POLICY GAMES AND THE OPTIMAL DESIGN OF CENTRAL BANKS

by

Andrew Hughes Hallett and Diana N. Weymark

Working Paper No. 02-W20

August 2002

DEPARTMENT OF ECONOMICS
VANDERBILT UNIVERSITY
NASHVILLE, TN 37235

www.vanderbilt.edu/econ

Policy Games and the Optimal Design of Central Banks

by

Andrew Hughes Hallett
Department of Economics,
Vanderbilt University,
Nashville, TN
a.hugheshallett@vanderbilt.edu

Diana N. Weymark
Department of Economics,
Vanderbilt University,
Nashville, TN 37235, USA
diana.weymark@vanderbilt.edu

August 2002

1. Introduction

Few studies of monetary delegation model the interaction between the government and the central bank as a game of strategy. Those that do allow for strategic policy formation, use non-cooperative simultaneous-move (Nash equilibrium) games to model the interaction between the fiscal and monetary authorities. In practice however, the institutional arrangements in many countries confer varying degrees of leadership on one of the policy authorities. In some cases, this is the result of statutory provisions or the institutional arrangements under which the central bank is required to operate. In other cases, it is simply a matter of convention or common practice built up over a number of years. But, in either event, a Nash equilibrium between the individual policy makers may not be the appropriate framework for analyzing the policies and performance of alternative central banking regimes.

Moreover, because most of the existing literature considers only the impact of monetary policy on inflation and output performance, it can offer no guidelines for choosing among regimes when there is some kind of leadership among the players; or when some of the institutional characteristics may be chosen by different players; or when fiscal policy with social equity (redistributional) objectives is being pursued by the government. In this article we analyze the implications of several alternative institutional configurations on economic performance.

No doubt there are many different institutional configurations that countries could employ in this context. For the purposes of this analysis we limit ourselves to just four representative alternatives that have practical counterparts in the real world. Our first case is represented by a two-stage game in which the government initially determines both the degree of independence and the conservatism of the central bank. Subsequently, in the second stage of the game, the government and the central bank move simultaneously in choosing fiscal and monetary policy. This constitutes our first case: it might be taken to represent the operation of the Federal Reserve System. We also include here a variant in which the central bank is free to choose a target inflation rate which is different from (less than) that preferred by the fiscal authorities. This is done to provide a second point of reference which will enable us to investigate the importance of target independence in the later stages of the paper.

Our second type of regime is one in which the government not only chooses the institutional design in the first stage of the game, but also exercises fiscal leadership in the second stage. This configuration might be taken as representative of the system under which the Bank of England now operates. In our third regime, we reverse the leadership roles and, in addition, grant the central bank target independence. In the first two regimes, by contrast, we assume that the inflation target pursued by the central bank coincides with that of the government. Our third case therefore captures some of the characteristics intended for the European central Bank. But as the degree of target independence is incomplete and because the degree of conservatism is still set by the government, it is probably more representative of the strong monetary leadership found in Switzerland or Germany before the Euro. Finally, we consider the case of simultaneously set monetary and fiscal policies, but in a world in which the government(s) can only choose the central bank's degree of independence. The central bank determines its own degree of conservatism. Here again we allow for the possibility that the inflation targets of the two policy authorities may differ. This regime captures the salient features of current practice at the European Central Bank.

2. Economic Structure

The model used in Weymark (2001) provides a useful framework for the present analysis. For purposes of exposition, we suppress potential spillover effects between countries and focus on the following three equations to represent the economic structure of any country:

$$\pi_t = \pi_t^e + \alpha y_t + u_t \tag{1}$$

$$y_t = \beta(m_t - \pi_t) + \gamma g_t + \epsilon_t \tag{2}$$

$$g_t = m_t + s(by_t - \tau_t) \tag{3}$$

where π_t is the inflation rate in period t, y_t is output growth in period t, and π_t^e

represents the rate of inflation that rational agents expect will prevail in period t, conditional on the information available at the time expectations are formed. The variables m_t, g_t, and τ_t represent, respectively, the growth in the money supply, government expenditures, and tax revenues in period t. The variables u_t and ϵ_t are random disturbances which are assumed to be independently distributed with zero mean and constant variance. The coefficients α, β, γ, s, and b are all positive by assumption. The assumption that γ is positive may be considered controversial.[1] However, short-run impact multipliers derived from Taylor's (1993) multi-country estimation provide empirical support for this assumption. [2]

According to (1), inflation is increasing in the rate of inflation predicted by private agents and in output growth. Equation (2) indicates that both monetary and fiscal policies have an impact on the output gap. The microfoundations of the aggregate supply equation (1), originally derived by Lucas (1972, 1973), are well-known. McCallum (1989) shows that aggregate demand equations like (2) can be derived from a standard, multiperiod utility-maximization problem.

Equation (3) describes the government's budget constraint. In the interests of simplicity, we allow discretionary tax revenues to be used for redistributive purposes only. Thus, in each period, the government must finance its remaining expenditures by selling government bonds to the central bank or to private agents.[3] We assume that

[1]Barro (1981) argues that government purchases have a contractionary impact on output. However, in contrast to those who argue that fiscal policy has little systematic or positive impact on economic performance, our model treats fiscal policy as important because (i) fiscal policy is used by governments to achieve includes redistributive objectives whose consequences need to be taken into account; and (ii) as Dixit and Lambertini (2001) point out, governments cannot precommit monetary policy with any credibility if fiscal policy is not also precommitted.

[2]For example, using Taylor's empirical results, Hughes Hallett and Weymark (2001) obtain short-run γ estimates of 0.57, 0.43, 0.60, and 0.58 for France , Germany, Italy, and the United Kingdom, respectively.

[3]Several variations which relax the restrictions on how fiscal policy may be financed are considered in Weymark (2001). Specifically, in one variation, bond financing is replaced by income taxes which can be used to finance both g_t and τ_t. In another variation, income taxes and newly-created general

there are two types of agents, rich and poor, and that only the rich use their savings to buy government bonds. In (3), b is the proportion of pre-tax income (output) that goes to the rich and s is the proportion of after-tax income that the rich allocate to saving. The tax, τ_t, is used by the government to redistribute income from the rich to the poor.

Using (1) and (2) to solve for π_t^e, π_t and y_t yields the following reduced forms:

$$\pi_t(g_t, m_t) = (1+\alpha\beta)^{-1}[\alpha\beta m_t + \alpha\gamma g_t + m_t^e + \frac{\gamma}{\beta} g_t^e + \alpha\epsilon_t + u_t] \tag{4}$$

$$y_t(g_t, m_t) = (1+\alpha\beta)^{-1}[\beta m_t + \gamma g_t - \beta m_t^e - \gamma g_t^e + \epsilon_t - \beta u_t]. \tag{5}$$

Equations (5) and (3) then imply

$$\tau_t(g_t, m_t) = [s(1+\alpha\beta)]^{-1}[(1+\alpha\beta+sb\beta)m_t - (1+\alpha\beta-sb\gamma)g_t - sb\beta m_t^e - sb\gamma g_t^e + sb\epsilon_t - sb\beta u_t] \tag{6}$$

3. Government and Central Bank Objectives

In this paper, we allow for the possibility that the government and a fully independent central bank may differ in their objectives in some significant way. In particular, we assume that the government cares about inflation stabilization, output growth, and income redistribution, whereas the central bank, if left to itself, would be concerned only with the first two objectives. We also assume that the government has been elected by majority vote, so that the government's loss function reflects society's preferences over alternative economic objectives.

Formally, the government's loss function is given by

$$L_t^g = \frac{1}{2}(\pi_t - \hat{\pi})^2 - \lambda_1^g y_t + \frac{\lambda_2^g}{2}[(b-\theta)y_t - \tau_t]^2 \tag{7}$$

where $\hat{\pi}$ is the government's inflation target, λ_1^g is the relative weight that the government assigns to output growth, and λ_2^g is the relative weight assigned to income

taxes are available to finance g_t and τ_t. However, the model's theoretical predictions are robust to these variations.

redistribution. The parameter θ represents the proportion of output that the government would, ideally, like to allocate to the rich. Galí and Monacelli (2002) have demonstrated that, under suitable assumptions, an objective function like (7) may be derived from the utility functions of individuals in a standard microfounded open economy model of the Obstfeld-Rogoff type. Demertzis et al (1999) have likewise shown that such a function would emerge out of the electoral process involving those agents. Hence, fiscal policy in this model will always be anchored in the microfoundations of voters' preferences, and may be considered "precommitted" in that sense.

The first term on the right of (7) reflects the government's concern with inflation stabilization. Specifically, the government incurs losses when actual inflation deviates from the government's or society's inflation target. The second term is intended to capture what many believe is a political reality for governments—namely, that voters reward governments for increases in output growth and penalize them for reductions in the growth rate.[4] The third component in the government's loss function reflects the government's concern with income redistribution. The parameter θ represents the government's ideal degree of income inequality. For example, in an economy in which there are as many rich people as poor people, an egalitarian government would set $\theta = 0.5$. Ideally, in this case, the government would like to redistribute output in the amount of $(b - 0.5)y_t$ from the rich to the poor.

We assume that the central bank has objectives which may differ from those of the government:

$$L_t^{cb} = \frac{1}{2}(\pi_t - \hat{\pi})^2 - (1-\delta)\lambda^{cb} y_t - \delta\lambda_1^g y_t + \frac{\delta\lambda_2^g}{2}[(b-\theta)y_t - \tau_t]^2 \qquad (8)$$

[4] In adopting a linear representation of the output objective, we follow Barro and Gordon (1983). In the monetary delegation literature, the output component in the government's loss function is more often represented as quadratic because the models employed typically preclude any stabilization role for monetary policy when the output term in the loss function is linear. In our model, the quadratic income redistribution term in the loss function allows monetary policy to play a direct role in output stabilization — the output variable being measured as deviations from full employment (capacity) output.

where $0 \leq \delta \leq 1$, and λ^{cb} is the weight that the central bank assigns to output growth. The parameter δ measures the degree to which the central bank is forced to take the government's objectives into account when formulating monetary policy. The closer δ is to 0, the greater is the independence of the central bank.

In (7) we have described $\hat{\pi}$ as the government's's inflation target. The fact that the same inflation target appears in (8) reflects our assumption that the central bank has instrument independence but not target independence. We relax that restriction below.

4. Four Policy Games

We characterize the strategic interaction between the government and the central bank as a two-stage non-cooperative game in which the structure of the model and the objective functions are common knowledge. Certain variations in institutional design are obtained by altering the assumptions about (i) which policy authority has control over the institutional parameters, δ and λ^{cb} in stage 1, and (ii) the timing of fiscal and monetary policy moves in stage 2. Our baseline is a game in which the government sets both δ and λ^{cb} in stage 1 and both policy authorities move simultaneously in stage 2. We then compare the outcomes associated with our baseline case to three alternatives. In one, we retain the baseline assumption for stage 1, but alter stage 2 to give the fiscal authority leadership in policy formation. Our second variation retains the stage 1 baseline assumptions, but switches the role of Stackelberg leader to the central bank in stage 2. Our third, and final, variant alters stage 1 by transferring control of λ^{cb} to the central bank, but retains the assumption of simultaneous policy moves in stage 2.

4.1 Simultaneous Moves — Government Chooses δ and λ^{cb}

In this section, we consider a situation in which the government chooses both of the institutional parameters, δ and λ^{cb} in the first stage of the game. In the second stage, the government and the monetary authority move simultaneously and set their policy instruments, given the δ and λ^{cb} values determined at the previous stage. Private agents understand the game and form rational expectations for future prices in the second stage. We consider two cases. In the first case, both government and central bank follow the same inflation target, $\hat{\pi}$, while in the second case the central bank's inflation target, $\hat{pi}^{cb}$ may differ from that of the government.

Case 1. The simultaneous-move game with coincident inflation targets can be described as follows:

Stage 1

The government solves the problem

$$\begin{aligned}\min_{\delta,\, \lambda^{cb}} \ \mathrm{E}\, L^g(g_t, m_t, \delta,\, \lambda^{cb}) \quad &= \quad \mathrm{E}\left\{\frac{1}{2}[\pi_t(g_t, m_t) - \hat{\pi}]^2 \ - \ \lambda_1^g[y_t(g_t, m_t)]\right.\\ &\left. + \ \frac{\lambda_2^g}{2}[(b-\theta)y_t(g_t, m_t) - \tau_t(g_t, m_t)]^2\right\} \qquad (9)\end{aligned}$$

where $L^g(g_t, m_t, \delta,\, \lambda^{cb})$ is (7) evaluated at $(g_t, m_t, \delta,\, \lambda^{cb})$, and E is the expectations operator.

Stage 2

(i) Private agents form rational expectations about future prices before the shocks u_t and ϵ_t are realized.

(ii) The shocks u_t and ϵ_t are realized and observed by the government and by the central bank.

(iii) The government chooses g_t, taking m_t as given, to minimize $L^g(g_t, m_t, \bar{\delta},\, \bar{\lambda}^{cb})$ where $\bar{\delta}$ and $\bar{\lambda}^{cb}$ indicates that these variables were determined in stage 1.

(iv) The central bank chooses m_t, taking g_t as given, to minimize

$$\begin{aligned}L^{cb}(g_t, m_t, \bar{\delta},\, \bar{\lambda}^{cb}) \quad &= \quad \frac{(1-\bar{\delta})}{2}[\pi_t(g_t, m_t) - \hat{\pi}]^2 \ - \ (1-\bar{\delta})\bar{\lambda}^{cb}[y_t(g_t, m_t)]\\ &+ \ \bar{\delta} L^g(g_t, m_t, \bar{\delta},\, \bar{\lambda}^{cb}). \qquad (10)\end{aligned}$$

The timing of this game is illustrated in Figure 1.

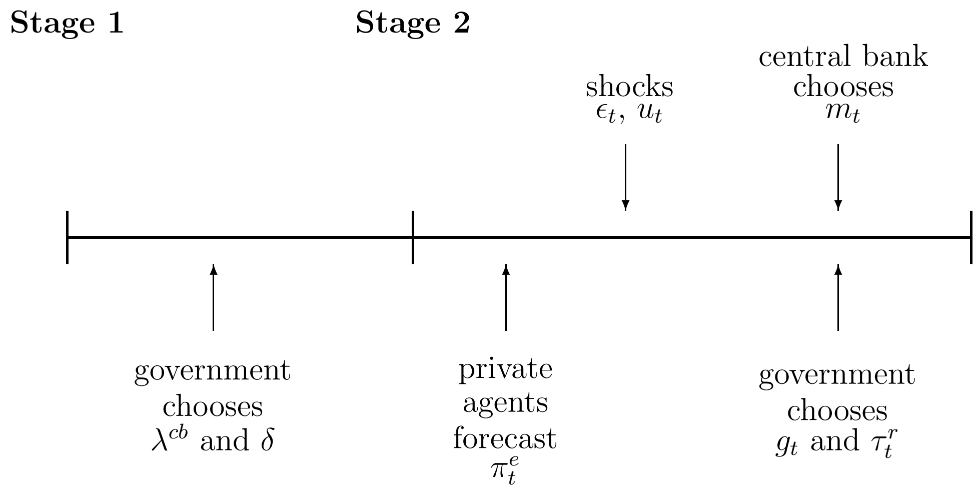

Figure 1: The Stages and Timing of the Simultaneous Move Game

This policy game can be solved by first solving the second stage of the game for the optimal money supply and government expenditure policies with δ and λ^{cb} fixed, and then solving stage 1 by substituting the stage 2 results into (9) and minimizing with respect to δ and λ^{cb}. The Nash equilibrium for stage 2 is

$$\begin{aligned} m_t(\delta, \lambda^{cb}) \;=\; & \frac{\beta\hat{\pi}}{(\beta+\gamma)} + \frac{(1-\delta)\beta[\alpha\gamma^2 s^2 \;+\; \beta\phi\lambda_2^g]\lambda^{cb}}{\alpha\lambda_2^g[\beta\phi+\delta\gamma\Lambda](\beta+\gamma)} + \frac{\delta\beta(1+\alpha\beta)\lambda_1^g}{\alpha[\beta\phi+\delta\gamma\Lambda]} \\ & - \frac{(1-\delta)\gamma^2\beta s^2\lambda_1^g}{(\beta+\gamma)[\beta\phi+\delta\gamma\Lambda]\lambda_2^g} - \frac{\epsilon_t}{(\beta+\gamma)} - \frac{(1-\beta\gamma+\gamma\theta-\gamma\theta s)u_t}{\alpha(\beta+\gamma)} \end{aligned} \tag{11}$$

$$\begin{aligned} g_t(\delta, \lambda^{cb}) \;=\; & \frac{\beta\hat{\pi}}{(\beta+\gamma)} + \frac{(1-\delta)\beta^2[\phi\lambda_2^g - \alpha\gamma s^2\;]\lambda^{cb}}{\alpha\lambda_2^g[\beta\phi+\delta\gamma\Lambda](\beta+\gamma)} + \frac{\delta\beta(1+\alpha\beta)\lambda_1^g}{\alpha[\beta\phi+\delta\gamma\Lambda]} \\ & + \frac{(1-\delta)\beta^2\gamma s^2\lambda_1^g}{(\beta+\gamma)[\beta\phi+\delta\gamma\Lambda]\lambda_2^g} - \frac{\epsilon_t}{(\beta+\gamma)} - \frac{(1+b\beta-\beta\theta+\beta\theta s)u_t}{\alpha(\beta+\gamma)} \end{aligned} \tag{12}$$

where

$$\phi \;=\; 1+\alpha\beta-\gamma\theta s \tag{13}$$

$$\Lambda \ = \ 1 + \alpha\beta + \beta\theta s. \tag{14}$$

The sign of the composite parameter Λ has no bearing on the results that follow: it is positive anyway. The results are, however, sensitive to the sign of ϕ. The parameter ϕ is perhaps most easily interpreted by noting that from (5) and (6)

$$\frac{\partial[(b-\theta)y_t - \tau_t]}{\partial g_t} = \frac{\phi}{(1+\alpha\beta)}. \tag{15}$$

The term $(b-\theta)y_t$ represents the transfer that the government would like to make to the poor. Equation (15) shows that the difference between the government's ideal transfer to the poor and actual transfer payment, τ_t^r, is positively (negatively) related to government expenditures when ϕ is positive (negative). The assumption that ϕ is positive therefore implies that increases in government expenditure make it more difficult for the government to achieve the optimal transfer. Because in this model, government expenditure is positively related to output growth, there is a conflict between government policies aimed at stimulating growth and those aimed at income redistribution when ϕ is positive. Although it is possible for ϕ to be negative, the implications of this are rather unappealing. In order for ϕ to be negative, the impact of government expenditure on output must be so large that the government can increase transfer payments without significantly reducing the funding available to finance its desired level of government expenditure. In this article, we restrict our analysis to the case in which ϕ is positive.

It is also assumed that the government and the central bank observe the white noise disturbances, u_t and ϵ_t, in the second stage before policies are chosen, but after private expectations have been formed. Although private agents cannot observe u_t and ϵ_t prior to forming expectations about future inflation rates, the characteristics of the institutions in place in the economy, represented by δ and λ^{cb}, are known to them with certainty. Under these conditions, it can be shown that (11) and (12) characterize a rational expectations equilibrium.

Taking the expectation of both sides of (11) and (12) to obtain m_t^e and g_t^e, respectively, and substituting the result, together with (11) and (12), into (4) and (5) yields the reduced-form solutions for π_t and y_t as functions of the institutional variables δ and λ^{cb}

$$\pi_t(\delta, \lambda^{cb}) \;=\; \hat{\pi} \;+\; \frac{(1-\delta)\beta\phi\lambda^{cb}}{\alpha[\beta\phi \;+\; \delta\gamma\Lambda]} \;+\; \frac{\delta[\beta\phi \;+\; \gamma\Lambda]\lambda_1^g}{\alpha[\beta\phi + \delta\gamma\Lambda]} \tag{16}$$

$$y_t(\delta, \lambda^{cb}) \;=\; \frac{-u_t}{\alpha}. \tag{17}$$

From (6), the reduced-form solution for τ_t is given by

$$\tau_t(\delta, \lambda^{cb}) \;=\; \frac{(1-\delta)\beta\gamma s(\lambda^{cb} - \lambda_1^g)}{[\beta\phi + \delta\gamma\Lambda]\lambda_2^g} \;-\; \frac{(b-\theta)u_t}{\alpha}. \tag{18}$$

Substituting (16) - (18) into (9), the government's stage 1 minimization problem can be expressed as

$$\min_{\delta, \lambda^{cb}} \;\; \mathrm{E}L^g(\delta, \lambda^{cb}) \;=\; \frac{1}{2}\left\{\frac{(1-\delta)\beta\phi\lambda_1^{cb}}{\alpha[\beta\phi \;+\; \delta\gamma\Lambda]} \;+\; \frac{\delta[\beta\theta + \Lambda\gamma]\lambda_1^g}{\alpha[\beta\phi + \delta\gamma\Lambda]}\right\}^2$$

$$+\; \frac{\lambda_2^g}{2}\left\{\frac{(1-\delta)\beta\gamma s(\lambda^{cb} - \lambda_1^g)}{[\beta\phi + \delta\gamma\Lambda]\lambda_2^g}\right\}^2. \tag{19}$$

Partial differentiation of (19) with respect λ^{cb} and δ now yields the first-order conditions for choosing δ and λ^{cb}:

$$\frac{\partial \mathrm{E}L^g(\delta, \lambda^{cb})}{\partial \lambda^{cb}} \;=\; \frac{(1-\delta)^2(\beta\phi)^2\lambda^{cb} + \delta(1-\delta)\beta\phi[\beta\phi + \gamma\Lambda]\lambda_1^g}{\alpha^2[\beta\phi + \delta\gamma\Lambda]^2}$$

$$+\; \frac{(1-\delta)^2(\beta\gamma s)^2(\lambda^{cb} - \lambda_1^g)}{\lambda_2^g[\beta\phi + \delta\gamma\Lambda]^2} \;=\; 0 \tag{20}$$

$$\frac{\partial \mathrm{E}L^g(\delta, \lambda^{cb})}{\partial \delta} \;=\; -\;\frac{\left\{(1-\delta)\beta\phi\lambda^{cb} + \delta[\beta\phi + \gamma\Lambda]\lambda_1^g\right\}\beta\phi[\beta\phi + \gamma\Lambda](\lambda^{cb} - \lambda_1^g)}{\alpha^2[\beta\phi + \delta\gamma\Lambda]^3}$$

$$-\; \left\{\frac{(1-\delta)(\beta\gamma s)^2[\beta\phi + \gamma\Lambda](\lambda^{cb} - \lambda_1^g)^2}{\lambda_2^g[\beta\phi + \delta\gamma\Lambda]^3}\right\} \;=\; 0 \tag{21}$$

It is evident that $[\beta\phi+\delta\gamma\Lambda]=0$ is not a solution to the minimization problem. When $[\beta\phi+\delta\gamma\Lambda]\neq 0$, (20) and (21) yield, respectively, (22) and (23):

$$\lambda_2^g(1-\delta)\phi\left\{(1-\delta)\beta\phi\lambda^{cb}+\delta[\beta\phi+\gamma\Lambda]\lambda_1^g\right\}\ +\ \alpha^2(1-\delta)^2\beta\gamma^2 s^2(\lambda^{cb}-\lambda_1^g)\ =\ 0 \tag{22}$$

$$\lambda_2^g\phi\left\{(1-\delta)\beta\phi\lambda^{cb}+\delta[\beta\phi+\gamma\Lambda]\lambda_1^g\right\}(\lambda^{cb}-\lambda_1^g)\ +\ \alpha^2(1-\delta)\beta\gamma^2 s^2(\lambda^{cb}-\lambda_1^g)^2\ =\ 0. \tag{23}$$

There are two solutions that satisfy both of the first-order conditions given above. By inspection, it is apparent that (22) and (23) are both satisfied when $\delta=1$ and $\lambda^{cb}=\lambda_1^g$. This solution characterizes a central bank that is fully dependent. When $\delta\neq 1$ and $\lambda^{cb}\neq\lambda_1^g$, then (22) and (23) imply the following relationship between δ and λ^{cb}

$$\delta\ =\ \frac{\beta\phi^2\lambda^{cb}\lambda_2^g\ +\ (\alpha\gamma s)^2\beta(\lambda^{cb}-\lambda_1^g)}{\beta\phi^2\lambda^{cb}\lambda_2^g\ +\ (\alpha\gamma s)^2\beta(\lambda^{cb}-\lambda_1^g)\ -\ \phi[\beta\phi+\gamma\Lambda]\lambda_1^g\lambda_2^g}, \tag{24}$$

or, equivalently,

$$\lambda^{cb}\ =\ \frac{(\alpha\gamma s)^2\lambda_1^g}{\phi^2\lambda_2^g+(\alpha\gamma s)^2}\ -\ \frac{\delta[\beta\phi+\gamma\Lambda]\phi\lambda_1^g\lambda_2^g}{(1-\delta)\beta[\phi^2\lambda_2^g+(\alpha\gamma s)^2]}. \tag{25}$$

The solution that yields the minimum loss for the government, as measured by the government's loss function (7), can be identified by using (19) to compare the expected loss that would be suffered under the alternative institutional arrangements. Substituting $\delta=1$ and $\lambda^{cb}=\lambda_1^g$ into (19) results in

$$\mathrm{E}L^g\ =\ \frac{(\lambda_1^g)^2}{2\alpha^2}. \tag{26}$$

Substituting (24) into the right-hand-side of (19) yields

$$\mathrm{E}L^g\ =\ \frac{(\lambda_1^g)^2}{2\alpha^2}\left\{\frac{(\alpha\gamma s)^2}{(\alpha\gamma s)^2+\phi^2\lambda_2^g}\right\}. \tag{27}$$

The preference parameter λ_2^g is nonnegative by assumption. For positive (nonnegative) values of λ_2^g, the value of (26) exceeds (equals) that of (27) which establishes that (24) is the solution to the government's loss minimization problem.

Case 2. As a small but important variant on our reference case, we can also allow for the possibility that the central bank may adopt its own inflation target , $\hat{\pi}^{cb}$. This gives the central bank target independence. In what follows, we assume that the central bank's inflation target would be lower than that of the government (i.e., $\hat{\pi}^{cb} < \hat{\pi}$). As in case 1, institutional parameters, δ and λ^{cb} are chosen by the government.

It is comparatively easy to rework the previous case, but allowing the central bank to adopt its own inflation target, $\hat{\pi}^{cb} < \hat{\pi}$ in (8) or (10). The expressions that emerge are somewhat more complicated however. Repeating the same steps as in case 1 we get

$$\pi_t(\delta, \lambda^{cb}) = \frac{(1-\delta)\beta\phi\hat{\pi}^{cb} + \delta(\beta\phi + \gamma\Lambda)\hat{\pi}}{[\beta\phi + \delta\gamma\Lambda]} + \frac{(1-\delta)\beta\phi\lambda^{cb} + \delta[\beta\phi + \gamma\Lambda]\lambda_1^g}{\alpha[\beta\phi + \delta\gamma\Lambda]} \quad (28)$$

$$y_t(\delta, \lambda^{cb}) = \frac{-u_t}{\alpha}. \quad (29)$$

$$\tau_t(\delta, \lambda^{cb}) = \frac{\alpha\beta\gamma s(1-\delta)(\hat{\pi}^{cb} - \hat{\pi})}{[\beta\phi + \delta\gamma\Lambda]\lambda_2^g} + \frac{(1-\delta)\beta\gamma s(\lambda^{cb} - \lambda_1^g)}{[\beta\phi + \delta\gamma\Lambda]\lambda_2^g} - \frac{(b-\theta)u_t}{\alpha}. \quad (30)$$

The new institutional parameters are then given by

$$\delta = \frac{(\beta\phi)^2\lambda^{cb}\lambda_2^g - \Omega(\hat{\pi}, \hat{\pi}^{cb})}{(\beta\phi)^2\lambda^{cb}\lambda_2^g - \Omega(\hat{\pi}, \hat{\pi}^{cb}) - \beta\phi([\beta\phi + \gamma\Lambda]\lambda_1^g\lambda_2^g} \quad (31)$$

and

$$\lambda^{cb} = \alpha(\hat{\pi} - \hat{\pi}^{cb}) + \frac{(\alpha\gamma s)^2\lambda_1^g}{[\phi^2\lambda_2^g + (\alpha\gamma s)^2]} - \frac{\phi[\beta\phi + \gamma\Lambda]\lambda_1^g\lambda_2^g\delta}{\beta(1-\delta)[\phi^2\lambda_2^g + (\alpha\gamma s)^2]} \quad (32)$$

where $\Omega(\hat{\pi}, \hat{\pi}^{cb}) = \alpha(\beta\phi)^2(\hat{\pi} - \hat{\pi}^{cb})\lambda_2^g + (\alpha\beta\gamma s)^2[\lambda_1^g - \lambda^{cb} + \alpha(\hat{\pi} - \hat{\pi}^{cb})]$.

Substituting (28) – (32) back into (7) yields exactly the same welfare losses for the government (and society) as in case 1: i.e., we get (27) again.

The results obtained here may now be compared to case 1, where there is no target independence. Various conclusions follow. First, there is no advantage (or disadvantage) in granting target independence to the central bank as far as society and its elected government are concerned.[5] The reason is that, if the central bank were (expected) to choose a lower inflation target than the government ($\hat{\pi}^{cb} < \hat{\pi}$), the government would then choose its institutional parameters to compensate. It is easy to check that $\partial\delta/\partial(\hat{\pi} - \hat{\pi}^{cb}) > 0$ for any value of λ^{cb}; or that, because of the extra term in $(\hat{\pi} - \hat{\pi}^{cb})$, the value of λ^{cb} in case 2 always exceeds that in case 1 for any value of δ. Consequently, any attempt by the central bank to systematically exploit target independence by setting its own inflation target would cause an optimizing government to reduce the degree of independence conferred on the bank and/or the degree of conservatism of those appointed to manage monetary policy. In comparison to case 1, inflation is always lower in case 2 and income inequality greater; output stability is the same in both cases. Clearly, the different institutional arrangements can result in the same welfare outcome. This result shows that granting central banks target independence will not, on its own, be welfare improving. However, the degree of target independence granted the central bank is not a matter of indifference. First, because target independence can alter the mix of outcomes, changes in the degree of target independence may benefit certain groups in society over others. Second, a central bank that unexpectedly imposes its own inflation target will inevitably appear — from society's perspective — to be too independent or too conservative in its policies. Such criticisms have been a matter of great concern to the ECB.

4.2 Fiscal Policy Leadership — Government Chooses δ and λ^{cb}

In this variation, we maintain the same constitutional structure (i.e., stage 1 is unchanged), but allow the government to exercise leadership with its fiscal policy while the central bank may be (but does not have to be) fully independent in pursuit of its objectives. Thus, the government still chooses the institutional parameters δ and λ^{cb} in the first stage of the game. But the second stage is a Stackelberg game in which the government takes on a leadership role. That means the government and the monetary

[5] That does not rule out the possibility that there may be some private advantage to the central bank.

authority set their policy instruments, given values for δ and λ^{cb} determined at the previous stage, in the knowledge that the second stage would be a Stackelberg game with fiscal leadership. Formally, this policy game can be described as follows:

Stage 1

The government solves the problem:

$$\min_{\delta,\,\lambda^{cb}} \; \mathrm{E}\, L^g(g_t, m_t, \delta,\, \lambda^{cb}) \quad = \quad \mathrm{E}\left\{\frac{1}{2}[\pi_t(g_t, m_t) - \hat{\pi}]^2 \; - \; \lambda_1^g [y_t(g_t, m_t)] + \frac{\lambda_2^g}{2}[(b-\theta) y_t(g_t, m_t) - \tau_t(g_t, m_t)]^2\right\} \tag{33}$$

where $L^g(g_t, m_t, \delta,\, \lambda^{cb})$ is (7) evaluated at $(g_t, m_t, \delta,\, \lambda^{cb})$, and E is the expectations operator.

Stage 2

(i) Private agents form rational expectations about future prices π_t^e before the shocks u_t and ϵ_t are realized.

(ii) The shocks u_t and ϵ_t are realized and observed by the government and by the central bank.

(iii) The government chooses g_t, before m_t is chosen by the central bank, to minimize $L^g(g_t, m_t, \bar{\delta},\, \bar{\lambda}^{cb})$, where $\bar{\delta}$ and $\bar{\lambda}^{cb}$ indicates that these variables were determined in stage 1.

(iv) The central bank chooses m_t, taking g_t as given, to minimize

$$L^{cb}(g_t, m_t, \bar{\delta},\, \bar{\lambda}^{cb}) = \frac{(1-\bar{\delta})}{2}[\pi_t(g_t, m_t) - \hat{\pi}]^2 - (1-\bar{\delta})\bar{\lambda}^{cb}[y_t(g_t, m_t)] + \; \bar{\delta} L^g(g_t, m_t, \bar{\delta},\, \bar{\lambda}^{cb}) \tag{34}$$

The timing of this game is illustrated in Figure 2.

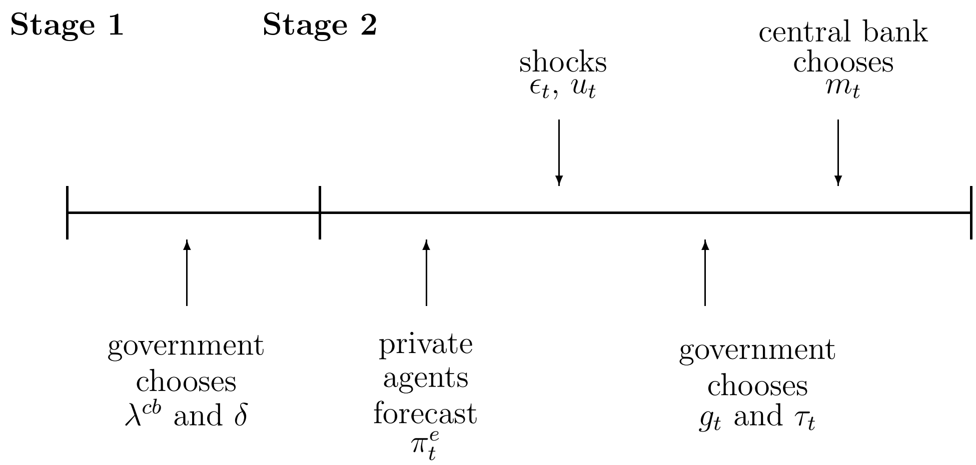

Figure 2: The Stages and Timing of the Government Leadership Game

This game can be solved by first solving the second stage of the problem for the optimal money supply and government expenditure policies with δ and λ^{cb} fixed, and then solving stage 1 by substituting the stage 2 results into (33) and minimizing with respect to δ and λ^{cb}. The equilibrium for the stage 2 leader-follower game is:

$$
\begin{aligned}
m_t(\delta, \lambda^{cb}) \;=\; & \frac{\beta\hat{\pi}}{(\beta+\gamma)} \;+\; \frac{(1-\delta)\beta[\beta(\phi-\eta\Lambda)\lambda_2^g \;+\; \alpha\gamma(\beta\eta+\gamma)s^2]\lambda^{cb}}{\alpha(\beta+\gamma)[\beta(\phi-\eta\Lambda)+\delta\Lambda(\beta\eta+\gamma)]\lambda_2^g} \\
& + \frac{\delta\beta[\beta\phi+\gamma\Lambda)\lambda_1^g}{\alpha(\beta+\gamma)[\beta(\phi-\eta\Lambda)+\delta\Lambda(\beta\eta+\gamma)]} \;-\; \frac{(1-\gamma\theta s)u_t}{\alpha(\beta+\gamma)} \\
& - \frac{(1-\delta)\beta\gamma s^2(\beta\eta+\gamma)\lambda_1^g}{(\beta+\gamma)[\beta(\phi-\eta\Lambda)+\delta\Lambda(\beta\eta+\gamma)]\lambda_2^g} - \frac{\epsilon_t}{(\beta+\gamma)} \qquad (35)
\end{aligned}
$$

$$
\begin{aligned}
g_t(\delta, \lambda^{cb}) \;=\; & \frac{\beta\hat{\pi}}{(\beta+\gamma)} \;+\; \frac{(1-\delta)\beta^2[(\phi-\eta\Lambda)\lambda_2^g - \alpha s^2(\beta\eta+\gamma)]\lambda^{cb}}{\alpha(\beta+\gamma)[\beta(\phi-\eta\Lambda)+\delta\Lambda(\beta\eta+\gamma)]\lambda_2^g} \\
& + \frac{\delta\beta[\beta\phi+\gamma\Lambda)\lambda_1^g}{\alpha(\beta+\gamma)[\beta(\phi-\eta\Lambda)+\delta\Lambda(\beta\eta+\gamma)]} \;-\; \frac{(1+\beta\theta s)u_t}{\alpha(\beta+\gamma)} \\
& + \frac{(1-\delta)(\beta s)^2(\beta\eta+\gamma)\lambda_1^g}{(\beta+\gamma)[\beta(\phi-\eta\Lambda)+\delta\Lambda(\beta\eta+\gamma)]\lambda_2^g} - \frac{\epsilon_t}{(\beta+\gamma)} \qquad (36)
\end{aligned}
$$

where

$$\eta \;=\; \frac{\partial m_t}{\partial g_t} \;=\; \frac{-\,\alpha^2\gamma\beta s^2 \;+\; \delta\phi\Lambda\lambda_2^g}{(\alpha\beta s)^2 \;+\; \delta\Lambda^2\lambda_2^g} \qquad (37)$$

$$\phi \;=\; 1+\alpha\beta-\gamma\theta s \tag{38}$$

$$\Lambda \;=\; 1+\alpha\beta+\beta\theta s. \tag{39}$$

Taking the mathematical expectation of both sides of (35) and (36) to obtain m_t^e and g_t^e, respectively, and substituting the result, together with (35) and (36), into (4) and (5) yields the reduced-form solutions for π_t and y_t as functions of the institutional variables δ and λ^{cb}

$$\pi_t(\delta,\lambda^{cb}) \;=\; \hat{\pi} \;+\; \frac{(1-\delta)\beta(\phi-\eta\Lambda)\lambda^{cb} \;+\; \delta[\beta\phi+\gamma\Lambda]\lambda_1^g}{\alpha[\beta(\phi-\eta\Lambda) \;+\; \delta\Lambda(\beta\eta+\gamma)]} \tag{40}$$

$$y_t(\delta,\lambda^{cb}) \;=\; \frac{-u_t}{\alpha}. \tag{41}$$

From (6), the reduced-form solution for τ_t is given by

$$\tau_t(\delta,\lambda^{cb}) \;=\; \frac{(1-\delta)\beta s(\beta\eta+\gamma)(\lambda^{cb}-\lambda_1^g)}{[\beta(\phi-\eta\Lambda) \;+\; \delta\Lambda(\beta\eta+\gamma)]\lambda_2^g} \;-\; \frac{(b-\theta)u_t}{\alpha}. \tag{42}$$

Substituting (40)–(42) into (33), the government's stage 1 minimization problem can now be expressed as

$$\begin{aligned}\min_{\delta,\lambda^{cb}} \;\; \mathrm{E}L^g(\delta,\lambda^{cb}) \;\;&=\;\; \frac{1}{2}\left\{\frac{(1-\delta)\beta(\phi-\eta\Lambda)\lambda^{cb} \;+\; \delta[\beta\phi+\gamma\Lambda]\lambda_1^g}{\alpha[\beta(\phi-\eta\Lambda) \;+\; \delta\Lambda(\beta\eta+\gamma)]}\right\}^2 \\ &\quad+\; \frac{\lambda_2^g}{2}\left\{\frac{(1-\delta)\beta s(\beta\eta+\gamma)(\lambda^{cb}-\lambda_1^g)}{[\beta(\phi-\eta\Lambda) \;+\; \delta\Lambda(\beta\eta+\gamma)]\lambda_2^g}\right\}^2.\end{aligned} \tag{43}$$

Partial differentiation of (43) with respect λ^{cb} and δ yields the first-order conditions

$$\begin{aligned}\frac{\partial \mathrm{E}L^g(\delta,\lambda^{cb})}{\partial\lambda^{cb}} \;\;&= \\ &\frac{[(1-\delta)\beta(\phi-\eta\Lambda)\lambda^{cb}+\;\delta[\beta\phi+\gamma\Lambda]\lambda_1^g](1-\delta)\beta(\phi-\eta\Lambda)}{\alpha^2[\beta(\phi-\eta\Lambda) \;+\; \delta\Lambda(\beta\eta+\gamma)]^2} \\ &\quad-\; \frac{(1-\delta)^2(\beta s)^2(\beta\eta+\gamma)^2(\lambda_1^g-\lambda^{cb})}{\lambda_2^g[\beta(\phi-\eta\Lambda) \;+\; \delta\Lambda(\beta\eta+\gamma)]^2} \;=\; 0\end{aligned} \tag{44}$$

$$\frac{\partial \mathrm{E}L^g(\delta,\lambda^{cb})}{\partial\delta} = \frac{\begin{array}{r}(1-\delta)\beta(\phi-\eta\Lambda)\lambda^{cb}+\delta[\beta\phi+\gamma\Lambda]\lambda_1^g\beta[\beta\phi+\gamma\Lambda]\\(\lambda_1^g-\lambda^{cb})\left\{\delta(1-\delta)\Lambda\Omega+(\phi-\eta\Lambda)\right\}\end{array}}{\alpha^2[\beta(\phi-\eta\Lambda)\ +\ \delta\Lambda(\beta\eta+\gamma)]^3}$$

$$-\ \frac{\begin{array}{r}(1-\delta)(\beta\eta+\gamma)(\beta s)^2[\beta\phi+\gamma\Lambda]\\\left\{(\beta\eta+\gamma)-(1-\delta)\beta\Omega\right\}(\lambda_1^g-\lambda^{cb})^2\end{array}}{\lambda_2^g[\beta(\phi-\eta\Lambda)\ +\ \delta\Lambda(\beta\eta+\gamma)]^3} = 0 \tag{45}$$

where $\Omega = \partial\eta/\partial\delta$.

It is evident that $[\beta(\phi-\eta\Lambda)+\delta\Lambda(\beta\eta+\gamma)] = 0$ is not a solution to the minimization problem. But when $[\beta(\phi-\eta\Lambda)+\delta\Lambda(\beta\eta+\gamma)] \neq 0$, (44) and (45) yield (46) and (47), respectively:

$$\begin{aligned}(1-\delta)(\phi-\eta\Lambda)\lambda_2^g\left\{(1-\delta)\beta(\phi-\eta\Lambda)\lambda^{cb}+\delta[\beta\phi+\gamma\Lambda]\lambda_1^g\right\}&\\ -\ (1-\delta)^2(\beta\eta+\gamma)^2(\alpha s)^2\beta(\lambda_1^g-\lambda^{cb}) = 0&\end{aligned} \tag{46}$$

$$\begin{aligned}&\left\{(1-\delta)\beta(\phi-\eta\Lambda)\lambda^{cb}+\delta[\beta\phi+\gamma\Lambda]\lambda_1^g\right\}(\lambda_1^g-\lambda^{cb})\\ &\qquad\qquad\left\{\delta(1-\delta)\Lambda\Omega+(\phi-\eta\Lambda)\right\}\lambda_2^g\\ &-(1-\delta)(\beta\eta+\gamma)(\alpha s)^2\beta\left\{(\beta\eta+\gamma)-(1-\delta)\beta\Omega\right\}(\lambda_1^g-\lambda^{cb})^2 = 0.\end{aligned} \tag{47}$$

There are two real-valued solutions that satisfy these two first-order conditions, and which fall within the permissible range for δ.[6] By inspection, it is apparent that (46) and (47) are both satisfied when $\delta = 1$ and $\lambda^{cb} = \lambda_1^g$. This solution characterizes a central bank that is fully dependent. The second solution is $\delta = \lambda^{cb} = 0$. In

[6]Because η is a function of δ, (47) is a quartic polynomial in δ. This polynomial has four distinct roots, of which only two are real-valued. We can discard the complex solutions as having no economic meaning. Details of the complete solution for these first-order conditions may be found in Appendix 1.

this case, the central bank is fully independent <u>and</u> concerned exclusively with the economy's inflation performance.

The solution that yields the minimum loss for the government, as measured by the government's loss function, can be identified by using (43) to compare the expected loss that would be suffered under the two alternative institutional arrangements. Substituting $\delta = 1$ and $\lambda^{cb} = \lambda_1^g$ into (43) results in

$$\mathrm{E}L^g \;=\; \frac{(\lambda_1^g)^2}{2\alpha^2}. \tag{48}$$

Substituting $\delta = \lambda^{cb} = 0$ into the right-hand-side of (43) yields

$$\mathrm{E}L^g \;= 0. \tag{49}$$

It is evident that when institutional arrangements are such that the government is the Stackelberg leader in the second stage policy game, the optimal central bank design — from society's point of view — is one in which the central bank is required to use monetary policy to achieve the government's chosen inflation target, ignoring output growth and social equality objectives, and is granted full independence to do so.[7] In the following section we show that central bank leadership does not provide as good a result from society's point of view, *even if* the government is able to impose its own inflation target, and we explain why in Section 5.

4.3 Monetary Policy Leadership — Government Chooses δ and λ^{cb}

In this section, we contrast the results of the last section, fiscal leadership, with the case where the central bank is granted leadership under the same constitutional arrangements. That is, when the government continues to choose the degree of monetary delegation (δ) and the general stance or conservatism of monetary policies (λ^{cb}). The words "is granted leadership" are significant because they indicate that there is a principal-agent relationship in which the government sets the parameters within

[7]Recall that $\hat{\pi} = \hat{\pi}^{cb}$ in this case. Since (49) shows an elected government would achieve $\mathrm{E}L^g = 0$, allowing $\hat{\pi}^{cb} < \hat{\pi}$ would not have generated any further improvements for society as a whole.

which the central bank must operate. The government is therefore responsible for determining the degree of delegation and the institutional arrangements that the central bank must observe — the relationship between the German government and the Bundesbank before the advent of the Euro is an example of such an arrangement. This differs from the case in which the central bank "assumes leadership and ultimate responsibility" for monetary policy. In that case, the government chooses the degree of delegation which makes monetary leadership possible, but all other aspects of monetary policy design (including the degree of conservatism and inflation targets) are subject to choice by the central bank. An arrangement of this sort would imply a much greater degree of target (as well as instrument) independence and is a reasonably good description of the role of the ECB in the Eurozone. We consider the implications of monetary leadership of this type in Section 4.4 below.

Whichever form of central bank leadership we study, a leadership role inevitably involves a certain degree of target independence. We therefore allow the central bank to choose it own inflation targets as follows:

$$L_t^{cb} = \frac{1}{2}(\pi_t - \hat{\pi}^{cb})^2 - (1-\delta)\lambda^{cb} y_t - \delta\lambda_1^g y_t + \frac{\delta\lambda_2^g}{2}[(b-\theta)y_t - \tau_t]^2 \tag{50}$$

where the central bank's inflation target, $\hat{\pi}^{cb}$, may now differ from the government's inflation target value $\hat{\pi}$.

When the central bank has full target independence and is the Stackelberg leader, the reduced-form solutions for π_t, y_t, and τ_t are:

$$\pi_t = \frac{[(\beta+\mu\gamma)\phi\hat{\pi}^{cb} + \delta\gamma(\Lambda-\mu\phi)\hat{\pi}}{(\beta+\mu\gamma)\phi + \delta\gamma(\Lambda-\mu\phi)} + \frac{(1-\delta)(\beta+\mu\gamma)\phi\lambda^{cb}}{\alpha[(\beta+\mu\gamma)\phi + \delta\gamma(\Lambda-\mu\phi)]} + \frac{\delta[\beta\phi+\gamma\Lambda]\lambda_1^g}{\alpha[(\beta+\mu\gamma)\phi + \delta\gamma(\Lambda-\mu\phi)]} \tag{51}$$

$$y_t = \frac{-u_t}{\alpha} \tag{52}$$

$$\tau_t = \frac{\alpha\gamma s(\beta+\mu\gamma)(\hat{\pi}-\hat{\pi}^{cb})}{[(\beta+\mu\gamma)\phi + \delta\gamma(\Lambda-\mu\phi)]\lambda_2^g}$$

$$+\ \frac{(1-\delta)\gamma(\beta+\mu\gamma)s(\lambda_1^g-\lambda^{cb})}{[(\beta+\mu\gamma)\phi\ +\ \delta\gamma(\Lambda-\mu\phi)]\lambda_2^g}\ -\ \frac{(b-\theta)u_t}{\alpha} \tag{53}$$

where $\mu\ =\ \dfrac{\partial g_t}{\partial m_t}\ =\ \dfrac{-\alpha^2\beta\gamma s^2\ +\ \phi\Lambda\lambda_2^g}{(\alpha\gamma s)^2\ +\ \phi^2\lambda_2^g}$.

Substituting (51)–(53) into the government's loss function (7), and differentiating with respect to λ^{cb} and δ yields the necessary first-order conditions:

$$\begin{aligned}\frac{\partial \mathrm{E}L_t^g}{\partial\lambda^{cb}} &= (1-\delta)\phi\lambda_2^g\left\{-\alpha\Gamma\phi(\hat{\pi}-\hat{\pi}^{cb})+\phi(1-\delta)\Gamma\lambda^{cb}+\delta[\beta\phi+\gamma\Lambda]\lambda_1^g\right\}\\ &\qquad -\ (\alpha\gamma s)^2\Gamma(1-\delta)\left[\alpha(\hat{\pi}-\hat{\pi}^{cb})+(1-\delta)(\lambda_1^g-\lambda^{cb})\right]\ =\ 0\end{aligned} \tag{54}$$

$$\begin{aligned}\frac{\partial \mathrm{E}L_t^g}{\partial\lambda^{cb}} &= \phi\lambda_2^g\Gamma\Sigma\left\{-\alpha(\beta+\mu\gamma)\phi(\hat{\pi}-\hat{\pi}^{cb})+\phi(1-\delta)\Gamma\lambda^{cb}+\delta[\beta\phi+\gamma\Lambda]\lambda_1^g\right\}\\ &\qquad -\ (\alpha\gamma s)^2\Gamma^2\Sigma\left[\alpha(\hat{\pi}-\hat{\pi}^{cb})+(1-\delta)(\lambda_1^g-\lambda^{cb})\right]\ =\ 0\end{aligned} \tag{55}$$

where

$$\Sigma\ =\ [\beta\phi+\gamma\Lambda](\lambda_1^g-\lambda^{cb})\ +\ \alpha\gamma(\hat{\pi}-\hat{\pi}^{cb})(\Lambda-\mu\phi)$$

$$\Gamma=(\beta+\mu\gamma).$$

There are two solutions that satisfy both of the first-order conditions given above. By inspection, it is apparent that (54) and (55) are both satisfied when $\delta=1$ and $\Gamma=0$. But when $0\le\delta<1$ and $\Gamma\neq 0$, then (54) and (55) imply the following relationship between δ and λ^{cb}

$$\delta\ =\ \frac{(\beta+\mu\gamma)\left\{\phi^2\lambda^{cb}\lambda_2^g\ +\ (\alpha\gamma s)^2(\lambda^{cb}-\lambda_1^g)\ -\ \alpha[\phi^2\lambda_2^g+(\alpha\gamma s)^2](\hat{\pi}-\hat{\pi}^{cb})\right\}}{(\beta+\mu\gamma)\left\{\phi^2\lambda^{cb}\lambda_2^g\ +\ (\alpha\gamma s)^2(\lambda^{cb}-\lambda_1^g)\right\}\ -\ \phi[\beta\phi+\gamma\Lambda]\lambda_1^g\lambda_2^g}. \tag{56}$$

It is straightforward to show that the government's expected losses are minimized by combinations of δ and λ^{cb} that satisfy (56). Substituting (56) into into the right-hand-side of (43) then yields

$$\mathrm{E}L^g\ =\ \frac{(\lambda_1^g)^2}{2\alpha^2}\left\{\frac{(\alpha\gamma s)^2}{(\alpha\gamma s)^2+\phi^2\lambda_2^g}\right\}. \tag{57}$$

Comparing (57) with (27) shows that the government's (and society's) expected loss is greater under central bank leadership than under government leadership. In fact, the loss under central bank leadership is identical to the loss incurred by the government in a simultaneous move regime.

Furthermore, we can also see that target independence has no impact on economic outcomes or government losses as long as the government can alter the degree of central bank conservatism to compensate for the difference between its own inflation target and that of the central bank. To see this, note that when the central bank is fully independent (i.e., $\delta = 0$), the optimal degree of central bank conservatism (from 56) becomes

$$\lambda^{cb} \; = \; \frac{(\alpha\gamma s)^2 \lambda_1^g}{(\alpha\gamma s)^2 \; + \; \phi^2 \lambda_2^g} \; + \; \alpha(\hat{\pi} - \hat{\pi}^{cb}). \tag{58}$$

So, if the central bank was (expected) to choose to be target conservative compared to the government, the government would relax the constitutional arrangements in the direction of less weight conservatism. That is the reason why this regime gives the same outcomes and performance as the simultaneous moves game of Section 4.1. The government therefore only runs into difficulties if it does not have sufficient power to change those constitutional arrangements or operating procedures.

Since the delegation of monetary policy is a matter which governments decide for themselves, and which requires certain constitutional provisions that cannot be changed very frequently, it is reasonable to assume that the government would retain control of the choice of δ. But the degree of conservatism adopted in the policies of the central bank is more in the nature of an operating procedure which might more easily be changed as circumstances require. Hence the most likely development, if the government cannot adjust λ^{cb} according to (58), is that the government continues to choose the degree of monetary delegation (δ) while the central bank assumes target as well as instrument independence and chooses λ^{cb}, $\hat{\pi}^{cb}$, and then the monetary policy. The implications of institutional arrangements of this sort are examined next.

4.4 Simultaneous Moves — Central Bank Chooses λ^{cb}

If the central bank is potentially independent (i.e., δ is small) and able to choose its own inflation target ($\hat{\lambda}^{cb}$), then it is artificial to suppose that the government would be able to impose its preferred degree of conservatism (λ^{cb}) on the central bank's operations at the same time. In this section, we allow the central bank to choose λ^{cb} in order to define the stance of monetary policy.

However, because the central bank can now choose all of the characteristics of monetary policy for itself, it is reasonable to assume the bank and the government would choose their policies separately but at the same time in stage 2; and also their preferred institutional arrangements separately, but simultaneously in stage 1. The government's objective function is given, as before, by (7). However, since it does not have monetary leadership at stage 2, the central bank would try to minimize

$$\begin{aligned} L^{cb} \;=\; & \frac{(1-\delta)}{2}(\pi_t - \hat{\pi}^{cb})^2 \;+\; \frac{\delta}{2}(\pi_t - \hat{\pi})^2 \;-\; (1-\delta)\lambda^{cb} y_t \;-\; \delta\lambda_1^g y_t \\ & + \frac{\delta\lambda_2^g}{2}[(b-\theta)y_t - \tau_t]^2 \end{aligned} \tag{59}$$

which converges to the monetary leadership case, (50), as $\delta \to 0$. The Nash equilibrium policies at stage 2 are then:

$$\begin{aligned} m_t(\delta, \lambda^{cb}) \;=\; & \frac{\delta\beta\gamma\Lambda\hat{\pi}}{(\beta+\gamma)[\beta\theta + \delta\gamma\Lambda]} \;-\; \frac{\alpha\beta(\gamma s)^2(1-\delta)(\hat{\pi} - \hat{\pi}^{cb})}{(\beta+\gamma)[\beta\theta + \delta\gamma\Lambda]\lambda_2^g} \;+\; \frac{\beta^2\phi[\delta\hat{\pi} + (1-\delta)\hat{\pi}^{cb}]}{(\beta+\gamma)[\beta\theta + \delta\gamma\Lambda]} \\ & + \frac{(1-\delta)\beta[\alpha(\gamma s)^2 + \beta\phi\lambda_2^g]\lambda^{cb}}{\alpha\lambda_2^g[\beta\phi + \delta\gamma\Lambda](\beta+\gamma)} \;+\; \frac{\delta\beta(1+\alpha\beta)\lambda_1^g}{\alpha[\beta\phi + \delta\gamma\Lambda]} \\ & - \frac{(1-\delta)\beta(\gamma s)^2\lambda_1^g}{(\beta+\gamma)[\beta\phi + \delta\gamma\Lambda]\lambda_2^g} \;-\; \frac{\epsilon_t}{(\beta+\gamma)} \;-\; \frac{(1-\gamma\theta s)u_t}{\alpha(\beta+\gamma)} \end{aligned} \tag{60}$$

$$g_t(\delta, \lambda^{cb}) = \frac{\delta\beta\gamma\Lambda\hat{\pi}}{(\beta+\gamma)[\beta\theta+\delta\gamma\Lambda]} + \frac{\alpha\beta^2\gamma s^2(1-\delta)(\hat{\pi}-\hat{\pi}^{cb})}{(\beta+\gamma)[\beta\theta+\delta\gamma\Lambda]\lambda_2^g} + \frac{\beta^2\phi[\delta\hat{\pi}+(1-\delta)\hat{\pi}^{cb}]}{(\beta+\gamma)[\beta\theta+\delta\gamma\Lambda]}$$
$$+ \frac{(1-\delta)\beta^2[\phi\lambda_2^g - \alpha\gamma s^2\ ]\lambda^{cb}}{\alpha\lambda_2^g[\beta\phi+\delta\gamma\Lambda](\beta+\gamma)} + \frac{\delta\beta(1+\alpha\beta)\lambda_1^g}{\alpha[\beta\phi+\delta\gamma\Lambda]}$$
$$+ \frac{(1-\delta)\beta^2\gamma s^2\lambda_1^g}{(\beta+\gamma)[\beta\phi+\delta\gamma\Lambda]\lambda_2^g} - \frac{\epsilon_t}{(\beta+\gamma)} - \frac{(1+\beta\theta s)u_t}{\alpha(\beta+\gamma)} \tag{61}$$

Substituting (60) and (61), and their expectations, back into the model yields the following outcomes:

$$\pi_t = \frac{\delta\gamma\Lambda\hat{\pi}}{[\beta\phi+\delta\gamma\Lambda]} + \frac{\beta\phi[\delta\hat{\pi}+(1-\delta)\hat{\pi}^{cb}]}{[\beta\phi+\delta\gamma\Lambda]} + \frac{(1-\delta)\beta\phi\lambda^{cb}}{\alpha[\beta\phi+\delta\gamma\Lambda]} + \frac{\delta[\beta\phi+\gamma\Lambda]\lambda_1^g}{\alpha[\beta\phi+\delta\gamma\Lambda]} \tag{62}$$

$$y_t(\delta, \lambda^{cb}) = \frac{-u_t}{\alpha} \tag{63}$$

$$\tau_t(\delta, \lambda^{cb}) = -\frac{(1-\delta)\beta\gamma s(\hat{\pi}-\hat{\pi}^{cb})}{[\beta\phi+\delta\gamma\Lambda]\lambda_2^g} - \frac{(1-\delta)\beta\gamma s(\lambda_1^g-\lambda^{cb})}{[\beta\phi+\delta\gamma\Lambda]\lambda_2^g} - \frac{(b-\theta)u_t}{\alpha}. \tag{64}$$

Moving back to stage 1, the first order conditions for the central bank's choice of λ^{cb} yield

$$\bar{\lambda}^{cb} = \frac{\delta\left\{(1-\delta)\beta(\alpha\gamma s)^2 - [\beta\phi+\gamma\Lambda]\phi\lambda_2^g\right\}[\alpha(\hat{\pi}-\hat{\pi}^{cb}) + \lambda_1^g]}{(1-\delta)\beta[\phi^2\lambda_2^g + \delta(\alpha\gamma s)^2]} \tag{65}$$

if $\delta \neq 1$. But the government's first order conditions for the choice of δ imply that the government would have preferred, conditional on δ,

$$\lambda^{cb*} = \frac{\begin{array}{l}(1-\delta)\alpha[\beta\phi^2\lambda_2^g + \beta(\alpha\gamma s)^2](\hat{\pi}-\hat{\pi}^{cb}) \\ \qquad + \left\{-\delta\phi[\beta\phi+\gamma\Lambda]\lambda_2^g + (1-\delta)\beta(\alpha\gamma s)^2\right\}\lambda_1^g\end{array}}{(1-\delta)\beta[\phi^2\lambda_2^g + (\alpha\gamma s)^2]}. \tag{66}$$

Two simple solutions are now obvious. If the government chooses $\delta = 0$, then $\bar{\lambda}^{cb} = 0$ follows. If, on the other hand, the government chooses $\delta = 1$, the central bank is indifferent about $\bar{\lambda}^{cb}$, so the government's preferred degree of conservatism $\lambda^{cb*} = \lambda_1^g + \alpha(\hat{\pi} - \hat{\pi}^{cb})$ would presumably prevail. In all other cases we need to solve (65) and (66) together to obtain δ. That yields four solutions when $\hat{\pi} \geq \hat{\pi}^{cb} : \delta = 1, \delta = 0$, $\delta > 1$, or $\delta < 0$. The latter two have no economic meaning, which implies that an optimizing government actually has only two solutions available:[8]

$$\begin{aligned} \delta &= 1 \quad \textit{and} \quad \bar{\lambda}^{cb} = \lambda_1^g + \alpha(\hat{\pi} - \hat{\pi}^{cb}) \\ \delta &= 0 \quad \textit{and} \quad \bar{\lambda}^{cb} = 0. \end{aligned} \tag{67}$$

Using these solutions, we can evaluate (7) to obtain

$$\begin{aligned} \mathrm{E}L_t^g &= \frac{(\lambda_1^g)^2}{2\alpha^2} \quad \text{when} \quad \delta = 1, \\ \text{and} \quad \mathrm{E}L_t^g &= \frac{(\hat{\pi} - \hat{\pi}^{cb})^2}{2} + \frac{(\gamma s)^2[\alpha(\hat{\pi} - \hat{\pi}^{cb}) + \lambda_1^g]^2}{2\phi^2\lambda_2^g} \quad \text{when} \quad \delta = 0. \end{aligned} \tag{68}$$

However, from (59), the central bank would achieve

$$\begin{aligned} \mathrm{E}L_t^{cb} &= \frac{[\alpha(\hat{\pi} - \hat{\pi}^{cb}) + \lambda_1^g]^2}{2\alpha} > 0 \quad \text{when} \quad \delta = 1 \\ \text{and} \quad \mathrm{E}L_t^{cb} &= 0 \quad \text{when} \quad \delta = 0. \end{aligned} \tag{69}$$

Hence, it is easy to see that the government would never choose $\delta = 1$ unless

$$(\hat{\pi} - \hat{\pi}^{cb}) > \left[\frac{\phi^2\lambda_2^g - (\alpha\gamma s)^2}{\alpha^2[\phi^2\lambda_2^g + (\alpha\gamma s)^2]}\right]^{1/2} \tag{70}$$

holds (a sufficient condition from (68)). That is, the government would not choose $\delta = 1$ unless the central bank threatened to be too ambitiously conservative with its inflation target; or if $\lambda_2^g \to 0$, in which case the government has no social or redistribution objectives. In all other cases, the government would rationally choose

[8] $\delta > 1$ or $\delta < 0$ would violate convexity axioms on the central bank's objective function, and would imply that the bank was either keener on the government's goals than even the government itself, or wanted to maximize the deviations from its own inflation target. Neither situation is at all likely.

$\delta = 0$. And (69) implies that the bank would, in its own interest, never want to lower its inflation target so far that the government ends up wanting to choose $\delta = 1$.

The upshot of this is that the central bank would have an incentive not to choose its inflation target $\hat{\pi}^{cb}$ too far below the government's target; but it would compensate for that by choosing a more conservative set of policies ($\bar{\lambda}^{cb} = 0$). The government for its part, would then always prefer a fully independent central bank. The outcomes of this regime would be more favourable to the central bank than in the other solutions. But they would be less favourable to the government than the fiscal leadership solution of section 4.2 since $\mathrm{E}L_t^g$ is always positive in (68). However, they would probably be more favourable than the other two institutional designs.[9] Thus, since the government presumably retains the right to determine what form of policy delegation takes place, this particular institutional arrangement would not be chosen if fiscal leadership were possible. But if fiscal leadership is not acceptable, then it is probably worthwhile to allow the central bank to choose its own degree of conservatism — as the Federal Reserve System does — rather than have a fixed value imposed by statute as in the ECB's case.

5. The Advantages of Fiscal Leadership

5.1 Central Bank Independence under Fiscal Leadership

Our results show that society's welfare, as measured by the inverse of (43), is maximized when there is fiscal leadership and the government appoints independent central bankers who are concerned only with the achievement of the mandated inflation target, and disregard the impact that their policies may have on output growth. However, our results also indicate that full central bank independence may be beneficial under more general conditions. When $\delta = 0$, $\beta\eta + \gamma = 0$ and (43) becomes

[9] It is straightforward to show that when $(\hat{\pi} - \hat{\pi}^{cb})$ is small, allowing the central bank to choose λ^{cb} is more favourable for the government (and society as a whole) than the regimes considered in sections 4.1 and 4.3, as long as $\phi^2\lambda_2^g(s^2+1) \geq (\alpha\gamma s)^2$. That inequality is certain to hold unless λ_2^g is very small.

$$\mathrm{E}L^g \;=\frac{1}{2}\left\{\frac{\lambda^{cb}}{\alpha}\right\}^2 \tag{71}$$

for any value of λ^{cb} when $\delta = 0$. Clearly therefore, an independent central bank always produces better results as long as it is more conservative than the government ($\lambda^{cb} < \lambda_1^g$) — compare (48) — irrespective of the latter's commitment to growth (λ_1^g) or to social equality (λ_2^g).

Notice that, in deriving our results, we have assumed that the central bank has instrument independence but not target independence. Consequently, the fact that $\mathrm{E}L^g = 0$ can be achieved by setting $\delta = \lambda^{cb} = 0$ indicates that it is instrument independence which matters. Target independence is ultimately irrelevant when there is fiscal leadership: neither target independence nor central bank leadership would reduce society's expected losses to zero.

5.2 Leadership vs. Simultaneous Moves

A more interesting question is whether fiscal leadership with an independent central bank generally produces better outcomes, from society's perspective, than those obtained in the simultaneous move game. In the simultaneous move game, the solution to the government's stage 1 minimization problem was:

$$\delta \;=\; \frac{\beta\phi^2\lambda^{cb}\lambda_2^g \;+\; (\alpha\gamma)^2\beta(\lambda^{cb}-\lambda_1^g)}{\beta\phi^2\lambda^{cb}\lambda_2^g \;+\; (\alpha\gamma)^2\beta(\lambda^{cb}-\lambda_1^g) \;-\; \phi[\beta\phi+\gamma\Lambda]\lambda_1^g\lambda_2^g}.$$

The optimal degree of conservatism for an independent central bank in this type of game can therefore be obtained by setting $\delta = 0$ to yield:

$$\lambda^{cb*} \;=\; \frac{(\alpha\gamma s)^2\lambda_1^g}{(\alpha\gamma s)^2+\phi^2\lambda_2^g} \tag{72}$$

It is now straightforward to show that (71) is always less than (27) as long as

$$\lambda^{cb} \;<\; \left[\lambda_1^g\lambda^{cb*}\right]^{1/2} \tag{73}$$

It is also evident that $\lambda^{cb*} \leq \lambda_1^g$ for $\lambda_2^g \geq 0$. Consequently, fiscal leadership with any value of λ^{cb} such that $0 \leq \lambda^{cb} < \lambda^{cb*}$ will produce better outcomes, from society's point of view, than any simultaneous move game between the central bank and the government. This is an important observation because many inflation targeting regimes, such as those operated by the Bank of England, the Swedish Riksbank, and the Reserve Bank of New Zealand, operate with fiscal leadership; while several others, notably the European Central bank and the US Federal Reserve System, are better characterized as being engaged in a simultaneous move game with their governments.

5.3 Sources of the Leadership Advantage

Substituting $\delta = 0$ and $\lambda^{cb} = 0$ into (40)–(42) shows exactly where the advantages of fiscal leadership come from. We get

$$\pi_t \;=\; \hat{\pi}, \quad y_t \;=\; \frac{-u_t}{\alpha}, \quad \tau_t \;=\; \frac{-(b-\theta)u_t}{\alpha} \tag{74}$$

as the final outcomes. By contrast, from (16)–(18), the optimal outcomes for the associated simultaneous move policy game are

$$\pi_t^* \;=\; \hat{\pi} \;+\; \frac{\alpha(\gamma s)^2}{[(\alpha\gamma s)^2 \;+\; \phi^2\lambda_2^g]} \tag{75}$$

$$y_t^* \;=\; \frac{-u_t}{\alpha} \tag{76}$$

$$\tau_t^* \;=\; \frac{\gamma s(\lambda^{cb*} - \lambda_1^g)}{\phi\lambda_2^g} \;-\; \frac{(b-\theta)u_t}{\alpha} \tag{77}$$

Comparing the two sets of outcomes we see that fiscal leadership eliminates inflationary bias and therefore results in a lower rate of inflation for any given $\hat{\pi}$.[10] The optimal outcome under fiscal leadership is also characterized by higher taxes and

[10]Notice that central bank independence alone implies a superior set of inflation outcomes. Setting $\delta = 0$ alone yields $\pi^* = \hat{\pi} + \lambda^{cb}/\alpha$ from (62), which is less than (75) if $\lambda^{cb}/\alpha < \alpha(\gamma s)^2/[(\alpha\gamma s)^2 + \phi^2\lambda_2^g]$. That inequality holds if $\lambda^{cb} < \lambda^{cb*}/\lambda_1^g$. Thus (73) is just a necessary, but not a sufficient condition for government leadership to produce lower inflation. But fiscal leadership can result in better welfare outcomes even if $\lambda^{cb*}/\lambda_1^g < \lambda^{cb} < [\lambda^{cb*}\lambda_1^g]^{1/2}$ because the social equality indicator is more satisfactory (even if inflation is not).

therefore more income redistribution or social equality.[11] Moreover, these improvements in inflation control and income distribution can be achieved with no loss in expected growth.

One of the central issues addressed in the policy coordination literature is whether there are institutional arrangements that yield Pareto improvements over the non-cooperative outcome.[12] When such institutions can be identified, they are viewed as a coordination device. In our model, fiscal leadership in the second stage of the policy game results in better outcomes for both policy authorities and is therefore an example of a rule-based form of policy coordination.[13]

6. Conclusions

Our results show that different institutional arrangements for the central bank and the fiscal authorities matter. Furthermore, our analysis indicates that fiscal leadership, with an independent central bank directed whose sole objective is inflation control, provides the best outcomes for society as a whole and also for the financial interests represented by the central bank. The reason for this is that this regime produces the greatest coordination between monetary and fiscal policies, and the benefits of this coordination outweigh any potential threat to the inflation target that fiscal dominance might have been expected to pose.

If fiscal leadership is not acceptable, then an independent central bank choosing its own degree of conservatism is the next best regime — provided that the central bank's inflation target is not too far from the government's target, and that the government has some social or redistribution objectives. Monetary leadership or imposed degrees

[11] Tax revenues are lower under the simultaneous move game because $\lambda^{cb*} < \lambda_1^g$. Redistribution is positively related to the amount of tax revenue because $(b - \theta)\mathrm{E}y_t^* = 0$, so that τ_t^* determines the amount of income redistribution actually achieved.

[12] See, for example, Currie, Holtham, and Hughes Hallett (1989); Currie (1990); and Currie and Levine (1991).

[13] See Currie (1990) for a discussion of the distinction between rule-based and discretionary, or *ad hoc*, forms of policy coordination.

of conservatism are not desirable when economic performance is affected by both fiscal and monetary policies.

We also find that target independence is ultimately unimportant. Instrument independence is the crucial feature, even under reasonable variations in the central bank's preferred degree of conservatism or inflation target. The reason for this is that greater conservatism or lower inflation targets generate a reaction from governments using fiscal policies or other policy instruments. Governments will therefore compensate — which makes it important that our models should take into consideration the strategic elements of fiscal or other policies, alongside their analysis of a suitable monetary framework. Although this has been shown in an extremely stylized manner here, through the choice of policy independence and conservatism parameters, recent experience in Europe bears out the practical importance of considering the interaction of fiscal and monetary when designing monetary institutions. In particular, the trend towards lower inflation targets, increased conservatism, and greater central bank independence in Europe has led to a compensating expansion in fiscal positions — to the point where the Stability Pact appears to be threatened in many of the larger economies.

References

Alesina, A. and R. Gatti (1995) "Independent Central Banks: Low Inflation at No Cost," *American Economic Review* **85**, 196-200.

Barro, R.J. (1981) "Output Effects of Government Purchases," *Journal of Political Economy* **89**, 1086-1121.

Barro, R.J. and D.B. Gordon (1983) "Rules, Discretion, and Reputation in a Model of Monetary Policy," *Journal of Monetary Economics* **12**, 101-21.

Currie, D.A. (1990) "International Policy Coordination," in D.T. Llewellyn and C. Milner (eds.) *Current Issues in International Monetary Economics*, Macmillan, London, 125-48.

Currie, D.A., G. Holtham, and A. Hughes Hallett, (1989) "The Theory and Practice of International Economic Policy Coordination: Does Coordination Pay?" in R. Bryant, D. Currie, J. Frenkel, P. Masson, and R. Portes (eds.) *Macroeconomic Policies in an Interdependent World*, International Monetary Fund, Washington DC.

Currie, D.A. and P. Levine (1991) "The International Coordination of Monetary Policy – A Survey," in C.J. Green D.T. Llewellyn (eds.) *Surveys in Monetary Economics*, Vol. 1, Cambridge MA and Oxford, Basil Blackwell.

Demertzis, M., A. Hughes Hallett, and N. Viegi (1999) "An Independent Central Bank Faced with Elected Governments," Discussion Paper No. 2219, Center for Economic and Policy Research, London.

Dixit, A.K. and L. Lambertini (2001) "Fiscal Discretion destroys Monetary Commitment," Unpublished Manuscript, Department of Economics, Princeton University.

Fischer, S. (1995) "Central Bank Independence Revisited," *American Economic Review* **85**, 201-6.

Galí, J. and T. Monacelli (2002) "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," Discussion Paper No. 3346, Center for Economic and Policy Research, London.

Hughes Hallett, A. and D.N. Weymark (2001) "The Cost of Heterogeneity in a Monetary Union," Discussion Paper No. 3223, Center for Economic Policy Research, London; Working Paper No. 01-W28, Department of Economics, Vanderbilt University, <www.vanderbilt.edu/econ>.

Lucas, R.E. (1973) " Some International Evidence on Output-Inflation Trade-Offs," *American Economic Review* **63**, 326-34.

Lucas, R.E. (1972) " Expectations and the Neutrality of Money," *Journal of Economic Theory* **4**, 103-24.

McCallum, B.T. (1989) *Monetary Economics: Theory and Policy,* Macmillan, New York.

Taylor, J.B. (1993) *Macroeconomic Policy in a World Economy: From Econometric*

Design to Practical Operation, W.W. Norton and Company, New York.

Weymark, D. N. (2001) "Inflation, Income Redistribution, and Optimal Central Bank Independence," Working Paper No. 01-W02R, Department of Economics, Vanderbilt University. <www.vanderbilt.edu/econ>.

Appendix 1
Solutions to (46) and (47)

The first-order condition (47) can be written as a quartic polynomial in δ. As a consequence, there are four solutions that simultaneously satisfy (46) and (47). By inspection, it is apparent that one of these solutions is $\delta = 1$ and $\lambda^{cb} = \lambda_1^g$. When $\delta \neq 1$ and $\lambda^{cb} \neq \lambda_1^g$, the first order conditions can be written

$$\begin{aligned}(\phi - \eta\Lambda)\lambda_2^g \left\{(1-\delta)\beta(\phi-\eta\Lambda)\lambda^{cb} \;+\; \delta[\beta\phi+\gamma\Lambda]\lambda_1^g\right\} & \\ -\,(1-\delta)(\beta\eta+\gamma)^2(\alpha s)^2\beta(\lambda_1^g - \lambda^{cb}) \;&=\; 0 \qquad \text{(A.1)}\end{aligned}$$

$$\begin{aligned}\left[\delta(1-\delta)\Lambda\frac{\partial\eta}{\partial\delta} \;+\; (\phi-\eta\Lambda)\right]\left\{(1-\delta)\beta(\phi-\eta\Lambda)\lambda^{cb} \;+\; \delta[\beta\phi+\gamma\Lambda]\lambda_1^g\right\}\lambda_2^g & \\ -\,(1-\delta)(\beta\eta+\gamma)(\alpha s)^2\beta\left[(\beta\eta+\gamma) \;-\; (1-\delta)\beta\frac{\partial\eta}{\partial\delta}\right](\lambda_1^g-\lambda^{cb}) \;&=\; 0. \qquad \text{(A.2)}\end{aligned}$$

But (A.2) can be expressed as

$$\begin{aligned}(A.1) \;+\; \delta(1-\delta)\Lambda\frac{\partial\eta}{\partial\delta}\left\{(1-\delta)\beta(\phi-\eta\Lambda)\lambda^{cb}+\delta[\beta\phi+\gamma\Lambda]\lambda_1^g\right\}\lambda_2^g & \\ +\,(1-\delta)^2(\beta\eta+\gamma)\frac{\partial\eta}{\partial\delta}(\alpha\beta s)^2(\lambda_1^g-\lambda^{cb}) \;&=\; 0. \qquad \text{(A.3)}\end{aligned}$$

Consequently, when $\delta \neq 1$ and (A.1) is satisfied, (A.2) becomes

$$\begin{aligned}\delta\Lambda\left\{(1-\delta)\beta(\phi-\eta\Lambda)\lambda^{cb}+\delta[\beta\phi+\gamma\Lambda]\lambda_1^g\right\}\lambda_2^g & \\ +\,(1-\delta)(\beta\eta+\gamma)(\alpha\beta s)^2(\lambda_1^g-\lambda^{cb}) \;&=\; 0. \qquad \text{(A.4)}\end{aligned}$$

Replacing η with (37) yields

$$(\phi - \eta\Lambda) \;=\; \frac{\alpha^2\beta s^2[\beta\phi + \gamma\Lambda]}{(\alpha\beta s)^2 + \delta\Lambda^2\lambda_2^g} \qquad \text{and} \qquad (\beta\eta + \gamma) \;=\; \frac{\delta\Lambda[\beta\phi + \gamma\Lambda]\lambda_2^g}{(\alpha\beta s)^2 + \delta\Lambda^2\lambda_2^g}. \tag{A.5}$$

It is evident that $(\beta\eta + \gamma) = 0$ when $\delta = 0$. Hence $\delta = \lambda^{cb} = 0$ is one solution that satisfies (A.1) and (A.4).

The remaining potential solutions can be found by substituting (A.5) into (A.4) and solving for δ (under the assumption that $\delta \neq 0$ and $\delta \neq 1$, since we have already examined those solutions). We obtain:

$$\delta^2 \;=\; \frac{-(\alpha\beta s)^2}{\Lambda^2\lambda_1^g\lambda_2^g}. \tag{A.6}$$

Consequently, there are only two real-valued solutions that satisfy the first-order necessary conditions: (i) $\delta = 1$ and $\lambda^{cb} = \lambda_1^g$, and (ii) $\delta = \lambda^{cb} = 0$.