

**THE FRIEDMAN RULE IN AN OVERLAPPING GENERATIONS MODEL:
SOCIAL SECURITY IN REVERSE**

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THE FRIEDMAN RULE IN AN OVERLAPPING GENERATIONS MODEL:
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The welfare gains from adopting a zero nominal interest policy depend on the implementation details. Here I focus on a government loan program that crowds out lending and borrowing and other money substitutes. Since money can be costlessly created the resources spent on creating money substitutes are a "social waste". Moving from an economy with strictly positive nominal interest rate to an economy with zero nominal interest rate will increase consumption by the amount of resources spent on lending and borrowing. But in general welfare will increase by more than that because consumption smoothing is better under zero nominal interest rate.

Key words: Welfare Cost of Inflation, Money Substitutes, Wealth Redistribution, Friedman Rule.

JEL CODES: E42, E52, E51, E58, H20, H21, H26

1. INTRODUCTION

The nominal interest rate is often viewed as a tax on real balances. Understanding the working of an economy with zero nominal interest is therefore crucial for estimating the welfare cost of this tax. Yet there is little agreement on exactly how a zero nominal interest - Friedman rule economy will operate. Will there be lending and borrowing at zero nominal interest? What will happen to the many substitutes for money that are currently used?

Aiyagari, Braun and Eckstein (ABE, 1998) is a good starting point. They use a cash-in-advance model of the type studied by Lucas and Stokey (1987). In their model there are cash and credit goods: cash goods are sold only for cash while credit goods are sold for costly credit. The distinction between credit good and cash good is endogenous and depends on the nominal interest rate. When the nominal interest rate declines more goods are bought for cash and resources used for the production of credit are saved. When the nominal interest rate is zero, all goods are "cash goods" and no resources are spent for creating credit goods. ABE show that in their model the welfare cost of inflation is the area under the demand for money curve as in Baily (1956) and Lucas (2000).

Following Lucas and Stokey (1983, 1987) ABE assume that transactions in the bonds market are costless. Thus a reduction in the nominal interest reduces the role of credit within the period but does not reduce the role of credit in smoothing consumption between periods. Here I allow money to replace between periods credit, in the same way that it replaces within period credit.

The implementation details of the zero interest rate policy are important. Friedman (1969) assumed that the government creates deflation by burning the receipts of lump sum taxes. Lucas and Stokey (1983) allow the government to offer loans at the initial date. The government (or the central bank) can therefore reduce the money supply

by burning the interest payments it receives. Both models assume an infinite horizon economy.

Here I use an overlapping generations, Baumol-Tobin type model. There are two assets: Indexed private bonds and real balances (money). A trip to the bank is required for changing the evolution of bonds. Changes in the evolution of real balances do not require a trip to the bank. Since smoothing consumption requires changes in asset holdings, money has an advantage in smoothing consumption.

Our private bonds and money are therefore not equivalent in the sense of Kocherlakota (2007). In his paper Kocherlakota assumes that private and public sectors have the same collection power and that money has no distinct transaction advantage over bonds. He concludes that "In reality, the collection power of the private and public sector may well differ, and money almost certainly does provide liquidity benefits that loans do not". In view of the equivalence theorem in his paper he argues that the issue of collection power is especially important for understanding the impact of government financing decisions. Here I relax both assumptions. I argue that the government has a better "collection technology" and I use the Baumol-Tobin idea to model the liquidity benefits of money over bonds.

Bonds here are analogous to gold that require resources to dig it, store it and carry it. Will money crowd out bonds in a Friedman-rule zero nominal interest rate equilibrium? The answer is in the positive when restricting attention to a steady-state equilibrium in an infinite horizon economy: There is no reason to go to the bank if you can write checks or pay in debit cards everywhere and your money in the checking account earns the same interest as bonds. This is true regardless of the implementation details.

In a more realistic overlapping generations model there is a difference between the two alternative implementation procedures. If we reduce the money supply by levying taxes, young agents who reach adulthood with no money will have to borrow money to

pay the taxes and to smooth consumption. This requires trips to the bank (both by lenders and borrowers) and such trips are waste of resources from the social point of view. I argue that implementing the Friedman rule by a government loan program may solve the need of young agents to borrow money from private individuals.

The question is whether the government has an advantage over the banking sector in making loans. Does it have better "collection technology"? Here is a tentative answer.

The government can use its tax collection system to implement the loan program in the following way. It gives an income contingent loan to each young adult. Each year the borrower makes a loan payment that depends on his current income. People who consistently make low income will not pay the loan in full: They will "partially default". We now increase consumption taxes and use the revenues to finance "partial defaults" by "poor" people and a reduction in the top marginal income tax rates.

This is a move towards a proportional consumption tax system with a rebate, in the form of "partial default", to low permanent-income people. Altig, Auerbach, Kotlikoff, Smetters and Walliser (2001) estimate that a switch to a proportional consumption tax would increase the steady state income by 9%. But low-income individuals will suffer a significant loss as growth fails to compensate for the decline in tax progressivity. Castaneda, Diaz-Gimenez and Rios-Rull (1998) estimated the moving to a proportional tax system in the US will increase output by 4.4% at the expense of a large increase in wealth inequality. Here the "partial default" by poor people mitigates the adverse effects on wealth distribution.

In addition to facilitating the move toward a flat consumption tax, a government loan program will crowd out many private activities including banking, rental, insurance and (wholesale and retail) trade. These sectors will contract and free resources for socially beneficial activities. In addition the loan program will facilitate consumption smoothing.

Since lending and borrowing are viewed here as money substitutes, I start by incorporating money substitutes in Baily's original framework. A multi-period analysis will follow.

2. BAILY (1956) WITH MONEY SUBSTITUTES

The literature distinguishes between the "transaction" role of money and the "store of value" role of money. Here I argue that when the nominal interest declines, money will crowd out both substitutes for the "transaction" function of money and substitutes for the "store of value" function.

Baily (1956) focuses on the "transaction" role of money.¹ He treats money as a "good" with zero cost of production and argues that the welfare cost of inflation is "a reasonably straightforward extension of the welfare cost of an excise tax". He explicitly rules out substitutes for money by assuming that "Bank deposits are not used as money or are negligible in amount" (page 94). Here I argue that once we allow for substitutes, there is no clear relationship between the welfare cost and the area under the demand for money curve.² To simplify, I use an analogous single period "real" (non-monetary) economy. A monetary version is in Eden (2005, chapter 3.3).

The economy lasts for one period. There are two goods: X and Y . We may think of X as "liquidity services" and Y as consumption good (corn). The representative agent

¹ Baily analyzed the welfare gains from reducing the steady state inflation rate to zero and the steady state nominal interest rate to the level of the real interest rate. Lucas (2000) argued that much of the welfare gains are obtained when reducing the nominal interest rate from the real interest rate level to zero (and reducing the inflation rate from zero to the appropriate negative level). There are important differences between the two and it may be useful distinguish between the "welfare cost of inflation" and "the welfare cost of the nominal interest rate". Here I focus on the nominal interest concept.

² ABE assume that there is no substitution between goods and this may explain the difference in results.

has a large endowment of Y . His utility function is: $U(x) + y$, where lower case letters denote quantities consumed. It is assumed that there is a satiation level z such that $U'(z) = 0$, $U'(x) > 0$ for $x < z$, $U'(x) < 0$ for $x > z$ and $U'' < 0$ everywhere.

The representative agent can produce "liquidity services" X by using money and paying an inflation tax of τ units of Y per unit of X . He can evade the tax payment by using "money substitutes". As in ABE the marginal cost of producing "liquidity services" by the credit technology is increasing with the amount produced. It is assumed that the cost of producing x units of "liquidity services" by the "money substitutes" (credit) technology is $C(x)$ units of Y where the cost function $C(x)$ has standard properties: $C(0) = 0$, $C' > 0$ and $C'' > 0$.

I borrow the terms from the tax evasion literature and use "legal production" for the money technology and "illegal (black market) production" for the "money substitute" technology. I use x_1 (x_2) to denote the quantity of X produced legally (illegally) and $x = x_1 + x_2$ to denote total production. The representative agent gets a lump sum transfer of g units of Y . This lump sum tax is equal to the per agent tax revenues. Since there are many identical agents in the economy, the individual agent cannot influence the size of g .

The representative agent solves the following maximization problem:

$$(1) \quad \max_{x_1, x_2} U(x_1 + x_2) - C(x_2) - \tau x_1 + g$$

Assuming $x_2 > 0$, the first order conditions are:

$$(2) \quad C'(x_2) = U'(x = x_1 + x_2) \leq \tau \text{ with equality if } x_1 > 0.$$

Figure 1 illustrates, using $x(\tau)$ and $x_2(\tau)$ to denote the solution to (2). The Figure can be used to show that the total amount produced $x(\tau)$ is a decreasing function but the

amount produced illegally $x_2(\tau)$ is an increasing function. It follows that the amount produced legally $x_1(\tau) = x(\tau) - x_2(\tau)$ is a decreasing function. When $\tau \geq \tau^*$ and the tax is above the intersection of the two curves, the amount produced is $x(\tau^*) = x_2(\tau^*)$ and the amount produced legally is zero.

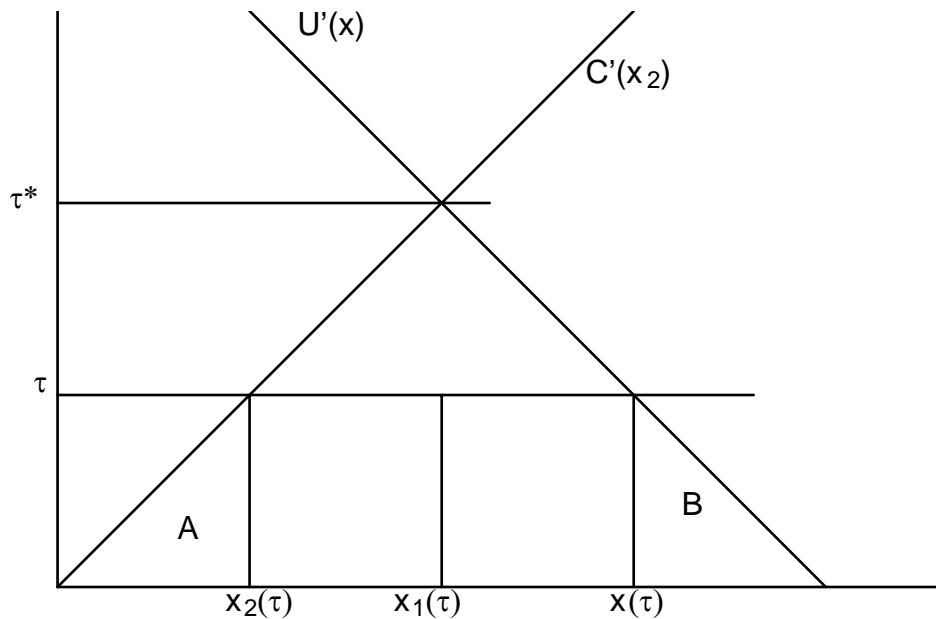


Figure 1: The solution to $x(\tau)$, $x_2(\tau)$ and $x_1(\tau) = x(\tau) - x_2(\tau)$

The welfare cost of the tax is:

$$(3) \quad U(z) - U(x(\tau)) + C(x_2(\tau))$$

The first component, $U(z) - U(x(\tau))$, is the area B in Figure 1. The second cost saving component, $C(x_2(\tau))$, is area A . It follows that:

Claim 1: The welfare cost of inflation is greater than the resources wasted on creating money substitutes (area A in Figure 1).

How is the welfare cost related to the area under the demand for money curve? To answer this question we add to Figure 1 the demand for "legal production", $m^d = x_1(\tau)$, that may be estimated by an econometrician who observes many points in the (x_1, τ) plane. The area under the demand for money (legal production) curve in Figure 2 is: $b + c$. The true welfare cost is the area: $a + c + d$.

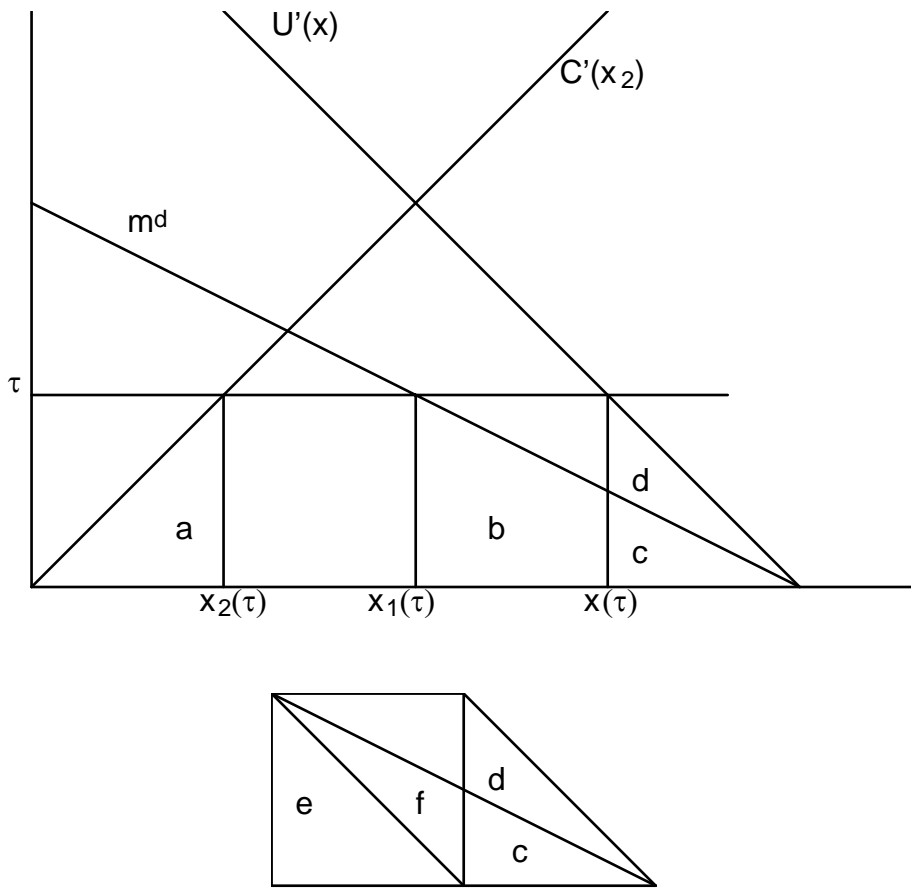


Figure 2: The true welfare cost ($a + c + d$) and the area under the demand for money curve ($b + c$).

The lower graph establishes: $b - (d + c) = e + f - (d + c) = e - c$. When $e = a$, the true welfare cost

is larger than the area under the demand for money curve.

With the help of the lower graph in Figure 2 we show that the area under the demand for money curve may be less than the true welfare loss. The area $e + f$ in the lower graph is equal to the area b in the main graph. The areas d and c are the same in

both graphs. Since $d = f$ and $a = e$, it follows that the difference between the true welfare loss and the area under the demand curve is c . Thus in this case, the area under the demand for money curve is less than the true welfare loss. It is also possible to draw the marginal cost curve in a way that will reverse this result by making the area a small. Thus the area under the demand for money curve may be larger or smaller than the true welfare cost.

Claim 1 can be used for measuring a lower bound of the welfare cost. Aiyagari, Braun and Eckstein (1998) estimated the cost of creating demand deposits and credit cards and argued that this is about 0.5% of GDP. Humphrey, Pulley, and Vesala (2000) estimate the social cost of a nation's payment system as 3% of its GDP. They argue that from the social point of view, debit card transactions are much cheaper than checks transactions and more generally electronic payment is cheaper than paper based payment (excluding cash which is assumed to have zero cost). But typically agents do not see the true social cost of making a payment because most banks do not charge for transactions if a minimum balance is held. In addition agents may prefer to make the payment by checks rather than by debit card because the floating period for checks is longer. It seems that the discrepancy between the social and the private cost will be eliminated once the Friedman rule of zero nominal interest rate is adopted. In a world with zero nominal interest rates banks are expected to charge per transaction rather than requiring minimum balances and individuals will not care about the length of the float period. We may therefore expect that the cost of the payment system will go down with the nominal interest rate.

Bergman, Gulborg and Segendorf (2007) found that the cost of cash and card payments in Sweden are about 0.4% of GDP. They found that credit card payments are expensive relative to debit card payments. We may expect that at a zero nominal interest rate, debit cards will replace credit cards.

3. THE MODEL

I now turn to the consumption-smoothing (asset) motive for holding money. I assume an overlapping generations economy. A new generation is born each period. Each generation lives for $T+1$ periods. I start with the case in which all agents are identical and can be represented by a single agent. The representative agent consumes and receives income starting from the second period of his life: in periods $t = 1, \dots, T$. The income (endowment) at age t is Y_t units of the consumption good. I focus on the steady state in which only the age of the agent matters and therefore the calendar time at which the agent was born is suppressed.

There are two assets in the economy: real balances (indexed money) and indexed bonds. A unit of real balances represents an obligation of the government to exchange it for one unit of consumption.³ An indexed bond represents an obligation by private agents to exchange it for a unit of consumption or a unit of real balances.

It may help to think about money as balances in a government-run interest-bearing checking account. The interest may be negative and is roughly equal to the negative of the rate of inflation in a fully articulated monetary model. The real interest rate on money is r_m and the real interest rate on indexed bonds is $r \geq r_m$.

As in Lucas and Stokey (1983), the representative agent may borrow real balances from the government (or the central bank) in the first period of his life. The amount that he may borrow must be less than L units. The amount of the loan is a choice variable and is denoted by d . The amount of real balances at age zero is thus:

³ This assumption is made for simplicity. It is different from the assumption used by standard models of fiat money. In these models there is no obligation on the part of the government to buy and sell money for goods. But our assumption is not unlike the obligation of the government to accept money as tax payment.

$$(4) \quad m_0 = d \leq L.$$

Each period the agent pays the interest rd on his initial loan. The agent pays the principal at the end of his life. In addition to the loan the agent gets a transfer payment from the government of g_t units of consumption at age t .

I use b_t to denote the real value of bonds held at age t . As in the Baumol-Tobin model, there is a fixed cost of α units of consumption when going to the bank (the bond market).⁴ When the agent does not go to the bank at age t his bond holdings will be $b_t = (1+r)b_{t-1}$. He may change the bonds balances if he goes to the bank. We may thus assume that the cost α occurs whenever $b_t \neq (1+r)b_{t-1}$ and therefore at age t the agent pays the cost:

$$(5) \quad TC_t(b_{t-1}, b_t) = \alpha \text{ if } b_t \neq (1+r)b_{t-1} \text{ and zero otherwise.}$$

The agent starts period t with $(1+r_m)m_{t-1}$ units of real balances and $(1+r)b_{t-1}$ units of indexed bonds. In addition he gets his endowment income and a transfer from the government. He chooses consumption, c_t and his end of trade portfolio, b_t and m_t subject to the following constraint:

$$(6) \quad b_t + m_t = Y_t + (1+r_m)m_{t-1} + (1+r)b_{t-1} - rd - c_t - TC_t(b_{t-1}, b_t) + g_t$$

At the end of his life the agent returns the principal:

$$(7) \quad m_T = d.$$

⁴ A trip to the bank will not be needed of-course in a world in which private contracts can be perfectly enforced and agents can pay in personal IOUs. We may therefore think of banks as specializing in enforcing loan contracts.

The representative agent's utility function is given by the strictly monotone and strictly quasi-concave function: $U(c_1, \dots, c_T)$. His maximization problem is:

$$(8) \quad \max_{d, c_t, b_t, m_t} U(c_1, \dots, c_T)$$

s.t. (4)-(7) and $b_0 = b_T = 0$; $m_t \geq 0$, $c_t \geq 0$

The population grows at the rate n . The number of Age t agents is a fraction γ_t of the number of age 1 agents. Thus,

$$(9) \quad \gamma_1 = 1; \quad \gamma_{t-1} = (1+n)\gamma_t$$

The market clearing conditions are:

$$(10) \quad \sum_{t=1}^T \gamma_t c_t = \sum_{t=1}^T \gamma_t (Y_t - TC_t)$$

$$(11) \quad \sum_{t=1}^T \gamma_t b_t = 0$$

Equation (10) is the clearing of the goods market and equation (11) is the clearing of the private bonds market. Here there is no "money market": Under the market clearing conditions, the government may play the role of a central clearing institution and deliver on its promise to exchange a unit of real balances for a unit of the consumption good.

But of-course, a Walrasian auctioneer can do it as well.

Note also that here we do not have a cash-in-advance constraint. A cash-in-advance constraint assumes that during a period of a given length, the trader cannot use his income (or part of it) to finance consumption. Here an agent that wants to consume

his income in each period can do it without holding money. Indeed if everyone wants to consume his income, money will not be used. See the introduction to Jovanovic (1982) for further discussion.

A steady state equilibrium is a vector $(d, c_1, \dots, c_T, b_1, \dots, b_T, m_1, \dots, m_T; g_1, \dots, g_T; r, r_m)$ such that (a) given $(g_1, \dots, g_T; r, r_m)$, the magnitudes $(d, c_1, \dots, c_T, b_1, \dots, b_T, m_1, \dots, m_T)$ solve (8) and (b) the market clearing conditions (10) - (11) are satisfied.

I evaluate welfare by the following planner's problem:

$$(12) \quad V(x) = \max_{c_t} U(c_1, \dots, c_T) \text{ s.t. } \sum_{t=1}^T \gamma_t c_t = x.$$

I now assume that L is sufficiently large and show the following Proposition.

Proposition 1: There exists equilibrium with $r = r_m = n$. In this equilibrium: (a) credit is not used ($b_t = 0$ for all t); (b) $\sum_{t=1}^T (1+n)^{1-t} g_t = 0$; (c) the utility of the representative agent is: $V\left(\sum_{t=1}^T \gamma_t Y_t\right)$.

In the proposed equilibrium the real rate of return on bonds is the same as the real rate of return on money and therefore the nominal interest rate is zero. I refer to this equilibrium as the "Friedman rule equilibrium". Part (b) says that the present value of the transfer payments must be zero. Part (c) says that the Friedman rule allocation is efficient.

Welfare cost: I now turn to compare welfare between two equilibria: The "initial" equilibrium with strictly positive nominal interest rate and the Friedman rule equilibrium with zero nominal interest rate. I use $(c_1^i, \dots, c_T^i; b_1^i, \dots, b_T^i)$ to denote the consumption and bonds holdings in the "initial" equilibrium and (c_1^F, \dots, c_T^F) to denote the consumption in the Friedman rule equilibrium. Part (b) in Proposition 1 and the market clearing condition (10) imply that under the Friedman rule aggregate consumption at time t is higher by aggregate spending on trading in the bonds market:

$$(13) \quad \sum_{t=1}^T \gamma_t (c_t^F - c_t^i) = \sum_{t=1}^T \gamma_t TC_t(b_{t-1}^i, b_t^i) = TC^i$$

Part (c) in Proposition 1 says that welfare in the Friedman rule equilibrium is:

$$U(c_1^F, \dots, c_T^F) = V(Y = \sum_{t=1}^T \gamma_t Y_t). \text{ I now turn to compute welfare in the initial equilibrium.}$$

I argue that in general, $U(c_1^i, \dots, c_T^i) < V(Y - TC^i)$. This says that the consumption allocation in the initial economy is not efficient relative to the resources that are devoted to consumption. As we shall see, this is because consumption smoothing is not efficient in the initial equilibrium.

I assume that as in the original Baumol-Tobin model, money may be used to smooth consumption between trips to the bank. The original model used continuous time. In our discrete time formulation, money is used if the agent does not go every period to the bank and hold a strictly positive amount of money between trips to the bank. Formally, money is used if there are ages h and k ($h < k$) such that (a) the representative agent chooses to go to the bank at ages h and k ; (b) he do not go to the bank at ages $h < t < k$ and therefore $b_t = (1+r)^{t-h} b_h$ for $h \leq t < k$ and $b_k \neq (1+r)^{k-h} b_h$; (c) he holds a strictly positive amount of money at some point during the above period: There exists an age index $h < j < k$ such that $m_j > 0$.

Claim 2: $U(c_1^i, \dots, c_T^i) \leq V(Y - TC^i)$ with strict inequality if money is used in the initial equilibrium.

Part (c) in Proposition 1 and Claim 2 imply:

Claim 3: $U(c_1^F, \dots, c_T^F) - U(c_1^i, \dots, c_T^i) \geq V(Y) - V(Y - TC^i)$ with strict inequality if money is used.

This says that from the planner's point of view, the utility gain from adopting the Friedman rule is greater than the utility gain from saving the resources spent on "going to the bank". Similarly to Claim 1, it implies that the saving of resources is a lower bound on the welfare gain.

Examples: To illustrate the working of the model, I now assume that $Y_t = 1$ when t is even and zero when t is odd. The utility function is: $\sum_{t=1}^{10} \beta^t U(c_t) = \sum_{t=1}^{10} \beta^t \ln(c_t)$, where $\beta = 0.96$. There is no population growth ($n = 0$) and the government provides an initial loan at a zero interest rate. The limit on the initial loan, L , is not binding.

Assuming that money is always held, the first order conditions for the consumer's problem (8) require:

$$(14) \quad U'(c_1) = (\beta R_m)^{j-1} U'(c_j),$$

where $R_m = 1 + r_m$. To derive (14) note that if the agent reduces his consumption at $t = 1$ by "one unit" he will suffer a loss of utility equal to: $U'(c_1)$ utils. He can use the accumulated unit of real balance to increase his consumption at age j by $(R_m)^{j-1}$ units and derive a benefit of $(\beta R_m)^{j-1} U'(c_j)$ utils. Since at the optimum such a deviation does

not increase the level of the objective function we must have that the pain is greater than the gain: $U'(c_1) \geq (\beta R_m)^{j-1} U'(c_j)$. When money is always held, the consumer can also increase consumption at $t=1$ and reduce it at age j . This will lead to $U'(c_1) \leq (\beta R_m)^{j-1} U'(c_j)$. Both inequalities lead to (14).

For the log utility function (14) implies:

$$(15) \quad c_j = (\beta R_m)^{j-1} c_1$$

I computed the equilibrium path of consumption and money holdings on Excel in the following way. We choose c_1 arbitrarily and then compute consumption at all ages according to (15). We keep guessing until the market clearing condition (10) is satisfied. We then work the financing. We assume $r_m \leq r = 0$ in all of the examples. We start with a guess about the size of the initial loan. We then use (6) to figure out the evolution of real balances assuming that the inflation tax payment $g_t = -r_m m_{t-1}$ is paid back to the consumer at age t as a lump sum. We search for the minimum size of the initial loan that satisfies the non-negativity constraint: $m_t \geq 0$ for all t .

Figure 3 illustrates the Friedman equilibrium with $r = r_m = n = 0$. Since $\beta R_m < 1$, consumption declines with age. The consumer starts with $c_1 = 0.6$ and reduces his consumption until it reaches 0.4 at the end of his life. To finance it the consumer takes a 0.75 loan from the government at date zero. The amount of money in his "bank account" fluctuates with income, but the moving two periods average declines until age 5 and then increases reaching 0.75 at the end of his life. At this point he pays his debt to the government.

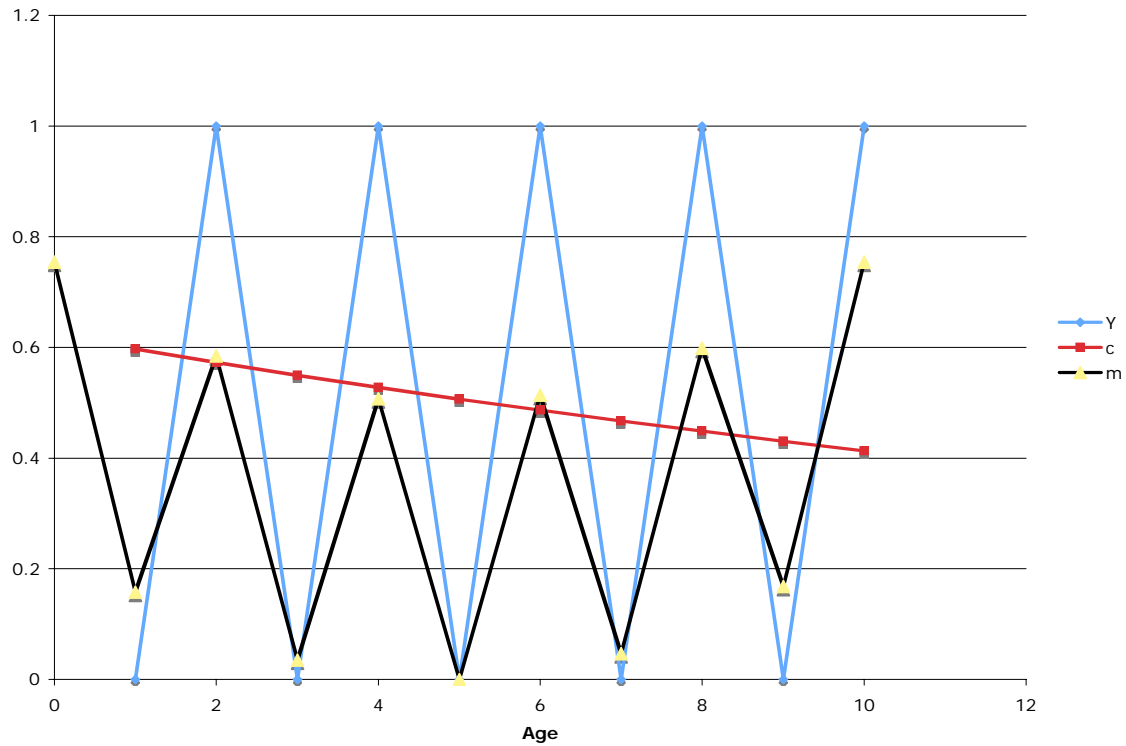


Figure 3: The evolution of income, consumption and real balances in the first best equilibrium;

$$\beta = 0.96, T = 10, R_m = R = 1, d = 0.75; \text{ No trips to the bank}$$

$$c_1 = 0.597, c_t = \beta R_m c_{t-1} \text{ for } t = 2, \dots, 10.$$

Figure 4 computes equilibrium for the case $R_m = 0.98$ assuming no trips to the bank. Consumption declines faster than under the Friedman rule. It starts at the level of 0.65 and ends at 0.37. It is financed by a 0.88 loan.

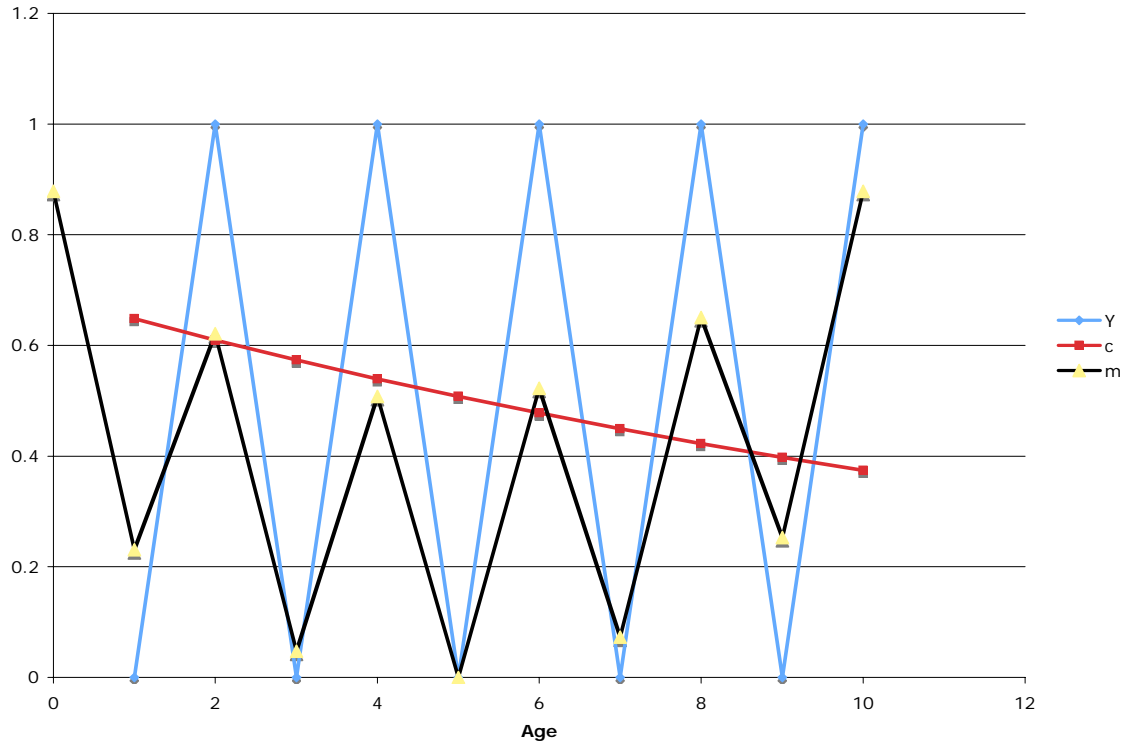


Figure 4: $\beta = 0.96$, $T = 10$, $R = 1$, $R_m = 0.98$; $d = 0.88$; No trips to the bank

$$c_1 = 0.65, c_t = \beta R_m c_{t-1} \text{ for } t = 2, \dots, 10.$$

To achieve the Friedman rule level of welfare we must increase consumption by 0.17% each period. This is a measure of the welfare cost of a nominal interest of 2% ($R_m = 0.98$). I also calculated the welfare cost of a nominal interest of 5% ($R_m = 0.95$). This is much larger and is equal to 1.07% of consumption.

I now allow trips to the bank at the cost of 0.1 per trip. To make such trips worthwhile, I assume that all the income of 5 units is received at the end of the consumer's life in period 10. The consumer makes the trips to the bank to reduce inflation tax payments. Indeed he gets the inflation tax back as a lump sum transfer but since those payments are taken as given he still tries to minimize the inflation tax payments.

Figure 5 assumes $R_m = 0.98$ and two trips to the bank at age 5 and age 10. Total consumption is now 4.8 reflecting the cost of the trips to the bank. From the individual

point of view, this cost is roughly equal to the reduction in the inflation tax payments. From the social point of view this is a waste of resources.

The consumer starts by borrowing from the government 2.276 units of real balances. He then borrows at age 5, 2.165 units. The cost of the trips to the bank is close to 4% of consumption. To achieve the first best we need to increase consumption by about 4.3%. Thus, most of the welfare cost is due to the cost of the trips to the bank.

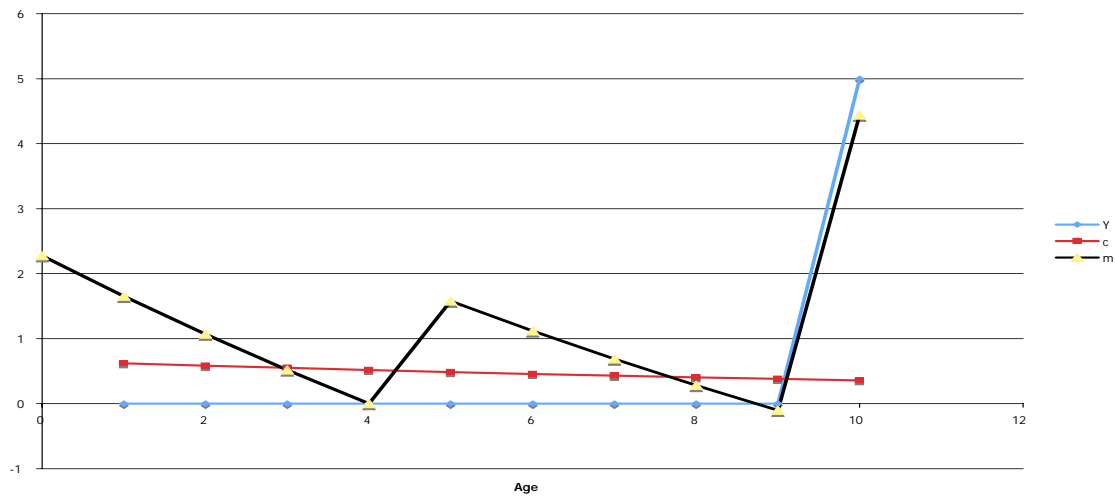


Figure 5: Income is received only at T; $R = 1$, $R_m = 0.98$; $d = 2.276$; At age 5 he borrows 2.165 units; Cost of going to the bank = 0.1 per trip

4. COLLECTION TECHNOLOGY

It was assumed that the government costlessly enforces loan contracts. I now turn to a more realistic case where contract enforcement is costly and there are administrative costs. This more realistic case is relevant for the important question of whether the government has an advantage over the banking sector in making loans.

Banks spend resources on trying to minimize the probability of bankruptcy by verifying collaterals and the financial status of the borrower. From a social point of view

some bankruptcies are transfers from the rich to the poor and therefore if income redistribution is desirable some of the resources spent on assessing the ability of the borrower to pay the loan are a "social waste". We will argue that even if the government does not want to redistribute wealth, it may still have an advantage in collecting loan payments because of its ability to levy taxes.

Some agents may choose to take the loan and move to another country. To reduce the incentives to do it, we should consider giving loans for say 10 years period. Thus a citizen is entitled to a loan of x units at the age of 20. Then if he pays it according to the schedule described shortly he will be entitled to another loan of y units at age 30 and so on. This may also address some problems of self-control discussed later. To simplify, I assume in what follows a single loan paid when the citizen reaches adulthood.

As before, I assume that agents live for $T+1$ periods but now there are many agents per generation. There are N types of agents indexed by: $h = 1, \dots, N$. One agent of each type is born in every period. Thus, there is no population growth and the distribution of the agents' characteristics is the same for all generations. As in the previous case, only the age matters. We will talk about agent (of type) h without specifying the date of his birth.

As in the previous section the government offers loans to young people but here the loan payments are contingent on income. A consumer who earns in the current period Y units and owes the government d units will pay

$$(16) \quad p(d, Y) = \tau(Y)Y \text{ if } d > \tau(Y)Y \text{ and } d \text{ otherwise.}$$

Here $\tau(Y)$ is the fraction of income paid when the principal is not small. When the principal is small the agent pays it and becomes debt free.

The principal that consumer h owes the government evolves according to:

$$(17) \quad d_t^h = (1+r)(d_{t-1}^h - p(d_{t-1}^h, Y_t^h))$$

The government imposes income and consumption taxes. An income of Y is taxed at the average rate of $\theta(Y)$. The consumption tax rate is κ . There are administrative costs for collecting taxes and loan payments from individuals.⁵

The cost of collecting income tax revenues of x units and loan payment revenues of y units from age t , type h agent is:

$$(18) \quad A_t^h(x, y),$$

where A_t^h is weakly increasing in both of its arguments. Mayshar (1991) assumes that the taxpayer devotes effort to paying taxes and sheltering income. We should therefore think of (18) as reflecting the true social cost, including the effort spent by the individual agent in filing tax returns, hiring accountants and so on.

Both income tax and loan payments create incentives to under report income. But the incentives to avoid loan payments are less than the incentives to avoid income tax of the same rate. To make this point I consider a young individual who gets a salary that is reported to the IRS by his employer. In addition to his salary the individual has done a one time 10,000 dollars consulting job in the current year. Assume further that he pays on the margin 20% of his income as loan payments and 20% as income tax. If he does not report the consulting job income he will reduce his income tax payment by 2000 dollars and his loan payment by 2000 dollars. But his net wealth will go up by only 2000 dollars, if eventually he pays the loan in full. This will be the case for individuals with high

⁵ See Slemrod and Yitzhaki (2002) for a survey of the literature on the administration costs of collecting taxes. Alt (1983) argues that collecting consumption taxes from businesses is simpler than collecting taxes from individuals. This is especially true for value added tax because of the incentives for self-policing. In what follows I abstract from the cost of collecting consumption taxes.

permanent income. The addition to net wealth will be 4000 dollars only if the individual has a low permanent income or plan to under report income consistently over time. The incentive to under report is thus less than in the case in which there are no loan payments and income tax is 40%. In this case the entire tax avoidance money represents an addition to wealth.

I therefore assume that the administration costs of collecting the loan payments from high permanent income people who pay their loan in full are zero. The administration costs of collecting the loan payments from low permanent income people who pay only part of their loan is equal to the costs of collecting these payments as income tax. Thus,

$$(19) \quad \begin{aligned} A_t^h(x,y) &= A_t^h(x,0) \text{ when } h \text{ pays the loan in full and} \\ A_t^h(x,y) &= A_t^h(x+y,0) \text{ otherwise; for all } x,y \geq 0. \end{aligned}$$

Finally, there are no lump sum transfers or taxes and the government spending is G units per period.

The consumer's asset evolution equation is now:

$$(20) \quad \begin{aligned} b_t^h + m_t^h &= \\ (1 - \theta(Y_t^h))Y_t^h + (1 + r_m)m_{t-1}^h + (1 + r)b_{t-1}^h - (1 + \kappa)c_t^h - TC_t(b_{t-1}^h, b_t^h) - p(d_{t-1}^h, Y_t^h) \end{aligned}$$

The market clearing conditions are now:

$$(21) \quad G + \sum_{h=1}^N \sum_{t=1}^T c_t^h = \sum_{h=1}^N \sum_{t=1}^T (Y_t^h - TC_t(b_{t-1}^h, b_t^h) - A_t^h[\theta(Y_t^h)Y_t^h, p(d_{t-1}^h, Y_t^h)]); \sum_{h=1}^N \sum_{t=1}^T b_t^h = 0$$

It is no longer clear that following the Friedman rule is optimal in an economy with costs of tax collection because of the ease of collecting inflation tax. This is especially true for developing countries. See Bordo and Vegh (2002). This is less of an issue for an economy like the US and I therefore assume that the Friedman rule is still optimal.

I now turn to the issue of financing the loan program. I treat the loans extended to young agents as expenses and the loan payments by older individuals as revenues. Using this convention, I show that the government budget must be balanced.

Claim 4: In a Friedman rule equilibrium with $r = r_m = n = 0$ we must have:

$$(22) \quad G + \sum_{h=1}^N d^h = \sum_{h=1}^N \sum_{t=1}^T (\theta(Y_t^h) Y_t^h + p(d_{t-1}^h, Y_t^h) + \kappa c_t^h).$$

An Illustration: I now consider an example in which a consumption tax is used both to finance "partial default" by the "poor" and a reduction in the income tax paid by the "rich". In this example there is no wealth redistribution.

Two types of agents are born in each period: "poor" and "rich". I start by assuming that the lowest annual earnings of the rich are higher than the highest annual earnings of the poor:

$$(23) \quad \min_i \{Y_i^R\} > \max_i \{Y_i^P\},$$

where the superscripts P and R denotes "poor" and "rich". Under this assumption the annual income of the poor is always below the median.

I assume $r = r_m = 0$ and $d^P = d^R = L$.⁶ The loan is equal to half of the total earnings of the poor. Loan payments are a flat rate of 25% of income so that the "poor" do not pay it in full. The rich pay their loan in full:

$$(24) \quad L = 0.5 \sum_{t=1}^T Y_t^P < 0.25 \sum_{t=1}^T Y_t^R$$

A consumption tax of 25% is used to finance the "partial default" by the "poor" and to finance a reduction in the top marginal income tax rate that is paid by the "rich".

Since the "rich" pay their loans in full the loan payments do not create an incentive to under report income (see the discussion that led to [19]). But the reduction in the income tax rate will encourage true reporting of income by the rich. The "poor" will treat the loan payments as an income tax and this has a discouraging effect on true reporting. On the whole, since the income tax paid by the "poor" is relatively small and since the cost of collecting consumption tax are relatively small, the total effect of the loan program on the administrative costs is likely to be negative.

The loan program has no effect on the distribution of wealth: The consumption tax paid by the "poor" finances their "partial default" and the consumption tax paid by the "rich" finances the reduction in their income tax payments. We now turn to compute the size of the loan L that can be financed by a 25% consumption tax rate.

From the IRS web site we find that in 2003 the average tax rate for the "poor" (bottom 50% of the income distribution) was only 3% and their adjusted gross income was: $Y^P = 879,735$ million dollars. I assume that over the life cycle consumption of the poor is roughly equal to their income. In 2003 the number of 20 years old was: $N = 4,118,942$.⁷ Assuming that half of those are "poor", the amount of the loan per 20 years old is:

⁶ Since $r = r_m$, $d^h = L$ is a solution to the agents' problems.

$L = \frac{0.5Y^p}{(\frac{1}{2})N} = 213,582$ dollars that is about 222,000 in terms of 2007 dollars.

A more realistic example that relaxes (24) will lead to a lower consumption tax rate for the same loan size. To illustrate this point, I assume three types of agents. A quarter of the population earns consistently above the median. A quarter earns consistently below the median and half are 50% of the time below the median and 50% of the time above the median. Assume further that only those who are consistently below the median do not pay their loan. In this case only a quarter of the population will "partially default" on their loan and the necessary consumption tax is only 12.5%.

6. SOME POLICY ISSUES

I now turn to discuss the effect of the loan program on labor supply, business loans and the relationship to social security.

Effect on labor supply: A loan payment has no effect on labor supply if it is returned in full. To see that note that the marginal loan payment for the rich in the above example is zero: If they earn an additional unit they will not increase the present value of their loan payments. There is also no wealth effect because the present value of the loan payments is equal to the amount of the loan.

Those who do not pay their loan in full will treat the loan payments as income tax. In the absence of a change in productivity, this increase in the marginal tax rate and wealth will reduce the labor supply of the "poor". But the adoption of the Friedman rule is likely to increase labor productivity. Some, for example, will use the loan to get higher

⁷ U.S Census Bureau, International data base. Table 094 (Midyear Population by age and sex) www.census.gov/cgi-bin/idbagg. We obtained the number of 20 years old by dividing the 20-24 category by 5.

education or to accumulate other types of human capital and some may buy a more reliable car and will come to work on time.

Time spent on executing transactions is expected to go down with the adoption of the Friedman rule. For example, selling a house to a buyer that does not need a mortgage is likely to be simpler. And of course, you do not need to go to the bank to sell bonds for money.

Thus agents will have more time to allocate between socially productive labor and leisure. This works in the direction of increasing labor supply.

For these reasons the effect of the loan program on the labor supply of the "poor" cannot be ascertained. The effect on the labor supply of the rich is likely to be positive because of the increase in productivity and the increase in the time available for socially productive activities.

Business loans: Evans and Jovanovic (1989) found that wealthier people are more likely to become entrepreneurs because they are less liquidity constrained. This is the apparent rationale used by the U.S. Small Business Administration to provide loans and loan guarantees to small business for start-up and expansion. Similar programs are present in other countries.

Our loan program relaxes liquidity constraints and may allow for poor people to become entrepreneurs. It may also incorporate or substitute existing programs aimed at facilitating start-ups.

The question is whether the loan program will create incentives to undertake projects with negative Expected Net Present Value (ENPV). If we allow for "partial default", an individual may undertake a project with negative ENPV because the government may take part of the losses: If as a result of failure his permanent income drops, he may not pay the loan in full. This problem is also present when making private loans. An individual may undertake a negative ENPV project because the collateral is not

sufficiently large and the bank assumes part of the losses. Stiglitz and Weiss (1981) analyze the adverse selection and moral hazard problems associated with asymmetric information in the market for loans. It seems that the government cannot solve these problems. It can only save some of the resources spent on selection (checking collaterals and the financial standing of the borrower) and the cost of bankruptcies. I leave these important issues to another paper.

Social security in reverse: The main objective of the social security system is to reduce the poverty level among the elderly. It has the following elements: (a) you pay during the working age and get the benefits in retirement; (b) the system transfers resources from the "rich" to the "poor". The objective of the loan program is to facilitate consumption smoothing. A loan program that allows for "partial default" may transfer resources from the "rich" to the "poor" but here you get the benefits at a young age and pay later.

It is sometimes argued that we need social security because many people lack "self-control". The lack of self-control is certainly a problem for children. Traditional economics assumes that all children become adults and once they become adults they gain self-control. Behavioral economics argues that some people never become adults in this sense.

It seems that self-control is endogenous to some degree. Parents that are over protective will tend to raise children that are less independent. Similarly, governments that are over protective may encourage myopic behavior.

The loan program discussed here is likely to encourage behavior that looks like "self control". For example, in the present system a young adult who drives an expensive car transmits a signal about the wealth of his parents or his own ability to make money. This signal may help him with the other sex. Once everyone gets a loan, such a behavior may signal the lack of self-control.

In any case, the implementation of the loan program may address the lack of self-control problem by making loans in various stages (say, some at age 20, some at age 30 and so on) and by offering options to buy annuities and other commitment devices.

7. CONCLUDING REMARKS

The question of the welfare cost of inflation is not a "straightforward extension of the welfare cost of an excise tax". In the excise tax case, there is no ambiguity about the implementation of a zero tax policy. In the monetary case, zero seigniorage policy can be done by imposing taxes or by offering loans to young individuals.

The literature has focused on the first tax alternative and considered the case in which only distortive taxes are possible. See for example, Phelps (1973), Kimbrough (1986), Correia and Teles (1996) and Chari, Christiano and Kehoe (1996). Here I focus on the second loan alternative. Unlike a tax, a loan that is paid in full does not distort behavior.

In general we expect the optimal policy to crowd out activities that the government can do better than the private sector. Most economists, Friedman being a prime example, believe that the private sector is in general more efficient than the government. But they still favor the government over the private sector in its money creation role. Here I extend this view and argue that the government may have an advantage over the private sector in making loans because it can levy taxes and because it may treat defaults made by "poor" people as part of its income redistribution policy.

We considered a Baumol-Tobin type model with two assets: indexed bonds and money. The difference between the two assets is in the ease in which one can affect their evolution. A trip to the bank is required for changing the evolution of bonds but the evolution of money can be changed without a trip to the bank. Since smoothing

consumption requires changes in asset holdings, this gives money an advantage in smoothing consumption.

Both assets may be used when the rate of return on bonds is higher than the rate of return on money. But only money will be used when the two assets promise the same rate of return. This is the case of zero nominal interest or the Friedman rule. Under the Friedman rule the allocation is efficient. Moving from an economy with strictly positive nominal interest rate to an economy with zero nominal interest rate will increase consumption by the amount of resources spent on trips to the bank. But in general welfare will increase by more than that because consumption smoothing is better under zero nominal interest rate.

We discussed collection technology issues that are relevant for the question of whether the government has an advantage in making loans. In the example we worked out, consumption tax finances both "partial default" by "poor" people and a reduction in the income tax paid by the rich. In this example, the implementation of the Friedman rule by a loan program does not change the distribution of wealth.

The income contingent loan program discussed here can be easily incorporated into the literature on flat consumption tax that followed Hall and Rabushka (1995). This literature estimates large gains from moving to a flat consumption tax but is worried about the adverse wealth redistribution effects. An income contingent loan that allows for "partial default" by people who make consistently low income (the truly "poor") is a natural addition to this system.

It is thus entirely possible that the loan program will allow for a tax system with less distortions. This is in sharp contrast to the literature that considers the implementation of the Friedman rule by imposing distortive taxes. In addition, implementing the Friedman rule by a government loan program will crowd out activities in many private sectors, especially in banking, rental, insurance and trade. The resources

saved can be used to produce product that are valued from the social point of view. I now elaborate.

Banking: The average bank loan in 1998 was about 30,000 dollars.⁸ Therefore it seems that a government loan of say 200,000 dollars will crowd out most of the loan activities of banks.

The zero nominal interest policy will reduce banks activities also on the liability side of their balance sheet. The commercial bank national average report (report, 1998) allocates 56% of the banks' non-interest expense to the liability side (demand deposits, saving deposits and time deposits). These expenses are mostly on processing checks and other paper based obligations. They also spend resources on moving balances from checking accounts to savings type accounts and vice versa. I expect that in a Friedman rule economy agents will have only one account (checking) and will use mostly debit cards and cash. There will be much less use of checks because zero nominal interest rate eliminates the benefit from the "float" period between the writing of a check and the actual payment.

Renting: Many people will use the loan to buy a house. Therefore, rental that is essentially used as substitute for loans will be crowded out.

Insurance: As pointed out by Bewley (1983) the initial loan may be used for self-insurance. Furthermore, since the loan payment depends on income, it has an insurance element. Thus, the loan program will crowd out some of the private insurance activities.

⁸ Calculated from the commercial bank national average report (report, 1998)

Wholesale and retail trade: The cost of trade will also be reduced because buyers will usually have enough money to carry out their desired transactions. This is the focus of Lagos and Wright (2005) who estimate the cost of 10% inflation to be between 3 to 5 percent of consumption.

On the whole, it seems that there are large potential benefits from implementing the Friedman rule by a loan program. Further research is needed of-course, before this theoretical paper can become a policy recommendation.

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APPENDIX

Proof of Proposition 1: I start by showing that in a Friedman rule equilibrium, when $r = r_m = n$, bonds will not be used (part [a]). To show it, consider a plan $\{d, c_t, b_t, m_t\}$ that satisfies all the constraint in (8) and uses costly credit that leads to $TC_t > 0$ for some t . Under the Friedman rule the plan $\{d^* = L, c_t^* = c_t + TC_t, b_t^* = 0, m_t^* = m_t + b_t\}$ is also feasible. Since the "star" alternative has more consumption at each date the original plan cannot be optimal. We have thus shown that trading in the bonds market cannot be optimal under the Friedman rule.

We now substitute $b_t = 0$ in the asset evolution equation (6). By forward substitution we then get a budget constraint that can be expressed in present values terms:

$$(A1) \quad \sum_{t=1}^T (1+n)^{1-t} c_t = \sum_{t=1}^T (1+n)^{1-t} (Y_t + g_t)$$

The budget constraint (A1) coincides with the market clearing condition (10) if and only if $\sum_{t=1}^T (1+n)^{1-t} g_t = 0$. Existence therefore requires that the present value of the transfer payments is zero. We have thus shown (b).

To show (c) note that the representative consumer's problem in a Friedman rule equilibrium is:

$$(A2) \quad \max_{c_t} U(c_1, \dots, c_T) \text{ s.t. (A1).}$$

The first order conditions for the problem (A2) require:

$$(A3) \quad \frac{U_t}{U_{t+1}} = 1 + n \text{ for all } t,$$

where U_t denotes the partial derivative of U with respect to consumption at age t . These are also the first order conditions for the planner's problem (12) and therefore the outcome is efficient. \square

Proof of Claim 2: When money is used in the initial equilibrium, there are indices $h < j < k$ such that $m_j > 0$, $b_t = (1+r)^{t-h} b_h$ for $h \leq t < k$ and $b_k \neq (1+r)^{k-h} b_h$. The first order conditions for the consumer's problem must be in this case:

$$(A4) \quad U_j(c_1^i, \dots, c_\tau^i) = (1+r_m)^{k-j} U_k(c_1^i, \dots, c_\tau^i);$$

$$(A5) \quad U_h(c_1^i, \dots, c_\tau^i) = (1+r)^{k-j} U_k(c_1^i, \dots, c_\tau^i)$$

To derive (A4) consider the following deviation from the optimal choice of the consumer in the initial equilibrium. The consumer reduces his consumption at age j by a unit, use it to increase his real balances and spend the accumulated amount of $(1+r_m)^{k-j}$ units at age k . The utility cost of reducing consumption at time j by a unit is $U_j(c_1^i, \dots, c_\tau^i)$. The utility gain of increasing consumption at age k by $(1+r_m)^{k-j}$ units is: $(1+r_m)^{k-j} U_k(c_1^i, \dots, c_\tau^i)$. Since at the optimum the agent cannot benefit from this deviation we must have:

$$(A6) \quad U_j(c_1^i, \dots, c_\tau^i) \geq (1+r_m)^{k-j} U_k(c_1^i, \dots, c_\tau^i)$$

Similarly, since $m_j > 0$ he can increase consumption at age j by a unit and reduce consumption at age k by $(1+r_m)^{k-j}$ units. Since at the optimum he cannot benefit from this deviation we must have:

$$(A7) \quad U_j(c_1^i, \dots, c_T^i) \leq (1 + r_m)^{k-j} U_k(c_1^i, \dots, c_T^i)$$

Conditions (A6) and (A7) imply (A4). The first order conditions to the planner's problem (12) require (A3). Since in the initial economy, $r_m < r$ the first order conditions (A4) and (A5) imply a violation of these first order conditions. Therefore a planner that can distribute $Y - TC^i$ units of consumption can improve on the initial equilibrium. \square

Proof of Claim 4: After forward substitution of (20) we get the budget constraint in terms of present values:

$$(A8) \quad d^h + \sum_{t=1}^T \left[(1 - \theta(Y_t^h)) Y_t^h - p(d_{t-1}^h, Y_t^h) - (1 + \kappa) c_t^h - TC_t(b_{t-1}^h, b_t^h) - (r - r_m) m_{t-1}^h \right] = 0$$

Summing over h leads to:

$$(A9) \quad \sum_{h=1}^N \left(d^h + \sum_{t=1}^T \left[(1 - \theta(Y_t^h)) Y_t^h - p(d_{t-1}^h, Y_t^h) - (1 + \kappa) c_t^h - TC_t(b_{t-1}^h, b_t^h) - (r - r_m) m_{t-1}^h \right] \right) = 0$$

Substituting $r = r_m$, $TC_t(b_{t-1}^h, b_t^h) = 0$ and the good market clearing condition in (A9) leads to:

$$(A10) \quad G + \sum_{h=1}^N \left(d^h - \sum_{t=1}^T \left[\theta(Y_t^h) Y_t^h + p(d_{t-1}^h, Y_t^h) + \kappa c_t^h - g_t^h \right] \right) = 0,$$

and to (22). \square