THE STRUCTURE AND PERFORMANCE OF THE WORLD MARKET IN A COBB-DOUGLAS EXAMPLE

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The Structure and Performance of The World Market in a Cobb-Douglas Example[1]

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Abstract: In an international trading economy where countries set tariffs strategically, modeled using a Cobb-Douglas example, this paper studies the relationship between the structure and the performance of the world market. Using new results from monotone comparative statics in a Shapley-Shubik market game, replication of such an international trading economy is studied. It is shown that, as the economy is replicated, the equilibrium converges monotonically towards the equilibrium of a competitive equilibrium model of international trade. The distributional implications of replication are also evaluated.

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1. Introduction

It used to be said that international trade theory was a showcase for the theory of general competitive equilibrium. A central assumption in trade theory at that time was that countries are price takers on world markets. Research in that framework gave rise to some of the fundamental results of international trade, not least the efficiency of free trade. However, in recent years the idea that countries have power on world markets, and hence that they interact strategically, has become central to the international trade theory. This shift in perspective has lead to greater scope for the analysis of situations where international trade in equilibrium may not be efficient.

The purpose of this paper is to investigate in a strategic setting the relationship between the structure of the world market and its performance in terms of world economic efficiency and equity. Of course, a central purpose of the existing literature is to analyze the implications for efficiency of opening up to international trade. But the approach taken in the past literature is to compare two discrete situations, one in which there is autarky (or, in the more recent literature, an equilibrium in which trade is restricted) and the other in which there is free trade. The novel approach taken in the present paper is to characterize the trade regime endogenously in terms of the underlying economic structure. Certainly, autarky can be considered in the present model, as can free trade. But each arises endogenously, as a feature of one particular underlying economic structure, as do the levels of openness in between.

The present paper introduces new tools in monotone comparative statics (Amir and Lambson 2000, Amir 2002, Amir and Bloch 2004) to the field of international trade. These tools have been developed specifically to address questions of how market structure determines market performance. The tools have been developed in the study of Cournot competition and Shapley-Shubik market games. This present paper introduces the tools to an international trade setting and shows how they can be used to yield new insights about the relationship between the structure of the world market and its performance.

The model that is developed in this paper blends Johnson's classic (1953-54) characterization of a tariff war with a model of bilateral oligopolies. Although it was not fully appreciated at the time of his writing, Johnson's characterization of a tariff war is in fact a static Nash equilibrium of a tariff game, where the tariff game can be thought of as a terms-of-trade driven Prisoner's Dilemma.[3] One model of bilateral oligopoly is due to Bloch and Ghosal (1997), which builds in turn on the market game originated by Shapley and Shubik (1977). A feature common to both Johnson and Bloch and Ghosal is that trade takes place in just two goods. A difference is that while in Johnson there are just two trading nations, in Bloch and Ghosal there are many traders. Adopting the terminology of the literature on market games, we would say that while in Johnson there is only one country on each "side of the market," in Bloch and Ghosal there may be any number. The model of this present paper adopts the underlying economic framework of Bloch and Ghosal's bilateral oligopoly to extend Johnson's tariff war model so that there is more than one country on each side of the world market.

The analysis of a tariff game in the present paper follows Amir and Bloch's analysis of a Shapley Shubik market game similar to that of Bloch and Ghosal. In the Cobb-Douglas example, the tariff game between countries on one side of the market is supermodular. To put this in more familiar terms, tariffs between countries on one side of the market are strategic complements; if one country lowers its tariff then it is a best response for all other countries on that side of the market to lower their tariffs as well. Consequently, the comparative statics properties of the model are found to be *monotone*. Comparative statics can then be carried out at two levels. We will first examine the effects of a change in the number of countries on one side of the market, holding constant the number of countries on the other side of the market. Combining the analysis of each side of the market separately, we will then consider the effects of a simultaneous increase in the number of countries on both sides of the market that holds them in the same proportion. This we will refer to as a *replication* of the international trading economy.[4]

The analysis of comparative statics in this extended model of Johnson's tariff war contributes to our understanding of the effects of international market structure on an international trading equilibrium in three different ways. First, the analysis highlights

[3] See Syropoulos (2002), which undertakes a comprehensive analysis of Johnson's model in a modern game theoretic setting and obtains new results.

[4] The idea behind increasing the number of countries is to understand how the structure of the market affects its performance. It is not intended to consider the question of how, say, trade between World Trade Organization (WTO) members is affected by the introduction of new members, although the framework of this paper could be developed in that direction.

conditions under which an increase in the number of countries affects the equilibrium terms of trade and volume of trade in a predictable manner. Extending Amir and Bloch, a key result provides conditions under which the trading equilibrium of the tariff game converges monotonically to the perfectly competitive equilibrium of the economy (the efficient outcome of free trade) as the economy is replicated. This provides a nice interface between the early international trade literature based on perfect competition and the newer literature based on strategic interaction between countries.

Second, it is possible to test the robustness of predictions made about strategic trade policy obtained in a partial equilibrium environment to the context of general equilibrium. While Brander and Spencer's (1984) model of strategic tariff setting has become a workhorse model for the analysis of strategic trade policy, it is criticized because it only allows analysis of strategic tariff setting in one sector, while trade in the numeraire good goes unfettered. The model of this present paper allows the extension of such strategic tariff policy analysis to a general equilibrium setting in which both sectors are subject to trade intervention simultaneously.

Finally, the model can be used to provide a new perspective on the equity implications of international trade. In particular, the more scarce is the good that a country exports in equilibrium, the worse off a country becomes when the economy is replicated. This surprising result occurs because although a country gains from the increase in competition to supply it with exports from the other side of the market, at the same time its own export becomes less scarce on world markets and the overall effect of replication is to undermine its power on world markets.

In terms of practical policy implications, the mechanics of the third (final) feature seem to be of greatest interest. Attention has focused recently on developing country complaints that they are badly served under the dispute settlement system of the WTO (see Bagwell, Mavroidis and Staiger 2005 for example). According to conventional wisdom, this is because developing countries are small, or the import elasticity of demand for their exports is assumed to be relatively high because they export similar or homogeneous goods (agricultural products, for example). Consequently, developing country power to retaliate using tariffs is weak. The model of the present paper suggests an alternative reason for why developing country tariff retaliation is weak (since all countries in the model are the same size and import substitution elasticities are equal ex ante). It is that there are many such developing countries relative to developed countries and so they compete more aggressively with one another to put goods on their side of the market, hence undermining their collective terms-of-trade.

It is important to emphasize that because the model presented in this paper is based on an example the results are only suggestive. Nevertheless, even though the model is based around trade in just two goods, and strong assumptions are made about functional forms, the results seem intuitively plausible and bring out more sharply effects that are likely to be present in more complex general models.

The paper proceeds as follows. In the next section, we will set up the basic model of the international market. Section 3 then sets up the tariff game which is played using the model set up in Section 2, and Section 4 defines equilibrium of that game. Section 5 introduces the Cobb-Douglas example. Characterization of the trading equilibrium and comparative statics are carried out in Section 6. Section 7 presents a discussion of the results and conclusions.

2. The Model of an International Trading Economy

We will work with an 'endowments model' of international trade that is standard except that there can be any number of countries on each side of the market. There are two sets of countries: manufacturers $A = \{1, 2, ..., a, ..., m\}$; and primary product producers $B = \{m+1, m+2, ..., b, ..., n+m\}$. There are m countries in the set A and there are n countries in the set B. Following notation from the matching literature, a will be used to denote the representative member of A and b will be used to denote the representative member of B. Where convenient, we will use i to refer to a country in either set A or in set B.

Each country in A has an endowment (normalized to unity) of a homogeneous manufactured good, referred to as Good 1, and each country in B has an endowment (also normalized to unity) of a homogeneous agricultural product, referred to as good 2. Goods 1 and 2 are the only two goods available.

Each country has an identical mass of atomistic consumers who are (domestic) price takers. Each consumer behaves non-strategically, and solves a standard consumption problem. The government of each country behaves as a benevolent dictator, setting tariffs in order to maximize the welfare of the representative citizen.[5]

Denote by x_{ij} the consumption of good $j \in \{1, 2\}$ in country i. The preferences of the representative consumer in country a over (x_{aj}) are specified by the utility function,

$$u_a = u^A(x_{a1}, x_{a2}).$$

Similarly, for the representative consumer in country b, preferences are defined over (x_{bj}) by the utility function,

$$u_b = u^B(x_{b2}, x_{b1}).$$

Following the literature on market games, utility functions are assumed to satisfy the following general properties:

1. The utility functions are twice continuously differentiable, strictly increasing and strictly concave.[6]

2. The utility functions satisfy the boundary conditions $\lim_{x_{aj}\to 0} \partial u^A/\partial x_{aj} = +\infty$, $\lim_{x_{bj}\to 0} \partial u^B/\partial x_{bj} = +\infty$, $j \in \{1, 2\}$.

3. The utility functions satisfy the symmetry assumption $u^A(x, y) = u^B(y, x)$.

The consumer in country i faces a budget constraint

$$\sum_{j=1}^{2} p_j(1 + \tau_i)x_{ij} = p_i + R_i, \tag{2.1}$$

where p_j, τ_i, $R_i = p_j\tau_i x_{ij}$ are, respectively, the world price of good j, the tariff set by country i on good j, and tariff revenue in country i, which as usual is returned to the consumer in a lump-sum. Without loss of generality, countries in A impose a zero tariff

[5]This framework could be extended to take account of governments' wider distributive/political concerns.

[6]The assumption here of strict concavity is slightly stronger than the usual assumption made in market games of strict quasi-concavity. It is made here to ensure that a well defined and smoothly varying solution exists to the consumer problem based on tariffs.

on Good 1 and countries in B impose a zero tariff on Good 2. For all other tariffs, let $\tau_i \in \mathbb{R}_+$; tariffs are non-negative.[7]

Denote the vector of tariffs set by all countries in A as $\boldsymbol{\tau}_a = \{\tau_1, ..., \tau_a, ..., \tau_m\}$ and the vector of tariffs set by all countries in B as $\boldsymbol{\tau}_b = \{\tau_{m+1}, ..., \tau_b, ..., \tau_{n+m}\}$. It will also be convenient to have notation for the tariffs of all countries in A *except* country a; $\boldsymbol{\tau}_{-a} = \{\tau_1, ..., \tau_{a-1}, \tau_{a+1}, ..., \tau_m\}$. The tariff vector $\boldsymbol{\tau}_{-b}$ is defined analogously. Finally, for the total tariff vector we have $\boldsymbol{\tau} = \{\boldsymbol{\tau}_a, \boldsymbol{\tau}_b\} = \{\tau_1, ..., \tau_m, \tau_{m+1} ..., \tau_{n+m}\}$.

3. The Tariff Game

The extensive form of the game is as follows. First, each country i simultaneously chooses an import tariff τ_i. Then, given world prices $\mathbf{p} = \{p_1, p_2\}$, and the tariff τ_i, the consumer in country i chooses x_{i1} and x_{i2} to maximize u_i subject to the budget constraint. This yields the usual excess demands and the indirect utility function. For country a, $v_a = v^A(\mathbf{p},\tau_a) = u^A\left(x_{a1}\left(\mathbf{p}, \tau_a\right), x_{a2}\left(\mathbf{p}, \tau_a\right)\right)$. Then, conditional on $\boldsymbol{\tau}$, markets clear and world prices $\mathbf{p}$ are determined.[8] These world prices will of course depend on tariffs i.e. $\mathbf{p} = \mathbf{p}(\boldsymbol{\tau})$. If equilibrium prices are unique, given tariffs, then the mapping $\mathbf{p}(.)$ is one-to-one. Then the indirect utility function can be written as a function only of tariffs.[9]

Finally, as this is an endowment economy, exports of Good 1 by a can be written as $e_a\left(\mathbf{p},\tau_a\right) = 1 - x_{a1}(\mathbf{p},\tau_a)$. Similarly, exports of Good 2 by b can be written as $e_b\left(\mathbf{p},\tau_b\right) = 1 - x_{b2}(\mathbf{p},\tau_b)$. Then for *total exports* we have $E_A\left(\mathbf{p}, \boldsymbol{\tau}_a\right) = \sum_{a \in A} e_a\left(\mathbf{p},\tau_a\right)$ and $E_B\left(\mathbf{p}, \boldsymbol{\tau}_b\right) = \sum_{b \in B} e_b\left(\mathbf{p},\tau_b\right)$. Also, it will be convenient to have notation for *exports by all countries in A except country a*; $E_{A-a}\left(\mathbf{p}, \boldsymbol{\tau}_{-a}\right) = E_A\left(\mathbf{p}, \boldsymbol{\tau}_a\right) - e_a\left(\mathbf{p},\tau_a\right)$. Symmetrically, $E_{B-b}\left(\mathbf{p}, \boldsymbol{\tau}_{-b}\right) = E_B\left(\mathbf{p}, \boldsymbol{\tau}_b\right) - e_b\left(\mathbf{p},\tau_b\right)$.

[7] This framework could easily be extended to allow for the possibility of import subsidies as well. In general, trade subsidies introduce a number of separate issues which we want to leave aside here; see Jackson (1989).

[8] As this is a general equilibrium model, prices are determined only up to a scalar, and so some normalization (e.g. choice of numeraire) must be made. This technical detail and others are dealt with below.

[9] Transfers are not allowed between countries. This is deemed to be a reasonable assumption in the trade literature, since although transfers do happen they are often constrained by extraneous factors. Thus, an equilibrium without transfers is often deemed to be a reasonable characterization of observed outcomes.

We can now derive an expression for world prices strictly in terms of tariffs. By world market clearing,

$$p_1 E_A(\mathbf{p}, \boldsymbol{\tau}_a) = p_2 E_B(\mathbf{p}, \boldsymbol{\tau}_b).$$

Without loss of generality, let $p_1 = p$ and let $p_2 = 1$. Rearranging, we have

$$p = \frac{E_B(\mathbf{p}, \boldsymbol{\tau}_b)}{E_A(\mathbf{p}, \boldsymbol{\tau}_a)}. \tag{3.1}$$

If each function $e_a(\mathbf{p}, \tau_a)$ and $e_b(\mathbf{p}, \tau_b)$ is continuous, the world market clearing condition implicitly defines a mapping from the vector of tariffs $\{\boldsymbol{\tau}_a, \boldsymbol{\tau}_b\}$ to p:

$$p = p(\boldsymbol{\tau}).$$

We are now able to see how the market game framework is being extended here to the context of an international trading economy. In a market game, a player is able to choose directly the quantity that he puts on the market. To make the analogy to the present international trade setting, a standard market game would model a situation in which each government were running a command economy and able to dictate the quantity exported by its country; the government of a could dictate e_a and b would dictate e_b directly. In the present model, each government can only affect the quantity that its country exports by affecting consumers' decisions through tariff setting. In all other respects, the underlying economic exchange that takes place is the same in a tariff game as in a market game.

There is an idea here that will appear unfamiliar in the context of a tariff game and it should be clarified. The idea is that a country manipulates import tariffs strategically in order to determine the quantity exported. Usually we emphasize the more obvious relationship between the import tariff and the quantity imported. But we know by the Lerner symmetry theorem that an export tax has an equivalent import tariff. The present paper uses the principle of the Lerner symmetry theorem to make a novel switch in focus from imports to exports, in order to highlight the link to the literature on market games. While the Lerner symmetry theorem is well know, it has not been exploited in a strategic setting such as this before. But it should be emphasized that the game is essentially equivalent to a standard tariff game.

4. Equilibrium and Efficiency

In a *trading equilibrium*,

$$E_A\left(p(\boldsymbol{\tau}),\tau_a\right) > 0 \text{ and } E_B\left(p(\boldsymbol{\tau}),\tau_b\right) > 0.$$

For any vector of tariffs $\{\boldsymbol{\tau}_a, \boldsymbol{\tau}_b\}$ in a trading equilibrium, the final allocation obtained by country a is given by

$$\left(x_{a1}\left(p\left(\boldsymbol{\tau}\right),\tau_a\right), x_{a2}\left(p\left(\boldsymbol{\tau}\right),\tau_a\right)\right) = \left(1 - e_a\left(p\left(\boldsymbol{\tau}\right),\tau_a\right), e_a\left(p\left(\boldsymbol{\tau}\right),\tau_a\right)p\left(\boldsymbol{\tau}\right)\right)$$

and the final allocation obtained by country b is given by

$$\left(x_{b2}\left(p\left(\boldsymbol{\tau}\right),\tau_b\right), x_{b1}\left(p\left(\boldsymbol{\tau}\right),\tau_b\right)\right) = \left(1 - e_b\left(p\left(\boldsymbol{\tau}\right),\tau_b\right), \frac{e_b\left(p\left(\boldsymbol{\tau}\right),\tau_b\right)}{p\left(\boldsymbol{\tau}\right)}\right).$$

Otherwise there is autarky.

This way of writing the final allocation that each country obtains in a trading equilibrium emphasizes the effect of tariff policy on exports.

We can now define an equilibrium in tariffs as a vector of tariffs $(\hat{\tau}_1, ..., \hat{\tau}_m, \hat{\tau}_{m+1}, ..., \hat{\tau}_{n+m})$ such that

(i) for any country $a \in A$, $\hat{\tau}_a$ maximizes

$$u^A\left(1 - e_a\left(p\left(\tau_a, \hat{\boldsymbol{\tau}}_{-a}, \hat{\boldsymbol{\tau}}_b\right),\tau_a\right), e_a\left(p\left(\tau_a, \hat{\boldsymbol{\tau}}_{-a}, \hat{\boldsymbol{\tau}}_b\right),\tau_a\right)p\left(\tau_a, \hat{\boldsymbol{\tau}}_{-a}, \hat{\boldsymbol{\tau}}_b\right)\right).$$

(ii) for any country $b \in B$, $\hat{\tau}_b$ maximizes

$$u^B\left(1 - e_b\left(p\left(\hat{\boldsymbol{\tau}}_a, \tau_b, \hat{\boldsymbol{\tau}}_{-b}\right),\tau_b\right), \frac{e_b\left(p\left(\hat{\boldsymbol{\tau}}_a, \tau_b, \hat{\boldsymbol{\tau}}_{-b}\right),\tau_b\right)}{p\left(\hat{\boldsymbol{\tau}}_a, \tau_b, \hat{\boldsymbol{\tau}}_{-b}\right)}\right).$$

Standard arguments can be used to prove existence of an equilibrium, which is just a Nash equilibrium of the tariff game.

Following standard definitions, *world welfare* is the sum of all national welfares, and *world efficiency* is given by

$$\max_{\boldsymbol{\tau}} \sum_{a\in A} u^A\left(p\left(\boldsymbol{\tau}\right), \boldsymbol{\tau}\right) + \sum_{b\in B} u^B\left(p\left(\boldsymbol{\tau}\right), \boldsymbol{\tau}\right).$$

5. The Cobb-Douglas Example

The preferences of the representative consumer in country a over (x_{aj}) are given by the following Cobb-Douglas utility function:

$$u_a = u^A(x_{a1}, x_{a2}) = (x_{a1})^{1-\alpha}(x_{a2})^{\alpha}. \tag{5.1}$$

The preferences of the representative consumer in country b over (x_{bj}) are given by the utility function,

$$u_b = u^B(x_{b2}, x_{b1}) = (x_{b2})^{1-\alpha}(x_{b1})^{\alpha}. \tag{5.2}$$

It is easily checked that (5.1) and (5.2) satisfy properties 1-3 introduced above. We can also verify that both goods are normal under Cobb-Douglas preferences.[10] Following Amir and Bloch (2004), Good 1 is a *normal good* for country a if and only if $\Delta_{a1} \equiv u_y^A u_{xy}^A - u_x^A u_{yy}^A > 0$. Good 2 is a *normal good* for country a if and only if $\Delta_{a2} \equiv u_x^A u_{xy}^A - u_y^A u_{xx}^A > 0$. By the symmetry of preferences, Good 1 is a *normal good* for country b if and only if $\Delta_{b1} \equiv u_x^B u_{xy}^B - u_y^B u_{xx}^B > 0$, and Good 2 is a *normal good* for country b if and only if $\Delta_{b2} \equiv u_y^B u_{xy}^B - u_x^B u_{yy}^B > 0$. As Amir and Bloch point out, this definition of normality requires the property that demand for the good is increasing in income *for all prices.*

Before undertaking a characterization of equilibrium, let us solve the model for the Cobb-Douglas preferences specified above under the assumption that the vectors of tariffs $\boldsymbol{\tau}_a$ and $\boldsymbol{\tau}_b$ are given. By doing this, we will be able to highlight a key property of the model under Cobb-Douglas that will help to highlight the link between a market game and a tariff game. The property is that e_a depends only on τ_a. In general, exports are given by $e_a(p(\boldsymbol{\tau}), \tau_a)$. Under Cobb-Douglas preferences, the model has the special feature that e_a can be written only as a function of τ_a; $e_a(\tau_a)$. This is because the terms in $p(\boldsymbol{\tau})$ cancel, as we shall see below. Thus, while the government cannot choose e_a directly as in a market game, under Cobb-Douglas preferences we can analyze in a tractable way how the government can set τ_a in order to choose e_a indirectly.

We will work out the problem of country a, the preferences of which are given by (5.1). The problem of country b is analogous. The consumer optimization problem gives

[10] From now on, we will write $\partial u^A(x_{a1}, x_{a2})/\partial x_{a1}$ as u_x^A, $\partial u^A(x_{a1}, x_{a2})/\partial x_{a2}$ as u_y^A, $\partial^2 u^A(x_{a1}, x_{a2})/\partial x_{a1}\partial x_{a2}$ as u_{xy}^A and so on. Symmetrically for $u^B(x_{a2}, x_{a1})$, $\partial u^B(x_{a2}, x_{a1})/\partial x_{a2}$ will appear as u_x^B and so on.

demands for the two goods:

$$x_{a1} = (1-\alpha)\left(\frac{p_1 + R_a}{p_1}\right); \tag{5.3}$$

$$x_{a2} = \alpha\left(\frac{p_1 + R_a}{p_2\,(1+\tau_a)}\right). \tag{5.4}$$

Using the fact that $R_a = p_2 \tau_{a2} x_{a2}$, we have

$$x_{a1} = \frac{(1-\alpha)\,(1+\tau_a)}{(1-\alpha)\,\tau_a + 1}.$$

Rewriting in terms of exports,

$$e_a\,(\tau_a) = 1 - \frac{(1-\alpha)\,(1+\tau_a)}{(1-\alpha)\,\tau_a + 1} = \frac{\alpha}{1 + (1-\alpha)\,\tau_a}. \tag{5.5}$$

Here we see how e_a depends only on τ_a. We can use $e_a\,(\tau_a)$ to calculate the response of exports to a change in tariffs:

$$\frac{de_a}{d\tau_a} = -\frac{(1-\alpha)\,\alpha}{(1+(1-\alpha)\,\tau_a)^2} < 0.$$

From this we see that $e_a\,(\tau_a)$ is everywhere decreasing in τ_a.

Choosing $p_1 = p$ and letting $p_2 = 1$, we can now derive an expression for the indirect utility function in terms of exports:

$$\begin{aligned} & u^A\,(1 - e_a\,(\tau_a)\,, e_a\,(\tau_a)\,p) \\ = & \left(\frac{(1-\alpha)\,(1+\tau_a)}{(1-\alpha)\,\tau_a + 1}\right)^{1-\alpha}\left(p\left(\frac{\alpha}{((1-\alpha)\,\tau_a + 1)}\right)\right)^{\alpha} \end{aligned}$$

So far we have been writing p as if it were parametric. We can now introduce the fact that, by (3.1), p depends on $\boldsymbol{\tau} = (\tau_1, ..., \tau_m, \tau_{m+1}, ..., \tau_{n+m})$;

$$\begin{aligned} p(\tau_1, ..., \tau_m, \tau_{m+1}, ..., \tau_{n+m}) &= \frac{E_B\,(\boldsymbol{\tau}_b)}{E_A\,(\boldsymbol{\tau}_a)} \\ &= \frac{\sum_{b\in B}\left(\frac{\alpha}{1+(1-\alpha)\tau_b}\right)}{\sum_{a\in A}\left(\frac{\alpha}{1+(1-\alpha)\tau_a}\right)} \end{aligned} \tag{5.6}$$

Note that $p_{\tau_a} > 0$ i.e. an increase in τ_a improves a's terms of trade. Analogously, $p_{\tau_b} < 0$; an increase in country b's tariff , τ_b, improves b's terms of trade.

Finally, it is easily verified that for the above set up world efficiency is maximized when all countries adopt free trade.

6. Characterization of Equilibrium

In principle, despite the simplifications we have made, the tariff game described here is quite complex. Each country must decide its strategy not just against the countries on the other side of the market but against the countries on its own side of the market as well. Following Amir and Bloch (2004), which builds in turn on Amir and Lambson (2000), we can simplify the problem by characterizing the tariff game played between countries on one side of the market, taking as given the tariffs of countries on the other side of the market. Given a simple characterization of this game, it is then straight forward to characterize the equilibrium of the tariff game between countries on both sides of the market.

6.1. Characterization of equilibrium for one side of the market

In the symmetric Cournot oligopoly analyzed by Amir and Lambson, the optimal quantity choice of an oligopolist only depends on the total output of the $(n-1)$ remaining firms. Amir and Bloch extend this approach to a market game setting in which there are two sides to the market as in the model of this present paper. They show that the optimal quantity placed on the market by a player on one side of the market depends on the quantity choices of the other $(n-1)$ players on the same side of the market, taking as given the quantity choices of players on the other side of the market. In this section we will show that the same approach as taken by Amir and Bloch can be extended to the present setting of a tariff game. We will hold the tariffs on one side of the market constant and analyze the tariff game played between countries on the other side of the market.

Formally, consider a symmetric tariff game, $\Gamma(\boldsymbol{\tau}_b)$, played by the manufactures producers in A, when the tariffs of primary product exporters in B are fixed at $\boldsymbol{\tau}_b$. Under Cobb-Douglas, the expression for preferences can be simplified to

$$\begin{aligned} u^A(\tau_a; \boldsymbol{\tau}_{-a}, \boldsymbol{\tau}_b) &= u^A\left(1 - e_a(\tau_a), e_a(\tau_a)\frac{E_B(\boldsymbol{\tau}_b)}{e_a(\tau_a) + E_{A-a}(\boldsymbol{\tau}_{-a})}\right) \\ &= (1 - e_a(\tau_a))^{1-\alpha}\left(e_a(\tau_a)\frac{E_B(\boldsymbol{\tau}_b)}{e_a(\tau_a) + E_{A-a}(\boldsymbol{\tau}_{-a})}\right)^{\alpha} \end{aligned} \tag{6.1}$$

where $E_A(\boldsymbol{\tau}_a) = e_a(\tau_a) + E_{A-a}(\boldsymbol{\tau}_{-a})$. We may then define a *manufacturer's reaction* *function* as

$$\hat{\tau}_a\left(\boldsymbol{\tau}_{-a}\right) \equiv \arg\max_{\tau_a} \left\{ u^A \left(1 - e_a\left(\tau_a\right), e_a\left(\tau_a\right) \frac{E_B\left(\boldsymbol{\tau}_b\right)}{e_a\left(\tau_a\right) + E_{A-a}\left(\boldsymbol{\tau}_{-a}\right)} \right) : \tau_a \in \mathbb{R}_+ \right\}, \quad (6.2)$$

where $\hat{\tau}_a\left(\boldsymbol{\tau}_{-a}\right)$ is a *best response tariff*. For any $\boldsymbol{\tau}_{-a}$ and $E_{A-a}\left(\boldsymbol{\tau}_{-a}\right)$, $\hat{\tau}_a$ is chosen in (6.2) to obtain the welfare maximizing level of e_a in (6.1).

Note for future reference (particularly for Proposition 2) that it is equally valid to express the best response tariff $\hat{\tau}_a$ as a function of $E_{A-a}\left(\boldsymbol{\tau}_{-a}\right)$, that is $\hat{\tau}_a\left(E_{A-a}\left(\boldsymbol{\tau}_{-a}\right)\right)$, rather than more compactly as $\hat{\tau}_a\left(\boldsymbol{\tau}_{-a}\right)$. This will make it possible to model the relationship between a responding country's exports, e_a, and the exports of all the countries in $A\backslash a$, expressed by the function $e_a\left(\hat{\tau}_a\left(E_{A-a}\left(\boldsymbol{\tau}_{-a}\right)\right)\right)$.

In the first step towards the characterization of equilibrium, let us first characterize the best response tariff function. It is clear from (6.2) that the best response tariff $\hat{\tau}_a\left(\boldsymbol{\tau}_{-a}\right)$ is related to $\boldsymbol{\tau}_{-a}$ through the impact of a change of $\boldsymbol{\tau}_{-a}$ on $E_{A-a}\left(\boldsymbol{\tau}_{-a}\right)$ and, in turn, the effect of a change in τ_a on e_a. Therefore, a convenient way to characterize the best response tariff function of the $\Gamma\left(\boldsymbol{\tau}_b\right)$ game is in terms of exports.

Lemma 1. *In the game $\Gamma\left(\boldsymbol{\tau}_b\right)$, for any $\boldsymbol{\tau}'_{-a} \neq \boldsymbol{\tau}_{-a}$, we have*

$$\frac{e'_a\left(\hat{\tau}'_a\left(\boldsymbol{\tau}'_{-a}\right)\right) - e_a\left(\hat{\tau}_a\left(\boldsymbol{\tau}_{-a}\right)\right)}{E'_{A-a}\left(\boldsymbol{\tau}'_{-a}\right) - E_{A-a}\left(\boldsymbol{\tau}_{-a}\right)} > -1.$$

This characterization of the reaction function says that $e_a\left(\tau_a\right)$ increases in response to an increase in $E_{A-a}\left(\boldsymbol{\tau}_{-a}\right)$. And since both $e_a\left(\tau_a\right)$ and $E_{A-a}\left(\boldsymbol{\tau}_{-a}\right)$ are decreasing in their arguments, the implication is that $\hat{\tau}_a$ decreases in response to a decrease in $\boldsymbol{\tau}_{-a}$; $\hat{\tau}_a\left(\boldsymbol{\tau}_{-a}\right)$ is an increasing function. Thus, the tariff game between countries in A is supermodular. The proof of Lemma 1 establishes that the condition depends on the normality of Goods 1 and 2 for manufacturers, which holds for Cobb-Douglas preferences as we noted above.

To see the intuition for Lemma 1, look at (6.1). As $E_{A-a}\left(\boldsymbol{\tau}_{-a}\right)$ is increased, through a reduction in some element of $\boldsymbol{\tau}_{-a}$, this brings about a reduction in the terms-of-trade of all countries in A, including country a, and hence brings about a reduction in the purchasing power of a's endowment. This may be thought of equivalently as a fall in a's income. Since Goods 1 and 2 are both normal, country a demands less of both goods because its income has fallen, and therefore exports more of its endowment; $e_a(\tau_a)$ is increased through a reduction in τ_a.

The next result shows that, since the $\Gamma(\boldsymbol{\tau}_b)$ game is supermodular, the Nash equilibria of the $\Gamma(\boldsymbol{\tau}_b)$ game are symmetric.

Proposition 1. *For any number of manufacturers, m, all Nash equilibria of the game $\Gamma(\boldsymbol{\tau}_b)$ are symmetric.*

The principle established in Lemma 1, that $e_a(\hat{\tau}_a(\boldsymbol{\tau}_{-a}))$ increases with $E_{A-a}(\boldsymbol{\tau}_{-a})$, underpins the symmetry of equilibrium established in Proposition 1. To see why the equilibrium must be symmetric, suppose to the contrary that an equilibrium exists in which Countries 1 and 2 in A set different tariffs; $\hat{\tau}_1 \neq \hat{\tau}_2$. And suppose without loss of generality that $\hat{\tau}_1 < \hat{\tau}_2$. Then it follows by (5.5) and the definition of $E_{A-a}(\boldsymbol{\tau}_{-a})$ that $E_{A-1}(\boldsymbol{\tau}_{-1}) < E_{A-2}(\boldsymbol{\tau}_{-2})$. We also require that, by definition, $E_{A-1}(\boldsymbol{\tau}_{-1}) + e_1(\hat{\tau}_1) = E_{A-2}(\boldsymbol{\tau}_{-2}) + e_2(\hat{\tau}_2) = E_A$, which in turn implies that $e_a(\hat{\tau}_2) < e_1(\hat{\tau}_1)$. But this contradicts the fact that, by Lemma 1, $e_a(\tau_a)$ is increasing in $E_{A-a}(\boldsymbol{\tau}_{-a})$ through the choice of best-response tariffs. Of course, a symmetrical result holds for the primary product producers in B.

The next result is stronger. It shows that the equilibrium of the tariff game between countries in A has a unique Nash equilibrium tariff $\hat{\tau}_a$ under Cobb-Douglas preferences, and that $\hat{\tau}_a$ is decreasing in m.

Proposition 2. *For any number of manufacturers, m, the tariff game $\Gamma(\boldsymbol{\tau}_b)$ admits a unique Nash equilibrium. Furthermore, the unique equilibrium tariff, $\hat{\tau}_a$, is decreasing in m and total exports E_A are increasing in m.*

In a symmetrical equilibrium, it must be the case that $e_a(\hat{\tau}_a) = E_{A-a}(\boldsymbol{\tau}_{-a}) / (m-1)$. Because (5.5) is a differentiable function, and because (6.1) is twice continuously differentiable, by implicit differentiation we may evaluate $\Delta \equiv \partial e_a(\hat{\tau}_a(E_{A-a}(\boldsymbol{\tau}_{-a}))) / \partial E_{A-a}(\boldsymbol{\tau}_{-a})$. If there exists more than one equilibrium, then it must be the case that $\Delta > 1/(m-1)$ at one or more equilibrium. The proof shows that when Good 1 is normal then this cannot happen because $e_a(\hat{\tau}_a(E_{A-a}(\boldsymbol{\tau}_{-a})))$ is increasing in $E_{A-a}(\boldsymbol{\tau}_{-a})$ but at a decreasing rate. Recall that an increase in $E_{A-a}(\boldsymbol{\tau}_{-a})$ reduces the purchasing power of country a, and hence causes a to demand less of Good 1 and hence export more. But also observe that Good 2 is now relatively more expensive, causing a's demand for Good 2 to fall at the margin. Hence, while the amount that a exports in order to obtain Good 2 increases, the rate of increase declines. Therefore, at equilibrium it must be the case that $\Delta < 1/(m-1)$, ruling out the possibility that there can be more than one equilibrium.

The result that E_a is increasing in m is easy to see, once it is realized that (starting at the unique equilibrium) country a responds to an increase in exports by all other countries in A by lowering its own tariff so that its own exports increase. It does not matter to country a whether the increase in $E_{A-a}(\boldsymbol{\tau}_{-a})$ comes about because one (or more) existing country in $A \backslash a$ increases its exports or because an additional country is added to A and that country's exports are positive.

This concludes our characterization of equilibrium for one side of the market. Our results have been derived for manufacturers, but all results extend directly to primary product producers as well. So we may now proceed to characterize equilibrium in both sides of the market simultaneously, thus characterizing general equilibrium.

6.2. Characterization of equilibrium for both sides of the market

So far we have defined a tariff game $\Gamma(\boldsymbol{\tau}_b)$, played by all countries in A taking as given the tariff vector $\boldsymbol{\tau}_b$. For this game, using the payoff function defined by (6.1), we have obtained a unique equilibrium tariff for the countries in A, $\hat{\tau}_a$, and shown that this tariff must be declining in the number of countries, m, in A. Symmetrically, we may define a tariff game $\Gamma(\boldsymbol{\tau}_a)$, played by all countries in B and taking as given the tariff vector $\boldsymbol{\tau}_a$. From Propositions 1 and 2, for any $\boldsymbol{\tau}_a$ there must exist a unique equilibrium tariff $\hat{\tau}_b$, which is decreasing in n.

Since we know that each side of the market sets a unique tariff in equilibrium, we may define a best response tariff function for each side of the market in terms of a unique tariff set by the other side of the market. In addition, we know that each tariff is a decreasing function of the number of countries on its own side of the market, and independent of the number of countries on the other side of the market. Thus, in general we have *overall tariff reaction functions* $\hat{\tau}_a(\tau_b, m)$ and $\hat{\tau}_b(\tau_a, n)$.

We are now able to solve explicitly for these equilibrium tariffs using the payoff functions (5.1) and (5.2).

Proposition 3. *The minimal equilibrium tariff of countries in A is*

$$\hat{\tau}_a = \frac{1}{m-1},$$

and the minimal equilibrium tariff of countries in B is

$$\hat{\tau}_b = \frac{1}{n-1}.$$

If $m = 1$ and/or $n = 1$ then no trading equilibrium exists. If $m \geq 2$ and $n \geq 2$ then there is a unique trading equilibrium.

It is well know that there is a continuum of Nash equilibria of a tariff game in which no trade takes place.[11] Clearly, if $m = 1$ and/or $n = 1$ then the tariff of one or both countries is prohibitive and there is no trading equilibrium. For $m \geq 2$ and $n \geq 2$, it is straight forward to solve for the trading equilibria of the tariff game on each side of the market and, in the process of doing so, verify that the equilibrium is unique. The general characterization of equilibrium is presented in the appendix. Let us here take a look at the specific solution for a country's best-response tariff function, and see how this gives rise to the equilibrium tariffs presented in Proposition 3.

We know from (5.5) that, rather than choose τ_a and obtain a resulting value of e_a, we can instead choose a value of e_a and solve for a value of τ_a that would implement e_a. More generally, for any feasible value of total exports E_A, we may solve for a vector of tariffs τ_a that would implement E_A. This property of the model is useful in deriving the best-response tariff function $\hat{\tau}_a(\boldsymbol{\tau}_{-a})$ for the game $\Gamma(\boldsymbol{\tau}_b)$. The same holds for the derivation of the best-response tariff function $\hat{\tau}_b(\boldsymbol{\tau}_{-b})$ for the game $\Gamma(\boldsymbol{\tau}_a)$.

To derive the best-response tariff function $\hat{\tau}_a(\boldsymbol{\tau}_{-a})$ of the game $\Gamma(\boldsymbol{\tau}_b)$, assume that we have arbitrary but feasible levels of exports E_B and E_{A-a}, with corresponding tariff vectors $\boldsymbol{\tau}_{-a}$ and $\boldsymbol{\tau}_b$. Fix the tariff vectors $\boldsymbol{\tau}_{-a}$ and $\boldsymbol{\tau}_b$ in the payoff function (6.1). To obtain the best response function, differentiate (6.1) and set the resulting expression equal

[11] This result depends on the assumption that tariffs are ad valorem; see Dixit (1987).

to zero in order to obtain the first order condition for the problem. Solving for $\hat{\tau}_a$, we thus obtain

$$\hat{\tau}_a = \frac{\sqrt{E_{A-a}(\boldsymbol{\tau}_{-a})\left(E_{A-a}(\boldsymbol{\tau}_{-a}) + 4\alpha(1-\alpha)\right)} - E_{A-a}(\boldsymbol{\tau}_{-a})}{2(1-\alpha)\, E_{A-a}(\boldsymbol{\tau}_{-a})}.$$

This root satisfies $\hat{\tau}_a \in \mathbb{R}_+$ and it is the unique positive root. Thus we have a unique best response function $\hat{\tau}_a(\boldsymbol{\tau}_{-a})$. Also note from this solution that it does not depend on the vector of tariffs $\boldsymbol{\tau}_b$. Using (5.5) and the fact that in a symmetrical equilibrium $E_{A-a} = (m-1)\, e_a$, we can solve for the unique equilibrium tariff $\hat{\tau}_a = 1/(m-1)$. The equilibrium solution for $\hat{\tau}_b$ is obtained analogously.

There are two features of the symmetric equilibrium tariffs that are worth highlighting. First, $\hat{\tau}_a$ depends only on m and $\hat{\tau}_b$ depends only on n. Using this property, the effects of a change in the number of countries on one side of the market can be analyzed in a tractable way. Second, as m is increased the countries in A behave in an increasingly competitive fashion, and indeed as $m \to \infty$ the equilibrium tariff approaches free trade. Countries in B respond in a corresponding way to an increase in n.

6.3. Entry of countries on one side of the market

Let us now focus on the trading equilibrium characterized above. We can perform comparative statics on the equilibrium, focusing in particular on the effect of changes in the number of countries on either side of the market. The basic framework for analysis is set up by substituting the equilibrium tariffs into the expressions for payoffs and terms of trade. As both these expressions are continuous, we can then perform comparative statics on them, differentiating in terms of m and n and evaluating the signs of the resulting expressions.

Substituting (symmetric) equilibrium tariffs $\hat{\tau}_a = 1/(m-1)$ and $\hat{\tau}_b = 1/(n-1)$ in (5.6), we obtain

$$p(m,n) = p(\hat{\tau}_a(m), \hat{\tau}_b(n)) = \frac{n(n-1)(m-\alpha)}{m(m-1)(n-\alpha)}$$

Using this expression and equilibrium tariffs in (6.1), we have

$$u^A(m,n) = u^A(\hat{\tau}_a(m), \hat{\tau}_b(n)) = \frac{m(1-\alpha)\left(\frac{n(n-1)\alpha}{m(n-\alpha)}\right)^{\alpha}\left(\frac{m(1-\alpha)}{m-\alpha}\right)^{-\alpha}}{m-\alpha}$$

The properties of equilibrium presented in the next result are easily obtained by performing comparative statics on $u^A(m, n)$.

Proposition 4. *The unique trading equilibrium exhibits the following comparative statics properties*:

(i) The aggregate export volume $E_A(\hat{\tau}_a; m)$ increases monotonically with m and converges to the efficient level as $m \to \infty$;

(ii) The level of $E_B(\hat{\tau}_b; n)$ increases monotonically with n and converges to the efficient level as $n \to \infty$;

(iii) $du^A(m, n)/dm < 0$ and is decreasing in the ratio of n to m;

(iv) $du^A(m, n)/dn > 0$ and is decreasing in the ratio of n to m.

An increase in n has the effect of bringing about a reduction in $\hat{\tau}_b$ and increasing exports of Good 2 to the world market. By inspection of $p(m, n)$, both of these effects improve country a's terms-of-trade and hence its welfare; see $u^A(m, n)$. While the effect of an increase in n on $u^A(m, n)$ is positive, it impact diminishes as n increases. This effect is easy to see by inspection of $u^A(m, n)$, and makes intuitive sense when it is realized that the effect is driven by an increase of x_{a2} in u^A, and x_{a2} is in turn valued less highly at the margin as n increases.

An increase in m has the opposite effect, of increasing exports of Good 1 to the world market and bringing about a reduction of $\hat{\tau}_a$. Both of these effects contribute to a reduction in a's terms-of-trade and hence welfare. The negative impact on welfare increases with m, since it is driven by a decrease of x_{a2} in u^A, and x_{a2} is in turn valued more highly at the margin as m increases.

6.4. Entry of countries on both sides of the market

We are now in a position to study the effects of simultaneous entry of countries on both sides of the market. Because the international market has two sides in our model, we can define any international market in terms of the ratio of countries on one side of the market to countries on the other side. For example, say that $m = 4$ and $n = 6$. Then we have $r = 3/2$. Now if we fix r then we can study the replication of the international economy by doubling m. Where the response of the economy in equilibrium to replication is monotonic, we can define replication simply in terms of an increase in m.

Proposition 5. *Assume an initial trading equilibrium for which $m \geq 2$, $n \geq 2$ and $r = n/m$.*

World efficiency implications of replication: The higher is m, the higher is world welfare; world welfare is maximized as $m \to \infty$.

Distributional implications of replication: There exists a value $r' > 1$ such that if $r = r'$ then a given increase in m will leave $u^A(m, rm)$ unchanged, if $r < r'$ then an increase in m will bring about an increase of $u^A(m, rm)$ and if $r > r'$ then an increase in m will bring about a decrease of $u^A(m, rm)$.

From the results that have already been established, the implications for world welfare of replication follow naturally. By Proposition 4, as the international trading economy is replicated, export volumes increase monotonically as equilibrium tariffs fall. Trade flows increase monotonically from one side of the market to the other and world welfare increases monotonically as well. Eventually, as m becomes large, equilibrium tariffs $\hat{\tau}_a = 1/(m-1)$ and $\hat{\tau}_b = 1/(rm-1)$ tend towards zero, and in the limit the outcome of world efficiency (free trade) is attained.

The distributional implications are more surprising but they can be understood as follows. The higher the value of r, the more scarce are manufactures (endowments in A) relative to primary products (endowments in B). Country a is able to exploit this scarcity because the import elasticity of demand for its export is relatively low in equilibrium. Consequently, country a sets a relatively high tariff in equilibrium compared to country b, and as a result $u^A(m, rm) > u^B(rm, m)$. If r is relatively high then the terms-of-trade effects of its relatively high tariff may be sufficient to ensure that $u^A(m, rm)$ is above its free trade level.

As m is increased this reduces the relative scarcity of country a's good, reducing a's equilibrium tariff. This has two effects on $u^A(m, rm)$. The static efficiency gains of tariff reduction increase $u^A(m, rm)$. On the other hand, the tariff reduction may reduce

a's terms-of-trade, reducing $u^A(m, rm)$. As m is increased, $u^A(m, rm)$ converges to its efficient (free trade) level. If at low levels of m, $r < r'$ then $u^A(m, rm)$ converges to its efficient free trade level from below; the positive effects on welfare of static efficiency gain from tariff reduction dominates. But, if $r > r'$ then $u^A(m, rm)$ converges to its free trade level from above; the negative effect on welfare of terms-of-trade loss dominates.

7. Summary and Conclusions

For a model in which the world market is characterized as having two sides, and the tariff game between countries on one side of the market is supermodular, the world economy responds in a predictable way to a change in its underlying structure. This offers an advance over conventional analysis in international trade which typically compares two 'snapshots,' one of autarky, or a Nash equilibrium in which trade is restricted, and the other of free trade. As explained in the Introduction, our framework yields insights in three areas of international trade. Each will be taken in turn and their implications discussed.

First, as the international trading economy is replicated, equilibrium tariff levels fall and trade volumes increase monotonically; equilibrium converges to the efficient free trade equilibrium. A complete characterization is presented of the relationship between market structure and market efficiency based on strategic interaction of country governments. The framework could be adapted to any situation in which the international market has two sides and the actions of agents on one side are strategic complements while actions across the sides of the market are strategically neutral.[12] It seems reasonable to suggest that the intuition underpinning this relationship extends to more complex environments in which the market has more sides as well.

Second, we see that the basic predictions made about strategic trade policy obtained in a partial equilibrium environment do extend to the context of general equilibrium. Brander and Spencer (1984) show that an oligopolistic market structure generates a motive for protectionism when firms in different countries compete in the supply of a (ho-

[12] Amir and Bloch (2004) show that for a strategic market game the results extend to a situation where actions across the two sides of the market are strategic substitutes. Preliminary work to generalize the results of this present paper suggest that the same holds in a general tariff game setting.

mogeneous) good on the same side of the market.[13] From the analysis of this present paper we can see that the same basic insight remains robust in a general equilibrium bilateral oligopoly structure. Here, countries on each side of the market compete to supply a homogeneous good to the other side of the market. Thus, both sides of the market are protected. The fewer competitors a country has in the supply of its good, the more "rents" it can collect from the other side of the market. The introduction of additional countries on one of the market has pro-competitive effects as in Brander and Spencer but, differently from Brander and Spencer, these are reaped by the countries on the other side of the market.

This brings us to the third and final feature of the model, that the more scarce is the good exported by a country in equilibrium, the worse off the countries on that side of the market become when the economy is replicated. The intuition is simple but surprising. When the ratio of countries on one side of the market to the other is high then the relatively small number of countries on one side of the market each can do better than under free trade. This result is reminiscent of the observation that Johnson (1953-4) made, and Syropoulos (2002) later proved, that in a two country world a sufficiently large country can 'win' a tariff war i.e. it does better even than under free trade while the smaller country does worse. In the present setting, where all countries are the same size, it is the countries on the scarce side of the market who win the tariff war while those on the abundant side lose. But here the effect is entirely strategic.

The analysis could be extended usefully in a number of directions. One would be to integrate intra-industry into the framework. A natural way to do this would be to assume that the goods on the manufacturing side of the market are horizontally differentiated while the agricultural good is homogeneous. This would enrich the results while preserving the feature of the model that the more numerous primary product producers undermine each others' terms of trade.

The model also offers a useful way to consider various different types of trade agreement in a framework where there are a larger number of countries on either side of the

[13] The oligopoly model of Brander and Spencer is embedded in a general equilibrium structure and there is a numeraire good in the model, the purpose of which is to pick up the general equilibrium effects of trade policy. But tariffs can only be imposed in one sector of the model and in this sense the policy analysis is partial equilibrium.

market. The literature on multilateral trade liberalization typically focuses on a situation where there are just two countries, one on either side of the market. The literature on preferential trade agreements typically focuses on models where there are three countries. Bagwell and Staiger (2002) present a general framework for analysis of the world trading system (and see Staiger 1995 for a comprehensive review of the literature). The present paper suggests that additional insights are revealed when a larger number of countries are introduced on either side of the market.

A. Appendix

A.1. Proof of Propositions

Proof of Lemma 1. Through the change of variable, $E_A(\tau_a) = e_a(\tau_a) + E_{A-a}(\boldsymbol{\tau}_{-a})$, we may view the objective of country a as being to choose τ_a in order to set $E_A \in [E_{A-a}(\boldsymbol{\tau}_{-a}), E_{A-a}(\boldsymbol{\tau}_{-a}) + 1]$ instead of $e_a \in [0,1]$. The corresponding payoff is thus given by

$$\begin{aligned} & \max\{\widetilde{u}^A(\tau_a, \boldsymbol{\tau}_{-a}) \\ = \; & \widetilde{u}^A\left(1 - E_A(\tau_a) + E_{A-a}(\boldsymbol{\tau}_{-a}), \left(1 - \frac{E_A(\tau_a)}{E_{A-a}(\boldsymbol{\tau}_{-a})}\right) E_B\right), \\ \tau_a \in \; & \mathbb{R}_+\}. \end{aligned}$$

Solving for the value of τ_a that maximizes $\widetilde{u}^A$, denote the optimal response by $\hat{\tau}_a(\boldsymbol{\tau}_{-a})$ and the corresponding level of total exports by $\hat{E}_A(\hat{\tau}_a(\boldsymbol{\tau}_{-a}))$. Since

$$\hat{E}_A(\hat{\tau}_a(\boldsymbol{\tau}_{-a})) = e_a(\hat{\tau}_a(\boldsymbol{\tau}_{-a})) + E_{A-a}(\boldsymbol{\tau}_{-a})$$

we have that

$$\frac{e'_a\left(\hat{\tau}'_a(\boldsymbol{\tau}'_{-a})\right) - e_a(\hat{\tau}_a(\boldsymbol{\tau}_{-a}))}{E_{A-a}(\boldsymbol{\tau}'_{-a}) - E_{A-a}(\boldsymbol{\tau}_{-a})} > -1$$

if and only if $\hat{\tau}_a(\boldsymbol{\tau}_{-a})$ is strictly increasing. To see why, first note that $\hat{E}_A(\hat{\tau}_a(\boldsymbol{\tau}_{-a}))$ is only increasing in $E_{A-a}(\boldsymbol{\tau}_{-a})$ if $e_a(\hat{\tau}_a(\boldsymbol{\tau}_{-a}))$ is increasing in $E_{A-a}(\boldsymbol{\tau}_{-a})$. Let $\partial\boldsymbol{\tau}_{-a}$ denote a change of a single element of the vector $\boldsymbol{\tau}_{-a}$ and let $\partial E_{A-a}(\boldsymbol{\tau}_{-a})/\partial\boldsymbol{\tau}_{-a}$ denote the change in $E_{A-a}(\boldsymbol{\tau}_{-a})$ that results from a change in a single element of the vector $\boldsymbol{\tau}_{-a}$. Then observe that, by 5.5), both $\partial e_a(\tau_a)/\partial\tau_a < 0$ and $\partial E_{A-a}(\boldsymbol{\tau}_{-a})/\partial\boldsymbol{\tau}_{-a} < 0$.

Therefore, if $\hat{\tau}_a(\boldsymbol{\tau}_{-a})$ is strictly increasing then a reduction in $\boldsymbol{\tau}_{-a}$ will bring about an increase in $E_{A-a}(\boldsymbol{\tau}_{-a})$ and a reduction in $\hat{\tau}_a$, which in turn will bring about an increase in $e_a(\hat{\tau}_a(\boldsymbol{\tau}_{-a}))$.

To establish that $\hat{\tau}_a(\boldsymbol{\tau}_{-a})$ is strictly increasing, we begin by obtaining the first order condition for $\widetilde{u}^A$:

$$\frac{\partial \widetilde{u}^A(E_A(\tau_a))}{\partial \tau_a} = \frac{\partial e_a(\tau_a)}{\partial \tau_a}\left(\frac{E_{A-a}(\boldsymbol{\tau}_{-a})E_B}{E_A(\tau_a)^2}u^A_{x_{a2}} - u^A_{x_{a1}}\right) = 0$$

Then

$$\frac{\partial^2 \widetilde{u}^A(E_A(\tau_a))}{\partial \tau_a \partial \boldsymbol{\tau}_{-a}} = \frac{\partial e_a(\tau_a)}{\partial \tau_a}\frac{\partial E_{A-a}(\boldsymbol{\tau}_{-a})}{\partial \boldsymbol{\tau}_{-a}}\times$$
$$\left(-\frac{E_A(\tau_a)}{E_B}u^A_{xx} + u^A_{xy} + \frac{E_{A-a}(\boldsymbol{\tau}_{-a})u^A_{xy} + u^A_y}{E_A(\tau_a)} - \frac{E_{A-a}(\boldsymbol{\tau}_{-a})E_B u^A_{xy}}{[E_A(\tau_a)]^2}\right)$$

Evaluating along the first order condition, this reduces to

$$\begin{aligned}\left[\frac{\partial^2 \widetilde{u}^A(E_A(\tau_a))}{\partial \tau_a \partial \boldsymbol{\tau}_{-a}}\right]_{\partial \widetilde{u}^A(E_A(\tau_a))/\partial \tau_a = 0} \quad = \quad & \frac{\partial e_a(\tau_a)}{\partial \tau_a}\frac{\partial E_{A-a}(\boldsymbol{\tau}_{-a})}{\partial \boldsymbol{\tau}_{-a}}\times \\ & [\frac{E_{A-a}(\boldsymbol{\tau}_{-a})}{E_A(\tau_a)}\{\frac{u^A_x}{u^A_y}\left(u^A_x u^A_{xy} - u^A_y u^A_{xx}\right) \\ & + \left(u^A_y u^A_{xx} - u^A_x u^A_{yy}\right)\} \\ & + \frac{e_a(\tau_a)}{E_A(\tau_a)}\left(u^A_y u^A_{xy} - u^A_x u^A_{yy}\right) \\ & + \frac{1}{E_A(\tau_a)}\left[u^A_y\right]^2].\end{aligned}$$

By (5.5) the first two terms on the right hand side are negative, and so their product is positive. As pointed out above, for the Cobb-Douglas function, (5.1),

$$\begin{aligned}\left(u^A_x u^A_{xy} - u^A_y u^A_{xx}\right) &> 0, \\ \left(u^A_y u^A_{xx} - u^A_x u^A_{yy}\right) &> 0,\end{aligned}$$

and $u^A_y > 0$. Therefore the right hand side is positive. $\square$

Proof of Proposition 1. Existence of a pure-strategy Nash equilibrium of the game $\Gamma(\boldsymbol{\tau}_b)$ follows from standard arguments. The domain, $\mathbb{R}_+$, of $u^A(\tau_a; \boldsymbol{\tau}_{-a}, \boldsymbol{\tau}_b)$, is a compact convex set, and $u^A(\tau_a; \boldsymbol{\tau}_{-a}, \boldsymbol{\tau}_b)$ is a continuous function from $\mathbb{R}_+$ into itself. Hence, the reaction functions are continuous single-valued functions and a pure-strategy Nash equilibrium exists by Brower's fixed point theorem.

Suppose, contrary to the proposition, that the game $\Gamma(\boldsymbol{\tau}_b)$ admits an asymmetric equilibrium; without loss of generality, say that countries 1 and 2 set equilibrium tariffs $\hat{\tau}_1$ and $\hat{\tau}_2$ respectively, where $\hat{\tau}_1 \neq \hat{\tau}_2$, giving rise to equilibrium exports $e_1(\hat{\tau}_1) \neq e_2(\hat{\tau}_2)$ and total exports E_A. Then clearly, $E_A - e_1(\hat{\tau}_1) \neq E_A - e_2(\hat{\tau}_2)$. However,

$$\begin{aligned}\hat{E}_A(\hat{\tau}_1(\boldsymbol{\tau}_{-1})) &= e_a(\hat{\tau}_1(\boldsymbol{\tau}_{-1})) + E_{A-a}(\boldsymbol{\tau}_{-1}) \\ &= e_a(\hat{\tau}_2(\boldsymbol{\tau}_{-2})) + E_{A-a}(\boldsymbol{\tau}_{-2}) = \hat{E}_A(\hat{\tau}_2(\boldsymbol{\tau}_{-2})) \\ &= E_A.\end{aligned}$$

This is a contradiction to Lemma 1. □

Lemma 2. *(Lemma 3, Amir and Bloch 2004) Let $b \geq a$ and $f : [0,a] \to [0,a]$ and $g : [0,b] \to [0,b]$ be two continuous functions. Let $\overline{x}_f$ and $\underline{x}_f$ be the largest and smallest fixed points of f, and let $\overline{x}_g$ and $\underline{x}_g$ be the largest and smallest fixed points of g. If $f(x) \leq g(x)$ for all $x \in [0,a]$, then $\overline{x}_g \geq \overline{x}_f$ and $\underline{x}_g \geq \underline{x}_f$.*

Proof of Lemma 2. We show that $\overline{x}_g \geq \overline{x}_f$ (the proof of $\underline{x}_g \geq \underline{x}_f$ is similar and is thus omitted). Since $f \leq g$, we have $g(\overline{x}_f) \geq f(\overline{x}_f) = \overline{x}_f$. Consider the function $G(x) \equiv g(x) - x$ on the restricted domain $[\overline{x}_f, b]$ and $g(b) \leq b$, we have $G(\overline{x}_f) = g(\overline{x}_f) - \overline{x}_f \geq 0$ and $G(b) = g(b) - b \leq 0$. By the intermediate value theorem applied to the continuous function G on $[\overline{x}_f, b]$, there is some $\widetilde{x} \in [\overline{x}_f, b]$ such that $G(\widetilde{x}) = 0$. This is equivalent to $g(\widetilde{x}) = \widetilde{x}$. Since $\widetilde{x} \geq \overline{x}_f$ and $\overline{x}_g$ is postulated to be the largest fixed point of g we have, a fortiori, $\overline{x}_g \geq \widetilde{x} \geq \overline{x}_f$. □

Proof of Proposition 2. Since the pure strategy Nash equilibria of the game $\Gamma(\boldsymbol{\tau}_b)$ are symmetric, in an equilibrium every country $a \in A$ sets the same equilibrium tariff $\hat{\tau}_a$ and, by (5.5), every country has the same level of exports $e_a(\hat{\tau}_a)$. Then at a symmetric equilibrium it must be the case that $e_a(\hat{\tau}_a, E_{A-a}(\hat{\boldsymbol{\tau}}_{-a})) = E_{A-a}(\hat{\boldsymbol{\tau}}_{-a}) / (m-1)$. Indeed, this is a necessary condition; only at this point is the responding country's exports equal to every other country's exports. And by (5.5), for $e_a(\hat{\tau}_a, E_{A-a}(\hat{\boldsymbol{\tau}}_{-a}))$ to be equal across all countries $a \in A$ it must be the case that the equilibrium tariff $\hat{\tau}_a$ must be the same for all $a \in A$.

In view of the symmetry of every Nash equilibrium (Lemma 1) and the differentiability of (6.2) (by (5.5) and the implicit function theorem), to show the uniqueness of Nash equilibrium it is sufficient to show that at every Nash equilibrium

$$\frac{\partial e_a\left(\hat{\tau}_a, E_{A-a}\left(\hat{\boldsymbol{\tau}}_{-a}\right)\right)}{\partial E_{A-a}\left(\hat{\boldsymbol{\tau}}_{-a}\right)} < \frac{1}{m-1}.$$

Since $e_a\left(\hat{\tau}_a, E_{A-a}\left(\hat{\boldsymbol{\tau}}_{-a}\right)\right)$ is continuous, for it to be the case that $e_a\left(\hat{\tau}_a, E_{A-a}\left(\hat{\boldsymbol{\tau}}_{-a}\right)\right) = E_{A-a}\left(\hat{\boldsymbol{\tau}}_{-a}\right) / (m-1)$ at more than one symmetric equilibrium, it must be true that $\partial e_a\left(\hat{\tau}_a, E_{A-a}\left(\hat{\boldsymbol{\tau}}_{-a}\right)\right) / \partial E_{A-a}\left(\hat{\boldsymbol{\tau}}_{-a}\right) > 1/(m-1)$ at one or more equilibrium. For brevity, write $\hat{e}_a$ instead of $e_a\left(\hat{\tau}_a, E_{A-a}\left(\hat{\boldsymbol{\tau}}_{-a}\right)\right)$.

But by the implicit function theorem, we have

$$\begin{aligned}
&\frac{\partial e_a\left(\hat{\tau}_a, E_{A-a}\left(\boldsymbol{\tau}_{-a}\right)\right)}{\partial E_{A-a}\left(\boldsymbol{\tau}_{-a}\right)} = \\
&\qquad \frac{E_b\left(2E_A 2E_{A-a}u_y - \left[E_A\right]^2\left(\hat{e}_a u_{xy} + u_y\right) + \hat{e}_a E_{A-a}E_b u_{yy}\right)}{\left[E_A\right]^4 u_{xx} - 2\left[E_A\right]^2 E_b E_{A-a}u_{xy} - 2E_A E_b E_{A-a}u_y + \left[E_b\right]^2\left[E_{A-a}\right]^2 u_{yy}} \\
&\qquad + \frac{\left(\left[E_A\right]^4 u_x - \left[E_A\right]^2 E_b E_{A-a}u_y\right)\frac{\partial^2 \hat{e}_a}{\partial \hat{\tau}_A \partial E_{A-a}}}{\frac{\partial \hat{e}_a}{\partial \hat{\tau}_a}\left(\left[E_A\right]^4 u_{xx} - 2\left[E_A\right]^2 E_b E_{A-a}u_{xy} - 2E_A E_b E_{A-a}u_y + \left[E_b\right]^2\left[E_{A-a}\right]^2 u_{yy}\right)}.
\end{aligned}$$

The first order condition implies that $E_B = \left[E_A\right]^2 u_x / \left(E_{A-a}u_y\right)$, and at a symmetric equilibrium $E_{A-a} = (m-1)\,\hat{e}_a$ and $E_A = m\hat{e}_a$. Using these facts, we see that $\partial e_a\left(\hat{\tau}_a, E_{A-a}\left(\boldsymbol{\tau}_{-a}\right)\right) / \partial E_{A-a} < 1/(m-1)$ if and only if

$$-u_{xx} + \frac{u_x}{u_y}u_{xy} + \frac{1}{\hat{e}_a}u_x > 0,$$

which is implied by $\left(u_x^A u_{xy}^A - u_y^A u_{xx}^A\right) > 0$. Hence there is a unique (and symmetric) Nash equilibrium for every m.

To prove that the unique equilibrium tariff $\hat{\tau}_a$ is decreasing in m, and that total exports E_A are increasing in m, consider the mapping $ER_m : \mathbb{R}_+ \to \mathbb{R}_+$ defined by

$$ER_m\left(\boldsymbol{\tau}_{-a}\right) = \frac{m-1}{m}\left[e_a\left(\hat{\tau}_a, E_{A-a}\left(\boldsymbol{\tau}_{-a}\right)\right) + E_{A-a}\left(\boldsymbol{\tau}_{-a}\right)\right].$$

It is easy to verify that $ER_m\left(\boldsymbol{\tau}_{-a}\right)$ maps $\mathbb{R}_+$ into itself and that ER_m is increasing in m for each $\boldsymbol{\tau}_{-a}$ and decreasing in $\boldsymbol{\tau}_{-a}$ for each m (by Lemma 2 and its proof). It can be shown that the fixed points of $ER_m\left(\boldsymbol{\tau}_{-a}\right)$ are the Nash equilibria of the game $\Gamma\left(\boldsymbol{\tau}_{-b}\right)$ and vice versa. Hence, by the foregoing $ER_m\left(\boldsymbol{\tau}_{-a}\right)$ has a unique fixed point. Moreover, by Lemma 3, this fixed point increases with m. Thus, $E_{A-a}(\hat{\boldsymbol{\tau}}_{-a})$ increases with m. For this to happen, by (5.5), each element of $\hat{\boldsymbol{\tau}}_{-a}$ must decrease with m (where all elements are identical in equilibrium, $\hat{\tau}_a$). Since $E_{A-a}(\hat{\boldsymbol{\tau}}_{-a})$ increases with m, for each m it follows by Lemma 1 that $e_a(\hat{\tau}_a, E_{A-a}(\hat{\boldsymbol{\tau}}_{-a}))$ is increasing with m, and so by definition E_A in equilibrium must increase with m. □

Proof of Proposition 3. Given Proposition 2, we can define, for any $\boldsymbol{\tau}_b$, the single valued mapping $\hat{\tau}_a(\boldsymbol{\tau}_b)$ that assigns to each tariff vector $\boldsymbol{\tau}_b$ the equilibrium tariff $\hat{\tau}_a$ of countries in A in the game $\Gamma(\boldsymbol{\tau}_b)$. Also, write $\hat{\tau}_a(\tau_b)$ for the equilibrium tariff of the game $\Gamma(\tau_b)$ in which all tariffs in the vector $\boldsymbol{\tau}_b$ are equal at τ_b. Given that u^A is strictly concave in τ_a, and jointly continuous in all tariffs, the mapping $\hat{\tau}_a(\tau_b)$ is a continuous function. Similarly, we may define $\hat{\tau}_b(\tau_a)$ as the equilibrium tariffs of countries in B when the countries in A set the tariffs listed in the vector $\boldsymbol{\tau}_a$. Also, write $\hat{\tau}_b(\tau_a)$ for the equilibrium tariff in the case where all tariffs in $\boldsymbol{\tau}_a$ are equal at τ_a. Now, consider the mapping $\tau : \mathbb{R}_+ \to \mathbb{R}_+$ where $\tau = \hat{\tau}_a \circ \hat{\tau}_b$. As $\hat{\tau}_b$ is independent of m and $\hat{\tau}_a$ is non-increasing in m, τ is also non-increasing in m. Given that τ is continuous and that its domain is of the form $\mathbb{R}_+$, we can invoke Lemma 2 to conclude that the extremal fixed points of τ are non-increasing in m.

As a fixed point τ_0 of τ satisfies $\hat{\tau}_a^0 = \hat{\tau}_a \circ \hat{\tau}_b\left(\hat{\tau}_a^0\right)$, it is clear that the pair $\left\{\hat{\tau}_a, \hat{\tau}_b\left(\hat{\tau}_a^0\right)\right\}$ are equilibrium tariffs. Conversely, every Nash equilibrium of the game is a fixed point of τ. Thus, the maximal fixed point of τ must induce an equilibrium outcome in which there is autarky. By standard arguments, there exists at least one fixed point of τ between the maximal fixed point and the zero tariff vector (free trade). The minimal fixed point of τ, call it $\underline{\tau}$, is interior and decreasing in m by the argument stated above.

The remainder of the proof is presented in the body of the paper. □

Proof of Proposition 4. Differentiation of $u^A(\hat{\tau}_a(m), \hat{\tau}_b(n))$ with respect to m yields

$$\frac{du^A(\hat{\tau}_a(m), \hat{\tau}_b(n))}{dm} = -\frac{(1+m-2\alpha)\,\alpha\,(1-\alpha)\left(\frac{n(n-1)\alpha}{m(n-\alpha)}\right)^{\alpha}\left(\frac{(1-\alpha)m}{m-\alpha}\right)^{-\alpha}}{(m-\alpha)^2} < 0$$

By inspection, $du^A(\hat{\tau}_a(m), \hat{\tau}_b(n))/dm$ is decreasing in n if and only if $n(n-1)\alpha/(m(n-\alpha))$ is increasing in n, which holds for all feasible values of α, m and n.

Differentiation of $u^A(\hat{\tau}_a(m), \hat{\tau}_b(n))$ with respect to n yields

$$\frac{du^A(\hat{\tau}_a(m), \hat{\tau}_b(n))}{dn} = \frac{(1-\alpha)\alpha^2 \left(\frac{n(n-1)\alpha}{m(n-\alpha)}\right)^{\alpha-1} \left(\frac{(1-\alpha)m}{m-\alpha}\right)^{-\alpha} (n^2 - 2n\alpha + \alpha)}{(m-\alpha)(n-\alpha)^2} > 0.$$

By inspection or by taking the second derivative, $du^A(\hat{\tau}_a(m), \hat{\tau}_b(n))/dn$ is decreasing in n. $\square$

Proof of Proposition 5. Efficiency implications of replication. In a trading equilibrium, by symmetry of the equilibrium world welfare is given by

$$mu^A(m, rm) + rmu^B(rm, m) = m\frac{m(1-\alpha)\left(\frac{n(n-1)\alpha}{m(n-\alpha)}\right)^{\alpha}\left(\frac{m(1-\alpha)}{m-\alpha}\right)^{-\alpha}}{m-\alpha} + rm\frac{rm(1-\alpha)\left(\frac{m(m-1)\alpha}{rm(m-\alpha)}\right)^{\alpha}\left(\frac{rm(1-\alpha)}{rm-\alpha}\right)^{-\alpha}}{rm-\alpha}$$

We will show that the above expression is globally increasing in m. Differentiating, we get

$$\frac{d\left(mu^A(m, rm) + rmu^B(rm, m)\right)}{dm} = \frac{1}{(m-\alpha)^2(rm-\alpha)^2} \times \left(m(1-\alpha)\left(r\alpha\left(\frac{r(rm-1)\alpha}{rm-\alpha}\right)^{\alpha-1}\left(\frac{m(1-\alpha)}{m-\alpha}\right)^{-\alpha}\Theta(m; r, \alpha) + r\left(\frac{(rm-1)\alpha}{rm-\alpha}\right)^{-\alpha}\alpha\left(\frac{(m-1)\alpha}{r(m-\alpha)}\right)^{\alpha-1}\Phi(m; r, \alpha)\right)\right)$$

where

$$\Theta(m; r, \alpha) = \left(r^2m^3 + m\alpha(1+2r) - (2-\alpha)\alpha^2 - rm^2\left(1 + r(2-\alpha) + \alpha^2\right)\right)$$

and

$$\Phi(m; r, \alpha) = \left(rm^3 + m\alpha(1+2r) - (2-\alpha)\alpha^2 - m^2\left(r + 2\alpha + (r-1)\alpha^2\right)\right)$$

The result is established by verifying that $\Theta(m; r, \alpha) > 0$ and $\Phi(m; r, \alpha) > 0$ for all feasible m, r and α.

We will show the existence of a value $r' > 1$ for which $du^A(rm_k)/dm \gtreqless 0$ for $r \lesseqgtr r'$. To do so, first observe that

$$\begin{aligned}\frac{du^A(rm_k)}{dm} &= \frac{du^A(m_k, rm_k)}{dm} + r\frac{du^A(m_k, rm_k)}{dn} \\ &= \frac{r(1-\alpha)^2\alpha^2(m(1-m(1-r))r-\alpha)\left(\frac{r(rm-1)}{rm-\alpha}\right)^{\alpha-1}\left(\frac{m(1-\alpha)}{m-\alpha}\right)^{-\alpha}}{(m-\alpha)^2(rm-\alpha)^2}\end{aligned}$$

We can see by inspection that $du^A(rm_k)/dm$ is monotonically decreasing in r. Now if we fix $r = 1$ we find that

$$\frac{du^A(rm_k)}{dm} = \frac{(1-\alpha)^2\alpha^2(m^2+\alpha-2m\alpha)\left(\frac{m(1-\alpha)}{m-\alpha}\right)\left(\frac{(m-1)\alpha}{m-\alpha}\right)^{\alpha-1-\alpha}}{(m-\alpha)^3} > 0$$

So there must exist a value $r' > 1$ for which a doubling of m has no effect on $du^A(m, rm)$.
$\square$

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