# FINANCE THY GROWTH: THE ROLE OF OCCUPATIONAL CHOICE BY ABILITY-HETEROGENEOUS AGENTS

by

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## FINANCE THY GROWTH:

# The Role of Occupational Choice By Ability-Heterogeneous Agents

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Abstract: This paper develops an overlapping-generations model of finance and growth with intrinsic heterogeneity in loanable fund conversion ability, where agents make occupational choice between becoming entrepreneurs and becoming workers. For a given ability distribution, a decrease in the number of entrepreneurs may create an occupational choice effect, enhancing the rate of growth of the economy, as the average conversion ability of the remaining entrepreneurs is higher. A change in ability distribution parameters may generate a permanent growth effect. Due to the presence of an occupational choice effect, a scale effect and general-equilibrium wage adjustments, however, financial-market thickness and income growth need not be positively correlated, in response to such distribution shifts. While both a reduction in the unit financial operation cost and an improvement in manufacturing productivity are growth enhancing, they have different effects on equilibrium prices and financial markup.

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## 1 Introduction

It is well-elaborated empirically and theoretically since at least Schumpeter (1911, 1939) and Bagehot (1915) that financial and real activities are inter-related. On the one hand, Goldsmith (1969), McKinnon (1973) and King and Levine (1993), among many others, have documented a positive relationship between financial depth (measured by financial intermediation ratios) and economic advancement (measured by either the level or the rate of growth of per capita output). Such a relationship is, however, not entirely uncontroversial. For example, Fernandez and Galetovic (1994) find much weaker positive correlation for the OECD countries (than for developing countries), and such a correlation is essentially zero when Japan is excluded from the OECD sample. Using data from twelve Latin American countries, De Gregorio and Guidotti (1992) conclude that the development of the real and the financial sectors are negatively related. On the other hand, based on time-series data from three developed economies (U.S., U.K. and Germany), Lehr and Wang (2000) show that financial innovation that reduces financial markup (measured by the loan-deposit interest rate spread) promotes economic growth. In the cross-country, cross-industry study of Cetorelli and Gambera (2001), however, such a relationship between financial markup and output growth is ambiguous. While there has been a growing literature addressing many interesting issues on finance and growth. theories towards resolving the two empirical puzzles mentioned above have remained largely unexplored.

Therefore, the main purpose of this paper is to develop a dynamic general-equilibrium model of finance and growth with heterogeneity in loanable fund conversion ability through which the interactions between the financial and the real sectors can be examined systematically.<sup>2</sup> We attempt to provide plausible answers to the following two questions motivated by the recent empirical literature mentioned above. Why are real and financial activities not necessarily positively correlated? What are the underlying factors influencing the long-run

<sup>&</sup>lt;sup>1</sup>For a critical review of the literature, the reader is referred to Becsi and Wang (1997).

<sup>&</sup>lt;sup>2</sup>Entrepreneurial heterogeneity is not only realistic (cf. Blanchflower and Oswald 1998), but yields important implications for the underlying financial activities, as observed and discussed by Evans and Jovanovic (1989) and Gertler and Gilchrist (1994).

movements in the financial markup? To address these issues, we must model the financial sector in such a way that it directly influences the behavior of the real sector and that it allows for an endogenous determination of both the loan and the deposit rates.<sup>3</sup>

Specifically, we use a general equilibrium overlapping-generations model with heterogeneous agents optimally choosing to become workers or entrepreneurs when they are young. Since it is not the purpose of this paper to study the formation of financial intermediation, we assume that borrowing and lending must all be financially intermediated. Workers work in their youth and deposit the entirety of their wage income in banks. Banks take deposits, employ labor and provide loans to borrowing entrepreneurs. Upon obtaining a loan from a bank, an entrepreneur of a particular ability type transforms the loan into a capital good, which can then be used together with labor to produce the final consumption good. Under forced savings and in the absence of liquidity constraints, agents' differential loanable fund conversion ability becomes the sole force determining the occupational choice between workers and entrepreneurs.<sup>5</sup>

Notably, occupational choice and the process of transforming loans into capital goods affect capital accumulation, financial activity and economic growth in such a way that the conventional positive relationship between financial and real activities may be upset. In particular, there are two conflicting effects: (i) an "occupational choice effect" as a result of individuals' decision on whether to become an entrepreneur and (ii) a "scale effect" as a result of positive Schumpeterian rents from uncompensated entrepreneurial knowledge spillovers in which the rate of growth is affected by the scale of production.<sup>6</sup> By allowing for endogenous

<sup>&</sup>lt;sup>3</sup>To our knowledge, the only paper separately determining the loan and the deposit rates within a dynamic general-equilibrium framework is Chen et al. (2000), which studies how moral hazard behavior induces credit rationing and affects the loan-deposit interest rate spread.

<sup>&</sup>lt;sup>4</sup>Those interested in endogenous formation of financial intermediation in an endogenous growth framework may refer to the now-classic papers by Greenwood and Jovanovic (1990) and Bencivenga and Smith (1991), among many others.

<sup>&</sup>lt;sup>5</sup>For simplity, our paper is abstracting from imperfect credit markets. For a discussion on the role of liquidity constraints on entrepreneurship, the reader is referred to Evans and Jovanovic (1989), Banerjee and Newman (1993), and Holtz-Eakin, Joulfaian and Rosen (1994).

<sup>&</sup>lt;sup>6</sup>As it will be elaborated in Sections 2 and 4 below, this Romer (1986)-type positive externality contrasts sharply with the negative participation externality considered by Becsi et al. (1999) and others in the finance

labor demand by banks, financial markup is tied to the equilibrium unit labor cost. Due to labor reallocation between the real and the financial sectors, financial markup need not be negatively related to economic growth, depending crucially on the origin of productivity enhancement.

The main findings of this paper can now be summarized as follows. First, when the occupational choice effect is strong and the degree of capital-skill complementarity is high, there is a positive relationship between financial-market thickness and economic growth in response to a uniform shift in the ability distribution. When the occupational choice effect is weak and the degree of capital-skill complementarity is low, a reversed relationship between the two economic indicators emerges. Second, the equilibrium properties concerning the relationship between economic growth and financial-market thickness in response to a local mean-preserving spread of a uniform ability distribution are exactly opposite to those in response to a uniform shift in the ability distribution under the same parametric regularities, as long as the degree of entrepreneurial knowledge spillovers is not too high. A dispersed distribution in this case may foster economic growth even with a thinner financial market, when the occupational choice effect is sufficiently strong and the degree of capital-skill complementarity is sufficiently high. Third and finally, while both a reduction in the unit financial operation cost and an improvement in manufacturing productivity are growth enhancing, they have different effects on equilibrium wages, interest rates and financial markep.

#### Related Literature

In the current literature of heterogeneous agents with occupational choice and economic growth, little has been done concerning how the distribution of agents may affect their occupational choice and economic growth. The primary purpose of this paper is to show in an economy with heterogeneous agents, the distribution of agents affects equilibrium wages and economic growth to a large degree. There is a small but growing literature on occupational choice with broadly defined finance-related activity, including Banerjee and Newman (1993), Galor and Zeira (1993), Aghion and Bolton (1997), Piketty (1997), De Gregorio and Kim (2000), LLoyd-Ellis (2000), LLoyd-Ellis and Bernhardt (2000), Ghatak and Jiang (2002), and literature.

Fender and Wang (2003), to name but a few. The present paper has the following features of importance contrasting with this previous literature. First, we emphasize on heterogeneity in loanable fund conversion ability rather than individual wealth (cf. Aghion and Bolton, Banerjee and Newman, Galor and Zeira, Ghatak and Jiang and Piketty) or labor skills (cf. De Gregorio and Kim, Fender and Wang, LLoyd-Ellis, and LLoyd-Ellis and Bernhardt). Second, we have a more complete financial sector with borrowing and lending activities with the loan and deposit rates separately determined, rather than focusing on liquidity constraints. Third, we explicitly model the process through which entrepreneurs transform loans into physical capital where uncompensated entrepreneurial knowledge spillovers is allowed.

Our result that the equilibrium growth rate depends crucially on the distribution of agents' ability is, to our knowledge, unique in the context of finance and growth. In our model, different distributions of borrowers' ability have different effects on the equilibrium growth rate. Here the growth rate is mainly driven by the effect of total capital stock and average conversion ability. For a given distribution, when there are less entrepreneurs in the economy, it is true that there are less entrepreneurs in the economy (the scale effect), but at the same time, the average conversion ability of the entrepreneurs is higher (the occupational choice effect). These two opposing effects make the resulting change in equilibrium wages and growth rate ambiguous. For a given family of distributions, a change in distribution parameters, either in mean or in dispersion measures, affects the equilibrium occupational choice, and hence returns to human capital as well as economic growth.

## 2 The Basic Environment

Time is discrete, indexed by t. In addition to an initially old generation at date t = 0, the economy consists of an infinite sequence of two-period lived overlapping generations. The population is constant over time, normalized to one. There are three theaters of activities, depicting the behaviors of workers, entrepreneurs and banks. Agents are endowed with one unit of labor only when they are young and value consumption only when they are old. In the absence of bequest motives, agents save the entirety of their income for consumption in the

second period of their lifetime. Call one born in period t a t-generation agent, who chooses to become a worker or an entrepreneur when young. A worker can be employed by a bank or a firm which is owned by a (t-1)-generation entrepreneur. All workers are paid the current competitive wage rate, and deposit the entire amount into banks for the purpose of future consumption. A young entrepreneur (i) borrows certain amount of consumption goods from a bank, (ii) devotes his labor to transform the consumption good into the capital good which lasts only one period, (iii) employs (t+1)-generation workers to utilize the capital good to undertake production, and, (iv) consumes the entire net profit as a Diamond-Yellin (1990) residual claimer. A bank's lone activity is to transform deposits into loans, a lá Gurley and Shaw (1955). The structure of the economy is conveniently depicted in Figure 1 where the timing of events is numerically ordered from 1 to 7.

### 2.1 Workers, Entrepreneurs and Occupational Choice

Agents of t-generation are born with the same level of human capital  $h_t$  but different ability  $\tau$  on transforming the consumption good into the capital good (which reflects the financial entrepreneurial ability). More specifically, an agent with ability  $\tau$  can transform x units of consumption good which he borrowed from bank into  $\tau^{\gamma}x$  units of capital good where  $\gamma > 0$ . We assume  $\tau \in [0, \tau^H]$ , where  $\tau^H$  can be infinity or a finite number. Denote  $F(\tau)$  as the associated cumulative distribution function. For a t-generation entrepreneur with ability  $\tau$ , if he borrows  $x_t$  units of consumption good when young and hires  $l_{t+1}$  units of (t+1)-generation workers, he is able to produce consumption good at date t+1 according to the technology given by,

$$y_{t+1}(\tau) = A\bar{k}_{t+1}^{\alpha-\theta} \cdot (\tau^{\gamma} x_t)^{\theta} \cdot (l_{t+1} h_{t+1})^{1-\alpha}, \tag{1}$$

where A > 0 is a productivity scaling parameter,  $h_{t+1}$  is the human capital level of the (t+1)-generation workers,  $\bar{k}_{t+1}$  is the amount of capital that an average entrepreneur has, and  $0 < \theta < \alpha < 1$ . More specifically, the amount of capital good available for production  $\tau^{\gamma}x_t$  depends on the entrepreneur's ability to transform the consumption good into the capital good. The production function displays decreasing returns to scale with respect to private

inputs, capital  $(\tau^{\gamma}x_t)$  and effective labor  $(l_{t+1}h_{t+1})$ . However, the presence of uncompensated entrepreneurial knowledge spillovers in the Romer (1986) convention restores constant return to scale with respect to all factors, usually referred to as social constant returns (cf. Benhabib, Meng and Nishimura 2000). Under this setup,  $\alpha - \theta$  measures both the degree of decreasing returns to scale to private inputs and the degree of entrepreneurial knowledge spillovers. This type of positive external effects is commonly seen in endogenous growth theory, which is very different from the conventional negative participation externality in finance (e.g., Besci at al. 1999).

Facing the competitive gross loan rate  $(\delta_{t+1})$  from date t to t+1 and expecting the competitive wage rate per unit of effective labor  $(\omega_{t+1})$  and the average capital stock  $(\bar{k}_{t+1})$  prevailed at date t+1, a t-generation entrepreneur with ability  $\tau$  chooses loan demand and labor demand to maximize the net profit:

$$\max_{\{x_t, l_{t+1}\}} \pi_{t+1}(\tau) = y_{t+1}(\tau) - \delta_{t+1} x_t - \omega_{t+1} h_{t+1} l_{t+1}. \tag{2}$$

Straightforward differentiation leads to the following loan and labor demand schedules:

$$x_t(\tau) = \frac{\theta y_{t+1}(\tau)}{\delta_{t+1}} \tag{3}$$

$$l_{t+1}(\tau) = \frac{(1-\alpha)y_{t+1}(\tau)}{\omega_{t+1}h_{t+1}}.$$
(4)

From (3) and (4), both demand functions are decreasing in their correspondent prices and increasing in the level of output, and one can easily see that the factor income shares of capital and effective labor are  $\theta$  and  $1 - \alpha$ , respectively. Substituting these derived demands into (1) and (2), we have:

$$y_{t+1}(\tau) = A_0 \tau^{\frac{\gamma \theta}{\alpha - \theta}} \bar{k}_{t+1} (\delta_{t+1}^{\theta} \omega_{t+1}^{1 - \alpha})^{-\frac{1}{\alpha - \theta}}$$

$$\tag{5}$$

$$\pi_{t+1}(\tau) = (\alpha - \theta)y_{t+1}(\tau), \tag{6}$$

where  $A_0 \equiv [A\theta^{\theta}(1-\alpha)^{1-\alpha}]^{\frac{1}{\alpha-\theta}}$ .

It is clear that entrepreneurs with higher ability hire more factors, produce more output, and earn higher profit. Hence, the higher  $\tau$  a t-generation agent has, the more likely he will

become an entrepreneur. Specifically, by becoming an entrepreneur, he gains a profit income of  $(\alpha - \theta)y_{t+1}(\tau)$  when old; by becoming a worker, he receives a wage income of  $\omega_t h_t$  and deposits it in a bank to collect the principal and interest  $r_{t+1}\omega_t h_t$  when old. Since  $y_{t+1}(\tau)$  is strictly increasing in  $\tau$ , there should be a unique cutoff point  $\tau_t^*$  such that agents with higher (lower) ability than  $\tau_t^*$  choose to become entrepreneurs (workers), as shown in Figure 2. Clearly, the cutoff point  $\tau_t^*$  must satisfy the following no-arbitrage condition between the two occupations, or, shortly, the occupational choice condition:

$$(\alpha - \theta)y_{t+1}(\tau_t^*) = r_{t+1}\omega_t h_t. \tag{7}$$

#### 2.2 Banks

We now turn to describing the perfectly competitive banking sector, where banks accept deposit from young workers and lend it to young entrepreneurs. At any time t, any worker can elect to form (and own) a bank, operating under a market deposit rate  $r_{t+1}$  to absorb loanable funds from young workers and a market loan rate  $\delta_{t+1}$  to provide loanable funds to potential young entrepreneurs. At time t+1, the bank receives the repayment from its borrowers and pays back to depositors. For simplicity, assume such a transformation from deposits into loans is undertaken purely by the bank employees, requiring no additional effort by the bank owner.

Specifically, the technology of transforming deposits into loans is Ricardian: in order to convert X units of deposits into X units of loans, a bank must employ  $X/\phi$  units of effective labor. That is, we assume zero reserve requirement (or, total deposits equal total loans) and a unit labor requirement  $1/\phi$  for bank operation. Parameter  $\phi$  serves as a measure of technology level of the financial sector. Under perfect competition, a bank must end up with zero profit, which implies the following equality must hold:

$$\delta_{t+1} = r_{t+1} (1 + \frac{\omega_t}{\phi}).$$
 (8)

The left-hand-side is the repayment that a bank receives at date t+1 for one unit of loan. The right-hand-side is the amount it has to pay which can have two interpretations depending

on the contract between the bank and its workers. One interpretation is that the bank uses the deposit to pay its employees and upon receiving their wage income, workers redeposit back into the bank. Alternatively, one may imagine that the bank promises its own employees that they will be paid  $r_{t+1}\omega_t$  per unit of effective labor when they become old via an implicit contract. While both yield the same zero-profit condition (8), we stick to the latter interpretation throughout the paper. That is, only workers at the final goods sector are depositors, under which it is much easier to maintain not only the balance sheet accounting but the equilibrium flow of funds.<sup>7</sup>

Under this setup, it is obvious that there must be a gap between deposit and loan rates to guarantee nonnegative profit for banks (i.e., a positive loan-deposit interest rate spread). Throughout the paper, we measure the financial markup at time t by  $\frac{\delta_{t+1}-r_{t+1}}{r_{t+1}}$ . From equation (8), the financial markup is simply bank's unit labor cost,  $\frac{\omega_t}{\phi}$ .

In our economy, there are three processes transforming from households' savings to final goods production: (i) a banking transactions process from deposits to loans (i.e., bank operation), (ii) a financial management process conducted by entrepreneurs converting converting from loans to capital goods (i.e., financial entrepreneurship), and (iii) a production process from capital (and labor) inputs to final goods (i.e., manufacturing activity). Under the special features of our model setup, it is important to recognize the first two processes as financial activities whereas the last as real activities. Recall that all entrepreneurial investments require external financing and that all banks have zero reserves. Under forced savings and perfect credit markets, it is convenient to measure the level of financial activity at time t by the mass of entrepreneurs,

$$\Gamma(\tau_t^*) = \int_{\tau_t^*}^{\tau^H} dF = 1 - F(\tau_t^*). \tag{9}$$

Thus, under our setup,  $\Gamma(\tau_t^*)$  captures financial-market thickness at time t.<sup>8</sup> It is obvious

<sup>&</sup>lt;sup>7</sup>Otherwise, a condition is needed to ensure that each bank would have enough funds to prepay wages to its own employees, which would create unnecessary complexity without providing any additional insights towards understanding the issues examined in this paper.

<sup>&</sup>lt;sup>8</sup>While this measure is the appropriate measure in our framework, it differs from the standard aggregate credit proxies of financial depth by Goldsmith and others. In order to discuss the empirical facts, we thus

that an increase in the frontier ability results in a thicker financial market. Moreover, for a given ability distribution, financial-market thickness is inversely related to the cutoff ability: a higher cutoff implies less entrepreneurial activities and hence a thinner financial market.

### 2.3 Human and Physical Capital Formation

To close the model, we must specify the formation of human and physical capital. Human capital evolves according to,

$$h_{t+1} = \bar{k}_t^{\beta} h_t^{1-\beta}. \tag{10}$$

This says that the (gross) rate of human capital accumulation  $(h_{t+1}/h_t)$  is driven by the average physical capital-human capital ratio  $(\bar{k}_t/h_t)$  – a scaling constant is omitted as it plays no additional role than A under our setting. This evolution pattern is justified if the economy exhibits capital-skill complementarity (cf. Griliches 1969) – a larger value of  $\beta$  means a greater degree of capital-skill complementarity. Throughout the paper, we assume that the production of human capital is relatively human-capital intensive, thereby requiring  $\alpha > \beta$ . If there is a positive relationship between individual's effort devoted to human capital accumulation and the (average) physical capital-human capital ratio in the spirit following Azariadis and Drazen (1990), the human capital evolution process essentially captures that in Lucas (1988) and Glomm and Ravikumar (1992).

Finally, the average physical capital stock is given by,

$$\bar{k}_{t+1} = \left( \int_{\tau_t^*}^{\tau^H} \tau^{\gamma} x_t(\tau) dF \right) \left( \int_{\tau_t^*}^{\tau^H} dF \right)^{-1}, \forall t \ge 0.$$
 (11)

The first term on the right-hand-side is the integral of the transformed capital goods for all types of entrepreneurs with ability exceeding the cutoff, whereas the second term is the mass of these entrepreneurs.

assume that our measure and the conventional proxies are positively correlated. A rigorous empirical reinvestigation is beyond the focus of the present paper and hence left for future research.

<sup>&</sup>lt;sup>9</sup>The Romer-Lucas convention is in sharp contrast with the imitation or the learning models where the gap between the individual and the frontier knowledge, rather than the average stock, matters.

## 3 Equilibrium

We are now ready to study the equilibrium of this economy. We first define the concept of equilibrium.

**Definition 1**: Given  $h_0$  and an initial distribution of individual entrepreneur's capital  $\tilde{k}_0$ , an equilibrium is a collection of quantity sequences  $\{\tau_t^*, \bar{k}_{t+1}, x_t(\tau), l_{t+1}(\tau), y_{t+1}(\tau), h_{t+1}\}_{t=0}^{\infty}$  together with a collection of price sequences  $\{\omega_t, \delta_{t+1}, r_{t+1}\}_{t=0}^{\infty}$  such that:

- (i) the occupational choice condition (7) holds for all t ≥ 0, where a t-generation agent with ability τ chooses to become a worker if τ < τ<sup>\*</sup><sub>t</sub> and to become an entrepreneur otherwise;
- (ii) given  $\omega_{t+1}, \delta_{t+1}, h_{t+1}$  and  $\bar{k}_{t+1}$ , t-generation entrepreneurs optimally choose their production plans with the triple  $\{x_t(\tau), l_{t+1}(\tau), y_{t+1}(\tau)\}$  satisfying (3), (4) and (5) for all  $\tau \geq \tau_t^*$  and for all  $t \geq 0$ ;
- (iii) banks earn zero profit so that (8) holds for all  $t \geq 0$ ;
- (iv) human capital evolves according to (10) for all  $t \geq 0$ ;
- (v) average physical capital prevailed satisfies (11) for all  $t \geq 0$ ;
- (vi) both loan and labor markets clear every period.

#### 3.1 Market Clearing Conditions

In this subsection, we derive conditions that characterize loan and labor market clearing.

The total demand for loan is simply the summation of individual entrepreneur's loan demands while the total supply of loan is more subtle. In particular, the total supply of loan is the aggregate wage income of, not all workers, but only those who work in the manufacturing sector. This can be best illustrated by regarding the payment arrangement between banks and their employees as a promised contract so that only manufacturing workers actually receive wage payment. Furthermore, for one unit of effective labor hired by the manufacturing sector

at time t, it costs  $\frac{\omega_t}{\phi}$  to pay for the required labor in the banking sector to transform deposits into loanable funds. Hence, the fraction of manufacturing workers to all workers is  $(1 + \frac{\omega_t}{\phi})^{-1}$ , so the loan market clearing condition can be written as:

$$\int_{\tau_{\star}^{*}}^{\tau^{H}} x_{t}(\tau) dF = \omega_{t} h_{t} \left[ \left( 1 + \frac{\omega_{t}}{\phi} \right)^{-1} \int_{0}^{\tau_{t}^{*}} dF \right], \forall t \ge 0.$$
 (12)

Notice that the term in the bracket on the right-hand-side is the total number of manufacturing workers at time t.

We can write down the labor market clearing condition in a similar fashion,

$$\int_{\tau_t^*}^{\tau^H} l_{t+1}(\tau) dF = \left(1 + \frac{\omega_{t+1}}{\phi}\right)^{-1} \int_0^{\tau_{t+1}^*} dF, \forall t \ge 0.$$
 (13)

Notice that under equation (13), labor market clears every period starting period 1, and it is the initial-period wage rate  $\omega_0$  which clears the labor market in period 0. More specifically,  $\omega_0$  must equate the labor supply in the manufacturing sector  $((1 + \frac{\omega_0}{\phi})^{-1} \int_0^{\tau_0^*} dF)$  and the labor demand which depends on the initial distribution of individual entrepreneur's capital  $(\tilde{k}_0)$ . It turns out that the wage dynamics in this model converges to a unique fixed point (to be shown formally in Section 3.4 below). Therefore, the initial wage rate and subsequently the initial capital distribution become inessential. Without loss of generality, we assume  $\omega_0$  as the given initial condition instead of  $\tilde{k}_0$ .

#### 3.2 Determination of the Cutoff Ability

To characterize the equilibrium, we first show that the cutoff ability  $\tau_t^*$  is time-invariant. To see this, let us start from the occupational choice condition (7) and work on the right-hand-side of the equation. We substitute  $r_{t+1}$  by  $\omega_t$  and  $\delta_{t+1}$  from banks' zero-profit condition (8) and utilize the loan market clearing condition (12) to eliminate  $\omega_t h_t$ . By expressing  $x_t(\tau)$  as a function of  $y_{t+1}(\tau)$  using equation (3), we can then rewrite the occupational choice condition as:

$$(\alpha - \theta)y_{t+1}(\tau_t^*) = \left[\theta \int_{\tau_t^*}^{\tau^H} y_{t+1}(\tau)dF\right] \cdot \left(\int_0^{\tau_t^*} dF\right)^{-1}, \forall t \ge 0.$$
 (14)

The left-hand-side of equation (14) is the profit that an entrepreneur with cutoff ability would earn. The first term on the right-hand-side of (14) says that a fraction  $\theta$  of the total output

at date t+1 is allocated to depositors whose mass is given by the second term,  $\int_0^{\tau_t^*} dF$ . Therefore, the right-hand-side is the amount of consumption when old if being a worker was chosen. Again, the equality states that agents with the cutoff ability must be indifferent between these two occupations. Combined with (5), equation (14) can be further simplified to,

$$\frac{\alpha - \theta}{\theta} \frac{\left(\tau_t^*\right)^{\frac{\gamma \theta}{\alpha - \theta}}}{\int_{\tau_t^*}^{\tau^H} \tau^{\frac{\gamma \theta}{\alpha - \theta}} dF} = \frac{1}{\int_0^{\tau_t^*} dF}, \forall t \ge 0, \tag{15}$$

which yields our first proposition.

**Proposition 1**: The cutoff ability  $\tau_t^*$  is time-invariant and is uniquely determined, depending positively on the frontier ability  $\tau^H$ .

*Proof.* The first and last parts of the proposition are trivial. For the second part, notice that as  $\tau^*$  increases from 0 to  $\tau^H$ , the left-hand-side of (15) strictly increases from zero to infinity whereas the right-hand-side strictly decreases from infinity to 1. Hence,  $\tau^*$  exists and is unique.  $\square$ 

Proposition 1 greatly simplifies our equilibrium analysis because we have one less state variable to worry about. More precisely, the state variables at time t are now summarized by the human capital of the young generation  $(h_t)$  and the distribution of the amount of physical capital owned by (t-1)-generation entrepreneurs  $(\tilde{k}_t)$ . Since we know how a  $\tau$ -entrepreneur's capital stock relates to that of the entrepreneur with the cutoff ability  $\tau_{t-1}^*$ , namely  $k_t(\tau) = (\frac{\tau}{\tau_{t-1}^*})^{\frac{\gamma \alpha}{\alpha-\theta}} k_t(\tau_{t-1}^*)$ , the second state variable can be fully represented by the average capital stock  $\bar{k}_t$  and the cutoff ability  $\tau_{t-1}^*$ . Therefore, if  $\tau^*$  is time-invariant,  $\bar{k}_t$  becomes a sufficient statistic of  $\tilde{k}_t$  and the pair  $(\bar{k}_t, h_t)$  fully describes the state at time t. Having said this, we naturally turn to studying the evolution of the state variables.

<sup>&</sup>lt;sup>10</sup>We would like to point out that the result on time-invariance cutoff ability is not robust to more general socially constant-return-to-scale production functions. Since factor shares are constant under Cobb-Douglas technology, the comparison between being an entrepreneur and being a worker is independent of tomorrow's states. This can be seen from equation (14) where all other time-dependent variables cancel out exactly from both sides.

### 3.3 Capital, Wage and Rental Dynamics

Since the evolution of human capital is simply captured by its law of motion (10), we focus primarily on the dynamics of physical capital. Notably,  $\bar{k}_{t+1}$  depends on the total savings at time t which in turn depends on the wage rate. Thus, it is useful to elaborate on the relationship between capital and the effective wage rate. Given  $(\bar{k}_t, h_t)$ ,  $\omega_t$  is uniquely determined at the value which equates the labor demand and supply in the manufacturing sector. From (4), (5) and (13), we can establish:

$$\omega_t^{\frac{1}{\alpha}} (1 + \frac{\omega_t}{\phi})^{-1} = \frac{\alpha - \theta}{\theta} [A(1 - \alpha)]^{\frac{1}{\alpha}} [\Omega(\tau^*)]^{-\frac{\theta}{\alpha}} (\tau^*)^{\frac{\gamma \theta}{\alpha - \theta}} (\frac{\bar{k}_t}{h_t}), \forall t \ge 1, \tag{16}$$

where  $\Omega(\tau^*) \equiv \left[\int_{\tau^*}^{\tau^H} \tau^{\frac{\gamma\alpha}{\alpha-\theta}} dF\right] / \left[\int_{\tau^*}^{\tau^H} dF\right]$ , which measures the average productivity of entrepreneurs and is unambiguously increasing in the cutoff ability  $\tau^*$  and decreasing in the frontier ability  $\tau^H$ . Notice that the left-hand-side of the equation is strictly increasing in  $\omega_t$  from 0 to infinity. Also notice that the state variable can be further simplified to the ratio of average physical capital to human capital,  $\frac{\bar{k}_t}{h_t}$ . When this ratio is large, labor becomes relatively scarce, so the equilibrium wage rate is higher. Equation (16) applies for all  $t \geq 1$ . This, again, is because  $\bar{k}_0$  is not a sufficient statistic of  $\tilde{k}_0$ . As we will see below, this economy converges to a unique steady state regardless of the initial condition. For the time being, we can simply presume that this equation holds for t=0 as well without loss of generality.

We can now calculate the total savings at time t which will be transformed into physical capital by t-generation entrepreneurs with different abilities. We begin by utilizing (5), (7) and (8) to express the output by a type- $\tau$  entrepreneur as:

$$y_{t+1}(\tau) = \left(\frac{\tau}{\tau^*}\right)^{\frac{\gamma\theta}{\alpha-\theta}} y_{t+1}(\tau^*) = \frac{1}{\alpha-\theta} \left(\frac{\tau}{\tau^*}\right)^{\frac{\gamma\theta}{\alpha-\theta}} \frac{\omega_t}{1 + \frac{\omega_t}{\phi}} \delta_{t+1} h_t.$$

This can then be combined with (3) to yield,

$$\frac{x_t(\tau)}{h_t} = \frac{\theta}{\alpha - \theta} \left(\frac{\tau}{\tau^*}\right)^{\frac{\gamma\theta}{\alpha - \theta}} \frac{\omega_t}{1 + \frac{\omega_t}{\phi}}.$$
 (17)

Substituting (17) into (11), we obtain the average capital stock next period as a function of this period effective wage:

$$\bar{k}_{t+1} = \frac{\theta}{\alpha - \theta} \frac{\omega_t}{1 + \frac{\omega_t}{4}} h_t \Omega(\tau^*) (\tau^*)^{-\frac{\gamma \theta}{\alpha - \theta}}, \forall t \ge 0.$$

which, together with the law of motion for human capital (10), implies the single state variable  $\{\frac{\bar{k}_t}{h_t}\}$  evolves according to,

$$\frac{\bar{k}_{t+1}}{h_{t+1}} = \frac{\theta}{\alpha - \theta} \Omega(\tau^*) (\tau^*)^{-\frac{\gamma \theta}{\alpha - \theta}} \frac{\omega_t}{1 + \frac{\omega_t}{\phi}} (\frac{\bar{k}_t}{h_t})^{-\beta}, \forall t \ge 0.$$
 (18)

Thus, as a result of factor substitution, an increase in the effective wage rate raises the physical-human capital ratio; by diminishing returns, an increase in the previous physical-human capital ratio reduces the current one.

More interestingly, the cutoff ability  $\tau^*$  generates two opposing effects on the physical to human capital ratio: a positive occupational choice effect through  $\Omega(\tau^*)$  and a negative scale effect captured by the term  $(\tau^*)^{\frac{-\gamma\theta}{\alpha-\theta}}$ . Intuitively, higher cutoff ability, on the one hand, implies an overall improvement of loanable fund conversion ability, thus enhancing the productivity and fostering capital formation. On the other hand, for a given distribution F, a higher cutoff ability means a smaller mass of agents involved in entrepreneurial activities, thereby discouraging capital formation. Note that when the transformation from loans into productive physical capital is independent of individual ability (i.e.,  $\gamma = 0$ ), both the scale effect and occupational choice effect vanish. That is, heterogeneity in entrepreneurs' ability in transforming loans into physical capital is essential to the presence of occupational choice and scale effects.

Since  $\frac{\bar{k}_t}{h_t}$  can be explicitly written as a function of  $\omega_t$  but not *vice versa*, it is easier to represent the behavior of this economy by the dynamics of the effective wage rate instead of the state variable. Substituting (16) into (18), we find the dynamics of the effective wage rate follows:

$$\frac{\omega_{t+1}^{\frac{1}{\alpha}}}{1 + \frac{\omega_{t+1}}{\phi}} = \frac{B(\tau^*)\omega_t^{\frac{\alpha-\beta}{\alpha}}}{(1 + \frac{\omega_t}{\phi})^{1-\beta}}, \forall t \ge 0, \tag{19}$$

where  $B(\tau^*) \equiv B_0[\Omega(\tau^*)]^{\frac{\alpha-\theta(1+\beta)}{\alpha}}(\tau^*)^{\frac{\beta\gamma\theta}{\alpha-\theta}}$ , with  $B_0 \equiv (\frac{\alpha-\theta}{\theta})^{\beta}[A(1-\alpha)^{1-\alpha}]^{\frac{1+\beta}{\alpha}}$ . It is clear that  $B(\tau^*)$  may be increasing or decreasing in  $\tau^*$ . Since given  $\omega_t$ ,  $\omega_{t+1}$  is uniquely determined from equation (19), the entire dynamic path of effective wage rates  $\{\omega_t\}_{t=1}^{\infty}$  is determined once  $\omega_0$  is given. Before discussing the first-order difference equation (19) in details, we would

like to point out that once  $\{\omega_t\}_{t=1}^{\infty}$  is known, the equilibrium dynamics of the whole economy is solved. To see this, first notice that for any  $t \geq 1$  and a given cutoff ability  $\tau^*$ , there is a one-to-one correspondence between  $\omega_t$  and  $\frac{\bar{k}_t}{h_t}$  from equation (16). Furthermore, we can combine (3) and (5) to write  $x_t$  as a linear function in  $\bar{k}_{t+1}$  and then use (11) to eliminate  $\bar{k}_{t+1}$ . This enables us to calculate the (gross) loan rate as as a function of the effective wage rate and the cutoff ability:

$$\delta_{t+1} = \theta(1-\alpha)^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}} [\Omega(\tau^*)]^{\frac{\alpha-\theta}{\alpha}} \omega_{t+1}^{-\frac{1-\alpha}{\alpha}}, \forall t \ge 0.$$
 (20)

Substituting (20) into (8), we obtain the (gross) deposit rate as follows:

$$r_{t+1} = \theta (1 - \alpha)^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}} [\Omega(\tau^*)]^{\frac{\alpha-\theta}{\alpha}} \omega_{t+1}^{-\frac{1-\alpha}{\alpha}} (1 + \frac{\omega_t}{\phi})^{-1}, \forall t \ge 0.$$
 (21)

Since  $\tau^*$  is time-invariant, the dynamics of either interest rate is determined once the profile of effective wage  $\{\omega_t\}_{t=0}^{\infty}$  is known. The inverse relationship between the effective wage rate and the rental (loan and deposit rates) conforms with the standard property of the factor price frontier (downward sloping).

We summarize these results as follows.

**Proposition 2**: Given  $\omega_0 > 0$ , the equilibrium dynamics of this economy  $\{\omega_t, \delta_t, r_t, \frac{\bar{k}_t}{h_t}\}_{t=1}^{\infty}$  are fully captured by the dynamical system (19), (20), (21), and (16).

#### 3.4 Balanced Growth Path

Next, we consider

**Definition 2**: Given  $h_0$  and  $\tilde{k}_0$ , a balanced growth equilibrium is an equilibrium where  $\{\bar{k}_{t+1}, x_t(\tau), y_{t+1}(\tau), h_{t+1}\}_{t=0}^{\infty}$  all grow at constant rates and  $\{\tau_t^*, l_{t+1}(\tau), \omega_t, \delta_{t+1}, r_{t+1}\}_{t=0}^{\infty}$  are all constant.

Under our setup, particularly equations (1) and (10), it is easily seen that if a balanced growth equilibrium exists, the equilibrium dynamics converges to a balanced growth path along which the stocks of physical and human capital grow at a common rate g, as do the amount

of loan and output. Since  $\{\delta_t, r_t, \frac{\bar{k}_t}{h_t}\}_{t=1}^{\infty}$  can be fully described by the wage rate, it suffices to show that the wage dynamics converges to a unique fixed point under any initial wage rate. For convenience, let us express the first-order difference equation (19) as:

$$\omega_{t+1} = \Psi(\omega_t), \tag{22}$$

where  $\Psi: \mathbf{R}_+ \to \mathbf{R}_+$  is a continuous function. It can be easily verified that  $\Psi(0) = \lim_{\omega_t \to \infty} \Psi(\omega_t) = 0$  from equation (19). Moreover,  $\Psi$  has a unique local maximum at  $\omega_t = \frac{\alpha - \beta}{\beta(1 - \alpha)} \phi$  and  $\Psi'(0) = \infty$  (as shown in the proof of Lemma 1 in the Appendix).

An example of the graph of function  $\Psi$  is provided in Figure 3. It is, however, only an example for two reasons. First,  $\Psi$  may not be concave before reaching its maximum as shown in the Figure, and therefore, we need to show that  $\Psi$  has a unique non-zero fixed point  $\omega$ . Lemma 1 assures that this indeed is the case. Second, it is possible that  $\omega \leq \frac{\alpha - \beta}{\beta(1-\alpha)}\phi$ , that is,  $\Psi$  intersects the 45-degree line before it starts to decrease. In this case, it should be clear that the wage converges to its fixed point  $\omega$  for any  $\omega_0 > 0$ . If  $\omega > \frac{\alpha - \beta}{\beta(1-\alpha)}\phi$  as shown in Figure 3, we must rule out the possibility of cycles to establish the convergence. This is further proved in Lemma 2.

**Lemma 1:**  $\Psi$  has a unique, locally stable fixed point  $\omega$ .

*Proof.* See Appendix.  $\square$ 

Although  $\omega$  is the only locally stable fixed point,  $\{\omega_t\}_{t=1}^{\infty}$  may not converge to  $\omega$  in the presence of cycles. The next lemma precludes such a possibility by applying the Sarkovskii Theorem.<sup>11</sup>

**Lemma 2:**  $\Psi$  has no periodic point.

*Proof.* See Appendix.  $\square$ 

By substituting  $\omega_{t+1} = \omega_t = \omega$  into (19), the equilibrium wage rate along the balanced

<sup>&</sup>lt;sup>11</sup>See Sharkovskii (1964).

growth path satisfies:

$$\Lambda(\omega) = \left(\frac{\alpha - \theta}{\theta}\right)^{\beta} \left[A(1 - \alpha)^{1 - \alpha}\right]^{\frac{1 + \beta}{\alpha}} \left[\Omega(\tau^*)\right]^{\frac{\alpha - \theta(1 + \beta)}{\alpha}} \left(\tau^*\right)^{\frac{\beta \gamma \theta}{\alpha - \theta}},\tag{23}$$

where  $\Lambda(\omega) \equiv \frac{\omega^{\frac{1-\alpha+\beta}{\alpha}}}{(1+\frac{\omega}{\phi})^{\beta}}$ , with  $\Lambda'(\omega) > 0$ . From equation (23), whether the effective wage is increasing or decreasing in the cutoff ability depends crucially on the relative magnitude of the occupational choice versus the scale effects as well as the degree of capital-skill complementarity (measured by  $\beta$ ). When the occupational choice effect is strong and the degree of capital-skill complementarity is high, the effective wage along the balanced growth path is decreasing in the cutoff ability.

Utilizing Lemmas 1 and 2, we can draw the main conclusion regarding the balance growth path in the following Proposition.

**Proposition 3**: Given  $\omega_0 > 0$ , the economy converges to a balanced growth path where all prices remain constant and  $\bar{k}_t$ ,  $h_t$  grow at a same constant rate,

$$g = [A(1-\alpha)^{1-\alpha}]^{\frac{1}{\alpha}} [\Omega(\tau^*)]^{\frac{\alpha-\theta}{\alpha}} \omega^{-\frac{1-\alpha}{\alpha}} - 1, \tag{24}$$

where  $\tau^*$  and  $\omega$  satisfies (15) and (23), respectively. The rate of economic growth depends positively on the cutoff ability  $\tau^*$  and negatively on the effective wage  $\omega$ .

Proof. From equations (16), (20), (21) and the Lemmas, it is clear that sequences  $\{\delta_t\}$ ,  $\{r_t\}$ ,  $\{\frac{\bar{k}_t}{h_t}\}$  converge as  $\{\omega_t\}$  converges to  $\omega$ . On the balanced growth path, since  $\frac{\bar{k}_t}{h_t}$  remains constant, k and h must grow at the same rate  $g = \frac{h_{t+1}}{h_t} - 1 = (\frac{\bar{k}}{h})^{\beta} - 1$ . By combining equations (16) and (23), g can be calculated as in (24). Since  $\Omega'(\tau^*) > 0$ , the characterization of the growth rate follows immediately.  $\square$ 

We have thus far solved the balanced growth equilibrium and are prepared to characterize the equilibrium properties to which we now turn.

## 4 Characterization of Balanced Growth Equilibrium

How would economic growth and credit market thickness be correlated along the balanced growth path? How would a change of productivity in the manufacturing sector or in banking sector affect the balanced growth path? These questions can now be answered based on comparative-static exercises with respect to exogenous shifts in A,  $\phi$  and  $\tau^*$  using the two key long-run relationships summarized by equations (23) and (24). Concerning the exogenous shifts that change the cutoff ability, we consider either a uniformly outward shift of the ability distribution function or an increase in the dispersion of the ability distribution function represented by a local mean-preserving spread.

Just how does either distributional change influence cutoff ability and financial-market thickness? Consider a uniformly rightward shift of the ability distribution function from a compact support  $[0, \tau^H]$  to  $[\lambda, \tau^H + \lambda]$ , where  $\lambda > 0$ . We then have

**Lemma 3:** A rightward shift in the ability distribution (an increase in  $\lambda$ ) raises both the cutoff ability  $\tau^*$  and financial-market thickness  $\Gamma$ .

*Proof.* See Appendix.  $\square$ 

Next, consider a change in the dispersion of the ability distribution. In order to obtain unambiguous results, we focus exclusively on a uniform distribution over a compact support  $[\eta \tau^H, (1-\eta)\tau^H]$  with constant density  $\frac{1}{(1-2\eta)\tau^H}$ , where  $\eta = 0$  restores the benchmark distribution support and a reduction in  $\eta$  represents a local mean-preserving spread. The following property can be established:

**Lemma 4:** For  $\theta \in (\frac{\alpha}{2}, \alpha)$ , a local mean-preserving spread of the uniform ability distribution (lower  $\eta$ ) raises the cutoff ability  $\tau^*$  and reduces financial-market thickness  $\Gamma$  unambiguously.

*Proof.* See Appendix.  $\square$ 

It may be noted that the condition in Lemma 4 restricts the degree of knowledge spillovers (measured by  $\alpha - \theta$ ) to be not too high.

Armed with Lemmas 3 and 4, we are now ready to conduct comparative-static analysis. We focus on explaining the results intuitively and relegate the formal proof in the Appendix. Table 1 summarizes the main findings concerning financial-market thickness and economic growth. If the occupational choice effect is sufficiently large and the degree of capital-skill

complementarity is sufficiently high, (23) suggests that the wage rate in effective units along the balanced growth path is decreasing in the cutoff ability. If the occupational choice effect is not too small, (24) indicates that there is a positive relationship between the cutoff ability and the rate of balanced growth of the economy. Thus, when the occupational choice effect and the degree of capital-skill complementarity are strong, higher financial-market thickness (higher  $\Gamma$ ) caused by a uniformly rightward shift of the ability distribution is associated with higher economic growth (higher g), confirming the Goldsmith-McKinnon proposition of finance and growth.

On the other hand, if the occupational choice effect is sufficiently small and the degree of capital-skill complementarity is sufficiently low, (23) implies the effective wage rate is now increasing in the cutoff ability. From (24), we must then have the rate of balanced growth depend negatively on the cutoff ability. In this case, a thicker financial market is accompanied by a staggered growth path of real activity. In the finance literature with negative participation externalities (e.g., Becsi et al. 1999 and papers cited therein), a thicker financial market may be harmful for the performance of the real sector due to a direct interactive crowding-out effect. Although our result resembles that in this previous literature, our model does not assume any negative participation externality and the result is entirely driven by endogenous occupational choice in the presence of positive entrepreneurial knowledge-spillover externality. Our findings may provide a plausible explanation for the ambiguity in the relationship between economic progress and financial development as discussed in De Gregorio and Guidotti (1992) and Fernandez and Galetovic (1994).

Furthermore, under a uniform distribution over a compact support of  $[\eta \tau^H, (1-\eta)\tau^H]$  with constant density  $\frac{1}{(1-2\eta)\tau^H}$ , a local mean-preserving spread of the ability distribution (lower  $\eta$ ) raises cutoff ability and results in a thinner financial market when the degree of entrepreneurial knowledge spillovers (measured by  $\alpha - \theta$ ) is not too high (more precisely,  $\frac{\alpha}{2} < \theta < \alpha$ ). Yet, such a dispersed distribution may still be growth-enhancing, as long as the occupational choice effect is strong enough to dominate the scale effect and the degree of capital-skill complementarity is sufficiently high (see Table 2).<sup>12</sup> That is, the equilibrium

<sup>&</sup>lt;sup>12</sup>One can also use (19) and (20) to show that in this case, the effective wage rate is decreasing with a more

properties concerning the relationship between economic growth and financial-market thickness in response to a local mean-preserving spread of the ability distribution turn out to be exactly opposite to those in response to a uniform shift in the ability distribution under the same parametric regularities.

We next turn to characterizing the different effects of an advancement in production technology and an improvement in banking efficiency. We summarize the results in Table 2. By examining (23), it is not difficult to show that along the balanced growth path, the effective wage rate ( $\omega$ ) and the financial markup ( $\frac{\delta-r}{r}$ , which equals  $\frac{\omega}{\phi}$  in balanced growth equilibrium) are decreasing in banking productivity ( $\phi$ ) but increasing in manufacturing productivity (A). The negative relationship between banking productivity and financial markup corroborates with empirical findings in Lehr and Wang (2000). Also, utilizing (20) and (21), the only effects of banking productivity on the interest rates are through the effective wage rate, implying  $\frac{d\delta}{d\phi} > 0$  and  $\frac{dr}{d\phi} > 0$ . Due to the presence of a direct effect, the effects of manufacturing productivity on loan and deposit rates are generally ambiguous. By differentiation and manipulation, however, it is possible to show that the direct effect is dominant in the loan rate so that  $\frac{d\delta}{dA} > 0$ , though the ambiguity of the sign of  $\frac{dr}{dA}$  remains.

Finally, notice from (24) that the balanced growth rate is inversely related to the effective wage rate and that while there is a positive direct effect of manufacturing productivity on the balanced growth rate g, banking productivity has no such direct influence. As a consequence, it is easy to sign  $\frac{dg}{d\phi} > 0$  and  $\frac{dg}{dA} > 0$ . That is, both an improvement in banking efficiency and an advancement in production technology promote economic growth. Yet, their effects on the financial markup are entirely opposite. Therefore, whether economic development is accompanied by a lower or higher financial markup depends crucially on the origin of the productivity increase. This provides a plausible explanation for the ambiguity of this relationship established in the cross-country, cross-industry study of Cetorelli and Gambera (2001).

Summarizing, we have the following main theorems:

dispersed ability distribution whereas the loan rate is increasing.

**Theorem 1**: The balanced growth equilibrium possesses the following properties in response to a uniform shift in the ability distribution:

- (i) when the occupational choice effect is strong and the degree of capital-skill complementarity is high, there is a positive relationship between financial-market thickness (high  $\Gamma$ ) and economic growth (high g);
- (ii) when the occupational choice effect is weak and the degree of capital-skill complementarity is low, there is a negative relationship between financial-market thickness (high  $\Gamma$ ) and economic growth (low g).

**Theorem 2**: Assume a uniform ability distribution with weak entrepreneurial knowledge spillovers such that  $\theta \in (\frac{\alpha}{2}, \alpha)$ . Then, in response to a local mean-preserving spread of the ability distribution, the balanced growth equilibrium possesses properties exactly opposite to those in response to a uniform shift in the ability distribution as described in Theorem 1. Moreover, a dispersed distribution may promote growth even with a thinner financial market, as long as the occupational choice effect and the capital-skill complementarity are sufficiently strong.

**Theorem 3**: In balanced growth equilibrium, both a reduction in the unit financial operation cost (higher  $\phi$ ) and an improvement in manufacturing productivity (higher A) are growthenhancing, but their effects on equilibrium wages, interest rates, and financial markup are generally different.

To the end, we would like to perform calibration exercises, illustrating that how the various ambiguous results established above can be obtained in response to changes in the underlying parameters from the benchmark case using the U.S. data. We focus on the case where the distribution of ability F is uniform. The main parameters to be calibrated are: A,  $\alpha$ ,  $\theta$ ,  $\gamma$ ,  $\beta$ ,  $\phi$ , and  $\tau_H$ .

Along a balanced growth path, we can rewrite (20) and (21) as:

$$\delta^* = \theta (1 - \alpha)^{\frac{1 - \alpha}{\alpha}} A^{\frac{1}{\alpha}} [\Omega(\tau^*)]^{\frac{\alpha - \theta}{\alpha}} \omega^{* - \frac{1 - \alpha}{\alpha}}$$

$$r^* = \theta (1 - \alpha)^{\frac{1 - \alpha}{\alpha}} A^{\frac{1}{\alpha}} [\Omega(\tau^*)]^{\frac{\alpha - \theta}{\alpha}} \omega^{* - \frac{1 - \alpha}{\alpha}} (1 + \frac{\omega^*}{\phi})^{-1}.$$

Under uniform distribution, we have:

$$\tau^* = \left(\frac{\theta}{\gamma\theta + \alpha}\right)^{\frac{\alpha - \theta}{\gamma\theta + \alpha - \theta}} \tau^H$$

$$\Gamma(\tau^*) = 1 - \frac{\tau^*}{\tau^H} = 1 - \left(\frac{\theta}{\gamma\theta + \alpha}\right)^{\frac{\alpha - \theta}{\gamma\theta + \alpha - \theta}}$$

$$\Omega(\tau^*) = \frac{\alpha - \theta}{\gamma\alpha + \alpha - \theta} \frac{(\tau^H)^{\frac{\gamma\alpha + \alpha - \theta}{\alpha - \theta}} - (\tau^*)^{\frac{\gamma\alpha + \alpha - \theta}{\alpha - \theta}}}{\tau^H - \tau^*}.$$

These together with (23) and (24) are the main equations used for our calibration analysis.

First, we adopt standard parameterization to set  $\alpha=0.3$  and  $\theta=0.2$  as in the standard endogenous growth literature. Next, we pin down the value of financial markup  $(\delta/r)$  by the ratio of one-year bank prime loan to T-bill rate, based on data from the Federal Reserve Board (1960-2002). We compute the average growth rate (g) from the Penn World Table (1961-2000) and the real wage rate  $(\omega)$  from the Bureau of Labor Statistics (1964-2002). Then, upon normalizing A=1 (since we can always rescale  $\tau_H$  to make A=1) and setting  $\gamma=1.5$ , we can calibrate the values of  $\beta$ ,  $\phi$ , and  $\tau_H$  to fit  $\delta/r$ , g and  $\omega$  using the U.S. data. This gives the U.S. benchmark values for  $\beta=0.0007$ ,  $\phi=41,066$ ,  $\tau_H=7,400,000$ ,  $\Gamma(\tau^*)=0.24$ , g=2.45%,  $\delta/r=1.35$  and  $\omega=14,373$ . This benchmark case falls into the strong capital-skill complementarity (strong  $\beta$ ) and strong occupational choice effect (strong  $\Omega$ ) category in that there is a positive relationship between financial market thickness and growth rate in response to changes in the ability distribution. However, if we change the values of the conversion parameter  $(\gamma)$  or the degree of capital-skill complementarity  $(\beta)$ , any ambiguous outcomes presented in Tables 1 and 2 can be produced (see Tables 3A and 3B, respectively).

## 5 Concluding Remarks

We have developed a dynamic general-equilibrium model of finance and growth with loanable fund conversion ability heterogeneity. Competitive banks earn zero profit by hiring workers to transform deposits into loans. By occupational choice, an agent may choose to become an entrepreneur, transforming loans into capital and producing the final good by employing young workers. We have shown that for a given ability distribution, a decrease in the number of entrepreneurs may create an occupational choice effect, enhancing the rate of growth of the economy, as the average conversion ability of the remaining entrepreneurs is higher. Moreover, for a given family of distributions, a change of distribution parameters with regard to its dispersion may generate a permanent growth effect. Due to the presence of an occupational choice effect, a scale effect and a general-equilibrium wage adjustment effect, financial-market thickness and income growth need not be positively correlated. In addition, production and banking technologies may have very different long-run implications for economic growth. Our paper has therefore promoted better understanding about the workings of financial markets in the process of economic development.

Our model predicts that a rightward shift of the ability distribution may lead to a negative relationship between financial-market thickness and economic growth, particularly when the capital-skill complementarity is low and the occupational choice effect is weak. This may be tested empirically using cross-country, cross-industry data. Furthermore, our model also suggests that an improvement in financial efficiency and an advancement in production technology can induce different relationships between financial markup and economic growth. It may therefore be interesting to separate these two types of productivity enhancements empirically to examine the relative magnitude of their effects on markup-growth relationship across countries and industries.

# Appendix

Proof of Lemma 1. Let  $\bar{\omega}$  be a fixed point of  $\Psi$ , then  $\bar{\omega}$  must satisfy  $\bar{\omega} = \Psi(\bar{\omega})$ . Replacing  $\omega_{t+1}$  and  $\omega_t$  in equation (19) with  $\bar{\omega}$  yields,

$$\bar{\omega}^{\frac{1}{\alpha}} (1 + \frac{\bar{\omega}}{\phi})^{-1} = B(\tau^*) [\bar{\omega}^{\frac{\alpha - \beta}{\alpha}} (1 + \frac{\bar{\omega}}{\phi})^{-(1-\beta)}]. \tag{A1}$$

Obviously,  $\bar{\omega} = 0$  is a fixed point of (A1). Assuming  $\bar{\omega} = \omega > 0$ , (A1) implies:

$$\omega^{\frac{1-\alpha+\beta}{\alpha}} (1+\frac{\omega}{\phi})^{-\beta} = (\frac{\alpha-\theta}{\theta})^{\beta} [A(1-\alpha)]^{\frac{1+\beta}{\alpha}} [\Omega(\tau^*)]^{\frac{\alpha-\theta(1+\beta)}{\alpha}} \tau^{*\frac{\beta\gamma\theta}{\alpha-\theta}}.$$
 (A2)

Since the right-hand-side of (A2) is a constant and the left-hand-side strictly increases in  $\omega$  from 0 to infinity,  $\omega$  (> 0) exists and is unique.

To prove that  $\omega$  is the only locally stable fixed point, it suffices to show that  $|\Psi'(\omega)| < 1$  and  $|\Psi'(0)| > 1$ . From equation (19),

$$\Psi'(\omega_t) = \frac{d\omega_{t+1}}{d\omega_t} = \frac{\left[ (\alpha - \beta)\phi - \beta(1 - \alpha)\omega_t \right] \left[ \omega_{t+1} \left( 1 + \frac{\omega_{t+1}}{\phi} \right) \right]}{\left[ \phi + (1 - \alpha)\omega_{t+1} \right] \left[ \omega_t \left( 1 + \frac{\omega_t}{\phi} \right) \right]}.$$
 (A3)

Therefore,

$$\Psi'(\omega) = \frac{(\alpha - \beta)\phi - \beta(1 - \alpha)\omega}{\phi + (1 - \alpha)\omega},$$

and it is easy to verify that  $|\Psi'(\omega)| < 1$ .

To show that  $\bar{\omega} = 0$  is an unstable fixed point, we utilize equation (19) again and rewrite equation (A3) as:

$$\Psi'(\omega_t) = \frac{\left[ (\alpha - \beta)\phi - \beta(1 - \alpha)\omega_t \right] \left[ B(\tau^*)^{\alpha} \left( 1 + \frac{\omega_{t+1}}{\phi} \right)^{1+\alpha} \right]}{\left[ \phi + (1 - \alpha)\omega_{t+1} \right] \left[ \omega_t^{1-\alpha+\beta} \left( 1 + \frac{\omega_t}{\phi} \right)^{1+\alpha(1-\beta)} \right]}.$$

Since  $\omega_{t+1} = 0$  when  $\omega_t = 0$ ,  $\Psi'(0) = \infty$ .  $\square$ 

Proof of Lemma 2: Since  $\Psi$  is continuous, Sarkovskii Theorem applies. Therefore, if we can show that  $\Psi$  has no two-period cycle, then the proof is completed.

Suppose  $\Psi$  has a periodic point  $\omega_p(>0)$  with prime period two. That is,  $\Psi(\omega_p) = k\omega_p$  and  $\Psi(k\omega_p) = \omega_p$  for some positive k and  $k \neq 1$ . From equation (19), this implies:

$$(k\omega_p)^{\frac{1}{\alpha}}(1 + \frac{k\omega_p}{\phi})^{-1} = B(\tau^*)(\omega_p)^{\frac{\alpha-\beta}{\alpha}}(1 + \frac{\omega_p}{\phi})^{-(1-\beta)}$$
(A4)

$$(\omega_p)^{\frac{1}{\alpha}} (1 + \frac{\omega_p}{\phi})^{-1} = B(\tau^*) (k\omega_p)^{\frac{\alpha - \beta}{\alpha}} (1 + \frac{k\omega_p}{\phi})^{-(1-\beta)}.$$
 (A5)

From (A4) and (A5), we get:

$$k^{\frac{1+\alpha-\beta}{\alpha}} = \left(\frac{1 + \frac{k\omega_p}{\phi}}{1 + \frac{\omega_p}{\phi}}\right)^{2-\beta},$$

which yields,

$$(1 - k^{-\frac{(1-\alpha)(1-\beta)}{\alpha(2-\beta)}})\omega_p = (k^{-\frac{1+\alpha-\beta}{\alpha(2-\beta)}} - 1)\phi.$$
(A6)

Since  $\operatorname{sign}(1-k^{-\frac{(1-\alpha)(1-\beta)}{\alpha(2-\beta)}})=-\operatorname{sign}(k^{-\frac{1+\alpha-\beta}{\alpha(2-\beta)}}-1)$  for any positive  $k\neq 1,\ \omega_p<0$  which contradicts our assumption. Therefore,  $\Psi$  has no two-period cycle and by Sarkovskii Theorem,  $\Psi$  has no cycle of any period.  $\square$ 

Proof of Lemma 3: Consider a uniformly rightward shift of the ability distribution function from a compact support of  $[0, \tau^H]$  to  $[\lambda, \tau^H + \lambda]$  with  $\lambda > 0$ . Then, this new distribution can be expressed as  $F(\tau - \lambda)$  with pdf  $f(\tau - \lambda)$ . The cutoff ability  $\tau_{\lambda}$  under the new distribution can be solved from,

$$\frac{\alpha - \theta}{\theta} \int_{\lambda}^{\tau_{\lambda}} dF(\tau - \lambda) = \int_{\tau_{\lambda}}^{\tau^{H} + \lambda} \left(\frac{\tau}{\tau_{\lambda}}\right)^{\frac{\gamma \theta}{\alpha - \theta}} dF(\tau - \lambda).$$

By changing variables, we get:

$$\frac{\alpha - \theta}{\theta} \int_0^{\tau_\lambda - \lambda} dF(\tau) = \int_{\tau_\lambda - \lambda}^{\tau^H} \left(\frac{\tau + \lambda}{\tau_\lambda}\right)^{\frac{\gamma \theta}{\alpha - \theta}} dF(\tau). \tag{A7}$$

Totally differentiating (A7) with respect to  $\lambda$  implies,

$$\frac{\alpha - \theta}{\theta} f(\tau_{\lambda} - \lambda) \left( \frac{d\tau_{\lambda}}{d\lambda} - 1 \right) = -f(\tau_{\lambda} - \lambda) \left( \frac{d\tau_{\lambda}}{d\lambda} - 1 \right) + \frac{\gamma \theta}{\alpha - \theta} \frac{1}{\tau_{\lambda}} \left[ \int_{\tau_{\lambda} - \lambda}^{\tau^{H}} \left( \frac{\tau + \lambda}{\tau_{\lambda}} \right)^{\frac{\gamma \theta}{\alpha - \theta} - 1} dF(\tau) - \frac{d\tau_{\lambda}}{d\lambda} \int_{\tau_{\lambda} - \lambda}^{\tau^{H}} \left( \frac{\tau + \lambda}{\tau_{\lambda}} \right)^{\frac{\gamma \theta}{\alpha - \theta}} dF(\tau) \right],$$

or, manipulating,

$$\frac{d\tau_{\lambda}}{d\lambda} = \frac{\frac{\alpha}{\theta} f(\tau_{\lambda} - \lambda) + \frac{\gamma \theta}{\alpha - \theta} \frac{1}{\tau_{\lambda}} \int_{\tau_{\lambda} - \lambda}^{\tau^{H}} \left(\frac{\tau + \lambda}{\tau_{\lambda}}\right)^{\frac{\gamma \theta}{\alpha - \theta} - 1} dF(\tau)}{\frac{\alpha}{\theta} f(\tau_{\lambda} - \lambda) + \frac{\gamma}{\tau_{\lambda}} \int_{0}^{\tau_{\lambda} - \lambda} dF(\tau)} > 0.$$
(A8)

Since the financial-market thickness measure under this new distribution becomes:  $\Gamma_{\lambda} \equiv 1 - \int_0^{\tau_{\lambda} - \lambda} f(\tau) d\tau$ , we have:

$$\frac{d\Gamma_{\lambda}}{d\lambda} = f(\tau_{\lambda} - \lambda) \left( 1 - \frac{d\tau_{\lambda}}{d\lambda} \right). \tag{A9}$$

It is clear that the second term in the denominator of (A8) can be rewritten as:

$$\frac{\gamma}{\tau_{\lambda}} \int_{0}^{\tau_{\lambda} - \lambda} dF(\tau) = \frac{\gamma \theta}{\alpha - \theta} \frac{1}{\tau_{\lambda}} \int_{\tau_{\lambda} - \lambda}^{\tau^{H}} \left( \frac{\tau + \lambda}{\tau_{\lambda}} \right)^{\frac{\gamma \theta}{\alpha - \theta}} dF(\tau)$$

which is unambiguously greater than the second term in the numerator of (A8), thus implying  $\frac{d\tau_{\lambda}}{d\lambda} < 1$  and, from (A9),  $\frac{d\Gamma_{\lambda}}{d\lambda} > 0$ .  $\square$ 

Proof of Lemma 4: Under uniform distribution over a compact support  $[\eta \tau^H, (1-\eta)\tau^H]$  with constant density  $\frac{1}{(1-2\eta)\tau^H}$ , equation (15) along a balanced growth path becomes,

$$T(\frac{\tau^*}{\tau^H}; \eta) \equiv \frac{\left(\frac{\tau^*}{\tau^H}\right) - \eta}{\left(\frac{\tau^*}{\tau^H}\right)} - \frac{1}{\gamma + \frac{\alpha - \theta}{\theta}} \left\{ \left[\frac{1 - \eta}{\left(\frac{\tau^*}{\tau^H}\right)}\right]^{1 + \frac{\gamma \theta}{\alpha - \theta}} - 1 \right\} = 0, \tag{A10}$$

which can be evaluated at the benchmark value of  $\eta = 0$  to yield,

$$\frac{\tau^*}{\tau^H} = \left(1 + \gamma + \frac{\alpha - \theta}{\theta}\right)^{\frac{-1}{1 + \frac{\gamma \theta}{\alpha - \theta}}}.$$
 (A11)

Straightforward differentiation of (A10) leads to  $\frac{\partial T}{\partial (\tau^*/\tau^H)} > 0$  and

$$\frac{\partial T}{\partial \eta} = \frac{1}{\left(\frac{\tau^*}{\tau^H}\right)} \left\{ \frac{1 + \frac{\gamma \theta}{\alpha - \theta}}{\gamma + \frac{\alpha - \theta}{\theta}} \left[ \frac{1 - \eta}{\left(\frac{\tau^*}{\tau^H}\right)} \right]^{\frac{\gamma \theta}{\alpha - \theta}} - 1 \right\}$$

$$= \frac{1}{(1 - \eta)\left(\frac{\tau^*}{\tau^H}\right)} \left\{ \left[ 1 + (1 + \gamma) \frac{\theta}{\alpha - \theta} \right] \left(\frac{\tau^*}{\tau^H}\right) - \frac{\gamma \theta}{\alpha - \theta} \eta - 1 \right\}.$$

Taking the limit  $\eta \to 0$  and utilizing (A11), the above expression reduces to,

$$\left(\frac{\partial T}{\partial \eta}\right)_{\eta \to 0} = \frac{1}{\left(\frac{\tau^*}{\tau^H}\right)} \left\{ \left[1 + (1+\gamma)\frac{\theta}{\alpha - \theta}\right] \left(\frac{\tau^*}{\tau^H}\right) - 1 \right\} \\
= \frac{1}{\left(1 + \gamma + \frac{\alpha - \theta}{\theta}\right)^{\frac{1}{1 + \frac{\gamma \theta}{\alpha - \theta}}} \left(\frac{\tau^*}{\tau^H}\right)} \left\{ \left[1 + (1+\gamma)\frac{\theta}{\alpha - \theta}\right] - \left(1 + \gamma + \frac{\alpha - \theta}{\theta}\right)^{\frac{1}{1 + \frac{\gamma \theta}{\alpha - \theta}}} \right\}.$$

Under  $\theta \in (\frac{\alpha}{2}, \alpha)$ ,  $(1 + \gamma) \left(\frac{\theta}{\alpha - \theta}\right) > \gamma + \frac{\alpha - \theta}{\theta}$ , it is sufficient to guarantee  $(\frac{\partial T}{\partial \eta})_{\eta \to 0} > 0$  and hence  $\left[\frac{d(\tau^*/\tau^H)}{d\eta}\right]_{\eta \to 0} < 0$ .

Next, under uniform distribution over a compact support  $[\eta \tau^H, (1-\eta)\tau^H]$ , the financial-market thickness measure along a balanced growth path is given by,

$$\Gamma(\tau^*; \eta) = \int_{\tau^*}^{(1-\eta)\tau^H} \frac{1}{(1-2\eta)\tau^H} d\tau = \frac{1}{1-2\eta} \left[ (1-\eta) - (\frac{\tau^*}{\tau^H}) \right], \tag{A12}$$

which is decreasing in  $\frac{\tau^*}{\tau^H}$  and increasing in  $\eta$ . Thus,  $(\frac{d\Gamma}{d\eta})_{\eta\to 0} = \frac{\partial\Gamma}{\partial(\tau^*/\tau^H)} \left[\frac{d(\tau^*/\tau^H)}{d\eta}\right]_{\eta\to 0} + (\frac{\partial\Gamma}{\partial\eta})_{\eta\to 0} > 0$ .  $\square$ 

Comparative Statics: We divide the proof into three parts by shifts in A,  $\phi$  and  $\tau^*$ .

Part I. Comparative statics with respect to changes in A: When A goes up, we can easily show:

(i)  $\frac{\partial \omega}{\partial A} > 0$ : using equation (23),

$$\frac{\partial \ln \omega}{\partial \ln A} = \frac{(1+\beta)(1+\frac{\omega}{\phi})}{(1-\alpha+\beta)+(1-\alpha)(1+\beta)\frac{\omega}{\phi}} > 0.$$

(ii)  $\frac{\partial \delta}{\partial A} > 0$ : using equation (20),

$$\frac{\partial \ln \delta}{\partial \ln A} = \frac{\beta}{(1 - \alpha + \beta) + (1 - \alpha)(1 + \beta)\frac{\omega}{\delta}} > 0.$$

(iii)  $\frac{\partial r}{\partial A} \geq 0$ : using  $r = \delta(1 + \frac{\omega}{\phi})^{-1}$ ,

$$\frac{\partial \ln r}{\partial \ln A} = \frac{\beta - (1+\beta)\frac{\omega}{\phi}}{(1-\alpha+\beta) + (1-\alpha)(1+\beta)\frac{\omega}{\phi}} \ge 0.$$

(iv)  $\frac{\partial g}{\partial A} > 0$ : using equation (24),

$$\frac{\partial \ln g}{\partial \ln A} = \frac{\beta}{(1 - \alpha + \beta) + (1 - \alpha)(1 + \beta)\frac{\omega}{A}} > 0.$$

Part II. Comparative statics with respect to changes in  $\phi$ : When  $\phi$  goes up, we have:

(i)  $\frac{\partial \omega}{\partial \phi}$  < 0: using equation (23),

$$\frac{\partial \ln \omega}{\partial \ln \phi} = -\frac{\alpha \beta(\frac{\omega}{\phi})}{(1 - \alpha + \beta) + (1 - \alpha)(1 + \beta)\frac{\omega}{\phi}} < 0.$$

(ii)  $\frac{\partial \delta}{\partial \phi} > 0$ : using equation (20),

$$\frac{\partial \ln \delta}{\partial \ln \phi} = -\left(\frac{1-\alpha}{\alpha}\right) \frac{\partial \ln \omega}{\partial \ln \phi} > 0.$$

(iii)  $\frac{\partial r}{\partial \phi} > 0$ : using  $r = \delta(1 + \frac{\omega}{\phi})^{-1}$ ,

$$\frac{\partial \ln r}{\partial \ln \phi} = \frac{\partial \ln \delta}{\partial \ln \phi} - \frac{\frac{\omega}{\phi}}{1 + \frac{\omega}{\phi}} \left( \frac{\partial \ln \omega}{\partial \ln \phi} - 1 \right) > 0.$$

(iv)  $\frac{\partial g}{\partial \phi} > 0$ : using equation (24),

$$\frac{\partial \ln g}{\partial \ln \phi} = \frac{\beta (1 - \alpha) \frac{\omega}{\phi}}{(1 - \alpha + \beta) + (1 - \alpha)(1 + \beta) \frac{\omega}{\phi}} > 0.$$

Part III. Comparative statics with respect to changes in  $\tau^*$ : Considering the effect of  $\tau^*$ , we first obtain:

$$\frac{\partial \ln \Omega(\tau^*)}{\partial \ln \tau^*} = \frac{\tau^* f(\tau^*)}{\Omega(\tau^*) \left(\int_{\tau^*}^{\tau^H} dF\right)^2} \left\{ -\tau^* \frac{\gamma_{\alpha}}{\alpha - \theta} \int_{\tau^*}^{\tau^H} dF + \int_{\tau^*}^{\tau^H} \tau^{\frac{\gamma_{\alpha}}{\alpha - \theta}} dF \right\}$$

$$= \frac{\tau^* f(\tau^*)}{\Omega(\tau^*) \int_{\tau^*}^{\tau^H} dF} \left[ \Omega(\tau^*) - \tau^* \frac{\gamma_{\alpha}}{\alpha - \theta} \right] > 0.$$

We can thus derive:

(i)  $\frac{\partial \omega}{\partial \tau^*} \geq 0$ : using equation (23),

$$\frac{\partial \ln \omega}{\partial \ln \tau^*} = \left[ \frac{\alpha (1 + \frac{\omega}{\phi})}{(1 - \alpha + \beta) + (1 - \alpha)(1 + \beta)\frac{\omega}{\phi}} \right] \left[ \frac{\beta \gamma \theta}{\alpha - \theta} - \frac{(1 + \beta)\theta - \alpha}{\alpha} \frac{\partial \ln \Omega(\tau^*)}{\partial \ln \tau^*} \right],$$

which is positive if  $\alpha > (1 + \beta)\theta$ , and is ambiguous if  $(1 + \beta)\alpha < \theta$ ;

(ii)  $\frac{\partial \delta}{\partial \tau^*} \geq 0$ : using equation (20),

$$\frac{\partial \ln \delta}{\partial \ln \tau^*} = \left(\frac{\alpha - \theta}{\alpha}\right) \frac{\partial \ln \Omega(\tau^*)}{\partial \ln \tau^*} - \left(\frac{1 - \alpha}{\alpha}\right) \frac{\partial \ln \omega}{\partial \ln \tau^*} \geqslant 0.$$

- (iii)  $\frac{\partial r}{\partial \tau^*} \geq 0$ : this is due to the previous two derivatives are ambiguous;
- (iv)  $\frac{\partial g}{\partial \tau^*} \geq 0$ : by straightforward differentiation,

$$\frac{\partial \ln g}{\partial \ln \tau^*} = \left(\frac{\alpha - \theta}{\alpha}\right) \frac{\partial \ln \Omega(\tau^*)}{\partial \ln \tau^*} - \left(\frac{1 - \alpha}{\alpha}\right) \frac{\partial \ln \omega}{\partial \ln \tau^*} \geqslant 0.$$

These together with Lemmas 3 and 4 complete the comparative static exercises.  $\Box$ 

## References

- [1] Aghion, Phillippe and Patrick Bolton (1997), "A Theory of Trickle Down Growth and Development," *Review of Economic Studies*, 64(2), 151-172.
- [2] Azariadis, Costas and Allan Drazen (1990), "Threshold Externalities in Economic Development," Quarterly Journal of Economics, 105(3), 501-526.
- [3] Bagehot, Walter (1915), Lombard Street: A Description of the Money Market, New York: E. P. Dutton and Company (original edition, 1873, Homewood, IL: R. D. Irwin).
- [4] Becsi, Zsolt and Ping Wang (1997), "Financial Development and Growth," *Economic Review*, 82(4), Federal Reserve Bank of Atlanta, 46-62.
- [5] Becsi, Zsolt, Ping Wang, and Mark Wynne (1999), "Costly Intermediation, the Big Push and the Big Crash," *Journal of Development Economics*, 59(2), 175-193.
- [6] Bencivenga, Valerie and Bruce Smith (1991), "Financial Intermediation and Endogenous Growth," *Review of Economic Studies*, 58(2), 195-209.
- [7] Banerjee, Abhijit V. and Andrew F. Newman (1993), "Occupational Choice, and the Process of Development," *Journal of Political Economy*, 101(2), 274-298.
- [8] Benhabib, Jess, Qinglai Meng and Kazuo Nishimura (2000), "Indeterminacy under Constant returns to Scale in Multisector Economies," *Econometrica*, 68(6), 1541-1548.
- [9] Blanchflower, David G., and Andrew J. Oswald (1998), "What Makes an Entrepreneur?," Journal of Labor Economics, 16(1), 26-60.
- [10] Cetorelli, Nicola and Michele Gambera (2001), "Banking Market Structure, Financial Dependence and Growth: International Evidence from Industry Data," *Journal of Finance*, 56(4), 617-648
- [11] Chen Been-Lon, Yeong-Yuh Chiang and Ping Wang (2000), "Credit Market Imperfections, Financial Activity and Economic Growth", Working Paper, Nashville, TN: Vanderbilt University.
- [12] De Gregorio, Jose and Pablo E. Guidotti, 1992, "Financial Development and Economic Growth," IMF Working Paper No.92-101.
- [13] De Gregorio, Jose and Se-Jik Kim (2000), "Credit Markets with Differences in Abilities: Education, Distribution, and Growth," *International Economic Review*, 41(4), 579-607.
- [14] Diamond, Peter A. and J. Yellin (1990), "Inventories and Money Holdings in a Search Economy," *Econometrica*, 58(4), 929-950.

- [15] Evans, Douglas and Boyan Jovanovic (1989), "An Estimated Model of Entrepreneurial Choice under Liquidity Constraints," *Journal of Political Economy*, 97(4), 808-827.
- [16] Fender, John and Ping Wang (1993), "Educational Policy in a Credit Constrained Economy with Skill Heterogeneity," *International Economic Review*, 44(3), 939-964.
- [17] Fernandez, David G. and Alexander Galetovic (1994), "Schumpeter Might Be Right-But Why? Explaining the Relation between Finance, Development, and Growth," Johns Hopkins University SAIS Working Paper in International Economics No.96-01.
- [18] Gertler, Mark and S. Gilchrist (1994), "Monetary Policy, Business Cycles and the Behavior of Small Manufacturing Firms," Quarterly Journal of Economics, 109(2), 209-340.
- [19] Ghatak, Maitreesh and Neville N. Jiang (2002), "A Simple Model of Inequality, Occupational Choice and Development," *Journal of Development Economics*, 62(4), 205-226.
- [20] Glomm, Gerhard and B. Rivikumar (1992), "Public Versus Private Investment in Human Capital: Endogenous Growth and Income Inequality," Journal of Political Economy, 100(4), 818-834.
- [21] Goldsmith, Raymond W. (1969), Financial Structure and Development, New Haven, CT: Yale University.
- [22] Greenwood, Jeremy and Boyan Jovanovic (1990), "Financial Development, Growth, and the Distribution of Income," *Journal of Political Economy*, 98(5), 1076-1107.
- [23] Griliches, Zvi (1969), "Capital-Skill Complementarity," Review of Economics and Statistics, 51(2), 465-468.
- [24] Gurley, John G. and Edward S. Shaw (1955), "Financial Aspects of Economic Development," *American Economic Review*, 45(4), 515-538.
- [25] Holtz-Eakin, Douglas, David Joulfaian and Harvey S. Rosen (1994), "Sticking It Out: Entrepreneurial Survival and Liquidity Contraints," *Journal of Political Economy*, 102(1), 53-75.
- [26] King, Robert G. and Ross Levine (1993), "Finance and Growth: Schumpeter Might Be Right," Quarterly Journal of Economics, 108(4), 717-738.
- [27] Lehr, Carol Scotese and Ping Wang (2000), "Dynamic Effects of Financial Intermediation over Business Cycle," *Economic Inquiry* 38(1), 34-57
- [28] Lloyd-Ellis, Huw (2000), "Public Education, Occupational Choice, and the Growth-Inequality Relationship," *International Economic Review*, 41(2), 171-201.

- [29] Lloyd-Ellis, Huw, and Dan Bernhardt (2000), "Enterprise, Inequality and Economic Development," *Review of Economic Studies*, 61(1), 147-168.
- [30] Lucas, Robert, E, Jr., 1988, "On the Mechanics of Economic Development," Journal of Monetary Economics, 22(1), 3-42.
- [31] McKinnon, Ronald, 1973, Money and Capital in Economic Development, Washington DC: Brookings Institute.
- [32] Piketty, Thomas, 1997, "The Dynamics of the Wealth Distribution and the Interest Rate with Credit Rationing," *Review of Economic Studies*, 64(1), 173-189.
- [33] Romer, Paul, 1986, "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, 94(6), 1002-1037.
- [34] Schumpeter, Joseph, 1911, *Theory of Economics Development*, Cambridge: Harvard University Press (Reprinted 1936).
- [35] Schumpeter, Joseph, 1939, Business Cycles: A Theoretical, Historical, and Statistical Analysis of the Capitalist Process, New York: McGraw-Hill.
- [36] Sharkovskii, A. N., 1964, "Coexistence of Cycles of a Continuous Map of a Line into Itself," *Ukrainian Mathematical Journal*, 16(1), 61-71.