UNIQUE INNEFICIENT PERFECT EQUILIBRIUM IN A STOCHASTIC MODEL OF BARGAINING WITH COMPLETE INFORMATION

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Unique Inefficient Perfect Equilibrium in a Stochastic Model of Bargaining with Complete Information*

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Abstract

We consider a two-player strategic bargaining model with discounting in which (i) the interim disagreement point in each period is stochastically determined at the beginning of the period, and (ii) the proposing player can delay in making an offer. Unlike many other bargaining models of complete information, in which inefficient perfect equilibrium outcomes are caused by the multiplicity of perfect equilibria, our model has a unique perfect equilibrium payoff in most of the cases. For some parameter values, the perfect equilibrium is inefficient since it has a stochastically delayed agreement. We show that both (i) and (ii) are also necessary for the unique inefficient equilibrium outcome in our model.

JEL Classification: C72 Noncooperative Games; C73 Stochastic and Dynamic Games; C78 Bargaining Theory.

Keywords: Negotiation, stochastic interim disagreement point, delay.

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1 Introduction

One of the reasons why Rubinstein's (1982) work has spurred extensive studies on bargaining lies in the fact that the alternating offer bargaining model with discounting has a unique, efficient perfect equilibrium featuring an immediate agreement. Despite its strong predictability, the Rubinstein model is unable to explain the phenomenon that many bargaining situations have inefficient outcomes with delayed agreements, or even with perpetual disagreements.

Studies have shown that incompleteness of information can cause inefficient outcomes in bargaining. Indeed, many bargaining models of incomplete information often have inefficient equilibria.[1] Many bargaining models of complete information also have inefficient equilibrium outcomes. For example, Chatterjee and Samuelson (1990) show that the possibility of simultaneous-offer can lead to inefficient equilibrium outcomes. Perry and Reny (1993) and Sákovics (1993) obtain inefficient outcomes in bargaining models with strategically timed offers. Haller and Holden (1990), Fernandez and Glazer (1991), and Busch and Wen (1995) identify endogenous interim disagreement payoffs as one possible source of inefficient equilibrium outcomes.[2] Busch, Shi and Wen (1998), and Manzini (1999) find that delay occurs in a bargaining model if one player can destroy the future value of the relationship between the two players.

Most of these models of complete information have one common feature: The multiplicity generates the inefficiency. Inefficient equilibria are supported by trigger strategies, as in the folk theorem for repeated games. The idea is that the player who deviates from a prescribed inefficient outcome would be punished by his worst perfect equilibrium available in the continuation game, which is made possible by the multiplicity of perfect equilibria. Under certain conditions, most of these models obtain "folk theorems": Any feasible payoff vector in which every player receives more than his lowest equilibrium payoff can be supported

[1] For an excellent review on this issue, see Osborne and Rubinstein (1990).

[2] The interim disagreement payoffs are the payoffs that players receive after a rejection and before the next proposal. It is equivalent to the disagreement payoff if the interim disagreement payoffs are constant overtime.

by a perfect equilibrium. Inevitably, these types of results lack of predictability due to the multiplicity of perfect equilibria.

In this paper, we present a strategic bargaining model with two new important features, comparing with Rubinstein's bargaining model. First, interim disagreement payoffs are stochastic. Second, the proposing player can delay making an offer. With these two features, we show that each player has a unique perfect equilibrium payoff in most of the cases. In some cases, the equilibrium is inefficient with a stochastically delayed agreement. We also prove that both of these features are necessary for the existence of such an inefficient equilibrium outcome in our model.

Our model fits to bargaining situations in which the bargaining environment changes over time. When a firm is contemplating to purchase another firm, or when two firms are contemplating to merge, they negotiate over the acquisition price, or exchange ratio of their stocks, respectively. During the negotiation, each firm continues to earn the profits that vary stochastically with the overall performance of the economy. Following the alternating proposing sequence, it is possible that a firm delays making an offer. When both firms have relatively high profits in the current period, the proposing firm is more likely to delay making an offer as the loss from temporary disagreement is relatively small. The perfect equilibrium in our model has the feature that two players will reach an agreement only when the realized interim disagreements are low.

Now we demonstrate the role of our second assumption in the model. In the Rubinstein bargaining model with variable interim disagreement points, the equilibrium outcome depends only on the sequence of the responding player's interim disagreement payoffs in the subsequent periods. The reason is, in each period the proposing player will offer the responding player the continuation payoff for the responding player which would result from his own rejection. For example, in a period where player 1 makes an offer and the interim disagreement point is (x_1, x_2), the continuation payoff for player 2 (the responding player) after his rejection can be written as $(1-\delta)x_2 + \delta v_2$, where δ is the discount factor and v_2 is player 2's continuation average payoff from the next period on. This implies that player 1 needs to make a generous offer to player 2 when x_2 is high. If x_1 is also high, then player 1 will be more reluctant to make an acceptable offer since player 1's opportunity cost is relatively high. Player 1's opportunity cost for making an acceptable offer is $(1-\delta)x_1+\delta v_1$, where v_1 is player 1's continuation payoff from the next period. If the value to be divided is normalized to 1, player 1 will not make an acceptable offer if

$$\begin{aligned} (1-\delta)x_1+\delta v_1 &> 1-[(1-\delta)x_2+\delta v_2] \\ \Leftrightarrow \qquad x_1+x_2 &> \frac{1-\delta(v_1+v_2)}{1-\delta}, \end{aligned} \tag{1}$$

which requires that both players' interim disagreement payoffs are high. In the Rubinstein bargaining model, inequality (1) does not hold when the interim disagreement point is below the bargaining frontier, since $v_1+v_2 \leq 1$. In our model, however, since the proposing player can delay making an offer, both v_1 and v_2 correspond to the proposing player's continuation payoff. As a result, it is possible that $v_1+v_2>1$. If so, inequality (1) holds even though $x_1+x_2<1$ so that the proposing player will not make an offer when the value of x_1+x_2 is relatively high.

We construct a two-player bargaining model that captures the two features mentioned above. Inequality (1) suggests that only the sum of the two players' interim disagreement payoffs matters to the proposing player's decisions in the model with transferable utility. Therefore, without loss of generality, we assume that both players have a common interim disagreement payoff in any period. This interim disagreement payoff takes one of two possible values, a high value and a low value, according to a simple stochastic process that is common knowledge. The low realization is normalized to 0. At the beginning of any period even before a player proposes, the interim disagreement payoff for that period is realized and observed by both players. Therefore, our model is of complete information. The two players offer and respond in the same way as in the Rubinstein model except that the proposing player may also delay making an offer for one period in our model.[3]

[3]Note that this assumption allows the proposing player to delay making an offer for more than one period.

Our model admits a unique perfect equilibrium for almost all the parameter values. The equilibrium outcome, however, varies with the variability of the interim disagreement payoff. When the high interim disagreement payoff is relatively small so the variation of disagreement payoffs is low, the two players will reach an agreement in the first period, which is an efficient outcome. On the other extreme, when the high interim disagreement payoff is so high that the players' expected interim disagreement point is strictly above the bargaining frontier, two players will never reach an agreement, which is also an efficient outcome. Most interesting is the case where the high interim disagreement payoff is in an intermediate range so that the variation of interim disagreement payoffs is high but the expected interim disagreement point is still below the bargaining frontier. In this case, the proposing player will not make an offer when the realization of the interim disagreement payoff is high, and will make an acceptable offer otherwise. Consequently, the equilibrium outcome involves a stochastically delayed agreement. Stochastic delay occurs even though the high interim disagreement point lies strictly below the bargaining frontier, in which case the outcome is inefficient.

Bargaining situations with stochastic disagreement points or with stochastic bargaining values have attracted much attention. Riddell (1981), Chun and Thomson (1990a,b), and Bossert, Nosal, and Sadanand (1996) analyze these issues within cooperative frameworks. Within a non-cooperative framework, Avery and Zemsky (1994) study a version of the Rubinstein bargaining model with a stochastic value of an asset to be traded. As our model predicts, they also derive an equilibrium with delay to an agreement. However, our model is crucially different from theirs in the following two aspects: (i) the proposing player may delay in making an offer for one period, and (ii) both players observe the realization of the current interim disagreement payoff. Avery and Zemsky (1994) specify that only the responding player, not the proposing player, observes a shock to the option value. In this aspect, Avery and Zemsky's (1994) model is of incomplete information. Jehiel and Moldovanu (1995) incorporate negative externalities into the bargaining model of complete information, and find that there is a unique perfect equilibrium outcome with delayed agreement.

The stochastic aspect of our model is similar to that of Merlo and Wilson (1995, 1998). They studied a bargaining model of complete information, in which both the value to be divided and the identity of the proposer in each period are determined stochastically.[4] Especially, Merlo and Wilson (1998) analyze a bargaining model with transferable utility like ours, and show that there is a unique, efficient perfect equilibrium that in some cases involves delayed agreement. There are two notable differences between our model and theirs. First, it is the interim disagreement payoff that varies stochastically in our model, whereas it is the value to be divided in theirs. As Busch, Shi, and Wen (1998) argue, this difference can be considered to be minor since it is the surplus that players bargain how to divide among them. However, delay to an agreement is more surprising in our model than theirs, since in our model the total value to be divided would not possibly increase by waiting. The second difference is more important. In our model, a proposer can retain the right to propose when he chooses not to propose in the current period, whereas in Merlo and Wilson (1998) who proposes in the next period does not depend on current actions. Delay to an agreement is caused by this feature and is inefficient in our model. On the other hand, delay is caused by the expectation that the total bargaining value may rise in the future, and hence is efficient in Merlo and Wilson (1998).

The rest of the paper is organized as follows. The next section presents our model. Sections 3 and 4 establish the existence and uniqueness of the perfect equilibrium, respectively. Section 5 investigates equilibrium properties. Section 6 concludes the paper.

2 The Model

Two players, 1 and 2, bargain over the allocation of a periodic value of 1. In any period prior to an agreement, after both players observe their interim disagreement payoffs (x, x) for the period, one of the players (called the proposing player) can either make an offer of how to share the value of 1 or keep quiet (make no offer at all). If the proposing player makes

[4] Eraslan and Merlo (1999) investigate the impact of majority voting rule in the bargaining model of Merlo and Wilson with more than two players.

an offer, then his opponent (called the responding player) can either accept or reject the standing offer, denoted by Y and N, respectively. If the offer is accepted, the game ends and each player enjoys a constant stream of the agreed-upon value forever. If the offer is rejected, the players receive their interim disagreement payoffs (x, x), and this process will repeat in the next period in which the two players switch their roles in bargaining, *i.e.*, the proposing player becomes the responding player and *vice versa*. If the proposing player chooses to be quiet, on the other hand, the two players will simply collect their interim disagreement payoffs (x, x) for the current period, and the same process will repeat in the next period *without* switching the players' roles in bargaining. By convention, player 1 is the proposing player in the first period. We assume that the two players are risk-neutral, expected-utility maximizers who discount the future with a common discount factor $\delta \in (0, 1)$ per period.

The interim disagreement payoff x in any period is an *i.i.d.* random variable which takes two possible values, 0 and d (≥ 0), with probabilities p and $1 - p$, respectively. At the beginning of any period before the proposing player makes an offer, both players observe the realization of x for the current period. Thus, this game has complete information. Figure 1 illustrates this bargaining model.

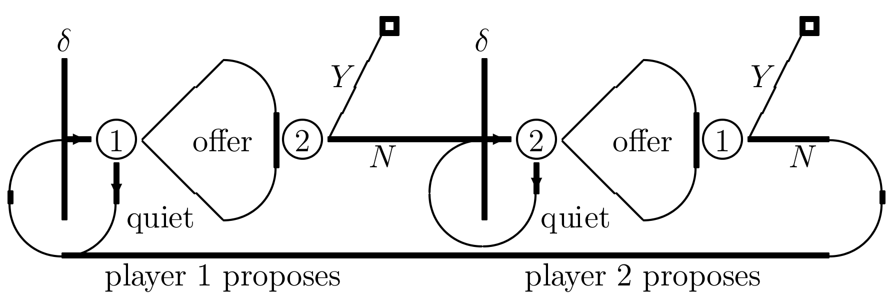

FIGURE 1. The schema of the bargaining game.

A history consists of all past proposing player's actions and realized interim disagreement payoffs. A player's strategy assigns an action to every possible history. Any strategy profile induces a unique outcome path, which consists of all realized interim disagreement payoffs before an agreement and the agreement itself (if there is any). An outcome path with an agreement in period T is denoted by

$$\pi(T) = (x(1), x(2), \ldots, x(T-1), (a_1, a_2), \{Y\}),$$

where $x(t) \in \{(d,d),\ (0,0)\}$ is the realized interim disagreement point of period t for $1 \le t < T$, and a_i is player i's share in the partition agreed upon in period T such that $a_1 + a_2 = 1$. By convention, T is set to be infinity if there is no agreement. The players receive their interim disagreement payoffs in every period until an agreement is reached, and continue to receive the agreed-upon share thereafter. The players' (non-expected) average discounted payoff vector associated with the outcome path $\pi(T)$ is

$$\begin{cases} (a_1, a_2) & \text{if } T = 1 \\ (1-\delta^{T-1}) \sum_{t=1}^{T-1} \delta^{t-1} x(t) + \delta^{T-1}(a_1, a_2) & \text{if } T \ge 2. \end{cases}$$

3 The Existence of Perfect Equilibrium

In this section, we establish the existence of perfect equilibrium. The proof for the existence is constructive. That is, we first provide a strategy profile and then verify the subgame perfection of the strategy profile. In what follows, we consider three distinct strategy profiles depending on the value of d. The strategy profiles we consider in Cases I and III are rather stationary in the sense that whether the proposing player makes an offer does not depend on the realization of the interim disagreement payoff in that period, but his offer does if he makes an offer. In Case II, whether the proposing player makes an offer depends on the realization of the interim disagreement payoffs. To classify these three cases, we define the following two critical values of d:

$$\underline{d} = \frac{1}{2(1+\delta p)}, \qquad \overline{d} = \frac{1}{2\delta(1-p)}. \tag{2}$$

It is straightforward that $0 < \underline{d} < 1/2 < \overline{d}$ for all $\delta \in (0,1)$ and $p \in (0,1)$.

Case I $0 \le d \le \underline{d}$**:** Consider the following strategy profile: In any period with the current interim disagreement payoff x (which equals either 0 or d), the proposing player demands a share of

$$b^*(x) = 1 - (1-\delta)x - \delta\frac{1-(1-\delta)(1-p)d}{1+\delta}. \tag{3}$$

The responding player rejects any offer if and only if the proposing player demands more than $b^*(x)$. According to this strategy profile, the proposing player's expected continuation payoff at the beginning of any period (before the interim disagreement payoff for the period is realized) is

$$EC \;=\; pb^*(0) + (1-p)b^*(d) \;=\; \frac{1-(1-\delta)(1-p)d}{1+\delta}. \tag{4}$$

Note that the last term in (3) is just δEC.

Now, we show that the strategy profile described above is a subgame perfect equilibrium. In any period, if the proposing player makes an offer, the current responding player will be the proposing player in the next period if he rejects the standing offer. Therefore, rejecting the offer would give the responding player a payoff of $(1-\delta)x + \delta EC = 1 - b^*(x)$. Hence, the responding player will reject the standing offer if and only if his share is less than $1 - b^*(x)$, or equivalently, if and only if the proposing player demands more than $b^*(x)$. Given the responding player's strategy, if the proposing player demands more than $b^*(x)$, his offer would be rejected, and he would be the responding player in the next period with the expected continuation payoff of $1-EC$. Therefore, the proposing player's payoff from making an unacceptable offer is $(1-\delta)x + \delta(1-EC)$, which is less than $b^*(x) = 1-(1-\delta)x - \delta EC$ since $x \leq d \leq \underline{d} < 1/2$. This means that the proposing player should make an acceptable offer if he makes an offer at all. If the proposing player keeps quiet, on the other hand, he would still be the proposing player in the next period with the expected continuation payoff of EC. Consequently, the proposing player's payoff would be $(1-\delta)x + \delta EC$, which is less than or equal to $b^*(x)$ for each $x = 0$ or d since

$$\begin{aligned} (1-\delta)x + \delta EC &\leq 1-(1-\delta)x - \delta EC \\ \Leftrightarrow \quad 2(1-\delta)x &\leq 1 - 2\delta EC \end{aligned}$$

$$
\begin{aligned}
\Leftrightarrow \qquad 2(1-\delta)d &\leq 1-2\delta EC \\
\Leftrightarrow \qquad d &\leq \frac{1-2\delta EC}{2(1-\delta)} = \underline{d}.
\end{aligned}
$$

Therefore, the proposing player should demand exactly $b^*(x)$ in his proposal.

Case II $\underline{d} \leq d \leq \overline{d}$: Consider the following strategy profile: In any period, the proposing player keeps quiet if $x = d$, and makes an offer such that his share is

$$
b^*(0) = 1 - \delta \frac{p + (1-\delta)(1-p)d}{1-\delta+2\delta p}, \tag{5}
$$

if $x = 0$. The responding player rejects the offer if and only if his share is less than

$$
1 - b^*(x) = (1-\delta)x + \delta \frac{p + (1-\delta)(1-p)d}{1-\delta+2\delta p}, \tag{6}
$$

for $x = 0$ and d. Notice that $b^*(x)$ (and also EC later) represents different values in the three different cases to minimize the number of the notations.

According to this strategy profile, the proposing player's expected continuation payoff in any period before x is realized is calculated as

$$
\begin{aligned}
EC &= p(1-\delta EC) + (1-p)[(1-\delta)d + \delta EC] \\
\Rightarrow \qquad EC &= \frac{p + (1-\delta)(1-p)d}{1-\delta+2\delta p}.
\end{aligned}
$$

Notice that the last terms of (5) and (6) are δEC.

Regardless of the realization of the current interim disagreement payoff, the responding player would obtain, by rejecting the standing offer (if an offer is made at all), $(1-\delta)x + \delta EC = 1 - b^*(x)$ as shown in (6). Thus, the responding player's strategies are the best responses to the proposing player's strategies in all subgames.

Now, when $x = d$, the proposing player obtains $(1-\delta)d + \delta EC$ by keeping quiet. If he makes an unacceptable offer, he would obtain $(1-\delta)d + \delta R$, where value R is the responding player's expected continuation payoff before x is realized,[5]

$$
R = p\delta EC + (1-p)[(1-\delta)d + \delta R],
$$

[5] We would like to thank Takako Fujiwara-Greve for pointing out the R value. $EC + R$ is the sum of the two players' expected payoffs, denoted by V, as calculated in Section 5.

$$\Rightarrow \qquad R \;=\; \frac{p\delta EC + (1-p)(1-\delta)d}{1-(1-p)\delta}.$$

Simple computation shows that $R < EC$ when $d \leq \overline{d}$. This implies that the proposing player would rather keep quiet than making an unacceptable offer. If the proposing player makes an acceptable offer, his payoff will be at most $b^*(d)$, which is less than or equal to $(1-\delta)d+\delta EC$ as $d \geq \underline{d}$. (Notice that $b^*(d) = (1-\delta)d+\delta EC$ if and only if $d = \underline{d}$.) Therefore, the proposing player would rather keep quiet than making an acceptable offer. In short, the proposing player should not make any offer in a period in which $x = d$.

If $x = 0$, on the other hand, the proposing player's payoffs from keeping quiet, from making an unacceptable offer, and from making an acceptable offer are δEC, δR, and $b^*(0)$, respectively. Since $EC > R$ and $d \leq \overline{d}$ yield $\delta R \leq \delta EC \leq b^*(0)$, the proposing player will demand exactly $b^*(0)$ in his proposal when $x = 0$ is realized.

Case III $d \geq \overline{d}$**:** Consider the following strategy profile: In any period, the proposing player keeps quiet. The responding player rejects the offer if and only if his share is less than $(1-\delta)x + \delta(1-p)d$, which is the responding player's continuation payoff following his rejection.

This strategy constitutes a subgame perfect equilibrium. The responding player's strategy is his best response for the same reason as in cases I and II. On the other hand, the proposing player's payoff from making an acceptable offer is at most $1-(1-\delta)x-\delta(1-p)d$, while those from making an unacceptable offer and from keeping quiet are both equal to $(1-\delta)x + \delta(1-p)d$. Since $d \geq \overline{d}$ implies

$$1-(1-\delta)x-\delta(1-p)d \leq (1-\delta)x + \delta(1-p)d,$$

for both $x = 0$ and $x = d \geq \overline{d}$, the proposing player should keep quiet in all periods.[6]

The following proposition summarizes the perfect equilibrium outcomes for the three cases.

[6]Since making an unacceptable offer and keeping quiet give the proposing player the same payoff, the strategy profiles in which the proposing player keeps quiet in some periods and makes unacceptable offers in the other periods are also subgame perfect. These equilibria involve no agreement and yield the same payoff vector.

Proposition 1 *For any $\delta \in (0,1)$, $p \in (0,1)$ and $d \geq 0$, the bargaining model has a perfect equilibrium. If $d \leq \underline{d}$, player 1 will make an acceptable offer in the first period. If $\underline{d} \leq d \leq \overline{d}$, player 1 will not make an offer until the interim disagreement payoff becomes 0 in which case player 1 will make an acceptable offer. If $d \geq \overline{d}$, player 1 will never make any offer or any acceptable offer.*

4 The Uniqueness of the Perfect Equilibrium

Following Shaked and Sutton's (1984) technique, we now derive the range of perfect equilibrium payoffs. It follows from Proposition 1 and our assumptions of the model that the set of perfect equilibrium payoffs is non-empty and bounded. Due to the symmetry, the range of perfect equilibrium payoffs depends on only whether a player is proposing or responding, and on the realization of the interim disagreement payoff. Let $M(x)$ and $m(x)$ be the maximum and minimum, respectively, of the proposing player's perfect equilibrium payoffs when the current interim disagreement payoff is x. Let M and m denote the expected values of $M(x)$ and $m(x)$, respectively, *i.e.*,

$$M = pM(0) + (1-p)M(d), \quad m = pm(0) + (1-p)m(d). \tag{7}$$

By definition, $M(x) \geq m(x)$ for $x = 0, d$, and hence $M \geq m$.

Now consider the proposing player's strategy in a period. If the proposing player chooses to be quiet, he will collect his interim disagreement payoff x and will still be the proposing player in the next period with an expected continuation payoff in between m and M. Therefore, the proposing player would receive neither less than $(1-\delta)x + \delta m$, nor more than $(1-\delta)x + \delta M$ if the proposing player does not make an offer at all in the current period. Alternatively, if the proposing player chooses to make an offer, the responding player in the current period will be the proposing player in the next period if he rejects the offer. The responding player will certainly reject any offer which gives him less than his lowest possible payoff after his rejection, $(1-\delta)x + \delta m$. Consequently, the proposing player cannot receive more than $1-(1-\delta)x-\delta m$. On the other hand, the responding player will certainly accept any offer which gives him more than his highest possible payoff after rejection, $(1-\delta)x+\delta M$. Thus, the proposing player would not receive less than $1-(1-\delta)x-\delta M$ in perfect equilibria.

In summary, since the proposing player *chooses* whether or not he makes an offer in the current period, $M(x)$ and $m(x)$ must satisfy the following inequalities for $x=0,\ d$:

$$\begin{aligned} M(x) &\leq \max\left\{(1-\delta)x+\delta M,\ 1-(1-\delta)x-\delta m\right\}, \\ m(x) &\geq \max\left\{(1-\delta)x+\delta m,\ 1-(1-\delta)x-\delta M\right\}. \end{aligned} \tag{8}$$

Since

$$\begin{array}{lrcl} & (1-\delta)x+\delta M & \leq\ (\geq) & 1-(1-\delta)x-\delta m \\ \text{if and only if} & (1-\delta)x+\delta m & \leq\ (\geq) & 1-(1-\delta)x-\delta M, \end{array}$$

for either $x=0$ or $x=d$, we need only check which of $(1-\delta)x+\delta M$ and $1-(1-\delta)x-\delta m$ is greater to find all possible cases. Given that x takes either 0 or d, it appears that there are four different cases to consider. However, we only have the following three cases since $(1-\delta)d+\delta M \leq 1-(1-\delta)d-\delta m$ implies $\delta M \leq 1-\delta m$. These three cases are shown to correspond to the three cases in the previous section, respectively.

Case A $(1-\delta)d+\delta M \leq 1-(1-\delta)d-\delta m$**:** In this case, inequality system (8) can be written as

$$\begin{aligned} M(0) &\leq 1-\delta m, \\ m(0) &\geq 1-\delta M, \\ M(d) &\leq 1-(1-\delta)d-\delta m, \\ m(d) &\geq 1-(1-\delta)d-\delta M. \end{aligned} \tag{9}$$

Then it follows from (7) that the expected values of $M(x)$ and $m(x)$ satisfy

$$\begin{aligned} M &\leq 1-(1-\delta)(1-p)d-\delta m, \\ m &\geq 1-(1-\delta)(1-p)d-\delta M. \end{aligned}$$

Consequently, we have

$$\frac{1-(1-\delta)(1-p)d}{1+\delta} \leq m \leq M \leq \frac{1-(1-\delta)(1-p)d}{1+\delta},$$

which yields that $M = m$. Now, substituting $M = m$ into (9) gives

$$\begin{aligned} 1-\delta m \;\leq\; m(0) \;&\leq\; M(0) \;\leq\; 1-\delta m, \\ 1-(1-\delta)d-\delta m \;\leq\; m(d) \;&\leq\; M(d) \;\leq\; 1-(1-\delta)d-\delta m, \end{aligned}$$

which imply that each player's equilibrium payoff is unique for each possible state. Substituting $M = m = [1-(1-\delta)(1-p)d]/(1+\delta)$ into the inequality that defines Case A, we have

$$2(1-\delta)d \leq 1 - \frac{2\delta[1-(1-\delta)(1-p)d]}{1+\delta},$$

which reduces to $d \leq \underline{d}$. In fact, EC in Case I equals the derived value of M and m. Since the perfect equilibrium described in the last section is the only strategy profile which gives the unique perfect equilibrium payoff, it is the unique perfect equilibrium except when $d = \underline{d}$, in which case the proposing player is indifferent between making an offer and keeping quiet when $x = d$.

Case B $\delta M \leq 1-\delta m$ **and** $(1-\delta)d + \delta M \geq 1-(1-\delta)d-\delta m$**:** In this case, inequality system (8) becomes

$$\begin{aligned} M(0) &\leq 1-\delta m, \\ m(0) &\geq 1-\delta M, \\ M(d) &\leq (1-\delta)d+\delta M, \\ m(d) &\geq (1-\delta)d+\delta m. \end{aligned} \tag{10}$$

It follows from (7) that the expected values of $M(x)$ and $m(x)$ satisfy

$$\begin{aligned} M &\leq p[1-\delta m] + (1-p)[(1-\delta)d+\delta M], \\ m &\geq p[1-\delta M] + (1-p)[(1-\delta)d+\delta m], \end{aligned}$$

which yield

$$\frac{p+(1-\delta)(1-p)d}{1-\delta+2\delta p} \leq m \leq M \leq \frac{p+(1-\delta)(1-p)d}{1-\delta+2\delta p}.$$

Therefore, we have $M = m = [p+(1-\delta)(1-p)d]/(1-\delta+2\delta p)$. Substituting the derived value of m and M into (10) gives us the uniqueness of the perfect equilibrium payoff for each state.

Next, we show that Case B corresponds to Case II in the last section. Substituting $M = m = [p + (1-\delta)(1-p)d]/(1-\delta+2\delta p)$ into the two inequalities that define Case B yields

$$\begin{aligned} \frac{2\delta[p+(1-\delta)(1-p)d]}{1-\delta+2\delta p} &\leq 1, \\ \frac{2\delta[p+(1-\delta)(1-p)d]}{1-\delta+2\delta p} &\geq 1-2(1-\delta)d, \end{aligned}$$

which reduce to $d \leq \overline{d}$ and $d \geq \underline{d}$, respectively. Therefore, together with the fact that EC in Case II equals the derived value of M and m, this finding implies that the perfect equilibrium described in the last section is the unique perfect equilibrium in Case II except at $d = \underline{d}$ and $d = \overline{d}$. When $d = \underline{d}$, the proposing player is indifferent between making an offer and keeping quiet when $x = 0$; whereas when $d = \overline{d}$, the proposing player is always indifferent between making an (unacceptable) offer and keeping quiet regardless of the realization of x.

Case C $\delta M \geq 1 - \delta m$**:** In this case, inequality system (8) becomes

$$\begin{aligned} M(0) &\leq \delta M, \\ m(0) &\geq \delta m, \\ M(d) &\leq (1-\delta)d + \delta M, \\ m(d) &\geq (1-\delta)d + \delta m. \end{aligned} \tag{11}$$

Then, the expected values of $M(x)$ and $m(x)$ satisfy

$$(1-p)d \leq m \leq M \leq (1-p)d,$$

which implies that $M = m = (1-p)d$. Substituting $(1-p)d$ for M and m in (11) yields the uniqueness of the perfect equilibrium payoff for each state.

Now, inequality $\delta M \geq 1 - \delta m$ reduces to $d \geq \overline{d}$ for the derived value of M and m, implying that Case C corresponds to Case III of the last section. Together with the fact that EC in Case III equals the derived value of M and m, we conclude that the perfect equilibrium described in the last section gives the unique equilibrium payoff for each player in Case III.[7]

The following proposition summarizes these findings.

[7] Equilibrium strategies are not unique in this case. For example, the strategy profile in which the proposing player makes unacceptable offers in some periods is also a perfect equilibrium.

Proposition 2 *When $d \neq \underline{d}$, each player has a unique perfect equilibrium payoff; when $d = \underline{d}$, the proposing player has a unique perfect equilibrium payoff. The equilibrium strategies are as described in Proposition 1.*

5 Properties of the Perfect Equilibrium

In contrast to most existing bargaining models with complete information, in which inefficiency results from the multiplicity of the equilibrium, we provide a model of complete information which has a unique inefficient equilibrium outcome as described in Case II.

The equilibrium outcome is quite intuitive. If the high interim disagreement payoff d is so small that $0 \leq d \leq \underline{d}$, the incentive to obtain the surplus from an agreement dominates any incentive to delay. The cost of delay is too high for both players. In the equilibrium, the proposing player makes acceptable offers in all subgames, including the original game itself. The two players reach an agreement immediately, as in the Rubinstein (1982) model. Indeed, the agreement itself closely resembles the Rubinstein solution, as EC in (4) is the Rubinstein solution when the disagreement point is $((1-p)d,\ (1-p)d)$, the expected interim disagreement point in our model.

On the other extreme, the players never reach an agreement if the interim disagreement payoff is large enough on average. More specifically, if the high interim disagreement payoff satisfies $d \geq \overline{d}$, or equivalently, the discounted average interim disagreement payoff satisfies $\delta(1-p)d \geq 1/2$, the players will simply collect their interim disagreement payoffs in every period. The perpetual disagreement does not mean inefficiency since the expected interim disagreement point is strictly above the bargaining frontier. Notice that in order to obtain the perpetual disagreement, the expected interim disagreement point has to be strictly above the bargaining frontier, so that the continuation payoff point is still above the bargaining frontier even when $x = 0$ is realized. In the equilibrium with no agreement, the continuation payoff when $x = 0$ is $\delta EC = \delta(1-p)d$ for both players. So, the continuation payoff point is above the bargaining frontier if and only if $\delta(1-p)d \geq 1/2$, or equivalently $d \geq \underline{d}$.

Stochastic delay occurs when the high interim disagreement payoff is in the intermediate range. If $\underline{d} \leq d \leq \overline{d}$, the proposing player will make an acceptable offer if and only if $x = 0$ is realized. Otherwise, the proposing player will delay an agreement to the next period. If $d < 1/2$, such a delay is obviously inefficient. The reason why such an inefficient outcome arises is that each player's payoff from an agreement should be at least as large as his continuation payoff, while the continuation payoff vector is above the bargaining frontier when $x = d$. The proposing player's continuation payoff is $(1-\delta)d + \delta EC = 1 - b^*(d)$, as seen from (6), which is obtained by delaying making an offer. As (6) shows, however, the responding player's continuation payoff (after his rejection) is also $1 - b^*(d)$. Since $b^*(d) \leq 1/2$ when $d \geq \underline{d}$, the continuation payoff vector is above the bargaining frontier.

Another interesting result is, even if the expected interim disagreement point is above the bargaining frontier, $(1-p)d > 1/2$, players will still reach an agreement when $x = 0$ is realized. It occurs when $1/2 < (1-p)d < 1/(2\delta)$. When $x = 0$ is realized, the two players must wait for one period in order to claim the expected interim disagreement payoffs. So unless the discounted expected interim disagreement point is above the bargaining frontier, *i.e.*, $\delta(1-p)d > 1/2$, both players collectively prefer an immediate agreement when $x = 0$. This result is different from the situation in which the interim disagreement point is fixed. In that case, there will be no agreement if a fixed interim disagreement point is above the bargaining frontier.

Figure 2 depicts the players' total expected payoff in the equilibrium. When $0 \leq d \leq \underline{d}$, the two players reach an agreement in the first period, and their total payoff, denoted by V, equals 1. The equilibrium outcome is efficient. When $d \geq \overline{d}$, the two players simply collect their interim disagreement payoffs and so their total expected equilibrium payoff equals $V = 2(1-p)d$, which is linearly increasing in d. When $\underline{d} \leq d \leq \overline{d}$, the total expected payoff in the equilibrium satisfies $V = p + (1-p)[2(1-\delta)d + \delta V]$. Therefore, we have

$$V = \begin{cases} 1 & \text{if } 0 \leq d \leq \underline{d} \\ [p + 2(1-\delta)(1-p)d]/[1-\delta(1-p)] & \text{if } \underline{d} \leq d \leq \overline{d} \\ 2(1-p)d & \text{if } d \geq \overline{d}. \end{cases}$$

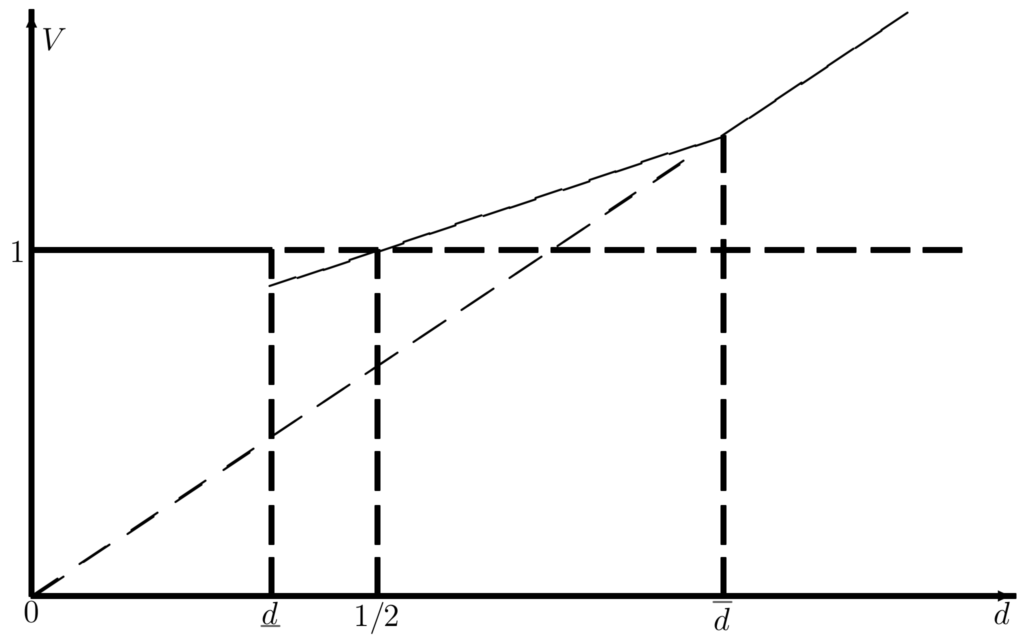

FIGURE 2. The players' total expected payoff.

Note that V takes two possible values at $d = \underline{d}$. The difference between these two values represents the responding player's loss if the proposing player changes his strategies from that described in Case I to that in Case II. At $d = \overline{d}$, V is single-valued since

$$\frac{p + 2(1-\delta)(1-p)\overline{d}}{1-\delta(1-p)} \;=\; 2(1-p)\overline{d} \;=\; \frac{1}{\delta}.$$

When $\underline{d} < d \leq \overline{d}$, V is increasing in d at a constant rate of $[2(1-\delta)(1-p)]/[1-\delta(1-p)]$ and $V = 1$ at $d = 1/2$, which implies that the equilibrium is inefficient when $\underline{d} < d < 1/2$. As p increases, $\underline{d}$ decreases and hence the range of d corresponding to the inefficient equilibrium expands. However, inefficiency itself decreases since the expected length of delay in equilibrium, $\sum_{t=1}^{\infty} t(1-p)^t p = (1-p)/p$, is shortened as p increases. Similarly, an increase in δ widens the range of d corresponding to the inefficient equilibrium by lowering $\underline{d}$. Also in this case, inefficiency in an equilibrium decreases as δ increases, since delay costs less to the players.

Now, we show that both of the variation of the interim disagreement point and the possibility for the proposing player to delay making an offer are necessary to yield the unique inefficient outcome in our bargaining model. First, if the proposing player can delay making an offer but the interim disagreement points are fixed, say at 0 for simplicity, then there will be no inefficient outcome. This can be considered as a special case of our model when $d = 0$ where only Case I applies. Intuitively, delaying an offer would not change the bargaining environment except for discounting for one period. Thus the proposing player would not delay in making an acceptable offer. The Rubinstein solution would be the unique perfect equilibrium in the model without stochastic interim disagreement points.

Second, stochastic disagreement points alone would not cause any inefficient outcome. Consider a modified version of our model such that the proposing player cannot delay making an offer. Whether or not the proposing player makes an offer, he will be the responding player in the next period. Let L and $L(x)$ (l and $l(x)$) denote the proposing player's (responding player's) expected equilibrium payoffs before and after the disagreement payoff x is revealed, respectively. Subgame perfection implies that, for $x = 0$ or d,

$$\begin{aligned} L(x) &= \max\{(1-\delta)x + \delta l,\ 1 - l(x)\}, \\ l(x) &= (1-\delta)x + \delta L. \end{aligned}$$

Parallel to Section 4, we can identify the following three cases:

$$\begin{aligned} &(\mathrm{A}') \qquad (1-\delta)d + \delta l \le 1 - (1-\delta)d - \delta L, \\ &(\mathrm{B}') \qquad \delta l \le 1 - \delta L \ \text{ and } \ (1-\delta)d + \delta l \ge 1 - (1-\delta)d - \delta L, \\ &(\mathrm{C}') \qquad \delta l \ge 1 - \delta L. \end{aligned}$$

Solving the equation system in these three cases, we find that

$$\begin{aligned} &(\mathrm{A}') \qquad L = \frac{1 - (1-\delta)(1-p)d}{1+\delta}, && \text{when } \ d \le \frac{1}{2}, \\ &(\mathrm{B}') \qquad L = \frac{p + (1-\delta)(1-p)[1 + \delta(1-p)]d}{(1+\delta)[1 - \delta(1-p)]}, && \text{when } \ \frac{1}{2} \le d \le \overline{d}, \\ &(\mathrm{C}') \qquad L = (1-p)d, && \text{when } \ d \ge \overline{d}. \end{aligned}$$

Therefore, the two players will immediately reach an agreement if $d \le 1/2$; the proposing player will make an acceptable offer when $x = 0$ and make an unacceptable offer when $x = d$

if $1/2 \leq d \leq \overline{d}$; and there will be no agreement if $d \geq \overline{d}$. Despite the similarity of this outcome to Proposition 1, the fundamental difference is that there is no *inefficient* delay. Therefore, stochastic interim disagreement points alone would not cause any inefficient delay.

6 Concluding Remarks

We considered a two-player strategic bargaining model with discounting in which (i) the interim disagreement point is stochastically determined either at $(0, 0)$ or at (d, d) in any period, and (ii) the proposing player can delay making an offer for one period. We showed that if d is in an intermediate range, the proposing player makes an (acceptable) offer if and only if the current interim disagreement payoff is 0. This outcome emerges even in the case where $d < 1/2$. Therefore, we concluded that the model has a unique, (stochastically) inefficient perfect equilibrium if the interim disagreement point varies below the bargaining frontier and the variation is large enough. As for the uniqueness, we should emphasize that the equilibrium strategy is unique when $0 \leq d < \overline{d}$ and $d \neq \underline{d}$. When $d \geq \overline{d}$, only the equilibrium payoff vector is unique. At $d = \underline{d}$, the model has two perfect equilibria where the proposing player has the same payoff, but the responding player does not.

We assumed that the interim disagreement payoff is stochastic, but not the bargaining value. In this aspect, our model is effectively equivalent to a bargaining model in which the interim disagreement payoff is fixed but the bargaining value is stochastic, as assumed in Avery and Zemsky's (1994) model. More generally, we may consider a model in which both interim disagreement payoff and bargaining value are stochastic. We can extend our model to this general case and obtain qualitatively the same results, paying attention to the difference between the bargaining values and interim disagreement payoffs. After all, what matters is this difference, the surplus from an agreement over disagreement.

Finally, the assumption that two players' interim disagreement payoffs are the same in any state is innocuous. As inferred from (1), it is the sum of the two players' interim disagreement payoffs that determines the equilibrium outcome. To see this claim in more detail, let us consider the case in which the interim disagreement point is $(0, 0)$ with the probability p and (d_1, d_2) with the probability $1 - p$. Here, either d_1 or d_2 can even take a negative value as long as $d_1 + d_2 > 0$. Our analysis will go through by replaying d with $(d_1 + d_2)/2$. As a result, if $\underline{d} \leq (d_1 + d_2)/2 \leq \overline{d}$, the proposing player makes an acceptable offer when the interim disagreement point is $(0, 0)$, while he makes no offer at all when the interim disagreement point is (d_1, d_2) in the subgame perfect equilibrium. Therefore, inefficient delay will still occur if the sum of the two players' high interim disagreement payoffs is in the intermediate range.

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