

OVERCONFIDENCE AND CONSUMPTION OVER THE LIFE CYCLE

by

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Overconfidence and consumption over the life cycle

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Abstract

Overconfidence is a widely documented phenomenon. In this paper, we study the implications of consumer overconfidence in a life-cycle consumption/saving model. Our main analytical result is a necessary and sufficient condition under which any degree of overconfidence concerning the mean return on savings can produce a hump in the work-life consumption profile. This condition is almost always met in the data. We show by simulations that overconfidence concerning the variance of the return can have little effect on the long-run average behavior of consumption over the life cycle, and that our basic conclusion is fairly robust with various realistic modifications to the baseline model. We interpret the general applicability of our analytical framework and discuss our numerical results in the light of aggregate consumption data.

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The chance of gain is by every man more or less overvalued, and the chance of loss is by most men undervalued.

The over-weening conceit which the greater part of men have of their own abilities, is an ancient evil remarked by philosophers and moralists of all ages.

—Adam Smith

1. Introduction

Humans can be overconfident in self-perceptions and overrate themselves on many positive personal traits. If John perceives he is better-looking than his classmates, he is not alone: Motley Fool guest columnist Whitney Tilson reports that 86% of his Harvard classmates feel they are better-looking than their peers (who is left to be worse looking?). If John thinks he can get a better grade than his peers, he is not alone either: University of Chicago professor Richard Thaler writes that on the first day of his class every student expects to get an above-the-median grade (half of them are always disappointed).

Overconfidence does not just belong to those elite school students. If you think you are safer and more skillful than your fellow drivers, you are not alone: in Svenson's (1981) study of Texas car drivers, 90% of those assessed believe they have above-average skills and 82% rank themselves among the top 30% of safe drivers. Anything you think you are better at or have better luck with than others, your peers are likely to think the same way: 70% of lawyers in civil cases believe their sides will prevail; doctors consistently overestimate their abilities in detecting certain diseases; parents feel their children are smarter than others'; lottery pickers bet that tickets they choose have greater odds to win than randomly selected ones; professional athletes and military personnel may even be trained to be overconfident and overoptimistic.

The presence of overconfidence in the business world is also well known. A large body of literature documents that managers are prone to the wishful thinking that projects they have command of are bound to succeed.¹ In a survey by Copper et al. (1988) of nearly 3,000 new business owners, 81% percent of those sampled believe their businesses

¹See Kidd and Morgan (1969), Langer (1975), Larwood and Whittaker (1977), Weinstein (1980), Bettman and Weitz (1983), March and Shapira (1987), Russo and Schoemaker (1992), and Malmendier and Tate (2003, forthcoming), among others.

have more than a 70% chance to succeed while 33% believe they will thrive for sure. In actuality, 75% of new ventures do not even survive the first five years.

The phenomena of overconfidence and overoptimism are widespread and have long been documented in the cognitive psychology and behavior science literature based on data from interviews, surveys, experiments, and clinical studies. Perhaps what is more overwhelming than the mere existence of overconfidence itself is the fact that the degree of overconfidence is rather persistent and generally does not wane over time. In the car-driver example, Camerer (1997) notes that even after suffering serious car accidents, drivers still rate themselves as above-average, and Bob Deierlein reports in a 2001 issue of *WasteAge* that more experienced drivers can develop a higher degree of overconfidence in their ability to avoid accidents but can in fact have accidents more frequently.

When it comes to investing, saving, and wealth, the phenomena of overconfidence and overoptimism are even more overwhelming. As we will survey in Section 2, the presence of overconfidence is a persistent phenomenon not only in stock and bond markets, but in other types of asset classes such as real estate and retirement savings. The evidence that we have already surveyed above indicates the general existence of overoptimism or overconfidence about one's ability in doing things, including making income.²

As the evidence on overconfidence has been accumulated, there is a surge of interest in understanding its consequence for issues of economic significance. A growing literature in finance examines the consequence of overconfidence in financial markets, as empirical evidence suggests that people persistently overestimate the average rate of return to their assets and underestimate uncertainty associated with the return. This recent strand of finance literature has focused on the implications of the underestimation of uncertainty associated with the asset return for short-run volatilities in financial markets, and shows that it can lead to high trading volumes and high turnover rates in asset markets, as well as speculative bubbles in asset prices.³ To the best of our knowledge, the implications

²A number of studies find that in many circumstances overconfidence can benefit the overconfident individuals themselves, and sometimes even the society as a whole. See, for instance, De Long et al. (1991), Kyle and Wang (1997), Hirshleifer and Luo (2001), and Berg and Lein (2005).

³See, for example, Benos (1998), Barberis et al. (1998), Odean (1998a, 1999), Hong and Stein (1999), Statman and Thorley (1999), Gervais and Odean (2001), Barber and Odean (2000, 2001, 2002), Daniel et al. (1998, 2001), Scheinkman and Xiong (2003), Deaves et al. (2004), Shiller (2005), and Englmaier (forthcoming).

of the overestimation of the average rate of asset return has not been explored in either the finance or the general economics literature.

This paper tends to take a first step in this direction. Our instinct is that consumer overconfidence may not only have important implications for trading and asset prices, but for consumption as well. We thus study the implications of consumer overconfidence in a general life-cycle consumption/saving model. For calibration purposes, and also to help draw a connection to the recent finance literature, we consider a standard version of the model in which a consumer has access to some asset that he can use to transfer resource across time and is overconfident about the asset return. The theoretical model can thus be made applicable to all types of asset classes, such as stocks and bonds, as well as retirement savings and housing, although the calibration of the model will be based on stock market participants, since the studies and sources of information that reveal consumer overconfidence in the other types of asset classes do not also allow us to quantify very precisely the degree of overconfidence in those areas.⁴

Our main finding is that overconfidence about the average asset return (income or wealth) can give rise to a hump-shaped work-life consumption profile, while overconfidence about the variance of the return (income or wealth) can have very little effect on the long-run average behavior of consumption over the work life. We view this result interesting for two reasons. First, it illustrates for the first time in the literature the potential relevance of overconfidence concerning expectations for some long-run economic behavior such as the behavior of life-cycle consumption. This complements the recent finance literature that emphasizes the relevance of overconfidence concerning variance for some short-run economic volatility such as volatility in trading volume, turnover rate, and asset price.

Second, the result also complements the existing studies on life-cycle consumption. A simple life-cycle model predicts that an agent's consumption is either increasing (if the agent is patient) or decaying (if the agent is impatient) monotonically over the life

⁴The model can also be recast to feature overconfidence about one's ability in making income. Alternatively, one can consider a situation as in Brunnermeier and Parker (2005) where an agent is overconfident or overoptimistic about future income, or simply about the present value of lifetime income, or wealth, so needs to revise consumption plans from time to time. Our theoretical result and basic intuition will go through under all of these alternative modeling choices.

cycle. Yet the work-life aggregate consumption observed from the data is usually hump-shaped, with peak consumption occurring between 45 and 55 years of age and with the ratio of peak consumption to consumption when first entering the workforce generally above 1.1.⁵ A number of assumptions have been made to modify the simple model to explain this feature of the data. In particular, Attanasio et al. (1999) and Gourinchas and Parker (2002) are able to reproduce this feature by making realistic assumptions about demographics and earnings.⁶ Our result adds to this literature by showing that overconfidence concerning expectations can be another mechanism for generating a hump-shaped work-life consumption profile. Our calibrated model produces a work-life consumption hump with both the age and the amplitude of the peak consumption comparable to those observed from aggregate consumption data.

Section 2 reviews some empirical evidence on overconfidence concerning various types of asset classes, savings, and wealth, along with some available explanations, in particular, an optimal-expectations story offered by Brunnermeier and Parker (BP) (2005).

To help isolate the effect of overconfidence concerning the expectation from the effect of overconfidence concerning the variance of asset returns (income or wealth) on life-cycle consumption, we consider in Section 3 a simple model that abstracts from uncertainty where an agent overestimates the rate of asset return at each age, in the spirit of BP (2005). We establish a proposition that shows that the overestimation of asset returns leads to a hump in the work-life consumption if and only if the elasticity of intertemporal

⁵See, for example, Attanasio et al. (1999), Browning and Crossley (2001), Gourinchas and Parker (2002), and Feigenbaum (2005).

⁶For other studies that also emphasize the roles of family size dynamics and labor-income uncertainty, see Tobin (1967), Browning et al. (1985), Attanasio and Browning (1995), Browning and Ejrnaes (2000), and Büttler (2001), and Nagatani (1972), Hubbard et al. (1994), and Carroll (1994, 1997), respectively. Other modifications invoked to reproduce a consumption hump include a “hand-to-mouth” assumption under which an agent simply consumes a constant fraction of his wage income that is hump-shaped, and assumptions about uncertain lifetime (e.g., Yaari, 1965, Büttler, 2001, Hansen and İmrohoroğlu, 2005, Feigenbaum, 2005), consumption-leisure substitutability (e.g., Heckman, 1974, Büttler, 2001, Bullard and Feigenbaum, 2005), consumer durables (e.g., Fernández-Villaverde and Krueger, forthcoming), and short-term planning (e.g., Caliendo and Aadland, 2007).

substitution in consumption is less than 1.⁷ We offer an informal description of the proof of the proposition and provide some intuition behind this result.

We report our model calibration and numerical results in Section 4. Here we also extend the baseline model to an environment with uncertainty where the actual rate of asset return follows some stationary stochastic process. We examine first the case in which the agent has an unbiased estimation of the mean return but underestimates uncertainty associated with the return, and then the case in which he both overestimates the mean return and underestimates the uncertainty, as in BP (2005). In the first case, the agent's work-life consumption profile is virtually flat, with some bumpy noise in the short run. In the second case, his work-life consumption profile is essentially the same as the one in the baseline model that abstracts from uncertainty. We conclude that it is the overestimation of the mean return that can give rise to a work-life consumption hump, while the underestimation of uncertainty has very little effect on the long-run average behavior of consumption over the work life. We conduct further robustness analysis here by expanding our simulated examples to incorporate some realistic features into the baseline model. In particular, we consider a hump-shaped earning profile due to age-dependent productivity, such as Feigenbaum's (2005) quartic polynomial estimate, and constant nonzero income stream during retirement, such as a pay-as-you-go social security program in the spirit of Feldstein (1985), with parameters chosen to match U.S. demographics and taxes. We find that our basic result is robust, and a work-life consumption hump continues to emerge with these realistic features incorporated.

We provide some concluding remarks in Section 5.

⁷Both the present paper and the paper by Caliendo and Aadland (2007) involve time-inconsistent dynamic programming problems: the life-cycler in our model solves many revising and replanning problems, each over the entire life span looking forward, while the short-sighted agents in their model face many short-horizon problems. Following the tradition in the life-cycle literature, their paper relies solely on the numerical illustration of a consumption hump, while both the location and the amplitude of the hump are quite sensitive to the length of the planning horizon, and aggregation over many short-term planners is crucial for producing a smoothed hump in the aggregate consumption profile. In contrast, our paper obtains both a closed-form solution (which helps establish the necessary and sufficient condition for a consumption hump) and numerical results, where the location of the hump is almost invariant to the degree of overconfidence and the amplitude of the hump is also fairly robust, and a smooth hump exists even in the individual consumption profile of a single life-cycler.

2. Additional evidence on overconfidence

The recent literature has documented a large body of empirical evidence on persistent overconfidence concerning various types of asset classes, savings, and wealth. The evidence reveals that people persistently believe that they have superior abilities in making income and good fortunes happen more often to them than to others, while they systematically overestimate the average rate of return to their assets and underestimate uncertainty associated with the asset return.⁸ An infamous example is Taylor and Brown's (1988) survey which indicates that only depressed people tend to become less overconfident and more realistic, even in activities like investing and saving.

Reports on overconfidence of American workers concerning their retirement savings frequently hit the headline news. For instance, according to Daniel Houston, a senior vice president for retirement and investor services at the Principal Financial Group Inc., a survey on retirement planning released in April 2005 indicates that workers are way too confident about the future performance of their retirement savings. The Employee Benefit Research Institute's annual retirement confidence survey conducted one year later and released in April 2006 reveals persistent overconfidence of workers concerning their retirement savings. Empirical studies suggest that overconfidence in pension plan performance is not just a trait of workers but of pension plan managers, and not just in the U.S. but in other countries as well (e.g., Gort, 2007, and the references therein).

The phenomenon of household overconfidence in their housing wealth is equally overwhelming. Robert Shiller reports homeowner overconfidence based on his 2003 survey of Los Angeles households (e.g., Shiller, 2004) and evidence worldwide (e.g., Shiller 2005). A recent nationwide survey conducted by the Boston Consulting Group in June 2007 shows that a vast majority of American homeowners continue to be overconfident that their houses will rise in value even in the face of turmoil in the real estate markets that includes record foreclosure numbers, mortgage rate increases and home price depreciation, and "appeared to feel that bad things happen to other people." Based on a data set of 81,943 house value estimates by the homeowners and their financial institution,

⁸See, among others, Presson and Benassi (1996), Odean (1998a, 1999), Barber and Odean (2002), Chuang and Lee (2003), Statman et al. (2004), Allen and Evans (2005), and Biais et al. (2005).

Agarwal (2007) finds that the homeowners overestimate their house value by 3.1%. Empirical evidence on homeowner overconfidence has been reported for other countries and regions as well (e.g., Wang et al., 2000, Wong et al., 2005, Wong, 2006).

When it comes to the securities markets, the phenomenon of persistent overconfidence is even more impressive. In a recent study based on a sample of nearly 15,000 individual investors surveyed by the Gallup Organization, Barber and Odean (2001) find that men estimate the rate of return to their investments by nearly 3 percentage points higher than the market average return and women by almost 2 percentage points higher. In another study of 78,000 individual investors, Barber and Odean (2000) also find substantial persistence of investor overconfidence, which results in high trading volume and high turnover rates in the face of repeatedly lower-than-expected realizations of asset returns. A Gallup poll conducted in 2001 indicates that, even after an unprecedented stock market bubble peak and subsequent burst, investors still remain overconfident and expect to beat the market return by more than 1.5 percentage points (Fisher and Statman, 2002).⁹ Based on a monthly survey of 350 financial market specialists, Deaves et al. (2005) find that even professional market analysts are persistently overconfident and the degree of their overconfidence even increases with their longevity (see also Atkins and Sundali, 1997, among others).

With the growing empirical evidence on persistent overconfidence, much attention has been paid to the question of why people are overconfident and experience does not lead them to become more realistic, especially in activities like investing and saving where results can be calculated ex post. Existing studies demonstrate that self-serving attribution bias (past successes tend to exacerbate overconfidence as people take too much credit for their successes, while past failures tend to be ignored as people blame their failures on forces beyond their control), confirmatory bias and cognitive dissonance (tendency to overweigh data confirming prior beliefs while to dismiss data contradicting prior beliefs), illusion of control or expertise, and forces related to evolution and tournaments or contests, can all contribute to generating persistent overconfidence throughout

⁹As Shiller (2005) notes, speculative bubbles were not new and had persisted over the entire last century, with pronounced peaks in 1901, 1929, 1966, and 2000; yet, even after serious bubbles in stock prices have popped, investors remain persistently overconfident.

the life cycle.¹⁰ Gervais and Odean (2001) find that, with attribution bias, people may even learn to become more overconfident rather than more realistic over time.

In an interesting study, Hvide (2002) demonstrates that overconfidence can be the equilibrium outcome if agents form beliefs pragmatically. In a more recent analysis, Brunnermeier and Parker (2005) show that overestimation of the mean of future income and underestimation of uncertainty associated with future income can be the outcomes of optimization by agents who choose subjective beliefs to maximize the average of their expected felicity over time. They provide a stereotypical scenario where optimal beliefs lead to an extremity of overconfidence under which future income is always perceived as certain, even if such belief is repeatedly contradicted by realizations. The agent merely observes one income realization at each age and believes he was unlucky, so he continues to be overconfident looking forward.

3. Model, analytical result, and some intuition

Time is continuous and begins at 0. An agent enters the workforce at $t = 0$, earns wage income at rate w during his work life, retires at $t = T$, and passes away at $t = \bar{T}$. When entering the workforce the agent is endowed with an initial stock of asset $S^*(0)$ which, without loss of generality, is assumed to be 0. The actual law of motion for the agent's asset position will be governed by the actual rate of return r^* to the asset. When making a consumption/investment plan the forward-looking agent will base his decision on an estimated rate of return r . At each time t the agent derives utility from his actual consumption $C^*(t)$ according to

$$U(C^*(t)) = \frac{C^*(t)^{1-\sigma} - 1}{1 - \sigma} \quad (1)$$

where σ is the inverse elasticity of intertemporal substitution in consumption. The agent has an instantaneous subjective discount rate $\rho > 0$.

We assume the agent is the sole member in the family, the dates of his retirement and death are both certain, the supply of his labor during the work life is inelastic (as he does not value leisure), the wage rate is constant during the work life, the good that

¹⁰See, among others, Odean (1998a, 1998b, and the references therein), Barber and Odean (1999), and Fellner et al. (2004). A classic example can be found in Shefrin and Statman (1985), where investors judge their performance by returns realized rather than returns accrued, and by holding “losers” and selling “winners” they persistently overestimate the rate of return to their assets.

he consumes is perishable, and the actual rate of return to his asset is equal to the discount rate. These assumptions effectively shut off all the channels for generating a hump-shaped life-cycle consumption profile that are already known in the literature.

The defining characteristic of our model is that the agent is overconfident and he overestimates his asset returns. Just as in the scenario described by Brunnermeier and Parker (2005), the agent merely observes one return realization at each age and believes he was unlucky, so he continues to be overconfident or overoptimistic looking forward. With the assumptions made in the last paragraph, overconfidence about the asset return is the only mechanism in the model that is potentially capable of producing a hump in the work life consumption.

It turns out that whether the agent's overestimation of the asset return can give rise to a hump in the work-life consumption depends on whether the elasticity of intertemporal substitution in consumption (σ^{-1}) is smaller than 1, or equivalently, whether the inverse of the elasticity (σ) is greater than 1.

Proposition 1 (The necessary and sufficient condition for a consumption hump)

Suppose that the agent overestimates the rate of return to his asset so that $r > r^ = \rho$. His consumption path during the work life is hump-shaped if and only if the inverse elasticity of intertemporal substitution in consumption σ is greater than 1.*

Proof. See Appendix.

The conclusion is fairly general. As long as the inverse elasticity of intertemporal substitution in consumption is greater than 1, a value near the lower end of empirical estimates (e.g., Kocherlakota, 1996) and well below the values commonly used in the life-cycle consumption literature, any degree of overestimation of the asset return will lead to a hump in consumption during the work life.

Why does overestimation of the rate of return to assets give rise to a hump-shaped work-life consumption profile if the inverse elasticity of intertemporal substitution in consumption is greater than unit? To help understand the intuition we first note that, because of his overconfidence, the dynamic optimization problem faced by the agent is time-inconsistent. In planning for his consumption and asset holding for the future, the agent relies on an estimated rate of asset return. Since he overestimates the rate

of return, his consumption plan so made is not sustainable and must be revised later. As we show in the appendix, a key step in the proof of the proposition is to establish a connection from the planned consumption paths to the actual consumption path. Another key step is to use the closed-form solution for the actual consumption profile so obtained to establish the claimed necessary and sufficient condition for a hump. The “only if” part of the proof in this step is fairly straightforward. The “if” part of the proof is more involved: the proof of existence of a peak consumption at an interior point during the work life is relatively easy, but to prove that the peak is unique and there is no other stationary point we carefully manipulate a sophisticated equation for solving a stationary point into a tremendously simplified one that allows us to fully explore the topological property of the profile.

Here we explain intuitively how the actual consumption path is related to the many planned consumption paths obtained in solving the time-inconsistent dynamic problem. The problem facing the agent is dynamically inconsistent, due to his overestimation of the asset return and thus his lifetime income, which renders his consumption plan made at any age for the future unsustainable. When the agent first enters the workforce, he plans to increase consumption gradually throughout his lifetime to capitalize on the difference between the estimated rate of asset return and the discount rate [see (18)]. The agent will follow the plan until he observes the actual return was lower than his estimated return for the period. While he believes he was just unlucky and hence continues to be overconfident looking forward, the agent does recognize that the rest of his original plan has become unsustainable. He thus revises down the rest of his old plan and makes a new plan for the remainder of his life span. Throughout his work life, the agent continuously adjusts down his plans made in the past—each being a monotonically increasing consumption path [see (18)]—and replans for the rest of his life time. This explains why his actual consumption (27) lies below his planned consumption (18), except at the time when the plan is made, at which the two coincide. As a solution to this time-inconsistent dynamic problem, the agent’s actual consumption constitutes the envelope of the many initial planned-values. Although the agent’s actual consumption also gradually increases through the early stage of his work life, it does so at a lower and lower pace than each of the planned paths, as the agent adjusts his estimated lifetime income lower and lower.

The fact that the agent’s actual consumption eventually peaks and turns around to make a hump (rather than ever increasing) has to do with the condition that the inverse elasticity of intertemporal substitution in consumption σ is greater than 1. To understand this, recall that σ governs the curvature of the agent’s utility function and therefore affects his willingness to shift consumption across time. If σ is small, marginal utility decreases slowly with the level of consumption and the agent is willing to capitalize on even small differences between his estimated rate of return to investment and the discount rate. If σ is big, marginal utility decreases rapidly with the level of consumption and a given addition to total utility requires a large income from investment. For $\sigma > 1$, the income effect dominates the substitution effect. At some point in time during his work life, the cumulative downward adjustments in the agent’s estimated lifetime income become large enough and drive the cumulative downward adjustments in his planned consumption paths so much that his actual consumption reaches a peak. From this point onward, his actual consumption declines monotonically as the agent ages and draws nearer to the end of his work life. This gives rise to a hump in his consumption during the working period.

4. Calibration and quantitative results

We describe in this section our model calibration and report our quantitative results.

We assume that the agent enters the workforce at age 25 (corresponding to the beginning point of time in the model), retires at age 65 (corresponding to $T = 40$), and passes away at age 80 (corresponding to $\bar{T} = 55$). We set the actual rate of return r^* to 7% in light of the long-term historical average real return in the U.S. equity market (e.g., Siegel, 1998, 1999)¹¹. To calibrate the degree of overconfidence, we draw on Barber and Odean (2001) who find that the estimation of asset returns by men is about 25% higher

¹¹Historical average real equity returns in most other industrialized countries have been about the same as in the U.S., as standard international data from Morgan Stanley Capital International suggest. It is the geometric average return that is referred to, and the arithmetic average return is considerably higher (e.g., Ibbotson, 2001). We could have also used the long-term historical average real return on U.S. Treasury bonds, which is about 3% to 3.5%. None of these choices would actually matter much for our results as it is just a normalization; what really matters is the degree of overconfidence, or, by what percent the estimated return is higher than the actual return. We tell our story using the equity market as a background since this is where the degree of overconfidence can be tightly calibrated.

than the market average return.¹² This implies a value of r equal to 8.75%. Our result is virtually invariant to the wage rate, a change in which shifts the work-life consumption path up or down in a parallel fashion but affects neither the age of peak consumption nor the ratio of peak consumption to consumption when first entering the workforce. In actuality, we set w to \$40,000.

Our analytical result in Section 3 shows that the inverse elasticity of intertemporal substitution in consumption is a key parameter, and it must be greater than 1 in order to have a hump-shaped work-life consumption profile. In our baseline calibration, we set σ to 3, a value close to what is commonly used in the literature, which is also the midpoint of two recent calibrations in the life-cycle macroeconomic models (e.g., Bullard and Feigenbaum, 2005, Feigenbaum, 2005).

Figure 1 displays under the baseline calibration the agent's actual consumption across his work life and some of his planned consumption paths. As is clear from the figure, the actual consumption path is the envelope of the numerous initial planned levels of consumption. As explained above, this envelope relationship is key to understanding why overestimation of asset returns can lead to a hump-shaped work-life consumption profile. As the figure illustrates, the actual consumption path is indeed hump-shaped, just as Proposition 1 predicts, with peak consumption occurring between 45 and 55 years of age, and with the ratio of peak consumption to consumption when first entering the workforce greater than 1.1. These numbers are comparable with those observed for aggregate consumption in the data.

Proposition 1 predicts that such a hump-shaped consumption profile should emerge for all $\sigma > 1$. Figure 2 displays the agent's work-life consumption profiles normalized to the level of his consumption when first entering the workforce under two alternative values of σ , 2 and 4, against the one under the baseline value 3. As is clear from the figure, all of the three consumption paths are hump-shaped, conforming to our analytical prediction. In the benchmark case, peak consumption occurs at 52 years of age, with the ratio of peak to initial consumption equal to 1.11. When we lower σ from 3 to

¹²Barber and Odean (2001) find that men estimate their asset returns by nearly 3 percentage points higher than the market average return that is about 12% during the period covered by the data. These numbers are nominal. We assume this overestimation is entirely due to overestimation of the real return rather than inflation. This translates into a 25% lower bound on overestimation of the real return.

2, the location of peak consumption shifts to 56 years of age, while the ratio of peak to initial consumption rises quite a bit, to 1.19. When we raise σ from 3 to 4, the location of peak consumption shifts to 50 years of age, while the ratio of peak to initial consumption declines somewhat, to 1.07. In all of these cases, the location and amplitude of peak consumption for the agent modeled here are consistent with those observed from aggregate consumption data.

The consumption hump in the above cases arises solely from the overestimation of the mean asset return, since uncertainty is deliberately abstracted away in the attempt to isolate the effect of this type of overconfidence. In order to examine the effect of underestimation of uncertainty on life-cycle consumption behavior, we extend the baseline model to an environment in which the actual asset return follows some stationary stochastic process. For simplicity, we assume that the actual rate of return to the agent's asset at each point in time is a random draw from a uniform distribution over the support [0%, 14%]. This choice of support imposes the largest volatility in the asset returns while still maintaining the limited-liability property of the asset and also ensuring a mean rate of return of 7% equal to the discount rate. As such, it gives uncertainty the greatest chance to make a difference on the life-cycle consumption profile.

We examine the case in which the agent has an unbiased estimation of the mean return but underestimates uncertainty associated with the return, as well as the case in which he both overestimates the mean return and underestimates the uncertainty. Just as in Brunnermeier and Parker (2005), the agent always perceives the future return to his asset as certain, although this belief may repeatedly be contradicted by realizations. With this extremity of overconfidence the effect of underestimation of uncertainty should be most pronounced. Nevertheless, as Figure 3 reveals, the work-life consumption profile in the former case is virtually flat, with some bumpy noise in the short run. In the latter case, as Figure 4 illustrates, the work-life consumption profile is essentially the same as the one in the baseline model without uncertainty where the agent only overestimates the mean return. We thus conclude that underestimation of uncertainty plays virtually no role in shaping the long-run average behavior of consumption over the work life, and it is the overestimation of the mean return that can give rise to a hump-shaped work-life consumption profile.

We do more robustness analysis by expanding our simulated examples to incorporate some realistic features into the baseline model. We begin by first replacing the constant work-life wage profile assumed in the baseline model by a more realistic, hump-shaped earning profile to capture the effect of age-dependent productivity. Specifically, we fit Feigenbaum's (2005) quartic polynomial estimate into the baseline model and then redo the simulations. Figure 5 displays the agent's work-life consumption path normalized to the level of his consumption when first entering the workforce with Feigenbaum's wage profile in force. It can be seen from the figure that the model with Feigenbaum's wages continues to produce a work-life consumption hump, with a similar location of peak consumption as in the baseline, and with the relative size of the hump slightly bigger than that in the baseline.

We next augment the model to allow for a constant nonzero income stream during retirement. In particular, we include a pay-as-you-go social security program in the spirit of Feldstein (1985) with parameters chosen to match U.S. demographics and taxes. To do so we append the state equations to include taxation and benefits so that the agent pays taxes during the working years and then receives benefits during retirement. We consider a pay-as-you-go program with taxes set to 10.6 percent of wage income that reflects the current OASI tax in the U.S., as well as a program with taxes set to 7.0 percent of wage income for compatibility with the early 1980's, before the massive rate increase in President Reagan's first term. The ratio of workers to retirees is set to 3 in both cases since this conforms to the U.S. experience over the last 25 years, which in turn implies that a pay-as-you-go program will provide a benefit per retiree at a given point in time that is three times as big as taxes collected per worker at that time. Figure 6 plots the agent's work-life consumption profile normalized to the level of his consumption when first entering the workforce with each of the two programs incorporated. In either case, a consumption hump continues to emerge during the working period, with a larger relative size and a slightly older age of peak consumption than in the case with no social security program.

We therefore conclude that a work-life consumption hump is a robust outcome of our model with these realistic features of the real world incorporated.

5. Concluding remarks

We have studied the consequence of overconfidence in a life-cycle consumption/saving model. We have shown that overconfidence concerning the mean return on savings can produce a work-life consumption hump while overconfidence about the variance of the return has little effect on the long-run average behavior of consumption over the life cycle, and that our basic conclusion is fairly robust with various realistic modifications to the baseline model. The necessary and sufficient condition established for the baseline model under which any degree of overconfidence about the mean return can produce a work-life consumption hump, that is, the inverse elasticity of intertemporal substitution in consumption is greater than 1, has almost always been validated by empirical estimates.

Caution should be exercised when taking the paper's quantitative results to the data. Our calibrated model generates a work-life consumption hump with the location and amplitude of peak consumption comparable to what have been observed for aggregate consumption in the data. Since the degree of overconfidence employed in simulating the quantitative results is calibrated based on stock market participants, the simulation results are particularly applicable to stockholders. If other agents in the economy have flat consumption profiles, the humps in the stockholders' consumption profiles would still give rise to a hump in the aggregate consumption profile, but the amplitude of peak consumption would be smaller in the aggregate profile, although the location of the peak consumption will be similar.

We have discussed how our analytical framework can be made applicable to any household that is overconfident about the future performance of its security holdings, its retirement savings, its housing value or other wealth, or any other properties it may own, or that is generally overconfident or overoptimistic about its ability or luck in generating income. We have reviewed some empirical evidence concerning such general existence of overconfidence or overoptimism. Yet the empirical studies and sources of information that reveal such evidence do not also allow us to quantify very precisely the degree of overconfidence in these areas, and this is why we have limited the calibration of our model to be based entirely on stock market participants. To fully assess the empirical promise of overconfidence for explaining the observed aggregate consumption hump calls for a more detailed examination in these areas. In light of our current finding that overconfidence may play a potentially important role in shaping life-cycle

consumption behavior, empirical research along these avenues should be elevated to the top of our research agenda.

Appendix

Proof of Proposition 1. We prove first the “only if” part of the proposition. Note that his overconfidence implies that the dynamic optimization problem faced by the agent is time-inconsistent. In planning his consumption and asset holding for the future, the agent relies on an estimated rate of return to his assets. Since he overestimates the rate of return, his consumption plan so made is not sustainable and must be revised later. The first step in the proof is to establish a connection from the planned consumption paths to the actual consumption path.

We first derive the planned consumption paths. At any time t_0 during his work life, the agent makes a consumption/investment plan for the rest of his lifetime by solving the following control problem

$$\max \int_{t_0}^{\bar{T}} e^{-\rho(t-t_0)} U(C(t)) dt \quad (2)$$

$$\text{s.t. } \dot{S}(t) = rS(t) + w - C(t), \quad \text{for } t \in [t_0, T] \quad (3)$$

$$\dot{S}(t) = rS(t) - C(t), \quad \text{for } t \in [T, \bar{T}] \quad (4)$$

$$S^*(t_0) \text{ given, } S(\bar{T}) = 0 \quad (5)$$

where $C(t)$ and $S(t)$ are the agent’s planned consumption and planned asset holding at t , and $\dot{S}(t)$ is the time derivative of $S(t)$. Note that the plan is made based on the agent’s estimated rate of return to his asset r and with his actual asset position at t_0 , denoted $S^*(t_0)$, taken as an initial condition.

The program defined by (2)-(5) is a two-stage fixed-endpoint control problem with a switch in the state equation from (3) to (4) at the agent’s retirement age T . We use the Maximum Principle for two-stage problems to solve this dynamic program. To begin, we define two multiplier functions, $\lambda_1(t)$ for $t \in [t_0, T]$ and $\lambda_2(t)$ for $t \in [T, \bar{T}]$, by two laws of motion and a matching condition

$$\dot{\lambda}_1(t) = -r\lambda_1(t), \quad \text{for } t \in [t_0, T] \quad (6)$$

$$\dot{\lambda}_2(t) = -r\lambda_2(t), \quad \text{for } t \in [T, \bar{T}] \quad (7)$$

$$\lambda_1(T) = \lambda_2(T) \quad (8)$$

where (8) is a continuity or matching condition that links the two multiplier functions at the switch point. Given (6)-(8), the solution to (2)-(5) must satisfy

$$e^{-\rho(t-t_0)}C(t)^{-\sigma} = \lambda_1(t), \quad \text{for } t \in [t_0, T] \quad (9)$$

$$e^{-\rho(t-t_0)}C(t)^{-\sigma} = \lambda_2(t), \quad \text{for } t \in [T, \bar{T}]. \quad (10)$$

Now, solve (6) and (7) to obtain $\lambda_1(t) = x_1 e^{-rt}$ for $t \in [t_0, T]$ and $\lambda_2(t) = x_2 e^{-rt}$ for $t \in [T, \bar{T}]$, respectively, where x_1 and x_2 are constants of integration. Using (8), we show that $x_1 = x_2$. Thus we can write the solution compactly as $\lambda(t) = x e^{-rt}$ for $t \in [t_0, \bar{T}]$. In consequence, we can also write (9) and (10) in a compact way,

$$e^{-\rho(t-t_0)}C(t)^{-\sigma} = x e^{-rt}, \quad t \in [t_0, \bar{T}]. \quad (11)$$

Solving (11) gives rise to

$$C(t) = y e^{gt + \frac{\rho}{\sigma} t_0}, \quad t \in [t_0, \bar{T}] \quad (12)$$

where $y \equiv x^{-1/\sigma}$ and $g \equiv (r - \rho)/\sigma$. It can be shown that, if $g = r$, then actual consumption will be monotonically increasing over time and a hump can never occur. Thus, the assumption that the consumption profile is hump-shaped implies that $g \neq r$.

We now proceed to pin down y . First, substituting (12) into (3) yields

$$S(t) = \left(d_1 + w \int^t e^{-rs} ds - y \int^t e^{(g-r)s + \frac{\rho}{\sigma} t_0} ds \right) e^{rt}, \quad \text{for } t \in [t_0, T] \quad (13)$$

where d_1 is a constant of integration. Evaluating (13) at $t = t_0$, using the initial condition $S(t_0) = S^*(t_0)$, and solving for d_1 , we get

$$S(t) = S^*(t_0) e^{r(t-t_0)} + w \int_{t_0}^t e^{r(t-s)} ds - y \int_{t_0}^t e^{(g-r)s + \frac{\rho}{\sigma} t_0 + rt} ds, \quad \text{for } t \in [t_0, T]. \quad (14)$$

Next, substituting (12) into (4) yields

$$S(t) = \left(d_2 - y \int^t e^{(g-r)s + \frac{\rho}{\sigma} t_0} ds \right) e^{rt}, \quad \text{for } t \in [T, \bar{T}] \quad (15)$$

where d_2 is a constant of integration. Evaluating (15) at $t = \bar{T}$, using the terminal condition $S(\bar{T}) = 0$, and solving for d_2 , we obtain

$$S(t) = y \int_t^{\bar{T}} e^{(g-r)s + \frac{\rho}{\sigma} t_0 + rt} ds, \quad \text{for } t \in [T, \bar{T}]. \quad (16)$$

Finally, evaluating (14) and (16) at $t = T$ and equating one another give rise to

$$y = \frac{(g-r) \left\{ S^*(t_0) e^{r(T-t_0)} - \frac{w}{r} \left[1 - e^{r(T-t_0)} \right] \right\}}{e^{r(T-\bar{T}) + g\bar{T} + \frac{\rho}{\sigma} t_0} - e^{(g-r + \frac{\rho}{\sigma}) t_0 + rT}}. \quad (17)$$

Substituting (17) into (12) and rearranging yield

$$C(t) = \frac{(g-r) \left[S^*(t_0)e^{-rt_0} - \frac{w}{r} (e^{-rT} - e^{-rt_0}) \right]}{e^{(g-r)\bar{T}} - e^{(g-r)t_0}} e^{gt}, \quad t \in [t_0, \bar{T}]. \quad (18)$$

This gives the agent's planned consumption path from the planning point t_0 onward.

We now obtain the agent's actual consumption path. We start to characterize the actual consumption by noting that the agent will actually follow his plan (18) at the initial instant t_0 when the plan is made. That is to say that his actual consumption at t_0 must satisfy (18) in which t is set to t_0 . We then note that t_0 is just an arbitrary point in time during his work life. This suggests that his actual consumption path throughout the work life must satisfy a relation characterized by replacing t_0 with t in (18)

$$C^*(t) = \frac{(g-r) \left[S^*(t)e^{-rt} - \frac{w}{r} (e^{-rT} - e^{-rt}) \right]}{e^{(g-r)\bar{T}} - e^{(g-r)t}} e^{gt}, \quad t \in [0, T] \quad (19)$$

where the law of motion for the agent's actual asset position $S^*(t)$ is governed by the actual rate of return to the asset r^* , rather than his estimated rate of return r ,

$$\dot{S}^*(t) = r^* S^*(t) + w - C^*(t), \quad \text{for } t \in [0, T]. \quad (20)$$

Equations (19)-(20), along with the initial condition, $S^*(0) = 0$, completely characterize the agent's actual consumption and actual asset position over his work life. It follows that the time derivative of $C^*(t)$ is

$$\dot{C}^*(t) = \frac{(g-r) \left[e^{(g-r)(\bar{T}-t)} C^*(t) + \dot{S}^*(t) - w e^{-r(T-t)} \right]}{e^{(g-r)(\bar{T}-t)} - 1}. \quad (21)$$

Substituting (20) into (21) yields

$$\dot{C}^*(t) = (g-r) \left\{ C^*(t) + \frac{r^* S^*(t) + w [1 - e^{-r(T-t)}]}{e^{(g-r)(\bar{T}-t)} - 1} \right\}. \quad (22)$$

Solving (19) for $S^*(t)$ and substituting the result into (22), we obtain a first order differential equation in $C^*(t)$

$$\dot{C}^*(t) = (g-r+r^*)C^*(t) + \frac{(g-r)(r-r^*)w [1 - e^{-r(T-t)}]}{r [e^{(g-r)(\bar{T}-t)} - 1]}. \quad (23)$$

The general solution to the differential equation (23) is

$$C^*(t) = \left[d + \frac{(g-r)(r-r^*)w}{r} \int^t \frac{1 - e^{-r(T-s)}}{e^{(g-r)(\bar{T}-s)} - 1} e^{-(g-r+r^*)s} ds \right] e^{(g-r+r^*)t}, \quad (24)$$

for some constant d . A particular solution is

$$C^*(t) = C^*(0)e^{(g-r+r^*)t} + \frac{(g-r)(r-r^*)w}{r} \int_0^t \frac{1-e^{-r(T-s)}}{e^{(g-r)(\bar{T}-s)}-1} e^{(g-r+r^*)(t-s)} ds. \quad (25)$$

Given $S^*(0) = 0$, (19) implies that

$$C^*(0) = \frac{(g-r)w(1-e^{-rT})}{r[e^{(g-r)\bar{T}}-1]}. \quad (26)$$

Substituting (26) back into (25) we obtain

$$C^*(t) = \frac{(g-r)w}{re^{-(g-r+r^*)t}} \left[\frac{1-e^{-rT}}{e^{(g-r)\bar{T}}-1} + (r-r^*) \int_0^t \frac{1-e^{-r(T-s)}}{e^{(g-r)(\bar{T}-s)}-1} e^{-(g-r+r^*)s} ds \right]. \quad (27)$$

This gives a closed-form solution to the agent's actual consumption during the work life.

We can then use (27) to establish the “only if” part of the proposition. We proceed by rewriting (27) as

$$C^*(t) = C_1^*(t) C_2^*(t), \quad (28)$$

where

$$C_1^*(t) \equiv we^{(g-r+r^*)t}, \quad (29)$$

$$C_2^*(t) \equiv \frac{(g-r)(1-e^{-rT})}{r[e^{(g-r)\bar{T}}-1]} + \frac{(g-r)(r-r^*)}{r} \int_0^t \frac{1-e^{-r(T-s)}}{e^{(g-r)(\bar{T}-s)}-1} e^{-(g-r+r^*)s} ds. \quad (30)$$

We note that $C_1^*(t)$ is monotone increasing in t if $g-r+r^* > 0$ but monotone decreasing in t if $g-r+r^* < 0$. By contrast, $C_2^*(t)$ is always monotone increasing in t since $r > r^*$. Hence, if $g-r+r^* \geq 0$, consumption would be monotone increasing across all $t \in [0, T]$ and no hump can exist. That is to say that $g-r+r^* < 0$ is a necessary condition for a hump in (27). This necessary condition can be rewritten as $(r-\rho)/\sigma - r+r^* < 0$. Since $r^* = \rho$, it simplifies to $(r-r^*)(1-\sigma)/\sigma < 0$, which is equivalent to $\sigma > 1$ given that σ is positive and $r > r^*$. This establishes the “only if” part of the proposition.

We prove now the “if” part of the proposition. Note that the assumed condition $\sigma > 1$ implies that $g \equiv (r-\rho)/\sigma < r$. We first evaluate (23) at $t = 0$ to get

$$\dot{C}^*(0) = (g-r+r^*)C^*(0) + \frac{(g-r)(r-r^*)w}{r} \frac{1-e^{-rT}}{e^{(g-r)\bar{T}}-1}. \quad (31)$$

Substituting (26) into (31) and combining terms, we obtain

$$\dot{C}^*(0) = \frac{g-r}{e^{(g-r)\bar{T}}-1} \frac{w}{r} (1-e^{-rT}) g > 0, \quad (32)$$

where the strict inequality holds since $g > 0$, as is implied by $r > r^* = \rho$, and $g < r$, which follows from $\sigma > 1$. We then evaluate (23) at $t = T$ to get

$$\dot{C}^*(T) = (g - r + r^*)C^*(T) < 0, \quad (33)$$

where the strict inequality holds since $C^*(T) > 0$ and $g - r + r^* = (r - \rho)/\sigma - r + r^* < 0$, which is implied by $r > r^* = \rho$ and $\sigma > 1$. Thus, with the agent's overestimation of his asset returns and a larger-than-unit inverse elasticity of intertemporal substitution in consumption, at the beginning of the work life the rate of growth in his consumption is strictly positive, while the growth rate of his consumption at the date of retirement is strictly negative. These together imply that his consumption during the working period must have one peak which lies strictly between date 0 and date T .

We next show that his consumption during the work life can have only one stationary point, so the peak is unique and there is no interior trough and thus the profile is actually hump-shaped. To help exposition, we define an auxiliary function $F(t)$ on $[0, T]$ by

$$F(t) \equiv \frac{1 - e^{-r(T-t)}}{e^{(g-r)(T-t)} - 1}. \quad (34)$$

In the light of (23), any stationary point t^* of the actual consumption profile during the work life must satisfy

$$C^*(t^*) = \frac{(g - r)(r - r^*)w}{-(g - r + r^*)r} F(t^*). \quad (35)$$

Evaluating the consumption profile (27) at the stationary point t^* , we have

$$C^*(t^*) = \frac{(g - r)w}{r e^{-(g-r+r^*)t^*}} \left[F(0) + (r - r^*) \int_0^{t^*} e^{-(g-r+r^*)s} F(s) ds \right]. \quad (36)$$

By virtue of (35) and (36), any stationary point must solve the following equation

$$F(t^*) = \frac{-(g - r + r^*)}{e^{-(g-r+r^*)t^*}} \left[\frac{F(0)}{r - r^*} + \int_0^{t^*} e^{-(g-r+r^*)s} f(s) ds \right]. \quad (37)$$

Applying integration by parts to the right hand side of (37) and manipulating, we can show that the equation reduces to

$$-\frac{F(0)}{\sigma} = \int_0^{t^*} e^{-(g-r+r^*)s} f(s) ds, \quad (38)$$

where the function f denotes the time derivative of the function F , that is, $f(s) \equiv \dot{F}(s)$.

Note that the left hand side of (38) is a strictly positive and constant real number. We can show that $f(s)$, and therefore the integrand inside the integral on the right hand side of (38), has the same sign as $h(s)$, for all $s \in [0, T]$, where

$$h(s) \equiv re^{(r-g)(\bar{T}-s)} - (r-g)e^{r(T-s)} - g. \quad (39)$$

Using (39), it is straightforward to verify that $h(T) > 0$, and that

$$\dot{h}(s) = r(r-g) \left[e^{r(T-s)} - e^{(r-g)(\bar{T}-s)} \right]. \quad (40)$$

We now break into cases.

CASE ONE: $rT \leq (r-g)\bar{T}$

We can show in this case that $\dot{h}(s) \leq 0$, for all $s \in [0, T]$, where the strict inequality holds for all $s \in (0, T]$, and for $s = 0$ as well except for the case in which $rT = (r-g)\bar{T}$. This combined with the fact that $h(T) > 0$ implies that $h(s) > 0$, and so $f(s) > 0$, for all $s \in [0, T]$. It follows that the function Φ defined by

$$\Phi(t) \equiv \int_0^t e^{-(g-r+r^*)s} f(s) ds \quad (41)$$

is strictly monotone increasing across $[0, T]$ and strictly positive on $(0, T]$. Consequently, there can exist at most one t^* that solves (38).

CASE TWO: $rT > (r-g)\bar{T}$

We proceed by first defining a useful time point

$$\hat{t} \equiv \frac{rT - (r-g)\bar{T}}{g}. \quad (42)$$

It can be verified for this case that \hat{t} so defined is strictly between date 0 and date T . We can then show that

$$\dot{h}(s) \begin{cases} > 0, & \text{for } s \in [0, \hat{t}), \\ \leq 0, & \text{for } s \in [\hat{t}, T], \end{cases} \quad (43)$$

where the equality holds only at $s = \hat{t}$.

Combining the facts that $h(T) > 0$ and that $\dot{h}(s) \leq 0$ for $s \in [\hat{t}, T]$, we conclude that $h(s) > 0$, and so $f(s) > 0$, for all $s \in [\hat{t}, T]$.

If $h(0) \geq 0$, then this and the fact that $\dot{h}(s) > 0$ for $s \in [0, \hat{t})$ imply that $h(s) \geq 0$, and so $f(s) \geq 0$, for all $s \in [0, \hat{t})$, where the strict inequality holds for $s \in (0, \hat{t})$, and the equality holds for $s = 0$ if $h(0) = 0$. Combining this and the above paragraph shows that the integrand inside the integral on the right hand side of (38) is strictly positive for all $s \in (0, T]$, and for $s = 0$ as well, except for the case in which $h(0) = 0$ then the integrand is 0 at time point 0. Thus, similarly as in CASE ONE, the function Φ defined in (41) is strictly positive and strictly monotone increasing across $(0, T]$, and thus there can exist at most one t^* that solves (38).

If $h(0) < 0$, then this and the facts that $\dot{h}(s) > 0$ for $s \in [0, \hat{t})$ and that $h(\hat{t}) > 0$ imply the existence of a unique $\tilde{t} \in (0, \hat{t})$ such that $h(s) < 0$ for $s \in [0, \tilde{t})$, $= 0$ for $s = \tilde{t}$, and > 0 for $s \in (\tilde{t}, \hat{t}]$. Combining this and the paragraph preceding the above one shows that the integrand inside the integral on the right hand side of (38) is strictly negative at all $s \in [0, \tilde{t})$, zero at $s = \tilde{t}$, and strictly positive at all $s \in (\tilde{t}, T]$. It follows that $\Phi(t)$ is strictly negative on $(0, \tilde{t}]$ and strictly monotone decreasing across $[0, \tilde{t})$ and, therefore, no point in $[0, \tilde{t}]$ can solve (38), given that the left hand side of that equation is a constant and strictly positive number. On the other hand, it follows also that $\Phi(t)$ is strictly monotone increasing with t for all $t \in (\tilde{t}, T]$, as the integrand becomes strictly positive in this range. Consequently, there can exist at most one t^* in $(\tilde{t}, T]$ that solves (38), and we have shown that T is not such a solution.

Combining the above cases proves the uniqueness of the stationary point in the agent's actual consumption profile during the work life. The existence of the peak and the uniqueness of the stationary point gives rise to a hump-shaped consumption path. Indeed, our above proof of the uniqueness can also be used to check separately that the consumption profile is strictly concave around the stationary point. To see this, note that we have shown that $f(t^*)$ must be strictly positive at the stationary point t^* . Taking the time derivative of (23) and evaluating the resultant equation at the stationary point t^* , we obtain

$$C^{*''}(t^*) = - \left[\frac{(r-g)(r-r^*)w}{r} \right] f(t^*) < 0. \quad (44)$$

This establishes the “if” part of the proposition.

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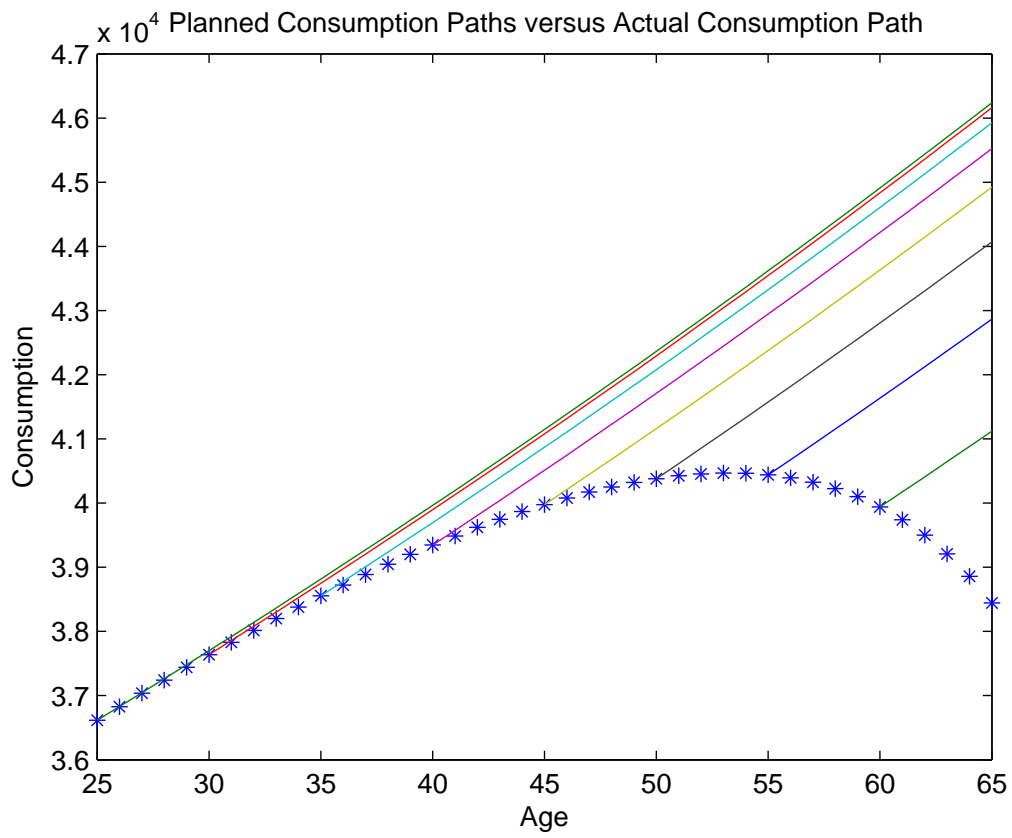


Figure 1. The actual work-life consumption path (line with stars) constitutes the envelope of many initial planned-values of consumption

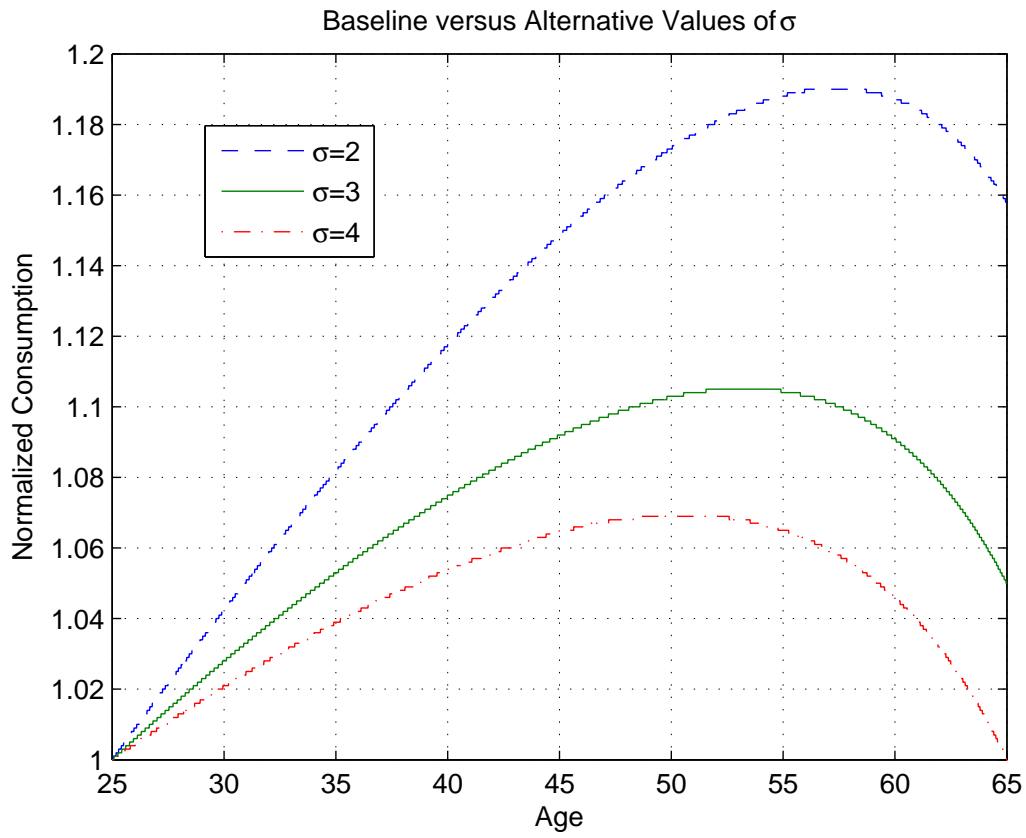


Figure 2. The work-life consumption profile normalized to the level of consumption when first entering the workforce: Under alternatives values of σ

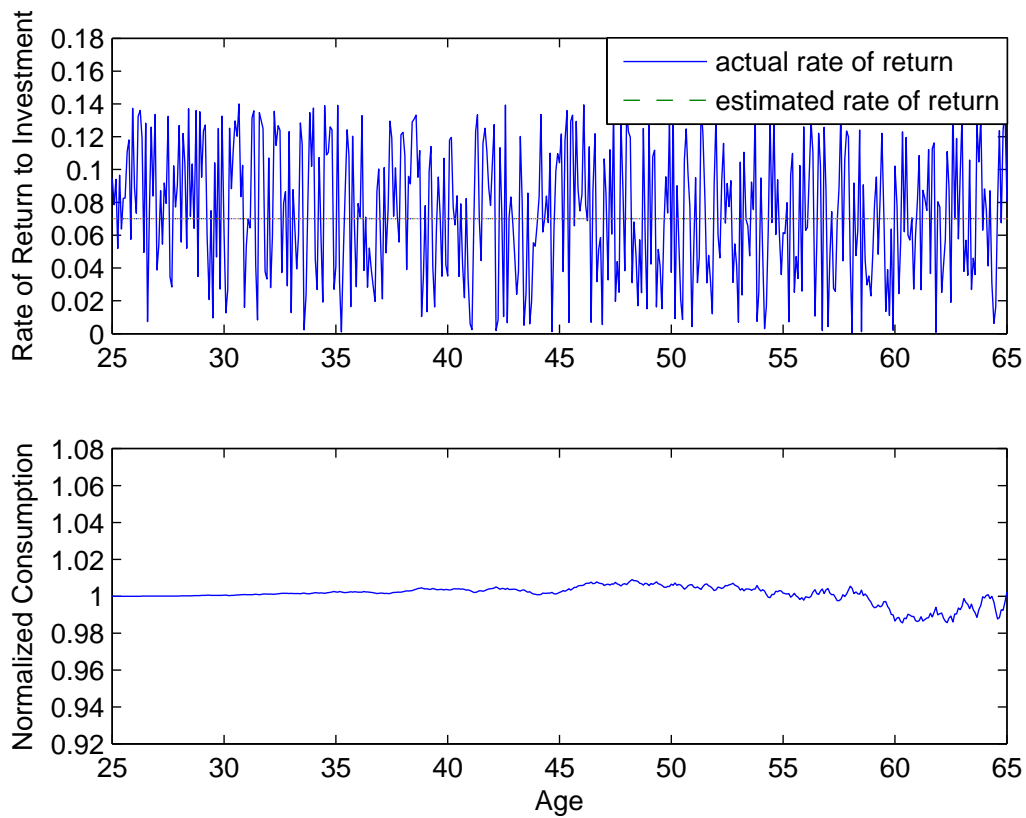


Figure 3. The work-life consumption profile normalized to the level of consumption when first entering the workforce (lower panel) under stochastic return to investment with underestimation of volatility (upper panel)

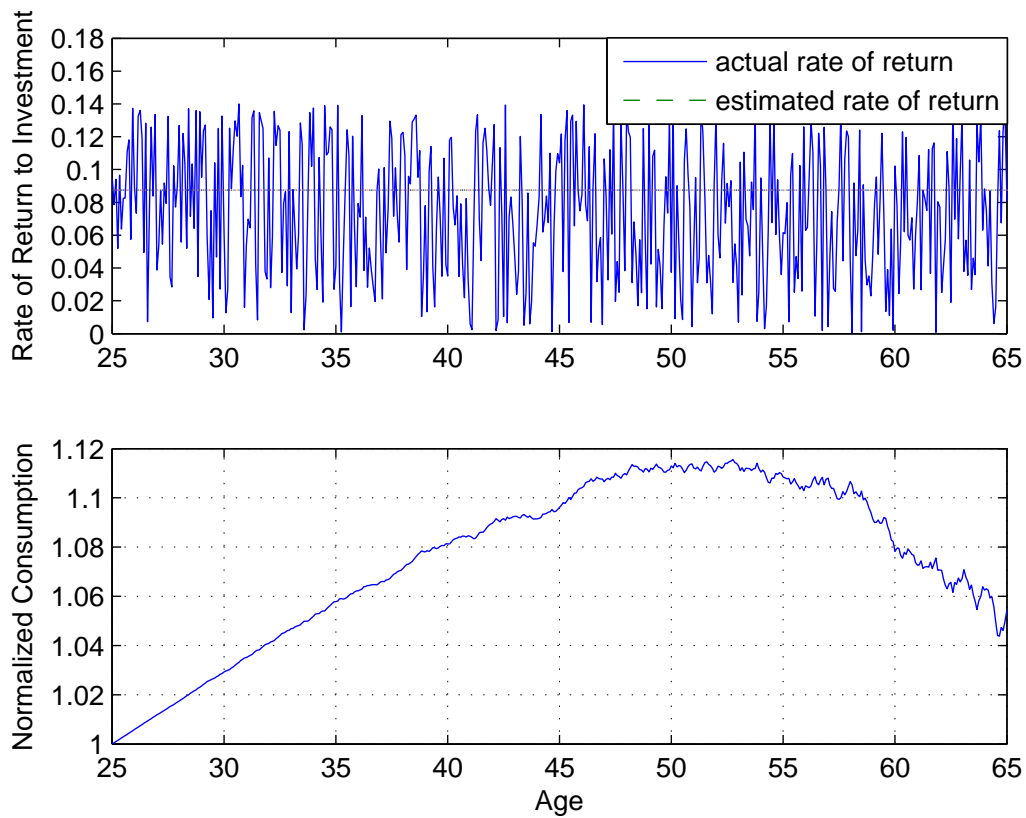


Figure 4. The work-life consumption profile normalized to the level of consumption when first entering the workforce (lower panel) under stochastic return to investment with overestimation of the mean return and underestimation of volatility (upper panel)

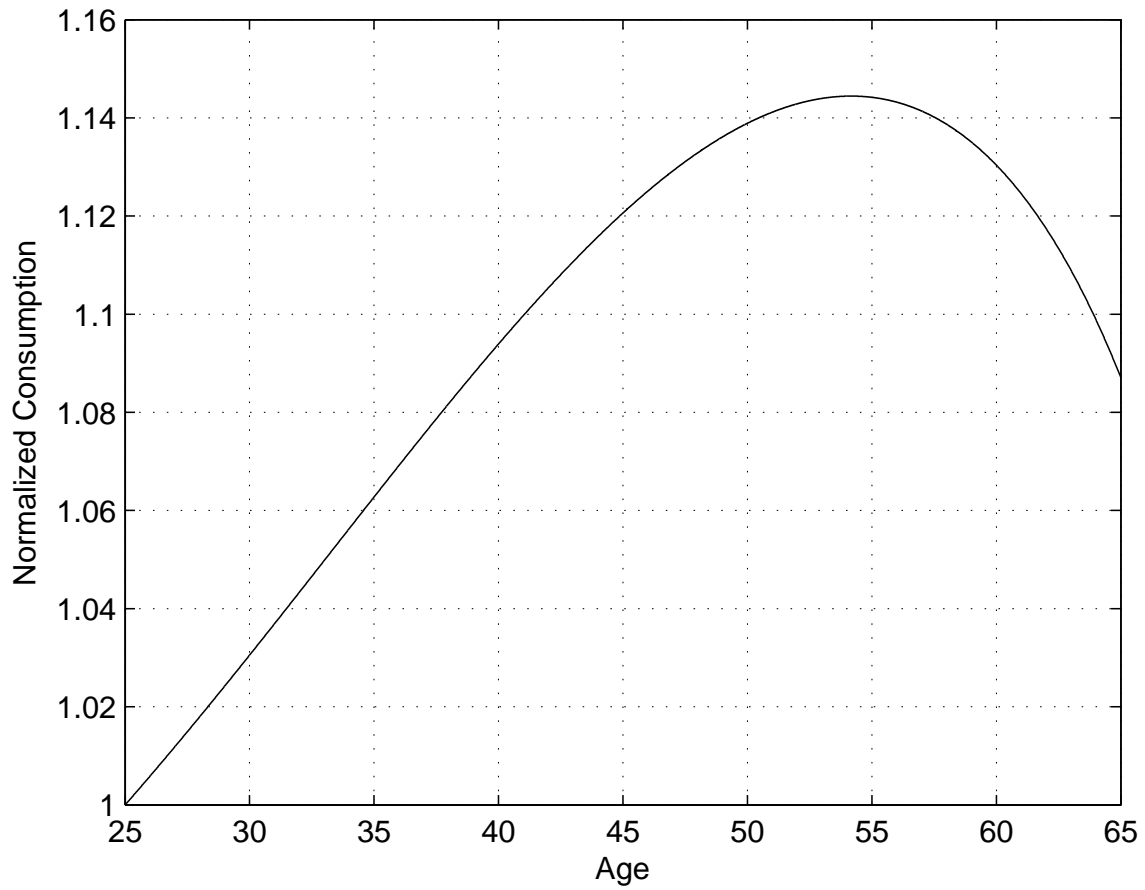


Figure 5. The work-life consumption profile normalized to the level of consumption when first entering the workforce: With Feigenbaum's (2005) quartic polynomial wage profile

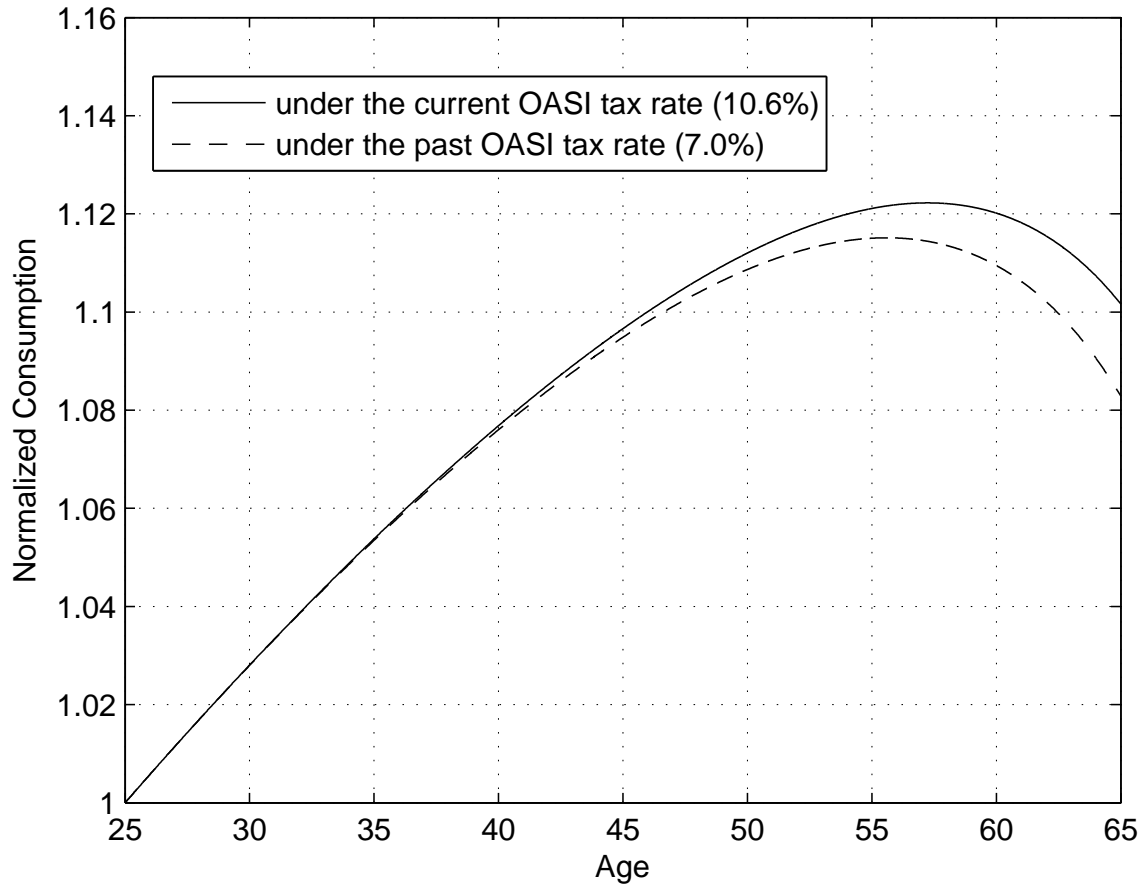


Figure 6. The work-life consumption profile normalized to the level of consumption when first entering the workforce: Under two alternative social security tax rates