

**EXTENSIVE AND INTENSIVE INVESTMENT OVER THE BUSINESS CYCLE**

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**Working Paper No. 09-W12**

September 2009

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# Extensive and Intensive Investment over the Business Cycle\*

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May 4, 2009

## Abstract

Investment of U.S. firms responds asymmetrically to Tobin's  $Q$ : Investment of established firms – ‘intensive’ investment – reacts negatively to  $Q$  whereas investment of new firms – ‘extensive’ investment – responds positively and elastically to  $Q$ . This asymmetry, we argue, reflects a difference between established and new firms in the cost of adopting new technologies. A fall in the compatibility of new capital with old capital raises measured  $Q$  and reduces the incentive of established firms to invest. New firms do not face such compatibility costs and step up their investment in response to the rise in  $Q$ . A composite-capital version of the model fits the data well using aggregates since 1900 and our new database of firm-level  $Q$ s that extend back to 1920.

## 1 Introduction

The distinction between the extensive and intensive margin in labor supply has been modeled at the individual-worker level (Cogan 1981) and at the aggregate level (Kydland and Prescott 1991) to understand why people move in and out of the labor market more wage-elastically than employed workers adjust their hours. A similar pattern seems to exist for investment as well in that fluctuations in aggregate Tobin's  $Q$  have significantly larger effects on the entry of firms and their investment than they do on the investment of incumbent firms.

The investment of listed firms in the United States is unrelated or even negatively related to aggregate  $Q$ . The negative relation between this ‘intensive’ investment and  $Q$  appears in investment data at both the aggregate and firm levels. In contrast, the

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\*We thank G. Alessandria, W. Brainard, R. Lucas, G. Violante and L. Wong for comments, R. Tamura for data, S. Bigio, L. Han, T. Islam, M. Jaremski, J. H. Kim, K. Livingston, H. Tretvoll and V. Tsyrennikov for research assistance, and the Ewing Marion Kauffman Foundation and the NSF for financial assistance.

investment of young and unlisted firms – ‘extensive’ investment – responds positively and highly elastically to  $Q$ . Inflows of venture capital in particular are highly elastic with respect to  $Q$ , as are more noisy measures of new firms’ investment such as IPOs.

In this paper we argue that the asymmetry in investment behavior originates in the cost of making new capital compatible with capital already in place. When new technology is embodied in capital, established firms will not invest in it as much as new firms will.<sup>1</sup> If the arrival of new technology precedes high- $Q$  periods, then a high  $Q$  is the result of high costs of adjustment that reflect this incompatibility.

Our model is most closely related to Kydland and Prescott (1991). They assume that people are heterogeneous in the costs of moving in and out of the labor market. We assume that new firms vary in how efficiently they can create capital. Both models give rise to aggregate adjustment costs. One difference is that in our model the aggregate cost of adjustment is not symmetric—its convexity as induced by the distribution of individual agents’ costs appears in the movement of new firms’ capital *into* the market, but not in its withdrawal. More generally, the model relates to others like Cho and Rogerson (1998) and Cho and Cooley (1993) that incorporate both the extensive and intensive margins into labor supply. But there are important other differences: Inelastic labor of incumbent workers in all of these models stems from curvature in the utility of leisure, with each additional unit forgone being more and more valuable. The analog of such a curvature in the utility function would be curvature in the incumbent firm’s production function sufficient to produce the asymmetric response.<sup>2</sup> But this would conflict with the near Gibrat Law that one sees in the data and with the large cross-sectional variance in the size distribution of firms seen in most industries. We therefore take a different approach, namely, to assume that there are hidden costs of implementing new technology that only incumbents face because they must make their old capital compatible with the new.

Compatibility costs are thus central to our model; Yorukoglu (1998) models their impact across steady states and finds that a fall in the compatibility of new and old capital raises the capital stock in young plants, lowers it in older ones, and shortens plant life. Our model extends this idea to time series as we fit it to 105 years of U.S. data. As in Greenwood, Hercowitz, and Krusell (1997) and Fisher (2006), we have two aggregate shocks: A TFP shock and a shock to the cost of capital, except that the latter applies only to incumbents. We then add idiosyncratic shocks to the generation of new ideas where the number of new ideas is proportional to the stock of human capital as in Chatterjee and Rossi-Hansberg (2008). Each period there is a quality distribution among the potential new projects, some of which are developed. The

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<sup>1</sup>This is because new firms rely less on traditional inputs and more on new ideas, which is consistent with Prusa and Schmitz (1991, 1994) who concluded that firms pursue less radical forms of innovation as they age.

<sup>2</sup>An extreme solution would be that of Campbell (1998) and Bilbiie *et al.* (2007) who assume that the firm’s size is fixed at its entering size. Bilbiie *et al.* in Sec. 5 briefly extend to allow variable plant size but do not study the issue of differential elasticity of extensive and intensive investment.

development of new projects has the interpretation of spin-outs. As do Franco and Filson (2006) and (implicitly) Prescott and Boyd (1987), we argue that incumbent firms are able to collect the rents of the spun-out projects through lower wages that their workers will accept, and that the equilibrium is efficient. This means that the equilibrium solves the planner’s optimum, and this is how we shall simulate it.

**Evidence on investment and  $Q$ .**—The patterns that we outline above can be seen in both micro and macro data. Here we summarize that evidence, first on the micro and then on the macro side.

*Compustat data.*—Standard and Poor’s *Compustat* database consists of public firms that we think of as incumbents. The investment rate of these listed firms depends positively on their own  $Q$ , but negatively on aggregate  $Q$ . This is seen in Figure 1 where the vertical axes measure the log of a firm’s physical investments relative to its capital stock.<sup>3</sup> The left panel shows a mildly positive relation between the investment rates of Compustat firms and their firm-specific  $Q$ ’s.<sup>4</sup> The right panel, on the other hand, shows a negative relation between the same Compustat firms’ investment rates and aggregate  $Q$ , measured as the annual averages of the firm-specific  $Q$ ’s in our sample. The horizontal axis in the right panel orders the observations by aggregate  $Q$ .

The same contrast emerges in another regression that uses the Compustat data displayed in Figure 1. Firm  $i$ ’s investment as a percentage of its capital stock at date  $t$  is denoted by  $(I/K)_{i,t}$ , aggregate  $Q$  is denoted by  $Q_t$  and firm  $i$ ’s deviation from it by  $Q_{i,t} - Q_t$ . The estimates and the t-ratios are

$$\ln \left( \frac{I}{K} \right)_{i,t} = 2.752 - 0.082Q_t + 0.015(Q_{i,t} - Q_t),$$

(230.9)      (-19.3)      (2.5)

with  $R^2=.008$  and 160,580 firm-level observations.

*Venture capital flows into young firms.*—The only systematic source of micro evidence on what we would regard as entering firms is Thompson’s *VentureXpert* sample of venture-backed firms. Venture capitalists (VCs) invest almost exclusively

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<sup>3</sup>The investment rate is measured as a firm’s annual expenditures on property, plant, and equipment (Compustat item 30) as a percentage of its total assets at the start of the year (item 6).

<sup>4</sup>We compute firm-specific  $Q$ ’s from 1962 to 2005 using year-end data from Compustat. We measure the numerator of  $Q$  as the value of a firm’s common equity at current share prices (the product of Compustat items 24 and 25), to which we add the book values of preferred stock (item 130) and long- and short-term debts (items 9 and 34). We use book values of preferred stock and debt in the numerator because prices of preferred stocks are not available on Compustat and we do not have information on issue dates for debt from which we might better estimate market value. We note that book values of these components are reasonable approximations of market values so long as interest rates do not vary excessively. We compute the denominator of  $Q$  in the same way except that book value of common equity (Compustat item 60) is used rather than its market value. Our micro-based measures of  $Q$  therefore focus primarily on the value of a firm’s outstanding securities and implicitly assume that the proceeds from these issues are fully applied to the formation of capital, both physical and intangible.

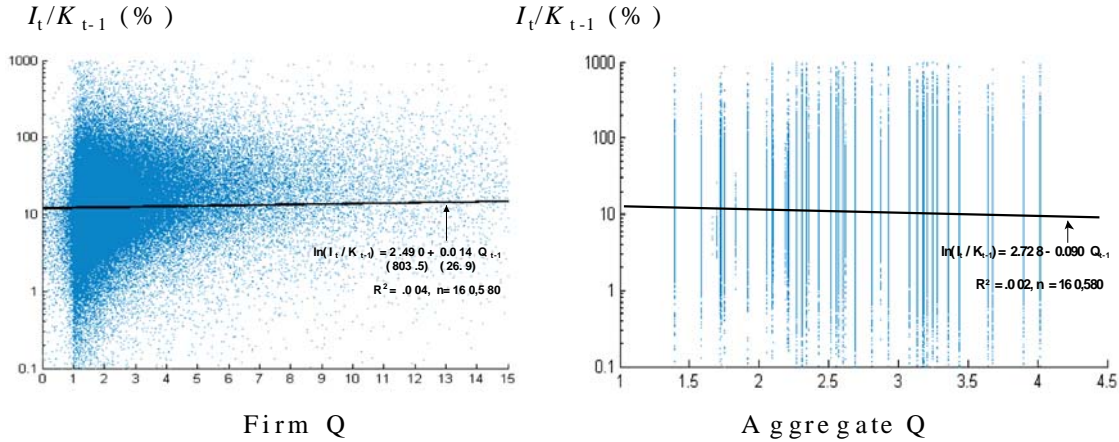


Figure 1: Firm -level investment rates and  $Q$ , Compustat sample, 1962-2006.

in young start-up firms. A young company receives funds from VCs in ‘rounds’ that typically are spaced 6 months to a year apart, and this goes on for several years until the company either succeeds (has an IPO or is acquired privately) or is ‘terminated’ by its controlling interests (typically the VCs). We pool all venture investment rounds made in a year, thus forming the series  $V$  that is then normalized by the economy-wide physical capital stock  $K$  and plotted in left panel Figure 2.<sup>5</sup> A company typically does not obtain fresh funds in an investment round until it has spent the funds that it received in the previous round, and therefore the money that these firms actually spend should track fairly closely (with perhaps a 3- to 6-month lead) to the series measuring the investment rounds.<sup>6</sup> Venture investment did not get off the ground until the 1980s. Figure 2 shows that since then VC investment has responded elastically to  $Q$ , quite in contrast to aggregate investment, which bears little relation to  $Q$ . The correlations with  $Q$  are 0.844 and 0.186, respectively.<sup>7</sup>

<sup>5</sup>Data on venture capital investment shown in the left panel of Figure 2 are from the “Venture Xpert Database” of Thompson Venture Economics, Inc., and represent flows over each calendar year from 1978 through 2005.  $K_t$  is measured as the year-end stock of private fixed assets from the detailed tables of the Bureau of Economic Analysis (BEA, 2006, Table 6.1, line 1).  $I_t$  is the annual flow of gross private fixed investment from the National Income and Product Accounts. For Tobin’s  $Q$ , we use fourth quarter observations underlying Hall (2001) for 1978-1999, and join them with estimates underlying Abel and Eberly (2008) for 1999 to 2005. These authors derive aggregate  $Q$  from the Federal Reserve Board’s *Flow of Funds* Accounts as the ratio of market-to-book value for tangible assets in the entire non-financial corporate sector.

<sup>6</sup>Even within a given round there are different types of investment ranging from seed financing all the way to buyouts and acquisitions. For the most part, however, the money goes to fund the activities of firms less than 10 years old (since founding). Occasionally VC funds are involved in taking mature companies public, but this is rare and the investment involved is small. Thus, while our series for VC-investment excludes new firms that are not backed by venture capital, it does exclude anything that one might call incumbent investment.

<sup>7</sup>With unlisted firms one cannot compute a firm-specific  $Q$  but can compute the  $Q$  that prevails for listed firms in particular sectors. Gompers *et al.* (2008) do this and show that VC investment

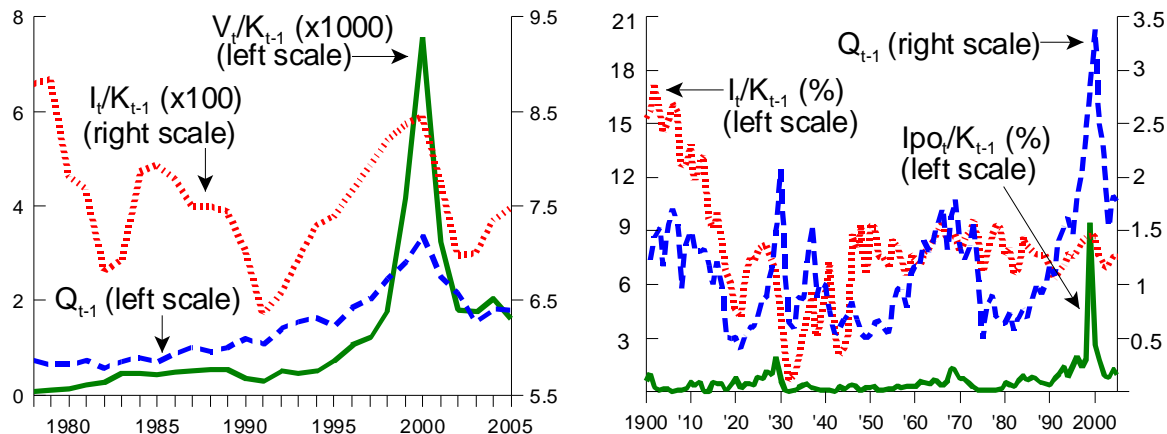


Figure 2: Venture investment, aggregate investment, and  $Q$ , 1978-2005.

*IPO volume.*—While VC investment leads investment spending of startups and young firms by 3-6 months, IPOs lag investment these days by several years and in the middle of the century by more than that – see Figure 1 of Jovanovic and Rousseau (2001a). While they are imperfect measure of investment by new firms, however, they represent the only century-long time series available.<sup>8</sup> A firm’s IPO value reflects the stock of assets that were put in place via the sequence of its past investments. We know, however, that the date of a firm’s IPO also coincides with a surge of investment, with investment on average rising by a factor of 1.4 around the time ( $\pm 2$  years) of an IPO (Chemmanur *et al.*, 2005, Figure 3), which would be expected since one motive for an IPO is to finance expansion. Another virtue of IPOs is that they track the value of assets financed from all sources, not just VCs. IPOs are shown along with  $Q$  and aggregate investment in the right panel of Figure 2.<sup>9</sup> Over the past 115 years,

in a portfolio firm responds positively and significantly to the listed firms’  $Q$ s in that firm’s sector.

<sup>8</sup>Data on incorporations stretch back even further but are dominated by businesses such as cleaners, corner delis, etc., and therefore do not have much to do with our model.

<sup>9</sup>In the right panel of Figure 2,  $K_t$  is the end-of-year stock of private fixed assets from the BEA (2006, Table 6.1, line 1) for 1925 through 2005. For 1900-1924, we use annual estimates from Goldsmith (1955, Vol. 3, Table W-1, col. 2, pp. 14-15) that include reproducible, tangible assets (i.e., structures, equipment, and inventories), and then subtract government structures (col. 3), public inventories (col. 17), and monetary gold and silver (col. 18). We then join the result with the BEA series.  $I_t$  for 1929-2005 is gross private domestic investment from the BEA (2006, Table 5.2.5, line 4), to which we join estimates from Kendrick (1961, Table A-IIb, column 5, pp. 296-7) for 1900-1928.  $IPO_t$  is measured as the aggregate year-end market value of the common stock of all firms that enter the University of Chicago’s Center for Research in Securities Prices (CRSP) files in each year from 1925 through 2005, excluding American Depository Receipts. The CRSP files include listings only from the New York Stock Exchange (NYSE) from 1925 until 1961, with American Stock Exchange and NASDAQ firms joining in 1962 and 1972 respectively. This generates large entry rates in 1962 and 1972 that for the most part do not reflect initial public offerings. Because of this, we linearly interpolate between entry rates in 1961 and 1963 and between 1971 and 1973, and assign these values to the years 1962 and 1972 respectively. For 1900-1924 we obtain market values of firms

the correlation coefficient between  $Q$  and IPO value is 0.574, while the correlation between  $Q$  and the rate of standard investment is 0.305. Restricting the time period to 1954-2005, the respective correlations are even further apart: 0.635 and 0.246.

## 2 Model

Our model allows investment to take place in new projects and in continuing projects. Continuing projects will all offer the same rate of return whereas new projects will be heterogeneous.

*Continuing projects.*—Continuing projects can be enlarged at the uniform rate of return  $1/q$ . For  $q$  units of the consumption good, a unit of new capital can be created via existing projects. Then if the capital created via existing projects is  $X$ , the total cost is  $qX$ .

*New projects.*—Many new potential projects are born each period. A project lasts one period. It requires  $\lambda$  units of the numeraire good as input, and as output it delivers  $\varepsilon\lambda$  units of capital in the following period. Projects vary in quality; new potential projects are born each period, and their quality is distributed with C.D.F.  $G(\varepsilon)$ . In an economy with capital stock  $k$ , the unnormalized distribution of new ideas is  $kG(\varepsilon)$ .<sup>10</sup> Each period, an available project must be implemented at once or not at all. Let  $\varepsilon_m$  be the marginal project implemented. The size of the average project is  $\lambda$ . Capital created by the new projects is

$$Y = \lambda k \int_{\varepsilon_m}^{\infty} \varepsilon dG(\varepsilon), \quad (1)$$

at a cost of

$$\begin{aligned} \text{TC} &= \lambda k [1 - G(\varepsilon_m)] \\ &\equiv h \left( \frac{Y}{k} \right) k. \end{aligned} \quad (2)$$

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that list for the first time on the NYSE using our pre-CRSP database of stock prices, par values, and book capitalizations (see Jovanovic and Rousseau 2001b, footnote 1, p. 1). For  $Q$ , we use the series derived from Hall (2001) and Abel and Eberly (2008) described in footnote 5 for 1950-2005, which we then take back to 1900 by ratio splicing the “equity  $Q$ ” measure underlying Wright (2004). Note that Hall’s measure of  $Q$  exceeds Wright’s by factor of more than 1.5 in 1950, when the splice occurs, producing  $Q$ ’s before 1950 that are considerably higher than Wright’s original estimates.

<sup>10</sup>Formally, the distribution  $G(\varepsilon)$  is closely related to Kydland and Prescott (1991, p. 67) where they model costs of moving an individual between the market and the household sector. In their model, however, the cost pertained to movement in either direction, whereas here they pertain only to moving capital into the market sector. Retiring capital is costless, it simply can be consumed.

These projects are ‘investment options’ but, in contrast to how they were modeled by Jovanovic (forthcoming, where, as here, the supply of options was also proportional to  $k$ ), they are heterogeneous in quality, but not storable.

The second definition in (2) is valid because by (1),  $\varepsilon_m$  depends only on the ratio  $Y/k$ . Therefore total cost is homogenous of degree 1 in  $Y$  and  $k$ , and we shall denote it by  $h\left(\frac{Y}{k}\right)k$ .

*Aggregate resource constraint.*—Aggregate output is  $zk$ . There is one capital good,  $k$ , but two ways to augment it: via continuing projects that deliver  $X$ , and via new projects that deliver  $Y$ . Thus  $k$  evolves as

$$k' = (1 - \delta)k + X + Y. \quad (3)$$

The resource constraint expresses aggregate consumption as

$$C = zk - qX - h\left(\frac{Y}{k}\right)k. \quad (4)$$

Since the form of  $h$  is fully determined by the constant  $\lambda$  and the distribution  $G$ , this is a consistency condition on the macro and the micro data. Note the asymmetric effect of  $q$  on investment costs: New projects do not require that new capital combine with old capital, and thus escape the compatibility costs that cause  $q$  to fluctuate.<sup>11</sup>

*One-sector representation.*—The model has a standard one-sector representation. Let  $I = X + Y$  and let  $y = Y/k$ ,  $x = X/k$ ,  $c = C/k$ , and  $i = x + y$ . Then

$$\gamma(i, q) \equiv \min_{0 \leq y \leq i} \{q(i - y) + h(y)\}$$

with the FOC

$$h'(y) = q \quad (5)$$

as illustrated in Figure 3. Then (3) and (4) become

$$\frac{k'}{k} = 1 - \delta + i \quad \text{and} \quad c = z - \gamma(i, q).$$

By the envelope theorem

$$\frac{\partial \gamma}{\partial q} = x \quad (6)$$

and

$$\frac{\partial \gamma}{\partial i} = q \quad \text{if } x > 0. \quad (7)$$

*The determination of investment.*—Figure 3 shows the supply of savings curve in blue. As we shall show, this curve is negatively sloped because, as seen from (4), a rise in  $q$  raises the cost of future consumption in terms of current consumption

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<sup>11</sup>This compatibility argument applies to physical capital and organization capital alike. Fitting new wiring or new equipment into an old building originally designed for something else is costly. Retraining workers who originally were trained to do something else is also costly.



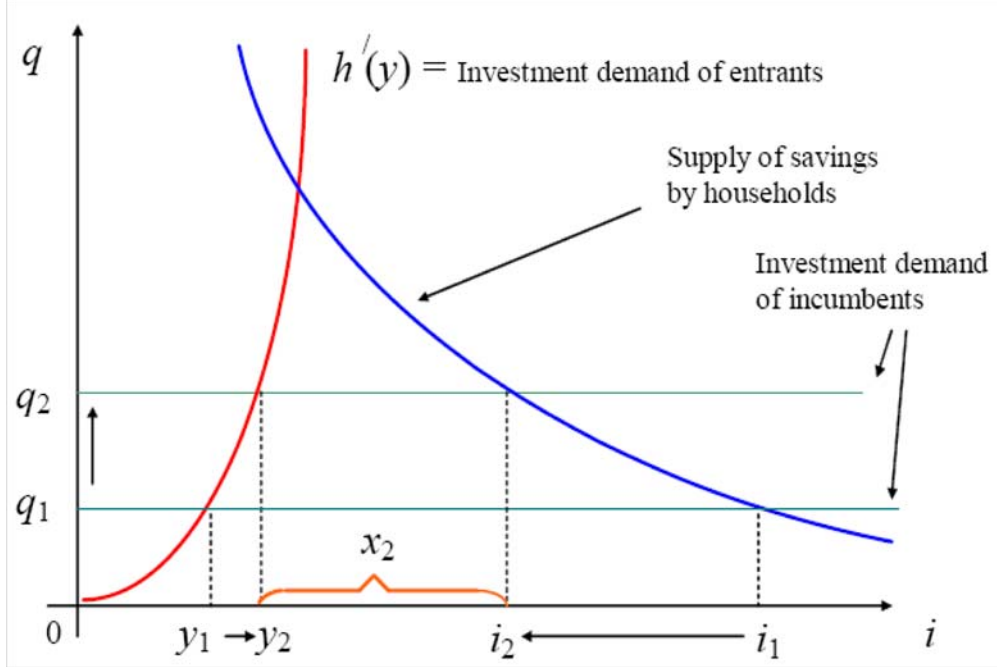


Figure 3: THE DETERMINATION OF  $y$  VIA (5).

thereby reducing the demand for future consumption and reducing investment. The investment rate of entering capital is fully determined by and increasing in  $q$ , as also shown. Incumbent investment will take up the slack between desired total investment and the investment of entrants. There will be a second shock,  $z$ , to the supply of savings and, hence, the residual incumbent investment will depend on both  $q$  and  $z$ .

Figure 3 illustrates the effect of a rise in  $q$  when  $z$  is held constant. The ‘interiority’ requirement that  $x > 0$  implies that we can determine  $y$  from the intersection of the entrants’ investment-demand curve  $h'(y)$  with  $q$ . As  $q$  rises while  $z$  stays fixed, two things happen: First, savings declines and with it total investment  $i$  must fall from  $i_1$  to  $i_2$ . Second, the supply of entrants rises from  $y_1$  to  $y_2$ , thereby crowding out even more incumbent investment.<sup>12</sup>

A rise in  $z$ , on the other hand, would shift the downward-sloping savings curve in Figure 3 to the right. This shift would cause both  $x$  and  $y$  to rise.

<sup>12</sup>These implications remain valid under a more general specification: Rewrite (4) as  $c = z - qx - h(y)$ . Thus  $q$  raises the costs of incumbents but not that of entrants. Suppose, instead, that the resource constraint were  $c = z - qx - q^\psi h(y)$  with  $\psi \neq 0$ . Then instead of (5) the FOC would be  $h'(y) = q^{1-\psi}$  and the conclusions illustrated in Figure 3 would remain qualitatively intact so long as  $\psi < 1$ . That is,  $y$  would be increasing in  $q$  and  $x$  would be decreasing in  $q$ . If, however,  $\psi = 1$ ,  $y$  would be a constant, independent of both  $q$  and  $z$ , and would solve the equation  $h'(y) = 1$ . But  $x$  would still be decreasing in  $q$ .

In terms of the micro interpretation of equations 1 and 2, they read

$$y = \lambda \int_{\varepsilon_m}^{\infty} \varepsilon dG(\varepsilon) \quad \text{and} \quad h(y) = \lambda [1 - G(\varepsilon_m)]. \quad (8)$$

Differentiating and simplifying leads to

$$h'(y) = \frac{1}{\varepsilon_m} \quad \text{and} \quad h''(y) = \frac{1}{\varepsilon_m^3} \frac{1}{g(\varepsilon_m)} > 0.$$

Since  $h$  is convex, (5) then implies that

$$\varepsilon_m = \frac{1}{q}. \quad (9)$$

Thus as we raise  $q$  in Figure 3, we draw in more new projects of ever lower quality.

## 2.1 The planner's problem

The economy has no external effects or monopoly power and equilibrium can be represented by a planner's problem. Preferences are  $E_0 \left\{ \sum_0^{\infty} \beta^t U(C_t) \right\}$ . Let  $s \equiv (q, z)$  be stochastic with transition function  $F(s', s)$ . If technological progress raises compatibility costs,  $z$  and  $q$  will be positively correlated. We shall therefore allow for such a correlation.

The state of the economy is  $(s, k)$ , but since returns are constant and preferences homothetic,  $k$  will not affect prices or investment rates. The planner's problem is to maximize the representative agent's expected utility by choosing the two kinds of investments  $X$  and  $Y$ . The planner has no other technology. Let preferences be

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

in which case the planner's value function must satisfy

$$V(s, k) = v(s) k^{1-\sigma}, \quad (10)$$

where

$$v(s) = \max_{i \geq 0} \left\{ \frac{(z - \gamma(i, q))^{1-\sigma}}{1-\sigma} + (1 - \delta + i)^{1-\sigma} v^*(s) \right\}, \quad (11)$$

and

$$v^*(s) = \beta \int v(s') dF(s', s).$$

But if  $z$  and  $q$  are positively persistent and mutually independent,  $v$  and  $v^*$  are both strictly increasing in  $z$  and strictly decreasing in  $q$ .

The FOC with respect to  $y$  is (6) and, in light of (7), the FOC with respect to  $i$  is

$$q(z - \gamma)^{-\sigma} = (1 - \delta + i)^{-\sigma} v^*(s). \quad (12)$$

To ensure that both FOCs hold with equality, we assume that  $h(0) = 0$ , which rules out the value  $y = 0$ , and that  $h'(y) > q_{\max}$  at a value of  $y$  too low to satisfy the demand for saving in any state  $s$ , which rules out  $x = 0$ .

We may rearrange (12) to obtain

$$i = \delta - 1 + (z - \gamma) \left( \frac{v^*}{q} \right)^{1/\sigma} \quad (13)$$

and we at once have the following properties of  $x$ ,  $y$ , and  $i$ :

1.  $y$  is increasing in  $q$  and independent of  $z$ , these follow from (5); and
2.  $i$  and  $x$  are decreasing in  $q$  and increasing in  $z$ . The claims about  $i$  follow from (6) and (13), and the claims about  $x$  then follow from property 1.

## 2.2 Decentralization

Although established firms find it harder than new firms to adopt technology, they do build new plants and start new ventures. Interestingly, however, when they do so, they establish semi-independent divisions. Gompers (2002, esp. table 6) and Dushnitsky and Shapira (2006) survey corporate VC activity. Nevertheless, implementing new ideas often requires that an employee leave the firm – a spinout. The microfoundations of spin-outs involve asymmetric information. Klepper and Thompson (2006) argue that spin-outs occur because people disagree over management decisions – people leave firms to develop their ideas on their own because coworkers would otherwise implement them suboptimally. Chatterjee and Rossi-Hansberg (2008) argue that the best ideas leave the firm. Finally, Franco and Filson (2006) argue that a firm’s span-of-control limit pushes some of its employees out to start their own firms using training they received on the job. The models differ, but they all treat entry essentially as a withdrawal of capital from existing firms, and that is also how we shall treat it.

We follow Prescott and Boyd (1987) and Franco and Filson (2006) and assume that workers accept lower pay in return for what they learn. Then  $kG(\varepsilon)$  is the distribution of idea qualities that a firm’s workers generate. Since the firm captures the rents via the lower wages that it pays, this gives it the correct accumulation incentives.  $X$  is the new capital that will stay in the firm and  $q$  is its unit cost common to all incumbents, while  $Y$  is developed by workers who leave, with  $h'(y)$  the marginal cost of doing new things and with new ideas varying in quality.<sup>13</sup> The

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<sup>13</sup>New opportunities for incumbents appear to be homogeneous relative to those of new firms: Dahlin *et al.* (2004) find that the variance in patent quality is higher among patents granted to independent inventors than it is among patents granted to corporations, and that inventions in the right tail of the quality distribution are more likely to originate with independent inventors.

best ideas are exploited first, hence  $h'$  is upward sloping. We now formalize this decentralization.

*A firm's decision problem.*—The firm maximizes its value. Since its  $k$  is predetermined, this amounts to maximizing the unit value of  $k$  which we define as  $w$ . It takes as given the next-period value of its capital, its ex-dividend unit value today being  $q$ . The firm's maximization problem is then

$$w(s) = \max_{i \geq 0} \{z - \gamma(i, q) + (1 - \delta + i)q\}. \quad (14)$$

This means that the firm's  $y$  must solve (5), but  $x$  must be obtained from the household savings decision and the identity that savings = investment =  $x + y$ . The linear technology for creating  $k$  yields no rents because the market price of capital adjusts to equal the average and marginal cost of creating it,  $q$ . In other words,  $x$  generates no rents. Since  $i = x + y$ , (14) reduces to

$$w(s) = z + q(1 - \delta) + \max_y \{qy - h(y)\}. \quad (15)$$

More explicitly, each unit of  $k$  entitles the firm to some new projects each period, and the firm's employees will spin out all those for which  $\varepsilon \geq \varepsilon_m = 1/q$  as derived in (9).

*The household's budget constraint.*—For a household, the only asset is shares of firms as in Lucas (1978). Firms distribute their profits to households, as well as the revenue from shares issued net of investment costs. Let the dividend be  $D(q, z)$  per share. The household's constraint is

$$qn' + c = (D(q, z) + q)n,$$

where  $n$  is the number of units of capital of the representative firm that the household owns and where  $c = C/k$ . It is helpful to write  $c$  as

$$c = (D - q\Delta)n,$$

where

$$\Delta = \frac{n' - n}{n}$$

is the household's rate of investment in the ownership of capital.

*The household's Bellman equation.*—The household's state is  $(n, s)$ . Exactly parallel to the planner's case, the household's lifetime value can be written as  $\psi(s)n^{1-\sigma}$ , where  $\psi(s)$  solves the Bellman equation

$$\psi(s) = \max_{\Delta} \left\{ \frac{(D(q, z) - q\Delta)^{1-\sigma}}{1 - \sigma} + (1 + \Delta)^{1-\sigma} \psi^*(s) \right\}, \quad (16)$$

and where

$$\psi^*(s) = \beta \int \psi(s') dF(s', s).$$

Assuming differentiability of  $v$  and using the envelope theorem, the FOC is

$$q(D + q\Delta)^{-\sigma} = (1 + \Delta)^{-\sigma} \psi^*(s). \quad (17)$$

*Equilibrium.*—We state the equilibrium conditions per unit of capital. We start with the household owning all the capital, i.e.,  $n = k$ . The rate at which the household grows its ownership of shares should equal the rate at which the firm grows its capital stock. Firms finance capital expansion by issuing  $i - \delta$  new shares which the public must choose to buy:

$$\Delta = i - \delta.$$

Firms' dividends equal their net revenue from all sources, which is profits plus the revenue from newly issued shares:

$$D = z - \gamma(i, q) + q(i - \delta).$$

If these two conditions are met, consumption will equal output net of investment

$$D + q\Delta = z - \gamma.$$

*Equilibrium and planner's optimum coincide.*—Substituting this information into (17) leads to

$$q(z - \gamma)^{-\sigma} = (1 - \delta + i)^{-\sigma} \psi^*(s). \quad (18)$$

From (12) we see that equilibrium and the optimum coincide if and only if  $v^*(s) = \psi^*(s)$  for all  $s$ . Now, if we substitute into (16) from the equilibrium conditions for  $D$  and  $\Delta$  we obtain

$$\psi(s) = \frac{(z - \gamma(i, q))^{1-\sigma}}{1 - \sigma} + (1 - \delta + i)^{1-\sigma} \beta \int \psi(s') dF(s', s),$$

which is the same as the expression that  $v(s)$  solves, from which we conclude that  $\psi(s) = v(s)$  for all  $s$ , which means that  $\psi^*(s) = v^*(s)$  for all  $s$ .

### 3 Estimating $G(\varepsilon)$

A cornerstone of the model as shown in equations (1) and (2) is the distribution  $G(\varepsilon)$  of new ideas. In this section we estimate  $G(\varepsilon)$  using micro-level data from 1920 to 2005 on the  $Q$ 's of IPO-ing and incumbent firms that we obtained from Compustat

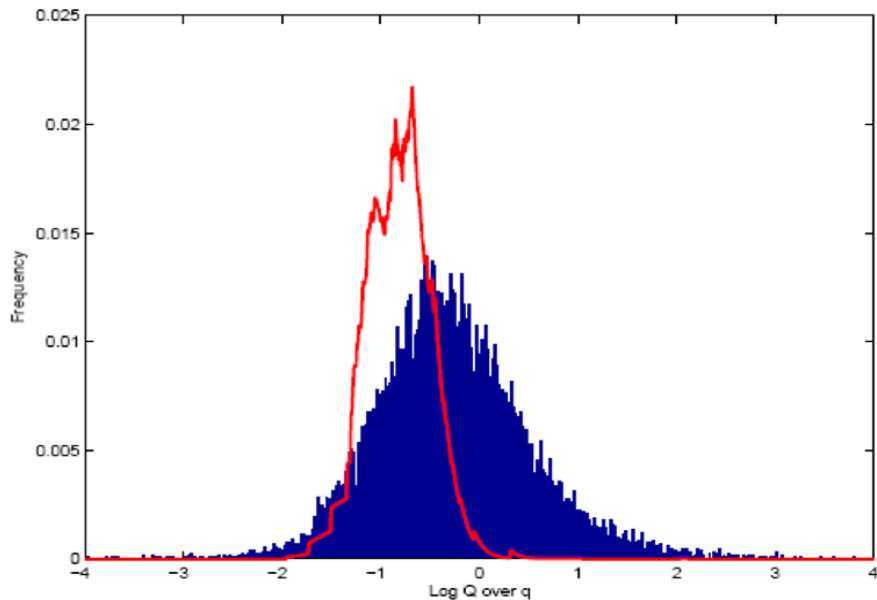


Figure 4: PREDICTED AND ACTUAL FREQUENCY DISTRIBUTION OF THE POOLED OBSERVATIONS ON  $Q_{i,t}/q_t$  USING DATA FROM 1920 TO 2005.

(1976-2005) and collected from the *Moody's Investors Manual* (1920-1975).<sup>14</sup> Implemented projects have (the time-invariant) distribution  $G(\varepsilon)$  with a time-varying truncation point given in (9). Let  $Q_{i,t}$  be firm  $i$ 's actual  $Q$  upon IPO. A firm uses  $\lambda$  units of the consumption good to generate  $\lambda\varepsilon$  expected units of the capital good. Then  $Q_{i,t} = q_t\varepsilon_{i,t}$ , where  $q_t$  is the value of installed capital per unit of consumption and  $\varepsilon_{i,t}$  is the quality of the  $i$ 'th IPO at date  $t$ . Therefore we need the counterpart of  $q_t$  from our micro-level data so it can be comparable to the  $Q_{i,t}$ . We construct  $\hat{q}_t$  as the value-weighted average  $Q$  of the incumbents at date  $t$ . We then compute

$$\hat{\varepsilon}_{i,t} = \frac{Q_{i,t}}{\hat{q}_t} \quad (19)$$

and check how well they fit the the actual IPO-ing-firms'  $Q_{i,t}$  distribution year by year, as well as the predicted number of IPOs given by (9), where  $\Phi$  is the standard normal CDF.

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<sup>14</sup>We compute firm-specific  $Q$ 's for 1976 to 2005 from Compustat data as described in footnote 4. For 1920-1975 we use a database of balance sheet items for individual companies that we collected from annual issues of *Moody's Investors Manual* (available on microfiche) as part of a five-year NSF project that yielded more than 59,000 firm-level observations. Using market values of common equity from CRSP and our backward extension of it, we compute  $Q$  similarly for 1920-1975 as we did for 1976-2005.

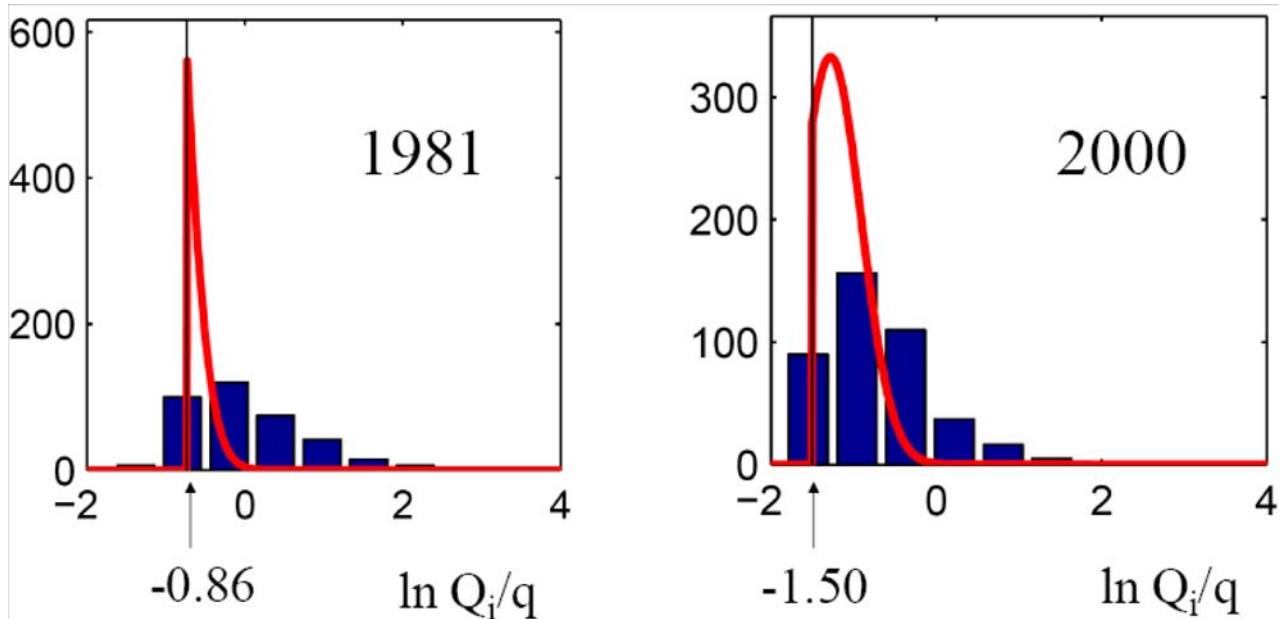


Figure 5: PREDICTED AND ACTUAL IPO DISTRIBUTIONS IN A TROUGH YEAR AND A PEAK YEAR.

The predicted frequency distribution of all the 1920-2005 observations (weighted each year by  $N_t$ ) is  $\left(\sum_{t=1920}^{2005} N_t\right)^{-1} \sum_{t=1920}^{2005} N_t p\left(\frac{Q}{q_t} \mid q_t\right)$ . Figure 4 plots the predicted and actual frequency distributions of  $\ln \hat{\varepsilon}_{i,t}$  pooled over  $t$ .

The estimated  $g(\varepsilon)$  is (i) to the left of the distribution of the pooled Compustat IPO-based  $\hat{\varepsilon}_t$  distribution and (ii) less dispersed. Let's discuss why each of these deviations arises. (i) The contribution to IPO value (which is how we measure  $Y$ ) of IPOs of a quality-level  $\varepsilon$  is value times the number of IPOs, i.e.,  $\varepsilon g(\varepsilon)$ . The peak of the product  $\varepsilon g(\varepsilon)$  occurs to the right of the model of  $g$ , at  $\mu_\varepsilon + \sigma_\varepsilon^2 = -1.14$  and its distribution is slightly right skewed. It is most responsive to  $q$  at a value to the right of the mean of  $\varepsilon$  and this is why the macro-based estimate of  $g(\varepsilon)$  places the mean of  $\varepsilon$  to the left of the mean of the data. (ii) The distribution of the  $Q_{i,t}$  is somewhat too dispersed to produce an adequate crescendo of implementations as aggregate  $q$  rises from unity to 3.4 as the aggregate data require. On the other hand, considering the small number of parameters, the model captures individual-year IPO distributions quite well. We consider a trough year, 1982, when  $\hat{q}_{1982} = 2.36$ , and a peak year, 2000, when  $\hat{q}_{2000} = 4.48$ . Then since  $Q \geq 1$ , (19) implies that  $\ln \hat{\varepsilon}_{1982} \geq -\ln 2.36 = -0.86$  and  $\ln \hat{\varepsilon}_{2000} \geq -\ln 4.48 = -1.5$ . This is shown in Figure 5.

## 4 Fitting the model to data

In our model,  $q$  is driven by hidden implementation costs. When we come to the data, this will mean that  $q$  represents a shock *relative* to the price of capital that the BLS measures and uses to construct its estimated stock of capital. Therefore measured  $Q$  equals the model's  $q$ . When compatibility problems cause reproduction costs of incumbents to rise, this will not enter their book values (i.e., the denominator of measured  $Q$ ), and this generates a measurement error that raises measured  $Q$ . The model normalizes the measured cost of capital to unity, which means that we implicitly assume that the price index for capital goods is correctly used by the BLS when constructing the capital stock numbers that we shall use here.

### 4.1 Construction of composite capital and other aggregates

Figure 2 does not measure the same concept for entrants as it does for incumbents. In both panels, the series  $I$  is aggregate investment in *physical* capital. On the other hand, the series  $V_t/K_{t-1}$  and  $IPO_t/K_{t-1}$  in Figure 2 track a broader concept of investment.

Some of the money that VCs spend goes on plant and equipment, but the rest is used to pay rent on office space, and on salaries, raw materials, etc. Similar remarks apply to the IPO series which measures the market value of firms entering the stock market, i.e., and that value reflects all the assets that those firms own, not just their physical capital.

Since we have no measures of physical-capital creation by entrants, we shall construct measures of composite-capital investment for incumbents (i.e., that reflect both physical and human capital components) and thereby put the comparison on an equal footing by measuring the investment of both groups in the same units. The composite-capital construction will tell the same story as did physical capital: Composite investment is unresponsive to  $Q$  whereas the composite investment of entrants responds elastically.

To fit the model's implications we shall need to measure  $q$ ,  $z$ ,  $X$ , and  $Y$ , and to construct  $k$ . The specification will need to satisfy the conditions that Hayashi and Inoue (1992, 'HI') display in their equations (2.8) and (2.11). Let  $k$  now denote the capital *composite* and let  $k'$  denote its next-period value. Aggregate consumption is

$$c = zk - q\xi(k' - (1 - \delta)k) + h \left( (1 - \xi) \frac{k' - (1 - \delta)k}{k} \right) k,$$

where  $\xi$  is the fraction of investment going to incumbents. This is the analogue of HI's profit function. Thus HI's condition (2.8) that profits depend only on the aggregate capital stocks  $k$  and  $k'$  is met, as is their degree-one-homogeneity condition (2.11). We retain (3) where  $X$  and  $Y$  now denote incumbent and entering investment



spending on broad capital. Letting  $m \equiv \frac{k'}{k} - 1$  denote the growth rate of aggregate capital, the decentralization (14) then reads

$$w = \max_{\xi, m} \{z - q\xi m - h([1 - \xi]m) + q(1 + m)\}.$$

The FOC with respect to  $\xi$  reads

$$q = h([1 - \xi]m). \quad (20)$$

Then (20) is the same as (5) since  $y = (1 - \xi)m$ . We now describe how we define and measure  $X$  and  $Y$  in the two-capital case and then use this composite-capital series to simulate the model.

We shall present the aggregation at the level of the price-taking firm and not at the level of the entire economy, similar to HI. The aim is to reduce the two-capital problem to the one-capital version stated in (14). Assume that  $k$  is the following aggregate of physical capital  $K$  and human capital  $H$ :

$$k = \phi(K, H). \quad (21)$$

The aggregator  $\phi$  corresponds to the aggregator  $\phi$  in HI's (2.8). The two capital stocks evolve as follows

$$K' = (1 - \delta)K + I_K \quad \text{and} \quad H' = (1 - \delta)H + I_H. \quad (22)$$

Each type of investment decomposes into two types

$$I_K = X_K + Y_K \quad \text{and} \quad I_H = X_H + Y_H \quad (23)$$

and the total cost of the combined investment is

$$\sum_{i \in \{K, H\}} p_i \left( q_t X_i + h_i \left( \frac{Y_i}{k} \right) k \right),$$

where  $h_K$  and  $h_H$  are the non-linear portions of the adjustment cost for  $K$  and  $H$ . Let

$$C(q, I_K, I_H) = \min_{(X_i, Y_i)_{i \in \{K, H\}}} \sum_{i \in \{K, H\}} p_i \left( q X_i + h_i \left( \frac{Y_i}{k} \right) k \right), \quad \text{s.t. (23)}. \quad (24)$$

If  $p_K$  and  $p_H$  are constant, the states are  $(s, K, H)$  where  $s = (z, q)$ .<sup>15</sup> Then letting

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<sup>15</sup>Implicitly we assume that whatever gives rise to the fluctuations in the  $K/H$  ratio over time does not cause the efficiency prices to change. If  $(p_K, p_H)$  were to change over time, we would need to add more states to the analysis, and the planner too would have additional states in the problem stated in Section 2.1.

$r(s)$  be the interest rate in state  $s$ , a risk-neutral firm's value would be<sup>16</sup>

$$W(s, K, H) = \max_{K', H'} \left\{ z\phi(K, H) - C(q, K' - (1 - \delta)K, H' - (1 - \delta)H) + \frac{1}{1+r(s)} \int W(s', K', H') dF(s', s) \right\}. \quad (25)$$

We now derive conditions under which the two-capital decision problem of a firm reduces to (14). As HI do, we too can break the maximization problem into two stages: The dynamic problem of choosing the scalar aggregate  $k$  over time, and the static problem of choosing  $K$  and  $H$  to minimize the total input cost subject to (21). Ignoring the proportionality constant define

$$\hat{C}(K, H, k') = \min_{K', H'} C(q, K' - (1 - \delta)K, H' - (1 - \delta)H) \quad \text{s.t.} \quad \phi(K', H') \geq k'. \quad (26)$$

**Lemma 1** *If*

$$\phi(K, H) = K^\alpha H^{1-\alpha}$$

and if the optimal  $X_K$  and  $X_H$  are positive<sup>17</sup> the solution to the problem in (26) is

$$K' = \lambda \frac{\alpha}{p_K} k' \quad \text{and} \quad H' = \lambda \frac{1-\alpha}{p_H} k' \quad (27)$$

where

$$\lambda = \left( \frac{p_K}{\alpha} \right)^\alpha \left( \frac{p_H}{1-\alpha} \right)^{1-\alpha}. \quad (28)$$

Since (27) and (28) also hold for  $(k, K, H)$ , we let  $c(q, k, k') = \hat{C}\left(\lambda \frac{\alpha}{p_K} k, \lambda \frac{1-\alpha}{p_H} k, k'\right)$ . Then since  $W$  is homogeneous of degree one in  $(K, H)$ , we have  $W\left(s, \lambda \frac{\alpha}{p_K} k, \lambda \frac{1-\alpha}{p_H} k\right) \equiv \hat{w}(s) k$ . Then the maximization problem (25) becomes

$$\begin{aligned} \hat{w}(s) k &= \max_{k'} \left\{ zk - c(q, k, k') + \frac{k'}{1+r(s)} \int \hat{w}(s') dF(s', s) \right\} \\ &= \max_{k'} \{zk - c(q, k, k') + qk'\} \end{aligned} \quad (29)$$

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<sup>16</sup>We shall in fact need to assume a trend in  $p_H$  relative to  $p_K$  which, to keep the notation simple, we shall ignore for the moment, but which eventually we shall bring into the calculated stock of  $h$ . If  $p_H$  has no trend,

$$h_t = \sum_{s=0}^{\infty} (1-\delta)^s \frac{I_{H,t-s}}{p_H},$$

whereas if  $p_H$  varies according to  $p_{H,t} = p_H (1 + g_H)^t$ , the true stock would be

$$h_t = \sum_{s=0}^{\infty} \left( \frac{1-\delta}{1+g_H} \right)^s \frac{I_{H,t-s}}{p_H}.$$

We shall infer below that  $g_H = 0.0155$ .

<sup>17</sup>This is a restriction on the  $X_i$  that parallels the development in Section 2 where we assumed that the optimal  $X$  is positive for all  $(z, q)$ . This will be true when  $h$  is steep enough, as portrayed in Figure 3.

because arbitrage over  $X$  forces  $r(s)$  to satisfy the equation  $MC_X = AC_X = q =$  marginal revenue from an additional unit of  $X$ , i.e.,

$$q = \frac{1}{1 + r(s)} \int \hat{w}(s') dF(s', s).$$

The firm therefore draws a zero return on all its units of  $X_K$  and  $X_H$ . Its rents derive from output  $zk$ , from the future value of the undepreciated portion  $(1 - \delta)k$ , and from the inframarginal units of  $Y_K$  and  $Y_H$ . Substituting into (29) and dividing by  $k$  we get

$$\hat{w}(s) = z + q(1 - \delta) + \sum_{i \in \{K, H\}} (qy_i - p_i h_i(y_i)),$$

where the  $y_i = Y_i/k$  solve the problem in (24). In order that  $w(s) = \hat{w}(s)$ , we must have

$$\sum_{i \in \{K, H\}} (qy_i - p_i h_i(y_i)) = \max_y \{qy - h(y)\},$$

which works if there are constants  $\theta_K$  and  $\theta_H$  summing to one such that the marginal conditions to the problem in (24) are also met at  $y_i = \theta_i y$  whenever  $h'(y) = q$ . This is true, e.g., if we choose  $h_i$  as follows:  $h_i(y_i) = \frac{\theta_i}{p_i} h\left(\frac{y}{\theta_i}\right)$ . In that case  $\hat{w}(s) = w(s)$ , and  $k$  as defined in (21) and  $k'$  as generated via the choice of  $(K', H')$  by the problem (25) are the same as the solutions that emerge in the problem defined in (14).

**Generating the series  $k$ .**—The definition (21) defines composite capital to be a function of  $K$  and  $H$ , and we now describe how these series were obtained and how they were combined into the index  $k$ , as well as how the investments  $x$  and  $y$  were calculated.

*The series  $K$ .*—The aggregate physical capital stock  $K_t$  is defined as the year-end stock of private fixed assets (see footnote 9). Then from (22) we calculate  $I_t = K_{t+1} - (1 - \delta)K_t$ . Both  $K_t$  and  $I_t$  are nominal series.

*The series  $H$ .*—We obtained raw data on educational achievement in the United States from worksheets underlying Turner *et al.* (2007). This source provided the number of low-skill and high-skill agents from 1901 to 2000, which we define as  $N_{U,t}$  and  $N_{S,t}$ . Our skill premium series is  $\frac{w_{S,t}}{w_{U,t}}$ , where  $w_{U,t}$  and  $w_{S,t}$  are wages per person at date  $t$ .<sup>18</sup> We then calculate the real stock of human capital  $H_t^R \equiv N_{U,t} + \frac{w_{S,t}}{w_{U,t}} N_{S,t}$ , and then convert  $H_t^R$  into nominal units by multiplying it by  $P_t$  (in our case the CPI) to get  $\hat{H}_t \equiv P_t H_t^R$ .

*Imposing stationarity on the capital-output ratio.*—We have no direct information on the unit price of  $h$ . Since we have assumed that  $p_K/p_H$  is constant and since the

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<sup>18</sup>Since the numbers of high- and low-skilled individuals ends in 2000 and their wage ratios end in 1995, we use the average annual growth rates of each over the proceeding five years to bring these series forward to 2005.

$k$  and the  $h$  series are already in nominal units, we shall infer the change in the unit price of  $h$  over the century. We want our shock  $z$  to have no trend. Since  $K_t$  grows as fast as output, we adjust the average growth of  $\hat{H}$  by  $g_H$  as explained in footnote 16, which is the exact amount needed to ensure that it grows as fast as the  $K$  series over the century.<sup>19</sup> We call the resulting series  $\tilde{H}_t$ . We calculate  $\tilde{I}_{H,t}$  from the  $\tilde{H}$  series as

$$\tilde{I}_{H,t} = \tilde{H}_{t+1} - (1 - \delta)\tilde{H}_t. \quad (30)$$

Moreover the series  $k$  obtained via (21) was normalized by a multiplicative constant to fit the average output-capital ratio.

*Decomposing  $k$  into incumbent and entering investment.*—In (3) we set  $X = X_H + X_K$  and  $Y = Y_H + Y_K$ , and we then define

$$I_H = X_H + Y_H \quad \text{and} \quad I_K = X_K + Y_K$$

to be composite investment by incumbents and entrants. We assume that the ratio  $X/Y$  is the same for physical and for human capital. That is,

$$\frac{X_K}{Y_K} = \frac{X_H}{Y_H}.$$

Then given data on the  $X_i$  and  $Y_i$ , we can then calculate

$$x = \frac{X_K + X_H}{k} \quad \text{and} \quad y = \frac{Y_K + Y_H}{k}.$$

*The series for  $Y$ .*—We cannot measure  $Y_H$  and  $Y_K$  separately, only their sum  $Y$ . We entertain two measures for  $Y_t$  that correspond to the series plotted in Figure 2 where the data sources are described as well. The first is total investment of venture capital funds and the second measure of  $Y$  is the real value of IPOs. IPO values relative to the capital stock are  $qy$ . This is the value of the composite capital stock brought into the stock market. Division by  $q$  yields  $y$ .

*Calculating  $z$ .*—Since output is  $zk$ , we measure  $z$  by the ratio of private output over the course of a given year to private capital at the start of that year.<sup>20</sup> Having obtained composite  $k$ , we then set  $z_t = GDP_t/k_t$  for all  $t$ .

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<sup>19</sup>Thus we first obtain the average growth differential over the century by solving for  $g_H$  the equation

$$\frac{\hat{H}_{2005}}{\hat{H}_{1900}} e^{105g_H} = \frac{K_{2005}}{K_{1900}}.$$

We then define

$$\tilde{H}_t = \hat{H}_t e^{(t-1900)g_H}.$$

This adjustment, for which  $g_H$  turns out to be 0.0155, ensures that  $Y/k = \hat{z}$  is trendless.

<sup>20</sup>Private output, defined as gross domestic product less government expenditures on consumption and investment, are from the Bureau of Economic Analysis (2006) for 1929-2005, to which we join Kendrick's (1961, Table A-IIb, pp. 296-7, col. 11) estimates of gross national product less government.

*Calculating  $q$ .*—In light of (14), we measure  $q$  by Tobin’s  $Q$ . The source of the  $Q_{i,t}$  for the IPOs and of  $q_t$  for the established firms that we used to estimate the distribution  $G(\varepsilon)$  of new ideas (in Section 3) was the Compustat data and our backward extension of it. We use the micro  $Q$ s to estimate  $G(\varepsilon)$  because the  $Q$ s of the IPOs are also from the micro data. It turns out, however, that the Compustat-based  $\hat{q}_t$  exceeds the aggregate  $\hat{q}$  displayed in Figure 2 by a factor of 2.4 and that the ratio of the two time series is trendless.<sup>21</sup> We therefore we generate  $y$  and  $h$  via (5) and (8) by using  $\hat{q}_t \equiv (2.4)q_t$ , where  $q_t$  is the aggregate series for the entire century.

*The price-earnings ratio.*—We also fit the economy-wide price-earnings ratio.<sup>22</sup> Assuming that  $(1 - \alpha)k$  goes to labor compensation, earnings are  $\alpha zk$  and therefore

$$\frac{\text{Price}}{\text{Earnings}} = \frac{q}{\alpha zk}.$$

## 4.2 Simulations of the model

We simulate the model over two time periods: 1901-2005 and 1978-2005. Parameters were chosen to be close to conventionally used values of  $\beta$ ,  $\delta$ , and  $\sigma$  while still fitting the historical averages of the rate of growth of output, investment by incumbent firms and entering firms respectively, and the price-earnings ratio. The simulations for both time periods were computed using the same set of parameters with the exception of  $\lambda$ , which changes because we use IPO investment for  $y$  over the 1901-2005 period and an alternate data series (i.e., VC investment) for 1978-2005. We assume that  $G(\varepsilon)$  is the log-normal distribution, with  $\ln \varepsilon \sim N(\mu_\varepsilon, \sigma_\varepsilon^2)$  which, together with  $\lambda$ , yields  $h(y)$  via (8). Table 1 reports the parameter values:

Table 1: Parameter values used

$\beta$	$\delta$	$\sigma$	$\mu_\varepsilon$	$\sigma_\varepsilon$	$\lambda_{1901-2005}$	$\lambda_{1978-2005}$	$\alpha$
0.95	0.065	4	-1.28	0.37	0.02	0.01	0.33

<sup>21</sup>The difference arises for two reasons. First, *Compustat* and our backward extension only cover firms that are listed on organized stock exchanges, while the aggregate measures of  $Q$  cover the entire non-financial corporate sector. Our sample is thus focused on larger and more successful firms. Second, our measure of  $Q$  is based on market and book values of a firm’s outstanding securities issues (see footnote 4), the proceeds of which are spent on physical capital and intangibles, while the aggregate measures reflect the holding of tangible capital only. Since intangibles probably form an important part of the forward-looking component of stock prices and our decentralization suggests that employees accept lower pay to develop human capital, our concept of  $Q$  would be expected to generate higher measured values than those based upon tangibles alone. The micro-based and macro series for  $Q$  are highly correlated with  $\rho = 0.92$  and their ratio does not vary much over the century, being 2.0 in 1921 and 2.2 in 2005.

<sup>22</sup>We obtain the economywide price-earnings ratio from worksheets underlying Shiller (2000), updated through 2005 and available from his website.

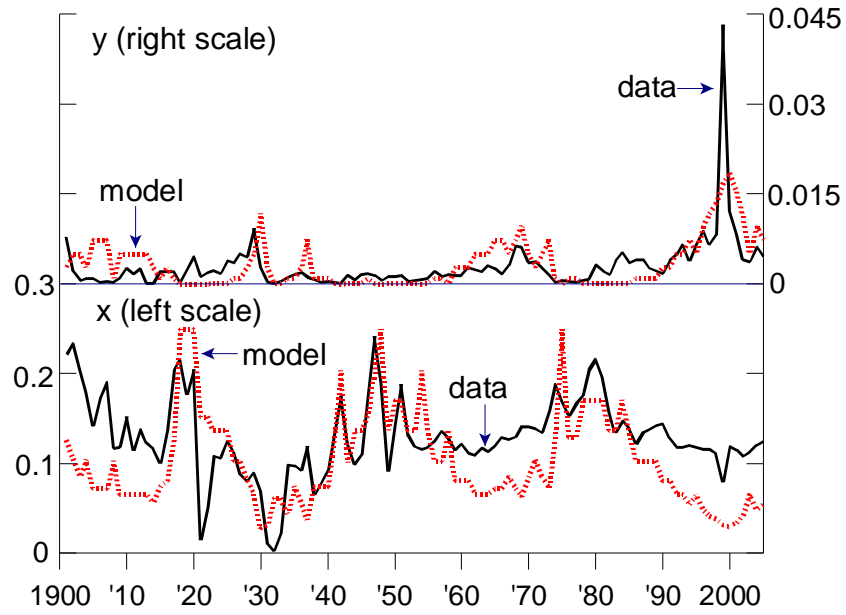


Figure 6: THE  $Y$  AND  $X$  SERIES AND THEIR SIMULATED VALUES (WITH COMPOSITE CAPITAL), 1901-2005.

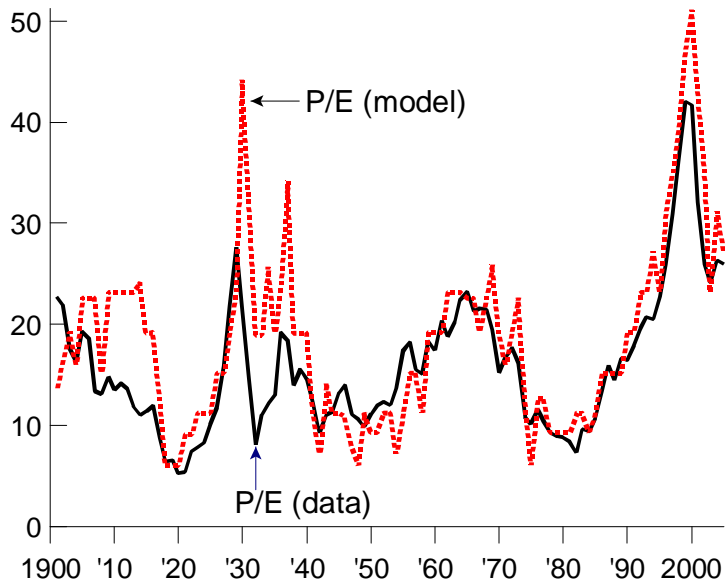


Figure 7: THE PRICE-EARNINGS RATIO AND ITS SIMULATED VALUES, 1901-2005.

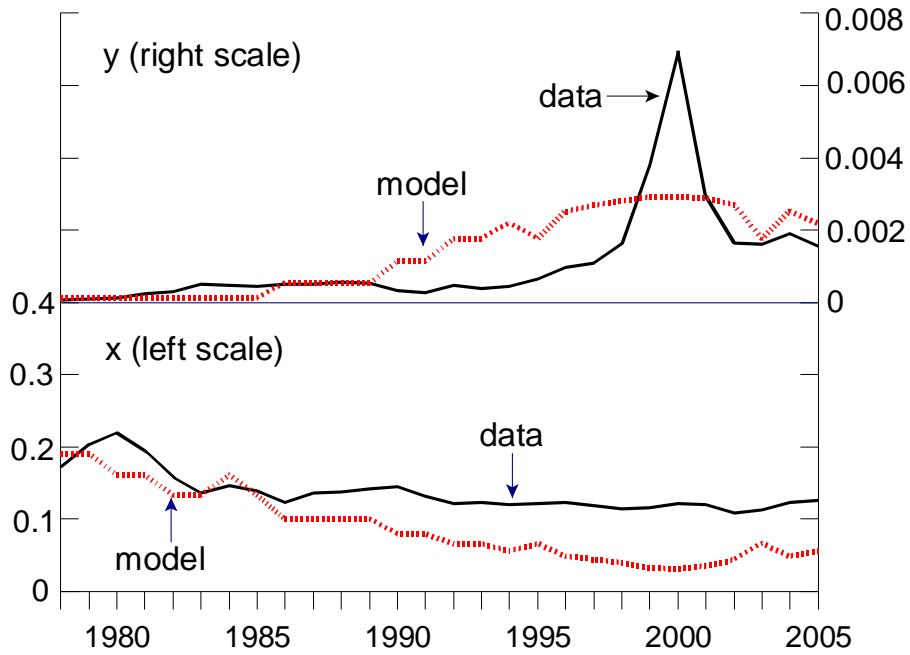


Figure 8: THE  $Y$  AND  $X$  SERIES AND THEIR SIMULATED VALUES (WITH COMPOSITE CAPITAL), 1978-2005.

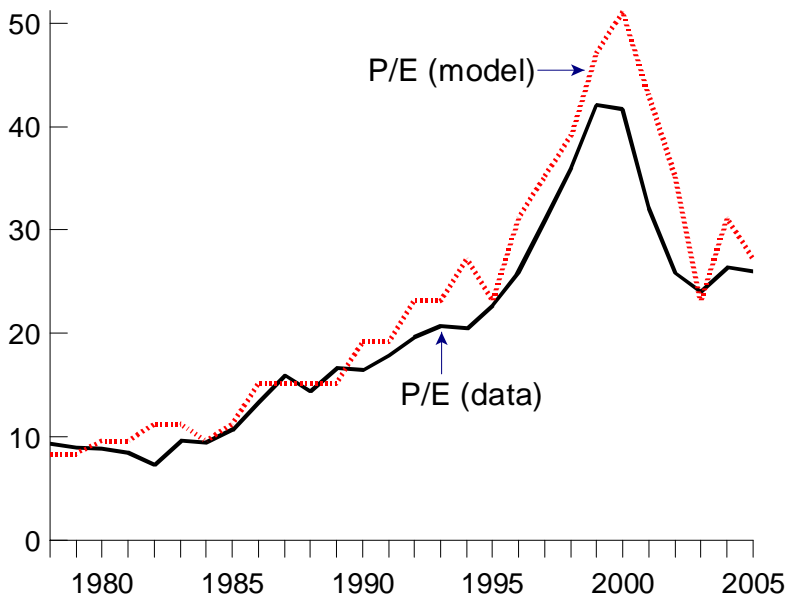


Figure 9: THE PRICE-EARNINGS RATIO AND ITS SIMULATED VALUES, 1978-2005.

A cornerstone of the model is  $h(y)$ , and given the values of  $\mu_\varepsilon$ ,  $\sigma_\varepsilon^2$  and  $\lambda$  we obtain an S-shaped supply curve of new projects as a function of  $q$ .

The model's predictions were generated by solving the planner's problem using the discretized  $(z, q)$  process obtained from the Tauchen-Hussey procedure applied to the data on  $q$  and  $z$  that we constructed using the procedures outlined in Section 4.1.

*Simulation results for 1901-2005.*—Figure 6 plots the model predictions and the data for 1901-2005. It has in the past been notoriously difficult to explain aggregate investment – we remind the reader that highly successful as it is, the RBC approach removes the HP trend and so takes the low-frequency variation in the data out. Yet the fit of our model seems to be quite close. The  $y$  series fits well too, except that we do not match all of the extreme spikes in the data. Roughly speaking, there is too much cross-sectional dispersion in  $Q_{i,t}$  to allow the model to capture the late-90s spike, though it does quite well with the one in the late 60s. Figure 7 plots the price-earnings ratio for the data and model and here the fit is quite good too.

*Simulation results for 1978-2005.*—The only data change here is the use of the VC-investment series in place of IPOs. The model parameters are the same as for 1901-2005 (see Table 1), except that  $\lambda$  is smaller. The latter is not surprising because VC investment is smaller than the IPO series. We also use the same values for  $z$  as we used for 1901-2005. The approximation is not as good due to the large drop in  $z$  at the start. Figure 8 plots the model prediction and the data for 1978-2005, while Figure 9 plots the price-earnings ratios. Again the fit is fairly close, but not as close as it is for the century as a whole.

### 4.3 Other evidence

A key further implication of the model is that the  $Q$ s of entering firms should exceed those of incumbents. The projects of the entering firms have the distribution  $G(\varepsilon)$  with a time-varying truncation point given in (9). The construction of the individual  $Q$ s is described at the beginning of Section 3, and the solid line in Figure 10 shows the annual value-weighted averages of these  $Q$ s from 1920 to 2005.

The dashed line in Figure 10 shows the value-weighted average  $Q$  of the incumbents. In 51 of the 86 years for which we have data on the  $Q$ s of IPOing firms, IPOs had an average  $Q$  greater than that of incumbents – often much greater. The time averages of the  $Q$ s over the 86 years are 3.02 for IPOing firms and 2.57 for incumbents with standard deviations of 1.52 and 1.04 respectively. This is consistent with our model, which also implies that the standard deviation of entrants should exceed that of incumbents.



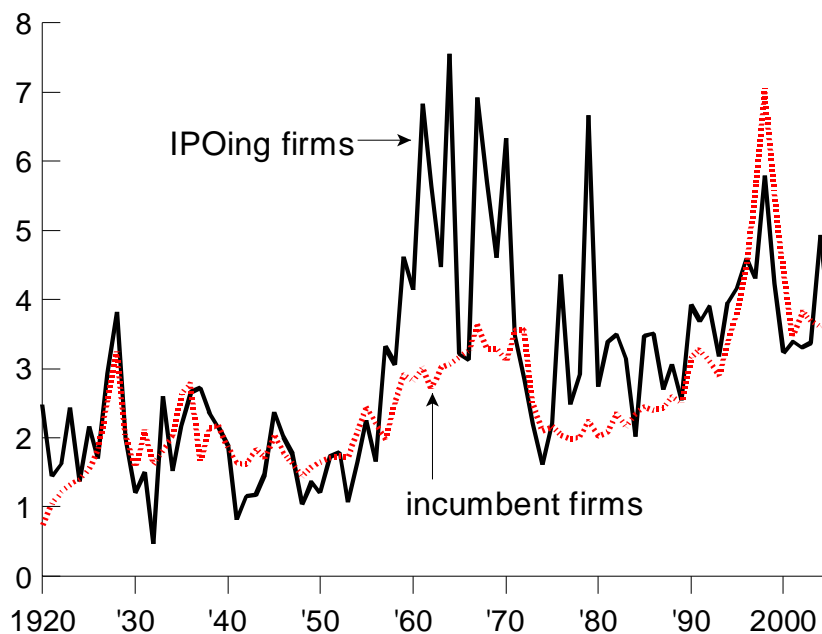


Figure 10: VALUE-WEIGHTED  $Q$ S FOR IPOING AND INCUMBENT FIRMS, 1920-2005.

## 5 Conclusion

As in the case of labor supply, we have found that extensive investment is more elastic than intensive investment. We analyzed various measures of investment by new firms and found that such investment responds to Tobin's  $Q$  much more elastically than does investment by incumbent firms, which responds negatively to movements in aggregate  $Q$ . We argue that this is because a high  $Q$  is a symptom of low compatibility of old capital with the new and, hence, of high implementation costs specific to incumbents alone. Entrants do not face compatibility problems because they start *de novo*.

Our model differs from the labor-macro models in the precise way in which it puts incumbents at a disadvantage. The analog of a curvature in the utility-of-leisure function would be sufficiently diminishing returns at the firm level which, given the observed cross-sectional variation in the size of firms, would not match the data. In our model, therefore, we opted for using new technology as a limit on incumbent firms through the implementation and compatibility costs that the adoption of new technology entails. New firms can escape this compatibility cost and therefore expand more elastically. In future work we hope to develop and test some of the labor-market implications of this way of looking at the business cycle.

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