# ESSAYS ON TWO-SIDED MARKETS WITH EXTERNALITIES

By

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To my departed father, rest in peace

and

To my family, infinitely supportive

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# Chapter I

# Introduction

This thesis consists of three independent chapters. However, each chapter, one way or another, examines the strategic incentives of market players under constrained situations that generate economic externalities. The second chapter considers a model of competition in which economic activity takes place through networks of bilateral interactions. The third chapter focuses on managerial incentives of a monopolistic retailer to maximize its profits in a market characterized by demand uncertainty. Finally, the last chapter focuses on incentive tradeoff between demand uncertainty and price discrimination for oligopolistic firms.

**Chapter II: Bargaining in a Network with Heterogeneous Buyers**

This chapter examines the effects of exogenously given network structure, which represents potential traders in an economy, on market outcomes and identifies the conditions that determine bargaining power of potential traders in a network with homogeneous sellers and heterogenous buyers. The first focus of this chapter is the network structures that allow perfectly competitive market interactions. In particular, we consider a benchmark solution that represents the competitive equilibrium outcome in networks context and characterize the network structures that support this competitive market outcome. We find, as opposed to earlier literature, that similar network structures may lead to different equilibrium outcomes. In our setting, not only positions of the agents in a network are crucial for the equilibrium outcome, so are the names (valuations) of the agents who capture those positions. Another focus of this chapter is the efficiency aspect the competitive markets in which there are communication restrictions between buyers and sellers. We provide a class of networks that ensures the efficient allocation of goods and show that any member of this class supports the competitive equilibrium outcome.

### Chapter III: Retail Assortment Planning Under Category Captainship

Retail assortment planning can have a tremendous impact on the retailer's bottom line performance. Recently, retailers have started to rely on their leading manufacturers for recommendations regarding the assortment to be offered to the consumers in a particular category, a trend often referred to as *category captainship*.

While retailers focus on many product categories, manufacturers usually focus on fewer categories and have superior understanding of the consumer trends in these particular categories. Thus, category captainship carries potential benefits for both the category captains and the retailers, mainly due to the elimination of information asymmetry. Category captains might be given access to crucial information such as sales data and pricing. This information allows captains to understand retail business better than their non-captain competitors. The category captains can leverage these insights to improve their own product marketing. On the other hand, captains often promise the retailers to grow retail categories and provide consumer insights which are not readily available to the retailers.

This chapter investigates the consequences of using category captains for assortment selection decisions. We develop a screening model where multiple manufacturers sell their products to consumers through a single retailer. We compare the models where the retailer selects the assortment in the category with a model where the retailer relies on a category captain for assortment decisions in return of a target contract. We show that while category captainship can provide significant benefits to the retailer and the category captain, it does not always benefit the non-captain manufacturers.

### Chapter IV: Price Discrimination in Quantity Competition

This chapter focuses on the incentive tradeoff between demand uncertainty and price discrimination. Markets that contain demand uncertainty and possibility of price discrimination create an incentive conflict for the firms operating in those markets. On one hand, firms that face uncertainty choose sub-optimal strategies, which results in profit losses, in order to smooth their strategies across different market outcomes. On the other hand, firms that face different variety of consumers tend to discriminate consumers by offering different prices in the hope of capturing higher surplus. Motivated by this tradeoff, the goal of the chapter is to better understand the consequences of exogenously enforced price discrimination. In particular, we consider a linear demand duopoly model in which two firms engage in quantity competition over two varieties of a product. The results of this chapter extends the standard Cournot and Stackelberg competition literatures by characterizing the equilibrium outcomes in the presence of multiple varieties. Moreover, our results provide intuition on whether the firms that engage in quantity competition choose to practice price discrimination or not. We show that a firm chooses not to practice price discrimination if the firm is the leader in the market and there is asymmetric price effect between varieties. In addition, we determine a crucial component for the price differences between the varieties in the equilibrium.

Chapter II

# BARGAINING IN A NETWORK WITH HETEROGENEOUS BUYERS

## Introduction

In many markets, much of the communication that is important for the economic activity take place through networks of bilateral interactions. While the nature of this interaction is negligible in large and competitive economies, it becomes a central determinant of the economic activity in highly non-competitive economic environments. This paper focuses on two-sided markets organized through a network that represents communication limitations on the potential traders.

Communication restrictions may take different forms such as social contacts, transportation costs, free trade agreements, technological compatibility, etc. Numerous examples in social and economic contexts have been provided to support the importance of networks.[1] Consider the U.S. housing market for example. In this market, not all buyers and sellers have access to the agents on the other side of the market. A network of housing market may represent the feasible houses for buyers and potential buyers for sellers. So, housing market is two sided and surrounded by communication restrictions between buyers and sellers.

In an economic environment with restricted communication, it is not surprising that the lack of ability to engage in trade may harm an agent. However, having relatively more connections alone may not guarantee a better outcome either. Networks can generate power differences among agents since they can create asymmetric positions in a market. A house seller, who negotiates with two buyers, will probably receive a higher bid when his house is the only one around compared to the situation when at least one of the buyers has an

[1] See Jackson (2008), Dutta and Jackson (2003), Jackson and Wolinsky (1996), and Jackson and Watts (1998) for surveys of related literature.

interest to another house. It is not immediately obvious what it means to be *well-connected* in a market with interdependent relations between buyers and sellers.

This paper examines the effects of exogenously given network structure on market outcomes and identifies the conditions that determine bargaining power of potential traders in a network with homogeneous sellers and heterogenous buyers. The setting is as follows. Each seller owns an identical indivisible good, which is worthless to him, and buyers value the good differently. Bargaining occurs simultaneously and in an alternating order. The network generates a potentially infinite horizon discrete time bargaining game. In each period, agents on the one side of the market simultaneously post prices that they are willing to accept, and then agents on the other side simultaneously announce their reservation prices. A buyer can buy a good from a seller, who is connected with the buyer in $G$, only if the buyer announces a price higher than the seller's posted price. If there are multiple feasible trade patterns, then a surplus maximizing mechanism determines the effective trade pattern. After some pairs trade at the posted prices, they leave the market, while the rest keep bargaining with alternating orders. The game is played repeatedly among the players who did not trade in previous periods until the market clears. Each agent has a common discount factor.

In a similar setting, Corominas-Bosch (2004) identifies *strong*, *weak*, and *even* agents in markets with homogenous buyers and sellers. She shows the conditions that are necessary and sufficient for a network structure to be *complete* enough so that the competitive market outcome still prevails. Our first main result carries this line of research to a step further and characterize the network structures that support competitive market outcomes in the presence of heterogeneous buyers. To do so, we first characterize the subgame perfect Nash equilibrium outcomes of small markets, i.e., the networks with at most two sellers and two buyers. We find, as opposed to Corominas-Bosch (2004), that similar network structures may lead to different equilibrium outcomes, especially in the presence of the *even* agents. In our setting, not only positions of the agents in a network are crucial for the equilibrium outcome, so are the names (valuations) of the agents who capture those positions. Later, we extend the small market exercises to more general network structures and characterize networks that support the competitive equilibrium.

Another focus of this paper is the efficiency aspect the competitive markets in which there are communication restrictions between buyers and sellers. While the two-sided network models with homogeneous buyers and sellers provide intuition regarding the meaning of being well-connected, they are silent on allocative efficiency. That is because, when buyers and sellers are homogeneous the question of who participates in trade becomes irrelevant. Homogeneity throws a veil over the efficiency properties of the buyer-seller networks. However, the question becomes relevant and important in the presence of heterogeneous agents. We provide a class of networks that ensures the efficient allocation of goods and show that any member of this class supports the competitive equilibrium outcome.

The literature on bargaining in markets is extensive. Stahl (1972), Rubinstein (1982), and Binmore (1987) introduce the fundamental models of two-player non-cooperative negotiations. Rubinstein and Wolinsky (1985, 1990), Gale (1987), and Binmore and Herrero (1988) consider homogeneous markets without communication restrictions, in order to identify the effects of various decentralized bargaining procedures on the competitive equilibrium price. The highlight of this line of research is that the equilibrium price is affected by information asymmetry, nature of market barriers, matching technology, and patience of the agents. Our analysis diverge from this literature by imposing restrictions on possible bilateral trades and relaxing the assumption that buyers are homogeneous. Kranton and Minehart (2001) focus on the efficient implementation of networks via centralized auction mechanism in a non-strategic sellers environment. Finally more recently, Polanski (2007), Manea (2008), and Abreu and Manea (2009) provide intuition on bargaining power of homogeneous agents in any market (not necessarily two sided) with communication restrictions. We restrict our attention to only two sided markets while relaxing the homogeneity assumption.[2]

[2] In addition, the models in this line of literature focus mainly on the market outcome in the limit and more appropriate for large markets. Our focus is more inline with the markets with smaller scale, e.g., housing market.

The results in this paper exploit connections to the structure of matchings in networks, including decomposition theorems for networks with perfect matchings, and general market properties like competitiveness and efficiency. The next section develops the model, while introducing the notation and the preliminary mathematical tools. We then consider thin markets with at most two sellers and two buyers to identify the intuition behind the general results. Then, we analyze the properties of the networks that generate an environment that is free enough for achieving competitive and efficient allocations. And finally in the last section, we discuss the implications of the model and conclude the paper.

## The Model

Consider a market with $|S|$ sellers $S = \{s_1, s_2, ..., s_{|S|}\}$ and $|B|$ buyers $B = \{b_1, b_2, ..., b_{|B|}\}$.[3] Each seller owns an identical indivisible good which is worthless for him. Each buyer wants to buy exactly one good. Sellers are homogeneous but buyers have different valuations for the good.[4] Let $v_i \in [\underline{v}, \overline{v}]$ denote the valuation of buyer $b_i$, where $\underline{v}$ and $\overline{v}$ are the lowest and highest valuations, respectively, in the market.

There are communication restrictions in the market. The potential trade partners in the market are represented by a *bipartite graph*. A non-directed bipartite graph, denoted as $G = (S, B, L)$, consists of a set of *nodes* formed by sellers in $S$ and buyers in $B$, and a set of *links* $L$.[5] Each link joins a seller with a buyer and can be represented as a subset of the cartesian product of $S$ and $B$, that is $L \subseteq S \times B$. An element of $L$, say a link from seller $s_i$ to buyer $b_j$, is denoted as $ij$.[6] In market terms, a link is a representation of possibility of trade. Thus, lack of a link between two agents is a restriction over their ability to exchange goods. We define a *network* of buyers and sellers as $(G, \mathbf{v})$, where $G$ is the underlying graph

[3]For any finite set $X$, $|X|$ represents the number of elements in $X$.

[4]Although we require different valuations, our results still apply if we include the possibility of having same valuation for some buyers. For simplicity, we drop such cases.

[5]We say that a node $v$ *belongs to a graph* $G = (S, B, L)$ if $v \in S \cup B$. A node $s$ is *adjacent* or *linked* to another node $b$ if there is a link joining the two.

[6]For the sake of notational consistency, we always write the seller first for any link. For instance, the link $xy$ means that the seller $x$ is connected to the buyer $y$.

of the market and $\mathbf{v} = (v_1, v_2, ..., v_{|B|})$ is the profile of buyers' valuation.[7]

A *path* in a graph is a sequence of nodes such that from each of its nodes there is a link to the next node in the sequence. A bipartite graph $G$ is *connected* if there exists a path linking any two nodes of the graph. We only consider the networks in which the underlying graph of the network is connected. If the graph is disconnected, we can apply all of our results to each disconnected component of the graph separately.

In our setting, trade can occur between a seller and a buyer only if they are linked with each other in a network. Thus, it is useful to introduce the following concepts. The *set of buyers who are linked with* $s$ in $G = (S, B, L)$ is denoted by $N_G(s) = \{b \in B \mid sb \in L\}$. We denote the *set of buyers who are collectively linked with the subset of sellers* $S' \subseteq S$ in $G$ as $N_G(S') = \bigcup_{i \in S'} N_G(s_i)$. Similarly, $N_G(b)$ stands for the neighbors of buyer $b$, and $N_G(B')$ is the set of sellers who are collectively linked to the subset of buyers $B' \subseteq B$ in $G$. A *subgraph* $G' = (S', B', L')$ of $G = (S, B, L)$ is a graph such that $S' \subseteq S$, $B' \subseteq B$, and the restriction of $L$ over $S' \cup B'$, denoted as $L' = L|_{S' \cup B'}$.

Bargaining occurs in an alternating order. In the first period, each seller simultaneously proposes the lowest price he is willing to accept for the good. After observing all the prices posted by the sellers, each buyer simultaneously announces the highest price she is willing to pay. A buyer can buy a good from a seller she is connected with in $G$ only if she announces a price higher than the seller's posted price. If there are multiple feasible trade patterns, then a surplus maximizing mechanism, which is defined in detail below, determines the effective trade pattern. After a buyer and a seller trade, they leave the game. In the second period, the remaining agents continue to bargain while preserving their positions in $G$, but this time buyers announce their prices first.

In period $t$, the network is represented by $G_t = (S_t, B_t, L_t)$ (with $G_1 = G$). If $t$ is odd, then sellers post their prices first, but if $t$ is even, buyers announce first. Let $p^t_{s_i}$ and $p^t_{b_i}$ denote the prices proposed at period $t$ by seller $s_i$ and buyer $b_i$, respectively. Given the actions of the

[7]Although they are slightly different concepts, throughout the paper, we use the terms "network" and "graph" interchangeably.

agents, we represent the set of agents who can be a part of the effective trade pattern in period $t$ by subgraph $\tilde{G}_t = (\tilde{S}_t, \tilde{B}_t, \tilde{L}_t)$. A seller $s_i$ is in $\tilde{S}_t$ if and only if $\{b \in N_{G_t}(s_i) \mid p_b^t \geq p_{s_i}^t\} \neq \emptyset$. Similarly, a buyer $b_i$ is in $\tilde{B}_t$ if and only if $\{s \in N_{G_t}(b_i) \mid p_{b_i}^t \geq p_s^t\} \neq \emptyset$.

The set of possible trade patterns in period $t$ is determined by the set of possible matchings in $\tilde{G}_t = (\tilde{S}_t, \tilde{B}_t, \tilde{L}_t)$. A *matching* in a network is a subset of links such that each agent in the network is connected to at most one link. A *maximum matching* is a matching that contains the largest possible number of links. The mechanism that chooses the effective trade pattern from the set of all feasible trade patterns uses a maximum matching in $\tilde{G}_t$. If there is more than one maximum matching in $\tilde{G}_t$, then the mechanism selects a matching with the highest *total surplus*, which is the summation of the valuations of the buyers in the selected matching. If there is more than one surplus maximizing matching, the mechanism picks one of them randomly. Notice that this procedure is well-defined because the set of all matchings at any time period is finite.

After a buyer and a seller trade, they leave the game. The remaining agents continue to bargain while preserving their positions in network. The game continues in this fashion until either all players trade or have no remaining connections. Each player discounts the future with the common factor $\delta$. If a buyer $b_i$ and a seller trade at price $p$ at period $t$, the seller receives a utility of $\delta^t p$ and the buyer $b_i$ receives $\delta^t (v_i - p)$. We are interested in subgame perfect Nash equilibrium payoffs of this game.

## Results for Small Markets

There are two types of heterogeneity in our model. The first type is driven by the network structure and the second one is by the differences in the valuations of the buyers. Bargaining power of an agent depends on possibly conflicting effects of these two forces. To simplify the exposition, throughout the rest of the paper we refer the advantage created by the former type of heterogeneity as *positional power* and that created by the latter as *valuational power*. Notice that any agent can have positional power depending on the network structure but only some of the buyers can enjoy valuational power. This implies that there is room for cases in which sellers with strong positional powers face with buyers with strong valuational powers. Next, we consider small markets where there are at most two sellers and two buyers. The results in this section are also the first step of the results in the general setting. All proofs are in the appendix.

*Case* $|S| = 1,\ |B| = 1$

If $G$ consists of only one connected pair, then agents engage in the alternating offers bargaining game of Rubinstein (1982), in which the unique equilibrium payoffs are $\frac{1}{1+\delta}v_i$ for the seller and $\frac{\delta}{1+\delta}v_i$ for the buyer. As the agents become perfectly patient (that is, $\delta \to 1$) they equally share the surplus.

*Case* $|S| = 2,\ |B| = 1$

In a market with one side is shorter than the other one, the agents in the short side have strong positions when both buyers and sellers are homogenous. However, due to the possibility of buyers with high valuational power, the name of the short side matters when there is heterogeneity among buyers. If the buyer side is short, buyers collect all the economic surplus because of the competition among sellers (if all sellers are competing). In particular, when there are two sellers and one buyer the unique equilibrium price is zero and the buyer gets her valuation as the equilibrium payoff. The situation in such a case is similar to the Bertrand competition where two homogenous firms undercut each other to capture all the demand. The following result is due to Corominas-Bosch (2004).

**Proposition 1** *When there are two sellers and one buyer in the market, there is a unique subgame perfect Nash equilibrium in which the good is sold at the price of zero.*[8]

*Case* $|S| = 1,\ |B| = 2$

When the seller side is short, there are two conflicting forces in the negotiation process. While the strong positional power of sellers pressure the prices upwards, buyers with high valuational powers can pressure the prices downwards since they can create more surplus.

[8]Throughout the paper, by *uniqueness*, we refer to the uniqueness in terms of payoffs not strategies.

As a result, sellers cannot collect all the surplus as the buyers did when the buyer side is short. Binmore (1985) is the first to characterize the equilibria of such a one seller two buyers bargaining game. The following proposition, see Osborne and Rubinstein (1990), summarizes the equilibria in this case.

**Proposition 2** *When there is one seller and two buyers in the market,*

***(i)*** *If $\underline{v} \geq \frac{\delta}{1+\delta}\overline{v}$, then the game has a subgame perfect Nash equilibrium, and in all the equilibria the good is sold (to high valuation buyer) at the price of $\delta\underline{v} + (1-\delta)\overline{v}$.*

***(ii)*** *If $\underline{v} < \frac{\delta}{1+\delta}\overline{v}$, then the game has a unique subgame perfect Nash equilibrium in which the good is sold (to high valuation buyer) at the price of $\frac{1}{1+\delta}\overline{v}$.*

Proposition 2 has to consider two cases because of the possibility of having a rival buyer whose valuation is so small that the players ignore the existence of such an option. The second part can be thought as if the seller has an outside option which is negligible. In such cases, the outside option does not affect the bargaining outcome. Thus, it is reasonable and costless to ignore such cases. We carry the following assumption throughout the rest of the paper: $\underline{v} > \frac{\delta}{1+\delta}\overline{v}$. Intuitively, this assumption ensures that there is rivalry among the neighbors of any given seller. In Proposition 2 terms, we ignore the case described in part *(ii)* and focus on the type of equilibria similar to the one in part *(i)*.

*Case* $|S| = 2$, $|B| = 2$

There are three possible connected networks. First, consider the complete network in which both sellers ($b_1$ and $b_2$) are connected to both buyers ($s_1$ and $s_2$). In the complete network, all the agents on one side of the market have symmetric network positions. Thus, no agent has a positional power. However, the existence of a buyer with valuational power, say $b_1$, destroys the possibility of an equilibrium in which one good is traded at a higher price. The competition among the sellers brings $b_1$'s price down to the sellers' outside option: to bargain with $b_2$ and collect $\frac{1}{1+\delta}v_2$. Indeed, Chatterjee and Dutta (1998) consider such a bargaining situation and show that both sellers proposing $\frac{1}{1+\delta}v_2$ is the unique equilibrium.[9]

[9] Chatterjee and Dutta (1998) use a random matching mechanism for tie breaking in their bargaining game.

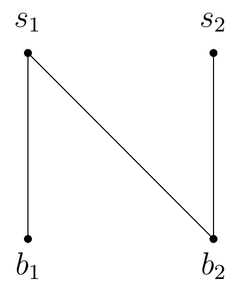



Figure 1: An asymmetric market with a powerful seller.

**Proposition 3** *There exists a unique subgame perfect Nash equilibrium in the market represented by the complete network with two buyers and two sellers. In this equilibrium, $b_1$ trades with $s_1$ and $b_2$ trades with $s_2$ at the price of $\frac{1}{1+\delta}v_2$.*

The two other possible connected networks when $|S| = 2$, $|B| = 2$ can be constructed by removing a link from the complete graph. In Corominas-Bosch (2004) setting these two networks generate the same equilibrium outcome. However, we show that in the presence of heterogenous buyers these two markets are fundamentally different.

Figure 1 represents the case where the buyer with low valuation, $b_2$, has a favorable network position. In this case, $b_1$ has valuational but not positional power and $b_2$ has positional but not valuational power. Because $b_1$ has only one link, the negotiation between $b_1$ and his neighbor $s_1$ is similar to the Rubinstein bargaining game. As his outside option, the most that $s_1$ can get by negotiating with $b_2$ is $\frac{1}{1+\delta}v_2$, which is less than what he can get by negotiating with $b_1$. Thus, $s_1$ prefers to ignore his link with $b_2$. In a sense, $s_1$ and $b_1$ engage in a Rubinstein bargaining game in which outside options are zero. In the equilibrium, $s_1$ collects $\frac{1}{1+\delta}v_1$ and $b_1$ receives $\frac{\delta}{1+\delta}v_1$. Similarly, $s_2$ and $b_2$ share the total surplus they create and receive $\frac{1}{1+\delta}v_2$ and $\frac{\delta}{1+\delta}v_2$, respectively.

However, their result still applies the game setting here. Only, the strategies that are off the equilibrium path need to be adjusted.

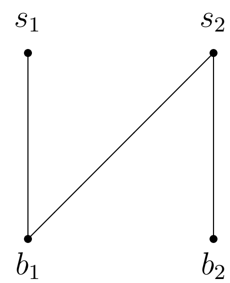



Figure 2: An asymmetric market with a powerful buyer.

**Proposition 4** *There exists a unique subgame perfect Nash equilibrium in the market represented by the network G in Figure 1. In this equilibrium, $b_1$ trades with $s_1$ at the price of $\frac{1}{1+\delta}v_1$ and $b_2$ trades with $s_2$ at the price of $\frac{1}{1+\delta}v_2$.*

Figure 2 represents the case where the buyer with high valuation also has a favorable network position. In particular, $b_1$ has both a strong position in the network and bargaining power due to the heterogeneity of buyers' valuations. Both of the sellers prefer to negotiate with $b_1$ simply because stakes are higher. Although $b_1$ benefits from the competition among sellers, she cannot capture all the surplus since $s_2$ competes only up his outside option. In the equilibrium, the competition brings the prices at which trades occur down to the second seller's outside option $\frac{1}{1+\delta}v_2$.

**Proposition 5** *There exists a unique subgame perfect Nash equilibrium in the market represented by the network G in Figure 1. In this equilibrium, $b_1$ trades with $s_1$ and $b_2$ trades with $s_2$ at the price of $\frac{1}{1+\delta}v_2$.*

Proposition 4 and 5 reveal the difference between seemingly similar two networks. These results suggest that if the buyers who have high reservation values are also the ones who have more connections, then the market outcome is similar to the one in the complete network. On the other hand, if the buyers who have low valuations are the ones who have more connections, then the market dynamics happen as if the market is segmented. In the next section, we carry our analysis into more general networks.

## Analysis

A competitive market is essentially free from any possible rigidities including communication restrictions. The restrictions over communication structure of the society plays an important role for divergence from the competitive market outcome. For instance, in the small market example with two buyers and two sellers, we showed that the existence of competition among sellers for the high valued buyer is crucial for the market price. It turns out that the results for small markets are representative for the dynamics in any two sided market.

### The Competitive Solution

In order to use the competitive equilibrium outcome as a benchmark, we first define the competitive solution in our context. Consider a market which consists of a set $S$ of sellers, each one endowed with a unit of an identical indivisible good, and a set $B$ of buyers, who values the good differently. If a seller trades with a buyer at price $p$, the seller gets utility $p$ and the buyer gets $v-p$, where $v$ is the valuation of the buyer for the good. The competitive equilibrium in such a market has the following characteristics: *(i)* if there are more sellers than buyers, the equilibrium price is zero and buyers get all the surplus, *(ii)* if there are more buyers than sellers, any price between the $(|S|+1)$th and $(|S|)$th highest valuations of $|B|$ buyers is an equilibrium price, and *(iii)* if the number of sellers is equal to the number of buyers, any price between zero and the minimum valuation of the buyers, $\underline{v}$, can be supported as the competitive price.

We select *the reference solution* that represents the competitive equilibrium of a market with heterogeneous buyers as follows.

- If $|S| > |B|$, the reference solution is the competitive solution: the equilibrium price is zero and all the surplus goes to buyers.

- If $|S| < |B|$, the reference solution is a selection of the competitive solution: the

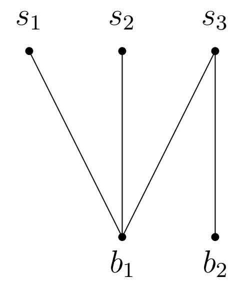



Figure 3: A network that does not support the reference solution.

equilibrium price is equal to the $(|S|+1)$th highest valuation of $|B|$ buyers, say $v^*$. All sellers receive the payoff $v^*$ and buyer $i$ receives $v_i - v^*$.

- If $|S| = |B|$, the reference solution is a selection of the competitive solution: the equilibrium price is equal $p = \frac{1}{1+\delta}\underline{v}$. All sellers receive the payoff $\frac{1}{1+\delta}\underline{v}$ and buyer $i$ receives $v_i - \frac{1}{1+\delta}\underline{v}$.

This selection of the reference solution is consistent with the one in Corominas-Bosch (2004) model. That is, if we assume that the buyers have homogeneous valuations the reference solution described above coincides with the reference solution described in Corominas-Bosch (2004). In addition, if we assume the market is consist of only one buyer and one seller, then we get the Rubinstein (1982) bargaining solution.

Not every communication structure allows a market to be competitive enough to achieve the reference solution. For example, consider the market consisting two buyers ($b_1$ and $b_2$ with valuations $v_1$ and $v_2$, respectively) and three sellers ($s_1$,$s_2$, and $s_3$), represented in Figure 3. There is a competition among the sellers due to the scarcity of buyers around and therefore the reference solution would give all the surplus to the buyers. That is, all buyers receive their valuations and all sellers get zero in the reference solution. However, $s_3$ receives a positive payoff in the equilibrium of the bargaining game described in section 3. To see why, observe first that both $s_1$ and $s_2$ will get a payoff of zero since they compete with each other to trade with $b_1$. If one of them gets a positive price the other would be able to undercut the accepted price. In such a case, $s_3$ is simply going to ignore his link with $b_1$ and bargain only with $b_2$. They will end up agreeing for price $\frac{1}{1+\delta}v_2$ in equilibrium.

The natural question to ask is then: what type of network structures support the reference solution? The next section seeks for an answer to this question.

**Competitive Networks**

Corominas-Bosch (2004) provides a decomposition algorithm, based on the well-known graph theory algorithm provided by Gallai and Edmonds, to characterize the networks that support the reference solution.[10] The results in this section heavily depend on this bipartite graph decomposition algorithm. Thus, first we describe the types of subgraphs that are identified by the Corominas-Bosch decomposition algorithm (henceforth, *CB-algorithm*) and briefly summarize the algorithm itself.

First, we need new definitions that are useful to describe networks. Let $G$ be the underlying bipartite graph of a market and $X$ be a subset of agents on one side of the market, that is $X \subseteq S$ or $X \subseteq B$. A set of nodes $X$ is *non-deficient* in $G$ if and only if $|N_G(X')| \geq |X'|$ for all $X' \subseteq X$. Similarly, a set of nodes $X$ is *strictly non-deficient* in $G$ if $|N_G(X')| > |X'|$ for all $X' \subset X$. Intuitively, non-deficiency requires that every subset of agents on one side of the market has enough neighbors to trade. The marriage theorem shows that non-deficiency is both necessary and sufficient for the existence of a matching saturating a given set of nodes.

**Theorem 1** *(*The marriage theorem*, Hall 1935)*[11] *There exists a matching in $G$ that saturates all the nodes in $X$ if and only if $X$ is non-deficient in $G$.*

The non-deficiency requirement may not be very plausible when there are more nodes on one side than the other. For such graphs, we relax the requirement only to the subsets of the nodes in the long side that has at most as many nodes as in the short side. Formally,

[10]See Lóvasz and Plummer (1986) for more details on the Gallai-Edmonds decomposition of graphs.

[11]Also known as the Hall's theorem. Not to be confused with the results shown by Gale and Shapley, which game theorists also refer to as "the marriage theorem", but which are not directly related.

in a graph $G = (S, B, L)$ with one side is longer, say $|S| > |B|$, the set of nodes $S$ is *almost non-deficient* in $G$ if for any subset $S_1 \subset S$ of size $|S_1| \leq |B|$ we have that $|N(S_1)| \geq |S_1|$. Almost non-deficiency mainly targets the networks in which one side of the market is longer. Basically, it requires non-deficiency whenever it is possible. The three types of graphs that are related to our analysis are defined as follows.

**Definition 1** *A bipartite graph $G = (S, B, L)$ is:*

- *of type $G^S$ if $|S| > |B|$ and $S$ is almost non-deficient.*
- *of type $G^B$ if $|S| < |B|$ and $B$ is almost non-deficient.*
- *of type $G^E$ if $|S| = |B|$ and there exists a perfect matching.*

It is easy to see that not every graph is one of the types described above. Figure 3 is an example of a graph which is not one of these types. Nevertheless, Corominas-Bosch (2004) showed that any bipartite graph can be decomposed into a union of subgraphs of types above and some extra links. Such a decomposition can be acquired by the CB-algorithm, which is summarized as follows. First, the algorithm checks all possible subgraphs for the existence of $G^S$ type subgraphs in an ascending order of size and separates them from the graph iteratively until no more $G^S$ type subgraphs can be found. Later, the algorithm repeats the same process in order to remove the $G^B$ type subgraphs. Finally, the remaining disconnected subgraphs are necessarily $G^E$ type.[12]

When buyers are homogenous in a market with communication restrictions, the network structure is the only source of heterogeneity. In such markets, similar network structures generate similar outcomes since names (valuations) of the buyers who capture the critical positions in the network do not matter. However, differences in buyers' valuations can destroy such an irrelevance and affect equilibrium outcomes significantly. We have already seen an example of two symmetric networks generating different equilibrium outcomes in Figure 1 and Figure 2.

[12] A very detailed description of the algorithm can be found in the appendix of Corominas-Bosch (2004).

In order to differentiate between similar network structures, we extend the CB-algorithm by decomposing $G^B$ and $G^E$ type subgraphs further into smaller subgraphs. To describe how these extensions works, we need new terminology and definitions. Let $N_G^+(v)$ be the neighbors of $v \in S \cup B$ who have more than one connection in network $G$, that is $N_G^+(v) = \{w \in N_G(v) : |N_G(w)| > 1\}$. In a $G^E$ type subgraph, we say that a buyer is *moderate* if she is connected to one seller only *or* she has the lowest valuation among all her competitors. Formally, we define a buyer $b_i$ of a $G^E$ type subgraph as moderate if $|N_G(b_i)| = 1$ or

$$|N_G(b_i)| > 1 \text{ and } v_i = \min \{v_j \mid b_j \in C_G(b_i)\}$$

where $C_G(b_i) = \bigcap_{s_j \in N_G^+(b_i)} N_G(s_j)$. The set of moderate buyers in a $G^E$ type subgraph is always non-empty since, in any graph of $G^E$ type, the buyer with the lowest valuation is by definition a moderate buyer.

Decomposition of the set of $G^E$ type subgraphs further into smaller subgraphs is as follows. First, find all moderate buyers and remove all links between the following agents: (i) sellers who are connected to the moderate buyer with the highest valuation and (ii) buyers who have valuations lower than the highest valued moderate buyer. Then, remove links between sellers who are connected to the moderate buyer with the second highest valuation and buyers who have valuations lower than the second highest valued moderate buyer. Continue removing links in this fashion.[13] At the end of this process, we may end up having more than one component. Label all the components with $G_k^E$ where $k$ is the subindex of the buyer with the lowest valuation in that component. Thus, the buyer with lowest valuation in $G_k^E$ is $b_k$ and her valuation is $v_k$. Notice that at the end of this process the buyer with the lowest valuation in $G_k^E$ has to be a moderate buyer. The number of moderate buyers determines the number of components. Intuitively, we remove all the redundant links from sellers' perspectives since being connected to a moderate buyer limits the minimum earning of a seller. Sellers who are connected to moderate buyers will never consider trading with buyers with lower valuations, which implies that the interaction in the market is local

[13]This process is well defined since there are only finite number of moderate buyers.

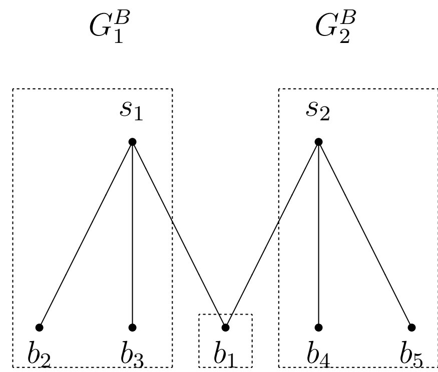



Figure 4: A $G^B$ type subgraph in which the non-uniqueness of the CB-algorithm matters.

even though the market is globally connected.

Moderate buyers have disadvantageous bargaining positions relative to their rivals. However, they can still obtain goods at the end of the bargaining process. In subgraphs of $G^B$ type, even though all buyers have weak positions in the network, some buyers are in worse bargaining situation than the others because of their poor valuational power. Such buyers cannot obtain goods due to the scarcity of the available goods. We need to differentiate such buyers from moderate buyers. We define a buyer $b_i$ as *soft* buyer in a $G^B$ type subgraph $G'$ if $v^*_{G'} \geq v_i$ where $v^*_{G'}$ is the $(|S'|+1)$th highest valuation of the buyers in $G'$. The main difference between soft and moderate buyers is that while moderate buyers are able to engage in trade, soft buyers cannot procure goods. In subgraphs of $G^S$ type there are no moderate or soft buyers since all buyers have strong positional advantages. We denote buyers who are neither moderate nor soft as *hard* buyers.

The CB-algorithm does not necessarily provide a unique decomposition in terms of network structure. However, the decomposition is unique up to the following degree: if an agent belongs to a subgraph of a certain type for a decomposition, then she belongs to the same type of subgraph for every decomposition.[14] This degree of uniqueness is not sufficient when the buyers have different valuations since the CB-algorithm does not take valuations

[14]This property is due to the uniqueness of a more general decomposition result in matching theory known as the Gallai-Edmonds decomposition. Please see Lóvasz and Plummer (1986) for details.

into account. Figure 4 provides an example in which the location of high valuation buyer matters. In the figure, suppose that the valuations of buyers follows a descending order, that is $v_1 > ... > v_5$. Because the valuations of the buyers are different, the price $b_1$ has to pay changes depending on the subgraph which algorithm puts her in. While $b_1$ pays a price of $v_2$ when she is a part of $G_1^B$, she only pays $v_4$ in $G_2^B$. Moreover, because $b_1$ engages in trade no matter which subgraph she ends in, the set of buyers who procure goods is affected by the structure of decomposition. In particular, this set is $\{b_1, b_4\}$ when $b_1$ is in $G_1^B$, but it is $\{b_1, b_2\}$ when $b_1$ is in $G_2^B$.

In order to avoid such cases, we further modify the CB-algorithm in the step where the algorithm identifies $G^B$ types of subgraphs as follows. Among all possible decompositions of $G^B$ type subgraphs obtained as a result of applying the CB-algorithm on $G$, we pick the decomposition which induces the maximum matching with the highest total surplus.[15] If there is more than one such decomposition, then we pick one of them randomly.

We start our analysis with the existence of a particular subgame perfect Nash equilibrium of the game for a general network. In this equilibrium, agents have similar incentives as they had in small markets discussion. While buyers with strong network positions can exploit their advantages fully, sellers with strong network positions can only extract partially since, in such situations, the positional power of sellers clashes with the valuational power of high valuation buyers.

**Proposition 6** *There exists a subgame perfect Nash equilibrium in the market represented by $G = (S, B, L)$ such that*

– *A seller $s_i$ in subgraph $G'$ gets a payoff of* $\begin{cases} 0, & \text{if } G' \text{ is a } G^S \text{ type subgraph;} \\ v^*_{G'}, & \text{if } G' \text{ is a } G^B \text{ type subgraph;} \\ \frac{1}{1+\delta} v_k, & \text{if } G' \text{ is a } G^E_k \text{ type subgraph;} \end{cases}$

[15]We look at the maximum matching with the highest total surplus after we apply the CB-algorithm.

– *A buyer $b_i$ in subgraph $G'$ gets a payoff of* $\begin{cases} v_i, & \textit{if } G' \textit{ is a } G^S \textit{ type subgraph;} \\ v_i - v^*_{G'}, & \textit{if } G' \textit{ is a } G^B \textit{ type subgraph;} \\ v_i - \frac{1}{1+\delta} v_k, & \textit{if } G' \textit{ is a } G^E_k \textit{ type subgraph;} \end{cases}$

*where $v^*_{G'}$ is the $(|S'|+1)$th highest valuation of the buyers in $G'$.*

The proof of Proposition 6 is by induction on the number of agents on both sides of the market. The small markets results provide that the proposition is true for markets in which there are at most two sellers and two buyers, which is also the first step of the proof. Later, we assume that the proposition holds for markets that have at most $n-1$ sellers and $n-1$ buyers and show that the proposition also holds for the markets that have at most $n$ sellers and $n$ buyers.

Essentially two forces play role in determining equilibrium outcome: critical network positions (positional power) and bargaining power due to the existence of heterogeneous buyers (valuational power). When there are more sellers than buyers in the market, these two forces work in the same direction and, therefore, buyers capture all the surplus. However, when there are more buyers than sellers in the market, positional power of sellers clashes with valuational power of some buyers. In such markets, hard buyers behave as if they all have the same valuation, which is the highest valuation of the soft buyers they are competing with. Thus, sellers capture some but not all of the surplus. If there are equal number of buyers and sellers in the market, the intuition is still true and sellers collect surplus depending on the moderate buyers they are connected to.

We know by Corominas-Bosch (2004) that when consumers are homogeneous a necessary and sufficient condition for a network to support the reference solution is that all separated small markets in the economy are of the same type. However, networks in Figure 1 and Figure 2 suggest that the intuition behind that result is no longer valid when buyers are allowed to have different valuations. Heterogeneity of buyers hinders competition in some economies which are unions of similar small markets. A network structure needs to have more properties than uniform decomposability. The next theorem shows that the intuition still prevails in the presence of heterogenous buyers but with additional conditions in which soft and moderate buyers play key roles.

**Theorem 2** *Let $(G, \mathbf{v})$ be a network which represents a two sided market. Then, $G = (S, B, L)$ supports the reference solution if and only if*

*(i) $G$ decomposes into subgraphs of a unique type,*

*(ii) (if applicable) the only moderate buyer is the buyer with the lowest valuation,*

*(iii) (if applicable) a buyer $b_i$ is a soft buyer if and only if $v^* \geq v_i$,*

*where $v^*$ is $(|S| + 1)$th highest valuation of $|B|$ buyers in the economy.*

Theorem 2 exploits the competition among agents. This intuition is clear especially when one side of the market is longer than the other one. The agents in the long side undercut each others' prices as much as they can in order to be a part of possible trades. However, when there are equal number of buyers and sellers in the market, the network does not necessarily create enough competition to support the reference solution. Figure 1 and Figure 2 demonstrate an example of this insufficiency. In the market represented by Figure 1, both buyers are moderate. Therefore, we can decompose the market into two parts by removing the link between $s_1$ and $b_2$. Intuitively, seller $s_1$ ignores his connection with $b_2$ and negotiates only with $b_1$ since stakes are a lot higher. However, in the market represented by Figure 2, the only moderate buyer is $b_2$ and she has the lowest valuation. We cannot decompose this market further into smaller markets since neither $b_1$ nor $s_2$ has incentives to ignore the link between them. Thus, in order to achieve the competitive outcome, a network structure has to be supported by a proper distribution of the valuations.

**Efficiency**

Communication networks give us information about which buyers can trade with which sellers. In other words, they determine the set of feasible allocation of goods. In our context, an allocation is simply a matching. Thus, given a network $G$, the set of all possible matchings in $G$ determines the set of feasible allocations in the economy.

A natural result of the restrictions imposed by a network structure over the market is that buyers who have the highest reservation prices are not necessarily the ones who procure goods from sellers. Therefore, restrictions created by a network structure may cause a loss of economic surplus. We define the economic surplus associated with a matching $M$ as the sum of the valuations of the buyers that receive a good in $M$. The focus of this section is on the *efficient* allocations, which are defined as the allocations yielding highest surplus in a given network. We say that a network is efficient if the allocation determined by the equilibrium of the game is an efficient allocation.

In a similar setting, Kranton and Minehart (2001) consider the effects of communication restrictions on the efficient allocation of goods via a centralized auction mechanism. Their model focuses on markets in which abundant buyers compete for goods of scarce sellers. Because the results in this section are closely related to ones in Kranton and Minehart (2001), we maintain the same assumption throughout the rest of this section.

Suppose that in a two sided market there are more buyers than sellers. In such a market, we define a network $G$ as *allocatively complete* if $G$ is almost non-deficient. We define a network $G$ as *least-link allocatively complete* (LAC) if $G$ is almost non-deficient with the minimum number of links. Intuitively, allocatively complete networks guarantee that any set of buyers with the size of sellers can obtain as many goods as they need. The LAC networks achieve this target with the most restrictive communication structure.[16]

Kranton and Minehart (2001) showed that allocatively complete networks are complete enough to support efficient allocations in the sense that they create the highest possible economic surplus for any distribution of prices. Our next result demonstrates that allocatively complete networks create economic environments that are not only complete enough to allocate goods efficiently but also competitive enough to support the reference solution.

**Theorem 3** *In a market where there are more buyers than sellers, a network $G$ supports the reference solution if and only if $G$ is allocatively complete.*

[16]Notice that by definition allocatively complete networks are of type $G^B$.

A direct implication of Theorem 2 and Theorem 3 leads us the following corollary, which is descriptive about the structure of the LAC networks.

**Corollary 1** *A network is least-link allocatively complete if and only if*

$$v^* \geq v_i \Leftrightarrow b_i \text{ is a soft buyer.}$$

An interpretation of the Theorem 3 is related to the fundamental theorems of welfare. In particular, Theorem 3 shows that markets that are characterized by allocatively complete networks function in such a way that efficient allocations are reached by non-cooperative behavior of the agents. Every efficient allocation of goods in these markets can be supported as the reference solution of our bargaining game.[17]

Allocatively complete networks are efficient for every distribution of valuations. However, for a given distribution of valuations, there may be efficient networks that are not allocatively complete. For instance, if we remove one of the links of a buyer in a LAC network, then the newly established network is not allocatively complete anymore. On the other hand, if the buyer whose link is removed happens to be a soft buyer in the original network, then the new network is still efficient since the equilibrium allocation of goods does not change. Theorem 3 shows us that this type of network does not support the reference solution. Thus, in highly restrictive markets, there is a divergence between individual and social objectives.

## Conclusion

This paper examines the role of communication restrictions on the divergence from competitive market outcome. In our setting, exogenously given networks represents the communication restrictions and the market prices are determined by the interactions between buyers and sellers.

[17] We need to adjust the priorities defined by our tie breaking mechanism in order to support all efficient allocations. For instance, consider the complete network. The tie breaking mechanism we selected leads to only one of the efficient allocations. However, by changing the priorities of the sellers in the mechanism we can support the rest of efficient matchings.

We first characterize the networks that are complete enough to support a natural selection of the Walrasian equilibrium when buyers have different reservation prices. While this characterization confirms the intuitions of previous literature on global markets in which the entire context matters, it also shows that in the presence of heterogeneous buyers the network structure has to be supported by appropriate distribution of reservation prices. An implication of this result is that if the buyers who have high reservation values are also the ones who have more connections, then the market tend to support the competitive outcome. On the other hand, if the buyers who have high valuations are likely to ignore most of their connections then the market is less likely to be competitive.

Our characterization of competitive networks has also implications on bargaining powers of agents in economies with restricted communication. Not surprisingly, an agent's bargaining ability depends on the position she is located in the network. While the network position is a global property of bargaining power that applies anonymously to all agents, it does not take individual characteristics (such as reservation prices) of agents into account. We contribute to this line of description of bargaining power by determining the importance of the reservation prices. In particular, we recognize buyers as *hard*, *moderate*, and *soft* depending on their valuations relative to their competitors. The bargaining ability generated by individual characteristics is a local property that is effected by both network structure and valuations of potential rivals.

We also identify that allocatively complete networks provide environments that are both competitive and efficient. For markets that are not allocatively complete, there is a tradeoff between individual objective and social objective. In such markets, there are individuals who have enough power to affect market prices.

## References

Abreu, D. and Manea, M. (2009) "Bargaining and Efficient in Networks", mimeo.

Binmore, K. (1985) "Bargaining and Coalitions" p. 269-304 in "Game-Theoretic Models of

Bargaining" by A. Roth, Cambridge University Press.

Binmore, K. (1987) "Perfect-Equilibria in Bargaining Models" in "The Economics of Bargaining" by K. Binmore and P. Dasgupta (eds.), Basil Blackwell, Oxford.

Binmore, K.G. and Herrero, M.J. (1988) "Matching and Bargaining in Dynamic Markets", *Review of Economic Studies* 55, 17-31.

Corominas-Bosch, M. (1999) "On Two-Sided Network Markets", Ph.D. Thesis, Universitat Pompeu Fabra, Barcelona.

Corominas-Bosch, M. (2004) "Bargaining in a Network of Buyers and Sellers", *Journal of Economic Theory*, 115, 35-77.

Chatterjee, K. and Dutta, B. (1998) "Rubinstein Auctions: On Competition for Bargaining Partners", *Games and Economic Behavior* 23 (2), 119-145.

Dutta, B, and Jackson, M. (2003) "On the Formation of Networks and Groups", in *Networks and Groups: Models of Strategic Formation*, Springer, Berlin, 2003.

Gale, D. (1987) " Limit Theorems for Markets with Sequential Bargaining", *Journal of Economic Theory* 43, 20-54.

Hall, P. (1935) "On Representatives of Subsets", *Journal of London Mathematics Society* 10, 26-30

Jackson, M. (2008) *Social and Economic Networks*, Princeton University Press.

Jackson, M. and Watts, A. (1998) "The Evolution of Social and Economic Networks", *Caltech WP*, 1044

Jackson, M. and Wolinsky, A. (1996) "A Strategic Model of Social and Economic Networks", *Journal of Economic Theory* 71 (1), 44-74.

Kranton, R. and Minehart, D. (2001) "A Theory of Buyer-Seller Networks", *American Economic Review* 91, 485-508.

Lóvasz, L. and Plummer, M.D. (1986) *Matching Theory*, North-Holland, Amsterdam.

Manea, M. (2008) "Bargaining in Stationary Networks", mimeo.

Osborne, M. and Rubinstein, A. (1990) *Bargaining and Markets*, Academic Press, San Diego.

Polanski, A. (2007) "Bilateral Bargaining in Networks", *Journal of Economic Theory* 134, 1, 557-565.

Rubinstein, A. (1982) "Perfect Equilibria of a Bargaining Model", *Econometrica* 50, 97-110.

Rubinstein, A. and Wolinsky, U. (1985) "Equilibrium in a Market with Sequential Bargaining", *Econometrica* 53, 295-328.

Rubinstein, A. and Wolinsky, U. (1990) "Decentralized Trading, Strategic Behaviour and the Walrasian Outcome", *Review of Economic Studies* 57, 1, 63-78.

Stahl, I. (1972) *Bargaining Theory*, Stockholm School of Economics, Stockholm.

## Appendix

For simplicity in the proofs, we refer to the actions of responders in which there is a possibility of agreement as *accept* and to the ones in which there is no possibility of agreement as *reject*.

**Proof of Proposition 4.** Notice that strategies must specify the distribution of proposed prices and the responses of the agents not only when the market is given by the graph $G$, but also when the market is given by any subgraph that results from $G$ after a pair trades and leaves the market. Two possibilities can happen: either one pair trades and two agents get isolated or one pair trades and the remaining two agents are still connected and can keep playing. The strategies followed in any of these subgraphs are simple. If agents get isolated, they automatically get zero and have no actions to choose. If a pair remains in the market, then the strategies are as in the two players alternating offers bargaining game. Let $p_{b_i}$ and $p_{s_i}$ be the offered prices by each agent.

*Existence:* If all agents are still in the initial graph $G$:

• when its their turn to propose, $s_1$ proposes $p_{s_1} = \frac{1}{1+\delta}v_1$ and $s_2$ proposes $p_{s_2} = \frac{1}{1+\delta}v_2$, $b_1$ proposes $p_{b_1} = \frac{\delta}{1+\delta}v_1$ and $b_2$ proposes $p_{b_2} = \frac{\delta}{1+\delta}v_2$,

• $s_2$ accepts $p_{b_2}$ if $p_{b_2} \geq \frac{\delta}{1+\delta}v_2$, $b_1$ accepts $p_{s_1}$ if $p_{s_1} \leq \frac{1}{1+\delta}v_1$.

The strategies followed by $s_1$ and $b_2$ when responding depend on the priorities they have, which is determined by the tie breaking matching mechanism.

• About seller $s_1$:

case a) the priority of $s_1$ is higher than that of $s_2$

$s_1$ accepts the maximum of the offered prices provided that $max_i\{p_{b_i}\} \geq \frac{\delta}{1+\delta}v_1$.

case b) the priority of $s_1$ is smaller than that of $s_2$

case b1) if $p_{b_2} \leq \frac{\delta}{1+\delta}v_1$ then $s_1$ accepts the maximum of the offered prices provided that $max_i\{p_{b_i}\} \geq \frac{\delta}{1+\delta}v_1$

case b2) if $p_{b_2} > \frac{\delta}{1+\delta}v_1$, then $s_1$ accepts $p_{b_2}$ when $p_{b_1} < \frac{\delta}{1+\delta}v_1$, otherwise (that is, $p_{b_1} \geq$

$\frac{\delta}{1+\delta}v_1$) he accepts $\min_i\{p_{b_i}\}$.

• About buyer $b_2$:

case a) the priority of $b_2$ is higher than that of $b_1$

$b_2$ accepts the minimum of the offered prices provided that $\min_i\{p_{s_i}\} \leq \frac{1}{1+\delta}v_2$.

case b) the priority of $b_2$ is smaller than that of $b_1$

case b1) if $p_{s_1} \geq \frac{1}{1+\delta}v_2$, $b_2$ accepts the minimum of the offered prices provided that $\min_i\{p_{s_i}\} \leq \frac{1}{1+\delta}v_2$

case b2) if $p_{s_1} < \frac{1}{1+\delta}v_2$, then: $b_2$ accepts $p_{s_1}$ when $p_{s_2} > \frac{1}{1+\delta}v_2$, otherwise (that is, $p_{s_2} \leq \frac{1}{1+\delta}v_2$) he accepts $max_i\{p_{s_i}\}$.

If there is only one pair of agents, $s_i$ and $b_i$, in the market, then:

• $s_i$ proposes $p_{s_i} = \frac{1}{1+\delta}v_i$, $b_i$ proposes $p_{b_i} = \frac{\delta}{1+\delta}v_i$

• $s_i$ accepts $p_{b_i}$ if $p_{b_i} \geq \frac{\delta}{1+\delta}v_i$, $b_i$ accepts $p_{s_i}$ if $p_{s_i} \leq \frac{1}{1+\delta}v_i$

Notice that although the strategies above are described for a more general case, our tie breaking mechanism gives a higher priority to the buyer with higher valuation. This implies that, in the equilibrium, $b_2$ cannot trade with $s_1$.

It can be checked that these strategies form a subgame perfect Nash equilibrium.

*Uniqueness (in terms of payoffs):* Call $M_{s_i}$, $m_{s_i}$ the supremum and infimum of subgame perfect Nash equilibrium for sellers in an $s$-game (respectively, $M_{b_i}$, $m_{b_i}$ for buyers in a $b$-game), when all four agents are still in the market, that is, when the market is imbedded in graph $G$. We will find inequalities in order to show that $M_{s_1} = m_{s_1} = M_{b_1} = m_{b_1} = \frac{1}{1+\delta}v_1$ and $M_{s_2} = m_{s_2} = M_{b_2} = m_{b_2} = \frac{\delta}{1+\delta}v_2$. Notice that we already know by existence that $M_{s_1} \geq \frac{1}{1+\delta}v_1$, $M_{b_1} \geq \frac{1}{1+\delta}v_1$, $m_{s_1} \leq \frac{1}{1+\delta}v_1$, $m_{b_1} \leq \frac{1}{1+\delta}v_1$, $M_{s_2} \geq \frac{1}{1+\delta}v_2$, $M_{b_2} \geq \frac{1}{1+\delta}v_2$, $m_{s_2} \leq \frac{1}{1+\delta}v_2$, and $m_{b_2} \leq \frac{1}{1+\delta}v_2$. We can now show that:

$$m_{s_1} \geq v_1 - \delta \max\{\frac{1}{1+\delta}v_1, M_{b_1}\} = v_1 - \delta M_{b_1}. \tag{1}$$

If $b_1$ rejects an offer from $s_1$, he may get $\frac{\delta}{1+\delta}v_1$ or $\delta M_{b_1}$. Thus, $s_1$ will never offer a price strictly smaller than $v_1 - \delta \max\{\frac{1}{1+\delta}v_1, M_{b_1}\}$ since he is sure to be accepted by buyer $b_1$ when he asks for $v_1 - \delta \max\{\frac{1}{1+\delta}v_1, M_{b_1}\}$. On the other hand, we can also show that:

$$M_{b_1} \leq v_1 - \delta \min\{\frac{1}{1+\delta}v_1, m_{s_1}\} = v_1 - \delta m_{s_1}. \tag{2}$$

By rejecting an offer from $b_1$, the minimum amount $s_1$ can get is either $\frac{\delta}{1+\delta}v_1$ or $\delta m_{s_1}$. Thus, to ensure an acceptance from $s_1$, the minimum $b_1$ has to offer is $\delta \min\{\frac{1}{1+\delta}v_1, m_{s_1}\}$. In other words, the maximum that $b_1$ can collect is less than or equal to $v_1 - \delta \min\{\frac{1}{1+\delta}v_1, m_{s_1}\}$.

We can rewrite inequalities (1) and (2) as

$$(1-\delta)m_{s_1} + \delta(m_{s_1} + M_{b_1}) \geq v_1$$
$$(1-\delta)M_{b_1} + \delta(m_{s_1} + M_{b_1}) \leq v_1$$

Notice that these two inequalities imply that $m_{s_1} \geq M_{b_1}$. We also know that $M_{b_1} \geq \frac{1}{1+\delta}v_1 \geq m_{s_1}$. Thus, it must be the case that $M_{b_1} = \frac{1}{1+\delta}v_1 = m_{s_1}$.

By using similar arguments, we can also show that $M_{s_2} = \frac{1}{1+\delta}v_2 = m_{b_2}$ must hold.

Now, consider the maximum payoff $s_1$ can collect. The following inequality has to hold

$$M_{s_1} \leq \max\{v_1 - \delta \min\{\frac{1}{1+\delta}v_1, m_{b_1}\}, v_2 - \delta \min\{\frac{1}{1+\delta}v_2, m_{b_2}\}\}. \tag{3}$$

To see why, notice that if $s_1$ offers a price strictly greater than the amount on the right hand side of the inequality, neither of the buyers will accept. Thus, the maximum payoff $s_1$ can get is bounded above by this amount. Because $m_{b_1} \leq \frac{1}{1+\delta}v_1$ and $m_{b_2} = \frac{1}{1+\delta}v_2$ we have $M_{s_1} \leq \max\{v_1 - \delta m_{b_1}, \frac{1}{1+\delta}v_2\}$. Notice that $v_1 - \delta m_{b_1} \geq \frac{1}{1+\delta}v_2$ since $v_1 - \delta m_{b_1} \geq v_1 - \delta\frac{1}{1+\delta}v_1 = \frac{1}{1+\delta}v_1 \geq \frac{1}{1+\delta}v_2$. Thus, $M_{s_1} \leq v_1 - \delta m_{b_1}$ has to hold. On the other hand, we also have a lower bound for the minimum amount $b_1$ can get.

$$m_{b_1} \geq v_1 - \delta \max\{\frac{1}{1+\delta}v_1, M_{s_1}\}. \tag{4}$$

By offering a price equal to the amount on the right hand side of the inequality $b_1$ can ensure $s_1$ to accept. Notice that this condition holds because of the tie breaking matching mechanism the game uses. Because $M_{s_1} \geq \frac{1}{1+\delta}v_1$ we have $m_{b_1} \geq v_1 - \delta M_{s_1}$. We can rewrite inequalities (3) and (4) as

$$(1-\delta)m_{b_1} + \delta(m_{b_1} + M_{s_1}) \geq v_1$$
$$(1-\delta)M_{s_1} + \delta(m_{b_1} + M_{s_1}) \leq v_1$$

These two inequalities imply that $m_{b_1} \geq M_{s_1}$. We also know that $M_{s_1} \geq \frac{1}{1+\delta}v_1 \geq m_{b_1}$. Thus, it must be the case that $M_{s_1} = \frac{1}{1+\delta}v_1 = m_{b_1}$. Using similar arguments we can also show that $M_{b_2} = \frac{1}{1+\delta}v_2 = m_{s_2}$ has to hold.

**Proof of Proposition 5.** As before, let $p_{b_i}$ and $p_{s_i}$ be the offered prices by each agent.

*Existence:* If all agents are still in the initial graph $G$:

• when its their turn to propose, sellers propose $p_s = \frac{1}{1+\delta}v_2$ and buyers propose $p_b = \frac{\delta}{1+\delta}v_2$,

• $s_1$ accepts $p_{b_1}$ if $p_{b_1} \geq \frac{\delta}{1+\delta}v_2$, $b_2$ accepts $p_{s_2}$ if $p_{s_2} \leq \frac{1}{1+\delta}v_2$,

• $b_1$ accepts the minimum of the offered prices provided that $\min_i\{p_{s_i}\} \leq \frac{1}{1+\delta}v_2$.

• The strategy followed by $s_2$ when responding depends on the priorities determined by the tie breaking matching mechanism.

case a) the priority of $s_2$ is higher than that of $s_1$

$s_2$ accepts the maximum of the offered prices provided that $max_i\{p_{b_i}\} \geq \frac{\delta}{1+\delta}v_2$.

case b) the priority of $s_2$ is smaller than that of $s_1$

if $p_{b_1} \leq \frac{\delta}{1+\delta}v_2$ then $s_2$ accepts the maximum of the offered prices provided that $max_i\{p_{b_i}\} \geq \frac{\delta}{1+\delta}v_2$.

if $p_{b_1} > \frac{\delta}{1+\delta}v_2$, then: $s_1$ accepts $p_{b_1}$ when $p_{b_2} < \frac{\delta}{1+\delta}v_2$, otherwise he accepts $\min_i\{p_{b_i}\}$.

If there is only one pair of agents, $s_i$ and $b_i$, in the market, then:

• $s_i$ proposes $p_{s_i} = \frac{1}{1+\delta}v_i$, $b_i$ proposes $p_{b_i} = \frac{\delta}{1+\delta}v_i$,

• $s_i$ accepts $p_{b_i}$ if $p_{b_i} \geq \frac{\delta}{1+\delta}v_i$, $b_i$ accepts $p_{s_i}$ if $p_{s_i} \leq \frac{1}{1+\delta}v_i$.

It can be checked that these strategies form a subgame perfect Nash equilibrium.

*Uniqueness (in terms of payoffs):* Call $M_{s_i}$, $m_{s_i}$ the supremum and infimum of subgame perfect Nash equilibrium for sellers in a $s$-game (respectively, $M_{b_i}$, $m_{b_i}$ for buyers in a $b$-game), when all four agents are still in the market, that is, when the market is imbedded in graph $G$. We will find inequalities in order to show that $M_{s_1} = m_{s_1} = M_{s_2} = m_{s_2} = M_{b_2} = m_{b_2} = \frac{1}{1+\delta}v_2$. Notice that these equalities also implies that $M_{b_1} = m_{b_1} = v_1 - \frac{1}{1+\delta}v_2$. By existence, we already know that $M_{s_1} \geq \frac{1}{1+\delta}v_2$, $M_{b_1} \geq v_1 - \frac{\delta}{1+\delta}v_2$, $m_{s_1} \leq \frac{1}{1+\delta}v_2$, $m_{b_1} \leq v_1 - \frac{\delta}{1+\delta}v_2$, $M_{s_2} \geq \frac{1}{1+\delta}v_2$, $M_{b_2} \geq \frac{1}{1+\delta}v_2$, $m_{s_2} \leq \frac{1}{1+\delta}v_2$, and $m_{b_2} \leq \frac{1}{1+\delta}v_2$. We can now show that:

$$m_{s_2} \geq v_2 - \delta \max\{\frac{1}{1+\delta}v_2, M_{b_2}\} = v_2 - \delta M_{b_2}. \tag{5}$$

If $b_2$ rejects an offer from $s_2$, he may get $\frac{\delta}{1+\delta}v_2$ or $\delta M_{b_2}$. Thus, $s_2$ will never offer a price strictly smaller than $v_2 - \delta \max\{\frac{1}{1+\delta}v_2, M_{b_2}\}$ since he is sure to be accepted by buyer $b_2$ when he asks for $v_2 - \delta \max\{\frac{1}{1+\delta}v_2, M_{b_2}\}$. Similarly, we can also show the lower bound of $s_1$'s payoffs:

$$m_{s_1} \geq \min\{v_1 - \delta \max\{v_1 - \frac{\delta}{1+\delta}v_2, M_{b_1}\}, v_2 - \delta \max\{\frac{1}{1+\delta}v_2, M_{b_2}\}\}. \tag{6}$$

The first element of the right hand side is the minimum amount that $s_1$ has to offer in order to ensure $b_1$'s acceptance and the second element is the amount to ensure $b_2$'s acceptance. Thus, $s_1$ will never offer a price less than the smallest of these two amounts. Equation (6) reduces to $m_{s_1} \geq \min\{v_1 - \delta M_{b_1}, v_2 - \delta M_{b_2}\}$ since we know that $M_{b_1} \geq v_1 - \frac{\delta}{1+\delta}v_2$ and $M_{b_2} \geq \frac{1}{1+\delta}v_2$. On the other hand, we can also show that:

$$M_{b_2} \leq v_2 - \delta \min\{\frac{1}{1+\delta}v_2, m_{s_2}\} = v_2 - \delta m_{s_2}. \tag{7}$$

By rejecting an offer from $b_2$, the minimum amount $s_2$ can get is either $\frac{\delta}{1+\delta}v_2$ or $\delta m_{s_2}$. Thus, to ensure an acceptance from $s_2$, the minimum $b_2$ has to offer is $\delta \min\{\frac{1}{1+\delta}v_2, m_{s_2}\}$. In other words, the maximum that $b_2$ can collect is less than or equal to $v_2 - \delta \min\{\frac{1}{1+\delta}v_2, m_{s_2}\}$.

Knowing this we can show that

$$M_{b_1} \leq v_1 - \delta \min\{\frac{1}{1+\delta}v_2, m_{s_2}, m_{s_1}\}. \tag{8}$$

If $b_1$ offers a price strictly smaller than $\min\{\frac{1}{1+\delta}v_2, m_{s_2}, m_{s_1}\}$ none of the sellers will accept. Similarly as above, seller $s_2$ will not accept. Knowing this, $s_1$ can get $\frac{\delta}{1+\delta}v_2$ or at least $\delta m_{s_1}$ by rejecting; therefore seller $s_1$ would not accept this price either. Moreover, $b_1$ cannot get a payoff as high as $v_1 - \delta \min\{\frac{1}{1+\delta}v_2, m_{s_2}, m_{s_1}\}$ by trading next period.

These inequalities together with $M_{s_1} \geq \frac{1}{1+\delta}v_2$, $M_{b_1} \geq v_1 - \frac{\delta}{1+\delta}v_2$, $m_{s_1} \leq \frac{1}{1+\delta}v_2$, $m_{b_1} \leq v_1 - \frac{\delta}{1+\delta}v_2$, $M_{s_2} \geq \frac{1}{1+\delta}v_2$, $M_{b_2} \geq \frac{1}{1+\delta}v_2$, $m_{s_2} \leq \frac{1}{1+\delta}v_2$, and $m_{b_2} \leq \frac{1}{1+\delta}v_2$ imply that $M_{s_1} = m_{s_1} = M_{s_2} = m_{s_2} = M_{b_2} = m_{b_2} = \frac{1}{1+\delta}v_2$.

**Proof of Proposition 6.** We prove the result by induction. First, we show that the result holds for a number of agents $|S| \leq t$, $|B| \leq t$ with $t = 2$.

*Step 1.* When $t = 2$, there are five possible graphs. We have already shown in Small Markets section that the result is true for all of these graphs.

*Step 2.* Now, suppose that the result is true for all graphs with at most $t = n - 1$ agents on one side of the market. That is, as the induction hypothesis, we assume that the result is true for graphs of size $|S| \leq n - 1$, $|B| \leq n - 1$. We are going to show that the result is true for any graph, say $G$, of size $|S| = |B| = n$.

The strategies must specify the actions of the agents for the graph $G$ and for any subgraph $G'$ of $G$ that results from removing a set of pairs of nodes from $G$. For proposers, an action is what price to propose, and for responders, it is what to do for any given distribution of prices, in any given graph $G'$. The strategy of a proposer depends on which subgraph she is in according to the extended CB-algorithm. The strategy of a responder depends on the subgraph and each distribution of prices. Strategies do not depend on past history.

If we are in a strict subgraph of $G$, that means at least one pair of agents has traded and left the market. Thus, the number of agents is strictly smaller than $n$ both in $S$ and $B$. By the induction hypothesis, we know that there exists a subgame perfect Nash equilibrium in this subgame. The equilibrium strategies will follow the ones in any such subgames.

If we are in $G$, that means no one has traded. Apply the extended CB-algorithm and identify the subgraphs of types $G^S$, $G^B$, and $G_k^E$. For the sake of brevity, call the subgames in which sellers are the proposers as $s$-game and the ones in which buyers are the proposers as $b$-game.

The price proposal in equilibrium is the following:

- In $G^S$ type subgraphs, all proposed prices are 0 (in a $s$-game or in a $b$-game).
- In $G^B$ type subgraphs, all proposed prices are $v^*_{G'}$ (in a $s$-game or in a $b$-game).
- In $G_k^E$ type subgraphs, all proposed prices are $\frac{1}{1+\delta}v_k$ in a $s$-game and $\frac{\delta}{1+\delta}v_k$ in a $b$-game.

For future reference, we denote this price proposal with $P$. If the price proposal is equal to $P$, then all responders accept (both in $s$-game and $b$-game). Notice that sellers in $s$-game have incentives to ask higher prices only while buyers in $b$-game would like to reduce the prices if they ever decide to deviate. Now, suppose that only one proposer deviates from the price proposal $P$. If it is a $s$-game and the seller who deviates belongs to a $G^S$ type subgraph, then all buyers in that subgraph accept 0. Similarly, if it is a $b$-game and the buyer who deviates belongs to a $G^B$ type subgraph $G'$, then all sellers in that subgraph accept $v^*_{G'}$. If it is a $s$-game and the deviating agent, say $s$, belongs to a $G_k^E$ type subgraph, then all neighbors of $s$ reject the proposal of $s$ and accept $\frac{1}{1+\delta}v_k$ (if they can, otherwise they reject all offers) while all the other buyers hold onto their earlier decisions (accept the same prices they did in $P$). On the other hand, if it is a $b$-game and the deviating agent, say $b$, belongs to a $G_k^E$ type subgraph, then all neighbors of $b$ reject the proposal of $b$ and accept $\frac{\delta}{1+\delta}v_k$ (if they can, otherwise they reject all offers) while all the other sellers hold onto their earlier decisions.

Until now, we have only specified what proposers should propose, what responders should do when they face the price proposal $P$, and how should responders react when they face some of the possible unilateral deviations. The remaining duty is to determine the reactions of responders in all the other cases. We define strategies so that if not all the possible number of pairs forms, then they follow the subgame perfect Nash equilibrium of the resulting subgraph (which we know exists by the induction step). If all agents reject, then the strategies will prescribe for proposers to propose price distribution $P$ and for responders to accept. Therefore we can conclude that given an action for all responders, the payoffs are immediately determined. For a given distribution of prices, the game is a one-shot game with a finite set of actions. This game must have at least one Nash equilibrium (possibly, in mixed strategies). We will define the strategies as follows: for a given distribution of prices, strategies will tell responders to play according to this Nash equilibrium. Notice that we may have a multiplicity of Nash equilibria. If this is the case, strategies must specify which of the several Nash equilibria will be played. Any specification will do the job.

It can be checked that these strategies construct a subgame perfect Nash equilibrium.

**Proof of Theorem 2.** Notice that when *(ii)* holds none of the $G^E$ type subgraphs can be decomposed further into smaller subgraphs. So, there cannot be two different prices in any $G^E$ type subgraph since the only way to have different prices in a $G^E$ type subgraph requires at least two moderate buyers. Furthermore, observe that if *(iii)* holds then there can be only one subgraph of type $G^B$, which is the graph itself. Then, by Proposition 6, we know that *(i)*, *(ii)*, and *(iii)* are sufficient conditions for a network $G$ to support the reference solution. Thus, we only need to show that if $G$ supports the reference solution then $G$ has to satisfy *(i)*, *(ii)*, and *(iii)*.

*Part (i). If $G$ supports the reference solution then $G$ decomposes into subgraphs which are all of the same type.*

Suppose that $G$ supports the reference solution but it decomposes into subgraphs with at least two different types when we apply the CB-algorithm. We have three cases to consider.

*Case I.* Suppose that $|S| > |B|$. Then, there must be a $G^S$ type subgraph in $G$. Otherwise, we would have a contradiction with $|S| > |B|$. Because $G$ supports the reference solution, all buyers receive their valuations and all sellers get 0. First, assume that there is also a

$G^B$ type subgraph in $G$, say $G_1 = (S_1, B_1, L_1)$. Because all buyers receive their valuations, they all must be matched with a separate seller in the equilibrium. Because $G_1$ is a $G^B$ type subgraph, by definition, $|N_{G_1}(B_1)| = |S_1| < |B_1|$ which contradicts with all buyers in $G_1$ being matched in the equilibrium. Thus, there can only be subgraphs of $G^S$ or $G^E$ type in $G$. Now, suppose that there is a $G^E$ type subgraph in $G$, say $G_2 = (S_2, B_2, L_2)$. By definition of $G^S$ type, there is no link between sellers in $G^S$ type subgraphs and buyers in $G_2$. We are going to show that there is a profitable deviation for a seller in $G_2$, say $s_i \in S_2$. Suppose, instead of asking zero, $s_i$ proposes a price of $\varepsilon > 0$. Then, a buyer in $G_2$ would accept the offer since there are $|S_2| - 1$ sellers proposing 0 price, $|S_2| = |B_2|$ buyers willing to accept it, and buyers in $G_2$ have no access to the sellers in $G^S$ type subgraphs. If the remaining buyer does not accept $\varepsilon$ she would have to share her valuation, roughly equally, with $s_i$ instead of receiving her valuation minus $\varepsilon$. Thus, if $G$ supports the reference solution and $|S| > |B|$, $G$ is a union of $G^S$ type subgraphs.

*Case II.* Suppose that $|S| < |B|$. Then, there must be a $G^B$ type subgraph in $G$. Otherwise, we would have a contradiction with $|S| < |B|$. Because $G$ supports the reference solution, all sellers receive payoff $v^*$ and all buyers receive their valuation minus $v^*$, where $v^*$ is equal to the $(|S| + 1)$th highest valuation of $|B|$ buyers. First, assume that there is also a $G^S$ type subgraph in $G$, say $G_1 = (S_1, B_1, L_1)$. Because all sellers receive a positive amount, they all must be matched with a separate buyer in the equilibrium. Because $G_1$ is a $G^S$ type subgraph, by definition, $|N_{G_1}(S_1)| = |B_1| < |S_1|$ which contradicts with all sellers in $G_1$ being matched in the equilibrium. Thus, there can only be subgraphs of $G^B$ or $G^E$ type in $G$. Now, suppose that there is a $G^E$ type subgraph in $G$, say $G_2 = (S_2, B_2, L_2)$. Because all sellers receive $v^*$, the total surplus captured by sellers is $|S|v^*$. Because all buyers receive their valuation minus $v^*$, the total surplus captured by buyers is $\sum_{i=1}^{|B|}(v_i - v^*)$. Then, we have

$$|S|v^* + \sum_{i=1}^{|B|}(v_i - v^*) = \sum_{i=1}^{|B|} v_i + (|S| - |B|)\, v^* < \sum_{i=1}^{|B|} v_i$$

since $|S| < |B|$. This inequality leads us a contradiction because the last part is the total value of all transactions. Thus, we can conclude that if $G$ supports the reference solution and $|S| < |B|$, then $G$ is a union of $G^B$ type subgraphs.

*Case III.* Suppose that $|S| = |B|$. If $G$ does not decompose into $G^E$ type subgraphs only, then it must have $G^S$ and $G^B$ type subgraphs, with all sellers getting $\frac{1}{1+\delta}\underline{v}$ and all buyers getting their valuations minus $\frac{1}{1+\delta}\underline{v}$. Let $G_3 = (S_3, B_3, L_3)$ be an arbitrary $G^S$ type subgraph in $G$. Because all sellers receive a positive amount in $G_3$, they all must be matched with a separate buyer in the equilibrium. Because $G_3$ is a $G^S$ type subgraph, by definition, $|N_{G_3}(B_3)| = |S_3| < |B_3|$ which contradicts with all buyers in $G_3$ being matched in the equilibrium. Thus, if $G$ supports the reference solution and $|S| = |B|$, then $G$ is a union of $G^E$ type subgraphs.

*Part (ii). If G supports the reference solution then (if applicable) the only moderate buyer is the buyer with the lowest valuation.*

If $G$ decomposes into subgraphs which are all of $G^S$ or $G^B$ type, then there is nothing to prove. So, suppose that $G$ decomposes into $G^E$ type subgraphs only. We will prove the contrapositive of the statement above. On the contrary, suppose that there is a moderate buyer $b_i$ who does not have the lowest valuation in the subgraph she belongs to and $G$ supports the reference solution. Let $G_i$ denotes the subgraph $b_i$ belongs to. By definition, there must be another moderate buyer in $G_i$ who has the lowest valuation, say $b_j$. Then, we can decompose $G_i$ further into at least two smaller subgraphs by using the process explained earlier in the text. Let $G_{ii}$ and $G_{ij}$ be the subgraphs $b_i$ and $b_j$ belongs to, respectively, at the end of this decomposition. It is straightforward to see that the equilibrium prices in these components will be $\frac{1}{1+\delta}v_i$ and $\frac{1}{1+\delta}v_j$, respectively. Thus, we have a contradiction since all trades in the reference solution occurs at a unique price.

*Part (iii). If $G$ supports the reference solution then (if applicable) a buyer $b_i$ is a soft buyer if and only if $v^* \geq v_i$.*

If $G$ decomposes into subgraphs which are all of $G^S$ or $G^E$ type, then there is nothing to prove. So, suppose that $G$ decomposes into $G^B$ type subgraphs only. First, we claim that there is only one subgraph, which is the graph itself. On the contrary, suppose that $G$ decomposes into at least two subgraphs of type $G^B$. Let $G'$ and $G''$ be two of those subgraphs. By Proposition 6, we know that the equilibrium prices in these subgraphs are $v^*_{G'}$ and $v^*_{G''}$, respectively, and because buyers have different valuations $v^*_{G'} \neq v^*_{G''}$. Then, we have a contradiction since all trades in the reference solution occurs at a unique price. If $b_i$ is a soft buyer in $G$, then $v^* \geq v_i$ by definition. The only remaining duty is to show that $b_i$ is a soft buyer in $G$ only if $v^* \geq v_i$. On the contrary, suppose that $v^* \geq v_i$ but $b_i$ is not a soft buyer. Thus, $b_i$ receives a good in the reference solution. Because only $|S|$ of the buyers can procure goods in the reference solution, there must be a buyer $b_j$ with valuation $v_j > v^*$ who does not receive any goods. Then, $b_j$ can have a profitable deviation by increasing her offer above $v^*$ since all seller would be willing to accept such an offer. This is a contradiction with the reference solution being an equilibrium outcome of the bargaining game.

**Proof of Theorem 3.** The necessity part of the theorem is a direct result of Theorem 2 since allocatively complete networks are of type $G^B$ and, by definition, all buyers who have valuations less than $v^*$ in $G$ are soft buyers. For the sufficiency, suppose that $G$ supports the reference solution but it is not allocatively complete. Because $G$ is not allocatively complete, there exists a subset $B' \subseteq B$ such that $|N_G(B')| < |B'|$. Then, the CB-algorithm would decompose $G$ into at least two subgraphs one of which involves only buyers in $B'$ and sellers in $N_G(B')$. This is a contradiction with $G$ supporting the reference solution because the existence of two subgraphs in the decomposition guarantees the existence of at least two different prices. Thus, if $G$ supports the reference solution then it is allocatively complete.

## Chapter III

# RETAIL ASSORTMENT PLANNING UNDER CATEGORY CAPTAINSHIP
(with Mumin Kurtulus)

### Introduction

The proliferation of products available to consumers and low margins due to intense competition are some of the challenges faced by the consumer goods retailers today. The assortment carried by a retailer can have a tremendous impact on its bottom line. In the late 80s, retailers started to segment products with similar characteristics into groups called categories and started to manage product categories as separate business units. Many retailers adopted the category management process which involves strategic management of the categories to maximize sales and profit while satisfying consumer needs. More recently, however, the scarcity of resources needed to manage categories and the increase in the number of product categories have led retailers toward a new trend. Retailers have started to rely on their leading manufacturers for strategic recommendations regarding key category management decisions such as category assortment. These leading manufacturers have often been referred to as *category captains* and the practice itself has been referred to as *category captainship* (Kurtulus and Toktay 2009).

Many retailers and suppliers have implemented category captainship and reported benefits as a result of their implementations (Progressive Grocer 2007 and 2008). For example, Coors Brewing serves as a category captain for a number of its retail clients in the alcoholic beverages category. The key insight provided by Coors Brewing company to one of their retail clients was that the retail chain's core shopper best matched the characteristics of premium light beer purchasers. However, the retailer was not able to convert its shoppers into premium light beer buyers. In addition, Coors recognized that ineffective beer merchandising and limited display support had resulted in flat market share and sales for the retailer.

In response, Coors developed an aggressive merchandising plan that included megadisplays, enhanced point of purchase materials, targeted zip code ads, and special pack-marketing programs. These strategies led to a 6% to 12% increase in store volume (Progressive Grocer 2007 and 2008).

Several retailers rely on General Mills' Small Planet subsidiary for recommendations in the natural/organic canned and packaged foods category. The category managers at Small Planet identified that consumers are confused about the placement of the organic and natural products. In addition, they did a study on what products perform well in a particular region and developed best practice planograms for the retailers for which they served as a category captain (Progressive Grocer 2007). For another retailer, Small Planet strategists found that placement of the organic/natural items was not a critical success factor when compared to things such as the type of consumer (heavy vs light), variety, and duration of shopping trip (Progressive Grocer 2007).

Both of these examples reveal that retailers can benefit from implementing category captainship in two ways. Category captains usually (1) provide consumer insights and/or (2) help retailers increase traffic into the category. Consumer insights such as the ones provided by Coors and General Mills in the examples above are not readily available to the retailers. These insights can help retailers offer an assortment that better matches consumer's needs. The category growth, on the other hand, is a result of traffic driving strategies such as consumer education, promotions, and in-store display strategies.

While many retailers and suppliers have reported benefits, category captainship practices have also been surrounded by controversies. In particular, category captains' potential bias in providing recommendations to their retailers has been an issue because these recommendations cover not only their own brands but also the brands of their rivals in the category (Desrochers et al. 2003). Even though the category captains are usually the biggest manufacturers in the category and have a significant interest in the categories they manage for retailers, their incentives may not be fully aligned with the retailer's objective of maximizing the performance of the entire category. Retailers have responded to these threats by setting some targets for their captains and continuously measuring their captain's performance in a scorecard (T. Kavanaugh, personal communication, March 4, 2008; ACNielsen 2005). However, it is not always possible for the retailers to detect biased recommendations. It has been reported that in some cases the category captains have taken advantage of their positions to disadvantage or exclude competing brands in the categories (Steiner 2001, Desrochers et al. 2003, Greenberger 2003, Klein and Wright 2006). This phenomenon, in general, has been referred to as *competitive exclusion*. Many category captainship implementations have been taken to court over category captainship misconduct and alleged competitive exclusion (Greenberger 2003).

Motivated by the controversies surrounding the category captainship practices, the goal of our research is to better understand the consequences of using category captains for assortment decisions. In particular, we answer the following questions: Is competitive exclusion (reduction in category variety) a valid concern for the retailers? What is the impact of category captainship on the retailer, the category captain, and the non-captain manufacturers? To answer these questions, we consider a supply chain model where multiple manufacturers sell to consumers through a single retailer. First, we consider a model where the retailer is responsible for selecting the category assortment. Then, we consider a model where the retailer delegates the assortment selection decision to one of its manufacturers in the category in return for making recommendations such that a certain target category profit is achieved. We assume that the retailer benefits from using a category captain for assortment decisions because the captain can (1) provide consumer insights not readily available to the retailer and (2) increase traffic into the category. Our results are based on a comparison of these two models.

Our results can be summarized as follows. First, we show that category captainship can be profitable for all involved parties (i.e., the retailer, the captain, and the non-captain manufacturers). Nevertheless, we also find that competitive exclusion remains a valid concern and is driven by both the captain's superior knowledge of consumers and ability to increase traffic into the category. At the same time, we show that the reduction in variety under category captainship is not always due to competitive exclusion. A retailer's lack of consumer insights might force the retailer to offer a suboptimal variety under retail category management. The category captain's insights can help the retailer to adjust the category variety to a level that can be higher or lower than the variety under retail category management. A possible reduction in category variety does not necessarily imply competitive exclusion but sometimes is simply due to an adjustment as a result of improved information about the consumers. Our results have implications regarding when and which categories the retailer should rely on a category captain for assortment selection decisions.

**Literature Review**

Our literature review focuses on three streams of research: (1) literature on category captainship; (2) literature on retail assortment planning; and (3) antitrust literature on category captainship. Next, we discuss the relevance and contribution of our work to these streams of research.

First, despite more than a decade of implementation, there has been limited academic research about category captainship. Only three papers address this topic (Niraj and Narasimhan 2003, Wang et al. 2003, and Kurtulus and Toktay 2009). Both Niraj and Narasimhan (2003) and Kurtulus and Toktay (2009) investigate the emergence of category captainship. Both papers consider a model with two manufacturers selling to the consumers through a single retailer. While Niraj and Narasimhan (2003) define category captainship as an exclusive information sharing alliance between the retailer and one of the manufacturers, Kurtulus and Toktay (2009) define category captainship as an alliance that involves retail pricing in an environment with limited shelf space. Niraj and Narasimhan's findings are in terms of the complementarity of the information available to each party whereas Kurtulus and Toktay show that the emergence of category captainship depends on the degree of product differentiation, the opportunity cost of shelf space and the profit sharing arrangement in the alliance. Wang et al. (2003) also model the category captainship as an alliance that involves retail pricing and investigate the consequences of using category captains. Wang et al. (2003) concludes that category captainship benefits the retailer and the category captain, but disadvantages the non-captain manufacturer.

The fundamental difference between our research and this stream of research on category captainship is that the category assortment has already been fixed in these papers and prices are considered as a decision variable whereas in our model, retail prices are fixed and category assortment is being considered as a decision variable. In addition, Kurtulus and Toktay (2009) also recognize the possibility of competitive exclusion when the retailer relies on a category captain for pricing recommendations. They identify the delegation of pricing decisions and the limited shelf space at the retailer as potential drivers of competitive exclusion. We contribute to this line of work by identifying other drivers of competitive exclusion. We find that the delegation of the assortment decisions can also lead to competitive exclusion when the category captain has private information about the consumers and ability to increase traffic into the category.

Second, there is a literature on retail assortment planning where the main focus is on retailer's optimal assortment selection (see Kök et al. 2006 for a review of this literature). Van Ryzin and Mahajan (1999) study the relationship between inventory costs and variety benefits in retail assortment. They determine the optimal assortment and provide insights on how various factors affect the optimal level of assortment variety. Various extensions to the model by van Ryzin and Mahajan have been considered. Hopp and Xu (2003) extend the model by assuming a risk-averse decision maker. Cachon et al. (2005) study retail assortment in the presence of consumer search. Cachon and Kök (2007) study assortment planning with multiple categories and consider the interaction between the categories. As in our model, Aydın and Hausman (2009) also focus on assortment decisions in a decentralized supply chain but their focus is on the use of slotting fees in coordinating the retailer's assortment decision. All of these papers focus on the retailer's optimal assortment. We contribute to this literature by investigating how retail assortment under category captainship may differ from that under retail category management.

Finally, some economists have voiced antitrust concerns related to category captainship (Steiner 2001, Desrochers et al. 2003, Leary 2003, Klein and Wright 2006). Some of these papers hypothesize that category captainship can result in competitive exclusion but offer no evidence for it. Our research contributes to the ongoing debate by offering theoretical support for the existence of competitive exclusion, but also identifying conditions under which such concerns are irrelevant.

## The Model

We consider a two-stage supply chain that consists of multiple manufacturers that are potential candidates for selling their products to consumers through a common retailer. We assume that each manufacturer sells only one product. The retailer faces the decision of which products to offer in the category. Let $n$ denote the number of products offered by the retailer. After being offered an assortment with $n$ products, a customer arriving at the retailer either purchases one of the $n$ products or does not purchase anything.

**Cost Structure.** We assume that all products are sold at the same retail price $r$. This assumption is reasonable in perfectly competitive markets where firms do not have any power over their pricing decisions. Retailers today operate in highly competitive marketplace and therefore have very little room for competing on price but more on assortment. Cachon et al. (2007) consider a similar model where all products have the same probability of appealing to a consumer and therefore it is optimal for the firm to choose the same retail price for their products. Shugan (1989) provides evidence for this assumption and indicates that the majority of flavors within a product line are sold at the same price in the ice cream industry. We also assume that all manufacturers offer the product to the retailer at the same wholesale price $w$. We define $m \doteq r - w$ to be the retailer's net profit margin. For simplicity, we also normalize the production cost at each manufacturer to zero so that each manufacturer's net profit margin is $w$.

In addition to the basic cost structure, we assume that the retailer incurs an operational cost $\sigma(n)$ associated with carrying a variety of products in the assortment. For clarity of exposition, we assume that $\sigma(n) = \beta n$, where $\beta > 0$.

**Demand Model.** We use a generic attraction market share type model introduced in Bell et al. (1975). The multinomial logit (MNL), which is commonly used in the operations literature to study assortment problems (e.g., van Ryzin and Mahajan 1999), is one example of an attraction type market share model. Let $v_i$ be the attraction of product $i$ to consumers. The attraction of a product may be a function of advertising, price, reputation of the company, the service given during and after purchase and other things that may play a role in the consumer's choice. For analytical tractability, we assume that all products in the category are identical, $v_1 = v_2 = ... = v_n = v$. In the Extensions section, we consider an extension that demonstrates how our results would change if the assumption of identical products was to be relaxed.

The consumers either select one of the $n$ identical products or the no-purchase option. Let $v_0$ be the attractiveness of the no-purchase option. We assume that $v_0$ is sufficiently small (i.e., $v_0 < \bar{v}_0$).[1] Let $q(n)$ be the probability that a consumer purchases one of the identical products offered in the category and $q_0(n)$ be the probability that an incoming customer selects the no-purchase option. Using the market share theorem in Bell et al. (1975), the market share (or alternatively, the purchase probability) for one of the products offered in the assortment is given by $q(n) = \frac{v}{v_0+nv}$. Similarly, the market share of the no-purchase option (or alternatively the probability of a consumer walking out of the store without a purchase) is given by $q_0(n) = \frac{v_0}{v_0+nv}$. Let also $\lambda$ denote the rate of consumers entering the store. Thus, the average demand rate for each product is given by $\lambda q(n)$ and the average

[1]This is a technical assumption that is used in some of our proofs and the exact value of $\bar{v}_0$ is given in the appendix. Under this assumption, the attractiveness of the no-purchase option is small enough, which implies that an incoming consumer's willingness to shop is high enough.

rate of consumers that do not purchase is given by $\lambda q_0(n)$.

**Information Structure.** We assume that the category captain has better information about the consumers' preferences. This is in line with the main motivation of the retailers for using category captains. We capture the information asymmetry through the attraction parameter $v$ in the demand model. While the retailer believes that the attraction parameter $v$ is either high ($v_H$) or low ($v_L$) with probabilities $\alpha$ and $1 - \alpha$, respectively, the captain knows the realization of $v$.

To summarize, we make a number of simplifying assumptions such as equal production, wholesale and retail prices for each product as well as assuming equally attractive products in the category. Similar assumptions have been used in Cachon et al. (2007). These assumptions ensure analytical tractability and allow us to focus on size of the assortment as opposed to the structure of the optimal assortment as is common in the operations literature (e.g., van Ryzin and Mahajan 1999 and Cachon et al. 2005). In addition, our goal is to investigate the changes in the size of the assortment and how the size of the assortment is impacted by the change in ownership of the assortment decisions. In the Extensions section, we discuss how relaxing some of our modeling assumptions would change our results.

## Analysis

We first consider the Retail Category Management (RCM) model where the retailer is responsible for selecting the variety of the retail assortment. Then, we consider the Category Captainship (CC) model where the retailer delegates the assortment decision to a leading manufacturer in return for a target profit. All proofs are in Appendix A. For convenience, we include a list of key notations used in the paper at the beginning of the appendix.

### Retail Category Management

In this scenario, given the wholesale and retail prices, the retailer decides how many items to include in the retail assortment in the face of uncertainty regarding the attractiveness parameter $v$. The retailer believes that the attractiveness for the products is either high, $v_H$, or low, $v_L$ with probability $\alpha$ and $1-\alpha$, respectively. The retailer selects the optimal variety $n$ by solving

$$\max_n \quad \alpha\frac{m\lambda n v_H}{v_0 + n v_H} + (1-\alpha)\frac{m\lambda n v_L}{v_0 + n v_L} - \beta n$$

where the first two terms are the expected revenue from sales and the last term captures the operational cost of managing variety. While in reality the parameter $n$ is discrete, in our paper we use an approximation where we treat the retailer's objective function as being continuous over $n$. Our approximation admits the following interpretation: If the continuous approximation suggests that the optimal variety for the retailer is $n_R = 12.3$ for example, we would round this to the nearest integer (12 in this case) to find the solution of the discrete optimization problem. This approximation is reasonable because our goal is to compare the variety under RCM and CC and investigate the directional effects rather than drawing conclusions regarding the variety to be offered in these scenarios separately. Let $n_R$ denote the optimal variety in the RCM scenario.

**Lemma 1** *There exists a unique variety level $n_R$ that maximizes retailer's expected profit.*

Let $n_R^H$ and $n_R^L$ be the optimal varieties as if the retailer knows the consumer type. First, it can be shown that $n_R^H < n_R^L$. The marginal revenue of adding another product for the retailer is higher when consumers are low type since the increase in probability of buying is higher for these consumers. This implies that the retailer would prefer a higher variety when consumers are low type and lower variety when the consumers are high type. Second, for $\alpha \in [0,1]$, the retailer's optimal variety under uncertainty $n_R$ is bounded above and below by $n_R^H$ and $n_R^L$, respectively. That is, $n_R^H = n_R < n_R^L$ when $\alpha = 1$ and $n_R^H < n_R = n_R^L$ when $\alpha = 0$. Therefore, the retailer's imperfect knowledge about the consumer's behavior forces the retailer to act as an expected profit maximizer and this results in a suboptimal category variety.

Let $\Pi_R$ and $\pi_R$ denote the expected profits of the retailer and the category captain, respectively, in the RCM scenario. We also define $\Pi_R^i$ and $\pi_R^i$ where $i \in \{L, H\}$ as the retailer's and the category captain's realized profits under RCM when consumers are $i$-type.

**Category Captainship**

In the category captainship scenario, the retailer assigns one of the manufacturers as the category captain and delegates the assortment selection decision to the category captain for two reasons.

First, the category captain has better information about consumer preferences. While a typical retailer sells a range of products that could fall in one of up to three to four hundred categories, a typical consumer goods manufacturer sells a smaller range of products and has better information about consumer preferences in particular categories. Manufacturers constantly conduct consumer studies that are used in guiding them while introducing new products and improving the existing products. For instance, the consumer insights provided by Coors Brewing and General Mills helped the retailers in adjusting their assortments to better match the consumers' preferences. We capture the captain's expertise and superior knowledge about consumers by assuming that the category captain knows the realization of the attraction parameter $v$ (i.e., whether consumers are H-type or L-type) while the retailer views this as uncertain. Better information about the parameter $v$ translates into an assortment that better matches consumers needs.

Second, the category captain can collaborate with the retailer and increase traffic into the category through consumer education, promotions, improved in-store displays and merchandising plans. The merchandising plan recommended by Coors Brewing is a good example that illustrates how a category captain can help its retailers drive traffic into the category. We capture this benefit to the retailer by assuming that the category captain increases the rate of consumers who would potentially shop in the category and denote this increase by $\Lambda$. The parameter $\Lambda$ captures the category captain's ability to stimulate demand.

The sequence of events in the category captainship scenario is as follows: At stage one, the retailer offers a category captainship contract. The category captain, in return, either accepts or rejects the contract. At stage two, if the contract is accepted, the category captain selects variety to be recommended to the retailer. We assume that if the category captain accepts the contract and cannot achieve the target profit goal set by the retailer, then the category captain pays a very high penalty to the retailer. If the category captain rejects the contract, the retailer updates its beliefs about the consumers' type and decides on variety of the assortment. We assume that once the category captain accepts the contract, no renegotiation or breach of contract takes place. If the contract could be renegotiated, then in cases where the category captain rejects the contract, both the retailer and the category captain could gain by renegotiation. Similarly, if a breach of contract was allowed, the retailer could renege on its promised actions after finding out that the category captain is making a positive surplus. Technically, we model the category captainship as a two stage screening game in which the (uninformed) retailer makes a take-it-or-leave-it offer to the (informed) category captain. We are interested in pure strategy perfect Bayesian equilibria.

One of the key steps in category captainship process is objective and target setting (or the so-called category scorecard). Retailers might set different objectives for different categories. While driving sales volume can be a very important performance metric for a traffic driver category such as soft drinks and fresh produce, profitability is usually the primary objective in most categories. We assume that the retailer delegates the assortment selection decision to the category captain in return for a fixed target category profit level $K$. However, in the Extensions section, we also consider an extension where the retailer's goal is to maximize sales in a category and offers a target sales contract to the category captain and discuss how our results would change when the retailer's goal is to maximize sales instead of profit.

A strategy profile is defined as $(K, (\phi^H, n^H), (\phi^L, n^L))$ where $K$ is the target profit level set by the retailer and $(\phi^i, n^i)$ is the category captain's strategy: $n^i$ is the variety level set by the category captain when the contract is accepted and consumers are $i$-type, and $\phi^i$ is a

$\{0,1\}$ dummy variable, with 0 and 1 representing rejection and acceptance of the retailer's offer, respectively. A strategy profile $(\widetilde{K}, (\widetilde{\phi}^H, \widetilde{n}^H), (\widetilde{\phi}^L, \widetilde{n}^L))$ is a perfect Bayesian equilibrium if and only if it satisfies the following conditions:

<u>*Retailer's Best Response:*</u>

$$\widetilde{K} \in \arg\max_{K} \left\{ \alpha \left[ \widetilde{\phi}^H \Pi_{TP}^H(\widetilde{n}^H, K) + (1 - \widetilde{\phi}^H)\Pi_R^H \right] + (1-\alpha) \left[ \widetilde{\phi}^L \Pi_{TP}^L(\widetilde{n}^L, K) + (1 - \widetilde{\phi}^L)\Pi_R^L \right] \right\}$$

where $\Pi_{TP}^i(n^i, K)$ is the retailer's profit with target profit (TP) when consumers are $i$-type and $\Pi_R^i$ is the retailer's optimal profit when consumers are $i$-type and the retailer sets variety with its updated beliefs (without the increase in traffic). The retailer's best response is such that if the category captain accepts the offer ($\phi^i = 1$), then the category captain recommends the variety but if the category captain rejects the offer ($\phi^i = 0$), then the retailer sets the variety after updating its beliefs.

<u>*Category Captain's Best Response:*</u> Let $\pi_{TP}^i(n^i, K)$ be the category captain's profit with target profit and $\pi_R^i$ be the category captain's profit under retail category management when consumers are $i$-type. The category captain accepts the offer (i.e., $\widetilde{\phi}^i = 1$) if and only if $\pi_{TP}^i(\widetilde{n}^i, \widetilde{K}) \geq \pi_R^i$ and recommends variety $\widetilde{n}^i \in \arg\max_n \{\pi_{TP}^i(n^i, \widetilde{K})\}$. The category captain rejects the contract (i.e., $\widetilde{\phi}^i = 0$) if and only if $\pi_{TP}^i(\widetilde{n}^i, \widetilde{K}) < \pi_R^i$.

<u>*Bayes Consistency of Beliefs:*</u> Let $P(\widetilde{\phi}^H, \widetilde{\phi}^L)$ be the probability that the consumers are $H$-type when the category captain's decision is $\widetilde{\phi}^H$ in the presence of $H$-type customers and $\widetilde{\phi}^L$ in the presence of $L$-type customers. Then, $P(1,1) = P(0,0) = \alpha$, $P(1,0) = 1$, and $P(0,1) = 0$. These conditions ensure that there is no bias in the retailer's beliefs.

The category captainship scenario is solved backwards: First, we assume that the category captain has already accepted the contract and consider the category captain's assortment selection problem. Then, given the category captain's variety response, we consider the retailer's target profit setting problem.

For a given target profit level $K$, the category captain who faces type $i \in \{L, H\}$ consumers solves the following problem at the second stage:

$$\begin{aligned} \max_{n} \quad & \frac{(\lambda + \Lambda)\, wv_i}{v_0 + nv_i} \\ \text{s.t.} \quad & \frac{(\lambda + \Lambda)\, mnv_i}{v_0 + nv_i} - \beta n \geq K. \end{aligned}$$

The category captain's profit is strictly decreasing in the variety offered to the consumers because each additional product in the category cannibalizes the demand for the category captain's product. However, the target profit constraint prevents the category captain from recommending its own product only. Therefore, the category captain recommends an assortment where the target profit level is binding. The following lemma characterizes the category captain's best response $n^i(K)$.

**Lemma 2** *There exists a unique best response $n^i(K)$ for $i \in \{L, H\}$ which is given by $n^i(K) = \frac{B^i(K) - \sqrt{(B^i(K))^2 - 4Kv_i v_0 \beta}}{2v_i \beta}$ where $B^i(K) = mv_i\,(\lambda + \Lambda) - v_0\beta - Kv_i$.*

At stage one, the retailer sets the target profit level $K$ in anticipation of the category captain's behavior at the second stage. If the category captain rejects the contract, then the retailer updates its beliefs about the type of consumers and then chooses the optimal variety.

In general, there are multiple equilibria in Bayesian games (Chu 1992). Essentially, two types of equilibria exist: (1) separating equilibrium and (2) pooling equilibrium. In a separating equilibrium (SE), the uninformed agent (the retailer) makes an offer such that the informed agent (the captain) reveals its type. In particular, the retailer sets a target profit level such that the category captain accepts the offer only if the consumers are H-type ($\phi^H = 1, \phi^L = 0$). In a pooling equilibrium (PE), the informed agent does not reveal its type. Both types accept the retailer's offer ($\phi^H = 1, \phi^L = 1$). Next, we characterize the target profits that lead to separating and pooling equilibria.

**Separating Equilibrium (SE).** If the retailer anticipates a separating equilibrium, the retailer selects the target profit that solves the following optimization problem,

$$\begin{aligned}
\max_{K} \quad & \alpha \Pi_{TP}^{H}\left(n^{H}\left(K\right)\right) + \left(1-\alpha\right)\Pi_{R}^{L} = \alpha K + \left(1-\alpha\right)\Pi_{R}^{L} \\
\text{s.t.} \quad & \pi_{TP}^{H}\left(n^{H}\left(K\right)\right) \geq \pi_{R}^{H} \\
& \pi_{TP}^{L}\left(n^{L}\left(K\right)\right) < \pi_{R}^{L}.
\end{aligned}$$

The retailer sets the target profit level $K$ in such a way that the category captain accepts the contract if consumers are H-type and delivers the required target profit. The category captain rejects the contract if consumers are L-type. We assume that the captain accepts the contract offer in case of indifference between rejecting and accepting. If the contract is rejected, the retailer concludes that consumers are L-type and sets the variety to maximize its profit under the RCM scenario knowing that consumers are L-type.

It is useful to define two auxiliary variety levels: $n_{TP}^{H}$ and $n_{TP}^{L}$. The variety level $n_{TP}^{i}$ represents the optimal variety in the best case scenario. That is, the retailer takes advantage of the additional traffic $\Lambda$ and knows that the consumers are $i$-type. Formally, $n_{TP}^{H}$ =arg $\max_n \Pi_{TP}^{H}(n)$ and $n_{TP}^{L}$ =arg $\max_n \Pi_{TP}^{L}(n)$. Let also $\overline{n}_{TP}^{i}$ be the variety level such that the category captain is indifferent between accepting and rejecting the contract offer when consumers are $i$-type. That is, $\pi_{TP}^{H}\left(\overline{n}_{TP}^{H}\right) = \pi_{R}^{H}\left(n_R\right)$ and $\pi_{TP}^{L}\left(\overline{n}_{TP}^{L}\right) = \pi_{R}^{L}\left(n_R\right)$.

**Lemma 3** *There exists an upper bound $\overline{\Lambda}$ such that for all $\Lambda \in [0, \overline{\Lambda}]$, the following holds*

$$0 < n_{TP}^{H}\left(\Lambda\right) < \overline{n}_{TP}^{H}\left(\Lambda\right) \leq \overline{n}_{TP}^{L}\left(\Lambda\right) \leq n_{TP}^{L}\left(\Lambda\right).$$

In the rest of the analysis, we focus on cases where $\Lambda \in [0, \overline{\Lambda}]$. This assumption is reasonable as it places an upper bound on the amount of additional traffic that the captain can drive to the category. The following proposition characterizes the target profit level and the variety offered to the consumers when the retailer offers a target profit that results in a separating equilibrium.

**Proposition 1** *In a separating equilibrium, the retailer offers* $K_{SE} = \left[\sqrt{(\lambda+\Lambda)\,m} - \sqrt{\beta \frac{v_0}{v_H}}\right]^2$. *The resulting variety is given by* $n_{TP}^H$ *if consumers are H-type and* $n_R^L$ *if consumers are L-type.*

The retailer offers a target profit that is accepted by the category captain if and only if the consumers are H-type in which case the category captain recommends variety $n_{TP}^H$. On the other hand, if the consumers are L-type, the category captain rejects the contract and retailer updates its beliefs about the consumer type. The retailer, in this case, sets the variety to $n_R^L$ which is the variety the retailer would have offered in the RCM scenario if the retailer knew that consumers are L-type. However, the retailer cannot take advantage of the additional traffic which could have been driven by the category captain in case the category captainship contract was accepted.

**Pooling Equilibrium (PE).** If the retailer anticipates a pooling equilibrium, the retailer selects the target profit that solves the following optimization problem

$$\begin{aligned} \max_{K} \quad & K \\ \text{s.t.} \quad & \pi_{TP}^H\left(n^H\left(K\right)\right) \geq \pi_R^H \\ & \pi_{TP}^L\left(n^L\left(K\right)\right) \geq \pi_R^L. \end{aligned}$$

The retailer sets the target profit level $K$ in such a way that the category captain accepts the contract no matter what the consumer type is and delivers the required target profit.

**Proposition 2** *In a pooling equilibrium, the retailer offers* $K_{PE} = (\lambda+\Lambda)m - \frac{\lambda m v_0}{v_0 + n_R v_L} - \beta\left(n_R + \frac{\Lambda}{\lambda}\left(\frac{v_0}{v_L} + n_R\right)\right)$. *The resulting variety is given by* $\frac{B_H(K_{PE}) - \sqrt{(B_H(K_{PE}))^2 - 4K v_H v_0 \beta}}{2 v_H \beta}$ *where* $B_H(K_{PE}) = m v_H\left(\lambda+\Lambda\right) - v_0\beta - K_{PE} v_H$ *if consumers are H-type and* $n_R + \left(\frac{\Lambda}{\lambda}\right)\left(\frac{v_0}{v_L} + n_R\right)$ *if consumers are L-type.*

The first term in the expression for $K_{PE}$ is the maximum achievable profit for the retailer (i.e., the profit if all incoming consumers would buy one of the products and the retailer makes margin $m$ on each product). The second term is the revenue loss due to the fact that some consumers do not purchase from the category and the last part is the operational cost of managing variety when the variety is set to $n_R + \left(\frac{\Lambda}{\lambda}\right)\left(\frac{v_0}{v_L} + n_R\right)$. The retailer wants to make sure that the category captain accepts the contract regardless of the consumer type. To do that, the retailer asks for a profit level that makes the category captain who faces L-type consumers indifferent between accepting and rejecting the contract. The retailer ensures the increase in customer rate because the category captain always accepts the contract but incurs a loss of surplus when consumers are H-type. Pooling type of equilibria are better for the retailer when the category captain can drive significant additional traffic to the category. We further investigate the conditions under which the pooling equilibrium is preferred in the next section.

## Results: Impact of Category Captainship

Our first result is about the effectiveness of the target profit contract in extracting consumer insights from the category captain. Comparing the retailer's profit in the separating and pooling equilibria yields the following proposition.

**Proposition 3** *For $\alpha \leq \bar{\alpha}$, there exists a threshold level $\Lambda^* \in [0, \bar{\Lambda}]$ such that for $\Lambda \leq \Lambda^*$, the retailer prefers the separating equilibrium and for $\Lambda > \Lambda^*$, the retailer prefers the pooling equilibrium.*[2]

The retailer has to make a tradeoff between the value of information (screening in the SE) and the value of additional traffic into the category (increase in $\Lambda$). If the value of information is greater than the value of additional traffic, which is the case for relatively low values of $\Lambda$, the retailer prefers screening the category captain. On the other hand, if the value of additional traffic is higher than the value of the category captain's private

[2] The condition $\alpha \leq \bar{\alpha} = \frac{m - \frac{\beta}{\lambda}\left(\frac{v_0}{v_L} + n_R\right)}{m - \frac{\beta}{\lambda}\left(\frac{v_0}{v_L} + n_R\right)\sqrt{\frac{v_L}{v_H}}}$ is an auxiliary assumption which shortens the proof of Proposition 3 substantially by eliminating some extreme possibilities. This assumption provides a sufficient condition for the existence of a threshold for every set of parameters. In particular, when this assumption holds, the increase in the pooling equilibrium profit of the retailer as a result of increase in $\Lambda$ is higher than the increase in the separating equilibrium profit.

information, the retailer prefers the pooling equilibrium. The threshold $\Lambda^*$ is the value where the retailer is indifferent between the separating and pooling equilibria.

Our next result is about the impact of category captainship on the variety offered to the consumers. Competitive exclusion refers to the phenomenon where the category captain takes advantage of its position and disadvantages the non-captain manufacturers in the category. There is an emerging debate on whether or not category captainship leads to competitive exclusion in the form of variety reduction in the categories where the retailers use category captains. These concerns are exacerbated by several antitrust cases concerning category captainship misconduct (Steiner 2001, Desrochers et al. 2003, Greenberger 2003, Leary 2003, Klein and Wright 2006). The following proposition sheds some light on the conditions under which competitive exclusion takes place.

Let $n_{CC}$ denote the variety offered to the consumers under the category captainship.

**Proposition 4** *If consumers are L-type, then $n_{CC} > n_R$. On the other hand, if consumers are H-type, there exists a threshold $\Lambda_1$ such that*

*(i) if $\Lambda \in [0, min\{\Lambda^*, \Lambda_1\})$, then $n_{CC} = n_{TP}^H < n_R$,*

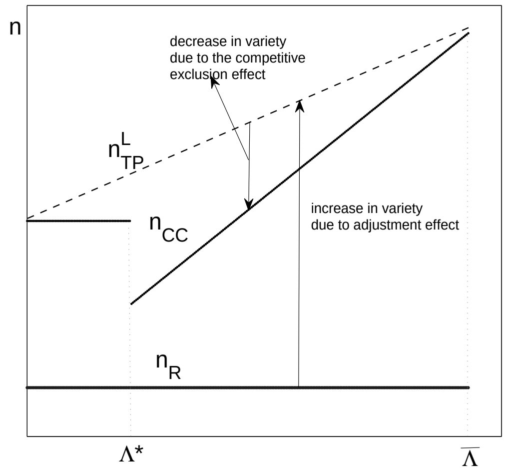
*(ii) if $\Lambda \in [min\{\Lambda_1, \Lambda^*\}, \Lambda^*)$, then $n_{CC} = n_{TP}^H \geq n_R$, and*

*(iii) if $\Lambda \in [\Lambda^*, \overline{\Lambda}]$, $n_{CC} < n_R$ and $n_{CC} < n_{TP}^H$.*

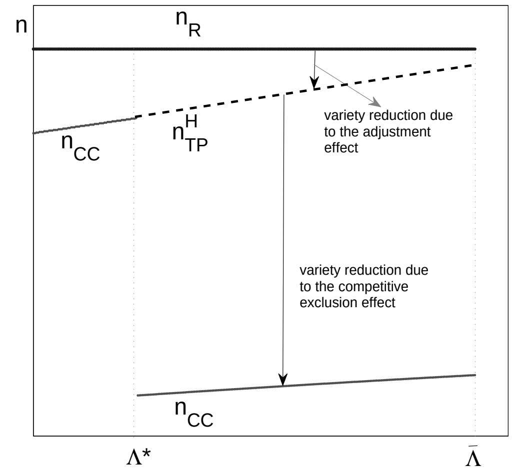
Proposition 4 suggests that the transition from retail category management to category captainship can increase or decrease the variety offered to the consumers. We find that this increase/decrease is due to two effects: (1) the *adjustment effect* and (2) the *competitive exclusion effect*. The *adjustment effect* can either increase or decrease the variety offered in the category and is due to the retailer's imperfect knowledge about consumers and the increased traffic into the category. In particular, the adjustment effect is a result of two potentially conflicting forces: (1a) the assortment-expanding effect of higher traffic created by the category captain, and (1b) the assortment-expanding or assortment-shrinking effect of better information about consumer preferences. When consumers are L-type, the adjustment effect suggests an increase in the assortment since both higher traffic and better information lead to assortment expansion. However, when consumers are H-type, the adjustment effect is ambiguous since higher traffic leads to assortment expansion but better information leads to assortment shrinking. The adjustment effect suggests a reduced variety only if the assortment-shrinking effect of better information dominates the assortment-expanding effect of additional traffic. The magnitude of the adjustment effect is measured by $|n_{TP}^L - n_R|$ when consumers are L-type and by $|n_{TP}^H - n_R|$ when they are H-type. The *competitive exclusion* effect, on the other hand, always reduces the variety offered in the category and is due to the category captain taking advantage of its position and reducing the variety to increase its share. The magnitude of the competitive exclusion effect is measured by $|n_{TP}^L - n_{CC}|$ when consumers are L-type and by $|n_{TP}^H - n_{CC}|$ when they are H-type.

The following two special cases delineate the drivers of the adjustment and competitive exclusion effects. First, when the category captain is used to drive additional traffic only (i.e., $H = L$ and $\Lambda > 0$), the variety under category captainship is always higher than the variety under the RCM, that is $n_{CC} > n_R$ (see Appendix B for proofs). The increase in the variety is entirely due to the assortment expanding effect of additional traffic. On the other hand, when the category captain is used for consumer insights only (i.e., $H > L$ and $\Lambda = 0$), the variety under category captainship can be higher or lower than the variety under RCM. If consumers are H-type, then $n_{CC} = n_{TP}^H < n_R$ whereas if the consumers are L-type, then $n_{CC} = n_{TP}^L > n_R$. In this case, the increase/decrease in variety is entirely due to the assortment-expanding/shrinking effect of better information. Therefore, we can conclude that while the adjustment effect can be driven by either asymmetric information or the category captain's ability to drive traffic into the category, the competitive exclusion effect is driven by both effects simultaneously.

Figure 5 illustrates the impact of the adjustment and competitive exclusion effects on the resulting variety for $\Lambda \in [0, \bar{\Lambda}]$ for both separating and pooling equilibrium cases. If




(a) variety when consumers are L-type

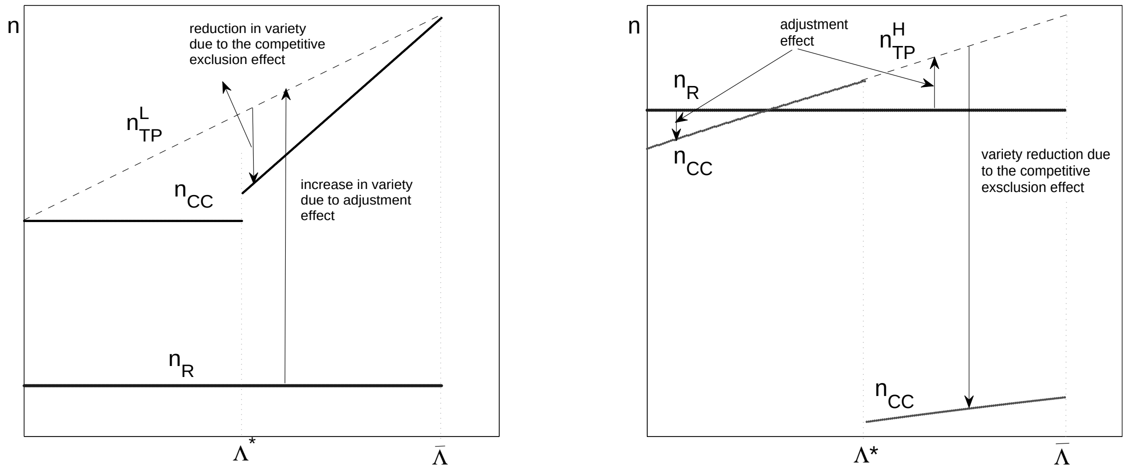



(b) variety when consumers are H-type

Figure 5: Comparison of equilibrium variety levels under the RCM and CC for $\lambda = 20$, $m = 5$, $w = 4$, $v_0 = 2$, $v_L = 2$, $v_H = 4$, $\alpha = 0.5$, $\beta = 1$.

the consumers are L-type (Figure 5a), the adjustment effect increases the variety offered in the category ($n_{TP}^L \geq n_R$), but the competitive exclusion effect reduces the variety ($n_{TP}^L \geq n_{CC}$). Because the magnitude of the adjustment effect is greater than the magnitude of the competitive exclusion effect ($|n_{TP}^L - n_R| > |n_{TP}^L - n_{CC}|$), the net effect is an increase in the variety offered to the consumers ($n_{CC} > n_R$). On the other hand, when the consumers are H-type (Figure 5b), both the adjustment and the competitive exclusion effects reduce the variety in the category and therefore, the net effect is a reduction in the category variety ($n_R \geq n_{CC}$). Notice that the adjustment effect reduces the variety in this case since, for the chosen parameter set, assortment-shrinking effect of better information dominates the assortment-expanding effect of additional traffic.

Figure 5 illustrates an example where $\min\{\Lambda^*, \Lambda_1\} = \Lambda^*$ and the adjustment effect always reduces the variety in the category when consumers are H-type (Figure 5b). Figure 6, on the other hand, illustrates a different example where $\min\{\Lambda^*, \Lambda_1\} = \Lambda_1$. In this example,




(a) variety when consumers are L-type

(b) variety when consumers are H-type

Figure 6: Comparison of equilibrium variety levels under the RCM and category captainship for $\lambda = 20$, $m = 5$, $w = 4$, $v_0 = 2$, $v_L = 2$, $v_H = 4$, $\alpha = 0.8$, $\beta = 1$.

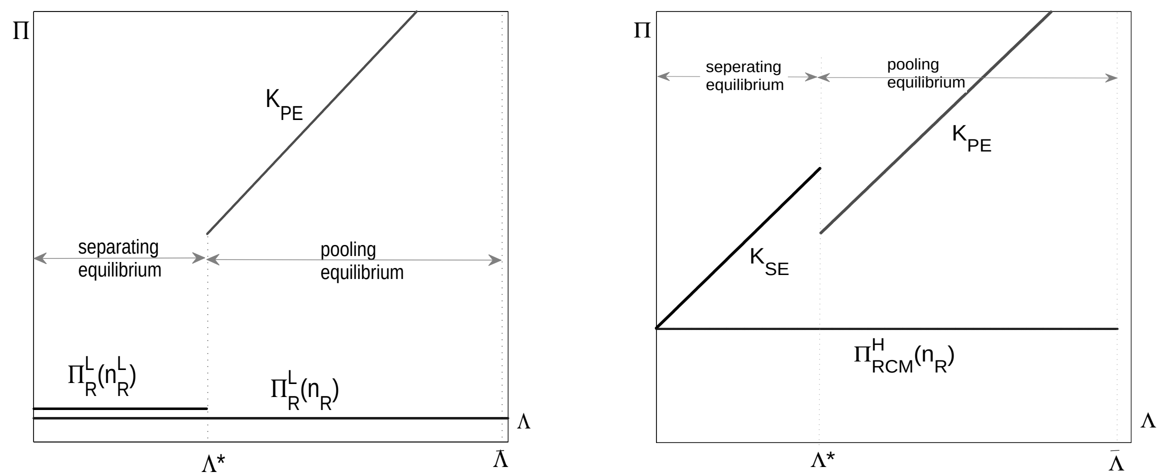
the adjustment effect can either increase or decrease the variety when consumers are H-type (Figure 6b). Because the competitive exclusion effect does not play a role when $\Lambda$ is small, the H-type consumers can observe an increase in variety under category captainship whereas for larger $\Lambda$'s ($\Lambda \in [\Lambda^*, \bar{\Lambda}]$) the reduction in variety is inevitable as the competitive exclusion effect dominates.

To summarize, Proposition 4 suggests that while category captainship can lead to reduction in the category variety, this reduction is not always due to competitive exclusion but can also be due to the adjustment effect. While the adjustment effect can increase or decrease the variety, the competitive exclusion effect always reduces the variety in the category.

### Impact of Category Captainship

In this section, we investigate the impact of category captainship on all the parties through numerical studies and summarize our results below.

**Impact on Retailer.** The retailer benefits from both the captain's superior consumer insights and the ability to drive traffic into the category and is always better off under category captainship by definition. Figure 7 illustrates the retailer's profit under both retail category management and category captainship as a function of category captain's traffic driving ability $\Lambda$. First notice that, irrespective of the consumer type, the retailer's profit is non-decreasing in the category captain's ability to drive traffic ($\Lambda$). Second, when consumers are L-type, the category captain rejects the contract if the retailer asks for a target profit $K_{SE}$, in which case the retailer infers that the consumers are L-type and does set the variety level to $n_R^L$. The gap between $\Pi_R^L(n_R^L)$ and $\Pi_R^L(n_R)$ measures the value of information for the retailer when the consumers are L-type.




(a) Retailer's profit when consumers are L-type (b) Retailer's profit when consumers are H-type

Figure 7: Comparison of equilibrium retailer profits under the RCM and category captainship for $\lambda = 20$, $m = 5$, $w = 4$, $v_0 = 2$, $v_L = 2$, $v_H = 4$, $\alpha = 0.7$, $\beta = 1$.

While the retailer benefits from the category captain rejecting the contract when the consumers are L-type, this is only the case for small values of $\Lambda$. As $\Lambda$ increases, the retailer prefers both types of category captains to accept the target profit contract because the benefit due to the additional traffic into the category exceeds the benefit of having perfect information about the consumers.

**Impact on Category Captain.** The category captainship agreement increases the size of the pie that is shared between the retailer and the category captain because of the category captain's private information about the consumers and its ability to drive additional traffic into the category. The question is how does the surplus created in the category captainship collaboration get split between the retailer and the category captain. Although we assume that the retailer has the power to offer a take-it-or-leave-it contract to the category captain, the retailer cannot always extract the entire surplus created in the category captainship collaboration as the category captain gets compensated for its private information when the consumers are H-type.

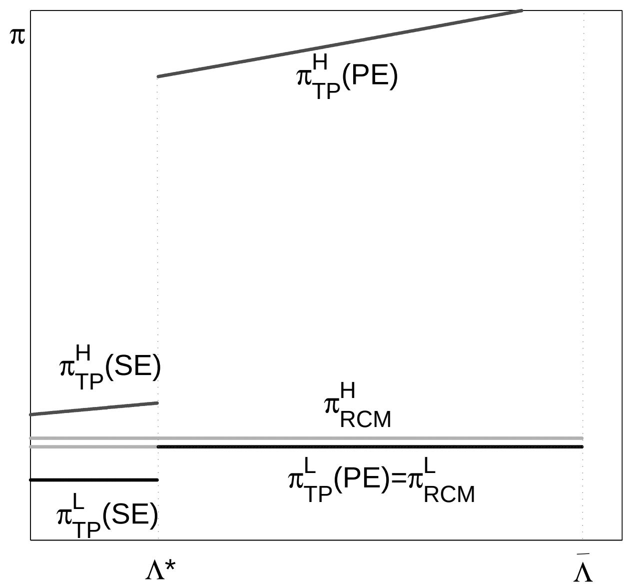



Figure 8: Comparison of equilibrium profits for the category captain under the RCM and category captainship for $\lambda = 20$, $m = 5$, $w = 4$, $v_0 = 2$, $v_L = 2$, $v_H = 4$, $\alpha = 0.7$,$\beta = 1$

Figure 8 illustrates the category captain's profit under retail category management and the category captainship scenarios. If the consumers are H-type, the category captain is better off in the category captainship under both separating and pooling equilibria (i.e.,

$\pi_{TP}^H(SE) \geq \pi_R^H$ and $\pi_{TP}^H(PE) \geq \pi_R^H$). The category captain is in an especially advantageous position when its ability to drive traffic is substantial (i.e., $\Lambda \in [\Lambda^*, \bar{\Lambda}]$). This is because the opportunity cost of disagreement is very high for the retailer. Moreover, incentives of the retailer and the category captain are somewhat aligned: increasing the category traffic benefits both. The category captain can afford to recommend a relatively low variety level and continue to meet the target profit because of the substantial additional traffic to the category.

On the other hand, when consumers are L-type, the category captain is either worse off under the category captainship with a separating equilibrium ($\pi_{TP}^L(SE) < \pi_R^L$ for $\Lambda \in (0, \Lambda^*)$) or indifferent with a pooling equilibrium ($\pi_{TP}^L(PE) = \pi_R^L$ for $\Lambda \in [\Lambda^*, \bar{\Lambda}]$). The category captain is worse off under category captainship for small $\Lambda$s because the retailer sets the variety as in the retail category management scenario with perfect information. The retailer increases the variety ($n_R^L > n_R$) which translates into a smaller market share and profit for the category captain. Recall that it would be prohibitively expensive for the category captain to accept the target profit contract when the consumers are L-type because a failure to deliver the target profit would result in a stiff penalty.

**Impact on non-Captain Manufacturers.** Due to our modeling assumptions, the impact of category captainship on a non-captain manufacturer is identical to the impact of category captainship on the category captain given that the non-captain manufacturer is offered to the consumers in both retail category management and category captainship scenarios. However, as indicated in Proposition 4, the transition to category captainship can both increase or decrease the variety offered to the consumers. If category captainship results in a broader assortment, then a non-captain manufacturer's chances of being included in the assortment improve, which in return increases the non-captain manufacturer's expected profit. If category captainship results in a narrower assortment, the non-captain manufacturer's expected profit suffers from the decrease in the probability of being included in the assortment. Therefore, we conclude that depending on the resulting variety, category captainship can either hurt or benefit the non-captain manufacturers in the category.

## Extensions

Our modeling choices were driven by our main research question which is to compare the resulting variety under retail category management and category captainship and provide some insight on whether category captainship results in competitive exclusion. In this section, we focus on the simplifying assumptions of our model to determine whether these assumptions limit the realism of our model and obstruct the ability to generate insights that can be generalized. In what follows, we provide the highlights of the key insights from these extensions but the extensive analyses are available in an online supplement.

### Category Captain Selection

Our model is silent on the retailer's category captain selection problem because we assume that the manufacturers are identical. To this end, we consider two extensions of our original model to gain some insights on the drivers of the category captain selection decision. In the first extension, we assume that one of the manufacturers offers a product which has a higher attraction compared to the other products in the category. In the second extension, we assume that one of the manufacturers can drive more traffic into the category as compared to the other manufacturers. In both extensions, we compare the retailer's profit when the manufacturer with higher attractiveness product or ability to drive more traffic is assigned as category captain to the retailer's profit when one of the other manufacturers serves as category captain.

In the first extension, we assume that one of the manufacturers offers a product with attractiveness $v_i + \delta$ and the other manufacturers offer products with attractiveness $v_i$. We keep all the other assumptions regarding the cost and information structure as in our original model. First, consider the case where the high attractiveness manufacturer is assigned as the category captain. Let $K_{SE}(\delta)$ and $K_{PE}(\delta)$ be the retailer's profit under the separating and pooling equilibria when the high attractiveness manufacturer (with attractiveness $v_i + \delta$) is selected as the category captain. Notice that $K_{SE}(0) = K_{SE}$ and $K_{PE}(0) = K_{PE}$ because, when $\delta = 0$, the model we consider in this extension is the same as our original model. It is also straightforward to show that both $K_{SE}(\delta)$ and $K_{PE}(\delta)$ are increasing in $\delta$. Thus, we can conclude that the retailer prefers an assortment that includes the product with attractiveness $v_i + \delta$ over an assortment that excludes it.

Now, suppose that the retailer chooses a manufacturer other than the high attractiveness manufacturer as the category captain. We consider the following two cases: (i) the category captain includes the high attractiveness manufacturer's product in the assortment and (ii) the category captain excludes the high attractiveness manufacturer's product. Case (ii) corresponds to our original model (or, model with $\delta = 0$). Case (i), on the other hand, results in exactly the same best response function for the category captain in the model where the high attractiveness manufacturer is assigned as a category captain. Thus, the retailer's equilibrium profit is the same as before (i.e., $K_{SE}(\delta)$ in the separating equilibrium and $K_{PE}(\delta)$ in the pooling equilibrium) if the category captain includes the high attractiveness manufacturer's product in the assortment. The only difference between case (i) and the case where the high attractiveness manufacturer is the category captain is the profit of the category captain.

In summary, if one of the manufacturers with low attractiveness is selected as the category captain then, in general, the category captain's incentives are toward excluding the manufacturer with a high attractiveness product. However, in equilibrium, the retailer sets the target profit high enough to ensure that the category captain includes the product with high attractiveness in the recommended assortment. Therefore, we conclude that the retailer is indifferent between selecting the high attractiveness manufacturer or any of the other manufacturers as the category captain as long as the high attractiveness product is included in the assortment.

In the second extension, we assume that one of the manufacturers can increase the category traffic by $\tilde{\Lambda}$ whereas all the other manufacturers can increase the category traffic by $\Lambda$. We assume that $\tilde{\Lambda} > \Lambda$. All of the cost and information structure assumptions in this extension are the same as the ones in our original model. Let $K_{SE}(\Lambda)$ and $K_{PE}(\Lambda)$ denote the retailer's equilibrium profits. By Lemma 5 (which is in the appendix), we know that both $K_{SE}(\Lambda)$ and $K_{PE}(\Lambda)$ are increasing functions of $\Lambda$. Thus, we conclude that the retailer is better off by choosing the manufacturer that can drive the most traffic into the category as the category captain.

To summarize, our results in the extensions described above suggest that the retailer prefers to choose a manufacturer who is able to put something unique to the table while considering the category captain selection problem. In the first extension described above, the manufacturer with high attractiveness product is not at an advantage because the high attractiveness product can be included in the category regardless of whether the high attractiveness manufacturer is selected as the category captain or not. What matters for the retailer is whether the high attractiveness manufacturer is included in the category or not: the retailer is better off when the manufacturer with high attractiveness product is included in the assortment. On the other hand, if a manufacturer has a unique characteristic such as being able to increase traffic more than the other manufacturers as in the second extension described above, then the retailer would prefer that manufacturer as the category captain over the other manufacturers.

### Implementing Category Captainship with Target Sales Contract

While target profit is one of the most commonly used measures in the retailer's category captainship scorecard, category sales is another important measure that retailers are interested in. Target sales type of measures are particularly important in destination categories that are used by the retailers to drive traffic into the store. In general, a retailer can use a target sales contract either (i) as a tool to *maximize profit* or (ii) as a tool to *maximize sales*. If the retailer uses the target sales contract as a tool to maximize profit, both target profit and target sales contracts lead to the same outcome. Intuitively, the reason for this equivalence is that the objective of the retailer is the main determinant of the equilibrium. If, on the other hand, the retailer's objective is to maximize its sales by using target sales contract, then our results about competitive exclusion would change. In the RCM scenario, the retailer selects the variety level to maximize expected sales, that is

$$\max_{n} \qquad \alpha \frac{\lambda n v_H}{v_0 + n v_H} + (1-\alpha) \frac{\lambda n v_L}{v_0 + n v_L}$$

Notice that the retailer's expected sales are increasing in $n$ and the retailer selects the maximum available variety, which we define as $N$.

In the category captainship scenario, the category captain has an incentive to reduce the variety since its profit is decreasing in variety. However, anticipating this incentive, the retailer would not allow a decrease in the already existing variety since such a decrease would hurt the retailer's sales. The retailer sets the target sales level to induce the category captain to recommend an assortment with $N$ products in it. Since the solution is on the boundary, both separating and pooling equilibria lead to the recommended variety level $N$. The retailer and the category captain would be better off under the category captainship scenario due to the additional traffic driven in the category. Therefore, if the retailer's objective is to maximize sales, information asymmetry plays no role and hence there is no room for competitive exclusion in this setting.

In practice, a retailer's objective often lies somewhere between the two extremes of either maximizing profit or maximizing sales in the category. Different categories have different objectives for different retailers. If a retailer's primary concern is maximizing profit (sales), then the retailer is better off using target profit (sales) contract. However, retailers should be aware that competitive exclusion is alive when the retailer seeks profit maximization, but not effective when the retailer's objective is to maximize sales. Finally, notice that as the retailer's operational cost parameter $\beta$ approaches zero, the difference between these two objectives fades away since profit and sales maximization problems become equivalent.

### Multiple Manufacturers Each Selling Multiple Products

Our model assumes that each manufacturer offers one product only. However, in practice manufacturers usually offer multiple products in a single category. In this section, we explore the drivers of competitive exclusion in a model with two manufacturers each selling multiple products through a single retailer. We assume that both the category captain and the retailer have perfect information about the consumers. In the retail category management, the retailer decides on variety for both manufacturers and in the category captainship scenario, the category captain makes a recommendation on the variety for its own and competitor's products (assortment mix) in return for a target profit level. For simplicity, as in our original model, we assume that all of the products offered by the first and second manufacturers have the same attractiveness $v$ and the retailer has perfect information about the consumers. These assumptions imply that all of the products are perfect substitutes from the retailer's point of view and therefore the retailer cares only about the total variety in the category and not about the assortment mix. Our analysis suggests that competitive exclusion is possible even in the absence of asymmetric information simply because the captain's incentives are toward recommending an assortment that includes more of its products while the retailer only cares about the total variety in the category. This extension allows us to identify another possible lever (i.e., assortment mix decision) through which the category captain excludes the competitors' products from the assortment in addition to asymmetric information.

### Manufacturers with Nonidentical Attractiveness

Our assumption of identical manufacturers prevents us from drawing conclusions regarding which products/brands are more likely to be included or excluded from the category. Next, we consider an extension where we relax our assumption of identical manufacturers and assume that the products are ordered such that $v_1 > v_2 > v_3 > ... > v_N$ and the first manufacturer is assigned as category captain. We keep all of the cost structure assumptions the same as in our original model. First, we consider the model without asymmetric information. We find that the retailer's and the category captain's recommended equilibrium assortments are both in the most attractive set (i.e., include only the most popular products in the assortment) and the variety under category captainship always increases due to the adjustment effect. While the competitive exclusion effect does not reveal itself in this extension due to the absence of asymmetric information, we show that the inclusion of asymmetric information into this model could result in competitive exclusion. Our numerical analysis (which is described in detail in the Online Supplement) suggests that depending on the category captain's traffic driving abilities, the category captainship might result in different types of products being excluded from the assortment. In particular, we find that if the category captain's traffic driving abilities are limited (i.e., the retailer prefers separating equilibrium), the category captain excludes the products with low attractiveness. On the other hand, if the category captain's traffic driving abilities are significant (the retailer prefers pooling equilibrium), the category captain might exclude some of the high attractiveness products. The intuition is as follows: Under the separating equilibrium, the equilibrium target profit forces the category captain to recommend an assortment which includes the most attractive products but allows exclusion of the less attractive products. On the other hand, under pooling equilibrium, the category captain can drive significant additional traffic into the category, which improves the captain's bargaining position against the retailer. Therefore, the captain can replace some of the most attractive products with less attractive ones in the assortment.

## Conclusions and Discussions

We consider a stylized two stage supply chain model where multiple manufacturers sell to the consumers through a single retailer. The goal of our research is to investigate the impact of a recent trend in the consumer goods supply chains where retailers rely on a leading manufacturer in a category for recommendations regarding the assortment to be offered to the consumers. Retailers benefit from category captain's (1) superior knowledge about the consumers and/or (2) ability to drive additional traffic into the category. Our results are along these two dimensions.

| Reliance on traffic driving \ Reliance on knowledge | no | yes |
|---|---|---|
| no | • No value in category captainship | • The retailer can extract valuable information but cannot extract the entire surplus<br>• The category captain is paid an information rent<br>• The variety offered to the consumers may increase or decrease as a result of the adjustment effect |
| yes | • The retailer can extract the entire surplus<br>• The category captain is indifferent<br>• The variety offered to the consumers increases under category captainship as a result of the adjustment effect | • The retailer can extract valuable information but can not extract the entire surplus<br>• The category captain is paid an information rent<br>• The variety offered to the consumers may decrease as a result of competitive exclusion |

Figure 9: Summary of results table.

The overall conclusion of our research is that while using category captains for category management can be an excellent value proposition for retailers, the consequences of using category captains should be better understood by retailers. We find that the consequences of using category captains may differ depending on what the category captains are used for. Figure 9 summarizes our results along two dimensions in a simple two by two matrix. First, a retailer should continue implementing retail category management in established categories where traffic is stable and consumer behavior is well understood. Second, categories where the retailer needs to increase traffic and consumer behavior is well understood are perfect candidates for category captainship implementations. The retailers can expand the product offering in these categories as a result of the increased traffic into the category.

Third, in categories where retailers need consumer insights only, retailers can use category captains who have a better understanding of the consumers but should be aware that the category captains should be rewarded for providing insights. Depending on consumer preferences, the category captainship implementation may result in either an expanded or a narrower assortment when compared to the assortment under retail category management. The possible reduction in variety is entirely due to the adjustment effect. Finally, categories where retailers rely on their category captains to drive traffic and provide consumer insights are also suitable for category captainship implementation but retailers need to compensate the category captain for both its superior information and its traffic driving abilities. The variety in the category can either increase or decrease as a result of the adjustment effect. The competitive exclusion effect, however, always reduces the variety offered to the consumers. The overall increase/decrease in the category variety depends on the magnitude of the adjustment and competitive exclusion effects.

Our results have a number of implications regarding the implementation of category captainship in practice. First, we find that while the concerns regarding the category captainship misconduct in the form of competitive exclusion are definitely valid, a reduction in variety under category captainship is not always due to competitive exclusion but sometimes due to the adjustment effect. In particular, expected profit maximizing behavior forces the retailer to offer a suboptimal variety in the category under retail category management. This is not desirable for the retailers as excess variety eats up precious retail shelf space while little variety may lead to lost consumers in the category. The category captain's additional consumer insights help the retailer to adjust its variety to better satisfy consumer's needs. While this adjustment takes place irrespective of the category captain's traffic driving abilities, competitive exclusion takes place when the category captain is capable of driving significant traffic into the category. This is because the category captain is in a stronger position against the retailer in this case. This is one explanation as to why the competitive exclusion effect is difficult to detect in practice as it is not clear whether a reduction in category variety is due to competitive exclusion or it is simply due to an adjustment.

Second, our research has some implications regarding the implementation of category captainship via target profit versus target sales contracts. On one hand, we find that if the main goal of the retailer is to maximize profit, the category captainship implementation might result in competitive exclusion whereas if the retailer's goal is to maximize sales, then competitive exclusion does not take place. On the other hand, the implementation of target profit level contract requires that retailers share sensitive information such as competitor's margin and operational cost data with the category captain, therefore, posing some implementation challenges. However, target sales is relatively easier to track and does not require sensitive information sharing. Therefore, retailers implementing category captainship via a target profit should be aware that maximizing profit in a category comes at a cost. Retailers have to make a tradeoff between maximizing profit (as opposed to maximizing sales) and the possible adverse effects of competitive exclusion and sharing sensitive information if target profit is to be used.

Third, our research suggests that while the retailer and the category captain can benefit from category captainship, contrary to the common belief, the non-captain manufacturers can also be better off under category captainship. In practice, many manufacturers get frustrated and fear competitive exclusion when they hear that a major competitor has been selected by a retailer to serve as a category captain. While the fear of competitive exclusion is valid in some instances, we find that the variety in the category might actually increase after implementing category captainship and the non-captain manufacturers can benefit from this variety increase as well as the increase in the traffic to the category.

Finally, our model only captures the short term benefits of category captainship because we model category captainship as a one shot game. However, category captainship can also have adverse effects on the retailers in the long run. While we are not able to capture the potential long term adverse effects of category captainship to the retailer, our model can be used to derive implications regarding some of these long term effects. For example, competitive exclusion can lead to monopolization in the category, which in the long run can result in price increases and reduction in category variety. While price spikes will lead to consumer dissatisfaction, a reduction in the category variety can also result in consumer dissatisfaction because consumers almost always prefer more variety to less (e.g., Broniarczyk et al. 1998 and Hoch et al. 1999). Retailers take measures such as assigning co-captains in categories to verify their category captain's recommendations to avoid biased recommendations but as our research points out, it might be very difficult to separate the variety reduction which is due to the adjustment and competitive exclusion effects. Therefore, we conclude that the retailers should balance the short term benefits and the potential long-term adverse effects while evaluating the pros and cons of category captainship practices.

## References

ACNielsen. 2005. Step Four: Set Performance Targets and Measure Progress with a Category Scorecard in *Consumer-Centric Category Management : How to Increase Profits by Managing Categories based on Consumer Needs*. Wiley, John & Sons.

Aydın, G. and W. H. Hausman. 2009. The Role of Slotting Fees in the Coordination of Assortment Decisions. *Production and Operations Management*, 18:635-652.

Bell, D.E., R.L. Keeney, and J.D.C. Little. 1975. A Market Share Theorem. *Journal of Marketing Research*. Vol. 12. No 2. 136-141.

Broniarczyk S.M., W.D. Hoyer and L. McAlister. 1998. Consumers' Perceptions of the Assortment Offered in a Grocery Category: The Impact of Item Reduction. *Journal of Marketing Research*, Vol. XXXV (May 1998), 166-176.

Cachon G.P., A.G. Kök. 2007. Category Management and Coordination of Categories in Retail Assortment Planning in the Presence of Basket Shoppers. *Management Science*. Vol. 53. No.6. 934-951.

Cachon, G.P., C. Terwiesch, Y. Xu. 2005. Retail Assortment Planning in the Presence of Consumer Search. *Manufacturing & Service Operations Management*. Vol. 7, No. 4, 330-346.

Cachon, G.P., C. Terwiesch, and Y. Xu. 2007. On the Effects of Consumer Search and Market Entry in a Multi-product Competitive Market, *Marketing Science*. Vol. 27, Issue 3 (May 2008), 461-473.

Chu, W. 1992. Demand Signalling and Screening in Channels of Distribution. *Marketing Science*, Vol. 11, No. 4. (Autumn, 1992), pp. 327-347.

Desrochers, D.M., G.T. Gundlach, and A.A. Foer. 2003. Analysis of Antitrust Challanges to Category Captain Arrangements, *Journal of Public Policy* & *Marketing*, Vol.22, No.2 (Fall), 201-215.

Greenberger, R.S. 2003. UST Must Pay $ 1.05 Billion To a Big Tobacco Competitor. *Asian Wall Street Journal*. New York, NY: Jan. 15, p.A.8.

Hoch, S.J., E.T. Bradlow, and B. Wansink. 1999. The Variety of an Assortment. *Marketing Science*, Vol.18, No.4, 527-546.

Hopp, W.J., X. Xu. 2003. A Joint Marketing/Operations Model of Differentiated Product Decisions. Northwestern Univerity.

Klein, B. and J.D. Wright. 2006. Antitrust Analysis of Category Management: Conwood v. United States Tobacco. George Mason University Law and Economics Research Paper Series.

Kök, A.G., M.L. Fisher, and R. Vaidyanathan. 2006. Assortment Planning: Review of Literature and Industry Practice. *Retail Supply Chain Management*, Eds. N Agrawal and S.A. Smith. Kluwer Publishers.

Kurtulus, M., L.B. Toktay. 2009. Category Captainship versus Retailer Category Management under Limited Retail Shelf Space. Forthcoming in *Production and Operations Management*.

Leary, T. B. 2003. A Second Look at Category Management. Federal Trade Commision Report. Available at www.ftc.gov.

Niraj, R, C. Narasimhan. 2003. Vertical Information Sharing in Distribution Channels. Washington University Working Paper.

Progressive Grocer. 2007. Category Captains 2007: Armed and Ready, Progressive Grocer, Nov. 15, 2007.

Progressive Grocer. 2008. Category Captains 2008, Progressive Grocer, Nov. 15, 2008.

Shugan, S.M. 1989. Product Assortment in a Triopoly. *Management Science*. Vol. 35, No.3, 304-321.

Steiner, R.L. 2001. Category Management - A Pervasive, New Vertical/Horizontal Format, *Antitrust*, 15 (Spring), 77-81.

van Ryzin, G., S. Mahajan. 1999. On the Relationship Between Inventory Costs and Variety Benefits in Retail Assortment. *Management Science*, Vol.45, No.11. 1496-1509.

Wang Y, J.S. Raju, S.K. Dhar. 2003. The Choice and Consequences of Using a Category Captain for Category Management. The Wharton School, University of Pennsylvania.

## List of Key Notations

| List of Notations | |
|---|---|
| $n$ | number of variants offered at the retailer store. |
| $n_{CC}$ | number of variants offered to the consumers under the category captainship. |
| $n_R$ | the optimal variety in the RCM scenario. |
| $n^i_{TP}$ | the optimal variety when the retailer takes advantage of the additional traffic $\Lambda$ and knows that the consumers are $i$-type. |
| $n^i_R$ | the optimal variety in RCM scenario when the retailer knows that the consumer type is $i \in \{H, L\}$. |
| $\Lambda^*$ | the value of $\Lambda$ that makes the retailer indifferent between the separating and pooling. |
| $\Lambda_1$ | the value of $\Lambda$ that makes $n^H_{TP} = n_R$. |
| $\Pi_R$ | the retailer's expected optimal profit in the RCM scenario. |
| $\pi_R$ | the category captain's expected optimal profit in the RCM scenario. |
| $\Pi^i_{TP}$ | the retailer's profit under TP when the consumer type is $i \in \{H, L\}$. |
| $\pi^i_{TP}$ | the category captain's profit under TP when the consumer type is $i \in \{H, L\}$. |
| $\Pi^i_R$ | the retailer's profit when consumers are $i$-type and retailer sets variety as if in the RCM scenario (i.e., without the increase in consumer rate). |
| $\pi^i_R$ | the category captain's profit when consumers are $i$-type and retailer sets variety as if in the RCM scenario (i.e., without the increase in consumer rate). |

# Appendix

## A. Proof of Lemmas and Propositions

**Proof of Lemma 1:** The retailer's expected profit is strictly concave in the variety level $n$ because the second derivative of the objective function is equal to

$$\lambda m \left[ -\alpha \frac{2\frac{v_0}{v_H}}{\left(\frac{v_0}{v_H}+n\right)^3} - (1-\alpha)\frac{2\frac{v_0}{v_L}}{\left(\frac{v_0}{v_L}+n\right)^3} \right] < 0.$$

Thus, the problem has a unique maximum, denoted by $n_R$. The maximum is determined by $\lambda m[\alpha f_H + (1-\alpha) f_L] = \beta$ where $f_i = \frac{\frac{v_0}{v_i}}{\left(\frac{v_0}{v_i}+n_R\right)^2}$ for $i \in \{H, L\}$.■

The result in the following lemma is used in the proof of some our results.

**Lemma 4** $v_0 < \overline{v}_0 = n_R\sqrt{v_H}\sqrt{v_L}$ *implies the following inequalities*

$$v_0 < \frac{\lambda m v_L}{\beta} < min\{\frac{\lambda m v_H}{\beta}, \frac{\lambda m v_L}{\beta}\left[\frac{v_H}{v_H - v_L}\right]\}$$

**Proof of Lemma 4:** It is easy to see that $\frac{\lambda m v_L}{\beta} < \frac{\lambda m v_H}{\beta}$ and $\frac{\lambda m v_L}{\beta} < \frac{\lambda m v_L}{\beta}\left[\frac{v_H}{v_H - v_L}\right]$ since $v_H > v_L$. It is enough to show that $v_0 < \overline{v}_0$ implies $v_0 < \frac{\lambda m v_L}{\beta}$. Suppose that $v_0$ has the biggest possible value under the presumption of the lemma, i.e. $v_0 = n_R\sqrt{v_H}\sqrt{v_L}$. Because, by Lemma 1, $\lambda m[\alpha f_H + (1-\alpha) f_L] = \beta$, we have $v_0 < \frac{\lambda m v_L}{\beta}$ if and only if $0 < v_L + 2\sqrt{v_H}\sqrt{v_L}$. Because the last inequality is always true, we can conclude that $v_0 < \overline{v}_0$ implies $v_0 < \frac{\lambda m v_L}{\beta}$.

■

**Proof of Lemma 2:** For given target profit level $K$, the category captain who faces $i \in \{H, L\}$ type consumers solves the following problem at the second stage:

$$\begin{aligned} \max_n \quad & \frac{(\lambda + \Lambda)\, w v_i}{v_0 + n v_i} \\ \text{s.t.} \quad & \frac{(\lambda + \Lambda)\, m n v_i}{v_0 + n v_i} - \beta n \geq K \end{aligned}$$

Because the objective function is decreasing and the constraint is increasing in $n$, the optimal solution is determined by the constraint $\frac{(\lambda+\Lambda)mnv_i}{v_0+nv_i} - \beta n = K$.

We can rewrite the equation as $A^i n^2 - B^i n + C = 0$ where $A^i = \beta v_i$, $B^i = (\lambda + \Lambda) mv_i - \beta v_0 - Kv_i$, and $C = -Kv_0$. The quadratic equality has two roots which are given by $n_1 = \frac{B^i - \sqrt{(B^i)^2 - 4A^i C}}{2A^i}$ and $n_2 = \frac{B^i + \sqrt{(B^i)^2 - 4A^i C}}{2A^i}$. Because the category captain prefers the variety as small as possible, $n_2$ cannot be a best response. Thus, there is a unique best response for each type which is given by the smaller of the roots $n^i(K) = \frac{B^i(K) - \sqrt{(B^i(K))^2 - 4Kv_i v_0 \beta}}{2v_i \beta}$. ■

**Proof of Lemma 3:** By solving $\pi^H_{TP}\left(\overline{n}^H_{TP}\right) = \pi^H_R(n_R)$ and $\pi^L_{TP}\left(\overline{n}^L_{TP}\right) = \pi^L_R(n_R)$, we can easily show that $\overline{n}^H_{TP}(\Lambda) = n_R + \frac{\Lambda}{\lambda}\left(\frac{v_0}{v_H} + n_R\right)$ and $\overline{n}^L_{TP}(\Lambda) = n_R + \frac{\Lambda}{\lambda}\left(\frac{v_0}{v_L} + n_R\right)$. Moreover, by solving the corresponding maximization problems, we can calculate $n^H_{TP}(\Lambda) = \arg\max_n \Pi^H_{TP}(n) = \sqrt{\frac{(\lambda+\Lambda)m}{\beta}\frac{v_0}{v_H}} - \frac{v_0}{v_H}$ and $n^L_{TP}(\Lambda) = \arg\max_n \Pi^L_{TP}(n) = \sqrt{\frac{(\lambda+\Lambda)m}{\beta}\frac{v_0}{v_L}} - \frac{v_0}{v_L}$. We prove the lemma in four steps.

*(i)* $n^H_{TP}(\Lambda) > 0$ for all $\Lambda \geq 0$

*Proof of (i)*: Take any $\Lambda \geq 0$. Then, $n^H_{TP}(\Lambda) = \sqrt{\frac{(\lambda+\Lambda)m}{\beta}\frac{v_0}{v_H}} - \frac{v_0}{v_H} > 0$ if and only if $\frac{(\lambda+\Lambda)mv_H}{\beta} > v_0$. By lemma 4, $v_0 < \frac{(\lambda+\Lambda)mv_H}{\beta}$ holds and, therefore, $n^H_{TP}(\Lambda) > 0$.

*(ii)* $\overline{n}^H_{TP}(\Lambda) > n^H_{TP}(\Lambda)$ for all $\Lambda \geq 0$

*Proof of (ii)*: Let $\varphi_H(\Lambda) = n^H_{TP}(\Lambda) - \overline{n}^H_{TP}(\Lambda) = \sqrt{\frac{(\lambda+\Lambda)m}{\beta}\frac{v_0}{v_H}} - \frac{v_0}{v_H} - n_R - \frac{\Lambda}{\lambda}\left(\frac{v_0}{v_H} + n_R\right)$. First, show that $\varphi_H(\Lambda)$ is concave. We rearrange $\varphi_H(\Lambda)$ in polynomial form: $\varphi_H(\Lambda) = -ax^2 + bx$ where $a = \left(\frac{1}{\lambda}\right)\left(\frac{v_0}{v_H} + n_R\right)$, $b = \sqrt{\frac{m}{\beta}\frac{v_0}{v_H}}$, and $x = \sqrt{\lambda + \Lambda}$. Because the coefficient on $x^2$ is negative, $\varphi_H(\Lambda)$ is concave. We are interested in the values of $\Lambda$ that satisfy $\varphi_H(\Lambda) = 0$. That is, $\sqrt{\lambda+\Lambda}\left[\sqrt{\frac{m}{\beta}\frac{v_0}{v_H}} - \sqrt{\lambda+\Lambda}\left(\frac{1}{\lambda}\right)\left(\frac{v_0}{v_H} + n_R\right)\right] = 0$. This equation has two roots: $\Lambda^1_H = -\lambda$ and $\Lambda^2_H = \lambda\left[\frac{\lambda m}{\beta} f_H - 1\right]$. If both of the roots are negative then $\varphi_H(\Lambda)$ is negative for all $\Lambda \geq 0$ and, therefore, $n^H_{TP}(\Lambda) < \overline{n}^H_{TP}(\Lambda)$. It is clear that $\Lambda^1_H < 0$. We claim that $\Lambda^2_H < 0$, or equivalently $\lambda m f_H < \beta$. The first order condition for the variety level in RCM is $\lambda m[\alpha f_H + (1-\alpha) f_L] = \beta$. Then, $\Lambda^2_H < 0 \Leftrightarrow v_0 < n_R\sqrt{v_H}\sqrt{v_L} = \bar{v}_0$. Because the latter inequality holds, we have $\Lambda^2_H < 0$ and $n^H_{TP}(\Lambda) < \overline{n}^H_{TP}(\Lambda)$.

*(iii)* $n_{TP}^{L}(\Lambda) \geq \overline{n}_{TP}^{L}(\Lambda)$ for all $\Lambda \in [0, \overline{\Lambda}]$

*Proof of (iii)*: Let $\varphi_L(\Lambda) = n_{TP}^{L}(\Lambda) - \overline{n}_{TP}^{L}(\Lambda) = \sqrt{\frac{(\lambda+\Lambda)m}{\beta}\frac{v_0}{v_L}} - \frac{v_0}{v_L} - n_R - \frac{\Lambda}{\lambda}\left(\frac{v_0}{v_L} + n_R\right)$. As before, $\varphi_L(\Lambda)$ is concave and $\varphi_L(\Lambda) = 0$ has two roots: $\Lambda_L^1 = -\lambda$ and $\Lambda_L^2 = \lambda\left[\frac{\lambda m}{\beta} f_L - 1\right]$. Next, we show that $\Lambda_L^2 > 0$. It is sufficient to show that $\lambda m f_L > \beta$. By using an argument similar to the one in the previous case, we can show that the above inequality holds since $v_0 < \overline{v}_0$ and $\lambda m[\alpha f_H + (1-\alpha) f_L] = \beta$. Because $\varphi_L(\Lambda)$ is concave, $\varphi_L(\Lambda)$ is positive between two roots. Therefore, $n_{TP}^{L}(\Lambda) \geq \overline{n}_{TP}^{L}(\Lambda)$ for all $\Lambda \in [0, \overline{\Lambda}]$ where $\overline{\Lambda} = \Lambda_L^2$.

*(iv)* $\overline{n}_{TP}^{L}(\Lambda) \geq \overline{n}_{TP}^{H}(\Lambda)$ for all $\Lambda \geq 0$

*Proof of (iv)*: Take any $\Lambda \geq 0$. Then, $\overline{n}_{TP}^{L}(\Lambda) > \overline{n}_{TP}^{H}(\Lambda)$ is true if and only if $n_R + \frac{\Lambda}{\lambda}\left(\frac{v_0}{v_L} + n_R\right) > n_R + \frac{\Lambda}{\lambda}\left(\frac{v_0}{v_H} + n_R\right)$. Because $v_H > v_L$ we have $\overline{n}_{TP}^{L}(\Lambda) > \overline{n}_{TP}^{H}(\Lambda)$ for all $\Lambda > 0$. If $\Lambda = 0$ then $\overline{n}_{TP}^{L}(\Lambda) = \overline{n}_{TP}^{H}(\Lambda) = n_R$. By (i), (ii), (iii), and (iv), we conclude that $n_{TP}^{L}(\Lambda) \geq \overline{n}_{TP}^{L}(\Lambda) \geq \overline{n}_{TP}^{H}(\Lambda) > n_{TP}^{H}(\Lambda) > 0$. ■

**Proof of Proposition 1:** We rewrite the retailer's problem as follows

$$\begin{aligned} \max_{K} \quad & \alpha K + (1-\alpha)\,\Pi_R^L \\ \text{s.t.} \quad & n_{TP}^{H} \geq n^H(K) \text{ and } n^L(K) > n_R + \frac{\Lambda}{\lambda}\left(\frac{v_0}{v_L} + n_R\right) \end{aligned}$$

We use the Karush-Kuhn-Tucker (KKT) method to solve this optimization problem

$$\begin{aligned} \mathfrak{L} = \quad & \alpha K + (1-\alpha)\,\Pi_R^L - \mu_1\left[n^H(K) - n_{TP}^{H}\right] - \mu_2\left[n_R + \frac{\Lambda}{\lambda}\left(\frac{v_0}{v_L} + n_R\right) - n^L(K)\right] \\ \text{s.t.} \quad & \mu_1 \geq 0 \text{ and } \mu_2 \geq 0 \end{aligned}$$

Here we use $\mu_2 \geq 0$ as an auxiliary assumption. Later, we relax it to $\mu_2 = 0$ since the second constraint cannot be binding.

The first order KKT conditions are:

$$\frac{\partial \mathfrak{L}}{\partial K}: \quad \alpha - \mu_1 \frac{\partial n^H(K)}{\partial K} + \mu_2 \frac{\partial n^L(K)}{\partial K} = 0 \tag{A1}$$

$$\frac{\partial \mathfrak{L}}{\partial \mu_1}: \quad n_R \geq n^H(K)\ ;\ \mu_1 \geq 0\ ;\ \mu_1\left[n_{TP}^H - n^H(K)\right] = 0 \tag{A2}$$

$$\frac{\partial \mathfrak{L}}{\partial \mu_2}: \quad n^L(K) \geq n_R + \frac{\Lambda}{\lambda}\left(\frac{v_0}{v_L} + n_R\right)\ ;\ \mu_2 \geq 0\ ;\ \mu_2\left[n^L(K) - n_R - \frac{\Lambda}{\lambda}\left(\frac{v_0}{v_L} + n_R\right)\right] = 0 \tag{A3}$$

There are two possible cases:

(i) $\mu_1 = 0$ and $\mu_2 = 0$: In this case, equation A1 reduces to $\alpha = 0$. Because $\alpha > 0$, there is no solution.

(ii) $\mu_1 > 0$ and $\mu_2 = 0$: Only one of the constraints is binding. Because $\mu_1 > 0$, A2 ensures that $n^H(K_{SE}) = n_{TP}^H$. From A1, we conclude that this case is possible only if $\mu_1 = \frac{\alpha}{\frac{\partial n^H(K_{SE})}{\partial K}} > 0$.

By definition, $n^H(K)$ is smaller than $n_{TP}^H$ (since $n^H(K)$ is the smaller root) and, therefore, we have to have $\frac{\partial n^H(K_{SE})}{\partial K} > 0$. Because the condition in (ii) always holds, we have a unique separating equilibrium in which the retailer's target profit satisfies $n^H(K_{SE}) = n_{TP}^H$. By solving the equation, we get

$$K_{SE} = (\lambda + \Lambda)m + \beta\frac{v_0}{v_H} - 2\sqrt{(\lambda + \Lambda)m\beta\frac{v_0}{v_H}} = \left[\sqrt{(\lambda + \Lambda)\, m} - \sqrt{\beta\frac{v_0}{v_H}}\right]^2$$

In the equilibrium, the category captain accepts the contract offer and recommends the variety level $n_{TP}^H$ when consumers are H-type, and otherwise, the category captain rejects the offer and the retailer chooses the variety level $n_R^L$. ■

**Proof of Proposition 2:** We rewrite the retailer's problem as follows

$$\begin{aligned} &\max_K \ K \\ \text{s.t.} \quad & n_{TP}^H \geq n^H(K) \text{ and } n_R + \frac{\Lambda}{\lambda}\left(\frac{v_0}{v_L} + n_R\right) \geq n^L(K) \end{aligned}$$

The first order KKT conditions are as follows:

$$\frac{\partial \mathfrak{L}}{\partial K}: \quad 1-\mu_1\frac{\partial n^H(K)}{\partial K}-\mu_2\frac{\partial n^L(K)}{\partial K}=0 \tag{A4}$$

$$\frac{\partial \mathfrak{L}}{\partial \mu_1}: \quad n^H_{TP}\geq n^H(K)\ ;\ \mu_1\geq 0\ ;\ \mu_1\left[n^H_{TP}-n^H(K)\right]=0 \tag{A5}$$

$$\frac{\partial \mathfrak{L}}{\partial \mu_2}: \quad n_R+\frac{\Lambda}{\lambda}\left(\frac{v_0}{v_L}+n_R\right)\geq n^L(K)\ ;\ \mu_2\geq 0\ ;\ \mu_2\left[n_R+\frac{\Lambda}{\lambda}\left(\frac{v_0}{v_L}+n_R\right)-n^L(K)\right]=0 \tag{A6}$$

There are four possible cases:

(i) $\mu_1=0$ and $\mu_2=0$: This case is not possible since the equation A4 reduces to $1=0$.

(ii) $\mu_1>0$ and $\mu_2=0$: In this case, only one of the constraints is binding. Because $\mu_1>0$, A5 ensures that $n^H(K)=n^H_{TP}$. Notice that $\Pi^H_{TP}(n)>\Pi^L_{TP}(n)$ for all $n>0$. And, moreover, $n^H_{TP}=\arg\max_n \Pi^H_{TP}(n)$. Thus, $K=\Pi^H_{TP}\left(n^H_{TP}\right)>\Pi^L_{TP}\left(n^L_{TP}\right)>\Pi^L_{TP}\left(n^H_{TP}\right)$ which contradicts the assumption that the category captain rejects any target profit level it cannot deliver. In this case, the category captain who faces L-type consumers cannot deliver the desired profit.

(iii) $\mu_1=0$ and $\mu_2>0$: Because $\mu_2>0$, A6 ensures that $n^L(K_{PE})=n_R+\frac{\Lambda}{\lambda}\left(\frac{v_0}{v_L}+n_R\right)$. From the equation A4 we can conclude that this case is possible only if $\mu_2=\frac{1}{\frac{\partial n^L(K_{PE})}{\partial K}}>0$. Because $n^L(K)$ is an increasing function, $K_{PE}$ is always a solution.

(iv) $\mu_1>0$ and $\mu_2>0$: Because $\mu_1>0$ and $\mu_2>0$, A5 and A6 ensure that $n^H(K)=n^H_{TP}$ and $n^L(K)=n_R+\frac{\Lambda}{\lambda}\left(\frac{v_0}{v_L}+n_R\right)$, respectively. By (ii), we know that $n^H(K)=n^H_{TP}$ leads to a contradiction. Thus, this case is not possible.

Because the condition in (iii) always holds, we have a unique pooling equilibrium in which the retailer's target profit satisfies $n^L(K_{PE})=n_R+\frac{\Lambda}{\lambda}\left(\frac{v_0}{v_L}+n_R\right)$. By solving the equation, we get

$$K_{PE}=(\lambda+\Lambda)m-\beta\left(\frac{v_0}{v_L}+n_R\right)\frac{\Lambda}{\lambda}-\frac{\lambda m v_0}{v_0+n_R v_L}-\beta n_R.$$

In the pooling equilibrium, both types accept the contract offer. However, the category captain chooses the variety level $n^H(K_{PE}) = \frac{B^H(K_{PE})-\sqrt{(B^H(K_{PE}))^2-4Kv_Hv_0\beta}}{2v_H\beta}$ when consumers are H-type and $n^L(K_{PE}) = n_R + \frac{\Lambda}{\lambda}\left(\frac{v_0}{v_L} + n_R\right)$ otherwise.

Before we start the proof of the Proposition 3, it is useful to prove the following lemma.

**Lemma 5** $\frac{\partial K_{SE}(\Lambda)}{\partial \Lambda} > \frac{\partial K_{PE}(\Lambda)}{\partial \Lambda} > 0$

**Proof of Lemma 5:** We use the following derivatives: $\frac{\partial K_{SE}}{\partial \Lambda} = m - \frac{\sqrt{m\beta \frac{v_0}{v_H}}}{\sqrt{\lambda+\Lambda}}$ and $\frac{\partial K_{PE}}{\partial \Lambda} = m - \frac{\beta}{\lambda}\frac{v_0}{v_L} - \frac{\beta}{\lambda}n_R$. First, recall that $n_R^L$ is an upper bound for $n_R$. We show that $\frac{\partial K_{PE}(\Lambda)}{\partial \Lambda} > 0$ even we assume that $n_R = n_R^L$. If we replace $n_R$ with its upper bound, we get $\frac{\partial K_{PE}(\Lambda)}{\partial \Lambda} > \sqrt{m}\left[\sqrt{m} - \sqrt{\frac{\beta}{\lambda}\frac{v_0}{v_L}}\right]$. Because $n_R^L = \sqrt{\frac{\lambda m}{\beta}\frac{v_0}{v_L}} - \frac{v_0}{v_L} > 0$ we have $\sqrt{m} > \sqrt{\frac{\beta}{\lambda}\frac{v_0}{v_L}}$ and therefore $\frac{\partial K_{PE}(\Lambda)}{\partial \Lambda} > 0$. Now, consider $\frac{\partial K_{SE}}{\partial \Lambda} = m - \frac{\sqrt{m\beta \frac{v_0}{v_H}}}{\sqrt{\lambda+\Lambda}}$. Because $n_{TP}^H = \sqrt{\frac{(\lambda+\Lambda)m}{\beta}\frac{v_0}{v_H}} - \frac{v_0}{v_H}$, we can rewrite $\frac{\partial K_{SE}(\Lambda)}{\partial \Lambda} = m - \frac{\beta}{\lambda+\Lambda}\frac{v_0}{v_H} - \frac{\beta}{\lambda+\Lambda}n_{TP}^H$. Because $n_{TP}^H < n_R$, $\Lambda > 0$, and $v_H > v_L$, we can conclude that $\frac{\partial K_{SE}(\Lambda)}{\partial \Lambda} > \frac{\partial K_{PE}(\Lambda)}{\partial \Lambda}$. ■

**Proof of Proposition 3:** Let us define $\Omega(\Lambda) = \alpha K_{SE}(\Lambda) + (1-\alpha)\Pi_R^L - K_{PE}(\Lambda)$. First, we are going to show that $\Omega(\Lambda)$ is positive for $\Lambda = 0$.

*Step 1.* $\Omega(0) > 0$

*Proof of Step 1:* Rewrite $\Omega(0) = \alpha[K_{SE}(0) - K_{PE}(0)] + (1-\alpha)\left[\Pi_R^L - K_{PE}(0)\right]$. By definition, $\Pi_R^L \geq K_{PE}(0)$. Let $\overline{K}$ be the profit level such that $n^H(\overline{K}) = n^L(K_{PE}(0))$. Because $n^H(K) = n^L(K')$ implies $K > K'$ for all $K$ and $K'$, we have $\overline{K} > K_{PE}(0)$. Because $n^H(K_{SE}(0)) = n_{TP}^H$, we have $K_{SE}(0) \geq \overline{K}$. Then, $K_{SE}(0) > K_{PE}(0)$. Hence, $\Omega(0) > 0$.

Next, we show that the slope of $\Omega(\Lambda)$ is always negative for sufficiently small $\alpha$.

*Step 2.* For all $\alpha < \overline{\alpha}$, $\frac{\partial \Omega(\Lambda)}{\partial \Lambda} < 0$.

*Proof of Step 2:* By taking the derivative with respect to $\Lambda$, we get $\frac{\partial \Omega(\Lambda)}{\partial \Lambda} = \alpha\frac{\partial K_{SE}}{\partial \Lambda} - \frac{\partial K_{PE}}{\partial \Lambda}$. By Lemma 5, we know that $\frac{\partial K_{SE}(\Lambda)}{\partial \Lambda} > \frac{\partial K_{PE}(\Lambda)}{\partial \Lambda} > 0$. Let $\alpha(\Lambda)$ be such that $\frac{\partial \Omega(\Lambda)}{\partial \Lambda} = 0$ holds. That is,

$$\alpha(\Lambda) = \frac{\frac{\partial K_{PE}}{\partial \Lambda}}{\frac{\partial K_{SE}}{\partial \Lambda}} = \frac{m - \frac{\beta}{\lambda}\frac{v_0}{v_L} - \frac{\beta}{\lambda}n_R}{m - \frac{\sqrt{m\beta\frac{v_0}{v_H}}}{\sqrt{\lambda+\Lambda}}}.$$

Observe that $\alpha(\Lambda)$ is a monotonically decreasing function of $\Lambda$. Because $\Omega(\Lambda)$ is defined over the range $\left[0, \overline{\Lambda}\right]$, $\alpha\left(\overline{\Lambda}\right)$ gives us the lower bound of $\alpha(\Lambda)$. Notice also that $0 < \alpha(\Lambda) < 1$ since $\frac{\partial K_{SE}(\Lambda)}{\partial \Lambda} > \frac{\partial K_{PE}(\Lambda)}{\partial \Lambda} > 0$. Hence, if

$$\alpha < \overline{\alpha} = \alpha\left(\overline{\Lambda}\right) = \frac{m - \frac{\beta}{\lambda}\left(\frac{v_0}{v_L} + n_R\right)}{m - \frac{\beta}{\lambda}\left(\frac{v_0}{v_L} + n_R\right)\sqrt{\frac{v_L}{v_H}}}$$

then we have $\frac{\partial \Omega(\Lambda)}{\partial \Lambda} < 0$. Note that if there is no information asymmetry (i.e., $v_L = v_H$), then $\alpha(\bar{\Lambda}) = 1$ which implies that the proposition holds for any $\alpha$.

Step 2 implies that there is a unique solution $\Lambda^*$ such that $\Omega(\Lambda^*) = 0$. Therefore, the retailer prefers the separating equilibrium if $\Lambda \leq \Lambda^*$ and the pooling equilibrium otherwise.

■

**Proof of Proposition 4:** First, suppose that consumers are H-type.

*(i) and (ii)*: Take any $\Lambda < \Lambda^*$. By Proposition 3, we know that the players play the separating equilibrium. In the equilibrium, by Proposition 1, $n_{CC} = n_{TP}^H$. By Lemma 3, $n_{TP}^H < \overline{n}_{TP}^H(\Lambda) = n_R + \frac{\Lambda}{\lambda}\left(\frac{v_0}{v_H} + n_R\right)$. Thus, $n_{CC} = n_{TP}^H < n_R$ when $\Lambda = 0$. Let $\Lambda_1$ be such that $n_{TP}^H(\Lambda_1) = n_R$. Then, $\Lambda_1 = \frac{\beta}{m f_H} - \lambda$. Because $n_{TP}^H(\Lambda)$ is an increasing function of $\Lambda$ and $n_R$ is constant, we know that such a $\Lambda_1$ exists. We have two cases: $\Lambda^* \geq \Lambda_1$ and $\Lambda^* < \Lambda_1$. If $\Lambda^* \geq \Lambda_1$, then we have $n_{CC} = n_{TP}^H < n_R$ when $\Lambda \in [0, \Lambda_1)$ and $n_{CC} = n_{TP}^H \geq n_R$ when $\Lambda \in [\Lambda_1, \Lambda^*)$. On the other hand, if $\Lambda^* < \Lambda_1$ we have $n_{CC} = n_{TP}^H < n_R$ for all $\Lambda \in [0, \Lambda^*)$. Then, we can conclude that $n_{CC} = n_{TP}^H < n_R$ when $\Lambda \in [0, min\{\Lambda_1, \Lambda^*\})$ and $n_R \leq n_{CC} = n_{TP}^H$ when $\Lambda \in [min\{\Lambda_1, \Lambda^*\}, \Lambda^*)$.

*(iii)*: Take any $\Lambda \in \left[\Lambda^*, \overline{\Lambda}\right]$. By Proposition 3, we know that the players play the pooling equilibrium. The category captain accepts the contract offer and chooses the variety at $n_{CC} = n^H(K_{PE})$.

Now, let $\overline{K}$ be the profit level such that $n^H(\overline{K}) = n^L(K_{PE})$. By definition of the profit function, the retailer produces more profit (with the same level of the variety) when the consumers are H-type, i.e. $n^H(\overline{K}) = n^L(K_{PE}) \Rightarrow \overline{K} > K_{PE}$. Because $\Pi^H_{TP}\left(n^H_{TP}\right) \geq \overline{K}$, we have $\Pi^H_{TP}\left(n^H_{TP}\right) > K_{PE}$ and, therefore, $n_H\left(K_{PE}\right) = n_{CC} < n^H_{TP}$.

Notice that when the category captain chooses $n^H(K_{PE})$ the resulting profit for the retailer has to be $\Pi^H_{TP}\left(n^H(K_{PE})\right) = K_{PE}$ in the equilibrium. It is enough to show that $n_R \leq n_{CC} = n^H(K_{PE})$ can never happen. On the contrary, suppose that $n_R \leq n_{CC} = n^H(K_{PE})$. Then, we must have $\Pi^H_{TP}\left(n_R\right) \leq \Pi^H_{TP}\left(n^H(K_{PE})\right) = K_{PE}$. If we write down the closed form solutions of $\Pi^H_{TP}\left(n_R\right)$ and $K_{PE}$ we get the following condition after simplifying algebra

$$\lambda m\left[\frac{v_0}{v_L}\frac{\frac{v_0}{v_H}+n_R}{\frac{v_0}{v_L}+n_R}-\frac{v_0}{v_H}\right] \leq \Lambda\left[m\frac{v_0}{v_H}-\frac{\beta}{\lambda}\left(\frac{v_0}{v_L}+n_R\right)\left(\frac{v_0}{v_H}+n_R\right)\right]$$

First, consider the right hand side of the inequality. We claim that

$$m\frac{v_0}{v_H}-\frac{\beta}{\lambda}\left(\frac{v_0}{v_L}+n_R\right)\left(\frac{v_0}{v_H}+n_R\right) < 0.$$

Suppose that is not the case. Then, $\lambda m\frac{\frac{v_0}{v_H}}{\left(\frac{v_0}{v_H}+n_R\right)\left(\frac{v_0}{v_L}+n_R\right)} \geq \beta$ has to hold, which contradicts with $\beta = \lambda m[\alpha f_H + (1-\alpha)f_L]$. Therefore, the right hand side of the inequality is negative.

Now, consider the left hand side of the inequality. It is not hard to see that $\frac{v_0}{v_L}\frac{\frac{v_0}{v_H}+n_R}{\frac{v_0}{v_L}+n_R} - \frac{v_0}{v_H} > 0$ since $v_H > v_L$. Because both $\lambda m > 0$ and $\Lambda \in \left[\Lambda^*, \overline{\Lambda}\right]$, we can conclude that the inequality never holds. Thus, we have $K_{PE} < \Pi^H_{TP}\left(n_R\right)$ and, therefore, $n_{CC} = n^H(K_{PE}) < n_R$.

Now, suppose that consumers are L-type. Then, in any separating equilibrium, the category captain who faces with L-type consumers rejects the contract offer and the retailer makes the variety decision after updating its belief on the consumer type. The optimal variety for the retailer in this case is $n_{CC} = n^L_R$. Because $n^L_R > n_R$ for any $\Lambda \leq \bar{\Lambda}$, we have $n_{CC} > n_R$. The category captain chooses the variety at $n_{CC} = n_R + \frac{\Lambda}{\lambda}\left(\frac{v_0}{v_L}+n_R\right)$ in the pooling equilibrium. It is clear that $n_{CC} > n_R$ for any pooling equilibrium. Thus, if the consumers are low type, $n_{CC} > n_R$. ■

## B. Special Cases

**I.** $H = L$ **and** $\Lambda > 0$**.** The retailer's problem in the RCM case reduces to $\max_n \lambda m[\frac{nv}{v_0+nv}] - \beta n$ where $v_H = v_L = v$. It is straightforward to show that the optimal variety for this problem is $n_R = \sqrt{\frac{\lambda m}{\beta}\frac{v_0}{v}} - \frac{v_0}{v}$.

Because there is no uncertainty and the category captain can drive additional traffic into the category the retailer always prefers to the category captainship practice. The retailer will choose its optimal target profit level as if it is maximizing a problem similar to the one above: $\max_n (\lambda + \Lambda)m[\frac{nv}{v_0+nv}] - \beta n$. The optimal solution for this problem is $n_{CC} = \sqrt{\frac{(\lambda+\Lambda)m}{\beta}\frac{v_0}{v}} - \frac{v_0}{v}$. Clearly, $\Lambda > 0$ implies $n_{CC} > n_R$.

**II.** $H > L$ **and** $\Lambda = 0$**.** By Proposition 3, we know the retailer prefers the separating equilibrium. The resulting variety in the separating equilibrium is given by $n_{TP}^H$ if consumers are H-type and $n_R^L$ if they are L-type.

Recall, by Lemma 3, that $\overline{n}_{TP}^L(\Lambda) \geq \overline{n}_{TP}^H(\Lambda) > n_{TP}^H(\Lambda)$. These inequalities reduce to $\overline{n}_{TP}^L(0) = \overline{n}_{TP}^H(0) = n_R > n_{TP}^H(0)$ when $\Lambda = 0$. It is also straightforward to see from Lemma 1 that $n_R^L > n_R$ since the solution for $n_R^L$ corresponds to the case where the retailer maximizes its RCM profit as if $\alpha = 0$. Therefore, we can conclude that $n_R^L > n_R > n_{TP}^H$ when $H > L$ and $\Lambda = 0$.

## C. Supplement for Extensions

**Category Captain Selection.** When all the manufacturers are homogenous, the question of which manufacturer becomes the category captain is not meaningful. In order to gain some insights on which manufacturers are better suited to become category captains, we consider an extension of our original model where one of the manufacturers differs from the other manufacturers. First, in Model IA, we consider a model where one of the manufacturers offers a product with higher attractiveness compared to the other products in the category. Second, in Model IB, we consider a model where one of the manufacturers can drive more traffic into the category when compared with the other manufacturers in the category.

*IA: Manufacturers with Nonidentical Attractiveness:*

In this model, we assume that one of the manufacturers offers a product with higher attractiveness compared to the other manufacturers' products. We keep all the other assumptions regarding the cost and information structure as in our original model. We capture the difference between the attractiveness levels with parameter $\delta$; that is, one of the manufacturers offers a product with attractiveness $v_i + \delta$ and the remaining manufacturers offer products with attractiveness $v_i$. Suppose that in the category captainship scenario, in addition to setting the target profit for the category captain, the retailer also faces the decision of which manufacturer to designate as the category captain. We compare the model where the retailer chooses the high attractiveness manufacturer (with attractiveness $v_i + \delta$) as the category captain with the model where the retailer chooses one of the other manufacturers (with attractiveness $v_i$) as the category captain to understand the drivers behind retailer's category captain selection problem.

First, we consider the model where the retailer designates the high attractiveness manufacturer as the category captain. The category captain solves the following maximization problem for a given target profit $K$:

$$\begin{array}{ll} \max\limits_{n} & \dfrac{(\lambda + \Lambda)\, w(v_i + \delta)}{v_0 + nv_i + \delta} \\ \text{s.t.} & \dfrac{(\lambda + \Lambda)\, m(nv_i + \delta)}{v_0 + nv_i + \delta} - \beta n \geq K \end{array}$$

The category captain's unique best response $n_\delta(K)$ is given by

$$n_\delta(K) = \frac{B_\delta(K) - \sqrt{(B_\delta(K))^2 - 4Kv\beta v_0 + 4\beta v((\lambda + \Lambda)m - K)\delta}}{2v\beta}$$

where $B_\delta(K) = (\lambda + \Lambda)\, mv - \beta v_0 - Kv - \beta\delta$.

The optimal assortment selection of the category captain decreases in $\delta$. The retailer sets its target profit level to ensure that $n^H_{TP} = n_\delta(K_{SE})$ in the separating equilibrium and

$n_R + (\frac{\Lambda}{\lambda})(\frac{v_0}{v_L} + n_R) = n_\delta(K_{PE})$ in the pooling equilibrium. The resulting profits for the retailer under the separating and pooling equilibria are as follows:

$$
\begin{aligned}
K_{SE}(\delta) &= (\lambda+\Lambda)m + \beta\left(\frac{v_0+\delta}{v_H}\right) - 2\sqrt{(\lambda+\Lambda)m\beta\frac{v_0}{v_H}} \\
K_{PE}(\delta) &= (\lambda+\Lambda)m - \beta\left(\frac{v_0}{v_L} + n_R\right)\frac{\Lambda}{\lambda} - \frac{\lambda m(\lambda+\Lambda)v_0}{\delta\Lambda + (\lambda+\Lambda)(v_0+n_R v_L)} - \beta n_R.
\end{aligned}
$$

It is easy to show that both $K_{SE}(\delta)$ and $K_{PE}(\delta)$ are increasing in $\delta$. Notice also that $K_{SE}(0) = K_{SE}$ and $K_{PE}(0) = K_{PE}$ where $K_{SE}$ and $K_{PE}$ are the target profit levels when the manufacturers offer identical products (i.e., as in our original model). Thus, we can conclude that the retailer prefers an assortment that includes the product with attractiveness $v_i + \delta$ over an assortment that excludes it.

Now, suppose that the retailer chooses a manufacturer other than the high attractiveness manufacturer as the category captain. We consider the following two cases: (i) the category captain includes the high attractiveness manufacturer's product in the assortment and (ii) the category captain excludes the high attractiveness manufacturer's product. The latter case is identical to the model in the original manuscript or, equivalently, to the model we consider above with $\delta = 0$. In the former case, the category captain solves the following maximization problem for a given target profit level $K$:

$$
\begin{aligned}
\max_{n} \quad & \frac{(\lambda+\Lambda)\, w v_i}{v_0 + n v_i + \delta} \\
\text{s.t.} \quad & \frac{(\lambda+\Lambda)\, m(n v_i + \delta)}{v_0 + n v_i + \delta} - \beta n \geq K
\end{aligned}
$$

Notice that, this problem produces the exact same best response function $n_\delta(K)$ we found above since the category captain's constraint, which remains the same in both cases, determines the solution. Thus, the retailer collects the same profit if the category captain includes the high attractiveness manufacturer's product in the assortment. The only difference between this case and the case where the high attractiveness manufacturer is the category captain is the profit of the category captain.

If the retailer chooses a manufacturer other than the high attractiveness manufacturer, the category captain has an incentive to exclude the high attractiveness manufacturer from the assortment since higher attractiveness of rival's product hurts the category captain's profit. However, anticipating this incentive, the retailer will set the target profit level high enough that the category captain will have to include the manufacturer with high attractiveness in the assortment. Therefore, we conclude that the retailer is indifferent between selecting the high attractiveness manufacturer or any of the other manufacturers as a category captain as long as the high attractiveness product is included in the assortment.

*IB: Manufacturers with Nonidentical Ability to Increase Category Traffic:*

In this model, we assume that one of the manufacturers can drive more traffic into the category than the other manufacturers. All of the cost and information structure assumptions in this extension are the same as the ones in our original model. We assume that while one of the manufacturers can increase the category traffic by $\tilde{\Lambda}$, all other manufacturers can increase the category traffic by $\Lambda$ (where $\tilde{\Lambda} > \Lambda$). As before, suppose that in the category captainship scenario, in addition to setting the target profit for the category captain, the retailer also faces the decision of which manufacturer to designate as the category captain. We compare the model where the retailer chooses the manufacturer who can increase category traffic by $\tilde{\Lambda}$ as the category captain with the model where the retailer chooses one of the other manufacturers (i.e., category traffic increases by $\Lambda$).

First, consider the model where the retailer designates the manufacturer that can drive $\tilde{\Lambda}$ as the category captain. The category captain solves the following maximization problem for a given target profit $K$:

$$\begin{aligned} \max_{n} \quad & \frac{\left(\lambda + \tilde{\Lambda}\right) w v_i}{v_0 + n v_i} \\ \text{s.t.} \quad & \frac{\left(\lambda + \tilde{\Lambda}\right) m n v_i}{v_0 + n v_i} - \beta n \geq K \end{aligned}$$

This problem is same as the one we consider in the original manuscript. Thus, the equilibrium profits are

$$
\begin{aligned}
K_{SE}(\tilde{\Lambda}) &= \left[\sqrt{\left(\lambda+\tilde{\Lambda}\right)m}-\sqrt{\beta\frac{v_0}{v_H}}\right]^2 \\
K_{PE}(\tilde{\Lambda}) &= (\lambda+\tilde{\Lambda})m-\frac{\lambda m v_0}{v_0+n_R v_L}-\beta\left(n_R+\frac{\tilde{\Lambda}}{\lambda}\left(\frac{v_0}{v_L}+n_R\right)\right).
\end{aligned}
$$

By Lemma 5 (which is in the appendix of original manuscript and states that $\frac{\partial K_{SE}(\Lambda)}{\partial \Lambda} > \frac{\partial K_{PE}(\Lambda)}{\partial \Lambda} > 0$), we know that both $K_{SE}(\tilde{\Lambda})$ and $K_{PE}(\tilde{\Lambda})$ are increasing functions of $\tilde{\Lambda}$. Thus, the retailer is better off by choosing a category captain that can drive more traffic into the category.

The results in models IA and IB suggest that the retailer prefers to choose a manufacturer who is able to put something unique to the table while considering the category captain selection problem. In Model IA, the manufacturer with high attractiveness is not at an advantage because the first product can be included in the category regardless of whether the high attractiveness manufacturer is selected as a category captain or not. What matters for the retailer in Model IA is whether the high attractiveness manufacturer is included in the category or not: the retailer is better off if the manufacturer with high attractiveness product is included in the assortment. On the other hand, if a manufacturer has a unique characteristics such as being able to increase traffic more than the other manufacturers as in Model IB, then the retailer would prefer that manufacturer over the other manufacturers.

**Multiple Manufacturers Each Selling Multiple Products.** Consider a two stage supply chain model where two competing manufacturers, each offering multiple products, are selling their products to the consumers through a single retailer. For simplicity, we assume that every product is equally attractive, both manufacturers' production costs are normalized to zero, both manufacturers sell to the retailer at the wholesale price $w$, and the retail price of each product is $r$. Let $v$ denote the attractiveness a product (which is assumed to be same for each product) and $v_0$ be the attractiveness of the no-purchase option. Notice that these assumptions imply that the products of the manufacturers are perfect substitutes from the retailer's point of view. The consumers can either buy one of the products by the first or second manufacturer or decide to leave without a purchase. Let also $n_1$ and $n_2$ denote the number of products offered by the first and second manufacturers, respectively. We assume that both manufacturers' product offerings are finite, that is, $n_1 \leq \bar{n}_1$ and $n_2 \leq \bar{n}_2$. We also define the retailer's profit margin as $m = r - w$.

Given these assumptions, when the retailer offers $n_1$ and $n_2$ products from the first and second manufacturers, respectively, in the category, the average total demand for the first and second manufacturers are

$$q_1 = \lambda \frac{n_1 v}{v_0 + (n_1 + n_2)v} \quad \text{and} \quad q_2 = \lambda \frac{n_2 v}{v_0 + (n_1 + n_2)v}.$$

In retail category management scenario, the retailer sets the variety levels $n_1$ and $n_2$ by solving the following optimization problem:

$$\max_{n_1, n_2} \lambda m \frac{(n_1 + n_2)v}{v_0 + (n_1 + n_2)v} - \beta(n_1 + n_2) \tag{1}$$

where the first part is net profit from sales and the second part is the cost of managing variety for the retailer. Let $n = n_1 + n_2$ be the total number of products to be offered in the category. Then, the retailer's problem in (1) coincides with the retailer's problem in our original model. When all the products have equal margins and equal attractiveness, we get multiple solutions of the type $n_1 + n_2 = constant$ since manufacturers' products are perfect substitutes from the retailer's point of view. The optimal solution for (1) requires the following first order condition to hold.

$$n_R = n_{1R} + n_{2R} = \sqrt{\frac{\lambda m}{\beta} \frac{v_0}{v}} - \frac{v_0}{v}. \tag{2}$$

Suppose that the retailer decides to use some allocation rule where the retailer allocates a fraction $\tau \in [0,1]$ to the first manufacturer's products and $1-\tau$ to the second manufacturer's products. Then, the retailer's choice regarding the mix of products is as follows: $n_{1R} = \min\{\bar{n}_1, \tau n_R\}$ and $n_{2R} = \min\{\bar{n}_2, (1-\tau)n_R\}$. Because $\bar{n}_1 \leq \tau n_R$ would generate a trivial outcome (solution would be on the boundary), we focus on the more interesting cases where $\bar{n}_1 > \tau n_R$.

As in our original model, we assume that the category captain can drive additional traffic into the category and increase the rate of consumers into the category by $\Lambda$. Because there is no uncertainty and the category captain can drive additional traffic into the category, the retailer always prefers implementing category captainship. The retailer chooses its optimal target profit level as if it is maximizing a profit similar to the one in (1). This is because when there is no asymmetric information the retailer can achieve its first best (i.e., the retailer can extract the entire surplus from implementing category captainship and leave the captain indifferent) by using target profit contract (see the special case for symmetric information in appendix B in the paper). The retailer's problem is

$$\max_{n_1,n_2} \; (\lambda+\Lambda)m\frac{(n_1+n_2)v}{v_0+(n_1+n_2)v} - \beta(n_1+n_2). \tag{3}$$

The optimal solution to this problem needs to satisfy the following condition:

$$n_C = n_{1C} + k_{2C} = \sqrt{\frac{(\lambda+\Lambda)m}{\beta}\frac{v_0}{v}} - \frac{v_0}{v}.$$

Notice that $n_C \geq n_R$, which is due to the adjustment effect. Now suppose that the retailer's first best choice would follow the same arbitrary allocation rule as before. That is, to allocate the total variety where a fraction $\tau$ is allocated to the first manufacturer and $1-\tau$ is allocated to the second manufacturer. The retailer's optimal assortment mix choice if the retailer could drive the additional traffic to the category itself would be given by $n_{1C} = \min\{\bar{n}_1, \tau n_C\}$ and $n_{2C} = \min\{\bar{n}_2, (1-\tau)n_C\}$.

Suppose that the first manufacturer is assigned as the category captain. The category captain's profit in the category captainship game is

$$\pi_C(n_1, n_2) = (\lambda + \Lambda)w\frac{n_1 v}{v_0 + (n_1 + n_2)v}$$

The category captain has an incentive to decrease $n_2$ and increase $n_1$ as much as possible. Therefore, the category captain recommends that all of his $\bar{n}_1$ products are offered and in addition $n_C - \bar{n}_1$ of the non-captain manufacturer's products are offered. There are two cases: $n_C - \bar{n}_1 \geq (1-\tau)n_C$ and $n_C - \bar{n}_1 < (1-\tau)n_C$. Remember that $(1-\tau)n_C$ is the allocation of the non-captain manufacturer's products to be included in the retailer's first best assortment. In the former case, there is no room for competitive exclusion. In the equilibrium of this case, the category captain recommends all of its products (i.e., $\bar{n}_1$) and $n_C - \bar{n}_1$ of the second manufacturer's products. Because $n_C - \bar{n}_1 \geq (1-\tau)n_C$, the second manufacturer benefits from the category captainship practice. However, in the latter case, the category captain's decision results in exclusion of the second manufacturer's products from the assortment since $n_C - \bar{n}_1 < (1-\tau)n_C$. Furthermore, in this case, if $(1-\tau)n_C > n_C - \bar{n}_1 \geq (1-\tau)n_R$, then the second manufacturer would benefit from the category captainship practice. More generally, it is straightforward to see that $\pi_C(\bar{n}_1, n_C - \bar{n}_1) \geq \pi_C(\tau n_C, (1-\tau)n_C)$ and $\Pi_C(\bar{n}_1, n_C - \bar{n}_1) = \Pi_C(\tau n_C, (1-\tau)n_C)$ where $\Pi_C$ denotes the retailer's profit under the category captainship practice. That is, when the optimal choice of the assortment requires $n_C$ products in the category, the retailer is indifferent between any assortment mix $(n_1, n_2)$ as long as $n_1 + n_2 = n_C$ but the category captain prefers to include all of its products in the assortment and none of its rivals. This implies that unless the incentives of the retailer and the category captain are perfectly aligned (that is, $\tau = 1$) there is room for competitive exclusion.

**Manufacturers with Nonidentical Attractiveness.** Consider a supply chain model that consists of multiple manufacturers that are potential candidates for selling their differentiated products to consumers through a common retailer. As in our original model, each manufacturer offers one product only. Let $N = \{1, 2, ..., n\}$ denote the set of manufacturers. The retailer faces the decision of which manufacturers' brands to offer to its consumers. Let $S \subseteq N$ denote the subset of variants that retailer decides to include in the retail assortment. A customer either purchases one of the variants in $S$ or does not purchase anything. Let variant 0 represent the no-purchase option for the consumers with attractiveness $v_0$. We assume that $v_1 > v_2 > v_3 > ... > v_n$.

Given the choice set $S$ and the no-purchase option 0, let $q_i(S)$ denote the market share of manufacturer $i$'s product. Then, $q_i(S) = \frac{v_i}{V_S}$ where $V_S = v_0 + \sum_{j \in S} v_j$ according to our demand model in the paper. Let $\lambda$ denote the rate of customers entering the store, $w$ denote the wholesale price (same for all products), and $m$ denote the retailer's net profit margin from a product in the category. We normalize the manufacturers' production costs to zero. In addition, we assume that the retailer incurs an operational cost $\sigma(\phi_S) = \beta\phi_S$ where $\phi_S$ denotes the number of products in the assortment set $S$.

In the RCM scenario, the retailer decides on an assortment set $S$ to maximize its profit:

$$\max_S \ \lambda m \sum_{i \in S} \frac{v_i}{V_S} - \beta\phi_S \tag{4}$$

**Result 1** *Retailer's optimal assortment is in the attractive assortment set* $P = \{\{\}, \{1\}, \{1, 2\}, ..., \{1, 2, .., n\}\}$.

**Proof.** First, consider the retailer's profit when the retailer offers assortment $S$ (which is arbitrary), that is

$$\Pi^b(S) = \frac{\lambda m(V_S - v_0)}{V_S} - \beta\phi_S. \tag{5}$$

Now, suppose that the retailer adds one more product, say product $j$ with attractiveness $v_j$, to the already existing assortment $S$. That is the retailer offers assortment $S_j = S \cup \{j\}$. Let us denote the retailer's profit with assortment $S_j$ as $\Pi^b(v_j)$. Then, the retailer's profit with assortment $S_j$ is given by

$$\Pi^b(S_j) = \frac{\lambda m v_j}{V_S + v_j} - \beta + \frac{\lambda m(V_S - v_0)}{V_S + v_j} - \beta\phi_S \tag{6}$$

Let $h^b(v_j) = \Pi^b(S_j) - \Pi^b(S)$ be the difference in the profit of the retailer with and without the product $j$. If $h^b(v_j)$ is positive, then it is profitable to add product $j$ to the assortment. By substituting (5) and (6) we get

$$h^b(v_j) = \frac{\lambda m v_j v_0}{V_S(V_S + v_j)} - \beta \tag{7}$$

From differentiation,

$$\frac{\partial h^b(v_j)}{\partial v_j} = \frac{\lambda m v_0 v_j}{(V_S + v_j)^2}. \tag{8}$$

Because $h^b(v_j) > 0, \forall v_j \in [0, \infty)$, we can conclude that $h^b(v_j)$ is monotonically increasing in $v_j$ on the interval $[0, \infty)$. This implies that if the retailer decides to add a product to the already existing assortment, the retailer will add the product with the highest $v_j$ (among the remaining products) to the assortment. ■

In the category captainship scenario, the retailer delegates the assortment selection decision to the category captain (i.e., the first manufacturer). Both the category captain and the retailer have the same information about the consumers (i.e., symmetric information) but the category captain can increase the rate of customers purchasing from the category. As in our original model, we denote this increase by $\Lambda$.

**Result 2** *In the subgame perfect Nash equilibrium, the category captain's recommended assortment is in the attractive assortment set $P$.*

**Proof.** The category captain solves

$$\begin{aligned} \max_{S \subseteq N} \quad & (\lambda + \Lambda) \frac{w v_1}{V_S} \\ \text{s.t.} \quad & m(\lambda + \Lambda) \sum_{i \in S} \frac{v_i}{V_S} - \beta \phi_S \geq K \end{aligned}$$

First, notice that the category captain's profit is decreasing in the number of products offered in the category. Second, if there are two sets of products with same number of products offered in each, the category captain chooses the one with the lowest $V_S$. Because the category captain's profit is decreasing in $V_S$, the solution for the optimization problem is determined by the constraint. Because all the products have different attractiveness levels, each assortment set $S$ with the same $\phi_S$ has a unique $V_S$. Because the number of feasible assortment sets are finite, the category captain's solution exists. Let $S(K)$ be the optimal assortment recommended by the category captain when the retailer's target profit is $K$. In the equilibrium, the retailer will consider each of $V_S$ possibilities and choose the target that is equal the highest possible profit. Let this profit be $K^*$. We claim that $S(K^*) \in P$. Suppose not. Then, we can proceed as in the proof of Result 1 and show that there is contradiction with $S(K^*)$ being in the most attractive set. ■

Notice that this result does not claim that the recommended assortment is in the attractive assortment set $P$ for any target profit level $K$. The category captain's recommended assortment with target profit level may not be in the attractive assortment set $P$ for arbitrary $K$. However, for $K$ large enough, the recommended assortment is in the attractive assortment set $P$. Since the retailer pursues the largest possible $K$, in the equilibrium the category captain's recommendation of assortment lies in $P$.

While the competitive exclusion effect does not reveal itself in the extension described above due to the absence of asymmetric information, the inclusion of asymmetric information into this model would result in competitive exclusion. In a separating equilibrium, since the information would be fully revealed, the competitive exclusion would not be a major concern.

We conjecture that the recommended assortment will be in the attractive set. In a pooling equilibrium, however, the competitive exclusion would become a serious issue because the retailer would set its target to ensure that the low type category captain is indifferent between accepting and rejecting the contract. In this case, the high type captain might be able to deliver the target profit set by the retailer with multiple different assortments. If that is the case, the captain would prefer recommending an assortment that has the lowest $V_S$, which would be a deviation from retailer's preferences toward an assortment in the most attractive assortment set. Therefore, in a pooling equilibrium, we conjecture that the recommended assortment might not be in the attractive set.

We conducted a numerical study to confirm our intuition. We assume that $v_i = v + \delta_i$ with $\delta_1 > \delta_2 > ... > \delta_N$. As in our original model, we assume that while the category captain knows the consumer type, the retailer's prior beliefs are such that $v = v_H$ with probability $\alpha$ and $v = v_L$ with probability $1 - \alpha$. We assume that there are eight manufacturers that want to sell their product to consumers through the retailer, that is $N = 8$. We use the following parameter set in our numerical study: $\lambda = 100$, $m = 5$, $w_1 = 4$, $v_0 = 12$, $v_H = 5$, $v_L = 2$, $\alpha = 0.5$, $\beta = 10$, and $\delta_1 = 5$, $\delta_2 = 4.5$, $\delta_3 = 4.2$, $\delta_4 = 4$, $\delta_5 = 3.5$, $\delta_6 = 3$, $\delta_7 = 2.4$, $\delta_8 = 2$. With this set of parameters, it is optimal for the retailer to offer the six products with the highest attractiveness in the RCM scenarios. Under category captainship, the assortment outcome depends on the parameter $\Lambda$ which measures the category captain's traffic driving ability. For example, for $\Lambda = 10$, the retailer prefers separating equilibrium. In this case, if the category captain accepts the category captainship contract (i.e., if consumers are L type), the retailer offers the all products in the category. The additional traffic allows the retailer to expand its assortment offering. If the category captain rejects the contract (i.e., the consumers are H type), the retailer infers that consumers are H type and offers the five products with the highest attractiveness.

If, on the other hand $\Lambda = 20$, the retailer prefers pooling equilibrium. In this case, if the consumers are L-type, the category captain recommends an assortment with the six most popular products and if the consumers are H-type, the category captain recommends an assortment that include the first, third and seventh products only. This confirms our intuition that under a pooling equilibrium the recommended assortment is not in the popular assortment set and some of the popular products might be excluded from the assortment.

To summarize, we conclude that depending on the type of equilibria that will be preferred by the retailer, the category captainship might result in different types of products being excluded from the assortment. If the captain's traffic driving abilities are limited (i.e., the retailer prefers separating equilibria), the category captain will exclude the products with low attractiveness whereas if the captain's traffic driving abilities are significant (the retailer prefers pooling equilibrium), the captain might exclude the high attractiveness products.

## Chapter IV

# PRICE DISCRIMINATION IN QUANTITY COMPETITION

### Introduction

Many firms operate in markets that are subject to demand uncertainty. Also, many firms operate in markets with different variety of consumers. Naturally, markets that contain both of these structural elements create an incentive conflict for the firms. On one hand, firms that face uncertainty choose sub-optimal strategies, which results in profit losses, in order to smooth their strategies across different market outcomes.[1] On the other hand, firms that face different variety of consumers tend to discriminate consumers by offering different prices in the hope of capturing higher surplus.[2]

Examples of firms competing in markets that have demand uncertainty and possibility of price discrimination can be seen in various industries such as passenger transportation, hotels, and automobile rentals.[3] Especially, passenger transportation industries (airlines, trains, buses, etc.) where firms compete for seats offered for a specific route are good examples of such settings. For instance, airline tickets are sold in unit quantity and it is a common practice that airlines price discriminate. There are also different type of consumers (e.g., people who only fly in business class no matter what the ticket price is, or people who can go with either business or economy, depending on the price) in the airline transportation market as well as the demand uncertainty.

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[1]For instance, in a Stackelberg environment, Anand and Goyal (2009) examine the incentives of competing firms when the information about the demand uncertainty is asymmetric. Their model predicts that firms in this environment choose sub-optimal quantities (comparing with the complete information benchmark) and make less profit in the presence of information asymmetry.

[2]For example, Hazledine (2006) and Kutlu (2009) extends the standard quantity competition models by introducing price discrimination to an exogenously determined number of markets.

[3]While these industries have other characteristics of price discrimination, such as the airlines use other type of travel restrictions (e.g., Saturday-night stay-overs) on discount fares and advance purchase requirements, they seem to satisfy the spirit of our assumptions.

This paper focuses on the incentive tradeoff between demand uncertainty and price discrimination. Motivated by this tradeoff, the goal of this paper is to better understand the consequences of exogenously enforced price discrimination. In particular, we consider a linear demand duopoly model in which two firms engage in quantity competition over two varieties of a product. In our quantity competition setting, the question of how prices are determined if firms do not set them directly arises naturally. A general answer to these types of questions is provided by Kreps and Scheinkman (1983), who showed that the quantity competition outcome is equivalent to the outcome of a two-stage game in which firms decide on production capacity in the first stage and subsequently compete in prices. Our setting fits better to the environments in which the total capacity is relatively inflexible vis-à-vis price changes. By changing the timing of the model, we consider the incentives of the duopoly firms under both Cournot and Stackelberg settings. Depending on the timing of the model we find and compare the relevant equilibrium outcomes.[4]

An important aspect of our model is that we allow demand interdependence between two varieties of the same product. We incorporate such an interdependence by allowing three different types of consumers in the demand model: loyal consumers for a variety and switchers, who can potentially purchase both of the varieties. The existence of three types of consumers produce a demand behavior where the cross price effects between varieties are asymmetric. Such a demand behavior is consistent with the empirical results in the marketing literature. For example, Blattberg et al. (1995) study the empirical generalizations on promotion effects and show that cross-promotion effects are asymmetric and promoting a higher-priced (higher quality) brand impacts a lower-priced (lower quality) brand more so than the reverse. This phenomenon, which is known as the asymmetric price effect, is documented by Blattberg and Wisniewski (1989) and has been extensively studied in the literature.[5]

[4]For instance, we find perfect Bayesian equilibria for the Stackelberg game with asymmetric information whereas we only need the standard Nash equilibrium for the full information Cournot game.

[5]See for example Sethuraman et al. (1999) and the references therein.

There is a small literature on the theory of price discrimination in oligopolistic settings. Borenstein (1985) examines third degree price discrimination in the Bertrand model. Holmes (1989) compares the impact of the third degree price discrimination on monopoly and duopoly outcomes when a market can be split into two independent markets. Corts (1998) identifies a situation in which price discrimination is a prisoners' dilemma for duopoly firms. There is also a recently developed literature of price discrimination in oligopolistic quantity competition settings. In particular, Hazledine (2006) and Kutlu (2009) examine the second degree price discrimination in the Cournot and Stackelberg competition models, respectively. This paper diverges from the literature by allowing asymmetric cross price effects between markets. In addition, we examine the equilibrium behavior of the duopoly firms in the presence of demand uncertainty, which is generally not the case for the models in the literature. Another stream of papers directly related to this paper are located in the revenue management literature, which can be summarized as the use of market segmentation and assigning quantity limits on each fare to generate maximum profit.[6] Dana (1999) presents an oligopoly model of price discrimination with uncertain demand in which competition increases the dispersion of prices. We diverge from this line of research by allowing firms to compete in a market where two different varieties of the same product exist.

This paper extends the standard Cournot and Stackelberg competition literatures by characterizing the equilibrium outcomes in the presence of multiple varieties. Our results provide intuition on whether the firms that engage in quantity competition choose to practice price discrimination. In the earlier literature, the results of Hazledine (2006) and Kutlu (2009) are conflicted about the optimal quantity behavior of the firms. While Hazledine (2006) shows that all firms choose to price discriminate in the Cournot setting, Kutlu (2009) points out that only the follower price discriminates in the Stackelberg setting. We contribute to this line of research by showing that a firm chooses not to practice price discrimination when the firm is the leader in the market and the cross price effects between varieties are

[6] See Dana (1999) for a nice review of the revenue management literature.

one sided.[7] If the leadership is not established in the market or the cross price effect is not one sided, then price discriminating is a dominant strategy for both firms. We also show that the existence of switcher type of consumers is crucial for the price differences between the varieties in the equilibrium.

## Demand Model

There is a product characteristics $x$ and the unit line $[0,1]$ is the characteristics space. There are two variants, which are represented by the points $B \in [0,1]$ (business variant) and $E \in [0,1]$ (economy variant) in the characteristic space, with $B \neq E$. For simplicity, we assume that the variants are located on the boundaries of the characteristic space, i.e., $B = 0$ and $E = 1$. Three types of consumers exist: (i) business loyal ($L_B$), (ii) economy loyal ($L_E$), and (iii) switcher ($S$). Business loyal type of consumers purchase either one unit of the variety $B$ or nothing, economy loyal type of consumers purchase either one unit of the variety $E$ or nothing, and switcher type of consumers purchase either one unit of $B$ or $E$, or nothing.

There is a continuum of consumers with type $k \in \{L_B, L_E, S\}$, who are differentiated by their most preferred characteristic points, distributed according to uniform distribution over the characteristic space. Each consumer purchases one unit of the variant which offers the greatest utility. The utility of the consumer with type $k$, who is located at $x$ (where $x$ corresponds to his most preferred point) and purchasing the variant $i \in \{B, E\}$, is given by

$$U_i^k(x) = v_i^k - \theta_i^k P_i - td(x,i) \tag{1}$$

where $v_i^k$ representing the reservation value of the type $k$ consumer from the consumption of the variant $i$, $t$ is a positive constant, and $d(x,i)$ is the ideological distance between the consumer located at $x$ and the variant $i$. The parameter $\theta_i^k$ measures the price sensitivity of

[7] We say the cross price effect is one sided if the change in the price of a variety effects the demand of the other quantity but not *vice versa*.

the type $k$ consumer who purchases the variant $i$. Hence the first two terms in (1) can be seen as being common to all consumers. By contrast, the last term differs across consumers and it measures a consumer's disutility from not buying the ideal variant. We assume that the loyal consumers receive a negative utility by consuming the product that they are not loyal to. That is, $v_i^{L_j} < 0$ for $i, j \in \{B, E\}$ and $i \neq j$. Figure 10 represents this market structure. While the switchers decide whether to buy the business variant or the economy variant, the loyal consumers decide whether to buy the variant that they are loyal to or not to buy at all. In particular, a business loyal consumer, who is located at $x$, solves

$$U^{L_B}(x) = \max\{0, U_B^{L_B}(x)\} = \max\{0, v_B^{L_B} - \theta_B^{L_B} P_B - tx\}$$

and an economy loyal consumer, who is located at $x$, solves

$$U^{L_E}(x) = \max\{0, U_E^{L_E}(x)\} = \max\{0, v_E^{L_E} - \theta_E^{L_E} P_E - t(1-x)\}$$

whereas a switcher consumer, who is located at $x$, solves

$$U^S(x) = \max\{0, U_B^{L_B}(x), U_E^{L_E}(x)\} = \max\{0, v_B^S - \theta_B^S P_B - tx, v_E^S - \theta_E^S P_E - t(1-x)\}$$

when facing the prices of $P_B$ and $P_E$.[8] In order to avoid boundary issues, we assume

$$t > \max\{V_B^{L_B}, V_E^{L_E}, V_B^S, V_E^S\}$$

throughout the rest of the paper, where $V_i^k = v_i^k - \theta_i^k P_i$ for $k \in \{L_B, L_E, S\}$ and $i \in \{B, E\}$.

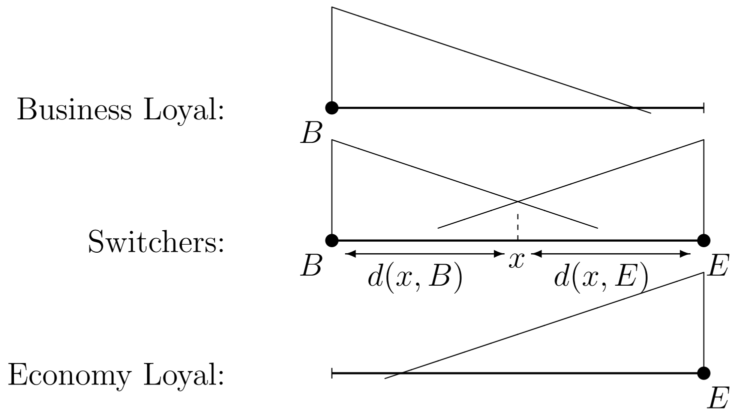


Figure 10: Demand Model

We can define the *market space* of the variant $i$ from the $k$-type consumers as

$$M_i^k = \{x \in [0,1] : U_i^k(x) \geq \max\{0, U_j^k(x)\}\}$$

where $i \neq j \in \{B, E\}$ and $k \in \{L_B, L_E, S\}$. Then, the *demand for variant $i$ from $k$-type consumers* is

$$Q_i^k = \begin{cases} \frac{v_i^k}{t} - \frac{\theta_i^k}{t} P_i, & \text{if } k = L_i, \\ \frac{t + v_i^k - v_j^k}{2t} - \frac{\theta_i^k}{2t} P_i + \frac{\theta_j^k}{2t} P_j, & \text{if } k = S, \text{ and} \\ 0, & \text{if } k = L_j. \end{cases}$$

There is a dependence (i.e., $M_B^S + M_E^S > 1$) between the two variants' demands when the prices are *low*. However, when the prices are *high* enough, the two markets are separated (i.e., $M_B^S + M_E^S < 1$). In particular, if

$$\theta_B^S P_B + \theta_E^S P_E > v_B^S + v_E^S - t$$

then the demand for each variant is independent from the price of the other variant and

[8] Notice that $d(x, B) + d(x, E) = 1$ since we assume that $B$ and $E$ are located on the two edges of the unit interval.

equal to

$$Q_i^k = \begin{cases} \frac{v_i^k}{t} - \frac{\theta_i^k}{t} P_i, & \text{if } k \in \{L_i, S\} \text{ and} \\ 0, & \text{if } k = L_j. \end{cases}$$

Throughout the rest of the paper, we restrict our attention to the more interesting case where there is a dependence between the two variants' demands. That is, we consider only the set of prices that satisfy the inequality $\theta_B^S P_B + \theta_E^S P_E \leq v_B^S + v_E^S - t$.

Total demand for the variant $i$ is simply the sum of the demand for variant $i$ from each consumer type and equal to

$$Q_i = \sum_k Q_i^k = \frac{t + v_i^S - v_j^S + 2v_i^L}{2t} - \frac{\theta_i^S + 2\theta_i^L}{2t} P_i + \frac{\theta_j^S}{2t} P_j$$

where $\frac{t+v_i^S - v_j^S + 2v_i^L}{2t}$ is the demand intercept, $\frac{\theta_i^S + 2\theta_i^L}{2t}$ measures the own price effect of the variant $i$ whereas $\frac{\theta_j^S}{2t}$ measures the cross price effect. The existence of different price sensitivities leads to a demand function with different cross price effects. In particular, the effect of a change in the price charged to the consumers close to the variant $B$ is different than the effect of a change in the price charged to the consumers close to the variant $E$. Such a demand behavior is consistent with the empirical results in the marketing literature, which names this phenomenon as *asymmetric price effect*.[9] The inverse demand functions obtained through these demand functions are

$$P_i = \delta_i - \gamma_{ii} Q_i - \gamma_{ij} Q_j$$

for $i \in \{B, E\}$, where

$$\delta_i = \frac{(t + v_i^S - v_j^S + 2v_i^L)\theta_j^L + (t + v_i^L + v_j^L)\theta_j^S}{\theta_i^S \theta_j^L + \theta_i^L \theta_j^S + 2\theta_i^L \theta_j^L},$$

[9] See Blattberg and Wisniewski (1989), Sethuraman et al. (1999), and the references therein.

$$\gamma_{ii} = t\frac{\theta_j^S + 2\theta_j^L}{\theta_i^S\theta_j^L + \theta_i^L\theta_j^S + 2\theta_i^L\theta_j^L}, \quad \text{and} \quad \gamma_{ij} = t\frac{\theta_j^S}{\theta_i^S\theta_j^L + \theta_i^L\theta_j^S + \theta_i^L\theta_j^L}.$$ [10]

Notice that the existence of loyal consumers ensures that the own price effect is greater than the cross price effect, i.e., $\gamma_{ii} > \gamma_{ij}$. We assume that $\delta_i = \delta_j = \delta$ throughout the rest of the paper. This assumption is only for convenience. It does not play any crucial role for the results of the paper, however, it simplifies the analysis substantially.

## Benchmarks

We focus our analysis on the markets in which two firms engage in quantity competition. Depending on the timing of the setup, we can have either a model of Cournot competition or Stackelberg competition. In particular, we consider a situation in which a leader firm ($L$), who is already providing both varieties to a market as a monopoly, faces a follower ($F$). There is no cost of entrance for the follower. For simplicity, we assume that firms have common marginal cost $c$, which is constant for all levels of production and *small* compared to the demand intercept, i.e., $\delta > c$. Each firm has to decide on allocating their total production to the two varieties, which is denoted by $q^k = (q_B^k, q_E^k)$ where $k \in \{L, F\}$. We consider two cases: (i) the follower *cannot* observe the quantity allocation decision of the leader (Cournot competition) and (ii) the follower *can* observe the quantity allocation decision of the leader before deciding on its quantity allocation (Stackelberg competition). The inverse demand function of the variety $i \in \{B, E\}$ is given by the expression in (2), where $Q_i = q_i^L + q_i^F$ denotes the total quantity supplied by two firms.

### Cournot Benchmark

We first consider that the firms play a simultaneous move game and know the shape of the demand for each variety. In the first stage, the leader decides how much quantity to supply for each variety, i.e., $q^L = (q_B^L, q_E^L)$. In the second stage, *without observing* the choices

[10] For convenience, throughout the rest of the paper, we use $i$ and $j$ for generic names of the two different varieties. That is, $i, j \in \{B, E\}$ and $i \neq j$.

of the leader, the follower decides on its allocation $q^F = (q_B^F, q_E^F)$. Because there is no information transition from the first stage to the second this setting technically corresponds to a simultaneous move game.

When the leader and the follower chose allocations $q^L$ and $q^F$, respectively, the profit for the firm $k \in \{L, F\}$ under the Cournot competition is

$$\Pi^k = \max_{q^k} \sum_{i \in \{B,E\}} (P_i - c) q_i^k = \sum_{i \in \{B,E\}} \left[\delta - c - \gamma_{ii} Q_i - \gamma_{ij} Q_j\right] q_i^k,$$

which leads to the first order conditions

$$\frac{\partial \Pi^k}{\partial q_i^k} = \delta - c - \gamma_{ii} Q_i - \gamma_{ij} Q_j - \gamma_{ii} q_i^k - \gamma_{ji} q_j^k = 0$$

for $i \in \{B, E\}$ and $k \in \{L, F\}$. The solution to these first order conditions determines the reaction functions and the equilibrium quantity choices of the firms for each market.

**Proposition 1** *When the firms engage in Cournot competition under complete information, the unique Nash equilibrium strategies of the firms, the market prices, and the firm profits are*

$$q_i^k = \tfrac{\delta - c}{2} f_j \quad \textit{and} \quad \Pi^k = \tfrac{(\delta-c)^2}{2}\left[\tfrac{f_1 + f_2}{3}\right],$$
$$P_i = c[\gamma_{ii} f_j + \gamma_{ij} f_i] + \delta[1 - \gamma_{ii} f_j - \gamma_{ij} f_i]$$

*where $i \in \{B, E\}$, $k \in \{L, F\}$, and*

$$f_j = \frac{3\gamma_{jj} - 2\gamma_{ij} - \gamma_{ji}}{4\gamma_{ii}\gamma_{jj} - (\gamma_{ij} + \gamma_{ji})^2 + \frac{(\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji})}{2}}.$$

This benchmark result is consistent with the results in the previous literature. First, it is straightforward to see that $q_i^k > 0$ for $i \in \{B, E\}$ and $k \in \{L, F\}$ because $\gamma_{ii} > \gamma_{ij}$ and $\gamma_{ii} > \gamma_{ji}$, which is a result of common demand intercept assumption and the existence of the loyal consumers. Second, when there is only one variety, the results in Proposition 1 coincide with the original Cournot model outcome. That is, when the own price effect of the variety $j$ approaches to infinity ($\gamma_{jj} \to \infty$), the equilibrium quantity decisions of the firms approach to the Cournot equilibrium outcome ($q_i^k \to \frac{\delta - c}{3}$). Similarly, if there are no cross price effects (i.e., $\gamma_{ij} = 0$), the solutions in Proposition 1 coincide with the classical Cournot model results in which two firms compete in two independent markets. Third, the results in Proposition 1 are also consistent with the ones in Hazledine (2006) when the number of markets in Hazledine (2006) is restricted to two. If the cross price effects are asymmetric in such a way that the quantity of a variety affects the price of the other variety but not vice versa, then the solutions in Proposition 1 coincide with the ones in Hazledine (2006). In particular, when $\gamma_{11} = \gamma_{22} = \gamma_{21} = 1$ and $\gamma_{12} = 0$, the results in Proposition 1 reduce to $q_1^k = \frac{2(\delta - c)}{7}$ and $q_2^k = \frac{q_1^k}{2} = \frac{\delta - c}{7}$, which is identical to the ones in Hazledine (2006) under the restriction that there are two markets.

Proposition 1 suggests that unless the cross price effect is one sided (i.e., $\gamma_{ij} = 0$ and $\gamma_{ji} > 0$), firms that are engaged in Cournot competition under complete information prefer to supply both varieties while differentiating their prices. A comparison of the equilibrium prices between two varieties reveals the following corollary.

**Corollary 2** $\theta_j^S > \theta_i^S$ *implies* $P_j^C > P_i^C$.

where $P_j^C$ is the complete information Cournot game equilibrium price for the $j$-th variety. Intuitively, as long as the own price effects dominate the cross price effects (i.e., $\gamma_{ii} > \gamma_{ij}$), the market price is higher for the variety that has higher price sensitivity of the switchers (i.e., $\theta_i^S$). This intuition holds for all possible non-negative levels of the own price effects.

**Stackelberg Benchmark**

In this section, we focus on the situation that the leader and the follower engage in a Stackelberg type of competition where they move sequentially, rather than simultaneously.

In particular, we consider the same setting as in the Cournot section but change the timing of the game. The leader decides on allocation strategy $q^L$ in the first stage. However this time, in the second stage, the follower decides allocation $q^F$ only *after observing* the strategy choice of the leader.

We find the subgame perfect Nash equilibrium by solving the game backwards, starting with the follower's problem. The profit for the follower when the allocation choices of the leader and the follower are $q^L$ and $q^F$, respectively, is

$$\Pi^F = \max_{q^F} \sum_{i\in\{B,E\}} (P_i - c)q_i^F = \sum_{i\in\{B,E\}} \left[\delta - c - \gamma_{ii}Q_i - \gamma_{ij}Q_j\right] q_i^F,$$

which leads to the following first order conditions:

$$\frac{\partial \Pi^F}{\partial q_i^F} = \delta - c - \gamma_{ii}Q_i - \gamma_{ij}Q_j - \gamma_{ii}q_i^F - \gamma_{ji}q_j^F = 0$$

for $i \in \{B, E\}$. The solution to these first order conditions determines the follower's reaction functions for each market. Let $q^{F*} = (q_1^{F*}, q_2^{F*})$ be the best response strategy of the follower, which solves the first order conditions of the follower's problem. By anticipating best response strategy of the follower, the leader solves

$$\Pi^L = \max_{q^L} \sum_{i\in\{B,E\}} (P_i - c)q_i^L = \sum_{i\in\{B,E\}} \left[\delta - c - \gamma_{ii}(q_i^L + q_i^{F*}) - \gamma_{ij}(q_j^L + q_j^{F*})\right] q_i^L$$

to find its optimal allocation. The allocation that solves the first order conditions of the leader simultaneously characterizes the unique equilibrium, which is summarized in Proposition 2.

**Proposition 2** *When the firms engage in Stackelberg competition under complete information, the unique subgame perfect Nash equilibrium strategies of the firms, the market prices,* *and the firm profits are*

$$q_i^L = \tfrac{\delta - c}{2} g_j \quad \textit{and} \quad q_i^F = \tfrac{\delta - c}{2} h_j,$$
$$\Pi^L = \tfrac{(\delta - c)^2}{2}[\tfrac{h_1 + h_2}{2}] \quad \textit{and} \quad \Pi^F = \tfrac{(\delta - c)^2}{2}[\tfrac{h_1 + h_2}{4}],$$
$$P_i = c[\gamma_{ii}\tfrac{g_j + h_j}{2} + \gamma_{ij}\tfrac{g_i + h_i}{2}] + \delta[1 - \gamma_{ii}\tfrac{g_j + h_j}{2} - \gamma_{ij}\tfrac{g_i + h_i}{2}]$$

*where*

$$g_j = \frac{\gamma_{jj} - \gamma_{ij}}{\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}.$$

The results in Proposition 2 are also consistent with the results in the earlier literature. First, when there is only one variety, the results in Proposition 2 coincide with the original Stackelberg model outcome. That is, when the own price effect of the second variety approaches to infinity ($\gamma_{jj} \to \infty$), the equilibrium quantity decisions of the firms approach to the Stackelberg equilibrium outcome ($q_i^I \to \frac{\delta - c}{2}$ and $q_i^E \to \frac{\delta - c}{4}$). Similarly, if there are no cross price effects (i.e., $\gamma_{ij} = 0$), the solutions in Proposition 2 coincide with the classical Cournot model results in which two firms compete in two independent markets. Second, the results in Proposition 2 are also consistent with the ones in Kutlu (2009) when the number of *bins* in Kutlu (2009) is restricted to two. If the cross price effects are asymmetric in such a way that the quantity of a variety affects the price of the other variety but not vice versa, then the solutions in Proposition 2 coincide with the ones in Kutlu (2009). In particular, when $\gamma_{11} = \gamma_{22} = \gamma_{21} = 1$ and $\gamma_{12} = 0$, the results in Proposition 2 reduce to $q_1^k = \frac{\delta - c}{2}$ and $q_2^k = \frac{\delta - c}{6}$, which is identical to the ones in Kutlu (2009) under the restriction that there are only two varieties.

Finally, Proposition 2 suggests that firms that are engaged in Stackelberg competition under complete information prefer to supply both varieties while differentiating between prices as long as the cross price effects are small as compared to the own price effects. A comparison of the equilibrium prices between two varieties reveals a result similar to

Corollary 2,

**Corollary 3** $\theta_j^S > \theta_i^S$ *implies* $P_j^S > P_i^S$

where $P_j^S$ is the complete information Stackelberg game equilibrium price of the $j$-th variety. Intuitively, as long as the own price effects dominate the cross price effects (i.e., $\gamma_{ii} > \gamma_{ij}$ for $i, j \in \{1, 2\}$), the market price is higher for the variety for which the price sensitivity of the switchers is higher. This intuition holds for all possible non-negative levels of the own price effects. Corollary 3 together with Corollary 2 highlight, perhaps surprisingly, that the order of move is not crucial for the differences between equilibrium market prices of the varieties. What matters ultimately is the price sensitivity of the switchers.

## Analysis with Incomplete Information

In the previous section, we consider the effects of multi variety on the quantity competition under complete information and found the benchmark results. When firms are completely informed about the demand, the existence of a second variety increases the equilibrium profits of both firms as compared to their profits in one standard quantity competition profits, thanks to the firms' ability to price discriminate. However, when one firm has incomplete information about the demand, the benchmark results are likely to move towards to the standard quantity competition outcomes since the firms will choose sub-optimal quantities in such situations. The surplus generating effects of price discrimination may be eliminated by the loss generating effects of sub-optimal quantity choices. We first examine the effects of incomplete information on the Cournot benchmark and later consider its effects on the Stackelberg benchmark.

### Cournot Competition with Incomplete Information

We model the effects of multi variety on the competition under incomplete information via an asymmetric information Cournot game. In particular, we relax the symmetric information assumption and consider the Cournot duopoly model in which the inverse demand given by

$$P_i = \tilde{\delta} - \gamma_{ii} Q_i - \gamma_{ij} Q_j.$$

The intercept $\tilde{\delta}$ is random and equal to $\delta^h$ (high demand) with probability $\alpha$ and to $\delta^l$ (low demand) with probability $1 - \alpha$. Furthermore, we assume that information is asymmetric: the leader $L$ knows the true value of $\tilde{\delta}$, but the follower $F$ only knows the distribution of $\tilde{\delta}$. All other aspects of the game are common knowledge.

Naturally, the leader may want to choose a different quantity allocation if the demand is high than if it is low. Let $q^{Lh}$ and $q^{Ll}$ denote the leaders's quantity allocation choices when the demand state is high and low, respectively. The leader will choose $q^{Lk}$ to solve

$$\Pi^{Lk} = \max_{q^{Lk}} \sum_{i \in \{B,E\}} \left[\delta^k - c - \gamma_{ii} Q_i - \gamma_{ij} Q_j\right] q_i^{Lk}$$

for $k \in \{h, l\}$. The follower should anticipate that the leader will tailor its quantity allocation according to the demand state and solve

$$\Pi^F = \max_{q^F} \alpha \left[\sum_{i \in \{B,E\}} (P_i^h - c) q_i^F\right] + (1-\alpha) \left[\sum_{i \in \{B,E\}} (P_i^l - c) q_i^F\right]$$

so as to maximize expected profit, where $P_i^k = \delta^k - \gamma_{ii} Q_i - \gamma_{ij} Q_j$ is the price of the variety $i \in \{B, E\}$ when the demand state is $k \in \{h, l\}$.

The first order conditions to these optimization problems are

$$\begin{aligned} \frac{\partial \Pi^{Lk}}{\partial q_i^{Lk}} &= \delta^j - c - \gamma_{ii} Q_i - \gamma_{ij} Q_j - \gamma_{ii} q_i^{Lk} - \gamma_{ji} q_j^{Lk} = 0, \text{ and} \\ \frac{\partial \Pi^F}{\partial q_i^F} &= \mu - c - \gamma_{ii} E[Q_i] - \gamma_{ij} E[Q_j] - \gamma_{ii} q_i^F - \gamma_{ji} q_j^F = 0 \end{aligned}$$

where $E[Q_i] = q_i^F + E[q_i^L] = q_i^F + \alpha q_i^{Lh} + (1-\alpha) q_i^{Ll}$ and $\mu = \alpha\delta^h + (1-\alpha)\delta^l$. The first order conditions of the firms determine their reaction functions. By solving the reaction functions of the firms simultaneously, we get the Bayesian-Nash equilibrium outcome, which is summarized in the following proposition.

**Proposition 3** *When the firms engage in Cournot competition under incomplete information, the unique Bayesian-Nash equilibrium strategies of the firms in each market, the market prices, and the firm profits are*

$$q_i^{Lk} = \tfrac{\mu-c}{2} f_j + (\delta^k - \mu) h_j \quad \text{and} \quad q_i^F = \tfrac{\mu-c}{2} f_j,$$

$$\Pi^{Lk} = \tfrac{(\delta^k-c)(\mu-c)}{2}\left[\tfrac{f_1+f_2}{3}\right] \quad \text{and} \quad \Pi^F = \tfrac{(\mu-c)^2}{2}\left[\tfrac{f_1+f_2}{3}\right],$$

$$P_i^k = c[\gamma_{ii} f_j + \gamma_{ij} f_i] + \delta^k[1 - \gamma_{ii} f_j - \gamma_{ij} f_i] + \mu[\gamma_{ii}(h_j - f_j) + \gamma_{ij}(h_i - f_i)]$$

*where* $k \in \{h, l\}$ *and*

$$h_j = \frac{2\gamma_{jj} - \gamma_{ij} - \gamma_{ji}}{4\gamma_{ii}\gamma_{jj} - (\gamma_{ij} + \gamma_{ji})^2}.$$

Proposition 3 shows that both firms prefer to supply both varieties, however, the leader provides different quantities for different varieties, whereas the follower smooths out its quantity allocation between the varieties. The leader takes advantage of the information asymmetry by supplying more whenever the demand is high and less whenever it is low. Notice that the expected quantity (i.e., $E[q_i^L] = \alpha q_i^{Lh} + (1-\alpha) q_i^{Ll}$) that the leader provides for a variety is equal to the quantity provided by the follower for the same variety. Thus, the leader and the follower earn the same profit in expectation. However, the existence of asymmetric information leads to sub-optimal quantity choices for both firms. By comparing the prices for two varieties, we can also conclude that the intuition in Corollary 2 is no more valid when one of the firms has superior knowledge about the demand state. In the presence of information asymmetry, not only the cross price effects but also the own price effects play an important role in determining the difference between the price levels of the varieties.

### Stackelberg Competition with Incomplete Information

We now consider the effects of multi variety on the competition under incomplete information when the firms choose their quantity allocations sequentially. As in the previous section, we relax the symmetric information assumption and consider a Stackelberg competition model with the inverse demand given by (2). We maintain all the modeling assumptions in the previous section but change the timing of the model. Technically, this setting is a signalling game in which the (informed) leader signals the demand state to the (uninformed) follower by choosing its quantity allocation. Due to the nature of the game, we are interested in the perfect Bayesian equilibria. As common in games of imperfect information, we focus on separating and pooling equilibria. In this game, the leader has an incentive to mislead the follower about the state of demand only when the true demand is high. Thus, it is reasonable to expect that, in a separating equilibrium, for some set of parameters the low type leader will sacrifice some profit for separation. Similarly, in a pooling equilibrium, the leader will possibly prefer to decrease the competitiveness of the follower by maintaining information asymmetry at the cost of losing sales when the demand is high.

Because the leader moves first and knows whether the demand is high or low, the leader's strategy choice generates a signal about the true state of the demand. A strategy $q^L = (q_1^L, q_2^L)$ for the leader specifies a quantity allocation for each possible level of $\tilde{\delta}$. A strategy for the follower specifies a quantity allocation in response to the leader's allocation choice. Because the follower does not know the true intercept $\tilde{\delta}$, the follower must form some conjectures (or beliefs) about $\tilde{\delta}$ on the basis of the leader's choice of quantity allocation. Define point beliefs $\tilde{\delta} = b(q^L)$, which assigns a unique type of the leader (level of intercept) to each quantity allocation choice of the leader. Given the beliefs $b(\cdot)$, the expected profit for the follower when the leader and the follower chose allocations $q^L$ and $q^F$, respectively, is

$$\Pi^F = \sum_{i \in \{B,E\}} (P_i - c) q_i^F = \sum_{i \in \{B,E\}} \left[ b(q^L) - c - \gamma_{ii} Q_i - \gamma_{ij} Q_j \right] q_i^F. \tag{2}$$

Expected profit for the leader, who knows that the intercept of the demand is $\tilde{\delta}$, chooses the allocation $q^L$, and takes as the given strategy $q^{F*}$ of the entrant, is

$$\Pi^L = \sum_{i\in\{B,E\}} (P_i - c)q_i^L = \sum_{i\in\{B,E\}} \left[\tilde{\delta} - c - \gamma_{ii}(q_i^L + q_i^{F*}) - \gamma_{ij}(q_j^L + q_j^{F*})\right] q_i^L. \tag{3}$$

As common in Bayesian games, there are multiple perfect Bayesian equilibria. We focus on two types of equilibria: (i) separating equilibrium and (ii) pooling equilibrium.

**Separating Equilibrium.** In a separating equilibrium, the leader chooses a distinct allocation in each demand state. The allocation decision of the leader shapes the follower's inferences about the underlying demand. In the equilibrium, the follower correctly infers the true state of the demand from the leader's choice of allocation.

We can construct a candidate separating equilibrium as follows. Consider first the decision problem of the follower, which is given by (2). Because the follower infers the true state of the demand in a separating equilibrium and the leader knows that the follower can infer the true state of the demand, the firms will make their decisions as if they are engaged in a complete information Stackelberg competition. However, in order to ensure the existence of the separating equilibrium, the equilibrium profits have to satisfy individual rationality and incentive compatibility constraints.

As before, let $q^{Lh}$ and $q^{Ll}$ be the quantity allocation of the leader when the demand intercept is high and low, respectively. Similarly, let $q^{Fh}$ and $q^{Fl}$ be the quantity allocation of the follower when the demand intercept is high and low, respectively. Because in a separating equilibrium the follower correctly infers the true state of the demand, the first order conditions for the follower's problem, which are derived from the optimization problem in (2), are

$$\frac{\partial \Pi^{Fk}}{\partial q_i^{Fk}} = \delta^k - c - \gamma_{ii}Q_i - \gamma_{ij}Q_j - \gamma_{ii}q_i^{Fk} - \gamma_{ji}q_j^F = 0$$

when the demand is $k \in \{h, l\}$. Let $q^{Fk*}$ be the reaction function of the follower, which solves the first order conditions of the follower's problem. Then, the $k$-type leader's optimal decision must emerge as a solution to the optimization program

$$\Pi^{Lk} = \max_{q^{Lk}} \sum_{i \in \{B,E\}} \left[\delta^k - c - \gamma_{ii}(q_i^{Lk} + q_i^{Fk*}) - \gamma_{ij}(q_j^{Lk} + q_j^{Fk*})\right] q_i^{Lk}$$

together with the non-negativity and incentive compatibility constraints. We first find the solution to the unconstrained case and later verify indeed this solution satisfies the incentive compatibility conditions, hence it is an equilibrium. By finding the first order conditions for the leader's problem and solving them simultaneously, we get the equilibrium strategies in Proposition 2.

The only remaining issue is to determine the conditions under which the allocations provided in Proposition 2 are indeed equilibrium allocations. To do that, we need to ensure that these strategies satisfy the incentive compatibility and non-negativity constraints. Because $g_j > 0$ and $h_j > 0$, the solutions are all non-negative. Thus, we just need to consider the incentive compatibility constraints, which are shown to be equivalent to the following inequalities after a tedious algebra

$$\frac{\delta^h - c}{\delta^l - c} \geq 1 + \bar{\gamma} \geq \frac{\delta^l - c}{\delta^h - c}$$

where $1 + \bar{\gamma} = 2\frac{g_1+g_2}{h_1+h_2}$. While the inequality on the left ensures that the high type leader does not find profitable to mimic the low type, the one on the right ensures the opposite. Notice that the inequality on the right is never binding because $\delta^k > c$ for $k \in \{h, l\}$. Thus, the low type leader never finds it profitable to mimic the high type.

A reasonable belief structure ought to satisfy the following intuitive essentials in order to support the equilibrium. If the quantity choices of the leader were "high enough" in both markets, the follower ought to infer that the demand state is high; similarly, if the quantity choices of the leader were "low enough" in both markets, the follower would infer that the demand state is low. The leaders allocation strategy increases in the probability the leader ascribes to high demand.

The above formulation embeds the notion of a perfect Bayesian-Nash equilibrium in the following sense: (i) the leader's actions (the optimal order quantities) are a best response to what the leader knows at that point (the realized demand state), what the follower optimizes, and to the leader's own conjecture on the follower's beliefs. (ii) the follower's optimization is in turn a best response to what the follower knows at that stage (the leader's order quantity) and the follower's beliefs on the actual demand state. (iii) the follower's actual beliefs and the leader's conjectures on the follower's beliefs coincide.

**Proposition 4** *When the firms engage in Stackelberg competition under incomplete information, there exist a separating equilibrium if* $\frac{\delta^h - c}{\delta^l - c} \geq 1 + \bar{\gamma}$ *and the firms play*

$$q_i^{Lk} = \tfrac{\delta^k - c}{2} g_j \quad \textit{and} \quad q_i^{Fk} = \tfrac{\delta^k - c}{2} h_j,$$

$$Pr(b(q^L) = \delta^h) = \begin{cases} 1, & q^L \geq q^{Ll}; \\ 0, & \textit{otherwise.} \end{cases}$$

*where* $k \in \{h, l\}$ *and* $Pr(b(q^L) = \delta^h)$ *is the follower's belief that the leader is the high type.*[11] *In this equilibrium, the market prices and the profits of the firms are*

$$\Pi^{Lk} = \tfrac{(\delta^k - c)^2}{2}[\tfrac{h_1 + h_2}{2}] \quad \textit{and} \quad \Pi^{Fk} = \tfrac{(\delta^k - c)^2}{2}[\tfrac{h_1 + h_2}{4}],$$

$$P_i^k = c[\gamma_{ii}\tfrac{g_j + h_j}{2} + \gamma_{ij}\tfrac{g_i + h_i}{2}] + \delta^k[1 - \gamma_{ii}\tfrac{g_j + h_j}{2} - \gamma_{ij}\tfrac{g_i + h_i}{2}].$$

Because the information is fully revealed in a separating equilibrium, the parties achieve the full information game outcome. However, this equilibrium outcome has to deter the high type leader to mimic the low type. The high type leader would not mimic the low type only if the asymmetry about the market demand (which is measured by $\frac{\delta^h - c}{\delta^l - c}$) is large enough, or in other words, when the information valuation is high for the leader. In the equilibrium, the follower believes that the demand is high whenever the follower observes that the leader chooses a quantity greater than $q^{Ll}$, which is the amount that the leader would choose if

[11] We define $q^L \geq q^{Lh}$ as $q_i^L \geq q_i^{Lh}$ for all $i \in \{1, 2\}$ and $q_i^L > q_i^{Lh}$ for at least one of the varieties.

the demand was low. So, the follower plays safe and chooses to be an aggressive competitor rather than a soft one whenever the demand state is still ambiguous after observing the leader's signal.

**Pooling Equilibrium.** In a pooling equilibrium, the leader chooses the same allocation at each demand state so that the follower cannot infer the underlying demand from the allocation decision of the leader. On the negative side, the high type leader incurs a cost of loss sales since the high type leader has to be less aggressive competitor in order to pool. On the positive side, the high type leader receives the benefit of competing with a less aggressive follower, which increases the profit of the leader.

A candidate pooling equilibrium can be constructed as follows. Consider first the decision problem of the follower, which is given by (2). Because the leader's strategy does not reveal any information to the follower, the first order conditions for the follower's problem are

$$\frac{\partial \Pi^F}{\partial q_i^F} = \mu - c - \gamma_{ii} E[Q_i] - \gamma_{ij} E[Q_j] - \gamma_{ii} q_i^F - \gamma_{ji} q_j^F = 0$$

for $i \in \{B, E\}$. The solution to these first order conditions determines the reaction functions of the follower for each variety. Let $q^{E*} = (q_1^{E*}, q_2^{E*})$ be the reaction function of the follower. In a pooling equilibrium, both types of the leader must play the same strategy so that the follower cannot infer the true demand state from the leader's strategy. The low type leader does not have an incentive to mimic the high type. Because the high type must play the same strategy as the low type, the leader's optimal decision must emerge as a solution to

$$\Pi^{Lk} = \max_{q^L} \sum_{i \in \{B,E\}} \left[ P_i^k(q^{Ll}, q^{E*}) - c \right] q_i^{Ll} \tag{4}$$

together with the non-negativity and incentive compatibility constraints. As in the previous section, we first find the solution to the unconstrained case and later verify that the solution satisfies the incentive compatibility conditions, hence it is an equilibrium. By deriving the first order conditions for the leader's problem and solving them simultaneously, we find a candidate for pooling equilibrium, which is summarized in Proposition 5 below.

It is straightforward to show that the solutions satisfy the non-negativity conditions. It is also immediate that the low type does not have any incentives to mimic the high type since doing so would result in a more aggressive follower. So, we only need to consider the incentive compatibility constraint of the high type leader, which is shown to be equivalent to the following inequality after a tedious algebra

$$\frac{\delta^h - c}{\delta^l - c} \leq 1 + \hat{\gamma} \tag{5}$$

where $\hat{\gamma} = \frac{\bar{\gamma}-(1-\alpha)^2}{\alpha\bar{\gamma}+(1-\alpha^2)}$. This inequality ensures that the high type leader finds it attractive to mimic the low type. The leader prefers to mimic the low type only if the information asymmetry about the market demand is low.

**Proposition 5** *When the firms engage in Stackelberg competition under incomplete information, there exist a pooling equilibrium if* $\frac{\delta^h-c}{\delta^l-c} \leq 1+\hat{\gamma}$ *and the firms play*

$$q_i^{Lk} = \tfrac{\delta^l-c}{2}g_j - \tfrac{\mu-\delta^l}{2}[g_j + g_{ij}] \quad \text{and} \quad q_i^F = \tfrac{\mu-c}{2}h_j + \tfrac{\mu-\delta^l}{2}[g_j + g_{ij}]$$

$$Pr(b(q^L) = \delta^h) = \begin{cases} 1, & q^L \geq q^{Ll}; \\ \alpha, & q^L = q^{Ll}; \\ 0, & otherwise. \end{cases}$$

*where* $k \in \{h, l\}$ *and* $Pr(b(q^L) = \delta^h)$ *is the follower's belief that the leader is the high type. In this equilibrium, the market prices and the profits of the firms are*

$$\Pi^{Ll} = \tfrac{(\mu-c)^2}{2}[\tfrac{h_1+h_2}{2}] - \alpha(\delta^l - c)\tfrac{\delta^h-\delta^l}{2}[g_1+g_2],$$

$$\Pi^{Lh} = \tfrac{(\mu-c)^2}{2}[\tfrac{h_1+h_2}{2}] + [\alpha(\delta^h - c) - (\delta^l - c)]\tfrac{\delta^h-\delta^l}{2}[g_1+g_2],$$

$$\Pi^{Fk} = \tfrac{(\mu-c)^2}{2}[\tfrac{h_1+h_2}{4}] + \alpha(\delta^l - c)\tfrac{\delta^h-\delta^l}{2}[g_1+g_2] + [\alpha\tfrac{\delta^h-\delta^l}{2}]^2[g_1+g_2],$$

$$P_i^k = c[\gamma_{ii}\tfrac{g_j+h_j}{2} + \gamma_{ij}\tfrac{g_i+h_i}{2}] + \delta^k[1 - \gamma_{ii}\tfrac{g_j}{2} - \gamma_{ij}\tfrac{g_i}{2}] + \mu[1 - \gamma_{ii}\tfrac{h_j}{2} - \gamma_{ij}\tfrac{h_i}{2}].$$

In a pooling equilibrium, the leader chooses the same allocation under either demand state and the follower cannot infer any demand information from the allocation choice of the leader. On one hand, the high type leader incurs a loss in profit since the high type leader has to supply less amount of products for both variety in order to mimic the low type. However, because no information revelation takes place when the high type leader mimics the low type, the high type leader gains a surplus since the follower becomes less aggressive competitor in the high demand state. The gains from having a less aggressive competitor is higher compared to profit losses due to loss sales when the information asymmetry is small. Thus, the high type leader prefer not to mimic the low type only if there is a big enough increase in demand from the low demand state to the high demand state.

As in the Cournot case, by comparing the prices for two varieties, we can conclude that the intuition in Corollary 3 is also no more valid when one of the firms has superior knowledge about the demand state. In the presence of information asymmetry, not only the cross price effects but also the own price effects play an important role in determining the gap between the market prices.

## Conclusion

We extend the standard quantity competition models of duopoly by allowing firms to compete in two varieties of a homogenous product simultaneously. In particular, we consider both Cournot and Stackelberg models under complete and incomplete information assumptions and characterize the equilibrium outcomes. Our findings shed light to the conflicting results in the recent literature by characterizing the conditions under which both of the duopoly firms practice price discrimination. Furthermore, we show that the order of move is not a crucial element of the equilibrium market price differences. What matters is the existence of consumers who care only about the differences in prices and are otherwise indifferent between the varieties.

This paper also contributes to the line of research on equilibrium price dispersion. In this literature, Dana (1999) extends the equilibrium price dispersion model of Prescott (1975) to monopoly and imperfect competition, and he finds that demand uncertainty and the perishable nature of the assets are sufficient for a firm to price discriminate. We show that unless the cross price effect is one sided and the market leadership is determined, the duopoly firms choose to operate in both markets if they are competing in quantities. Thus, we can conclude that competition in markets with asymmetric cross price effects and demand uncertainty is sufficient for firms to price discriminate.

## References


Anand, K. S. and Goyal, M. (2009), "Strategic Information Management Under Leakage in a Supply Chain", *Management Science*, Vol. 55 (3), 438-452

Blattberg, R. C. and Wisniewski, K. J. (1989), "Price Induced Patterns of Competition", *Marketing Science*, 8, Fall, 291-309

Blattberg, R. C., Briesch, R., and Fox, E. (1995) "How Promotions Work", *Marketing Science*, 14, 3, G122-G132

Sethuraman, R., Srinivasan, V., and Kim, D. (1999), "Asymmetric and Neighborhood Cross-Price Effects: Some Empirical Generalizations", *Marketing Science*, 18, 1, 23-41

Borenstein, S. (1985), "Price Discrimination in Free-Entry Markets", *The RAND Journal of Economics*, 16, 380-397

Corts, K. (1998), "Third-degree Price Discrimination in Oligopoly: All-out Competition and Strategic Commitment", *The RAND Journal of Economics*, 29, 306-323

Dana, J.D, Jr. (1999), "Equilibrium Price Dispersion under Demand Uncertainty: The Roles of Costly Capacity and Market Structure", *The RAND Journal of Economics*, 30, 4, 632-660

Daughety, A.F. and Reinganum, J.F. (1994), "Asymmetric Information Acquisition and Behavior in Role Choice Models: An Endogenously Generated Signaling Game", *International Economic Review*, 35, 4

Hazledine, T. (2006), “Price Discrimination in Cournot-Nash Oligopoly”, *Economics Letters*, 93, 413-420

Holmes, T. J. (1989), “The Effects of Third-Degree Price Discrimination in Oligopoly”, *American Economic Review*, 79, 244-250

Kutlu, L. (2009), “Price Discrimination in Stackelberg Competition”, *Journal of Industrial Economics*, 57, 2, 364-364

Prescott, E.C. (1975), “Efficiency of the Natural Rate”, *Journal of Political Economy*, 83, 4, 1229-1236

Stole, L.A. (2007), *Price Discrimination and Competition*, M. Armstrong and R. Porter (ed.), Handbook of Industrial Organization, vol. 3, Elsevier, chapter 34, 2221-2299