RELAXING TAX COMPETITION THROUGH PUBLIC GOOD DIFFERENTIATION

by

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ABSTRACT: This paper argues that, because governments are able to relax tax competition through public good differentiation, traditionally high-tax countries have continued to set taxes at a relatively high rate even as markets have become more integrated. The key assumption is that there is variation in the extent to which firms can use public good provision to reduces costs. We show that, in a setting where tax competition promotes efficiency, governments are able to use this variation to relax the forces of tax competition, which reduces efficiency. In this environment, a 'minimum tax' counters the relaxation of tax competition, thereby enhancing efficiency, and 'split the difference' tax harmonization also enhances efficiency.

KEYWORDS: asymmetric equilibrium, core-periphery, tax competition, tax harmonization.

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1. Introduction

While the literature on international tax competition has focused mainly on the fall in tax rates on capital across countries, attention has recently been drawn to the fact that some countries have continued to tax at higher rates than others. For example, Baldwin and Krugman (2004) comment with reference to European nations that 'it has always been the case that tax rates have been higher in the core than the periphery.' Acknowledging the crudeness of the approximation, Baldwin and Krugman present data to show that capital tax rates in the 'core' countries France, Germany, Italy and Benelux have always been higher than tax rates in the poorer periphery countries Ireland, Greece, Portugal and Spain. In 1982 the effective average tax rate (EATR) in the core was 42 percent compared to 31 percent in the periphery, while in 2003 the EATR in the core was 31 percent compared to 23 percent in the periphery. These figures support the general observation that, while tax rates have fallen between 1982 and 2003, convergence between the core and periphery rates has been limited.[4]

The purpose of this paper is to formalize one possible explanation for the limited convergence in capital tax rates. We will investigate how governments are able to relax the forces of tax competition by offering different levels of public goods. Our model features a world in which tax competition has desirable efficiency properties but where policy-failure makes a fully efficient equilibrium unattainable. *Relaxed tax competition* characterizes a situation in which the difference between the level of taxes across countries is greater than under efficiency.[5] Our general idea builds on the well established notion that firms are able to relax price competition by offering goods with different characteristics. We propose a

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[4]These averages are calculated from data provided by Devereux and Griffith (2003), and made available at http://www.ifs.org.uk/corptax/internationaltaxdata.zip. These data focus specifically on taxation of capital. We will leave aside issues of commodity taxation, which are synthesized by Lockwood (2001).

[5]The literature on tax competition has focused on three different situations. The first concentrates on the presence of a 'fiscal externality,' whereby lowering the tax rate attracts capital to the jurisdiction. As a result, each government has an incentive to engage in wasteful competition for capital. The second, following Tiebout (1956), focuses on situations where competition among independent governments is like competition among firms and has desirable efficiency properties. The third setting combines features of the other two. On the one hand, competition introduces efficiency-enhancing incentives. On the other hand, such incentives operate in an environment characterized by market- or policy-failures that make a fully efficient equilibrium unattainable. The present paper is placed in the third setting. See Wilson (1999) and Wilson and Wildasin (2004) for comprehensive surveys of the tax competition literature.

simple explanation for why tax rates have not converged more in Europe and elsewhere as markets have become more integrated. Our explanation is that the impact of public good provision on cost reduction varies across firms, and governments are able to use this fact to relax the forces of tax competition.[6]

The idea that there is variation in public good requirements by firms has been adopted to investigate various related ideas: Brueckner (2000) considers Tiebout/tax competition; Hoyt and Jensen (2001) consider the capitalization of public education quality into house prices in the presence of tax competition; Justman, Thisse and van Ypersele (2002) consider fiscal competition when public good quality varies. While each of these papers makes an important contribution, none of them focus on relaxed tax competition, and tax coordination in an environment where tax competition can be relaxed. We will continue the discussion of how the present paper relates to these papers just cited and to the wider literature in Section 6 below.

Casella and Feinstein (2002) describe the same variation in public good requirements that we have in mind: "[Public goods] can be given a physical interpretation - roads, airports, infrastructure - or ... they can be more abstract - laws and legal enforcement, rules and conventions, standards and regulations, currency and language. The key feature is that preferences over the specific realization of the public good are not homogeneous among all market participants, but depend on the individual's position within the market." For example, in the textiles and apparel market, at the "top" of the market there is 'haute couture,' consisting of the leading innovators in the industry. These firms make extensive use of international travel and communications networks, employ highly educated and trained workers, and rely on intellectual property laws to safeguard returns on the designs that they produce. At the "bottom" end of the market there are so-called 'sweat shops' that employ local and relatively low skilled workers, source inputs locally, and tend to copy rather than create the designs that they use, and therefore do not rely on intellectual property

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[6]Tax rates and incomes in some countries traditionally regarded to be on the periphery of Europe have risen recently (Baldwin and Krugman 2004), leading to suggestions that traditional 'core-periphery' distinctions may no longer be valid. But at the same time, the arrival of new countries on the 'periphery' with capital tax rates lower than the core have prompted new concerns about the differences in tax levels and new questions about appropriate policy responses (EUbusiness 2004). There are undoubtedly features beyond the scope of our model which play an additional role in shaping core-periphery relationships.

protection.[7]

Our model is one where there are insufficient constitutional constraints on the intrinsic 'pressures and temptations of office' exercised through excessive taxation, but where tax competition between governments supplements the constitutional constraints and promotes efficiency (Brennan and Buchanan 1980). The policy-failure that we highlight in this setting is that levels of public good provision are inflexible relative to levels of taxation. Coupled with the feature that there is variation in public good requirements by firms, the result is that the equilibrium outcome must be inefficient. Through the characterization of tax competition in this environment, the model yields insights which may be useful in understanding contemporary patterns of capital taxation.

To capture governments' self-serving interests, our model focuses on bureaucrats in two countries who benefit personally from the budget they control and, as a result, face incentives to pursue activities that increase the size of the budget (Niskanen 1971). This approach is unsatisfactory in that it leaves unmodelled the incentive structures that motivate bureaucrats, and ignores the mandate of elected politicians to serve the electorate. Yet it has become increasingly influential, as Edwards and Keen (1996) point out: 'The British government, for example, resisted the European Commission's initial proposals for indirect tax coordination on the grounds that without them "[t]he pressure on tax rates would in general be downwards, providing an essential antidote to the in-built pressures for increased public expenditure and taxation"'' (as quoted by Edwards and Keen 1996, who in turn quote UK Treasury 1988). Within this framework, the effects of variation in public good requirements can be analyzed in a straight-forward way.[8]

The first main result of the paper (Proposition 2) characterizes *relaxed tax competition*. Under the assumption that public good provision is different across the two countries,

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[7]Casella and Feinstein (2002) do not focus on tax competition.

[8]Consideration of the balance of interests between government self-interest and the electoral mandate in the present setting is beyond the scope of this paper. Edwards and Keen (1996), Mintz and Tulkens (1996) and Rauscher (1996, 1998) examine "Leviathan models" in other settings, where governments are concerned in part with maximizing the size of the public sector. All four papers assume that governments retain some degree of 'benevolence,' perhaps caused by re-election concerns that are not formally modeled. For other work on tax competition in which governments are revenue maximizers with no benevolent tendencies, see Kanbur and Keen (1993), Hoyt (1995, 1999), Keen and Katsogiannis (2003), and Devereux, Lockwood and Redoano (2006).

Proposition 2 shows that the more effective the public good at reducing firms' costs, the greater the difference in tax levels at equilibrium; i.e the more relaxed is tax competition. (While the impact on cost of the public good varies across firms, we have a parameter k in the model that varies the effectiveness of the public good at reducing costs across all firms in the same proportion.)

The second main result (Proposition 3) establishes existence of a unique asymmetric subgame perfect equilibrium, in which levels of public good provision are determined as well as taxes. In equilibrium, if the public good reduces firms' costs, then tax competition must be relaxed. The firms who care relatively more that taxes are low than that public good provision is high locate in the country that taxes at a relatively low level and provides no public goods. The remaining firms locate in the other country, which taxes at a higher level. Moreover, the relaxation of tax competition implies that full efficiency is not achieved; under-provision of the public good occurs as a direct result.[9]

A key feature of the equilibrium is that the more effective the public good at reducing costs, the more tax competition is relaxed, and the more wasteful is the outcome. Governments' self-seeking objectives are the motivation behind the inefficient outcome. The greater the forces of tax competition between governments, the more these self-seeking tendencies are constrained. But the more effective the public good at reducing firms' costs (i.e. the higher is k) the more governments are able to relax tax competition. Under relaxed tax competition, an inefficiently large share of firms locate in the low tax country. This in turn reduces the incentive of the high tax country to provide the public good, thus compromising the efficiency enhancing effects of competition.[10]

The third main result of the paper (Proposition 4) addresses the question of how a minimum tax (a lower bound on taxes) would be set in this environment. In a standard basic model with no variation in public good requirements and homogeneous firms, governments would agree to set a minimum tax at a level sufficiently high to extract all surplus. Under

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[9]The novelty of the result lies partly in the strategy of proof. Also note that the equilibrium is unique in pure strategies only up to a re-labelling of countries and their governments. There must also exist at least one mixed strategy Nash equilibrium. We do not consider mixed strategy Nash equilibria for reasons discussed below.

[10]Rothstein (2005) discusses a related set of effects; see p. 22 of his paper.

relaxed tax competition, the effect of a minimum tax is more subtle since one government taxes at a lower rate than the other. As the minimum tax is increased, this reduces the difference between taxes, reversing the relaxation of tax competition and increasing efficiency i.e. it raises the total surplus available for distribution.

Efficiency is increased because raising the minimum tax makes the low-tax country less attractive to firms and induces some of them to locate in the other country. Thus, while rents initially increase for both countries as the constraint imposed by the minimum tax begins to bind, eventually the minimum tax reduces the rent made by the low-tax country. Proposition 4 thus defines a *non-renegotiable minimum tax frontier* as the set of minimum taxes for which neither government can obtain higher rent by a change in the minimum tax without the other government having to accept lower rent. This, in turn, can be used to place an upper bound on the minimum tax that the low tax country would voluntarily agree to.

The surprising conclusion is that, while the minimum tax counters the relaxation of tax competition, thereby raising the surplus available for distribution, it does not increase Pareto efficiency. This is because governments select a minimum tax at which they both gain at the expense of firms. Therefore, our conclusions about the imposition of a minimum tax are not as optimistic as those of Kanbur and Keen (1993) who find that a minimum tax does improve Pareto efficiency.[11] We will also discuss 'split the difference' tax harmonization, the imposition of which increases efficiency in an intuitive way.

A simplifying step that we take is to assume that the public good in our model is a pure public good. This gives rise to an apparent difference between our model and previous models of tax competition, which focus on a publicly provided service or congestible public good (see Wilson 1986 for an example of the former, Brueckner 2000 for an example of the latter). However, in Appendix A2 we rework the analysis for an extension of our model that includes congestion costs. We show that, while congestion costs do make a quantitative difference, the qualitative conclusions of our results remain robust in the extended model.

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[11]Unlike Kanbur and Keen, our purpose is not to analyze the effects of variation in country size on tax competition. While country size does vary in our analysis, this is a feature of equilibrium and not an exogenous variable as in Kanbur and Keen. Our model differs from Kanbur and Keen's in other ways; see Section 6 for further discussion.

It should be pointed out that the model presented in this paper is in no sense a general one and the results are only suggestive. There are just two countries, and we make strong assumptions about functional forms. Yet the results seem intuitively plausible and bring out sharply effects that are likely to be present in more complex general models.

The paper proceeds as follows. In Section 2 the basic model is set up. In Section 3 the efficient solution is solved for under the assumption that taxes and levels of public good provision are set by a planner. Section 4 models a game of tax competition between countries, characterizing a non-cooperative equilibrium. In Section 5, policies of tax coordination are considered. Section 6 places the paper's contribution to the literature and draws conclusions.

2. The Model

There are two countries, A and B, each of which has a government that sets the level of public good provision, x_A and x_B respectively, and the tax level, τ_A and τ_B respectively for its country. There is a set of firms, each of which is able to sell a single unit of a good in the market. We will first specify the behavior of firms, after which we will turn to governments. Finally, we will set out the sequence of events in the policy-setting game.

In the absence of the public good, each firm incurs a private cost c to produce a unit of the good that it sells and deliver it to market. But the public good provides a technology which reduces a firm's cost of production (or delivery to market).[12] The size of x_i captures the extent of public good provision in country $i \in \{A, B\}$. The expression $kx_i{}^\theta$ captures the overall cost reducing impact across all firms in country i, where $k > 0$ and $0 < \theta < 1$ are parameters. The parameter θ ensures that the impact of the public good is declining at the margin as we should generally expect. The parameter k determines the overall impact of public good provision on profitability. Note that, under the present specification, use of the public good generates no congestion externalities within the country and no spillovers to other countries.[13]

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[12] For some types of public good such as intellectual property protection it is more appropriate to think of the public good reducing the ex ante expected cost of production. This interpretation is broadly consistent with our analytical framework but our model is deterministic.

[13] As mentioned in the introduction, the analysis of congestion costs is carried out in Appendix A2.

Firms are not strategic. They simply take taxes and levels of public good provision as given and locate in the country where they make the highest profits.[14] Each firm is able to sell its single unit for price p. The set of firms is distributed uniformly on $[0, 1]$. The profit function for the firm at $s \in [0, 1]$ is given by

$$\pi_s = p - c - \tau_i + skx_i^{\theta}. \tag{2.1}$$

To focus the analysis on location decisions we shall assume that $p - c$ is fixed at a sufficiently high level for all firms to make non-negative profits no matter where they locate.[15]

The (technological) position of a firm s reflects the extent to which public good provision reduces its costs. Thus the cost-reducing impact of the public good on an individual firm is given by skx_i^{θ}. For a given increase in public good provision, the further a firm is to the right of the interval the greater is the cost-reducing impact of the public good on the firm's production. If the firm at s locates in country i it must pay a tax τ_i. The tax can be thought of as a lump sum tax or a sales tax (since each firm produces and sells only a single unit of the good).[16]

Each firm takes τ_A, τ_B, x_A and x_B as given, choosing between A and B on the basis of where it makes the highest profits. If $x_A \neq x_B$ then without loss of generality we assume that $x_A < x_B$.[17] In that case a firm may find it profitable to locate in the country with higher taxes if the cost reducing effect of the public good dominates.

For given τ_A, τ_B, x_A and x_B we can calculate the position in $[0, 1]$ of the marginal firm $\hat{s}$ that is just indifferent between locating in A and B. That is, the firm $\hat{s}(\tau_A, \tau_B)$ makes

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[14]Each firm must choose between one country or the other. There are no multinational firms in our model and so we do not consider instances where a firm can avoid paying taxes by locating part of its production activity in a low tax country. Our model could be extended to consider certain types of multinational enterprise by allowing one firm to purchase the output of another and use that output as an intermediate input in its own production. However, such an extension would not change the basic insights of our model. See Gresik (2001) for a review of the literature on taxing multinational firms. See also Mintz and Smart (2004).

[15]The price that each firm receives for the good that it sells could be made to vary across firms without affecting the results.

[16]In principle a firm at $s \in [0, 1]$ could change its position in the interval. Perhaps it could make an investment that enabled it to make better use of the public good. While this possibility is interesting, we do not analyze it in the present paper.

[17]In Section 4.2 we will show that there exists a unique subgame perfect equilibrium in which one government must set a higher tax than the other. Then $x_A < x_B$ is just a choice of labelling.

the same profits in either country;

$$\tau_A - \hat{s}kx_A{}^\theta = \tau_B - \hat{s}kx_B{}^\theta.$$

Then $\hat{s}$ also gives the share of firms in A and $1 - \hat{s}$ gives the share of firms in B. We impose the necessary restrictions to ensure that the marginal firm must belong to the $[0, 1]$ interval. First, solve the above expression for $\hat{s}$ and hence define the function[18]

$$\hat{s}\left(\tau_A, \tau_B, x_A, x_B\right) = \frac{\tau_B - \tau_A}{k\left(x_B{}^\theta - x_A{}^\theta\right)}. \tag{2.2}$$

Then $\hat{s}$, the share of firms in Country A, is defined as follows:

$$\hat{s} = \begin{cases} \hat{s}\left(\tau_A, \tau_B, x_A, x_B\right) \text{ if } \hat{s}\left(\tau_A, \tau_B, x_A, x_B\right) \in [0, 1]\, ; \\ 1 \text{ if } \hat{s}\left(\tau_A, \tau_B, x_A, x_B\right) > 1; \\ 0 \text{ if } \hat{s}\left(\tau_A, \tau_B, x_A, x_B\right) < 0. \end{cases}$$

If $\left(\tau_B - \tau_A\right)/k\left(x_B{}^\theta - x_A{}^\theta\right) \in [0, 1]$ it is easy to check that all firms $s \in [0, \hat{s})$ make higher profits in A than in B and all firms $s \in (\hat{s}, 1]$ make higher profits in B than in A. For the firms $s \in (\hat{s}, 1]$, the difference in the tax $\tau_B - \tau_A$ is dominated by the lower costs brought about by higher public good provision. Clearly, the higher is τ_B the smaller is the share of firms that finds it profitable to locate in B.

If $x_A = x_B$ then $\hat{s}$ as given by (2.2) is undefined. However, $x_A = x_B$ implies that the public good offered by the governments is homogeneous, and so firms can be thought of as responding in the manner of consumers in a Bertrand price setting game. So we borrow the usual Bertrand assumptions to define the distribution of firms between countries. If $x_A = x_B$ then all firms locate in the country with the lowest taxes:

$$\hat{s} = \begin{cases} 0 \text{ if } \tau_A < \tau_B; \\ 1 \text{ if } \tau_A > \tau_B; \\ \frac{1}{2} \text{ if } \tau_A = \tau_B. \end{cases}$$

The *rents to office*, r_A, of Government A are given by the function $r_A = \tau_A\hat{s} - x_A$. The rents to office, r_B, of Government B are given by $r_B = \tau_B\left(1 - \hat{s}\right) - x_B$, where all policy variables take non-negative values. From the rent functions it is evident that the level of public good provision by a government also determines its cost; a level of public

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[18]Parameter values k and θ will be suppressed throughout from general functional notation.

good provision x_i costs x_i to provide. In cases where $\hat{s}$ is defined by (2.2), $r_A\left(\tau_A, \tau_B\right)$ and $r_B\left(\tau_A, \tau_B\right)$ are given as follows:

$$
\begin{aligned}
r_A\left(\tau_A, \tau_B, x_A, x_B\right) &= \frac{\tau_A\left(\tau_B - \tau_A\right)}{k\left(x_B{}^\theta - x_A{}^\theta\right)} - x_A; \\
r_B\left(\tau_A, \tau_B, x_A, x_B\right) &= \tau_B\left(1 - \frac{\left(\tau_B - \tau_A\right)}{k\left(x_B{}^\theta - x_A{}^\theta\right)}\right) - x_B.
\end{aligned}
$$

Otherwise, in situations were $\hat{s} = 0$, $r_A = -x_A$ and $r_B = \tau_B - x_B$ and where $\hat{s} = 1$, $r_A = \tau_A - x_A$ and $r_B = -x_B$.[19]

To summarize, in terms of their technological requirements for public good provision, firms' positions are fixed in the interval $s \in [0, 1]$, but each firm is able to choose its preferred country to maximize profits. Each government, on the other hand, is able to choose its level of taxation and public good provision but obviously its country (A or B) is fixed.

3. Efficiency

In this section we adapt a standard definition of efficiency to the context of the present model. The notion of efficiency determines the maximum level of surplus available for distribution to the agents in the model. We make the standard assumption that a planner chooses taxes τ_A and τ_B and public good levels x_A and x_B on behalf of the governments to maximize total surplus. Given the planner's choices, it is possible to use (2.2) to solve for the marginal firm $\hat{s}$, and so $\hat{s}$ can be used in the definition of efficiency.

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[19] A 'partial equilibrium' interpretation is given to the assumption that governments can make negative rents; either that the model focuses on specific sectors within a larger economy, or that there is an unmodelled international capital market from which governments can borrow. We do not give greater prominence to this point because governments make positive rents in equilibrium.

Definition 1. A *plan*, consisting of a pair of taxes $\boldsymbol{\tau}^E = \left(\tau_A^E, \tau_B^E\right) \in \mathbb{R}_+^2$ and a public good allocation $\mathbf{x}^E = \left(x_A^E, x_B^E\right) \in \mathbb{R}_+^2$, is *efficient* if, for all other pairs of taxes $\boldsymbol{\tau} = (\tau_A, \tau_B) \in \mathbb{R}_+^2$ and public good allocations $\mathbf{x} = (x_A, x_B) \in \mathbb{R}_+^2$, it holds that

$$r_A\left(\tau_A^E, \tau_B^E, x_A^E\right) + r_B\left(\tau_A^E, \tau_B^E, x_B^E\right) + \int_0^{\hat{s}} \pi_s\left(\tau_A^E, x_A^E\right) ds + \int_{\hat{s}}^1 \pi_s\left(\tau_B^E, x_B^E\right) ds$$

$$\geq \quad r_A\left(\tau_A, \tau_B, x_A\right) + r_B\left(\tau_A, \tau_B, x_B\right) + \int_0^{\hat{s}} \pi_s\left(\tau_A, x_A\right) ds + \int_{\hat{s}}^1 \pi_s\left(\tau_B, x_B\right) ds.$$

Under Definition 1, a pair of taxes and a public good allocation is efficient if it entails the largest possible surplus for division between the two governments and the firms. The planner's problem can be represented in the form

$$
\begin{aligned}
\max_{\tau_A, \tau_B, x_A, x_B} \Omega\left(\tau_A, \tau_B, x_A, x_B\right) &= r_A\left(\tau_A, \tau_B, x_A\right) + r_B\left(\tau_A, \tau_B, x_B\right) \\
&\quad + \int_0^{\hat{s}} \pi_s\left(\tau_A, x_A\right) ds + \int_{\hat{s}}^1 \pi_s\left(\tau_B, x_B\right) ds \\
&= (p - c) - x_A - x_B + \frac{1}{2}\left(k x_B{}^\theta - \hat{s}^2 k\left(x_B{}^\theta - x_A{}^\theta\right)\right). \quad (3.1)
\end{aligned}
$$

The first term, $(p - c)$, measures the net private revenues across all firms that are independent of public good provision under the planner. The terms $-x_A$ and $-x_B$ reflect the costs (to society) of providing the public good in each of the countries. The first term in the parentheses, $k x_B{}^\theta / 2$, reflects the impact on total output across all firms if all firms locate in B. The second term in the parentheses reflects the loss of total surplus that results if a proportion $\hat{s}$ of firms locates in A. This loss comes about because, for all firms, output is increasing in public good provision and public good provision is lower in A than in B.

We will now characterize efficiency in our first result.[20]

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[20]The proofs of all results are contained in Appendix A1.

Proposition 1. *There exists an efficient plan* $\boldsymbol{\tau}^E = \left(\tau_A^E, \tau_B^E\right)$, $\mathbf{x}^E = \left(x_A^E, x_B^E\right)$ *where* $\tau_A^E = \tau_B^E$, $x_A^E = 0$, $x_B^E = \left(\frac{1}{2}\theta k\right)^{\frac{1}{1-\theta}}$ *and* $\hat{s} = 0$.

Note that the level of public good provision is higher in one country than the other. It is efficient for the planner to only provide the public good in one country and induce all firms to locate there; $\hat{s} = 0$. This is achieved by setting $\tau_A^E = \tau_B^E$.[21]

4. Competition in Taxes and Public Good Provision

In this section we examine the outcome of competition for firms between governments within the framework of a two-stage game. The basic idea is to model the way that each of the governments, in attempting to induce firms to locate in its country, competes over taxes and the levels of public good provision.

In Stage 1 of the game, the two governments noncooperatively and simultaneously choose (as pure strategies) levels of public good provision $x_A \in \mathbb{R}_+$ and $x_B \in \mathbb{R}_+$ respectively. Then in Stage 2 the governments, having observed the levels of public good provision, choose (as pure strategies) levels of taxation $\tau_A \in \mathbb{R}_+$ and $\tau_B \in \mathbb{R}_+$ respectively. This order of events is regarded to reflect the idea that taxes can be changed relatively easily once the level of public good provision has been chosen, while a change in the level of public good provision requires modification of the infrastructure through which it is provided.[22] Once the governments' decisions have been taken, firms take taxes and levels of public good provision

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[21]In fact, there is a continuum of efficient solutions given by $\tau_A^E \geq \tau_B^E$. But the solution presented in Proposition 1 is the same as the unique solution obtained from the model with congestions costs in the limit as the congestion cost goes to zero. Therefore, unlike the efficient solutions in which $\tau_A^E > \tau_B^E$, it may be compared with the efficient solution when there are (positive) congestion costs.

In the presence of congestion costs $\hat{s} > 0$, and the planner achieves this by setting $\tau_B^E > \tau_A^E$. Interestingly, the property that $x_A^E = 0$ is preserved even in the presence of congestion costs; the planner can reduce congestion costs (to society) by inducing the less efficient firms to locate in A, even though no public goods are provided there.

Under alternative specifications, some public good may be provided in both jurisdictions. For example, if we had assumed ad valorem or specific taxation then the planner's solution would have to take account of the marginal effect of the tax on production within each jurisdiction as well. But the main point, that $x_B^E > x_A^E$, would be preserved.

[22]It will be assumed throughout that mixed strategies in tax rates are not available to governments. This is generally deemed to be an acceptable assumption in the applied literature on policy setting in a perfect-information environment. Some work in related settings has allowed for mixed strategies. For example, Justman et al (2002) consider mixed strategies in their model of fiscal competition.

as given and choose their geographical locations (i.e. A or B) to maximize profits. We refer to this whole process, including both stages, as a *tax competition game* and we solve for an equilibrium in taxes and public good provision using backwards induction.[23]

4.1. Stage 2: The Tax Subgame

We will next solve for Stage 2, where the levels of public good provision by the two governments are taken as fixed at (non-negative) levels x_A and x_B.

For given levels of public good provision x_A and x_B, a strategy τ_A^* of Government A is a *best response tax* against a strategy τ_B when it maximizes $r_A(\tau_A, \tau_B)$. A *Nash equilibrium in taxes* is a pair (τ_A^*, τ_B^*) for which τ_A^* is a best response to τ_B^* and vice-versa.

We will start with the case where $x_A < x_B$. We need the following lemma to establish best response taxes in this situation.

Lemma 1. *Assume that x_A and x_B are fixed, with $0 \leq x_A < x_B$. For given τ_B, the unique tax that maximizes $r_A(\tau_A, \tau_B)$ is*

$$T_A(\tau_B) = \frac{\tau_B}{2}.$$

For given τ_A, the unique tax τ_B that maximizes $r_B(\tau_A, \tau_B)$ is

$$T_B(\tau_A) = \frac{\tau_A}{2} + \frac{k\left(x_B^{\theta} - x_A^{\theta}\right)}{2}.$$

Lemma 1 determines tax reaction functions, which are illustrated in Figure 1. We see that, for fixed levels of public goods, optimal tax rates are strategic complements. Government A's reaction function is derived by rearranging the first order condition for the

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[23] We are not the first to model interjurisdictional competition in tax and spending levels as a two stage game; this approach has been taken previously by Hoyt and Jensen (2001) among others. As Kreps and Scheinkman (1983) argue in their study of firm behavior, the appropriateness of the set-up, or the game context, is essentially an empirical matter. Certainly, it seems reasonable to argue that levels of public good provision are more difficult to change than taxes and so these are set in the first stage because governments can more easily commit to them. This parallels the familiar argument that firms can more easily commit to the capacity for production than prices. Then in the second stage governments announce taxes in the same way that firms announce prices.

maximization of r_A. The reaction function shows that Government A's best response depends only on the level of τ_B.

Government B's reaction function is more interesting. For any τ_A, the level of τ_B that maximizes r_B is increasing in k. To see why, look at the first order condition for maximization of r_B;

$$
\begin{aligned}
\frac{dr_B}{d\tau_B} &= 1 - \hat{s} - \tau_B \frac{\partial \hat{s}}{\partial \tau_B} \\
&= 1 - \frac{\tau_B - \tau_A}{k\left(x_B{}^\theta - x_A{}^\theta\right)} - \frac{\tau_B}{k\left(x_B{}^\theta - x_A{}^\theta\right)} = 0.
\end{aligned}
$$

From the first order condition it is easy to see that r_B is strictly concave. It also becomes clear that $dr_B/d\tau_B$ is increasing in k. Look first at $\hat{s}$; the second term in the expression above. Assuming values of τ_A, τ_B, and $x_A < x_B$ that imply $\hat{s} \in (0,1)$,

$$
\frac{\partial \hat{s}}{\partial k} = -\frac{\tau_B - \tau_A}{k^2\left(x_B{}^\theta - x_A{}^\theta\right)} = -\frac{\hat{s}}{k} < 0.
$$

An increase in k results in a decrease in $\hat{s}$. Intuitively, the greater the positive impact of the public good on profits, the higher Government B can set its tax τ_B above τ_A and still attract a given share of firms $1 - \hat{s}$ to its country.[24]

Looking now at the third term of the first order condition and differentiating with respect to k, we see that

$$
\frac{\partial^2 \hat{s}}{\partial \tau_B \partial k} = -\frac{1}{k^2\left(x_B{}^\theta - x_A{}^\theta\right)} < 0. \tag{4.1}
$$

So if Government B increases its tax this induces firms to move to A, i.e. $\partial \hat{s}/\partial \tau_B = \left(k^2\left(x_B{}^\theta - x_A{}^\theta\right)\right)^{-1}$, but this effect is dampened by an increase in k. For higher k, Government B's loss in share of firms due to an increase in τ_B is more limited. It is due to these two combined effects that an increase in k increases Government B's best response tax for any given τ_A. It is through these two effects that governments are able to relax tax competition, and tax competition is increasingly relaxed as a result of an increase in k.

We now characterize equilibrium taxes and the equilibrium share $\hat{s}$ of firms between countries. We will say that *tax competition is relaxed* when $(\tau_B^* - \tau_A^*) - \left(\tau_B^E - \tau_A^E\right) > 0$.

---

[24]The parameter θ affects the impact of the public good on profits in a similar but more complex way. This will be discussed further below.

Proposition 2. (*Relaxed Tax Competition*). *Assume that x_A and x_B are fixed.*

For $x_A = x_B$, both governments provide the same level of public good and there exists a unique equilibrium in which $\tau_A^* = \tau_B^* = 0$.

For $x_A \neq x_B$ assume that $x_A < x_B$. Then there exists a unique subgame equilibrium point in taxes for which $\tau_A^*(x_A, x_B) < \tau_B^*(x_A, x_B)$:

$$\tau_A^*(x_A, x_B) = \frac{1}{3}k\left(x_B^{\,\theta} - x_A^{\,\theta}\right);$$
$$\tau_B^*(x_A, x_B) = \frac{2}{3}k\left(x_B^{\,\theta} - x_A^{\,\theta}\right).$$

The larger is k, the more tax competition is relaxed. At $\tau_A^*(x_A, x_B; k)$ and $\tau_B^*(x_A, x_B; k)$, the share of firms locating in Country A is given by $\hat{s} = 1/3$.

From Proposition 2, tax competition is more relaxed the larger is x_B relative to x_A, and the higher is k; $\tau_B^* - \tau_A^* = k\left(x_B^{\,\theta} - x_A^{\,\theta}\right)/3$ (since $\tau_A^E = \tau_B^E$, this only requires that the gap between τ_B^* and τ_A^* is increasing in k). These features of the equilibrium are seen from Figure 1, which shows that the intercept of Government B's reaction function $T_B(\tau_A)$ is increasing in $x_B^{\,\theta} - x_A^{\,\theta}$ and k. Consequently, the equilibrium tax levels τ_A^* and τ_B^* increase as either $x_B^{\,\theta} - x_A^{\,\theta}$ or k are increased.

As x_A is reduced relative to x_B, Country A becomes less attractive to firms that locate in B. So Government B is able to raise its tax, making higher rents from each firm while holding its share of firms constant. At the same time, this makes Country B less attractive to firms in A, so Government A is able to raise its tax and make higher rents from each firm while holding its share of firms constant.[25]

If $x_A = x_B$ then public good provision is the same across countries and we effectively have Bertrand tax competition which leads to an outcome in which $\tau_A^* = \tau_B^* = 0$. Because x_A

---

[25]The introduction of congestion costs tends to work against the relaxation of tax competition, i.e. congestions costs make tax competition more intense. But the relaxation of tax competition is not eliminated even as congestion costs are made to be large. In the presence of congestion costs, the planner sets taxes at a higher rate in B than in A in order to induce firms to move to A, alleviating the congestion cost on firms in B; $\tau_B^E - \tau_A^E$ is increasing in the congestion cost. But the congestion cost does not affect the difference in equilibrium tax rates since Governments A and B do not care about the distribution of the congestion costs; $\tau_B^* - \tau_A^*$ is not affected by the congestion cost. Therefore, tax competition becomes less relaxed with an increase in the size of the congestion cost, but does not converge to zero; $(\tau_B^* - \tau_A^*) - (\tau_B^E - \tau_A^E) > 0$. See Appendix A2 for the analysis.

is sunk, for any positive tax level it is a dominant strategy for each government to undercut the other in setting taxes and in doing so attract all firms to its country. Recall that the share of firms that locates in each country is indeterminate in such an equilibrium, but because taxes are zero the share of firms that locates in each country makes no difference to rents; thus $r_A = r_B = -x_A$.

It is interesting to note from Proposition 2 that the share of firms locating in Country B is relatively large, at $1 - \hat{s} = 2/3$, even though B sets a higher tax in equilibrium. We might have expected to see the high-tax country attracting a relatively small share of firms but this is not the case. A higher level of public good provision can have a cost-reducing impact sufficiently large as to make location in Country B more profitable for a majority of firms, despite higher taxation there.[26]

4.2. Stage 1: Level of public good provision

We now solve Stage 1, which determines the level of public good provision by the respective governments. To do this, we must drop the assumption that $x_A \leq x_B$. In looking for Government A's best response to x_B, we must evaluate $r_A(x_A, x_B)$ for $x_A < x_B$, $x_A = x_B$ and $x_A > x_B$. The same applies for Government B.

Using the equilibrium values for τ_A^* and τ_B^* from Proposition 2 in $r_A = \tau_A \hat{s} - x_A$, Government A's rent function is defined as follows:

$$r_A(x_A, x_B) = \begin{cases} k\left(x_B^{\theta} - x_A^{\theta}\right)/9 - x_A & \text{if } 0 \leq x_A < x_B; \\ -x_A & \text{if } 0 \leq x_A = x_B; \\ 4k\left(x_A^{\theta} - x_B^{\theta}\right)/9 - x_A & \text{if } 0 \leq x_B < x_A. \end{cases} \tag{4.2}$$

For Government B,

$$r_B(x_A, x_B) = \begin{cases} 4k\left(x_B^{\theta} - x_A^{\theta}\right)/9 - x_A & \text{if } 0 \leq x_A < x_B; \\ -x_B & \text{if } 0 \leq x_A = x_B; \\ k\left(x_A^{\theta} - x_B^{\theta}\right)/9 - x_B & \text{if } 0 \leq x_B < x_A. \end{cases} \tag{4.3}$$

A level of public good provision x_A^* of Government A is a *best response against a level of public good provision* x_B, denoted $BR_A(x_B)$, when it maximizes $r_A(x_A, x_B)$. A *Nash equilibrium*

---

[26]In the presence of congestion costs, the share of firms that locate in A is increasing in the size of the congestion cost as one would expect. However, Country B attracts a larger share of firms than Country A even when congestion costs are large. See Appendix A2 for further details.

in levels of public good provision is a pair (x_A^*, x_B^*) where x_A^* is a best response against x_B^* and vice-versa.

We will now state our existence-and-characterization-of-equilibrium result.

Proposition 3. *(Unique Asymmetric Equilibrium) Assume that governments play a tax competition game.*

1. *There exists a unique subgame perfect equilibrium in pure strategies.*

2. *The equilibrium has the property that one country, say A, provides a smaller amount of the public good than the other, B.*

3. *The subgame perfect equilibrium is determined by the levels of public good provision*

$$x_A^* = 0,$$
$$x_B^* = \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}},$$

and taxes are (uniquely)

$$\tau_A^* = \frac{1}{3}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}},$$
$$\tau_B^* = \frac{2}{3}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}.$$

4. *In the (pure strategies) subgame perfect equilibrium, public good provision in Country B is inefficiently low: $x_B^E = \left(\frac{1}{2}\theta k\right)^{\frac{1}{1-\theta}} > x_B^* = \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}}.$*

Proposition 3 shows that while Country B provides the public good at a positive level, A provides none at all. Also note that, although taxation is higher in B than in A, taxation in A is nevertheless positive. Thus Country A has a degree of monopoly power and is able to collect rents due to the fact that firms must locate in one country or the other in order

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to produce. Finally, the result shows that the equilibrium level of public good provision is inefficient.[27]

In Section 4.1 we argued that tax competition becomes more relaxed the greater the difference between x_B and x_A, which suggests that Government A has an incentive to reduce x_A relative to x_B in Stage 1 so that it can raise taxes in Stage 2. Proposition 3 shows formally that this effect does indeed operate to the point where Government A provides no public goods at all. It seems reasonable to argue that such an effect would operate under more general specifications than ours, although for more complex models public good provision may not be driven all the way to zero in A.

Proposition 3 also shows that in equilibrium the opposing forces on x_B balance at a positive level $x_B^* = \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}}$. Tax competition is more relaxed when x_B is increased, enabling Government B to raise τ_B while holding its share of firms constant, potentially increasing rents. But of course this increases the cost of provision, which works on rents in the opposite direction. The effect of a change in k is clear. As k is increased this increases x_B^* because public good provision has a bigger impact on firms' profits and therefore on government rents through taxation.[28]

It is easy to check that both governments make positive rents in equilibrium. For Country A this is immediately obvious because it collects taxes from a positive share of firms but has no costs of public good provision. For Country B we use the equilibrium values for τ_B^* and x_B^* in the expression for Government B's rents to obtain, in reduced form, $r_B = \left(\frac{4k}{9}\right)^{\frac{1}{1-\theta}} \left[\theta^{\frac{\theta}{1-\theta}} - \theta^{\frac{1}{1-\theta}}\right]$. To see that $r_B > 0$ for all $\theta \in (0,1)$ note that $\lim_{\theta \to 0} \theta^{\frac{\theta}{1-\theta}} = 1$ while $\lim_{\theta \to 0} \theta^{\frac{1}{1-\theta}} = 0$ and $\lim_{\theta \to 1} \theta^{\frac{\theta}{1-\theta}} = \lim_{\theta \to 1} \theta^{\frac{1}{1-\theta}} = 1/e$, with $\theta^{\frac{\theta}{1-\theta}}$ decreasing monotonically from 1 to $1/e$ as θ is varied from 0 to 1, and $\theta^{\frac{1}{1-\theta}}$ increasing monotonically from 0 to $1/e$ as θ is varied from 0 to 1. This makes intuitive sense if we think of the outcome as oligopolistic, where both governments are able to choose quantities and prices (here taxes) at which they

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[27]In the proof we show that the equilibrium in pure strategies must be asymmetric in that one government sets public good provision above the level of the other. We prove that this equilibrium exists and is unique subject to a re-labelling of countries. We then choose to label Countries A and B as before, as the countries of low and high level public good provision respectively.

[28]An increase in the congestion cost has a negative impact on x_B^*; see Appendix A2.

make non-negative rents.[29]

We can now determine which government makes higher rents. Using equilibrium values from Proposition 3, we know that $r_A = \frac{k}{9} \left(\frac{4\theta k}{9} \right)^{\frac{\theta}{1-\theta}}$ and $r_B = \left(\frac{4k}{9} \right)^{\frac{1}{1-\theta}} \left[\theta^{\frac{\theta}{1-\theta}} - \theta^{\frac{1}{1-\theta}} \right]$. From this we have that $r_A \gtreqqless r_B$ if and only if $\frac{1}{4\theta} \left(\frac{4\theta k}{9} \right)^{\frac{1}{1-\theta}} \gtreqqless \left(\frac{1}{\theta} - 1 \right) \left(\frac{4\theta k}{9} \right)^{\frac{1}{1-\theta}}$ or, equivalently, if and only if $\theta \gtreqqless \frac{3}{4}$.[30]

Finally, and this is a point worth noting, we are now able to see why the level of public good provision is suboptimal under relaxed tax competition. This suboptimality arises because more firms locate in Country A under relaxed tax competition than under efficiency. As a result, the marginal benefit to a policy-setter of providing the public good is lower, whether this policy-setter is the planner or the government. If $\tau_A = \tau_B$ were arbitrarily fixed at Stage 2, then Government B's incentive to set x_B is identical to that of the planner, and it would set $x_B = x_B^E$. Conversely, if the planner were constrained to set taxes $\tau_A^* = \frac{1}{3}k \left(\frac{4}{9}\theta k \right)^{\frac{\theta}{1-\theta}}$ and $\tau_B^* = \frac{2}{3}k \left(\frac{4}{9}\theta k \right)^{\frac{\theta}{1-\theta}}$, the outcome of relaxed tax competition, then the planner's solution to the level of public good provision would be $x_B = x_B^*$.

As a final point, note that both x_B^E and x_B^* go to 0 as k goes to 0. This is plausible since the public good becomes less effective at saving costs as k tends to zero so no firm will pay for it, and so no country will (or should) provide it. Thus, as the public good becomes less effective across the distribution at reducing firms' costs, and consequently as tax competition becomes less relaxed, the equilibrium of the tax competition game converges towards the efficient solution.[31]

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[29]We conjecture that this property, governments making positive rents in equilibrium, would hold for a more general specification for the profit function in that the term kx_i^{θ} could be replaced by a general function $b(x_i; \theta, k)$, with $b(\cdot)$ concave in x_i and $\partial b / \partial k > 0$.

[30]As mentioned above, the broader effects of changes in θ are not directly relevant to the focus of our analysis and so they are discussed in Appendix A3.

[31]When congestion costs are introduced to the model, as they are increased this has the effect of bringing about a reduction both in x_B^E and x_B^*. But the (negative) effect on x_B^E is larger than on x_B^*. Tax competition becomes less relaxed as the congestion cost is increased. More firms are induced to locate in A both in the planner's solution and in equilibrium. But the effect is more muted under tax competition than under the planner's solution because governments care about their rents and not the overall social cost of congestion. So there exists a level of congestion costs at which $x_B^E = x_B^*$. But the solution is not efficient since the equilibrium share of firms across jurisdictions does not correspond to efficiency. And as congestion costs are increased equilibrium does not converge to efficiency; to the contrary, $x_B^E > x_B^*$.

5. Policies of Tax Coordination

The two most commonly advanced proposals for tax policy coordination are the setting of a minimum tax and tax harmonization. We will now consider each in turn as applied in the context of relaxed tax competition, taking a minimum tax first.

5.1. A Minimum Tax

We now examine the imposition of a minimum tax of the kind considered by Kanbur and Keen (1993). If governments agree to set a minimum tax, denoted by μ, then they agree to a common lower bound for taxes. We characterize the *non-renegotiable minimum tax frontier* as the set of minimum taxes for which, given a minimum tax: (i) neither government can obtain higher rent by a change in the minimum tax without the other government having to accept lower rent; (ii) both governments obtain higher rents than with no minimum tax.[32] Given any minimum tax on the frontier, the two governments would not jointly agree to renegotiate to any other minimum tax or to abolish the minimum tax.[33]

A minimum tax only imposes a binding constraint if $\mu \geq \tau_A^*$. On the other hand, μ can be set sufficiently high to ensure that tax rates are equalized. By inspection of (2.2), it is clear that if the constraint sets a minimum such that $\tau_A = \tau_B$ then all firms locate in Country B. Since rents for A are zero if the share of firms that locates in A is zero, a value of μ higher than the value required to ensure $\tau_A = \tau_B$ cannot yield higher rents for A than with no minimum tax. Therefore, we may restrict attention to μ that lies between τ_A^* and a value that ensures $\tau_A = \tau_B$.[34]

An issue that arises is whether a minimum tax should be applied when countries are ex-ante symmetric; that is, when $x_A = x_B$. Here we take the view that the primary motivation for a minimum tax is to reduce the difference between tax levels only when countries would

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[32]The notion of the non-renegotiable minimum tax frontier is related to the Pareto efficient frontier. The key difference is that the non-renegotiable minimum tax frontier is defined by the outcome of strategic interactions between the two governments and, as we shall see, is not Pareto efficient.

[33]We will not discuss the determination of the specific minimum tax that is implemented on the frontier because this would depend on factors beyond the scope of our model.

[34]For reasons that will become clear, τ_B^* does not impose the upper bound on μ, unlike in Kanbur and Keen (1993).

otherwise set different taxes in equilibrium, motivated by the fact that they provide public goods at different levels. When countries provide public goods at the same level, arguably this motivation for a minimum tax does not apply. Thus, we maintain the approach taken throughout the paper that if $x_A = x_B$ in Stage 1 then tax competition between governments in Stage 2 is characterized by standard Bertrand competition, and taxes are competed to zero.[35]

We now formalize a minimum tax under the assumption that $x_A < x_B$.[36] Let μ be set at a level ε above A's equilibrium tax under relaxed tax competition;

$$\mu = \tau_A^* + \varepsilon = \frac{1}{3}k\left(x_B^{\theta} - x_A^{\theta}\right) + \varepsilon.$$

Let τ_A^{μ} be the tax that Government A sets in the presence of the minimum tax. By the concavity of r_A in τ_A, the best Government A can do in the presence of the minimum tax is to set $\tau_A^{\mu} = \mu$. The tax set by Government B is determined by the reaction function $T_B(\tau_A) = \left(\tau_A + k\left(x_B^{\theta} - x_A^{\theta}\right)\right)/2$ as $\tau_B^{\mu} = \frac{2}{3}k\left(x_B^{\theta} - x_A^{\theta}\right) + \frac{1}{2}\varepsilon$. We can now see that if $\varepsilon = \frac{2}{3}k\left(x_B^{\theta} - x_A^{\theta}\right)$, then $\tau_A^{\mu} = \tau_B^{\mu}$. Therefore, we restrict attention to $\varepsilon \in \left[0, \frac{2}{3}k\left(x_B^{\theta} - x_A^{\theta}\right)\right]$. To agree upon a minimum tax, the governments must effectively agree upon a value for ε.

There are similarities here to Kanbur and Keen's (1993) approach to the analysis of a minimum tax. However, an issue that Kanbur and Keen do not need to address is how the introduction of the minimum tax affects the sequence of events because their game only has a single period. The minimum tax is imposed before tax setting takes place within that period, bringing about a constrained equilibrium. In our model, the imposition of a minimum tax constraint raises the extra issue of whether the constraint is anticipated before the level of public good provision is fixed. From an abstract standpoint, it seems natural to argue that the imposition of the constraint is fully anticipated when levels of public good

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[35]It has alternatively been argued that the primary purpose of a minimum tax is to limit socially wasteful competition between the governments, and that this applies when countries are ex ante symmetrical as well. This is the spirit in which a minimum tax is applied in Keen and Marchand (1997), for example. In our framework, a minimum tax acts in a similar spirit to reduce waste. But it works in a different way, by limiting the extent to which tax competition can be relaxed.

[36]The case where $x_B < x_A$ is analogous. In demonstrating equilibrium we take the same approach as in Section 4.2, initially dropping the assumption that $x_A < x_B$. After it is established that in equilibrium one government must set public good provision at a higher level than the other then the assumption that $x_A < x_B$ may be adopted without loss of generality.

provision are determined. But from a more practical policy-motivated point of view it could be argued that proposals for a minimum tax take place after public good provision has been fixed. The context we have in mind here is the current call for a minimum tax in the newly expanded EU. In the following we will make the assumption that the minimum tax is not anticipated.[37]

Let us now assume that the governments set the levels of public good provision simultaneously and noncooperatively at Stage 1 as if no minimum tax were to be imposed, anticipating instead that the game would proceed straight to Stage 2 in which tax setting would take place. After levels of public good provision are fixed in Stage 1, the governments are then unexpectedly granted the opportunity to agree upon a minimum tax. After the minimum tax is agreed upon, the game then proceeds to Stage 2, at which point governments set taxes simultaneously and noncooperatively (but now subject to the minimum tax).

Writing the respective levels of public good provision under the unanticipated minimum tax constraint as x_A^μ and x_B^μ we therefore have $x_A^\mu = x_A^* = 0$ and $x_B^\mu = x_B^* = \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}}$. Using $x_A^\mu = x_A^* = 0$, $x_B^\mu = x_B^* = \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}}$, $\tau_A^\mu = \mu = \tau_A^* + \varepsilon = \frac{1}{3}k\left(x_B^\mu\right)^\theta + \varepsilon$ and $\tau_B^\mu = \frac{2}{3}k\left(x_B^\mu\right)^\theta + \frac{1}{2}\varepsilon$ in the expressions for $\hat{s}$, r_A, and r_B, (that is 2.2, 4.2 and 4.3), we obtain the following reduced form expressions for government rents. To emphasize that rents are being derived under the minimum tax, we shall write these as $r_A^\mu(\varepsilon)$ and $r_B^\mu(\varepsilon)$ respectively:

$$r_A^\mu(\varepsilon) = \frac{1}{9}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}} + \frac{1}{6}\varepsilon - \frac{\varepsilon}{2k\left(\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}\right)};$$

$$r_B^\mu(\varepsilon) = \frac{4}{9}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}} - \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}} + \frac{2}{3}\varepsilon + \frac{\varepsilon^2}{4k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}}.$$

We now characterize the non-renegotiable minimum tax frontier.

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[37]In Appendix A4 we assume instead that the minimum tax is anticipated. In fact, our findings are qualitatively similar. Differences will be noted where relevant.

Proposition 4. *Fix $x_A^\mu = 0$ and $x_B^\mu = \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}}$ and fix a minimum tax $\mu = \frac{1}{3}k\left(x_B{}^\theta - x_A{}^\theta\right) + \varepsilon$. Then Government A maximizes $r_A\left(\varepsilon\right)$ by setting $\tau_A^\mu = \frac{1}{3}k\left(\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}\right) + \varepsilon$ and Government B maximizes $r_B\left(\varepsilon\right)$ by setting $\tau_B^\mu = \frac{2}{3}k\left(\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}\right) + \frac{1}{2}\varepsilon$. A minimum tax is on the non-renegotiable minimum tax frontier if $\varepsilon \in \left(\frac{1}{6}k\left(\frac{4}{9}k\theta\right)^{\frac{\theta}{1-\theta}}, \frac{1}{3}k\left(\frac{4}{9}k\theta\right)^{\frac{\theta}{1-\theta}}\right)$. If the minimum tax is on the non-renegotiable minimum tax frontier, then both governments make higher rents than with no minimum tax and any change in the minimum tax yields strictly higher rents for one government but strictly lower rents for the other government. Any minimum tax for which $\varepsilon \in \left(0, \frac{1}{3}k\left(\frac{4}{9}k\theta\right)^{\frac{\theta}{1-\theta}}\right)$ increases efficiency.*

The implications for efficiency of a minimum tax reported in Proposition 4 are easy to deduce using (2.2) and (3.1). First note that x_A^μ and x_B^μ are fixed. So, by inspection of (3.1), the only way that efficiency can change under the imposition of the minimum tax is by a change in $\hat{s}$. Since the imposition of the minimum tax reduces the difference between taxes, by (2.2), the share of firms that locates in B increases with the minimum tax. Due to the fact that public good provision is higher in B, this increases efficiency. However, note that the imposition of the minimum tax does not increase Pareto efficiency because, while one or more governments is able to extract additional surplus through higher taxation, this is achieved at the expense of firms' profits (see the profit function of the firm 2.1; in it, $p - c$ and x_i are fixed, while τ_i increases).[38] This result accords with Kanbur and Keen's (1993) analysis, which shows that total surplus may be increased through the imposition of a minimum tax.[39] But it adds the twist that the additional surplus generated by the minimum tax may be expropriated by self-seeking governments if they have to power to achieve this.

While obviously the assumption that the minimum tax is unanticipated is restrictive, we see now why it is useful. By holding the level of public good provision constant, we are able to see the direct effect on taxes and hence rents of introducing the minimum tax. Using (2.2) it is possible to check that while A benefits from being able to set higher taxes, it loses firms as ε is increased. As ε is increased above $\varepsilon = \frac{1}{6}k\left(\frac{4}{9}k\theta\right)^{\frac{\theta}{1-\theta}}$, the loss to A from

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[38]We have not formally analyzed the effect of a minimum tax in the presence of a congestion cost but intuitively the logic of Proposition 4 extends to that setting too.

[39]Note that full efficiency cannot be achieved because x_B^μ is fixed at x_B^*; the level of efficiency associated with x_B^E could not be achieved under an unanticipated minimum tax.

the migration of firms to B is greater than the gain from being able to tax each firm at a higher rate. From Appendix A4 it is clear that this effect carries over to the situation where governments anticipate the introduction of the minimum tax.

It is important to note that the result presented in Proposition 4 concerning efficiency is *not* robust to dropping the assumption that the minimum tax is unanticipated. The analysis presented in Appendix A4 shows that Government B's incentive to compete in public good provision (by offering the public good at a higher level than Government A) is reduced by the fact that Government A is limited in the extent to which it is allowed to set its tax lower than B's. Consequently, while more firms locate in the high-public-good country, the fact that x_B^μ decreases monotonically with ε dominates and efficiency is reduced by the minimum tax as a result. This effect is not captured by Kanbur and Keen (1993), since they do not consider the determination of public good provision.

5.2. Tax Harmonization

We consider tax harmonization that restricts taxes in such a way that it moves them towards efficient levels; Mintz and Tulkens (1996). In the present setting, this is equivalent to 'split the difference' tax harmonization used by Baldwin and Krugman (2004) among others. This is different from a minimum tax in the sense that it imposes a ceiling as well as a floor to the level of taxation. This notion of tax harmonization is appropriate in a framework such as the one we are studying here, where the difference in levels of taxation is excessive (i.e. the result of relaxed tax competition) and generates inefficiency.

For consistency with the analysis of the minimum tax carried out above, we will maintain the assumption that policy coordination is unanticipated and so the level of public good provision is fixed at $x_A = 0$ in A and $x_B = \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}} < x_B^E$ in B. From (3.1), tax harmonization involves a complete equalization of tax rates. The outcome from tax harmonization is that all firms locate in Country B. Rents collected by B increase while rents collect by A decrease. The level of efficiency is higher than under the minimum tax because all firms locate in B, where public good provision is higher, while under the minimum tax some firms

locate in A.[40]

Overall, then, tax harmonization appears to have more favorable efficiency implications than a minimum tax. The distributive effects are more mixed. Government B makes higher rents than without harmonization and higher than with the minimum tax. This is essentially because more firms locate in B. Firms make higher profits due to lower taxation and higher public good provision. But Government A generally makes lower rents under tax harmonization.

6. Related Literature and Conclusions

It was mentioned in the introduction that the tax competition literature focuses on three different situations. We will now consider the literature in each of these areas in turn and how the present paper relates to it.

The first area of focus is where there is a fiscal externality created by the fact that governments must rely on taxation of mobile capital while labor (also referred to as residents) is immobile. This is sometimes referred to as 'basic' or 'standard' tax competition. Asymmetric equilibria have been studied before in such tax competition models; see for example Wilson (1987, 1991). In Wilson (1991), one country is larger in the sense that it is endowed with more labor (the immobile factor). Then the larger country sets its tax rate closer to (but still below) the social optimum, and higher than the smaller country, but at the cost of allowing some of its (also larger) capital endowment to move to the other country.[41] The asymmetry of tax levels across countries results from the assumed asymmetry of endowments

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[40]If we drop the assumption that tax harmonization is unanticipated and if taxes are equalized then Government B's incentive to set x_B is identical to that of the planner, and it sets $x_B = x_B^E$. The qualitative conclusions are the same as when tax harmonization is unanticipated but efficiency is higher. In the model with congestion costs, efficiency does not imply that tax rates must be equalized. So providing that tax harmonization brings B's tax closer to its efficient level from above, and brings A's tax closer to its efficient level from below, then efficiency is increased. In that case, as in the case with no congestion costs, the effect would be to counter the relaxation of tax competition, and induce Government B to provide the public good at the efficient level. Rents collected by B would increase and rents collected by A would decrease relative to the equilibrium under relaxed tax competition.

[41]Wilson (1987) studies a model of Heckscher-Ohlin trade and tax competition, where one jurisdiction is endowed with more capital than the other. Consequently, public good provision is above the efficient level in one jurisdiction and inefficiently low in the other.

while in our work, in equilibrium, the asymmetry of country size and the difference in tax levels across countries are co-determined. In standard tax competition, the imposition of a minimum tax increases efficiency but unlike in our paper this is because tax competition is wasteful.[42]

The second situation analyzed in the literature on tax competition is where competition promotes efficiency. Tiebout (1956) was the first to discuss the idea that competition between jurisdictions may promote efficiency by citizens sorting themselves into jurisdictions composed of those with a similar preferences for public good provision and hence a similar willingness to pay. Brueckner (2000) sets up a framework for the consideration of Tiebout issues (differences in tastes for public services) and tax competition within a unified framework. The model of the present paper shares the feature of Tiebout-tax competition that there is variation in firms' public good requirements. Another common feature is that governments' objectives are entirely self-serving in that they are profit/rent maximizing but are constrained by competition. However, following the structure of Kanbur and Keen's (1993) tax competition model, in our model a firm cannot be disaggregated into its capital and labor inputs. Consequently, the results of the present paper contrast with those of the Tiebout-tax competition literature in that more capital (i.e. a bigger share of firms) locates in the country with higher taxation. Also, in contrast to Tiebout/tax competition where there is no policy failure, the policy-failure in our model does allow governments to have market power and this underpins the difference in outcome that efficiency is not achieved in equilibrium.[43]

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[42]Keen and Marchand (1997) show that raising the minimum tax also increases efficiency due to the effect on the composition of public spending. Questions concerning the composition of spending are beyond the scope of the present paper.

[43]Oates and Schwab (1988) show that majority rule can select the efficient outcome when there is interjurisdictional competition for mobile resources. Black and Hoyt (1989) show how the process by which jurisdictions bid for firms may promote efficiency. The promotion of efficiency within the context of competition has also been discussed by Boadway, Cuff and Marceau (2002), Boadway, Pestieau and Wildasin (1989) among others. Lockwood and Makris (2006) show that wasteful tax competition can be offset through the political process.

In this present paper each government presides over a country whereas in much of the tax competition literature governments preside over jurisdictions more broadly defined.[44] So in the present paper it seems reasonable to take the number of countries as given. Alternatively, seeking parallels with the literature on Tiebout-tax competition, market failure is created by the fact that costs of public good provision are sunk, and this notionally creates barriers to entry of new countries, enabling existing countries to make positive rents. The underlying structure of our model is developed by Shaked and Sutton (1982) and Sutton (1991 Chapter 3 in their model of vertical product competition between firms.[45]

A related idea to ours is explored by Hoyt and Jensen (2001). They too borrow the idea from the literature on vertical product differentiation and apply the analogy to the level of public good provision within the context of tax competition. However, their main focus is on the capitalization into house prices of the quality of public school provision. While tax competition is a feature of their model, they do not develop the idea of relaxed international tax competition as we do here. And their setting is essentially within the nation so issues of tax coordination are not discussed.

Justman, Thisse and van Ypersele (2002) also study a related idea that under fiscal competition regions can segment the market for industrial location by offering infrastructure services that are differentiated by quality. They identify a fiscal agglomeration property, which motivates an asymmetric equilibrium in which one jurisdiction offers a subsidy but offers public goods at a low level while another jurisdiction offers a higher level of public good provision but charges a positive 'entry fee' to the jurisdiction. However, they do not consider tax competition nor cooperation over taxes, and do not compare their equilibrium outcome to efficiency. Hence they do not characterize the relationship between the effectiveness at

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[44]Thus, while the issues that we investigate are similar to the problems of fiscal federalism investigated by Arnott and Grieson (1981), Gordon (1983) and Wilson (1986), the range of policy options that we consider are more limited than under federalism, mirroring more closely an international setting.

[45]In the sense of Sutton (1991 Chapter 3), one might say that we have a model of vertical public good differentiation, in which expenditure on public goods may be thought of as a sunk cost. This is not to be confused with vertical tax competition discussed by Dahlby and Wilson (2003) or Keen and Kotsogiannis (2003), for example, which relates to competition between governments at the 'federal' and 'state' levels. Wilson and Janeba (2005) show that a country's decentralization level serves as a strategic tool under tax competition which may improve welfare. See also Devereux, Lockwood and Redoano (2006), who consider the interaction of horizontal and vertical tax competition.

reducing costs of public good provision and the degree to which tax competition is relaxed.[46]

Some models of tax competition obtain asymmetry of outcomes as a consequence of increasing returns to scale. In that setting, industrial concentration creates 'agglomeration rent' (Baldwin and Krugman 2004). Firms benefit from the externalities of location in the core and the government is able to extract some of this surplus through higher taxation. These externalities play essentially the same role as the public good externalities in the model of this present paper (see also Kind, Knarvik and Schelderup 2000 and Ludema and Wooton 2000).

While we relate the predictions of our model to patterns of international capital taxation, a looser interpretation could be extended to explain the pattern of taxation across states in a federation. For example, the variation in tax rates across states in the US has attracted significant media attention, with the spotlight focused on discrepancies between states where taxes and public good provision are relatively high, like Massachusetts, and those where taxes and public good provision are at low levels, such as Alabama. Our model, while focused on international taxation, puts forward a way of understanding these patterns of variation in taxation across states as well, characterizing a situation where federal transfers between states are imperfect.

Although our model can explain in static terms why taxes and public good provision may be higher in one country than another, it is silent on the dynamics of how taxes have evolved over time. While some commentators have taken evidence of falling taxes across all countries to suggest that tax rates will eventually converge, our model suggests that the long run equilibrium will exhibit differentiation in tax levels across countries. An agenda for future research is to explain how average tax rates fall over time as markets become more integrated while still maintaining a stable differential between the core and the periphery.

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[46]A framework of horizontal (as opposed to vertical) product differentiation has also been adapted in previous work to the context of tax competition; see Justman, Thisse and van Ypersele (2005) for a recent contribution and review of the literature; see also Groenert, Wooders and Zissimos (2006).

A. Appendix

A.1. Proof of Propositions

Proof of Proposition 1. We first derive the efficient solution under the assumption that $x_A < x_B$. We will then show that the efficient solution cannot arise when $x_A = x_B > 0$.

Differentiate the planner's problem (3.1) to obtain the first and second order conditions for an interior efficient solution; that is, a solution in which $x_A < x_B$ and $\hat{s} \in (0, 1)$ by (2.2). We shall see from these first and second order conditions that the efficient solution is in fact obtained at $\hat{s} = 0$, and it will be obvious that the efficient solution cannot occur at $\hat{s} = 1$.

First, substitute the right hand side of (2.2) into (3.1) to obtain

$$\max_{\tau_A, \tau_B, x_A, x_B} \Omega\left(\tau_A, \tau_B, x_A, x_B\right) = (p - c) - x_A - x_B + \frac{1}{2}\left(kx_B^{\theta} - \frac{(\tau_B - \tau_A)^2}{k\left(x_B^{\theta} - x_A^{\theta}\right)}\right)$$

Then, under the assumption that $x_A < x_B$, it is easy to see that the first and second order conditions for τ_A are as follows:

$$\frac{\partial\Omega\left(\tau_A, \tau_B, x_A, x_B\right)}{\partial\tau_A} = \frac{\tau_B - \tau_A}{k\left(x_B^{\theta} - x_A^{\theta}\right)} = 0;$$

$$\text{and } \frac{\partial^2\Omega\left(\tau_A, \tau_B, x_A, x_B\right)}{\partial\tau_A^2} = -\frac{1}{k\left(x_B^{\theta} - x_A^{\theta}\right)} < 0. \tag{A.1}$$

Admitting corner solutions in taxes also requires that $\tau_B < \tau_A$. But in that case the outcome is the same as for $\tau_B = \tau_A$ because, by definition, $\hat{s} = 0$.

Next we have the same thing for τ_B:

$$\frac{\partial\Omega\left(\tau_A, \tau_B, x_A, x_B\right)}{\partial\tau_B} = -\frac{\tau_B - \tau_A}{k\left(x_B^{\theta} - x_A^{\theta}\right)} = 0;$$

$$\text{and } \frac{\partial^2\Omega\left(\tau_A, \tau_B, x_A, x_B\right)}{\partial\tau_A^2} = -\frac{1}{k\left(x_B^{\theta} - x_A^{\theta}\right)} < 0. \tag{A.2}$$

Again, admitting corner solutions in taxes also requires that $\tau_B < \tau_A$. The second order conditions in (A.1) and (A.2) show that $\Omega\left(\tau_A, \tau_B, x_A, x_B\right)$ is concave in τ_A (holding τ_B constant) and τ_B (holding τ_A constant). From the first order condition, the efficient solutions for taxes is $\tau_A^E = \tau_B^E$.

Now we introduce the efficient condition for x_A and x_B. Take x_A first and so fix $x_B > x_A$:

$$\frac{\partial \Omega\left(\tau_A, \tau_B, x_A, x_B\right)}{\partial x_A} = -1 - \frac{\theta x_A^{\theta-1}\left(\tau_B - \tau_A\right)^2}{2k\left(x_B^\theta - x_A^\theta\right)^2} < 0$$

$$\text{and } \frac{\partial^2 \Omega\left(\tau_A, \tau_B, x_A, x_B\right)}{\partial x_A^2} = -\frac{x_B^{\theta-2}\theta((\theta-1)x_B^\theta + (\theta+1)x_A^\theta)(\tau_B - \tau_A)^2}{2k\left(x_B^\theta - x_A^\theta\right)^3}. \quad \text{(A.3)}$$

Next take x_B and so fix x_A. Then for any $x_B > x_A$:

$$\frac{\partial \Omega\left(\tau_A, \tau_B, x_A, x_B\right)}{\partial x_B} = -1 + \frac{\theta x_B^{\theta-1}}{2k}\left(k^2 + \frac{\left(\tau_B - \tau_A\right)^2}{\left(x_B^\theta - x_A^\theta\right)^2}\right) = 0 \text{ and}$$

$$\frac{\partial^2 \Omega\left(\tau_A, \tau_B, x_A, x_B\right)}{\partial x_B^2} =$$

$$-\frac{\theta\left(x_B\right)^{\theta-2}\left((1-\theta)k^2\left(x_B^\theta - x_A^\theta\right)^3 + \left((1+\theta)x_B^\theta - (1-\theta)x_A^\theta\right)\left(\tau_B - \tau_A\right)^2\right)}{2k\left(x_B^\theta - x_A^\theta\right)^3} < 0. \quad \text{(A.4)}$$

Condition (A.3) shows that $\Omega\left(\tau_A, \tau_B, x_A, x_B\right)$ is everywhere declining in x_A and therefore achieves a maximum when $x_A = 0$ given $x_B > 0$. The second order condition cannot be signed unambiguously but this does not matter given that the first order condition is unambiguously negative.

Condition (A.4) shows that $\Omega\left(\tau_A, \tau_B, x_A, x_B\right)$ is concave in x_B and ensures a unique efficient level. It is immediate from (A.1) and (A.2) that the efficient level of taxation is obtained when $\tau_A^E = \tau_B^E$. Using $\tau_A^E = \tau_B^E$ in (A.3), $\partial\Omega\left(\tau_A, \tau_B, x_A, x_B\right)/\partial x_A = -1$ and $\partial^2\Omega\left(\tau_A, \tau_B, x_A, x_B\right)/\partial x_A^2 = 0$ so $\Omega\left(\tau_A, \tau_B, x_A, x_B\right)$ is maximized with respect to x_A at $x_A = 0$. Using $\tau_A^E = \tau_B^E$ in (A.4), setting $\partial\Omega\left(\tau_A, \tau_B, x_A, x_B\right)/\partial x_A = 0$ and solving in terms of x_B^E we have that $x_B^E = \left(\frac{1}{2}\right)^{\frac{1}{1-\theta}}(\theta k)^{\frac{1}{1-\theta}}$. In addition, it is clear by inspection that $\partial^2\Omega\left(\tau_A, \tau_B, x_A, x_B\right)/\partial x_B^2 < 0$ for any $x_A < x_B$. Thus we have characterized the efficient solution as $\tau_A^E = \tau_B^E$, $x_A = 0$ and $x_B^E = \left(\frac{1}{2}\right)^{\frac{1}{1-\theta}}(\theta k)^{\frac{1}{1-\theta}}$ under the assumption that $x_A < x_B$.

It remains to show that efficiency cannot be increased by setting $x_A = x_B > 0$. In that case, the value of $\hat{s}$ depends on the value of τ_A relative to τ_B: If $\tau_A > \tau_B$ then, by (2.2), $\hat{s} = 0$; if $\tau_A < \tau_B$ then $\hat{s} = 1$; if $\tau_A = \tau_B$ then by assumption $\hat{s} = \frac{1}{2}$. Take each case in turn.

Suppose first that efficiency is achieved for $x_A = x_B$ and $\tau_A > \tau_B$. By (2.2), $\hat{s} = 0$ and so by (3.1),

$$\Omega\left(\tau_A, \tau_B, x_A, x_B\right) = p - c - x_A - x_B + \frac{1}{2}\left(kx_B^\theta\right).$$

But efficiency could be increased by reducing x_A; contradiction.

Next suppose that efficiency is achieved for $x_A = x_B$ and $\tau_A < \tau_B$. By (2.2), $\hat{s} = 1$ and so by (3.1),

$$\Omega\left(\tau_A, \tau_B, x_A, x_B\right) = p - c - x_A - x_B + \frac{1}{2}\left(kx_A{}^{\theta}\right).$$

But now efficiency could be increased by reducing x_B; contradiction.

Finally, suppose that efficiency is achieved for $x_A = x_B$ and $\tau_A = \tau_B$. By (2.2), $\hat{s} = \frac{1}{2}$ and so by (3.1),

$$
\begin{aligned}
\Omega\left(\tau_A, \tau_B, x_A, x_B\right) &= p - c - x_A - x_B + \frac{1}{2}\left(\frac{1}{2}kx_A{}^{\theta} + \frac{1}{2}kx_B{}^{\theta}\right) \\
&= p - c - x_A - x_B + \frac{1}{2}\left(kx_B{}^{\theta}\right)
\end{aligned}
$$

where the second equality follows because $x_A = x_B$. But this is the same outcome as for $x_A = x_B$ and $\tau_A > \tau_B$, and for that case we saw that it was possible to increase efficiency by reducing x_A; contradiction. $\square$

Proof of Lemma 1. Fix $0 \le x_A < x_B$. To solve for τ_A^*, fix τ_B. We want to solve

$$\max_{\tau_A} r_A\left(\tau_A, \tau_B\right) = \frac{\tau_A\left(\tau_B - \tau_A\right)}{k\left(x_B{}^{\theta} - x_A{}^{\theta}\right)} - x_A.$$

First, looking at the second order condition, we see that

$$\partial^2 r_A / \partial \tau_A^2 = -2 / \left(k\left(x_B{}^{\theta} - x_A{}^{\theta}\right)\right) < 0,$$

so $r_A\left(\tau_A, \tau_B\right)$ is everywhere concave with respect to τ_A. Setting the first order condition $\partial r_A / \partial \tau_A = \left(-2\tau_A^* + \tau_B\right) / \left(k\left(x_B{}^{\theta} - x_A{}^{\theta}\right)\right)$ equal to zero and rearranging in terms of τ_A^* obtains $\tau_A\left(\tau_B; x_A, x_B, k\right) = \tau_B/2$.

To solve for τ_B^*, fix τ_A. Now we want to solve

$$\max_{\tau_B} r_B\left(\tau_A, \tau_B\right) = \tau_B\left(1 - \frac{\left(\tau_B - \tau_A\right)}{k\left(x_B{}^{\theta} - x_A{}^{\theta}\right)}\right) - x_B.$$

Again, looking at the second order condition first, we see that

$$\partial^2 r_B / \partial \tau_B^2 = -2 / \left(k\left(x_B{}^{\theta} - x_A{}^{\theta}\right)\right) < 0,$$

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so $r_B(\tau_A, \tau_B)$ is concave with respect to τ_B. Setting the first order condition $\partial r_B / \partial \tau_B = 1 + (\tau_A - 2\tau_B^*) / \left(k \left(x_B{}^\theta - x_A{}^\theta \right) \right)$ equal to zero and rearranging in terms of τ_B obtains the result. $\square$

Proof of Proposition 2. For $x_A = x_B$ both governments provide the same level of public goods and we effectively have a standard Bertrand equilibrium in homogeneous products. Then $\hat{s} = 1/2$.

For $x_A < x_B$, by Lemma 1 for given τ_B, $r_A(\tau_A, \tau_B)$ is maximized by $\tau_A^* = \tau_B/2$. For given τ_A, $r_B(\tau_A, \tau_B)$ is maximized by $\tau_B^* = \tau_A/2 + k \left(x_B{}^\theta - x_A{}^\theta \right)/2$. Solving simultaneously obtains the reduced form expressions for $\tau_A^*(x_A, x_B; k)$ and $\tau_B^*(x_A, x_B; k)$.

Using $\tau_A^*(x_A, x_B; k) = k \left(x_B{}^\theta - x_A{}^\theta \right)/3$ and $\tau_B^*(x_A, x_B; k) = 2k \left(x_B{}^\theta - x_A{}^\theta \right)/3$ in $\hat{s} = (\tau_B - \tau_A) / k \left(x_B{}^\theta - x_A{}^\theta \right)$ obtains $\hat{s} = 1/3$. $\square$

Proof of Proposition 3: To determine Government A's set of best responses, we investigate the properties of $r_A(x_A, x_B)$. It is clear by inspection of (4.2) that $r_A(x_A, x_B)$ achieves a minimum at $x_A = x_B$. So we can rule out $x_A = x_B$ from $BR_A(x_B)$. Now observe that if $0 \leq x_A < x_B$ then $r_A = k \left(x_B{}^\theta - x_A{}^\theta \right)/9 - x_A$ so $r_A(x_A, x_B)$ is everywhere downward sloping and convex over this range. Consequently, $x_A = 0$ maximizes $r_A(x_A, x_B)$ for $0 \leq x_A < x_B$. If on the other hand $0 \leq x_B < x_A$, then $r_A = 4k \left(x_A{}^\theta - x_B{}^\theta \right)/9 - x_A$, and $r_A(x_A, x_B)$ is everywhere strictly concave. Differentiating once, setting the result equal to zero and rearranging, we find that $r_A(x_A, x_B)$ has a unique maximum at $x_A = \left(\frac{4}{9}\theta k \right)^{\frac{1}{1-\theta}}$. Thus $BR_A(x_B) \in \left\{ 0, \left(\frac{4}{9}\theta k \right)^{\frac{1}{1-\theta}} \right\}$. Because $r_B(x_A, x_B)$ has the same functional form as $r_A(x_A, x_B)$, it follows that $BR_B(x_A) \in \left\{ 0, \left(\frac{4}{9}\theta k \right)^{\frac{1}{1-\theta}} \right\}$; see (4.3). Recall that $r_A(x_A, x_B)$ and $r_B(x_A, x_B)$ achieve a minimum at $x_A = x_B$. So $\left(0, \left(\frac{4}{9}\theta k \right)^{\frac{1}{1-\theta}} \right)$ is the only set of mutual best responses and must therefore be a Nash equilibrium. Clearly, there are two Nash equilibria; $\left(0, \left(\frac{4}{9}\theta k \right)^{\frac{1}{1-\theta}} \right)$ and $\left(\left(\frac{4}{9}\theta k \right)^{\frac{1}{1-\theta}}, 0 \right)$. But we may now assume, without loss of generality, that $x_A < x_B$. Then $(x_A^*, x_B^*) = \left(0, \left(\frac{4}{9}\theta k \right)^{\frac{1}{1-\theta}} \right)$ is the unique Nash equilibrium. Using these values to solve for equilibrium taxes from Proposition 2, we have that $\tau_A^* = \frac{1}{3}k \left(\frac{4}{9}\theta k \right)^{\frac{\theta}{1-\theta}}$ and $\tau_B^* = \frac{2}{3}k \left(\frac{4}{9}\theta k \right)^{\frac{\theta}{1-\theta}}$. Thus we have the result.

Finally, differentiate the planner's problem (3.1), to obtain the first order condition; $d\Omega / dx_B = \frac{1}{2}\theta k x_B^{\theta-1} - 1$. Setting this equal to 0 and solving for x_B obtains x_B^E. $\square$

Proof of Proposition 4. To see that $r_A^\mu(\varepsilon)$ is concave in ε, differentiate $r_A^\mu(\varepsilon)$ once with respect to ε to obtain

$$\frac{dr_A^\mu(\varepsilon)}{d\varepsilon} = \frac{1}{6} - \frac{\varepsilon}{k\left(\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}\right)}.$$

Clearly, $dr_A^\mu(\varepsilon)/d\varepsilon > 0$ as $\varepsilon \to 0$ and $dr_A^\mu(\varepsilon)/d\varepsilon < 0$ as ε becomes large. Also, $dr_A^\mu(\varepsilon)/d\varepsilon$ declines monotonically with ε. The unique value of ε that maximizes r_A^μ is $\varepsilon = \frac{1}{6}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$. By definition, a minimum tax for which $\varepsilon < \frac{1}{6}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$ cannot be on the frontier because both governments make higher rents by increasing ε to $\varepsilon = \frac{1}{6}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$; thus we have defined the lower bound of the non-renegotiable minimum tax frontier.

By definition, the minimum tax on the frontier must yield higher rents for both governments than no minimum tax. Because $r_B^\mu(\varepsilon)$ increases monotonically with ε, B makes higher rent with any minimum tax than with no minimum tax. However, $r_A^\mu(\varepsilon)$ declines monotonically with ε for $\varepsilon > \frac{1}{6}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$. Therefore, a level of $\varepsilon > \frac{1}{6}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$ must exist at which $r_A^\mu(\varepsilon) = r_A^\mu(0)$. It is easy to establish that $r_A^\mu(0) = \frac{2}{3}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$. Then $\varepsilon = \frac{1}{3}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$ is the unique level of $\varepsilon > \frac{1}{6}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$ at which $r_A^\mu(\varepsilon) = r_A^\mu(0)$; thus we have defined the upper bound of the non-renegotiable minimum tax frontier. $\square$

A.2. The Model with Congestion Costs

Congestion costs can be introduced in a simply way, following Ahlin and Ahlin (2006). Define as $\hat{s}_c$ the firm that is just indifferent between locating in either country when congestion has a negative impact on profits. Specifically, we will say that if firm $s \in [0,1]$ locates in Country A then its profits are reduced in proportion to overall share of firms that locate in A, $\phi\hat{s}_c$, where $\phi \geq 0$ is a parameter. Similarly, if s locates in Country B then its profits are reduced by $\phi(1-\hat{s}_c)$. Then the profit function for the firm at $s \in [0,1]$ is given by

$$\pi_s = \begin{cases} p - c - \tau_A + skx_A^{\,\theta} - \phi\hat{s}_c & \text{if } s \text{ locates in } A \\ p - c - \tau_B + skx_B^{\,\theta} - \phi(1-\hat{s}_c) & \text{if } s \text{ locates in } B \end{cases}.$$

Then

$$\tau_A - \hat{s}_c kx_A^{\,\theta} - \phi\hat{s}_c = \tau_B - \hat{s}_c kx_B^{\,\theta} - \phi(1-\hat{s}_c).$$

First, solve the above expression for $\hat{s}_c$ and hence define the function

$$\hat{s}_c(\tau_A, \tau_B, x_A, x_B; \phi) = \frac{\tau_B - \tau_A + \phi}{k\left(x_B^{\,\theta} - x_A^{\,\theta}\right) + 2\phi} \tag{A.5}$$

Then $\hat{s}_c$ is defined as follows:

$$
\hat{s}_c =
\begin{cases}
\hat{s}_c\left(\tau_A, \tau_B, x_A, x_B; \phi\right) & \text{if } \hat{s}_c\left(\tau_A, \tau_B, x_A, x_B; \phi\right) \in [0,1]; \\
1 & \text{if } \hat{s}_c\left(\tau_A, \tau_B, x_A, x_B; \phi\right) > 1; \\
0 & \text{if } \hat{s}_c\left(\tau_A, \tau_B, x_A, x_B; \phi\right) < 0.
\end{cases}
$$

For $x_A = x_B$,

$$
\hat{s}_c =
\begin{cases}
0 & \text{if } \tau_A < \tau_B; \\
1 & \text{if } \tau_A > \tau_B; \\
\frac{1}{2} & \text{if } \tau_A = \tau_B.
\end{cases}
$$

The *rents to office* functions for Governments A and B remain as $r_A = \tau_A \hat{s} - x_A$ and $r_B = \tau_B\left(1 - \hat{s}\right) - x_B$ respectively.

First, let us consider the efficient solution with the congestion cost. The congestion cost enters the expression for efficiency through firms' profits, so that (3.1) becomes

$$
\begin{aligned}
\Omega \;=\; & p - c + \frac{1}{2}\left(kx_B^{\theta} - 2\left(x_A^{\theta} + x_B^{\theta} + \phi\right)\right. \\
& \left. + \frac{\left(\tau_B - \tau_A + \phi\right)\left(\tau_B - \tau_A - 3\phi\right)}{k\left(x_B^{\theta} - x_A^{\theta}\right) + 2\phi} - \frac{2\phi\left(\tau_B - \tau_A + \phi\right)^2}{\left(k\left(x_B^{\theta} - x_A^{\theta}\right) + 2\phi\right)^2}\right).
\end{aligned}
$$

From the first order conditions for efficiency,

$$
\tau_B^E = \tau_A^E + \phi - \frac{4\phi^2}{k\left(x_B^{\theta} - x_A^{\theta}\right) + 4\phi}.
$$

Note that if $x_B^{\theta} = x_A^{\theta}$ then $\tau_B^E = \tau_A^E$ and if $x_B^{\theta} > x_A^{\theta}$ then $\tau_B^E > \tau_A^E$. Using this relation for efficient taxes in the expression for $\hat{s}_c$, we have

$$
\hat{s}_c = \frac{2\phi}{k\left(x_B^{\theta} - x_A^{\theta}\right) + 4\phi}.
$$

Thus, as one should expect, in the presence of the congestion cost it is no longer efficient for the planner to induce all firms to locate in the same country.

Using this relation for efficient taxes in the first derivative of Ω with respect to x_A,

$$
\frac{d\Omega}{dx_A} = -1 + \frac{2\theta k\phi^2 x_A^{\theta-1}}{\left(k\left(x_B^{\theta} - x_A^{\theta}\right) + 4\phi\right)^2}.
$$

From this we can see that, providing k is not too large, $d\Omega/dx_A < 0$ and so $x_A^E = 0$. Even in the presence of the congestion cost, under which some firms are induced to move to A in

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order to avoid the congestion cost, it is efficient for the planner to provide the public good only in B. Because the firms in the interval $[0, \hat{s}_c)$ are relatively unproductive in their use of the public good, the planner does not find it worth making the public good available in the country where they locate. If k is large then it is possible to have $d\Omega/dx_A > 0$ but checking endpoints reveals that $x_A = 0$ maximizes Ω and so $x_A^E = 0$ holds nevertheless.

Using the relation for efficient taxes in the first derivative of Ω with respect to x_B,

$$\frac{d\Omega}{dx_B} = -1 + \frac{1}{2}\theta k x_B^{\theta-1}\left(1 - \frac{4\phi^2}{\left(k\left(x_B^{\theta} - x_A^{\theta}\right) + 4\phi\right)^2}\right).$$

The second term in the brackets determines the impact of the congestion cost on x_B^E. Clearly, for $\phi > 0$ it is not possible to obtain a general closed-form solution for x_B^E by setting this first order condition to zero (for $\phi = 0$ we obtain the solution shown in Proposition 1). But, by inspection of the first derivative, x_B^E is lower in the presence of the congestion cost. The intuition is that, since the congestion cost induces some firms to move to A, the value to society of providing the public good to (fewer) firms in B is reduced.

Let us now solve for the tax subgame in the presence of the congestion cost. Solving as in Section 4.1, we obtain the following. For $x_A = x_B$, both governments provide the same level of public good and there exists a unique equilibrium in which $\tau_A^* = \tau_B^* = \phi$. For $x_A \neq x_B$ we assume as usual that $x_A < x_B$. Then there exists a unique subgame equilibrium point determined by the taxes

$$\begin{aligned}
\tau_A^*\left(x_A, x_B\right) &= \frac{1}{3}k\left(x_B^{\theta} - x_A^{\theta}\right) + \phi; \\
\tau_B^*\left(x_A, x_B\right) &= \frac{2}{3}k\left(x_B^{\theta} - x_A^{\theta}\right) + \phi.
\end{aligned}$$

At $\tau_A^*\left(x_A, x_B; k\right)$ and $\tau_B^*\left(x_A, x_B; k\right)$, the share of firms locating in Country A is given by

$$\hat{s}_c = \frac{1}{3}\left(1 + \frac{\phi}{k\left(x_B^{\theta} - x_A^{\theta}\right) + 2\phi}\right).$$

The congestion cost acts to push firms towards Country A. If $\phi = 0$ then $\hat{s}_c = 1/3$ as we should expect. And $\hat{s}_c$ is increasing in ϕ. Also, as we shall see, x_B is decreasing in ϕ (both in absolute terms and relative to x_A) the effect of which contributes further to an increase in $\hat{s}_c$. Note also that $\hat{s}_c$ is bounded from above at $\frac{1}{2}$.

Tax competition is relaxed in the presence of the congestion cost as well. From the above solutions we have

$$\left(\tau_B^* - \tau_A^*\right) - \left(\tau_B^E - \tau_A^E\right) = \frac{1}{\frac{12}{k\left(x_B^\theta - x_A^\theta\right)} - \frac{9}{k\left(x_B^\theta - x_A^\theta\right) + \phi}} > 0. \tag{A.6}$$

We can see that there is a larger difference between taxes in equilibrium than under efficiency. Note that, for given x_A and x_B, the difference is decreasing in ϕ; the effect of a fall in x_B (relative to x_A) in response to an increase in ϕ tends to reinforce this effect. Thus, tax competition becomes *less relaxed* as ϕ is increased. The reason is that the congestion effect serves to make the countries more similar, by bringing x_B closer to x_A (to be shown below) and by increasing $\hat{s}_c$. But note that the right hand side does not converge to zero as ϕ becomes large (providing $x_B > x_A$).

Finally, we can now solve for the equilibrium level of public good provision in Stage 1. With congestion costs, the first order condition for the maximization of rents in Country A becomes

$$\frac{dr_A}{dx_A} = -1 + \frac{1}{9}\theta k x_A^{\theta-1}\left(-1 + \frac{\phi^2}{\left(k\left(x_B^\theta - x_A^\theta\right) + 2\phi\right)^2}\right)$$

It is straight forward to verify that $dr_A/dx_A < 0$. This is immediate when $\phi = 0$. Now observe that the second term in the interior brackets on the right hand side converges to $1/4$ as ϕ becomes large. So the sum of the terms in the brackets must always be negative. Therefore, $x_A^* = 0$ in equilibrium, as with no congestion costs (Proposition 3).

Also,

$$\frac{dr_B}{dx_B} = -1 + \frac{1}{9}k\theta x_B^{\theta-1}\left(4 - \frac{\phi^2}{\left(k\left(x_B^\theta - x_A^\theta\right) + 2\phi\right)^2}\right)$$

The second term in brackets determines the impact of the congestion cost on x_B^*. We can see by inspection that x_B^* is positive and decreasing in the size of ϕ (equal to the value in Proposition 3 for $\phi = 0$).

Notice that the negative impact of ϕ is larger in $d\Omega/dx_B$ then in dr_B/dx_B. That is, the (negative) term in ϕ in the brackets is bigger in $d\Omega/dx_B$ than in dr_B/dx_B. This implies that for large ϕ there may actually be over-provision of the public good; $x_B^* > x_B^E$. This stands to reason. As remarked above, the congestion cost induces more firms to locate in Country A. In equilibrium the difference in taxes remains constant. Country B tries to offset the

loss of firms, and hence rents, by providing the public good at a higher level. This implies that there must exist a level of ϕ at which $x_B^E = x_B^*$. In other words, as ϕ increases and tax competition becomes less relaxed, public good provision also tends towards the efficient level. But note that $\hat{s}_c$ does not correspond to efficiency at $x_B^E = x_B^*$, so it cannot be said that the congestion cost completely neutralizes the effect of relaxed tax competition. Moreover, (as noted above) $x_B^* > x_B^E$ for larger values of ϕ, so there is no convergence to efficiency.

A.3. The Role of θ in the Model

This discussion continues from the discussion of the effect of k on x_B^* in Section 4.2. The effect of θ on x_B^* is less obvious than the effect on k. While for k relatively large, x_B^* is monotonically increasing in θ, for k relatively small the effect on x_B^* of an increase in θ is ambiguous. To show the ambiguity, in Figure 2 we illustrate r_B under the assumption that $k = 1$ (i.e. relatively small) and that all equilibrium values other than x_B^* hold; $\tau_A = \tau_A^*$, $\tau_B = \tau_B^*$, $x_A^* = 0$ and consequently $\hat{s} = \frac{1}{3}$. Using these values, it is easy to work out that $r_B = \frac{4}{9}kx_B^\theta - x_B$. Figure 2 illustrates how r_B varies with x_B for $\theta = \frac{1}{10}$, $\theta = \frac{1}{4}$ and $\theta = \frac{2}{3}$. We see that for each value of θ there is a unique value x_B^* that maximizes r_B. Moreover, x_B^* *increases* as θ is increased from $\theta = \frac{1}{10}$ to $\theta = \frac{1}{4}$ but x_B^* *decreases* as θ is increased form $\theta = \frac{1}{4}$ to $\theta = \frac{2}{3}$. The reason can be seen most clearly by inspection of the first derivative of the rent function, $dr_B/dx_B = \frac{4}{9}\theta kx_B^{\theta-1} - 1$. An increase in θ has two conflicting effects on the first term. While an increase in θ tends to increase $\frac{4}{9}\theta kx_B$, an increase in θ tends to decrease $x_B^{\theta-1}$ (for fixed k and x_B). Moreover, the negative second effect increases non-linearly with θ. To put this another way, an increase in θ reduces the curvature of r_B everywhere but also reduces the initial gradient of r_B in the neighborhood of $x_B = 0$. Thus x_B^* may be first increasing then decreasing in θ. However, it is also easy to see that k may be set large enough so that the first term is monotonically increasing in k for $\theta \in (0, 1)$. In that case x_B^* is monotonically increasing in θ just as it is monotonically increasing in k.

The effect of θ on x_B^E is very similar, for reasons that are closely related. Observe, by differentiating the planner's problem (3.1), that $d\Omega/dx_B = \frac{1}{2}\theta kx_B^{\theta-1} - 1$. We can see by analogy that, for relatively low k, x_B^E is first increasing then decreasing in θ. As for x_B^*, it is possible to set k sufficiently large so that x_B^E is monotonically increasing in θ.

We are also able to see quite clearly the effect of θ on the suboptimality of public good provision. We do this by calculating the ratio of the level of public good provision at equilibrium and efficient levels in Country B; $x_B^*/x_B^E = \frac{8}{9}^{\frac{1}{1-\theta}}$. Observe that $x_B^*/x_B^E \to \frac{8}{9}$ as $\theta \to 0$ and $x_B^*/x_B^E \to 0$ as $\theta \to 1$. We noted above that the effect of an increase in θ on x_B^* and x_B^E may be ambiguous. Recall from Figure 2, for example, that an increase in θ could bring about an increase in x_B^* and x_B^E at θ relatively close to 0 but a decrease in x_B^* and x_B^E at θ relatively close to 1. From Proposition 3 it becomes evident that there is a systematic effect of θ on x_B^* relative to x_B^E in spite of the ambiguous effect of θ on the levels of x_B^* and x_B^E.

A.4. Minimum Tax Anticipated

In the following, we show that even when the minimum tax is anticipated, rents for the respective governments have the same qualitative characterization as in Section 5.1 where public good provision was fixed. Specifically, $r_B(0, x_B^\mu(\varepsilon))$ is monotonically increasing in ε while $r_A(0, x_B^\mu(\varepsilon))$ is concave in ε with a unique optimum that defines the lower bound of the non-renegotiable minimum tax frontier.

We assume that the imposition of the minimum tax is anticipated before the start of Stage 1, so each government takes the minimum tax into account when determining the level of public good provision. Thus, the minimum tax is agreed upon after which the sequence of events is exactly as in Section 4. Best response taxes with the minimum tax are as follows: if $x_B > x_A$ then $\tau_A^\mu = \frac{1}{3}k\left(x_B{}^\theta - x_A{}^\theta\right) + \varepsilon$ and $\tau_B^\mu = \frac{2}{3}k\left(x_B{}^\theta - x_A{}^\theta\right) + \frac{1}{2}\varepsilon$; on the other hand if $x_A > x_B$ then $\tau_A^\mu = \frac{2}{3}k\left(x_A{}^\theta - x_B{}^\theta\right) + \frac{1}{2}\varepsilon$ and $\tau_B^\mu = \frac{1}{3}k\left(x_A{}^\theta - x_B{}^\theta\right) + \varepsilon$. If $x_A = x_B$ then $\tau_A^\mu = \tau_B^\mu = 0$. But now the levels of public good provision x_A and x_B are determined optimally in Stage 1. Using these expressions for τ_A^μ and τ_B^μ, Government A's rent function is defined as follows for $\varepsilon \in \left[0, \frac{2}{3}k\left|x_B{}^\theta - x_A{}^\theta\right|\right]$:

$$r_A^\mu(x_A, x_B; \varepsilon) = \begin{cases} \frac{1}{9}k\left(x_B{}^\theta - x_A{}^\theta\right) - x_A + \frac{1}{6}\varepsilon - \frac{1}{2k\left(x_B{}^\theta - x_A{}^\theta\right)}\varepsilon^2 & \text{if } 0 \le x_A < x_B; \\ -x_A & \text{if } 0 \le x_A = x_B; \\ \frac{4}{9}k\left(x_A{}^\theta - x_B{}^\theta\right) - x_A + \frac{2}{3}\varepsilon + \frac{1}{4k\left(x_A{}^\theta - x_B{}^\theta\right)}\varepsilon^2 & \text{if } 0 \le x_B < x_A. \end{cases} \quad \text{(A.7)}$$

For Government B,

$$r_B^{\mu}(x_A, x_B; \varepsilon) = \begin{cases} \frac{4}{9}k\left(x_B{}^{\theta} - x_A{}^{\theta}\right) - x_B + \frac{2}{3}\varepsilon + \frac{1}{4k\left(x_B{}^{\theta} - x_A{}^{\theta}\right)}\varepsilon^2 & \text{if } 0 \leq x_A < x_B; \\ -x_B & \text{if } 0 \leq x_A = x_B; \\ \frac{1}{9}k\left(x_A{}^{\theta} - x_B{}^{\theta}\right) - x_B + \frac{1}{6}\varepsilon - \frac{1}{2k\left(x_A{}^{\theta} - x_B{}^{\theta}\right)}\varepsilon^2 & \text{if } 0 \leq x_B < x_A. \end{cases} \tag{A.8}$$

As was the case for when there was no minimum tax, when it maximizes $r_A^{\mu}(x_A, x_B; \varepsilon)$ a level of public good provision $x_A^{\mu}(\varepsilon)$ of Government A is a *best response against a level of public good provision* $BR_A(x_B; \varepsilon)$.[47] A *Nash equilibrium in levels of public good provision* is a pair $(x_A^{\mu}(\varepsilon), x_B^{\mu}(\varepsilon))$ where $x_A^{\mu}(\varepsilon)$ is a best response against $x_B^{\mu}(\varepsilon)$ and vice-versa.

The characterization of equilibrium is technically the same as discussed in Section 4.2 for the case with no minimum tax; see the appendix for details. The equilibrium is asymmetric, with one government providing no public good and the other providing the public good at a positive level. As before, w.o.l.o.g. we let A be the country with no public good provision in equilibrium; $x_A^{\mu}(\varepsilon) = 0$. The top panel of Figure 3 shows a plot of $x_B^{\mu}(\varepsilon)$ while the bottom panel shows $1 - \hat{s}(0, x_B^{\mu}(\varepsilon))$ for $k = 1$ (and $\theta = \frac{1}{2}$) as ε is varied.[48] Note from the bottom panel that $1 - \hat{s}(0, x_B^{\mu}(\varepsilon))$ is increasing in ε and $1 - \hat{s}(0, x_B^{\mu}(\varepsilon)) = 1$ for $\varepsilon = \frac{2}{3}k\left(x_B{}^{\theta} - x_A{}^{\theta}\right) = \frac{5}{36}$. Also note that all values for $\varepsilon = 0$ correspond to equilibrium values given in Proposition 3. Thus, at $\varepsilon = 0$, $x_B^{\mu}(0) = x_B^{\mu} = \left(\frac{2}{9}\right)^2$. The top panel shows that $x_B^{\mu}(\varepsilon)$ decreases monotonically with ε until the point where $1 - \hat{s}(0, x_B^{\mu}(\varepsilon)) = 1$. Government B's incentive to compete in public good provision (by offering the public good at a higher level than Government A) is reduced by the fact that Government A is limited in the extent to which it is allowed to set its tax lower than B's.

Turning to Figure 4, we see that for $k = 1$, $\theta = \frac{1}{2}$, rents for the respective governments have the same qualitative characterization as in Section 5.1 where public good provision was fixed. Thus, as claimed, $r_B(0, x_B^{\mu}(\varepsilon))$ is monotonically increasing in ε while $r_A(0, x_B^{\mu}(\varepsilon))$ is concave in ε. The non-renegotiable minimum tax frontier is shown in Figure 4 as the interval $\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon})$. The upper and lower bounds, $\underline{\varepsilon}$ and $\overline{\varepsilon}$, are defined in the same way as in Proposition 4. For $\varepsilon < \underline{\varepsilon}$ both governments would agree to implement a higher minimum tax. But for $\varepsilon > \overline{\varepsilon}$, Government A makes higher rent with no minimum tax.

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[47]Note the distinction we make between the best response function and rent function with and without the minimum tax; the functions are shown to be dependent on the parameter ε in the former case.

[48]We have written $\hat{s}(\tau_A, \tau_B, x_A, x_B)$ in the form $\hat{s}(0, x_B^{\mu}(\varepsilon))$ to represent the fact that taxes $\tau_A = \tau_A^{\mu}$ and $\tau_B = \tau_B^{\mu}$ have been determined as functions of $x_A^{\mu}(\varepsilon) = 0$ and $x_B^{\mu}(\varepsilon)$.

Minimum Tax Anticipated: Characterization of Equilibrium. To determine Government A's set of best responses with the minimum tax, we investigate the properties of $r_A^\mu (x_A, x_B; \varepsilon)$. First we check the range $0 \leq x_A < x_B$, over which $r_A^\mu (x_A, x_B; \varepsilon) = \frac{1}{9}k \left(x_B^{\ \theta} - x_A^{\ \theta} \right) - x_A + \frac{1}{6}\varepsilon - \varepsilon^2/2k \left(x_B^{\ \theta} - x_A^{\ \theta} \right)$. Taking the first derivative, we have $dr_A/dx_A = -1 - \theta x_A^{\theta-1} \left(2k^2 + 9\varepsilon^2/ \left(x_B^{\ \theta} - x_A^{\ \theta} \right)^2 \right)/18k < 0$; $r_A^\mu (x_A, x_B; \varepsilon)$ is everywhere downward sloping over the range $0 \leq x_A < x_B$. Now note that $r_A^\mu (x_A, x_B; \varepsilon) > -x_A$ at $x_A = x_B > 0$ for $\varepsilon \in \left(0, \frac{2}{3}k \left(x_B^{\ \theta} - x_A^{\ \theta} \right) \right)$, and $r_A^\mu (x_A, x_B; \varepsilon) = -x_A$ at $x_A = x_B > 0$ for $\varepsilon = \frac{2}{3}k \left(x_B^{\ \theta} - x_A^{\ \theta} \right)$. We can conclude that $r_A^\mu (x_A, x_B; \varepsilon) \geq -x_A$ for all ε as $x_A \to x_B$ from below. Consequently, $x_A = 0$ maximizes $r_A^\mu (x_A, x_B; \varepsilon)$ for the range $0 \leq x_A < x_B$ and $x_A = 0$ dominates $x_A = x_B$. Thus $x_A^\mu (\varepsilon) = 0$ is the best response over the range $0 \leq x_A < x_B$.

If on the other hand $0 \leq x_B < x_A$, then $r_A^\mu (x_A, x_B; \varepsilon) = \frac{4}{9}k \left(x_A^{\ \theta} - x_B^{\ \theta} \right) - x_A + \frac{2}{3}\varepsilon + \varepsilon^2/4k \left(x_A^{\ \theta} - x_B^{\ \theta} \right)$. Taking the first derivative, we have

$$dr_A/dx_A = -1 + \frac{4}{9}k\theta x_A^{\theta-1} \left(1 - 9\varepsilon^2/16 \left(x_A^{\ \theta} - x_B^{\ \theta} \right)^2 \right).$$

We cannot solve explicitly for $x_A^\mu (\varepsilon)$ over the range $0 \leq x_B < x_A$ without specifying θ. However, by specifying values of ε we can obtain a characterization of $x_A^\mu (\varepsilon)$. To illustrate, fix ε at its maximum admissible value $\varepsilon = \overline{\varepsilon} = \frac{2}{3}k \left(x_B^{\ \theta} - x_A^{\ \theta} \right)$, and substitute this into the first derivative. We have $dr_A/dx_A = -1 + \frac{5}{12}k\theta x_A^{\theta-1}$. Setting the result equal to zero and solving, we have $x_A^\mu (\overline{\varepsilon}) = \left(\frac{5}{12}\theta k \right)^{\frac{1}{1-\theta}}$. Then, following the same logic as in Section 4.2 preceding Proposition 3, and using the assumption that $x_A < x_B$, we have that $(x_A^\mu (\overline{\varepsilon}), x_B^\mu (\overline{\varepsilon})) = \left(0, \left(\frac{5}{12}\theta k \right)^{\frac{1}{1-\theta}} \right)$ is the unique Nash equilibrium. Taxes are obviously the same across countries for $\varepsilon = \overline{\varepsilon}$, at $\tau_A^\mu = \tau_B^\mu = k \left(\frac{5}{12}\theta k \right)^{\frac{\theta}{1-\theta}}$ and $\hat{s} = 1$.

Notice that $x_A^\mu (\overline{\varepsilon}) = \left(\frac{5}{12}\theta k \right)^{\frac{1}{1-\theta}} < x_A^\mu (0) = \left(\frac{4}{9}\theta k \right)^{\frac{1}{1-\theta}}$. More generally, by the implicit function theorem we know that $x_A^\mu (\varepsilon)$ may be treated as a continuous function of ε. It can be established that $x_A^\mu (\varepsilon)$ is inversely related to ε as ε is varied over the interval $\varepsilon \in [0, \overline{\varepsilon}]$ for $0 \leq x_B < x_A$. Following, once again the, same logic as in Section 4.2 we have that $(x_A^\mu (\varepsilon), x_B^\mu (\varepsilon)) = (0, BR_B (0; \varepsilon))$ is the unique Nash equilibrium (given that $x_A < x_B$).[49]

We want to go one step further, and investigate the behavior of $r_A (0, BR_B (0; \varepsilon); \varepsilon)$ and $r_B (0, BR_B (0; \varepsilon); \varepsilon)$ as ε is varied in order to determine the non-renegotiable minimum tax frontier. While this cannot be done at a general level, it can be done for the specific

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[49]By the same arguments as in Section 4.2, $BR_B (0; \varepsilon) \neq 0$.

value $\theta = \frac{1}{2}$, which we believe to be generally illustrative. We maintain the assumption that $x_A < x_B$ and solve for $x_B^\mu(\varepsilon)$. This root is very cumbersome to write down, and since $x_A^\mu(\varepsilon) = 0$ is a dominant strategy for Government A we jump straight to the equilibrium value:

$$x_B^*(\varepsilon) = 512(-2)^{1/3} k^8 - (-2)^{2/3} \phi^{2/3} + 16k^4 \left(-2187(-2)^{1/3} \varepsilon^2 + 2\phi^{1/3}\right) / \left(1944\phi^{1/3}\right)$$

where

$$\phi = -8192k^{12} + 839808k^8\varepsilon^2 - 14348907k^4\varepsilon^4 + 59049\sqrt{-768k^{12}\varepsilon^6 + 59049k^8\varepsilon^8}.$$

This solution for $x_B^\mu(\varepsilon)$ is illustrated for $k = 1$ in Figure 3 and used to define the non-renegotiable minimum tax frontier illustrated in Figure 4.

References

[1] Ahlin, C. and P. Ahlin (2006); "Hotelling Was Right about Congestible Goods" Vanderbilt University Typescript.

[2] Arnott, R. and R.E. Grieson (1981); "Optimal Fiscal Policy for a State or Local Government." *Journal of Urban Economics*, 9: 23-48 (January).

[3] Baldwin, R. and P. Krugman (2004); "Agglomeration, Integration and Tax Harmonization." *European Economic Review* 48(1): 1-23.

[4] Black, D. and W.H. Hoyt (1989): "Bidding for Firms." *American Economic Review*, 79(5): 1249-1256.

[5] Boadway, R., K. Cuff and N. Marceau (2002); "Inter-jurisdictional Competition for Firms," *International Economic Review*, 43(5): 761-782.

[6] Boadway, R. P. Pestieau and D.E. Wildasin (1989); "Non-cooperative Behavior and Efficient Provision of Public Goods," *Public Finance*, 44(1): 1-7.

[7] Brennan, G. and J.M. Buchanan, (1980); *The Power To Tax: Analytical Foundations of a Fiscal Constitution*, Cambridge University Press, Cambridge.

[8] Brueckner, J.K. (2000); "A Tiebout/Tax-Competition Model." *Journal of Public Economics*, 77: 285-306.

[9] Casella, A. and J.S. Feinstein (2002); "Public Goods in Trade: On the Formation of Markets and countries." *International Economic Review* 43: 437-462.

[10] Dahlby, B., and L.S. Wilson (2003); "Vertical Fiscal Externalities." *Journal of Public Economics*, 87: 917-930.

[11] Devereux, M.P. and R. Griffith (2003); "Evaluating Tax Policy for Location Decisions." *International Tax and Public Finance*, 10: 107-126.

[12] Devereux, M.P., B. Lockwood and M. Redoano (2006); "Horizontal and Vertical Indirect Tax Competition: Theory and Some Evidence from The USA." *Journal of Public Economics* doi:10.1016/j.jpubeco.2006.07.005.

[13] Edwards, J. and M. Keen (1996); "Tax Competition and Leviathan." *European Economic Review* 40(1): 113-134.

[14] EUbusiness (2004); "Irish PM rejects calls for EU corporate tax harmonization." Available online at http://www.EUbusiness.com, published on 14 May 2004.

[15] Gordon, R. and J.D. Wilson (1986); "An Examination of Mulitjurisdictional Corporate Income Taxation under Formula Apportionment," *Econometrica*, 54(6): 1357-1373.

[16] Gresik, T. (2001); "The Taxing Task of Taxing Transnationals." *Journal of Economic Literature* 39: 800-838. Reprinted in *Petroleum Regulation within Stable States*, by Solveig Glomsrod and Petter Osmundsen eds,. Ashgate Press, 2005.

[17] Groenert, V., M. Wooders and B. Zissimos (2006) "Competition in Standards and Taxes." Vanderbilt University typescript.

[18] Hoyt, W.H. (1995); "Leviathan and Local Tax Policies: Do Local Governments Maximize Revenue?" Working Paper E-185-95, Center for Business and Economic Research, University of Kentucky.

[19] Hoyt, W.H. (1999); "Leviathan, Local Government Expenditures and Capitalization." *Regional Science and Urban Economics*, 29: 155-171.

[20] Hoyt, W.H. and R.A. Jensen (2001); "Product Differentiation and Public Education." *Journal of Public Economic Theory*, 3(1): 69-93.

[21] Justman, M., J-F. Thisse and T. Van Ypersele (2002); "Taking the Bite out of Fiscal Competition." *Journal of Urban Economics*, 52: 294-315.

[22] Justman, M., J-F. Thisse and T. Van Ypersele (2005); "Fiscal Competition and Regional Differentiation," *Regional Science and Urban Economics*, 35(6): 848-861.

[23] Kanbur, R. and M. Keen (1993); "Jeux Sans Frontieres: Tax Competition and Tax Coordination When Countries Differ in Size." *American Economic Review*, 83(4): 877-892.

[24] Keen, M. and C. Kotsogiannis (2003); "Leviathan and Capital Tax Competition in Federations," *Journal of Public Economic Theory*, 5(2): 177-199.

[25] Keen, M. And M. Marchand (1997); "Fiscal Competition and the Pattern of Public Spending." *Journal of Public Economics*, 66: 33-53.

[26] Kind, H.J., K. Knarvik and G. Schelderup (2000); "Competing for Capital in a 'Lumpy' World" *Journal of Public Economics*, 78: 253-274.

[27] Kreps, D. and J.A. Scheinkman (1983); "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes," *The Bell Journal of Economics*, 14(2): 326-337.

[28] Lockwood, B. (2001); "Tax Competition and Tax Coordination Under Destination and Origin Principles: A Synthesis." *Journal of Public Economics*, 81: 279-319.

[29] Lockwood, B. and M. Makris (2006); "Tax Incidence, Majority Voting and Capital Market Integration." *Journal of Public Economics*, 90: 1007-1025.

[30] Ludema, R.D. and I. Wooton (2000); "Economic Geography and the Fiscal Effects of Regional Integration," *Journal of International Economics*, 52: 331-357.

[31] Mintz, J. and M. Smart (2004): "Income Shifting, Investment and Tax Competition: Theory and Evidence from Provincial Taxation in Canada." *Journal of Public Economics*, 88: 1149-1168.

[32] Mintz, J. and H. Tulkens (1986); "Commodity Tax Competition Between Member States of a Federation: Equilibrium and Efficiency." *Journal of Public Economics*, 29: 133-72 (March).

[33] Mintz, J. and H. Tulkens (1996); "Optimality Properties of Alternative Systems of Taxation of Foreign Capital Income." *Journal of Public Economics*, 60: 373-399.

[34] Niskanen, W. (1971) *Bureaucracy and Representative Government.* Chicago: Aldine.

[35] Oates, W.E. and R.M. Schwab (1988); "Economic Competition Among countries: Efficiency Enhancing or Distortion Inducing?" *Journal of Public Economics*, 35: 333-354.

[36] Rauscher, M. (1996); "Interjurisdictional Competition and the Efficiency of the Pubic Sector: The Triumph of the Market over the State?" Working Paper 732. Kiel: Institute of World Economics.

[37] Rauscher, M. (1998); "Leviathan and Competition Among countries: The Case of Benefit Taxation." *Journal of Urban Economics* 44 : 59-67.

[38] Rothstein, P. (2005); "Fiscal Cooperation and the Permission to Tax." Washington University typescript.

[39] Shaked, A. and J. Sutton (1982); "Relaxing Price Competition Through Product Differentiation," *The Review of Economic Studies*, 49(1): 3-13.

[40] Sutton, J. (1991); *Sunk Costs and Market Structure*, MIT Press, Cambridge.

[41] Tiebout, C.M. (1956); "A Pure Theory of Local Expenditures." *Journal of Political Economy*, 64: 416-24.

[42] U.K. Treasury (1988); "Taxation in the Market: A Market-Based Approach." UK Treasury Office, London.

[43] Wilson, J.D. (1986); "A Theory of Interregional Tax Competition." *Journal of Urban Economics*, 19: 296-315.

[44] Wilson, J.D. (1987); "Trade, Capital Mobility, and Tax Competition." *Journal of Political Economy*, 95: 835-56.

[45] Wilson, J. (1991); "Tax Competition with Interregional Differences in Factor Endowments." *Regional Science and Urban Economics*, 21: 423-51.

[46] Wilson, J.D. (1999); "Theories of Tax Competition." *National Tax Journal*, 52(2): 269-305.

[47] Wilson, J.D. and E. Janeba (2005); "Decentralization and International Tax Competition." *Journal of Public Economics*, 89: 1211-1229.

[48] Wilson, J.D. and D.E. Wildasin (2004) "Tax Competition: Bane or Boon?" *Journal of Public Economics* 88: 1065-1091.
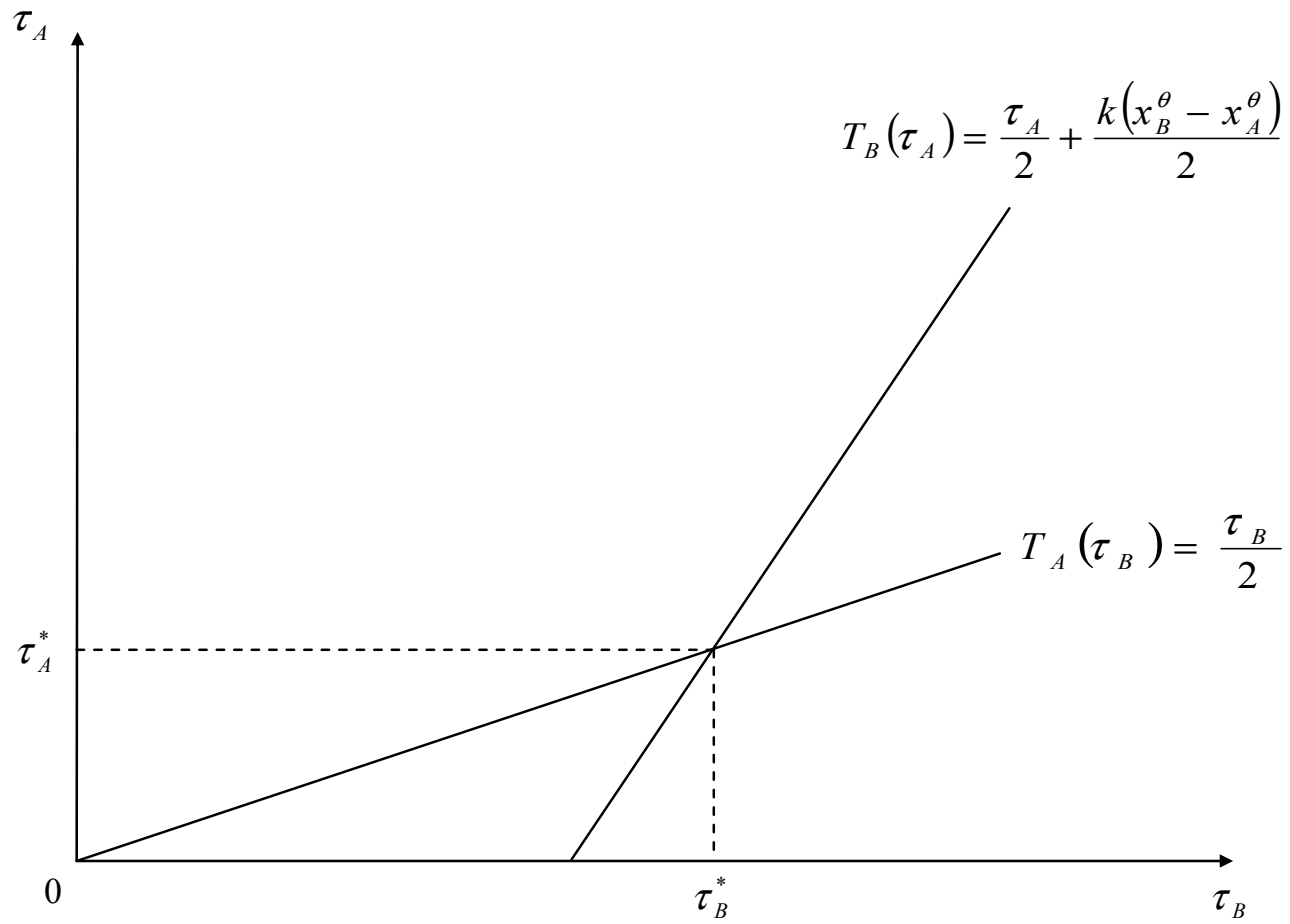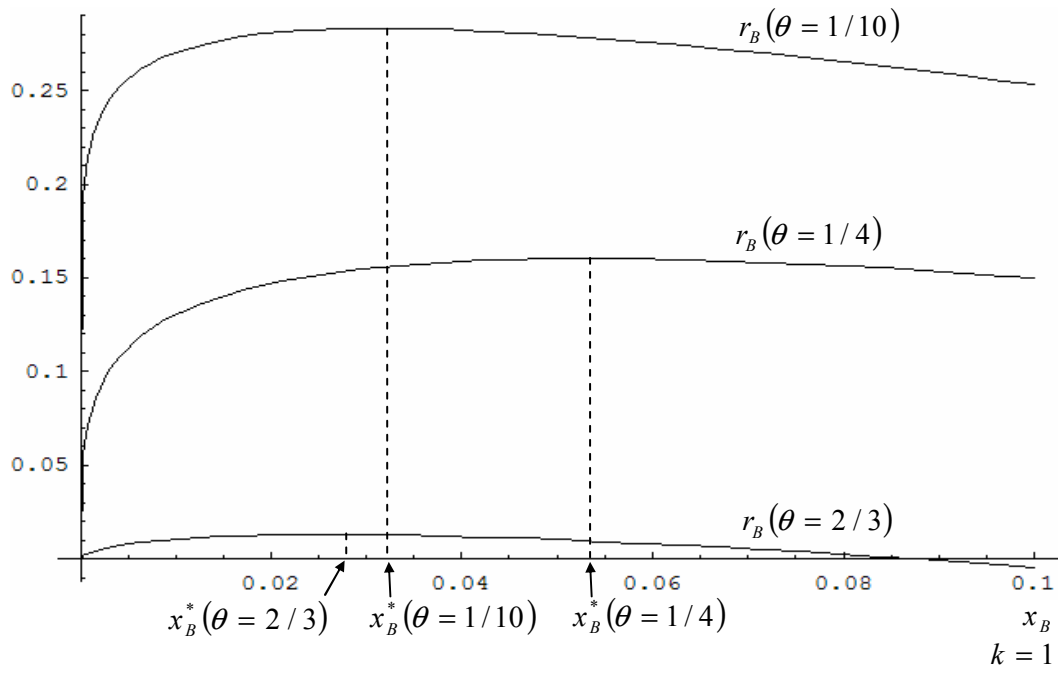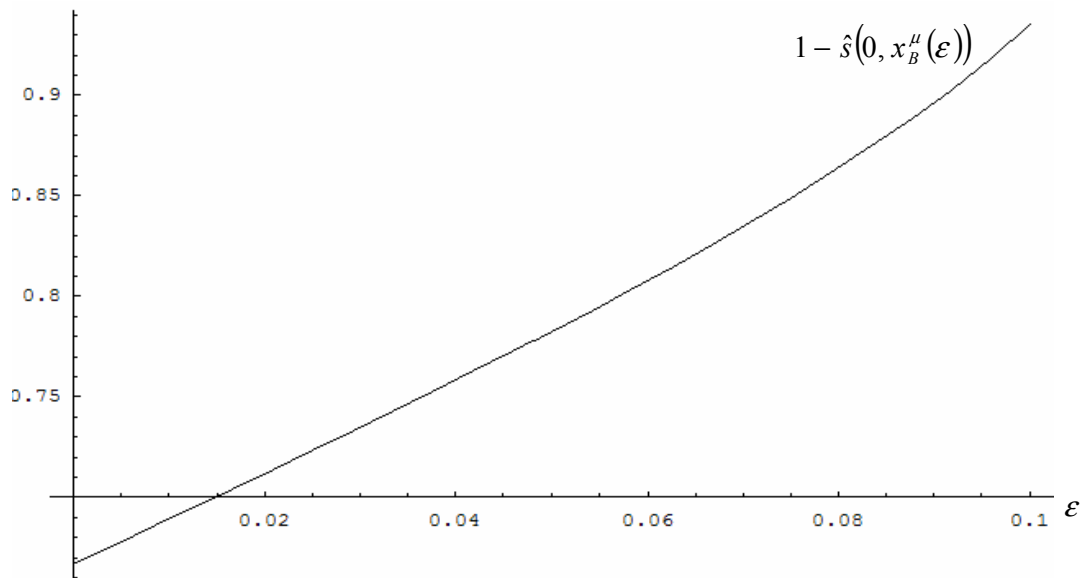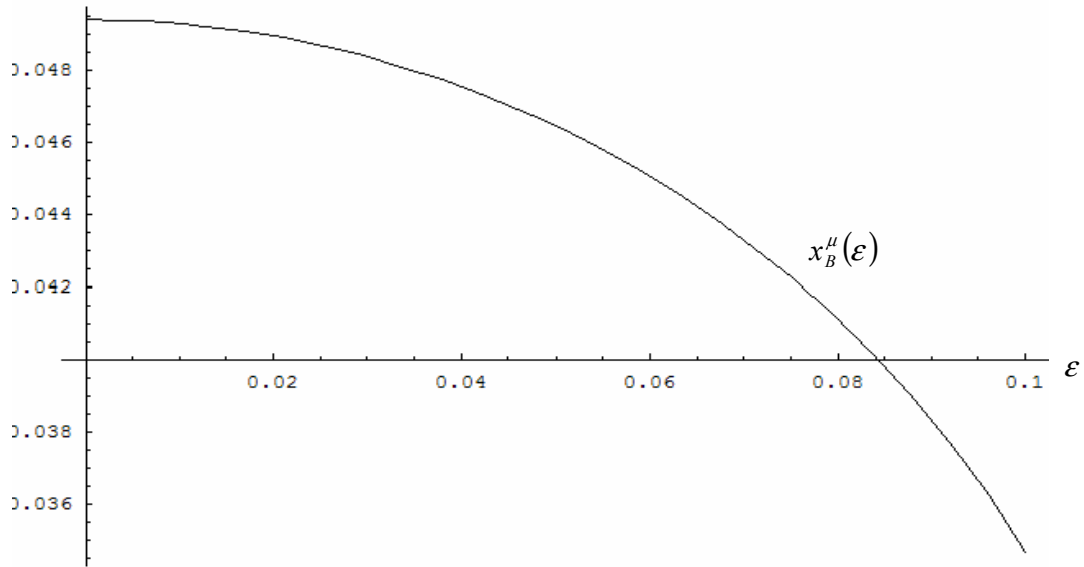
Figure 1

Figure 2

$r_B(\theta = 1/10)$

$r_B(\theta = 1/4)$

$r_B(\theta = 2/3)$

0.25

0.2

0.15

0.1

0.05

0.02   0.04   0.06   0.08   0.1

$x_B^*(\theta = 2/3)$ $x_B^*(\theta = 1/10)$ $x_B^*(\theta = 1/4)$

x_B

$k = 1$

Figure 3

$x_B^\mu(\varepsilon)$

$1 - \hat{s}\big(0, x_B^\mu(\varepsilon)\big)$

Figure 4