FRAMING AN ECOLOGICAL PERSPECTIVE ON MATHEMATICS TEACHERS' COLLABORATIVE SENSEMAKING AS PROFESSIONAL DEVELOPMENT

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Nadav Ehrenfeld

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Approved:

Ilana S. Horn, Ph.D.

Noel D. Enyedy, Ph.D.

A. Susan Jurow, Ph.D.

Barbara S. Stengel, Ph.D.

This dissertation is dedicated to the memory of my father Ami Ehrenfeld (1951-2015)

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INTRODUCTION

Over the last two decades, researchers have explored and documented ways to support mathematics teachers in working towards more cognitively and socially ambitious pedagogical goals. The resulting studies challenge traditional models of professional development (PD) that tend to be acontextual in nature and divorced from the rhythms and settings of teachers' practices. Instead, they portray quality PD as collaborative and situated in teachers' instructional contexts (Ball & Cohen, 1999; Borko et al., 2008; Horn, 2005; Horn & Garner, 2022; Kazemi & Hubbard, 2008; Wilson & Berne, 1999). In this dissertation, I refer to these designs as *Collaborative Sensemaking as Professional Development* (CSPD).

Researchers who study teacher CSPD typically adopt sociocultural, situated, and situative perspectives (Greeno, 2006; Lave & Wenger, 1991; Vygotsky, 1980), all of which underscore the importance of context in interaction and learning. However, it is not always clear which contexts warrant careful attention and which are overlooked. This theoretical and analytical gap was made clear by several literature reviews in the research fields of teacher professional conversations (Lefstein et al., 2019), teacher learning (Opfer & Pedder, 2011), and mathematics teachers' professional development (Goldsmith et al., 2014). As these reviews suggest, studies of teacher PD most often consider the immediate interactional context of learning (*microsystems*), sometimes consider the institutional context (*mesosystems*), and seldom acknowledge either the multiple experiences teachers have in remote settings (*exosystems*) or the sociopolitical, economic, and racial contexts of the PD (*macrosystems*). **Some broader and deeper contexts are accounted for by many researchers. However, there is no theoretical consensus or compelling framework that guides researchers of teacher PD towards being explicit and**

mindful of the contexts they account for and ignore. My goal here is to offer one possible framing.

I refer to this framing as *ecological* to highlight the quality of looking at learning as shaped by an interactive set of environments (Barron, 2006; Bronfenbrenner, 1979; Cobb et al., 2003a). When discussing *teacher learning ecologies*, I follow Cobb et al.'s (2003a) definition of a learning ecology as "a complex, interacting system involving multiple elements of different types and levels" (p. 9). Studies of instructional reforms tend to focus on a single PD or on school, obscuring the ways teacher learning spans multiple experiences and communities, all of which are essential in providing teachers with access to resources and expertise (Coburn et al., 2012; Morel & Coburn, 2018; Penuel et al., 2007). In this dissertation, I focus on a teachers' multiple professional experiences, as a specific aspect of teacher PD that is understudied and yet central to PD from teachers' perspectives. My empirical question then is: *What is the role of teachers' previous professional experiences in their collaborative sensemaking?*

Paying more attention to previous professional experiences in PD conversations raises questions regarding what these resources are and where they come from (*scope*), how they are used across contexts in conversations (*interconnectedness*), and when they arise in teachers' sensemaking (*temporality*). These types of questions highlight the need to frame teacher PD in deeper and wider ways. In the first conceptual part of the dissertation, I build on sociocultural, ecological, and complexity theories to work towards clarifying what an ecological framing of teacher PD could entail. I do that by operationalizing the notions of *scope, interconnectedness,* and *temporality*. Within my conceptual framework, I refer to the range of professional settings (e.g., PD workshops or conferences) attended by teachers in the team as the *exosystem*. Respectively, I refer to the experiences and resources from the exosystem as *exo-resources*.

In the second empirical part of the dissertation, I use data from Project SIGMa (Horn & Garner, 2022) to illustrate the ecological framework and to explore three sub-research questions (SRQs). Project SIGMa was a research-practice partnership with a PD organization, and, as a central part of our joint work, we used video-based conversations to support secondary mathematics teacher teams in improving their practice. In Chapter 4, I ask, *How do teachers invoke and use exo-resources in collaborative sensemaking?* (SRQ1). This chapter builds on two case studies from different teacher teams to illustrate the transformative potential of exo-resources in supporting teachers' collaborative reasoning. Thus, this chapter offers a proof-of-concept — exo-resources can constitute a core part of teachers' sensemaking as they engage in instructional change.

Identifying episodes of teams who invoked exo-resources as a part of their sensemaking still leaves open questions about how common this was in our data set and what kinds of exo-resources were salient for our participants. To this end, Chapter 5 zooms-out of the case study perspective to portray a bird's eye view on the exosystem. Looking across nine video-based conversations of the same two teams, I ask, *What exo-resources are invoked by teachers in collaborative sensemaking, and where do they originate from?* (SRQ2). In doing so, I illustrate how common and diverse the phenomenon of referencing exo-resources was across the two teams. Findings also show that while the exo-resources of the two teams differed, they could be described with the same six categories: PD workshops, conferences, PD organizations, online resources, research & policy, and curricula. These categories provide a framework for designers and facilitators who want to take seriously the practice of inviting, acknowledging, and building PD conversations on teachers' prior knowledge and experiences. Finally, this chapter reveals that most exo-resources in this study were aligned with visions of ambitious mathematics teaching.

This finding strengthens the notion that the two teams' exosystems were potentially a rich source of contributions when invoked and recognized as important.

While these empirical chapters illustrate that exo-resources and exosystems matter in teacher PD and we should pay more attention to them, I then theorize the temporal dimension of teachers' learning more carefully to offer a generative framework to the field. In the last empirical chapter, Chapter 6, my goals are twofold. First, I seek to understand phases within learning trajectories in the PD to move beyond linear notions of progress in learning about teaching. That is, instead of focusing on temporal issues as questions of duration, I investigate qualitative aspects of time (Erickson, 2004; Gunderson & Holling, 2002), distinguishing different types of learning at different phases. Second, tying this back to the notions of scope and interconnectedness, I aim to understand how these phases are shaped by different resources and levels in teacher learning ecologies. Bringing the two goals together, I ask, *How do different levels of teachers' learning ecologies interact in different phases of their learning?* (SRQ3). Building on the notion of temporality from the conceptual framework, I describe the teams' learning in the PD through the cyclic phases of *problematization*, *reorganization*, *growth*, and *conservation*. Findings show that the first team focused on concept development through reorganization and growth. In contrast, the second team focused on conceptual change across all four phases of learning. Additional findings reveal phases in which video-based reflections were more salient to teacher learning, and phases in which the role of exo-resources was more central. These findings provide numerous implications for teacher education.

In the concluding Chapter 7, I discuss the affordances of looking at teacher learning trajectories through the lens of scope, interconnectdness, and temporality. I argue that it is a productive expansion of current sociocultural perspectives on teacher PD, in the sense that it is

complex enough to capture and explain processes of teacher learning, and yet simple enough to be useful for researchers and teacher educators.

CHAPTER 1

Literature Review on Teacher Professional Development and Theoretical Framework: Sociocultural, Ecological, and Complexity Theories

Research on teacher PD tends to be divorced from the multiple rhythms and settings of teachers' practice. It often focuses on the direct effect of PD interventions, where teacher learning itself is not the main object of study but rather an indicator of the effectiveness of specific programs (Goldsmith et al., 2014; Opfer & Pedder, 2011). A typical linear pathway perspective for teacher PD assumes (often implicitly) that teachers attend PD where they develop their knowledge and beliefs, which, in turn, changes their teaching practices and eventually improves students' learning (Figure 1; Clarke & Hollingsworth, 2002).

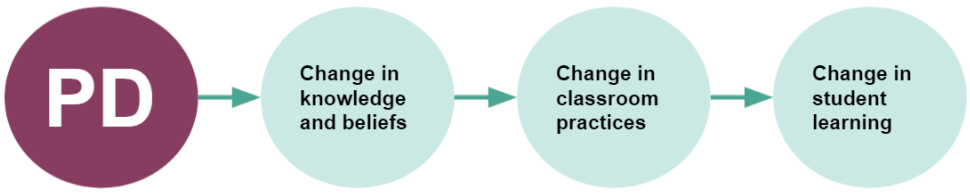

Figure 1: Typical linear pathway perspective on teacher PD

Studying PD through linear pathway lenses can be useful, and they remain central in research on teacher learning for good reasons. These reasons are (1) knowledge accumulation, (2) policy, and (3) logistics. First, from a knowledge accumulation perspective, linear models offer visible and relatively fast ways to study specific programs or refine general guidelines for effective PD (e.g., Desimone, 2009). These lists provide a pragmatic toolkit of "core features" of professional development. On the other hand, critics of such lists of core features pointed out that

they lend themselves to a binary view of variables of PD as absent or present (Opfer & Pedder, 2011). A binary perspective on features of PD is problematic because it overlooks their specifications and interactions, which are likely highly consequential for teachers' learning. In addition, we are unable to predict teacher learning in PD based on these characteristics alone (Asterhan & Lefstein, 2020; Opfer & Pedder, 2011; Borko, 2019), adding empirical evidence to their insufficiency as a guide to designing and facilitating PD.

Second, from a policy and accountability perspective, schools, districts, and researchers, are under pressure to improve achievement measures, a framework that privileges linear and causal research over contextual or ecological inquiry of how and why change occurs (Hill et al., 2013; Opfer & Pedder, 2011; Sztajn, Borko & Smith, 2017). Indeed, increased large scale testing in the U.S. provides yearly data that makes such measures for PD "success" accessible and convenient (Hill et al., 2013), especially in mathematics. In the case of researchers, they are encouraged by funding agencies to write grant proposals that make visible the direct impact of their own designs, making it less likely that they will position their PD as merely another component in teachers' professional lives.

From a logistical perspective, it is more convenient for researchers to study teacher learning in single PD activities, usually led by their own teams, than to investigate the ongoing learning of teachers with relation to the multiple contexts they engage with. Lefstein et al. (2020) critique this tendency by comparing it to a drunk looking for their keys under the streetlamp rather than where they lost them; just because a location is illuminated does not make it the right place to look.

In contrast to the focus on direct effects of single activities, from teachers' perspectives, their experiences in PD are always connected to a web of other learning experiences. Linear

models tend to simplify these interconnected influences and consider them a methodological problem. For example, in their essay on studying PD impacts, Wayne et al. (2008) discuss the variety of PD activities that teachers engage with as ambient PD. They explain:

> Teachers participate in a variety of PD activities each year because of mandates, incentives, or personal initiative. Teachers may be part of informal groups at their schools that serve PD needs. Teachers will presumably continue to participate in all these PD experiences regardless of their status in the study, except to the extent researchers are able to negotiate special arrangements. We refer to this PD as the ambient PD, to indicate that it pervades the context in which the study takes place. *The existence of ambient PD is problematic* because an experiment measures impact as the difference in outcomes between the treatment group and the control group. To the extent that the content of the ambient PD overlaps with the content of the study PD, the difference in outcomes between the treatment and the control conditions may be reduced. [...] One way to address such a problem is to conduct one's study in districts where the content of the study PD is least likely to overlap with the content of the ambient PD; another is to increase the intensity of the study PD to sharpen the contrast with the ambient PD. Through selection of the study context, one can also avoid selecting contexts in which the ambient PD would contradict the study PD (pp 472-473, emphasis added).

In contrast to the concern about teachers' learning ecologies as a methodological problem, I argue that appreciating these interconnected complexities, rather than avoiding or simplifying them, would better position teacher educators to support learning within PD initiatives. However, understanding how teachers make sense of PD and incorporate changes into their instruction requires frameworks that invite situated web-like logics. In this dissertation,

I offer one possible framework by bringing forth the three aforementioned dimensions of *scope, interconnectedness, and temporality*.

In the rest of this chapter, I discuss sociocultural, ecological, and complexity theories, as the theoretical foundation of this framework. First, I discuss the ways sociocultural research on teacher learning resists some of the constraints that characterize linear models, and instead privileges contexts and conversations, and positions teachers as agentic learners and sensemakers (Borko, 2004; Horn & Garner, 2022; Lefstein & Snell, 2013). For these reasons, this work is rooted in a sociocultural perspective on teacher learning (Horn & Garner, 2022). Then, I argue that the three aforementioned dimensions of scope, interconnectedness, and temporality are *inherently* overlooked when using linear models, *can be* overlooked by sociocultural perspectives, and are *at the heart* of ecological and complexity theories. For this reason, the conceptual framework is also strongly informed by the latter two theories.

1.1 Sociocultural Theories

At the center of sociocultural perspectives on learning is the notion that learning is social and interaction plays a key role in learning processes (Borko, 2004; Greeno, 2006; Horn & Garner, 2022; Sfard, 2008). Studies that take a sociocultural lens with a focus on teacher conversations include both investigations of teacher learning on-the-job and more structured learning opportunities in PD settings. Notably, this analytic distinction is a spectrum rather than two bodies of literature with well-defined boundaries. In fact, CSPD settings that are at the center of this dissertation, are located exactly in this intersection. On the one hand, they are collaborative and situated in teachers' instructional context, hence to some extent, on-the-job. On the other hand, while these activities might physically be within school space, they are typically initiated and facilitated by researchers rather than the participating teachers, hence to some extent, PD.

Sociocultural research of teacher conversations in PD settings includes different types of interventions, such as Japanese Lesson Study (Dudley, 2013; Huang et al., 2019), teacher video-based conversations (Borko et al., 2008; Horn & Garner, 2022; van Es & Sherin, 2010), and Professional Learning Communities (PLCs; Grossman et al., 2001; Popp & Goldman, 2016). Researchers operationalize learning in a variety of ways to answer the questions of how and what teachers learn from an interactionist perspective: They detect specific learning moments within meetings; follow processes of collaborative and individual sensemaking and knowledge construction; and underscore conditions that are productive for learning (Lefstein et al., 2019).

Research on learning on-the-job is rooted in sociological traditions of school as a workplace (Lortie, 1975), anthropological approaches of learning within communities (Lave & Wenger, 1991), and organizational psychology (Feldman and Pentland, 2003). On-the-job perspectives typically represent learning as ongoing, inevitable, and situated in the multiple spaces of everyday life. An early, well-known piece is Little's (1990) study on the persistence of privacy in teachers' everyday activities. In the study, Little elaborated on the extent to which teachers work collaboratively in different activities, suggesting that the activities rarely extend to direct commentary on practice, and are thus only mildly beneficial for teaching (see also Horn et al., 2017). Extending this work, researchers continued to inquire into teacher learning through interaction within everyday contexts of their work. For example, Horn (2005) followed mathematics teachers in two high-school teams, recognizing three main processes of everyday learning on-the-job: First, teachers learned by collectively assigning local meanings to instructional resources. Second, teachers learned by collaboratively constructing informal language categories. And third, the ways that teachers shared classroom events and practices in

collegial talk significantly affected the quality of learning opportunities provided by their conversations.

More recently, building on Vygotsky's (1978) ideas of concept development as emerging from the interplay between formal and lived concepts, Horn and Garner (2022) theorized that learning of ambitious mathematics instruction can productively be viewed as *a situated conceptual change*. Their approach strongly takes into consideration how teachers' understandings are embedded in webs of social meaning — specifically, the fact that ambitious mathematics teaching most often goes against the institutional logics of schooling. Concepts, in this sense, are not abstract pieces of information or prescriptions to implement. Rather, they are "teachers' narratives about given teaching practices, along with their related conditions and consequences… pointing to the ways concepts span across people, activities, and contexts" (Horn & Garner, 2022, p. 48; see also Hall & Horn, 2012; Hall & Jurow, 2015).

Understanding concepts for teaching as distributed across people, activities, and contexts, calls attention to the conceptual infrastructure used for the representation, organization, flow, and production of knowledge (Erickson, 1986; Hall & Horn, 2012; Hall & Jurow, 2015; Horn, 2005; Jurow et al., 2019). Horn and Kane (2015) and Horn and Garner (2022) suggested a framework for key conceptual resources that, together, form the conceptual infrastructure shaping how teachers make sense of teaching. These included: *onto-epistemic stances*, *representational infrastructure*, *problem frames*, *institutional logics* and *activity structures* (see Table 1).

Conceptual Resource	Definition
Onto-epistemic stances and claims	Positions and statements about what can be known by them and by students, how they come to know it, and why it matters (Garner, 2018; Hall & Horn, 2012)
Representations of practice	Implicit or explicit aspects of communication that make different aspects of teaching more or less visible (Hall & Horn, 2012; Little, 2003)
Problem frames	Implicit or explicit aspects of communication about problems of practice that make different ways to understand a situation count as legitimate (Bannister, 2015; Goffman, 1974; Louie, 2016; Vedder-Weiss et al., 2018)
Structures and rituals of schooling	Social organization of institutional logics (Horn & Garner, 2022)
Activity structures	The patterned ways (formalized or improvised) tasks get carried out in teachers' work (Horn & Garner, 2022; Horn & Little, 2010)

Table 1: Framework of Conceptual Infrastructure in Teacher Conversations (adapted from Horn & Kane, 2015; Horn & Garner, 2022)

Building on this framework, Horn and Kane (2015) provided evidence that limited engagement with conceptual resources in teacher workgroups results in limited learning opportunities. These findings and others imply that teacher collaborative sensemaking significantly changes depending on the resources teachers build on. It illuminates the need to recognize additional resources that are salient for teachers within professional interactions. Similarly, Horn and Garner (2022) explained:

> Identifying the conceptual resources teachers draw on is important for supporting teachers' concept development, as they offer potential levers for transforming teachers' understandings towards ambitious and equitable instruction — ones that live outside individual teachers… (p. 77)

Building on these findings and arguments, this dissertation features another type of conceptual resource that opens up new units of analysis for the study of teacher conversations. As Hall and Jurow (2015) argued in their discussion of conceptual practices, "to understand concepts and conceptual change, one must seek different units of analysis and processes that play out at multiple scales in time, setting, and social participation" (p. 187). Specifically, I focus on the ways teachers invoke lived experiences and resources from remote settings as external conceptual resources, as well as their potential to transform the local representational infrastructure and problem frames, towards teachers' concept development.

My focused (and more modest) goal is to theorize these external conceptual resources, what they are, where they are invoked from (scope), how they are being used (interconnectedness), when (temporality), and towards what ends. However, this task calls upon a more ambitious goal, which is framing the PD in the context of teacher learning ecologies. To do so, I next turn to ecological and complex theories.

1.2 Ecological Theories

In line with sociocultural traditions of learning, Horn and Garner (2022) called for reflecting on how we consider teachers' contextual realities in the analysis of teacher learning. Indeed, studying teacher learning "in context" could mean many things. *Ecological theories* foreground the fact that learners are simultaneously involved in many settings, and learning is always shaped by the relation between these settings, as well as by the social contexts in which they are

embedded (Barron, 2006; Bronfenbrenner, 1979; Cobb et al., 2003a). Cobb et al.'s (2003a) defined *learning ecology* as "a complex, interacting system involving multiple elements of different types and levels… complexity that is a hallmark of educational settings" (p. 9). Their argument was that educational research designs should ideally result in a greater understanding of learning ecologies. Toward this goal, for example, Cobb et al. (2003b) analyzed interconnections across teacher learning communities by focusing on *boundary objects* (Star & Griesemer, 1989) such as state standards or district pacing guides around which teachers organize their collaborative sensemaking. The authors pointed out that these resources do not carry ready-made meanings across contexts but instead constitute a starting point for local meanings to emerge.

Urie Bronfenbrenner was a co-founder of the Head Start program, and his work mostly examined how U.S. society supported the development of children and families (Shelton, 2019). The goal of his framework was to theorize the ways people develop within and across changing settings "in both the immediate and more remote environment" (p. 11). Bronfenbrenner defined human development as follows:

> the process through which the growing person acquires a more extended differentiated, and valid conception of the ecological environment, and becomes motivated and able to engage in activities that *reveal* the properties of, *sustain*, or *restructure* that environment at levels of similar or greater complexity in form and content. (p. 27, emphasis added)

Given that ambitious mathematics instruction is still counter-cultural —a disruption — to the realities of most schools in the United States (Horn & Garner, 2022; Louie, 2017a), an extended understanding of teacher learning ecologies — and of ways to act within it and upon it — becomes crucial for teachers committed to changing instruction in inclusive ways.

A main contribution of Bronfenbrenner's ecological framework is the four analytic levels of *micro-, meso-, exo-, and macro- systems.* (Note that users of these ideas often adapt the micro- meso- macro- trio and exclude the exosystem and its resources.) *Microsystems* represent immediate settings in which people engage. The *mesosystem,* rather than a layer that surrounds the microsystem, represents the relationships between two or more settings. For example, for a child, it can be the relation between home and school. The *exosystem,* central to this work, consists of settings in which the focal person is not actively involved, but others who interact with them are. Following our example of a child, if we consider the relations between home and school as a mesosystem, then the parents' workplace or social lives could be considered part of their exosystem, even if they never physically attended these settings. For teachers, if we consider the school as a mesosystem, then PD settings attended by their peers would become part of the exosystem of their learning ecology. The *macrosystem* relates to the larger social, cultural, economic, racial and political structures within which learning is taking place, including the different types of identities they invite or discourage.

Importantly, operationalizing the four levels in the case of teacher learning should only be considered as a first step towards studying them as interconnected. According to reviews of literature by Tudge et al. (2009, 2016), many wrongly see Bronfenbrenner's framework as a theory about the direct-linear influence of context on development, which means they use it to "simplify what is complex" (Tudge et al., 2016, p. 429) by asking questions about the direct effects of individual factors (i.e., a "reductionist" or "mechanist" paradigm). In contrast, Bronfenbrenner's four levels are meant for exploring how different factors act synergistically towards multiplicative outcomes.

1.3 Complexity Theories

In line with ecological theories, a complexity theory orientation seeks to identify systems that interact towards the emergence of teacher professional learning (Opfer & Pedder, 2011). *Complex*, as Opfer and Pedder explain, is a category different from *simple* or *complicated* problems. Simple problems involve a few variables, interactions, and possible outcomes. Complicated problems involve many variables and interactions, and yet their outcomes are predictable. Cochran-Smith et al. (2014) described complicated systems as wholes that are equal to the sums of their parts. In contrast, complex systems are more than the sum of their parts, with their complexity emerging from the relationships between their components. While the outcomes of complex problems (such as the outcomes of teacher PD) are not predictable, they are also not random. This frames teacher learning as unpredictable and yet highly patterned (Opfer & Pedder, 2011), and these patterns "can be generalized [even] across highly contextualized instances" (Opfer & Pedder, 2011, p. 381).

Complexity theories also guide us to look at phases of learning beyond linear progress. Just as linear and simplistic trends in cognitive psychology prompted the development of Bronfenbrenner's (1979) ecological framework, traditional environmental theories led to the development of Gunderson and Holling's (2002) adaptive cycles in explaining environmental changes. In traditional environmental ecology, processes of change within ecosystems (such as forests) were described as linear with two phases: *growth* towards an end point or climax, and *conservation*, the state the system would reach if not disturbed. In the conservation phase, the level of resources in use is high, and the connectedness of components is high (see Figure 2). This means external variability has low influence on the system. However, in adaptive cycles, the climax, or the conservation phase, becomes a transition phase in a continuous cycle,

proceeding through phases of *release* (sometimes referred to as creative destruction, or collapse), *reorganization*, and then again growth and conservation. In the forest example, the phase of release can be thought of as triggered by forest fires or drought. Then, in the reorganization phase, nutrients become available for new pioneer species to capture opportunities towards the following phase of growth. As I later illustrate, adaptive cycles can also describe phases of teachers' learning.

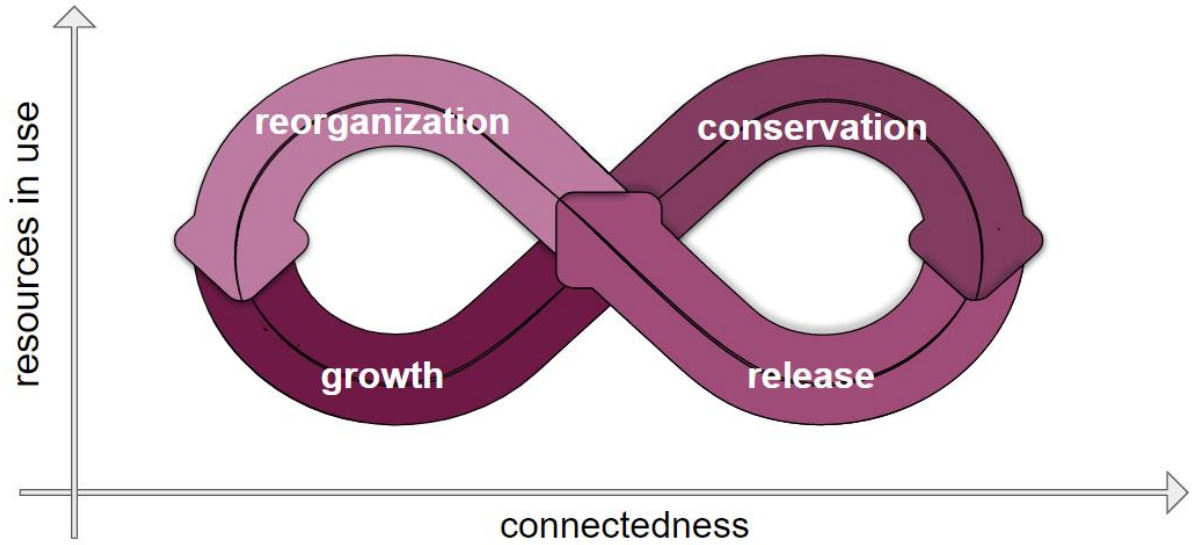

Figure 2: The four phases of the adaptive cycles

Note. The cycle reflects changes in two properties: (1) The y-axis represents accumulated resources in use (originally described by Gunderson and Holling as *potential*); (2) The x-axis represents connectedness of elements within the system. The behavior of loosely connected elements is largely influenced by external variability. The behavior of highly connected elements is mostly influenced by their inward relations, which strongly mediate external variability.

In the next chapter, I present the conceptual framework. For each of the three dimensions of scope, interconnectedness, and temporality, I first elaborate on their affordances, and then suggest ways to operationalize them for research of teacher PD. Later on, I illustrate them with empirical examples from Project SIGMa.

CHAPTER 2

Conceptual Framework: Scope, Interconnectedness, and Temporality

To frame an ecological perspective on teacher CSPD, I bring forth the notions of its scope, interconnectedness, and temporality (see Table 2). The framework presented here is rooted in and contributes to sociocultural research on teacher learning (Borko, 2004; Horn & Garner, 2022; Lefstein et al., 2019) by adopting elements of ecological models of learning (Bronfenbrenner, 1979; Cobb et al., 2003a; Erickson, 2004; Nasir et al., 2020), as well as concepts from complex systems theories (Opfer & Pedder, 2011; Gunderson & Holling, 2002).

Elements	**Inspired by…**	**Affords…**	**Operationalized by…**
Scope	Bronfenbrenner (1979); Cobb et al. (2018)	A lens for naming and distinguishing scopes and contexts of teacher learning	Micro- meso- exo- and macro- levels
Interconnectedness	Clarke & Hollingsworth (2002); Cobb et al. (2003b)	A lens for understanding connections between immediate PD settings and broader contexts	The ways teachers invoke lived experiences and external conceptual resources from remote settings
Temporality	Erickson (2004); Gunderson & Holling's (2002); Horn & Garner (2022)	A lens for understanding learning trajectories in the PD beyond linear progress.	Concept development within Adaptive Cycles of release, reorganization, growth, and conservation

Table 2: Suggested elements of an ecological perspective on teacher learning

2.1 Operationalizing Scope: Beyond the Microsystem of Teacher PD

As I reviewed, Bronfenbrenner's framework (1979) includes four interacting structures of environment: microsystem, mesosystem, exosystem, and macrosystem, with a later emphasis on time (Bronfenbrenner, 2005; Xia et al., 2020). To clarify, the point of this section is *not* that researchers need to follow and document teachers' activity across all activities all the time *nor* that each analysis needs to include all possible contexts and scopes. Rather, it is a call to be aware of and explicit about contexts that might be salient and yet overlooked in designs for and analyses of teacher learning. Towards this goal, I re-consider Bronfenbrenner's four levels in the case of teacher CSPD conversations.

2.1.1 Microsystems & Mesosystems

In this work, the main *microsystem* of interest is, of course, the CSPD setting. The main *mesosystem* of interest is *the school institutional mesosystem*. I refer to these levels in the analysis by using the aforementioned framework for understanding the conceptual framework of teacher conversations by attending to the conceptual resources of *onto-epistemic stances, representations of practice, problem frames, institutional logics,* and *activity structures* (Horn & Garner, 2022; Horn & Kane, 2015).

2.1.2 Exosystems

In this work, the *exosystem* represents professional settings external to the school, attended by one or more teachers in a team, but not necessarily by all. These settings include, for example, PD workshops, conferences, and teachers' previous schools. As I will argue and illustrate, when teacher teams collaborate, they often invoke these external conceptual resources (from now on to be called exo-resources) and can significantly transform their conversations and learning.

2.1.3 Macrosystems

In CSPD settings, the *macrosystem* represents the social, racial, economic, and other political structures within which the teachers and students live and work, and within which the school operates. Consolidating macrosystems in frameworks of teacher learning is crucial to support researchers in integrating issues such as power dynamics, class realities, and racial tensions more seriously into analysis of teacher conversation, and consequently, into teacher collaborative sensemaking. In this work I attend to macrosystems insofar that the teachers explicitly discuss them, and with regards to the histories and political baggage that the conceptual resources carry.

2.1.4 Summary of Scopes and Contexts

Figure 3 represents the suggested scope of an ecological perspective on teacher learning. Figure 4 illustrates possible contexts in the study of teacher PD when incorporating these levels. To reiterate, my claim is *not* that every study of teacher learning must include all possible aspects of teachers' learning ecologies. Rather, attempts to look at subsystems must be understood as partial (Bronfenbrenner, 1979; Opfer & Pedder, 2011), and employing more complex perspectives on teacher learning would extend our ability to explain it and better support teachers (Clarke & Hollingsworth, 2002; Cochran-Smith et al., 2014; Horn, 2005; Opfer & Pedder, 2011). Importantly, distinguishing and naming contexts should only be considered as a first step towards studying them as interrelated. The next section on interconnectedness elaborates on this goal.

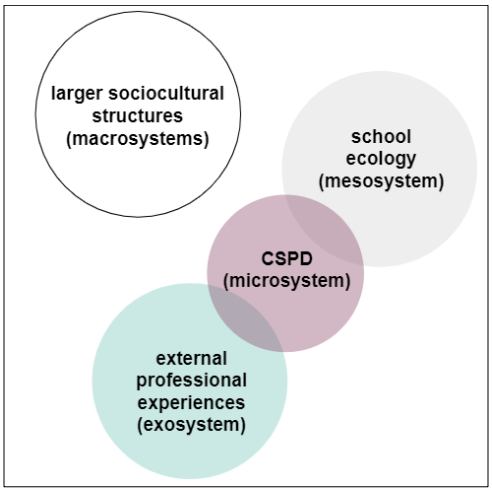

Figure 3: Suggested scope of an ecological perspective on teacher learning in CSPD

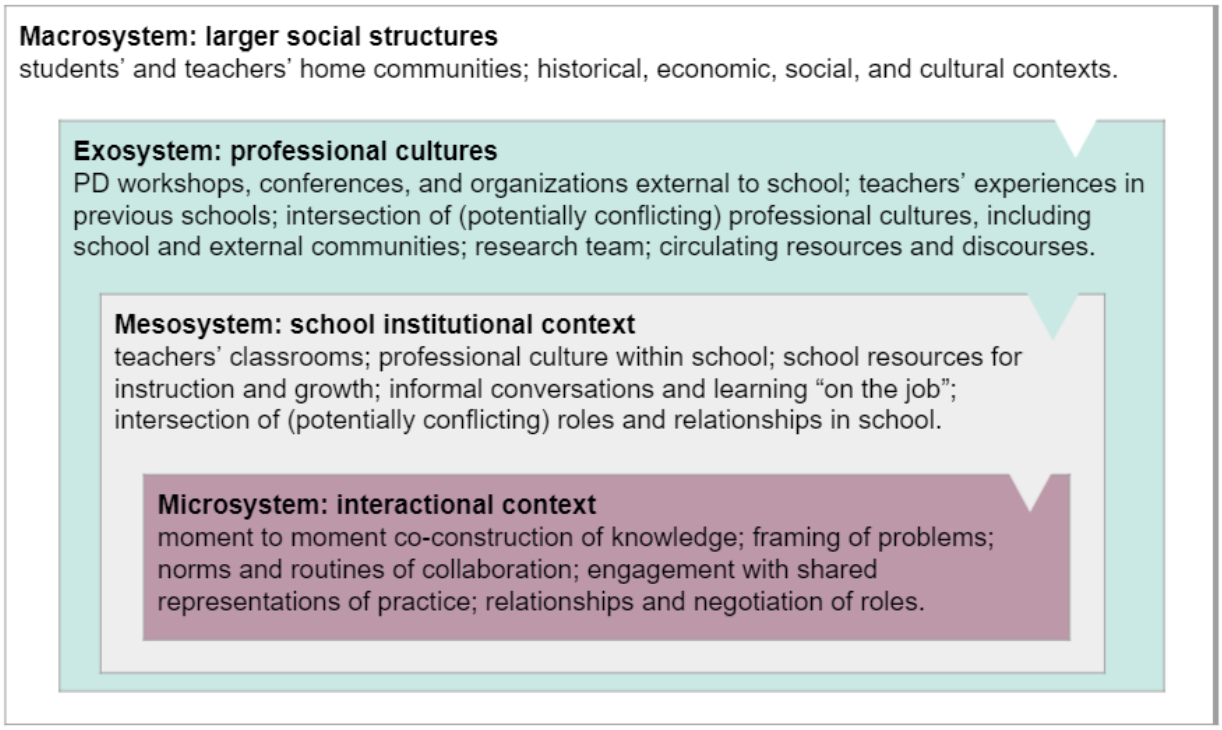

Figure 4: Possible contexts for the study of teacher conversations in CSPD

2.2 Operationalizing Interconnectedness: Invoked Experiences and Exo-Resources

Emphasizing interconnectedness guides us to identify specific patterns of interaction across resources and levels of activity that explain teacher learning (Cobb et al., 2003b; Opfer & Pedder, 2011). It requires a shift away from questions about the direct effects of individual factors. I suggest studying the interconnections across the micro- meso- exo- and macro- levels of the learning ecology by focusing on the ways teachers invoke experiences and resources as a specific type of boundary object (Akkerman & Bakker, 2011; Cobb et al., 2003; Pinto & Koichu, 2021; Star & Griesemer, 1989). As Akkerman and Bakker (2011) reviewed, on the one hand, claims about boundary objects and learning in the literature are widespread and appealing, but on the other hand, they are often "general in nature, and the literature hardly explicates how or what kind of learning is taking place" (p. 133). Here, attending to teachers' experiences and resources includes uncovering the connections across teachers' reflections on their teaching practices through the video based CSPD design (*microsystem*) with their experiences and resources in different professional settings (*exosystems*), reconciled with their school professional culture and institutional practices (*mesosystems*), which include the racial and economic realities of life in their instructional realities (*macrosystems*).

Focusing on interconnectedness across different levels makes visible how teachers incorporate new ideas into their sensemaking. It also makes visible tensions and contradictions across institutional logics, different professional experiences, and the goals of various PD, all of which are ubiquitous in teacher learning (Horn & Garner, 2022; Yamagata-Lynch & Haudenschild, 2009). Practice-wise, the goal of attending to interconnectedness is to acknowledge and connect teacher learning resources in a way that is timely and responsive to teachers' learning ecologies. Such responsiveness is essential in CSPD settings, and besides

scope and interconnectedness, it also requires a rich conceptualization of teacher learning trajectories across longer time periods.

2.3 Operationalizing Temporality: Concept Development within Adaptive Cycles

Adaptive cycles are a metaphor that helps us managing learning trajectories in a way that is not simply about linear progress. I center temporality in the sense of time that Erickson (2004) referred to as *kairos,* which in modern Greek means opportunity. It is the qualitative aspect of time as humanly experiences. It is not simply some duration, or the sequential *chronos*, but rather "a brief strip of the *right time*." (p. 7, originally italicized). In the case of CSPD, a focus on temporality helps us think about different phases in the teachers' learning trajectories and how they are supported by different resources (see Table 3).

Phase	**Forest Analogy**	**Teacher PD**
Release	Forest fires, drought, or intense pulses of grazing.	**Problematization** of institutional practices and teaching norms.
Reorganization	Nutrients become available for new pioneer species to capture opportunities.	**Navigating** tensions between institutional logics and teachers' pedagogical goals. Renewal, **planning**, and **recruitment** of new arrangements and practices
Growth	Competitive processes lead to a few species becoming dominant, potentially previously suppressed vegetation.	**Experimenting** with the new professional arrangement.
Conservation	Nutrient and biomass resources become bound with existing vegetation preventing others from utilizing them.	New practices are **consolidated** and teachers are proficient with a new professional arrangement.

Table 3: Four Phases of Adaptive Cycles in Forests and Teacher PD

Growth is the longest and slowest phase and represents what Gunderson and Holling

called incremental learning, where teachers are experimenting with new teaching arrangements, adapting or rejecting practices according to their needs and sense of agency, until the arrangement becomes relatively stable. Whether these are teachers who are socialized into mathematics teaching based on memorization of facts and procedures or teachers who are introduced to more ambitious forms of teaching, *conservation* represents the phase when conceptual practices are consolidated, and teachers become more proficient with their professional arrangement. *Release* represents the problematization of current institutional logic and teaching practices, and often involve external resources and agents. These can be professional organizations, PD workshops, conferences, and teachers' experiences in previous schools (i.e., the exosystem of teacher learning ecologies). In the case of video-based PD, video-based reflection can also disrupt existing teaching and learning arrangements. Then, *reorganization* can represent navigating tensions between institutional logics and teachers' pedagogical goals, inherent conditions of teacher learning and the nature of teacher knowledge (Horn & Garner, 2022). Just as the reorganization phase in the forest is where nutrients become available for new pioneer species, teachers' reorganization phase is about renewal, planning, and recruitment of new arrangements and practices. Gunderson and Holling (2002) describe this phase as "the engine of variety and the generator of new experiments" (p. 74). At this point, *transition to growth* (experimenting with the new professional arrangements) and back to *conservation* may represent what Gunderson and Holling called *transformational learning*, and in the case of teacher PD what Horn and Garner (2022) described as *conceptual and cultural change.*

Importantly, adaptive cycles also guide teacher educators' responsiveness to different phases of teacher learning. In the case of teacher learning, the x-axis in Figure 2 represents

connectedness of resources and teaching arrangements within the system. Within the reorganization and growth phases, when resources and teaching arrangements are loosely connected, invoked exo-resources are expected to be more salient in promoting change. In contrast, within the conservation and release phases, internal processes such as video-based reflection are more likely to be salient in promoting change.

CHAPTER 3

Methods

A Qualitative Analysis of Teacher Conversations and Learning Ecologies

In this chapter, I first describe the research context, including the larger project, data collection procedures, the teachers, and focal cases for this dissertation. I then elaborate on the data analysis procedures used to examine the role of teachers' previous professional experiences in their collaborative sensemaking. Finally, I explain how the different phases of analysis map to the sub-research questions about how teachers invoke their experiences and resources in conversation, what resources they invoke, where the resources originate from, and in which phases of their learning these episodes occur.

3.1 Research Context: Project SIGMa and Video-based Formative Feedback

This study is part of a larger research-practice partnership, Supporting Instructional Growth in Mathematics (Project SIGMa; Horn & Garner, 2022) where the research team collaborated with a Professional Development Organization (PDO) to support the participating teachers' development of ambitious and equitable mathematics instruction. Through this partnership, we worked with teachers from six schools. All participating teachers had five or more years of experience and were affiliated with the PDO. Together, we co-developed a video-based formative feedback (VFF) intervention to provide teachers with timely information about their classroom instruction and help them make sense of problems of practice.

The five-step VFF cycle unfolded as follows (see Figure 5): (a) the teacher identified a question of interest (e.g., how well are my questions surfacing student thinking?); (b) the day of

the lesson, the teacher identified four focal student groups to record; (c) the research team documented the lesson; (d) the team reviewed the video and audio with the teacher's question in mind, identifying clips that support co-inquiry; (e) the teacher, their school-based colleagues, and members of the research team met to view and discuss the clips in light of the teacher's question. Using the conceptualization of professional development I introduced earlier, I consider the VFF as CSPD, since it builds on teacher community and dialogue to respond to teachers' instructional puzzles.

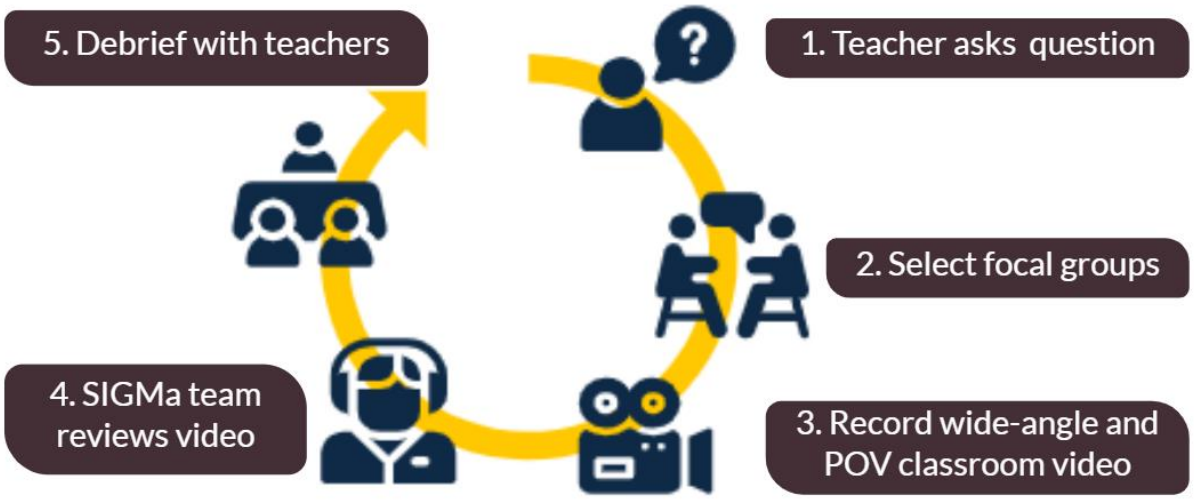

Figure 5: The five-step video-based formative feedback (VFF) cycle

3.2 Data Collection: Recorded Lessons Coupled with Debrief Conversations

During the 2017–2018 and 2018-2019 school years (Year 1 and Year 2 of our partnership), we worked with six school-based teacher teams, each ranging from two to five people. We visited and filmed teachers in each team one to six times over the course of the year. To film lessons, we used two cameras. Camera 1, a tablet camera on a robot tripod (Swivl), captured the whole class with a focus on the teachers' movements. Camera 1 also captured conversations from four student groups through four separate microphones placed at their tables. Camera 2, a point-of-

view camera (GoPro), was mounted on the focal teacher's head, shoulder, or chest to approximate what they saw as they moved through the classroom interacting with students. In addition to these recordings, our classroom data included fieldnotes, lesson artifacts, photos of whiteboards and student work. The data also included fieldnotes about or recordings of conversations with the teachers before and after instruction, as well as texts and email exchanges with the teachers about the classroom activities.

To film debrief conversations, we used the same wide-lens tablet camera and a recording of the researchers' laptop screen to document what teachers and researchers watched at any given time. In addition, debrief data included fieldnotes, photos of whiteboards when used, and fieldnotes about or recordings of informal conversations with the teachers before and after the formal debrief. All 32 debrief conversations were initially transcribed by an external transcription service and then finalized by Project SIGMa team members. In this dissertation transcription notations are (.) for pause in talk, (*text*) for nonverbal activity, *text* for emphasized talk, tex- for word cut off, and […] when transcript was shortened for brevity.

All in all, the Project SIGMa dataset consisted of 32 VFF cycles across two academic years (Year 1: 2017-2018, Year 2: 2018-2019), where each VFF cycle included a recorded lesson coupled with a debrief conversation about that lesson. We interviewed all teachers 2-3 times per year. Finally, during the 2019-2020 school year (Year 3), we conducted Member Check visits. The goal of these visits was to look for evidence of sustained learning beyond the time of the intervention. During these visits, due to limitations in our human subjects agreements, we did not film classrooms, but rather took fieldnotes and interviewed teachers for final updates and reflections.

3.3 The PDO Teachers: An Unusual Case of Well-Supported and Experienced Teachers

Yin (2014) distinguished between the study of extreme (unusual) cases and common cases. The study of extreme cases reveals important insights about phenomena of interest, and the study of common cases captures everyday situations in a way that is more generalizable. As a group, our partner teachers represented extreme cases. Although they taught in under-resourced urban schools, they were experienced and professionally well-supported by the PDO. The PDO provided teachers with approximately 100 hours of high-quality PD per year. These included monthly daylong PDs, funds to attend professional conferences, and other activities such as the video-based PD we offered.

These experiences make this a useful case to explore questions about the role of teachers' previous professional experiences in collaborative sensemaking, the overarching theme of this dissertation. Given that only about 40% of mathematics high-school teachers in the United States participate in more than 12 hours of mathematics focused PD annually (Banilower et al., 2018), I do *not* claim that findings in this study are typical. Yet, the fact that Project SIGMa's teachers had such a rich network of PD support made their previous experiences more visible within the VFFs, which, in turn, made them useful cases for theorizing teacher learning ecologies. As Horn and Garner (2022) explain, this unique setting offers a window into the possibilities of teacher learning.

3.4 Focal Cases: Rees and Noether Teams

Within this larger group of well-supported and experienced secondary math teachers, I focus on the Rees and Noether teams. These two school-based teams had a similar leading concern, which remained relatively stable across our partnership. Both teams had the explicit goal of promoting student collaboration. This goal typically included a focus on teaching conceptual math content

and supporting social inclusion (see Table 4). In addition, the nature of resources referenced by participants was relatively similar.

School	Teachers	VFFs	Leading Concern	Examples for Resources Invoked by Participants
Rees	Ezio & Veronica	3	Conceptual math content with social inclusion	IMP **curriculum** TRU teaching **framework** Previous **VFFs** Grouping strategies from PD **workshops** Questioning strategies from a **conference**
Noether	Brad, Marisa, Grag & Abigail	6	Conceptual math content with social inclusion	Problem-based **curriculum** District teaching **framework** Previous **VFFs** Methods for student groups from PD **workshops** Planning strategies from a **conference**

Table 4: Overview of Rees and Noether teams

Notwithstanding their similarities, the two teams perceived their institutional contexts in significantly different ways. For example, at Rees, tensions between the teachers' personal commitments and school practices were significantly more contradictory. In contrast, at Noether, between Year 1 and Year 2 of our partnership, one focal teacher (Brad) was appointed as department chair. This shift implied that he had greater agency around issues like curriculum design and even purchasing classroom furniture to better support student collaboration. Another difference between the two teams was the length of our partnership. By the end of Year 1, one of the two teachers from Rees moved schools and our partnership ended. However, our work with Noether continued to Year 2, with a one-time Member Check visit in Year 3.

The similarities between the two teams' pedagogical goals and external networks of support, coupled with the differences between their institutional contexts, make them generative

cases for comparison. Understanding tensions and coherence is significant for studying and supporting teachers' learning (Cobb et al., 2018; Horn & Garner, 2022; Yamagata-Lynch & Haudenschild, 2009).

All in all, the primary data for this dissertation include video recordings and transcripts of nine VFF cycles of the two teams (see Data Collection section). As Table 5 describes, Year 1 with the Rees team included three VFF cycles. As Table 6 describes, Year 1 and Year 2 with the Noether team included six VFF cycles. Secondary data include interviews with participating teachers, fieldnotes of PD sessions, and VFF video data of other teams on the larger project.

Year of Study	**VFF**	**Focal Teacher**	**Inquiry Question/Topic**	**Class**
Year 1	Rees VFF 1 Dec 2017	Ezio	How are the group dynamics in terms of math talk and collaboration?	8th grade math
	Rees VFF 2 Feb 2018	Veronica	Teacher's questions and students access to the content	7th grade math
	Rees VFF 3 May 2018	Ezio	Did facilitation surface and/or address students' thinking?	8th grade math

Table 5: Overview of Rees VFF Debrief Conversations

Year of Study	VFF	Focal Teacher	Inquiry Question/Topic	Class
Year 1	Noether VFF 1 Dec 2017	Brad	Questioning and helping students construct arguments	Algebra 1
	Noether VFF 2 Feb 2018	Abigail	Facilitating student-student talk	Algebra 1
	Noether VFF 3 May 2018	Brad	Teacher's feedback and how groups use it	Algebra 1
Year 2	Noether VFF 4 Dec 2018	Marisa	Does math come from the students or the teacher?	Geometry
	Noether VFF 5 Mar 2019	Greg	How does technology support (or not) student collaboration?	Computer Science
	Noether VFF 6 May 2019	Brad	Teacher's feedback and how groups use it. What happens if he doesn't answer students' questions?	Statistics

Table 6: Overview of Noether VFF Debrief Conversations

3.5 Data Analysis: Analyzing Teacher Conversations by Focusing on Scope, Interconnectedness, and Temporality

To analyze the role of teachers' previous experiences in collaborative sensemaking, I built on video recordings of conversations from Project SIGMa and used the analytical tools described in the conceptual framework: micro- meso- exo- macro- levels (*scope*), exo-resources as boundary objects (*interconnectedness*), and concept development in adaptive cycles (*temporality*). I elaborate on the unfolding logic of inquiry in the following phases as linear for the sake of clarity. However, as is common in qualitative analyses, the process was iterative in nature and I refined earlier phases as findings emerged (Horn & Kane, 2015; Strauss & Corbin, 1998). After describing how this work relied on the larger research team's routines (Phase 0), I explain the

three phases of analysis, corresponding to the three sub-research questions (SRQs). In Phase 1, I asked *how do teachers invoke and use exo-resources in collaborative sensemaking?* (SRQ1). In phase 2, I asked *what exo-resources are invoked by teachers in collaborative sensemaking and where do they originate from?* (SRQ2). In Phase 3, I asked, *how do different levels of teachers' learning ecologies interact in different phases of their learning?* (SRQ3).

The larger research project had several secondary strands of analysis, led by different team members. As part of the team data collection and analysis routines, we kept an inventory for each strand of analysis so that team members who encounter relevant episodes could index them. In this way, our team collaboratively flagged instances of teachers' explicit references to resources and professional experiences.

This initial inventory resulted in a spreadsheet pointing to 17 instances of teachers across the six teams explicitly discussing resources from other PDs in VFF conversations. Discussions with the research team about the meaning of these instances were consolidated into a conference paper written with Katherine Schneeberger McGugan, Sammie Marshall, and Brette Garner (Ehrenfeld et al., 2020). The preliminary inventory collected by the team and ideas in the conference papers formed the base for this data analysis.

Before selecting Rees and Noether schools as my focal cases, we extended the preliminary inventory to a more rigorous one across all six teacher teams. This work was done by me and an undergraduate research assistant, Jessica Moses. In the summer of 2021, Jessica and I reviewed all 32 VFF debriefs and met weekly to organize our findings. Across the 32 video-based conversations, we identified and documented 126 occurrences of participants invoking experiences and resources from remote professional settings (e.g., a practice they learned in a PD or experiences in a previous school).

3.5.1 Phase 1: Case Studies of Exo-Resources in conversations

The preliminary inventory collected by the SIGMa team (Phase 0) formed the base for Phase 1. After further reviewing videos and transcripts of the nine VFF debriefs in the Rees and Noether dataset, I chose two exo-resource for an extended analysis (grouping practices in the case of Rees, and a problem-based curriculum in the case of Noether) based on two criteria: (1) teacher invoked them across conversations and (2) they were salient to key conceptual learning opportunities (Enyedy et al., 2015). Finally, building on episodes which illustrated key learning opportunities, for each exo-resource I chose one episode for a deeper video analysis by using interaction analysis (IA) methods (Hall & Stevens, 2015; Hall & Horn, 2012; Horn & Kane, 2015; Jordan & Henderson, 1994).

As Jordan and Henderson (1994) explain, "no method is without theoretical assumptions*"* (p. 40), and this work shares with IA the assumption that learning is a distributed, ongoing, and social process. In addition, Jordan and Henderson describe that a goal of IA is to "look for the mechanisms through which participants assemble and employ the social and material resources inherent in their situations for getting their mutual dealing done" (p. 42). In line with this goal, I looked closely at the two episodes, both alone and with other researchers (including in Project SIGMa meetings and in Rogers Hall's IA Lab) and asked a variety of analytical questions towards answering SRQ1: *How do teachers invoke and use exo-resources in collaborative sensemaking?* I organized the analytical questions according to different connections between Bronfenbrenner's (1979) four levels of the environment:

- *Micro-exo connections*: How do resources interact with the local conceptual infrastructure (that is, representations of teaching and framing of problems of practice)?

- *Micro-meso-exo connections*: How do teachers negotiate relations between resources and the institutional context? To what extent do teacher perceive the resources and the institutional context as coherent or contradictory?
- *Micro-meso-exo-macro connections*: How do teachers negotiate relations between resources and institutional contexts in light of larger social, racial and economic contexts?

Phase 1 resulted in the two extended case studies of exo-resources in use, which are presented in Chapter 4.

3.5.2 Phase 2: Bird's-eye View on Rees and Noether Teams' Exosystems

The goal of Phase 2 was to elaborate and categorize the preliminary inventory of exo-resources used by the two teams to answer SRQ2: *what exo-resources are being invoked by teachers in collaborative sensemaking, and where do they originate from?* Within research of teacher learning, this sub-research question echoes Lieberman and Grolnick's (1996) argument that it is important to focus on how participants are involved with ideas and where these ideas are coming from.

First, Jessica, the undergraduate RA, and I documented each occurrence of a resource according to four aspects that constitute its use in conversation:

- the *setting* in which teachers originally engaged with the resource (e.g., a PD workshop or a previous school),
- the *mediating tool* of this resource (e.g., a curriculum or a teaching framework),
- the *pedagogic elements* that the resource entails (e.g., an instructional practice), and

- the teachers' *pedagogical reasoning* (Horn, 2005) that the resource supported in the conversation (e.g., suggestion of how to promote student talk or how to disrupt the labeling of some students as less capable learners).

For example, a teacher could mention a certain PD (setting) in which they learned about a certain teaching framework like Complex Instruction (mediating tool) which included a certain teaching practice like assigning students with group roles (pedagogic element) which they decided to use for certain reasons (pedagogic reasoning). See Appendix 1 and Appendix 2 for the full inventory of Rees and Noether teams' exo-resources according to this organization.

Second, I coded and clustered the external resources into six emerging categories: PD workshops, teacher conferences, online resources and textbooks, research and policy resources, experiences with different curricula, and experiences with our partner PDO. The visual organization of data according to these six categories, presented in Chapter 5, is inspired by visuals in a previous work by Brigid Barron (2006) that focuses on the learning ecologies of students and the contexts of their technological fluency development.

Finally, I characterized the overall nature and the mathematics education conversations these resources are part of. I did it by examining the extent to which these practices, frameworks, texts, and lessons materials contradicted or affirmed visions of ambitious mathematics instruction. Horn and Garner (2022) portrayed these visions as a telos that details what changes as teacher learn to teach in these ways. They focused on three main shifts with regards to the organization of activities, what class sounds like, and who belongs to math class. Understanding teachers' exosystem as contradicting or affirming ambitious mathematics instruction enabled an interpretation of how the our partnership was positioned within teachers' broader learning ecologies.

3.5.3 Phase 3: Analyzing the Temporality of Rees and Noether Teachers' Learning by Using the Adaptive Cycles Framework

Phase 3 explicitly adds the dimension of temporality to the overall analysis. The goal of this phase was to understand learning trajectories across all meetings we had with each team in the PD. Within this phase of data analysis, I also moved away from centering the exo-resources as the primary focus of analysis. While I still acknowledged them as potentially powerful, here I treated them as one part of the ecological system. I asked: *How do different levels of teachers' learning ecologies interact in different phases of their learning?* (SRQ3).

In line with the operationalization of temporality from the conceptual framework, I described the teams' learning at the PD through the lens of the Gunderson and Holling's (2002) adaptive cycles (see Chapter 2). For each of the phases of *release*, *reorganization*, *growth*, and *conservation,* I asked whether and where do I recognize conversations that are associated with this phase. First, for the phase of release, I searched for and analyzed instances where teachers problematize either institutional practices and norms, or aspects of their own instruction. Second, for reorganization, I focused on instances where teachers discussed planning and recruitment of new ideas, including the tensions between institutional context and their own pedagogical goals. Third, for growth, I focused on instances where teachers tried out relatively new ideas and professional arrangements (in and out of classroom.) Finally, for the phase of conservation, I focused on evidence that teachers became proficient with new professional arrangements, to the extent that these arrangements were consolidated and stable within teachers' routine practices.

Notably, I quickly learned that instances that represent the four phases were usually mixed across the data, that many episodes can be seen as representing multiple phases, and that phases were never really "done" but more or less salient at different points of the conversations.

In other words, while these categories are analytically distinct, real life is messier. I tried to consider this messiness in my analysis and be explicit about it. This nuance in and of itself led to some interesting findings. For example, when the Rees team mostly coordinated resources in light of institutional contexts, I considered it *reorganization*. When the Rees team mostly tried out these new resources, I considered it *growth*. However, I noticed that while the main focus of the formal video-based conversations became the growth, our informal conversations with the teachers were still a space for teachers' sensemaking about their reorganization, highlighting how formal and informal dimensions of the VFF cycles were interrelated.

Finally, I explored the guidance provided by adaptive cycles to consider the influence of external variability on different phases of learning. Adaptive cycles suggests that within the reorganization and growth phases, invoked external resources will be more salient. In contrast, in the conservation and release phases, internal processes such as video-based reflection will be more salient. The temporal analysis conducted in Phase 3 is reported in Chapter 6.

CHAPTER 4

Two Case Studies of Exo-Resources in Conversation:

The Potential of Exo-Resources in Supporting Teachers' Collaborative Sensemaking

In this chapter, I study how experiences and resources are invoked and used by teachers in collaborative sensemaking. I build on two case studies to illustrate the transformative potential of exo-resources in conversations about instruction. I argue that in these two cases invoking exo-resources opened up teacher conversations towards more ecological teacher reasoning. By ecological reasoning, I mean that invoking these experiences and resources invited teachers to consider new aspects of teaching at different levels and their mutual interaction (e.g., classroom interaction, activity design, unit design, classroom norms, school teaching culture, institutional practices, and even the school neighborhood). Conversation 1 took place in Rees VFF 1, when teachers discussed different grouping strategies that had been introduced in two PD workshops: Purposeful Grouping (PG) and Random Grouping (RG) workshops. Conversation 2 took place in Noether VFF 3, when teachers discussed a problem-based curriculum that was used in a teacher's previous school. Table 7 provides an overview of the two VFF debrief conversations and of the exo-resources at the center of analysis.

	Case 1: **Rees Midle-School**	**Case 2:** **Noether High-school**
VFF/Date	Rees VFF 1 December 2017	Noether VFF 3 May 2018
Teachers	Ezio and Veronica	Brad, Marisa, Abigail and Greg
Researchers	Patty, Lani and Nadav	Lani and Nadav
Exo-resources recruited from…	PG and RG workshops	Marisa's previous school
Mediating tool of exo-resource	Grouping practices	Problem-based curriculum
Pedagogic element of Exo-resource	Grouping strategies	Designing for student collaboration

Table 7: Overview of the two case studies

4.1 Conversation 1: Rees Team Discuss Grouping Strategies

Conversation 1 comes from a debrief conversation at Rees Middle School. The teachers, Ezio and Veronica, discussed two different groupwork structures they encountered and deliberated over which was a better fit for their teaching. Specifically, Ezio invoked experiences in two different PD workshops: school-based PG workshop, where he learned about purposeful grouping that involved putting high and low achieving students in each group (i.e., structured heterogeneous grouping), and RG workshop, where he learned to group students randomly (Horn, 2012; Liljedahl, 2014). Comparing the two resources created a space that supported a *framing* of the problem of grouping as directly related to institutional tracking practices (and implicitly, to gentrification processes in their school's neighborhood.) This extended frame was rooted in a *representation* of the work of teaching as entailing the navigation of school structures.

In contrast to the linear view of teacher professional development presented in Chapter 1 (see Figure 1), an ecological perspective on the case of Ezio and Veronica (see Figure 6) highlights that teacher learning is an iterative process that involves activities and resources from different settings such as experiments in the classroom, experiences in PD workshops, informal conversations, instructional resources, and navigation of institutional structures (e.g., tracking), and broader social changes (e.g., gentrification processes).

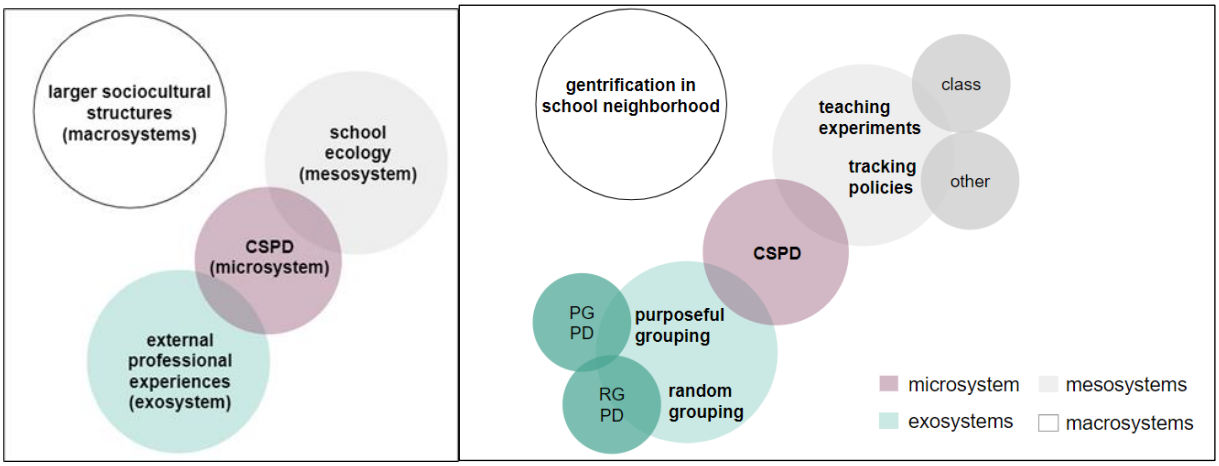

Figure 6. Overview of scope and exo-resources in Conversation 1 (Rees)

Note. On the left is the general model for scopes and contexts of teacher conversations suggested in Chapter 2. On the right is the application of this model to Case 1.

4.1.2 Rees Team Context: Long Years of Collegial Relationship and Shared Commitments

Several years before our partnership began[1] Ezio and Veronica moved to Rees High-school from the same previous school. Veronica moved first. She was displaced from their old school as the youngest teacher in the department, with less than 5 years of teaching experience at the time. Ezio joined her shortly after, with more than 15 years of experience. Ezio was proud of the work

[1] Exact number of years not provided to protect the teachers' identities

he did in their old school, in a neighborhood he described as “a bad area.” However, he decided to change schools because he felt “stagnant” and needed new challenges. In an interview, he shared he had felt like there was not nothing left for him to learn there.

When they re-joined forces at Rees, Ezio and Veronica had a strong collegial relationship and collaborated as much as their schedules allowed. This collaboration strengthened when they joined the PDO as a team, which bought them an official shared planning time during their school day. Veronica described her participation in the PDO as “career changing,” mainly through being exposed to many ideas in conferences and in the monthly PDO meetings. She also described social media as meaningful to her professional growth. Ezio shared how joining the PDO offered him an outlet to learn and try new things. More specifically, it made him question his previous lecture style of teaching that he described as one that had appealed only to a certain type of students.

As a team, Ezio and Veronica shared two main goals. First, they wanted to change their lessons to support more student collaboration. In an interview in Fall 2017, they described supporting student collaboration as a topic they tried to grow the most that year. Ezio described not having a lot of experience with student small groups and mentioned he was working on leading students to the answers instead of simply giving answers. Veronica described groupwork as being “outside her comfort zone,” and yet she was willing to take “big risks” and try new practices in her classroom.

Second, Ezio and Veronica felt a commitment to work towards making their school more equitable. Both teachers were frustrated by the distribution of resources within school, which they felt was favoring affluent families. These families were either newcomers to the school’s neighborhood, or students who were recruited to school’s magnet program from other

neighborhoods. For example, Veronica made sure to tell all students about new electives that were being opened (e.g., robotics) in case only the more affluent families were aware of them. Ezio highlighted his goal of teaching all students to persevere in math, because he believed that otherwise, when his students faced their first challenge, they would be "done with math," which meant they would "never get back into that math again, because they've mentally just given up on math." (Ezio, Rees VFF 1)

More generally, Ezio, the focal teacher in Case 1, had a strikingly warm rapport with his students. We learned that students showed respect and admiration for him, even years after they had graduated. As Veronica shared in the debrief conversation: "The kids love him; there is so much evidence of that. Your kids are coming back. They had him eight, nine, ten years ago. They come back to be like, this is where I'm at" (Veronica, Rees VFF 1).

4.1.3 Classroom Context: Ezio's Eighth Grade

The lesson filmed for the debrief featured 90 minutes of animated talk, laughter, and wooden manipulatives in action. It left a strong impression on the visiting researchers as a vibrant and joyful environment to learn and do math. The lesson included 31 eighth graders and was built around two mathematical tasks: The first task was a visual patterns activity (see Figure 7), in which the patterns represented a linear function that students had to reconstruct (around 30 minutes). The second task was the Tower of Hanoi (around 60 minutes). Across the two tasks, Ezio wanted to let students experience the difference between working with linear functions (as in the visual patterns activity) and nonlinear functions (as in the Tower of Hanoi). The episode discussed here took place after watching a clip of students working on the first task.

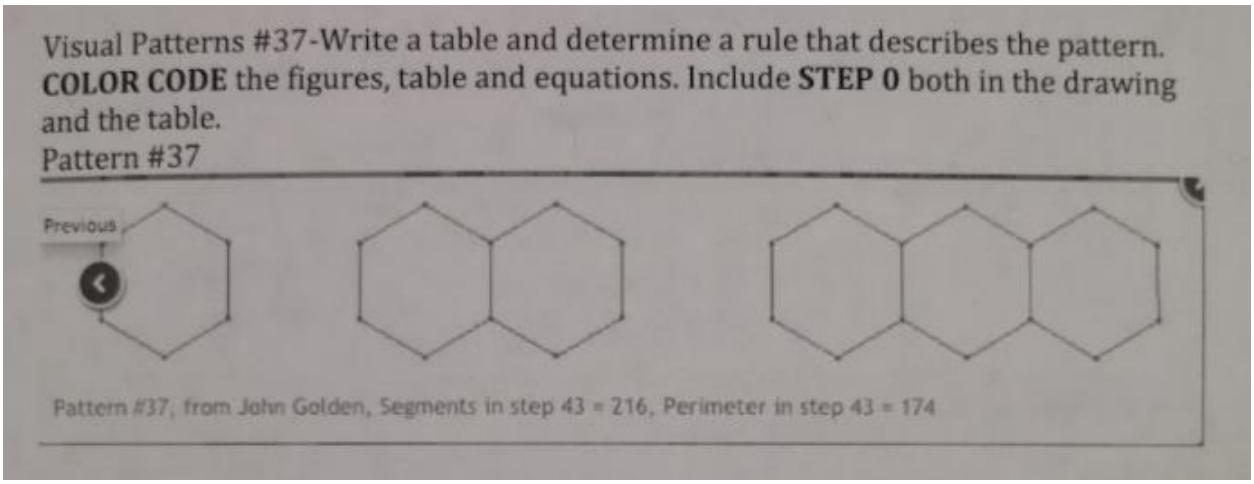

Figure 7: Visual patterns activity

4.1.4 Debrief Context: Group Dynamics

Ezio's inquiry centered around student group dynamics. In preparing for the debrief over the four days following Ezio's lesson, the research team carefully listened to and watched classroom materials. This resulted in us recognizing, on the one hand, consistent and vital student participation, and on the other, limited opportunities for students to make sense of the mathematics undergirding the task. A deeper analysis we conducted on this lesson in Buenrostro and Ehrenfeld (2019) strengthened these impressions. This mix of strengths and limitations affirmed Ezio's statements about himself as being new to groupwork facilitation.

4.1.5 Random and Purposeful Grouping as Exo-Resources in the Debrief Conversation

Within the VFF debrief, Patty facilitated the conversation with the teachers. Lani and I primarily filmed and took fieldnotes but, on occasion, we participated as well. In this part of the conversation, Patty played a video clip to discuss Ezio's topic of interest: group dynamics. After watching a clip of students working on the first activity, Ezio became concerned that not all students were contributing to their group conversations. For example, Ezio said, "I guess what concerned me— like I really didn't hear Edward" [00:12:28]. Veronica and Ezio then discussed

whether providing more structures to distribute participation would remedy the problem. For example, Ezio mentioned a structure where the student who is taking notes cannot talk, and Veronica generalized that into the overall idea of assigning students group roles: “My question, I'm wondering if they were more structured roles, like you had said, would we have heard more conversation?” [00:16:02] As they brainstormed, Veronica also suggested a procedure where students discuss the problem before they start writing [00:17:55]. At this point, Patty prompted the teachers to elaborate on their interest in group dynamics:

[00:18:50]

Patty: Is there anything about the group dynamics that you're interested in?

Ezio: so yeah like uh (.) At least in RG workshop, I really did not agree with the random grouping.

Veronica: but you are doing random grouping.

Ezio: I am doing ...

Patty: This is random grouping.

Ezio: Yeah. I let the computer pick it out. I've been trying it out. We got a PG workshop (.)

Patty: mm-hmm

Ezio: a couple years ago and at least what they said made sense, where it's purposeful ...

Veronica: Purposeful grouping.

Ezio: Yeah, like a high low ...

Patty: Yep.

Ezio: There was a structure to everything. and uh—

Patty: so you're wondering if-

Ezio: I was wondering, I didn't agree with RG workshop but I wanted to try it out to see.

[00:19:50]

Ezio responded to Patty's prompt by sharing his experience from the two aforementioned RG and PG workshops. He recalled how, at first, he "did not agree" with random grouping [00:19:09, 00:19:46]. On the contrary, purposeful grouping "made sense" to him [00:19:30]. Veronica, for her part, seemed surprised by Ezio's initial dismissal of random grouping as a practice he disagreed with, clarifying that he *was* doing random grouping. Ezio then explained that he was experimenting with it, and then again, that he did not agree with it, but wanted to "try it out." Notably, in contrast to the underlying assumptions of linear models of teacher learning, where learning starts at a specific PD and ends at the classroom practice, Ezio narration represents his work of teaching as entailing classroom experiments, agency with regards to instructional practices, and coordinating two visions of teaching from different PD workshops.

4.1.6 Micro-Exo-Meso Connections: Negotiating the Exo-Resources and Institutional Practices

As the conversation progressed, Ezio and Veronica continued to reason about these two practices with relation to their classroom experiences, and this time, also in light of their concerns about tracking being part of their school's institutional practices:

[00:19:55]

Ezio: One thing I do fear, so like what's bothering me in this school, we do. Okay unofficially, unofficially we track kids.

Patty: Yeah.

Veronica: Officially.

Ezio: No, unofficially. [inaudible]

Patty: Yeah, well, we talked about this, right? Last time[2]

Ezio: So I've had kids tell me, "Oh, we're in the dumb class." They know, they already have that label

Lani: Is this one of the groups of kids that is in the “dumb class”?

Ezio: No, no.

Veronica: No.

Lani: Okay.

Veronica: like in a PG workshop, when you purposefully group, the kids automatically know.

Lani: Yeah

Patty: Yeah, which they do.

Veronica: Whereas if you randomly group, they don't know.

Lani: Right

Ezio: Right, so I don't want to subconsciously be telling kids, "Oh, I think you're awesome” or whatever

Patty: Yeah, yeah.

Ezio: That's the one thing I did like about the random grouping.

[00:20:50 *Classroom phone rings and Veronica walks away.*]

[2] To clarify the formal/informal issue, Ezio and Veronica explained that the school doesn't call it tracking but classrooms are not heterogeneous and are de facto tracked by things like ESL status and electives. Their ESL students are often not newcomers, but rather students who were placed in ESL as kindergartners or first graders and have not met the district’s exit criteria, which Ezio and Veronica described as “difficult tests.”

In this last excerpt, Ezio and Veronica continued broadening the unit of their analysis, brokering meso-exo connections as they started to attend to school structures. At this point, their framing of the problem of supporting student groupwork shifted from searching for "tips and tricks" (Horn et al., 2017) to connecting the problem to tracking structures in their school and describing how tracking make students feel "dumb." Ezio and Veronica experimented with the two approaches and noticed that purposeful grouping amplified the consequences of tracking, in the shape of labeling kids as "dumb" or "awesome," while random grouping disrupts them. Their sensemaking about the problem of supporting student collaboration spanned two practices they hadlearned in two PD workshops (*exosystem*), classroom experiments and school structures (*mesosystem*), all of which they negotiated through video representations of the classroom, interactional representations of teaching, and dynamic framing of the problem of practice (the CSPD *microsystem*). Their negotiation of classroom practices and institutional practices of tracking was also deeply rooted in their understanding of economic and social processes within their school's gentrifying neighborhood (the *macrosystem*).

4.1.7 Conversation 1 Summary: The Exo-Resources Supported Extended Representation of Teaching and Framing of Problems of Practice

Representations of practice are a part of the conceptual infrastructures, that make different aspects of teaching more or less visible (Horn & Kane, 2015; Little, 2003). The video representations of Ezio's classroom made visible the need to better support students' participation. The episode contained several different problem framings. First, the problem was framed around quick solutions like group roles and rules. Then, Ezio's comparison of the two grouping practices, including his account of his experiences with them in the PD workshops, supported an extended framing. This new framing was rooted in a representation of teaching as

inherently involving judgments between contrasting practices, entailing experiments, judgment, and resistance to school practices. The extended framing and representation supported teachers in reasoning more ecologically about their actions (see Figure 8), including stressing their goal to disrupt what they saw as unethical tracking in Rees Middle School.

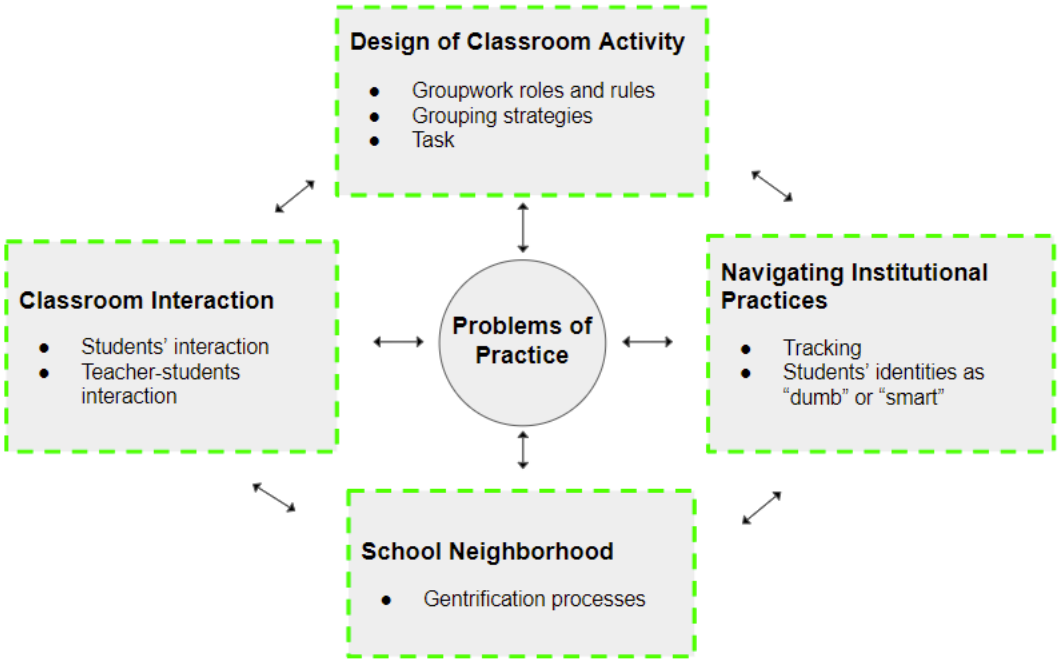

Figure 8: Ecological reasoning about problems of practice in Conversation 1 (Rees)

4.2 Conversation 2: Noether Team Discuss Student Collaboration Norms

The second conversation is from a debrief conversation at Noether High School. The teachers discussed ways of giving feedback to student groups and other aspects of supporting students' collaboration. During the discussion, Marisa invoked her experiences teaching a problem-based curriculum in her previous school (see Figure 8). In this case, the appearance of the resource afforded a *framing* of the problem of leveraging students' agency as a problem of classroom culture, which extended a previous, narrower framing around moment-to-moment feedback. The

new framing was rooted in an extended *representation* of teaching as taking place within departmental norms.

Similarly to Conversation 1, an ecological perspective on Conversation 2 (see Figure 9) highlights that teacher learning is an iterative process that involves activities and resources from different settings such as experiments in the classroom, experiences in previous schools, curricular resources, and navigation of institutional structures

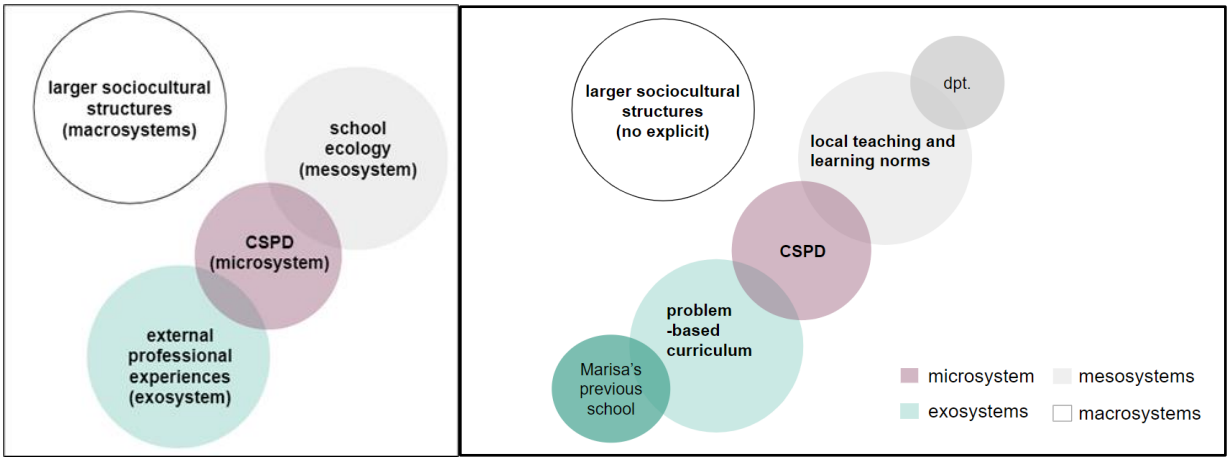

Figure 9: Overview of scope and exo-resources in Conversation 2 (Noether)

Note. On the left is the general model for scopes and contexts of teacher conversations suggested in Chapter 2. On the right is the application of this model to Case 2.

4.2.1 Noether Team Context: Supportive Team with a Variety of Backgrounds and Experiences

The Noether team included four teachers: Brad, Marisa, Greg, and Abigail. The four teachers had different backgrounds and years of experience. For Brad, Noether was his first teaching job. He was there for over five years, and at the end of the first year of our partnership, he was appointed department chair. Marisa was new to the school and was Brad's collaborative planning partner. One aspect of this transition she often mentioned was that in her previous school she taught with a problem-based curriculum and greatly enjoyed this experience. Abigail arrived at Noether the same year as Brad, after teaching at another school for two years. She had a close relationship

with Brad and Greg, who both described her as someone who helped them become better teachers. Greg was the most experienced teacher on the team with more than 20 years experience, all at Noether. Similarly to Ezio from Rees, he shared with us that at some point in his career he was tired of PD workshops, and he stopped improving his teaching. Then, after years of teaching in the same way, as well as being isolated in the department, he met the teachers on the current team. Meeting them made him passionate about learning and improving again. He said: “I do want to improve, and to be able to do that, I have to find people that I could be able to communicate with, and that's what Brad and Abigail and Marisa [...], that's what they have done.” Brad himself was inspired by Greg’s commitments, as a veteran teacher, to shift his teaching and focus more on students’ thinking and discussions. In sum, the team was supportive and collaborative, committed to improvement, and included a variety of backgrounds and experiences.

4.2.2 Classroom Context: Brad’s Algebra 1

The lesson filmed for the debrief featured almost 60 minutes of mathematical work. As in Ezio’s lesson, it left a general impression on the visiting researchers as being a joyful learning environment where students feel comfortable and supported. The lesson included 34 students, which Brad described as having a variety of levels of prior knowledge. Brad’s mathematical goal was to have students graph quadratic functions. The lesson plan included a review of the content, a groupwork activity (see Figure 10), and finally a whole class discussion. In the groupwork activity, students were working in groups of four. This structure was new for Brad and the students, who usually worked individually and in pairs. The task that students worked on included two main (planned) challenges that many of the groups had difficulties with and needed Brad’s help. The first challenge was that one of the graphs extended past the 20X20 grid that

Brad provided. The second challenge was that the vertex x-coordinate of that graph was a decimal. The episode discussed here took place after watching a clip of students working on that task and calling Brad from help.

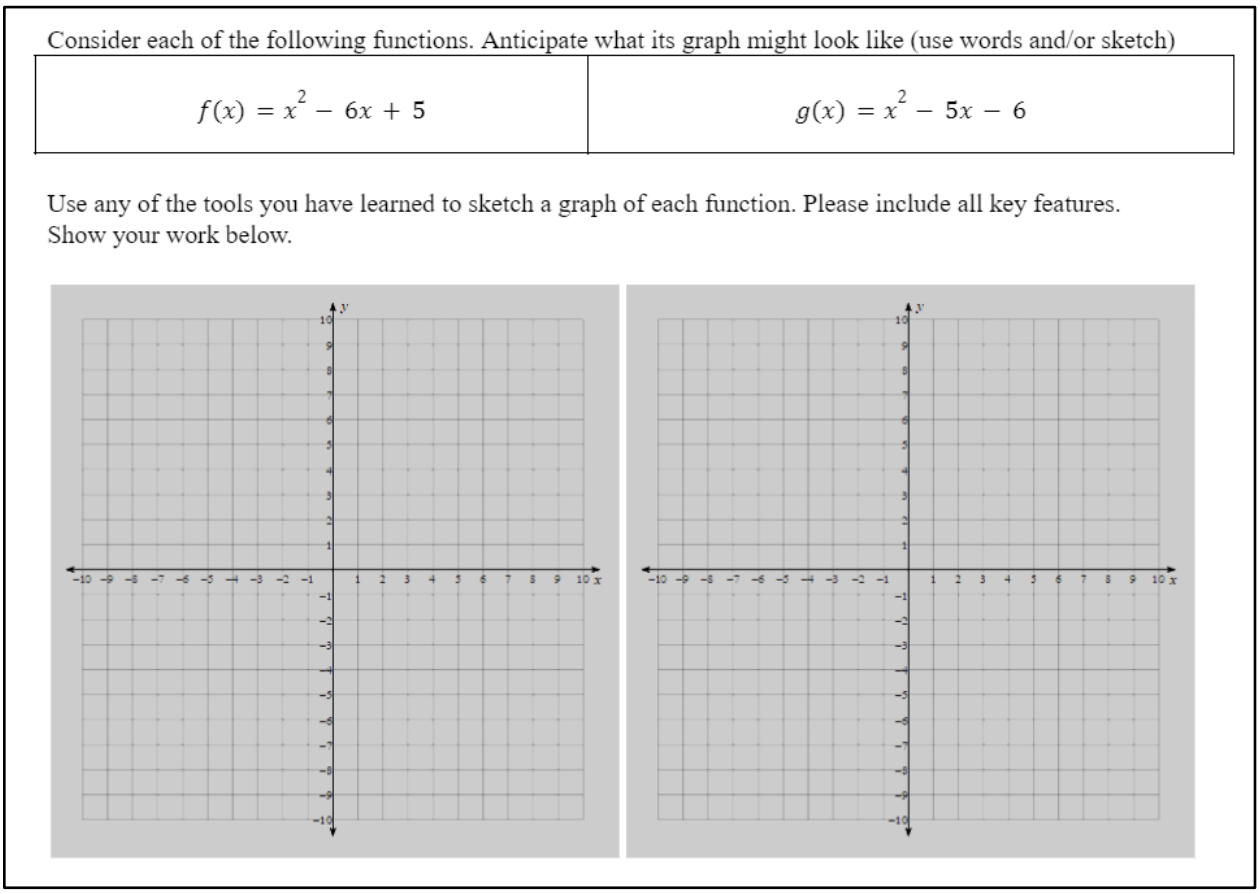

Figure 10: Graphing Quadratic Functions Task

4.2.3 Debrief Context: Brad's Groupwork Monitoring

Brad's inquiry centered around the feedback that he gave student groups and the ways they took it up. In preparing for the debrief over the days following Brad's lesson, the research team carefully listened to and watched classroom materials. This resulted in us recognizing, on the one hand, several episodes of sustained mathematical conversations, and on the other hand, several areas for potential growth in Brad's groupwork facilitation. Brad's feedback to groups tended to start by probing students' thinking, often by having a rich mathematical conversation with them,

and then ended by answering their questions in a close ended fashion. These impressions were strengthened in a deeper analysis we conducted on this lesson in Ehrenfeld and Horn (2020). They affirmed Brad's description of himself and the students as being new to groupwork activities.

4.2.4 Marisa's Experience with a Problem-Based Curriculum as an Exo-Resource

The first clip we reviewed in the VFF debrief featured a conversation between Brad and a group of four students that the research team thought was typical of this lesson. In the clip, the students asked Brad several questions. Specifically, they were confused about the line of symmetry (the vertex x-coordinate) being a decimal.

Adar: …this graph does not have a line of symmetry.

Brad: It doesn't? Are you sure?

Ginn: It doesn't?

Adar: Yes.

Ginn: It could be here.

Adar: That's the box though.

Brad: Is it possible that the axis of symmetry doesn't fall on a nice numbered point?

Ginn: No, yeah, it is.

Brad: Or is it an integer? I should say, better yet. Is it possible that the axis of symmetry is not an integer?

Ginn: Yeah, I think it is. I don't know.

Brad: Alright, so I want you guys to always be sort of aware that some parabolas will be really nice and line up with beautiful coordinates but other times they won't.

The researchers shared this clip as an example of Brad probing students' thinking, and yet eventually providing the group with the information they needed. Brad's reaction to the clip started with him noticing that one pair of students was involved (Ginn and Adar), and the other pair was disengaged. He was worried that it was not a good grouping of students. He also noticed in the transcript we printed that he was speaking a lot. He said, while laughing, "You can see speaker two [Brad] has a lot of talking going on." Nadav pointed out that the students had different opinions about whether the vertex x-coordinate could be decimal and asked: "What can we do to take this disagreement between them, that they would take it as an object to figure it out between them and not to turn to you?" Brad responded by wondering if there was a better question he could have asked rather than, "Are you sure?" By opening this question to the team, Brad framed the problem of supporting student groupwork as a problem of moment-to-moment response in teacher-student interaction.

In response, Lani asked, "What do you think 'are you sure' communicates?" Brad acknowledged it indeed implied, "You're wrong and that you need to check your work," and then all participants brainstormed different responses. For example, Greg suggested, "Why do you think that?" and Marisa suggested, "How did you figure that out?"

Returning to the idea of directing students to each other, Nadav then asked, "What would be the goal of such feedback?" and suggested a possible goal to make sense of the problem with

one another. By saying that, Nadav framed the problem slightly differently, as related to the teacher's general goal or stance towards his role.

Then, in a relatively long turn of talk of 1:41 minutes [00:27:13 - 00:28:54], Marisa elaborated on that idea describing her experience of teaching a problem-based curriculum in her previous school.

> [00:27:13]
>
> I'm looking forward to, hmm, possibly next year having a, a problem based curric- well, I don't know if we're going to teach geometry- what we're going to teach, but if we do teach geometry, if we could possibly use the problem-based curriculum, because one of the reasons I like *that* particular curriculum, is because it's, in groups from day one.
>
> [00:27:30] *Every* day students are working in groups, that's the culture of the classroom that's built up. so, they're, creating the meaning, from doing the problems and developing the mathematics, by, by the problem solving that they're doing, and the teacher's kind of just there facilitating, and you're walking around the whole time asking the questions and guiding if they need it but it's all coming from the students all the time.
>
> [00:27:57]

In the first part of this turn, Marisa described her experience in a classroom where students had mathematical agency and saw themselves and their peers as resources of mathematical knowledge. She attributed it to the problem-based curriculum, and more specifically, to structures such as daily groupwork "from day one." Importantly, she described the consequences of this curriculum in terms of classroom culture when saying "that's the culture of the classroom that's built up." By doing so, she reframed the problem of supporting student collaboration as

requiring not only improvising the right feedback but also as requiring the teacher to foster a collaborative classroom culture.

4.2.5 Micro-Exo-Meso Connections: Negotiating Exo-Resources and Institutional Contexts

Importantly, Marisa did not mention the idea of a collaborative classroom culture as an abstract ideal, but rather, as she continued, she contrasted her experience of teaching the problem-based curriculum with the teaching and learning norms at Noether.

> [00:27:57]
>
> So I think it's hard when your classes is kind of a mix and, I mean this because we don't have a good textbook and we don't— but we're doing a lot of direct instruction; we're doing group work but, we're going kind of back and forth and always the default I think is for the student to be dependent on the teacher as the source of information (.) and trying to wean them off of that is a lot harder um when— I mean you do it both ways and it's always easier for them just to ask you 'what's the answer, what's the answer.' [00:28:30] Um so (.) the problem-based curriculum — and I'm sure there's others that are really good — but because from day one (.) they are taught that you are the source of the knowledge— I mean the student— it's coming from the student as opposed to the teacher. I think that (.) it's easier to do this kind of thing. In a class that has that from the start, that culture is established.
>
> [00:28:54]

Marisa contrasted her experience of teaching the problem-based curriculum in her former school with their current teaching situations at Noether, where they are "going kind of back and forth" [00:28:14] between direct instruction and groupwork. She perceived that students in Noether

classrooms got frustrated when teachers did not hand over answers, as evidenced by her narration of students saying, ”What's the answer, what's the answer” [00:28:30] and “Well, just tell me, just tell me what it is” [00:29:26].

After Marisa’s long turn of talk, others also acknowledged that the problem of supporting students’ collaboration involve working against classroom norms that were cultivated for years, and that even with a problem-based curriculum, students would always struggle the first few times they were required to count on each other to solve problems if they were not used to it.

4.2.6 Conversation 2 Summary: The Exo-Resources Supported Extended Representation of Teaching and Framing of Problems of Practice

In Noether team’s debrief, the teachers discussed the focal teacher, Brad’s, groupwork facilitation and ways to provide feedback to student groups. The initial framing of the problem of providing feedback to student groups focused on different responses teachers could provide. This framing was rooted in the representation of teaching as the improvisation of moment-to-moment interaction.

This framing shifted when Marisa shared her previous experience of teaching the problem-based curriculum. Drawing on that experience, Marisa supplemented this representation of teaching with issues of designing a learning environment, thus reframing the problem of supporting students’ collaboration. The new frame shifted from the *particularities of the teaching interactions* to the *development of a classroom culture*. It moved the conversation away from framing the teaching problem as a transient and technical task to a complex long term one, yet still positioned the teachers as having the power to address it.

Implicit in this reframing was the contradiction between Noether’s institutional norms and the teachers’ ambitious cognitive and social goals. This implied that in Noether’s current

reality, teachers' differing responses were not enough; teachers needed to engage in the work of reorganizing their classroom environment to disrupt current norms. In this case, Marisa pointed to the need to set collaborative classroom norms from the first day with the appropriate curricular materials to support this work. These new frames supported the teams' ecological reasoning about their problems (see Figure 11), making it a meaningful resource that enriched the conceptual infrastructure and supported new learning opportunities in this conversation.

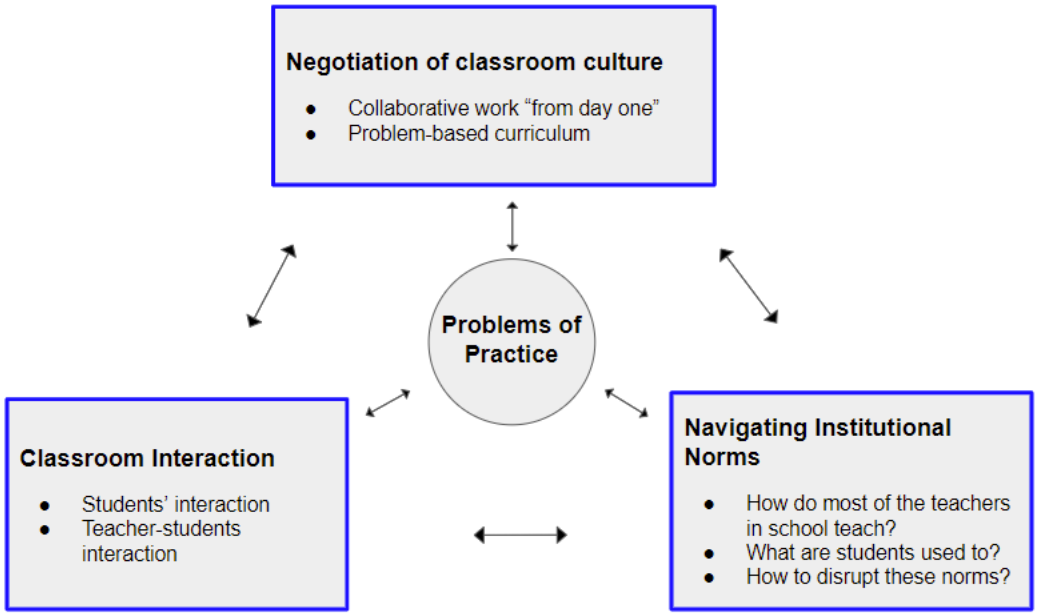

Figure 11: Ecological reasoning about problems of practice in Conversation 2 (Noether)

4.3 Summary and Discussion Across the Cases: Invoking Exo-Resources Supported More Ecological Teacher Reasoning About Instruction

Across the two cases in this chapter, my main claim is that the experiences and resources were invoked and used in ways that supported more ecological teacher reasoning. In other words, invoking these experiences and resources invited teachers to consider new aspects of teaching and the relations among them. Figure 12 summarizes ecological ways of reasoning across the

two episodes. Conversation 1 started by thinking about classroom interaction, where some students were excluded from the group conversation. They then moved to think about grouping as an aspect of activity design and ended up connecting these two frames to institutional practices (tracking) and to gentrification processes in the school neighborhood. Similarly, Conversation 2 started by considering classroom interaction, where students heavily drew on the teacher as a main (and sometimes only) source of knowledge. They then moved to consider aspects of classroom culture across time, and the institutional norms of authority structures in the classrooms. Developing such ecological views of teaching is essential teacher learning of ambitious and equitable instruction (Horn & Garner, 2022).

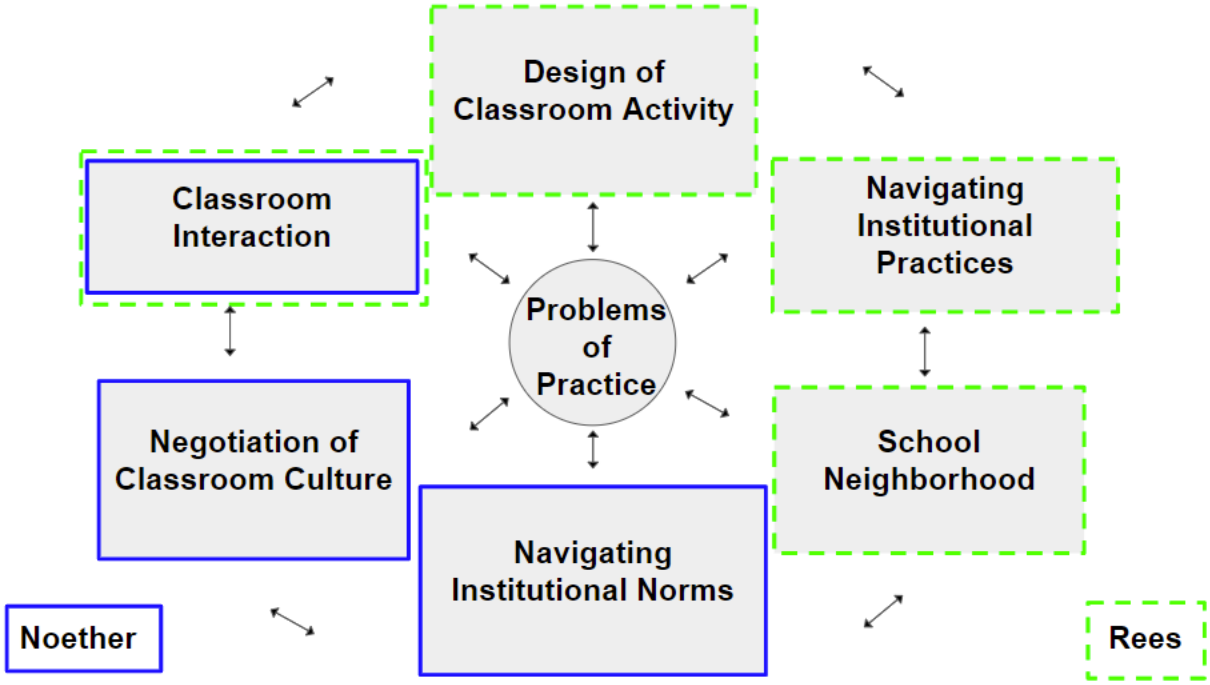

Figure 12: Ecological reasoning about problems of practice across the two cases

analytically, this chapter used an exo-centered lens. That is, I focused on the exo-resources of grouping strategies (in Conversation 1) and the problem-based curriculum (in Conversation 2). I then examined how the exo-resources interacted with other resources from

different levels, eventually looking at interactions across all four environmental levels (the micro-, exo-, meso-, and macro- systems). Table 8 summarizes the analyses across the two cases and four levels.

With relation to micro-exo connections, the cases illustrate how the teachers invoked and took up exo-resources to create more productive problem frames rooted in richer representations of practice. In turn, these framings and representations supported broadening teachers' units of analysis in thinking about teaching, which, in turn, supported their ecological reasoning illustrated in Figure 9. With relation to micro-meso-exo connections, the cases illustrate different conditions and coordination of the exo-resources with the local school context. Conversation 1 involved teachers' explicit concern that informal tracking policies were harming students by implicitly labeling them as "smart" of "dumb." Conversation 2 involved teachers' explicitly acknowledging that, in the current teaching culture in their department (including their own), they were unable to enact the instructional practices they aspired to. With relation to micro-meso-exo-macro connections, issues of social, racial, and economic realities in the United States were not explicitly discussed across the two cases. They were, however, discussed in interviews and other informal conversations. Conversation 1 involved a related background story of teachers' reasoning about the school's gentrified neighborhood, and their concern that the principal was creating informal tracks and unfair distribution of resources based on middle-class (newcomers) parents' demands.

Case	Micro-Exo Connections		Micro-Exo-Meso Connections	Micro-Exo-Meso-Macro Connections
	Relationship between resources and conceptual infrastructure		Brokering resources and institutional context	Sensemaking in light of larger social, racial and economic contexts
	Framing afforded by the resource…	Rooted in a Representation…		
Case 1	*Framing* the problem of grouping strategies around the labeling of students by institutional practices	*Representation* of the work of teaching that makes visible experiments, judgment, and resistance.	Mostly contradictory: exo-resources used for disruption of institutional practices	Explicitly connected to gentrification processes in their school's neighborhood
Case 2	*Framing* of the problem of providing feedback to student groups as embedded in classroom norms	*Representation* of the work of teaching that makes visible the negotiation of classroom culture	Partially contradictory: exo-resource was used to disrupt teaching and learning norms	Was not explicitly connected

Table 8: Summary of Analysis of The Two Case Studies

To conclude, by showing *how teachers invoked and used exo-resources in these two conversations,* this chapter illustrated some ways that the exosystem and exo-resources can be meaningful for teacher collaborative sensemaking. Furthermore, it highlighted the interconnectedness of the exosystem with other scopes and contexts of teacher learning ecologies. In the next chapter, I further ground and illustrate the notions of the exosystem and exo-resources. To do so, I move from looking at one conversation of each team, to looking across all their VFF conversations, portraying a bird's eye view on their exosystems.

CHAPTER 5

A Bird's-Eye View on Rees and Noether Teams' Exosystems: The Types and Origins of Exo-Resources Across the Dataset

> *Each word tastes of the context and contexts in which it has lived its socially charged life […] it is not, after all, out of a dictionary that the speaker gets their words!* (Bakhtin, 1982, pp. 293-294)

In Chapter 4, I described two case studies that illustrated the way exo-resources can contribute to teachers' ecological sensemaking. In this chapter, I zoom out from the case studies to portray a bird's eye view on Rees and Noether teams' exosystems as they were reflected across all their VFF debrief conversations. I ask what resources were invoked within these conversations and where they originated. The results of this chapter point to three main insights. First, the recruitment of exo-resources was a commonplace feature in VFF conversations. Second, the bird's eye view on these teams provided six categories for describing the exo-resources, potentially granting teacher educators with a practical tool for inviting and building PD conversations on teachers' broader knowledge and experiences. Finally, most exo-resources in this study were aligned with visions of ambitious mathematics teaching, strengthening the notion that the teams' exosystems constituted a substantial fund of knowledge relevant to the PD goals.

5.1 The Recruitment of Exo-Resources was a Commonplace Feature of Conversation in Project SIGMa

Across all 32 conversations of six mathematics teacher teams in Project SIGMa, exo-resources were often referenced by the participants, including ideas from PD workshops, professional conferences, and the PDO, along with insights from previous VFFs. Across the 32 video-based conversations, we identified 126 such mentions: 91 invoked by the teachers and 35 by the facilitators. On average, there were approximately four occurrences of exo-resources per VFF, of which three occurrences were initiated by the teachers and one by the facilitator. This consistency signals that the reference of exo-resources was commonplace in VFF conversations in Project SIGMa. In the case of the Rees team, across three VFFs, 20 exo-resources were referenced (6.6 per VFF on average). In the case of the Noether team, across six VFFs, 26 exo-resources were referenced (4.3 per VFF on average). Of course, this does not necessarily mean that each of these mentions provided meaningful learning opportunities. However, as I illustrated in Chapter 4, they were potentially a meaningful part of the conversation. Consider Ball and Cohen's (1999) description of the way that many PD efforts fall short from the teacher's perspective:

> Participation in modal staff development is the professional equivalent of yo-yo dieting for many teachers. Workshop handouts, ideas, and methods provide brief sparks of novelty and imagination, most squeakily practical. But most teachers have a shelf overflowing with dusty vinyl binders, the wilted cast-offs of staff development workshops (p. 4).

Indeed, many ideas and methods introduced to teachers are forgotten on a dusty shelf (or in a never opened virtual folder). In contrast, the ones the SIGMa teachers invoked were, by

definition, the ones that left some mark on their sensemaking or practice, and "stuck" with them to some extent. For this reason, among others, attention to these resources is worthwhile.

Appendix 1 and Appendix 2 render the full inventory of exo-resources in each of Rees' three VFF cycles and Noether's six VFF cycles. For illustration, Table 9 renders the inventory of exo-resources in the two case study conversations from Chapter 4, Rees VFF 1 and Noether VFF 3. As discussed in the methods section, SIGMa teachers are not typical cases of PD participants but rather extreme cases. That is, our partnership with the teachers was based on them being part of the PDO, which provided them with 100 hours of professional development annually. In contrast, among typical mathematics high-school teachers, only 40% participate in more than 12 hours of mathematics focused PD annually (Banilower et al., 2018). On the one hand, this means that these inventories are probably more elaborate than we would see in most teacher conversations. On the other hand, as Horn and Garner (2022) explain, this unique setting offers a window into understanding possibilities for teacher learning. In regard to questions about external resources and teacher learning, these cases offer a glimpse into how a rich exosystem, within the context of a PD design that centers teacher sensemaking, can invite teachers to invoke these experiences and resources, as they integrate them into their practice. In other words, rather than evidence of what there is in the general case of teacher conversations, the extreme case helps us imagine what could be.

VFF	Resource Recruited from…	Mediating Tool of the Resource	Pedagogic Element Discussed	Illustrative Quote
Rees VFF 1 Dec 2017	North Conference Session	Probing questions	Teacher impact on student agency	Veronica: you know Cathy Humphreys. I went to her talk at North conference…
	Common Core State Standards	First standard (Perseverance)	Task choice	Ezio: I want them to learn how to persevere. It is one of the standards… I totally agree with that standard.
	RG workshop	Group roles	Using group roles to promote inclusiveness	Ezio: At RG workshop when we did this, the person holding the marker wasn't allowed to talk.
	RG workshop	Random Grouping	Grouping strategies	Ezio: At least in RG workshop, I really did not agree with the random grouping.
	PG workshop	Purposeful Grouping	Grouping strategies	Ezio: We got PG workshop a couple years ago and at least what they said made sense…
	North Conference Session	Concrete representations of fractions	Task choice and enactment	Ezio: It was so awesome …. She folded it in half… I wanted to tie that with the Tower of Hanoi
	Research team (Patty)	IMP curriculum	Task choice and enactment: use more simple problem	Patty: the Tower of Hanoi is in the IMP curriculum and it's in the unit for seniors

	Twitter	Foxes and sheeps task	Task choice and enactment: circulate back to unsolved problems	Veronica: I was thinking of foxes and sheep…
Noether VFF 3 May 2018	PDO	National Boards	Describing the focal class	Brad: they are actually like the most fun class because there's the most collaborative... They're actually what I'm focusing my national boards on
	Marisa's previous school	Problem-based curriculum	Designing for student collaboration	Marisa: so the problem-based curriculum… because from day one they are taught that you are the source of the knowledge… it's coming from the student as opposed to the teacher.
	Research team (Nadav)	5 Practices	Groupwork Monitoring	Nadav: like the five practices, like don't talk with them… monitor and write yourself where they are so you can segment.

Table 9: Inventory of exo-resources in Rees VFF 1 and Noether VFF 3.

5.2 Six Categories Describing Types of Exo-Resources

This list only begins to name… and only hints at the blizzard of guidance and regulation that falls on teachers (Cohen, 2011, p. 58)

Up to this point, I have argued that the exo-resources and the exosystem exist, they matter, and we should pay more attention to them. While the exo-resources invoked by teachers in the Rees and Noether teams differed, they can be described by the same six categories: (1) PD workshops,

(2) professional conferences, (3) online resources and textbooks, (4) research and policy resources, (5) experiences with different curricula, and (6) experiences with the PDO .

To illustrate these categories, I draw on examples from nine Rees and Noether VFF debrief conversations. Figure 13 and Figure 14 represent the exo-resources referenced by the Rees and Noether teams respectively, clustered by the six aforementioned categories of settings and contexts from which the resources were recruited. These figures are inspired by visuals in previous papers that focus on the learning ecologies of students (Barron, 2006), teachers (Louie, 2017b), and PD facilitators (Morel & Coburn, 2018).

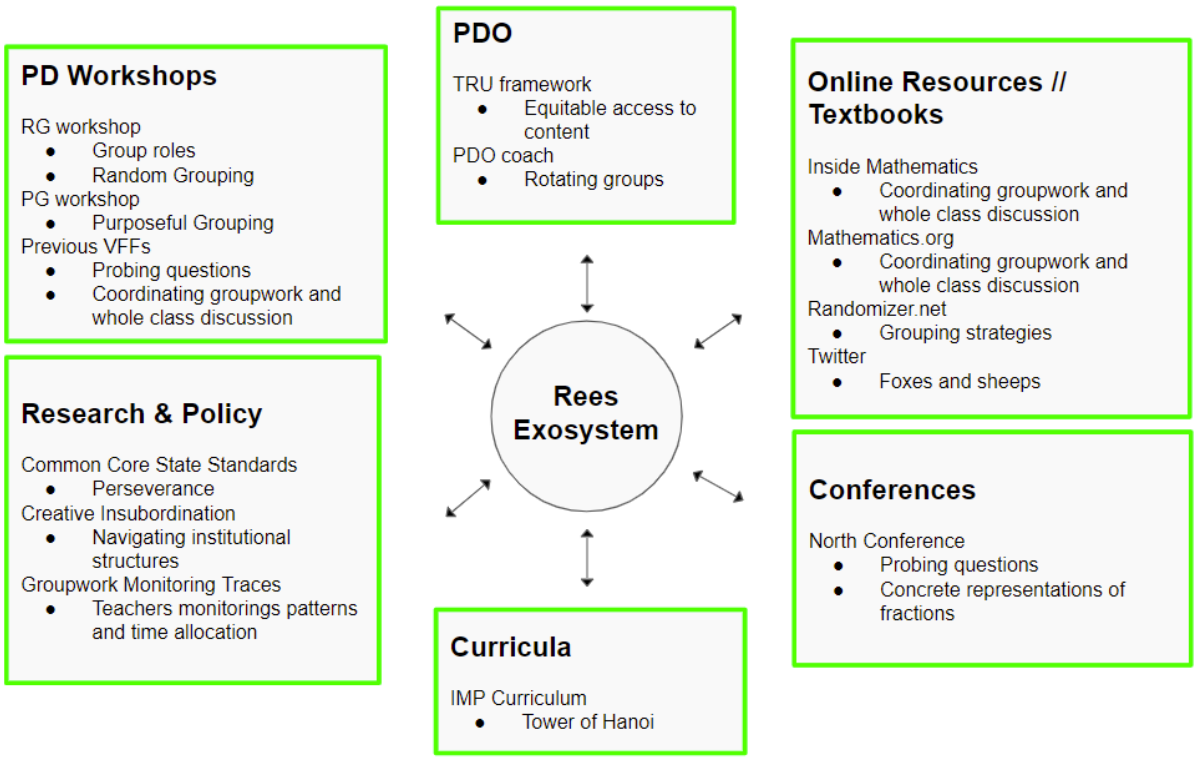

Figure 13: Bird's-eye view of the Rees team exosystem

Note: the six rectangles represent the six categories of settings and contexts from which the exo-resources were recruited. Within each category, items detail the specific settings and pedagogic elements taken from them in the Rees team VFF conversations.

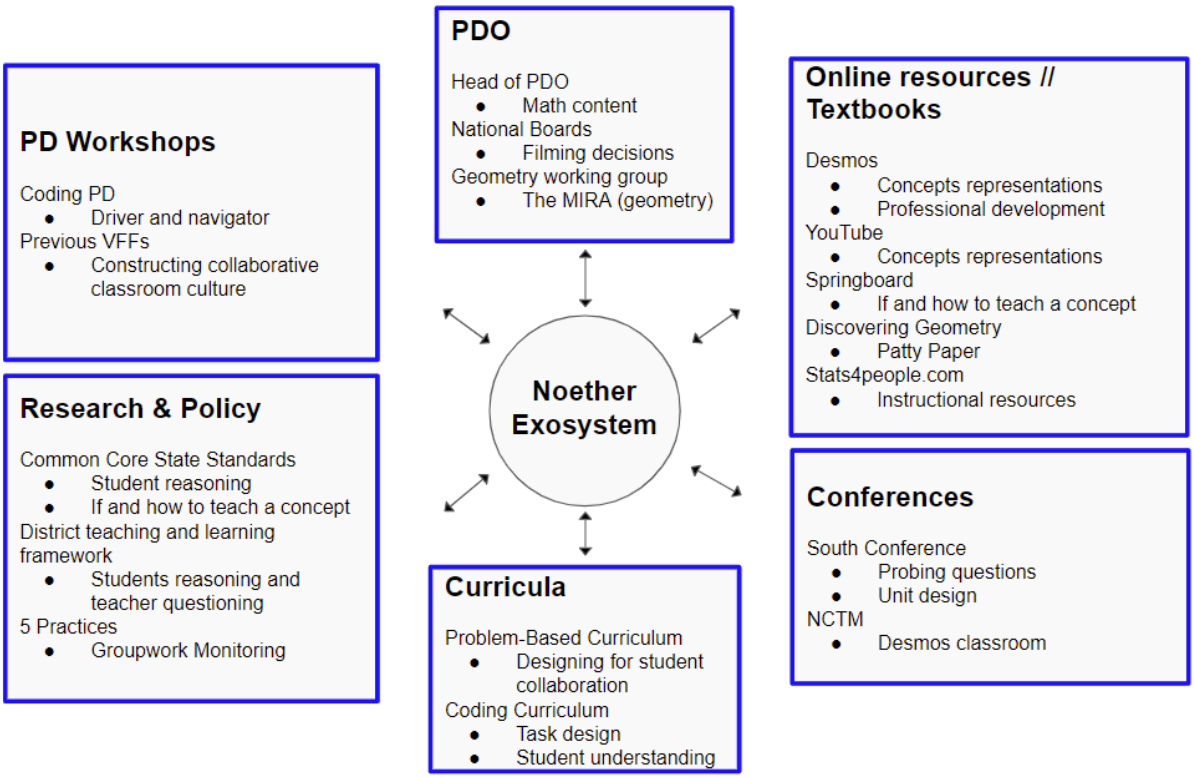

Figure 14: Bird's-eye view of the Noether team exosystem

Note: the six rectangles represent the six categories of settings and contexts from which the exo-resources were recruited. Within each category, items detail the specific settings and pedagogic elements taken from them in the Noether team VFF conversations.

5.2.1 PD Workshops

Both Rees and Noether teachers invoked instructional resources and experiences from various PD workshops they engaged in. Within this category, I also include our own PD intervention, the VFFs.

Ezio and Veronica's discussion of grouping strategies in Rees VFF 1 (Conversation 1 in Chapter 4) illustrated this category. In that example, the teachers discussed two grouping practices they learned in two different workshops and compared them. In addition, toward the end of that debrief conversation, Ezio referenced again one of these workshops, saying that "there were a lot of good ideas at the RG workshop and little by little, I'm trying to implement

them" [01:27:25]. From the perspective of designers and facilitators of PD, simply knowing what "good ideas" Ezio refers to, could help facilitators support Ezio in making sense of them and incorporating them into his routine practices.

5.2.2 Conferences

Both teacher teams referenced conferences, discussing sessions they attended, and specific takeaways from them. The takeaways were either pedagogical (like probing questions), a specific task, or related to curricular design.

For example, at the beginning of Noether VFF 6, Brad (the focal teacher) introduced the team to the lesson we filmed and were about to discuss. He shared that this lesson was part of a five-week unit of statistics within Geometry that he and Marisa planned. The new unit design provided Brad and Marisa with an opportunity to experiment with shifting their classrooms into small groups. When describing the origin of the statistics unit, Brad explained how this idea originated from a session he attended at the South Conference.

> This [unit] was inspired by, actually, a session I went to two years ago at South Conference, which was done by Kaila Milrad and Diego Ginat. They title themselves the Stats People [...] their kids were really struggling in the SAT because the SAT had changed to have like 30% data analysis and problem solving and a lot of that statistics they hadn't seen until, maybe, some kids got to AP Stats [...] [so they decided] let's just have every 9th or 10th grader take statistics in Geometry. So me and Marisa are piloting that this year.

After answering some questions from the team, Brad ended his introduction by sharing the Stats People website. He described how he and Marisa built their unit using that website and complimented the session presenters. Brad's introduction illustrated how his learning

simultaneously involved several settings including teacher conferences. Another illustration for the role of conferences within the teachers' professional development comes from Veronica, who said in an interview:

> I feel like [the conferences] that's what I have to bring to the table right now, it's like I've been exposed to so much ideas. The other math teachers just don't have the opportunity to see, like I've been to so many conferences in the last three years. (Veronica, February 2018 interview).

Similar to the "good ideas" that Ezio mentioned he learned in a PD workshop, I contend that knowing what Veronica meant when she talked about "so much ideas" would have helped facilitators to respond to her sensemaking and incorporate them into her routine teaching practices.

5.2.3 Professional Development Organizations

All Project SIGMa teachers were associated with our partner PDO, and indeed, within VFF conversations participants often referenced the organization. For instance, when they asked the researchers to use the TRU framework (Schoenfeld et al., 2019) as a lens for looking at their class, this pointed back to a PDO session introducing the framework as a tool for teacher reflection and growth. In addition, teachers invoked advice they received from the PDO's coach; their PDO content work groups; their certification process with the National Board through the PDO; and the PDO leader as a math content authority. In sum, VFF conversations reflected the centrality of the PDO community in the teachers' professional life.

Elaborating on the TRU framework example, a PDO leader introduced the teachers to the TRU framework in the PDO monthly meetings in September 2017 and then the teachers used it (by design) in every following monthly meeting. Veronica, as the focal teacher in Rees VFF 2

(February 2018), requested that her VFF inquiry focus on two component of the framework: (1) equitable access to content, and (2) agency, ownership, and identity. In turn, Patty used this information to locate relevant video clips from Veronica's filmed lesson, as well as to open and frame the conversation.

Patty: Um okay, so your questions. What you wanted us to think about were-

Veronica: So it's these… The equitable access to content [and] agency. Those are the two things that I was looking at really carefully. Is this fair for everybody, and are you held accountable for being in the room essentially?

Patty: Okay.

Veronica: Is that how you interpreted my question? [*laughs*]

Patty: No. Equitable access, right? Is everyone able to— Can everyone enter into the—

Veronica: Everyone gets a turn. Yeah, yeah, so equitable access.

Lani: Yeah.

Patty: Are students able to access the content from multiple entry points is one way to think about it.

Veronica: Right, and what students. Can everybody enter?

Patty: Mm-hmm

Veronica: um and then— Yeah, I think that's pretty much together. Yeah, those two go together in my opinion.

Patty: Okay. We're going to talk about this…

Within this short introduction to Rees VFF 2, Patty asked Veronica to explicate what she had meant by equitable access to content and agency. On one level, the introduction helped framing

the conversation in a more nuanced way around Veronica's interests and commitments. On another level, given the centrality of the framework in the PDO monthly meetings, invoking its terms, helped Veronica integrate her learning in the PDO with her instruction through the VFF conversation.

5.2.4 Research and Policy

VFF participants mentioned a variety of research and policy frameworks for teaching and learning. Teachers most often referenced policy frameworks and researchers-facilitators typically introduced research frameworks.

The teams referenced two policy frameworks. First, in the Noether team, the teachers referenced the district teaching and learning framework. In this episode, they discussed students' reasoning and the teacher's questioning. Second, both teacher teams referenced the Common Core State Standards (CCSS; National Governors Association, 2010), which were their state's framework for math instruction. In these episodes, the teachers discussed the notion of students' perseverance in problem solving, students' reasoning, and students' conceptual understanding.

For example, in Rees VFF 1, Ezio explained his choice of using the Tower of Hanoi task by invoking CCSS's first standard: "Make sense of problems and persevere in solving them" (National Governors Association, 2010).

Ezio: I think the Tower of Hanoi is complex enough, at least in a middle school. I want them to learn how to persevere. It is one of the standards.

Patty: Persevere, okay.

Ezio: I totally agree with that standard. I don't feel like I do a good job of teaching them how to persevere.

Patty: Okay.

Ezio: Over those obstacles. Tower of Hanoi at least gives me the opportunity to focus on that.

Patty: Yeah. I will share that the Tower of Hanoi is in the IMP curriculum and it's in the unit for [high-school] seniors.

Ezio's inquiry in Rees VFF 1 centered around group dynamics while solving problems. As we learned during the VFF, his thinking about group dynamics was strongly connected to his goal to teach students how to persevere. Indeed, this connection between problem solving and perseverance is apparent in the CCSS. Within the conversation we learned that Ezio was familiar with this standard, agreed with it, did not feel like he was doing a good job implementing it, and saw it as a personal goal and commitment. Interestingly, Patty's response also involved an exo-resource, when she shared that in the IMP curriculum the Tower of Hanoi is part of a unit for high-school seniors. By invoking IMP, Patty communicated that the problem is challenging and students needed either a more simple version of this problem or more resources to engage with it. Within this same conversation about Ezio's task choice, Ezio also invoked a third exo-resource. In short, he said that he was inspired from a conference session that dealt with showing students the power of exponential function. This example, while illustrating policy frameworks as exo-resources, also shows how in some cases teachers used resources from different categories altogether.

Researchers-facilitators typically invoked the research frameworks. Patty introduced the Rees team with the notion of *creative insubordination* (Gutiérrez, 2016); Lani introduced the Rees team to *monitoring routines* and shared visual monitoring traces (Ehrenfeld & Horn, 2020); And Nadav referenced *5 Practices* (Stein et al., 2008) in a conversation with the Noether team.

As illustrated in the previous section about the PDO, teachers invoked the TRU framework (Schoenfeld et al., 2019) in several cases.

5.2.5 Online Resources and Textbooks

Teachers and researchers on both teams discussed online resources and textbooks from which they learned and applied different ideas in their classrooms. For example, in the Conferences section above, I presented an excerpt from Noether VFF 6, where Brad introduced a statistics unit he and Marisa designed. After discussing the unit and how it was inspired from a conference session of the Stats People, Brad ended his introduction by describing how he and Marisa built the unit by using the Stats People website. He shared the website and said:

> if you go to the stats4people.com, they have [...] just open resource of everything they've ever done. They're the most sharing people I've ever met in my life, and just so quality their instruction [...] 180 days of regular stats and 150 days of AP stats a lesson per day with a blog and like a solution key that goes with it, and they're just next level. I'm trying to get there.

In this example, as in others, Brad made public his own ways of allocating instructional resources, which functioned in the conversation in several ways: First, it provided the other teachers access to this specific website with those specific resources. Second, Brad brought into discussion the activities that were used in this specific lesson and their rationale. Third, he modeled agency with regards to unit planning and instructional resources, illustrating how he changed the original unit plan with what he saw as quality instructional material. Finally, Brad framed the practice of collaboration and sharing of resources as an ideal that he is "trying to get" at. As a leader in this team, who is also the department chair, this framing most likely

strengthened the norms of collegiality in a team that was already supportive and committed to improvement.

In addition to websites with focused instructional resources, teachers also used textbooks and social media. For example, Veronica mentioned in an interview that "the exposure to like, Twitter for math teachers, that's insanely helpful on a daily basis for just affirmation and confirmation (Veronica, February 2018 interview). In sum, online resources and textbooks are yet another category of exo-resources that teachers used for their ongoing learning and invoked in collaborative sensemaking.

5.2.6 Curricula

Teachers and researchers on both teams discussed their experiences with different curricula. In Rees, Patty mentioned her experience with the IMP curriculum to discuss Ezio's choice of teaching the Tower of Hanoi in eighth grade. The Noether teachers mentioned two curricula: a coding curriculum (which also involved a related PD), and the problem-based curriculum which I elaborated on in Chapter 4.

To briefly reiterate, at Noether VFF 3, the teachers discussed ways of supporting students' collaboration. During the discussion, Marisa invoked her experiences teaching a problem-based curriculum in her previous school. In Chapter 4, I argued that the appearance of this exo-resource afforded a framing of the problem of leveraging students' agency as a problem of classroom culture, which extended a previous, narrower framing around moment-to-moment feedback (e.g., what could the teacher say differently?).

5.2.7 The Categories as a Framework for Designers and Facilitators

These six categories grant a practical tool for designers and facilitators wishing to center teachers' prior experiences as a driver of teacher learning. For example, the six categories

provide designers with guidance for questions they can directly ask in a background interview or in a written survey. For facilitators, being familiar with these aspects of teachers' prior knowledge and experience is important both in terms of realizing their potential within conversations, and for understanding teachers' overall learning trajectories.

5.3 Most Exo-Resources Were Aligned with Visions of Ambitious Mathematics Teaching

Notwithstanding the six categories above, exo-resources can be part of different mathematics education conversations, which might imply different visions of teaching. In this inventory, most resources in both teams were aligned with an ambitious vision of math teaching and learning that is collaborative and based on student dialogue, aims at conceptual math understanding, and foregrounds inclusiveness of all students.

This matters for two main reasons. First, the inventory uncovers the potential that the exosystem had for supporting the work of SIGMa facilitators with the teacher teams (to whichever extent it was fulfilled). Second, the inventory uncovers a potential tension between the teachers' exosystems and their school mesosystems. On the one hand, a broad consensus in the mathematics education research community advocates for ambitious instruction (Sztajn, Borko, & Smith, 2017) as an ideal telos for mathematics teacher learning (Horn & Garner, 2022). On the other hand, ambitious mathematics instruction is counter to most school cultures in the United States, where memorization of facts and procedures illustrated by the teacher are still the main emphases of instruction, and mathematical smartness is framed as being fast and accurate in these procedures (Horn, 2007; Louie, 2017a). Within teacher learning ecologies, tensions between institutional logics and PD efforts are ubiquitous and of particular importance

when supporting teachers towards ambitious instruction (Horn & Garner, 2022). An ecological perspective guides us to explore how these tensions are brokered within CSPD.

5.4 Limitations

An inventory of resources, in and of itself, does not precisely reflect the nature of the teachers' conversations or learning trajectories, and might even be deceptive. Teacher may introduce resources in conversations for the sake of critiquing them or for comparing between seemingly contradicting perspectives on teaching. For example, in Noether VFF 4 teachers discussed a certain curriculum and its lab guide, problematizing online activities where students "can just try things and see if they got it wrong or right" whereas if they got it right, they move on and sometimes without really understanding what they did. The teachers discussed assessments during groupwork monitoring in these situations, and one of them concluded that "that mentality is not beneficial-- it's kind of harmful almost." The Rees case study from Chapter 4 illustrated another example of invoking resources and challenging them. Teachers discussed two grouping strategies they learned in two PD workshops. They discussed what made sense for them, what they found harmful, and compared their usefulness considering their specific goals. These examples show that an inventory of resources does not necessarily represent how it was used, let alone the team's visions of good teaching.

The unique case of our partner teachers, while offering a window into rich teacher conversations, also poses a limitation. The fact that these teachers were part of the PDO ensured that they participated in a variety of PD initiatives. The PDO guided the teachers toward conferences and workshops that align with their visions of ambitious mathematics instruction. The volume of PD and its quality, while enabling investigation into the role of teachers' previous professional experiences, also calls for carefulness in generalizing these findings.

5.5 Conclusions

> Consider Bronfenbrenner's (1979) distinction between types of activities:
>
> Activities differ in the extent to which they invoke objects, people, and events not actually present in the immediate setting. Such invocation may be accomplished through conversation, storytelling, fantasy, pictorial representation, or a variety of other media. To the extent that activities refer to events occurring in other places at other times, they reflect an expansion of the actor's phenomenological world beyond the immediate situation (Bronfenbrenner, 1979, p. 47)

Indeed, as I illustrated in Chapters 5 and 6, teachers in Project SIGMa often invoked ideas and experiences from external settings and contexts, such as the six categories described above. I argue that attention to these episodes is worthwhile for several reasons. First, these instances were common and diverse in VFF conversations. Second, teachers invoked ideas and experiences that are almost by definition salient to them, even if introduced or experienced months or years before. Third, as illustrated in Chapter 4, episodes of conversations that build on exo-resources might have a significant influence on teachers' reasoning. Fourth, exploring and categorizing these resources as I have done in this chapter opened opportunities for designers and facilitators of teacher PD to be more attuned and responsive to teachers' previous professional experiences. Fifth, because these instances are rooted in teachers' previous professional experiences, the type of pedagogical inquiry they promote has more potential to be sustainable as a way of collaboration over time, after facilitators step away. Sixth, thinking about inclusion of such conceptual resources, what they are and where they come from (*scope*), how they are used in context and conversation (*interconnectedness*), and when (*temporality*), highlights the need to frame teacher PD in deeper and wider ways. The sixth and final point is the main concern of the

following chapter, with a focus on the temporal aspects of the teachers' learning. First, I de-center the exo-resources and re-center a more thoroughgoing ecological framing of teacher CSPD. Second, I pay specific attention to different phases in teachers' learning trajectories and how attention to these phases (i.e., to the temporality of learning) might offer teacher educators a tool for acknowledging and providing different resources for teacher learning at different times.

CHAPTER 6

The Temporality of Rees and Noether Teachers' Learning Across the Partnership

לַכֹּל, זְמָן; וְעֵת לְכָל-חֵפֶץ, תַּחַת הַשָּׁמָיִם (קהלת ג' א')

To everything there is a season, and a time to every purpose under the heaven (Ecclesiastes 3:1)

In Chapter 4, I illustrated the affordances of looking at lived experiences and exo-resources by asking *how* teachers invoke and use them in conversation. The two case studies in Chapter 4 revealed the transformative potential of exo-resources towards more ecological teacher reasoning. Then, in Chapter 5, I made concrete the notion of the exosystem of teacher learning ecologies, asking *what* exo-resources were in use and *where* they originated from. Now, in Chapter 6, I move forward from the exo-centered perspective of Chapters 5 and 6, re-centering the overall ecological frame with a specific focus on the temporality of learning. I ask, *How do different levels of teachers' learning ecologies interact in different phases of their learning?*

Implied in this question is my goal to understand learning trajectories in the PD beyond linear progress from point A to point B. I operationalized temporality in Chapter 2 with the lens of Gunderson and Holling's (2002) adaptive cycles (Table 3). To reiterate, the adaptive cycles include four phases of release, reorganization, growth, and conservation. For the purposes of this study, these phases constitute a cyclic lens on learning trajectories (Figure 15). To briefly reiterate, for teacher PD, **release** represents problematization of institutional practices and teaching norms; **reorganization** represents navigating tensions between institutional logics and teachers' pedagogical goals as well as planning forward; **growth** represents experimenting with the new professional arrangement; and **conservation** represents the phase when new practices

are consolidated, and teachers are proficient with the new professional arrangement. Adaptive cycles also provide guidance with regards to phases when the influence of external variability of resources is high (reorganization and growth). My conjecture is that in the reorganization and growth phases, invoked external resources will be more salient, and for the conservation and release phases, internal processes such as video-based reflection will be more salient (see Figure 15).

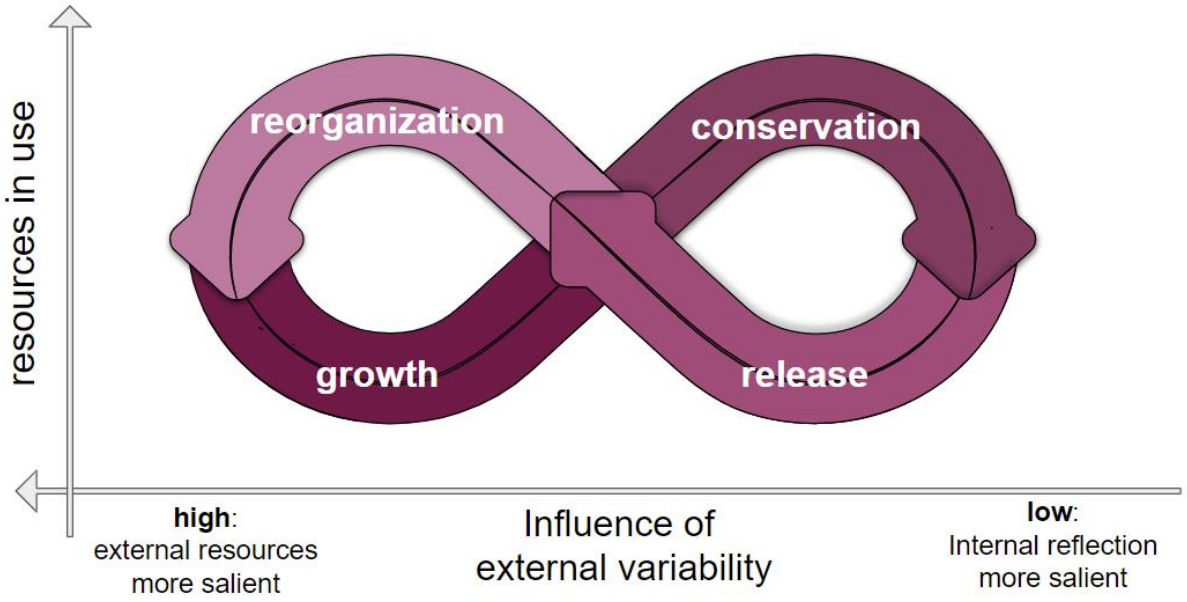

Figure 15: Internal reflection and external resources in adaptive cycles

Note. The x-axis represents the influence of external variability. Within the reorganization and growth phases, invoked exo-resources are expected to be more salient in promoting change. In contrast, within the conservation and release phases, internal processes such as video-based reflection are more likely to be salient in promoting change.

In the rest of this chapter, I look at our partnership with the two teams through the lens of these four phases. First, I show that our work with the Rees team centered on the reorganization and growth phases. Then, in contrast to Rees, I show that our work with the Noether team can be

seen as a full adaptive cycle across all four phases. I illustrate these temporal aspects of the CSPD while also paying attention to the ways they were shaped by different resources and levels in teachers' learning ecologies. I conclude by considering implications to designers' and facilitators' responsiveness to teacher learning by using the lens of temporality.

6.1 Rees: PD Focused on the Reorganization and Growth Phases

In this section, I use the lens of the adaptive cycles to describe and illustrate the Rees team's phases of learning within our partnership, showing that they focused on reorganization and planning new teaching arrangements towards experimentation and growth. Using the terminology of the adaptive cycles, the phase of **release —** the problematization of their school routines and of their own teaching — mostly happened before our intervention. Specifically, Ezio and Veronica had grown concerned about the unofficial tracking in their department. For them, tracking was not just about differentiating levels of math classrooms but extended to other activities (such as electives) and created what Ezio called a "sharp divide" between different groups of students. In other words, as was evidenced in Rees VFF 1, Ezio and Veronica started our partnership with relatively clear articulations of issues that bothered them in their classrooms as well as in their school.

The phase of **reorganization** represents the generating of ideas to address these problems, navigating tensions between institutional practices and their personal commitments. Then, the phase of **growth** involves experimenting with new ideas and instructional practices. In what follows, I show that our work with Rees focused on these two phases. The following two sections illustrate this argument with episodes from Rees VFF 1-3.

6.1.1 Reorganization: Navigating Institutional Practices and Planning New Classroom Arrangements

For Ezio and Veronica, acting toward their goal of making their school more equitable included actions outside of the classroom, such as conversations with the principal and the other teachers, and actions within the classroom, such as changing their lessons to become more inclusive and collaborative. These two interrelated goals were salient across our work together.

Much of our work with Ezio and Veronica focused on supporting them in navigating tensions between institutional norms and their pedagogical commitments, with a focus on reorganizing their classrooms around student collaboration. Within the formal video-based conversations, I see this phase mostly in Rees VFF 1. In the later VFF cycles, the formal video-based conversations mostly focused on experimenting with new practices, which I later describe as the phase of **growth. (**Notably, even within these later VFF cycles, the informal "backstage" was constantly a sensemaking space for their challenges outside of the classroom.)

Rees VFF 1 was described in depth in Chapter 4. To briefly reiterate, Ezio and Veronica were explicit about rejecting some of their school's practices, both with regards to tracking and other distribution of resources. They felt that these norms served the school's recruitment and accountability measures but did not serve all students well, especially those from lower socioeconomic backgrounds. These students were typically from families of old-timers in the school's gentrified neighborhood.

School-level concerns came up frequently in Rees VFF 1, as well as in Ezio and Veronica's interviews and other informal conversations we had with them. For instance, in Rees VFF 1, Ezio and Veronica made a strong connection between their grouping strategies and their institutional concerns about stigmatizing groups of students. When it came to grouping

strategies, they noticed that purposeful grouping amplified the consequences of tracking in the shape of labeling kids as “dumb” or “awesome” — even if teachers did not explicitly state their grouping scheme — while random grouping disrupted it.

Ezio and Veronica considered ambitious instruction to be “outside of their comfort zone.” Rees VFF 1 illustrated their preliminary reasoning and recruitment of ideas towards supporting more equitable student collaboration (i.e., reorganization phase). Then, the following two VFFs were mostly focused on experimenting with and consolidating instructional practices such as random grouping, re-grouping, and rotating groups (i.e., growth phase).

Notwithstanding the useful analytic distinction between the two phases, the reorganization challenges in light of the school context were never fully resolved. Even though the formal video-based conversations shifted to focus on experimenting with new instructional practices, Ezio and Veronica continued to mention their concerns about school in interviews and informal conversations. The following example illustrates such an informal conversation between Patty and Ezio.

6.1.2 Rees VFF Informal “Backstage”

For Rees VFF 3, a few days elapsed between Ezio’s filmed lesson and the lesson debrief. In those days, the research team attended our partner PDO’s Saturday's PD (as we always did). When Patty met Ezio at the that PD, he described how he was emotionally consumed by the continuous struggles at his school: confrontations with administration and advocating for students but not feeling heard. In her fieldnotes from the conversation, Patty described supporting Ezio in making sense of a reality in which, in Patty’s words:

> middle-class, white parents will always ensure that they are heard and that their kids will receive what they need but that is not the case for our parents (low-income, immigrant, monolingual).

Furthermore, Patty responded by sharing a variety of experiences and resources to navigate the situation. In her notes, she described sharing her experiences as an educator and parent. First, she acknowledged that advocating for students and families whose voices are silenced often involves certain professional risks. Then she provided more resources. For example, she shared that it often only took a few vocal and persistent parents to create a movement for the kids. In addition, Patty described to Ezio Rochelle Gutiérrez's argument of why teachers need political knowledge, and she later sent him Gutiérrez's article about creative insubordination (Gutiérrez, 2016).

This example calls attention to seeing the VFF cycle — and PD in general — as an activity that happened both on the formal "front stage" (the video-based conversation) and on the informal "backstage" (Goffman, 1959). Indeed, Patty was extremely responsive to Ezio's teaching and learning situation. In addition to providing these resources, Patty also noted in her fieldnotes that, because she learned how bad Ezio felt, she "switched gears" in her debriefing plans. Instead of featuring video clips that questioned his group facilitation, she decided to try and identify positive moments of peer-to-peer interaction to lead a lighter debrief.

6.1.3 Growth: Experimenting with and Consolidating Practices

Notwithstanding the fact that the reorganization phase was never really "done," but rather shifted to the informal backstage, Rees VFF 2 and VFF 3 focused on Ezio and Veronica experimenting with new pedagogical arrangements in their classroom. To illustrate this phase, I describe an

example from each VFF. Because of their similar nature, I then further elaborate only on one of the examples.

In the first example, Veronica incorporated the aforementioned random grouping into her teaching routine. In this case, Veronica implemented it by the book, with limited flexibility and discretionary judgment. Trying out these new practices constituted experimenting. The debrief offered her an opportunity to hone her flexibility and consider overriding the randomness to attend to the specific teaching situation, which allowed her to consolidate some of her understandings of her experiment.

In the second example, Ezio incorporated the instructional practices of rotating-groups, another experiment. This new practice, or exo-resource, introduced to Ezio and Veronica by their PDO coach. The debrief offered teachers several opportunities to consolidate the practice. The idea was that when groups were rotating, they left (by design) their whiteboard scribes to the following groups to reflect on. However, in watching the clips of his classroom, Ezio realized that when he addressed the groups, he talked to the students as if they wrote the scribes next to them, which resulted in some type of miscommunication.

The two examples illustrate experimenting and consolidating, to some extent, new instructional practices, which is the heart of the growth phase. Next, I elaborate on the first example, showing in more detail how experimenting and consolidating unfolded in the interactions.

Rees VFF 2 Context. Rees VFF 2 took place in February 2018, and Veronica was the focal teacher. Veronica devoted most of that 7th grade math lesson to an "always, sometimes, never" groupwork task, where students received 15 cards with mathematical statements they had to sort as *always*, *sometimes*, or *never* true. During that activity, students worked in ten groups of

three around the classroom walls. For the debrief, Veronica asked the research team (Patty and Lani) to discuss students' group conversations, and in particular, their equitable access to content. As veronica mentioned in her email to Patty and Lani prior to the debrief, the aforementioned TRU Math framework inspired her choice of these topic.

Pedagogical Judgment When Revisiting Random Grouping. The two short episodes illustrate Veronica's growth phase, where she experimented with random grouping and implemented it in a ritual manner, by the book, with limited flexibility. The episodes illustrate opportunities to develop Veronica's notion of this practice, and more generally, her sense of agency when implementing any given practice. In the first episode, 24 minutes into the conversation, Patty asked Veronica about one of the groups. In response, Veronica responded with an unsatisfied face, and Lani asked for the meaning of her facial expression:

Patty: We have the blue group. Do you remember which was the blue group?

Veronica: Kim, Eli, and Katy.

Lani: What's the face mean? [00:24:38]

Veronica: [*laughing*] Sometimes when you draw a random card group, you get a group where you're just like, "If I had set up the groups, I wouldn't have put those kids together." It makes me question actually, if. one, if the cards weren't shuffled when I started passing them out. Kim and Katy are sitting together, and then they *mag*ically end up in the same group together.

Lani: Are they friends?

Veronica: They are friends, yeah. Yeah. and uh—

Patty: It was mostly social interactions, right?

Veronica: Mostly social interaction. And so like— that's like a pitfall, one, of random grouping and, two ... Yeah, when you do random grouping. …

Up to this point, Veronica explained her dissatisfaction with the two friends, Kim and Katy being together at the same group. She attributed it to a pitfall of the random grouping strategy. At this point, Lani shared her own teaching experience with random groups, suggesting an alternative through a replay of her own practice (Horn, 2005):

Lani: so I'd sort the cards, but then I'd be like, "No, Kim and Katy."

Veronica: Right, you would do some-

Lani: "You guys are going to have fun, but I want you to do math today." Then just I'd just like publicly switch it.

Veronica: Yeah.

Lani: They knew that any time that-

Veronica: You could override the randomness.

Lani: I could. I could use my judgment and be like, [Veronica: yeah] "This isn't going to go well."

Veronica: Yeah, right.

In this short episode Lani provided Veronica with guidance how to implement a specific exo-resource, random grouping. On another level, this was also a guidance of Veronica towards greater agency, flexibility, and judgment when she is using any external practice in her particular teaching situation. At the end of the conversation, Veronica specifically mentioned this insight about the random grouping to be a significant takeaway from the VFF cycle. I see this as an example of concept development within the growth phase since Veronica moved away from "the

enactment of pre-determined practices" toward honing pedagogical judgments "which links practice to [her] understandings." (Horn & Garner, 2022, p. 58).

6.1.4 Conservation: Instructional Growth with No Evidence for a Stable Phase

To conclude, on the one hand, Ezio and Veronica demonstrated instructional growth and concept development. On the other hand, given Veronica's example from VFF 2 (random grouping) and Ezio's example from Rees VFF 3 (rotating groups and re-grouping), the end of our partnership after Rees VFF 3, we cannot describe these new instructional arrangements as stable. Using the terminology of the adaptive cycles, Ezio and Veronica did not reach the phase of **conservation**. Rather, our work with the Rees team was centered on the reorganization and growth phases. In contrast, the case of the Noether team illustrates such a full adaptive cycle.

6.2 Noether: Full Adaptive Cycle Toward Conceptual Change

In this section I use the lens of the adaptive cycles to describe and illustrate the Noether team phases of learning within our partnership, which included all four phases towards almost a full cycle. I focus my analysis in this chapter on the sub-team of Noether that includes Brad and Marisa as daily common-planning partners.

6.2.1 Release: Acknowledging Where Instruction Falls Short

In contrast to the Rees team, the Noether team did not show signs of arriving at our partnership with a sense of urgency, neither with regards to their institution nor their teaching. However, using the terminology of the adaptive cycles, a phase of **release,** a problematization of some aspects of their instruction, and of institutional norms, emerged as a result of the video-based reflection. The three VFFs in Noether Year 1 often included problematization of the teachers' practices. By problematization in this case, I mean that we discussed central aspects of ambitious

instruction aligned with the teachers' stated goals, and yet, were still not recognized by them as possible avenues of growth in their teaching. These aspects included: involving all students in the group, refraining from providing students with answers, and directing students to each other as resources of mathematical knowledge. I illustrate this with an episode from Noether VFF 1 that focused on involving all students in the group.

6.2.1.1 Debrief Overview

In Noether VFF 1, inspired by two rubrics in the county's teaching framework, Brad asked the research team to focus on his questioning and other techniques to support student participation. The Algebra 1 lesson was about linear equations, with students practicing how and when to use slope-intercept and point-slope formulas. The activities in this lesson included short pairwork time and lengthy whole class discussions. When Lani asked Brad what image he had in mind for good student argumentation, he described a vision of talk as leading to good mathematical thinking, logic and reasoning. Clip 1 included a student talking to the whole class and revising his thinking. Clip 2 featured two students talking about "arguing" and trying to make sense of how their peer discussion needs to look. Clip 3 then more explicitly focused on Brad's support of student collaboration. (For more on this conversation, see Horn, 2020.)

6.2.1.2 Supporting Students' Collaboration

Clip 3 was projected 22:45m into the conversation and showed Brad coming over to a pair of students who were working on a slope-formula task. Brad approached the pair and talked only with one student. Then, while probing her thinking, he led her to the answer by asking, for example, "Do you think the point-slope [formula] we could use for either of those equations? Which of the two equations might be useful to use point-slope [formula]?"

After watching the clip Brad said:

> I think that I kind of led her and told her, “Do you think point slope would work?”, like a yes or no answer. She kind of had her head down most of the time and wasn't really taking a second to think about what information she knew. (...) I guess I was kind of just trying to... You know you're supposed to use-point slope here, right? Which one would help? “Well we don't know the y-intercept.” And then I kind of just said, "Would point slope-work?"

On the one hand, this turn of talk illustrates that Brad noticed his strong cues which eventually shifted to telling the student the answer. On the other hand, it also illustrates that during the observation of the clip Brad did not explicitly show concern about the other student’s disengagement, which Lani then pointed out:

> Lani: I'm wondering like, if you could rewind time and think about... You said one thing, like, I would try to ask who's leading the questions and ... But I wonder if there are moves that you could incorporate so that he was more involved, ‘cause his body language was kind of like [Brad: yeah yeah yeah] back.
>
> Brad: Yeah yeah. If he was just gonna let her do it and then copy it

In this exchange, Lani first pointed out how the interaction between Brad and one student excluded the second student from the conversation, then Brad acknowledged it was indeed the case.

After this exchange, Lani and Marisa both suggested practical solutions to direct students to work with each other and see each other as a resource. Lani suggested that Brad could explicitly not talk to student groups during the first few minutes of the task, which would “make” students turn to their partners. Marisa invoked her experience with teaching a problem-based

curriculum and suggested that he use group roles —specifically a resource manager, which would "make" the students discuss the questions before asking the teachers. Brad concluded the VFF debrief by saying that it made him think about shifting from a management perspective of keeping students on task to what he termed an "equity standpoint" on who he is speaking to and at what times.

Noether VFF 1-3 in Year 1 of our partnership included several such episodes of release. As in the above example, reflection on video resource was significant in this phase, as well as the researchers' help to notice and problematize classroom events, policy frameworks such as the CCSS and the county's teaching framework, and Marisa's previous experience with a problem-based curriculum.

6.2.2 Reorganization: Renewal and Recruitment of New Teaching Arrangements

Toward the end of Year 1, it became evident that Marisa and Brad were not only reflecting on and problematizing their practice, but also considering ways to **reorganize** their teaching arrangements. Noether VFF 3 signaled this shift and was described in depth in Chapter 4. As in Noether VFF 1, Brad was the focal teacher in Noether VFF 3, and the conversation focused on ways to support student collaboration. To briefly reiterate, the research team pointed out the gap and possibility of further directing students' questions to each other within the group. Like Noether VFF 1, Marisa then invoked her experiences teaching a problem-based curriculum in her previous school. In this VFF, she built on her previous experience to suggest that the constraints of supporting student collaboration are stemming from the culture of their classrooms and school. This framing situated their inquiry within the institutional context and suggested future actions that tied together curricular design, classroom instruction, and institutional constraints.

In later interviews, we found further evidence that these discussions pushed Brad and Marisa to reorganize their classroom and units. For example, in retrospect, at the Member Check interview in Year 3 Brad said that:

> I remember you guys came and observed somebody and it was like we had been very new into doing group work. [...] And I just felt like the kids needed to talk more and needed to work together more and that I just needed to change things up… (Brad's Member Check interview, February 2020)

Indeed, in the following year, Brad and Marisa used their common planning time to restructure their curriculum and to experiment with new lesson structures.

6.2.3 Growth: Experimenting with Groupwork

During Year 2, we visited Noether at the beginning of the year to film Marisa (Noether VFF 4), mid-year to film Greg (Noether VFF 5), and at the end of the year to film Brad (Noether VFF 6). Marisa's VFF 4 did not feature any groupwork. However, later that year, Brad and Marisa restructured their geometry classrooms to have students work in small groups daily over a five-week unit on statistics. This was the context for Noether VFF 6. Important to consider with this configuration, recall that at this point, Brad had become department chair and therefore had greater agency in curriculum design. Brad and Marisa took advantage of the new design to "dive in full" into groupwork experimentation in the ongoing way Marisa pushed to in the previous year. Consider Brad's introduction to the other teachers at the beginning of Noether VFF 6:

> I've had these kids in these groups for the last two weeks. They've been in groups of four, even though me and Marisa have barely done groupwork [before the unit.] We did it once in a while if the lesson may have called for it, but the last two weeks since we started Statistics, every single day has been group work, the entire time.

As Brad explained, at the time of VFF 6, Brad and Marisa taught the Statistics unit for two-weeks in small groups. The topic of Brad's focal lesson was one- and two-variable quantitative data. The general lesson plan included asking students to measure their shoe length and investigate the results with relation to their handspan measurements from the previous week. The goal of the lesson was to discuss both descriptive statistics (concepts such as mean, median, range, standard deviation) and the correlation between the two variables.

VFF Video Clips Context. In terms of the VFF Video Clips, the fact that Brad and Marisa reorganized their classrooms in small groups enabled us to discuss groupwork monitoring in more complex ways than in Year 1, where they "barely done groupwork" (as Brad said). Notably, before the lesson, Brad's asked the research team (Nadav and Katherine) to focus on his feedback to the groups and the way they use it. Later on, right after the lesson ended, he also asked Nadav and Katherine to look at "what happens when I leave a group without giving them an answer?" Clip 1 afforded seeing what happened when Brad redirected the students to each other and moved on the next group. Clip 2 then further suggested the need to build Brad's feedback on students' previous conversation.

Clip 1: Affirming Brad's New Practices of Directing Students to Each Other. In the beginning of the lesson, Brad explained to the whole class that the student with the biggest handspan in each group should be the only one who asks questions. (This practice echoed Marisa's suggestion from Year 1.) Explaining the rationale to his students, Brad further said: "I need to see you talking to each other. Come with questions as a group." Then, unlike in Year 1, when individual students asked Brad questions, he constantly redirected them, saying things like:

- "Who is the big handspan person in the group? Ask him"
- "Did you ask your group? I bet someone in your group know" S: "I bet you know..."

- "Cool, so talk about this as a group"
- "I'm telling you they have more resources than I do"
- "I feel like your handspan is not the largest. Did you guys talk about it? [S: no] Why won't you talk about it and I'll come back in a few minutes."

Similarly, in Clip 1, a student asked Brad about center measures. Brad recognized that she did not talk about it with the group and directed them to discuss it amongst themselves. In the debrief, we listened to the group audio after the teacher left the group, we were able to show him that the group figured out the problem, together, without his help. Even more, once students found the center measures, they continued to talk to each other as they moved on to questions about the data spread. As Katherine mentioned to Brad, "That move that you had made [redirection] kind of started this like catalyst of them working together, using each other." Affirming Brad's new redirection practices illustrates an opportunity for Brad to experiment and consolidate these new practices within the growth phase.

Clip 2: Developing Pedagogical Judgment When Interacting with Groups. While Clip 1 illustrated how Brad approached groups after they *did not* talk about their question with each other, Clip 2 illustrated how Brad approached groups after they *did* talk about their question before calling him. The clip started with a conversation students had before asking Brad, "How would we describe the spread?" In this conversation, it was clear that they were familiar with the concepts of range and standard deviation as two options to describe the spread of their handspan data. However, they had some confusion in calculating the standard deviation and their result did not make sense to them. When Brad approached the group he first asked if they had talked about spread before asking him, as he routinely did throughout class. When they confirmed they did, Brad immediately moved to remind them and reteach them the two options to describe spread.

In the debrief, we listened to the students' conversation before Brad came, and then to Brad's interaction with the group. This clip made it transparent that Brad was repeating what the students already discussed without knowing what they had talked about. Brad acknowledged that, suggesting he could have asked, "What about spread confused you?" and Katherine suggested "What was your conversation about?" While reflecting on Clip 1 affirmed Brad's new redirection practices, reflecting, and discussing on Clip 2 opened up new learning opportunities for Brad and the other teachers with regards to these new practices. This example illustrates another way of experimenting and consolidating new teaching arrangements within the growth phase.

This clip apparently left a strong impression on Brad. In the Member Check visit a year later (Year 3), Lani and Katherine visited Brad's classroom, and then discussed their observation with him. During this conversation, Lani mentioned seeing Brad coming over to a group and saying, "What'd you guys talk about so far?" In response, Brad referenced the discussion from Noether VFF 6:

> I would not have said that if it wasn't for watching that video at [Noether VFF 6] where I gave that group the answer, without having heard from them. I swear that's my number one take away from the video observations is that group, remember the standard deviation range, all that stuff. And I'm just like, I didn't give them a chance to say what they had talked about and I just told them the answer. And so that is... Yeah, no, and that is the biggest thing that you guys' research has changed for me is they have talked or if they haven't talked, that's their fault and they should talk. And I used to just make sure that person called me over and then I would answer their question kind of, but I still wouldn't necessarily have a debrief on what they talked about.

Summary. Brad's classroom design and interaction in Noether VFF 6 exhibited experimenting with groupwork including significant changes compared to all of his and Marisa's previous VFFs in Year 1 and Year 2. These changes echoed Marisa's previous experiences with the problem-based curriculum that she invoked frequently. These experiences and resources supported them in planning and teaching a five-week unit of statistics within their geometry classes. Using the framework in Figure 15, their experiments with these ideas represented a phase of **growth** towards the following phase of consolidating the new arrangements into relatively stable practices.

6.2.4 Conservation: New Teaching Arrangements Consolidated into a Stable Practice

Until now, I showed how Brad and Marisa's learning trajectory spanned across the three phases of release, reorganization, and growth. My argument in this last section is that Year 3 illustrated a full adaptive cycle towards conceptual change. The VFF video resources, together with their shared analysis, exo-resources like curricula and conferences, and institutional resources such as Brad becoming the department chair, all supported Brad and Marisa in the adoption of new structures, new practices, and concept development around the notion of groupwork.

In February 2020 of Year 3, Lani and Katherine visited Brad's classroom for a Member Check. (At this point, Marisa had moved to a new school for family reasons.) Coincidentally, they arrived a day after Brad received new group tables to replace his individual desks with obstructive arm trays that made groupwork difficult. He had completely built the lessons they observed around student collaboration, and Lani and Katherine noted in their fieldnotes that students seemed used to it (which Brad confirmed in the interview). Importantly, our team also noticed several instructional moves we had discussed during past VFFs: (1) quiet circulation in

the classroom listening to groups; (2) asking what students were talking about; (3) directing students to each other rather than giving them answers; (4) using student roles.

In the interview after the Member Check observations, Brad recalled this learning process:

> I remember you guys came and observed somebody and it was like we had been very new into doing group work. [...] And I just felt like the kids needed to talk more and needed to work together more and that I just needed to change things up [...] So I changed last year in the fall with my AP Stats. So, I did all year group work, fall to spring and then Geometry. I only started last spring. So they're used to working in groups, they're used to some of the kids being frustrated, not being able to ask me questions. And some of them, always in the first like August, September, they're used to being AP students and just taking in information and being motivated students where they just like, "Teacher, tell me and I'll reproduce." So it takes some of the kids a little bit of time. But ultimately when they start reflecting on statistics is really hard to do by yourself. That even though highest level kids that are used to just being A students and don't need the other three end up valuing group work by the end of the year. So... [Lani: That's awesome.] So I've just dove in full on to having all my classes work in groups.

Overall, there was a lot of evidence in Brad's classrooms that he has developed more strategies, structures, and awareness about setting up and supporting effective groupwork. Even more, Brad recalled how these arrangements started with problematization, or what I call here **release**, when he "remember you guys came and observed… I just needed to change things up." They then shifted to reorganization and experimentations, during which he and the kids got used

to work in groups, and side by side with their frustrations, students are seeing the affordances. Eventually Brad seemed to arrive at a stable phase, which I argue is a new **conservation** phase**.**

6.3 Discussion: Comparing Rees and Noether Learning From Adaptive Cycles Perspective

From a quantitative perspective on time, the partnership with Rees was one year long, and the partnership with Noether was two years long, plus a Member Check visit in the third year. From a qualitative perspective on time, which I called temporality, I described the different phases of the two teams' concept development trajectories through the lens of the adaptive cycles in the contexts of their learning ecologies.

With regards to the Rees team, in Chapter 5, we looked at an episode from Rees VFF 1, where Ezio and Veronica reflected on processes of their professional learning across a variety of activities and connected them to their current teaching situation. In particular, they developed a stance that problematized some of their school practices (mostly tracking, but also other distributions of resources they deemed unequitable), as well as problematizing their own instruction. For them, a **release** phase took place before our intervention, and exo-resources (such as the PDO) were significant in this process. Within the timeline of the research project, throughout the three VFF cycles, Ezio and Veronica were mainly concerned with establishing new professional arrangements, experimenting with them, and coordinating their work (within and outside of the classroom) with school norms and policies. In other words, our work with them centered on the **reorganization** and **growth** phases. In addition, there is evidence that at the end of our partnership their new teaching arrangements did not yet stabilize and reached another phase of **conservation**.

In contrast to Rees, I described our work with the teachers at Noether as spanning almost across all four phases. Unlike the teachers at Rees, the teachers at Noether started their career within spaces that were relatively progressive and as such these spaces valued student-centered pedagogies of the start. Brad started his career with our partner PDO, and Marisa was teaching with a problem-based curriculum from her first teaching job. Greg's professional trajectory was indeed more similar to Ezio's, both in the number of years of experience and in the role that the PDO played in shifting their thinking about teaching. Notably, because Abigail left school after year 1, and Greg represents a learning trajectory that is similar in nature to Ezio and Veronica, I focused my analysis in this chapter on Brad and Marisa.

For Noether, I used the metaphor of **release** to describe the phase of acknowledging and reflecting on the gap between the teachers' pedagogical goals, and the ways their teaching often played out in the classroom and fell short with regards to student learning. Then, transitions from release to **reorganization** represents navigating tensions between institutional norms and teachers' pedagogical goals towards renewal. A main institutional resource in this phase was Brad becoming the department chair. At this point, the transition from reorganization to **growth** included experiments with the new visions of collaborative learning, towards a well-established new arrangement we found in a Member Check in Year 3, that can be described as nearing **conservation**, or what Horn and Garner (2022) described as *conceptual and cultural change*.

Temporal analysis and comparison between the Rees and Noether VFF cycles resulted in several insights. First, while the phases in Figure 15 are analytically distinct, in real life, they are interrelated. For example, while Ezio and Veronica were experimenting in the growth phase, they kept making sense of their school contexts and coordinating meso- with exo- resources (reorganization phase). Relatedly, this analysis also highlighted how formal and informal

dimensions of the VFF cycles were interrelated, as when Patty's conversation with Ezio shaped her facilitation of the debrief. This insight extends the meaning of responsiveness in CSPD, from facilitation of the formal video-based conversation to the ongoing formal and informal communication and relationship building. Finally, responsiveness to different phases of teacher learning calls for different support. As a general guidance, these examples suggest that in the conservation and release phases, internal reflection has more potential to promote change. This was particularly evident in the case of Noether VFFs in Year 1, where problematization mainly stemmed from reflection on video representations of teaching. In addition, these examples suggest that in the reorganization and growth phases, acknowledging and providing a variety of external resources has more potential to promote change. Within the reorganization phase, the focus should be on reconciling these resources with school context, and in the growth phases on experimenting and consolidating them into practice.

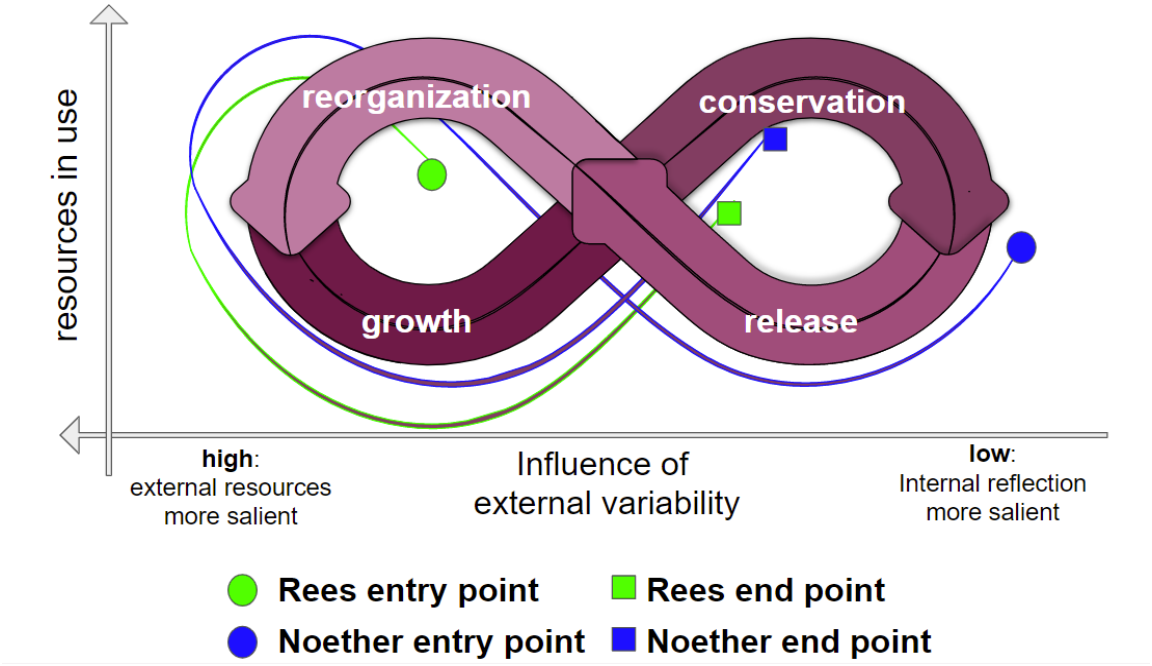

Figure 16: Revisiting Rees and Noether adaptive cycles

CHAPTER 7

Discussion

Collaborative Sensemaking from the Perspective of Teachers' Learning Ecologies

In this dissertation, I offered an ecological framing of teacher professional development that shifts away from (often tacit) linear models. I outlined and illustrated three concepts worthy of attention when considering ecological models of professional learning and practice: scope, interconnectedness, and temporality. I argue that this framing preserves the strengths of sociocultural research on teacher PD while expanding it to go beyond the immediate contexts of interaction or the institutions teachers work in. As Nasir et al. (2020) claimed, the sociocultural focus on local (immediate) interactions often obscures broader dynamics with other settings, experiences, institutions, and systems of power. While Nasir et al.'s argument reflects widespread acceptance of the importance of considering students' learning ecologies, the field has yet to engage issues of teachers' learning ecologies equally and as systematically.

To reiterate, my claim is *not* that every study of teacher learning must include all possible aspects of teachers' learning ecologies. Rather, I claim that attempts to look at teacher PD need to be understood as partial, researchers of teacher PD should be explicit about contexts they attend to and overlook, and employing ecological perspectives on PD would extend our ability to understand and thus be more responsive to teacher learning (Clarke & Hollingsworth, 2002; Cochran-Smith et al., 2014; Horn, 2005; Horn & Garner, 2022; Opfer & Pedder, 2011).

Attending to the scope, interconnectedness, and temporality of teacher learning in the analysis of VFFs helped uncover the ways teachers made sense of instructional resources and

incorporated changes into their systems of instruction. Clearly, there is more than one way to operationalize the three dimensions of this framework. The ways I operationalized them were strongly informed by analytical tools from ecological and complexity theories (i.e., micro- meso- exo- macro- levels, boundary objects, and adaptive cycles). Even if using tools from other theories, this framework can help guide researchers towards being explicit and mindful of which contexts they account for and which they ignore, how they do that, and why.

To illustrate and refine the ecological framework, I built on data from Project SIGMa and asked: *what is the role of teachers' previous professional experiences in their collaborative sensemaking?* I divided this overarching question into three smaller and more manageable sub-research questions. In Chapter 4, I asked *how do teachers invoke and use exo-resources in collaborative sensemaking?* (SRQ1). This chapter built on two case studies: one of the Rees team discussing grouping strategies and a second of the Noether team discussing a problem-based curriculum. The two case studies offered a proof-of-concept that exo-resources exist in teacher conversations and can be central to teachers' sensemaking. More specifically, these cases illustrated the transformative potential of exo-resources to support more ecological teacher reasoning.

In Chapter 5, I made the notions of exo-resources and exosystem more concrete. I looked at the two focal teams across the length of our partnership and asked, *what exo-resources are being invoked by teachers in collaborative sensemaking and where do they originate from?* (SRQ2). This analysis illustrates how common and diverse the phenomenon of referencing exo-resources was across the two teams. I found that, while the exo-resources of the two teams differed, they could be described with the same six categories: PD workshops, conferences, PD organizations, online resources, research & policy, and curricula. This framework can be used by

designers and facilitators for inviting, acknowledging, and building PD conversations on teachers' prior knowledge and experiences.

In the final empirical chapter, Chapter 6, I focused on *temporality* in teacher PD taking a qualitative perspective on time (Erickson, 2004; Gunderson & Holling, 2002). I asked *how do different levels of teachers' learning ecologies interact in different phases of their learning?* (SRQ3). Using the lens of the adaptive cycles, I described the teams' learning at the PD through the phases of *release*, *reorganization*, *growth*, and *conservation*. First, I found that the Rees team, Ezio and Veronica, entered the CSPD with a deep concern about tracking and intentions to improve specific (related) aspects of their teaching. The work with Rees focused on navigating their institutional contexts, planning and recruitment of new arrangements, and experimenting with these new professional arrangements. Making sense of a variety of exo-resources was central to these learning phases. In contrast to the Rees team, I found that the Noether team's learning spanned all four phases. Year 1 was influential in clarifying their intentions to improve aspects of their teaching, as well as their concerns about teaching and learning norms in school. Within the phases of conservation and release in Noether Year 1, video-based reflection was central. Teachers became more attuned to aspects of their instruction that fell short and to problems with their institutional norms. Only then did they shift to planning and recruitment of new arrangements, experimenting with these, and consolidating them. As with the Rees team, acknowledging and providing a variety of external resources was central to these two latter phases: reorganization and growth.

7.1 Implications for Research and Practice

Studying PD through linear pathway lenses can be useful. For researchers, it offers a way to study specific programs or refine general guidelines for effective PD (e.g., Desimone, 2009).

Other stakeholders — such as district decision makers or funding agencies — might find linear models appealing for their visible and relatively fast results. In turn, researchers are encouraged to write grant proposals that make visible the direct impact of their own PD designs. This dissertation suggested why and how researchers can resist these institutional constraints. Most importantly, attending to the interconnected nature of our PD to teachers' learning ecologies is essential in understanding how teachers make sense of PD and incorporate changes into their instruction.

The case studies in Chapter 4 illustrate the potential of building on teachers' experiences and resources in collaborative conversations to support more ecological reasoning. Chapters 5 and 6 then provided two practical tools for designers and facilitators who wish to do so. The six categories of exo-resources provided in Chapter 5 can be used in two main ways. First, they offer a framework for PD designers to inquire about the teachers' exosystem through interviews, questionnaires, or the collective PD discussions to be used strategically in the new PD endeavor. Second, the framework supports PD facilitators to develop their sensitivity and judgment in responding to resources and experiences that teachers invoke, recognizing their importance in sensemaking. The framework of temporality in Chapter 6 is another tool for developing responsiveness in PD facilitation. While both exo-resources and video-based reflection on instruction can be helpful in all phases, the findings imply that within the phases of conservation and release, video-based reflection is likely to be most influential. This could be seen in Noether Year 1 when the team reflected on videos of student collaboration from Brad's lesson. The reflection allowed for the recognition of several gaps between the teachers' pedagogical goals and their department teaching and learning norms. In contrast, acknowledging and providing a variety of exo-resources in the reorganization and growth phases is likely to be more influential

than reflection. For example, when the Rees team reorganized their classroom to support more student collaboration, engaging with a variety of practices such as random grouping, purposeful grouping, rotating groups, and re-grouping, allowed them to compare, contrast, and experiment with the new practices toward concept development.

7.2 Limitations of the Study

The unique case of our partner teachers enabled a rich investigation of teacher collaborative sensemaking in light of their learning ecologies. However, it also called for carefulness in generalizing these findings when looking at more common cases of mathematics teachers.

First, our partner teachers were part of a well-resourced organization which provided them professional development opportunities, which were unusual both in their quantity and quality. It is likely that teachers with fewer PD opportunities would invoke less exo-resources in their collaborative conversations. From a systemic perspective, this potential limitation further illustrates the necessity of providing teachers with meaningful PD experiences. From a PD facilitation perspective, even if they invoke fewer exo-resources, teachers always have previous teaching and learning experiences, and this study points to the need to incorporate these experiences in their learning.

Second, our partner teachers mostly invoked resources that are aligned with ambitious and equitable mathematics instruction. However, teachers could bring in exo-resources that clash with the ideas promoted in a PD. As discussed in Chapter 6, teachers have different entry points to PD initiatives. If the teachers' vision of good teaching stands in sharp contrast to ambitious instruction, it is not likely that simply providing teachers with resources with which to experiment (reorganization and growth phases) would be useful in promoting change. Rather, a

video-based reflection on practice would like serve as a more useful first phase in helping teachers to identify gaps or problems in their teaching environment.

Finally, I mostly discussed the implications of ecological framing on PD for educational research and PD providers. Further research would be needed to uncover its affordances for other stakeholders as well. For example, it can provide insights for partnerships that design PD as part of larger systems for teacher improvement (e.g., Cobb et al., 2018). I suggest the framework and empirical findings as a departure point for future work and acknowledge that further research to consolidate what it means to study, design, and facilitate CSPD from an ecological perspective is much needed.

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Appendices

Appendix 1: Rees Exo-Resources Across Three VFF Cycles

VFF	Resource Recruited from…	Mediating Tool of the Resource	Pedagogic Element Discussed	Illustrative Quote
Rees VFF 1 Dec 2017	North Conference Session	Probing questions	Teacher impact on students agency	Veronica: you know [name]. I went to her talk at North Conference…
	Common Core State Standards	First standard (perseverance)	Task choice	Ezio: I think the Tower of Hanoi is complex enough, at least in a middle school. I want them to learn how to persevere. It is one of the standards… I totally agree with that standard.
	RG workshop	Group roles	Using group roles to promote inclusiveness	Ezio: At RG workshop when we did this, the person holding the marker wasn't allowed to talk.
	RG workshop	Random Grouping	Grouping strategies	Ezio: At least in RG workshop, I really did not agree with the random grouping.
	PG workshop	Purposeful Grouping	Grouping strategies	Ezio: We got PG workshop a couple years ago and at least what they said made sense…
	North Conference Session	Concrete representations of fractions	Task choice and enactment	Ezio: It was so awesome …. She folded it in half…

				I wanted to tie that with the Tower of Hanoi
	Research team (Patty)	IMP curriculum	Task choice and enactment: use more simple problem	Patty: the Tower of Hanoi is in the IMP curriculum and it's in the unit for seniors
	Twitter	Foxes and sheeps task	Task choice and enactment: circulate back to unsolved problems	Veronica: I was thinking of foxes and sheep…
Rees VFF 2 Feb 2018	PDO	TRU Framework	Equitable access to content	Veronica: The equitable access to content… Is this fair for everybody, and are you held accountable for being in the room essentially
	Presumably RG workshop	Random Grouping	Grouping strategies	Veronica: Mostly social interaction. And so like—That's like a pitfall, one, of random grouping
	Unspecified	Social Justice Circles	A student status	Veronica: We do social justice circles. When it gets to B, he always says a joke.
	Previous Rees VFF	Ezio's classroom video from VFF 1/3	Teacher questions to the groups	Patty: It reminds me of Ezio's class in that you're asking, they're probing, they're thinking, but they're not sure where they need to go necessarily.

	SIGMa Team Research	Monitoring Traces (Ehrenfeld & Horn, 2020)	Teacher walking patterns and time allocation across groups	Lani: So these are the seven groups. This is time. You can see the circulation pattern right.
pre-debrief (informal conversation at MfA LA lunch)	Gutiérrez (2016) by research team (Patty)	Creative Insubordination	Navigating institutional structures	From Patty's field notes: I shared with him briefly Rochelle's argument of why teachers need political knowledge whose video I had just watched that morning. He seemed interested and so I agreed to send him Rochelle's piece on Creative Insubordination
Rees VFF 3 May 2018	Online	[URL].org	Coordinating groupwork and whole class discussion	Patty: I was just looking inside [URL].org where they have these public lessons, right?
	PDO coach	Rotating Groups	Enhance cognitive demand during groupwork	Veronica: That was a nuance we added in… actually based on A's recommendation… Have them reflect one more layer.
	Online	[URL].net (random grouping virtual tool)	Grouping strategies	Ezio: That might have just been a coincidence. Patty: That might have just been a coincidence because you did the [URL].net.

Previous Rees VFF	Veronica's classroom video from VFF 2/3	Coordinating groupwork and whole class discussion	Veronica: And so, for me, when I watched the video, I was like, "I should have stopped the class."
Research team (Patty)	IMP Curriculum	Coordinating groupwork and whole class discussion	what I'm hearing Veronica say in that what I've read, I actually had done when I did IMP, right? Is when you have something so filled like this…
Research team (Patty)	Inside Mathematics	Coordinating groupwork and whole class discussion	Patty: I just wanted to ... I don't know if it's gonna connect, but I did come across this really interesting ... One of the public lessons on inside mathematics.

Appendix 2: Noether Exo-Resources Across Six VFF Cycles

VFF	Resource Recruited from…	Mediating Tool of the Resource	Pedagogic Element Discussed	Illustrative Quote
Noether VFF 1 Dec 2017	District	District teaching and learning framework	Students reasoning and teacher questioning	Brad: from the teaching and learning framework… quality and purpose of questioning and discussion techniques
	State	Common Core State Standards	Students mathematical argumentation	Brad: having them explain reasoning and justify all the things that Common Core wants the students to be able to do
	Marisa's previous school	Problem-based curriculum	Designing for student collaboration	Marisa: One of the things I like about the problem-based curriculum is that… the only person who is allowed to question the teacher is the resource manager
	South Conference	Probing questions	Dropping a question on them and walking away	Brad: I went to a session in South conference that was about that, was about providing students with feedback…
Noether VFF 2 Feb 2018	Online	Desmos	How to teach and represent imaginary zeros	Brad: and if you would show a graph of of like it on desmos; if they would see that, like "Oh okay. It's not crossing the x-axis, but still something's going on
	Online	YouTube video about	How to teach and represent imaginary zeros	Greg: Abigail brought up something on YouTube where it kind of showed

		imaginary zeros		them, well if you put it in a different dimension
	NCTM Conference	Desmos classroom	Teacher Professional Development	Lani: Who's applying to be a desmos fellow? …I went to a session at NCTM on desmos classroom. Have any of you that before?
	Springboard	Textbook	If and how to teach synthetic division	Abigail: and then looking at the springboard textbook that we're supposed to be teaching out of, and they had it still.
	Common Core State Standards	polynomial standards	How to teach and represent imaginary zeros	Lani: It's hard while we're looking at the Common Core standards to see what the polynomial standards were. I have them. None of this stuff is on there.
	PDO	Head of PDO	Math content	Abigail: Is it meaningful in terms of the graph at all? Lani: No, we asked head of PDO. [laughter]
	PDO	National Boards	Logistics but also identity	Brad: Do you know when in May? Lani: Around PD time… Brad: National boards are due right around then
Noether VFF 3 May 2018	PDO	National Boards	Describing the focal class	Brad: they are actually like the most fun class because there's the most collaborative... They're actually what I'm focusing my national boards on

	Marisa's previous school	Problem-based curriculum	Designing for student collaboration	Marisa: so the problem-based curriculum… because from day one they are taught that you are the source of the knowledge… it's coming from the student as opposed to the teacher.
	Research team (Nadav)	5 Practices	Groupwork Monitoring	Nadav: like the five practices, like don't talk with them… monitor and write yourself where they are so you can segment.
Noether VFF 4 Dec 2018	Michael Cera's Patty Paper Geometry	Patty paper activity	Adapting the task so students construct the math	Marisa: we had revised the activity. The original activity that's taken from Michael Cera's Patty Paper Geometry, it didn't have them coming up with the angle relationship.
	Marisa's previous school	Problem-based curriculum	Designing for student collaboration	Marisa: I feel like it was different when I had problem-based curriculum and I was like, I set up the group norms from day one and we were working in groups everyday the whole year
	PDO Geometry working group	The MIRA mirror	Tools for student geometry exploration	Nadav: But then also Brad and I in the geometry work group we learned also… and some other tools like the MIRA.
	Research team (Lani)	Discovering Geometry	Tools for student geometry exploration	Lani: Do you think the kids got that they were doing an investigation? …I mean I taught through I think two

				different versions of Discovering Geometry… and the first original version that I got trained to teach out of was like super open ended and I loved it
Noether VFF 5 Mar 2019		Abigail's powerpoint	Lesson plan	Greg: I'm using Abigail's PowerPoint, so everything good about the lesson will be due to Abigail. Anything bad about the lesson is because of me, and maybe Coding PD, the curriculum.
	Coding PD	Coding curriculum	Lesson plan	
	Coding PD	driver and navigator	Student collaborative programming	Greg: Yeah every four minutes. That was from Coding PD, but we did it when I went to the PD they said okay this is pair programming.
	Coding PD lab guide		Eliciting student thinking	Joseph: I think we rely on the lab guide… I'm learning more and more that with these activities where they can just try things and see if they got it wrong or right, they don't, sometimes they don't really need to understand what they're doing.
Noether VFF 6 May 2019	South Conference	Stats People	Unit design	Brad: me and Marisa made a decision that we were going to do a five-week statistics unit at the end of geometry. This was inspired by actually a session I went to

				two years ago at South Conference…
	Online	Stats4People .com	Instructional resources	Brad: Stats people, if you go to the Stats4People.com, or the Stats Medic, singular, .com they have the most resources just open resource of everything they've ever done
	Springboard	Textbook	Unit design	Joseph: This is all ... this five week unit is not part of Springboard, right?
	Marisa's previous school	Problem-based curriculum	Designing for student collaboration	Nadav: … [In] the previous onversations like Marisa was telling us about the when she was teaching the problem-based curriculum, how students from day one were training to see each other as a resource…

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