USING TAYLOR RULES AS EFFICIENCY BENCHMARKS

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Abstract

Using Taylor Rules As Efficiency Benchmarks

In this article, benchmark Taylor rules are obtained as the solution to a dynamic programming problem in which interest rates are chosen to minimize the discounted sum of observed inflation and output variations. The properties of these benchmark rules are used to derive efficiency conditions that are amenable to estimation. Estimated efficient ranges for the coefficients in the benchmark rule are used to characterize efficient classes of rules for Canada, France, Germany, Italy, the United Kingdom, and the United States, and to assess the efficiency of the monetary policies implemented in these countries from the early 1980s onwards.

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1. Introduction

Taylor's (1993a) suggestion that a simple interest rate rule could serve as a guide to formulating good monetary policy has resulted in a large and growing literature concerned with the practical aspects of monetary management. According to Taylor's rule, monetary policy can be characterized in terms of the response of interest rates to a weighted sum of inflation and output variation. Taylor (1993a, 1999a) has found his rule to be a good description of monetary policy in the United States from the mid-1980s onwards and Stuart (1996) finds that a very similar response function describes interest rate policy in the United Kingdom. Recently, Clarida, Galí, and Gertler (1998) have used a forward-looking version of Taylor's rule to characterize interest rate policy in France, Germany, Italy, Japan, the UK, and the US.

Svensson (1997) has shown that feedback rules of the type proposed by Taylor can be obtained as the first-order condition to a dynamic optimization problem. Because the structural models for which Taylor rules are optimal policy responses are very parsimonious, Taylor rules and simple rules like it are generally viewed as approximations to more complex optimal feedback rules. It is of course possible to specify more complex models and derive more complex interest rate rules. However, this approach is viewed as relatively unattractive because our knowledge of the economy's true structure is imperfect and, as Levin, Wieland, and Williams (1999) have shown, more complex feed-back rules tend to be less robust across models than simpler rules.

Taylor (1999a) and Nelson (2000) use Taylor rules to compare different monetary policy regimes in the United States and the United Kingdom, respectively. This application of Taylor rules can be justified on the grounds that most monetary policies can be characterized in terms of the ex post relationship between the observed nominal interest rate, inflation, and output outcomes they generate.[1] In this article, I use

[1]Orphanides (1998, 2000) points out that because the real-time data available to monetary authorities at the time policy is formulated is often noisy and subject to significant revision over time, monetary authroities generally respond to a broader set of variables than those included in the Taylor rule. Nevertheless, as noted above, many studies have shown that estimated Taylor rules fit the

Taylor rules to characterize the monetary policies of six countries and try to determine whether the monetary policies implemented in these countries can be regarded as having been relatively efficient.

In the literature, the relative efficiency of simple, sub-optimal rules is usually judged by comparing the performance of simple rules to that of the fully optimal rule generated by the researcher's preferred model. This is the approach employed by Rudebusch and Svensson (1999). Given the lack of consensus about the underlying economic structure, an alternative approach is to identify the simple rule that performs best for an appropriate generic class of models. This approach, which was originally proposed by McCallum (1988), is the one I have adopted in this article.[2] In order to identify the best simple rule, I use Svensson's (1997) reduced-form equations and, following Taylor's (1999b) example, interpret them as reasonable approximations to a generic class of economic models. Solving the policy authority's dynamic optimization problem using this approximate model then yields the best simple rule for models of this class. This rule provides the efficiency benchmark against which the policies that countries have actually employed are judged.

The argument in favour of policy guidelines that have relatively simple functional forms is most often made on the grounds that they are more robust than more complex, fully optimal rules. However, there is at least one other compelling reason for employing simple benchmark rules. There is now a general consensus that if monetary policy is to be implemented successfully, then the policy must be credible. In order

observed behaviour of monetary authorities reasonably well. In the subsequent discussion, when I say that the policy authority has employed a particular monetary rule, this should be interpreted as meaning that the monetary authority behaves as if it has adopted this policy, whether or not this was its conscious objective.

[2]Onatski (2001) and Onatski and Stock (2001) also take McCallum (1988) as their starting point. However, Onatski and Stock explicitly characterize the nature of the policy authority's uncertainty (model uncertainty and shock uncertainty) and use robust control methods to characterize policy rules that minimize the risk of producing very bad outcomes. For a critique of the application of robust control methods to monetary policy, see Sims (2001).

for policy to be credible, the central bank must be seen to do what it says it will do. Using simple benchmark rules to characterize monetary policy provides a convenient way for the central bank to communicate its policy stance to the public and thereby facilitates reputation-building. One of the challenges that central banks face is choosing a good benchmark rule from among the infinite number of possibilities. Ball (1999a,b), McCallum (1998), and Taylor (1993a), among others, have provided either theoretical or practical reasons for favouring a particular simple rule. Typically, the recommendations made on the basis of theory are not country-specific. My approach has the advantage that it generates country-specific efficiency criteria that can be used to reduce the feasible set of benchmark rules by eliminating particular classes of rules. The classes of rules I focus on are the pure price rule, in which the interest rate is expressed only as a function of inflation variation, and the nominal income rule, in which equal weight is given to inflation and output variation in determining the interest rate.

Unlike actual interest-rate response functions, efficient (benchmark) Taylor rules cannot be estimated directly. The reason for this is that the weights in efficient Taylor rules are functions of the policy authority's behavioural parameters, which are not generally observable or amenable to estimation. However, by extending a method of analysis introduced by Ball (1999a), I am able to derive cross-coefficient constraints and theoretical bounds on the efficient weights that can be calculated from estimates of the reduced-form parameters alone. These theoretical bounds are used to calculate efficient ranges for the relative and absolute weights in the Taylor rule for all permissible values of the policy authority's behavioural parameters.[3] An interest-rate rule is deemed to be efficient if the intersection between the 95% confidence intervals associated with every observed weight and its efficient counterpart is non-empty.

[3]Allowing the policy authority's behavioural parameters to vary in this way raises the possibility of parameter instability in the estimating equations. A discussion of the steps taken to determine whether policy invariance poses an empirically significant problem in this study may be found in Section 4.

The countries included in this study are Canada, France, Germany, Italy, the United Kingdom, and the United States. In order to determine whether the interest rate policies employed in these countries were reasonably good ones, I compare the characteristics of annual benchmark rules with the interest rate policies actually employed. In the theoretical model, the transition function of the policy authority's programming problem is described as a first-order difference equation in order to obtain an optimal response function that, like the original Taylor rule, contains no lagged endogenous variables. In order to preserve consistency between the theoretical results and their empirical application, the endogenous variable may be lagged only once in the relevant estimation equation. Because the statistically significant lag-length is generally positively related to the frequency of the data employed, annual data is used to estimate country-specific reduced-form equations and interest-rate response function in this study.[4] The Taylor rules I estimate therefore describe the monetary policy implemented in each country in terms of the ex post average annual relationship between the domestic interest rate, inflation, and the output gap that the policy generated.

The rest of this article is organized as follows. A modified version of Svensson's (1997) dynamic model is introduced in Section 2 and used to derive a benchmark generic Taylor rule. In Section 3, necessary conditions for efficiency that can be estimated are derived from the theoretical model. Estimation of the representative equations for the six countries included in this study is undertaken in Section 4. Country-specific efficient ranges for the relative weight on output variation are also reported in this section. The conditions under which pure price and nominal income rules are useful benchmark rules are discussed in Section 5. In Section 6, Taylor rules are estimated for each country. The efficiency criteria derived in Section 3 are then used to evaluate the estimated interest-rate rules. A brief summary of the results obtained may be found in Section 7.

[4]Using quarterly data, Rudebusch and Svensson (1999) have found that the statistically determined transition function is a fourth-order difference equation.

2. The Benchmark Taylor Rule

For the purposes of identifying a benchmark Taylor rule, I use a modified version of the model employed by Svensson (1997) to represent a simple approximation to a variety of more complex structural models. The economic structure is summarized by the following reduced-form equations:

$$\pi_{t+1} = \alpha_1 \pi_t + \alpha_2 y_t + \varepsilon_{t+1} \tag{1}$$

$$y_{t+1} = \beta_1 y_t - \beta_2 (i_t - \pi_t) + \mathbf{b}_3 \mathbf{x}_t + \eta_{t+1} \tag{2}$$

where π_t is the inflation rate in period t, y_t is the output gap, i_t is the nominal interest rate, and $\mathbf{x}_t = (x_{1t}, x_{2t}, x_{3t}, ...)$ is a column-vector of exogenous and predetermined variables that have an impact on the magnitude of the output gap. The variables ε_{t+1} and η_{t+1} represent random disturbances to inflation and the demand for goods, respectively, which are not contemporaneously observable. All variables are expressed, in logarithms, as deviations from their long-run equilibrium values.[5] As in Svensson's original model, each time period t is assumed to have a duration of one year.

The model employed by Svensson is a special case of (1) and (2) in which $\alpha_1 = 1$ and $\mathbf{b}_3 = 0$.[6] Removing some of the restrictions that Svensson originally imposed in his model allows (1) and (2) to represent a wider class of models and also accommodates country-specific differences in economic structure. For example, the addition of the $\mathbf{x}_t$ vector to (2) allows variables such as exchange rates, which Ball (1999b) has found to be important for efficient interest rate management in open economies, to be introduced into the model. I follow Clarida, Galí and Gertler (1998) and allow

[5] In order to ensure that (1) exhibits long-run consistency, it is assumed that the policy authority chooses its inflation target π^* to coincide with the long-run equilibrium inflation rate, which, for the purposes of this article, is defined as the measured inflation rate at which the output gap is zero.

[6] McCallum (1997) has pointed out that imposing the restriction $\alpha_1 = 1$ leads to dynamic inconsistency in this model when the policy authority sets interest rate policy to minimize the variation of nominal income. Tests conducted as part of the empirical application discussed in Section 4, strongly rejected this parameter restriction for every country in the sample.

for country-specific determinants of the output gap by including predetermined and exogenous variables in the $\mathbf{x}_t$ vector.[7]

One potentially controversial feature of the model used here is that (1) describes a backward-looking Phillips curve. Recently, the desire to use an aggregate supply equation that can be derived from an explicit microeconomic optimization problem has led some authors to use a 'New Keynesian' Phillips curve in place of (1). In this new version of the Phillips curve, expected future inflation either replaces or supplements expected current inflation as a determinant of the current inflation rate.[8] In this study, I chose not to incorporate future expected inflation into (1) for several reasons. First, as Mishkin (1999) has pointed out, the models from which forward-looking Phillips curves are derived have the implication that the policy authority need not act pre-emptively to control inflation. However, one of the lessons that policy-makers learned from the experiences of the period under study was precisely that pre-emptive action was necessary given the lags in the economy's response to policy changes. Second, the empirical evidence on the significance of expected future inflation as a determinant of the current inflation rate is mixed and the results seem to be quite sensitive to the estimation method used. Using quarterly US data, Fair (1993) and Fuhrer (1997) obtain estimates for the forward-looking expectations component that are not significantly different from zero; other estimates for the US range between statistically significant coefficients of 0.28 to 0.42.[9] Overall, the empirical results indicate that the coefficient on the forward-looking expectation component is low and this, together with Levin, Wieland, and Williams' (1999) finding that the inclusion of a forward-looking inflation element does not significantly improve the performance of their simple rules suggests that (1) is a parsimonious reduced-form representation of a reasonable generic structural model.[10]

[7]In this study, decisions about which variables to include in the $\mathbf{x}_t$ vector were made on a purely empirical basis. The method used to identify the components of $\mathbf{x}_t$ is described in Section 6.1

[8]See, for example, Rotemberg and Woodford (1997,1999) and Svensson (2000).

[9]Rudebusch (2000) provides a summary of the estimation results obtained in a variety of studies.

[10]Levin, Wieland, and Williams (1999) used US data in their study. It is possible that the US

Following Svensson (1997), I assume that the policy authority's objective is to stablilize inflation around the long-run inflation target π^* and the output gap around zero. The policy authority's one-period loss function is then given by:

$$L(\pi_t, y_t) = \frac{1}{2}\left\{(\pi_t - \pi^*)^2 + \lambda y_t^2\right\} \tag{3}$$

where λ is the relative weight assigned to output stabilization. With period-by-period losses given by (3), the policy authority's intertemporal loss function is:

$$\mathrm{E}_t \sum_{\tau=0}^{\infty} \delta^\tau L(\pi_{t+\tau}, y_{t+\tau}) \tag{4}$$

where δ is the policy authority's discount factor, and E_t denotes that the expectation of future losses is conditioned on the information available at time t.

Given that the policy authority views the short-term interest rate i_t as its control variable, the policy authority's objective is to set i_t so as to minimize (4). From (1) and (2) it is evident that the policy authority faces a two-period control lag. Following Svensson (1997), the policy authority's problem can be formulated as

$$V(\pi_{t+1|t}) = \min_{y_{t+1|t}} \left\{\frac{1}{2}\left[(\pi_{t+1|t} - \pi^*)^2 + \lambda y_{t+1|t}^2\right] + \delta \mathrm{E}_t V(\pi_{t+2|t+1})\right\} \tag{5}$$

subject to

$$\pi_{t+2|t+1} = \alpha_1 \pi_{t+1} + \alpha_2 y_{t+1}$$

where the notation $z_{t+1|t}$ denotes the value that the variable z is expected to take on in period $t+1$ conditional on the information available in period t. Once the optimal value of $y_{t+1|t}$ has been obtained, the optimal level of i_t can be inferred from (2).

Because the period loss function (3) is quadratic and the constraint is linear, $V(\pi_{t+1|t})$ must be a quadratic polynomial. Let $V(\pi_{t+1|t})$ be given by

$$V(\pi_{t+1|t}) = k_0 \ + \ k_1(\pi_{t+1|t} - \pi^*) \ + \ \frac{k_2}{2}(\pi_{t+1|t} - \pi^*)^2. \tag{6}$$

results are not representative and that expected future inflation may be of greater importance in determining the rate of inflation in other countries.

Using (6) to replace $V(\pi_{t+2|t+1})$ in (5) and taking the derivative of the expression in braces with respect to $y_{t+1|t}$ results in the first-order condition

$$y_{t+1|t} = -\frac{\delta\alpha_2 k_1}{\lambda} - \frac{\delta\alpha_2 k_2}{\lambda}\left[\pi_{t+2|t} - \pi^*\right] \tag{7}$$

where

$$k_1 = \frac{-\lambda\delta\alpha_1 k_2(1-\alpha_1)\pi^*}{\lambda(1-\delta\alpha_1)+\delta\alpha_2^2 k_2} \tag{8}$$

$$k_2 = \frac{[\delta\alpha_2^2 - \lambda(1-\delta\alpha_1^2)] + \sqrt{[\delta\alpha_2^2 - \lambda(1-\delta\alpha_1^2)]^2 + 4\delta\alpha_2^2\lambda}}{2\delta\alpha_2^2}. \tag{9}$$

Details of the solutions for k_1 and k_2 are provided in Appendix 1.

From (1), $\pi_{t+2|t}$ can be expressed as

$$\pi_{t+2|t} = \alpha_1^2\pi_t + \alpha_1\alpha_2 y_t + \alpha_2 y_{t+1|t}. \tag{10}$$

Substituting (10) into (7) reveals that the solution to (7) is

$$y_{t+1|t} = -\frac{\delta\alpha_2 k_1}{\lambda+\delta\alpha_2^2 k_2} - \frac{\delta\alpha_2 k_2\alpha_1^2\,\pi_t}{\lambda+\delta\alpha_2^2 k_2} - \frac{\delta\alpha_2^2 k_2\alpha_1\, y_t}{\lambda+\delta\alpha_2^2 k_2} + \frac{\delta\alpha_2 k_2\,\pi^*}{\lambda+\delta\alpha_2^2 k_2}. \tag{11}$$

Substituting (11) into (2) and solving for the interest rate i_t yields the generic benchmark Taylor rule

$$i_t - \pi_t = \bar{K} + g_1\,[\pi_t - \pi^*] + g_2\, y_t + \mathbf{g}_3\,\mathbf{x}_t \tag{12}$$

where

$$\bar{K} = \frac{\delta\alpha_2\,[k_1 + k_2(\alpha_1^2-1)\pi^*]}{\beta_2(\lambda+\delta\alpha_2^2 k_2)} \tag{13}$$

$$g_1 = \frac{\delta\alpha_2 k_2\alpha_1^2}{\beta_2(\lambda+\delta\alpha_2^2 k_2)} \tag{14}$$

$$g_2 = \frac{[(\alpha_1+\beta_1)\delta\alpha_2^2 k_2 + \lambda\beta_1]}{\beta_2(\lambda+\delta\alpha_2^2 k_2)} \tag{15}$$

$$\mathbf{g}_3 \;=\; \frac{\mathbf{b_3}}{\beta_2}. \tag{16}$$

The benchmark Taylor rule reduces to Taylor's original two-parameter rule when $\bar{K} = \mathbf{b}_3 = 0$.[11]

For a policy authority willing to commit itself to particular values of δ and λ, the benchmark rule can be calculated directly from (12)-(16) once the reduced-form parameters have been estimated. For the outside observer, the assessment of observed interest rate policy is not quite as straightforward because the policy authority's choice of δ and λ is generally private information (which the policy authority may or may not have an incentive to reveal truthfully). Nevertheless, (12)-(16) can be used to establish a set of necessary conditions that must be fulfilled for a simple interest-rate rule to be a useful efficiency benchmark.[12] Specifically, it is possible to establish bounds on the coefficients in (12) by constructing efficient ranges for the individual coefficients and for the cross-coefficient constraint on g_1 and g_2.

3. Derivation of the Efficiency Conditions

In the context of the model described in the foregoing section, efficiency requires the coefficient g_2 to be an affine function of g_1. In particular, (14) and (15) imply that a necessary condition for efficiency is characterized by the cross-coefficient constraint

$$g_2 \;=\; \frac{\beta_1}{\beta_2} \;+\; \frac{\alpha_2}{\alpha_1}\, g_1. \tag{17}$$

The efficiency condition (17) can be expressed in terms of the relative weight $\gamma = g_2/g_1$. This form of the cross-coefficient constraint turns out to be useful for identifying broad classes of benchmark rules. The relative Taylor weight $\gamma = g_2/g_1$ implied by (14) and (15) is given by

[11] One set of parameter restrictions for which Taylor's original two-parameter rule is also the benchmark rule is $\mathbf{b}_3 = 0$ and $\alpha_1 = 1$.

[12] The simple benchmark rule that is obtained as the solution to (5) is referred to as efficient rather than optimal to emphasize the fact that minimization of the policy authority's loss function does not necessarily mean that the social optimum has been achieved.

$$\gamma \;=\; \frac{[(\alpha_1+\beta_1)\delta\alpha_2^2 k_2+\lambda\beta_1]}{\delta\alpha_2 k_2\alpha_1^2}. \tag{18}$$

The efficient range for γ is composed of all of the values of γ which satisfy (18) given α_1, α_2, and β_1. The efficient range for γ can be obtained from (18) by allowing λ to vary from zero to infinity for all possible values of δ (i.e., $0<\delta<1$).[13] The boundary value of the efficient range that is associated with $\lambda=0$ is easily determined. From (9) it is apparent that $k_2=1$ when $\lambda=0$. Substituting $k_2=1$ and $\lambda=0$ into (18) yields the boundary value $\gamma=\alpha_2(\alpha_1+\beta_1)/\alpha_1^2$. It will be established below that $\gamma=\alpha_2(\alpha_1+\beta_1)/\alpha_1^2$ is the lower bound of the efficient range for γ when β_1 is positive.[14]

The boundary value of the efficient range when λ approaches infinity is a little trickier to determine. The simplest way to proceed is to begin by dividing the numerator and denominator of (18) by k_2 to obtain

$$\gamma \;=\; \frac{[(\alpha_1+\beta_1)\delta\alpha_2^2+(\lambda/k_2)\beta_1]}{\delta\alpha_2\alpha_1^2}. \tag{19}$$

In Appendix 2, it is demonstrated that in the limit, as λ approaches infinity, the value of k_2 converges to the positive constant $(1-\delta\alpha_1^2)/\delta\alpha_2^2)$. Under the assumption that $0<\alpha_1\le 1$, $\alpha_2>0$, and $0<\delta<1$, γ increases without bound as λ goes to infinity when β_1 is positive. The empirically relevant efficient range for γ is therefore characterized by:

$$\gamma \;\ge\; \alpha_2(\alpha_1+\beta_1)/\alpha_1^2 \quad \text{for} \quad \beta_1\ge 0. \tag{20}$$

The efficient range for γ identifies the relative magnitudes of g_1 and g_2 that are

[13] Ball (1999a) identifies ranges for the weights g_1 and g_2 that are necessary for efficiency *given* the assumption that $\delta=1$. With $\delta=1$, the policy authority does not discount the future at all. If $\delta\ne 1$, then the weights in an efficient Taylor rule need not fall in the ranges identified by Ball. Ball also restricts his analysis to two-parameter Taylor rules by imposing the a priori restrictions $\mathbf{b}_3=0$ and $\alpha_1=1$.

[14] Theoretically, $\gamma=\alpha_2(\alpha_1+\beta_1)/\alpha_1^2$ is the upper bound of the range when β_1 is negative, but this case is not likely to be empirically relevant.

permissible in efficient interest-rate rules. However, fulfillment of this criterion is not, by itself, evidence of efficiency. In the context of the model employed here, efficiency also requires that all of the generalized Taylor rule coefficients fall within their individual efficient ranges. Using (9), (14), and (15), it is straightforward to show that, for fixed δ, g_1 and g_2 are strictly decreasing in λ when $0 < \alpha_1 \leq 1$, $\alpha_2 > 0$, and $0 < \delta < 1$. Allowing λ to vary from 0 to ∞ in (14) and in (15) yields the following efficient ranges for g_1 and g_2

$$g_1 = [\,0, \alpha_1^2/\alpha_2\beta_2\,] \tag{21}$$

$$g_2 = [\,\beta_1/\beta_2, (\alpha_1 + \beta_1)/\beta_2\,]. \tag{22}$$

Notice that because of the impact of the lagged output gap on inflation in (1), $\lambda = 0$ does not imply that $g_2 = 0$. Substituting $k_2 = 1$ and $\lambda = 0$ into (15) shows that $g_2 = (\alpha_1 + \beta_1)/\beta_2$ when $\lambda = 0$.

The efficient values of $\mathbf{g}_3$ are independent of λ and δ and therefore can be obtained directly from (16). Owing to the complexity of the expression for $\bar{K}$ given by (13), a general characterization of the efficient range of this term is not illuminating. However, it is perhaps worth pointing out that, in the limit, as λ approaches infinity, the value of $\bar{K}$ goes to zero.

4. Estimated Efficient Ranges

Taylor (1993b) identifies three broad classes of rules which are of particular interest. These interest-rate rules are (1) a pure price rule in which the weight on output variation is set equal to zero, (2) a nominal income rule in which the weight on inflation variation is set equal to the weight on output variation, and (3) a 'general' rule (referred to in this article as a variable-weight rule) in which the weights on inflation variation and output variation may differ and the rule is not a pure price rule. The relative weight $\gamma = g_2/g_1$ can be used to distinguish between the three types of interest-rate rules. The pure price and nominal income rules are characterized by $\gamma = 0$ and $\gamma = 1$, respectively, while a general rule is one for which γ is equal to a value other than 0 or 1. The parameters in (1) and (2) determine to which of these classes a country's benchmark interest rate rule belongs. Identifying the classes from which each country's benchmark interest rate rule may be chosen therefore entails estimating the coefficients in (1) and (2) and then using these to calculate country-specific efficient ranges for γ.

4.1 Estimation of the Reduced-Form Parameters

In order to preserve consistency between the theoretical and the estimated efficiency criteria, equations (1) and (2) were estimated using annual data for each country in the sample. The estimation period for Canada, France, the United Kingdom, and the United States begins in 1975 and ends in 1996. The estimation period for Germany also begins in 1975 but it ends in 1995. The German data set was truncated at 1995 because including 1996 introduced serious end-point problems. Equations (1) and (2) did not find strong support in the Italian data. For Italy, the quality of the estimation results deteriorated steady as the sample was extended beyond 1992. Rather than drop Italy from the sample altogether, I elected to include the Italian results for the period of best fit, which is 1973-92. For countries other than Italy, the year 1975 was chosen as a starting point to eliminate possible estimation problems associated with the abandonment of the Bretton Woods system in early 1973. Unfortunately, choosing 1975 as the initial date does not eliminate other sources of structural disturbance, such as the impact of the OPEC oil price increases which strongly influenced short-term Phillips curve relationships in most countries until the early 1980s. Furthermore, for the European countries, the financial turmoil surrounding the ratification of the Maastricht treaty in late 1992 appears to have caused some temporary changes in structural relationships. In the German data, the impact of German unification is also clearly discernable.

In order to keep the estimation equations as close to their theoretical counterparts as possible, dummy variables were used to deal with the above-mentioned changes in structure.[15] Every effort was made to avoid introducing structural dummies after 1982 to ensure that the estimated parameter values correspond to the time period which Taylor identifies as being associated with interest rate policies that follow Taylor rules. The variables used for the estimations were obtained from the International Monetary Fund's International Financial Statistics. Following Taylor's (1993a) example, the output gap was calculated as the deviation of the natural log of annual real GDP from its trend which, for the purposes of this study, is assumed to be deterministic and linear.[16] All other variables were pre-tested for order of integration using Augmented Dickey-Fuller tests. Perron's (1989) procedure was applied in those cases where structural change in the data generating process was suspected. The null hypothesis of a unit root was rejected at a significance level of at least 10% for all of the non-output variables needed to estimate (1) and (2).[17] The presence of a significant deterministic trend was rejected at the 5% level for these variables.[18]

[15]The dummy variables employed are described in detail in Appendix 3.

[16]In a more recent article, Taylor (1999a) uses a Hodrick-Prescott (HP) filter to obtain a quarterly GDP trend series for the United States. The likelihood that the results obtained here might be sensitive to the construction of the output gap was assessed using a Wald test on the slope coefficient obtained by regressing the standardized linear-trend gap on the standardized HP gap. The test results indicate that, for all countries except Canada, there is no significant difference in the two annual output gap series. In Canada's case, the linear-trend gap results in larger output gap values than does the HP gap. The decision to use the linear-trend gap for annual Canadian GDP data is supported by Serletis (1992).

[17]Note that the unit root hypothesis could not be rejected at the 10% level for the French nominal interest rate. However, it is the real interest rate that is needed to estimate (2), and this (composite) variable is I(0) at the 10% level.

[18]As noted above, the sample period is characterized by a number of significant changes in the economic environment. For most of the countries in this study, the 1980's were a transition period in which countries were wrestling with the results of the oil price increases. Perron's (1989) Model A captures the impact of the oil price shocks in the form of a shift in the mean of the inflation and/or interest rate processes in France, Italy, the United Kingdom, and the United States. The significance of this shift may very well decline as the sample period lengthens with the passage of time.

Table 1

Parameter Estimates for Equation (1)

	$\hat{\alpha}_1$	$1.96\ \hat{\sigma}_{\alpha_1}$	$\hat{\alpha}_2$	$1.96\ \hat{\sigma}_{\alpha_2}$
Canada	0.4964	0.1615	0.1324	0.0883
1982-96	(6.4580)		(3.1500)	
France	0.9810	0.1336	0.2933	0.2417
1981-94	(15.6500)		(2.5862)	
Germany	0.5841	0.2297	0.1979	0.0963
1975-95	(5.3652)		(4.3882)	
Italy	0.7008	0.1831	0.6615	0.3935
1981-92	(8.2714)		(3.6308)	
U.K.	0.4900	0.0865	0.1943	0.1505
1981-96	(11.953)		(2.7246)	
U.S.A.	0.5062	0.1694	0.1820	0.1662
1982-95	(6.3355)		(2.3206)	

The parameter estimates obtained using OLS to estimate (1) and (2) are reported in Tables 1 and 2, respectively. Details of the variable definitions, unit root tests, and the estimation results may be found in Appendix 3.

The dates given in column 1 of Table 1 identify the time periods over which the estimated values, $\hat{\alpha}_1$ and $\hat{\alpha}_2$, are free of structural changes. In Table 2, these dates specify stable periods for $\hat{\beta}_1$ and $\hat{\beta}_2$. In each table, the t-statistic associated with the parameter estimate is given in parentheses below the estimated value. Because the data set is small, it seems advisable not to rely too heavily on point estimates alone. For this reason, the 95% margin of error is provided in the column immediately to the right of each estimate.[19]

The country-specific components of $\mathbf{x}_t$ that were used to estimate (2) are described

[19] The 95% confidence interval for $\hat{\alpha}_i$ is given by $\hat{\alpha}_i \pm 1.96\hat{\sigma}_{\alpha_i}$ for $i = 1, 2$.

TABLE 2

Parameter Estimates for Equation (2)

	$\hat{\beta}_1$	1.96 $\hat{\sigma}_{\beta_1}$	$\hat{\beta}_2$	1.96 $\hat{\sigma}_{\beta_2}$	$\hat{\beta}_{31}$	1.96 $\hat{\sigma}_{\beta_{31}}$	$\hat{\beta}_{32}$	1.96 $\hat{\sigma}_{\beta_{32}}$
Canada	0.9386	0.1062	0.7311	0.2991	-0.1192	0.0588		
1983-96	(18.838)		(5.2084)		(-4.3245)			
France	0.9693	0.2668	0.4204	0.1650	-0.9121	0.3410	0.0559	0.0423
1981-94	(7.9179)		(5.5516)		(-5.8293)	0.0804	(2.8755)	
Germany	0.6315	0.1155	0.8563	0.2892	0.2597	0.2073	-0.0668	0.0384
1975-89	(11.812)		(6.3956)		(2.7056)		(-3.7586)	
Italy	0.6628	0.2001	0.4037	0.1457	-0.5990	0.3222		
1976-92	(7.1038)		(5.9444)		(-3.9880)			
U.K.	0.6663	0.2493	0.1468	0.1012	-1.5219	0.5278	-0.5155	0.3386
1975-96	(5.6662)		(3.0746)		(-6.1123)		(-3.2278)	
U.S.A.	0.4480	0.2466	0.8541	0.2750	-0.4838	0.3330	-0.3888	0.1797
1982-96	(3.8708)		(6.5169)		(-3.0960)		(-4.6114)	

TABLE 3

Country-Specific Components of $\mathbf{x}_t$

Canada	$x_{1t} = q^{us}_{t-1}$ = lagged Canada/US real exchange rate x_{2t} = none
France	$x_{1t} = \pi^{ger}_{t-2}$ = German inflation rate, lagged two periods $x_{2t} = \Delta e^{fus}_{t-1}$ = lagged % Δ nominal franc/dollar exchange rate
Germany	$x_{1t} = \Delta Y^{us}_{t-2}$ = US output growth, lagged two periods $x_{2t} = \Delta e^{gus}_{t-1}$ = lagged % Δ nominal dmark/dollar exchange rate
Italy	$x_{1t} = \pi^{ger}_{t-1}$ = lagged German inflation x_{2t} = none
U.K.	$x_{1t} = \pi^{ger}_{t-1}$ = lagged German inflation $x_{2t} = \Delta Y^{ger}_{t-1}$ = lagged German output growth
U.S.A.	$x_{1t} = \pi^{ger}_{t-1}$ = lagged German inflation $x_{2t} = y^{ger}_{t-1}$ =lagged German output gap

in Table 3. The variables representing x_{1t} and x_{2t} were chosen by the following method. I used the characteristics of each country to identify a set of variables that might be expected to have a significant influence on output and/or inflation. For instance, in Canada's case, US output, prices, and interest rates, and the Canada/US exchange rate were all likely candidates. For European countries like France and Italy, German output, prices, and interest rates, as well as the value of the domestic currency relative to the dmark were in the intial variable set. I then ran a series of regressions for each country and retained only those variables whose coefficients were significant at the 5% level. For both the US and Canada, only the dummy variable associated with x_{1t} was found to be significant at the 5% level.

4.2 Country-Specific Efficient Ranges

By definition, the relative weights associated with pure price rules and nominal income

TABLE 4

Efficient Ranges for γ

	point estimates	95% CI for lower bound of $\hat{\gamma}$
Canada	$[0.7710, \infty]$	$[-0.0326, 1.5746]$
France	$[0.5944, \infty]$	$[0.1677, 1.0211]$
Germany	$[0.7051, \infty]$	$[-0.0123, 1.4225]$
Italy	$[1.8366, \infty]$	$[0.3642, 3.3090]$
U.K.	$[0.9357, \infty]$	$[0.2019, 1.6695]$
U.S.A.	$[0.6777, \infty]$	$[0.0542, 1.3012]$

rules are $\gamma = 0$ and $\gamma = 1$, respectively. The efficient ranges for γ that are consistent with the model employed in this article are summarized in Table 4. The lower bounds of the efficient ranges shown in the second column of this table were caculcated as $\hat{\gamma} = \hat{\alpha}_2(\hat{\alpha}_1 + \hat{\beta}_1)/\hat{\alpha}_1^2$ using the point estimates reported in Tables 1 and 2. The third column in Table 4 gives the 95% confidence interval for the estimated lower bound of the efficient range for each country.[20]

It is immediately apparent that the pure price rule ($\hat{\gamma} = 0$) is only included in the 95% confidence interval for Canada and Germany. The nominal income rule ($\hat{\gamma} = 1$), on the other hand, falls within the 95% confidence interval for all countries and is excluded from the range calculation on the basis of point estimates only for Italy. The properties of efficient relative weights and the conditions under which benchmark rules can be characterized as pure price and nominal income rules are discussed in greater detail in Section 5.

The efficient range of the relative weight γ is useful for identifying which classes

[20]The 95% confidence interval for $\hat{\gamma}$ was obtained using the asymptotic standard error of the estimator $\hat{\gamma} = \hat{\alpha}_2(\hat{\alpha}_1 + \hat{\beta}_1)/\hat{\alpha}_1^2$ and the critical value $t_c = 1.96$. The confidence limits and intervals reported in Tables 5,6, 9, and 10 were calculated in a similar manner.

of interest-rate rules are potentially efficient rules. Although an infinite number of combinations of g_1 and g_2 satisfy this cross-coefficient constraint, only those values of g_1 and g_2 that fall within their individual efficient ranges, given by (21) and (22), respectively, are permissible in an efficient rule. Efficient ranges for the individual generalized Taylor rule coefficients are reported in Section 6 (see Table 10), where they are used to evaluate the efficiency of estimated annual interest-rate rules.

4.3 Parameter Invariance

The efficient ranges reported in Table 4 were obtained by allowing the policy authority's relative weight on output variation, λ, to vary over its permissible range. The construction of these efficient ranges is therefore based on the assumption that changes in the value of λ have no impact on the estimated values of the parameters needed to calculate them. The problem of policy-based parameter invariance, often referred to as the Lucas critique, arises because (1) and (2) are reduced-form equations whose parameters may be composites of the economy's structural (invariant) parameters and the policy authority's behavioural parameter, λ. The validity of the calculated efficient ranges clearly depends on the extent to which the Lucas critique represents a significant empirical problem in this study.

An empirical approach to dealing with the issue of potential parameter invariance has been suggested by Hendry (1988). This approach regards parameter invariance as a theoretical possibility which may or may not be of empirical significance in the context of a particular study. Ericsson and Irons (1995) illustrate a method of testing for the empirical significance of the Lucas critique which is particularly appropriate in this study, given size of the data set and the constraints that the theoretical structure places on the specification of the estimating equations. The idea behind their methodology is to test whether changes in the processes generating the explanatory variables lead to significant changes in the parameter estimates. The test methodology is composed of two steps. The first step involves careful modelling of the individual processes generating the variables included in the estimating equation. In the second step, these marginal processes are introduced into the original estimating equation. An F-test is then used to determine whether introducing information about how a particular variable changes over time has a significant impact on the parameter estimate associated with that variable. Failure to reject the null hypothesis (that the estimated parameters are statistically invariant) is interpreted as empirical support for the assumption of policy-based parameter invariance. The results obtained by applying this test to each of the countries included in this study indicate that all of the parameters estimated on the basis of (1) and (2) are statistically invariant for Canada, France, the United Kingdom, and the United States. The results for Germany and Italy are less satisfactory. In the case of Italy, the null hypothesis is rejected for two coefficient estimates, $\hat{\alpha}_1$ and $\hat{\beta}_1$. The results for Germany are even weaker with the null hypothesis being rejected for the parameters $\hat{\alpha}_1$, $\hat{\beta}_1$, and $\hat{\beta}_{32}$. These results indicate that the values reported for Germany and Italy in Tables 1 and 2 must be interpreted with caution. Details of the estimated marginal processes and the invariance test results are provided in Appendix 3.

5. Efficient Classes of Rules

Simple benchmark rules are useful not only for assessing the efficacy of monetary policy, but also as tools for communicating the central bank's policy stance to the public. A central bank that wishes to emphasize its commitment to inflation control may want to use a pure price rule in which the interest rate is expressed as a function of inflation variation alone. A nominal income rule, in which inflation and output variation are given equal weight, may be useful for communicating to the public that the central bank is equally concerned with the economy's inflation and output performance. In this section I investigate the conditions under which simple rules that can be described as pure price rules or nominal income rules are likely to be good benchmark rules for each of the countries in the sample.

5.1 Efficient Pure Price Rules

The results reported in Table 4 indicate that the pure price rule ($\gamma = 0$) is included in the 95% confidence interval for Canada and Germany. In a pure price rule, g_2 is set equal to zero. In contrast to variable-weight rules, which are efficient for a wide variety of economic structures and policy authority preferences, pure price rules are efficient only under very special circumstances. Because an efficient Taylor rule must satisfy (17), pure price benchmark rules must fulfill

$$g_1 \;=\; \frac{-\alpha_1\beta_1}{\alpha_2\beta_2}. \tag{23}$$

According to (23), efficient weights for pure price rules are independent of the values of λ and δ. Efficient pure price weights, together with their 95% confidence intervals, are reported in Table 5 for Canada and Germany. (Pure price rules are excluded from the feasible set of benchmark rules for all other countries in the sample.)

Because the coefficients α_1, α_2, β_1, and β_2 are generally positive, (23) indicates that the efficient weight on inflation variation in a pure price rule must be negative. However, $g_1 < 0$ does not necessarily mean that efficient monetary policy is characterized by a negative relationship between the real interest rate and observed (contemporaneous) inflation. Using (1) and expressing the real interest rate as $i_t - \pi^e_{t+1|t}$, the pure price rule, $i_t - \pi_t = g_1\pi_t$, can be written

$$i_t - \pi^e_{t+1|t} \;=\; (1 + g_1 - \alpha_1)\pi_t \;-\; \alpha_2 y_t. \tag{24}$$

Substituting (23) into (24), it is straightforward to show that the partial derivative of (24) with respect to inflation is positive for $g_1 > \alpha_1 - 1$. For both Canada and Germany, the relevant 95% confidence intervals for $\hat{g}_1$ and $\hat{\alpha}_1$ contain values for which this condition is satisfied. It is also apparent that an efficient pure price rule ensures that the real interest rate is negatively related to the output gap.

Substituting $\gamma = 0$ into (18) and solving for λ as a function of δ yields

$$\lambda \;=\; \frac{\alpha_1\alpha_2^2\delta(\alpha_1 + \beta_1)}{\beta_1^2 - \beta_1(\alpha_1 + \beta_1)(1 - \delta\alpha_1^2)}. \tag{25}$$

Table 5

Efficient Pure Price Rules

	$\hat{g}_1$	95% CI for $\hat{g}_1$	Range for λ	95% CI for λ
Canada	-4.81	$[-9.28, -0.34]$	$[-0.0282, 0)$	$[-0.1630, 0.1066]$
Germany	-2.24	$[-4.80, 0.31]$	$[-0.2560, 0)$	$[-0.2875, 0.0315]$

Equation (25) describes all of the combinations of λ and δ for which the pure price rule is efficient, given the reduced-form parameters α_1, α_2, and β_1. Policy authorities whose preferred combinations of λ and δ do not satisfy (25) should use either a nominal income rule or an appropriate variable-weight rule rather than a pure price rule to characterize their interest-rate policies. Equation (25) provides the policy authorities with an easy way to check whether a pure price rule is appropriate. In addition, (25) can be used to describe the conditions under which pure price rules will be efficient. Substituting estimated values of α_1, α_2, and β_1 into (25) and allowing δ to vary from 0 to 1 identifies the range of λ values for which efficient pure price rules are feasible.

The values of λ for which pure price rules are efficient are reported in the last two columns of Table 5. The fact that the point estimates of the efficient ranges for λ contain no permissible values of λ for either country reflects the exclusion of $\hat{\gamma} = 0$ from the point estimates of the efficient ranges for γ as reported in Table 4. The 95% confidence intervals for λ indicate that a pure price rule may be a useful benchmark rule if the Canadian and German policy authorities are, respectively, at least 9.4 and 31.7 times more concerned about price variablility than about output variability.

5.2 Efficient Nominal Income Rules

A nominal income rule is defined as $g_1 = g_2$ (i.e., $\gamma = 1$). It follows from (17) that an efficient nominal income rule must fulfill

$$g_1 = g_2 = \frac{\alpha_1 \beta_1}{(\alpha_1 - \alpha_2)\beta_2}. \tag{26}$$

Table 6

Efficient Nominal Income Rules

	$\hat{g}_1 = \hat{g}_2$	95% CI for $\hat{g}_1 = \hat{g}_2$	Range for λ	95% CI for λ
Canada	1.75	$[1.09, 2.41]$	$(0, 0.0087]$	$[-0.0247, 0.0421]$
France	3.29	$[1.31, 4.30]$	$(0, 0.2776]$	$[-1.9791, 2.5343]$
Germany	1.12	$[0.66, 1.58]$	$(0, 0.0537]$	$[-0.0318, 0.1392]$
Italy	29.28	$[-195.18, 253.74]$	$[-0.0340, 0)$	$[-0.2638, 0.1958]$
U.K.	7.52	$[0.32, 14.72]$	$(0, 0.0047]$	$[-0.0250, 0.0302]$
U.S.A.	0.82	$[0.20, 1.44]$	$(0, 0.0471]$	$[-0.1098, 0.2040]$

Efficient nominal income weights calculated according to (26), together with their 95% confidence intervals, are reported for each country in Table 6.

As is the case with pure price rules, nominal income rules are efficient only for special combinations of the policy authority's preference parameter and discount rate. Setting $\gamma = 1$ in (18) and solving for λ as a function of δ yields

$$\lambda = \frac{\alpha_2\delta(\alpha_1 - \alpha_2)[\alpha_1^2 - \alpha_2(\alpha_1 + \beta_1)]}{\beta_1(\alpha_1 - \alpha_2) - \alpha_1\delta[\alpha_1^2 - \alpha_2(\alpha_1 + \beta_1)]}. \tag{27}$$

The range of values of λ for which efficient nominal income rules are feasible can be obtained by letting δ range from 0 to 1 in (27). If a policy authority should choose a value of λ outside this range, there is no permissible value of δ for which a nominal income rule will be efficient. The values of λ for which efficient nominal income rules exist are given in the last two columns of Table 6.

The statistics given in the last column of Table 6 show that nominal income rules cannot be excluded from the feasible set of efficient rules at the 5% level of significance for any country. It is also evident that the conditions under which nominal income rules may be efficient vary markedly among the six countries. In the United Kingdom, the policy authority must be at least 33 times more concerned about inflation variation than output variation if a nominal income rule is to be efficient. In France, by contrast, a nominal income rule may be efficient when the policy authority is more concerned about output variation than price variation (i.e., when $1 < \lambda \leq 2.5$). In Canada and Germany, concern for inflation variation must be, respectively, at least 24 and 7 times as great as concern for output variation if a nominal income rule is to be efficient.[21] For Italy and the United States nominal income rule efficiency requires that the policy authority regard inflation variation as being at least 5 times as important as output variation.

The last column in Table 6 identifies the range of λ values for which efficient nominal income rules can be found. I now show that when the policy authority's preferred value of λ exceeds the upper bound of this range, the efficient interest-rate rule is characterized by $\gamma > 1$ for every possible value of δ. From (18), we know that the efficient γ is a function of λ and δ. Tables 1 and 2 show that $0 < \alpha_1 < 1$, $\alpha_2 > 0$, and $\beta_1 > 0$ for all countries in the sample. This has the following implications for the functional relationship between γ, λ, and δ. Using (9) and (18), it is straightforward to establish that for every $\delta \in (0, 1)$, γ is an increasing, strictly concave, unbounded from above function of λ. It also follows from (9) and (18) that γ is a decreasing function of δ for each $\lambda > 0$. Let γ_0 denote the minimum value of γ for $\delta \in (0, 1)$ and $\lambda \geq 0$. It then follows from (18) that $\gamma = \gamma_0$ for all $\delta \in (0, 1)$ when $\lambda = 0$. From these observations, it can be inferred that if δ is increased, λ must also be increased in order to keep γ constant when $\gamma > \gamma_0$. These properties of the efficient relative weight γ can be used to identify the values of λ for which γ must exceed unity. Let $\bar{\lambda}$ be the supremum of the values of λ that satisfy (27) for some $\delta \in (0, 1)$. Because λ is increasing in δ when $\gamma = 1$, $\bar{\lambda}$ can be calculated by taking the limit as δ goes to 1 in (27). It follows from this observation and the fact that γ is increasing in λ that the efficient γ must exceed unity for any value of λ greater than $\bar{\lambda}$. The relationship

[21] The fact that the 95% confidence intervals for the values of λ associated with efficient pure price rules and nominal income rules overlap for Canada and Germany indicates that there is a range of λ values for which these countries may choose to use either pure price or nominal income benchmark rules.

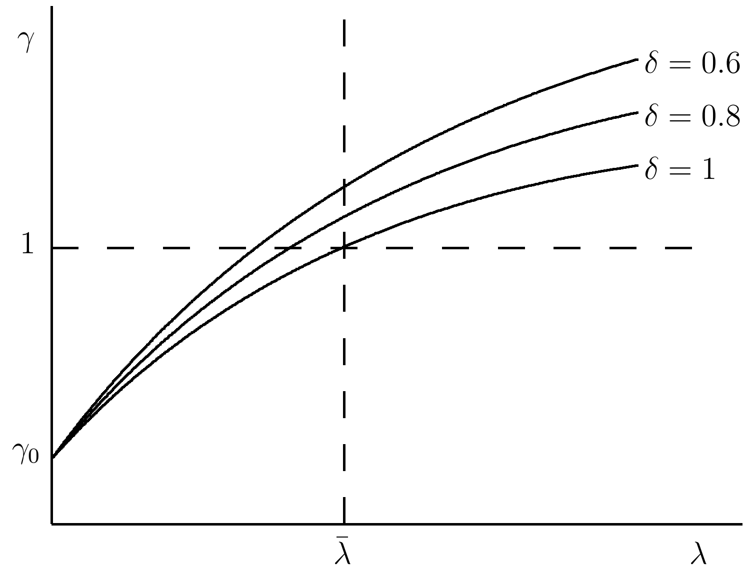

FIGURE 1

between the efficient values of γ, λ, and δ is illustrated in Figure 1.

Taylor (1999b) studies the robustness of simple rules by comparing the performance of a small set of rules in a variety of different models. All of the simple rules Taylor chooses have a value of $\gamma < 1$. The foregoing discussion implies that similar annual interest-rate rules are likely to perform well in the US only if the Fed's preference parameter λ does not significantly exceed 2.04.

6. Estimated Interest-Rate Rules

6.1 Coefficient Estimates

Two types of annual Taylor rules were estimated for each country in the sample — a simple, two-parameter Taylor rule and a more general interest-rate rule similar in form to the theoretical benchmark (12). The results obtained are reported in Tables 7 and 8. The Schwarz Criterion (SC), given in the last column of Tables 7 and 8, was used to determine whether the simple two-parameter Taylor rule or a more general formulation that more closely resembles the benchmark rule (i.e., a generalized Taylor Rule) provides the better characterization of the monetary policy actually implemented in each country. The results show that monetary policy is better represented by a generalized Taylor rule for Canada, the United Kingdom, and the United States, and by a (two-parameter) Taylor rule for France, Germany, and Italy.

The results reported in Table 7 were obtained using the estimating equation

$$i_t - \pi_t = g_0 \ + \ g_1\pi_t \ + \ g_2 y_t \ + \ \omega_t. \tag{28}$$

Dummy variables were introduced into the estimating equations for Canada and the United States to allow for the possible impact of significant monetary policy events in each country. In Canada this event was the adoption of inflation targeting in 1991 and in the United States, Greenspan's replacement of Volcker as Chairman of the Board of Governors of the Federal Reserve System in August 1987. For both Canada and the US the coefficient estimates were obtained using the equation

$$i_t - \pi_t = g_0 \ + \ (g_1 + g_1^d D)\pi_t \ + \ (g_2 + g_2^d D) y_t \ + \ \omega_t \tag{29}$$

with $D = 0$ over the period 1983-90 for Canada and 1982-87 for the US. The dummy was set equal to one over the period 1991-96 for Canada and 1988-96 for the US. In the Canadian case, the dummy was not significant for inflation or output at the 5% level whereas the dummy was significant for both of these variables in the US estimation.

The generalized Taylor rule coefficients given in Table 8 were estimated using the equation

$$i_t - \pi_t \ = \ g_0 \ + \ g_1\pi_t \ + \ g_2 y_t \ + \ g_{31} x_{1t} \ + \ g_{32} x_{2t} \ + \ \phi_t \tag{30}$$

with the country-specific variables x_{1t} and x_{2t} as described in Table 3. For Canada and the United States dummy variables analogous to those employed in estimating (29) were used for all variables in (30); in both cases, the dummies were found to be significant only for the variables included in the $\mathbf{x_t}$ vector.

Table 7

Taylor Rule Estimates

		$\hat{g}_1$	1.96 $\hat{\sigma}_{g_1}$	$\hat{g}_2$	1.96 $\hat{\sigma}_{g_2}$	SC
Canada:	83-96	-0.5165	0.8015	0.5254	0.2598	-8.5181
		(-1.4357)		(4.5048)		
France:	81-96	-0.2145	0.2204	0.3888	0.3462	-8.3440
		(-2.1021)		(2.4255)		
Germany:	75-95	0.0335	0.3108	0.1809	0.1364	-9.0519
		(0.2325)		(2.8639)		
Italy:	81-92	-0.1892	0.1765	-0.6454	0.5089	-8.9596
		(-2.4244)		(-2.8683)		
U.K.:	81-96	0.1176	0.4287	0.3282	0.2114	-7.9836
		(0.6374)		(2.7548)		
U.S.A.:	82-87	0.7719	0.1407	-0.0079	0.3676	-6.6585
		(12.2217)		(-0.0479)		
	88-96	0.2228	0.4319	0.5626	0.2678	
		(1.1495)		(4.6802)		

A striking feature of the results reported in Table 7 is that the point estimate of g_1 associated with the best-fitting interest-rate rule is negative for Canada, France, Italy, and the United Kingdom; however, positive values for $\hat{g}_1$ are excluded from the 95% confidence interval only for Italy. In Table 8, positive values for $\hat{g}_1$ are excluded from the 95% confidence intervals for Canada, France, and the United Kingdom.[22] In the literature, negative $\hat{g}_1$ is generally viewed as evidence that the monetary authority allowed the real interest rate to decrease during inflationary periods and, in so doing,

[22] There is some evidence that the sign of the inflation response coefficient $\hat{g}_1$ is quite sensitive to changes in the specification of the interest rate rule used to characterize monetary policy. Clarida, Galí, and Gertler (1998), for example, use a policy rule in which the interest rate responds to expected future inflation and obtain positive point estimates of g_1 for the countries in their sample.

TABLE 8

Generalized Taylor Rule Estimates

		$\hat{g}_1$	1.96 $\hat{\sigma}_{g_1}$	$\hat{g}_2$	1.96 $\hat{\sigma}_{g_2}$	$\hat{g}_{31}$	1.96 $\hat{\sigma}_{g_{31}}$	$\hat{g}_{32}$	1.96 $\hat{\sigma}_{g_{32}}$	SC
Canada:	83-90	-0.6879	0.5211	0.6609	0.2419	0.1659	0.0680			-9.3093
		(-2.9860)		(6.1788)		(5.8277)				
	91-96	-0.6879	0.5211	0.6609	0.2419	-0.0235	0.0254			
		(-2.9860)		(6.1788)		(-2.0893)				
France:	81-96	-0.3530	0.3515	0.5539	0.5156	0.3425	0.8457	0.0104	0.0767	-8.1109
		(-2.2104)		(2.3646)		(0.8913)		(0.2976)		
Germany:	80-95	-0.0145	0.4148	0.2061	0.1756	0.0339	0.2909	0.0202	0.0532	-8.7744
		(-0.0771)		(2.5635)		(0.2566)		(0.8351)		
Italy:	81-92	-0.3712	0.3735	-0.4565	0.6078	0.3978	0.7222			-8.9363
		(-2.2921)		(-1.7319)		(1.2704)				
U.K.:	81-96	-0.5339	0.5044	0.5656	0.3740	1.3105	1.0215	0.5319	0.4176	-8.3341
		(-2.3583)		(3.3670)		(2.8585)		(2.8379)		
U.S.A.:	82-87	0.2864	0.4147	0.6109	0.4686	0.9131	0.6469	0.1198	0.2803	-9.4842
		(1.5625)		(2.9448)		(2.7665)		(0.9664)		
	88-96	0.2864	0.4147	0.6109	0.4686	-0.2398	0.5329	0.1198	0.2803	
		(1.5625)		(2.9448)		(-1.0177)		(0.9664)		

destabilized the economy.[23] However, in the context of the model used here, $\hat{g}_1$ does not measure the response of the real interest rate to contemporaneously observed inflation. The real interest rate is usually defined as the difference between the nominal interest rate in a given period and *expected*, rather than contemporaneous, inflation. Because $\hat{g}_1$ measures the responsiveness of the nominal interest rate to contemporaneous inflation, rather than expected future inflation, the coefficient estimates I have obtained do not provide direct information about the relationship between the real interest rate and inflation. Taking expectations of (1) conditional on the information available in period t and substituting the result into (12) yields

$$i_t - \pi_{t+1|t} \ = \bar{K} + \ (1 + g_1 - \alpha_1)\pi_t \ + \ (g_2 - \alpha_2)y_t \tag{31}$$

It is evident from (31) that a positive relationship between the real interest rate and contemporaneous inflation only requires $g_1 > (\alpha_1 - 1)$ rather than $g_1 > 0$.[24] For Italy, the point estimate of g_1 reported in Tabe 7 satisfies this condition. Although the point estimates reported for Canada, France, and the UK in Table 8 do not satisfy the condition $g_1 > (\alpha_1 - 1)$, the 95% confidence intervals for $\hat{g}_1$ include values that do.

6.2 Determining the Efficiency of Interest-Rate Policy

The country-specific estimates of g_1 and g_2 can be used to obtain estimates of the relative weight γ. Point estimates of γ together with their 95% confidence intervals are given in Table 9. In each case, the $\hat{g}_1$ and $\hat{g}_2$ values used to construct $\hat{\gamma}$ were those from the best-fitting Taylor rule estimates from Table 7 (T7) or generalized Taylor rule estimates from Table 8 (T8), as indicated in column two.

[23]Taylor (1999b) has argued that negative values of $\hat{g}_1$ cause the aggregate demand function to be upward-sloping and are therefore destabilizing.

[24]Because Clarida, Galí, and Gertler (1998) estimate a policy rule in which the nominal interest rate is set in response to expected future inflation, rather than contemporaneous inflaton, their estimate of g_1 must be positive in order ensure that the real interest rate is increased in response to expected increases in inflation.

Table 9

Estimates of γ

	Source	$\hat{\gamma}$	95% CI for $\hat{\gamma}$	Efficient
Canada	T8	-0.9580	$[-1.5996, -0.3164]$	no
France	T7	-1.8125	$[-3.9134, 0.2884]$	yes
Germany	T7	5.4064	$[-41.314, 52.126]$	yes
Italy	T7	3.4109	$[-0.9945, 7.8163]$	yes
U.K.	T8	-1.0594	$[-1.9953, -0.1235]$	no
U.S.A.	T8	2.1330	$[-0.8974, 5.1634]$	yes

Comparing the interval estimates for $\hat{\gamma}$ with the efficient ranges given in Table 4 reveals that France, Germany, Italy, and the United States fulfill the necessary condition for efficiency given by (18) in that some or all of the estimated 95% interval for $\hat{\gamma}$ is contained in the 95% confidence interval for the efficient range. For Canada and the United Kingdom, the 95% confidence interval for $\hat{\gamma}$ lies below the confidence interval for the efficient range indicating that monetary policy in these two countries was insufficiently responsive to output variation over the sample period.

In order for a simple interest-rate rule to be judged efficient, the values of g_1 and g_2 that characterize it must satisfy the cross-coefficient constraint (18) and also fall within the theoretical efficient ranges given by (21) and (22). According to the results reported in Table 9, the monetary policies implemented in France, Germany, Italy, and the United States satisfy the cross-coefficient constraint and are therefore potentially efficient. To determine whether the second efficiency criterion is fulfilled, the actual values of g_1 and g_2 must be compared with estimates of the theoretical efficient ranges derived in Section 3. Point estimates of the efficient values of g_1, g_2, g_{31}, and g_{32}, together with their 95% confidence intervals are reported in Table 10.

A summary of the results obtained by comparing the characteristics of each ocuntry's estimated actual and efficient interest-rate rule is presented in Table 11. The

Table 10

Efficient Generalized Taylor Rule Coefficients

		g_1	95% CI for g_1	g_2	95% CI for g_2
Canada:	82-96	[0.0000, 2.5455]	[−2.1668, 7.2578]	[1.2837, 1.9627]	[0.0.7253, 2.7984]
France:	81-96	[0.0000, 7.8048]	[0.0000, 14.282]	[2.3057, 4.6392]	[1.9934, 6.2208]
Germany:	80-95	[0.0000, 2.0133]	[−0.5164, 4.5430]	[0.7375, 1.4196]	[0.4741, 1.9512]
Italy:	81-92	[0.0000, 1.8391]	[0.0000, 3.3448]	[1.6418, 3.3778]	[0.9905, 4.5872]
U.K.:	81-96	[0.0000, 8.4177]	[−0.1076, 16.943]	[4.5388, 7.8767]	[0.7965, 13.663]
U.S.A.:	82-96	[0.0000, 1.6484]	[−0.0767, 3.3735]	[0.5245, 1.1172]	[0.2042, 1.5973]
		g_{31}	95% CI for g_{31}	g_{32}	95% CI for g_{32}
Canada:	83-90	−0.1631	[−0.2285, −0.0977]		
France:	81-96	−2.1696	[−2.9459, −1.3933]	0.1330	[0.0437, 0.2223]
Germany:	80-95	0.3033	[0.0425, 0.5641]	−0.0780	[−0.1199, −0.0361]
Italy:	81-92	−1.4838	[−2.0168, −0.9508]		
U.K.:	81-96	−10.367	[−15.964, −4.7700]	−3.5116	[−6.6351, −0.3881]
U.S.A.:	82-96	−0.5802	[−0.9680, −0.1924]	0.4552	[0.3126, 0.5978]

Note: The 95% confidence intervals for the estimated efficient ranges of g_1 and g_2 were obtained as the union of the point estimate of each range and the 95% confidence intervals for the estimated lower (g_2) and upper (g_1 and g_2) bounds. Although theory requires efficient values of g_1 to be non-negative, the confidence interval for the efficient range of g_1 was not truncated at 0 when the 95% confidence interval around the estimated upper bound included negative numbers.

Table 11

Efficiency Checklist

			$\hat{g}_1$	$\hat{g}_2$	$\hat{g}_{31}$	$\hat{g}_{32}$	$\hat{\gamma}$
Canada:	82-90	T8	$\cap$	$\cap$	$> g_{31}$		$< \gamma$
	91-96	T8	$\cap$	$\cap$	$\cap$		$< \gamma$
France:	81-96	T7	$< g_1$	$< g_2$	$\cap$	$\neq 0$	$\cap$
Germany:	80-95	T7	$\cap$	$< g_2$	$\neq 0$	$\neq 0$	$\cap$
Italy:	81-92	T7	$\cap$	$\cap$	$\neq 0$		$\cap$
U.K.:	81-96	T8	$\cap$	$< g_2$	$> g_{31}$	$> g_{32}$	$< \gamma$
U.S.A.:	82-87	T8	$\cap$	$\cap$	$> g_{31}$	$\cap$	$\cap$
	88-96	T8	$\cap$	$\cap$	$\cap$	$\cap$	$\cap$

column headings $\hat{g}_1$, $\hat{g}_2$, $\hat{g}_{31}$, $\hat{g}_{32}$, and $\hat{\gamma}$ represent the parameter values associated with the policy rule actually implemented. The symbols below these column headings indicate how the paramenter estimates from the implemented rule compares with the estimated efficient values for each parameter. The type of rule implemented by each country is given in column two.

The efficiency of alternative monetary policies is usually evaluated by comparing the combinations of inflation and output variation associated with a given policy to an efficient policy frontier. This method of assessment, which was originally introduced by Taylor (1979), evaluates the efficiency of the monetary policy as a whole. In this study, I use an efficiency criterion that allows the individual components of the monetary policy employed to be evaluated so that the source(s) of inefficiency can be identified. For the purposes of this analysis, a parameter is considered to be efficient if the intersection between the 95% confidence interval for the efficient parameter range (for $\hat{g}_1$, $\hat{g}_2$, and $\hat{\gamma}$) or the efficient parameter value (for $\hat{g}_{31}$ and $\hat{g}_{32}$) and the 95% confidence interval for the parameter value associated with the implemented interest-rate rule is non-empty. Finding that the intersection between the two 95%

confidence intervals is non-empty indicates that the null hypothesis of parameter efficiency cannot be rejected at the 5% level of significance. In the table, non-empty intersections are denoted by the symbol $\cap$.

In those cases where there is no intersection between the two 95% confidence intervals, there are two possible sources of inefficiency. One possibility is that the policy authority may be using a Taylor rule or a generalized Taylor rule with $g_{3i} = 0$ for all i when theory indicates that a generalized Taylor rule with $g_{3i} \neq 0$ for some i is needed for efficiency. This circumstance is denoted by the symbol $\neq 0$, meaning that the efficient value of the parameter in question is significantly different from zero. The second possibility is that the implemented interest-rate rule is of the correct form, but violates one or more of the efficiency criteria derived in Section 3. In this case the nature of the inefficiency is described in terms of the relative positions of the two confidence intervals. For example, the entry $< g_1$ under the column heading $\hat{g}_1$ for France indicates that the 95% confidence interval for France's inflation-response coefficient lies everywhere below the 95% confidence interval for that parameter's efficient range. Consequently, the null hypothesis that the French response to inflation variation is efficient must be rejected at the 5% level of significance.

It is evident from Table 11 that there is only one country whose interest-rate policy satisfies the efficiency criteria. In particular, the results suggest that efficient interest-rate policies were implemented by the U.S. Federal Reserve over the period 1988-96. The source of inefficiency prior to that period appears to have been a tendency on the part of the Federal Reserve to overreact to German inflation. For the other countries considered, there are a variety of sources of inefficiency. According to Table 11, the policy authorities in Canada, France, Germany, and the U.K. appear to have responded too little to the output gap when setting their interest rate policies. The results also suggest that a smaller response to changes in the US/Canada real exchange rate would have improved the efficiency of Canadian interest rate policy in the 1982-90 period, whereas a stronger response to changes in the value of the franc relative to the US dollar would have been of benefit to France. For Germany, a stronger response to changes in US output and the value of the Dmark relative to the US dollar would have improved the efficiency of interest rate policy. Greater efficiency could have been achieved in Italy and the UK if German inflation had been given more weight in the interest rate rule, and the UK would also have benefitted from an interest rate policy that was responsive to changes in German output.

According to the criteria employed to evaluate the efficiency of interest rate policies in this study, only the United States appears to have used an efficient interest rate rule. These results support the commonly held view that the Federal Reserve's policies were very successful in reducing inflation during the first half of the 1980s and then, subsequently, in maintaining both strong economic growth and price stability. However, there are a number of reasons that the analysis undertaken here may yield results that reflect more favourably on the Federal Reserve's policies than on the policies implemented by the other central banks in the sample. First, the absence of forward-looking variables in (1) and (2), results in benchmark rules that contain no forward-looking terms. However, Clarida, Galí, and Gertler (1998) found that forward-looking reaction functions dominate backward-looking response functions as descriptions of the interest rate policies implemented in France, Germany, Italy, and the United Kingdom, so the results obtained for these countries may be sensitive to the exclusion of forward-looking variables.[25] Second, the loss function I have used may better describe the Fed's objectives than those of other monetary authorities during the period under study.

6.4 An Exercise in Revealed Preference

In the model employed in this article, a policy authority's choice of interest-rate policy depends on the behavioural parameters δ and λ. When the interest-rate policy

[25] Clarida, Galí, and Gertler (1998) also found forward-looking rules to be better representations of the Fed's policy response. However, Levin, Williams, and Wieland (1999) found that forward-looking variables do not enhance the robustness of simple interest-rate rules for the US, so the results I have obtained for the US are less likely to be sensitive to the exclusion of forward-looking variables than those obtained for the European countries in this study.

Table 12

Behavioural Parameters for the U.S.

	$\delta \to 0$	$\delta = 0.2$	$\delta = 0.4$	$\delta = 0.6$	$\delta = 0.8$	$\delta = 0.9$	$\delta = 1$
λ	0	0.03	0.06	0.09	0.11	0.14	0.16

implemented is efficient, there is a well-specified functional relationship between the relative weight γ and the two behavioural parameters. Consequently, for countries implementing efficient rules, the estimated values of γ can be used to determine the combinations of δ and λ that are consistent with the observed interest-rate policy. Given that the U.S. policy rule has been found to be efficient over the period 1988-96, the estimation results can be used to identify the values of δ and λ implied by the Federal Reserve's interest-rate policy.

Substituting (9) into (18) and rearranging, results in

$$\delta\alpha_1^4\alpha_2\gamma^2 + \alpha_1^2\left[\Omega - 2\alpha_2^2\delta(\alpha_1+\beta_1)\right]\gamma + (\alpha_1+\beta_1)^2\delta\alpha_2^3 - \beta_1^2\lambda\alpha_2 - \alpha_2(\alpha_1+\beta_1)\Omega = 0 \tag{32}$$

where $\Omega = \beta_1\left[\delta\alpha_2^2 - \lambda(1-\delta\alpha_1^2)\right]$. Solving (32) for λ as a function of δ and substituting the estimated values of α_1, α_2, β_1, and γ for the United States into this expression yields

$$\lambda = \frac{0.0265\delta}{0.2036 - 0.0428\delta}. \tag{33}$$

The continuum of combinations of λ and δ that is consistent with U.S. interest-rate policy over the period 1988-96 is obtained by allowing δ to vary from 0 to 1 in (33). A subset of these combinations is reported in Table 12.

It is evident from the combinations of δ and λ given in Table 12 that the weight assigned to output variation in the Federal Reserve's loss function is quite low, reaching a maximum of $\lambda = 0.16$ when $\delta = 1$. The fact that all of the $[\delta, \lambda]$ combinations described by (33) are consistent with $\hat{\gamma} = 2.133$ indicates that, on their own, estimated Taylor rule weights provide only limited information about the attitude of the policy authority towards inflation and output stabilization.

7. Conclusion

Uncertainty about the true structure of the macroeconomy and the nature of the monetary transmission mechanism has led to the search for monetary policy rules that are robust to changes in model specification. There is growing evidence that simple rules like the one proposed by Taylor (1993) characterize monetary policies that achieve good results across a variety of model specifications. However, not all simple rules are equally good benchmarks against which to judge the effectiveness of monetary policy. Monetary authorities are therefore faced with the challenge of identifying the most appropriate benchmark rule from among the set of feasible alternatives.

In this article I have used the efficient Taylor rule generated by a standard, generic economic model to derive efficiency conditions that can be used to identify efficient classes of benchmark rules and to evaluate the monetary policies that were employed in six countries from the early 1980s onwards. Efficient ranges for the individual Taylor rule coefficients and for the cross-coefficient constraint on inflation and output variation were estimated for Canada, France, Germany, Italy, the United Kingdom, and the United States. A number of conclusions follow from the empirical analysis. Pure price rules are in the feasible set of efficient rules only for Canada and Germany. Nominal interest rate rules, on the other hand, may be efficient for all six countries. Both of these classes of rules are efficient only for very special combinations of the policy authority's behavioural parameters. The empirical results also indicate that of the six countries included in this study, the United States was the only country to have implemented an efficient interest-rate policy.

Although the Federal Reserve is generally acknowledged to have managed the US economy very well since the mid 1980s, it is somewhat surprising that the Fed should be the only central bank to meet the efficiency criteria used in this study. A priori, one would not expect that the Bundesbank, with its high degree of credibility and policy independence, systematically implemented suboptimal policies. In studies of the sort conducted here, there is always the possibility that the conclusions are sensitive to the structure of the model. There are a number of reasons to view the results obtained here with caution. First, the fact that several of the coefficient estimates for Germany and Italy failed the invariance test indicates that the benchmark ranges for Taylor rule coefficients for these two countries, as well as the conclusions reached on the basis of these estimates, are unreliable. Second, although there is evidence that expectations about future inflation may not be important in determining the US inflation rate, it is not clear that this is also true for the other countries in this study. Because the benchmark Taylor rule is sensitive to the underlying model structure, an efficient rule based on a traditional Phillips curve may not be the appropriate benchmark for some of the countries in the sample. Finally, during the sample period, European monetary authorities were occupied with the transition to a single currency. European monetary policy was constrained by repeated speculative attacks against individual currencies and also by some of the provisions of the Maastricht treaty. Although quadratic loss functions are the most common way of representing the objectives of policy authorities, the results from this study suggest that loss functions of this form may not describe the objectives of European monetary authorities very well. On the other hand, the results obtained here provide some indirect support for using a quadratic loss function to describe the Federal Reserve's policy objectives.

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Appendix 1. Determination of k_1 and k_2

Solutions for k_1 and k_2 can be obtained by applying the envelope theorem to (5) and (6). Using (6) to replace $V(\pi_{t+2|t+1})$ in (5) and taking the derivative of the expression in braces with respect to $\pi_{t+1|t}$ yields

$$V_\pi(\pi_{t+1|t}) \;=\; (\pi_{t+1|t} - \pi^*) + \delta\alpha_1\{k_1 + k_2(\pi_{t+2|t} - \pi^*)\}. \tag{A.1}$$

Using (1) and (9), $\pi_{t+2|t}$ can be expressed as

$$\pi_{t+2|t} \;=\; \frac{-\delta\alpha_2^2 k_1}{\lambda + \delta\alpha_2^2 k_2} \;+\; \frac{\delta\alpha_2^2 k_2 \pi^*}{\lambda + \delta\alpha_2^2 k_2} \;+\; \frac{\lambda\alpha_1}{\lambda + \delta\alpha_2^2 k_2}\,\pi_{t+1|t}. \tag{A.2}$$

Substituting (A.2) into (A.1) results in

$$V_\pi(\pi_{t+1|t}) \;=\; \left[1 \;+\; \frac{\delta\alpha_1^2\lambda k_2}{\lambda + \delta\alpha_2^2 k_2}\right](\pi_{t+1|t} - \pi^*) \;+\; \frac{(\alpha_1 - 1)\delta\lambda k_2}{\lambda + \delta\alpha_2^2 k_2}\,\pi^* \;+\; \frac{\lambda\delta\alpha_1(k_1 + \alpha_0 k_2)}{\lambda + \delta\alpha_2^2 k_2}. \tag{A.3}$$

Differentiating the conjectured solution for $V(\pi_{t+1|t})$, given by (6), with respect to $\pi_{t+1|t}$ yields

$$V_\pi(\pi_{t+1|t}) \;=\; k_1 + k_2(\pi_{t+1|t} - \pi^*). \tag{A.4}$$

Using (A.3) to identify the coefficients in (A.4) produces

$$k_1 \;=\; \frac{-\lambda\delta\alpha_1 k_2(1-\alpha_1)\pi^*}{\lambda(1-\delta\alpha_1) + \delta\alpha_2^2 k_2} \tag{A.5}$$

$$k_2 \;=\; 1 \;+\; \frac{\delta\alpha_1^2\lambda k_2}{\lambda + \delta\alpha_2^2 k_2} \tag{A.6}$$

It is evident from (A.5) and (A.6) that solving for k_2 identifies both k_1 and k_2. Rearranging (A.6) yields the quadratic polynomial

$$\delta\alpha_2^2 k_2^2 \;+\; [\lambda - (\lambda\alpha_1^2 + \alpha_2^2)\delta]k_2 \;-\; \lambda \;=\; 0. \tag{A.7}$$

Solving (A.7) for k_2 yields

$$k_2 \;=\; \frac{[\delta\alpha_2^2 - \lambda(1-\delta\alpha_1^2)] \;+\; \sqrt{[\delta\alpha_2^2 - \lambda(1-\delta\alpha_1^2)]^2 \;+\; 4\delta\alpha_2^2\lambda}}{2\delta\alpha_2^2}. \tag{A.8}$$

Only the positive root of (A.7) is a solution for k_2 because, from (A.6), k_2 must equal 1 for all non-zero values of δ and α_2 when $\lambda = 0$; this condition is not satisfied by the negative root.

Appendix 2. Limiting values of k_1 and k_2

The limiting value of k_1 as λ approaches ∞ can be established quite easily using (8) once the limiting value of k_2 is known. However, finding the limit of k_2 as λ increases without bound is not as straightforward. The derivation below outlines the method I used to identify the limiting values of k_1 and k_2.

Redefining variables in (9) to simplify the expression for k_2 results in

$$k_2 \;=\; \frac{A - B\lambda \;+\; \sqrt{(A-B\lambda)^2 + 4B\lambda}}{2A} \tag{A.9}$$

where $A = \delta\alpha_2^2$ and $B = (1-\delta\alpha_1^2)$. For $\lambda > A/B$, (A.9) can be expressed as

$$k_2 \;=\; \frac{\sqrt{(A-B\lambda)^2 + 4B\lambda} - \sqrt{(A-B\lambda)^2}}{2A} \tag{A.10}$$

Expanding the expressions under each of the square root signs in (A.10) yields

$$k_2 \;=\; \frac{\sqrt{A^2 + C\lambda + B^2\lambda^2} - \sqrt{A^2 + D\lambda + B^2\lambda^2}}{2A} \tag{A.11}$$

where $C = -2B(A-2)$ and $D = -2AB$. The limit of k_2 as λ approaches infinity can be expressed as

$$\lim_{\lambda\to\infty} k_2 = \frac{1}{2A}\left[\lim_{\lambda\to\infty} f(\lambda)\right] \tag{A.12}$$

where $f(\lambda) = \sqrt{A^2 + C\lambda + B^2\lambda^2} - \sqrt{A^2 + D\lambda + B^2\lambda^2}$. It follows from the definition of $f(\lambda)$ that

$$f(\lambda)\left[\sqrt{A^2 + C\lambda + B^2\lambda^2} + \sqrt{A^2 + D\lambda + B^2\lambda^2}\,\right] \;=\; (C-D)\lambda. \tag{A.13}$$

Using (A.13), (A.12) can be written

$$\lim_{\lambda\to\infty} k_2 = \frac{(C-D)/2A}{\lim_{\lambda\to\infty}\left[\frac{2B\lambda+C}{2\sqrt{A^2+C\lambda+B^2\lambda^2}} + \frac{2B\lambda+D}{2\sqrt{A^2+D\lambda+B^2\lambda^2}}\right]}. \tag{A.14}$$

Now, C and D are constants so the denominator on the right-hand-side of (A.14) is given by

$$\lim_{\lambda\to\infty} \frac{2B\lambda}{\sqrt{A^2+C\lambda+B^2\lambda^2}} = \lim_{\lambda\to\infty} \frac{2\sqrt{B^2\lambda^2}}{\sqrt{A^2+C\lambda+B^2\lambda^2}} = \lim_{\lambda\to\infty} \frac{2}{\sqrt{\frac{A^2}{B^2\lambda^2}+\frac{C}{B^2\lambda}+1}} = 2. \tag{A.15}$$

It follows from (A.15) and the definitions of A, B, C, and D, that

$$\lim_{\lambda\to\infty} k_2 = \frac{(C-D)}{4A} = \frac{(1-\delta\alpha_1^2)}{\delta\alpha_2^2}. \tag{A.16}$$

The limiting value of k_1 can now be obtained quite easily from (8). Minor manipulation of (8) yields

$$k_1 = \frac{\delta(1-\alpha_1)\pi^*}{\frac{(1-\delta\alpha_1)}{k_2} + \frac{\delta\alpha_2^2}{\lambda}}. \tag{A.17}$$

Using (A.16), it follows directly from (A.17) that

$$\lim_{\lambda\to\infty} k_1 = \frac{\delta(1-\alpha_1)\pi^* \lim_{\lambda\to\infty} k_2}{(1-\delta\alpha_1)} = \frac{(1-\delta\alpha_1^2)(1-\alpha_1)\pi^*}{\alpha_2^2(1-\delta\alpha_1)}. \tag{A.18}$$

It is evident from (A.16) and (A.18) that for given δ, k_1 and k_2 both converge to finite values as λ increases without bound.

Appendix 3. Details of Empirical Procedures

A3.1 Estimation of Equations (1) and (2)

Equations (1) and (2) were estimated for each country in the sample using annual data obtained from the International Monetary Fund's International Financial Statistics (IFS) on CD-ROM. For each country, the output gap y_t was constructed as the deviation of the natural log of real GDP (IFS line 99b.r) from its linear trend value in period t. For all countries except Germany, the inflation rate π_t is measured as the change in the natural log of the GDP deflator from period $t-1$ to period t. National GDP deflators were constructed as the ratio of nominal GDP (IFS line 99b.c) to real GDP as reported in the IFS statistics. The Consumer Price Index (IFS line 64) is used to measure German inflation because real GDP figures are not available for Germany prior to 1979. The interest rate employed in the estimations is a short-term market rate (IFS line 60bs for France and 60b for all other countries). Two types of dummy variables are employed, step dummies and pulse dummies. Step dummies are identified as SD and pulse dummies by PD, the dates following these letters indicate the years for which the value of the dummy is set equal to 1. For example, SD7579 indicates that the step dummy has a value of 1 from 1975 to 1979, inclusive, and a value of zero in evey other year. Pulse dummies are used to deal with outliers and are set equal to 1 only in the year the outlier occurred. indicated All other, country-specific variables used in estimating equation (2) are defined below, immediately following the equation in which they appear.

Estimation results for (1) and (2) are reported below on a country-by-country basis. In each case, variables without a superscript are domestic variables. Foreign variables are identified by a superscript composed of the first three letters of the relevant country's name. In the case of the United States and the United Kingdom, the superscript is composed of the intials US and UK, respectively. The estimation period for each country is given in parentheses beside the country name. The coefficient of determination R^2 is given immediately following each equation. The F-statistics associated with Lagrange multiplier tests for first and second-order serial correlation are also reported. Neither the first-order statistic, LM(1), nor the second-order statistic, LM(2), is significant at the 10% level for any country.

1. *Canada (1975-1996)*

$$\pi_t = \underset{(3.5564)}{0.013606} + \underset{(5.3572)}{0.030769}\,\text{SD7981} + \underset{(6.4581)}{0.496440}\,\pi_{t-1} + \underset{(3.1480)}{0.132408}\,y_{t-1}$$

$$R^2 = 0.941842 \qquad \text{LM(1)} = 0.0019 \qquad \text{LM(2)} = 0.0319$$

$$\begin{aligned} y_t = & \underset{(2.0241)}{0.012089} + \underset{(18.8376)}{0.938622}\,y_{t-1} + \underset{(5.7809)}{1.817638}(\text{SD8182})(i-\pi)_{t-1} \\ & - \underset{(5.2084)}{0.731140}(i-\pi)_{t-1} - \underset{(5.3949)}{0.103863}\text{SD8182} - \underset{(4.3245)}{0.119246}\,q^{us}_{t-1}\text{SD8390} \end{aligned}$$

$$R^2 = 0.978682 \qquad \text{LM(1)} = 0.0040 \qquad \text{LM(2)} = 0.5422$$

The variable q^{us} is the real Can/US exchange rate calculated using the Canadian and US GDP deflators and the average bilateral Can/US exchange rate (IFS line rf).

2. *France (1975-1994)*

$$\pi_t = -\underset{(1.8181)}{0.008442} + \underset{(2.4450)}{0.014131}\text{SD7880} + \underset{(15.6500)}{0.980997}\pi_{t-1} + \underset{(2.5862)}{0.293297}y_{t-1}$$

$$R^2 = 0.956566 \qquad \text{LM(1)} = 1.1982 \qquad \text{LM(2)} = 0.7075$$

$$\begin{aligned} y_t = & \underset{(6.3031)}{0.043040} + \underset{(7.917920)}{0.969312}y_{t-1} - \underset{(2.3338)}{0.805115}(\text{SD7580})y_{t-1} \\ & - \underset{(5.5516)}{0.420430}(i-\pi)_{t-1} - \underset{(5.8293)}{0.912145}\pi^{ger}_{t-2} + \underset{(2.8755)}{0.055866}\Delta e^{us}_{t-1} \end{aligned}$$

$$R^2 = 0.892232 \qquad \text{LM(1)} = 0.3981 \qquad \text{LM(2)} = 2.3183$$

The variable Δe^{us} is the change in the natural log of the average nominal France/US exchange rate (IFS line rf).

3. Germany (1975-1995)

$$\pi_t = \underset{(2.9272)}{0.0111584} + \underset{(5.3652)}{0.584158\pi_{t-1}} + \underset{(4.3382)}{0.197923 y_{t-1}}$$

$$R^2 = 0.795353 \qquad \text{LM}(1) = 2.8107 \qquad \text{LM}(2) = 1.3212$$

$$\begin{aligned} y_t = {} & \underset{(1.8456)}{0.009774} + \underset{(11.8118)}{0.631462 y_{t-1}} - \underset{(6.3956)}{0.856270(i-\pi)_{t-1}} \\ & + \underset{(2.7056)}{0.259669\Delta Y^{us}_{t-2}} - \underset{(3.7586)}{0.066758\Delta e^{us}_{t-1}} + \underset{(8.8869)}{0.055489\text{SD9092}} \end{aligned}$$

$$R^2 = 0.969949 \qquad \text{LM}(1) = 0.0071 \qquad \text{LM}(2) = 0.0166$$

The variables ΔY^{us} and Δe^{us} denote the change in the natural logarithm of US real GDP and the change in the natural log of the average Germany/US nominal exchange rate (IFS line rf), respectively.

4. Italy (1973-1992)

$$\begin{aligned} \pi_t = {} & \underset{(1.8290)}{0.016422} + \underset{(4.5825)}{0.161620\text{SD7380}} + \underset{(8.2714)}{0.700819\pi_{t-1}} \\ & + \underset{(3.6308)}{0.661509 y_{t-1}} - \underset{(3.7122)}{0.873030(\text{SD7380})\pi_{t-1}} - \underset{(3.7879)}{1.236094(\text{SD7379}) y_{t-1}} \end{aligned}$$

$$R^2 = 0.963075 \qquad \text{LM}(1) = 1.0512 \qquad \text{LM}(2) = 0.8403$$

$$\begin{aligned} y_t = {} & \underset{(4.9140)}{0.031946} + \underset{(7.1038)}{0.662790 y_{t-1}} - \underset{(5.9444)}{0.403751(i-\pi)_{t-1}} \\ & - \underset{(3.9880)}{0.0598952\pi^{ger}_{t-1}} - \underset{(5.9666)}{0.057373\text{PD75}} \end{aligned}$$

$$R^2 = 0.885715 \qquad \text{LM}(1) = 0.0891 \qquad \text{LM}(2) = 2.2659$$

5. United Kingdom (1975-1996)

$$\pi_t = \underset{(5.9378)}{0.022138} + \underset{(11.9527)}{0.490036\pi_{t-1}} + \underset{(2.7246)}{0.0194312 y_{t-1}} + \underset{(8.9847)}{2.541687(\text{SD7580}) y_{t-1}}$$

$$R^2 = 0.960926 \qquad \text{LM}(1) = 0.1893 \qquad \text{LM}(2) = 0.4003$$

$$y_t = \underset{(5.7983)}{0.061433} + \underset{(5.6662)}{0.666267 y_{t-1}} - \underset{(3.0746)}{0.146825(i-\pi)_{t-1}} - \underset{(6.1123)}{1.521862\pi^{ger}_{t-1}} - \underset{(3.2278)}{0.515498\Delta Y^{ger}_{t-1}}$$

$$R^2 = 0.884930 \qquad \text{LM}(1) = 1.2712 \qquad \text{LM}(2) = 2.6031$$

6. United States (1975-1996)

$$\pi_t = \underset{(4.2851)}{0.016689} + \underset{(4.8616)}{0.024307\text{SD7781}} + \underset{(6.3355)}{0.506153\pi_{t-1}} + \underset{(2.3206)}{0.181972 y_{t-1}} - \underset{(2.1264)}{0.015952\text{PD96}}$$

$$R^2 = 0.929046 \qquad \text{LM}(1) = 1.0396 \qquad \text{LM}(2) = 0.8335$$

$$y_t = \underset{(6.2078)}{0.0404026} + \underset{(3.8708)}{0.447961 y_{t-1}} - \underset{(6.6169)}{0.854082(i-\pi)_{t-1}} - \underset{(3.0960)}{0.483859\pi^{ger}_{t-1}} - \underset{(4.6114)}{0.388817 y^{ger}_{t-1}} + \underset{(3.2164)}{0.718136(\text{SD7581})(i-\pi)_{t-1}}$$

$$R^2 = 0.875167 \qquad \text{LM}(1) = 0.0010 \qquad \text{LM}(2) = 2.3053$$

A3.2 Invariance Tests

The invariance tests conducted using the procedure described in Section 4.2 of the main text are summarized in Tables A3.1 and A3.2. The variables used to model the marginal processes in (3) and (4) are given in Table A3.1. Note that marginal processes are specified only once. Variables used as regressors for more than one country are specified in the section pertaining to their country of origin if they are used for domestic estimation or, if used only for foreign countries, in the section pertaining to the first country for which the variable is used.

The results of the invariance tests for the parameters α_1, α_2, β_1, β_2, β_{31}, and β_{32} are presented in Table A3.2. The calculated values of the test statistic and the distributions of the statistic under the null hypothesis are reported. An asterisk appended to the test statistic in Table A3.2 indicates rejection of the null hypothesis that the estimated parameter value is invariant at the 5% level of significance.

A3.3 Unit Root Tests

The results of the unit root tests undertaken are reported in Table A3.3. Augmented Dickey-Fuller tests were used for variables which did not contain any apparent structural breaks. F-tests were used to determine the appropriate form of the test equation. In those cases where visual inspection of the data suggested the presence of structural change, Perron's (1989) procedure was employed. In particular, Perron's Model A was used to allow for a shift in either the unit root process or the time trend. Perron has shown that the critical values of the test statistic depend on the time period in which the structural break occurs. The year of the break and the proportion λ of the total observations occurring prior to the break are given in the third column of Table A3.3. The absence of an entry in this column indicates that there was no apparent break in the data over the sample period.

The test statistics obtained on the basis of Augmented Dickey-Fuller tests and Perron's test procedure are given in the last column of the table under the heading ADF/ADFP. It is evident from the reported results that all of the variables employed in estimating (3) and (4) are I(0) at a level of significance of at least 10%. In Table A3.3, significance of the test statistic at the 1% and 5% level is denoted by ** and *, respectively. Where there are no asterisks, the significance level of the test statistic is 10%. The presence of a deterministic time trend was rejected at the 5% level of significance for all variables.

Table A3.1

Marginal Processes

		Regressors[a]	R^2
Canada:	π_{t-1}	π_{t-2}, SD7981, SD8689YT_{t-1}	0.929097
	y_{t-1}	y_{t-2}, SD8292, SD8292YT_{t-1}, SD8292YT^3_{t-1}	0.942910
	$(i-\pi)_{t-1}$	$(i-\pi)_{t-2}$, $(i-\pi)^{us}_{t-1}$, SD8182$(i-\pi)^{us}_{t-1}$, PD80	0.770946
	q^{us}_{t-1}	q^{us}_{t-2}, Δq^{us}_{t-2}, π^{us}_{t-3}, SD9195	0.859076
France:	π_{t-1}	π_{t-2}, y_{t-2}, SD8292π_{t-2}, PD94	0.954917
	y_{t-1}	y_{t-2}, SD7579YT_{t-1}, SD8790YT_{t-1}, SD9294YT_{t-1}	0.829176
	$(i-\pi)_{t-1}$	$(i-\pi)_{t-2}$, SD7881(t-1), SD7881	0.912695
	Δe^{us}_{t-1}	Δe^{us}_{t-2}, $\Delta e^{ger/us}_{t-1}$	0.874298
Germany:	π_{t-1}	π_{t-2}, π_{t-3}, SD7981, PD86	0.922911
	y_{t-1}	y_{t-2}, SD7579, SD8285, SD9092	0.941247
	$(i-\pi)_{t-1}$	$(i-\pi)_{t-2}$, i^{us}_{t-1}, i^{us}_{t-2}, PD86, SD7580(t-1), SD9092	0.919230
	ΔY^{us}_{t-2}	i^{us}_{t-3}, YT^{us}_{t-2}, PD82, PD91	0.762180
	Δe^{us}_{t-1}	SD7881, SD7881(t-1), PD86	0.613645
Italy:	π_{t-1}	π_{t-2}, SD7380π_{t-2}, SD7380π^{ger}_{t-2}	0.907503
	y_{t-1}	y_{t-2}, $(i-\pi)_{t-1}$, SD7980, SD8890	0.944798
	$(i-\pi)_{t-1}$	$(i-\pi)_{t-2}$, SD8092π_{t-2}, PD92, PD75	0.943125
U.K.:	π_{t-1}	π_{t-2}, PD76, SD7980	0.906777
	y_{t-1}	y_{t-2}, SD8188, SD8188YT_{t-1}, SD9192	0.880105
	$(i-\pi)_{t-1}$	$(i-\pi)_{t-2}$, PD76, SD8090	0.919834
	ΔY^{ger}_{t-1}	SD8591ΔY^{ger}_{t-2}, SD7576(t-1), SD7576, SD8182, PD93	0.771210
U.S.A.:	π_{t-1}	π_{t-2}, π^{jap}_{t-2}, SD7781, SD8791	0.895109
	y_{t-1}	y_{t-2}, y^{jap}_{t-3}, Δe^{jap}_{t-2}, PD82	0.884877
	$(i-\pi)_{t-1}$	$(i-\pi)_{t-2}$, g^{ger}_{t-2}, SD7581, SD7581(t-1)	0.870782

[a]The variable YT_{t-1} is the trend value of the natural logarithm of real GDP at time $t-1$ and $e^{ger/us}_{t-1}$ is the natural logarithm of the DMark cost of one US dollar at time $t-1$. All other variables in the table are as defined in Section A3.1.

Table A3.2

Results of Invariance Tests

	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_{31}$	$\hat{\beta}_{32}$
Canada	0.6065	0.6585	2.2230	2.6700	0.9197	
	F(2,15)	F(4,13)	F(4,11)	F(3,11)	F(4,9)	
France	0.7066	0.5033	2.6200	1.4300	0.7013	1.7879
	F(4,10)	F(4,10)	F(4,7)	F(3,9)	F(4,8)	F(2,9)
Germany	8.1250*	1.7541	5.2069*	2.3687	2.3423	10.1126*
	F(4,13)	F(4,12)	F(2,10)	F(5,8)	F(4,8)	F(4,7)
Italy	4.2509*	1.2995	7.9041*	0.5894	1.4539	
	F(4,9)	F(4,8)	F(4,9)	F(3,11)	F(4,10)	
U.K.	1.3805	1.1566	0.7504	0.6986	1.7876	1.2447
	F(2,15)	F(4,12)	F(4,11)	F(2,14)	F(4,10)	F(3,13)
U.S.A.	0.6885	0.2189	2.2571	2.3773	0.5222	1.5000
	F(3,13)	F(4,10)	F(4,9)	F(4,10)	F(4,11)	F(4,10)

TABLE A3.3

Unit Root Tests

	var.	lags	break/λ	ADF/ADFP
Canada	π	0		$\tau_\mu = -2.6352^*$
	i	1	$1992/\lambda = 0.9$	$\tau_\lambda = -3.8287^*$
	q^{us}	1		$\tau_\mu = -3.7908^*$
France	π	0		$\tau = -1.7342$
	$i - \pi$	0	$1989/\lambda = 0.27$	$\tau_\lambda =$ -3.6447
	Δe^{us}	0		$\tau = -3.0588^{**}$
Germany	π	0		$\tau_\mu = -2.7336$
	i	0		$\tau_\mu = -4.3628^{**}$
	ΔY^{us}	0		$\tau_\mu = -3.4638^*$
	Δe^{us}	0		$\tau = -3.0048^{**}$
Italy	π	0	$1983/\lambda = 0.55$	$\tau_\lambda = -5.8452^{**}$
	i	0		$\tau_\mu = -2.8172$
U.K.	π	0	$1980/\lambda = 0.27$	$\tau_\lambda = -4.3998^{**}$
	i	1	$1979/\lambda = 0.23$	$\tau_\lambda = -4.2279^*$
	ΔY^{ger}	0		$\tau = -2.0055^*$
U.S.A.	π	0	$1981/\lambda = 0.31$	$\tau_\lambda = -3.7160$
	i	1		$\tau_\mu = -2.9393^\dagger$
	π^{jap}	0		$\tau_\mu = -8.4581^{**}$
	Δe^{us}	0		$\tau = -3.1183^{**}$

[†]In order to achieve this result, the beginning of the sample period was expanded to 1960. All other results reported in the table correspond to the estimation periods for each country as noted in Section A3.1.