

THREE ESSAYS ON MULTI-ROUND PROCUREMENT AUCTIONS

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Dissertation

Submitted to the Faculty of the
Graduate School of Vanderbilt University
in partial fulfillment of the requirements

for the degree of

DOCTOR OF PHILOSOPHY

in

Economics

December, 2006

Nashville, Tennessee

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To my mother Heying Lu and my father Yangquan Ji

and

To my wife Fei Liu

AKNOWLEDGEMENT

I am very grateful to my advisor, Professor Tong Li. He spent a great deal of time on guiding my work. He not only inspired economic insights, but also offered technical help. In addition, he helped me transfer from Indiana University to Vanderbilt University to continue my dissertation research, and supported my research in summer 2005 financially. Without his guidance and generous help, the completion of my dissertation would not have been possible. I develop my dissertation from a joint work with Professor Li. I enjoyed working with him.

I would like to thank my dissertation committee: Professors Rick Bond, Andrew Daughety, Yanqin Fan and Jennifer Reinganum. They all read each chapter of my dissertation very carefully and provided a lot of useful suggestions.

I would like to thank M. Beuchel, G. Curson, R. Emmert, M. Jenkins, M. Omran and A. Sullivan at Indiana Department of Transportation for providing the data analyzed in the dissertation and answering my questions.

Finally, I would like to thank my dearest wife, Fei Liu, for always having supported me in my life.

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CHAPTER I

INTRODUCTION

1 Introduction

My dissertation contributes to auction studies in the field of Industrial Organization. An auction is a selling mechanism. To a seller, the motivation for using an auction as a selling mechanism is that bidders have one-sided private information. Because of the asymmetric information and the induced uncertainty, auctions help sellers to find the buyers who value their goods the most. From a social planner's point of view, auctions also can help allocate resources efficiently and achieve the social optimum.

In practice, the implication of research on auctions is three-fold. First, plenty of goods and services are traded by auctions in the real world nowadays. For example, the U.S. traded volume through auctions will reach \$660 billion according to the International Data Corporation 2002 report. Here are a few categories from an economist's point of view. (1) Consumer goods such as art, wine, fish and flower are often traded through auctions. (2) Public goods and utilities such as highway projects, electricity and oil drilling projects are often auctioned as contracts by the government. (3) Financial assets such as stocks, bonds and securities are also good examples. (4) Goods sold by online auctions are even more diverse. For example, you can bid for Staples coupons on eBay.com. Studies of auctions can help evaluate mechanisms in various auctions and improve the efficiency of the auctions.

Second, auctions offer a testing ground for economic theory, for example, game

theory with incomplete information. Economic theories are often much more developed than empirics. One reason is lack of data. The other reason is the intensive computational burden incurred when we estimate structural parameters. On one hand, various auctions in practice provide economists easily accessible datasets. On the other hand, the structural econometric approach, which is an important tool for my dissertation research and which I will elaborate on later, helps us establish a link between game-theoretic models and empirical analyses.

Third, the Internet greatly expands the auction market and accelerates its development. Demand for professional auction design is higher and higher. A recent example is the design for the auctions of the 3G mobile phone licenses in UK in 2000. The auctions of mobile phone licenses across the world are only the most famous new auction markets. This presents another reason for economists to develop theories and analyze data to guide auction practice.

My dissertation is motivated by an interesting feature that I observed from the procurement auctions organized by the Indiana Department of Transportation (INDOT): many of these auctions are held with multiple rounds. This feature is attributed to the use of secret reserve prices in these auctions. Prior research has indicated that auctions with reserve prices sometimes lead to no transaction if no bidder can propose a price better than the reserve price.¹ However, there are still chances of trade if bidders' values for the unsold objects change. Thus the seller can continue auctioning the unsold objects from the previous auctions. Previous research, however, has not paid much attention to this feature in auctions. My dissertation offers thorough theoretical and empirical analyses on

¹See, for example, Elyakime, Laffont, Loisel and Vuong (1997), Bajari and Hortasu (2003), and Li and Perrigne (2003).

multi-round procurement auctions with reserve prices.

According to the 1992 US Census of Construction Industries a total of \$35.3 billion was spent during 1992 on highway and street construction activities. Auctions are adopted by departments of transportation in many states to sell construction contracts to firm contractors. However, auction mechanisms are heterogeneous across states. For example, the INDOT adopts a secret reserve price and announces it after the contract is successfully awarded. The California Department of Transportation adopts a secret reserve price, but never announces it. The Texas Department of Transportation uses an announced reserve price, but the reserve price is not binding. Heterogeneous policies motivate my research into government practice to help evaluate and improve the procurement auction process.

My dissertation uses three chapters to study multi-round procurement auctions with reserve prices. In the first chapter, I develop a static model with non-forward looking bidders. I make this assumption to simplify the analysis and to accommodate the flexibility of allowing for changes of bidders' private cost distributions across stages. I first propose a game-theoretic bidding model in multi-round procurement auctions with secret reserve prices and evaluate how the release of the auctioneer's reserve price affects bidders' bidding behavior and auctioneer's expected payment. Then I provide various reduced-form analyses on the INDOT data to validate the model.

In the second chapter, I maintain the assumption of myopic bidders and carry out a structural econometric analysis on the multi-round procurement auction data from the INDOT. Using the structural estimates, I evaluate how the release of the reserve price affects the government's expected payment through a counterfactual analysis.

In the third chapter, I introduce dynamic features into the model by assuming that

bidders are forward looking and their private cost distributions do not change across stages. I propose a dynamic bidding model. Then I solve a bidder's dynamic control problem to obtain the symmetric Markov perfect equilibrium. Next, I develop a two-step estimation approach to conduct a structural econometric analysis on the dynamic multi-round procurement auction game. Then I use this approach to analyze the data from the INDOT. Lastly, using the estimates of the inferred cost distribution, I evaluate the reserve price policy through counterfactual analysis.

To model the multiple stages and secret reserve price, I focus the first-price sealed-bid auction model on a simple environment -- the independent private value (IPV) paradigm. I also restrict my attention to bidders' strategic changes over stages, while assuming that the government's reserve price is exogenous and private over stages. This assumption, although restrictive, is consistent with the data.² While the model focuses on the procurement auctions that are low-bid auctions, as it is motivated by the procurement data from the INDOT, it can be readily extended to high-bid auctions.

My model yields some interesting predictions and implications. First, the bidding prices uniformly decline over stages, because of the information about the secret reserve prices revealed in the previous stage. This prediction holds regardless of whether the bidders are forward looking or not. Second, bidders mark up higher when placing bids with forward looking than without forward looking. This reflects the bidders' opinion of winning today versus winning later. Third, I study the reserve price release policies in the static model by simulation. Under some specifications, a secret reserve price is better than a public reserve price, and vice versa.

²According to the officials at the INDOT, generally no change is made in the engineer estimate (as the reserve price) after a round of unsuccessful auction and in practice there were very few changes made. Hence the

My structural analyses recover the inferred costs, the use of which allows us to evaluate the reserve price policy in the current procurement auction mechanism. I conduct counterfactual analyses. I find that on one hand when the bidders are not forward looking, to help the INDOT save budget, the use of a secret reserve price is better than a public reserve price. On the other hand, the result depends on the discount factor, in other words the bidders' attitude about the future, when the bidders are forward looking. When the discount factor is low, the use of a secret reserve price is better. When the discount factor is high enough, the use of a public reserve price is better.

My dissertation contributes to the study of structural auction models and the policy of procurement auctions. First, its theoretical finding has potential to explain why secret reserve prices are widely used in practice. Second, it is the second study of dynamic structural econometric auction models. Jofre-Bonet and Pensendorfer (2003) give the first. Lastly, it offers insights into the highway procurement auctions policies.

2 Multi-Round Auctions and Reserve Prices: A Survey

Since Vickrey's (1961) seminal work, studies of auction theories have grown rapidly. See surveys by Milgrom (1985, 1987), McAfee and McMillan (1987), Wilson (1992) and Klemperer (1999). The reserve price, as an important policy instrument in auctions, has drawn a lot of attention. Reserve prices, if binding, may lead to no trade. Multi-round auctions can therefore be attributed to the use of reserve prices in auctions. I investigate research on these issues one by one. I also provide a short survey on studies of auctions in a dynamic setting as well as studies of highway procurement auctions.

reserve price is the government's knowledge rather than its dynamic strategy.

2.1 Reserve Prices in Auctions

There are various reasons for a seller to use a reserve price in auctions. Most importantly, setting a reserve price can protect the seller from a profit loss. Basically, there are two reserve price policies, namely a public reserve price when it is announced and a secret reserve price when it is kept private to the seller. The focus of my dissertation is on secret reserve prices as they are used in the INDOT auctions. Secret reserve prices are widely used. Use of secret reserve prices in auctions has been studied in empirical work. Hendricks, Porter and Wilson (1994) study the Outer Continental Shelf auctions. Ashenfelter (1989) examines wine and art auctions. Elyakime, Laffont, Loisel and Vuong (1994, 1997) and Li and Perrigne (2003) find secret reserve prices are used in timber auctions. Bajari and Hortacsu (2003) study eBay coin auctions with endogenous entry and empirically compare the two reserve price policies in these auctions.

Theoretical work in studying secret reserve prices, however, has been limited, with exceptions such as Vincent (1995) using risk aversion to explain the use of secret reserve prices in a common value paradigm and Li and Tan (2000) in an independent private value paradigm. They show that in the presence of risk aversion, using a secret reserve price is better for the seller than using the optimal public reserve price for single-round auctions under some conditions. Alternative explanations have also been provided through the seller's objectives other than maximizing profits such as maximizing the expected sales as in Elyakime, Laffont, Loisel and Vuong (1994). Hiding reserve prices may also help the auctioneer deter collusion from bidders as explained in Ashenfelter (1989). In addition, as argued in Bajari and Hortacsu (2003), secret reserve prices may be used to encourage participation in auctions with entry.

The theoretical study of the seller's optimal reserve price strategy is often in a relatively simple environment. See e.g., Riley and Samuelson (1981) find the optimal reserve price in first-price sealed-bid auctions within IPV paradigm, and Laffont and Maskin (1980) solve for the optimal public reserve price in second-price auctions. Elyakime, Laffont, Loisel and Vuong (1994) also find the optimal secret reserve price in a one-shot first-price auction. The empirical study of auction data in complicated environment, on the other hand, often treats the seller's reserve as exogenous and concentrates on analyzing bidding. See e.g., Bajari and Hortacsu (2003) and Jofre-Bonet and Pesendorfer (2003)'s study of repeated games of highway auctions.

2.2 Multi-Round Auctions

Studies of multi-round auctions have been quite limited. Elyakime, Laffont, Loisel and Vuong (1997) study a two-round auction game where the first round is conducted as a first-price sealed bid auction with a secret reserve price, and if the object is not sold, the second round is conducted through bargaining between the seller and the bidder with the highest bid from the first round. Horstmann and LaCasse (1997) propose a common value second-price bidding model in which the seller is assumed to know the true common value and has the option of holding the auction for a one-time resale. The seller announces a reserve price for screening inferior bids but does not guarantee a sale in the first round auction. Skreta (2004) proposes a new concept, namely non-commitment, which is the same as in my research. Under non-commitment, if no trade takes place, the seller cannot commit not to try to sell the object in the second period. Skreta studies a two-period auction model. The seller can implement a revenue maximizing allocation rule by running a

Myerson auction with buyer-specific cutoffs in each period. The reserve price decreases overtime if no trade takes place. Evidently, these models do not fit with the multi-round procurement auctions organized by the INDOT as these auctions can be held for more than two rounds if the project is not sold in the previous round, and the government does not strategically choose to re-auction the project.

2.3 Dynamic Auction Models

The literature on empirical estimation of auctions has largely focused on a static auction setting. Paarsch (1992), Laffont, Ossard and Vuong (1995), and Guerre, Perrigne and Vuong (2000) among others develop empirical approaches in static auction games. Elyakime, Laffont, Loisel and Vuong (1994, 1997) and Li and Perrigne (2003) empirically analyze the use of secret reserve prices in static auction games. There is little empirical work on dynamic auction games with an exception of Jofre-Bonet and Pesendorfer (2003). Jofre-Bonet and Pesendorfer develop an estimation approach of a dynamic auction game with capacity constraint as the state variable to analyze the California highway auctions. Laffont and Robert (1999) and Donald, Paarsch and Robert (2002) also analyze finitely repeated auctions. Recently, Athey and Bagwell (2006) study dynamic auctions with persistent private information.

2.4 Applications in Highway Procurement Auctions

As one of the main applications in the empirical auction literature, highway procurement auctions have inspired a great deal of research. Early studies are Feinstein, Block and Nold (1985) and Porter and Zona (1993) who study issues of bidder collusion.

Bajari (1997) studies asymmetry between bidders. For recent structural work in analyzing procurement auctions, see Bajari and Ye (2002) for detecting collusion, Hong and Shum (2002) for assessing the winner's curse, Jofre-Bonet and Pesendorfer (2003) for capacity constraints in dynamic procurement auctions, Krasnokutskaya (2002) for the effect of unobserved auction heterogeneity in an asymmetric first-price IPV paradigm, and Li and Zheng (2005) for entry and competition effects.

It is worth noting that unobserved auction heterogeneity in procurement auctions has been documented (e.g. Krasnokutskaya (2002) and Li and Zheng (2005)). Failing to control for the unobserved auction heterogeneity can cause severe bias in structural estimation, and hence result in misleading policy evaluations and recommendations. My structural approach takes into account the unobserved auction heterogeneity.

3 Structural Econometric Approach: A Survey

In contrast to the large number of theoretical auction studies, fewer empirical studies have attempted to validate theoretical auction models using real auction data. A possible reason for this gap between theoretical and empirical work arises from the computational difficulties due to the nonlinearity and numerical complexity associated with the estimation of structural econometric models. Most empirical studies concentrate on testing some implications of the theory of auctions using reduced-form econometric models, such as linear regressions. See the recent survey by Laffont (1997).

The main methodology of the reduced-form approach is to test the comparative statics predictions from the theoretical models, without directly recovering the parameters of bidders' value distribution (so called underlying structure). The advantage of this

approach is that it is computationally easy. The disadvantage is that it is hard to interpret the estimates because it lacks connection to the theoretical auction model. Without primitive parameters, it is also hard to further simulate alternative mechanisms in comparison to the current mechanism. Therefore further analysis calls for the structural econometric approach.

My dissertation mainly uses the structural approach. Nevertheless I use reduced-form when it is necessary to present a fast and intuitive test of the theoretical model as well. In subsequent sections, I provide a brief survey on the auction literature in a static setting on one hand as it is well developed in auction studies. I present the literature on a broader class of empirical industrial organization models in a dynamic setting on the other hand as there have been very few empirical studies of dynamic auction models.

3.1 Literature on Static Models

The earliest structural static econometric auction models are Paarsch (1989, 1992) who estimates econometric models that are closely derived from theory. In recent years, we have seen various studies that have well established this area. I briefly review the methods that have been very useful in estimation and testing.

One strand of methods is within a parametric framework. This line of research specifies the distribution of the unobserved private values in some finite dimensional parameter space. Then it uses observables to derive a likelihood function or moment conditions. Structural parameters are estimated by methods in the family of maximum likelihood or generalized method of moments (GMM). Since the boundary of the observed equilibrium bids often depends on the parameters of the distribution of the latent values,

the standard maximum likelihood approach is inappropriate. Donald and Paarsch (1993) propose a piecewise maximum likelihood estimation method to overcome this difficulty. However, using likelihood and moment conditions of bids requires the computation of the equilibrium bidding strategy which is complicated as a closed-form solution is often not attainable. As a resolution, Laffont, Ossard and Vuong (1995) propose the simulated non-linear least squares estimation method using winning bids. Li and Vuong (1997) further extend this method to the framework of using all bids. This method does not require to solve the equilibrium bids and has great computational advantage. Recently, Hong and Shum (2002) propose a monotone quantile estimator for first-price sealed-bid auctions including both a private and a common value component. Li (2005) proposes a method of simulated moments for the first-price sealed-bid auctions with entry and binding reserve price.

The other strand of research is within a nonparametric framework. Guerre, Perrigne and Vuong (2000) show that the latent values are non-parametrically identified from the observed bids in the first-price IPV auction models. They establish a general two-step non-parametric framework to recover the distribution of the values. In the first step, it estimates the distribution of bids. In the second step, relying on the first order condition for the optimal bids, it obtains the inferred values and uses these values to non-parametrically fit the distribution of values. The implication of this method is two-fold. First, the estimates obtained from this method are robust as it is non-parametric. Second, it has computational advantage because it does not require the computation of the equilibrium bids. The main disadvantage of this method is that the rates of convergence are slower than \sqrt{N} as is typical of non-parametric methods. In addition, if there are many covariates, it is

impractical. Following this line of research, Elyakime, Laffont, Loisel and Vuong (1994, 1997) analyze the first-price IPV model with a secret reserve price. Li, Perrigne and Vuong (2000, 2002) propose non-parametric identification and estimation methods for the first-price affiliated private value auction models and conditionally independent private information auction models. Athey and Haile (2002) provide more non-parametric identification and testing results for a variety of auction models. Hong and Shum (2003) propose a non-parametric test of common values versus private values. Athey and Haile (2005) provide an excellent survey on the non-parametric structural approach.

3.2 Literature on Dynamic Models

In contrast to the well-established structural approach in analyzing static auction games, there is far less structural empirical work on dynamic auction games. There are a few exceptions. Laffont and Robert (1999) and Donald, Paarsch and Robert (2002) analyze finitely repeated auctions. Laffont and Robert consider a sequence of auctions in which an identical object is sold at each stage. Their model generates complex intra-day dynamics that are applied to data on eggplant auctions. Donald, Paarsch and Robert consider a model in which a finite number of objects are sold in a sequence of ascending-price auctions. They estimate the model using data on the sales of Siberian timber-export permits. Jofre-Bonet and Pensendorfer (2003) estimate a dynamic repeated highway procurement auction game with capacity constraint as the observed state variable. They propose a two-step estimation method. In the first step, it estimates the distribution of bids. In the second step, it recovers the cost distribution relying on the first order condition of bids. A complication in estimating a dynamic model is to approximate the value function

numerically.

Jofre-Bonet and Pensendorfer's approach builds partially on the two step approach that Elyakime, Laffont, Loisel and Vuong (1994) and Guerre, Perrigne and Vuong (2002) develop for static models. The main contribution of their paper is to extend the estimation method to dynamic auction games. In their paper, the dynamic auction model seeks to find a symmetric Markovian equilibrium which shares the property as in Rust (1987). The conditional independence assumption in the data generating process is necessary to adopt the framework of Markov dynamic decision processes. See Rust (1994) for a detailed survey on the structural estimation of Markov decision processes. See also Pakes (1994) for a survey on the literature of estimation in dynamic games.

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CHAPTER II

A GAME OF MULTI-ROUND AUCTION WITH SECRET RESERVE PRICE AND NON-FORWARD LOOKING BIDDERS

1 Introduction

In this chapter, I propose a game-theoretic bidding model in multi-round procurement auctions with secret reserve prices and evaluate how the release of the auctioneer's reserve price affects bidders' bidding behavior and auctioneer's expected payment. Then I carry out various reduced-form econometric analyses on the multi-round procurement auction data from the INDOT to validate the model.

To analyze the complexity of multiple stages and secret reserve price, I establish my first-price sealed-bid auction model in a simple environment -- the independent private value (IPV) paradigm. Meanwhile, I assume that the government's reserve price is exogenous and private over stages. This assumption, although restrictive, is consistent with my data. It also permits me to focus on bidders' strategic changes over stages

Although my model for the procurement auctions focuses on low-bid auctions which is motivated by the procurement data from the INDOT, it can be easily generalized to analyze high-bid auctions. My model yields some interesting predictions and implications. First, the bidding prices uniformly decline over stages, because of the information about the secret reserve prices revealed in the previous stage. Second, under some conditions, hiding the secret reserve price is better for the government than announcing it. This result provides an explanation as to why secret reserve prices are commonly used in auctions from a new perspective.

The implication of the reduced-form analyses is two-fold. First, it validates the important assumptions in the model, such as the exogeneity of reserve prices and number of potential bidders. Second, it tests first the equilibrium bidding monotonically decreasing in cost and second the bid in the second round is lower than in the first round.

Studies of multi-round auctions have been quite limited. A few existing studies are all limited to a two-round auction model with various seller's second-round options. See Elyakime, Laffont, Loisel and Vuong (1997), Horstmann and LaCasse (1997), and Skreta (2004). Apparently, these two-round models do not fit with the multi-round procurement auctions organized by the INDOT as these auctions can be held for more than two rounds if the project is not sold in the previous round, and the government does not strategically choose to re-auction the project.

This chapter is organized as follows. In Section 2, I present the data to motivate the model. In Section 3, I construct the model of multi-round procurement auctions with secret reserve prices, and solve the Bayesian Nash equilibrium. I also investigate the implications from my model. In Section 4, I compare the effects of different information release policies. In Section 5, I conduct a reduced-form econometric analysis of the data. Section 6 concludes. All technical proofs are included in the Appendix.

2 Data

My dissertation analyzes a data set of highway auctions held by the INDOT. The INDOT lets highway construction contracts through auctions. The auctions are held as first-price sealed-bid auctions where the INDOT reserve prices are unknown to bidders. Each contract specifies the construction work on highways within Indiana undertaken by

the winner of the auction. The winner of each auction performs projects described in the contract and is paid by the government. The prices bid by all participants are the amount that they ask for compensation.

An auction proceeds as follow. The INDOT posts the notice to contractors to invite bids five weeks prior to the bidding day. The notice includes simple information such as the type of projects in each contract, date of completion requested, and the length or area of the projects. Bid proposals and plans for the contracts that consist of more information on characteristics of the projects are also available upon request. Next, with the advent of the bidding day, each bidder submits a sealed bid to an electronic bidding system knowing that the government has a secret reserve price. Finally, on the bidding day, the received bids are unsealed and ranked by the government publicly. If the lowest bid in the auction is lower than the reserve price, the contract is then awarded to the bidder. Otherwise the contract will be readvertised and reauctioned in the following month. This feature makes the data unique.

The INDOT lets four types of construction work: road work, bridge work, traffic facilities and highway maintenance. I select one specific type of bridge work, which is called bridge rehabilitation, to analyze for two reasons. First, there exists large heterogeneity across different auctions. The characteristics of bridges are relatively more observable to econometricians among all work. Second, among all bridge work, bridge rehabilitation work not only reveals most characteristics to econometricians but also occurs most frequently.

The sample analyzed in this paper is from INDOT monthly lettings from September 1996 to December 2004. For each auctioned contract, I have the following

observations: the identity of each bidder, all bids, the reserve price, the number of bidders, the number of projects, the length of projects, the number of working days (or the completion date), the DBE goal and the structure of the bridge.¹ Before I exclude the lettings whose descriptive variables are missing, I have 37 lettings that have two rounds of auctions. In 34 lettings, the contractors in the second round are a subset of the contractors in the first round. There are three lettings, however, all of which have one single bidder in the first round and one new single bidder in the second round. I exclude them from my sample. I also exclude from the sample the lettings whose descriptive variables are missing. As a result, my final sample consists of 273 lets and 1428 bids in total. Among the 273 lets, 243 were sold in the first round that involves 1261 bids. There are 30 lets unsold in the first round (near 12.5%) but sold in the second round with totally 167 bids in both rounds, and 102 bids in the first round, and 65 bids in the second round, respectively.

Table 2.1 and Table 2.2 give the description of the variables and the summary statistics of the data. On average, DBE percentage is 7.52 which means 7.52% of the total value of the contract is operated by DBE firms. DBE is regulated by the government hence it is not the choice of bidders. The average number of working days for completing the bridge work is around 138. The average length of the projects is 79.21 meters (about 260 feet). Intuitively, the longer a project takes and/or the longer the bridge is, the more work needs to be done and hence the higher cost it could result in. The average number of projects in each contract is 1.18, meaning that there can be multiple projects on vicinity

¹DBE (short for disadvantaged business enterprise program) is committed by the INDOT to implement to ensure nondiscrimination in the award and administration of USDOT-assisted contracts. DBE goal is expressed as a percentage. This percentage, when applied to the total federal highway construction funds received by the INDOT during the year, represents the amount of dollars that DBE firms working on INDOT contracts as prime contractors, subcontractors, or truckers should receive. Hence in a particular letting, the primary contractor if not a DBE firm, has the responsibility for contracting all ready, willing and able DBE firms who express a desire to work on any of the pay items of the contract; and must subcontract at least as

sites. Multiple projects could potentially affect the capacity as well as the share ability of the facilities of the firms. 38% of the bridges have a steel structure, with the rest having structures of concrete, wood and others.

The summary statistics also reveal several important features of the data. On average, the number of bidders in the first round is 4.99 whereas the number of bidders in the second round is 2.23. Second, the average reserve price is \$855,615 and the average bid is \$839,506 for those with only one round. The former is greater than the latter meaning that the secret reserve price is effectively binding. On the other hand, they are very close. Third, if I concentrate on the auctions with two rounds, I find that on average, the bid is \$638,917 in the first round and \$588,992 in the second round, with a difference of \$49,925. This indicates that bids on average are lower in the second round than in the first round.

3 The Model for Multi-Round Auctions with Secret Reserve Prices

In this section, I propose a game-theoretic model for multi-round procurement auctions with secret reserve prices, and derive the corresponding Bayesian-Nash equilibrium.

3.1 Setup of the Game

The government lets a single and indivisible contract to firm contractors. There are N potential contractors who are interested in bidding for the contract. Each potential bidder is risk-neutral with a disutility equal to his private cost c . The government has an engineer estimate that is secret and serves as a reserve price in that the lowest bid has to be below it to become the winning bid. Because of the secret reserve price, it is possible for a

much as the required percentage of the total value to one or more DBE firms.

project not to be awarded in an auction. If this is the case, the project will be re-auctioned later. Thus the game has multiple stages.

The government's secret reserve price r_0 is drawn from a distribution $G(\cdot)$ with support $[\underline{c}, \bar{c}]$ where $\underline{c} \geq 0$. $G(\cdot)$ is twice continuously differentiable and has a density $g(\cdot)$ that is strictly positive on the support. Potential bidders draw their private costs independently at stage j from a common distribution denoted $F_j(\cdot)$ with support $[\underline{c}, \bar{c}]$ and the corresponding density $f_j(\cdot)$ that is strictly positive on the support.² Thus I focus on the independent private value paradigm. When forming his bid, each bidder knows his private cost c , but does not know r_0 as well as others' private costs. On the other hand, each bidder knows that r_0 is drawn from $G(\cdot)$ and all private costs are independently drawn from $F_j(\cdot)$. $G(\cdot)$ and $F_j(\cdot)$ are common knowledge to all bidders. As a result, all bidders are identical *a priori* and the game is symmetric.

More specifically, the game can be characterized in the following order. In the first stage, the government has an engineer estimate that serves as the reserve price. The reserve price is kept fixed and secret until the contract is sold. It is exogenous in that it is not related to the government's optimal and strategic decision. As a result, I can focus on the strategic changes of the bidders' strategies across stages. Without knowing the reserve price, all N potential bidders participate in the game in the first stage and submit their bids. At the end of the first stage, all bids are opened, ranked and released. The reserve price, however, is not made public until after the contract is sold out. If the lowest bid, which requests the

²While we assume that $G(\cdot)$ and $F_j(\cdot)$ have a common support for simplicity, our approach can be readily generalized to the general case where $G(\cdot)$ and $F_j(\cdot)$ have different supports.

least compensation of cost from the government, is lower than the reserve price, the contract is awarded to the associated bidder and the game ends. Otherwise, the game continues to the next stage.

In the second stage, there are two main changes. First, each contractor re-draws his private cost from a common distribution $F_2(\cdot)$, which in general can be different from $F_1(\cdot)$. This is a key assumption in my model, and will be labeled as the random cost replacement assumption hereafter. This assumption implies that each bidder's cost in one auction round can be different from his in another. Being endowed with the lowest cost in the first round does not mean being endowed with the lowest in other rounds. This assumption can be used to justify my observation that in most of the auctions in my data, the actual bidders of the second round are a subset of the bidders in the first round. Moreover, the assumption that each bidder re-draws his private cost in a different round is reasonable. Each firm can participate in several different auctions in one month. They may lose in some auctions while winning in others. In a later round, the firm's private cost for the same project can change from the previous round because the firm may face different capacity constraints and may have different opportunity costs.

Another important feature of my model is that there is a Bayesian updating on the reserve price from the bidders. Specifically, when an unsold project is re-auctioned, though the engineer's estimate is still kept secret, bidders have more information about this secret reserve price in this round than in the preceding one as they know the lowest bid from the preceding round. Therefore, they take this lowest bid into their strategy calculation as additional information as they know the secret reserve price has to be below this lowest bid. If a potential bidder's private cost he re-draws in this new round is above the lowest bid

from the preceding round, he will not submit his bid. Thus this lowest bid plays a similar role to that of a public reserve price in screening bids. Thus, though I assume that there is no entry problem in the first round in that all potential bidders submit their bids, in the subsequent rounds, a potential bidder will not submit his bid if his private cost is higher than the lowest bid he observes from the preceding round. As a result, the actual bidders in the subsequent rounds must be a subset of the potential bidders of the first round.

3.2 The Bayesian-Nash Equilibrium Bidding Strategy

Denote a bidder's cost at the j -th stage c_j and the associated bidding strategy b_j . I focus on the symmetric increasing Bayesian-Nash bidding equilibrium. Define the equilibrium bidding function as $b_j = \beta(c_j)$ such that $\beta'(\cdot) > 0$. I also use s_j^* to denote the lowest bid in the j -th stage.

In this chapter, I assume that bidders solve their bidding strategies stage by stage without considering possible future rounds at the current round.³ Under this assumption and the random cost replacement assumption, I can derive the bidder's Bayesian-Nash equilibrium across stages as follows.

Proposition 1 *The Bayesian-Nash equilibrium strategies are*

$$\beta_1(c) = c + \frac{\int_c^{\bar{c}} [1 - F_1(x)]^{N-1} [1 - G(\beta_1(x))] dx}{[1 - F_1(c)]^{N-1} [1 - G(\beta_1(c))]}, \quad (1)$$

for the first round and

³This assumption rules out forward-looking bidders. On the other hand, it has a generality in that it allows for different private cost distributions across different rounds, while one has to assume the same private cost distribution across stages in a dynamic game with forward-looking bidders.

$$\beta_j(c) = c + \frac{\int_c^{s_{j-1}^*} [1 - F_j(x)]^{N-1} [G(s_{j-1}^*) - G(\beta_j(x))] dx}{[1 - F_j(c)]^{N-1} [G(s_{j-1}^*) - G(\beta_j(c))]} \quad (2)$$

for the j -th reauction round respectively.

3.3 Comparing Bidding Strategies across Stages

Expressions (1) and (2) indicate that the bidding strategies differ from stage to stage in my model, and the interval over which the bidding strategy is defined also changes over stages. For instance while the first round equilibrium is defined on $[\underline{c}, \bar{c}]$, the second round equilibrium is defined on the interval $[\underline{c}, s_1^*]$, which is truncated from above compared to the first round. While the secret reserve price is not revealed, the rejected lowest bid from the previous round gives bidders information that the secret reserve price is below this bid; bidders will not submit their bids above this lowest bid. Intuitively, this would make bidders bid more aggressively and reduce their bids over stages. This is indeed the case, as shown in the next proposition.

Proposition 2 *In the multi-round auction model, the equilibrium markup and bid in stage j is less than or equal to the equilibrium markup and bid in the previous stage everywhere on $[\underline{c}, s_{j-1}^*]$.*

A few remarks follow. First, this proposition shows that the equilibrium bidding strategies are indeed decreasing from stage to stage if the contract is not sold out. Moreover, the reduction is universal on the whole common interval. Second, this result is established allowing the private cost distributions to vary across stages. Thus it is a strong prediction from the model as it is robust to the change of the bidders' cost distribution over stages. Third, this result is empirically testable and can be used for testing rationality of bidders in

real auctions.

3.4 Numerical Examples

To explore more properties of the bidding functions across stages, I give some numerical examples. I specify different distributions and vary the number of potential bidders. As the analytical solutions are in general not attainable, I numerically solve for equilibrium bids. Without loss of generality, I illustrate $\beta_1(c)$ and $\beta_2(c)$. The bidding functions under different specifications are depicted in Figure 2.1 -- 2.3.

The depicted curves reinforce two main findings from the theoretical model. First, the bidding functions are strictly increasing. Second, the bids in the second round are everywhere below the bids in the first round on the common support. I conduct a large number of numerical specifications and these findings are generally consistent.

The graphs also reveal some other interesting patterns. First, the bids are negatively related to the number of potential bidders in every round. This is simply because of the *competition effect*. Second, the disparity between $\beta_1(c)$ and $\beta_2(c)$ are affected by two factors. On one hand, it is affected by the number of potential bidders. The difference between them shrinks as the number of potential bidders increases. This is reasonable because as the number of potential bidders increases, the *competition effect* becomes more intense, which makes a bidder's mark-up in every auction round small and converging. On the other hand, it is affected by the lowest bid in the first round of unsold auction. The smaller this lowest bid is, the larger the difference between $\beta_1(c)$ and $\beta_2(c)$. I label this effect as *boundary effect*. In the first round, the boundary condition is at the upper bound of the cost distribution, while in the second round the boundary condition is at the previous

lowest bid (upper bound of a truncated distribution). The lower the truncated bound is, the lower the maximum possible bid in the second round is. Hence the *boundary effect* tends to enlarge the difference in bidding across stages.

4 Government's Information Revelation

In this section I compare welfare impacts of different reserve price release policies from the government. Motivated by the INDOT data feature, my model has focused on the use of the secret reserve price by the government. Alternatively, the government can make the engineer's estimate public and use it as a public reserve price. In this scenario, the government can find no bids submitted if all bidders' private costs are above the public reserve price. If I maintain the random cost replacement assumption, then the government can re-auction the project in the next round with the same public reserve price. As a result, under the random cost replacement assumption, the multi-round feature can be accommodated by both secret and public reserve prices. It would be interesting to compare the welfare implications of these two mechanisms and gain insights on why secret reserve prices are used in auctions.

4.1 Multi-Round Auctions with Public Reserve Prices

I maintain all the assumptions made in Section 2.1 except that now the reserve price r_0 is public. In the j -th round, the Bayesian-Nash equilibrium strategy, as shown in Riley and Samuelson (1981), is given by

$$\beta_{pj}(c) = c + \frac{\int_c^{r_0} [1 - F_j(x)]^{N-1} dx}{[1 - F_j(c)]^{N-1}} \quad (3)$$

for $c < r_0$; for a potential bidder whose private cost is above r_0 , he will not bid.

If all potential bidders' private costs are above r_0 , no bids are submitted at the current round, and the project can be re-auctioned in the next round. As in the secret reserve price case, I assume that the set of potential bidders remains the same across stages. At each round, however, a bidder's bidding strategy as defined in (3) may change because of the new private cost he re-draws from $F_j(\cdot)$.

4.2 The Comparison of Mechanisms

I compare the government's ex ante expected payments under the two reserve price policies, assuming that bidders re-draw their private costs at each round, and the government will re-auction the unsold contract in the next round until it is sold out. Since it is infeasible to make such a comparison generally as the ex ante expected payments in these two cases do not have closed form expressions in general, I conduct some simulation studies by assuming that the private cost distribution remains the same across stages, and by considering some commonly used functional forms for the private cost distribution and the reserve price distribution such as the uniform and exponential distributions.

I specify different distributions and vary the number of potential bidders to carry out a group of simulations. I then compare the government's expected expenditures under the two mechanisms.

I first plot the simulated expected expenditure as a function of the reserve price. As can be seen from the graphs, under some specifications, the secret reserve price policy

dominates the public reserve price in that the (ex post) expected expenditure under the secret reserve price is below that under the public reserve price. Some other specifications, however, yield the opposite findings. I further compute the ex ante expected expenditures by integrating out the reserve prices and report them in Table 2.3, which reveals that the expected expenditure under secret reserve prices is sometimes lower and sometimes higher than under public reserve prices.

The graphs also reveal some other interesting patterns. First, the (ex post) expected expenditure as a function of either a secret or public reserve price is almost increasing with the reserve price⁴. This is reasonable because the higher is the reserve price, the less restrictive is the auction to the bidders. The acceptable bids are high when the reserve prices are high. Second, the effect of the number of bidders is complicated. On one hand, the larger the number of bidders, the closer the two curves are because of the competition that tends to offset the different effects of different reserve prices on bidding. This competition effect is in analogy to that in the earlier numerical results on equilibrium bids between stages. On the other hand, the minimum bid in the previous round is lowered by the intensity of competition in the case of a secret reserve price. This is a factor that drags down the bids in the auctions with secret reserve prices, which does not exist in the public reserve price mechanism. The boundary effect favors the secret reserve price. It affects the position of the intersection and the difference of the two curves. Consequently, the boundary effect tends to enlarge the favorable range for the secret reserve price and increase the distance between the two curves in its favorable range. The net effect is determined by the combination. It seems from the graphs that the competition effect often

⁴The sampling variation resulting from simulations causes the small fluctuations on the curve; otherwise the curve could be more monotone.

dominates.

5 Reduced-Form Empirical Analyses

In this section, I provide a preliminary analysis of my data, trying to relate my theoretical model to the data by justifying some assumptions, and to test some predictions from the model.⁵

5.1 Exogeneity of Number of Potential Bidders in the First Round and the Reserve Price

In my model, I assume that there is no entry in the first round. In other words, the set of potential bidders is identical to that of actual bidders. To justify this assumption, I take a look at the number of bidders in the first round and test the exogeneity of this variable. To this end, I use both the Poisson model and the negative binomial model as the number of bidders is a count variable. Using all data in the first round including both sold lettings and unsold lettings, I estimate both models. Since in my data set, I do not have any auction that has no bidder participation, the number of bidders in my data is truncated from zero. Thus I use the truncated Poisson and negative binomial models.

The ML estimation results of both models are reported in Table 2.4-2.5. The results show that no covariates used in the regression are statistically significant in explaining the number of bidders. Thus, the number of actual bidders can be treated exogenous and considered the same as the number of potential bidders.

Another important assumption in the theoretical model is that the government's reserve price is exogenous in that it is not related to the number of bidders and does not

⁵While our model is general enough to allow for possibility of infinite rounds, we can only focus on analyzing

change across different rounds. To test the exogeneity of the reserve price in my data, I run a regression of the logarithm of the reserve price on a set of covariates including the number of bidders. From the results reported in Table 2.6, I can see that interestingly, both the number of bidders and the round-two dummy are not significant in the regression.⁶ That both the number of bidders and round-two dummy have no effect on the reserve price provides support for the exogeneity of the reserve price.

5.2 Regression Analysis of Bids

There are two empirically testable implications about the equilibrium bids from my theoretical model. First, it can be easily verified that the Bayesian-Nash equilibrium strategies given by (1) and (2) are monotone decreasing with the number of potential bidders. Intuitively, the larger the number of bidders, the more competitive the auction. The competition drives the bidders to bid more aggressively. Second, the theoretical model predicts that the equilibrium bids are lower in the second stage.

To test these two implications, I run a pooled regression of the logarithm of bids on a set of covariates. To allow for structural change in bid over the two auction rounds, which is indicated by the theoretical model, I include the round-two dummy variable and its interactions with other variables. I report the regression results in Table 2.7. It turns out that the number of bidders is strongly significant and negatively related to bids. The round-two dummy and some interactive terms are strongly significant, meaning that there exists structural change in bid across auction stages. Furthermore, I calculate the marginal effect of the round-two dummy on the bids. The marginal effect is -\$53004 and strongly

the first two rounds because of our data limitation.

⁶round-two is a dummy variable equal to 0 when an auction is in the first round and 1 when an auction is in

significant, meaning that on average the bidders tend to lower their bids in the second round by \$53,004 which is about 8.3% of the project value. This result is quite close to the outcome in the summary statistics. These findings offer support to my theoretical model.

The R^2 of the pooled regression is 0.51, indicating that on one hand the model fits moderately well, on the other hand I may ignore some unobserved auction heterogeneity. To further ascertain the existence of unobserved auction heterogeneity, I conduct a random-effect panel data analysis using only the first round auction data, as the auction data have a panel feature. I report the regression results in Table 2.8. The results strongly indicate that there exists unobserved auction heterogeneity as the error variance from the unobserved heterogeneity accounts for 95% of the total error variance. Hence it calls for controlling the unobserved heterogeneity in the structural inference.

6 Conclusion

In this chapter, I study multi-round auctions with secret reserve prices. My model yields some predictions that can be empirically tested, such as that the equilibrium bids decline uniformly over various stages. Also, my simulation study of the model demonstrates that depending on the specifications of the underlying distributions, the auctioneer may be better off by keeping the reserve price secret, which is the case in my data that motivates my study. Thus my model has the potential to be used to explain why, in some real world auctions, secret reserve prices are used.

This chapter offers insight on the use of secret reserve prices in multi-round auctions and the strategic changes in bidders' bidding strategies. It is worth noting, on the other hand, that my model is a static model with non-forward looking bidders. I make this

the second round.

assumption to simplify the analysis and to accommodate the flexibility of allowing for changes of bidders' private cost distributions across stages. Alternatively, one can introduce dynamic features into the model by assuming that bidders are forward looking and their private cost distributions do not change across stages. In chapter IV, I propose a dynamic model of multi-round auctions with secret reserve prices and develop a structural approach.

APPENDIX

1. Lemma 1

In the first stage, bidder i 's probability of winning is $[1 - F(c_{i1})]^{N-1}[1 - G(b_{i1})]$; while in the j -th stage, bidder i 's probability of winning is $\frac{[1 - F_j(c_{ij})]^{N-1}[G(s_{j-1}^*) - G(b_{i1})]}{G(s_{j-1}^*)}$.

Proof Bidder i wins the auction if his bid is less than the other $N - 1$ bids as well as the reserve price. The probability of winning can be described by the probability of the occurrence of the following event, $\Pr(b_{i1} < \min_{k \neq i}(b_{k1}) \text{ and } b_{i1} < r_0)$, which is a joint probability. Because the pair wise independence of agents, it is a product of $\Pr(b_{i1} < r_0)$ and $\Pr(b_{i1} < b_{k1}), \forall k \neq i$. At equilibrium if the bidders play the symmetric bidding strategy, then $\beta(c_{i1}) < \beta(c_{k1})$ implies $c_{i1} < c_{k1}$. Hence it follows that

$$\Pr(c_{i1} < c_{k1}) = 1 - \Pr(c_{k1} > c_{i1}) = 1 - F_1(c_{i1}),$$

$$\Pr(b_{i1} < r_0) = 1 - \Pr(r_0 < b_{i1}) = 1 - G(b_{i1}).$$

In the j -th stage, the information from previous auction rounds enables the bidders to form a Bayesian updated belief of r_0 . Therefore the probability that b_{ij} is less than r_0 is contingent on the past information set Λ_{j-1} , i.e., $\Pr(b_{ij} < r_0 | \Lambda_{j-1})$, where s_{j-1}^* is the lowest bid from the previous auction round. Henceforth $r_0 < s_{j-1}^*$, is the information set Λ_{j-1} . bidders bid as if they saw r_0 drawn from a truncated distribution $G(r | r < s_{j-1}^*)$. It then leads to the following result

$$\Pr(b_{ij} < r_0 \mid \Lambda_{j-1}) = 1 - \Pr(r_0 < b_{ij} \mid r_0 < s_{j-1}^*) = 1 - \frac{G(b_{ij})}{G(s_{j-1}^*)} = \frac{G(s_{j-1}^*) - G(b_{ij})}{G(s_{j-1}^*)}.$$

Similarly, the probability of winning is based on $\Pr(b_{ij} < \min_{k \neq i}(b_{kj}))$ and $b_{ij} < r_0$, a joint probability. Thus the result immediately follows.

2. Proof of Proposition 1

Define $\beta(\cdot)$ as the symmetric increasing Bayesian-Nash equilibrium bidding strategy. Since it is the same function for each bidder, we can suppress the subscript i . We index the strategy in J -th stage by β_j . We solve the game stage by stage to obtain the separate equilibrium. In the first stage, the bidder chooses b_1 to maximize his expected payoff

$\pi_1 = (b_1 - c_1)[1 - F_1(\beta_1^{-1}(b_1))]^{N-1}[1 - G(b_1)]$. The first order condition is as follows

$$\begin{aligned} & [1 - F_1(\beta_1^{-1}(b_1))]^{N-1}[1 - G(b_1)] - (b_1 - c)[1 - F_1(\beta_1^{-1}(b_1))]^{N-1}g(b_1) \\ & - (b_1 - c)[1 - G(b_1)](N-1)[1 - F_1(\beta_1^{-1}(b_1))]^{N-2}f_1(\beta_1^{-1}(b_1))\frac{1}{\beta'(\beta_1^{-1}(b_1))} = 0, \end{aligned}$$

where $b_1 = \beta_1(c)$ and $c = \beta_1^{-1}(b_1)$. After we replace b_1 with the function of c , we get the differential equation for β_1

$$\begin{aligned} & [1 - F_1(c)]^{N-1}[1 - G(\beta_1)]\beta_1'(c) - (\beta_1 - c)[1 - F_1(c)]^{N-1}g(\beta_1)\beta_1'(c) \\ & - (\beta_1(c) - c)[1 - G(\beta_1(c))](N-1)[1 - F_1(c)]^{N-2}f_1(c) = 0. \end{aligned}$$

Further algebra turns the differential equation into

$$\frac{d}{dc} \{ \beta_1(c)[1 - F_1(c)]^{N-1}[1 - G(\beta_1(c))] \} = c \frac{d}{dc} \{ [1 - F_1(c)]^{N-1}[1 - G(\beta_1(c))] \}.$$

Using the boundary condition $\beta_1(\bar{c}) = \bar{c}$, we can solve the equation above and get

$$\beta_1(c) = c + \frac{\int_c^{\bar{c}} (1 - F_1(x))^{N-1} [1 - G(\beta_1(x))] dx}{[1 - F_1(c)]^{N-1} [1 - G(\beta_1(c))]}.$$

In the j -th reaction stage, the bidder maximize his expected payoff in the j -th round as follows

$$\text{Max}_{b_j} (b_j - c_j) \frac{[1 - F_j(\beta_j^{-1}(b_j))]^{N-1} [G(s_{j-1}^*) - G(b_j)]}{G(s_{j-1}^*)}.$$

The corresponding first order condition is

$$\begin{aligned} & [1 - F_j(\beta_j^{-1}(b_j))]^{N-1} \frac{[G(s_{j-1}^*) - G(b_j)]}{G(s_{j-1}^*)} - (b_j - c) [1 - F_j(\beta_j^{-1}(b_j))]^{N-1} g(b_j) \frac{1}{G(s_{j-1}^*)} \\ & - (b_j - c) \frac{[G(s_{j-1}^*) - G(b_j)]}{G(s_{j-1}^*)} (N-1) [1 - F_j(\beta_j^{-1}(b_j))]^{N-2} f_j(\beta_j^{-1}(b_j)) \frac{1}{s'(\beta_j^{-1}(b_j))} = 0. \end{aligned}$$

In equilibrium, we obtain the differential equation for β_j

$$\begin{aligned} & [1 - F_j(c)]^{N-1} [G(s_{j-1}^*) - G(\beta_j)] \beta_j'(c) - (\beta_j - c) [1 - F_j(c)]^{N-1} g(\beta_j) \beta_j'(c) \\ & - (\beta_j(c) - c) [G(s_{j-1}^*) - G(\beta_j(c))] (N-1) [1 - F_j(c)]^{N-2} f_j(c) = 0. \end{aligned}$$

Then it can be written as

$$\frac{d}{dc} \{ \beta_j(c) [1 - F_j(c)]^{N-1} [G(s_{j-1}^*) - G(\beta_j)] \} = c \frac{d}{dc} \{ [1 - F_j(c)]^{N-1} [G(s_{j-1}^*) - G(\beta_j)] \}.$$

The boundary condition is different here. It involves participation decision of the bidder's entry to the j -th round auction occurs only if a bidder's private cost in j -th round is less than s_{j-1}^* . Hence the above strategy is conditional on that the private cost is less than s_{j-1}^* .

As a result, the boundary condition is $\beta_j(s_{j-1}^*) = s_{j-1}^*$ for this stage. Integrate over $[c, s_{j-1}^*]$, using the boundary condition, we can get the following

$$\beta_j(c) = c + \frac{\int_c^{s_{j-1}^*} [1 - F_j(x)]^{N-1} [G(s_{j-1}^*) - G(\beta_j(x))] dx}{[1 - F_j(c)]^{N-1} [G(s_{j-1}^*) - G(\beta_j(c))]}.$$

3. Proof of Proposition 2

By the uniqueness of symmetric Bayesian Nash equilibrium solution of (1) and (2), we have

$$\begin{aligned} & \frac{1}{G_{j-2}^*} (\beta_{j-1} - c) [1 - F_{j-1}(c)]^{N-1} [G_{j-2}^* - G(\beta_{j-1})] \\ & \geq \frac{1}{G_{j-2}^*} (\beta_j - c) [1 - F_{j-1}(c)]^{N-1} [G_{j-2}^* - G(\beta_j)] \end{aligned}$$

where G_{j-2}^* is short for $G(s_{j-2}^*)$, and

$$\begin{aligned} & \frac{1}{G_{j-1}^*} (\beta_j - c) [1 - F_j(c)]^{N-1} [G_{j-1}^* - G(\beta_j)] \\ & \geq \frac{1}{G_{j-1}^*} (\beta_{j-1} - c) [1 - F_j(c)]^{N-1} [G_{j-1}^* - G(\beta_{j-1})]. \end{aligned}$$

Since $s_{j-1}^* < s_{j-2}^* < \bar{c}$, it immediately follows that

$$(\beta_{j-1} - c) [G_{j-2}^* - G(\beta_{j-1})] \geq (\beta_j - c) [G_{j-2}^* - G(\beta_j)], \quad (\text{A.1})$$

$$(\beta_j - c) [G_{j-1}^* - G(\beta_j)] \geq (\beta_{j-1} - c) [G_{j-1}^* - G(\beta_{j-1})]. \quad (\text{A.2})$$

Furthermore, from (A.1) we can get

$$(\beta_{j-1} - c) [G_{j-2}^* + G_{j-1}^* - G_{j-1}^* - G(\beta_{j-1})] \geq (\beta_j - c) [G_{j-2}^* + G_{j-1}^* - G_{j-1}^* - G(\beta_j)],$$

that is

$$\begin{aligned} & (\beta_{j-1} - c) [G_{j-1}^* - G(\beta_{j-1})] + (\beta_{j-1} - c) (G_{j-2}^* - G_{j-1}^*) \\ & \geq (\beta_j - c) [G_{j-1}^* - G(\beta_j)] + (\beta_j - c) (G_{j-2}^* - G_{j-1}^*). \end{aligned} \quad (\text{A.3})$$

From (A.2) and (A.3), we have

$$(\beta_{j-1} - c)(G_{j-2}^* - G_{j-1}^*) \geq (\beta_j - c)(G_{j-2}^* - G_{j-1}^*).$$

Therefore it follows that $\beta_j(c) \leq \beta_{j-1}(c)$.

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Table 2.1 Variables and Number of Observations

number of lets with one round		243
number of lets with two round		30
Variable	Description	NOBS
characteristics of lets		
Dbe	DBE percentage goal	273
Time	number of working days needed	273
Np	number of projects	273
Steel	whether the bridge is of steel structure	273
Length	length of the bridge(s), meter	273
Prices		
Rp	government's engineer estimate	273
bid1	for lets with one round	1261
bid1&2	for lets with two round (both rounds)	167
bid1-2	bid in the first round auction that is unsold	102
bid2-2	bid in the second round	65
participation		
Nb	number of potential bidders	273
nb2	number of bidders in 2nd round	30

Table 2.2 Summary Statistics

Variable	mean	std.dev.	min	max
Dbe	7.52	3.16	0	15
Time	137.53	65.76	20	451
Np	1.18	0.60	1	5
Steel	0.38	0.49	0	1
Length	79.21	82.27	3.22	607.31
Rp	855614.8	895489.2	70671.35	6742284
bid1	839506	869855.5	65325.78	6684512
bid1&2	619485.5	398895.8	94853	2230051
bid1-2 (first round)	638917.3	427400.6	94853	2230051
bid2-2 (second round)	588992.4	350553.3	97637.2	1505183
Nb	4.99	1.98	1	10
nb2	2.23	1.01	1	5

Table 2.3 Results of Simulation Study

$F(c)$	$G(r)$	N	public rp	secret rp
exp (1)	exp (1)	4	0.0388	0.0341
exp (1)	exp (1)	7	0.024	0.0228
exp (1)	exp (1)	10	0.0174	0.0172
exp(2)	exp (1)	10	0.0306	0.0281
exp(0.5)	exp (1)	10	0.0092	0.0096
exp(0.3)	exp (1)	10	0.0056	0.0062
exp(0.3)	exp(1)	7	0.0082	0.0086
unif [0,1]	unif [0,1]	4	0.2911	0.2662
unif [0,1]	unif [0,1]	7	0.2032	0.1958
unif [0,1]	unif [0,1]	10	0.1554	0.1543
unif [0,0.3]	unif [0,1]	10	0.0519	0.0586
unif [0,0.5]	unif [0,1]	10	0.0838	0.0886
unif [0,0.7]	unif [0,1]	10	0.1138	0.1174

exp: exponential distribution, mean in parentheses

unif: uniform distribution, bounds in brackets

Table 2.4 Poisson Regression Model of NB

nb (Dept. Var.)	Coef.	Std. Err.
dbe	0.0161	0.01
time	5.07e-04	4.31e-04
np	-0.0505	0.0513
steel	0.042	0.0608
length	3.59e-04	3.82e-04
_cons	1.42*	0.104

Observations: 273 Log likelihood: -567.24

restricted log likelihood -573.42

chi2(d.f.=5) = 12.37 p-value = 0.03

left truncated data, at nb=0

*: significant at 5%

Table 2.5 Negative Binomial Model of NB

nb (Dept. Var.)	Coef.	Std. Err.
dbe	0.016	0.01
time	5.08e-04	5.21e-04
np	-0.0504	0.0653
steel	0.042	0.073
length	3.59e-04	3.37e-04
_cons	1.42*	0.13

Observations: 273 Log likelihood: -567.24

left truncated data, at nb=0

*: significant at 5%

Table 2.6 OLS Estimates of Regression of Reserve Prices

log(rp) (Dept. Var.)	Coef.	Std. Err.
nb	-0.011	0.011
round-two	-0.12	0.12
dbe	0.045*	0.011
time	0.005*	0.0005
np	0.204*	0.060
steel	0.273*	0.073
length	0.0024*	0.0004
_cons	11.84*	0.149
Number of Observations: 273		$R^2 = .52$

*: significant at 5%

Table 2.7 OLS Estimates of Regression of Bids

log(bid) (Dept. Var.)	Coef.	Std. Err.
round-two	-0.826*	0.4113
nb	-0.020*	0.0068
dbe	0.037*	0.0053
time	0.0048*	0.0002
np	0.191*	0.0387
steel	0.272*	0.0337
length	0.0024*	0.0002
nb*round-two	0.0504	0.0481
dbe*round-two	0.006	0.0197
time*round-two	-0.001	0.0016
np*round-two	0.681*	0.1560
steel*round-two	-0.340	0.2276
length*round-two	0.0005	0.0005
_cons	11.927*	0.0809
marginal effect ($b_2 - b_1$)	-5.3e+04*	3.1e+03
Number of Observations: 1428		$R^2 = 0.51$
*: significant at 5%		

Table 2.8 Random Effect Analysis of Bids

log(bid) (Dept. Var.)	Coef.	Std. Err.
nb	-0.0214	0.016
dbe	0.039*	0.011
time	0.0047*	0.0005
np	0.201*	0.060
steel	0.265*	0.073
length	0.0024*	0.0005
_cons	11.91*	0.14
$\sigma_u^2 = 0.5346$		$\sigma_\varepsilon^2 = 0.1179$

$\rho = 0.9536$ (fraction of variance due to u_i)

Number of Groups: 273

Model: $\log(\text{bid}_{il}) = x_{il}\beta + u_l + \varepsilon_{il}$

*: significant at 5%

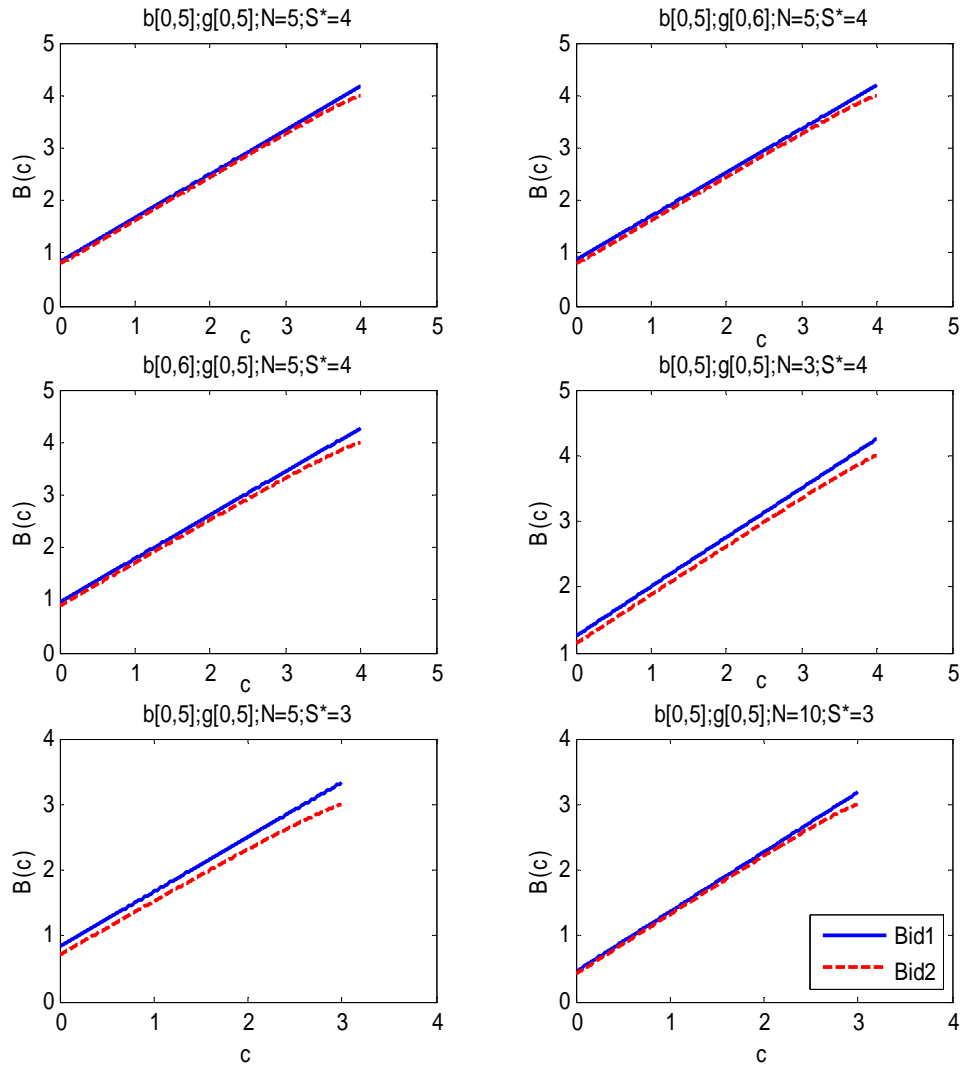


Figure 2.1 Bidding Functions of the First Two Stages in Multi-Round Auctions with Secret Reserve Prices: Uniform Distributions

Note: The contractors' private cost and the governmental reserve price are uniformly distributed. In each subplot, b refers to contractor, g refers to government, N refers to number of contractors and S^* refers to the lowest bid in the first round of an unsold contract. We maintain the convention of notations throughout the figure set 1.

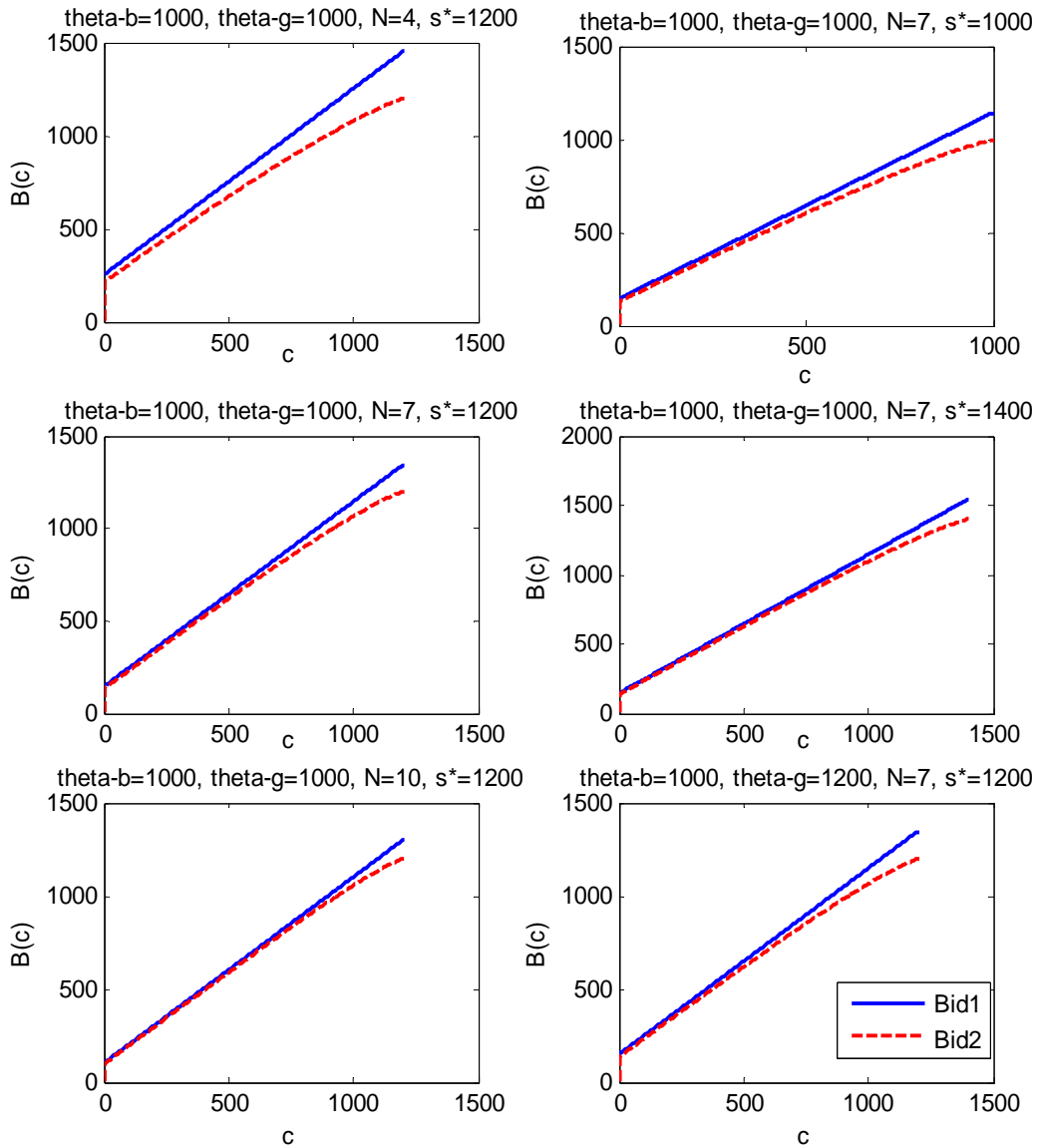


Figure 2.2 Bidding Functions of the First Two Stages in Multi-Round Auctions with Secret Reserve Prices: Exponential Distributions

Note: The contractors' private cost and the governmental reserve price are exponentially distributed.

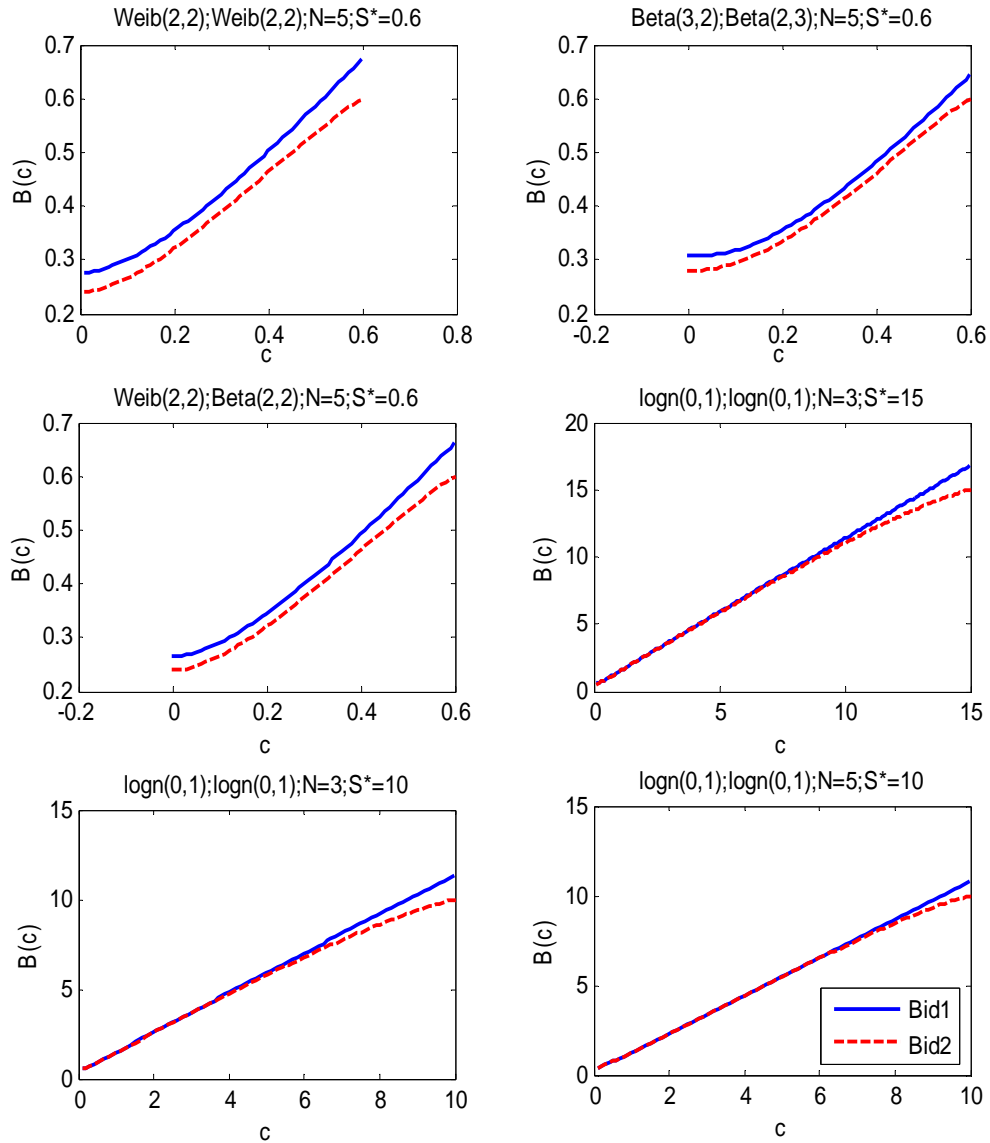


Figure 2.3 Bidding Functions of the First Two Stages in Multi-Round Auctions with Secret Reserve Prices: Mixed Distributions

Note: Weib refers to weibull distribution; Beta refers to Beta distribution; logn refers to log normal distribution. In each subplot, the contractor's distribution is put in the first place.

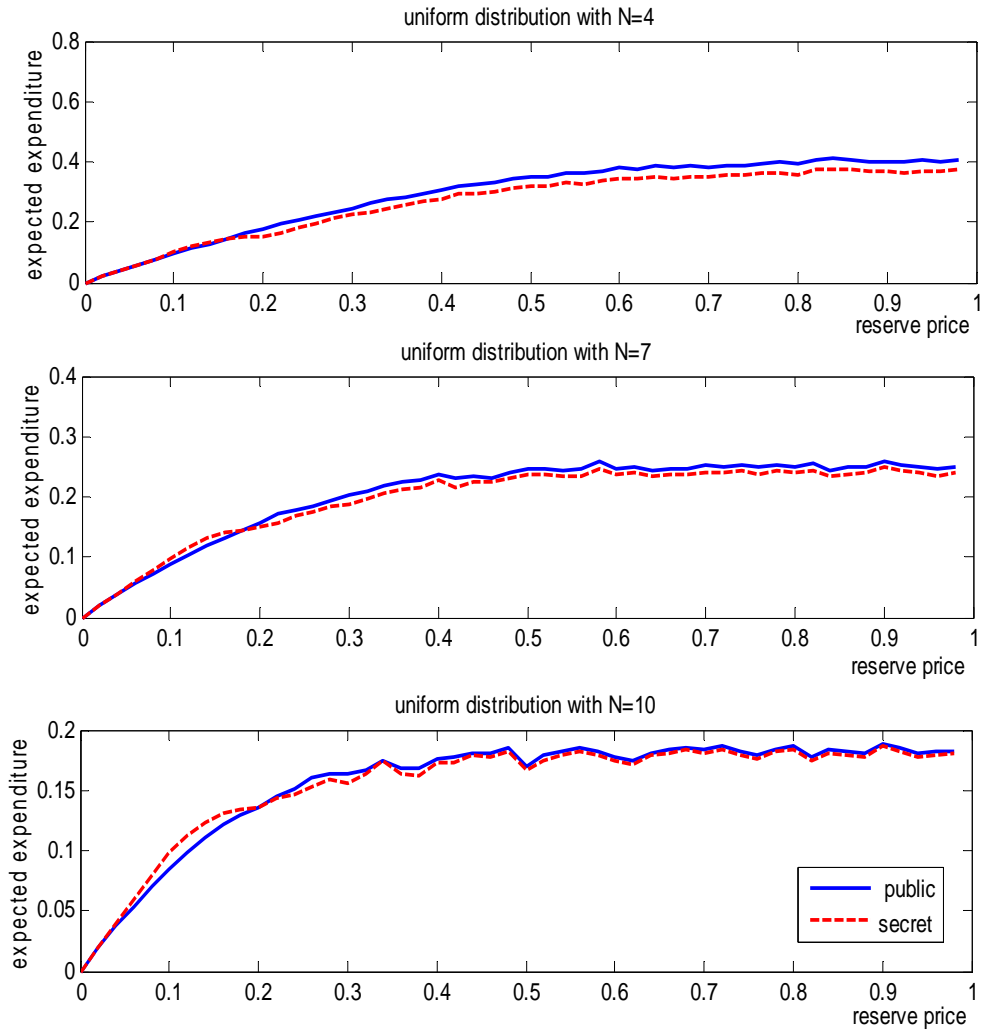


Figure 2.4 The Comparison of Governmental Expenditures under Uniform Cost Distributions

Note: solid lines represent (ex post) governmental expenditures under public reserve prices, dash lines are under secret reserve prices.

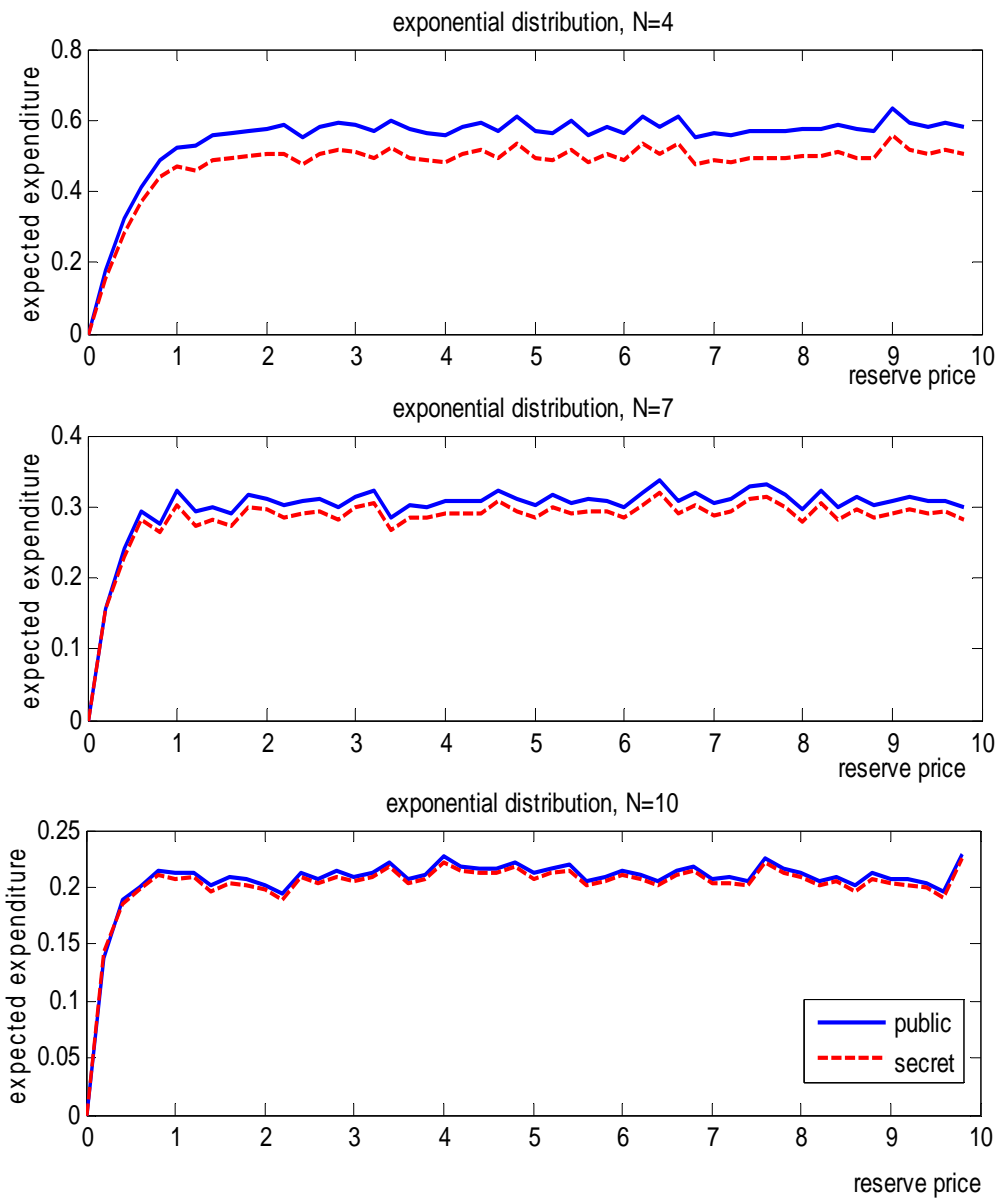


Figure 2.5 The Comparison of Governmental Expenditures under Exponential Cost Distributions

CHAPTER III

STRUCTURAL ANALYSIS OF MULTI-ROUND PROCUREMENT AUCTIONS WITH NON-FORWARD LOOKING BIDDERS

1 Introduction

In this chapter, following the game-theoretic model in Chapter II, I carry out a structural econometric analysis on the multi-round procurement auction data from the INDOT. Using the structural estimates, I evaluate how the release of the reserve price affects the government's expected payment through counterfactual analysis.

To analyze the procurement auction data, and in addition to provide an empirical framework within which the multi-round model with secret reserve prices can be analyzed, I develop a structural model from the theoretical model that I propose. My structural approach takes into account of the unobserved auction heterogeneity, the existence of which in procurement auctions has been documented (e.g. Krasnokutskaya (2002) and Li and Zheng (2005)). Failing to control for the unobserved auction heterogeneity can cause severe bias in structural estimation, and hence result in misleading policy evaluations and recommendations.

I adopt the method of simulated moments (MSM) to estimate the underlying structural parameters. This approach provides a unified framework within which some interesting hypotheses can be tested, in addition to the computational advantage in obtaining consistent estimates. For example, we can test whether the private cost distribution varies across stages. I use my structural approach to analyze the INDOT data. Using the structural estimates, I carry out a counterfactual analysis by simulating the

auctions with different government's reserve price release policies in the multi-round scenario. I find that the government could save more than \$13,000 (or about 2.5% of the project value) on average on a representative bridge work auction by hiding the engineer estimate rather than disclosing it. Hence the use of secret reserve price may be a good policy in practice in procurement auctions.

This chapter is organized as follows. In Section 2, I provide a structural econometric framework for analyzing multi-round auction data. In Section 3, I apply the structural approach to analyze the INDOT data. In Section 4, I use counterfactual analysis to evaluate the government's reserve price policy. Section 5 concludes. All technical proofs are included in the Appendix.

2 Structural Inference of Multi-Round Auction Models

2.1 The Parameterization of the Structural Model

Based on the theoretical auction model, there are three primitives, namely, the government's reserve price distribution $G(\cdot)$ and the private cost distributions $F_j(\cdot)$, $j=1,2$. $F_1(\cdot)$ can be in general different from $F_2(\cdot)$. Nonparametrically, $G(\cdot)$ can be identified from the observed reserve prices as they are assumed to be random draws from $G(\cdot)$. Moreover, following Guerre, Perrigne and Vuong (2000) and Li and Perrigne (2003), it can be verified that $F_1(\cdot)$ is identified over its entire support $[\underline{c}, \bar{c}]$ by the observed bids in the first round, and $F_2(\cdot)$ is identified over $[\underline{c}, s_1^*]$ by the observed bids in the second round (see the appendix for a discussion). In this paper, however, I adopt the parametric approach because I only observe 30 auctions in the second round, which makes

nonparametric estimation problematic.

In an econometric framework, asymptotic statistical inference is based on a large number of auctions. Let L_1 be the number of auctions in the first round, L_2 be the number of auctions in the second round. Some auctions in L_1 did not result in a sale and went into the second round. For the ℓ -th auction at the j -th round, let $G_\ell(\cdot)$ and $F_{j\ell}(\cdot)$ denote each primitive distribution respectively with corresponding densities $g_\ell(\cdot)$ and $f_{j\ell}(\cdot)$, $j=1,2$. Assume that $G_\ell = G(\cdot | x_\ell, u_\ell, \gamma)$ and $F_{j\ell} = F(\cdot | x_\ell, u_\ell, \theta_j)$, where x_ℓ is a vector of variables that I use to control for the observed auction heterogeneity, and u_ℓ is a scalar variable that represents the unobserved auction heterogeneity, both affecting the government's reserve price as well as the bidders' costs, γ is a vector of unknown parameters in $\Gamma \subset \mathbb{R}^K$, and θ is a vector of unknown parameters in $\Theta \subset \mathbb{R}^K$. I assume that u is independent of x , and has a distribution $W(\cdot | \sigma)$ with $w(\cdot | \sigma)$ being the density function, where σ is a vector of unknown parameter in $\Sigma \subset \mathbb{R}^m$.

Conditional on both observed and unobserved heterogeneity x and u , I specify the reserve price distribution and the cost distribution as exponential as follows

$$g_\ell(r | x_\ell, u_\ell, \gamma) = \frac{1}{\exp(x_\ell \gamma + u_\ell)} \exp\left(\frac{-r}{\exp(x_\ell \gamma + u_\ell)}\right), \quad (1)$$

$$f_{j\ell}(c | x_\ell, u_\ell, \theta_j) = \frac{1}{\exp(x_\ell \theta_j + u_\ell)} \exp\left(\frac{-c}{\exp(x_\ell \theta_j + u_\ell)}\right) \quad j=1,2 \quad (2)$$

where $c \in (0, \infty)$ and $r \in (0, \infty)$. By including the intercept in x , I normalize the unobserved heterogeneity term u such that $E[u] = 0$. I assume that $u \sim N(0, \sigma^2)$, where σ^2 is an unknown parameter.

2.2 Structural Equilibrium Solutions

Next I need to solve the theoretical auction model for equilibrium solutions with the above specified distributions. The Bayesian Nash equilibrium bidding strategy in the first round is given as follows, which is a closed-form solution

$$\beta_1(c) = c + \frac{1}{\frac{N-1}{\exp(x_\ell \theta_1 + u_\ell)} + \frac{1}{\exp(x_\ell \gamma + u_\ell)}}. \quad (3)$$

The Bayesian Nash equilibrium bidding strategy in the second round, which is a solution to the equation given below

$$\beta_2(c) = c + \frac{\int_c^{s_1^*} \left[\exp\left(\frac{-z}{\exp(x_\ell \theta_2 + u_\ell)}\right) \right]^{N-1} \left[\exp\left(\frac{-\beta_2(z)}{\exp(x_\ell \gamma + u_\ell)}\right) - \exp\left(\frac{-s_1^*}{\exp(x_\ell \gamma + u_\ell)}\right) \right] dz}{\left[\exp\left(\frac{-c}{\exp(x_\ell \theta_2 + u_\ell)}\right) \right]^{N-1} \left[\exp\left(\frac{-\beta_2(c)}{\exp(x_\ell \gamma + u_\ell)}\right) - \exp\left(\frac{-s_1^*}{\exp(x_\ell \gamma + u_\ell)}\right) \right]}, \quad (4)$$

does not have a closed form.

2.3 Estimation and Testing for Changes of Private Cost Distributions

In my auction data, at round j , $j=1,2$, we observe reserve prices, number of potential bidders and a set of auction heterogeneities (r_ℓ, N_ℓ, x_ℓ) . We also observe bids in round 1 and round 2, respectively. My estimation of the structural parameters is based on the likelihood function of r given in (1) and the moment conditions of $b_{j\ell}$ ($j=1,2$), where $b_{j\ell}$ denotes the i -th bid in the ℓ -th auction at the j -th round. Specifically, from (3) we obtain the moment condition

$$\begin{aligned}
E[b_{1i\ell} | N_\ell, g_\ell(\cdot), x_\ell, u_\ell] &\equiv m_1(b_{1i\ell}, x_\ell, u_\ell, \gamma, \sigma; \theta_1) \\
&= E[c | N_\ell, g_\ell(\cdot), x_\ell, u_\ell] + \frac{1}{\frac{N_\ell - 1}{\exp(x_\ell \theta_1 + u_\ell)} + \frac{1}{\exp(x_\ell \gamma + u_\ell)}} \quad (5) \\
&= \exp(x_\ell \theta_1 + u_\ell) + \frac{1}{\frac{N_\ell - 1}{\exp(x_\ell \theta_1 + u_\ell)} + \frac{1}{\exp(x_\ell \gamma + u_\ell)}}
\end{aligned}$$

for the equilibrium bids $b_{1i\ell}$ in the first auction round. Similarly, from (4) we can obtain

$$E[b_{2i\ell} | c \leq s_1^*, N_\ell, g_\ell(\cdot), x_\ell, u_\ell] \equiv m_2(b_{2i\ell}, x_\ell, u_\ell, \gamma, \sigma; \theta_2) \quad (6)$$

for the equilibrium bids $b_{2i\ell}$ in the second round, where $m_2(b_{2i\ell}, x_\ell, u_\ell, \gamma, \sigma; \theta_2)$ does not have a closed form because the second round bidding function does not have a closed form. Note (1), (5) and (6) are all conditional on u_ℓ which is not observed. For estimation, however, we need to obtain conditions that depend only on observables. In order to derive such conditions, we integrate u_ℓ out of (1), (5) and (6) and get the followings

$$g_\ell(r | x_\ell, \gamma, \sigma) = \int_{-\infty}^{\infty} \frac{1}{\exp(x_\ell \gamma + u_\ell)} \exp\left(\frac{-r}{\exp(x_\ell \gamma + u_\ell)}\right) \cdot w(u_\ell | \sigma) du_\ell, \quad (7)$$

$$\begin{aligned}
E[b_{1i\ell} | N_\ell, g_\ell(\cdot), x_\ell] &\equiv M_1(b_{1i\ell}, x_\ell, \gamma, \sigma; \theta_1) \\
&= \int_{-\infty}^{\infty} m_1(b_{1i\ell}, x_\ell, u_\ell, \gamma, \sigma; \theta_1) \cdot w(u_\ell | \sigma) du_\ell, \quad (8)
\end{aligned}$$

$$\begin{aligned}
E[b_{2i\ell} | c \leq s_1^*, N_\ell, g_\ell(\cdot), x_\ell] &\equiv M_2(b_{2i\ell}, x_\ell, \gamma, \sigma; \theta_2) \\
&= \int_{-\infty}^{\infty} m_2(b_{2i\ell}, x_\ell, u_\ell, \gamma, \sigma; \theta_2) \cdot w(u_\ell | \sigma) du_\ell. \quad (9)
\end{aligned}$$

The parameters of primary interests are γ , θ_1 , θ_2 and σ . I estimate them using (7), (8) and (9).

2.3.1 A Two-Step Estimation Approach

I adopt a two-step estimation strategy. In the first step, I recover γ and σ using likelihood function (7) to get $\hat{\gamma}$ and $\hat{\sigma}$. In the second step, I estimate θ_1 and θ_2 using moment conditions (8) and (9) as well as the estimates $\hat{\gamma}$ and $\hat{\sigma}$.

Since I fully specify the distribution of the reserve price and we observe reserve prices, in the first step, γ and σ can be efficiently estimated by maximum likelihood (ML) approach. A complication arises from the feature that there is no closed form likelihood function because of the integration with respect to u_ℓ . Thus, I use a simulated maximum likelihood (SML) estimation approach (Gourieroux and Monfort (1996)). Specifically, the SML estimator is defined by

$$(\hat{\gamma}, \hat{\sigma})_{SML} = \arg \max_{\gamma, \sigma} \sum_{\ell=1}^{L_1} \log \left[\frac{1}{S} \sum_{s=1}^S \frac{g_\ell(r | x_\ell, u_\ell^s, \gamma) w(u_\ell^s | \sigma)}{\phi(u_\ell^s)} \right]. \quad (10)$$

As indicated in (10), I use the importance sampling technique in the numerical integration. The importance density function is the standard normal $\phi(\cdot)$. I draw S of u_ℓ^s s from $\phi(\cdot)$ in simulation, where S is sufficiently large compared to the sample size L_1 . As $S, L_1 \rightarrow \infty$ and $\sqrt{L_1}/S \rightarrow 0$, the SML estimator is asymptotically equivalent to the ML estimator (Gourieroux and Monfort (1996)).

In the second step, I separately estimate θ_1 and θ_2 , using moment conditions (8) and (9), respectively, and the estimates $\hat{\gamma}$ and $\hat{\sigma}$ obtained in the first step. Again because of the presence of the unobserved heterogeneity, I propose a method of simulated moments estimator (MSM). Let $Y_{j\ell}(\theta_j) = b_{j\ell} - M_j(b_{j\ell}, x_\ell, \hat{\gamma}, \hat{\sigma}; \theta_j)$, ($j=1,2$). We need to simulate $M_j(b_{j\ell}, x_\ell, \hat{\gamma}, \hat{\sigma}; \theta_j)$, hence I draw u_ℓ^s from $w(\cdot | \hat{\sigma})$ and define

$$y_{j\ell}^s(\theta_j) = b_{j\ell} - M_j(b_{j\ell}, x_\ell, u_\ell^s, \hat{\gamma}, \hat{\sigma}; \theta_j).$$

For each ($j = 1, 2$), a MSM estimator can be defined by

$$\hat{\theta}_{jMSM} = \arg \min_{\theta_j} \left(\sum_{\ell=1}^{L_j} \frac{1}{n_\ell} \sum_{i=1}^{n_\ell} \left(x_{j\ell} \frac{1}{S} \sum_{s=1}^S y_{ji\ell}^s(\theta_j) \right) \right)' \Omega \left(\sum_{\ell=1}^{L_j} \frac{1}{n_\ell} \sum_{i=1}^{n_\ell} \left(x_{j\ell} \frac{1}{S} \sum_{s=1}^S y_{ji\ell}^s(\theta_j) \right) \right) \quad (11)$$

where Ω is a $K \times K$ symmetric positive-definite weighting matrix.

Additional difficulty in computation arises from the fact that the simulated $M_j(b_{ji\ell}, x_\ell, \hat{\gamma}, \hat{\sigma}; \theta_j)$ involves the Bayesian-Nash equilibrium strategy, which is especially cumbersome for $j = 2$, because it does not have a closed form solution. I follow Elyakime, Laffont, Loisel and Vuong (1994) to numerically recover the bidding function by a recursive procedure. Starting from the boundary condition, the equilibrium bidding strategy can be numerically solved in a recursive manner. Note that the resulting MSM estimator is consistent given that the first-step estimators $\hat{\gamma}$ and $\hat{\sigma}$ are consistent.

Noting the complexity involved in my two-step estimation procedure, I use bootstrap to obtain variance-covariance matrices of the estimates. Because of the panel feature of the auction data, I adopt a block bootstrap (e.g. Andrews (2002)) to obtain the standard errors for my two-stage MSM estimator.

2.3.2 Testing for Cross-Stage Change of Private Cost Distributions

In the previous section, I develop a framework of estimating the structural model of multi-round auctions separately round by round, allowing for the underlying cost distributions to change across two rounds. It would be interesting to test whether the underlying cost distributions change or not across stages. If it turns out that the distributions do not change, it means that bidders re-draw their costs from the same distribution across stages. Moreover, in this case, I can more efficiently estimate the

private cost distribution parameters by jointly estimating both auction rounds.

I propose a formal test following Andrews and Fair (1988), who extend the Chow test of structural changes in classical linear models (Chow (1960)) to test structural changes in nonlinear models. The null hypothesis here is $H_0 : \theta_1 = \theta_2$, which is the case of testing for pure structural change (see Andrews and Fair (1988)). The Wald test statistic is applicable to my MSM estimator, which can be implemented as follows. First, the MSM in (MSM) is implemented as discussed previously. Let $\pi_1 = L_1 / L$ and $\pi_2 = L_2 / L$. Then, the Wald test statistic is given by

$$W = L(\hat{\theta}_1 - \hat{\theta}_2)' (\hat{V}_1 / \pi_1 + \hat{V}_2 / \pi_2 - 2\hat{V}_{12} / \sqrt{\pi_1 \pi_2})^- (\hat{\theta}_1 - \hat{\theta}_2)$$

where \hat{V}_1 and \hat{V}_2 are the estimated asymptotic variances matrices of $\hat{\theta}_1$ and $\hat{\theta}_2$ respectively, \hat{V}_{12} is the matrix of the estimated asymptotic covariances between $\hat{\theta}_1$ and $\hat{\theta}_2$

The general inverse $(\cdot)^-$ of covariance term in the middle equals the regular inverse $(\cdot)^{-1}$ with probability going to one as $L \rightarrow \infty$. W follows a χ^2 distribution with the dimension of the structural parameter vector θ as its degrees of freedom.

3 Results

In this section, I apply my structural econometric approach to analyze the data from the INDOT, so as to uncover the underlying private cost and reserve price distributions. By concentrating on a specific type of bridge work, I choose a set of observed covariates $x = \{ \text{dbe, time, np, steel, length, intercept} \}$. First I use the two-stage estimation to estimate the model under unobserved auction heterogeneity. Then I test the cross-stage change of private cost distributions. Lastly, I conduct a robustness check.

3.1 SML and MSM Estimates for the Structural Parameters and the Unobserved Heterogeneity

The parameters of the reserve price distribution γ and the parameter of the unobserved heterogeneity σ can be jointly estimated based on (SML). I draw a large sample, namely $S = 1000$, of u_ℓ^s s from $N(0,1)$, i.e, $\phi(u_\ell)$, and adopt importance sampling to implement the SML. Furthermore, I gain the standard errors through bootstrap. The results are reported in Table 3.1.

Next we estimate parameters in private cost distributions θ_j (for $j = 1, 2$) based on (11). To gain the simulated moments, I recursively solve for equilibrium bids and calculate the numerical integration. I simulate u_ℓ from $N(0, \hat{\sigma}^2)$. Here the number of u_ℓ^s s that I draw is $S = 100$, a number relatively smaller than the one I use in implementing SML, as an MSM estimator is consistent for any fixed number of simulations (Gourieroux and Monfort (1996)). Furthermore, I use the identity matrix as the weighting matrix. Using bootstrap, I obtain the standard errors of the estimates. Moreover, as we need to incorporate the variation from the estimation of $\hat{\gamma}$ and $\hat{\sigma}$, I jointly resample the auction data including both reserve prices and bids and repeat the SML estimation and MSM estimation simultaneously. The results of the estimation are reported in the first four columns of Table 3.2.

The results indicate that all variables that I pick up have significant effects on private costs. Evaluated at the sample mean of the observed and unobserved auction characteristics, the mean private cost is about \$641,000. Increases in the length of the bridges and the time needed to accomplish the projects raise private costs, and in turn increase bids, as expected. Specifically, holding all the other factors constant, increasing

the length of the project by one meter (or 3.28 feet) will increase the mean private cost by 0.21% or about \$1,350. One more working day needed for a project will increase the mean private cost approximately by 0.42% or \$2,700. Furthermore, rising in the DBE percentage results in higher private cost. This is reasonable because higher DBE percentage increases the primary contractor's transaction cost in a project by finding and subcontracting partial work to a DBE firm. More specifically, one unit increase in DBE will increase the mean private cost by about 4% or slightly more than \$25,700. An interesting pattern shows different effects of the number of projects (np) on the government's reserve price and the bidders' costs. Increasing the number of projects involved in one contract tends to raise the government's valuation of the work, but to lower the bidders' costs. This is because a bidder, while undertaking the projects, will consider the economic scale of taking multi-projects on multi-sites in neighborhood that reduces his cost. The government may not take the effect of economic scale into account since it does not assume the work anyway. This explains why we see a negative effect of np on the private costs, but a positive effect on the reserve price. Moreover, one unit increase in the number of projects can save the firm's private cost on average by about 3.9% which is slightly less than \$25,000. Bridges of a steel structure cause about \$180,000 more than bridges of other structures on the mean private cost. Furthermore, the estimate of the unobserved heterogeneity parameter is strongly significant, meaning that there exists unobserved auction heterogeneity in my data set.

3.2 Testing for Cross-Stage Change of Private Cost Distributions

To implement the test, I estimate the bidding equations separately round by round

and obtain $\hat{\theta}_1$ and $\hat{\theta}_2$. I then compute the W statistic, which is 1.45. Thus the null hypothesis is not rejected at a 5% significance level. It implies that a bidder re-draws his private cost from the same cost distribution across different auction rounds.

In this case, I re-estimate θ in view of $\theta_1 = \theta_2 = \theta$ by utilizing this restriction in the MSM estimation to obtain a more efficient estimate. The results are reported in the last two columns of Table 9. In sections that follow I use these estimates for inference.

3.3 Robustness Check

I take the estimate of the parameter of the unobserved auction heterogeneity $\hat{\sigma}$ from the first-stage estimation for granted in the second-stage, assuming that the unobserved auction heterogeneities are from the same distribution for both the auctioneer and the bidders. Although this is mainly for simplifying computation (particularly for those of the second round auctions), I can empirically check its validity. I estimate the parameter of the unobserved heterogeneity in (11) jointly with the parameters of the private cost distribution and obtain $\tilde{\sigma}$. I then compare $\hat{\sigma}$ and $\tilde{\sigma}$. It turns out that $\hat{\sigma}$ and $\tilde{\sigma}$ are very close (0.054 and 0.055), which validates the assumption.

4 Counterfactual Analysis

In this section we investigate welfare impacts of different reserve price release policies on the government. Motivated by the INDOT data feature, my model has focused on the use of the secret reserve price by the government. Alternatively, the government can make the engineer's estimate public and use it as a public reserve price. In this scenario, the government can find no bids submitted if all bidders' private costs are above the public

reserve price. If I maintain the random cost replacement assumption, then the government can re-auction the project in the next round with the same public reserve price. As a result, under the random cost replacement assumption, the multi-round feature can be accommodated by both secret and public reserve prices. It would be interesting to compare the welfare implications of these two mechanisms using a counterfactual analysis. Such a comparison allows us to evaluate the INDOT's auction mechanism and assess the efficiency of its current reserve price policy. Moreover, it offers insight on why secret reserve prices are used in auctions. Since I have uncovered the underlying structural elements, I can conduct simulations under the two different reserve price release policies and compare the government's payment under the two different scenarios.

I construct a representative auction by setting all observed characteristics at the sample means of the corresponding covariates. The simulation results are reported in Table 3.3. The expected procurement cost is \$537,689 under the public reserve price, \$524,048 under the secret reserve price. The difference is strongly significant. The INDOT on average can save \$13,641, or 2.5% of the project value, on a typical bridge work auction by adopting a secret reserve price, thereby saving millions of dollars of budgets on all highway projects yearly. On the other hand, the difference in the probability of no sale is 10%. The INDOT undergoes a no sale risk of 10% greater by adopting a secret reserve price. However, it comes with a large standard error and therefore statistically insignificant. Moreover, in practice the highway contracts are often sold out within two rounds, the cost saved by adopting the secret reserve price outweighs the no sale risk caused. Hence my findings indicate that the use of secret reserve price may be a good policy in practice in

procurement auctions.¹

5 Conclusion

I develop a structural approach to analyze the INDOT data. The structural approach recovered the distributions of the reserve prices and the private cost. The estimates for structural parameters allow us to conduct counterfactual analyses. I find that the INDOT could have significantly saved budgets by adopting a secret reserve price rather than using a public reserve price.

The empirical analysis of this chapter offers insights into the use of secret reserve prices in multi-round auctions and the strategic changes in bidders' bidding strategies. It is worth noting, on the other hand, that this structural analysis is based on the static model with non-forward looking bidders. The advantage of the static model is that it accommodates the flexibility of allowing for changes of bidders' private cost distributions across stages. However, the conclusions from the structural analysis and the counterfactual analysis do not necessarily hold if the bidders are forward looking. This stimulates my research in a new chapter. One can introduce dynamic features into the model by assuming that bidders are forward looking and their private cost distributions do not change across stages. In the next chapter, I propose a dynamic model of multi-round auctions with secret reserve prices and develop a structural approach.

¹McAfee and McMillan (1992) have argued that secret reserve prices can be used for preventing collusions.

APPENDIX

1. Derivation of the equilibrium bids

For the exponential distribution, we have

$$F(c) = 1 - \exp(-c / \exp(x\theta + u)), \quad G(r) = 1 - \exp(-r / \exp(x\gamma + u)),$$

$$1 - F(c) = \exp(-c / \exp(x\theta + u)), \quad 1 - G(r) = \exp(-r / \exp(x\gamma + u)).$$

Substituting them into equation (1) in Chapter 3, I get

$$\begin{aligned} \beta_1(c) &= c + \frac{\int_c^\infty [\exp(-x / \exp(x\theta + u))]^{N-1} \exp(-\beta_1(x) / \exp(x\gamma + u)) dx}{\exp[-c / \exp(x\theta + u)]^{N-1} \exp(-\beta_1(c) / \exp(x\gamma + u))} \\ &= c + \frac{\int_c^\infty \exp(-[(N-1)x / \exp(x\theta + u) + \beta_1(x) / \exp(x\gamma + u)]) dx}{\exp(-[(N-1)c / \exp(x\theta + u) + \beta_1(c) / \exp(x\gamma + u)])}. \end{aligned}$$

I use contraction mapping to solve it. We start with a conjecture of $\beta_1(c)$, say $\beta_1^0(c)$, which is the left hand side function. Then we substitute it into the right hand side to compute. This yields the right hand side function $\beta_1^1(c)$. If my conjecture is right, then $\beta_1^0(c) = \beta_1^1(c)$. Otherwise replace my conjecture with $\beta_1^1(c)$ and start iteration until the left hand side function equals the right hand side function, say $\beta_1^i(c) = \beta_1^{i+1}(c)$. Start with $\beta_1^0(c) = c$, calculate right hand side function as

$$\begin{aligned} \beta_1^1(c) &= c + \frac{\int_c^\infty \exp(-[(N-1)x / \exp(x\theta + u) + x / \exp(x\gamma + u)]) dx}{\exp(-[(N-1)c / \exp(x\theta + u) + c / \exp(x\gamma + u)])} \\ &= c - \frac{1}{(N-1)/\gamma + 1/\theta} \\ &\quad \cdot \frac{\exp(-[(N-1)x / \exp(x\theta + u) + x / \exp(x\gamma + u)])}{\exp(-[(N-1)c / \exp(x\theta + u) + c / \exp(x\gamma + u)])} \Big|_c^\infty \\ &= c + \frac{1}{(N-1) / \exp(x\theta + u) + 1 / \exp(x\gamma + u)}. \end{aligned}$$

Then with $\beta_1^1(c)$, we compute $\beta_1^2(c)$. It follows that $\beta_1^2(c) = \beta_1^1(c)$,

$$\beta_1^2(c) = c + \frac{1}{(N-1)/\exp(x\theta+u) + 1/\exp(x\gamma+u)}.$$

Therefore the solution is given by $\beta_1^2(c)$, *i.e.*

$$\beta_1(c) = c + \frac{1}{(N-1)/\exp(x\theta+u) + 1/\exp(x\gamma+u)}.$$

2. Nonparametric Identification

2.1. The First Stage of Multi-Round Procurement Auction

In real auctions, on one hand reserve prices are observed and can be used to identify the distribution of the reserve price $G(\cdot)$. On the other hand, costs are bidders' private information which is not observed to econometricians. Instead, the econometricians can observe bids. At issue is with the observation of bids and reserve prices, whether we can identify the distribution of the latent private costs. Because the equilibrium bidding strategy relates the observed bids b_1 to the unobserved private costs c_1 which are random, bids are also random. Denote the distribution of bids in the first stage by $H_1(\cdot)$ with support $[\underline{b}_1, \bar{b}_1]$ which is twice continuously differentiable and has a density $h_1(\cdot)$ that is strictly positive on the support. Therefore the identification problem of the multi-round auction model with a secret reserve price reduces to whether the bidders' private cost distribution in every auction round is uniquely determined from the observed bids. Depending on the equilibrium relationship between b_1 and c_1 , we can show that $H_1(b) = F_1(\beta_1^{-1}(b))$ and $h_1(b) = f(\beta_1^{-1}(b)) / \beta'(\beta_1^{-1}(b))$ for all $b \in [\underline{c}, \beta_1(\bar{c})]$. Further, with some algebra the first order condition of the bidders' problem in the first stage gives us the

following inverse bidding function for all $c \in [\underline{c}, \bar{c}]$

$$c_1 = \psi_1(b_1, H, G, N) = b_1 - \frac{1}{(N-1)\lambda_1(b_1) + \mu_1(b_1)}, \quad (\text{A.1})$$

where $\lambda_1(b_1) = h_1(b_1)/(1 - H_1(b_1))$, $\mu_1(b_1) = g(b_1)/(1 - G(b_1))$ are hazard rates. Equation (A.1) expresses the private cost c_1 as a function of the equilibrium bid b_1 , its distribution $H_1(\cdot)$, its density $h_1(\cdot)$, the government's reserve price distribution $G(\cdot)$ with its corresponding density $g(\cdot)$, and the number of potential bidders N . Specifically, equation (A.1) states that if the observed bids b_1 is the equilibrium bid, as is assumed in the structural econometric approach, then the bidders' private costs c_1 corresponding to b_1 must satisfy such a relation. Let $H_{1N}(\cdot)$ denote the joint distribution of (b_{11}, \dots, b_{1N}) .

Proposition 1 For $N = 2, \dots$, let $H_{1N}(\cdot)$ and $G(\cdot)$ be two distributions with respective support $[\underline{b}_1, \bar{b}_{1N}]^N$ and $[\underline{c}, \bar{c}]$ with $\underline{c} = \psi_1(\underline{b}_1, H_1, G, N)$ and $\bar{c} = \psi_1(\bar{b}_1, H_1, G, N)$. There exist a pair of distribution $[F_1, G]$ of bidders and the government's reserve price with common support $[\underline{c}, \bar{c}]$ such that $H_{1N}(\cdot)$ and $G(\cdot)$ are the corresponding distributions of the equilibrium bids and reserve prices in the j -th stage of the multi-round procurement auction with independent private costs and a secret reserve price if and only if for $N = 2, \dots$,

A1. $H_{1N}(b_{11}, \dots, b_{1N}) = \prod_{i=1}^N H_{1N}(b_{1i})$

A2. *the function $\psi_1(\cdot, H_1, G, N)$ defined in equation (A.1) is strictly increasing on $[\underline{b}_1, \bar{b}_1]$,*

and its inverse is differentiable on $[\underline{c}, \bar{c}] \equiv [\psi_1(\underline{b}_1, H, G, N), \psi_1(\bar{b}_1, H, G, N)]$.

In addition, when $F_j(\cdot)$ and $G(\cdot)$ exist, they are unique with support $[\underline{c}, \bar{c}]$ and satisfy

$F_1(c) = H_1(\psi_1^{-1}(c, H, G, N))$ for all $c \in [\underline{c}, \bar{c}]$. Meanwhile, $\psi_1(\cdot, H, G, N)$ is the quasi inverse of the equilibrium bidding strategy in the sense that $\psi_1(b, H, G, N) = \beta_1^{-1}(b, F, G, N)$ for all $b \in [\underline{c}, \beta_1(\bar{c})]$. Proposition 1 indicates that the multi-round procurement auction model with a secret reserve price is nonparametrically identified. The identification is achieved without any parametric assumptions.

2.2. The Reauction Stage of Multi-Round Procurement Auction

In a reauction stage, the information released from the previous stage raises a new difficulty because the lowest bid from the previous stage serves as a binding announced reserve price. For illustration, we solve the identification problem in stage j . Different from stage 1, the binding pseudo reserve price s_{j-1}^* introduces a truncation because a potential bidder with a private cost higher than s_{j-1}^* does not bid. Let b_{ji}^* denote the equilibrium bid of the i -th actual bidder, $i = 1, \dots, N_j^*$, and $H_j^*(\cdot)$ be its distribution. Thus $H_j^*(b^*) = \Pr(\beta_j(c) \leq b^* \mid c \leq s_{j-1}^*) = F_j(c) / F_j(s_{j-1}^*)$ where $c = \beta_j^{-1}(b^*)$. Differentiating with respect to b^* gives the conditional density $h_j^*(b^*) = [f_j(c) / F_j(s_{j-1}^*)] / \beta_j'(c)$ for all $b \in [\underline{c}, \beta_j(s_{j-1}^*)]$. Hence with some algebra the first order condition of the bidders' problem in stage j gives us the following inverse bidding function for all $c \in [\underline{c}, s_{j-1}^*]$

$$\begin{aligned} c_j &= \psi_j(b_j^*, H_j^*, G, N, F_j(s_{j-1}^*)) \text{ if } c_j \leq s_{j-1}^* \\ &= b_j^* - \frac{1}{(N-1)\lambda_j^*(b_j^*) + \mu_j(b_j \mid s_{j-1}^*)}, \end{aligned} \quad (\text{A.2})$$

where $\lambda_j^*(b_j^*) = h_j^*(b_j^*) \cdot F_j(s_{j-1}^*) / (1 - H_j^*(b_j^*) \cdot F_j(s_{j-1}^*))$ and

$\mu_j(b_j | s_{j-1}^*) = g(b_j)/(G(s_{j-1}^*) - G(b_j))$ are quasi hazard rates. Equation (A.2) is the analog of (A.1), but involves $F_j(s_{j-1}^*)$, which is unknown. This complicates the identification of the model. The next result solves the identification problem.

Proposition 2 For $N = 2, \dots$, let $H_j^(\cdot)$ and $G(\cdot)$ be two distributions with respective support $[\underline{b}_j, s_{j-1}^*]$ and $[\underline{c}, \bar{c}]$, and $\pi(\cdot)$ be a discrete distribution. There exist a pair of distribution $[F_j, G]$ of bidders and the government's reserve price with common support $[\underline{c}, \bar{c}]$ such that (1) $H_j^*(\cdot)$ is the truncated distribution of the equilibrium bids and $G(\cdot)$ is the reserve price distribution in the j -th stage of the multi-round procurement auction with independent private costs, a secret reserve price and a binding information bound $s_{j-1}^* \in [\underline{c}, \bar{c}]$, and (2) $\pi(\cdot)$ is the distribution of the number of actual bidders N_j^* if and only if for $N = 2, \dots$,*

A1. $\pi(\cdot)$ is Binomial with parameters $(N, 1 - F_j(s_{j-1}^))$, where $0 < F_j(s_{j-1}^*) < 1$*

A2. The observed bids are i.i.d. as $H_j^(\cdot)$ conditionally upon N_j^* and $\lim_{b \uparrow s_{j-1}^*} h_j^*(b) = +\infty$*

A3. the function $\psi(\cdot, H_j^, G, N, F_j(s_{j-1}^*))$ defined in equation (A.2) is strictly increasing on*

$[\underline{b}_j, s_{j-1}^]$, and its inverse is differentiable on*

$[\underline{c}, s_{j-1}^] \equiv [\psi_j(\underline{b}_j, H_j^*, G, N, F_j(s_{j-1}^*)), \psi_j(s_{j-1}^*, H_j^*, G, N, F_j(s_{j-1}^*))]$.*

Moreover, if conditions A1-A3 hold, then $F_j(s_{j-1}^*)$ is unique while $F_j(\cdot)$ is uniquely defined on $[\underline{c}, s_{j-1}^*]$ as $F_j(\cdot) = F_j(s_{j-1}^*) \cdot H_j^*(\psi_j^{-1}(\cdot, H_j^*, G, N, F_j(s_{j-1}^*)))$. In addition, $\psi_j(\cdot, H_j^*, G, N, F_j(s_{j-1}^*))$ is the quasi inverse of the equilibrium bidding strategy in the sense that $\psi_j(b, H_j^*, G, N, F_j(s_{j-1}^*)) = \beta_j^{-1}(b, F, G, N)$ for all $b \in [\underline{c}, \beta_j(s_{j-1}^*)]$. Proposition

2 parallels Proposition 1. In particular, it indicates that the multi-round procurement auction model with a secret reserve price is nonparametrically identified on $[\underline{c}, \beta_j(s_{j-1}^*)]$. Because of the truncation of the observed bids, a nonparametric approach prevents us from identifying the entire distribution of private costs.

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Table 3.1 SML Estimates of Reserve Prices Distribution and Unobserved Heterogeneity

Variable	Coef.	Std. Err.
dbe	0.0514*	0.0018
time	0.0052*	0.0002
np	0.299*	0.0082
steel	0.300*	0.0106
length	0.00212*	0.0001
_cons	11.469*	0.0923
σ^2	0.054*	0.0016

*: significant at 5%

Table 3.2 MSM Estimates of Private Distribution

Variable	Separate Estimates				More Efficient Estimates	
	θ_1	Std. Err	θ_2	Std. Err	θ	Std. Err
dbe*	0.0398	0.0010	0.0393	0.0037	0.0401	0.0009
time*	0.0044	0.0001	0.0041	0.0005	0.0042	0.0001
np*	-0.0343	0.0008	-0.0394	0.0077	-0.0388	0.0009
steel*	0.2947	0.0098	0.2440	0.0529	0.2809	0.0071
length*	0.0021	0.0001	0.0019	0.0002	0.0021	0.0001
_cons*	12.1394	0.0336	16.8904	2.2696	12.2670	0.0654

σ^2 (robustness check): 0.0548

*: significant at 5%

Table 3.3 The Comparison of Policies by Simulations

difference in		difference in	
government's payment	Std. Err	probability of no sale	Std. Err
<i>(public – sec ret)</i>		<i>(public – sec ret)</i>	
13641*	5876	- 0.1	0.06

CHAPTER IV

A DYNAMIC ANALYSIS OF MULTI-ROUND AUCTION MODEL WITH FORWARD LOOKING BIDDERS

1 Introduction

In this chapter, I continue to study the interesting multi-round feature observed in the procurement auctions organized by the Indiana Department of Transportation. This feature is attributable to the use of secret reserve prices in these auctions. The previous chapters have obtained various predictions and implications from the theoretical model and results from the structural empirical analyses on the INDOT data. However, these results are all based on a critical assumption: bidders are non-forward looking. As I proposed at the end of the last chapter, what if bidders are forward looking? In this chapter, I construct a dynamic bidding model in multi-round procurement auctions with secret reserve prices to analyze the dynamic bidding behavior when the bidders foresee the possibility of a reauction round in the future.

In a dynamic setting, a bidder's control problem is to optimally choose bid in an infinite period horizon, because he considers the possibility of the future reauctioning. The state variables in the dynamic auction game include a bidder's belief about the secret reserve price and his private cost. In the first auction round, the belief of the secret reserve price is the upper bound of the reserve price distribution because there is no information about the reserve price. In a reauction round, it is represented by the lowest bid from the previous auction round if the auction round fails. The state of the belief of the reserve price is common knowledge to all bidders and also observed to the econometrician. Private costs,

on the other hand, are bidders' private information and unobserved to the econometrician. The conditional independence assumption, as will be discussed subsequently, allows us to derive a Markov Perfect equilibrium (MPE) in the presence of an unobserved state.

When bidders' private cost distribution does not change and cost redraws are independent across period, the dynamic model in this paper readily accommodates my previous static multi-round auction model. The previous model is a special case in which bidders are not forward looking. Without being forward looking, bidders place bids in every round after they obtain their draws of new private costs. I prove that when the discount factor in my dynamic auction model is zero, I am back to the case without forward looking. My model yields some interesting predictions and implications. First, bidding prices uniformly decline over stages, because of the information about the secret reserve prices revealed in the previous stage. Numerical results show that the value function increases in the state variable, namely the belief about the reserve price. Second, my bidding model predicts that in the presence of a future bidding possibility, bidders bid less aggressively and therefore they increase their current bids.

There has been little empirical work on dynamic auction games. Existing studies on empirical estimation of auctions has largely restricted to a static auction setting. See Paarsch (1992), Laffont, Ossard and Vuong (1995), and Guerre, Perrigne and Vuong (2000) among others. An exception is Jofre-Bonet and Pesendorfer (2003) who first study the estimation approach of a dynamic auction game with capacity constraint as the state variable to analyze California highway auctions. In this chapter, my dissertation offers the second study of structural dynamic auction models.

To analyze the procurement auction data, and in addition to provide an empirical

framework within which the dynamic multi-round model with secret reserve prices can be analyzed, I develop a structural model from the theoretical model that I propose. My structural approach takes into account of the unobserved auction heterogeneity as I did in the model with non-forward looking bidders.

I adopt a two step estimation method. As the two-step estimation approach in the literature, my approach is computationally easy to implement. Two step estimation methods based on first order condition are well known in the literature. Although many studies apply the two step method, most of them are on static auction games.¹ Estimation of dynamic auction games remains limited with an exception of Jofre-Bonet and Pendorfer (2003). I distinguish my research from Jofre-Bonet and Pendorfer in the following ways. First, I analyze an auction game with multiple rounds and single auction object instead of a repeated auction game with different auction objects in different periods. Second, I explicitly estimate the reserve price distribution from the data. Third, I control for the unobserved heterogeneity when I estimate the distribution of bids.

I use my structural approach to analyze the INDOT data. Using the structural estimates, I carry out a counterfactual analysis by simulating the auctions with different government's reserve price release policies in the multi-round scenario. I find that whether using a secret reserve price or using a public reserve price depends on the bidder's attitude about the future. When the bidders are forward looking and their discount factor is sufficiently large, announcing the reserve price can be better than keeping it secret.

This chapter is organized as follows. In Section 2, I construct the model of dynamic multi-round procurement auctions with secret reserve prices, and solve the Markov Perfect

¹See Elyakime, Laffont, Loisel and Vuong (1994), Guerre, Perrigne and Vuong (2000) and Athey, Levin and Seira (2004) for applications in various auction studies.

equilibrium. I also investigate the implications from my model. In Section 3, I provide a two step estimation method for analyzing dynamic multi-round auction data. In Section 4, I apply the structural framework to analyze the INDOT data. In Section 5, I use counterfactual analysis to evaluate the government's reserve price policy. Section 6 concludes. All technical proofs are included in the Appendix.

2 The Model for Dynamic Multi-Round Auctions

In this section, I propose a game-theoretic model for dynamic multi-round procurement auctions with secret reserve prices, and derive the symmetric MPE.

2.1 The Stage Game

In the first stage, the government offers a single indivisible contract for sale. The events occur in the following order. First, the characteristics of the projects contained in the contract are revealed to bidders. An exogenous engineer estimate r_0 is kept secret and fixed throughout the auction rounds. The value of r_0 is drawn by the government from a distribution $G(\cdot)$. Second, N potential bidders draw their costs of conducting the contractual highway work privately and independently from a common distribution $F(\cdot)$. Third, bidders may submit bids. Lastly, the contract is awarded to the lowest bidder if the bid is at the same time lower than the reserve price. Bids are released regardless of whether the contract is awarded or not, while the reserve price is disclosed only if the contract is awarded.

If the contract is unsold, the auction proceeds to a reauction stage. In a reauction stage t , the government reauctions the same contract from the previous round. The reserve

price is kept secret and the same as before. Each bidder readjusts his private information and obtains a new random draw of c_t . Observing information from the previous round, bidders update their beliefs about the reserve price. Specifically, the lowest bid from the previous round denoted by s_t serves as a truncated upper bound of the reserve price. The upper bound of the reserve price s_t is a random variable to the bidders and changes over periods. It is used by bidders when forming new bids. Therefore it is a common state variable in the dynamic game. Only bidders with costs lower than the bound choose to submit bids. The game continues until the contract is awarded.

Note that from an econometric point of view, s_t is a state variable observed to both bidders and the econometrician. However, each individual bidder's cost realization c_t is not observed by the other bidders and the econometrician. Therefore I distinguish the observed and the unobserved state respectively. For further discussion, see Rust (1987). In Section 2.3, I give assumptions on the statistical properties of the state variables.

2.2 The Dynamic Structure

The common priors include the distribution of c , $F(\cdot)$ and the distribution of r_0 , $G(\cdot)$ on the support $[\underline{c}, \bar{c}]$. I use lower case to denote the corresponding density functions. The transition of states is a Markov process. Denote transition probability density function as $q(s_{t+1} | s_t)$ and transition function as $\omega(s_t)$ such that $\omega(s_t) = E(s_{t+1} | s_t)$. I will give the specific functional form of the transition function while I discuss the distribution of optimal bids. As a result, all bidders are identical a priori and the game is symmetric.

In order for me to adopt the framework of Markov dynamic decision processes,

conditional independence of the observed state and the latent cost is crucial in the data generating process. The role of conditional independence assumption is discussed in details in Rust (1987). I explain it in our context in the next section. I seek to find a symmetric MPE for an infinite period dynamic control problem. Conditional on the current state, the value function $V(\cdot)$ and the policy function $\beta(\cdot)$ are independent of time index.

2.3 The Value Function

In period t , bidder i chooses b to maximize his discounted expected payoff. The discount factor is denoted by δ ($0 < \delta < 1$). The value function is defined by the following equation

$$W_i(c_t, b_{-it}, s_t) = \max_b [(b - c_t) \Pr(i | b, b_{-it}, s_t) + \delta \Pr(\text{none} | b, b_{-it}, s_t) \int W_i(c_{t+1}, b_{-it+1}, s_{t+1}) p(c_{t+1}, s_{t+1} | c_t, s_t) dc_{t+1} ds_{t+1}] \quad (1)$$

Since we cannot observe c , in the subsequent analysis I use the ex ante value function which integrates out cost. In order for this to work, I make the conditional independence assumption, following the discussion in Rust (1987).

$$\mathbf{CI} \quad p(c_{t+1}, s_{t+1} | c_t, s_t) = q(s_{t+1} | s_t) \cdot f(c_{t+1} | c_t)$$

CI implies that given the previous state, the current observed state and private cost are independent. Note however, CI in our context is different from Rust (1987).² In addition, we make the following assumption.

$$\mathbf{A1} \quad c_{t+1} \perp c_t$$

A1 means while they are forward looking, future cost is random and not dependent

²Rust makes CI as $p(c_{t+1}, s_{t+1} | c_t, s_t) = q(s_{t+1} | s_t) \cdot f(c_{t+1} | s_{t+1})$, which implies that c_t is noise superimposed on the s_t process. This does not fit our case.

upon their current random draws.³ Bidders replace their costs in every round because of reasons such as opportunity cost and capacity constraint. A1 and CI imply the following conditional independence assumption adaptable to our context.

$$\mathbf{CI} \quad p(c_{t+1}, s_{t+1} | c_t, s_t) = q(s_{t+1} | s_t) \cdot f(c_{t+1})$$

Furthermore, we define the following ex ante value function

$$V_i(b_{-it}, s_t) = \int W_i(c_t, b_{-it}, s_t) f(c_t) dc_t. \quad (2)$$

From here on, as I seek to find a symmetric bidding strategy, I suppress subscript i and the dependence of the value function on the bidding strategies of the other bidders in the value function. With assumption CI', combining (1) and (2) leads to the following recursive equation

$$V(s_t) = \int \max_b [(b - c_t) \Pr(i | b, s_t) + \delta \Pr(\text{none} | b, s_t) \times E(V(s_{t+1}) | s_t)] \times f(c_t) dc_t. \quad (3)$$

In equation (3), s represents the belief about the reserve price. When s is larger, bidders are more optimistic about the reserve price. Furthermore, because of the symmetry of the bidding strategies and the value functions across bidders, the value functions of all bidders are uniformly higher. Hence intuitively, the value function is increasing in s . We will obtain the value function numerically to verify our intuition.

2.4 The Bid Distribution

Because the equilibrium bidding strategy relates the observed bids b to the unobserved private costs c which are random, bids are also random. Throughout my

³Statistical tests we perform in the data offer support for the random replacement assumption. The correlation of the ranks of the bidders in the first round and in the second round is about 0.30. Further test of the correlation of same bidders' bids across auction rounds shows that the correlation is 0.17.

analysis, I denote the distribution of bids b , $H(\cdot)$ which has a support $[\underline{b}, \bar{b}]$.

Furthermore, I can express the probability terms using the distributions of bids and reserve prices. Bidder i 's probability of winning the auction is defined as follows

$$\begin{aligned}\Pr(i | b, b_{-i}, s_t) &= \Pr(b < \min(b_{-i}) \text{ and } b < r_0 | s_t) \\ &= [1 - H(b)]^{N-1} \frac{G(s_t) - G(b)}{G(s_t)}.\end{aligned}$$

Denote $-i$ as the other bidders and $\min(b_{-i}) = b_{(N-1)}$ as the smallest order statistic in the other $N-1$ bids. Denote its distribution function as $M(\cdot)$ with corresponding density function $m(\cdot)$. The expression of the distribution of the order statistic $\min(b_{-i}) = b_{(N-1)}$ is $M(u) = 1 - [1 - H(u)]^{N-1}$, with density $m(u) = (N-1)[1 - H(u)]^{N-2} h(u)$.

The following lemma gives the probability of the event that no bidder wins the contract in period t conditional on the state s_t and greatly simplifies the computation burden in my subsequent estimation method.

Lemma 1 *The probability of the event that no bidder wins the contract given the state s in period t can be represented by*

$$\Pr(\text{none} | b, b_{-i}, s_t) = 1 - \Pr(i | b, b_{-i}, s_t) - \int_{\underline{c}}^{s_t} M(v) \frac{g(v)}{G(s_t)} dv.$$

Now I give the functional form of the transition probability density function and the motion function respectively. The transition probability function is given by the following equation

$$q(s_{t+1} | s_t) = N \cdot [1 - H(s_{t+1})]^{N-1} \cdot h(s_{t+1}) / [1 - (1 - H(s_t))^N].$$

The motion function is given by the following equation

$$\omega(s_t) = E(s_{t+1} | s_t) = \int s_{t+1} \cdot q(s_{t+1} | s_t) ds_{t+1}.$$

2.5 Markov Perfect Equilibrium

Next, I derive the MPE bidding strategies using the distribution function of bids. For easy exposition, I use s to denote the current period state and s' to denote the next period state. The first order condition is given by the following expression

$$\begin{aligned}
0 = & [1 - H(b)]^{N-1} \frac{[G(s) - G(b)]}{G(s)} - (N-1)(b-c)[1 - H(b)]^{N-2} \frac{[G(s) - G(b)]}{G(s)} h(b) \\
& - (b-c)[1 - H(b)]^{N-1} \frac{g(b)}{G(s)} + \delta E(V(s') | s) \\
& \cdot \{-(N-1)[1 - H(b)]^{N-2} \frac{G(b)}{G(s)} h(b) + [1 - H(b)]^{N-1} \frac{g(b)}{G(s)} \\
& + \frac{1}{G(s)} \int_c^s m(b)g(v)dv\}.
\end{aligned}$$

Multiplying the equation by $G(s)/\{[1 - H(b)]^{N-1}[G(s) - G(b)]\}$, defining hazard rates for bid and reserve price $\lambda(b) = h(b)/[1 - H(b)]$ and $\mu(r | s) = g(r)/[G(s) - G(r)]$ respectively, I get the following

$$\begin{aligned}
0 = & 1 - (N-1)\lambda(b)(b-c) - \mu(b | s)(b-c) \\
& + \delta E(V(s') | s) \cdot \{-(N-1)\lambda(b) \frac{G(b)}{G(s) - G(b)} + \mu(b | s) \\
& + \frac{m(b)G(s)}{[1 - H(b)]^{N-1}[G(s) - G(b)]}\}.
\end{aligned}$$

Some algebra yields

$$b = c + \frac{1}{(N-1)\lambda(b) + \mu(b | s)} + \delta E(V(s') | s). \quad (4)$$

Note the bidder's markup includes two terms. The second term on the right hand side is the markup in the current period, the third term is the discounted markup in the future.

Proposition 1 *Under the monotone hazard rate assumption, equilibrium bids in the*

dynamic model are increasing in the discount factor δ .

Equation (4) has the following implications. First, as stated in the above proposition, equilibrium bids are increasing in δ . The larger the discount factor is, the more patient the agents are about the future, therefore the higher markup the bidders will add to their costs while placing bids. Second, the larger the state s is, the higher the bound of the truncation, in other words the less restrictive (or competitive) the secret reserve price to the bidders. Therefore intuitively, bidders increase bids when s is high. With unknown function $V(\cdot)$, it is not easy to show it analytically. However, I will show it by the numerical result through my empirical analyses.

Proposition 2 *If $\delta = 0$ and the private cost distribution does not change, the following two results hold.*

(i) bidders' dynamic control problem is equivalent to non-forward looking problem. One can solve the game stage by stage to get the separate equilibrium.

(ii) In the non-forward looking case, the equilibrium bid in stage t has the following expression

$$b = c + \frac{1}{(N-1)\lambda(b) + \mu(b|s)}. \quad (5)$$

The implication of the above proposition is two-fold. First, it shows that the dynamic model with forward-looking accommodates the static model without forward looking, when private cost distributions do not change across stages. No forward looking means that bidders do not care about the same auction in the future. Second, with forward looking, bidders add more to the costs when they bid in the current period. The second result is stated in the corollary below. The additional part as seen in equation (4) explains the future possibility of winning this auction.

Corollary 1 *When the private cost distributions are the same and the monotone hazard rate assumption holds, bidders bid more when they are forward looking than when they are not forward looking.*

The intuition is when a bidder is forward looking, he claims that his type is higher than his actual type by adding a future expectation to it. In other words, he bids less aggressively. In the context of procurement auctions, he bids more.

Proposition 3 *In the multi-round dynamic auction game, at equilibrium at the same state level on the whole common state space, the bid function in one period is less than or equal to the bid function in the previous period, i.e., $\beta_t(c) \leq \beta_{t-1}(c)$ for all c on the common support.*

This result is consistent with the result derived from the non-forward looking case. It is also consistent with the two-round auction data and supported by the reduced-form regression analysis (see Chapter 2).

3 Estimation Method

This section gives a discussion of my estimation method of the dynamic multi-round auction model. First, I describe the two-step estimation approach based on the distribution of equilibrium bids and the first order condition of optimally chosen bids. Then I describe the identification of the distribution function of privately known costs briefly.

3.1 Estimation Approach

I observe data on bids, contract characteristics, number of potential bidders and bidders' state variable. My objective is to infer private costs. I propose a two step

estimation method which is computationally easy in that it does not require solving the equilibrium bid functions. In the first step, I estimate the distribution of equilibrium bids. In the second step, based on the equilibrium first order condition, I obtain the private costs and its distribution. This method, as the other two-step methods seen in auction literature, assumes that the observed bids are generated by equilibrium play and satisfy the first order condition of equilibrium bids.

Two step estimation methods based on the first order condition are well known in the literature. See Elyakime, Laffont, Loisel and Vuong (1994), Guerre, Perrigne and Vuong (2000) and Athey, Levin and Seira (2004) for applications in various auction studies. Although many studies apply the two step method, most of them are on static auction games. Estimation of dynamic auction games remains limited with an exception of Jofre-Bonet and Pendorfer (2003).

From the first order condition, I have derived equation (4) for the optimal bids. This leads to the following equation for the private cost

$$\varphi(b | s) = b - \frac{1}{(N-1)\lambda(b) + \mu(b | s)} - \delta E(V(s') | s). \quad (6)$$

Equation (6) provides an explicit expression of the private costs that involves bid, the hazard function of bid, the hazard function of reserve price and the value function. Parallel to the former analysis of the markup of equilibrium bids, equation (6) states that the cost equals the bid minus a mark down. The mark down has two parts. The first part accounts for the level of competition in the current auction round. The second part accounts for the future discounted profit if firm i wins the contract. Equation (6) can be used to recover the distribution of private costs.

In order to infer the distribution of costs, I need estimators for the functions

appearing in the right hand side of (6). These functions are: the bid hazard function, the secret reserve price hazard function, the discount factor, the transition function and the value function. I discuss my strategy to find the appropriate estimators for them as follows. First, an estimator of the bid distribution function can be directly obtained from the data on bids and observed state variable and other heterogeneity. Therefore the bid hazard function can be obtained. Second, the secret reserve price hazard function can also be obtained directly from the data on reserve prices. Third, I choose different discount factors and examine how sensitive the estimates are to variations in the discount factor. Fourth, the transition function of the state is the distribution function of the random smallest order statistic from the truncated bid distribution on $[\underline{b}, s]$. In other words, $\omega(s)$ is drawn from a probability distribution function defined by $[1 - [1 - H(\cdot)]^N] / [1 - [1 - H(s)]^N]$. Hence the transition of state is estimated along with the distribution function of bids. Finally, I need to recover the value function. The value function is defined in equation (3). However, equation (3) involves latent cost variables and endogenous decisions of other bidders. I give my method of how to recover the value function later.

3.2 Reserve Price and Bid Distributions

Based on the theoretical auction model, I parameterize two distributions: the reserve price distribution $G(\cdot)$ and the bid distribution $H(\cdot)$. In an econometric framework, asymptotic statistical inference is based on a large number of auctions. Let L be the number of auctions. For the ℓ -th auction, let $G_\ell(\cdot)$ and $H_\ell(\cdot)$ denote each primitive distribution respectively with corresponding densities $g_\ell(\cdot)$ and $h_\ell(\cdot)$, $j = 1, 2$. Assume that $G_\ell = G(\cdot | x_\ell, u_\ell, \gamma)$ and $H_\ell = F(\cdot | x_\ell, u_\ell, \theta)$, where x_ℓ is a vector of variables that I

use to control for the observed auction heterogeneity, and u_ℓ is a scalar variable that represents the unobserved auction heterogeneity, both affecting the government's reserve price as well as the bidders' bids, γ is a vector of unknown parameters in $\Gamma \subset \mathbb{R}^K$, and θ is a vector of unknown parameters in $\Theta \subset \mathbb{R}^K$. I assume that u is independent of x , and has a distribution $W(\cdot | \sigma)$ with $w(\cdot | \sigma)$ being the density function, where σ is a vector of unknown parameters in $\Sigma \subset \mathbb{R}^m$.

Conditional on both observed and unobserved heterogeneity x and u , I specify the reserve price distribution and the bid distribution as exponential as follows

$$g_\ell(r | x_\ell, u_\ell, \gamma) = \frac{1}{\exp(x_\ell \gamma + u_\ell)} \exp\left(\frac{-r}{\exp(x_\ell \gamma + u_\ell)}\right), \quad (7)$$

$$h_\ell(b | x_\ell, u_\ell, \theta) = \frac{1}{\exp(x_\ell \theta + u_\ell)} \exp\left(\frac{-b}{\exp(x_\ell \theta + u_\ell)}\right), \quad (8)$$

where $b \in (0, \infty)$ and $r \in (0, \infty)$. By including the intercept in x , I normalize the unobserved heterogeneity term u such that $E[u] = 0$. I assume that $u \sim N(0, \sigma^2)$, where σ^2 is an unknown parameter.

3.3 The value function

Note the estimation of the bid hazard function and the transition function is based on the distribution of observed bids from the first step. The key idea is if I can express the value function in terms of the distribution of bids, then I can get an estimator of the value function. The following proposition states that the value function can be represented as a recursive equation involving the bid distribution function.

Proposition 4 *Given the distribution of the equilibrium bids, the value function can be*

represented as follows

$$\begin{aligned}
 V(s) = & \int_b^s \frac{\Pr(i | b, s)}{(N-1)\lambda(b) + \mu(b | s)} h(b) db \\
 & + \delta \left[\int_b^s [\Pr(\text{none} | b, s) + \Pr(i | b, s)] h(b) db \right] \times E(V(s') | s).
 \end{aligned} \tag{9}$$

The representation of the value function in equation (9) contains two parts. The first part accounts for the bidder's current expected profits. The second part accounts for the bidder's sum of discounted expected future profits. The proof of the proposition is based on two observations. First, I may write the probability of winning as a function of the distribution of bids by other bidders, ignoring dependence of other bidders' bids on cost draws. Thus each bidder's dynamic game is reduced to a single agent dynamic decision problem where each bidder maximizes the discounted sum of future payoffs using the equilibrium bid distribution associated with other bidders. Still, this single agent dynamic decision problem does involve the latent cost. My second observation is that the first order condition of optimal bids gives an explicit expression of the bidder's costs in terms of his equilibrium bids and the equilibrium bids distribution. Substituting this expression into the value function yields an expression involving the distribution and the density of equilibrium bids only.

Next I develop numerical methods to approximate the value function based on equation (9). Furthermore, my methods are computationally easy for two reasons. First, the symmetric Markovian strategies require that bidders with the same state follow the same bidding strategy and the observed state variable s is a common random variable for all bidders. Therefore the ex ante value function $V(s)$ is common to all bidders in the same period. Second, the transition function $\omega(s)$ is continuous in s and therefore the value

function is continuous in s as well.

Specifically, I numerically evaluate the expected current period payoff as follows

$$A(s) = \int_{\underline{b}}^s \frac{[1 - H(b)]^{N-1} \frac{[G(s) - G(b)]}{G(s)}}{(N-1)\lambda(b) + \mu(b|s)} h(b) db.$$

Complexity arises from the computation of the integration. I adopt Monte Carlo sampling method to evaluate the integration as I obtain an estimator of the distribution of bids $h(b)$. I draw a large number of b^j ($s = 1, 2, 3 \dots J$) from $H(b|s)$ on the support $[\underline{b}, s]$. The integration is the expectation of the integrand approximated by the sample analogue

$$A(s) = \frac{1}{J} \sum_{j=1}^J H(s) \left[\frac{[1 - H(b^j)]^{N-1} \frac{[G(s) - G(b^j)]}{G(s)}}{(N-1)\lambda(b^j) + \mu(b^j|s)} \right].$$

Using Lemma 1, I can show that the probability terms in equation (recursive value) sum to $H(s) - \frac{H(s)}{G(s)} \int_{\underline{c}}^s M(v)g(v)dv$. Then I numerically evaluate the discounted sum of future payoffs similarly by the Monte Carlo sampling approach. The probability terms in the bracket in equation (9) are defined as $B(s)$. The following lemma establishes the numerical method of evaluating $B(s)$ which greatly simplifies the computation.

Lemma 2 *It can be shown that the probability term $B(s)$ can be calculated by the following equation*

$$B(s) = \frac{H(s)}{I} \sum_{i=1}^I (1 - H(u^i))^{N-1},$$

where u^i is drawn from $G(u|s)$ on the support $[\underline{c}, s]$.

Finally, I iteratively solve for $V(s)$ based on the recursive equation in proposition 3 and the numerical method. To evaluate the value function at any value s , start with $s^0 = s$

and obtain $A(s^0)$ and $B(s^0)$. Then obtain $s^1 = \omega(s^0)$ by the transition function. $V(s^0)$ is given by $V(s^0) = A(s^0) + \delta B(s^0)E(V(s^1) | s^0)$. Take a step further, calculate $A(s^1)$, $B(s^1)$ and s^2 , and then substitute $A(s^1) + \delta B(s^1)V(s^2)$ for $V(s^1)$. After the 1-st substitution, I get $V(s^0) = A(s^0) + \delta B(s^0)A(s^1) + \delta^2 B(s^0)B(s^1)E(V(s^2) | s^1)$. Stop at the m -th iterative substitution when the last term that involves s^m dies out. Sum up all terms to get $V(s^0)$.

3.4 The Distribution of Private Costs

With the distribution function of bids at hand, we can infer bidders' private costs. To see how I infer the distribution of costs, notice the following. First, there is a relationship between the distribution function of costs and the distribution function of bids given by $F(c | s) = H(b | s)$. Second, the inverse of the bid function conditional on state variables, $c = \varphi(b | s)$, is given in equation (6). Thus, using these two relationships, I can specify my estimator of the cost distribution function as

$$F(c | s) = \int_{\{b | \varphi(b | s) \leq c\}} dH(\cdot | s). \quad (10)$$

Given that φ is invertible, the estimator can be written as $F(c | s) = H(\varphi^{-1}(c | s) | s)$.

Standard errors of estimates are calculated using the delta method.

Before I end this section, I briefly discuss the identification issue. In a dynamic structural auction model, I try to identify two primitives: the private cost distribution function $F(\cdot)$ and the discount factor δ . In my model, knowing the random bids can not jointly identify the distribution of costs and the discount factor, which is similar to other dynamic estimation approaches (see Rust (1994)). However, given δ , I can identify the

distribution of cost which is the same as Jofre-Bonet and Pensendorfer (2003).

4 Results

This section presents the estimation results. I first report the estimates of the bid distribution function under control of unobserved heterogeneity. The estimation method is simulated maximum likelihood approach. I then discuss the estimates of the value function. Lastly, I discuss the inferred costs.

4.1 Estimates of the Reserve Price and Bid Distribution Functions

The parameters of the reserve price distribution γ and the parameter of the unobserved heterogeneity σ can be jointly estimated based on (7). I draw a large sample, namely $S = 1000$, of u_ℓ^s s from $N(0,1)$, *i.e.*, $\phi(u_\ell)$, and adopt importance sampling to implement the SML. Furthermore, I gain the standard errors through bootstrap. The results are reported in Table 3.1 (see Chapter 3).

Next I estimate parameters in bid distributions θ based on (8). I simulate u_ℓ from $\hat{\theta}$. Here the number of u_ℓ^s s that I draw is $S = 1000$. Using bootstrap, I obtain the standard errors of the estimates. The results of the estimation are reported in Table 4.1.

The results indicate that all variables that I pick up have significant effects on bids. I evaluate the effects at the mean of the bid distribution which is \$516,210. Increases in the length of the bridges and the time needed to accomplish the projects raise private costs, and in turn increase bids, as expected. Specifically, holding all the other factors constant, increasing the length of the project by one meter (or 3.28 feet) will increase the mean bid by 0.23% or about \$1,187. One more working day needed for a project will increase the

mean bid approximately by 0.53% or \$2,633. Furthermore, rising in the DBE percentage results in higher bid. This is reasonable because higher DBE percentage increases the primary contractor's transaction cost in a project by finding and subcontracting partial work to a DBE firm. More specifically, one unit increase in DBE will increase the mean bid by about 4% or slightly more than \$20,600. The number of projects (np) has a positive effect on bids. One unit increase in the number of projects can increase the mean bid by 28% which is slightly more than \$144,000. Bridges of a steel structure cause about \$139,000 more than bridges of other structures on the mean bid. Furthermore, the estimate of the unobserved heterogeneity parameter is strongly significant, meaning that there exists unobserved auction heterogeneity in my data set.

4.2 Estimates of Value Function

Empirically, the value function depends on the state variable and the auction heterogeneity. I can approximate the value function for each auction ℓ . I depict the value function in figure 4.1 at the mean of the auction characteristics and $\delta = 0.95$ for illustration.

The empirical results reveal some features of the value function. First, from the graph the value function is increasing in the state s . This finding reinforces my former intuition. The higher the bidder's state, the larger the value function is. The state s represents the bidder's belief of the reserve price. A high belief of the reserve price leads to a high markup. Finally such an optimistic bidding results in a high value. Second, I try different values of the discount factor δ . The results of the value function do not differ much. To find the reason, I decompose the contribution to the value function into $A(s)$ and

$B(s)$. $A(s)$ is the current payoff. $B(s)$ is the probability of winning in the future. I find that the first part $A(s)$ accounts for a dominant part of contribution. $B(s)$ is a very small number on the other hand. In other words, in symmetric bidding bidder i has a much stronger belief that he wins in the current round. Most of his bidding efforts are paid to the current round. Hence the discount factor does not affect the value function dramatically.

4.3 Estimates of Costs

To illustrate the estimates of the distribution, I fix the value of the auction characteristics at the sample means and $\delta = 0.95$. I first depict the bid function. Then I discuss the markup and depict the distribution function of costs with 95% confidence interval.

Figure 4.2 depicts the equilibrium bids versus costs. The bid function is estimated using equation (4). The bid function is plotted by fixing the auction characteristics at the sample mean and state variable at the upper bound of the reserve price and varying the cost. In addition to the bid function, the 45 degree line is reported. As is evident in the figure, the bid increases with the cost.

The markup denotes the difference between the bid and the cost of a bidder. In the figure, the markup is the distance between the bid and the 45 degree line. Note that the bid line is almost parallel to the 45 degree line. This further strengthens that bidders weigh dominantly on the current proportion of the markup as I have shown in the analysis of the value function. In particular, the current proportion is constant in the exponential specification. To some extent, it reflects bidders' opinion of winning today versus winning later.

Figure 4.3 depicts the distribution function of costs. The distribution function is obtained by using equation (10). The distribution function is reported at the sample mean of the auction characteristics. I also compute the standard errors by delta method. The dash lines represent the 95% confidence interval.

Note that in order to obtain the estimates of private costs, we have to fix the discount factor. Next I change the discount factor to gain different sets of inferred costs. Correspondingly, we can examine how the discount factor affects the inferred costs. I try different values of the discount factor and find that the larger the discount factor, the smaller the inferred costs. This is reasonable because increasing the discount factor results in greater markup thereby leading to smaller inferred costs. Figure 4.4 illustrates the distributions of costs with $\delta = 0.3$ and $\delta = 0.9$ respectively. The distribution function associated with $\delta = 0.9$ stochastically dominates the distribution function associated with $\delta = 0.3$.

5 Counterfactual Analysis

In this section, I conduct counterfactual analysis. Secret reserve prices are used in the INDOT highway auctions. After I recover the cost distribution, I can estimate the procurement cost under the use of public reserve price by simulation. I find that with use of public reserve price, the INDOT slightly save some costs.

Motivated by the INDOT data feature, my model has focused on the use of the secret reserve price by the government. Alternatively, the government can make the engineer's estimate public and use it as a public reserve price. In this scenario, the government can find no bids submitted if all bidders' private costs are above the public

reserve price. Thus the government can re-auction the project in the next round with the same public reserve price. As a result, under the random cost replacement assumption, the multi-round feature can be accommodated by both secret and public reserve prices. It would be interesting to compare the welfare implications of these two mechanisms using a counterfactual analysis. Such a comparison allows us to evaluate the INDOT's auction mechanism and assess the efficiency of its current reserve price policy. Since I have uncovered the underlying cost distribution, I can conduct simulations under the two different reserve price release policies and compare the government's payment under the two different scenarios. However, as the cost distribution depends on the discount factor, I vary the discount factor to conduct simulations.

I construct a representative auction by setting all observed characteristics at the sample means of the corresponding covariates. The simulation of the secret reserve price can be done directly with use of the estimated bid distribution. Hence it does not involve the cost distribution, neither the discount factor. However, the simulation of the public reserve price involves the cost distribution. I vary the discount factor. For each value of the discount factor, I obtain the corresponding simulation result of the public reserve price.

I report some of the results of my simulation in Table 4.2. My simulation produces several interesting findings. First, using a public reserve price, the expected procurement cost is decreasing in the discount factor.⁴ This is consistent with my previous finding that the inferred costs decrease in the discount factor. Because the cost distribution function with a larger discount factor dominates the distribution with a smaller discount factor, using the former leads to a lower winning bid in simulation than using the latter, keeping all the other factors constant. Second, it reveals the effects of the two reserve price policies

on the procurement cost, the implication of which is two-fold. First, the lower the discount factor, the greater the advantage by keeping the reserve price secret. Second, as the discount factor increases to a sufficient value, we see a completely opposite result: announcing the reserve price public is better than keeping it secret. The cutoff value of the discount factor is about 0.9 according to the simulation.

On average, when $\delta = 0.1$, the INDOT can save about \$12,000 on a typical bridge work auction by adopting a secret reserve price. This number is comparable to the number 13,641 in the third chapter in view of $\delta = 0$. Hence our finding indicates that the use of secret reserve price may be a good policy in practice in procurement auctions when bidders are not forward looking or not so forward looking (the discount factor is low). However, as the bidders care more and more about the future, secret reserve price policy loses its advantage. The reason is simple. When they are forward looking, as we have seen earlier, the bidders increase the markups. Furthermore, the greater the value of the discount factor, the higher the markups.

6 Conclusion

In this chapter, I study multi-round auctions with secret reserve prices in a dynamic framework. This chapter is an extension of my previous chapter on multi-round auctions with secret reserve price in a static framework. I prove that the static model is a special case of the dynamic model in which the discount factor is zero. My model yields some predictions that can be empirically tested, such as that the equilibrium bids decline uniformly over various stages. Also, in the dynamic auction model, because of forward looking bidders may increase their current bids.

⁴ It would be strictly monotone had there been no simulation error.

I develop a structural approach to analyze the INDOT data. The structural approach recovered the distributions of the reserve prices and the private cost. The estimates for structural parameters allow us to conduct counterfactual analyses. I find that whether using a secret reserve price or using a public reserve price depends on the bidder's attitude about the future. In particular, the INDOT could have saved budgets by adopting a public reserve price rather than using a secret reserve price when the bidders are forward looking and have a sufficient large discount factor. This chapter offers insights into the use of reserve prices in multi-round auctions with forward looking bidders and the strategic changes in bidders' bidding strategies.

APPENDIX

1. Proof of Lemma 1

$$\begin{aligned}
\Pr(\text{none} | b, b_{-i}, s) &= \Pr[(r_0 < b < \min(b_{-i}) | s) \cup (b > \min(b_{-i}) > r_0 | s)] \\
&= [1 - H(b)]^{N-1} \frac{G(b)}{G(s)} + \frac{1}{G(s)} \cdot \int_{s > b_{(N-1)} > v} m(u) \cdot g(v) dudv \\
&= [1 - H(b)]^{N-1} \frac{G(b)}{G(s)} + \frac{1}{G(s)} \int_{\underline{c}}^s \int_v^b m(u) g(v) dudv \\
&= [1 - H(b)]^{N-1} \frac{G(b)}{G(s)} + \frac{1}{G(s)} \int_{\underline{c}}^s [M(b) - M(v)] g(v) dv \\
&= [1 - H(b)]^{N-1} \frac{G(b)}{G(s)} + M(b) - \frac{1}{G(s)} \int_{\underline{c}}^s M(v) g(v) dv \\
&= [1 - H(b)]^{N-1} \frac{G(b)}{G(s)} + 1 - [1 - H(b)]^{N-1} \\
&\quad - \frac{1}{G(s)} \int_{\underline{c}}^s M(v) g(v) dv \\
&= 1 - [1 - H(b)]^{N-1} \frac{G(s) - G(b)}{G(s)} - \frac{1}{G(s)} \int_{\underline{c}}^s M(v) g(v) dv.
\end{aligned}$$

Thus the result immediately follows.

2. Proof of Proposition 1

The monotone hazard rate is standard in auction literature, it requires that $f(\cdot)/(1 - F(\cdot))$ is an increasing function. This is the same for $G(\cdot)$. This is equivalent to $g'[1 - G] \geq -g^2$.

With a few steps of algebra, it can be shown that $g'[G(s) - G] \geq -g^2$. This implies that $\mu(\cdot | s)$ is increasing too. The implicit function in equation (4) is an increasing function.

Therefore we conclude that it is an increasing function in c and δ .

3. Proof of Proposition 2

When $\delta = 0$, the dynamic control problem is reduced to the following

$$W(c, b_{-it}, s_{t-1}) = \max_b [(b - c) \Pr(i | b, b_{-it}, s_{t-1})],$$

which implies that the optimal bidding strategy can be solved stage by stage.

The problem in stage t is

$$\max (b - c) [1 - S(b)]^{N-1} \frac{G(s) - G(b)}{G(b)}.$$

Defining hazard rates as for bid and reserve price $\lambda(b) = h(b)/[1 - H(b)]$ and $\mu(r | s) = g(r)/[G(s) - G(r)]$ respectively, the first order condition implies the representation in the proposition.

4. Proof of Corollary 1

Note in non-forward looking case, I can generally distinguish the distributions of bids and therefore the bidding functions from stage to stage.

If the private cost distributions are the same, we get the same functional forms for the equilibrium bids. With the monotone hazard rate assumption, bid is an increasing function in c and $\delta E(V(s') | s)$. Because $c + \delta E(V(s') | s) > c$, the bid with forward looking is larger than the bid without forward looking.

4. Proof of Proposition 3

We derive the First order condition using the cost distribution. Define $b = \beta(c)$. Then we have

$$\begin{aligned}
0 &= [1 - F(c)]^{N-1} \frac{[G(s) - G(b)]}{G(s)} - (N-1)(b-c) \\
&\quad \cdot [1 - F(c)]^{N-2} \frac{[G(s) - G(b)]}{G(s)} \frac{f(c)}{\beta'(c)} \\
&\quad - (b-c)[1 - F(c)]^{N-1} \frac{g(b)}{G(s)} + \delta EV(s' | s) \\
&\quad \cdot \{ -(N-1)[1 - F(c)]^{N-2} \frac{G(b)}{G(s)} \frac{f(c)}{\beta'(c)} + [1 - F(c)]^{N-1} \frac{g(b)}{G(s)} \\
&\quad + \frac{1}{G(s)} \int_c^s m(b)g(v)dv \}.
\end{aligned}$$

The distribution of smallest order statistic $b_{(N-1)}$ is corresponding to the smallest order statistic $c_{(N-1)}$. Therefore, I have

$$m(\beta(c)) = (N-1)[1 - S(\beta(c))]^{N-2} s(\beta(c)) = (N-1)[1 - F(c)]^{N-2} \frac{f(c)}{\beta'(c)}. \quad (\text{A.1})$$

Substituting this into (A.1) result in the following

$$\begin{aligned}
0 &= [1 - F(c)]^{N-1} \frac{[G(s) - G(b)]}{G(s)} - (N-1)(b-c) \\
&\quad \cdot [1 - F(c)]^{N-2} \frac{[G(s) - G(b)]}{G(s)} \frac{f(c)}{\beta'(c)} \\
&\quad - (b-c)[1 - F(c)]^{N-1} \frac{g(b)}{G(s)} + \delta EV(s' | s) \\
&\quad \cdot \{ -(N-1)[1 - F(c)]^{N-2} \frac{G(b)}{G(s)} \frac{f(c)}{\beta'(c)} + [1 - F(c)]^{N-1} \frac{g(b)}{G(s)} \\
&\quad + (N-1)[1 - F(c)]^{N-2} \frac{f(c)}{\beta'(c)} \}.
\end{aligned}$$

Multiplying by $G(s)\beta'(c)$ and rearranging, we get the following

$$\begin{aligned}
0 &= [1 - F(c)]^{N-1} [G(s) - G(b)]\beta'(c) - (N-1)(b-c - \delta EV(s' | s)) \\
&\quad \cdot [1 - F(c)]^{N-2} [G(s) - G(b)]f(c) \\
&\quad - (b-c - \delta EV(s' | s))[1 - F(c)]^{N-1} g(b)\beta'(c).
\end{aligned}$$

Note that $b = \beta(c)$. We can rewrite the above equation as

$$\begin{aligned}
& \frac{d}{dc} \{ \beta(c) [1 - F(c)]^{N-1} [G(s) - G(b)] \} \\
&= (c + \delta EV(s' | s)) \frac{d}{dc} \{ [1 - F(c)]^{N-1} [G(s) - G(b)] \}.
\end{aligned}$$

Integrating over $[\underline{c}, s]$ and using the boundary condition $\beta(s) = s$, we obtain

$$\beta(c) = c + \delta V + \frac{\int_{\underline{c}}^s [1 - F(x)]^{N-1} [G(s) - G(\beta(x))] dx}{[1 - F(c)]^{N-1} [G(s) - G(\beta(c))]} .$$

In order to compare b_t and b_{t-1} , I add subscript to s to distinguish bidding strategies in different periods. At the symmetric MPE, b_t is the optimal choice in period t . Therefore I have the following inequality

$$\begin{aligned}
& \frac{1}{G(s_t)} \{ (b_t - c) [1 - F(c)]^{N-1} (G(s_t) - G(b_t)) + \delta V \\
& \cdot \left([1 - F(c)]^{N-1} G(s_t) + \int_{\underline{c}}^{s_t} [M(b_t) - M(v)] g(v) dv \right) \} \\
& \geq \frac{1}{G(s_t)} \{ (b_{t-1} - c) [1 - F(c)]^{N-1} (G(s_t) - G(b_{t-1})) + \delta V \\
& \cdot \left([1 - F(c)]^{N-1} G(s_t) + \int_{\underline{c}}^{s_t} [M(b_{t-1}) - M(v)] g(v) dv \right) \}.
\end{aligned}$$

Similarly, b_{t-1} is the maximizer in period $t - 1$. The following inequality holds

$$\begin{aligned}
& \frac{1}{G(s_{t-1})} \{ (b_{t-1} - c) [1 - F(c)]^{N-1} (G(s_{t-1}) - G(b_{t-1})) + \delta V \\
& \cdot \left([1 - F(c)]^{N-1} G(s) + \int_{\underline{c}}^{s_{t-1}} [M(b_{t-1}) - M(v)] g(v) dv \right) \} \\
& \geq \frac{1}{G(s_{t-1})} \{ (b_t - c) [1 - F(c)]^{N-1} (G(s_{t-1}) - G(b_t)) + \delta V \\
& \cdot \left([1 - F(c)]^{N-1} G(s_{t-1}) + \int_{\underline{c}}^{s_{t-1}} [M(b_t) - M(v)] g(v) dv \right) \}.
\end{aligned}$$

Rewrite the second inequality as

$$\begin{aligned}
& (b_{t-1} - c)[1 - F(c)]^{N-1} (G(s_{t-1}) - G(s_t) + G(s_t) - G(b_{t-1})) \\
& + \delta V \cdot ([1 - F(c)]^{N-1} G(s_{t-1}) + \int_c^{s_t} [M(b_{t-1}) - M(v)]g(v)dv \\
& + \int_{s_t}^{s_{t-1}} [M(b_{t-1}) - M(v)]g(v)dv) \\
\geq & (b_t - c)[1 - F(c)]^{N-1} (G(s_{t-1}) - G(s_t) + G(s_t) - G(b_t)) \\
& + \delta V \cdot ([1 - F(c)]^{N-1} G(s_t) + \int_c^{s_t} [M(b_t) - M(v)]g(v)dv \\
& + \int_{s_t}^{s_{t-1}} [M(b_t) - M(v)]g(v)dv).
\end{aligned}$$

Decompose the above inequality and use the first inequality to get the following inequality

$$\begin{aligned}
& (b_{t-1} - c)[1 - F(c)]^{N-1} (G(s_{t-1}) - G(s_t)) \\
& + \delta V \int_{s_t}^{s_{t-1}} [M(b_{t-1}) - M(v)]g(v)dv \\
\geq & (b_t - c)[1 - F(c)]^{N-1} (G(s_{t-1}) - G(s_t)) \\
& + \delta V \int_{s_t}^{s_{t-1}} [M(b_t) - M(v)]g(v)dv.
\end{aligned}$$

Note that both sides of the inequality are the same increasing function that can be written as $\Pi(b_{t-1}) \geq \Pi(b_t)$, which implies that $b_{t-1} \geq b_t$.

5. Proof of Proposition 4

First, I may write the probability of winning as a function of the distribution of bids by other bidders, ignoring dependence of other bidders' bids on cost draws. Thus each bidder's dynamic game is reduced to a single agent dynamic decision problem where each bidder maximizes the discounted sum of future payoffs taking as given the equilibrium bid distribution associated with other bidders. Still, this single agent dynamic decision problem does involve the latent cost. My second observation is that the first order condition of optimal bids gives an explicit expression of the bidder's costs in terms of his equilibrium bids and the equilibrium bids distribution. Substituting this expression into the value

function yields an expression involving the distribution and the density of equilibrium bids only.

6. Proof of Lemma 2 Following Lemma 1, one can easily calculate and get $B(s)$.

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Table 4.1 SML Estimates of bid Distribution and Unobserved Heterogeneity

Variable	Coef.	Std. Err.
dbe	0.0403*	0.0012
time	0.0051*	0.0002
np	0.284*	0.0077
steel	0.274*	0.0066
length	0.0023*	0.0001
_cons	11.533*	0.0636
σ^2	0.054*	0.0016

*: significant at 5%

Table 4.2 Counterfactual Analysis on the Reserve Price Policy

δ	0.1	0.3	0.5	0.7	0.9
d^*	-11992	-8229	-10326	-4498	509

*expected procurement cost under the secret reserve price minus the expected procurement cost under the public reserve price

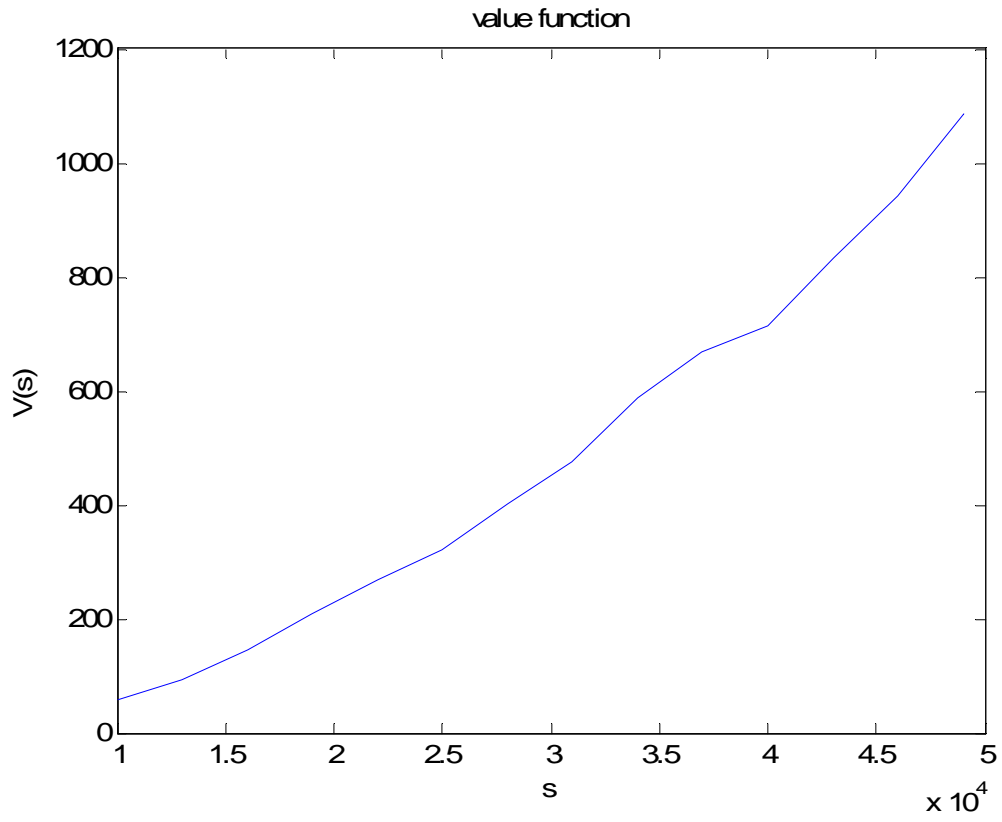


Figure 4.1 The Value Function

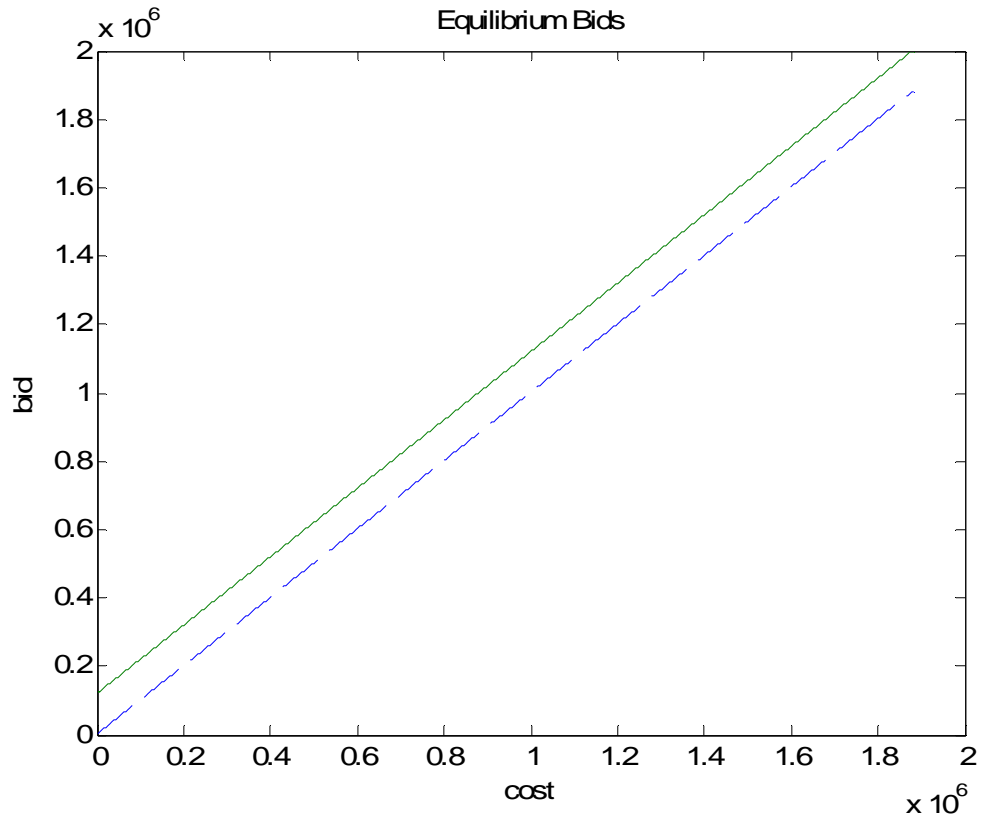


Figure 4.2 The Bidding Function

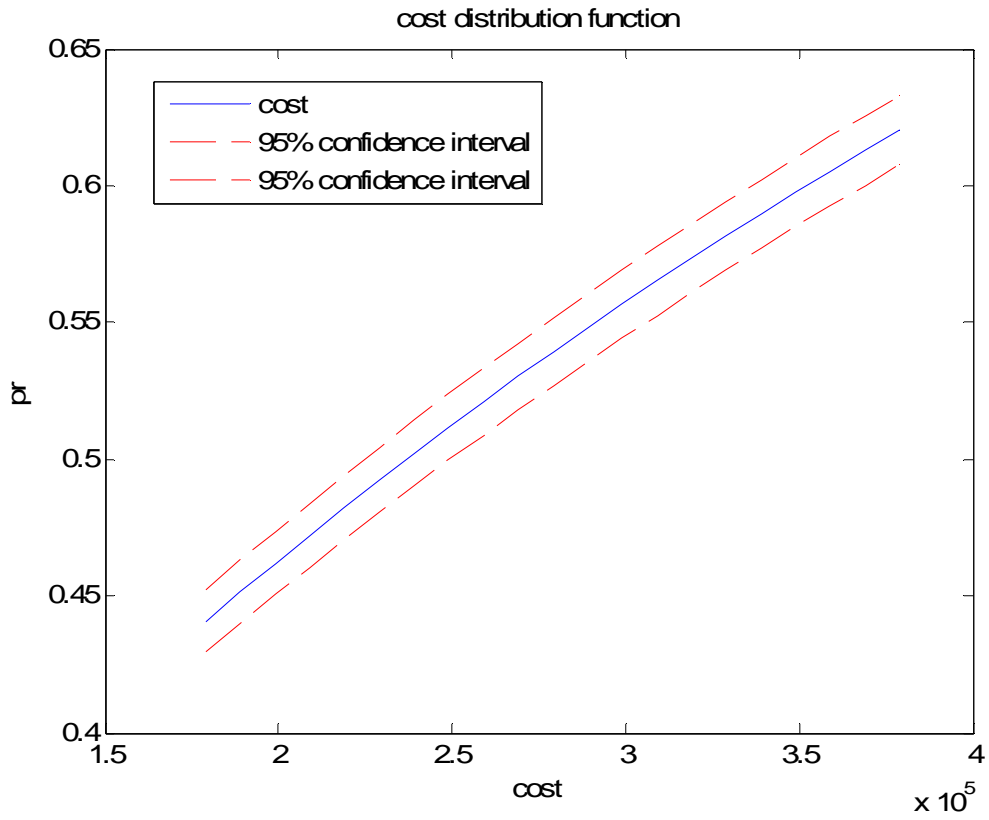


Figure 4.3 The Distribution of Costs

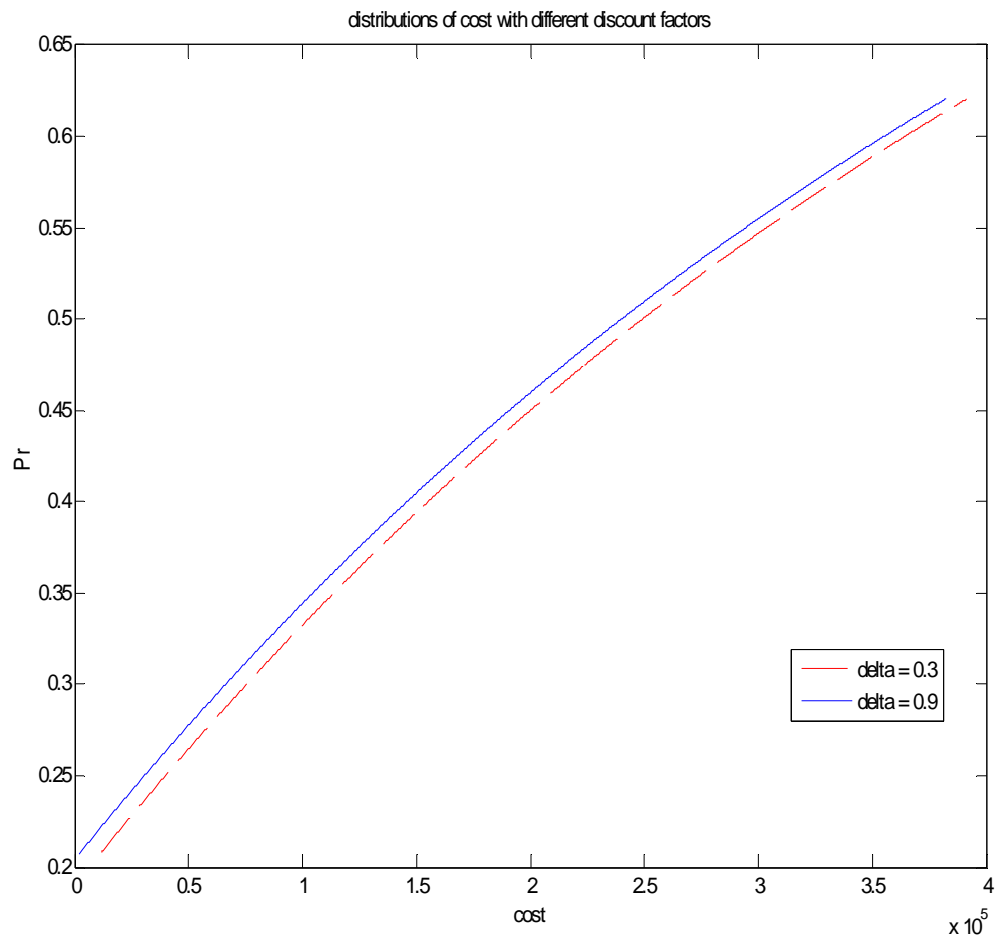


Figure 4.4 Distribution of Costs with Different Discount Factor