 148) were too vague or otherwise did not fit the coding scheme.

Figure 3

Model summarizing current study findings.



Notes. This figure summarizes tested pathways and so an absence of an arrow does not necessarily indicate an absence of a relationship (e.g., analyses were not conducted for relations between patterning and numeracy variables nor how parent education related to beliefs). Broken lines indicate relations for which there is mixed evidence.

DISCUSSION AND IMPLICATIONS

The current study contributes to a more comprehensive theory of early math development as it is the first study to integrate how parents' knowledge about early math development relates to the math support that they provide to their preschool-aged children. The current study also provides insight into how the Parent Early Numeracy Socialization (PENS) model can be expanded to include parents' early patterning support, child-specific patterning beliefs, and knowledge about early patterning development, allowing for a parent socialization model for early math development that moves beyond the numeracy subdomain. In the next sections, I discuss the nature and role of parents' knowledge about early math development, their child-specific math beliefs, and their socioeconomic status in their efforts to support their preschoolers' math development.

Parents' Knowledge about Math Development

Parents' knowledge about early patterning and numeracy development was positively related to their numeracy and patterning support respectively, but only their knowledge about early patterning development remained a unique predictor after other parent-child variables were controlled for. This suggests that parents who know more about which patterning skills most typical preschoolers can develop engage their preschoolers in related activities and discussions more frequently than parents with less knowledge. Notably, this includes activities and discussions around more complex, developmentally appropriate early patterning skills which parents focus on infrequently (Zippert et al., 2020; Zippert & Rittle-Johnson, 2020). The match hypothesis (Hunt, 1961) holds that parents who are more accurate at estimating their child's skills are better at supporting their child's growth and development. Similarly, the current findings may indicate that parents who are more knowledgeable about which patterning skills preschoolers can learn are better at identifying and engaging their child in developmentally appropriate activities that can further support the development of their child's patterning skills.

There are several potential explanations for the differences in the role of parents' knowledge about early patterning versus early numeracy development. This could potentially be because the measure of parents' knowledge about early numeracy development was only somewhat reliable (less so than their knowledge about early patterning development). Another could be because parents had more varied patterning support than numeracy support and knew less about patterning than numeracy development on average. Overall, parents' knowledge about early patterning development, which has not been studied previously, seems to play an important role in their efforts to support their children's patterning development

It is important to understand whether parents' knowledge about early math development varies with their socioeconomic status and their child-specific math beliefs as it could be an important pathway through which their SES affects their math beliefs and support and their children's math skills (e.g., Casey et al., 2018; DeFlorio & Beliakoff, 2015; Rittle-Johnson et al., 2013; Saxe et al., 1987; Starkey et al., 2004; Thompson et al., 2017; Vandermaas-Peeler et al., 2009; Zippert & Rittle-Johnson, 2020). This also provides insight into how parents' knowledge about early math development should be incorporated into the PENS model.

Socioeconomic Status and their Knowledge about Early Math Development

The current study is the first to examine which aspects of parents' socioeconomic status (SES) relate to and may potentially be sources of variability in their knowledge about early math development. This is important given that it allows for specific theorization about the role of socioeconomic status and the development of more targeted efforts to ameliorate the effects of SES.

First, consider how indicators of parents' financial resources relate to their knowledge about early math development. A previous study found that parents' knowledge about early math development was related to whether they were eligible for state or federally subsidized preschool or could pay tuition for private school (Deflorio & Beliakoff, 2015). Specifically, they found that parents with more financial resources (those who were not eligible for state or federally subsidized preschool) had more accurate knowledge about early math development than parents with less financial resources. However, in the current study, parents' knowledge about early math development did not vary by whether they reported that they received financial assistance for their child to attend preschool. There are several potential reasons for these differing results. For instance, the studies may have covered different ranges of SES. Deflorio and Beliakoff's (2015) low SES sample was recruited from state or federally subsidized schools that exclusively serve families with low SES and so may have only or mostly included parents whose incomes are close to the poverty line. This may not be true of parents in the current study who were only asked to report whether they received any financial assistance. Alternatively, a third variable might explain Deflorio and Beliakoff's finding. For instance, it could be factors related to the type of school that children attend that relates to parents' knowledge about early math development rather than their financial resources per se. Parents with lower and higher SES in the previous study were recruited from different types of schools (i.e., none were recruited from schools that serve a mix of SES) which may vary in the frequency and effectiveness of parent-educator communication about early math. Indeed, more frequent and more effective parent-educator communication is often observed among parents whose children attend private schools and who have higher SES (Hossain, 2021; Murray et al., 2015). Parents in the current study were not recruited based on schools and so there might not be a systematic difference in school type among parents who reported receiving financial assistance for their child to attend schools and parents who do not receive any assistance. Notably, parents' knowledge about early math development also did not vary by their approximate household income in the current study, a second indicator of their financial resources.

Next, consider whether parents' knowledge about early math development varies with their education, which the current study was the first to examine. Parents' knowledge about early numeracy but not patterning development was related to their highest educational attainment. Interestingly, parents' knowledge about early numeracy development was negatively related to their highest educational attainment suggesting that parents with graduate education tended to underestimate which numeracy skills are within the developmental range for typical preschoolers. Parents with only a bachelor's degree and parents without a bachelor's degree did not differ substantially in their knowledge about early numeracy development. One potential reason for this is that parents with a graduate degree may value formal schooling more and overestimate the role of formal schooling for early numeracy development (i.e., think that children only develop certain numeracy skills with formal schooling), and thus underestimate children's skills before formal schooling. Previous studies have examined parents' beliefs about the role of school versus home in preschoolers' math development but have not examined whether these differ among parents with and without a graduate education (e.g., Deflorio & Beliakoff, 2015). Another potential reason for the negative relation between parents' highest educational attainment and their knowledge about early numeracy development is that parents with a graduate degree are more likely to work full time and therefore less likely to be able to spend time developing knowledge about children's numeracy skills than parents with less education. Indeed, most parents with a graduate degree worked full time, and parents who worked full time had significantly lower knowledge about early numeracy development than parents who worked part-time or were unemployed. This does not seem to be driven by the amount of time that parents spend doing numeracy activities with their children as parents with a graduate education report more frequent and complex numeracy support than other parents.

Overall, the current findings suggest that parents' knowledge about early numeracy development is not a pathway through which their SES affects their support given that it was negatively related to their educational attainment in the current study while parents' support is often positively related to their educational attainment (e.g., Gaylord et al., 2020; Thompson et al., 2017) and was unrelated to indicators of their financial resources. Similarly, parents' knowledge about early patterning development does not seem to be a pathway through which their SES affects their support (Zippert & Rittle-Johnson, 2020) given that it was unrelated to multiple indicators of their SES.

Parents' Child-specific Math Beliefs and their Knowledge about Early Math Development

The current study was the first to examine whether parents' knowledge about early numeracy and patterning development relates to their child-specific numeracy and patterning beliefs and might potentially be a source of variability in their beliefs. Indeed, parents' knowledge about early numeracy and patterning development was often positively correlated with their child-specific numeracy and patterning beliefs in the current study, similar to previous findings about parents' knowledge about child development and their parenting beliefs like maternal self-efficacy (e.g., Albarran & Reich, 2014). Further, parents' knowledge about early numeracy and patterning development was predictive of their child-specific numeracy and patterning beliefs even after controlling for related parent-child demographic characteristics. Thus, parents' knowledge about early math development may be an important source of variability in their child-specific math beliefs. For instance, parents with more accurate knowledge about the math skills that most preschoolers can learn may value their preschoolers' development of these skills more than other parents. Likewise, knowing which math skills preschoolers can learn might lead parents to ask their children to demonstrate these skills and may lead to them having a more accurate perception of their children's math skills. At the same time, parents' beliefs may influence their knowledge. Future research should examine the effect of changing parents' knowledge about early math development on their child-specific math beliefs to test for causal relations.

Parents' Socioeconomic Status

In alignment with previous correlational research, parents' highest educational attainment was a unique, positive predictor of their math support (e.g., Casey et al., 2018; Gaylord et al., 2020). This may be an indication that parents with different levels of education view and approach the academic socialization of their children in distinct ways (Elliott & Bachman, 2018; Lareau et al., 2011; Sonnenschein et al., 2016). Indeed, there is some evidence that parents with different levels of education hold different numeracy beliefs (see Douglas et al., 2021 for a review) though little is known about how parents' approaches to providing numeracy support relate to their education. Notably, parents' income was not a unique predictor of the frequency or complexity of their math support. Parental investment and household production models (e.g., Duncan et al., 2014) would suggest that parents with higher income can invest more financial resources into their children's academic development such as by purchasing math books or games. Thus, it might be that certain types of parent-child math engagement vary with income while the overall frequency of parent-child math engagement does not vary with income as parents with less income may circumvent these challenges by engaging their children in activities that do not require purchases. However, it is also worth considering the possibility that the effect of income on the overall frequency and complexity of parents' math support is only evident when considering financial hardship or poverty. This would align with family stress and family process models which posit that higher parental financial stress or adversity negatively impacts their engagement with their children (Cooper et al., 2010; McLoyd, 1990). Indeed, previous studies have found that parents' income-to-needs ratio and eligibility for state or federally funded preschools such as having household income below the federal poverty line (variables not measured in the current study) are related to their math support (Casey et al., 2018; Deflorio & Beliakoff, 2015; Vandermaas-Peeler et al., 2009). Overall, the current study helps elucidate which aspects of parents' socioeconomic status influence their math support by providing evidence for the predictive nature of parents' educational attainment, but not their unadjusted income level.

Parents' Child-specific Math Beliefs

As proposed in the Parent Early Numeracy Support model (Douglas et al., 2021), parents' child-specific numeracy beliefs were uniquely and differentially predictive of their numeracy support in the current study. Their value of their child's numeracy development and their perception of their child's current numeracy abilities were consistently positive predictors of their numeracy support in alignment with previous correlational research (Skwarchuk et al., 2014; Susperreguy et al., 2020; Uscianowski et al., 2020; Zippert & Ramani, 2017; Zippert & Rittle-Johnson, 2020), while their perception of their child's numeracy interest was not. Notably, parents' perception of their child's interest in patterning and current patterning abilities were consistent, positive predictors of their patterning support whereas their value of their child's patterning development was inconsistently related to their support. Differences in the role of parents' value of their child's numeracy versus patterning development might be due to numeracy being a more well-known and central component of math than patterning. Thus, parents may engage their children in numeracy activities to support their numeracy development regardless of their child's interest whereas they may view patterning as more optional and allow their child's interest to guide whether and how they provide patterning support.

The current study was the first to operationalize parents' expectations for their children's early math development in a way that aligns with the expectancy-value theory which is a well-studied socialization model that is often used with school-aged children (Eccles et al., 1983; Eccles & Wigfield, 2020). The current study's findings suggest that parents' expectations for their preschoolers' math development (at least the current operationalization) were often negatively related to their support when controlling for other variables. This would suggest that parents with higher expectations for their child's early math development do less of the math activities that we asked about (which correspond to skills that typically developing children can be learning). Perhaps, parents with higher expectations do other, more advanced math activities with their children that we did not ask about or encourage their children to engage in math activities more independently and do less of the activities that we ask about. Alternatively, perhaps parents who do not think their children will do well at math (i.e., have low expectations for their children's math development) do more math activities with their children to try to circumvent this. Meta-analyses have revealed that parents' expectations play a key role in their children's (math) achievement (Daucourt et al., 2021; Fan & Chen, 2001) so it is important to understand whether and how they relate to parents' efforts to support their children's math development. However, the current negative relations should be interpreted with caution given that parents' expectations positively predict their support when their other child-specific math beliefs are not controlled for.

Overall, parents' child-specific math beliefs were often uniquely and differentially related to their math support and accounted for a substantial percentage of the variance (29-34%) in their numeracy and patterning support, providing supportive evidence that they are important components of the home math environment.

Limitations and Future Directions

Despite five rounds of measurement development and pilot testing, a limitation of the current study is that the measures of parents' knowledge about early numeracy and patterning development were only somewhat reliable, suggesting that additional measure development research is needed. Notably, in one of our five rounds of piloting of the measure, we found that the measures were substantially more reliable after parents received information about early math development (i.e., Cronbach's alpha increased by about .20). Another limitation is that although the sample consisted of participants from several states across the US, its socioeconomic status was not representative of the US population and only a few parents were at the ends of the income spectrum which potentially reduced the study's ability to detect income-related differences.

A third limitation is that although the regression models accounted for a substantial percentage of the variance in parents' math support, the amount of variance explained indicates that the current study did not consider some variables that play an important role in predicting parents' math support. For instance, the PENS model includes two additional categories of parental beliefs that were not measured in the current study (parents' numeracy beliefs about themselves and parents' beliefs about numeracy in general; see Douglas et al., 2021 for a review). Previous research also highlights additional variables like parent-educator communication which could potentially play a key role in their math support (Lin et al., 2019).

Finally, the current study only provides correlational evidence about the potential role of parents' knowledge about early math development in their efforts to support their children's math development. Future research should examine the malleability of parents' knowledge about early patterning development and the effect of increasing this knowledge on their math support. Increasing parents' patterning support might be important given previous findings that parents rarely mentioned patterning spontaneously when asked about their math support and reported supporting their children's patterning development less often than other aspects of their math development (Cannon & Ginsburg, 2008; Zippert et al., 2020; Zippert & Rittle-Johnson, 2020). The current finding that very few parents referenced information that they received from schools or other resources when sharing the basis of their knowledge about early math development coupled with previous findings that parents are interested in receiving information about math (Sonnenschein et al., 2021) suggests that parents might benefit from receiving information about early math development.

Conclusion

The current study provides insight into the nature and role of parents' knowledge about early math development. In particular, parents' knowledge about early numeracy development was positively related to their value of their child's numeracy development and their perception of their child's current numeracy abilities and interests as well as the frequency of their numeracy support. Similarly, parents' knowledge about early patterning development was positively related to their value of and expectations for their child's patterning development and their perception of their child's current patterning abilities and interests. Parents' knowledge about early patterning development was also positively correlated with and predictive of both the frequency and complexity of their patterning support. Thus, parents' knowledge about early patterning development may play an important role in their efforts to support their children's math development.

Appendix A

Development of Measure of Parents' Knowledge about Early Numeracy and Patterning Support

The measure of parents' knowledge about early math development was adapted from a previously used measure (DeFlorio & Beliakoff, 2015) whose instruction was "These following questions concern children's mathematical development during the preschool years. Which of the following abilities or skills do you believe typical children have developed before their 5th birthday?". See table A1 for details about the final measure's items including the origin of each item. The previously used measure had 23 items. It included 13 items on numeracy skills, with six items being about skills that are within the developmental range for most five-year-olds and seven that are beyond the developmental range for most five-year-olds. The previous measure also included two items on patterning skills (both within the developmental range) and eight items on spatial skills (four within and four beyond). DeFlorio and Beliakoff (2015) only included 14 of their 23 items in their analyses (the remaining items were dropped because their study focused on SES differences and the excluded items had no SES differences). The revised measure was developed based on their full measure.

Sixteen items were initially added to measure a wider variety of numeracy and patterning skills and to make the measure similar across subscales. Specifically, five numeracy, eight patterning, and three spatial items were added. New items were based on previous research on children's early math skills including developmental learning trajectories (e.g., Claessens & Engel, 2013; Clements & Sarama, 2014; Common Core State Standards Initiative, 2010; National Research Council, 2009; Litkowski et al., 2020; Rittle-Johnson et al., 2017; Rittle-

Johnson et al., 2019). Nine items (8 numeracy, 1 spatial) were dropped primarily because they were ambiguous or were very similar to other items (e.g., "Use a computer with age-appropriate software to learn math concepts" did not focus on a specific numeracy, patterning, or spatial skill). Ten items were also revised for clarity. The instruction was expanded with the addition of a sentence that reads "Please select "yes" for each skill that you think most children in the United States correctly master by age five. Please select "no" for each skill that you do not think most children in the United States correctly master by age five.".

After the first round of edits, the measure included 30 items (10 numeracy, 10 patterning, and 10 spatial). Within each subscale, there were seven items on skills children typically develop by age five (within the developmental range) and three on skills children typically do not develop by age five (beyond the developmental range). This version of the measure was piloted with 161 parents. The measure's reliability according to Cronbach's alpha was low across all items overall, but questionable to acceptable for subscales when considering within and beyond items separately. Further analyses focused on the items that are within the developmental range for most typically developing preschool-aged children given that the measure focuses on this age range. One item measuring a patterning skill within the developmental range and three spatial items measuring spatial skills within items were flagged for poor item fit (item-total correlation below .1) and/or nonsignificant factor loading from Confirmatory Factor Analyses (CFA).

This version of the measure was piloted simultaneously with 36 parents (pilot 2) and again a second time with a subset of 21 of the same parents (pilot 3). Items were the same at both time points and identical to pilot 1. However, for pilots 2 and 3, the question was revised to ask about "most" instead of "typical children". Similar to pilot 1, the measure's reliability as estimated by Cronbach's alpha was low across all items overall and questionable to acceptable for subscales when considering within and beyond items separately for both rounds of piloting. Statistics were better for pilot 3 (after parents received some information designed to change their knowledge about early numeracy or patterning development). Across pilots 2 and 3, one numeracy-within item was flagged twice for poor item fit and/ or zero variance and two other numeracy within items were flagged once. Two patterning within items were flagged for poor item fit and/ or nonsignificant factor loading including one that was flagged in pilot 1. Three spatial items were flagged for poor item fit or zero variance including two which were flagged in pilot 1. Based on these three rounds of piloting, one numeracy item was revised to increase its difficulty (from counting 10 objects to counting 15 objects). Two patterning items about skills within the developmental range were added in case any patterning items needed to be dropped in further analyses. One of the new patterning items focused on a skill that was not previously included in the measure (copying a repeating pattern). Thus, the new version of the measure included 32 items.

The revised measure was piloted again with 45 parents (pilot 4) with a slightly modified instruction: “Please select "Yes" for each skill that you think over 50% of children in the United States can correctly do themselves by their 5th birthday. Otherwise, select "No".” Again, reliability was low overall and questionable to acceptable for subscales when considering within and beyond items separately. Three numeracy within and three patterning within items were flagged for poor item fit, nonsignificant factor loadings, and/ or zero variance including four which were flagged in previous rounds of piloting. Six of the seven spatial within items were flagged for poor item fit and/or nonsignificant factor loading including four that were flagged in previous rounds. Thirteen items (five numeracy, four patterning, and three spatial) were revised. The revisions included changes to wording to increase the clarity of nine items as well as to increase the difficulty of 2 items (e.g., “name the numerals from 1 to 5” was changed to “name the written numbers from 1 to 10”).

The revised measure was piloted with another 46 parents (pilot 5; see Table 2). Again, reliability was low across all items and questionable to acceptable for subscales when considering within and beyond items separately. Items that had poor item fit included one numeracy within, four patterning within including one which was previously flagged, and three spatial within items which were all previously flagged.

The measure was piloted again with 344 parents to create the final version of the measure. Again, reliability was low across all items and for the numeracy-within subscale. However, all other subscales had acceptable reliability. One numeracy within (“Count a row of 15 objects (for example, count 15 plastic worms)” and one pattern-within item (“Sort a set of objects into 3 groups based on color such as red, blue, and green”) which were previously flagged had poor item fit and were dropped from further analyses. A series of CFAs were conducted for numeracy items and patterning items. For each subscale, a one-factor model with all items and a two-factor model with all within items on one factor and all beyond items on the second factor were conducted. For the two-factor model, the factors were allowed to correlate as were the items measuring the same skill. All items loaded well onto their subscales. Both two-factor models had adequate model-data fit (see Table A2) and had significantly better model-data fit than the one-factor models, $\chi2(20)=106.648$, $p < .001$ for numeracy and $\chi2(27)=353.443$, $p < .001$ for patterning.

Table A1

Details About the Measure of Parents' Knowledge about Early Math Development

Order	Item	Specific Skill	Learning Trajectory Level	Question Origin	Previous Version
			Numeracy		
Within Developmental Range					
1	Count a row of 15 objects (for example, count 15 plastic worms)	Object Counting/One-to-One Correspondence	Counter (10)	Deflorio & Beliakoff (2015)	Count a row of 10 objects (for example, count ten plastic worms)
13	Counts out the correct number of things when asked for a specific number of things up to 10 (for example, gives 6 cookies when asked for 6 cookies)	Cardinality	Counter And Producer (10+)	Created based on previous research (e.g., Clements et al., 2014)	Give the correct number of things when asked for a specific number of things up to 10 (for example, gives 6 cookies when asked for 6 cookies)
14	Name the written numbers from 1 to 10 (for example, points to the 9 when asked "where is the number nine?")	Numeral ID	Functional Numeral User	Deflorio & Beliakoff (2015)	Name the numerals from 1 to 5 (for example, points to the 1 when asked "where is the numeral one?")
16	Solve small addition or subtraction problems presented with objects (for example, 3 blocks and 2 blocks is ___ blocks)	Simple Arithmetic	Find Result +-	Deflorio & Beliakoff (2015)	Solve small addition or subtraction problems presented with objects, such as 3 blocks and 2 blocks = __ blocks
23	Tell which of two spoken numbers between one and ten is bigger (for example, says "five" in response to "Which is bigger, five or two?")	Symbolic Magnitude Comparison	Counting Comparer (10)	Created based on previous research (e.g., Clements et al., 2014)	Tell which of two numbers between one and ten is bigger (for example, says "five" in response to "Which is bigger, five or two?")
25	Tell which of two written numbers between one and ten is bigger (for example, points to the written number 9 when shown the written numbers 2	Symbolic Magnitude Comparison	Counting Comparer (10)	Created based on previous research (e.g., Clements et al., 2014)	Tell which of two numerals between one and ten is bigger (for example, points to the numeral 9 when shown the numerals 2 and 9 and asked "Which is bigger")

	and 9 and asked "Which is bigger")				
29	Answer questions by adding or subtracting small numbers (for example, says "three" in response to "If you have four stickers and then you give me one of your stickers, how many stickers would you have left?")	Simple Arithmetic	Find Result +-	Created based on previous research (e.g., Clements et al., 2014)	Answer questions by adding or subtracting small numbers (for example, says "three" in response to "If you have four stickers. How many stickers would you have if you give me one sticker)
Beyond Developmental Range					
8	Solve single-digit addition or subtraction problems presented on flashcards (for example, 5 + 3 = ___)	Advanced Arithmetic	Numbers In Numbers +/-	Deflorio & Beliakoff (2015)	Solve single-digit addition or subtraction problems presented on flashcards, such as 5 + 3 = __
11	Recite number words from 1 to 100	Rote Counting - 100	Counter To 100	Deflorio & Beliakoff (2015)	Recite numbers from 1 to 10
27	Name the written numbers from 1 to 100 (for example, says the word "ninety-three" when shown the written number 93)	Numeral ID - 100	Decade Number Identifier	Created based on previous research (e.g., Clements et al., 2014)	Name the numerals from 1 to 100 (for example, says the word "ninety-three" when shown the written number 93)
Pattern					
Within Developmental Range					
2	Continue a pattern of cubes (for example, blue, blue, red, red, blue, blue, red, red, _______, _______, _______, _______)	Extend	Pattern Extender	Created based on previous research (e.g., Clements et al., 2014)	NA
3	Use colored beads to make a simple pattern, such as a "blue-purple" pattern	Create	Patterner	Deflorio & Beliakoff (2015)	Use colored beads to make a simple pattern, such as "blue-purple"
4	Figure out what should come next in a simple pattern (for example: clap, stomp, clap,	Extend	Pattern Extender AB	Created based on previous research (e.g.,	Figure out what should come next in a simple alternating pattern (for example: clap, stomp, clap, stomp, _______, _______)

	stomp, _______, _______)			Clements et al., 2014)	
5	Sort a set of objects into 3 groups based on color such as red, blue, and green	Sort/Similarities Differences	Pre-Explicit Patterner 2	Created based on previous research (e.g., Clements et al., 2014)	Sort a set of objects into 2 groups based on color such as red and blue
10	Identify two patterns that follow the same rule made with different materials (for example, a block-block-ball pattern and a sun-sun-moon pattern are similar)	ID Rule	Pattern Unit Recognizer	Created based on previous research (e.g., Clements et al., 2014)	NA
18	Fill in the missing part of a pattern made of repeating objects (for example: circle, square, square, circle, square, _____, circle, square, square)	Missing	Pattern Fixer	Created based on previous research (e.g., Clements et al., 2014)	NA
22	Make the same kind of simple pattern in their bracelet as their friend's bracelet, but using different colors (for example, your child makes a yellow-green pattern to match a friend's red-blue pattern)	Abstract	Pattern Unit Recognizer	Created based on previous research (e.g., Clements et al., 2014)	Make the same kind of simple alternating pattern in their bracelet as their friend's bracelet, but using different colors (for example, your child makes a yellow-green pattern to match a friend's red-blue pattern)
31	Makes a repeating pattern (for example, makes a clap, spin, snap, clap, spin, snap pattern)	Create	Patterner	Created based on previous research (e.g., Clements et al., 2014)	NA
32	Copy a pattern someone else makes in the same way (for example, your child beats a drum in a loud-soft pattern just like you do)	Copy	Pattern Duplicator	Created based on previous research (e.g., Clements et al., 2014)	NA

Beyond Developmental Range					
12	Use number patterns to solve problems (for example, fills in the blanks when presented with 26, 22, 18, __, 10, __)	Growing ID Pattern Rule/Unit	Beginning Arithmetic Patterner	Created based on previous research (e.g., Clements et al., 2014)	NA
20	Figure out what comes next in a growing pattern that subtracts two cubes for each stair in a staircase (for example, 9 cubes, 7 cubes, 5 cubes, 3 cubes, ____)	Growing Extend	Pattern Unit Recognizer	Created based on previous research (e.g., Clements et al., 2014)	NA
30	Describe even numbers (for example, 2, 4, 6, 8, 10) as "skipping" every other number on a 100s chart	Growing ID Pattern Rule/Unit	Pattern Unit Recognizer	Created based on previous research (e.g., Clements et al., 2014)	NA
			Spatial		
Within Developmental Range					
6	Arrange sticks in order of increasing length (for example, long, longer, longest)	Measurement - Serial Order	Length Comparer Direct	Deflorio & Beliakoff (2015)	Arrange sticks in order of increasing length
9	Cut apart a rectangle to make two squares	Recognize Shapes	Shape Recognizer - All Rectangles	Created based on previous research (e.g., Clements et al., 2014)	NA
17	Name the following shapes: circle, triangle, and square	Geometry - ID Shapes	Shape Recognizer - Typical	Deflorio & Beliakoff (2015)	Same as current
19	Measure the length of a pencil using a string	Measurement - with Objects	End To End Length Measurer	Deflorio & Beliakoff (2015)	Same as current
21	Put a circular puzzle piece in the circular space	Geometry - Shape Matcher	Shape Matcher - Identical,	Created based on previous research	NA

			Orientation, Sizes	(e.g., Clements et al., 2014)	
24	Describe the properties of shapes (for example, says “It has three sides and three points so it’s a triangle.”)	Recognize Shapes	Shape Recognizer - Circle, Square, Triangle	Created based on previous research (e.g., Clements et al., 2014)	NA
26	Understand location words such as "under", "on", "next to", and "behind"	Spatial Orientation	Local Framework User	Deflorio & Beliakoff (2015)	Understand spatial words such as "under", "on", "next to", and "behind"
Beyond Developmental Range					
7	Use shapes to construct a larger geometric shape, such as using right triangles to construct a square	Geometry - ID Shapes	Parts Of Shapes Identifier	Deflorio & Beliakoff (2015)	Use pattern blocks to construct a larger geometric shape, such as using 2 right triangles to construct a square
15	Measure the angles of a triangle	Geometry - Measure Angles	NA	Deflorio & Beliakoff (2015)	Measure the angles of a triangle
28	Measure the width of a sheet of paper using a ruler (for example, uses a ruler to determine that a paper is 5 centimeters long)	Measurement - With Ruler	Length Unit Relator and Repeater	Deflorio & Beliakoff (2015)	Measure the width of a sheet of paper using a ruler

Table A2
Comparison of Confirmatory Factor Analysis Models of the Knowledge Measure

Model	df	Maximum Likelihood Ratio Chi-Square [a]	NNFI [b]	CFI [c]	RMSEA [d]	AGFI [e]
Numeracy						
1 Factor Model	35	200.85***	0.478	0.592	0.117	0.81
Correlated 2-factor model [f]	32	43.34	0.904	0.932	0.047	0.911
1 Factor Model (Within Only)	9	20.55*	0.75	0.85	0.089	0.918
Pattern						
1 Factor Model	54	226.08***	0.639	0.704	0.096	0.821
Correlated 2-factor model [f]	40	93.09***	0.872	0.907	0.062	0.922
1 Factor Model (Within Only)	20	39.72**	0.89	0.92	0.05	0.95

Notes. *** $p < .001$ ** $p < .01$ * $p < .05$
[a] Non-significant Maximum Likelihood Ratio Chi-Square indicates a good model fit
[b] NNFI ≥ 90 indicates adequate model fit (Coroiu et al., 2018; Kline, 2005).
[c] CFI ≥ 0.90 indicates adequate model fit (e.g., Awang, 2012; Coroiu et al., 2018; Kline, 2005).
[d] RMSEA ≤.08 indicates adequate model fit (Coroiu et al., 2018, Browne & Cudeck, 1993).
[e] AGFI ≥ .90 (Cornell Statistical Consulting Unit).
[f] Items measuring similar skills were allowed to correlate (i.e., two simple arithmetic items, two magnitude comparison items, two extend pattern items, two create pattern items and two abstract pattern items).

Appendix B

Predicting Parents' Math Support, Controlling for their Child's Age

Four linear regression analyses similar to those in the main text were conducted. The first regression block in these analyses included child age, and then the following three blocks were identical to the three blocks described in the main text. Results are presented in Table B1. Note that these analyses which include eight predictors are underpowered since a post hoc analysis revealed that the sample size only yields 78% power to detect the effect of eight variables. Overall, findings were similar to the analyses without child age. Notably, child age was not a unique predictor of the frequency or complexity of parents' early math support, except for the complexity of parents' numeracy support.

Table B1
Linear regression predicting parents' numeracy and patterning support, controlling for seven parent-child variables including child age.

Variables (Final Block)	Numeracy Support				Patterning Support			
	Frequency [a]		Complexity [b]		Frequency [c]		Complexity [d]	
	β	t	β	t	β	t	β	t
Child age	0.08	1.84	0.11	2.62**	0.03	0.70	0.03	0.72
Educational attainment	0.13	2.79**	0.19	4.10***	0.14	3.12**	0.17	3.81***
Household income	0.08	1.77	0.02	0.55	0.03	0.75	0.04	0.99
Value of child math ability	0.19	3.85***	0.15	2.90**	0.15	2.90**	0.13	2.50†
Expectation for child's math development	-0.10	-2.03†	-0.16	-3.08**	-0.19	-3.88***	-0.23	-4.69***
Perception of child current math ability	0.43	6.50***	0.49	7.18***	0.31	4.20***	0.31	4.31***
Perception of child math interest	0.14	2.16†	0.18	1.50	0.26	3.78***	0.29	4.24***
Knowledge about early math development	0.03	0.72	0.02	0.41	0.15	3.41***	0.16	3.72***

Notes. Parents' beliefs and knowledge about numeracy and patterning were used in numeracy and patterning models respectively.
[a] $R^2 = .44$, $F(8, 335) = 35.25$, $p < .001$. [b] $R^2 = .43$, $F(8, 335) = 32.09$, $p < .001$. [c] $R^2 = .45$, $F(8, 335) = 35.56$, $p < .001$. [d] $R^2 = .47$, $F(8, 335) = 38.80$, $p < .001$.
*** $p < .001$. ** $p < .0125$. † $p < .05$.

Appendix C

Reliability and Validity of Parents' Knowledge about Math Skills (Beyond Developmental Range Subscale)

Given that parents' knowledge about math skills that are within the typical developmental range for preschoolers was negatively related to their knowledge about skills that are beyond the typical developmental range (see Table C1) and that the overall measure had low reliability, I report on parents' knowledge about these two categories of skills separately. Specifically, analyses on the reliability and validity of the subscale measuring parents' knowledge about skills that are within the developmental range are reported in the main text while analyses on the reliability and validity of the subscale measuring parents' knowledge about skills that are beyond the developmental range are reported in this appendix.

Table C1

Correlations among the subscales of the measure of parents' knowledge about early math development

Subscale	1	2	3	4	5
1. Numeracy within	-				
2. Numeracy beyond	$-.25^{***}$	-			
3. Numeracy overall	$.73^{***}$	$.47^{***}$	-		
4. Patterning within	$.40^{***}$	$-.31^{***}$	$.15^{**}$	-	
5. Patterning beyond	$-.19^{***}$	$.61^{***}$	$.25^{***}$	$-.33^{***}$	-
6. Patterning overall	$.28^{***}$	.08	$.32^{***}$	$.78^{***}$	$.31^{***}$

Reliability

Cronbach alpha and Kuder-Richardson Formula 20 were computed for items focused on numeracy skills that are beyond the developmental range and revealed acceptable reliability (α = .66 and KR20 = .66). Likewise, Cronbach alpha and KR20 were computed for patterning items that are beyond the developmental range and revealed acceptable reliability ($\alpha = .76$ and KR20 = .76).

Validity

Convergent Validity. As a measure of convergent or concurrent validity, I examined whether each subscale was significantly related to parents' beliefs about their child's corresponding ability. I found evidence of convergent validity for both parents' knowledge about early patterning and about early numeracy. Specifically, parents' knowledge about early patterning development (average of items corresponding to patterning skills within the developmental range except for the previously identified item) was significantly correlated with their perception of their child's patterning abilities, $r(342) = -.43$, $p < .001$. Parents' knowledge about early numeracy development (average of items corresponding to numeracy skills within the developmental range except for the previously identified item) was also significantly correlated with their perception of their child's numeracy abilities, $r(342) = -.33$, $p < .001$.

Discriminant Validity. However, I did not find evidence of discriminant validity for parents' knowledge about early numeracy development and early patterning development. Specifically, parents' knowledge about early numeracy development was correlated with their perception of their child's literacy abilities, $r(343) = -.45$, $p < .001$, as was their knowledge about early patterning development, $r(343) = -.52$, $p < .001$.

Descriptive Analyses

Parents accurately classified 38% of the items about numeracy skills that are beyond the developmental range for most five-year-old children ($M = .38$, $SD = .37$). Their average accuracy on this subscale was not substantially skewed but was platykurtic as indicated by a kurtosis statistic that is less than -1 (skew = .47, SE = .13; kurtosis = -1.28). Parents accurately classified a larger percentage of the items about patterning skills that are beyond the developmental range for most five-year-old children (*M* = .54, *SD* = .41). Their average accuracy on this subscale was not substantially skewed but was platykurtic as indicated by a kurtosis statistic that is less than -1 (skew = -.16, SE = .13; kurtosis = -1.54).

Appendix D

Exploring Parents' Expectations about Early Math Development and Their Math Support

Parents' expectations for their children's numeracy and patterning development were unique, positive predictors of their numeracy and patterning support, except for the complexity of their patterning support when their other child-specific numeracy and patterning beliefs are not controlled for (see Table D1). Additionally, a composite of parents' child-specific math beliefs including their expectations was a unique, positive predictor of their math support (see Table D2). Interestingly, parents' knowledge about early numeracy development was a unique, positive predictor of the frequency of their numeracy support when their expectations for their child's numeracy development were the only child-specific beliefs controlled for.

Table D1

Linear regression predicting parents' math support by their expectations for their child's math development, knowledge about math development, and socioeconomic status.

Variables (Final Block)	Numeracy Support				Patterning Support			
	Frequency [a]		Complexity [b]		Frequency [c]		Complexity [d]	
	β	t	β	t	β	t	β	t
Educational attainment	0.30	5.75***	0.37	7.03***	0.33	6.42***	0.36	7.09***
Household income	0.06	1.09	0.00	-0.08	0.00	0.07	0.03	0.09
Expectation for child's math development	0.25	5.15***	0.17	3.60***	0.15	3.09**	0.15	2.38†
Knowledge about early math development	0.13	2.72**	0.11	2.17†	0.31	6.34***	0.32	6.58***

Notes. Parents' beliefs and knowledge about numeracy and patterning were used in numeracy and patterning models respectively.
[a] $R^2 = .19$, $F(4, 339) = 21.01$, $p < .001$. [b] $R^2 = .19$, $F(4, 339) = 19.18$, $p < .001$. [c] $R^2 = .22$, $F(4, 339) = 24.32$, $p < .001$. [d] $R^2 = .24$, $F(4, 339) = 26.40$, $p < .001$.
*** *p < .001.* ** *p < .0125.* †*p < .05.*

Table D2

Linear regression predicting parents' math support by their child-specific math beliefs (composite), knowledge about math development, and socioeconomic status.

Variables (Final Block)	Numeracy Support				Patterning Support			
	Frequency [a]		Complexity [b]		Frequency [c]		Complexity [d]	
	β	t	β	t	β	t	β	t
Educational attainment	0.19	4.05***	0.27	5.58***	0.22	4.69***	0.25	5.47***
Household income	0.05	1.20	-0.01	-0.18	0.00	-0.06	0.00	0.06
Child-specific math beliefs	0.52	11.69***	0.46	9.81***	0.46	10.16***	0.44	9.69***
Knowledge about early math development	0.05	1.13	0.03	0.68	0.19	4.29***	0.20	4.57***

Notes. Parents' beliefs and knowledge about numeracy and patterning were used in numeracy and patterning models respectively.

[a] $R^2 = .38$, $F(4, 339) = 52.88$, $p < .001$. [b] $R^2 = .33$, $F(4, 339) = 43.78$, $p < .001$. [c] $R^2 = .38$, $F(4, 339) = 24.32$, $p < .001$. [d] $R^2 = .39$, $F(4, 339) = 54.83$, $p < .001$.

*** *$p < .001$.*

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