

**USING REPAYMENT DATA TO TEST ACROSS MODELS OF
JOINT LIABILITY LENDING**

by

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Working Paper No. 02-W27

December 2002

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Abstract

Spurred by its successful delivery of credit to poor borrowers in diverse areas of the developing world, joint liability lending has caught the imagination of development theorists and practitioners. Various theories have arisen to explain why joint liability group-based lending can be an improvement over traditional individual-based lending. Here we exploit the idea that if a model were true, the repayment rate would vary in a systematic way with various covariates. We thus use observed repayment rates to test across four representative and oft-cited models of joint liability lending. The theoretical part of this paper develops the models' implications for repayment and derives new ones, signing the derivative of a project choice, monitoring, default, or selection equation. For example, we find that several models imply that higher correlation of output can raise the observed repayment rate, and in some the ability to act cooperatively leads to lower repayment rates. More generally, the models agree on some dimensions and conflict on others. The empirical part uses survey data from 262 Thai joint liability groups of the Bank for Agriculture and Agricultural Cooperatives (BAAC) and from 2880 households of the same villages. Nonparametric, univariate tests and multivariate logits are used to evaluate the predictions of the models. A hybrid, partially linear procedure is used to understand any differences in the results between the two types of tests. We find that the Besley and Coate model of limited enforcement is strongly supported in the more rural, poorer region of Thailand covered by the data. In the more prosperous region, closer to Bangkok, support is found for the Stiglitz model of moral hazard and the Ghatak model of adverse selection.

*We thank Jonathan Conning, Steve Levitt, Derek Neal, Sombat Sakuntasathien, seminar participants at Chicago, Southern California, Vanderbilt and at NEUDC 2001, ESSFM 2002 and the 2002 European Economic Association meetings for valuable input. Financial support from the National Institute of Health and the National Science Foundation (including its Graduate Research Fellowship Program) is gratefully acknowledged. All errors are ours.

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1 Introduction

Joint liability lending is a potential breakthrough strategy in economic development. International conferences and a proliferation of such lending contracts in a wide range of countries are evidence of this. The hope is that this tool will allow credit to be extended to impoverished borrowers lacking physical collateral. For example, without land title, borrowers cannot take out individual loans. The premise of group, joint liability lending is that if one borrower cannot repay a loan, then other members of a joint liability group will do so. This type of contract can be used to lend to new or expanded enterprise. Economic development will follow, or so the story goes.

In turn, various theoretical papers have been written to explore the key mechanism that gives group loans an advantage over individual loans. Specifically, these papers pinpoint conditions under which joint liability contracts are optimal relative to individual liability contracts. But they take different stands on the underlying economic environment and the problem which groups are imagined to try to overcome. These obstacles include moral hazard, monitoring, adverse selection, and limited enforcement, among others. Thus we have in hand a variety of logical possibilities that might account for the use of joint liability contracts.

The empirical side of research in joint liability lending has lagged relatively far behind. A few evaluation studies do exist. They offer mixed results. More to the point, there is little empirical work that views the data through the lens of theory.¹ This paper attempts bridge the gap between theoretical and empirical work.

Our approach, naturally enough, is to use the explicit structure suggested by the different models to distinguish them in the data. Specifically, we use as springboards to the data four unique and widely cited papers. Two of these papers highlight moral hazard problems which joint liability lending and monitoring can mitigate: Stiglitz (1990) and Banerjee, Besley, and Guinnane (1994). One focuses on an environment of limited contract enforcement: Besley and Coate (1995). The fourth shows how adverse selection of borrowers can be partially overcome by joint liability contracts: Ghatak (1999).

Again, most of the literature focuses on conditions under which joint liability contracts are optimal relative to individual liability contracts, typically with an endogenous, market-clearing interest rate. For example, in Stiglitz (1990) and in Ghatak (1999), an increase in the degree of joint liability allows profit-maximizing lenders to lower the interest rate and induce safer project choice or draw in safer borrowers. In a competitive equilibrium, lenders still make zero profits but borrower welfare is enhanced through joint liability.

Direct tests involve regressing the relative prevalence of joint liability versus individual liability contracts against the covariates suggested by the models. In a companion paper, Ahlin and Townsend (2000), we use this direct approach to test Holmstrom and Milgrom (1990) and the related Prescott and Townsend (2002). We find, consistent with the theory, that village wealth heterogeneity predicts group, joint liability contracts, and further that there is a U-shaped relationship of average village wealth. Curiously, we do not find that positive correlation in project returns is a force for individual, relative performance contracts.

In this paper we adopt an indirect, but equally telling, approach. Specifically, we test

¹Wydick (1999) provides a notable exception to this.

the models' implications for borrower repayment rates, *given* the joint liability contract is in use. This exploits the idea that if a model were true, the repayment rate would vary in a systematic way with various covariates. We thus use observed repayment rates as the key dependent variable. This strategy is justified given the fundamental role the probability of repayment plays in each model's setup and results. It is a useful key for unlocking and examining the mechanics of each of the models. We therefore turn our attention from contract choice per se to those internal mechanics. That is, in this paper we take as given that joint liability contracts are optimal and are what we observe, and focus on the models' implications for repayment.

Apart from choice of contract, there is also the decision of the agents whether or not to borrow at all, and if so, how much. Three of the four models do not deal at all with selection of agents into borrowing groups or loan size. An important exception is the Ghatak model, where agents choose between borrowing and an outside alternative. While the original model does not allow loan size to vary, here we extend it by allowing loan size to be determined in part by borrower choice.

All four types of models that we use are rich in predictions regarding the determinants of the group repayment rate. As noted, the theoretical contribution of our paper is the exploration these determinants within each existing model. We do this both with variables already included in the models, as published in the literature, and also with variables we can introduce in a general way. For example, the interest rate is a key variable in all four models, while loan size appears only in one, and correlation of borrower output appears in none. In our approach, we introduce loan size and correlation into the models where this can be done with generality, and then explore the effects of all three variables on repayment rates. Other variables considered in some if not all the models, or introduced in this paper, include borrower productivity, screening ability, the ease of monitoring, the degree of cooperation, the availability of outside additional credit, and penalties for default.

Though tempted, we do not try other major innovations on the model front. That is, we accept for now that the current models have their limitations or shortcomings. For example, the models are static and involve borrowing groups of fixed size (two). Our goal here instead is to evaluate these current, widely-used theories by a confrontation with the data. Hopefully the insights provided can be used in future research, including the construction of revised models.

As noted implicitly, each of the models relies on a different mechanism to generate the predicted repayment rate. In the Stiglitz model of moral hazard in project choice, the key equation is one that captures the indifference of borrowers between safe and risky projects, a so-called Project Switch line. In the moral hazard model of Banerjee, Besley, and Guinnane, the key equation is a Monitoring Equation which determines the resources a non-borrowing member is willing to put into supervising and penalizing a potentially deviating borrowing member. In the limited enforcement model of Besley and Coate, the key equation is a Default Curve that determines the joint project return thresholds below which there will be failure to repay. Finally, the adverse selection model of Ghatak features a classic selection equation, that is, a threshold risk type below which safer borrowers will refuse to participate.

We thus focus on how repayment rates are expected to vary according to each model's predictions, that is, to vary with the above-mentioned covariates *ceteris paribus*. We find

similarities and differences across models. For example, we show that higher interest rates² lead to lower repayment rates in all four models. Similarly, higher borrower human capital improves repayment rates in each of the four. Outside credit options lower repayment in two of the four with no predictions in the others. On the other hand, cooperation among borrowers hurts repayment rates in two models and helps in another. Repayment is decreasing in the joint liability payment in two models, and increasing in a third. The same is true for covariability. Finally there are predictions that are special and crucial to each of the models separately: screening in the model of adverse selection and informal penalties in the model of limited enforcement, for example.

Our empirical strategy is to use semi-parametric and non-parametric methods to estimate the key conditional probabilities of repayment of each model, each as a function of the covariates. We thus find the empirical counterparts to the derivatives suggested by the models. Specifically, we run multi-variate logits, in effect a particular semi-parametric single-index model, thus imposing the form of the distribution function and the requirement that the covariates interact with one another linearly and additively. Logits should be viewed as a common approximation to what would be predicted by each of models, each with its own explicit specification of a likelihood function. In a more non-parametric if univariate direction, we compute conditional mean values of repayment when the data are ordered and stratified by values of each of the covariates, one at a time. We also examine objects that are closer to suggested derivatives by running non-parametric regressions, that is locally linear weighted least squares regressions in neighborhoods of each distinct value of the covariate, again one at a time. Finally, we reconcile potential divergence between the multi-variate logits and both the non-parametric univariate procedures, by extracting from the repayment rate and each covariate the influence of all the other covariates, using the double residuals from linear regressions in the non-parametric, locally linear regressions.

The joint liability groups that we evaluate in this paper are those of the Bank for Agriculture and Agricultural Cooperatives, the BAAC, a development bank operating under the auspices of the Government of Thailand. It has a charter which requires targeting rural medium income farmers and operating procedures which impose a schedule of interest rates and limit loan size. The BAAC is not profit maximizing, unlike the banks of the theoretical models. Instead the BAAC draws an explicit subsidy from the government. We take advantage of that in our empirical procedure.

The data used in this paper were acquired in a relatively large cross-sectional survey designed by one of the authors (See Townsend et al (1997)). Up to two BAAC joint liability groups were interviewed in 192 villages, spread out across two distinct regions of the country, the more developed Central region near Bangkok and the poorer, semi-arid Northeast. In addition, data were gathered from 15 households and the headman in each village, giving us important background and control variables, and the data for various instruments.

Our findings offer general support for the four types of models under consideration: human capital increases repayment and outside options decrease repayment. More interesting perhaps are the findings that distinguish the models. The moral hazard model of Stiglitz

²Here we are not imposing a zero-profit condition on the lender, as do all of the models except Besley and Coate. The reason has to do with the lending institution in our data, discussed in this section and again in more detail in section 2.

and the screening model of Ghatak do best in the wealthier, central region in the prediction that the level of the joint liability payment will decrease repayment and that covariability in borrower outcomes will increase repayment. The limited enforcement model of Besley and Coate does best in the poorer Northeast in its prediction that penalties will increase repayment. The results on cooperation support in part the prediction of BBG and BC that repayments should decrease with cooperation. We note in particular that the higher is the fraction of family related members in a group, the lower are repayment rates. This result should serve to challenge the notion that groups succeed because of their ability to access and make use of social collateral.

Indeed the paper can be viewed as a critique of the presumed premise that there is a unified force, invariate over regions or customers, that can be captured in a fixed formula by altruistic, or profit maximizing, lenders. Instead policy advice on the targeting of lending would need to be informed by the interaction of theory with data that we propose here.

2 Background on the BAAC

The Bank for Agriculture and Agricultural Cooperatives is a government-operated development bank in Thailand. It was established in 1966 and is the primary formal financial institution serving rural households. By its own estimates, it serves 4.88 million farm families, in a country with just over sixty million inhabitants, about eighty percent of which live in rural areas. In the data here, BAAC loans constitute 34.3% of the total number of loans, but we include in this total loans and reciprocal gift giving from friends, relatives, and moneylenders (see Kaboski and Townsend 1998). Indeed, commercial banks in the sample here have only 3.4% of total loans, and provide loans to only about 6% of the household sample. Occasionally a village will have established a local financial institution, but typically these are small and constitute on average only 12.8% of total loans. Informal loans, though 39.4% of the total, are also small in size.

The point then is that the BAAC faces little competition from other financial intermediaries, especially those capable of generating large loans. It is thus not realistic or desirable to impose a zero profit constraint as if markets were in competitive equilibrium. Many BAAC branches do not make profits, that is, cannot and do not adjust their lending to the local environment and the local clients. Indeed the BAAC as a whole receives a non-trivial government subsidy.

The point is two-fold. First, we see little variation in interest rates across the joint liability groups of our sample. There is an exogenous, pre-specified, unified national schedule mapping loan size into interest rates. For example in 1997, at the time the data used here were collected, all loans under 60,000 baht carried a 9% interest rate, while loans between 60,000 baht and 1,000,000 baht charged 12.25% interest rates. Thus except for the highest loan amounts, we should see virtually no variation.

The second point is that in this regulated context, though competing for deposits, the BAAC charges low, below market interest rates on its loans. Its subsidy dependency index of 35%, as estimated by Yaron (Townsend and Yaron 2001), is the amount that would be necessary to raise the average on-lending rate in order to break even. That is, under its charter the BAAC is responsible for the well being of farmers and those in rural areas, and it

carries out that responsibility by charging a lower interest rate to small clients. Note that is the opposite of what one might have anticipated – smaller loans must surely be more costly to administer and hence should carry higher rates.

We are thus dealing with a bank that does not attempt to break even by adjusting interest rates based on risk or other group and location specifics. Interestingly, the BAAC interest rate policy was changed subsequent to the collection of our data, after the Asian crisis and with the advice of outside agencies. The changes specify higher rates for riskier borrowers according to a formula based on the number of past defaults. The latter would make interest rates a monotone increasing function of default, rather than having defaults increasing with the interest rates, as predicted by all the models here. That is, one could not identify from the positive association of interest and defaults either of these two scenarios. But note again that this new policy was implemented, in principal, well after the data we use were collected. Juxtaposition of the more recent policy with the old, initial policy only reaffirms the peculiar, if exogenous, nature of the latter. Further, to be safe we weight the lowest rate much more than the highest rate in the group when constructing a group average interest rate – the highest rate very well may reflect a previous penalty.

Loan size is determined in part by BAAC policy as well. Initial loans are small. Farmers work their way up the ladder toward larger loans as they gain seniority, so to speak. The BAAC sets the maximum loan size for any farmer to an estimated 60% of the revenue sale of crop, though of course farmers are free to borrow less. This can dramatically limit loan size for subsistence farmers and those who eat much of their crop. Thus it is plausible that much of the sample is credit-constrained in the usual sense. Loans can also be larger if the government has targeted a particular area for expansion, or if the government wishes to be lenient to those experiencing adverse events (see Townsend and Yaron 2001 for examples). The point then is that there is some exogenous policy variation in loan size across borrowers groups. On the other hand loans can depend on credit officer assessment of the client and on good repayment histories. This could make loan size increase with assessed safety, and hence decrease with the observed, ex post probability of default. If anything, then, this kind of endogeneity would bias the tests toward a negative relationship between loan size and default. But the data say the opposite, if anything, that is that default increases with loan size (though in general not statistically significantly). This agrees with several, but not all, of the models' predictions.

The BAAC requires some kind of collateral for all loans, but it allows smaller loans to be backed with social collateral in the form of joint liability. Thus loans underwritten by a BAAC group do not in principle require land or other physical collateral, only the promise that individual members be jointly liable. Loans larger than 50,000 baht must be backed by an asset such as land. Any particular loan is classified as a group-guaranteed or individual loan, and the appropriate collateral box checked off on the loan form. In practice, however, the BAAC asks about individual landholdings and may even require the deed for "safekeeping". Indeed, the BAAC will give a farmer two to three opportunities to repay before restricting loans to that farmer, and ultimately to the rest of the group. Legal proceedings can be implemented as well, against the defaulting member or other members of the group. Thus repayment by some members for other members may be seen as a last resort and the likelihood that the group will have to pay rather than the defaulting member is seemingly mitigated by that member's landholdings. We can thus take the degree of joint

liability to be proportional to the fraction of the group which are landless.³

We emphasize that the BAAC is not a universal bank. BAAC clients are more educated and wealthier than the typical rural household for example, particularly so for those taking out individual loans. However, this paper does not seek to explain among all rural households of the sample who borrows and who does not. We also leave aside almost entirely decisions about whether to borrow from the BAAC as an individual or with a group, and whether to borrow from commercial banks or other lenders instead of or in addition to the BAAC. Rather, with one exception, this paper follows the models in taking as given the selection of some of the rural population into BAAC groups and then focusing on the potential inner workings of those groups themselves. The major exception is our examination of Ghatak's adverse selection model, with its implication that group members are more risky than those who take the outside option.

3 Overview of Theories and the Empirical Strategy

The key⁴ to understanding each of the four types of models is, naturally enough, the derivation of the probability that the group will repay its loan, or equivalently, the residual probability that the group will default. This is also the probability as a function of observables of whether a particular group in our sample would have defaulted on its obligations to the outside lender, the BAAC. The determinants of these repayment rates differ rather radically across the models.

Stiglitz (1990) is representative of moral hazard models. That is, the agents in a joint liability group receive a loan and then take an action which determines repayment, an action that is not seen by the outside lender. In Stiglitz that action is the choice of the riskiness of the investment project financed with the loan. A conflict of interest arises, because the borrower only pays off the loan when the project succeeds. This gives the agent a greater incentive to choose risky projects. But in a joint liability group, the borrower with a successful project may have to repay another member's loan, if another member has a failed project. This causes each member to care a little more about the safety of projects chosen. Specifically one enumerates the utility consequences of members jointly choosing safe or risky projects and derives an indicator function for the choice of safe projects. A so-called Switch Line characterizes the locus of indifference. That indicator function, or Switch Line, is the key determinant of repayment rates.

Banerjee, Besley, and Guinnane (1994, hereafter BBG) is representative of how costly

³The groups in our data may be unrepresentative of all groups in this particular, in the sense that groups with few landed households may have dispersed upon default and not appear in our data. Thus we may be losing some of the upper tail of landless groups, which would reduce the variation of our measure of joint liability in the data. We still find the sign to be significant. More generally, there can be some variation in default that our data do not capture, namely the upper tail of groups who have defaulted and gone out of existence. This only makes our job in this paper harder, as we estimate results from a subset of the range of outcomes.

⁴As noted in the introduction, the approach in this paper is not a direct test of the paper's theorems, which generally show conditions under which the joint liability contract is optimal relative to the individual liability contract. Instead, here we focus on the models' determinants of the repayment rate, given that the joint liability mechanism is in place.

monitoring may help to overcome the same moral hazard problem. More precisely, costly verification of actions is necessary to implement penalties. The conflict of interest between a borrowing member and the outside lender falls on the second monitoring member who is otherwise responsible for paying off the loan. More precisely, at an increasing cost of effort, the monitoring member can inflict yet higher penalties on the borrowing member in the event of default, giving the borrowing member an incentive to choose a safer project. On the margin, with the incentive thus internalized, the marginal cost of increased monitoring is equated to the marginal benefit of increased monitoring, namely the decreased likelihood of facing the joint liability payment at the endogenously chosen level of project risk. This first order condition or Monitoring Equation is the key to the characterization of repayment rates.

Ghatak (1999) is representative of models with an adverse selection problem. Again, investment projects differ in their likelihoods of success, but an agent here cannot choose his project type. That is, some agents are inherently more risky than others, but the outside lender cannot discern who is who. Thus a common interest rate will have to cover (at a given outside subsidy) the overall level of risk in the pool of borrowers, with high risk borrowers in effect cross-subsidized by safer ones. In fact with an outside, if low, return option, safe borrowers will drop out of the pool, leaving an otherwise riskier overall set, and in Ghatak, (but not here) a higher equilibrium rate is the result. In this context, local knowledge about the characteristics of other borrowers and joint liability gives each member an incentive to match with a similar risk type. As Ghatak shows, this would allow a lower interest rate. Here though we focus on the selection equation which determines the cut-off risk type, below which safe potential borrowers choose the outside option and do not participate. The repayment rate of any particular group is, in so far as the lender or econometrician is concerned, the average across risk types of those left in the borrowing pool. This Selection Equation then is the key to characterizing the model's implications for observed default as a function of observables.

The Besley and Coate (1995, hereafter BC) model of strategic default or limited enforcement is distinctly different from the others in that there is no moral hazard and no adverse selection problem. Rather, joint liability members are endowed with a uniform risky project financed with loans. Further, while each other model assumes repayment always occurs if the borrower's project is successful, in BC borrowers choose whether or not to repay based on ex post project outcomes. Here the incentive to repay depends on the outcome of other group members and on presumed official and unofficial penalties for default. Both types of penalties are assumed increasing in project outcome. Thus can one partition the space of joint project outcomes with a Default Curve, below which the group will opt not to repay. This is the key to repayment in the model; that is, the probability of repayment is the likelihood that project outcomes fall in the region above the curve.

The Switch Line in Stiglitz, the Monitoring Equation in BBG, the Selection Equation in Ghatak, and the Default Curve in BC are thus the sources of restrictions on the data. For example in Stiglitz one can characterize how the Switch Line, or point of indifference in project choice, will move with observables such as the interest rate, loan size, and joint liability payment. That is, a change in the interest rate will be associated with a change in the utility return from choosing the safe project, and similarly for the risky project, and the movement in the difference in utilities between these two will move the point of indif-

ference. Thus the key to whether increasing interest rates will raise or lower the repayment rates, *ceteris paribus*, lies in the derivative of a utility difference. We show in fact that the derivative with respect to the interest rate, can be signed uniformly negatively under mild, non-parametric restrictions, for example, that utility be concave and that both projects have expected returns that are concave in the amount of capital loaned. The order of magnitude of the (negative) derivative can vary however, so there should be no presumption at this level that the frequency of repayment rates should fall uniformly, that is, linearly with the interest rate. Likewise, the derivative depends non-parametrically not only on preferences and technology but also on other variables, such as loan size and joint liability payment. Repayment probabilities are not linearly additive in the covariates given the structure of the model.

We take some liberty in modifying the models when this can be done with minimal assumptions, in order to test unexamined dimensions. In the Stiglitz model, for example, one can sign the effect on repayment of a certain zero-one conceptual experiment, of allowing joint liability borrowers to play non-cooperatively.⁵ The requirement that joint project choices be a Nash equilibrium among borrowers places an additional term in the equation of indifference. Indeed we show it pushes the project Switch Line downward, so that risky projects would be chosen more often, *ceteris paribus*. One can also introduce productivity differences across groups. On the assumption that the production function can be decomposed multiplicatively into a piece related to the risk factor and a piece related to productive inputs, say loaned capital and human capital, we are able to sign the derivative of the utility difference with respect to human capital. Similarly, one can allow project returns to be correlated, parameterize the degree of that correlation, take a derivative, and determine that correlations in returns actually increases the likelihood of repayment.

This same strategy is applied to the other models. In BBG the riskiness of the project is determined by the Monitoring Equation. Thus we can see how riskiness will change with movement in the joint liability payment, for example, by totally differentiating that equation with respect to both the probability of success, call it p , and the degree of joint liability, call it q . Increasing q increases the benefit of monitoring without affecting the cost of monitoring, that is, of implementing a given project choice. Thus repayment rates are increasing in q . Increasing the interest rate r , however, leaves the benefit of monitoring unchanged while increasing the cost of monitoring. That is, more monitoring effort must be exerted to enforce a given project choice, since the higher interest rate skews the borrower's incentives toward risky choices. Repayment rates thus decrease with r . For both of these results, it is sufficient to assume that the cost of monitoring is convex, that the expected project return is concave in p , and that the expected return is increasing in p at a rate which is less than the interest rate, so that the borrower is tempted to take a risky project.

Similarly, in BC the Nash equilibrium of the repayment game determines the critical values of income from the project returns, which determine whether the group will pay off their loans. These critical values form the Default Curve in the space of joint project returns. Below the Default Curve, the cost of repayment, the interest rate, is higher than the benefit, avoiding official and unofficial penalties. Conversely, above the Default Curve,

⁵Stiglitz implicitly assumes the borrowers can enforce costlessly project choice agreements, while BBG and BC involve non-cooperative games.

Table 3a - Repayment Implications

Entries with a '‡' are the result of our own extensions of the authors' original models.

Variable	Effect on Repayment			
	Stiglitz	BBG	BC	Ghatak
interest rate r	↓	↓	↓	↓
loan size L	↓	↓‡	no pred.	↑‡ ^a
liability payment q	↓ ^b	↑	no pred.	↓
productivity H	↑‡	↑‡	↑‡	↑‡
screening	no pred.	no pred.	no pred.	↑
positive correlation	↑‡	no pred.	↓‡ ^c	↑‡
cost of monitoring	no pred.	↓	no pred.	no pred.
cooperative behavior	↑‡	↓‡ ^d	↓‡ ^e	—
outside credit options	↓‡	no pred.	no pred.	↓‡
official penalties	no pred.	no pred.	↑	no pred.
unofficial penalties	no pred.	no pred.	↑	no pred.

^aIf borrowers cannot borrow as much as they would like.

^bUnder assumption A3, section 5.1

^cUnder certain regularity conditions on the correlation.

^dIf $M'(c) \leq 1$.

^eIf $c_u(Y, \Lambda) \geq \Lambda$.

repayment does happen because increased penalties mean benefits of repaying are higher than costs. Thus the probability of repayment is moved up or down by variation in interest rates, in official and unofficial penalties, and in the ability to play cooperatively, since these all shift the Default Curve. It is also affected by productivity differences and correlation across project returns, which leave the Default Curve unaffected but shift the probability mass enclosed by it.

In Ghatak, one differentiates with respect to covariates such as the interest rate the critical cutoff probability that determines the pool of residual risky borrowers. The average risk in the residual pool, which determines the expected repayment rate of the group we observe, moves in the same direction as the cutoff probability that determines the pool.

Table 3a summarizes the results of this exercise; the details are contained in section 5. As is evident, some of the predictions are common across the models. Increased interest rates lower the probability of repayment, and increased productivity raises that probability, for example. If any of the models are to have some validity, these should be found in the data. There are also implications that are peculiar to specific models. An increased cost of monitoring lowers the repayment rate in BBG, for example. Similarly, increased unofficial penalties and an enhanced possibility for screening raise the probability of repayment in the BC and Ghatak models, respectively. Each of these can be seen as a crucial test of these models, separately.

We emphasize however the lines in Table 3a which reveal sign reversals across the models. An increased joint liability payment increases repayment in BBG, for example, but decreases

repayment in Ghatak. The intuition is that increased joint liability raises the marginal benefit of monitoring in the BBG model, making increased monitoring and reduced risk worth while. But in Ghatak an increase in the joint liability payment makes borrowers pay more on average, at a given interest rate. This makes the outside option more attractive, increasing the risk of the residual pool. Likewise, cooperative behavior increases repayment in the Stiglitz model but lowers repayment in BBG and BC. Under moral hazard, Nash non-cooperative behavior makes it harder for members jointly to choose the safe project. But under costly enforcement, ex ante cooperative agreements can mitigate the threat that especially severe unofficial penalties will be implemented ex post, making default more likely.

Our empirical strategy thus focuses on testing in various ways whether or not the particular covariates, vector $X = (X_1, \dots, X_j, \dots, X_M)$, move monotonically with repayment history R . As noted, the key objects coming from the theories are the probabilities of repayment for group g ⁶ as functions of covariates, $P(R^g = 1|X^g)$.⁷ A different approach from this paper's would be to test across the models using maximum likelihood estimation. That is, one could parameterize preferences, utility functions, the distributions of shocks, the values of project risk, and so on, then maximize the likelihoods for each model separately by choice of these parameters. One attractive feature of this approach is that the likelihoods for each model are identical up to the function $P(R = 1|X)$:

$$\prod_{g=1}^G P(R^g = 1|X^g)^{R^g} [1 - P(R^g = 1|X^g)]^{1-R^g}. \quad (1)$$

Note⁸ however that the sign restrictions inherent to each model would be loaded automatically into the probabilities and in a sense forced onto the data. Though it would be possible to compare likelihoods across non-nested models as in Vuong (1989), for example, we prefer here to get a direct look at the monotonicities as predicted by the models.

Our first approach to testing the monotonicity predictions is the most structural. It involves making two simplifying assumptions on the models themselves. First we assume that for each model, $P(R^g = 1|X^g)$ can be written as a function $P(\beta'X^g)$, where β is an $M \times 1$ vector of parameters and X^g is an $M \times 1$ vector containing group g 's values for the M covariates. Clearly this restricts the covariates to enter repayment probabilities as

⁶To be more precise, two of the models, Stiglitz and Ghatak, deliver the probability that one individual in the (homogeneous) group succeeds and repays, p say. The probability that both succeed and repay is then p^2 . The probability that only one succeeds and repays his own obligation and some of his partner's is $2p(1-p)$. Finally, the probability that neither succeed or repay anything is $(1-p)^2$. However one classifies the middle outcome, where only one succeeds and the full group debt may not be repaid, the group repayment rate is monotonically increasing in p , being either p^2 or $p(2-p)$. Thus we focus in the theory just on p . Empirically, our measure of default would likely encompass the middle case, since it involves a penalty that is most often imposed on the defaulting individual, whether or not others in the group defaulted.

⁷Our data on R are binary-valued. Details are in sections 4 and 6.1.

⁸In practice, our data on R reflect whether, *in the history of the group*, any blemishes in the repayment record have occurred, as will be described in section 4 and 6.1. Thus we would have to control for group age in the likelihood 1. Since we have data on the age of the group, O^g say, we could do this by substituting each term $P(R^g = 1|X^g)$ in the likelihood 1 with the same term raised to the power O^g , $P(R^g = 1|X^g)^{O^g}$. Of course, this is equivalent to assuming the static models that deliver $P(R|X)$ are repeated every year with no feedback across years and no change in covariates. Ideally, the models and the data would be dynamic, but our more modest goal in this paper is to test the current state of the theory, which involves static models.

a linear combination while leaving the function P unrestricted. This is the single-index model, studied by Ichimura (1993) among others, and potentially computationally complex to estimate. Our second assumption is that $P = \Lambda$, that is the probability function is logistic. This is the logit model, easily estimated by maximum likelihood. Of course, since our data on repayment R are binary-valued, expected repayment equals the probability of repayment – $E(R^g|X^g) = 1 * P(R^g = 1|X^g) + 0 * P(R^g = 0|X^g)$ – and so we are finding the covariates which make for expected repayment.

The second, bivariate approach is the least structural – namely a simple test of means. We order each of the covariates X_j in the data X_j^g across groups g from low to high, pick an intermediate cutoff value, say \hat{X}_j , and then test whether the means of the repayment variable are significantly different across groups with values above and below \hat{X}_j , and if so with what sign. For robustness, we move the cutoff value across the ordered domain from low to high, forming all possible two-bin partitions, with the requirement that we have a minimal sample size in each bin and that we do not put into separate bins observations that have equal values for the covariate. The latter requirement is especially necessary for discrete-valued covariates, such as indices of sharing within the group.

The third approach is also bivariate. Here we use locally linear regression techniques to allow for more local variation, computing the expected value of repayment as it varies with each (continuous-valued) covariate X_j . These regressions calculate an expected repayment rate at each value X_j^g of the covariate X_j using only the 80% of the sample closest to X_j^g . That is, with a local neighborhood centered at a value in the domain of the covariate, with a width large enough to capture 80% of the sample, we compute fitted values from a weighted least squares regression, using the tri-cube weighting function (see Cleveland 1979). These conditional expected values are plotted in figures with bootstrap standard error bands.

In many instances, the two univariate approaches, tests of means and locally linear regressions, are consistent with the multivariate logits. However, this is not the case for all variables. To sort these cases out, our final approach is to return to locally linear regressions, but with linear multivariate controls. That is, for each covariate X_j , we first extract the influence of the other covariates from both R and X_j by regressing each linearly on the vector $(X_1, \dots, X_{j-1}, X_{j+1}, \dots, X_M)$. We then use the error terms, the orthogonalized versions of R and X_j , the double residuals so to speak, in a locally linear regression as above. Almost without exception, these tests confirm the results of the multivariate, logit specification.

4 Data

The data used in this paper are from the Townsend Thai data base, in particular from a large cross section of 192 villages, conducted in May 1997. The survey covers two contrasting regions of Thailand. The central region is relatively close to Bangkok and enjoys a degree of industrialization, as in the province of Chachoengsao, and also fertile land for farming, as in the province of Lopburi. The Northeast region is poorer and semi-arid, with the province of Srisaket regarded as one of the poorest in the entire country and the province of Buriram offering a transition as one moves back west toward Bangkok. Each of these two provinces was chosen because at least one county, or amphoe, had been sampled repeatedly in the national level socioeconomic survey, thus allowing some background and comparison for the

survey here. In practice we split the sample and compare results for the northeast and central provinces separately.

Within each province, twelve subcounties, or tambons, were chosen under a stratified random sampling scheme. The stratification was based on an analysis of satellite imagery that allowed an oversampling of forested zones, to ensure variation in agricultural productivity and the timing of good and bad years. Within each tambon, a cluster of 4 villages was selected, and within each village fifteen households were administered a Household instrument. There are thus 2280 households in the household data base. Of key importance for the paper here, in each village as many BAAC borrowing groups as possible were interviewed, up to two. In all we have data on 262 groups, 62 of which are the only groups in their respective village. We call this instrument the BAAC survey. Each group designates an official leader, and the leader responded to questions on behalf of the group.

Tables 6.1a and 6.1b display means and standard deviations in the data. Among the questions asked was whether in its history the group had suffered a higher interest penalty for failure to repay and whether one member had ever repaid loans for another. We also asked about the dispersion of interest rates and loan sizes within each group, the age, education, and landholdings of each member, the number of members in the group, which members had close relatives in the same group, and when the group was founded. We also have measures of cooperation within each group, as measured by the amount of sharing within the group between closely related members and unrelated members, respectively, and by the number of decisions that are made by the group as opposed to individual farmer members; and of cooperation in the village by a poll in the household survey which asked which village in the tambon enjoys the highest degree of cooperation among villages. We also have measurements of the outside availability of funds in a village, as measured by the same poll and its questions about availability and quality of institutions. We also know from the household survey the fraction of households in the village borrowing from commercial banks and village funds and we know the average household wealth in each village. We have measures of the covariance of individual member returns, as measured by the coincidence among households in the village of good years (and bad years) within the last five and by occupational homogeneity, and measures of the risk of future returns. Proxies for the costliness of monitoring come from the number of members with a close relative in the group, number of members who live in the same village, and again the occupation of each member. Unofficial penalties are proxied by answers to questions to households on what would happen if a borrower did not pay back a loan. Screening is proxied by whether members say there are others who would like to join the group but cannot.

5 Theories and Implications

In this section we examine in turn three main types of theories: moral hazard, limited enforcement, and adverse selection. Moral hazard is represented by Stiglitz (1990) and Banerjee et al (1994, abbreviated BBG); limited enforcement by Besley and Coate (1995, abbreviated BC); and adverse selection by Ghatak (1999). We present the basic setup of each model and develop the implications of key variables on repayment rates. In some cases, we extend models to develop predictions along new dimensions.

There are common themes running through the models, and we attempt to adopt common notation wherever possible. All the models have in common that there are two members of the single group considered, and the framework is always static. Both members of a group face the same contract terms.⁹

Three of the four models are similar. They focus on the borrowers affecting distribution of output through their project choice (Stiglitz and BBG) or through their risk-type (Ghatak). The fourth model, BC, considers a fixed distribution of output, and focuses on what happens after output is realized. This distinction allows three of the models to be mapped into similar notation. In what immediately follows, therefore, we restrict attention to Stiglitz, BBG, and Ghatak.

Let L denote the amount of capital lent to the borrower. Loan size is fixed (normalized to 1) in the original BBG and Ghatak models, but we will consider what happens when it is allowed to vary. Borrowers are expected to pay back at gross interest rate r (inclusive of principal) rL when they succeed. Any liable group member is expected to pay qL when he has the funds and his partner fails. Thus q denotes the degree of joint liability.

Borrowers are characterized by (Ghatak) or choose (Stiglitz, BBG) a risk level for the project they will undertake. In these models, the distribution of output is binomial. Thus risk is fully captured by a probability of achieving the high output return, p . The higher is p , the less risky the project. Since the low output return is normalized to zero, p is also called the probability of success. In Ghatak and BBG, p is taken from a real interval: $p \in [\underline{p}, 1]$, where \underline{p} is a parameter. Stiglitz is dichotomous: $p \in \{p_R, p_S\}$, where both p_R and p_S are parameters.

The risk-type p also affects how much output is achieved upon success. It is the only determinant in the original Ghatak and BBG models. In Stiglitz and our modified versions of Ghatak and BBG, L also affects successful output. Successful output can then be written $Y(p, L)$, and expected output is then $pY(p, L)$. We sometimes add an additional production factor H as a determinant in all four models. H has interpretations as human capital or land.

Borrower expected payoffs are similar in the three models. Utility when project output is zero is normalized to zero. Thus the only payoff comes when a borrower succeeds. However, depending on whether or not his partner also succeeds, the borrower repays rL or $(r + q)L$. Thus payoffs take the basic form

$$U(p, p', r, q, L) = pp'U[Y(p, L) - rL] + p(1 - p')U[Y(p, L) - rL - qL], \quad (2)$$

where p is the borrower's probability of success and p' is his partner's probability of success. With probability pp' , both succeed and the borrower repays rL . With probability $p(1 - p')$, the borrower succeeds but his partner fails, so he pays $(r + q)L$. Two of the models differ slightly from this specification. In Ghatak and BBG, utility is just linear: $U(x) = x$. Further, in BBG, a group consists of one borrower and one cosigner, who does not borrow. Thus only the cosigner faces the possibility of a joint liability payment, and $q = 0$ in the borrower's payoff. But the cosigner has some means of penalizing the borrower, and this penalty, call it c , is subtracted from the borrower payoff when imposed.

⁹In general, the theory we examine does not take into account intra-group heterogeneity, though this clearly exists in the data. The exception is Ghatak, who shows intra-group heterogeneity in risk-type does not exist in equilibrium.

Unlike the other three models' binomial distribution of output, BC employ a continuous distribution of realizable output, denoted by $F(Y)$. F is unaffected by borrower choice or type. The only decision of the borrowers is whether or not to repay after output is realized. By contrast, borrowers always repay when they can in the other three models. There the borrower's choice is in what kind of project (Stiglitz, BBG) or whether or not to borrow (Ghatak).

5.1 Moral Hazard: Stiglitz 1990

The Stiglitz 1990 model shows how joint liability lending can alleviate the moral hazard issues involved in lending to those with no collateral, and thus limited liability. In this context, when project choice is unobservable the borrower has incentives to choose a high-risk project, since this lowers expected interest payments. Joint liability can increase group incentives for safe project choice by making a borrower willing to encourage a partner to choose a safer technology. Though joint liability contracts introduce greater variance in payoffs than would be observed in the individual liability case, Stiglitz shows in his main result that this effect is second-order and there is some positive degree of joint liability that increases welfare.

In this section we show that the model contains the following basic predictions¹⁰ involving group repayment rates, under certain assumptions.

- The repayment rate is decreasing in the interest rate r .
- The repayment rate is decreasing in the loan size L .
- The repayment rate is decreasing in the joint liability payment q , as long as safe projects involve opposite borrower outcomes more often than risky projects do.
- The existence of outside options decreases repayment probabilities by enabling borrowers to increase loan size L .
- Cooperative behavior increases repayment rates relative to non-cooperative behavior, enabling borrowers to circumvent free-riding on each other's safe behavior.
- Increased correlation between borrower output increases repayment rates.
- Higher borrower productivity, through education say, increases repayment rates.

There is assumed to be exactly one source of credit (or a lender who can restrict borrowers from tapping other sources) for a population of borrowers who lack capital. This lender offers contracts to borrowers specifying the amount of capital loaned per individual ($L > 0$), the gross interest rate charged ($r > 0$), and the degree of joint liability payment rate ($q > 0$). Here we take r , L , and q as given.

Groups consist of two borrowers. Each chooses a risky or safe technology, producing output $Y(p_R, L)$ with probability p_R or $Y(p_S, L)$ with probability p_S , where p_S and p_R are

¹⁰Note that all these predictions are true *ceteris paribus*.

parameters, and $0 < p_R < p_S < 1$. Failure using either technology results in zero output. It is assumed that

$$p_S Y(p_S, L) > p_R Y(p_R, L), \quad (\text{A1})$$

and that

$$Y \text{ is strictly increasing, concave, and twice continuously differentiable (in } L). \quad (\text{A2})$$

Thus the safe project has a higher expected return, and is preferred by a risk neutral lender.¹¹ We will sometimes invoke the following assumption when exploring determinants of the repayment rate:

$$p_S(1 - p_S) \geq p_R(1 - p_R). \quad (\text{A3})$$

This ensures that safe projects involve opposite borrower outcomes more often than risky projects do.¹²

A borrower repays zero upon failure, rL upon success, and an additional qL upon his own success and his partner's failure. Let U be utility of consumption, assumed strictly increasing. Normalize $U(0) \equiv 0$. We make the following standard assumption on U :

$$U \text{ is strictly concave, and twice continuously differentiable.} \quad (\text{A4})$$

Expected utility of a borrower who chooses technology i and whose partner chooses technology j , call it V_{ij} , can be written similarly to equation 2 as follows:

$$V_{ij} = p_i p_j U[Y(p_i, L) - rL] + p_i(1 - p_j)U[Y(p_i, L) - rL - qL] - v(L), \quad i, j \in \{R, S\}. \quad (3)$$

The first term represents the expected payoff from both borrowers succeeding, while the second represents the expected payoff from borrower i succeeding and borrower j failing. The third term represents the effort cost as a function of L , which will be immaterial to the discussion here since it does not vary with project risk.

The ability of the two borrowers to make a binding agreement on project choice is seen as key to mitigating the moral hazard problem. Restricting attention to symmetric choices, the pair will choose safe projects if and only if this gives them each higher expected utility, and the reverse for risky projects. The paper calls the curve along which borrowers are indifferent between safe and risky choices the "**Switch Line**". Along the Switch Line, it must be that $V_{SS} = V_{RR}$; equivalently, using equation 3,

$$p_S^2 U[Y(p_S, L) - rL] + p_S(1 - p_S)U[Y(p_S, L) - rL - qL] = p_R^2 U[Y(p_R, L) - rL] + p_R(1 - p_R)U[Y(p_R, L) - rL - qL]. \quad (4)$$

¹¹Returns are assumed independent across projects. Correlated project returns are introduced in section 5.1.5.

¹²Note that the joint liability payment is only charged when borrowers realize different outcomes. This makes this assumption potentially crucial. Since $p(1 - p)$ is a parabola maximized at $p = 1/2$, assumption A3 is equivalent to the condition that p_S be closer to $1/2$, or $|p_S - 1/2| \leq |p_R - 1/2|$.

This expression is the key to understanding repayment determinants in the Stiglitz model. If the contract terms, technology and utility parameters, and environmental assumptions are such that the left side of equation 4 is less than the right side, risky projects are chosen, and the probability of repayment is p_R . If the reverse is true, repayment occurs with probability p_S as safe projects are chosen. Thus differential effects on safe versus risky expected payoffs are the driving force behind repayment predictions here.

If equation 4 is to be satisfied, the following must be true of the output function $Y(p, L)$.

Lemma 1. *For any L that can satisfy equation 4, it must be true that $Y(p_R, L) > Y(p_S, L)$, that is, risky output is greater than safe output.*

Proof: see Appendix A. Lemma 1 holds because if projects with a lower success rate are ever to be chosen, they must deliver higher output when successful. This is important for later results.

The Stiglitz paper goes further by assuming that the expected payoff of risky projects increases faster in loaned funds than that of safe projects, at least at $q = 0$:

$$\frac{\partial\{p_S U[Y(p_S, L) - rL]\}}{\partial L} < \frac{\partial\{p_R U[Y(p_R, L) - rL]\}}{\partial L}. \quad (\text{A5})$$

This assumption guarantees that r and L are inversely related along the Switch Line (for q small enough), as pictured in figure 1. It also guarantees that if (L, r) fall to the right of this curve, the pair will choose risky projects, and otherwise safe, as we show in proposition 1 below.¹³

The main result of the Stiglitz paper is that given cooperative behavior among borrowers, increasing joint liability (q) from zero to slightly above it results in an improvement in borrower welfare, holding lender profits constant. We do not attempt a direct test of this result, but rather focus on repayment implications. These are generated from the indifference condition 4. For reasons discussed in section 2, we also discard the assumption made by Stiglitz that the lender makes no profits in equilibrium. We can then examine how differences in interest rate and loan size affect repayment.

5.1.1 Checking r , L , and q

Consider a group g , with contract specifying loan size L_g , interest rate r_g , and joint liability payment q_g . If p_g is group g 's probability of repayment, then

$$p_i(r_g, L_g, q_g) = p_R + (p_S - p_R)1\{V_{SS}(r_g, L_g, q_g) \geq V_{RR}(r_g, L_g, q_g)\}, \quad (5)$$

where $1\{\cdot\}$ represents the indicator function. This is equivalent to saying that groups left of the Switch Line in figure 1 choose safe projects, and that those right of the Switch Line choose risky projects. We now show that higher values for r_g , L_g , and q_g all lead to lower repayment rates.

¹³Because safe projects result in repayment of interest more often, and involve repayment in times of lower output (see lemma 1), a reduction in the interest rate increases the safe payoff more than the risky one. Assumption A5, however, directly ensures that an increase in loan size makes risky projects more attractive relative to safe ones. Thus on the Switch Line curve of indifference, decreases in the interest rate must be accompanied by increases in loan size.

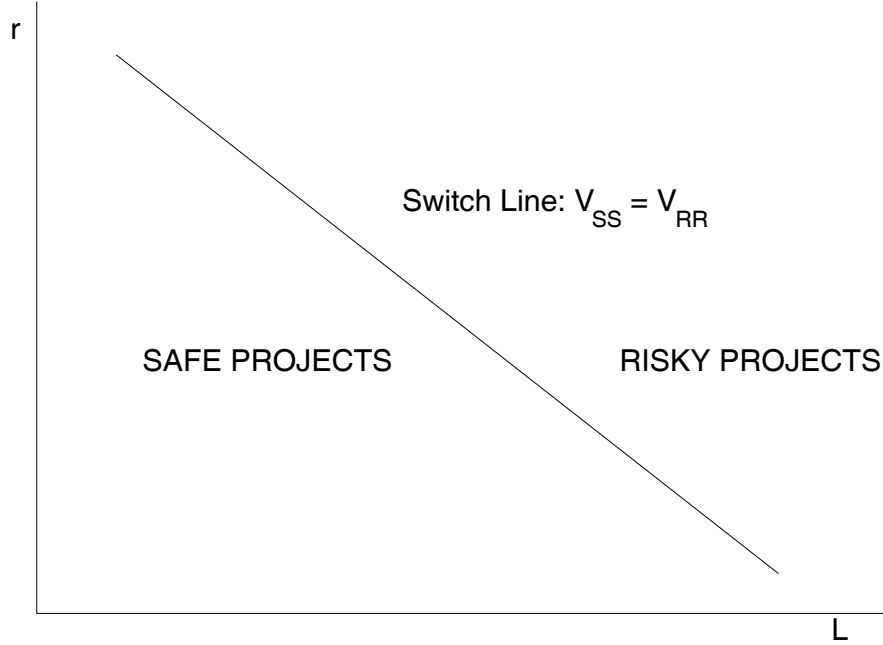


Figure 1: The Switch Line. To the left safe projects are chosen, to the right risky ones.

Proposition 1. *Under assumptions A2, A4, and A5, for q small enough, the group repayment rate is lower for groups with higher r and L .*

Proof: see Appendix A.

The intuition for proposition 1 is that increases in L make risky projects relatively more attractive due to assumption A5, and that increases in r hurt safe projects relatively more because safe projects lead to paying interest more often and in times of lower output. The result is also clear graphically from figure 1. Fixing L (r), for every r (L) below the switch line safe projects are chosen, and vice versa.

The effect of q_g can be found similarly from equation 5.

Proposition 2. *Under assumptions A2, A3, and A4, the group repayment rate is lower for groups with higher q .*

Proof: see Appendix A.

Intuitively, since the joint liability payment is paid more often (under assumption A3) and during times of lower income under safe project choice, the safe payoff is hurt more than the risky one by an increase in q . In terms of figure 1, a higher q shifts the Switch Line left in r - L space, diminishing the region in which safe projects are chosen.

To measure these variables, we use data on group loan sizes, interest rates charged, and the percent of the group which are landless, to proxy the degree of joint liability. These measures are discussed in more detail in section 6.1.

5.1.2 Checking outside options

The Stiglitz paper implicitly assumes that all groups are capital-constrained, that is, would like to borrow more at the interest rate and loan terms they are currently borrowing. In this context, outside loan opportunities at contract terms similar to those of the BAAC would simply allow some of these groups to augment their loan amount L_g . The extra capital would increase the relative attractiveness of risky projects, in some cases possibly changing the project choice of the borrowers to risky if they are pushed beyond the Switch Line. *Thus outside loan options would decrease repayment.* We measure outside borrowing opportunities by the percent of households in the village where the group is resident that borrow from village banks or commercial banks, and by a poll measuring availability of institutions. These are more fully described in section 6.1.

5.1.3 Subtracting cooperation

The BAAC data contain extensive information regarding social interaction within the group and the village. These can be used to proxy the extent to which groups are behaving cooperatively.

In a departure from the Stiglitz model, assume borrowers cannot mutually enforce technology choices. In this setting, they will both choose safe projects only if neither can gain by deviation to a risky project. Thus a Nash equilibrium where both choose safe projects exists when

$$V_{SS} \geq V_{RS}. \quad (6)$$

This ensures that each borrower weakly prefers to operate a safe project given his partner is choosing a safe one. Now V_{RS} can be written using equation 3 as

$$V_{RR} + (V_{RS} - V_{RR}) = V_{RR} + p_R(p_S - p_R)\{U[Y(p_R, L) - rL] - U[Y(p_R, L) - (r + q)L]\}. \quad (7)$$

Substituting V_{RS} from equation 7 into 6 gives

$$V_{SS} \geq V_{RR} + p_R(p_S - p_R)\{U[Y(p_R, L) - rL] - U[Y(p_R, L) - (r + q)L]\} \quad (8)$$

as necessary for both borrowers to choose safe projects. Recall that under cooperative behavior, the condition for safe project choice was $V_{SS} \geq V_{RR}$. Clearly 8 is a stronger condition, as we now show.

Proposition 3. *The group repayment rate is higher for groups acting cooperatively.*

Proof: see Appendix A.

One can draw a new Switch Line for non-cooperative groups, shifted down as compared to the cooperative Switch Line. This shift accounts for the stricter condition needed for safe projects to be chosen in a non-cooperative game.¹⁴ This line is pictured in figure 2.

¹⁴It turns out there is a segment of r - L space where the Nash equilibrium is not unique, that is, both risky and safe project choices are non-cooperative equilibria. Analogous to the argument in the text, one

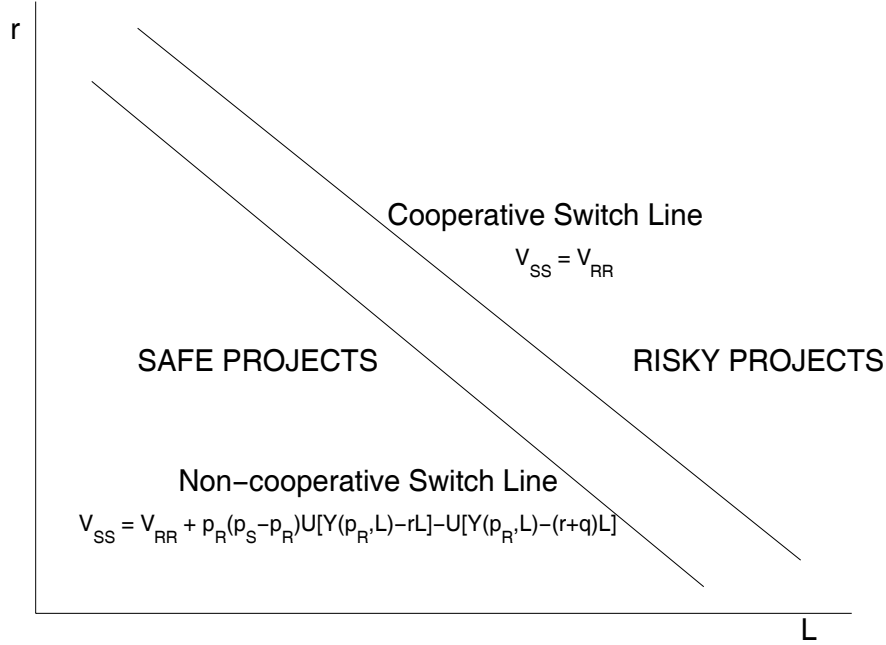


Figure 2: Cooperative and non-cooperative Switch Lines. To the left safe projects are chosen, to the right risky ones.

5.1.4 Adding borrower productivity

We next examine differences in borrower productivity, writing output as a function of an additional factor, H : $Y(p, L, H)$. Empirically, H is proxied by group average education levels and by group average landholdings. It seems plausible that both higher human capital and more land would increase the marginal return of a given amount of capital loaned from the BAAC.

Here we use an assumption on output separability:

$$Y(p, L, H) = z(p)F(L, H), \quad (\text{A6})$$

can show that for risky project choices to be a non-cooperative equilibrium, we must have

$$V_{SS} \leq V_{RR} + p_S(p_S - p_R)\{U[Y(p_S, L) - rL] - U[Y(p_S, L) - (r + q)L]\}. \quad (9)$$

Condition 9 is very similar to condition 8. They differ in the long second term, when p_S replaces p_R . It turns out this second term is bigger in 9 than in 8. This is because

$$U[Y(p_S, L) - rL] - U[Y(p_S, L) - (r + q)L] > U[Y(p_R, L) - rL] - U[Y(p_R, L) - (r + q)L]$$

due to the concavity of U and the result that $Y(p_R, L) > Y(p_S, L)$. (Actually this result comes from lemma 1 modified so that it applies to L that satisfy not equation 4 but rather condition 9 at equality. The proof would be nearly identical.) Therefore, there are (r, L, q) combinations that satisfy both 8 and 9, where both safe projects and risky projects are non-cooperative equilibria. Whatever groups are assumed to do in the multiple-equilibria space, it is still true that more of r - L space leads to safe project choice for a cooperative group than for a non-cooperative group.

where F is assumed strictly increasing in both arguments. With this assumption, we can show the following.

Proposition 4. *Under assumptions A1, A4, and A6, for q small enough, the group repayment rate is higher for more productive groups (groups with higher H).*

Proof: see Appendix A.

This result exists in part because safe projects have higher expected returns (assumption A1), and thus due to separability (assumption A6), higher expected marginal product in H . In addition, the safe project payoff is augmented in times of lower output than the risky project payoff. Relative to figure 1, groups with higher H have higher Switch Lines, and thus more of the contract space leads to safe project choice.

5.1.5 Adding correlation

A final modification to the model introduces correlation in borrower output realizations.¹⁵ Let the joint probability distribution for the returns of a borrowing group choosing projects that succeed with probability p_i and p_j be

	j Succeeds	j Fails
i Succeeds	$p_i p_j + \epsilon$	$p_i(1 - p_j) - \epsilon$
i Fails	$p_j(1 - p_i) - \epsilon$	$(1 - p_i)(1 - p_j) + \epsilon$

Here ϵ can be any number provided it is not so large or small that it causes any of the cells to exceed one or fall below zero. Note that $\epsilon = 0$ is the zero-correlation case assumed thus far. It is easily verified that $\epsilon > 0$ implies positive correlation and $\epsilon < 0$ negative correlation. Further, one can show that any joint distribution of output that preserves p_i and p_j , respectively, as the individual probabilities of success, must take this form.¹⁶

However, for each (p_i, p_j) pair there may be a different ϵ . The Stiglitz model allows for two such values, $\epsilon(p_R, p_R)$ and $\epsilon(p_S, p_S)$.¹⁷ These two values could differ, indicating different types of covariance across safe and risky projects. In this section, however, we use natural assumptions to tie $\epsilon(p_R, p_R)$ and $\epsilon(p_S, p_S)$ to a single underlying parameter that reflects the degree of overall correlation. The first variation is the simplest:

$$\epsilon(p, p') = \epsilon, \quad \forall p, p'. \tag{A7}$$

In this case, the same probability mass is added to the events where both succeed and fail, whether the projects are safe or risky. Note that $\epsilon \leq \min\{p_R(1 - p_R), p_S(1 - p_S)\}$ must hold for all cells in the distribution matrix to stay within $[0, 1]$. The second variation is the assumption that

$$\epsilon(p, p') = \epsilon * \min\{p'(1 - p), p(1 - p')\}. \tag{A8}$$

¹⁵A similar modification to a model of strategic default is analyzed by Armendariz de Aghion (1999).

¹⁶This follows from the fact that the entries in row one (column one) must add to p_i (p_j), and the entries in row two (column two) must add to $1 - p_i$ ($1 - p_j$).

¹⁷Since $p_i, p_j \in \{p_R, p_S\}$ and Stiglitz restricts attention to symmetric project choices, these are the only two combinations.

Assumption A8 ensures that both the safe and risky joint project distributions have the same *correlation coefficient*, equal to ϵ , as can easily be verified.¹⁸

Denote the two values for $\epsilon(p_S, p_S)$ and $\epsilon(p_R, p_R)$ ϵ_S and ϵ_R , respectively. Payoffs (gross of the cost of effort) can be written incorporating ϵ_R and ϵ_S in the following way:

$$V_{kk} = (p_k^2 + \epsilon_k)U[Y(p_k, L) - rL] + [p_k(1 - p_k) - \epsilon_k]U[Y(p_k, L) - (r + q)L], \quad (10)$$

for $k \in \{R, S\}$. A group g will now choose projects and probability of repayment according to:

$$p_g(r_g, L_g, q_g, \epsilon_{g,S}, \epsilon_{g,R}) = p_R + (p_S - p_R)1\{V_{SS}(r_g, L_g, q_g, \epsilon_{g,S}) \geq V_{RR}(r_g, L_g, q_g, \epsilon_{g,R})\}. \quad (11)$$

Next we show that under assumption A7, and under assumption A8 with one additional assumption, an increase in ϵ increases probability of repayment.

Proposition 5. *Under assumption A4 and the type of covariance expressed in assumption A7, or under assumptions A3, A4, and the type of covariance expressed in assumption A8, the group repayment rate is higher for groups with higher project return correlation (higher ϵ).*

Proof: see Appendix A.

Proposition 5 implies that groups with higher correlation will have Switch Lines shifted up, so that more of the contract space leads to safe projects. This somewhat surprising result contrasts with the assumptions of most of the empirical literature that addresses correlation, which assumes it is bad for repayment and finds supporting evidence in the data. However, this model makes clear that correlation can tilt payoffs in favor of safe projects.

In the empirical section, we measure covariance by coincidence of economically bad, or good, years across villagers, and by occupational homogeneity within the group, as discussed in more detail in section 6.1.

5.2 Moral Hazard: Banerjee, Besley, Guinnane 1994

BBG also focus on the moral hazard problem engendered by lending to borrowers lacking collateral under a limited liability constraint. As in Stiglitz, the borrowers' key choice is whether to engage in risky or safe projects.

BBG tackle a larger question than we are concerned with in this paper. They examine cooperatives that can internally lend some of their own funds as well as borrow from an outside source. We modify their model by shutting down internal lending options exogenously (as does the majority of the micro-credit literature) and only consider capital as coming from an outside source.¹⁹ In this section, we present the details of the model crucial to understanding the determinants of repayment behavior.

¹⁸Both assumptions are made in more general terms than necessary here, since the Stiglitz model narrows attention from all possible (p, p') combinations down to two, (p_S, p_S) and (p_R, p_R) . However, the generality is so that the same assumptions can be made in the context of the Ghatak model, which allows for a wide range of (p, p') combinations.

¹⁹However, the departure from their paper is probably negligible, since one of their propositions shows that groups will not use internal funds if the outside lender has a lower opportunity cost of funds than do group members. This is quite possible in the rural data we examine: various distortions have likely prevented institutional lenders from taking full advantage of the investment opportunities there.

We show that the model contains the following basic predictions²⁰ involving group repayment rates, under certain assumptions.

- The repayment rate is decreasing in the interest rate r .
- The repayment rate is decreasing in the loan size L .
- The repayment rate is increasing in the joint liability payment q .
- Non-cooperative behavior increases repayment rates relative to cooperative behavior, by introducing the use of cheap penalties to enforce safe project choices.
- The repayment rate is decreasing in the cost of monitoring.
- Higher borrower productivity increases repayment rates.

BBG consider groups of two risk-neutral agents, in which only one borrows and the other monitors. This asymmetry captures the idea that at any given time, only some members of a typical group will want loans, while all members remain liable for these loans.

The setup is as follows: the monitor first chooses a level of monitoring effort, then the borrower makes his project choice based on the penalty that may be imposed on him. The greater is the monitoring effort, the greater the penalty that can be imposed. Let c denote the disutility of the penalty in monetary terms. Then monitoring that enables a penalty of c to be inflicted on the borrower costs $M(c)$ to the monitor. We assume

M is strictly increasing, strictly convex, and twice continuously differentiable. (A9)

We turn first to the borrower's choice: a probability of success $p \in [\underline{p}, 1]$, where $\underline{p} > 0$ is a parameter of the model. The borrower's payoff (gross of any penalties) is $p[Y(p) - r]$, where r is the gross interest he will pay and $Y(p)$ is successful output as a function of p .²¹ Let $E(p) \equiv pY(p)$ be the expected output of project p ; then the borrower's payoff can be written $E(p) - pr$ (again, gross of any penalties). It is assumed that

$$0 < E'(p) < r \text{ and } E''(p) \leq 0. \quad (\text{A10})$$

Assumption A10 guarantees²² that safer projects have higher expected output. The socially optimal choice is thus the safest project, $p = 1$, since this maximizes expected surplus. However, since $E'(p) < r$, the borrower's payoff $E(p) - pr$ is decreasing in p . So $p = \underline{p}$ is privately optimal.

If the monitor can impose penalty c , say, then he can implement project choice p if and only if the following incentive constraint is satisfied:

$$E(p) - pr \geq E(\underline{p}) - \underline{p}r - c. \quad (12)$$

²⁰Note that all these predictions are true *ceteris paribus*.

²¹In BBG, loan size is fixed; L is therefore dropped from the notation in this section. We allow loan size to vary in section 5.2.4.

²²The actual assumption made by BBG corresponding to A10 is that $0 < E'(p) < \rho$, where ρ is the lender's cost of funds and is connected to the interest rate through a zero-profit condition. For our derivations, however, we discard the zero-profit condition, as discussed in section 2. Therefore, we make the assumption directly on the interest rate r .

The left side is the payoff at p , while the right side is the deviation payoff, which includes setting $p = \underline{p}$ and incurring penalty c . Since it is costly for the monitor to threaten penalties, constraint 12 will hold at equality in equilibrium if the monitor implements p . It thus defines a function $c(p, r)$ which gives the penalty c needed to enforce a given probability of success:

$$c(p, r) = E(\underline{p}) - E(p) + r(p - \underline{p}). \quad (13)$$

Note that $c(\underline{p}, r) = 0$ and

$$c_p(p, r) = r - E'(p), \quad (14)$$

which is strictly positive by assumption A10. Thus the monitor must threaten a higher penalty to enforce a safer project choice.

The monitor's problem is to choose which project risk level p to enforce so as to maximize

$$(1 - p)(-q) - M[c(p, r)]. \quad (15)$$

This payoff includes the joint liability payment q paid with probability $(1 - p)$ (when the borrower fails), and monitoring costs of implementing p .²³ Maximization of payoff 15 yields a first-order condition that is key to the repayment predictions of the BBG model:

$$q = M'[c(p, r)]c_p(p, r). \quad (16)$$

We will call this the **Monitoring Equation**. Here q represents the return to monitoring, since this is the payment that can be saved if the borrower succeeds. The right-hand side represents the costs of monitoring. These are affected both by how costly it is to increase penalties, $M'(c)$, and by how acute the moral hazard problem is, that is, how sharply penalties must increase to induce the borrower to choose a safer project, $c_p(p, r)$. One can check that the second-order condition is (strictly) satisfied because $M(\cdot)$ is strictly increasing and convex, by assumption A9; and because $E''(p) \leq 0$, by assumption A10, which ensures that $c_{pp}(p, r) \geq 0$.

Equation 16 is graphed in figure 3. The left side is a horizontal line at q . The right side is a strictly increasing (for the same reasons that guarantee the second-order condition), not necessarily linear function that crosses q exactly once. This intersection gives the unique equilibrium p .²⁴ Thus this model delivers a probability of repayment that varies with the parameters and variables according to equation 16.²⁵

5.2.1 Checking q

From the first-order condition 16, it is clear that groups with higher degrees of joint liability q exhibit higher probabilities of repayment p . This is represented in figure 3 by an upward shift of the horizontal line.

²³The implicit assumption is that the monitor has wealth enough to make the liability payment in any state of the world.

²⁴In this section, we will assume an interior solution in all derivations.

²⁵BBG is unique (at least among the models we consider) in relating underlying variables not only to repayment rates, but also to the amount of monitoring taking place within the group. That is, in addition to the probability of repayment p , an equilibrium monitoring level c emerges from r , q , and other variables and parameters. In principle, implications for monitoring and default could be tested jointly. Tests and theory involving these predictions can be found in an earlier working paper draft.

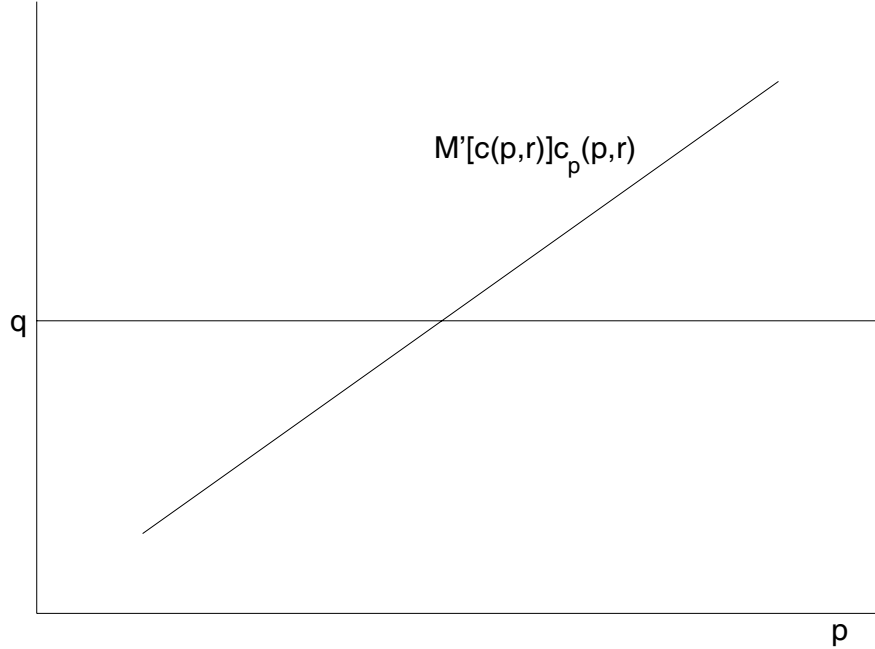


Figure 3: Determination of p .

Proposition 6. *Under assumptions A9 and A10, the group repayment rate p is higher for groups with higher joint liability payment q .*

Proof: see Appendix B.

Interestingly, this prediction is opposite to that of the Stiglitz model (see proposition 2). BBG emphasize the fact that greater liability engenders more intense group pressure to perform well. Stiglitz, on the other hand, accounts for the fact that the joint liability payment is like an additional tax on success, since it is paid only when the borrower who pays it is successful. In BBG, however, the joint liability payment creates no adverse incentives since the monitor always is able to make the payment.

5.2.2 Checking Cost of Monitoring

Cost of monitoring affects repayment probabilities exactly as q does, at least when we consider proportional shifts. Assume that the monitor of group g faces cost of monitoring

$$M_g(c) \equiv \kappa_g m(c), \quad \kappa_g > 0. \tag{A11}$$

Groups with higher values for κ_g will repay less often, as we now show.

Proposition 7. *Under assumptions A9, A10, and A11, the group repayment rate is higher for groups with lower cost of monitoring κ .*

Proof: see Appendix B.

Empirically, costs of monitoring are proxied by variables measuring percent of the group living in the village, homogeneity of occupations in the group, and the percent of relatives in the group, as discussed in section 6.1.

5.2.3 Checking r

Higher interest rates lead unambiguously to lower repayment rates, shifting up the sloped curve in Figure 3. This is because a higher interest rate raises the monitor's marginal cost of increasing p , by tilting borrower incentives further toward risky projects.

Proposition 8. *Under assumptions A9 and A10, the group repayment rate is lower the higher is interest rate r .*

Proof: see Appendix B.

5.2.4 Adding L

In this section we introduce as a factor of production loaned capital L and find that as in the Stiglitz model, a higher loan leads to a lower repayment rate. We again rely on assumption A6 of section 5.1.4, which allows us to break Y multiplicatively into components relating to probability p and loan size L , modified here to suppress the argument H :

$$Y(p, L) = Y(p, 1)F(L),$$

where $F(1)$ is normalized to 1.²⁶ We continue to let $E(p) \equiv pY(p, 1)$, so that expected output satisfies

$$pY(p, L) = pY(p, 1)F(L) = E(p)F(L). \quad (17)$$

We make standard assumptions about $F(L)$:

$$F \text{ is strictly increasing and concave, twice continuously differentiable, and } F(0) = 0. \quad (\text{A12})$$

The amount due by the borrower upon success is now rL , while the joint liability payment due from the monitor when the borrower fails is qL . Thus the borrower's expected payoff becomes, modifying equation 17,

$$E(p)F(L) - prL \quad (18)$$

and the monitor's payoff becomes

$$(1 - p)(-qL) - M[c(p, r, L)]. \quad (19)$$

The analogue to assumption A10 is now

$$0 < E'(p)F(L) < rL \text{ and } E''(p) \leq 0. \quad (\text{A13})$$

²⁶Note that $Y(p, 1)$ is equivalent to the $Y(p)$ we use in earlier sections, in which the second argument was suppressed since loan size L was assumed fixed at 1.

This assumption ensures that $p = 1$ is socially optimal but $p = \underline{p}$ is privately optimal, since the borrower's expected payoff is decreasing in p .²⁷ The monitor's first-order condition, the analogue to Monitoring Equation 16, becomes

$$qL = M'[c(p, r, L)]c_p(p, r, L). \quad (20)$$

We are now able to show that an increase in loan size decreases the repayment rate.

Proposition 9. *Under assumptions A9, A12, and A13, the group repayment rate is lower the higher is the loan size L .*

Proof: see Appendix B.

Intuitively, a higher loan size has two opposite effects. It increases the monitor's liability and thus his payoff to monitoring. It also increases the expected interest cost to the borrower more than his expected output, giving him greater incentive for risky project choice. The latter effect dominates, leading to lower probability of success.

5.2.5 Adding Borrower Productivity

Effects of borrower productivity can be analyzed in a manner similar to the preceding section. We introduce an additional factor of production called H , but consider loan size fixed as in previous sections. Again we use assumption A6 of section 5.1.4 to write, suppressing the argument L ,

$$Y(p, H) = Y(p, 1)G(H),$$

where $G(1)$ is normalized to 1. We continue to let $E(p) \equiv pY(p, 1)$, so that expected output satisfies

$$pY(p, H) = pY(p, 1)G(H) = E(p)G(H).$$

We make weaker assumptions on $G(H)$ than were made on $F(L)$ in section 5.2.4:

$$G \text{ is strictly increasing and differentiable.} \quad (A14)$$

The borrower's expected payoff becomes, modifying equation 17, $E(p)G(H) - pr$; the monitor's payoff is unchanged from equation 15.

The analogue to assumption A10 is now

$$0 < E'(p)G(H) < r \text{ and } E''(p) \leq 0. \quad (A15)$$

This ensures that $p = 1$ is socially optimal but $p = \underline{p}$ is privately optimal.²⁸ The monitor's first-order condition becomes

$$q = M'[c(p, r, H)]c_p(p, r, H). \quad (21)$$

We now show that the repayment rate is increasing in borrower productivity.

²⁷Note that this inequality is a condition on L and $F(L)$ as well as on $E(p)$ and r . For example, it will not hold for low L under Inada-like assumptions on $F(L)$. Thus the moral hazard problem would come into play only for high enough loan sizes.

²⁸Clearly this inequality will not hold for high enough H as long as $G(H)$ is not bounded. Thus the moral hazard problem disappears for productive enough groups.

Proposition 10. *Under assumptions A9, A14, and A15, the group repayment rate is higher the higher is borrower productivity H .*

Proof: see Appendix B.

Intuitively, higher productivity increases the payoff of safe projects relative to risky projects, without changing the interest cost of any project. The moral hazard problem is softened, making monitoring more effective and leading to a higher probability of repayment.

5.2.6 Adding cooperation

The model can also be modified in a direction that involves cooperative rather than adversarial behavior. We find, paradoxically, that groups acting cooperatively are likely to repay less often. An interesting implication of this result is that lenders would prefer to lend to groups behaving non-cooperatively. If they can distinguish between groups acting cooperatively and non-cooperatively, it may well be that the ability of a group to side-contract will be the very thing that keeps it from obtaining credit. This type of social capital can thus have perverse consequences.

In the baseline model, a monitor expends resources to enforce a project choice by the borrower. In this section we assume the monitor and borrower can enforce any joint agreement on project choice *costlessly*, as in Stiglitz. This implies they will simply maximize joint surplus.²⁹ The problem becomes to choose p in order to maximize

$$p[Y(p) - r] - (1 - p)q,$$

the sum of net payoffs of the borrower and monitor. The first order condition with respect to p is³⁰ $E'(p) - (r - q) = 0$.³¹ In this case also, assumption A10 that $E''(p) \leq 0$ guarantees the second order condition is met.

To facilitate comparison with equation 16 of the non-cooperative case, the first order condition can be rewritten

$$q = r - E'(p), \tag{22}$$

graphed in Figure 4. We also reproduce Monitoring Equation 16 here, with $c_p(p, r)$ substituted in from equation 14:

$$q = M'[c(p, r)][r - E'(p)]. \tag{23}$$

The comparison evidently hinges on whether $M'(c)$ is greater than or less than one. If it is less than one, the upward sloping curve in Figure 3 is lower than the one in Figure 4, and the resulting p is higher under non-cooperation. If it is greater than one, p is higher under cooperation.

²⁹This is due to utility being transferable. The maximized surplus can then be redistributed among the group members to achieve the desired distribution.

³⁰Recall that $E(p) \equiv pY(p)$.

³¹Here $(r - q)$ is a measure of the degree of *group* moral hazard: it is the amount paid by the group only when the borrower is successful, while q is paid by the group regardless. To see this, note that the group pays $r = q + (r - q)$ when the borrower succeeds and q when he does not.

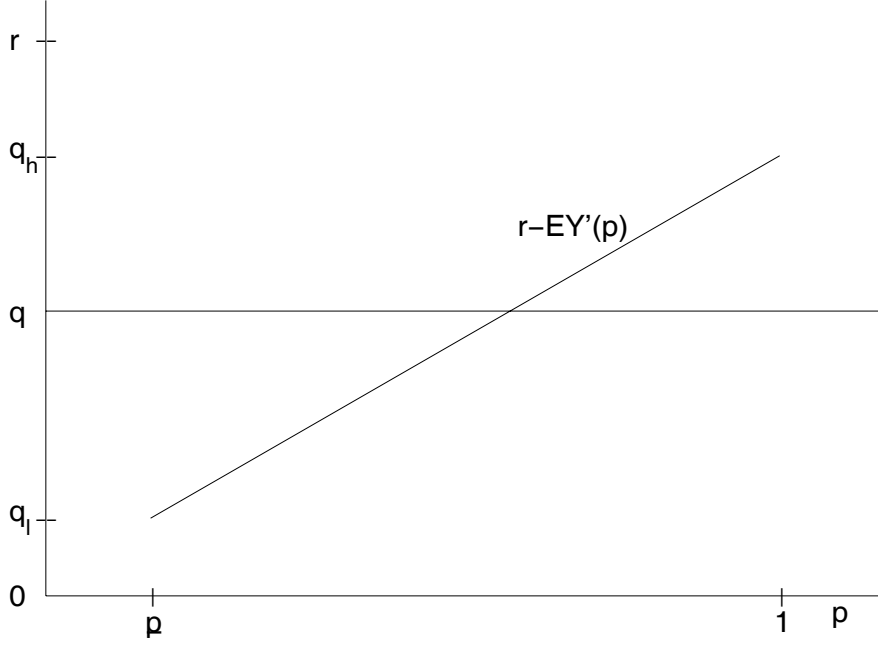


Figure 4: Determination of p under costless cooperation.

Proposition 11. *Under assumption A10, the group repayment rate is lower for groups that can cooperate and enforce side-contracts if $M'(c) < 1$ and higher for groups that can enforce side-contracts if $M'(c) > 1$.*

Proof: see Appendix B.

Intuitively, the cooperative setup is isomorphic to the non-cooperative case where $M(c) = c$ and thus $M'(c) = 1$. Here the monitor can apply penalties that affect the borrower's payoff at a one-for-one cost to the monitor's own payoff. This results in maximization of joint surplus, just as in the cooperative case.³² If the marginal cost of penalizing is greater than one, that is $M'(c) > 1$, then the monitor monitors less than in the surplus-maximizing case, and non-cooperation results in a lower repayment rate. If $M'(c) < 1$, imposing penalties is cheaper (on the margin) and the monitor will enforce a higher repayment rate under non-cooperation.

There is good reason to think that in general $M'(c) \leq 1$. This would effectively be true if the monitor had at his disposal not only penalties, but also the ability to commit to pay a bonus conditional on project choice.³³ Given these two options, he would use penalties up to the point where their marginal cost reached one. Beyond that, any further incentives would

³²A natural interpretation of the non-cooperative case with $M(c) = c$ is that the monitor can commit to paying a bonus to the borrower contingent on his project choice. This is a reward rather than the penalty discussed in the text, but the effect on the borrower's incentive constraint is the same, as is the cost to the monitor if $M(c) = c$. In particular, the monitor's payoff is $(1 - p)(-q) - c$ and the borrower's incentive constraint is $E(p) - pr + c \geq E(p) - pr$.

³³This is not assumed in the model, but is described in the preceding footnote.

be through bonus payments,³⁴ since this becomes the cheaper option. In this scenario, $M'(c)$ is bounded above at one. Cooperatively behaving groups would then have probabilities of repayment the same as or lower than, but never higher than, those of non-cooperative groups.

This prediction is counter that of the Stiglitz model. In that model, cooperation improves repayment by enabling the group to circumvent free-riding of one member on his partner's safe behavior. In this model, non-cooperation brings the monitor to use cheap penalties to enforce a higher probability of repayment than is optimal from the group's perspective. The common idea that social capital leads to better-behaving groups may thus be turned on its head. Cooperative groups will limit their efforts to repay, while non-cooperative groups may observe some group members, who prefer safe behavior, exerting overly harsh penalties on other members who prefer more risky behavior. These stricter penalties will lead to better repayment records.

5.3 Strategic Default: Besley, Coate 1995

In Besley and Coate 1995 (call it BC), project choice is fixed and homogeneous across borrowers, but the lender cannot fully enforce repayment. Borrowers decide whether or not to repay after realizing project returns, by comparing the repayment amount with the severity of penalties imposed by the lender, and perhaps the community. BC show that joint liability can increase repayment rates relative to individual liability, but may not in the absence of sufficiently strong social penalties. We take joint liability as given and examine how repayment rates respond to various determinants in the model.

In this section we show that the model contains the following basic predictions³⁵ involving group repayment rates, under certain assumptions.

- The repayment rate is decreasing in the interest rate r .
- The repayment rate is increasing in the severity of official and unofficial penalties.
- Non-cooperative behavior increases repayment rates relative to cooperative behavior, by introducing the use of sub-optimally severe penalties to enforce repayment.
- The repayment rate is decreasing in the amount of correlation between borrower output.
- Higher borrower productivity increases repayment rates.

Groups consist of two borrowers. Output is distributed for each independently according to $F(Y)$, which is assumed strictly increasing over its support $[0, Y_{max}]$. Each borrower takes out a loan and owes a gross interest payment of r .³⁶ Borrower returns are realized and then repayment decisions are made. In particular, the borrowers decide simultaneously whether or not to repay in a first stage. If the decision is not unanimous, the borrower who decided

³⁴This relies on there being no fixed costs of imposing penalties or making bonus payments. The former is true since M is convex under assumption A9.

³⁵Note that all these predictions are true *ceteris paribus*.

³⁶Loan size does not vary in this model and is normalized to one.

in the first stage to repay can revise his decision in a second stage. He can then choose to repay for himself and his partner, or not to repay at all.

Joint liability implies that if the lender does not receive the full amount $2r$ from the group, he imposes a penalty of $c^o(Y_i)$ on borrower i , for $i = 1, 2$. Here $c^o(Y_i)$ is the *official* penalty measured in terms of the single good. It is assumed that

$$\begin{aligned} c^o(Y_i) \geq 0 \text{ is continuous, strictly increasing, and unbounded,} \\ \text{and } c^o(Y_i) < Y_i \text{ when } Y_i > 0. \end{aligned} \tag{A16}$$

In other words, the lender penalizes more severely when output is higher, but never as severely as outright confiscation.

There are also unofficial penalties, imposed by the jointly liable borrower and the community on a borrower who in the first stage decides to default when his partner decides to repay. In particular, an *unofficial penalty* function $c^u(Y_i, \Lambda_j)$ gives the additional penalty (in terms of the single good) on delinquent borrower i , as a function of his output realization Y_i and Λ_j .

$$\Lambda_j \equiv \min\{c^o(Y_j) - r, r\} \tag{24}$$

is the loss to borrower j resulting from i 's truancy, derived as follows. If j ultimately decides to bail out his partner, he loses r relative to the case where i repaid; if he decides not to repay at all, he saves his own debt of r but loses $c^o(Y_j)$. His loss is just the minimum of these two, since at the second stage of the game he will minimize his loss.

It is assumed that the unofficial penalty function

$$c^u(Y_i, \Lambda_j) \text{ is continuous and strictly increasing in } Y_i, \text{ and } \Lambda_j, \text{ for } \Lambda_j, Y_i \geq 0. \tag{A17}$$

Thus the delinquent borrower is punished more the more he hurt the other borrower and the more output he realized. Further,

$$\text{If } \Lambda_j \leq 0 \text{ or } Y_i = 0, \text{ then } c^u(Y_i, \Lambda_j) = 0. \tag{A18}$$

Penalties are not imposed if the output realization was minimal or if the jointly liable borrower was not hurt by his partner's default. The repayment game is pictured in Figure 5.

To characterize the equilibria, it is helpful to define some points of indifference between repayment and default. Using assumption A16 and ignoring unofficial penalties for the moment, we can be sure there will be a critical value for output, above which repaying some amount X is less costly than incurring official penalties, and below which it is less costly to default. One can define a function $\underline{Y}(X) \equiv (c^o)^{-1}(X)$ which gives this critical minimum output realization for which the borrower prefers to repay X instead of incurring the official penalty. Note that $\underline{Y}(X)$ is increasing in X : an indifferent borrower would be willing to repay a higher amount only when he would otherwise face greater official penalties, which come with higher output.

A similar function $\hat{Y}(X, Y_j)$ can be defined that gives the minimum output level for borrower i to be willing to repay X , given his partner realized output Y_j . It differs from $\underline{Y}(X)$ in that it accounts for both official and unofficial penalties. Its dependence on Y_j is

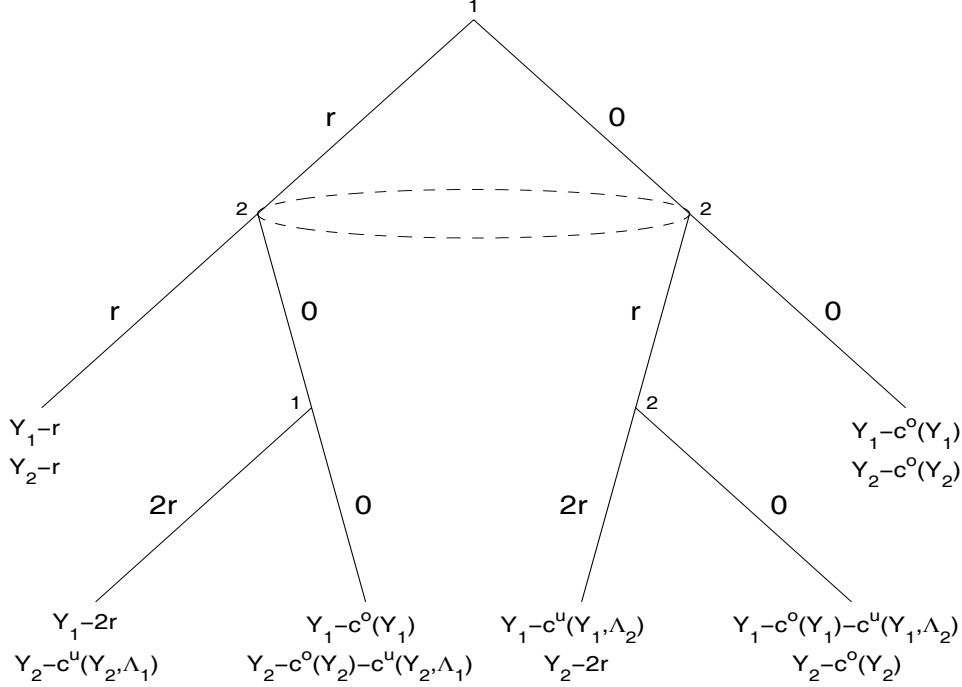


Figure 5: **Repayment game.** Borrowers simultaneously choose in the first stage whether to repay 0 or r . If the choice is not unanimous, the borrower who had decided to repay chooses in the second stage whether to repay 0 or $2r$.

due to the fact that unofficial penalties depend on the loss Λ_j to the other borrower, which in turn depends on Y_j . $\hat{\underline{Y}}(r, Y_j)$ is defined to satisfy

$$r = c^o(\hat{\underline{Y}}) + c^u[\hat{\underline{Y}}, \Lambda(r, Y_j)]. \quad (25)$$

At $\hat{\underline{Y}}$, it is equally costly to pay r and to suffer official and unofficial penalties; above $\hat{\underline{Y}}$, both official and unofficial penalties increase, making it strictly better to pay r .

The subgame perfect equilibria break down into the following cases.

- If at least one of the two borrowers realizes output $Y_i \geq \underline{Y}(2r)$, repayment will take place. This borrower will ensure that repayment happens, even paying for the whole group in the second stage if necessary. He does this because the official penalty he would incur if the group defaults is greater than $2r$.
- If both borrowers realize output $Y_i \in [\underline{Y}(r), \underline{Y}(2r))$, $i = 1, 2$, then repayment may or may not happen. If one is not repaying, the other will not either, refusing to bail the group out since his output realization warrants penalties less than $2r$: $Y_i < \underline{Y}(2r)$. If one is repaying r , the other will also pay r . Choosing not to would bring both kinds of penalties (since his partner has not been successful enough to want to bail him out), and the official penalty alone is greater than r : $Y_i \geq \underline{Y}(r)$. Thus there are two possible equilibria here.

- If both borrowers realize output $Y_i < \underline{Y}(r)$, $i = 1, 2$, repayment will not happen. Default is the best decision at every node. This is because official penalties are less than r ; and there are no unofficial penalties, by assumption A18, since the loss Λ_j is negative (see equation 24).
- The final case is where one realizes output $Y_j \in [\underline{Y}(r), \underline{Y}(2r))$ and the other $Y_i < \underline{Y}(r)$. In this case, group default remains an equilibrium, for the same reasoning as in the second case. But in a certain subcase, repayment is also an equilibrium, made possible by unofficial penalties. From assumption A18 we know that positive unofficial penalties are imposed if unanimity is not reached in the first stage, since $\Lambda_j = c^o(Y_j) - r > 0$.³⁷ Thus if $Y_i \in [\hat{Y}(r, Y_j), \underline{Y}(r))$, borrower i prefers repaying to suffering both kinds of penalties, and thus both borrowers repaying in the first stage is an equilibrium. Of course for $Y_i < \hat{Y}(r, Y_j)$, group default remains the only equilibrium as borrower i prefers not to repay even when faced with both kinds of penalties.

We assume along with BC that

$$\text{Repayment occurs when both de fault and repayment equilibria exist.} \quad (\text{A19})$$

Summarizing the four cases above under this assumption, the unique subgame perfect equilibrium results in default only when either both borrowers realize $Y_i < \underline{Y}(r)$, $i = 1, 2$, or one realizes output $Y_j \in [\underline{Y}(r), \underline{Y}(2r))$, $j = 1, 2$, and the other $Y_i < \hat{Y}(r, Y_j)$, $i \neq j$. Repayment occurs otherwise. The repayment rate p then equals

$$p = 1 - [F(\underline{Y}(r))]^2 - 2 \int_{\underline{Y}(r)}^{\underline{Y}(2r)} F(\hat{Y}(r, Y)) dF(Y). \quad (26)$$

The second term captures the case where both borrowers realize output below $\underline{Y}(r)$. The third term captures the case where one borrower realizes $Y \in [\underline{Y}(r), \underline{Y}(2r))$ and the other below $\hat{Y}(r, Y)$; either borrower can end up with less, hence the term is multiplied by two.

The set of joint output realizations leading to default is pictured in Figure 6, in which $a \equiv \underline{Y}(r)$, $b \equiv \underline{Y}(2r)$, and Y_{max} is normalized to one. The case where $\underline{Y}(2r) < Y_{max}$ is pictured.³⁸ Repayment happens everywhere in the unit square except in box A and in some parts of the AB boxes. In particular, there is a curve running through the AB boxes and the point (a, a) , symmetric about the 45-degree line, below which repayment does not happen. (Two examples are pictured in Figure 6, dashed and dotted.) This curve represents $\hat{Y}(r, Y)$, and thus its placement depends on unofficial penalties; the stronger they are, the lower it is.³⁹

³⁷We ignore the measure-zero event of $Y_i = 0$.

³⁸This is with little loss of generality, since the probability mass assigned to $Y \in [\underline{Y}(2r), Y_{max}]$ can be negligible.

³⁹More precisely, the curve through the lower right AB box of Figure 6 represents $Y_j = \hat{Y}(Y_i)$ (suppressing the argument r). It starts at (a, a) because the loss to borrower i is zero there (at a he is by construction indifferent between paying r and suffering official penalties) so no unofficial penalties are imposed on borrower j . The cutoff thus starts at a , the amount corresponding to official penalties only. From (a, a) , $Y_j = \hat{Y}(Y_i)$ is strictly decreasing because the loss to borrower i increases as Y_i increases, raising unofficial penalties on borrower j , who is then willing to repay with lower and lower output realizations. The line $Y_j = \hat{Y}(Y_i)$ ends

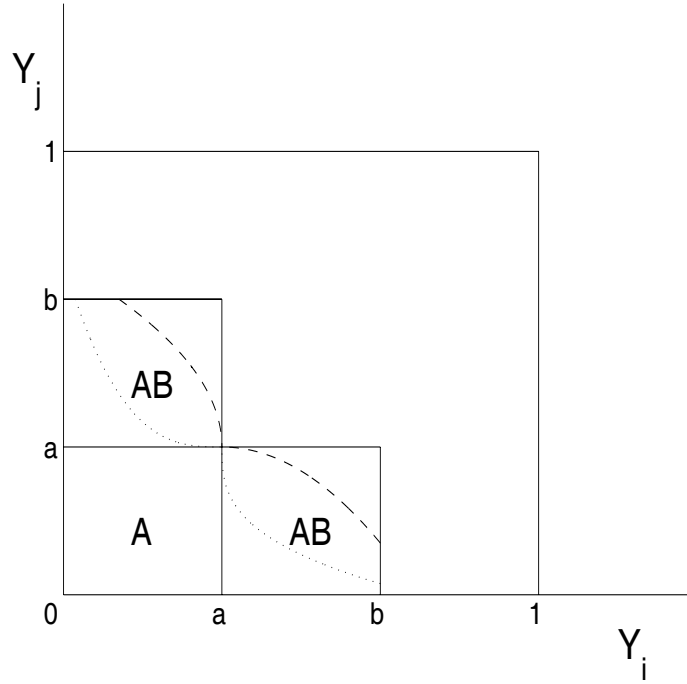


Figure 6: **Regions of non-repayment.** (Note that $a \equiv \underline{Y}(r)$, $b \equiv \underline{Y}(2r)$, and Y_{max} is normalized to one.) Repayment never occurs if joint output realizations fall in box A, or in boxes AB below the \hat{Y} (or \hat{Y}^{-1}) lines. Two examples are drawn here, dashed and dotted. More generally, the only restriction on the curve through the lower AB box, for example, is that it start at (a, a) and strictly decrease to (b, z) , where $z > 0$. The curve through the upper AB box must be its reflection about the 45-degree line, due to borrower symmetry.

5.3.1 Checking r

Equation 26 can be differentiated without difficulty to see that p decreases with r . We use a more instructive graphical argument, modifying Figure 6 to illustrate an increase in r in Figure 7. There two groups are pictured, identical except that they face interest rates r_1 and $r_2 > r_1$, respectively. Since $\underline{Y}(r)$ is increasing in r , it must be that $\underline{Y}(r_2) > \underline{Y}(r_1)$ and $\underline{Y}(2r_2) > \underline{Y}(2r_1)$. This corresponds to $a_2 > a_1$ and $b_2 > b_1$ in Figure 7. Clearly, the higher interest rate of group 2 leads to a larger region of non-repayment. The intuition is that it raises the cost of repayment without affecting (or even lowering, in the relevant range of output) the cost of default.

Proposition 12. *Under assumptions A16, A17, A18, and A19, the group repayment rate is lower for groups with higher r .*

somewhere at (b, z) , where $z > 0$. We know it ends at $Y_i = b$, since for $Y_i \geq b$, borrower i is willing to bail out borrower j , and default no longer results. We know that $z > 0$ because the cutoff $\hat{Y}(Y_i)$ can never reach zero. This is because at $Y_j = 0$, both unofficial and official penalties are zero, so repayment of $r > 0$ would never be optimal. Continuity of both penalty functions in Y_j puts the cutoff output level for borrower j strictly above zero, no matter how large Y_i may be. The line through the upper left AB box corresponds analogously to $Y_i = \hat{Y}(Y_j)$, giving the symmetry about the 45-degree line.

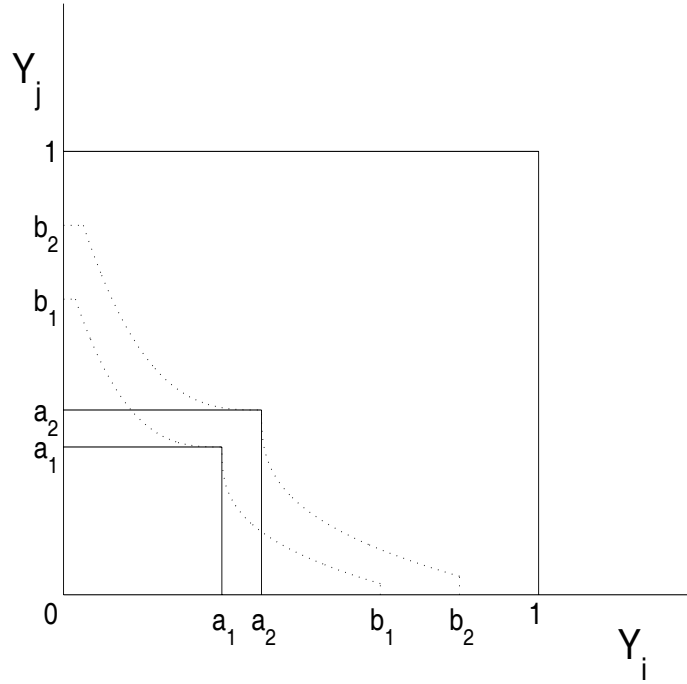


Figure 7: Comparison of group 1 and group 2. (Recall that $a \equiv \underline{Y}(r)$ and $b \equiv \underline{Y}(2r)$.)

Proof: see Appendix C.

5.3.2 Checking official and unofficial penalties

Not surprisingly, repayment improves as the general severity of either kind of penalty increases. This is because the cost of default is higher if penalties are stiffer, while the cost of repayment does not vary with penalties.

Consider first two groups identical except that they face different official penalties, $c_1^o(Y)$ and $c_2^o(Y)$, respectively, where $c_2^o(Y) < c_1^o(Y)$ when $Y > 0$. The stronger penalties of group one will lead to $\underline{Y}_1(r) < \underline{Y}_2(r)$ and $\underline{Y}_1(2r) < \underline{Y}_2(2r)$, as we show formally in the proof of proposition 13. Figure 7 illustrates this comparison as well, showing that a greater area of joint output realizations lead to default for group two.

Proposition 13. *Under assumptions A16, A17, A18, and A19, the group repayment rate is higher for groups with stronger official penalties.*

Proof: see Appendix C.

Next consider two groups, identical except in facing unofficial penalties $c_1^u(Y, \Lambda)$ and $c_2^u(Y, \Lambda) < c_1^u(Y, \Lambda)$ when $\Lambda, Y > 0$. Figure 6 illustrates this case well, with the dotted line representing group one and the dashed line group two. Both groups face the same official penalties, and so have the same values for a and b and the same A and AB squares. However, the curve through the AB squares will be lower for group one than for group two, as we show formally in the proof of proposition 13, since it faces stronger unofficial penalties.

Proposition 14. *Under assumptions A16, A17, A18, and A19, the group repayment rate is higher for groups with stronger unofficial penalties.*

Proof: see Appendix C.

Our data on official and unofficial penalties both come from the HH survey. The former comes from a poll measuring quality of institutions in the village. The latter comes from penalties for default in the village, specifically, the percentage of loans that would be met with denial of credit from all village sources, not just the current lender, upon default. These are described in more detail in section 6.1.

5.3.3 Checking borrower productivity

As in the other models, more productive groups repay more frequently. The intuition in BC is that higher output increases the cost of default, since official and unofficial penalties increase in output, but does not change the cost of repayment, r .

Consider groups 1 and 2, identical that except output is distributed according to F_1 and F_2 , respectively. Let group 2 borrowers' output distribution first-order stochastically dominate that of group 1 borrowers: $F_1(Y) \geq F_2(Y)$ for all Y . Group 2 borrowers have less chance of realizing output below any given cutoff, and are thus more productive.⁴⁰ Figure 6 is again a useful reference. Specifically, groups 1 and 2 have the exact same penalty functions and interest rates. Thus they have the same set of joint output realizations leading to default, for example the area below the dotted curves. However, the groups differ in the *probability* mass assigned to this area (not pictured in the graph). Not surprisingly, the less productive group has a higher probability of falling in this region.

Proposition 15. *Under assumptions A16, A17, A18, and A19, the group repayment rate is higher for more productive groups, where higher productivity means first-order stochastic dominance.*

Proof: see Appendix C.

The intuition of the proof is to transform the problem from output space, in which both groups have the same region of default but different probability mass assigned to this region, to percentile space, into any given subset of which both groups have the same probability of falling. However, the region of default in percentile space for group 2 is wholly contained within the region of default for group 1.

5.3.4 Adding cooperation

How do things change if the group can behave cooperatively rather than playing a non-cooperative game with unofficial penalties? As in section 5.2.6, we find that groups acting cooperatively are likely to repay less often, pointing to a potentially perverse effect of social capital.

⁴⁰First-order stochastic dominance is a perfectly general way to incorporate borrower productivity. It includes, for example, the case where output is multiplicative in an error term and a productivity term. For example, let output be written as $Y = G(H)\varepsilon$, with ε distributed according to Q , where $Q(0) = 0$ and Q is strictly increasing on its support $[0, \bar{\varepsilon}]$. Further, let the two groups be identical except that $H_2 > H_1$. Then $F_1(Y) = Q[Y/G(H_1)] \geq Q[Y/G(H_2)] = F_2(Y)$, with strict inequality for any $Y \in (0, G(H_2)\bar{\varepsilon})$.

Since utility is transferable, a group acting cooperatively will maximize the sum of their payoffs. Specifically, they will repay if and only if $Y_1 + Y_2 - 2r$ is at least as great as $Y_1 + Y_2 - c^o(Y_1) - c^o(Y_2)$, i.e.

$$c^o(Y_1) + c^o(Y_2) \geq 2r. \quad (27)$$

In words, for repayment to be optimal the sum of official penalties must be greater than the group repayment amount.

Note that if either borrower realizes output at least as great as $\underline{Y}(2r)$, condition 27 is satisfied and repayment happens. If both borrowers realize output strictly less than $\underline{Y}(r)$, condition 27 is violated and default occurs. The remaining case is where one borrower, i say, realizes Y_i in $[\underline{Y}(r), \underline{Y}(2r))$. Then if borrower j realizes $Y_j \geq \underline{Y}[2r - c^o(Y_i)]$, condition 27 is satisfied and repayment occurs;⁴¹ conversely, $Y_j < \underline{Y}[2r - c^o(Y_i)]$ leads to default. Subtracting from one the measure of the cases leading to default, the repayment rate works out to be

$$p_c = 1 - [F(\underline{Y}(r))]^2 - 2 \int_{\underline{Y}(r)}^{\underline{Y}(2r)} F[\underline{Y}(2r - c^o(Y))] dF(Y). \quad (28)$$

Note the similarity to the repayment rate without cooperation, call it p_{nc} , expressed in equation 26. Subtracting one equation from the other gives

$$p_{nc} - p_c = 2 \int_{\underline{Y}(r)}^{\underline{Y}(2r)} \{F[\underline{Y}(2r - c^o(Y))] - F[\hat{\underline{Y}}(Y)]\} dF(Y), \quad (29)$$

where the dependence of $\hat{\underline{Y}}$ on r is suppressed.

Examination of equation 29 reveals that the effect of cooperation on repayment rates depends on how severe are the unofficial penalties that cooperation renders unused. If unofficial penalties are severe, $\hat{\underline{Y}}(Y)$ is low and $p_{nc} > p_c$; and vice versa. This can be seen graphically in Figure 8. The default region for the cooperative case is below the dash-dotted line, which represents $\underline{Y}[2r - c^o(Y)]$. In general, this curve need not be linear, but does strictly decrease from $(0, b)$ to $(b, 0)$, passing through (a, a) and symmetric about the 45-degree line.⁴² The default region under non-cooperation and relatively severe (weak) unofficial penalties is represented by the dotted (dashed) curve, for example. As discussed in section 5.3, this curve is low when unofficial penalties are strong, making repayment optimal even for low output realizations.

Thus the strength of unofficial penalties determines the comparison. To make explicit the conditions under which one line is always above or below the other, we weaken assumption A18 in a benign way:

$$\text{If } \Lambda \leq 0, \text{ then } c^u(Y, \Lambda) = 0. \quad (\text{A20})$$

⁴¹In this case, the penalty for borrower i is $c^o(Y_i)$ and the penalty for borrower j is at least $c^o[\underline{Y}(2r - c^o(Y_i))] = 2r - c^o(Y_i)$. (Recall that $\underline{Y} \equiv (c^o)^{-1}$.) Thus the sum of penalties will be at least $2r$.

⁴²The graph is drawn under the assumption of linear official penalties: $c^o(Y) = \lambda_o Y$. Note that this makes $a = r/\lambda_o$ and $b = 2r/\lambda_o = 2a$. Under this assumption, the curve $\underline{Y}[2r - c^o(Y)]$ is a straight line from $(0, b)$ to $(b, 0)$, passing through (a, a) , as represented in the figure. If official penalties are not linear, the curve must still pass through $(0, b)$ and $(b, 0)$, since $\underline{Y}[2r - c^o(b)] = \underline{Y}(0) = 0$, and through (a, a) , since $\underline{Y}[2r - c^o(a)] = \underline{Y}(r) = a$.

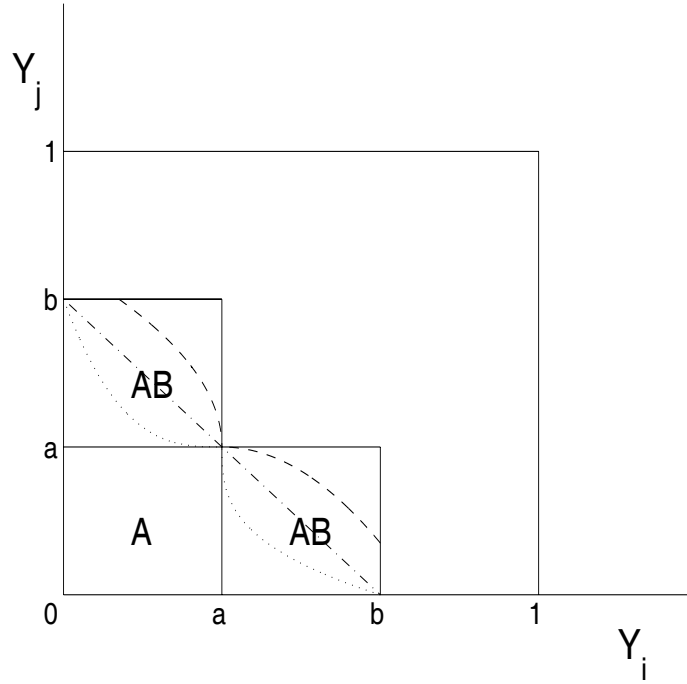


Figure 8: Default under cooperation occurs for output realizations below the dash-dotted line. Default under non-cooperation and relative severe (weak) unofficial penalties occurs below the dotted (dashed) line. (Recall that $a \equiv \underline{Y}(r)$ and $b \equiv \underline{Y}(2r)$.)

The difference between assumptions A18 and A20 is slight: in the latter, positive unofficial penalties may be imposed even on a borrower who realizes zero output.⁴³ Assumption A20 thus allows the possibility that the cutoff $\hat{Y}(Y)$ may reach zero if Y is high enough, since even at zero output, penalties may be imposed that exceed r . It thus allows us to portray the dotted line as always below the dash-dotted line in Figure 8, and in particular, as dropping all the way to $(b, 0)$.⁴⁴ Under assumption A20, there is a simple sufficient condition for cooperation to lower the repayment rate.

Proposition 16. *Under assumptions A16, A17, A19, and A20, the group repayment rate is lower for groups acting cooperatively if $c^u(Y, \Lambda) > \Lambda$ for all $Y \geq 0$ and $\Lambda > 0$ and higher for groups acting cooperatively if $c^u(Y, \Lambda) < \Lambda$ for all $Y \geq 0$ and $\Lambda > 0$.*

Proof: see Appendix C.

⁴³This may seem troubling if unofficial penalties are viewed as confiscation of output. However, confiscation does not seem to exhaust what BC have in mind; they do not impose an appropriate limited liability constraint, for example $c^o(Y) + c^u(Y, \Lambda) \leq Y$.

⁴⁴Under assumption A18, the default curve for non-cooperation could never be always below the default curve for cooperation, since the former always descends to $(b, 0)$ while the latter descends to some (b, z) for some $z > 0$ (see the discussion of Figure 6 in section 5.3).

We could obtain a similar result to the one we derive here without modifying assumption A18. Since the conditions would be substantially more complicated without adding anything substantive, we use assumption A20.

Proposition 16 rests on the fact that the cooperative setup is isomorphic to the non-cooperative case where $c^u(Y, \Lambda) = \Lambda$. Under this specification, the unofficial punishment for default fits the damage from default exactly, resulting in maximization of joint surplus.

Thus, very similar to BC (see the discussion in section 5.2.6), cooperation decreases repayment if unofficial penalties are severe. The idea here is that borrowers would like to commit ex ante not to use penalties to force repayment when borrower i 's cost of repaying is more than the borrower j 's benefit from a non-delinquent partner, and vice versa. They would like to because either borrower is ex ante equally likely to end up with the low output realization as the high one. When the borrowers cannot commit, however, the borrower who realizes higher output will use the penalties at his disposal to force repayment ex post, even when the cost to the low-output borrower is higher than what the high-output borrower stands to gain. Thus severe penalties push the repayment rate above its optimal amount.⁴⁵

5.3.5 Adding correlation

Finally, we examine how correlation between borrower project returns affects repayment. This is an easier task in the other models, where output takes on one of two values. Here, borrower returns are distributed according to some general distribution function, F . Thus there are a multitude of ways to introduce correlation. We aim for as general an approach as possible.

Recall that the support of F is $[0, Y_{max}]$; here we normalize Y_{max} to 1. Let f be the density associated with F , and assume it is continuous and strictly positive on its support. In the baseline BC model, returns are independent across the borrowers and thus the joint density function is $f(Y_i)f(Y_j)$. In this section, we are interested in constructing a generalized joint density function $\phi(Y_i, Y_j)$ that preserves the unconditional output distributions but allows for correlation. The restrictions this places on $\phi(Y_i, Y_j)$ make this a potentially complicated task. To simplify then, we first parametrize $\phi(Y_i, Y_j)$ in the following way:

$$\phi(Y_i, Y_j) = f(Y_i)f(Y_j) + \kappa \left[g(Y_i, Y_j) - \int_0^1 g(Y_i, Y_j) dY_i - \int_0^1 g(Y_i, Y_j) dY_j + \int_0^1 \int_0^1 g(Y_i, Y_j) dY_i dY_j \right]. \quad (30)$$

This is a useful parametrization in that it allows us to reframe our choice of $\phi(Y_i, Y_j)$ as a choice of positive integer κ and function $g(Y_i, Y_j)$, which must only satisfy continuity. It is also a perfectly general parametrization, as we now show.

Lemma 2. *Every continuous joint density $\phi(Y_i, Y_j)$ that preserves f as the unconditional density for Y_i and Y_j can be written as in equation 30 for some continuous function $g(Y_i, Y_j)$ and some integer κ arbitrarily close to zero. Every continuous function $g(Y_i, Y_j)$ generates, through equation 30, for κ close enough to zero, a continuous joint density $\phi(Y_i, Y_j)$ that preserves f as the unconditional density for Y_i and Y_j .*

Proof: see Appendix C.

⁴⁵Here we mean optimal for the group, given the contract.

This lemma thus allows us to parametrize $\phi(Y_i, Y_j)$ without loss of generality and to generate all permissible such joint densities. For ease of exposition, define

$$\gamma(Y_i, Y_j) \equiv g(Y_i, Y_j) - \int_0^1 g(Y_i, Y_j) dY_i - \int_0^1 g(Y_i, Y_j) dY_j + \int_0^1 \int_0^1 g(Y_i, Y_j) dY_i dY_j, \quad (31)$$

so that $\phi(Y_i, Y_j) = f(Y_i)f(Y_j) + \kappa\gamma(Y_i, Y_j)$. Essentially, $\gamma(Y_i, Y_j)$ is the added (or subtracted) mass, relative to the zero-correlation case, at a point (Y_i, Y_j) . Note that it must add zero net mass to any horizontal or vertical slice of the unit square. If it added more, say, then the particular Y_i or Y_j value corresponding to the horizontal or vertical slice would have increased in probability mass, altering one borrower's unconditional distribution of output.⁴⁶ Of course, in any given slice of the square, some segments may be raised and others lowered, but the net addition along the slice must be zero. The form $\gamma(Y_i, Y_j)$ takes guarantees this property holds for any choice of continuous $g(Y_i, Y_j)$.

We will think of $\gamma(Y_i, Y_j)$ as carrying the basic structure of the correlation, uniform across groups, and the parameter κ as carrying the intensity of the correlation, varying across groups. To support the latter interpretation, we now show that the correlation of borrower returns increases (in absolute value) as κ increases.

Lemma 3. *The covariance under joint density $\phi(Y_i, Y_j)$ defined in equation 30 increases (in absolute value) with κ .*

Proof: see Appendix C.

We turn now to the basic structure of correlation. Lemma 2 makes clear that there are many ways to introduce correlation; all that is required is a continuous function $g(Y_i, Y_j)$. Our approach is to parametrize $g(Y_i, Y_j)$ as generally as possible, while imposing some structure on the correlation in the form of symmetry.⁴⁷ Consider sets $\{\alpha_1, \alpha_2, \dots, \alpha_N\}$ and $\{\beta_1, \beta_2, \dots, \beta_N\}$, where each $\alpha_k, \beta_k > 0$ and $N \geq 1$, and assume

$$g(Y_i, Y_j) = - \sum_{k=1}^N \beta_k |Y_i - Y_j|^{\alpha_k}. \quad (A21)$$

This formulation of the basic correlation structure $g(Y_i, Y_j)$ has the nice property that it subtracts mass in proportion to some polynomial function of the distance between two returns.⁴⁸ The more disparate the output realizations, the more mass is subtracted, while if they are identical, clearly no mass is subtracted: $g(x, x) = 0$. Intuitively, and as we will show formally, this should lead to positive correlation. Assumption A21 clearly encompasses very simple examples like absolute difference $-|Y_i - Y_j|$ and squared difference $-(Y_i - Y_j)^2$. More generally, we conjecture that it can approximate to an arbitrary degree of accuracy any

⁴⁶The comparison is clear with the earlier, simpler example in section 5.1.5 and below in section 5.4.6.

⁴⁷No perfectly general results are available. It is possible to introduce strong positive correlation in an irrelevant area of the distribution, that of high returns, and small negative correlation in the area of low returns, creating a situation of net positive correlation coupled with higher repayment rates. The reverse could be done just as easily.

⁴⁸It is correct that this $g(Y_i, Y_j)$ is everywhere subtracting mass, while it would be necessary to add mass in some places in order to add net zero mass to each slice of the unit square. However, the $\gamma(Y_i, Y_j)$ constructed by equation 31 from this $g(Y_i, Y_j)$ automatically takes care of this.

continuous function $f(|Y_i - Y_j|)$ that is monotonically decreasing in $|Y_i - Y_j|$. This therefore appears to be a general way of introducing symmetric correlation.

Transforming the $g(Y_i, Y_j)$ of assumption A21 into $\gamma(Y_i, Y_j)$ using equation 31 gives

$$\gamma(Y_i, Y_j) = \sum_{k=1}^N \beta_k \left[\frac{Y_i^{\alpha_k+1} + (1 - Y_i)^{\alpha_k+1} + Y_j^{\alpha_k+1} + (1 - Y_j)^{\alpha_k+1} - 2/(\alpha_k + 2)}{\alpha_k + 1} - |Y_i - Y_j|^{\alpha_k} \right]. \quad (32)$$

This is the actual mass that will be added (or subtracted) at each point (Y_i, Y_j) .⁴⁹ For the simple cases of absolute difference and squared difference, $\gamma(Y_i, Y_j)$ boils down to

$$Y_i^2 - Y_i + Y_j^2 - Y_j + 2/3 - |Y_i - Y_j|, \quad \text{and} \quad 2Y_i Y_j - Y_i - Y_j + 1/2,$$

respectively.

We next calculate the covariance under $\phi(Y_i, Y_j) = f(Y_i)f(Y_j) + \kappa\gamma(Y_i, Y_j)$ and find it equal to⁵⁰

$$Cov(Y_i, Y_j) = \kappa \int_0^1 \int_0^1 Y_i Y_j \gamma(Y_i, Y_j) dY_i dY_j.$$

Using expression 31 for $\gamma(Y_i, Y_j)$, carrying out some detailed integration, and simplifying, the covariance can be written

$$Cov(Y_i, Y_j) = \kappa \sum_{k=1}^N \frac{\alpha_k \beta_k}{2(\alpha_k + 1)(\alpha_k + 2)(\alpha_k + 4)}. \quad (33)$$

As expected, the covariance is strictly positive, since $\alpha_k, \beta_k > 0$.

The goal then is to determine whether the repayment rate is higher or lower for groups with higher covariance, that is a higher value for κ . Graphically, Figure 6 is a useful reference. Specifically, groups that are identical except for their value of κ have the exact same penalty functions and interest rates, and thus the same Default Curve and output space leading to default (for example the area below the dotted curves). However, the groups may differ in the *probability* mass assigned to this area due to their different degrees of correlation.

The result we show is that if unofficial penalties severe enough, groups with higher correlation have lower repayment rates. Graphically, as unofficial penalties get more intense, in the limit only box A of Figure 6 results in default. Higher correlation would clearly seem to increase the probability of falling into box A, since $|Y_i - Y_j|$ is on average smaller in that box than in the overall support. This we show to be true given the specific structure on correlation of assumption A21.

⁴⁹Note that though our base function g required only the distance $|Y_i - Y_j|$, we end up with a distorted function γ that requires the individual values Y_i and Y_j . This is because adding mass proportional only to the distance $|Y_i - Y_j|$ would alter the unconditional distributions. To see this, one can compare the vertical slices of the unit square when $Y_i = 0$ and $Y_i = 1/2$, respectively. The distance $|Y_i - Y_j|$ on the former slice varies from zero to one, and on the latter slice from zero to one half. Clearly, if the mass added strictly increases with distance $|Y_i - Y_j|$, it cannot sum to zero over both of these slices. In other words, $2 \int_0^{1/2} \gamma(x) dx = \int_0^1 \gamma(x) dx$ is impossible if γ is strictly increasing. Thus corrections are made to γ through equation 31 to add more weight than the distance term alone would imply, near the boundaries of the square.

⁵⁰See the proof of lemma 3 for the simple derivation.

Proposition 17. *Under assumptions A16, A17, A18, A19, and A21, if unofficial penalties are severe enough, the group repayment rate is lower for groups with higher covariance of returns, that is, higher κ .*

Proof: see Appendix C.

The intuition is simple: increased correlation makes it more likely that when one borrower realizes low returns, the other does also, and thus repayment for each other becomes less likely. Even under our assumption on covariance, no simple result appears to be available without restricting attention to box A of Figure 6.⁵¹ Thus the prediction of the BC model regarding correlation is in general unclear. However, if social penalties are strong enough and correlation takes the symmetric structure analyzed here, the repayment rate decreases with correlation.

5.4 Adverse Selection: Ghatak 1999

Ghatak 1999 shows how joint liability can take advantage of the information borrowers have about each other, but the outside lender does not have, to draw into the market relatively safe borrowers who would be excluded under individual liability contracts. The model is similar to those of Stiglitz and BBG, except that riskiness of borrower projects is exogenous, while the decisions of whether to borrow and with whom are endogenized. With respect to matching, Ghatak shows that borrowers form homogeneous groups. Given this, he shows that joint liability effectively varies the interest rate by risk of borrower, reducing the cross-subsidization from risky to safe borrowers.

In this section we show that the model contains the following basic predictions⁵² involving *expected*⁵³ group repayment rates, under certain assumptions.

- The expected repayment rate is decreasing in the interest rate r .
- The expected repayment rate is increasing in the loan size L , if groups are credit-constrained.
- The expected repayment rate is decreasing in the joint liability payment q .
- The existence of outside credit decreases expected repayment because it enhances the outside option more than the payoff of borrowing from this lender.

⁵¹To see this, consider the case where $\underline{Y}(2r) \equiv b = 1$ and unofficial penalties are nonexistent, so that boxes A and AB lead to default. The area of repayment is then the box complementary to boxes A and AB, in the upper right corner of the support. The same reasoning underlying Proposition 17 would give that this box must increase in mass under an increase in correlation, since $|Y_i - Y_j|$ is on average smaller there than in the overall support. In this case, higher correlation would be associated with higher repayment. This case we do not explore because the required assumptions, that b is near 1 and that unofficial penalties are weak, must be made jointly on $F()$, r , c^o , and $c^u()$, and are thus less general.

⁵²Note that all these predictions are true *ceteris paribus*.

⁵³Each of the other models delivers a deterministic probability of repayment p as a function of observed characteristics. This model is different from the other three in that observing all characteristics does not generally pin down p , but rather restricts it to some interval, as will be demonstrated. Our best guess then for the repayment rate of a group with certain characteristics is the expectation of p conditional on it falling in the appropriate interval. Hence we use the term "expected" repayment rate.

- Cooperative behavior has no effect on the expected repayment rate relative to non-cooperative behavior.
- Increased correlation between borrower output increases the expected repayment rate.
- Higher borrower productivity, through education say, increases the expected repayment.
- The ability to screen raises the expected repayment rate.

A continuum of potential borrowers is assumed, each with an exogenously given project. The project pays off output $Y(p)$ with probability p and gives zero output otherwise. Ghatak assumes that

$$pY(p) = E, \quad \forall p \in [\underline{p}, 1]. \quad (\text{A22})$$

Thus attention is restricted to agents with projects having the same expected return. The higher is an agent's probability of success p , or 'risk-type', the lower is his risk (measured by variance, for example). There is a density $g(p) > 0$ of borrowers at each type $p \in [\underline{p}, 1]$, for some $\underline{p} \in (0, 1)$. As in all the models except Stiglitz, borrowers are risk-neutral.

A lender offers a borrower one unit of capital through a joint liability contract involving a group of size two. Each borrower is free to choose his partner, freely observing type. The lender, however, observes neither borrower type nor amount of output, but only whether there is at least some output, that is, whether each project has succeeded or failed. Thus the lender contracts on the binary outcomes of success or failure.⁵⁴ Further, limited liability prevents the lender from receiving any payment from a borrower who fails. Thus the Denote r as the interest rate, that is, the amount paid whenever and q the joint liability payment, . Thus a borrower of type p who pairs with one of type p' has expected payoff of

$$E - pr - p(1 - p')q, \quad (34)$$

This is exactly equation 2 specialized for linear utility, loan size L normalized to 1, and assumption A22 incorporated. The interest rate is paid whenever the borrower succeeds and the joint liability payment whenever the borrower succeeds and his partner does not. Clearly the payoff is increasing in p' : all borrowers prefer to be matched with safer borrowers. The key point is that it increases in p' more steeply the higher is p . Intuitively, safe borrowers benefit relatively more from safe partners. This is because safe borrowers succeed more often, and are thus in the position of being able to bail out their partners more often.

Using this insight, Ghatak shows that *homogeneous matching is the only equilibrium*. First, it is an equilibrium, since no one would want to deviate from it. Consider a borrower of type p (matched with another p) trying to lure a borrower of type $p' > p$ away from his group (consisting of another p'). The p -borrower would be willing to pay up to the increase in expected payoff that would result from matching with a p' rather than a p , namely

$$[E - pr - p(1 - p')q] - [E - pr - p(1 - p)q] = p(p' - p)q.$$

⁵⁴Under perfect information, the lender would vary the interest rate based on fully observed borrower risk, eliminating cross-subsidization of risky borrowers by safe ones, and thus the adverse selection problem.

The borrower of type p' , however, would need to be compensated for his loss in expected payoff from being matched with a borrower of type p rather than p' , which equals

$$[E - p'r - p'(1 - p')q] - [E - p'r - p'(1 - p)q] = p'(p' - p)q.$$

Clearly no trade can occur, since the loss to the safer borrower, $p'(p' - p)q$, is greater than the gain to the riskier one, $p(p' - p)q$. Second, it is a unique equilibrium, in the sense that in *any* equilibrium, the measure of homogeneous groups is one.⁵⁵ If there were an equilibrium in which this were not the case, there would be some (p, p') combination with $p' > p$ and a positive measure of groups containing borrowers of types p and p' , respectively. But by reasoning similar to the above, this is not an equilibrium. At least two borrowers of type p' would find it optimal to abandon their riskier p -partners for those of their own type, and the partners they left could not offer attractive enough payments to keep them.

Homogeneous matching means the payoff of a borrower of type p will be the following modified version of expression 34 in equilibrium:

$$E - pr - p(1 - p)q. \tag{35}$$

Note that the derivative of the payoff with respect to p is $-[r + q(1 - 2p)]$. As long as

$$q \leq r, \tag{A23}$$

this derivative is strictly negative for $p \in [\underline{p}, 1)$. Thus, the safer an agent's type, the lower his payoff from borrowing and undertaking the project.

The key choice of an agent is whether to produce with borrowed capital or to exercise an outside option and produce using labor only. It is assumed that one unit each of capital and labor are required to carry out the project. Each potential borrower is endowed with one unit of labor, which could be used by itself to produce \underline{u} , and no capital. Capital is available from a lender who offers borrowers take-it-or-leave-it contracts. Ghatak assumes that the lender's cost of capital plus the borrower's opportunity cost of labor, \underline{u} , do not exceed expected output of a borrower's project. Thus lending to all borrowers is socially optimal, since the expected return to their unit of labor, when combined with the loaned capital, is greater than the alternate uses of the labor and capital. The model takes no stand on whether the outside option is the same occupation, but plied without (loaned) capital; or a different occupation altogether, such as wage labor. Regardless, since the outside option pays \underline{u} , agents will borrow if and only if payoff 35 is greater than \underline{u} . Given that the payoff of borrowing is declining in p , there exists a cutoff type, call it \hat{p} , such that borrowers of type $p > \hat{p}$ will not find it optimal to borrow, and all others will borrow. Assuming \hat{p} is in the interior, that is $\hat{p} \in (\underline{p}, 1)$, then \hat{p} must satisfy

$$E - \hat{p}r - \hat{p}(1 - \hat{p})q = \underline{u}. \tag{36}$$

Ghatak shows that increasing q to slightly above zero, with r being lowered to maintain zero profits for the lender, raises \hat{p} and thus draws in more borrowers. We do not test

⁵⁵Our argument for this is slightly different from the paper's, which assumes at this point that there are an even number of borrowers of each type.

this directly. Rather, as above, we take $q > 0$ as given and analyze the determinants of the expected repayment rate in the model. As in previous sections, we do not impose a zero-profit constraint. The reasons are discussed in section 2.

At this point it is clear that the Ghatak model differs from the other three in that p does not vary with any covariates. Rather, p is given to a group exogenously. What varies is whether or not agents choose to borrow. However, for the econometrician who does not observe the group's p but does observe other characteristics, there is an induced relationship between observed characteristics of the group and the expectation they will repay. This is because the observed characteristics determine \hat{p} through equation 36. In turn, \hat{p} narrows the support from which the group can be drawn to $[\underline{p}, \hat{p}]$ and gives a distribution for p which equals $g(p)/G(\hat{p})$.⁵⁶ The likelihood of observing repayment outcome $R \in \{0, 1\}$, given \hat{p} , is then

$$\int_{\underline{p}}^{\hat{p}} p^R [1 - p]^{1-R} \frac{g(p)}{G(\hat{p})} dp. \quad (37)$$

Let \tilde{p} be defined as $E(p|p \leq \hat{p})$, or equivalently,

$$\tilde{p} \equiv \int_{\underline{p}}^{\hat{p}} p \frac{g(p)}{G(\hat{p})} dp. \quad (38)$$

It is clear that the likelihood 37 can be rewritten in terms of \tilde{p} as⁵⁷

$$\tilde{p}^R (1 - \tilde{p})^{1-R}. \quad (39)$$

Therefore, this model gives rise to exactly the same likelihood (expression 1 of section 3) as the other models, if we interpret \tilde{p} as the probability the group will repay.

This implies that the corresponding theoretical results we need involve signing $\partial \tilde{p} / \partial x$ as the method for finding the effect of variable x on the group's (expected) probability of repayment. From its definition, it is clear that \tilde{p} is monotonically increasing in \hat{p} , strictly so since $g(p)$ is assumed strictly positive on its support. Further, \tilde{p} only depends on \hat{p} and g . Therefore, to sign the effect of a variable x on expected repayment rate, $\partial \tilde{p} / \partial x$, it is sufficient to sign $\partial \hat{p} / \partial x$ and attribute the same sign to $\partial \tilde{p} / \partial x$, as long as x has no effect on g . Thus we focus on $\partial \hat{p} / \partial x$ in the following sections.

5.4.1 Checking r and q

We begin by examining the effects of r and q on \hat{p} . Total differentiation of equation 36 shows that both partial derivatives, $\partial \hat{p} / \partial r$ and $\partial \hat{p} / \partial q$, are negative. This implies that the higher a group's r or q , the lower its cutoff type and thus the lower its expected repayment rate. The intuition is that higher r or q lower the payoff of borrowing relative to the outside option, and thus safer potential borrowers are pushed away.

Proposition 18. *Under assumptions A22 and A23, the expected group repayment rate is lower for groups with higher r and q .*

Proof: see Appendix D.

⁵⁶This is just the conditional density for $p < \hat{p}$.

⁵⁷If $R = 1$, likelihood 37 becomes \tilde{p} ; if $R = 0$, it becomes $1 - \tilde{p}$.

5.4.2 Adding loan size

In this section we introduce as a factor of production loaned capital L and find that contrary to the Stiglitz and BBG models, a higher loan size can be associated with higher expected repayment rates. This happens when all borrowers are sufficiently constrained on the intensive margin, that is cannot borrow all they would like. If some borrowers are not constrained, conditions exist that guarantee the exact opposite relationship.

As in section 5.2.4, we use assumption A6 of section 5, which allows us to break Y multiplicatively into components relating to probability p and loan size L . We reproduce it here, modified to suppress the argument H :

$$Y(p, L) = Y(p, 1)F(L),$$

where $F(1)$ is normalized to 1.⁵⁸ We continue to let

$$E(p) \equiv pY(p, 1),$$

so that expected output satisfies

$$pY(p, L) = pY(p, 1)F(L) = E(p)F(L) = EF(L), \quad (40)$$

where the first equality uses assumption A6, the second uses the definition of $E(p)$, and the final equality is due to assumption A22. We here make similar assumptions to A12 that

$$F \text{ is strictly increasing and concave, differentiable, } \lim_{L \rightarrow \infty} F'(L) = 0, \text{ and } \lim_{L \rightarrow 0} F'(L) = \infty. \quad (\text{A24})$$

In addition, the interest and joint liability payments are now assumed to be proportional to L : rL and qL , respectively. The borrower payoff analogous to 35 now becomes

$$EF(L) - prL - p(1 - p)qL. \quad (41)$$

Using this modified payoff, the new indifference equation that defines the cutoff type \hat{p} , analogous to 36, becomes

$$EF(L) - \hat{p}rL - \hat{p}(1 - \hat{p})qL = \underline{u}. \quad (42)$$

Incorporating loan size into the model turns up some important issues. In particular, the group has its own preference over how much capital is loaned to it.⁵⁹ To see this, note that the loan size that maximizes payoff 41 must solve

$$EF'(L) - pr - p(1 - p)q = 0. \quad (43)$$

⁵⁸Note that $Y(p, 1)$ is equivalent to the $Y(p)$ we use in earlier sections, in which the second argument was suppressed since loan size L was assumed fixed at 1.

⁵⁹The original model has groups making a binary choice between borrowing some predetermined amount and not borrowing at all.

Given concavity and Inada conditions on F , there is clearly a unique, interior optimum for L , call it L^* , determined by

$$L^*(p; q, r, E) = F'^{-1}([pr + p(1-p)q]/E). \quad (44)$$

It is straightforward to show that L^* is strictly decreasing in p ,⁶⁰ that is, safer groups prefer smaller loans. Thus, among the set of groups that obtained their desired loan amount, we would expect to see a *negative* relationship between p and L .

However, it is almost certainly unrealistic to assume all groups obtain their desired loan amount. The lender in our data, the BAAC, instead has a formula for deciding the maximum loan amount, based on previous revenues; and of course the borrower is free to borrow less. Thus a much better assumption is that the lender specifies to each borrower the interest rate and the maximum loan he can borrow, and the borrower chooses whether or not to borrow, and if so, whether to borrow the maximum amount or any amount less.⁶¹ This mechanism implies that the observed loan is the *minimum* of the borrower's most preferred loan and the lender's maximum loan for this particular borrower. As in previous analysis, we continue to assume the lender's offer of (maximum) loans varies exogenously across groups. Let $h(L)$ be the density of loan offers and $H(L)$ be the associated distribution function. We assume $H(L)$ is independent of $G(p)$, the distribution of types, which is in line with the model's assumption that the lender cannot observe risk type.⁶²

We are concerned with finding the best guess for type p , given that we observe a loan size of L . Note that if we observe L , it must be that the group is of type $p \in [\underline{p}, \hat{p}]$. This is from the standard selection equation 42, from which we know any $p > \hat{p}$ will find the payoff to borrowing less than \underline{u} . However, it must also be that

$$EF'(L) - pr - p(1-p)q \geq 0.$$

This second fact comes from the ability of the borrower to take a smaller loan than the lender offers. This implies that a borrower will never end up with a loan larger than his optimum, one that gives a negative derivative to the payoff. He could do better by reducing to his optimal loan amount. We define \bar{p} to be the type for whom L is the optimal loan, or in other words,

$$EF'(L) - \bar{p}r - \bar{p}(1-\bar{p})q = 0. \quad (45)$$

This is equivalent to saying $L^*(\bar{p}) = L$. Now since the derivative on the left-hand side of 45 must be non-negative as we argued, and since it is decreasing in p , we must have that $p \in [\underline{p}, \bar{p}]$. Combining this with the earlier conclusion of this paragraph, we see that $p \in [\underline{p}, \min\{\hat{p}, \bar{p}\}]$.

⁶⁰This is because F' and thus F'^{-1} are strictly decreasing functions; and the derivative of the argument of F'^{-1} is $(r + q - 2pq)/E$, which is strictly positive for $p \in [\underline{p}, 1)$ under assumption A23 that $q \leq r$.

⁶¹If the lender knew everything about the group except their type, it could offer a menu of contracts with differing L and r , using borrower preference over L to accomplish some screening. However, we assume it must offer a single contract to a given borrower. The lender in our data does not offer a tradeoff between r and L , but offers for the most part a fixed r and a maximum L .

⁶²The lender in our data, the BAAC, did not condition contract terms on risk. Only recently has it begun to, for example by varying interest rates with default history, but this was significantly after our data were collected.

It turns out that completely general monotonicity results are not obtainable since both \hat{p} and \bar{p} can go in opposite directions with L depending on how large L is and on the distributions G and H . However, under certain assumptions, results can be obtained. In particular, we will find conditions under which $\hat{p} < \bar{p}$, and vice versa, and proceed from there.

It will be helpful to write

$$\underline{u} = EF(0) + \chi. \quad (46)$$

Here if $\chi = 0$, then \underline{u} just equals the borrowing payoff 41 at $L = 0$. If $\chi > 0$, then \underline{u} is greater than the borrowing payoff at $L = 0$. One interpretation for χ is the fixed cost of borrowing, it being the only wedge between the payoff to borrowing as $L \rightarrow 0$ and the outside option payoff. If the outside option corresponds to a different occupation altogether, then χ is the premium paid to this occupation over the occupation corresponding to borrowing (farming, in the BAAC context) with no capital.

Note that if $\chi = 0$ in equation 46, *everyone* would want to borrow. The Inada conditions on F give that at $L = 0$, payoffs are strictly increasing in L ; and since the payoff at $L = 0$ equals the outside option when $\chi = 0$, everyone could do better than the outside option with at least a small loan. This of course changes the spirit of the model from extensive-margin credit rationing to intensive-margin credit constraints.

We proceed by determining whether \hat{p} or \bar{p} is bigger. One way to answer this is to sign the expression

$$EF(L) - \bar{p}rL - \bar{p}(1 - \bar{p})qL - \underline{u}, \quad (47)$$

which is the difference between the borrowing payoff and the outside option for $p = \bar{p}$. If this expression is negative, then the outside option is preferable to borrowing, which implies that $\bar{p} \notin [p, \hat{p}]$, and thus that $\hat{p} < \bar{p}$. If it is positive, the reverse holds. Incorporating equations 45 and 46, expression 47 can be rewritten as

$$EF(L) - EF'(L)L - EF(0) - \chi,$$

which is negative as long as $F(L) - F(0) - LF'(L) < \chi/E$. We define

$$\Gamma(L) \equiv F(L) - F(0) - LF'(L) \quad (48)$$

for any $L > 0$. Differentiating equation 48 gives that $\Gamma'(L) = -LF''(L)$, which is strictly positive by assumption A24. Since Γ is strictly increasing, it can be inverted. The condition then for expression 47 to be negative then reduces to

$$L < \Gamma^{-1}(\chi/E). \quad (A25)$$

This assumption guarantees that $\hat{p} < \bar{p}$. Since Γ , and thus Γ^{-1} , is increasing, it holds when χ is large enough relative to loans. Intuitively, a low L means that a relatively safe borrower finds L optimal (since $L^*(p)$ is decreasing in p), so \bar{p} is high; a low L also means the borrowing payoff is not that high and only the most risky types would find it higher than the outside payoff (depending on how large χ is), so \hat{p} is low. The reverse happens for a high L , though \hat{p} does not move monotonically with L .

Note also that $\lim_{L \rightarrow 0^+} \Gamma(L) = 0$.⁶³ Thus

$$\lim_{\chi \rightarrow 0^+} \Gamma^{-1}(\chi/E) = 0,$$

or in other words, as χ goes to zero or below, no positive loan size can satisfy assumption A25. Credit rationing could only exist then on the intensive margin, since all would want to borrow some amount. However, if $\chi > 0$, there is an interval of loan sizes that satisfy A25.⁶⁴

Assumption A25 also guarantees that everyone who borrows, those in $[\underline{p}, \hat{p}]$, receive loans smaller than their optimal amount. In other words, they are credit-constrained. This is evident because L is optimal for type \bar{p} , so loans larger than L are optimal for all types below \bar{p} ,⁶⁵ including those in $[\underline{p}, \hat{p}]$. Since everyone is constrained under that assumption, a higher loan size means higher payoffs and safer borrowers being drawn in.

Proposition 19. *Under assumptions A6, A22, A23, A24, and A25, the expected group repayment rate is higher for groups with higher L .*

Proof: see Appendix D.

In contrast to the Stiglitz and BBG models, here an increase in loan size increases expected repayment rates, as long as groups are sufficiently credit-constrained. The intuition is that bigger loans scale up the payoff of borrowing relative to the outside option and thus draw in safer borrowers.

When assumption A25 does not hold, that is when $L \geq \Gamma^{-1}(\chi/E)$, we have that $\bar{p} \leq \hat{p}$. In this case, observing L tells us that the group must be of a type within $[\underline{p}, \bar{p}]$. However, it turns out that the expected repayment rate is not as simple as $E(p|p \leq \bar{p})$. This is because there is probability mass concentrated at $p = \bar{p}$. For all types $p < \bar{p}$, we know for certain they were offered exactly L as their loan size. This is because any group who takes less than the lender offers ends up with their optimal loan size, and we know that L is not the optimal loan size for $p < \bar{p}$. However, a borrower with $p = \bar{p}$ could have been offered any loan L or higher, and reduced the actual loan to his optimum loan size, L . The probability of getting a loan greater than L , which is $1 - H(L)$, is much higher than that of getting a loan exactly L , which is $h(L)$. Thus analyzing how $E(p|p \leq \bar{p})$ varies with L is complex.

We begin with the probability of observing L , $P(L)$:

$$P(L) = G(\bar{p})h(L) + g(\bar{p})[1 - H(L)]. \quad (49)$$

The first term is the probability of a group of type less than \bar{p} and a loan of exactly L . The second term is the probability of a group of type \bar{p} and a loan greater than L (of which the

⁶³For $L > 0$, note that $0 < LF'(L) < F(L) - F(0)$, where the inequalities comes from assumption A24. The first inequality is because F is increasing, the second is because F is concave and can be shown by a Taylor expansion $F(0) = F(L) - LF'(L) + F''(\tilde{L})L^2/2$, where $\tilde{L} \in (0, L)$. Now as $LF'(L)$ is sandwiched by two terms approaching zero as $L \rightarrow 0$, clearly it does also, and thus it is clear that $\lim_{L \rightarrow 0^+} \Gamma(L) = 0$.

⁶⁴For example, if $F(L) = L^\alpha$, then $\Gamma(L) = (1 - \alpha)L^\alpha$. Assumption A25 reduces to

$$L < \left[\frac{\chi}{(1 - \alpha)E} \right]^{1/\alpha}.$$

Clearly, the right-hand side is zero for $\chi = 0$ and greater than zero for $\chi > 0$.

⁶⁵Recall that $L^*(p)$ is decreasing in p .

group would only accept L). Given this, the probability of group being risk type p given observation L , is

$$P(p|L) = \begin{cases} 0 & , p > \bar{p} \\ \frac{g(\bar{p})[1-H(L)]}{P(L)} & , p = \bar{p} \\ \frac{g(p)h(L)}{P(L)} & , p < \bar{p} \end{cases} \quad (50)$$

where $P(L)$ is defined in equation 49. Using this conditional density, we calculate the conditional expectation, $E(p|L)$ as

$$E(p|L) = \int_{\underline{p}}^1 pP(p|L)dp.$$

Note, however, that the conditional density should be treated as if there is a mass point at $p = \bar{p}$, since the density there is multiplied by $1 - H(L)$, representing a positive probability event, and everywhere else by $h(L)$, representing a zero-probability event. The conditional expectation then works out to be, using equation 49,

$$E(p|L) = \frac{\bar{p}g(\bar{p})[1 - H(L)] + E(p|p \leq \bar{p})G(\bar{p})h(L)}{G(\bar{p})h(L) + g(\bar{p})[1 - H(L)]}, \quad (51)$$

which can be rewritten as

$$E(p|L) = E(p|p \leq \bar{p}) + [\bar{p} - E(p|p \leq \bar{p})] \frac{g(\bar{p})[1 - H(L)]}{G(\bar{p})h(L) + g(\bar{p})[1 - H(L)]}. \quad (52)$$

Note that if $g(\bar{p}) \approx 0$ or $[1 - H(L)] \approx 0$, $E(p|L)$ is roughly equal to $E(p|p \leq \bar{p})$, similar to previous sections except with \bar{p} substituted for \hat{p} . The two conditions noted correspond to either type \bar{p} being very rare or to loans greater than L being very rare, either of which make us certain \bar{L} was the lender's offer, not the borrower's reduced counteroffer. Analysis would be very simple if either held, as \bar{p} declines monotonically in L , since a higher L implies a lower p that finds L optimal. However, if these conditions do not hold, then our expectation is closer to type \bar{p} , which is like a mass point.

Apparently, $\bar{E}(p|L)$ as expressed in equation 52, for example, need not vary monotonically with L . Certainly \bar{p} and thus $E(p|p \leq \bar{p})$ are monotonically decreasing in L . However, the *weight* assigned to \bar{p} versus $E(p|p \leq \bar{p})$ depends positively on $g(\bar{p})$ and $[1 - H(L)]$. In particular, one might imagine a case in which as \bar{p} decreases with L , a narrow spike in the density might be encountered, giving a very high value for $g(\bar{p})$ and thus shifting weight significantly toward \bar{p} , even so much as to increase $E(p|L)$. However, we do know that $E(p|L)$ is *coarsely* decreasing, in the sense that it falls back down to \underline{p} as L gets large enough. This is because $E(p|L)$ is bounded between two functions, \bar{p} and $E(p|p \leq \bar{p})$, that are monotonically decreasing in L toward \underline{p} .

We can also place restrictions on G and H to guarantee that $E(p|L)$ is monotonically decreasing in L . Without loss of generality, we rewrite $G(p)$ and $H(L)$ as

$$G(p) = A_G e^{f_G(p)}; \quad H(L) = 1 - A_H e^{-f_H(L)}, \quad (53)$$

where⁶⁶ clearly both A_G and A_H must be strictly positive. One can also show that $f'_G(p)$ and $f'_H(L)$ must be strictly positive. The following assumption on the two functions will guarantee monotonicity:

$$f''_G(p) \geq 0; \quad f''_H(L) \geq 0. \quad (\text{A26})$$

These imply that the density of loan offers $h(L)$ must be strictly decreasing and at least as convex as an exponential distribution, while the density of types $g(p)$ must be strictly increasing and at least as convex as an exponential function. Note that uniform does not satisfy either distribution's requirements, since it is neither increasing nor decreasing. However, distributions arbitrarily close to the uniform distribution do.⁶⁷ Using this assumption, we are able to prove

Proposition 20. *Under assumptions A6, A22, A23, A24, the converse of A25,⁶⁸ and A26, the expected group repayment rate is lower for groups with higher L.*

Proof: see Appendix D.

Note that if χ is close enough to zero, then the converse of assumption A25 is sure to hold. Thus with a small fixed cost of borrowing, or under the alternate interpretation of χ , a small difference in payoff between the outside occupation and the occupation associated with borrowing at zero capital, we are more likely to see an inverse relationship between loan size and repayment rate.

5.4.3 Adding borrower productivity

Effects of borrower productivity are analyzed as before by introducing an additional factor of production called H, which can be thought of as the borrower's human capital or land. We find that as in all other models, higher productivity leads to a higher repayment rate. It makes both the borrowing option and the outside option more attractive, but the former by more under reasonable assumptions.

We use the separability assumption A6 of section 5, which allows us to break Y multiplicatively into three components, relating to probability p , productivity H , and loan size L . We reproduce it here:

$$Y(p, L, H) = Y(p, 1, 1)F(L)G(H),$$

where $F(1)$ and $G(1)$ are normalized to 1.⁶⁹ Since loan size of borrowing groups will not vary in this section, we fix it at 1. We continue to let

$$E(p) \equiv pY(p, 1, 1),$$

⁶⁶This is without loss of generality, since $f_G(p)$ simply equals $\ln[G(p)/A_G]$, and similarly for $f_H(L)$.

⁶⁷For example, $f_g(p) = a_g p$, where $a_g > 0$ is a constant, implies the density of risk types is $g(p) = a_g A_g e^{a_g p}$. Note that as a_g approaches zero, with A_g adjusted accordingly, this density approaches the uniform distribution on $[p, 1]$. Similar arguments hold for $H(L)$.

⁶⁸In other words, we assume that $L \geq \Gamma^{-1}(\chi/E)$.

⁶⁹Note that $Y(p, 1, 1)$ is equivalent to the $Y(p)$ we use in earlier sections, in which the other arguments were suppressed since H and L were fixed at 1.

so that expected output satisfies

$$pY(p, 1, H) = pY(p, 1, 1)F(1)G(H) = E(p)G(H) = EG(H), \quad (54)$$

where the first equality uses assumption A6, the second uses the definition of $E(p)$ and normalization of $F(1)$, and the final equality is due to assumption A22. The borrower payoff analogous to 35 now becomes

$$EG(H) - [pr + p(1 - p)q]. \quad (55)$$

We will compare payoff 55 to the outside payoff,

$$\underline{u} = EF(0)G(1) + \chi,$$

written here exactly as in equation 46, though here we include $G(1)$ which is normalized to one. If χ has the interpretation as a fixed cost of borrowing, the natural generalization is that

$$\underline{u}(H) = EF(0)G(H) + \chi, \quad (A27)$$

as long as the fixed cost of borrowing does not vary with productivity H . On the other hand, if the fixed cost does vary with H , or if χ is not a fixed cost but rather the wedge between the non-borrowing occupation and the borrowing occupation at zero capital, then $\underline{u}(H)$ should be written differently. A second natural generalization is that

$$\underline{u}(H) = EF(0)G(H) + \chi G(H) = G(H)\underline{u}. \quad (A28)$$

Clearly this second generalization involves the outside option being augmented by productivity to a greater degree than in the first generalization.

Under either assumption, and using modified payoff 55, the new indifference equation that defines the cutoff type \hat{p} , analogous to 36, becomes

$$EG(H) - \hat{p}r - \hat{p}(1 - \hat{p})q = \underline{u}(H). \quad (56)$$

Total differentiation with respect to \hat{p} and H allows us to prove the following:

Proposition 21. *Under assumptions A6, A22, A23, A27, and assuming G and F are strictly increasing and G is differentiable⁷⁰, the expected group repayment rate is higher for groups with higher H . Under the same set of assumptions with A28 substituted for A27, the expected group repayment rate is higher for groups with higher H .*

Proof: see Appendix D.

As in all the other models, an increase in borrower productivity increases expected repayment rates. The intuition is that the increased productivity makes both borrowing and the outside option more attractive. Under assumptions A27 and A28, the net outside payoff is augmented by at most $G(H)$. However, it is the borrowing payoff *gross of interest payments* (see payoff 55) that is augmented by $G(H)$, leaving the net borrowing payoff to

⁷⁰Differentiability is a convenient but unnecessary assumption here.

be augmented by a factor greater than $G(H)$. Thus borrowing is made relatively more attractive with higher productivity, and higher types are drawn in. Of course, if the productivity augmented the outside payoff more steeply than the borrowing payoff, the result of proposition 21 could easily be overturned.⁷¹

5.4.4 Checking outside options

The effect of outside credit options, i.e. a second lender, on repayment depends crucially upon how this access to credit benefits each occupation. If \underline{u} is taken to be the payoff from wage labor, it is plausible to assume that the existence of a second lender does not affect \underline{u} . The second lender does augment the payoff from borrowing from the primary lender, however, if borrowers are credit-constrained from the primary lender. Then the availability of outside credit would potentially increase the borrowing payoff and draw in safer borrowers, leading to an interesting externality between lenders.

On the other hand, consider the case where both payoffs correspond to the same occupation, but one involves paying a fixed cost and acquiring use of some capital, while the other involves using no capital. In this case, the infusion of extra capital would help the outside payoff more, if the production function exhibits diminishing returns to capital. Of course, whether borrowers would ever make use of the second lender given they had chosen not to make use of the first would depend on the fixed cost and the distribution of loan offers from the second lender. If this fixed cost was low enough, no credit-constrained borrower would refuse some extra capital, and those not already borrowing would be affected more. Thus the outside payoff would increase and safe borrowers would leave the borrowing pool (of the primary lender, whom we observe).

Evidently, either result could obtain. We focus on the simpler interpretation of the outside payoff, assuming that it corresponds to wage labor, for example, and in particular that it is not altered by additional credit sources.

$$\textit{Additional sources of capital do not affect } \underline{u}. \tag{A29}$$

Partly for simplicity, we also assume the same terms of credit exist from the second lender, and that every borrower has some chance of getting positive credit.

$$\begin{aligned} &\textit{The second lender offers the same conditions as the primary one,} \\ &\textit{and every type has a positive probability of getting offered at least } L' > 0. \end{aligned} \tag{A30}$$

Finally, we assume that the existence of a second lender does not change the primary lender's unconditional distribution of loan offers; and that the primary lender is the one approached first, with the secondary lender fulfilling only demand not satisfied by the primary lender's offers.

⁷¹If \underline{u} represents the payoff to farming without capital and without paying the fixed cost χ , or if productivity is measured by landholdings or land quality, say, assumptions A27 or A28 make very good sense. That is, it is easy to believe the gross borrowing payoff is augmented by at least as much as the non-borrowing payoff. However, if \underline{u} represents a different occupation, wage labor say, and productivity is measured by education, it could be that the outside payoff is augmented by productivity to a greater degree than the borrowing payoff. In this case the result could be overturned.

Proposition 22. *Under assumptions A6, A22, A23, A24, A29, and A30, the expected group repayment rate is higher for groups with outside credit options (and strictly so under assumption A25), as long as the fixed cost of borrowing from the second lender is low enough.*

Proof: see Appendix D.

In contrast to the Stiglitz model, here the existence of a second lender increases expected repayment rates. It makes the occupation that requires capital more attractive and thus draws in safer borrowers. The intuition is exactly the same as for the result on (primary lender's) loan sizes, another differing prediction across the two models. This result goes counter to the received wisdom that multiplicity of creditors (who are not coordinating) is harmful for repayment rates. It suggests there can be a positive force, the relaxing of village credit constraints and the drawing in of safer borrowers (affecting intensive *and* extensive margins). Thus there can be a "Big Push" effect as the amount of lending increases.

5.4.5 Checking cooperation

The distinction between cooperation and non-cooperation is irrelevant in the model. In both cases, the homogeneous group formation equilibrium should emerge. This is because the equilibria are robust to side payments, which were exactly the technique used to determine equilibrium assortative sorting. The only difference might arise from a coordination failure in a non-cooperative game, but this clearly depends on the formulation of such a game, and it is not robust to renegotiation in the form of the above side payments. Therefore, it can be said that *this model predicts there will be no difference between cooperatively- and non-cooperatively behaving groups.*

5.4.6 Adding correlation

Next we consider correlation of output between borrowers. We return to the baseline model in which loan size and borrower productivity are normalized to one. With probability p , output is $Y(p)$, and it is zero otherwise. As we argued in section 5.1.5 in the context of the Stiglitz model, there is a unique way to introduce correlation which preserves the same individual probabilities of success, for a given pair of borrowers, (p, p') . It is reproduced here:

	2 Succeeds	2 Fails
1 Succeeds	$pp' + \epsilon$	$p(1 - p') - \epsilon$
1 Fails	$p'(1 - p) - \epsilon$	$(1 - p)(1 - p') + \epsilon$

By inspection, each row and column adds to the correct individual probability of success or failure. Of course $\epsilon = 0$ is the zero-correlation case, while $\epsilon > 0$ introduces positive correlation. Recall that the above is the unique joint distribution, *given a (p, p') pair* of individual probabilities. However, there are many (p, p') combinations, and each could in theory have its own ϵ : $\epsilon(p, p')$.

For the off-diagonal cells of the matrix to be positive, we need ϵ to be less than $p(1 - p')$ and $p'(1 - p)$. As p or p' approach one, ϵ is forced to zero, which is not interesting. To avoid this problem, we modify the support innocuously to exclude those with probability of success near 1:

$$\exists \bar{p} \in (\underline{p}, 1), \text{ such that } \forall p \in (\bar{p}, 1], g(p) = 0. \tag{A31}$$

This assumption merely changes the support from $[\underline{p}, 1]$ to $[\underline{p}, \bar{p}]$, ensuring that all borrowers' face at least some uncertainty.

We will consider exactly the same two natural forms for the function $\epsilon(p, p')$ that we did in the Stiglitz case, both of which give some uniformity to the correlation structure across all project pairings. We assume $\epsilon(p, p')$ to be constant in assumption A7 of section 5.1.5:

$$\epsilon(p, p') = \epsilon, \quad \forall p, p'.$$

Since we are using the same constant ϵ for every (p, p') combination, it must be less than every value of $p(1 - p')$. One can verify that $\epsilon \leq \underline{p}(1 - \bar{p})$ is necessary (and sufficient if $\epsilon \geq 0$) to ensure no cell in the distribution matrix of every possible (p, p') pairing exceeds one or falls below zero. The second assumption A8 has the property of giving any homogeneous group the same *correlation coefficient* over project returns, equal to ϵ :

$$\epsilon(p, p') = \epsilon * \min\{p'(1 - p), p(1 - p')\}.$$

It is clear that any $\epsilon \in [0, 1]$ is allowable, that is, keeps each cell in the distribution matrix of every possible (p, p') pairing from exceeding 1 or falling below zero. For non-homogeneous groups, that is those for whom $p \neq p'$, ϵ is not the correlation coefficient, but something closely related: it is the correlation coefficient expressed as a fraction of the maximum possible correlation possible, given individual probabilities of p and p' .⁷² When $p = p'$, then the maximum possible correlation coefficient is 1, and so ϵ equals the correlation coefficient. This formulation is arguably the most general, being the unique way of affecting each potential group's (appropriately normalized) correlation coefficient symmetrically.

Before we examine how these assumptions affect the choice to borrow, we first must verify that the shape of $\epsilon(p, p')$ does not overturn Ghatak's homogeneous matching result. This is done for both assumptions used in the proof of Proposition 23 below.⁷³ Given that

⁷²Note that for non-homogeneous groups, it is a theoretical impossibility to have a correlation coefficient of one. In general, two binomial variables with different individual probabilities of success can never be perfectly positively correlated. The further apart are the two probabilities, the lower the maximum correlation coefficient. One can calculate that the maximum correlation coefficient across two projects (p, p') , call it $\bar{\rho}(p, p')$, is equal to

$$\bar{\rho}(p, p') = \min\left\{\sqrt{\frac{p(1 - p')}{p'(1 - p)}}, \sqrt{\frac{p'(1 - p)}{p(1 - p')}}\right\}.$$

Further, the correlation coefficient corresponding to the $\epsilon(p, p')$ of assumption A8 is equal to $\epsilon * \bar{\rho}(p, p')$. Thus ϵ has the interpretation, described in the text, as the fraction of the maximum possible correlation, and it can vary freely from zero to one. Of course, it is clear from the formula that $\bar{\rho}(p, p) = 1$, so for homogeneous groups, ϵ is just the correlation coefficient.

⁷³The basic Ghatak result is thus robust to the introduction of correlation between borrowers' project returns, where the correlation affects all possible matches evenly. As an aside, this model suggests another dimension along which matching would occur. Not only would type- p agents match with other type- p agents, but those whose project returns are relatively highly correlated would also want to match. Imagine that borrowers of each type p are partitioned into two, such that within each partition returns are perfectly correlated, while across partitions they are independent. Then not only would homogeneous matching across p -types occur, but also within each type p , borrowers would only match with those perfectly correlated with themselves. This is because the payoff of matching with an uncorrelated borrower is $E - pr - p(1 - p)q$, while

homogeneous matching still obtains, the borrower payoff analogous to 35 now becomes

$$E - pr - [p(1 - p) - \epsilon(p, p)]q. \quad (57)$$

The new indifference equation that defines the cutoff type \hat{p} , analogous to 36, becomes

$$E - \hat{p}r - [\hat{p}(1 - \hat{p}) - \epsilon]q = \underline{u} \quad (58)$$

under assumption A7, and

$$E - \hat{p}r - \hat{p}(1 - \hat{p})(1 - \epsilon)q = \underline{u} \quad (59)$$

under assumption A8. Total differentiations with respect to \hat{p} and ϵ allow us to prove the following:

Proposition 23. *Under assumptions A22, A23, A31, and either A7 or A8, the expected group repayment rate is higher for groups with higher project return correlation (higher ϵ).*

Proof: see Appendix D.

The idea here comes from the property of the model that positive payoff only occurs when the borrower is successful. Correlation shifts the weight in this state of the world toward the sub-state where the borrower's partner is successful, away from the sub-state where the borrower's partner fails. Thus it raises the payoff and draws in safer borrowers.

5.4.7 Checking screening

Finally, we examine how the ability to screen and sort affects the expected repayment rate in the model. Specifically, we will do away with the assumption that borrowers know each other's types. Instead, they only know their own type and the population distribution of types. This leads them to form groups in some sense randomly.

Recall that when borrowers do know each other's types, the cutoff probability \hat{p} must satisfy equation 36, reproduced here:

$$E - \hat{p}r - \hat{p}(1 - \hat{p})q = \underline{u}.$$

The average type of those who borrow is $E(p|p \leq \hat{p})$, which was called \tilde{p} .

Now let \hat{p}' be the highest value for p satisfying

$$E - \hat{p}'r - \hat{p}'(1 - \hat{p}')q = \underline{u}, \quad (60)$$

where $\hat{p}' \equiv E(p|p \leq \hat{p}')$. As we will argue, \hat{p}' describes the marginal borrower under the assumption borrowers do not know each other's types; therefore, a borrower of type p will borrow only if $p \in [p, \hat{p}']$.⁷⁴

it is $E - pr$ when matching with a perfectly correlated borrower. In equilibrium, groups would be identical in probabilities of success and also have higher correlation than a random selection of borrowers. Of course, in our simple example, the joint liability payment would never be paid; if this was internalized by the lender, the interest rate would be at the individual liability level.

⁷⁴Actually, this is the best case scenario, in which the most people possible borrow. If equation 60 has multiple solutions (which is possible for a density with sufficiently pronounced spikes) any of the solutions can be an equilibrium cutoff type, depending on borrower expectations.

To check this is an equilibrium, we verify that everyone is optimizing if the set of borrowers is identical to $[\underline{p}, \hat{p}']$. Note the payoff for anyone who chooses to borrow in this scenario is

$$E - pr - p(1 - \tilde{p}')q.$$

The expression involves \tilde{p}' because anyone who chooses to borrow will take the expectation over all types who are borrowing, which is all types $p \in [\underline{p}, \hat{p}']$. This payoff is strictly decreasing in p , at rate $-[r + (1 - \tilde{p}')q]$. Therefore, optimal behavior can be described by a cutoff type: those above find the outside option better, while those below find borrowing optimal. The fact that \hat{p}' satisfies 60 means that this cutoff is \hat{p}' . Therefore, we have an equilibrium.

The remaining question is how \hat{p}' compares to \hat{p} . We now show that it is lower:

Proposition 24. *Under assumptions A22 and A23, the expected group repayment rate is higher for groups with the ability to screen.*

Proof. Note that since $\hat{p}' > \tilde{p}'$,

$$E - \hat{p}'r - \hat{p}'(1 - \hat{p}')q > E - \hat{p}'r - \hat{p}'(1 - \tilde{p}')q = \underline{u}, \quad (61)$$

where the equality comes from equation 60. The inequality says that the payoff for the marginal type would be greater under homogeneous matching than under random matching, because it would be matched with the best, rather than the average, of the borrower pool.⁷⁵ Now from equation 36,

$$E - \hat{p}r - \hat{p}(1 - \hat{p})q = \underline{u}.$$

Combining this with 61, we get that

$$E - \hat{p}'r - \hat{p}'(1 - \hat{p}')q > E - \hat{p}r - \hat{p}(1 - \hat{p})q. \quad (62)$$

Recall that the payoff from (homogeneous-matching) borrowing, $E - pr - p(1 - p)q$, is decreasing in p , under assumption A23. Applying this to inequality 62, which states the payoff from $p = \hat{p}'$ is greater than from $p = \hat{p}$, we have that $\hat{p}' < \hat{p}$. Therefore, the ability to screen and sort increases the expected repayment rate. ■

The intuition for this result is that in the absence of screening, marginal borrowers would match with riskier borrowers. This would lower their expected payoffs and drive them out of the market. We examine this prediction using two dummy variables from the BAAC survey to proxy screening, as described in section 6.1.

6 Empirical Results

In this section we discuss our results using data from Thai BAAC groups and the villages where they are located described in section 4 to test predictions summarized in section 3

⁷⁵The riskiest types, on the other hand, get higher payoffs in the absence of screening, because they expect to match with the average, rather than the worst, of the borrower pool.

and shown in detail in section 5. First we describe the data, in section 6.1 and tables 6.1a and 6.1b. In section 6.2, we begin by looking at simple cross-tabulations using the central variables of each of the four models, summarized in table 6.2a. We also present logit results for the variables, summarized in table 6.2b. We carry out a detailed comparison of our results with the extant empirical repayment literature, in appendix E. Finally, all the information is summarized, along with the theoretical predictions, in table 6.2c. The evidence is discussed along with additional locally linear regression tests attempting to harmonize any divergent predictions from the univariate, nonparametric tests and the multivariate logits. Conclusions regarding the models are drawn.

6.1 Variable descriptions

To test the models' predictions involving repayment probabilities, the dependent variable on which we will focus our analysis is a binary dummy from the BAAC survey. It equals one if the BAAC has ever, in the history of the BAAC lending program. *Annual* default rates are much lower, XX% for group-guaranteed loans in 1997. The difference in these figures is that our variable asks whether default has occurred *at any time during the life of the group*. For example, consider a ten-year-old group (ten is close to the median group age in our data) with an annual default probability of 3%. Then the probability of defaulting at least once over ten years is $1 - (.97)^{10} = 26.3\%$. Imposing an interest rate penalty is one of the first remedial actions in a dynamic process the BAAC uses with delinquent group-guaranteed borrowers. This variable thus measures a mild form of default, since repayment ultimately may have occurred. The BAAC survey contains another valuable measure of repayment, that is, whether members have ever repaid BAAC loans for another member. However, only about one tenth responded positively to that, so estimates are not as clear and not reported here.

The independent variables used are summarized in Tables 6.1a and 6.1b. Table 6.1a contains several control variables that are not featured in any of the four models.

LNYRSOLD is the log-age of the group. It is crucial to control for the age of the group, because the dependent variable measures whether default has ever happened in the history of the group. If we think of the event as having some probability p of occurring each year, then clearly groups with a longer history have a greater chance of having run into problems. The effect of group age is non-linear. Results under inclusion of terms for age and age squared, rather than log of age, are similar and not reported here.

VARIBLTY is a village-wide measure of risk. It is equal to the village average each household's coefficient of variation in next year's expected income. It is not a featured variable in any of the theoretical models, but could readily be introduced. Our measure is taken from the household survey, where households are asked how much they will earn if next year is a good year (Hi), how much if bad (Lo), and how much they expect to earn (Ex). We assume a distribution of income over two of these mass points, Hi and Lo, as do

Table 6.1a - Independent Variables

Variables marked with an asterisk are taken or constructed from the household-level survey, HH. All others are from the group-level survey, BAAC.

Variable	Description	Mean	(σ)
<i>Control:</i>			
LNYSOLD	Number of years group has existed (Log)	11.4 ^a	(8.5)
VARIBLTY*	Village average coefficient of variation for next year's expected income	0.30	(0.09)
WEALTH*	Village average wealth (million 97 Thai baht)	1.1	(2.1)
MEMBERS	Number of members in the group	12.3	(5.1)
<i>Fundamentals:</i>			
r	Average interest rate faced by the group	10.9	(2.0)
L	Average loan size borrowed by the group (thousand 97 Thai baht)	18.7	(18.3)
q	Percent landless in the group	0.06	(0.15)

^aHere the mean and standard deviation are for age, not log of age.

the models. It is not difficult to calculate the coefficient of variation⁷⁶ to equal

$$\sigma/Ex = \sqrt{Hi/Ex - 1} \sqrt{1 - Lo/Ex}.$$

This quantity is calculated for each villager in the HH survey, and the village average is used. Thus it is a measure of average riskiness of occupation in a given village.

WEALTH is another village average variable. Villagers were asked detailed questions about assets of all types – ponds, livestock, appliances, and so on – as well as liabilities. Date of purchase was used to estimate current value after depreciation. These different types of wealth were aggregated for each villager, then averaged across villagers. The unit of measure is one hundred thousand 1997 Thai baht. We control for wealth so that we do not get confounding effects from other variables that are correlated with wealth. An example of these are the availability of outside borrowing options from commercial banks and village banks, the former being positively correlated and the latter negatively correlated with wealth in the data.

Number of MEMBERS is also controlled for. Groups in our data range in size from five to thirty seven, with eleven being the median. However, each model we consider fixes group size at two.⁷⁷ There is theoretical reason to believe group size may affect repayment. As

⁷⁶Let a be the probability of realizing output "Hi" and $1 - a$ be the probability of realizing output "Lo". Given that the expected value must be "Ex", we know $aHi + (1 - a)Lo = Ex$. This gives that $a = (Ex - Lo)/(Hi - Lo)$. The variance σ^2 is, using definitions, $a(Hi - Ex)^2 + (1 - a)(Ex - Lo)^2$. Substituting in our expression for a in terms of Hi, Lo, and Ex, the variance simplifies to

$$\sigma^2 = (Hi - Ex)(Ex - Lo).$$

Dividing by $(Ex)^2$ and taking the square root, we calculate the household's coefficient of variation to be $\sqrt{Hi/Ex - 1} \sqrt{1 - Lo/Ex}$.

⁷⁷Ghatak does generalize this in an appendix.

Table 6.1b - Independent Variables

Variables marked with an asterisk are taken or constructed from the household-level survey, HH. All others are from the group-level survey, BAAC.

Variables marked with a † might also be included under the next set of variables; variables marked with a ‡ might also be included under the previous set.

Variable	Description	Mean	(σ)
<i>Productivity:</i>			
AVGLAND	Average landholdings of group members (rai)	23.6	(15.7)
EDCATION	Index of group average education levels	3.1 ^a	(0.32)
<i>Screening:</i>			
SCREEN	Do some want to join this group but cannot?	0.39	
KNOWTYPE	Do group members know the quality of each other's work?	0.94	
<i>Covariance:</i>			
COVARBTY*	Measure of coincidence of economically 'bad' years across villagers	0.28	(0.16)
HOMOCCUP†	Measure of occupational homogeneity within the group	0.87	(0.24)
<i>Cost of Monitoring:</i>			
LIVEHERE	Percent of group living in the same village	0.88	(0.22)
RELATPCT†	Percent of group members having a close relative in the group	0.58	(0.36)
<i>Cooperation:</i>			
SHAREREL	Measure of sharing among closely related group members	2.1	(1.6)
SHAREUNR	Measure of sharing among unrelated group members	1.5	(1.4)
BCOOPPCT*	Percent in tambon naming this village best in the tambon for "cooperation among villagers"	0.25	(0.11)
JOINTDCD	Number of decisions made collectively	0.37	(0.91)
<i>Outside Credit Options:</i>			
PCGMEM*‡	Percent in village claiming Production Cooperative Group membership	0.08	(0.16)
CBANKMEM*	Percent in village claiming to be clients of a commercial bank	0.28	(0.18)
<i>Penalties for default:</i>			
BINSTPCT*‡	Percent in tambon naming this village best in the tambon for "availability and quality of institutions"	0.27	(0.19)
SNCTIONS*	Percent of village loans where default is punishable by informal sanctions	0.10	(0.11)

^aSee text for the interpretation of this education index.

groups increase in size, monitoring may become harder but the social penalties at the group's disposal may also increase. Generalization of the models we consider may be feasible along the dimension of group size, but it is not attempted in this paper.

Key variables explicitly included in some or all models are the interest rate r , loan size L , and joint liability payment q . Data on groups' loan sizes and interest rates come from a BAAC survey question asking about the highest and lowest loan size and interest rate experienced by any member of the group over the past year. We take these high (hi) and low (lo) figures and use a weighted average $(lo + 0.1 * hi)/1.1$. The high end is only slightly weighted since the upper tail is often quite long and unrepresentative of the group as whole.

Our strategy for proxying the degree of joint liability q exploits the compromise the BAAC makes between individual liability and joint liability. One option the BAAC has toward the end of the process of reclaiming a delinquent loan is to seize assets of the borrower or guarantors, most often land. This fact leads to some variation in the actual degree of liability. If all group members own land, then there is less of a chance that a guarantor will in the end have to pay rather than the borrower himself, since the BAAC can take his land. If on the other hand some members of the group are landless, then the effective degree of joint liability can be thought of as higher, since it is more likely a guarantor will have to repay if a landless borrower defaults. Thus our approach will be to proxy joint liability by a group characteristic available from the BAAC survey, namely the percent of the group who do not own land. This is a clear source of variation and also measures the likeliness of having to pay for a defaulting member, given the BAAC can confiscate land of defaulters.

Productivity shifters include AVGLAND, the average amount of land per group member, measured in rai,⁷⁸ and EDUCATION, average educational attainment in the group. EDUCATION was constructed as follows. The leader was asked to classify each member into one of four categories: no schooling; some schooling, but below P4; P4; and higher than P4 schooling. The majority of borrowers have P4 schooling, the minimum level required by the Thai government. Our measure attempts to summarize this information, but in a necessarily somewhat arbitrary way. It is equal to $1 * (\text{Pct of group with some schooling, but below P4}) + 3 * (\text{Pct of group with P4 schooling}) + 5 * (\text{Pct of group with higher than P4 schooling})$. The exact specification of this seems to make little difference in the empirical results.

Next are two dummy variables from the BAAC survey associated with screening, SCREEN and KNOWTYPE. For KNOWTYPE, groups were asked if members know the quality of each other's work. This speaks to the assumption of Ghatak's model that borrowers know each other's types, which is a necessary prerequisite for screening to occur. For SCREEN, groups were asked whether there are borrowers who would like to join their group but cannot. In the Ghatak model, every group except those of type $p = \underline{p}$ has riskier types below them who would like to move up to safer partners. Thus if groups are sorting under the mechanism specified in the model, they should answer yes. This dummy then measures the degree to which groups are actually screening.

Measures of covariance are included next. COVARBTY is a village-level measure taken from the HH survey. Villagers were asked which of the previous five years were the best and worst economically, respectively. Our variable is constructed as the probability that two respondents chosen at random from the same village reported the same year as worst. If N_v

⁷⁸One rai is approximately equal to 0.4 acres.

is the number of villagers in village v and s_{vy} is the share of villagers in village v who named year y as the worst, this probability is equal⁷⁹ to

$$\frac{(\sum_{y=1}^5 s_{vy}^2) - 1/N_v}{1 - 1/N_v}.$$

Our second proxy for covariance is a measure of occupational homogeneity from the BAAC survey, equal to the probability two randomly chosen group members have the same occupation. It is constructed exactly the same as above. Clearly, this should be positively associated with covariance of output in the group. A cautionary note is that it can also be a measure of the cost of monitoring, those of the same profession being easier to monitor.

Cost of monitoring is further measured by the percent of the group living in the same village, LIVEHERE, and the percent of group members who have a close relative in the group, RELATPCT, both from the BAAC survey. Recall that the model that focuses on cost of monitoring, BBG, inextricably ties monitoring to imposing penalties, making the degree of relatedness a mixed signal. It can also be thought of as a measure for cooperation.

Further measures of cooperation are captured by variables, SHAREREL and SHAREUNR, measuring sharing; a poll of villagers in the HH survey, BCOOPPCT; and JOINTDCD, which counts on how many key productive decisions does some subset of the group, rather than the individual farmer, have the final say. To construct the sharing variables, we utilize six yes/no questions asked each group in the BAAC survey: whether sharing of rice, helping with money, helping with free labor, coordinating to transport crops, coordinating to purchase inputs, and coordinating to sell crops has occurred in the past year. In our measure, we exclude the sharing of rice, since this may indicate not only sharing but a regional character reflected in the predominance of rice farming. The index is thus the number of yes responses to the remaining five questions. These should be correlated with the degree to which the group can make binding agreements. The same set of questions was asked twice, regarding relatives and non-relatives, respectively, within the group. These lead to SHAREREL and SHAREUNR, respectively.

BCOOPPCT comes from a poll of villagers in the HH survey. Each is asked which village in his tambon (subcounty) enjoys the best cooperation among villagers. The percentage of villagers in the HH survey naming the village in which a group is resident is the measure we use. Finally, JOINTDCD counts the number of the following three decisions on which some or all group members, as opposed to the individual farmer, have the final say: which

⁷⁹It is calculated as follows. The number of different pairs of villagers in village v with N_v respondents is

$$\binom{N_v}{2} = N_v(N_v - 1)/2.$$

Similarly, if N_{vy} is the number of year- y respondents in village v , then the number of different pairs of v villagers in which both respondent indicated year v is N_{vy} choose 2, or $N_{vy}(N_{vy} - 1)/2$. Since the question is asked about the previous five years, the measure works out to be

$$\frac{\sum_{y=1}^5 N_{vy}(N_{vy} - 1)/2}{N_v(N_v - 1)/2} = \frac{(\sum_{y=1}^5 N_{vy}^2) - N_v}{N_v^2 - N_v} = \frac{(\sum_{y=1}^5 s_{vy}^2) - 1/N_v}{1 - 1/N_v},$$

where $s_{vy} = N_{vy}/N_v$ is the share of villagers in village v who named year y as the worst.

crops to grow, pesticide and fertilizer usage, and production techniques. Thus it is intended to capture joint choice of project, as is assumed in Stiglitz's model of cooperation.

Measures of outside borrowing opportunities are captured in the variables PCGMEM and CBANKMEM, taken from the HH survey.⁸⁰ They give the percent of villagers in the group's village who are members of a production cooperative group (PCG) or commercial bank, respectively. PCGs are village-run organizations that collect regular savings deposits from members and offer loans after a member has met some threshold requirement involving length of membership, amount deposited, or both. Often the maximum available loans from these institutions are small, possibly one fifth the size of BAAC loans, and the interest rates are similar or slightly higher (see Kaboski and Townsend, 1998). There do exist PCGs large enough to offer loans as large as BAAC loans. Occasionally joint liability is used with these loans.⁸¹ Commercial banks are conventional lenders, requiring collateral. We restrict attention to these two non-BAAC lenders because they probably offer contracts closest to that of the BAAC. Of the two, PCG loans are much closer to BAAC group loans than are commercial bank loans.

A more indirect measure of outside loan options, BINSTPCT, comes from a poll of villagers in the HH survey. Specifically, it is the percent of villagers in the tambon (sub-county) naming the group's village best in terms of availability and quality of institutions. This refers at least in part to lending institutions; thus this captures outside loan availability. The measure probably also captures to some degree the legal infrastructure, which is related to the official penalties the BAAC can impose on borrowers.

Finally, official and unofficial penalties are proxied by BINSTPCT, described in the preceding paragraph, and SNCTIONS, respectively. SNCTIONS comes from the HH survey and is constructed from a question asking villagers what the penalties for default on their current loans are. We use the percent of loans in the village that have penalties extending beyond the direct participants in the loan agreement. Specifically, we count loans for which the borrower reports that under default, he cannot borrow again from this lender *and other* lenders, or that reputation in the village is damaged. This captures very directly a form of unofficial penalties – denial of future credit – that extends not only beyond the BAAC's power to punish, but also the power of the group members themselves.

6.2 Nonparametric and logit results

First we report on some simple non-parametric comparisons that give evidence on how the repayment rate varies with each independent variable separately. Logit results are then reported. Both are done for the whole sample of BAAC groups, as well as by region.⁸²

⁸⁰In measuring outside borrowing opportunities, we shy away from using *actual borrowing by the group* from other sources. This is not for lack of data, but because borrowing can be a sign of repayment problems. It is not uncommon to borrow in order to repay one BAAC debt and enable another. Instead we use measures of *loan availability in the village* from alternate sources in the village. This is presumably much less a function of a single group's repayment problems than is that group's current loans from other sources.

⁸¹PCGs also might be thought of as a measure of cooperation in the village, since they are essentially completely run by villagers, though formation is often spurred by local community development workers. They may measure to some degree the unity and civic pride of the community.

⁸²Recall there are two regions represented in the sample. The more rural and poorer northeast region includes provinces Buriram and Srisaket, while the central region includes Chachoengsao and Lopburi, closer

Each non-parametric comparison is performed across two subsamples that partition the whole sample, one with high values of a given independent variable and the other with low values. If the repayment rate varies monotonically with this independent variable, we should expect to find differences in average repayment rate across the subsamples.⁸³

There is inherent arbitrariness in setting the cutoff value of the independent variable that will partition the sample into groups with high and low values, with obvious exceptions such as dummy variables. To address this, we perform the comparison using *all* two-subsample partitions that satisfy two conditions. First, since the partition must divide groups based high and low values of the independent variable, we only consider partitions in which every group in one subsample has a *strictly* higher value for the independent variable than every group in the other subsample.⁸⁴ Second, we only consider partitions that leave at least fifteen groups in each subsample. The number fifteen is arbitrary, but setting a minimum subsample size ensures some chance for significant mean differences across subsamples in each partition.

Results of the comparisons are listed in Table 6.2a. The first two columns give the results when considering the northeast groups only, the second two give those of the central groups, and the third two give the results for the whole sample. For each of these samples, there are two columns. The first contains the percentage of partitions in which the high subsample yielded significantly *higher* mean repayment rates at a 90% confidence level; the second contains the percentage of partitions in which the high subsample yielded significantly *lower* mean repayment rates at a 90% confidence level. Partitions that produce the first or second of these results give evidence that the repayment rate is monotonically increasing or decreasing, respectively, with the independent variable. The final three columns of the table record the total number of partitions for each sample satisfying the two criteria we imposed, as stated above.

These comparisons are all univariate in the sense that the sample is partitioned based on high and low values of a single independent variable. In principle, it would be possible and desirable to partition using multiple independent variables, in order to identify partial effects. However, the number of combinations of tests and the data requirements grow very quickly as the number of independent variables grows, so we report here the simple univariate comparisons.

Logit regressions allow us to add a multivariate dimension at the expense of the simplifications mentioned in section 3. Results on all variables simultaneously are listed in

to Bangkok.

⁸³Needless to say, partitioning into more than two subsamples and comparing across each combination of subsamples is also possible. However, for simplicity of reporting and interpreting the results, among other reasons, we focus our attention on partitions of the sample into two subsamples.

⁸⁴Note that due to the requirement of strict inequality, there are fewer partitions that satisfy this condition the smaller the number of values the independent variable can take on. For example, SHAREREL, the measure of sharing among group members, is an index equalling the sum of five dummy variables. Thus there will be at most five partitions of the sample based on SHAREREL that satisfy strict inequality in SHAREREL across the subsamples. (These involve setting 0 as low and 1-5 as high, setting 0-1 as low and 2-5 as high, and so on.)

If we only required weak inequality, the number of partitions would in general increase the smaller the number of values the independent variable takes on. This is because ties are allowed across the high and low subsamples, and thus there is an indeterminacy that can be resolved in multiple ways.

Table 6.2a - Univariate Mean Comparisons

For each independent variable, the sample was partitioned into two subsamples as many ways as possible satisfying 1) no subsample had less than fifteen groups and 2) each group in one subsample had strictly higher values for the independent variable than each group in the other subsample. For each partition, a significant difference in the mean of the dependent variable across subsamples was tested for, at the 90% level. (Recall that the dependent variable equals one if the group has never had its interest raised as a penalty for late payment.) The table lists the percentage of partitions for a given independent variable producing significant mean differences, with the sign indicating a positive or negative relationship.

Variables that could be included in the next group are denoted by a †; in the previous group by a ‡.

	Percent of tests significant at 90%, with sign						Total Tests		
	Northeast groups		Central groups		All groups		NE	CE	All
<i>Control:</i>	(+)	(-)	(+)	(-)	(+)	(-)			
LNYSOLD	0%	83%	0%	90%	0%	85%	18	20	26
VARIBLTY	3%	3%	8%	0%	0%	0%	73	50	140
WEALTH	5%	0%	0%	49%	0%	12%	73	51	139
MEMBERS	0%	0%	0%	50%	0%	53%	9	14	17
<i>Fundamentals:</i>									
r	0%	5%	0%	0%	0%	28%	20	16	29
L	0%	3%	0%	0%	0%	9%	37	24	55
q	0%	0%	0%	95%	0%	96%	5	20	27
<i>Productivity:</i>									
AVGLAND	0%	0%	68%	0%	11%	0%	29	34	46
EDCATION	30%	0%	7%	0%	18%	0%	23	14	44
<i>Screening:</i>									
SCREEN	0%	0%	0%	0%	0%	0%	1	1	1
KNOWTYPE	—	—	—	—	0%	0%	0	0	1
<i>Covariance:</i>									
COVARBTY	0%	8%	38%	0%	10%	0%	49	32	71
HOMOCCUP†	0%	0%	0%	0%	0%	0%	11	28	46
<i>Cost of Monitoring:</i>									
LIVEHERE	0%	0%	20%	0%	38%	0%	13	30	42
RELATPCT†	0%	2%	0%	18%	0%	35%	44	39	57
<i>Cooperation:</i>									
SHAREREL	0%	0%	0%	25%	0%	0%	5	4	5
SHAREUNR	0%	0%	0%	67%	0%	50%	4	3	4
BCOOPPCT	0%	3%	0%	0%	0%	2%	60	38	108
JOINTDCD	33%	0%	—	—	100%	0%	3	0	3
<i>Outside Credit Options:</i>									
PCGMEM‡	0%	100%	0%	0%	0%	55%	9	6	11
CBANKMEM	0%	0%	0%	0%	0%	20%	13	9	20
<i>Penalties for Default:</i>									
BINSTPCT‡	0%	0%	8%	0%	0%	0%	63	37	108
SNCTIONS	55%	0%	0%	0%	0%	0%	33	25	51

Table 6.2b. There are 219 groups included in the regression incorporating both regions; 43 are excluded for missing data. There are 130 observations in the regression restricted to the northeast region, and 89 observations in the regression restricted to the central region.

A summary of predictions from the nonparametric tests and the logits is contained in Table 6.2c. There, we also attempt to summarize empirical findings in the literature. When existing studies conflict, we use two arrows. If of equal length, the two arrows indicate relatively equal evidence for both signs; if unequal, the longer one represents the effect with more evidence behind it. For more details on the studies reviewed, see appendix E. The final columns reproduce Table 3a, which summarizes the theoretical predictions of the four models. For more details on the predictions, see section 5 and Table 3a.

Our own results are summarized in the first three columns. The three columns represent results from the northeast region sample, the central region sample, and the combined sample, respectively. The pre-comma entry of each column represents the nonparametric, mean-comparison results of Table 6.2a. If 80% or more of the partitions showed means significantly different at 90%, three arrows are used; if 50% or more showed means significantly different at 90%, two arrows are used; if 20% or more showed means significantly different at 90%, one arrow is used. Otherwise the empty set symbol \emptyset is used. The post-comma entry of each column represents the logit results of Table 6.2b. A coefficient different from zero at 85%, 90%, and 95% significance is represented by one, two, and three arrows, respectively. A coefficient not different from zero at any of these significance levels is represented by \emptyset . In both the nonparametric and logit cases, the arrows point up if mean repayment was increasing in the independent variable and down if the reverse was true.

The final four columns summarize the theoretical predictions. Note that in most cases, the predictions are put on the same row as the thematic group heading, rather than on the row of the variables themselves. This is because we often use multiple variables to proxy what in the models is a single concept. The theoretical predictions do not vary across the different proxies. The exceptions to this rule in the table are the control variables, about which the models by definition have no predictions,⁸⁵ and the category including penalties for default. Since both the theory and our data do distinguish between official and unofficial penalties for default, we list the predictions for both proxies.

The chart treats existing empirical literature similarly. If evidence was found on a variable very similar to one we use, it is reflected on the same line as our variable. If evidence was found on variables that we do not use but serve as proxies for the same concepts we examine, it is reflected on the same line as the group heading. In some cases, the literature has examples of both.

In analyzing the results, we focus primarily on whole-sample results, since they contain the most data. Only if interesting and significant differences emerge from the regional results are they noted.

Of the **control variables**, LNYRSOLD performs utterly predictably with a significant negative correlation with repayment. This is clearly because our dependent variable involves default at any time in the history of the group, which is more likely for older groups. None of the others show up significantly, except MEMBERS, which is negative in the nonparametric tests but insignificantly positive in the logit. To reconcile this difference of outcome, we

⁸⁵Future research could extend these models to incorporate what in this paper are control variables.

Table 6.2b - Logit Results

Dependent Variable = 1 if BAAC has never raised the interest rate as a penalty, 0 if it has.

Significance in parentheses; 85, 90 and 95% denoted by *, **, and ***, respectively.

Variables that could be included in the next group are denoted by a †; in the previous group by a ‡.

	Northeast groups	Central groups	All groups
	N=130	N=89	N = 219
<i>Control:</i>			
LNYRSOLD	-1.55 (.002)***	-1.19 (.046)***	-0.94 (.001)***
VARIBLTY	0.67 (.877)	-7.74 (.122)*	-3.40 (.194)
WEALTH	7.88 (.521)	0.77 (.476)	0.22 (.789)
MEMBERS	-0.004 (.967)	0.097 (.192)	0.043 (.354)
<i>Fundamentals:</i>			
r	-0.044 (.759)	-0.356 (.212)	-0.106 (.296)
L	-0.017 (.414)	0.008 (.691)	-0.014 (.190)
q	-3.60 (.582)	-6.63 (.023)***	-3.40 (.020)***
<i>Productivity:</i>			
AVGLAND	-0.012 (.638)	-0.007 (.760)	-0.004 (.737)
EDCATION	1.82 (.053)**	0.61 (.595)	1.19 (.081)**
<i>Screening:</i>			
SCREEN	-0.94 (.130)*	1.31 (.111)*	-0.318 (.426)
KNOWTYPE	-1.45 (.226)	1.40 (.494)	-0.34 (.650)
<i>Covariance:</i>			
COVARBTY	2.03 (.334)	0.96 (.780)	2.05 (.131)*
HOMOCCUP†	1.72 (.215)	0.47 (.769)	0.44 (.598)
<i>Cost of Monitoring:</i>			
LIVEHERE	-0.78 (.675)	1.09 (.358)	0.85 (.306)
RELATPCT†	-1.49 (.097)**	-1.21 (.301)	-0.82 (.139)*
<i>Cooperation:</i>			
SHAREREL	0.55 (.195)	0.56 (.235)	0.46 (.068)**
SHAREUNR	-0.53 (.200)	-0.75 (.167)	-0.62 (.021)***
BCOOPPCT	-6.70 (.057)**	-5.07 (.366)	-1.98 (.403)
JOINTDCD	0.29 (.420)	1.50 (.043)***	0.49 (.064)**
<i>Outside Credit Options:</i>			
PCGMEM‡	-6.90 (.000)***	-0.93 (.768)	-3.89 (.001)***
CBANKMEM	-1.67 (.473)	0.23 (.920)	0.60 (.614)
<i>Penalties for Default:</i>			
BINSTPCT‡	3.58 (.061)**	3.86 (.271)	1.91 (.151)
SNCTIONS	12.30 (.005)***	0.12 (.972)	3.56 (.066)**

Table 6.2c - Summary of Results

Dependent Variable = 1 if BAAC has never raised the interest rate as a penalty, 0 if it has. Significance in the logit regression at 85, 90, and 95% denoted by one, two, and three arrows, respectively; similarly, significant mean differences at the 90% level in 20, 50, and 80% of the nonparametric, univariate tests denoted by one, two, and three arrows, respectively.

Variables that could be included in the next group are denoted by a †; in the previous group by a ‡.

Predictions that require us to make non-trivial assumptions are marked with a §.

Variable	NE (Nonparametric,Logit)	CE	All	Others' Results	S	BBG	BC	G
<i>Control:</i>								
LNYSOLD	↓↓↓,↓↓↓	↓↓↓,↓↓↓	↓↓↓,↓↓↓		np	np	np	np
VARIABLETY	∅,∅	∅,↓	∅,∅	↓↑	np	np	np	np
WEALTH	∅,∅	↓,∅	∅,∅	missing	np	np	np	np
MEMBERS	∅,∅	↓↓,∅	↓↓,∅	↑	np	np	np	np
<i>Fundamentals:</i>								
r	∅,∅	∅,∅	↓,∅	missing	↓	↓	↓	↓
L	∅,∅	∅,∅	∅,∅	↓	↓	↓	np	↑§
q	∅,∅	↓↓↓,↓↓↓	↓↓↓,↓↓↓	missing	↓§	↑	np	↓
<i>Productivity:</i>								
AVGLAND	∅,∅	↑↑,∅	∅,∅	↑				
EDUCATION	↑,↑↑	∅,∅	∅,↑↑	∅				
<i>Screening:</i>								
SCREEN	∅,↓	∅,↑	∅,∅	↑	np	np	np	↑
KNOWTYPE	∅,∅	∅,∅	∅,∅					
<i>Covariance:</i>								
COVARBTY	∅,∅	↑,∅	∅,↑	missing	↑	np	↓§	↑
HOMOCCUP†	∅,∅	∅,∅	∅,∅	↓				
<i>Ease of Monitoring:</i>								
LIVEHERE	∅,∅	↑,∅	↑,∅	↑	np	↑	np	np
RELATPCT†	∅,↓↓	∅,∅	↓,↓	↓↑				
<i>Cooperation:</i>								
SHAREREL	∅,∅	↓,∅	∅,↑↑		↑	↓§	↓§	—
SHAREUNR	∅,∅	↓↓,∅	↓↓,↓↓↓					
BCOOPPCT	∅,↓↓	∅,∅	∅,∅					
JOINTDCD	↑,∅	∅,↑↑↑	↑↑↑,↑↑					
<i>Outside Credit Options:</i>								
PCGMEM‡	↓↓↓,↓↓↓	∅,∅	↓↓,↓↓↓	↓↑	↓	np	np	↓
CBANKMEM	∅,∅	∅,∅	↓,∅					
<i>Penalties for Default:</i>								
BINSTPCT‡	∅,↑↑	∅,∅	∅,∅	missing	np	np	↑	np
SNCTIONS	↑↑,↑↑↑	∅,∅	∅,↑↑	↑	np	np	↑	np

look at the results of two locally linear regressions. The first involves MEMBERS as the independent variable and repayment as the dependent variable. In the second, we separately regress first MEMBERS, then repayment, on all the other independent variables. We then take the residuals from these linear regressions and relate them using the same locally linear regression technique.

The particular local linear regression technique we use is similar to Lowess (see for example Cleveland 1979). For each unique value of the independent variable, we calculate a fitted value of the dependent variable from a weighted least squares regression on a "nearby" subset of the total sample. Thus the choices are weights for the regression and a bandwidth which determines the subsample. The bandwidth $h(x)$ is set for each unique value x of the independent variable to ensure inclusion of the 80% of the data whose values x_g are closest to x .⁸⁶ The weighting function is the tri-cube weighting function:

$$w_g = \left(1 - \left(\frac{|x - x_g|}{1.0001h(x)}\right)^3\right)^3.$$

This function places more weight on observations located more closely to x . Standard errors at 90% confidence are calculated using the bootstrap method, that is, recreating 1000 samples from the original sample by sampling with replacement, and calculating the fitted values for each unique value x of the *original* sample, using the same technique described above on each of the bootstrapped samples. For each value x , the confidence interval is the 51st and 950th smallest fitted value from these samples.

The results are plotted in Figure 9. The thrust of these graphs is that the negative correlation appears to be an artifact of the univariate nature of the comparisons. Once we control for the other covariates, albeit in a linear fashion, any hint of a monotonic negative relationship is erased. If anything, the relationship takes on what may be an inverted-U shape, which could accord with the idea that repayment improves as risk-sharing capabilities increase, then declines as free-riding in a large group becomes problematic. In the case of this variable, MEMBERS, the reason for the difference of results is easily found in a significant, positive correlation between group size and age of group in our data. Older groups tend to have more members and also have a greater chance of having defaulted in their history. When group age is controlled for, MEMBERS no longer appears negative.

As for the contract variables, the results on q are most robust, appearing in both types of tests in the whole sample and the central region subsample. These results strongly favor Stiglitz and Ghatak over BBG, especially in the central region. This is an interesting result, perhaps paradoxical, given the popularity of these types of contract. But recall, the main idea in the two models that predict this decline is that increasing q allows a decrease in r , while here we hold r fixed and vary q .

All the models agree that repayment decreases with r . Both types of results bear this out to a small degree, though the logit coefficient is insignificant. Given that any bias on the coefficient is working in favor of a negative relationship, due to interest rate penalties for default, the lack of results is all the more telling. One potential cause is lack of variation in r across groups, due to rigid national BAAC policy on interest rates.

⁸⁶If there are clusters of observations at the boundary of the bandwidth with the same value for the independent variable, all are included. Thus potentially more than 80% of the sample is used.

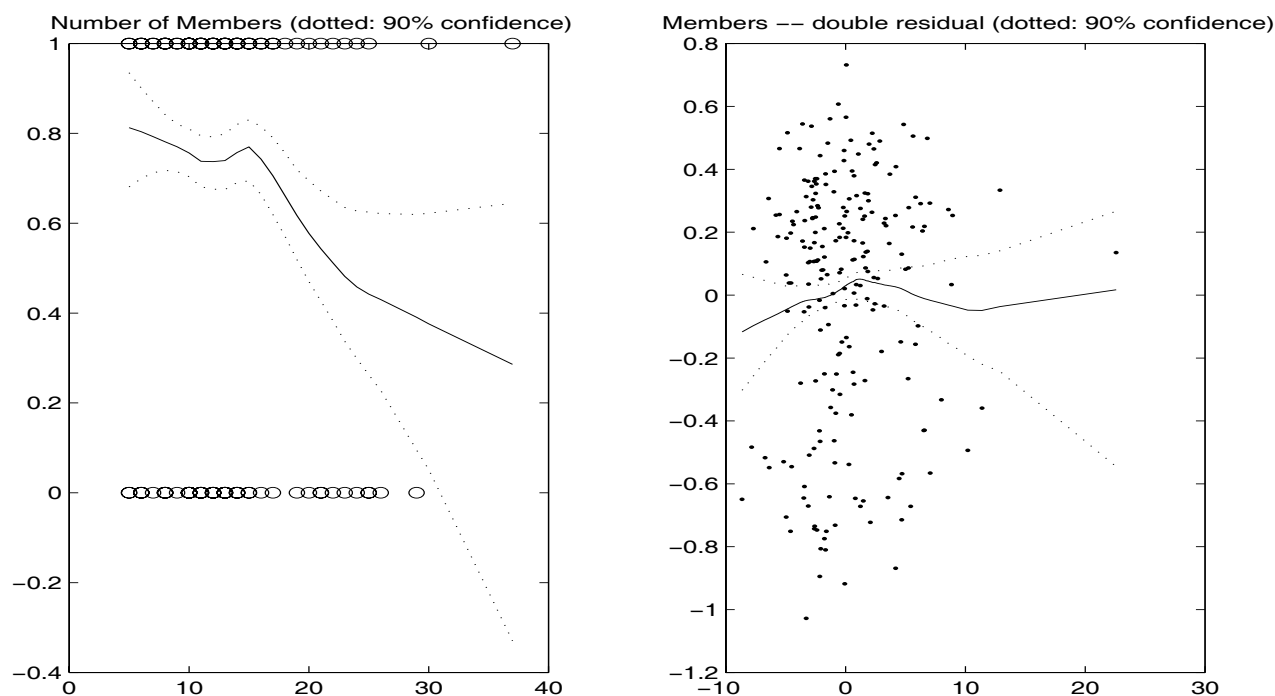


Figure 9: Locally linear regressions - Number of Members against Repayment.

There is also very slight evidence from the nonparametric tests that loan size L is a negative predictor of repayment. However, L could also be endogenous, an issue partly addressed in section 3. Lenders are often thought to increase loan size over time as groups show good performance. If the BAAC increases loan size for groups performing well, then high L would be associated with good repayment. In other words, the bias works in the opposite direction of the barely-insignificant results we get, increasing our confidence that the relationship is indeed negative. Overall, the results on r and L are not strong.

Productivity represented by our direct measure of education turns up evidence in favor of all four theories. This contrasts with previous empirical studies, in which the only measure of human capital used was a measure of literacy and the coefficient was not significantly different from zero. However, the logit produces a more significant positive relationship than the mean comparisons. As with MEMBERS described above, we resort to nonparametric, locally linear regressions for further evidence. The results from the regressions involving EDUCATION and repayment, and the residuals of each, are pictured in Figure 10. Here the effect in the double residual case seems close to what the logits give, that is a clear positive relationship, though in the locally linear regression there is a slight change of direction at the end. The absence of significant effects then in the mean comparisons seems likely to be due to the bivariate nature of the tests.

The dummies related to **screenieen groups that responded no to the question**. The one exception is in the central region logit, where SCREEN carries a positive and significant sign, as predicted by Ghatak. This is further evidence in that the Ghatak model seems to hold fairly well in the central region.

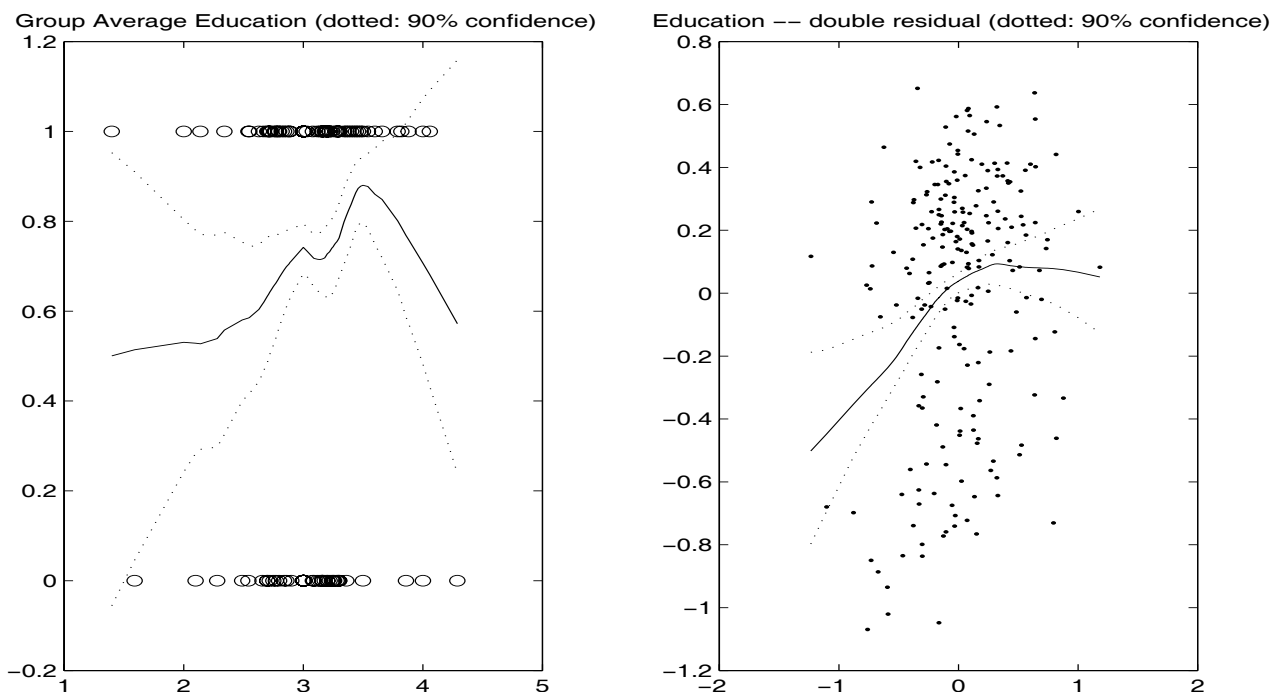


Figure 10: Locally linear regressions - Group Average Education against Repayment.

The direct measure of village **covariance of output**, COVARBTY, is a significant predictor of good repayment in the whole-sample logit, though not as much in the nonparametric tests. This result is in line with the predictions of Stiglitz and Ghatak. It is at odds with BC under certain assumptions as well as the empirical literature, which assumes high correlation should lead to lower repayment. The more indirect measure of covariance that measures homogeneity of occupations within the group, HOMOCCUP, is an insignificant predictor in each test. As with education, we report results from locally linear regressions in order to clarify the differences in results for COVARBTY between the mean comparisons and logits, in Figure 11. As above, the double residual version appears more in line with the logit result, yielding a steadily increasing curve, though the slope is small at first.

The **cost of monitoring** proxies show mixed performance. LIVEHERE, the percentage of group members living in the village is insignificant in the logits, but shows some signs of a positive relationship with repayment in the nonparametric comparisons. This agrees with the BBG model that lower costs of monitoring should improve repayment performance. Again, we use locally linear regressions of LIVEHERE against repayment, as well as the double residual version of the same pair, to understand the differences in results. The regression results are graphed in Figure 12. The second graph is closer to the logits in showing an weak, statistically insignificant positive relationship. Further, RELATPCT, the percent of members with a close relative in the group, which should also proxy the ease of monitoring,⁸⁷ is significantly and negatively associated with repayment, in both types of tests. This contradicts the cost of monitoring prediction of the BBG model. However, that

⁸⁷RELATPCT can also be taken as a proxy for cooperation.

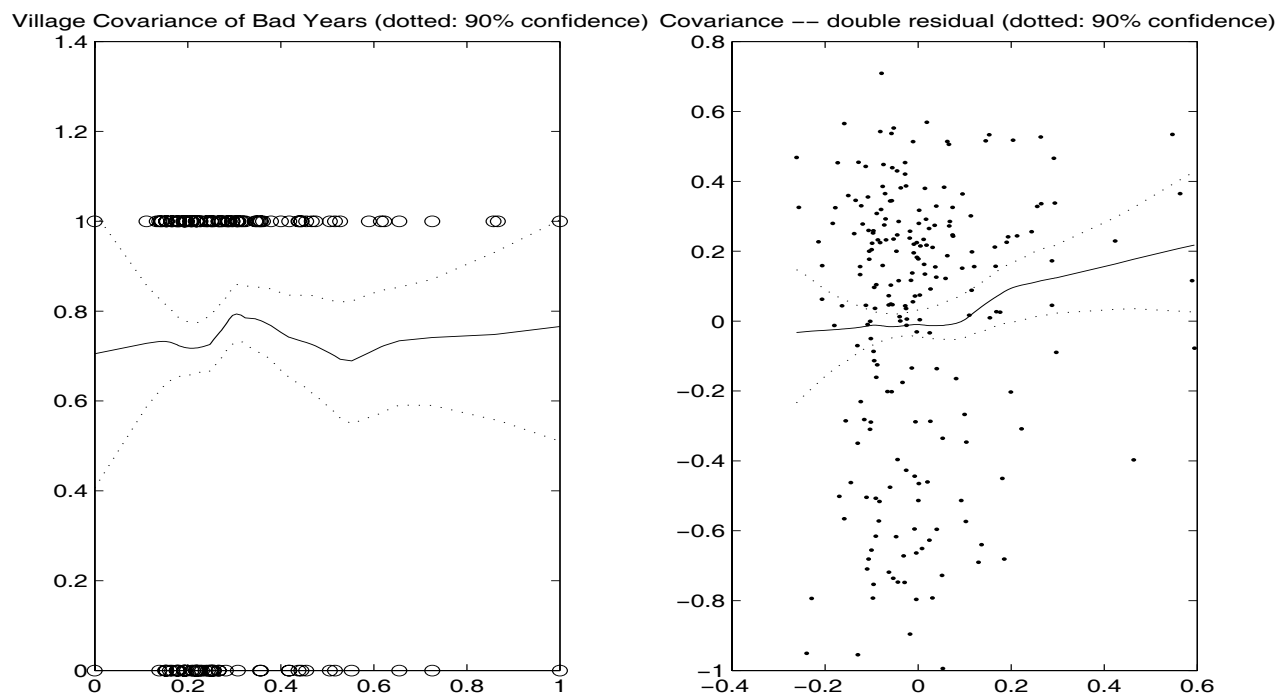


Figure 11: Locally linear regressions - Village covariability against Repayment.

model equates monitoring with the ability to impose penalties; it may be that imposing penalties is harder among relatives.

Penalties for default are crucial in the BC model, and their importance is upheld by these data, overwhelmingly so in the northeast region. We seem to be the first to document this, as the one paper in the literature to date that examined the issue found little effect. The unique measure for unofficial penalties – the exclusion of a delinquent borrower from future credit in his village – shows up very positively and significantly, overwhelmingly so in the poorer northeast region. Once again, the results of the univariate mean comparisons differ from that of the logits. We report the results from locally linear regressions of repayment on SNCTIONS, and the analogous double residual regression, in Figure 13. These results closely mimic the logits and mean comparisons: in both cases the multivariate tests show strong positive relationships, while the univariate tests are murky. Since our theoretical results are derived *ceteris paribus*, multivariate tests are more desirable. We also find a more or less direct measure of institutional capacity in the village to be a positive predictor of repayment in the northeast logits. This can be taken as a proxy for official penalties (which depend on the legal system), and thus provides further confirmation of BC.

The **availability of outside credit sources** seems to hurt repayment. The prevalence of PCGs, which are basically village-run savings and loan institutions, is negatively and very significantly associated with repayment in both types of tests, especially the logits. This result is extremely strong in the northeast region.⁸⁸ The fact that the loan size of

⁸⁸One worry is that the prevalence of PCGs may indicate that an area has been abandoned by institutional lenders. This would lead to a potential endogeneity problem, if bad repayment is affected by some village-

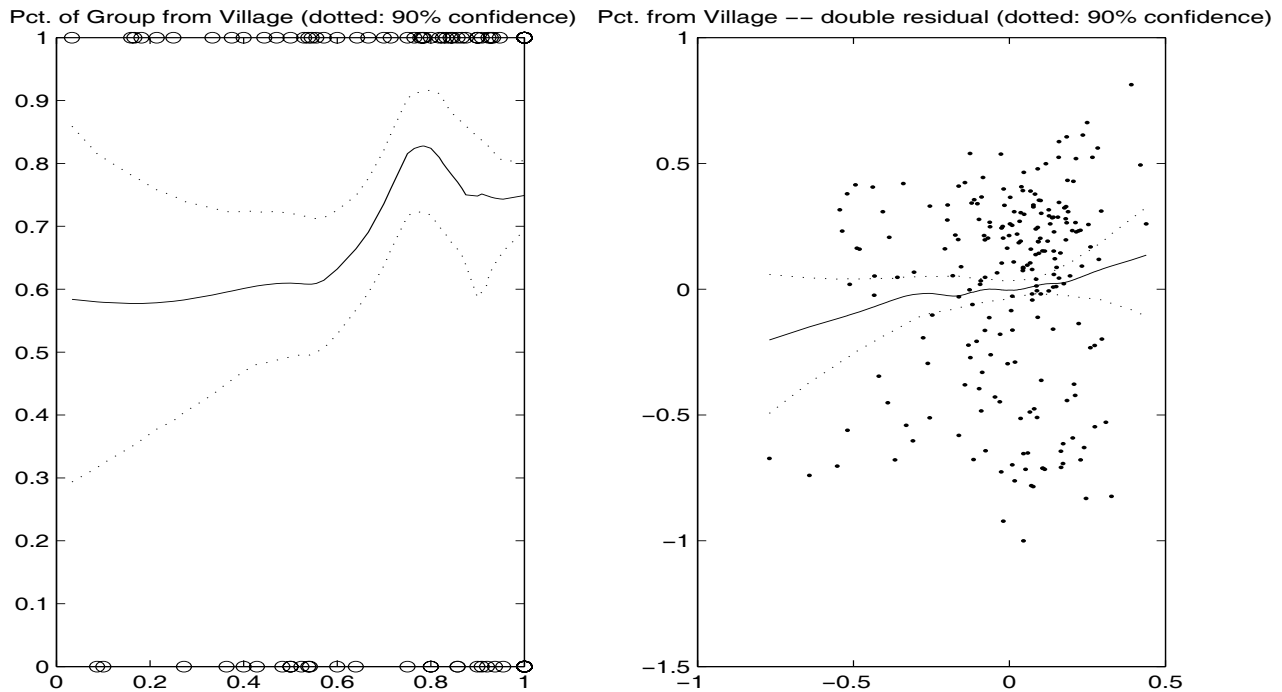


Figure 12: Locally linear regressions - Percent of Group from Village against Repayment.

the average PCG is about 10-20% that of a typical BAAC loan (though contract terms can be better and delay less) puts this result in greater relief, implying that access to even small amounts of outside credit can hurt repayment.⁸⁹ The evidence on commercial bank prevalence is weaker. It is significant in just 20% of the univariate tests, where it too is a negative predictor, and not at all in the logit. However, commercial banks typically serve a different segment of the capital market, borrowers with legal collateral. Thus it is not surprising the result is weak. Overall, the evidence appears strong that the presence of other lenders decreases repayment to the BAAC.⁹⁰

For robustness, we report both types of locally linear regressions for the PCG prevalence variable. The results are shown in Figure 14. There it is clear that controlling for the other covariates results in a much clearer negative relationship between PCGMEM and repayment, a fact that is evident to some degree by the stronger results in the logits as compared to the mean comparison tests.

Finally, we use rich data on sharing, relatedness, and decision-making in the group to level variable we do not observe but these institutional lenders do. However, this does not appear to be the case, as there appears to be a positive, not negative, correlation between village borrowing from the BAAC and PCG prevalence. We calculate a correlation coefficient of 0.072 between percentage of villagers' loans that come from the BAAC with percentage of villagers who are PCG members.

⁸⁹Another alternative explanation is that PCGs essentially proxy cooperation within the village, since they are village-run banks. Separating these two effects is difficult.

⁹⁰It should be noted that BINSTPCT, a measure of institutional availability, probably also reflects lending institutions to some degree. In this case, its positive correlation with repayment, at least in the northeast logit, would dampen our conclusion that outside borrowing options lower repayment.

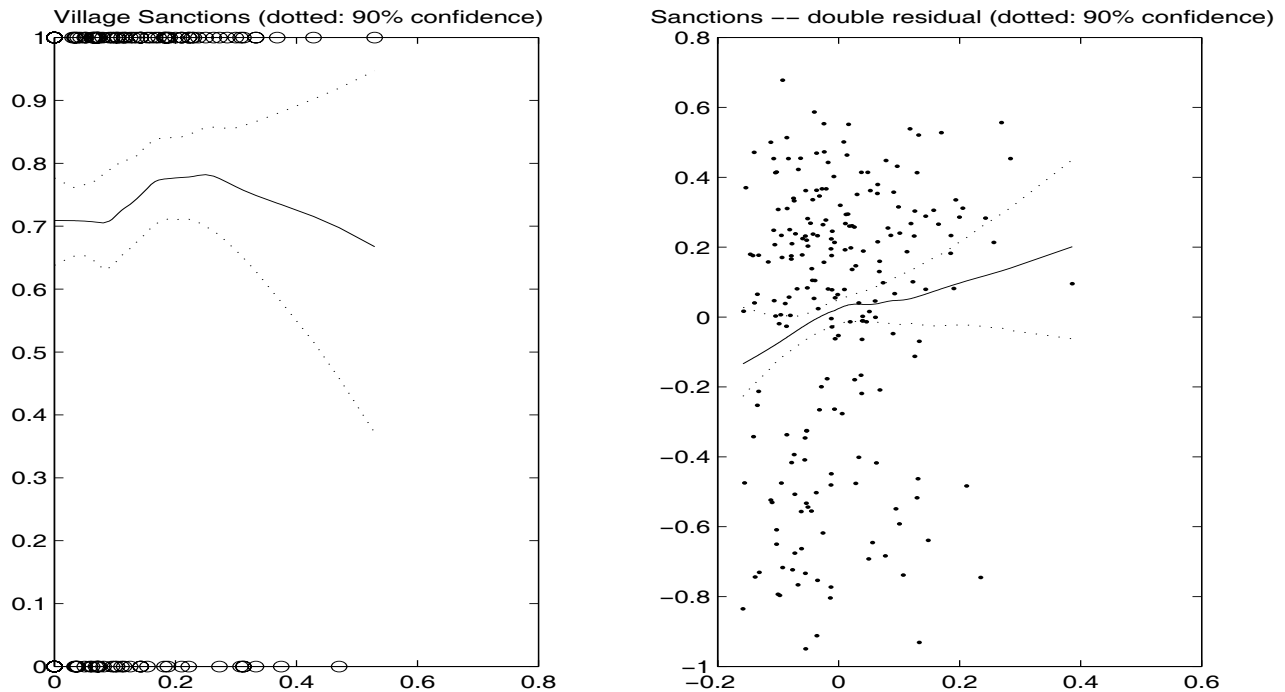


Figure 13: Locally linear regressions - Village Sanctions against Repayment.

clarify the puzzle surrounding the effect of **cooperation** on repayment. If anything, our results seem to tip the scale toward establishing cooperation’s negative effect on repayment rates (though not necessarily borrower welfare).

If we take the extent of sharing among group members as the proxy for cooperation, it appears cooperation is negatively associated with repayment. This is most clear from SHAREUNR, which measures sharing among non-relatives in the group. Both types of tests show significantly negative correlations. There is also some evidence from the univariate tests that SHAREREL, which measures sharing among relatives in the group, is a negative predictor of repayment, at least in the central region.

Oddly, SHAREREL becomes a positive and significant predictor of repayment in the multivariate logit. It is clear that this result is only due to the controlling for sharing among non-relatives, SHAREUNR. If we exclude sharing among non-relatives from an otherwise unaltered logit regression, the sign on SHAREREL becomes negative, though insignificant. Thus the result suggests that sharing per se within the group is bad for repayment; but holding fixed sharing among non-relatives, sharing among relatives is good for repayment. Explaining this result seems to require a theory far more precise than a casual invoking of “social capital”. One possible explanation is that the degree of sharing that takes place within the family shows the strength of the family structure. This may also be correlated with religious beliefs and values which have their own effect on repayment.⁹¹

⁹¹Religious beliefs can be an important determinant of repayment. In conversation with one of the authors, one group leader in the northeast said it is just a sense of honesty that leads villagers to repay. The answer to the next question – what makes them honest? – had nothing to do with horizontal reputational arguments,

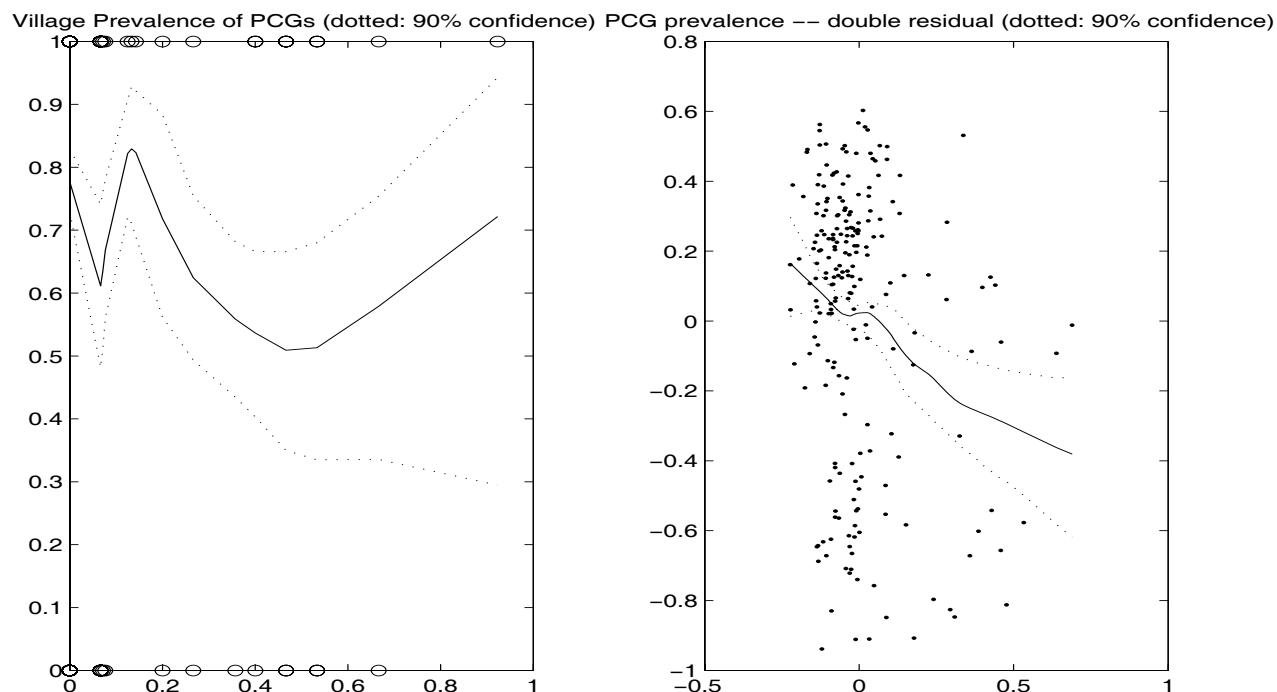


Figure 14: Locally linear regressions - PCG prevalence against Repayment.

The third measure of cooperation, BCOOPPCT, results from a poll of villagers as to which villages exhibit the most cooperation. It is always negative in sign but only significant in the northeast sample logit and in a few nonparametric tests on the northeast and complete samples. This adds slightly more evidence that cooperation is associated with lower repayment.

Two variables that are primarily classified in other categories, RELATPCT in cost of monitoring and PCGMEM in outside borrowing options, may also capture the degree of cooperation in the group. One might think it easier to make binding ex ante agreements among relatives, and the existence of PCGs is evidence of a certain level of self-enforcement in the village. These variables also show significant negative relationships with repayment in both kinds of tests on these data, as mentioned above.

Thus far there is strong evidence that cooperation hurts repayment, favoring the BBG and BC models over the Stiglitz model. However, one strong exception to this conclusion arises in JOINTDCD, which is a positive predictor of repayment in both the logit and the nonparametric tests.

Making sense of this apparent contradiction is difficult. Recall that JOINTDCD is the number of production decisions (regarding pesticides and fertilizers, for example, and which crops to grow) over which some or all group members have the final say. One possibility is that JOINTDCD does not measure cooperation in the sense of being able costlessly to *enforce* agreements from which individuals have incentives to deviate, but rather may reflect a transfer of knowledge and expertise. Under this interpretation, JOINTDCD could fit under but that Buddha would punish those who cheat.

the heading of borrower productivity, and a positive sign would thus match our other results and all the models' predictions. Varian (1990) examines incentives for the transfer of human capital between jointly liable borrowers.

A second intriguing possibility is that the ability to cooperate is heterogeneous across different types of actions, and that JOINTDCD reflects cooperation on *project choice* as opposed to cooperation on *punishment behavior*. This is arguably the case since JOINTDCD reflects cooperation in explicit production decisions. In this case, it would best serve as a measure of cooperation in a moral hazard model such as Stiglitz. Stiglitz is exactly the model that predicts a positive effect of cooperation.⁹² Under this interpretation, the other cooperation variables most likely measure cooperation in general or cooperation on punishment behavior, and hence are negative. What is attractive here is that the three models with predictions on cooperation can be reconciled to this otherwise odd result.

One fact that supports both interpretations is that the correlation of JOINTDCD with each of the other measures of cooperation is statistically insignificant. Thus JOINTDCD appears to be measuring a different type of cooperation or a different phenomenon entirely.

The overall conclusion that seems warranted is that cooperation is harmful for repayment. It may, however, be that cooperation in project choice is helpful, but cooperation in punishment strategies is not.

Table 6.2c is very useful in evaluating across the models. There are three kinds of variables. The first kind consists of variables that elicit predictions from multiple models, all of which agree. This includes r , productivity, and outside credit options. We find evidence, quite strong in two of the three cases, to support the model's unanimous predictions.

The second type of variable is the focal variable in exactly one of the models. This includes screening in the Ghatak model, ease of monitoring in BBG, and penalties in BC. BC is powerfully confirmed along this dimension, with official and unofficial penalties being quite strong predictors of repayment, especially in the northeast sample. Ghatak is given slight confirming evidence in the central sample and slight contradicting evidence in the ruralcludes those which the models disagree about. This includes L , q , covariance, and cooperation. Interestingly enough, all four models fail on exactly one of these variables. Ghatak predicts higher loan sizes increase repayment, while the empirical literature suggests otherwise. (Our results are not quite statistically significant on this variable, but agree in sign with the literature.) BBG predicts higher q increases probability of repayment, while our evidence argues strongly the other way (though not in the northeast region). BC predicts that higher covariance of borrower output generally lowers repayment. Our results from a direct measure of covariance suggest the reverse (again, not in the northeast region). Finally, Stiglitz predicts groups that can enforce ex ante agreements should repay more often, while our results on cooperation tend to point in the opposite direction. Given the non-definitive nature of the results in some cases and interpretations of the results in others, none of these errors can be considered the final word (with the possible exception of q and BBG). However, these are suggestive and begin to point us toward a set of stylized facts future models should aim to match.

If attention is restricted to the northeast region, it can be said definitively that BC is quite

⁹²Note that BBG is also a moral hazard model, but predicts the opposite effect of cooperation. However, like BC, BBG is a model of penalties and this drives the result on cooperation.

strongly upheld along key dimensions, in particular unofficial penalties, and not contradicted in any. In the central region, however, Stiglitz and Ghatak emerge unscathed, as BBG's prediction on q is contradicted and BC's prediction on correlation appears contradicted. It is quite possible, and not surprising, that different mechanisms are at work in the different regions, with joint liability potentially solving more of a selection and motivation problem in the central region and an enforcement problem in the northeast.

7 Conclusion

[UNDER CONSTRUCTION]

We have compiled and helped construct a theoretical framework through which to view repayment data of joint liability borrowing groups and to test between theories regarding them. Using this and rich data on group characteristics and the villages where they are located, four models were compared.

We find that the Besley and Coate model of social sanctions that prevent strategic default performs remarkably well, especially in the northeast region. The Ghatak model of screening by risk type is reasonably supported in the central region, though it has a mixed performance overall. The Stiglitz model's predictions are also borne out in the data, except its result that cooperation will be good for repayment. The Banerjee, et al model for the most part fails in explaining repayment determinants. A more detailed summary of our findings is found in the introduction.

One of the most striking aspect of the results for policy implications is the repeated confirmation of strong social ties – measured by sharing among non-relatives, cooperation, clustering of relatives, and village-run savings and loan institutions – having *adverse* effects on repayment performance. This result has not been seen in the previous literature, nor focused on in the theoretical models. On the contrary, social ties are seen as positive for group lending.

Clearly this idea must be qualified: social structures that enable penalties can be helpful for repayment, while those which discourage them can lower repayment. However, a higher repayment rate is not always synonymous with higher welfare. It can merely reflect the use of cheap penalties to enforce repayment when it is not (ex ante) pareto optimal for the group. Thus joint liability may flourish most in areas where social penalties are especially severe and borrowers are worse off than they would be if they were cooperating.⁹³

⁹³See the work of anthropologist Aminur Rahman on Grameen borrowers. Of course, if they still borrow, they should be better off doing so.

A Proofs from Section 5.1 (Stiglitz)

Proof of Lemma 1. We will show that if $Y(p_S, L) \geq Y(p_R, L)$, equation 4 cannot hold. For convenience, let $Y \equiv Y(p_R, L)$. Note that if $Y(p_S, L) \geq Y(p_R, L)$ and since U is increasing by assumption A4,

$$p_S^2 U[Y(p_S, L) - rL] + p_S(1 - p_S)U[Y(p_S, L) - rL - qL] \geq p_S^2 U(Y - rL) + p_S(1 - p_S)U[Y - rL - qL].$$

The left side of this inequality is just V_{SS} . Using this fact and the expression found in equation 4, we have that

$$V_{SS} - V_{RR} \geq (p_S^2 - p_R^2)U(Y - rL) + [p_S(1 - p_S) - p_R(1 - p_R)]U(Y - rL - qL),$$

or, rearranging,

$$V_{SS} - V_{RR} \geq (p_S^2 - p_R^2)[U(Y - rL) - U(Y - rL - qL)] + (p_S - p_R)U(Y - rL - qL) > 0.$$

This expression is greater than zero since $p_S > p_R$ and U is strictly increasing. Thus $Y(p_S, L) \geq Y(p_R, L)$ implies that $V_{SS} - V_{RR} > 0$, that is, equation 4 does not hold. ■

Proof of Proposition 1. Note that $V_{kk,r}$ is equal to

$$-p_k^2 L U'[Y(p_k, L) - rL] - p_k(1 - p_k) L U'[Y(p_k, L) - rL - qL],$$

where $k \in \{R, S\}$. Letting q approach zero, this expression converges to

$$-p_k L \{U'[Y(p_k, L) - rL]\},$$

since U is continuously differentiable. We then have that

$$V_{RR,r} - V_{SS,r}|_{q \rightarrow 0} = L \{p_S U'[Y(p_S, L) - rL] - p_R U'[Y(p_R, L) - rL]\} > 0. \quad (63)$$

The inequality holds since $p_S > p_R$; since by lemma 1, $Y(p_S, L) < Y(p_R, L)$; and since U is concave by assumption A4. Then by assumption A4 on the continuous differentiability of U , $V_{RR,r} - V_{SS,r}$ is also strictly positive for q small enough. Using this in equation 5, it is clear that there may be a cutoff r , such that for $r_g \leq r$, safe projects are chosen ($p_i = p_S$), and for $r_g > r$, risky projects are chosen ($p_i = p_R$).

Now $V_{kk,L}$ is equal to

$$p_k^2 \partial U[Y(p_k, L) - rL] / \partial L - p_k(1 - p_k) \partial U[Y(p_k, L) - rL - qL] / \partial L,$$

where $k \in \{R, S\}$. Again letting q approach zero, this converges to

$$p_k \partial U[Y(p_k, L) - rL] / \partial L,$$

since U is continuously differentiable. It is then direct from assumption A5 that

$$V_{SS,L} - V_{RR,L}|_{q \rightarrow 0} < 0. \quad (64)$$

This strict inequality also holds for q near zero due to the continuous differentiability of U and Y assumed in A4 and A2. Using this in equation 5, it is clear that there may be a cutoff L , such that for $L_g \leq L$, safe projects are chosen ($p_i = p_S$), and for $L_g > L$, risky projects are chosen ($p_i = p_R$). ■

Proof of Proposition 2. Note that

$$V_{kk,q} = -p_k(1 - p_k)(L)U'[Y(p_k, L) - (r + q)L], \quad k \in R, S. \quad (65)$$

Holding r and L constant, an increase in q has a negative effect on both risky and safe payoffs. We then have that

$$V_{RR,q} - V_{SS,q} = L\{p_S(1 - p_S)U'[Y(p_S, L) - (r + q)L] - p_R(1 - p_R)U'[Y(p_R, L) - (r + q)L]\}. \quad (66)$$

By lemma 1 and the concavity of U , we know that

$$U'[Y(p_S, L) - rL - qL] > U'[Y(p_R, L) - rL - qL].$$

Assumption A3 gives that $p_S(1 - p_S) \geq p_R(1 - p_R)$. Thus $V_{SS,q} - V_{RR,q} < 0$. Using equation 5, it is clear that there can exist some q , such that for $q_g \leq q$, group g chooses safe projects ($p_i = p_S$) and for $q_g > q$, group g chooses risky projects ($p_i = p_R$). In this sense, probability of repayment is lower the higher is q . ■

Proof of Proposition 3. We reproduce equation 5 that gives the repayment rate, call it p_c , for a group acting cooperatively:

$$p_c(r, L, q) = p_R + (p_S - p_R)1\{V_{SS}(r, L, q) \geq V_{RR}(r, L, q)\}.$$

The non-cooperative repayment rate, call it p_{nc} , is⁹⁴

$$p_{nc}(r, L, q) = p_R + (p_S - p_R)1\{V_{SS}(r, L, q) \geq V_{RR}(r, L, q)\} \\ + p_R(p_S - p_R)\{U[Y(p_R, L) - rL] - U[Y(p_R, L) - (r + q)L]\},$$

as is clear from Nash equilibrium condition 8. Now $0 < p_R < p_S$ and $U[Y(p_R, L) - rL] - U[Y(p_R, L) - (r + q)L] > 0$, since U is strictly increasing. Thus any (r, L, q) combination that results in $p_{nc} = p_S$ also results in $p_c = p_S$. However, there clearly exist (r, L, q) combinations that give $p_c = p_S$ but $p_{nc} = p_R$ (for example, any combinations on or just below the cooperative Switch Line). Therefore, all of (r, L, q) space that leads to safe project choice for non-cooperative groups does so for cooperative groups; and there is a positive measure of (r, L, q) space that leads to safe project choice for cooperative groups and risky project choice for non-cooperative groups. ■

⁹⁴This Assumes that groups acting non-cooperatively choose safe projects whenever such an equilibrium exists. Since there can exist values where both safe and risky project choices are equilibria, this assumption attributes the highest repayment rate possible to non-cooperative behavior.

Proof of Proposition 4. Rewriting equation 5 in new notation, we have that

$$p_g(r_g, L_g, H_g, q_g) = p_R + (p_S - p_R)1\{V_{SS}(r_g, L_g, H_g, q_g) \geq V_{RR}(r_g, L_g, H_g, q_g)\}, \quad (67)$$

where $1\{\}$ represents the indicator function and g indexes a group. Clearly the effect of H_g on p_g depends on whether V_{SS} or V_{RR} is affected more by a change in H_g .

Using equation 3, we calculate that

$$V_{kk,H} = Y_H(p_k, L, H) \{p_k^2 U'[Y(p_k, L, H) - rL] + p_k(1 - p_k)U'[Y(p_k, L, H) - (r + q)L]\}, \quad (68)$$

where $k \in \{R, S\}$ and the H subscript denotes the partial derivative. Rearranging, we get that

$$V_{kk,H} = p_k Y_H(p_k, L, H) \{ U'[Y(p_k, L, H) - (r + q)L] + p_k [U'[Y(p_k, L, H) - rL] - U'[Y(p_k, L, H) - (r + q)L]] \}, \quad (69)$$

Note that the entire second line approaches zero as q approaches zero, since U is continuously differentiable. We also have that

$$U'[Y(p_S, L, H) - (r + q)L] > U'[Y(p_R, L, H) - (r + q)L].$$

This is because U is concave and $Y(p_S, L, H) < Y(p_R, L, H)$ by lemma 1. Finally, we must show that $p_S Y_H(p_S, L, H) > p_R Y_H(p_R, L, H)$.

Using the separability assumption A6, we can write $p_k Y_H(p_k, L, H) = p_k z(p_k) F_H(L, H)$. Substituting A6 into assumption A1, we know that $p_S z(p_S) F(L, H) > p_R z(p_R) F(L, H)$, which directly implies that $p_S z(p_S) > p_R z(p_R)$. This is sufficient for

$$p_S Y_H(p_S, L, H) = p_S z(p_S) F_H(L, H) > p_R z(p_R) F_H(L, H) = p_R Y_H(p_R, L, H).$$

We have thus shown that $V_{SS,H} - V_{RR,H} > 0$ for q small enough. Using this in equation 67, it is clear that there may be a cutoff H , such that for $H_g \geq H$, safe projects are chosen ($p_i = p_S$), and for $H_g < H$, risky projects are chosen ($p_i = p_R$). In this sense, the probability of repayment is higher the higher is H . ■

Proof of Proposition 5. Under either covariance assumption, equation 11 can be written as

$$p_i(r_g, L_g, q_g, \epsilon_g) = p_R + (p_S - p_R)1\{V_{SS}(r_g, L_g, q_g, \epsilon_g) \geq V_{RR}(r_g, L_g, q_g, \epsilon_g)\}. \quad (70)$$

Clearly the effect of ϵ_g on p_i depends on whether V_{SS} or V_{RR} is affected more by a change in ϵ_g .

We first examine the covariance of assumption A7. Using equation 10, we calculate that

$$V_{kk,\epsilon} = U[Y(p_k, L) - rL] - U[Y(p_k, L) - (r + q)L], \quad (71)$$

where $k \in \{R, S\}$ and the ϵ subscript denotes the partial derivative. Clearly,

$$U[Y(p_S, L) - rL] - U[Y(p_S, L) - (r + q)L] > U[Y(p_R, L) - rL] - U[Y(p_R, L) - (r + q)L], \quad (72)$$

since U is concave and $Y(p_S, L) < Y(p_R, L)$ by lemma 1. Thus $V_{SS,\epsilon} - V_{RR,\epsilon} > 0$.

Alternatively, the covariance of assumption A8 leads to the partial derivative

$$V_{kk,\epsilon} = p_k(1 - p_k)\{U[Y(p_k, L) - rL] - U[Y(p_k, L) - (r + q)L]\}. \quad (73)$$

Assumption A3 gives us that $p_S(1 - p_S) \geq p_R(1 - p_R)$. This in addition to inequality 72 and the reasoning following it gives us that $V_{SS,\epsilon} - V_{RR,\epsilon} > 0$.

Next we use the fact that $V_{SS,\epsilon} - V_{RR,\epsilon} > 0$, which is true under both types of covariances as discussed in the preceding two paragraphs, in equation 70. It is clear that there may be a cutoff ϵ , such that for $\epsilon_g \geq \epsilon$, safe projects are chosen ($p_i = p_S$), and for $\epsilon_g < \epsilon$, risky projects are chosen ($p_i = p_R$). In this sense, the probability of repayment is higher the higher is ϵ . ■

B Proofs from Section 5.2 (BBG)

Proof of Proposition 6. Totally differentiating Monitoring Equation 16 with respect to p and q gives that

$$dq = [M''(c)c_p^2 + M'(c)c_{pp}]dp, \quad (74)$$

where subscripts denote partial derivatives and the arguments of c are suppressed. Under assumption A9, both $M'(\cdot)$ and $M''(\cdot)$ are strictly positive. Using assumption A10 in equation 14 gives that c_p , and thus c_p^2 , is *strictly* positive. Differentiating equation 14 gives that $c_{pp} = -E''(p)$, which is positive by assumption A10. Using these facts in equation 74 gives that $dp/dq > 0$. ■

Proof of Proposition 7. Making use of assumption A11, which parametrizes group cost of monitoring by κ , Monitoring Equation 16 can be rewritten

$$q/\kappa = m'(c)c_p. \quad (75)$$

Since $m(\cdot)$ inherits all the properties of $M(\cdot)$ under assumption A11, it is evident that a decrease in κ affects p the same as an increase in q . Thus $dp/d\kappa < 0$. ■

Proof of Proposition 8. Totally differentiating Monitoring Equation 16 with respect to p and r gives that

$$0 = [M''(c)c_p^2 + M'(c)c_{pp}]dp + [M''(c)c_p c_r + M'(c)c_{pr}]dr, \quad (76)$$

where subscripts denote partial derivatives and the arguments of c are suppressed. As argued in the proof of proposition 6, the bracketed term multiplying dp is strictly positive. Under assumption A9, both $M'(\cdot)$ and $M''(\cdot)$ are strictly positive. Using assumption A10 in equation 14 gives that c_p is strictly positive. Equation 13 can be differentiated to give that $c_r = p - \underline{p} \geq 0$, and again to give that $c_{pr} = 1$. Using these facts in equation 76 gives that $dp/dr < 0$. ■

Proof of Proposition 9. Totally differentiating modified Monitoring Equation 20 with respect to L and p gives that

$$qdL = [M''(c)c_p^2 + M'(c)c_{pp}]dp + [M''(c)c_p c_L + M'(c)c_{pL}]dL, \quad (77)$$

where subscripts denote partial derivatives and the arguments of c are suppressed. An argument analogous to that in the proof of proposition 6 shows that the bracketed term multiplying dp is strictly positive. Rearranging equation 77 makes clear that the sign of dp/dL will be the same as the sign of

$$q - M''(c)c_p c_L - M'(c)c_{pL}. \quad (78)$$

Note from equation 20 that $q = M'(c)c_p/L$. Substituting this into 78 and rearranging gives

$$M'(c)[c_p/L - c_{pL}] - M''(c)c_p c_L \quad (79)$$

as the expression that must be signed.

We know $M'(\cdot) > 0$ by assumption A9. Also, $c(p, r, L)$ can be defined analogously to equation 13 and shown to equal

$$c(p, r, L) = rL(p - \underline{p}) - F(L)[E(p) - E(\underline{p})]. \quad (80)$$

Differentiating gives that $c_p = rL - F(L)E'(p)$, strictly positive by assumption A13. Further, $c_{pL} = r - F'(L)E'(p)$. Using these expressions we can write

$$c_p/L - c_{pL} = E'(p)[F'(L) - F(L)/L].$$

Assumption A13 gives that $E'(p) \geq 0$. Since $F(0) = 0$ and F is strictly concave, from assumption A12, it is clear that

$$F'(L) < F(L)/L. \quad (81)$$

Thus the first term in the sum of expression 79 is negative.

Turning to the second term of expression 79, assumption A9 gives that $M''(\cdot) > 0$ and the previous paragraph argues that $c_p > 0$. Differentiating equation 80 gives that

$$c_L = r(p - \underline{p}) - F'(L)[E(p) - E(\underline{p})]. \quad (82)$$

Clearly at $p = \underline{p}$, c_L is zero. For c_L to be positive when $p > \underline{p}$, the condition needed is

$$F'(L) \leq \frac{r(p - \underline{p})}{E(p) - E(\underline{p})}. \quad (83)$$

Now $[E(p) - E(\underline{p})]/(p - \underline{p})$ is the average product (in terms of per unit expected output) of an increase in p . Since $E(p)$ is concave, by assumption A10, this average product is decreasing in p . Condition 83 contains the inverse of this average product, and therefore the bound is tightest as $p \rightarrow \underline{p}$. Note that as $p \rightarrow \underline{p}$, the right side approaches $r/E'(\underline{p})$. Now by assumption A13, $F(L)/L < r/E'(\underline{p})$; and expression 81 gives that $F'(L) < F(L)/L$. Putting

these two inequalities together gives that $F'(L) < r/E'(\underline{p})$. Therefore, expression 83 holds, with strict inequality as long as $p > \underline{p}$. This signs c_L as positive and the whole second term of expression 79 as negative. Thus dp/dL is negative, strictly so when $p > \underline{p}$. ■

Proof of Proposition 10. Totally differentiating modified Monitoring Equation 21 with respect to p and H gives that

$$0 = [M''(c)c_p^2 + M'(c)c_{pp}]dp + [M''(c)c_p c_H + M'(c)c_{pH}]dH, \quad (84)$$

where subscripts denote partial derivatives and the arguments of c are suppressed. An argument analogous to that in the proof of proposition 6 shows that the bracketed term multiplying dp is strictly positive.

Turning to the bracketed term multiplying dH , assumption A9 gives that $M'(\cdot)$ and $M''(\cdot)$ are strictly positive. Now $c(p, r, H)$ can be defined analogously to equation 13 and shown to equal

$$c(p, r, H) = r(p - \underline{p}) - G(H)[E(p) - E(\underline{p})]. \quad (85)$$

Differentiating gives that $c_p = r - G(H)E'(p)$, which is positive by assumption A15. Use of assumptions A14 and A15 and differentiation of equation 85 give that $c_H = -G'(H)[E(p) - E(\underline{p})] \leq 0$, and that $c_{pH} = -G'(H)E'(p) < 0$. These facts together sign the second bracketed term in expression 84 as strictly negative, and thus $dp/dH > 0$. ■

Proof of Proposition 11. Let p_c (p_{nc}) be the probability of repayment of a group acting cooperatively (non-cooperatively). From equation 22, p_c satisfies

$$E'(p_c) = r - q \quad (86)$$

under assumption A10. From equation 23, p_{nc} satisfies

$$E'(p_{nc}) = r - \frac{q}{M'[c(p_{nc}, r)]} \quad (87)$$

under the same assumption and assumption A9. Comparison of equations 86 and 87 gives that if $M'(c) < 1$, then $E'(p_{nc}) < E'(p_c)$. This and the concavity of $E(p)$, by assumption A10, imply $p_{nc} > p_c$. On the other hand, if $M'(c) > 1$, then $p_{nc} < p_c$. ■

C Proofs from Section 5.3 (BC)

Proof of Proposition 12. In this proof, we make use of figure 7. In particular, the joint output realizations leading to default for r_1 and $r_2 > r_1$ correspond to the respective areas below the two sets of dotted lines, under the assumptions listed. Clearly, this area is larger for r_2 in the figure as drawn, based on the fact that $a_2 > a_1$ and $b_2 > b_1$, and that the dotted line for $\hat{Y}(r_2, Y)$ is above the dotted line for $\hat{Y}(r_1, Y)$ for Y between a_2 and b_1 . We now justify those assertions.

To show that $\underline{Y}(r_2) > \underline{Y}(r_1)$ and $\underline{Y}(2r_2) > \underline{Y}(2r_1)$ (i.e. $a_2 > a_1$ and $b_2 > b_1$), it is sufficient that $\underline{Y}(X)$ is increasing. By definition, $\underline{Y}(X) \equiv (c^o)^{-1}(X)$. Since c^o is strictly increasing under assumption A16, so is $(c^o)^{-1}$, and thus $\underline{Y}(X)$.

Second, we show that $\hat{Y}(r_2, Y) > \hat{Y}(r_1, Y)$ for Y between a_2 and b_1 , which is the only relevant region. Note that for Y_i between a_2 and b_1 , the loss Λ_i equals $c^o(Y_i) - r_1$ under r_1 (since $Y_i < b_1 = \underline{Y}(2r_1)$) and $c^o(Y_i) - r_2$ under r_2 (since $Y_i < b_1 < b_2 = \underline{Y}(2r_2)$). Therefore in this region, $\hat{Y}(r_1, Y_i)$, call it \hat{Y}_1 , must satisfy (see equation 25)

$$c^o(\hat{Y}_1) + c^u[\hat{Y}_1, c^o(Y_i) - r_1] = r_1, \quad (88)$$

and $\hat{Y}(r_2, Y_i)$, call it \hat{Y}_2 , must satisfy

$$c^o(\hat{Y}_2) + c^u[\hat{Y}_2, c^o(Y_i) - r_2] = r_2. \quad (89)$$

Now if $\hat{Y}_2 \leq \hat{Y}_1$, then the left side of equation 89 is also no greater than the left side of equation 88, since c^o is increasing in Y , by assumption A16; c^u is increasing in both arguments, by assumption A17; and $c^o(Y_i) - r_2 < c^o(Y_i) - r_1$, since $r_2 > r_1$ by hypothesis. But since the right side of equation 89 is strictly greater than the right side of equation 88, we must have $\hat{Y}_2 > \hat{Y}_1$. ■

Proof of Proposition 13. The proof for official penalties follows a similar argument as in the proof of proposition 12, using figure 7. In particular, the measure of the area below each set of dotted lines is the probability of default under $c_1^o(Y)$ and $c_2^o(Y)$, respectively, under the assumptions listed. This area is larger for group two as long as $a_2 > a_1$, $b_2 > b_1$, and the dotted line for $\hat{Y}_2(r, Y)$ is above the dotted line for $\hat{Y}_1(r, Y)$ for Y between a_2 and b_1 . We now justify those assertions.

Recall that by definition, $\underline{Y}_1(r)$ and $\underline{Y}_2(r)$ (a_1 and a_2 , respectively) satisfy

$$c_1^o(a_1) = r = c_2^o(a_2). \quad (90)$$

Now if $a_2 \leq a_1$, then

$$c_2^o(a_2) \leq c_2^o(a_1) < c_1^o(a_1),$$

a contradiction to equation 90. (The first inequality is since c^o is increasing by assumption A16; the second is by hypothesis that $c_1^o(Y) > c_2^o(Y)$.) Thus $a_2 > a_1$; by a similar argument, $b_2 > b_1$.

Second, we show that $\hat{Y}_2(r, Y) > \hat{Y}_1(r, Y)$ for Y between a_2 and b_1 , which is the only relevant region. By an argument similar to that in the proof of proposition 12, for Y_i between a_2 and b_1 , the loss Λ_i equals $c_1^o(Y_i) - r$ for group one and $c_2^o(Y_i) - r$ for group two. Therefore in this region, $\hat{Y}_1(Y_i, r)$ and $\hat{Y}_2(Y_i, r)$, call them \hat{Y}_1 and \hat{Y}_2 , respectively, by construction must satisfy

$$c_1^o(\hat{Y}_1) + c^u[\hat{Y}_1, c_1^o(Y_i) - r] = r = c_2^o(\hat{Y}_2) + c^u[\hat{Y}_2, c_2^o(Y_i) - r]. \quad (91)$$

Now if $\hat{Y}_2 \leq \hat{Y}_1$, the far left side of equation 91 will be strictly greater than the far right side. Term by term, this is because

$$c_1^o(\hat{Y}_1) \geq c_1^o(\hat{Y}_2) > c_2^o(\hat{Y}_2),$$

where the first inequality is due to assumption A16 that c^o is increasing, and the second by the fact that $c_1^o(Y) > c_2^o(Y)$. Also,

$$c^u[\hat{Y}_1, c_1^o(Y_i) - r] \geq c^u[\hat{Y}_2, c_1^o(Y_i) - r] > c^u[\hat{Y}_2, c_2^o(Y_i) - r],$$

where the inequalities use assumption A17 that c^u is increasing in both arguments, and the latter inequality also uses the fact that $c_1^o(Y) > c_2^o(Y)$. Since $\hat{Y}_2 \leq \hat{Y}_1$ is contradictory, we must have that $\hat{Y}_2 > \hat{Y}_1$. ■

Proof of Proposition 14. The proof for unofficial penalties uses figure 6. In particular, since a and b depend only on the interest rate and official penalties, both groups have the same a and b , and the same A and AB squares. The only difference across groups is the unofficial penalty curves through the AB squares. Here we show that this line is lower for group one, in other words, that $\hat{Y}_2(r, Y) > \hat{Y}_1(r, Y)$ for Y between a and b .

Now $\hat{Y}_1(r, Y)$ and $\hat{Y}_2(r, Y)$, call them \hat{Y}_1 and \hat{Y}_2 , respectively, by construction must satisfy

$$c^o(\hat{Y}_1) + c_1^u[\hat{Y}_1, \Lambda(r, Y)] = r = c^o(\hat{Y}_2) + c_2^u[\hat{Y}_2, \Lambda(r, Y)]. \quad (92)$$

If $\hat{Y}_2 \leq \hat{Y}_1$, then the far left side of equation 92 will be strictly greater than the far right side. Term by term, this is because

$$c^o(\hat{Y}_1) \geq c^o(\hat{Y}_2),$$

where the inequality is due to assumption A16 that c^o is increasing. Also,

$$c_1^u[\hat{Y}_1, \Lambda(r, Y)] \geq c_1^u[\hat{Y}_2, \Lambda(r, Y)] > c_2^u[\hat{Y}_2, \Lambda(r, Y)],$$

where the first inequality uses assumption A17 that c^u is increasing in the first argument and the latter inequality uses the fact that $c_1^u(Y, \Lambda) > c_2^u(Y, \Lambda)$ when $\Lambda > 0$.⁹⁵ Since $\hat{Y}_2 \leq \hat{Y}_1$ is contradictory, we must have that $\hat{Y}_2 > \hat{Y}_1$. ■

Proof of Proposition 15. Equation 26 can be rewritten to express the probability of default, $1 - p$, as

$$1 - p = [F(a)]^2 + 2 \int_a^b F[\hat{Y}(Y)] dF(Y),$$

where $a \equiv \underline{Y}(r)$, $b \equiv \underline{Y}(2r)$, and the argument r is suppressed from \hat{Y} . This can be further manipulated to get

$$(1 - p)/2 = \int_0^a F(Y) dF(Y) + \int_a^b F[\hat{Y}(Y)] dF(Y). \quad (93)$$

⁹⁵Note $\Lambda > 0$ when Y is between a and b , where again $a = \underline{Y}(r)$ and $b = \underline{Y}(2r)$. This is because for $Y > a$, $c^o(Y) > r$. (Recall that $\Lambda \equiv \min\{c^o(Y) - r, r\}$.)

Next we transform equation 93 from an integral over output space to one over output-percentile space, since the measure over the latter space is the same for both groups. Specifically, let $z = F(Y)$, so that $dz = dF(Y)$ and $Y = F^{-1}(z)$. Making these substitutions in equation 93 allows us to write

$$(1 - p)/2 = \int_0^{F(a)} z dz + \int_{F(a)}^{F(b)} F[\underline{\hat{Y}}(F^{-1}(z))] dz. \quad (94)$$

Define Δ as the difference in probabilities of default (scaled by two): $\Delta \equiv (1 - p_1)/2 - (1 - p_2)/2$. Showing that $\Delta > 0$ will establish that group 1 has a higher probability of default.

In the case where $F_1(a) \leq F_2(b)$,⁹⁶ Δ can be written, using equation 94, as

$$\begin{aligned} \Delta = & \int_0^{F_2(a)} (z - z) dz + \int_{F_2(a)}^{F_1(a)} \{z - F_2[\underline{\hat{Y}}(F_2^{-1}(z))]\} dz \\ & \int_{F_1(a)}^{F_2(b)} \{F_1[\underline{\hat{Y}}(F_1^{-1}(z))] - F_2[\underline{\hat{Y}}(F_2^{-1}(z))]\} dz + \int_{F_2(b)}^{F_1(b)} F_1[\underline{\hat{Y}}(F_1^{-1}(z))] dz. \end{aligned} \quad (95)$$

The first integral is clearly zero. The fourth integral is clearly non-negative. For the second integral, note that $F_2^{-1}(z) > a$ for $z > F_2(a)$. Note that the cutoff $\underline{\hat{Y}}(X) < a$ when $X > a$, since both unofficial and official penalties will be imposed.⁹⁷ Thus, $\underline{\hat{Y}}(F_2^{-1}(z)) < a$. Clearly then, $F_2[\underline{\hat{Y}}(F_2^{-1}(z))] < F_2(a)$. Since z is ranging from $F_2(a)$ upwards, the second integral must be positive, strictly so if $F_1(a) > F_2(a)$. Considering the third integral, note that $F_1^{-1}(z) \leq F_2^{-1}(z)$, that is a given percentile z means a lower output for a group 1 member than a group 2 member. Since $\underline{\hat{Y}}$ is decreasing in Y , we know that $\underline{\hat{Y}}(F_1^{-1}(z)) \geq \underline{\hat{Y}}(F_2^{-1}(z))$. Hence, $F_1[\underline{\hat{Y}}(F_1^{-1}(z))] \geq F_2[\underline{\hat{Y}}(F_1^{-1}(z))] \geq F_2[\underline{\hat{Y}}(F_2^{-1}(z))]$, where the first inequality is from stochastic dominance, and the third integral is positive. Thus Δ must be positive.

The case where $F_1(a) > F_2(b)$ gives

$$\Delta = \int_{F_2(a)}^{F_2(b)} \{z - F_2[\underline{\hat{Y}}(F_2^{-1}(z))]\} dz + \int_{F_2(b)}^{F_1(a)} z dz + \int_{F_1(a)}^{F_1(b)} F_1[\underline{\hat{Y}}(F_1^{-1}(z))] dz. \quad (96)$$

Similar arguments establish Δ positive in this case also. Sufficient in both cases for Δ to be strictly positive is that $F_1(a) > F_2(a)$ (see also the discussion of the second integral of equation 95). ■

Proof of Proposition 16. Equation 29 makes clear that if

$$\int_{\underline{Y}(r)}^{\underline{Y}(2r)} F[\underline{Y}(2r - c^o(Y))] dF(Y) > \int_{\underline{Y}(r)}^{\underline{Y}(2r)} F[\underline{\hat{Y}}(Y)] dF(Y), \quad (97)$$

holds, then cooperation lowers the repayment rate, while if the reverse inequality holds, cooperation raises the repayment rate.

⁹⁶We know that $0 < F_2(a) \leq F_1(a) < F_1(b)$ and that $0 < F_2(a) < F_2(b) \leq F_1(b)$. However, it is not clear whether $F_1(a)$ or $F_2(b)$ is greater.

⁹⁷In other words, if one borrower realizes output $X > a$, then positive unofficial penalties can be imposed since $\Lambda(X) = \min\{c^o(X) - r, r\} > 0$. Thus the cutoff output level for the other borrower to repay, $\underline{\hat{Y}}(X)$, must be lower than a , the cutoff output level corresponding to official penalties alone.

Suppose first that unofficial penalties satisfy $c^u(Y, \Lambda) > \Lambda$ for $\Lambda > 0$. By inspection of inequality 97, it is sufficient for the result to show that $\underline{Y}[2r - c^o(Y)] > \hat{\underline{Y}}(Y)$, for $Y \in (\underline{Y}(r), \underline{Y}(2r))$. Note that $\hat{\underline{Y}}(Y)$ is generally defined⁹⁸ as the value satisfying

$$r = c^o[\hat{\underline{Y}}(Y)] + c^u[\hat{\underline{Y}}(Y), \Lambda(Y)]. \quad (98)$$

This ensures that at $\hat{\underline{Y}}(Y)$, the cost of repayment r equals the official and unofficial penalties for non-repayment, given the partner realized output Y . However, as $\Lambda(Y)$ approaches r , which occurs as $Y \rightarrow \underline{Y}(2r)$, the right side of equation 98 approaches some amount greater than r , since by hypothesis $c^u(Y, \Lambda) > \Lambda$. In this case, the equation cannot hold since the right side is higher than the left however low $\hat{\underline{Y}}(Y)$ is. Since repayment is less costly than default whatever output a borrower realizes, $\hat{\underline{Y}}(Y)$ is at the lower boundary, zero. Thus $\hat{\underline{Y}}(Y)$ is defined by equation 98 where possible, and otherwise equals zero.

Consider first the space over which equation 98 defines $\hat{\underline{Y}}(Y)$. Using this equation, the fact that $\Lambda(Y) = c^o(Y) - r > 0$ when $Y \in (\underline{Y}(r), \underline{Y}(2r))$, and the hypothesis that $c^u(Y, \Lambda) > \Lambda$ for $\Lambda > 0$, we can write

$$r = c^o[\hat{\underline{Y}}(Y)] + c^u[\hat{\underline{Y}}(Y), c^o(Y) - r] > c^o[\hat{\underline{Y}}(Y)] + c^o(Y) - r.$$

Rearranging and using the fact that $\underline{Y} \equiv (c^o)^{-1}$ and that $(c^o)^{-1}$ is a strictly increasing function since c^o is, we have that

$$\hat{\underline{Y}}(Y) < (c^o)^{-1}[2r - c^o(Y)] = \underline{Y}[2r - c^o(Y)].$$

Consider next the space over which $\hat{\underline{Y}}(Y)$ is at its lower bound of zero. Note that for Y over the entire interval $(\underline{Y}(r), \underline{Y}(2r))$, we have that $\underline{Y}[2r - c^o(Y)] > 0$. Thus $\hat{\underline{Y}}(Y) < \underline{Y}[2r - c^o(Y)]$ in this space as well.

The same basic argument can be repeated to show that for $c^u(Y, \Lambda) < \Lambda$, the reverse of inequality 28 holds. The only difference is that $\hat{\underline{Y}}(Y)$ will never reach the lower bound of zero, and so is defined by equation 98 over the whole interval $(\underline{Y}(r), \underline{Y}(2r))$. ■

Proof of Lemma 2. Take first a continuous joint density $\phi(Y_i, Y_j)$ that preserves unconditional density f for Y_i and Y_j . If $\phi(Y_i, Y_j) = f(Y_i)f(Y_j)$, then setting $\kappa = 0$ and $g(Y_i, Y_j) = 0$ works. Otherwise, take κ as strictly positive but arbitrarily close to zero. Then note that setting $g(Y_i, Y_j) = [\phi(Y_i, Y_j) - f(Y_i)f(Y_j)]/\kappa$ satisfies equation 30. This is because integrating out Y_i, Y_j , or both, produces zero; that is, the integrals in the brackets are zero. In the case of Y_i , we have

$$\int_0^1 g(Y_i, Y_j) dY_i = (1/\kappa) \int_0^1 [\phi(Y_i, Y_j) - f(Y_i)f(Y_j)] dY_i = (1/\kappa)[f(Y_j) - f(Y_j)] = 0,$$

where the last equality is due to the fact that $\phi(Y_i, Y_j)$ preserves unconditional density f for Y_j . The same kind of equation holds for Y_j . Finally, g is continuous because ϕ and f are.

⁹⁸In the exposition of section 5.3, this was the full definition. Assumption A18 kept $\hat{\underline{Y}}$ off the boundary of zero, by ensuring that all penalties were zero there. Here, however, we are under assumption A20 instead of A18, and a non-interior case must be defined.

Next take any continuous function $g(Y_i, Y_j)$. The function $\phi(Y_i, Y_j)$ defined using $g(Y_i, Y_j)$ in equation 30 is continuous because f and g are. Further, it is a joint density function as long it stays non-negative and integrates to one over Y_i and Y_j together. Finally, it preserves unconditional density f as long as it integrates over Y_i to $f(Y_j)$ and over Y_j to $f(Y_i)$.

Note that non-negativity is satisfied as long as κ is close enough to zero. This because $f(Y)$ is strictly positive and continuous, and thus $f(Y_i)f(Y_j)$ is bounded below by a strictly positive number on the (compact) unit square; and the bracketed term of equation 30 is bounded below on the unit square because $g(Y_i, Y_j)$ is continuous. Thus however negative the bracketed term is, κ can be set low enough to make the subtracted amount small compared to $f(Y_i)f(Y_j)$. Note also from integrating equation 30 with respect to Y_i that

$$\begin{aligned} \int_0^1 \phi(Y_i, Y_j) dY_i &= \int_0^1 f(Y_i) f(Y_j) dY_i + \kappa \left[\int_0^1 g(Y_i, Y_j) dY_i - \int_0^1 g(Y_i, Y_j) dY_i \right. \\ &\quad \left. - \int_0^1 \int_0^1 g(Y_i, Y_j) dY_i dY_j + \int_0^1 \int_0^1 g(Y_i, Y_j) dY_i dY_j \right] = f(Y_j). \end{aligned}$$

Similarly, $\int_0^1 \phi(Y_i, Y_j) dY_j = f(Y_i)$. Thus the unconditional densities are preserved. Finally, this fact implies that $\phi(Y_i, Y_j)$ must integrate to one over Y_i and Y_j together. ■

Proof of Lemma 3. First we calculate $\text{Cov}(Y_i, Y_j)$ under joint density $\phi(Y_i, Y_j)$. Using equation 30, equation 31, and the fact that $\text{Cov}(Y_i, Y_j)$ equals $E(Y_i Y_j) - E(Y_i)E(Y_j)$, we write it as

$$\begin{aligned} \text{Cov}(Y_i, Y_j) &= \int_0^1 \int_0^1 Y_i Y_j f(Y_i) f(Y_j) dY_i dY_j + \kappa \int_0^1 \int_0^1 Y_i Y_j \gamma(Y_i, Y_j) dY_i dY_j - E(Y_i)E(Y_j) \\ &= \kappa \int_0^1 \int_0^1 Y_i Y_j \gamma(Y_i, Y_j) dY_i dY_j. \end{aligned}$$

The sign of the covariance is the same as that of the double integral, since κ is positive, and the derivative of the covariance with respect to κ equals the double integral. Thus if the covariance is positive, a higher κ increases it; if it is negative, a higher κ decreases it. ■

Proof of Proposition 17. For convenience, define $a \equiv \underline{Y}(r)$ and $b \equiv \underline{Y}(2r)$. Modifying equation 26 to get the repayment rate in the case of generalized joint density function $\phi(Y_i, Y_j; \kappa)$ gives

$$p(\kappa) = 1 - \int_0^a \int_0^a \phi(Y_i, Y_j; \kappa) dY_i dY_j - 2 \int_a^b \int_0^{\hat{Y}(Y_i)} \phi(Y_i, Y_j; \kappa) dY_j dY_i, \quad (99)$$

where the dependence of \hat{Y} on r is suppressed. From equations 30 and 31, which write $\phi(Y_i, Y_j; \kappa) = f(Y_i)f(Y_j) + \kappa\gamma(Y_i, Y_j)$, we get that $d\phi/d\kappa = \gamma(Y_i, Y_j)$. Using this in equation 99 gives

$$dp/d\kappa = - \left[\int_0^a \int_0^a \gamma(Y_i, Y_j) dY_i dY_j + 2 \int_a^b \int_0^{\hat{Y}(Y_i)} \gamma(Y_i, Y_j) dY_j dY_i \right]. \quad (100)$$

This equation merely says that the effect of higher correlation on p is inversely related to the amount of mass that the introduced correlation adds to the region of default, for example, the area below the dotted line in Figure 6. Our strategy is to show that the first double integral (corresponding to box A) is strictly positive and that the second double integral is close enough to zero as unofficial penalties get sufficiently severe.

Carrying out the first integration in equation 100 using the expression for $\gamma(Y_i, Y_j)$ of equation 32 gives

$$\int_0^a \int_0^a \gamma(Y_i, Y_j) dY_i dY_j = 2a(1-a) \sum_{k=1}^N \beta_k \frac{1 - a^{\alpha_k+1} - (1-a)^{\alpha_k+1}}{(\alpha_k+1)(\alpha_k+2)} \equiv \mathcal{A},$$

say. Clearly \mathcal{A} is strictly positive as long as $1 - a^{\alpha_k+1} - (1-a)^{\alpha_k+1}$ is, since $\alpha_k, \beta_k > 0$ and $a \in (0, 1)$. Considering $1 - a^{\alpha_k+1} - (1-a)^{\alpha_k+1}$, note that it equals zero at $\alpha_k = 0$ and is continuous and strictly increasing in α_k .⁹⁹ Since $\alpha_k > 0$, this term, and hence \mathcal{A} , is strictly positive.

Call the second double integral in equation 100 \mathcal{AB} . Let $\underline{\gamma}$ be the minimum value $\gamma(Y_i, Y_j)$ (a continuous function) can take over its (compact) domain $[0, 1]^2$. Inspection of equation 100 reveals that

$$\mathcal{AB} \geq 2\underline{\gamma} \int_a^b \int_a^{\hat{Y}(Y_i)} dY_j dY_i. \quad (101)$$

Note that if $\underline{\gamma}$ were not negative, \mathcal{AB} would be positive and the proof would be complete ($dp/d\kappa$ signed as negative). We thus consider only the case where $\underline{\gamma} < 0$.

Next define

$$\epsilon \equiv \min\left\{\frac{\mathcal{A}}{2|\underline{\gamma}|b}, \frac{b-a}{2}\right\} \quad (102)$$

and assume unofficial penalties are strong enough to satisfy¹⁰⁰

$$c^o(\epsilon) + c^u(\epsilon, c^o(a+\epsilon) - r) \geq r. \quad (103)$$

Condition 103 implies that $\hat{Y}(a+\epsilon) \leq \epsilon$, ensuring that a borrower, i say, who realizes output ϵ and whose partner j realizes $a + \epsilon$ ¹⁰¹ will face total penalties of at least r . The point of indifference for borrower i , $\hat{Y}(a+\epsilon)$, is thus no higher than ϵ , since total penalties on him would only be higher with higher output.

As argued in section 5.3, $\hat{Y}(Y)$ is strictly decreasing from a to some $z > 0$ as Y increases from a to b . Using this and the fact that $\hat{Y}(a+\epsilon) \leq \epsilon$, we know that $\hat{Y}(Y) \leq a$ for

⁹⁹Its derivative with respect to α_k is $-\ln(a)a^{\alpha_k+1} - \ln(1-a)(1-a)^{\alpha_k+1}$. This is strictly positive because $a \in (0, 1)$.

¹⁰⁰Unofficial penalties this strong need not violate assumption A18 because $\epsilon > 0$ and $c^o(a+\epsilon) - r > 0$, since $c^o(a) = r$ by construction and c^o is strictly increasing. Thus such a severe unofficial penalty function is possible, requiring only that $c^u(Y, \Lambda)$ increase fast enough in either argument.

¹⁰¹Note that since $a < a + \epsilon < b$, the loss to borrower j equals $\Lambda(a + \epsilon) = c^o(a + \epsilon) - r$.

$Y \in [a, a + \epsilon]$ and $\hat{Y}(Y) \leq \epsilon$ for $Y \in [a + \epsilon, b]$.¹⁰² It follows that

$$\int_a^b \int_0^{\hat{Y}(Y_i)} dY_j dY_i \leq \int_a^{a+\epsilon} \int_0^a dY_j dY_i + \int_{a+\epsilon}^b \int_0^\epsilon dY_j dY_i = \epsilon(b - \epsilon) < b\epsilon.$$

Recalling that $\underline{\gamma} < 0$, this inequality can be rewritten as

$$2\underline{\gamma} \int_a^b \int_0^{\hat{Y}(Y_i)} dY_j dY_i > 2\underline{\gamma}b\epsilon.$$

Combining this with inequality 101 gives that $\mathcal{AB} > 2\underline{\gamma}b\epsilon$. Finally, equation 102 defining ϵ gives that $\epsilon \leq \mathcal{A}/[2|\underline{\gamma}|b]$. Combining these last two inequalities, given that $\underline{\gamma} < 0$, gives

$$\mathcal{AB} > 2\underline{\gamma}b\epsilon \geq 2\underline{\gamma}b\mathcal{A}/[2|\underline{\gamma}|b] = -\mathcal{A}. \quad (104)$$

Thus $\mathcal{A} + \mathcal{AB} > 0$ and $dp/d\kappa$, expressed in equation 100, is negative. ■

D Proofs from Section 5.4 (Ghatak)

Proof of Proposition 18. Total differentiation of the selection equation 36 gives the following derivatives:

$$\frac{\partial \hat{p}}{\partial r} = \frac{-\hat{p}}{r + q(1 - 2\hat{p})}$$

and

$$\frac{\partial \hat{p}}{\partial q} = \frac{-\hat{p}(1 - \hat{p})}{r + q(1 - 2\hat{p})}.$$

Note that under assumption A23 that $q \leq r$ and for $\hat{p} \in [\underline{p}, 1)$, both derivatives are negative. Clearly the numerators are negative. The denominators are strictly positive, since they are strictly decreasing in \hat{p} and equal to $r - q \geq 0$ when $\hat{p} = 1$. However, $\hat{p} < 1$ in the interior. ■

Proof of Proposition 19. Assumption A25 gives us that a group borrowing L must be in $[\underline{p}, \hat{p}]$, where \hat{p} is determined by the new selection equation 42. Total differentiation of equation 42 gives the derivative:

$$\frac{\partial \hat{p}}{\partial L} = \frac{EF'(L) - \hat{p}r - \hat{p}(1 - \hat{p})q}{r + q(1 - 2\hat{p})}.$$

As argued in the proof of proposition 18, the denominator is strictly positive under assumption A23. The numerator is strictly positive, for the following reasons. Note that $L^*(\underline{p}) = L$,

¹⁰²Graphically, we have merely pinned the Default Curve running through the lower right AB box in Figure 6 below the point $(a + \epsilon, \epsilon)$, and symmetrically for the upper right AB box.

by construction. Also, $\hat{p} < \bar{p}$, which is implied by assumption A25 as argued in the text, gives that $L^*(\hat{p}) > L$. This is because $L^*(p)$ is an increasing function under assumption A24, as argued in the text. Now if $L^*(\hat{p}) > L$, then the derivative with respect to L of the payoff function 41 at \hat{p} must be positive:

$$EF'(L) - \hat{p}r - \hat{p}(1 - \hat{p})q > 0.$$

Thus both numerator and denominator are strictly positive. ■

Proof of Proposition 20. The goal is to show that $E(p|L)$ is decreasing in L . Note that under the converse of assumption A25, $\bar{p} \leq \hat{p}$ as argued in the text, and thus $E(p|L)$ is as written in equation 51, for one. We now define $\Phi(L) \equiv h(L)/[1 - H(L)]$, the hazard rate for loan sizes; $Z(p) \equiv g(p)/G(p)$, the hazard rate for risk types when proceeding from high to low; and $Q(L) \equiv \Phi(L)/Z(\bar{p}(L))$. Using these, equation 51 can be rewritten

$$E(p|L) = \bar{p} \frac{1}{1 + Q(L)} + E(p|p \leq \bar{p}) \frac{Q(L)}{1 + Q(L)}, \quad (105)$$

after dividing numerator and denominator by $g(\bar{p})[1 - H(L)]$.

We now argue that \bar{p} is decreasing in L . Recall \bar{p} is defined as the type p for whom L is optimal. Thus \bar{p} must satisfy first-order condition 43, reproduced here:

$$EF'(L) = \bar{p}r + \bar{p}(1 - \bar{p})q.$$

Note that by assumption A24 guaranteeing concavity of F , the higher is L , the lower is the left-hand side. On the other hand, the derivative of the right-hand side with respect to \bar{p} is $r + q(1 - 2\bar{p})$, which is positive under assumption A23, as argued in the proof of proposition 18. Thus the higher is \bar{p} , the higher is the right-hand side. This proves that L and \bar{p} must be inversely related. The intuition is that the higher is L , the lower the cost of capital must be to justify L as optimal; and the cost of capital is decreasing in risk type.

Now if \bar{p} decreases in L , then so must $E(p|p \leq \bar{p})$. Thus both components of $E(p|L)$ in equation 105 that involve \bar{p} , $E(p|p \leq \bar{p})$ and \bar{p} itself, are decreasing in L . Note that these terms are averaged according to weights $Q(L)/[1 + Q(L)]$ and $1/[1 + Q(L)]$, respectively. As mentioned in the text, the only way for $E(p|L)$ not to decrease in L is for the weight on \bar{p} , $1/[1 + Q(L)]$, to increase with L . If this happened, even though both terms involving \bar{p} were decreasing with L , the weight on the larger term, \bar{p} , would be increasing with L , and thus $E(p|L)$ could be increasing. We rule this out by showing that the weight on \bar{p} , $1/[1 + Q(L)]$, is not increasing in L , or equivalently, $Q'(L) \geq 0$.

Recall that $Q(L) \equiv \Phi(L)/Z(\bar{p}(L))$. Sufficient conditions for $Q'(L) \geq 0$ are that $\Phi'(L) \geq 0$ and that $Z'(p)\bar{p}'(L) \leq 0$. As argued above, \bar{p} is decreasing in L , that is, $\bar{p}'(L) < 0$. Thus the second sufficient condition boils down to $Z'(p) \geq 0$. From their respective definitions,

$$\Phi'(L) = \frac{h'(L)[1 - H(L)] + [h(L)]^2}{[1 - H(L)]^2}$$

and

$$Z'(p) = \frac{g'(p)G(p) - [g(p)]^2}{[G(p)]^2}.$$

Rewriting these equations using the expressions of equations 53 that $G(p) = A_G e^{f_G(p)}$ and $H(L) = 1 - A_H e^{-f_H(L)}$, we get that

$$\Phi'(L) = \frac{f_H''(L)[A e^{-f_H(L)}]^2}{[1 - H(L)]^2}$$

and

$$Z'(p) = \frac{f_G''(p)[A e^{f_G(p)}]^2}{[G(p)]^2}.$$

These expressions make clear that assumption A26 guarantees non-negativity to both $\Phi'(L)$ and $Z'(p)$. Thus $Q'(L) \geq 0$, and we have the result. ■

Proof of Proposition 21. Under assumption A27, total differentiation of the selection equation 56 gives the derivative:

$$\frac{\partial \hat{p}}{\partial H} = \frac{G'(H)E[F(1) - F(0)]}{r + q(1 - 2\hat{p})},$$

using the fact that $F(1)$ is normalized to one. As argued in the proof of proposition 18, the denominator is strictly positive under assumption A23. The numerator is also strictly positive, since F and G are assumed strictly increasing.

Under assumption A28, total differentiation of the selection equation 56 gives the derivative:

$$\frac{\partial \hat{p}}{\partial H} = \frac{G'(H)[E - \underline{u}]}{r + q(1 - 2\hat{p})}.$$

As argued in the proof of proposition 18, the denominator is strictly positive under assumption A23. The numerator is also strictly positive, since G is strictly increasing by assumption and $E > \underline{u}$, by assumption that borrowers' expected output (their borrowing payoff gross of interest, which does not vary with p) exceeds their outside payoff. ■

Proof of Proposition 22. First take the case where the converse of assumption A25 holds. In this case, observing L tells us that the borrowing group must be of type $p \leq \bar{p}$ when there is only one lender, and thus $E(p|p < \bar{p})$ (from equation 52) is the best guess for p conditional on observing this L . This continues to be true even if there are outside options: in particular, equations 49 and 50 continue to hold, and thus the expression for $E(p|p < \bar{p})$ is unchanged. The same combinations of loan offers and types would lead us to observe this L : an offer greater than L and a type of \bar{p} or an offer of L and type greater than \bar{p} . In either case, since this is the primary lender, these types and offers would lead to observing L , and none others. (Of course, in the latter case, more funds would be applied for, but this would not affect probability of repayment.)

In the case where assumption A25 holds and there are no outside options, we know exactly that $p < \hat{p}$, and also that every type that borrows is constrained. The expected repayment rate is simply $E(p|p < \hat{p})$. If there are outside options, however, more borrowers

are drawn in. To see this, first note that every borrower we observe will apply for outside funds, if the fixed cost is low enough. Also, a positive fraction of each type that applies will receive funds. Now denote $\hat{p}' \equiv \hat{p}(\min\{L + L', [L + \Gamma^{-1}(\chi/E)]/2\})$.¹⁰³ Clearly $\hat{p}' > \hat{p}$. Therefore, given the existence of a second lender, we know that not only those types $p < \hat{p}$ will be borrowing, but also a positive measure of borrowers of type $p \in [\hat{p}, \hat{p}']$. These latter borrowers would not be borrowing if they only received L , but they received an extra offer of capital no less than L' that made them desire to borrow from both lenders. Clearly, the expected repayment rate will be strictly larger than $E(p|p < \hat{p})$, which it equaled in the absence of a second lender, since the extra borrowers have $p > \hat{p}$. ■

Proof of Proposition 23. Under assumption A7, the payoff of a borrower of type p who matches with a type p' equals $E - pr - [p(1 - p') - \epsilon]q$. Homogeneous matching will occur in this case. Consider a borrower of type p (matched with another p) trying to lure a borrower of type $p' > p$ away from his group (consisting of another p'). The p -borrower would be willing to pay up to the increase in expected payoff that would result from matching with a p' rather than a p , namely

$$[E - pr - p(1 - p')q + \epsilon q] - [E - pr - p(1 - p)q + \epsilon q] = p(p' - p)q.$$

The borrower of type p' , however, would need to be compensated for his loss in expected payoff from being matched with a borrower of type p rather than p' , which equals

$$[E - p'r - p'(1 - p')q + \epsilon q] - [E - p'r - p'(1 - p)q + \epsilon q] = p'(p' - p)q.$$

Clearly no trade can occur, since the loss to the safer borrower, $p'(p' - p)q$, is greater than the gain to the riskier one, $p(p' - p)q$. Following the argument of section 5.4, the measure of homogeneous groups is one in any equilibrium.

Knowing that groups are homogeneous, we can use equation 58, differentiating to get that

$$\frac{d\hat{p}}{d\epsilon} = \frac{q}{r + q(1 - 2\hat{p})}.$$

As argued in the proof of proposition 18, the denominator is strictly positive. The numerator is also positive. Thus this version of correlation increases expected repayment rates.

Assumption A8 gives a borrower of type p who matches with a type p' the payoff:

$$E - pr - [p(1 - p') - \epsilon * \min\{p(1 - p'), p'(1 - p)\}]q.$$

Homogeneous matching will occur in this case also. Consider a borrower of type p (matched with another p) trying to lure a borrower of type $p' > p$ away from his group (consisting of another p'). The p -borrower would be willing to pay up to the increase in expected payoff that would result from matching with a p' rather than a p , namely

$$E - pr - [p(1 - p')q - \epsilon p(1 - p')]q - \{E - pr - [p(1 - p) - \epsilon p(1 - p)]q = qp(p' - p)(1 - \epsilon).$$

¹⁰³The minimum here ensures that we are still in the range where \hat{p} is increasing with L .

The borrower of type p' , however, would need to be compensated for his loss in expected payoff from being matched with a borrower of type p rather than p' , which equals

$$\begin{aligned} E - p'r - [p'(1 - p') - \epsilon p'(1 - p')]q - \{E - p'r - [p'(1 - p) - \epsilon p(1 - p')]\}q \\ = q(p' - p)[p' + \epsilon(1 - p')]. \end{aligned}$$

Note that the loss to the safer borrower is increasing in ϵ , while the gain to the riskier borrower is decreasing in ϵ . Further, at $\epsilon = 0$ the gain to the riskier borrower, $p(p' - p)q$, is less than the loss to the safer borrower, $p'(p' - p)q$. Thus no trade can occur. Following the argument of section 5.4, the measure of homogeneous groups is one in any equilibrium.

Knowing that groups are homogeneous, we can use equation 59, differentiating to get that

$$\frac{d\hat{p}}{d\rho} = \frac{q\hat{p}(1 - \hat{p})}{r + q(1 - \epsilon)(1 - 2\hat{p})} > 0.$$

Following an argument similar to the one contained in the proof of proposition 18, the denominator is strictly positive. The numerator is also positive, since p is bounded away from zero and one. Thus this version of correlation increases expected repayment rates. ■

E Relation to Existing Work

Several studies have attempted to uncover the determinants of group repayment. We describe first the data and techniques used, and then the main results that emerge from this collection of papers, comparing to our own findings.

Wenner 95 uses data from 25 FINCA borrowing groups in Costa Rica. The data reflect 36 borrowing periods between 1985 and 1988 (evidently there is more than one observation for some groups) so that $N = 36$ in the regressions. Wenner has data that enable him to categorize groups into three types: those with no loan delinquency, those with internal delinquency only, and those with external delinquency. Internal delinquency means that a borrower did not meet his obligation to the group, though the group may have met its obligation to the lender. External delinquency means the group did not meet its obligation to the lender (and thus implies internal delinquency as well.) Binomial probit, a multinomial logit, and tobit procedures are run on the dependent variables. Independent variables measure group characteristics.

Sharma and Zeller 97 use a 1994 survey of 128 borrowing groups randomly selected from 41 villages across Bangladesh. Each is a customer of the programs BRAC, ASA, or RDRS. 868 loans whose due date is before the date of the survey are the observations. The dependent variable is the percent unpaid at the due date. Independent variables are characteristics of the group, community, and lender. Each independent variable is multiplied by the loan size.

Zeller 98 uses 1992 data on 146 borrowing groups from six different lenders in Madagascar. Four regions of the country were chosen, ensuring variation in agroecology. Villages were selected from these regions based on a stratified random sampling scheme which divided villages into those above and below the regional population median, and into three

categories based on distance from a national road. Within these villages, groups from six different group-based credit programs were interviewed. These include ODR-GCV, ACCS, KOBAMA, MALTO, FIFATA, and CIDR. In addition, community-level data were collected. The dependent variable is the percent repayment rate (no further description is given.) This is related using a tobit procedure to six community-level variables, two program-level variables, and twelve group-level variables.

Wydick 99 uses self-collected data from 137 groups in Western Guatemala which borrow from FUNDAP, an affiliate of ACCION International. Of the groups, 51 are considered urban and 86 rural. Three dependent dummy variables are used. The first measures misuse of borrowed funds, as reported by the group in interviews. The second measures common mutual help with repayment of loans when necessary, as reported by the group. The third is a measure of good repayment record, from the lending institution's records. Nine logit procedures are run, matching each dependent variable with the rural and urban subsamples and the whole sample. The independent variables are put in four categories: social ties, group pressure, monitoring, and control. Log-likelihood ratio tests are used to test the significance of each group of variables in each regression.

Results from these four papers are mixed in their confirmation of theory. We will use as an outline Table 3 to summarize the results.

It is quite an omission that none of the works to date have included the interest rate in the regressions, since it is a key variable in almost all joint liability models, which nearly unanimously agree that higher interest rates lead to lower repayment. Possibly this is due to lack of data or of sufficient exogenous variation in interest rates across groups. We find a slight amount of evidence that the interest rate is a negative predictor, as theory suggests.

The degree of joint liability q also seems missing from the existing work. This is not surprising since its empirical counterpart is fuzzy. Sharma/Zeller 97 use a variable somewhat similar to our measure of q : variance of landholdings. However, this is interpreted as diversification of risk by them. Their result is not significantly different from zero. Our measure of q as the percent landless in the group is unique.

Loan size is controlled for by Sharma/Zeller 97 only, who interact every independent variable in their specification (including the constant) with loan size. They find that increasing loan size is linked significantly to repayment problems, as it is in two of the theories we examine.¹⁰⁴

Borrower productivity in the form of average land owned or operated appear in Zeller 98 and Sharma/Zeller 97. Sharma/Zeller 97 find it predicts increased repayment in a significant way. Zeller finds no significant result. He also includes local transport costs, the density of input retailers, and a measure of literacy, which all can be taken as evidence for higher borrower productivity per loan. Of these, only the second is significant, and it is a positive predictor of repayment. Thus borrower productivity seems to be positive when significant. There are some counterexamples. Namely, Wenner 95 finds an infrastructure variable predicting delinquency, significantly in one specification, and Sharma/Zeller 97 find that the degree of irrigation in the village significantly predict repayment problems. Apart

¹⁰⁴Sharma/Zeller 97 also use a variable intended to measure the amount of rationing taking place: amount of loan granted divided by amount applied for. Rationing is positively associated with repayment, and its squared term is negatively so. In the relevant region, the effect of this variable is positive. The idea of rationing could be analyzed in the Stiglitz and Ghatak models, but we do not do so here.

from these infrastructure variables, though, the theories we consider in this paper are confirmed. Our data confirm them as well, but it is human capital, rather than average land, that more consistently predicts good repayment.

Covariance of borrowers' project returns is included by Sharma/Zeller 97, Zeller 98, and Wydick 99.¹⁰⁵ All measures essentially proxy covariance by occupational homogeneity. Each finds some evidence that positive covariance hurts repayment. In the Sharma/Zeller 97, it is measured by the percent of the group engaging in agriculture. They interpret this variance as a proxy for occupational diversification, since in the sample most groups have few agricultural workers. In Zeller 98, covariance is proxied by the coefficient of variation of holdings of "upland" within the group, which aims to capture lack of covariation through occupational dissimilarity. Wydick 99, it is a dummy variable equalling one if the whole group is in the same business, zero otherwise. Thus all three essentially proxy correlation by a measure of occupational homogeneity. All find high correlation is bad for repayment; all but Wydick 99 find significant coefficients, and Zeller 98 finds a significant quadratic relationship. This is of course at odds with what several of the theories we consider predict, and what our data turn up. As they do, we include a direct measure of occupational homogeneity. We also include a unique, *direct* measure of covariance of output (though at the village level). This latter could be a needed addition, because occupational similarities need not always imply that correlation is high. We find some significant evidence of a positive effect of covariance on repayment.

Various indirect measures of outside borrowing options have significant effects on repayment. This was found in Sharma/Zeller 97, where they are reflected in measures of village remoteness, mean village participation in group-based organizations, informal self-help and insurance groups, and the presence of a food-for-work program in the village. Remoteness and a food-for-work program, which the authors argue probably reflect the existence of few outside borrowing options, significantly predict good repayment. Insurance groups' presence predicts bad repayment significantly, also probably because these are outside loan options. The one opposing result is that village participation in group-based organizations (not necessarily the lender in question) predicts good repayment. This variable is probably positively correlated with outside borrowing options. There are also exceptions in Zeller 98, who found higher repayment for *more monetarized and accessible* regions, and in Wydick 99, who found no significant relationship with repayment and a dummy variable measuring outside loan availability. In short, the evidence relies mostly on proxies for outside loan options and reaches various findings, the majority suggesting outside options decrease repayment. We measure existence of other loan sources arguably more directly, as the prevalence in the village of commercial bank and village bank members, and find strong evidence that outside borrowing options hurt repayment.

Formalized screening arising from agreement on a group code, or some form of rules and regulations, significantly increased repayment in Wenner 95 and Zeller 98. Interestingly, Wenner 95 finds that *informal* screening based on reputation actually hurts repayment rates, significantly in some specifications. However, this effect reverses when formal credit rating is conditioned upon, in which case informal screening is also significantly associated with

¹⁰⁵Wydick uses a measure of occupational homogeneity to capture monitoring. We interpret here as also reflecting covariance.

good repayment.¹⁰⁶ Sharma/Zeller 97 find that groups, which form themselves and thus can screen informally, as opposed to being formed by an NGO officer, perform better. Zeller uses a similar measure but the coefficient is insignificant. On the whole, the results seem clearly to uphold screening as a mechanism that improves repayment. We find very little of significance on screening in our data. In the northeast, screening seems to be a negative predictor, while in the central region it shows up positively and significantly as theory predicts. These results seem to indicate that the Ghatak model performs well in the central region. Wydick 99, the other study broken down by region, does not include measures of screening, so we cannot compare.

Monitoring ideas are found in several of the papers. One makes the mistake of using direct measures of monitoring. In particular, Wenner 95 shows that visits by the lending institution are significant predictors of bad repayment. There appears to be a clear endogeneity problem: the lender may visit only groups that are in trouble. Wydick 99 does well in aligning with theory and avoiding this problem by focusing on the cost of monitoring rather than using a direct measure of the amount of monitoring. The variables he uses are average distance between borrowers' businesses, knowledge of each other's sales, and homogeneous occupations. The first two of these variables significantly predict repayment in the whole sample the way theory suggests; distance between businesses remains significant in the rural sample also. Homogeneous occupations is always insignificant in predicting repayment. Zeller 98 uses a measure of cohesiveness that includes relatedness, distance, and ethnicity, and finds it a significant positive predictor of repayment. On the other hand, a measure of relatives is used in Sharma/Zeller 97, who find as we do that relatedness is bad for repayment. We use variables very similar to Wydick's, plus one describing relatedness, and find that cost of monitoring variables have little predictive power in explaining repayment.

Results are mixed when it comes to cooperation's effect on repayment. Zeller 98 finds good repayment predicted significantly by a measure of homogeneity in social characteristics, such as ethnicity and religion; and by oral or written group rules, which may proxy side-contracting abilities. Wydick 99 finds results not significantly different from zero using variables measuring the length and depth of relationships between group members. As mentioned above, the degree of relatedness in a group was found to hurt repayment rates in Sharma/Zeller 97. They also find that mean village participation in group-based organizations helps repayment, while participation in informal self-help and insurance groups hurts repayment. The overall picture is thus very unclear. Our results seem to tip the balance toward cooperation hurting repayment, though this depends some on variable interpretation. In addition, we find very different effects resulting from cooperation within a family and without.

Wydick 99 is the only paper to consider penalties for default. These he measures with dummy variables measuring reported willingness to pressure others, reported difficulty in applying sanctions, perceived moral obligation, and testimony that repayment occurs to stay on good terms with the group. None of these turn out to be significantly different from zero in predicting *repayment*.¹⁰⁷ However, three of these are significant predictors of

¹⁰⁶The explanation for this may be that those with low formal credit ratings are the ones relying on informal screening, which can mitigate other factors that reduce repayment.

¹⁰⁷In fact, Wydick also includes group size as a fifth variable measuring the pressure a group can exert. This set of five variables passes a likelihood ratio test of joint significance in the urban sample, largely due

a dummy variable that equals one if group members report in the survey that there was no misuse of borrowed funds in the group. These three are willingness to pressure others, difficulty in applying sanctions, and perceived moral obligation. All three are significant in the rural sample, and all but willingness to pressure others in the whole sample. This evidence is strong, but it is hard to know how to relate this variable measuring misuse of borrowed funds and with the variable measuring repayment. Our own results find strong significance for a measure of penalties based on village-wide response to a member's default, in contrast to Wydick's focus on a group-based response. Like Wydick, we find the effect is strongest in the more rural sample. We also find a measure of official penalties to predict repayment in the rural sample. Variation in this aspect of enforcement is missing from previous studies.

Of the control variables, group size is included by Sharma/Zeller 97, who find it negative but insignificant; by Zeller 98, who finds it positive and significant; and by Wydick 99, who finds it a positive predictor, significant in the urban sample. In the logit, we too find group size positive and verging on significance in the central region, closer to Bangkok, but very near zero in the northeast. Group size actually shows up negatively in our central region nonparametric tests, but this is most likely due to the univariate nature of the tests and a strong correlation between group size and age.

None measure wealth of the village directly. Zeller 98 finds that monetarization is a significant and positive predictor of repayment, which may capture a positive wealth effect. On the other hand, Sharma/Zeller 97 find that the number of Food-for-Work programs, which are prevalent in the poorest areas, is a positive predictor of repayment. We use a detailed measure of average villager wealth and find results not significantly different from zero.¹⁰⁸

Measures of risk are also found in Zeller 98 and Sharma/Zeller 97. In the latter, it is the number of adverse shocks reported as having struck in the last eighteen months. In the former, it is the number of types of risks listed in the questionnaire to which the village representative says the village is susceptible. It is a negative and significant predictor of repayment in Zeller 98, but strangely, it shows up positive and significant in Sharma/Zeller 97. We measure village risk by calculating coefficients of variation on expected income, based on villagers' subjective assessments. The logit results show uncover a negative effect, significant in the central region and verging on significance in the whole sample.

to the significance of group size in predicting repayment.

We believe there is only tenuous theoretical support for including group size as a measure of the amount of pressure that can be exerted. A larger group has more potential punishers, but each one is hurt less on average by a single member's default and thus may punish less. Recall that the penalty function BC posit is increasing in the loss to a non-defaulting borrower.

¹⁰⁸Again, the exception is in the central region under the nonparametric tests, but as with members, village wealth is strongly positively correlated with group age.

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