ENGAGING CHILDREN IN TALK ABOUT MATHEMATICS: THE EFFECTS OF AN EARLY MATHEMATICS INTERVENTION

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To my sons:

Harrison, Taylor, Ellis, Isaiah, and Zavier.

I started because of you; I finished because of you.

I love you.

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INTRODUCTION

In the United States, there is a well-documented gap in mathematics performance between students from different economic backgrounds favoring students who do not come from economically disadvantaged homes (Coleman, 1966; Rampey, Dion, & Donahue, 2009). This gap is particularly relevant because of the growing importance of quantitative literacy to attaining a postsecondary education and future economic prosperity (Moses & Cobb, 2001; National Research Council, 1989, Riley, 1997). The mathematics performance gap between students from economically disadvantaged backgrounds and their more advantaged counterparts is evident at the national level, school level, and even before children start school. The mathematics knowledge students bring to school has important implications for their overall academic trajectories, favoring those with more competence. If the majority of students beginning school with less math knowledge come from low income homes, then a disproportionate number of these students will spend their K-12 career falling further behind their more advantaged peers. Perhaps for this reason, increased emphasis has been placed on improving the early math skills of children before they enter kindergarten.

Currently, large scale preschool programs predominately serving children from economically disadvantaged backgrounds are not successfully narrowing the observed performance gaps between this group and their peers (US-DHHS, 2010). There are several challenges to improving the effects of these programs. One challenge is the very small amount of time allocated to mathematics instruction in typical preschool classrooms (Winton & Buysse, 2005), despite this being an important correlate of early mathematics learning (Bodovski & Farkas, 2007). Another challenge to improving math outcomes is teacher preparedness for delivering math instruction, another important correlate of student math learning (Hill, Rowen & Ball, 2005). Yet, when surveyed, preschool teachers report feeling uncomfortable with mathematics and knowing little about the standards (Copley, 2004), acknowledge not knowing how to develop children's numeracy (Farran, Silveri, & Culp, 1991), and rank themselves as just mediocre in teaching math (Arnold, Fisher, Doctoroff, & Dobbs, 2002). In response to the need to better provide practitioners with the supports necessary to improve early childhood mathematics, early intervention research should focus on identifying strategies that are specific to early learning environments as well as to the needs of children from economically disadvantaged backgrounds.

Despite the early development of certain fundamental mathematical ideas among children of all backgrounds, children from low income backgrounds appear to have greater difficulty engaging in the formal math curriculum introduced in school (Griffin, Case & Siegler, 1994; Resnick, 1989). Children from all backgrounds engage in mathematics before they start school by inventing ideas and strategies about shape and number based on their early experiences (Baroody, 1987; Resnick, 1989). These informal mathematical understandings facilitate or hinder children's ability to engage in the formal curriculum introduced in school (Baroody, 1987; Clements, 2004; Griffin et al., 1994; Resnick, 1989). One hypothesis to explain observed differences in children's ability to connect their informal ideas about mathematics with the formal math curriculum is based on the language experiences children have before they begin school.

There is ample evidence to suggest that children from economically disadvantaged backgrounds often use language differently than how it is used by teachers, schools (Tough, 1982), and in math instruction (Orr, 1997). These differences in language usage lead to different mathematical representations and understanding (Orr, 1997; Resnick, 1989). There is further evidence that the difference in math ability between children from different economic backgrounds is not evident in non-verbal calculation tasks, but is very apparent in verbal tasks of reasoning (Jordan, Huttenlocher, and Levine, 1992). It is possible, then, the mathematics performance gap between students from different economic backgrounds might be addressed by in-school experiences that seek to provide young children with opportunities to use language in ways that better support their learning at school.

The relationship between language and learning has its roots in the sociocultural perspective (Vygotsky, 1978) and undergirds the rationale to develop children's math competencies through talk (National Council of Teachers of Mathematics, 2000). Language development and conceptual development are interdependent processes; development in one area facilitates development of the other (Carey, 2004). What is less understood is the process by which to guide young children's participation in the conversation about mathematics that leads to their mathematical language and conceptual development.

Math talk is often described as one process by which to guide children's participation in mathematical activity using language in ways that builds conceptual understanding. This study conceptualizes a math talk learning environment as an instructional process situated at the intersection of math content, social activity, and instruction. This study will focus the content of the math talk learning environment on number sense development because early number sense is the most important predictor of children's later success in elementary school mathematics

(Howell & Kemp, 2010; Jordan, Glutting, & Ramineni, 2010). The social activity of the math talk learning environment is based in playing games to generate student interest and motivation to focus on the math content (Griffin et al., 1994; Siegler, 2009). The instructional strategy to foster the math talk learning environment is guided participation in children's use of math language and ways of reasoning. Thus, for this study, a math talk learning environment is defined as guiding children's participation (instruction) in games (social activity) focused on number sense (content).

While math talk is grounded theoretically and supported by several national organizations, several assumptions have been made concerning its usefulness in preschool. The first assumption is that practitioners know what math talk is when, in fact, very little guidance exists on defining its principal components. The second assumption is that practitioners are familiar with strategies for developing children's math talk, particularly among children who have little or no experience with such language and ways of reasoning. Most resources on math talk focus on children in older grades and few provide instructions on how to develop this skill in preschool. The third assumption is that student participation in math talk will improve student learning. Two studies demonstrate evidence that talk is important to children's math development; however, neither study was specific to the direct effects of children using math talk. This study sought to examine these assumptions in an effort to inform classroom practices that might lead to an effective application of the math talk principle in the preschool classroom.

Data were collected from the field to explore strategies of engaging preschool children from economically disadvantaged backgrounds in a math talk learning activity. Recommendations from this exploratory work provide early childhood educators with specific descriptions, examples, and demonstrations illustrating the principles of math talk. Finally, a math talk intervention strategy was designed to test whether the components identified by this review lead to improved mathematics performance among preschool children from economically disadvantaged backgrounds.

Objectives

The objective of this study was to test the effects of a math talk intervention on the academic performance of preschool children from economically disadvantaged backgrounds. First, the study investigated whether children who engaged in math games (social interaction focused on developing number sense) would learn more math and reasoning skills than similar children who did not engage in these games. Second, this study examined whether there would be an additional learning benefit to children who engaged in math games with a focus on talk (guided participation in the use of language and ways of reasoning) to the playing of math games. This research sought to provide early childhood practitioners with specific strategies for improving the early math skills of children in programs serving economically disadvantaged populations. In addition, this research hopes to offer the early childhood research community empirical evidence for the use of math talk in preschool.

CHAPTER II

REVIEW OF THE LITERATURE

Background

Two nationally representative longitudinal studies conducted by the National Center for Education Statistics show the importance of mathematics achievement to students' future outcomes. Data from the National Education Longitudinal Study (NELS) reveal that 83% of students who took algebra I and geometry, compared to only 36% of students who did not take these courses, went on to college within two years of graduation (Riley, 1997). Data from the High School and Beyond (HS&B) study showed the highest level of math taken while in high school explained nearly 25% of the earnings gap ten years after high school graduation between those who grew up in low income and middle income homes (Rose and Betts, 2004). Students with a strong grasp of mathematics have an advantage in academics and in the job market. Yet, the gap in mathematics performance between students from economically advantaged and disadvantaged backgrounds is large in size and has remained relatively unchanged for decades.

The disparity between groups has persisted despite campaigns to improve pre-collegiate math and science for all students, particularly students from lower income homes who are typically underrepresented in those professional fields (see the United States federal statutes *Elementary and Secondary Education Act* and *No Child Left Behind Act*, and the National Science Foundation's *Educating Americans for the Twenty-First Century*). Ever since the Coleman Report in 1966, researchers have recognized that the scores of students from economically disadvantaged homes lag well behind those from more affluent backgrounds (Coleman, 1966). Almost forty years later, this trend is still evident across the K-12 years.

The mathematics performance gap is observable at the national level, school level, and even before children start school. Starting with the oldest children and working backwards across the grades, the next sections summarize research demonstrating the performance gap associated with family income as well as the relationship between students' later achievement and earlier mathematics performance.

The Mathematics Performance Gap at the National Level

National assessments taken in representative samples of fourth, eighth, and twelfth grade students show great variation in the performance between groups differentiated by parental income, in favor of students from more economically advantaged backgrounds (USDE, 2010). The National Assessment of Educational Progress (NAEP) is the only nationally representative and continuing assessment of student achievement in various subject areas. Results from the mathematics assessments administered over time show the size of the mathematics gap associated with family income is similar across all three grade levels tested (USDE, 2010). The gap size is equivalent to the number of points needed to move a student from one achievement level to the next higher level. In other words, the difference between groups in the number of points scored on the exam is enough to move a student who scored "basic" to "proficient" or from "proficient" to "advanced."

Children from families qualified for free and reduced lunch (the poverty criterion) are also differentiated by more than a standard deviation on the NAEP mathematics assessment from children of higher income families. Students from all backgrounds currently take an increased number of high school math credits and advanced placement courses than students in school in 1984 (USDE, 2007). Also fourth and eighth grade students from all income groups are scoring higher on the NAEP mathematics assessment than similar students in 1978 (twelfth grade students' scores remain the same as in 1978) (Rampey et al., 2009). The relative performance on the NAEP Mathematics Assessment between students from economically disadvantaged backgrounds and their peers, however, remains as large as it was more than a decade ago when NAEP first began collecting information on students' eligibility for the national free-and-reduced lunch program (USDE, 2010).

The Mathematics Performance Gap at the School Level

The observed performance gap between students from different economic backgrounds can be linked across their K-12 career. Student performance in mathematics at the end of high school is significantly related to the highest level of math coursework taken in high school (Rampey et al., 2009) and taking advanced mathematics courses in high school matters more to later student achievement than taking advanced or regular courses of another subject matter (Gamoran, 1987). Using the NELS dataset, Ingels and Dalton (2008) examined the mathematics performance gap across four income levels instead of the two used to report NAEP results (free and reduced lunch versus all others). They found that seniors from the highest income quartile took advanced mathematics courses at higher rates than seniors from lower income groups. In the 2004 cohort alone, forty percent of seniors from the highest income quartile enrolled in advanced mathematics courses compared to 14 percent of seniors from the lowest quartile and 21 percent of seniors from the middle two quartiles.

The number of high school math courses students take is significantly related to their middle school coursework and differs by students' economic backgrounds. Algebra has long been dubbed "the gatekeeper course" because taking it by the eighth grade puts students on track for a sequence of higher-level math classes important for qualifying them for post secondary work in science and math (Oakes, Ormseth, Bell, & Camp, 1990). In fact, 60% of the students who took calculus in high school had taken algebra in the 8^{th} grade (Riley, 1997). Yet, almost two times the number of students from economically advantaged homes enroll in middle school algebra than do students from economically disadvantaged homes. Using data from the NAEP, Shakrani (1996) showed that substantially twice as many of the former (33%) than the latter (15%) were taking eighth grade algebra.

The likelihood of being enrolled in eighth grade algebra is associated with students' math performance at the end of fifth grade. The Early Childhood Longitudinal Study, Kindergarten Class of 1998-1999 (ECLS-K) was funded to follow a nationally representative cohort of students from the start of their kindergarten year through the eighth grade. In a brief focused on the experiences of a cohort of children who progressed on schedule from first through eighth grade in schools located within the United States (approximately 80% of the original cohort), Walston and McCarroll (2010) reported on students' mathematics course enrollment. They found nearly four in 10 of the students were enrolled either in algebra or a higher mathematics course in the eighth grade, and those students taking more advanced math courses scored the highest on the ECLS-K mathematics assessment. Conversely, students who were enrolled in basic eighth grade math scored the lowest on the assessment and would thereby not be on track for the higher math courses in high school.

Students’ math achievement at the end of the fifth grade differs by family income and is due in part to differing educational experiences in elementary school. The educational experiences of students from economically disadvantaged backgrounds differ in important ways from those of their more advantaged peers, ways that are believed to be important contributors to their math performance at the end of elementary school. Using cross sectional data obtained through the National Science Foundation’s 1985 – 1986 National Survey of Science and Mathematics Education, Oakes et al. (1990) reported that children attending schools in economically disadvantaged neighborhoods had less extensive and less demanding programs available to them. High ability students in these schools had fewer opportunities, fewer resources, and fewer qualified teachers in mathematics than low-ability students in schools in economically advantaged neighborhoods (Oakes et al., 1990). Children in schools predominately serving children from economically disadvantaged homes, then, are less likely to have the access to enroll or the preparation to succeed in eighth grade algebra.

The Mathematics Performance Gap Before Children Start School

The mathematics performance gap between children from different economic backgrounds does not first appear in school, but is evident at school entry and has important implications for their overall academic achievement. West, Denton, and Hausken (2000) reported on the first year’s findings from the ECLS-K study in which children were assessed in the fall of their kindergarten year. The mathematics assessment consisted of five levels of increasing difficulty measuring children's procedural skills and conceptual knowledge. Regardless of income level, ninety-four percent of all first time kindergartners passed level one; virtually all children were capable of reading numerals, recognizing shapes, and counting to ten. The mathematics performance gap appeared at higher levels between children whose families did or did not receive welfare. Children from families not receiving welfare were also able to sequence numbers, name ordinal positions, and solve word problems. These findings suggest that, at this age, children's math knowledge may be relatively equal between groups in some mathematical content areas and more variable in others.

Implications of Poor Mathematics Knowledge at School Entry

Children's mathematics performance at school entry is linked to later mathematics achievement and places children on different learning trajectories, which has important implications for lower performing children who are generally from economically disadvantaged homes. Aunola, Leskinen, Lerkkanen, and Nurmi (2004) found Finnish children's math knowledge at school entry predicted their achievement at the end of the second grade. Bodovski and Farkas (2007) found the same relationship through the end of third grade for a nationally representative sample of American children. Moreover, both studies found children's learning trajectories differed by their math knowledge at school entry. Children with higher math performance when they started school learned more math over time while children who started school with less knowledge fell further behind. As the ECLS-K study demonstrated, children from economically disadvantaged backgrounds are far more likely to enter school with fewer early math skills than their more advantaged peers.

Early mathematics ability predicts more than later math skills and does more than place children on a higher or lower learning trajectory; early math appears to be the best predictor of later overall learning. A key study by Duncan et al. (2007) highlighted the importance of early math skills. Using data from six national datasets, Duncan et al. (2007) showed that, from among the significant predictors of later achievement, children's rudimentary mathematics skills mattered more than did early literacy skills or attentional skills (socioemotional behaviors and social skills were not significant predictors of later achievement).

Perhaps it is because of this pattern (whereby children who know more upon school entry, learn more) and the strong relationship between early math skills and later academic outcomes that many politicians and advocates of early childhood education are placing greater emphasis on early childhood outcomes, to address the performance gap when the gap is at its narrowest point. For example, in a speech to the Hispanic Chamber of Commerce in March of 2009, President Obama said that investing in early childhood initiatives would be the "first pillar" (p. 2) of reforming our schools:

> Studies show that children in early childhood education programs are more likely to score higher in reading and math; more likely to graduate from high school and attend college; more likely to hold a job and more likely to earn more in that job. For every dollar we invest in these programs, we get nearly $10 back in reduced welfare rolls, fewer health care costs, and less crime. (p. 2-3)

Comments like this one from the President draw on conclusions from earlier, smaller studies; there is surprisingly little evidence that the scale up of early childhood programs is making any sustainable effects on the academic achievement of children from economically disadvantaged backgrounds. Researchers acknowledge a closer investigation of the challenges to improving early math instruction was warranted.

Summary

Representative national studies conducted by the Department of Education all confirm a large and persistent achievement gap in mathematics between children whose families differ by income. Longitudinal studies demonstrate the relationship between postsecondary income earning and the highest math course taken, highest math course taken and taking algebra by the eighth grade, eighth grade algebra enrollment and fifth grade math achievement, and elementary math achievement with children's math skills when they enter kindergarten. Furthermore, these studies consistently show how these relationships differ by family income favoring students from more advantaged economic backgrounds. The patterns illuminated here have two important implications. The first implication is that children with poor math skills at school entry are disproportionately represented by children from economically disadvantaged backgrounds. The second implication is that school further disadvantages these children by placing them on a lower educational learning trajectory over the K-12 years. It is likely due to these implications that the academic outcomes of early childhood programs have received much attention, and challenges to addressing early childhood mathematics education have been closely investigated.

Current Challenges to Improving Early Childhood Mathematics Education

Findings from the first randomized study of the effects of Head Start have recently been released with depressing results. Head Start, created in 1965, is the largest national school readiness program in the United States serving children from families that meet 100% of the poverty level. The Head Start Impact Study (US-DHHS, 2010) was conducted with a nationally representative sample of 84 delegate agencies and included nearly 5,000 newly entering, eligible 3- and 4-year-old children who were randomly assigned to either a Head Start group that had access to Head Start program services or control group that did not have access to Head Start, but could enroll in other early childhood programs or non-Head Start services selected by their parents. Results from the study showed that Head Start children showed no sustainable effects through the end of first grade. In other words, by the end of first grade, there were no observable differences between children who did or did not participate in Head Start in their ability to excel in the formal curriculum introduced in elementary school. There are several challenges to improving the effects of programs like Head Start. When considering the challenges specific to the improvement of early childhood mathematics education, it is necessary to examine research from kindergarten and elementary school. The amount of time allocated for math instruction and teacher preparedness to engage children in mathematics are both necessary to achieve higher outcomes, but appear to be absent from today's typical preschool classrooms.

Time Spent in Instruction

Time in instruction has been significantly linked to students' math achievement in elementary and secondary grades (Barr & Dreeben, 1983; Brophy, 1986) as well as among kindergartners (Bodovski & Farkas, 2007). However, in preschool learning environments, very little time is dedicated to explicit instruction in any subject matter (Zill et al., 2001) and consequently little time is allocated specifically for mathematics instruction (Graham, Nash & Paul, 1997). Findings from the National Center for Early Development and Learning's Multi-State Study of Pre-K (NCEDL-MS) indicate that children were engaged in math activities for 6% of the school day (Winton & Buysse, 2005). The time in math instruction was less than the amount of time spent in gross motor (7%), science (8%), art/music (9%), social studies (13%), or literacy/writing (13%). For the remaining 44% of the time observed, children were not engaged in any learning activity.

When children are exposed to mathematics, it is typically integrated with other subjects, which has the possible effect of further watering down the direct effects of instruction. In response to these findings, the largest association of advocates of young children, the National

Association for the Education of Young Children (NAEYC), and the largest association of math educators, the National Council of Teachers of Mathematics (NCTM), issued a joint statement concerning early childhood mathematics instruction advocating for the explicit instruction of mathematics:

> In high-quality mathematics education for 3- to 6-year-old children, teachers and other key professional should… actively introduce mathematical concepts, methods, and language through a range of appropriate experiences and teaching strategies. (2002, p. 4)

Teacher Knowledge and Attitudes About Mathematics

Teachers' knowledge and attitudes have a great influence on children's math development. Hill et al. (2005) highlighted the critical importance of teachers' mathematical knowledge for teaching first and third grade mathematics and the relationship between that knowledge and student outcomes in mathematics. Teaching elementary school mathematics requires more than understanding mathematical content alone or knowing how to teach well alone. Hill et al. characterize *the work of teaching mathematics* as instruction that includes explaining concepts to students, interpreting students' responses, and facilitating group discussions about mathematics. The researchers developed an instrument to measure teachers' content knowledge for teaching mathematics. Teachers' scores on this measure predicted children's math achievement as strongly as children's background characteristics, such as socio-economic status. However, preschool teachers appear to be inadequately prepared to instruct mathematics (Lobman, Ryan, & McLaughlin, 2005). Primary school teachers with little content knowledge tend to depend on texts for content, de-emphasize verbal interactions about mathematical ideas, and overuse seatwork assignments (Brophy, 1991).

In addition to teacher knowledge about mathematics, teachers' attitudes toward mathematics have been linked to student achievement in mathematics. Beilock, Gunderson, Ramirez, and Levine (2010) showed that teachers of young children who possessed higher levels of mathematics anxiety may unintentionally pass on these negative feelings to their students. This is alarming when considering the "math anxiety" reported among preschool teachers. For example, many teachers reported being uncomfortable with mathematics and knowing little about the standards (Copley, 2004), reported not knowing how to develop children's numeracy (Farran et al., 1991), and ranked themselves as just mediocre in teaching math (Arnold et al., 2002).

To adequately prepare children from low income backgrounds for elementary school mathematics requires preparing preschool teachers with the knowledge, skills, and confidence to ground children's knowledge in the mathematical foundations required for future success. A case study of improving preschool math instruction chronicled one teacher's experiences as she learned to implement a new math curriculum (Ginsburg & Amit, 2008). Her experiences demonstrate teaching math to preschoolers is as multifaceted as teaching math in older grades. For example, she had to develop profound knowledge of the subject matter, alter her pedagogy in response to the changing needs and understandings of the children, and use multiple strategies to help children connect their everyday experience to abstract ideas. While teaching preschool mathematics might be as difficult as teaching mathematics in the older grades, the content and strategies used need to be specific to the developmental needs and understanding of preschool children.

The Need for Strategies Specific to Preschool

The Committee on Early Childhood Mathematics, NAEYC, NCTM, and education experts warn that prescribing an increase in time spent doing mathematics and an increased expectation of academic outcomes could cause many early childhood practitioners to lean towards inappropriate strategies imported from older grades. For example, Seo and Ginsburg (2004) represent the conflict in this way:

> We do not wish to pressure young children, to subject them to harsh forms of instruction, and to impose on them material they are not ready to learn. We do not want a "push-down curriculum" forcing young children to engage in developmentally inappropriate forms of written drill and practice in mathematics. Our desire to prepare children for school success (and to avoid school failure) thus clashes with our reluctance to impose inappropriate forms of teaching on young children. (p. 91)

In order to avoid the push down from older grades, the approach in early childhood must be carefully constructed around the skills and needs of younger children. In a paper where Farran laid out a prescription for the kinds of programs young children from poor families need, she argued, "Interventions for children with fragmented and disconnected early experiences must be organized and structured with parameters that children can grasp and depend on. Teachers must understand the necessity of creating enriching experiences that follow predictable patterns" (Farran, 2005, p. 279). For this reason, it is important to consider the specific mathematical skills and understandings very young children from economically disadvantaged backgrounds have formed before they come to school and how instruction might support later learning.

Summary

Due to the long standing performance gap in mathematics and the potential consequences for students who start school on the lower end of this gap, much attention has been drawn to programs that might help prepare children at risk for lower math achievement for the formal math curriculum. Research on large scale preschool programs serving children from economically disadvantaged backgrounds has not shown they have been effective in improving outcomes for their participants. An examination of elementary school research on important correlates of math achievement reveals that the time spent in instruction and teachers' knowledge and attitudes towards mathematics are critical to student achievement in mathematics. However, there are important challenges to improving early childhood mathematics that need to be addressed. In addition, answers to these challenges must be specific to the developmental needs of preschool children and to the particular population at risk of lower math achievement.

The Math Knowledge Children Come to School With

Children's basic understanding of counting, number, and arithmetic emerges before they begin attending school (Resnick, 1989; Song & Ginsburg, 1987). From birth to age five, young children develop informal ideas about number, geometry, and measurement (see Geary 1994 for a review). Children seem to acquire informal mathematics through such processes as spontaneous interactions with the environment, imitation of more capable peers or adults, and watching television (Song & Ginsburg, 1987). Formal mathematics represents the culturally constructed body of knowledge that children learn in school through interactions with teachers or more knowledgeable peers. There are several important challenges for all children to learning formal math, and various hypotheses explaining the differences observed in math performance by students' economic backgrounds. These next sections review the informal math knowledge most children come to school with, the challenges to learning the formal math they encounter in school, and specific challenges for children from economically disadvantaged backgrounds to connecting their informal mathematical knowledge and the formal mathematics curriculum they encounter in school.

Children's Informal Mathematics

The mathematical skills and strategies children formulate independent of instruction are referred to as informal mathematics, sometimes called "everyday" math (Song & Ginsburg, 1987). Through observation, experience, and everyday conversation, children develop and test theories about more and less, taking away, shape, size, location, pattern, and position (Ginburg, Lee, & Boyd, 2008). Children who have more opportunities to engage in informal mathematical tasks will develop a more flexible use of and concrete understanding of general math principles than will peers who have fewer opportunities. For example, helping to set the table for the correct number of family members or sharing cookies with a sibling so that each has the same amount are informal interactions about math that will help children form accurate ideas about number and quantity. Building towers with blocks or assisting an adult with measuring ingredients for a recipe aid children's conceptual understanding of comparisons and measurement. In the absence of experiences and discussions about number, quantity, shape, and/or measurement, children are less prepared to make important connections between their intuitive understanding about mathematical concepts and the symbols and rules presented in school (NAEYC/NCTM, 2002). Additionally, children may develop a sufficient wealth of informal mathematical knowledge that schools fail to connect with the formal mathematics curriculum (Clements, 2004; Resnick, 1989).

The Challenge of Formal Mathematics

Formal mathematical knowledge is taught in school (Geary, 1994; Resnick, 1989) and emphasizes written symbols, algorithms, and principles (Song & Ginsburg, 1987). Formal mathematics are usually those mathematical skills and concepts that children will not learn spontaneously and are the result of deliberate efforts by teachers and students (Geary, 1994; Resnick, 1989). For example, children may learn that the word "plus" means the summation of two or more numbers by hearing it in an informal setting. However, they are unlikely to learn that + is a symbol used to represent that word until they encounter this in school. Many early childhood math educators reason that understanding the formal math sentence 3 + 2 = 5 is a far simpler task for children who already have the informal understanding of the words three, two, five, plus, and altogether (for a review, see Geary, 1994). Conversely, understanding the same formal math sentence presents many difficult challenges for children who have not developed these informal mathematical ideas.

Children's ability to succeed in the elementary school math curriculum depends on their ability to overcome several challenges specific to the learning of formal mathematics. Whether learned informally through experience or formally through instruction, children must come to understand the functional and abstract nature of quantity. The arbitrary word *three* can be applied to any set of concrete or abstract things (3 houses, 3 cookies, 3 ideas). They learn that the word *three* is not connected to the objects being counted, but instead represents *how many* is in the set. Children must come to understand that each number in the counting series is related to the number before it and after it; each number represents one more than the number before it and one less than the number after it. Children must learn that when you count 4 cookies, that the last number stated (i.e., four) represents the group of cookies, including those that were tagged one, two, and three.

Another challenge to learning formal mathematics is understanding quantity in decontextualized terms. In informal settings, children are more likely to hear numbers connected to objects, such as "I put two plates on the table. I need one more to have three." However, the formal code "What is two and one more?" is *context free*. Research shows that young children capable of answering one question in context struggle to answer the same question stated formally (Hughes, 1983). For example, a child who is able to answer the question, "You ate three cookies and then one more; how many cookies did you eat altogether?" might not be able to answer the question, "What is three plus one more?"

A third, but related, challenge for children learning formal mathematics is the language that is used. Sometimes children might understand a mathematical concept, but not a teacher's description of the procedures she is asking students to use (Carpenter, Fennema, Franke, Levi, & Empson, 1999). A teacher might ask a child to *add* them together, *take away all* of them, or give *each* child *one* cup. The child receiving these instructions might be able to do each of these tasks, but be unfamiliar with the math terminology. Moving beyond the specific vocabulary a teacher might use, the syntactical structure of a mathematical question also needs to be considered. A single math problem can be asked in several different ways, but require the same procedure to solve it (Ellis had 5 cookies and ate 2 of them, so how many are left? Ellis has 2 cookies, how many more does he need to have 5? Ellis has 2 cookies. Tom has 5 cookies. How many more cookies does Tom have than Ellis). All of these challenges pose difficulties for children from all backgrounds in making appropriate connections between what they understand informally about shape and number and what teachers or peers are saying in class or doing in written form.

Challenges for Children from Economically Disadvantaged Backgrounds

While the abstraction of mathematics, the decontextualization of formal mathematics, and the language of mathematics pose challenges to all children, the latter might potentially cause additional challenges for children from economically disadvantaged backgrounds. Using words and communicating knowledge to others are learned skills; skills important to classroom learning (Farran, 1982). Tough (1982) argues that differences in children's language skills are due to the differences in children's experiences with how language is used. Parents who are disposed to reason with children about their behavior, share narratives describing their day, express ideas about activities, and encourage children to think about their experiences are shaping how their children observe, compare, and reason. Children who come to school with these skills will, in general, perform better in school.

While children from different backgrounds are certain to have had multiple and diverse experiences with language and reasoning, children from economically disadvantaged homes often have not experienced using language congruous with that of their teachers and ways of reasoning highly valued at school (Cazden, 2001; Tough, 1982). There is evidence that differences in how children of all ages use language are related to their ability to integrate their informal math knowledge with the formal math being taught in school (Orr, 1997; Resnick, 1989). In an experimental school established to use action research to develop courses, curriculum, and teaching strategies for an economically representative body of high school students, the faculty traced specific ways that nonstandard English usage led to misunderstandings in several content areas. Orr (1997), author and co-founder of the school, describes a divergence between speakers of standard and nonstandard English in the use of *function* words (prepositions, conjunctions, and relative pronouns). This divergence, or *linguistic* *interference*, appeared to be related to a divergence in conceptual understanding. Thus, the basic perceptions, intuitive understandings, and ways of reasoning about quantitative comparisons appeared to be shaped by the language children used.

The linguistic interference experienced by speakers of nonstandard English was especially pronounced in their math and science performance. For example, students' different usage of the prepositions *as* and *than* resulted in misrepresentations of subtraction and division. Nonstandard English speakers would often say, "three is two times less than six" instead of "three is half as much as six." However, the former way of reasoning could lead to thinking that "three is two times less than nine." Differential usage of the prepositions as, than, by, to, from, of, between, among, and so forth affected students' representations of addition, subtraction, multiplication, division, distance, motion, and magnitude comparisons. However, interventions designed to focus solely on language development to address children's educational disadvantage fail to meet the needs of children from poor and low income backgrounds. Farran (1982) argue the children's disadvantage is not a lack of language, but a lack of experience in *using* language in ways that support learning.

This principle can also be observed in preschool children's math development. Jordan and colleagues (1992) demonstrated that preschool children from different economic backgrounds performed similarly on non-verbal calculation tasks, but significantly different on verbal tasks of math reasoning. Preschool children from different economic backgrounds engage in similar informal mathematical activities during free play (Seo & Ginsburg, 2004) and employ similar strategies to solve basic math problems (Ginsburg & Pappas, 2004). Such research suggests that children from economically disadvantaged backgrounds possess sufficient informal mathematical competence, but less experience in using mathematical language in ways that support their formal mathematical development. It would stand to reason then that an appropriate intervention of early childhood mathematics for preschool children from economically disadvantaged backgrounds would focus on providing opportunities to hear and use language in ways that shape their mathematical reasoning.

Summary

The mathematical understandings children derive from everyday play or interactions with more knowledgeable individuals are informal mathematics. The mathematical concepts, symbols, and vocabulary taught in school are formal mathematics. Many educators of early childhood mathematics theorize that children who are able to connect their informal math knowledge with the formal curriculum presented in schools are better able to excel in elementary school mathematics. Formal mathematics learning possesses challenges for children from all backgrounds; however, more challenges are likely to be experienced by children from poverty. Evidence shows that children from economically disadvantaged backgrounds do not lack basic math concepts and skills, yet appear to show less proficiency in verbal reasoning skills. It is possible that children from disadvantaged backgrounds would benefit from in-school experiences focused on providing opportunities to use language in ways that better support learning in school.

Theoretical Framework

The Relationship Between Language and Learning

The construction of knowledge and understanding is thought of as an inherently social activity mediated by language. Vygotsky (1978) conceptualized social interaction as being at the core of the developmental process. The child's interactions with other people, notably those who are more advanced and capable, mediate the child's encounters with the world. Making sense of that world, or cognitive development, occurs as the child participates in the cultural life of the community and uses the tools of that community. The language used and the way language is used by the community become the language and ways of reasoning by the individual. This perspective of cognitive development can be applied to the learning of mathematics. As children use the tools of the formal mathematics community, such as its vocabulary and ways of reasoning, and thereby internalize new knowledge.

The process by which social interaction leads to new knowledge is described by Carey (2004) as "bootstrapping." According to the bootstrapping concept, learning new words in context, even without fully understanding them, creates new structures. These structures then provide an opportunity for the novice to make assumptions about new ideas. As the novice interacts with an individual or community that understands both the new concept and what the novice is already familiar with, the novice's assumptions are clarified. Eventually this interaction helps the novice map relations between the novel words learned and previously understood concepts. At first, the bits of knowledge that a novice acquires within the new structures may be disconnected from each other or some bits might be misrepresented altogether. The interaction within the cultural community plays a critical role by allowing the novice attempts at using the tools of that community. It is through trying to use the knowledge – such as what happens when an individual is *talking* about new concepts – that he or she begins to make connections, correct representations, and gain new knowledge.

Math knowledge is conceptual understanding (what a child comprehends). Talking about math requires *using* that knowledge. *Math talk*, then, is the use of domain specific vocabulary for the purpose of reasoning mathematically. The National Research Council (NRC) in collaboration with the NCTM outlined a set of curriculum and pedagogical standards for mathematical talk (language and ways of reasoning) that is believed to lead to language and conceptual development:

- Modeling: representing worldly phenomenon by mental constructs, often visual or symbolic, that capture important and useful features
- Optimization – finding the best solution (least expensive or most efficient) by asking "what if" and exploring all possibilities
- Symbolism – extending natural language to symbolic representation of abstract concepts in an economical form that makes possible both communication and computation
- Inference – reasoning from data, from premises, from graphs, from incomplete and inconsistent sources
- Logical analysis – seeking implications of premises and searching for first principles to explain observed phenomena
- Abstraction – singling out for special study certain properties common to many different phenomena

(NRC, 1989, p. 31)

A math talk learning environment is designed by teachers to provide opportunities for the use of math talk. The purpose of this activity is to *co-construct* mathematical understandings. Whereas it is possible for a child to engage in math talk alone (e.g., while reasoning to him or herself which shape comes next in a pattern), engaging in a math talk learning environment requires two or more individuals to construct knowledge and understanding together.

Components of a Math Talk Learning Environment

The social interaction required to engage children in talking about math concepts in a way that leads to their language and conceptual development involves intentional planning on the part of the teacher. The teacher must consider the content of the lesson, the activity in which children will participate, and the mode of instruction that fosters interactions about the math content. Content, activity, and instruction are the three components of a math talk learning environment. This section describes each component and what occurs at the intersections among them.

The content. Mathematical content is the subject matter teachers are trying to impart to children. The NCTM has worked for almost two decades to define and articulate appropriate mathematics curriculum and standards. In 2006, the NCTM released *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*. The focal points recommended for prekindergarten are number and operations, geometry, and measurement. While early experiences with all three topics are relevant, developing a strong sense of number prior to first grade is critical for later mathematical competency (Griffin, et al., 1994; Jordan et al., 2010). Number sense (also referred to as 'numerosity', 'number competence', 'numerical proficiency', or 'mathematical proficiency') (Howell & Kemp, 2010) includes being able to count, state the cardinal number, perform one-to-one correspondence, subitize, recognize numerals, manipulate a number line, compare magnitude, and compose numbers.

Number sense is highly related to later student success in the formal math curriculum. Griffin et al. (1994) argued that children from low income backgrounds were less likely than their more advantaged peers to begin school with the central conceptual structures required to compare quantity, for example. These central conceptual structures, such as a mental number line, could be developed through interventions designed to give children practice with using number lines. Jordan and colleagues (2010) found children's number sense at the beginning of first grade made a significant and unique contribution to math achievement at the end of third grade, over and above both age and cognitive factors. They defined number sense as the ability to flexibly count, judge numerical magnitude, and perform simple addition and subtraction calculations in three contexts (non-verbally, story format, and decontextualized). In pilot work investigating relationships between components of early number sense and standardized measures of mathematics, Howell and Kemp (2010) found few children at the end of the preschool year were able to use number flexibly or design mental strategies for solving mathematical tasks, skills that were significantly related to scores on standardized math measures.

Collectively, this research suggests three important points. The first is that number sense should occupy a substantial portion of the intellectual content on which preschool children focus. The second point is that number sense can be influenced through experience. The work by Griffin et al. (1994) in particular demonstrates that these experiences should be deliberate in nature, not perfunctory or marginal. The third point is that multiple experiences are necessary for children to develop a flexible use of count and number. Howell and Kemp (2010) report that preschools often focus on counting skills (rote counting) without giving adequate opportunities for children to learn the principles of counting (e.g., three is one less than four and two more than one).

The social activity. The social activity used to engage children in mathematical content is also important to young children's learning. Games provide an opportunity to practice mathematical principles, communicate mathematically, and make connections between informal ideas and more exact representations. Games also engage children's interests (Randel, Morris, Wetzel, & Whitehill, 1992) and engender intersubjectivity (Rogoff, 1990), both of which lead to greater student effort towards a given task (Paas, Renkl, & Sweller, 2003).

When children manipulate objects in games and use mathematics to execute the rules of the game, children practice mathematics and come to understand numbers and the quantities they represent. When children use board games, they are practicing count, one-to-one correspondence, numeral recognition. They come to learn and understand why 13 is larger than nine, and later, that it is larger by a value of four. When they play with dice, they not only practice subitizing (instantly recognizing quantity), but are also developing the skills to compose and decompose quantities (three dots and three more dots are six altogether). Several successful interventions of early mathematics used games to focus children's attention on count and number. For example, a series of intervention studies by Siegler and colleagues tested whether playing linear board games with preschool children from low-income backgrounds would improve their number sense (Booth & Siegler, 2006; Ramani & Siegler, 2008; Siegler & Opfer, 2003). The researchers theorized:

> [B]oard games provide multiple cues to both the order and the magnitude of numbers. The greater the number in a square of the game, the greater (a) the number of discrete movements of the token the child has made, (b) the number of number names the child has spoken, (c) the number of number names the child has heard, (d) the distance the child has moved the token, and (e) the amount of time that has passed since the game

> began. The linear relationships between numerical magnitudes and these kinesthetic, auditory, visuospatial, and temporal cues provide a broadly based, multimodal foundation for a linear representation of numerical magnitudes. (Siegler, 2009, p. 3)

Siegler and colleagues found children who played the linear board game focused on number significantly improved on all measures of number sense. The number sense scores of children who played a similar game focused on color rather than number did not change from pretest to posttest. In one study, the number sense of children who played the number game grew to be equivalent to a group of children from middle class backgrounds who did not participate in the intervention. The findings suggest that children can construct deeper understandings of number relationships through playing games focused on number.

Two reasons why games are successful in influencing student learning are that they capture children's interest and provide a context for intersubjectivity. When the joint attention of teacher and learner is focused on the child's interests, language and concepts are better remembered by the child. For example, in an experimental study, toddlers were better able to remember the names of toys when they were told its name while they were in play with those toys; conversely, children who were told the name of the same toy while they were attending to another toy did not remember the names or concepts they were told (Tomasello & Farrar, 1986). Providing instruction for children when they are engaged in their own interests, rather than having to redirect children's attention, increases the likelihood children will learn more efficiently.

Intersubjectivity provides the grounds for communication and at the same time supports the extension of children's understanding (Vygotsky, 1978). In a study of 54 four and five-year-old children from middle-class backgrounds, generating explanations, regardless of the presence of a listener, improved learning of a task, but generating explanations for a listener led to more flexible knowledge that could be transferred to novel problems. In addition, more children attempted to give an explanation and the explanations were more detailed when an adult was listening (Rittle-Johnson, Saylor, & Swygert, 2008). Authors noted that, despite the difficulty in eliciting clear verbal explanations from young children, attempting to explain to a listener still had moderate effects. The importance of intersubjectivity, at the preschool level at least, might be underestimated; joint attention appears to increase children's motivation to communicate and attempting to communicate appears to improve cognition.

The instructional process. Using games focused on count and number might be adequate for practicing children's early number sense, but together they are not enough to bring about the dynamics and interactions that optimize learning. Students do not automatically begin talking about mathematics in meaningful ways; exposure to mathematical talk, that is the language and the ways of reasoning, does not guarantee that students will understand or use this kind of talk (Rittenhouse, 1998). Essential to the challenge of engaging very young children in relevant mathematical experiences is the role that teachers play in stimulating interactions that support language and conceptual development. Drawing from the literature on guided instruction and the literature on developing children's mathematical talk in the older grades, teachers must be skillful at scaffolding children's talk, transferring responsibility of game play to children, and moving between different teachers roles.

Helping students learn to talk with one another to build ideas involves a process called scaffolding (Vygotsky, 1978). Scaffolding implies intentionally *locating* what children can do alone and *moving* children's thinking and language into what they might do with assistance. Scaffolding is not easy for teachers to implement – it requires giving attention to individual children, taking the perspective of individual children to understand what he or she is trying to say or do or what he or she misunderstands, determining the level of mathematical understanding that follows next, and selecting tasks or representations that will clarify novel concepts and language. Bransford, Brown, and Cocking (2000) list the following goals of scaffolding children's talk and reasoning:

- Motivate or enlist the child's interest related to the task;
- Simplify the task to make it more manageable and achievable for a child;
- Provide some direction in order to help the child focus on achieving the goal;
- Clearly indicate differences between the child's work and the standard or desired solution;
- Reduce frustration and risk; and
- Model and clearly define the expectations of the activity to be performed

Scaffolding children's talk and reasoning in these ways helps children engage in mathematical activity at a level just above what they would be able to do alone; however, as children become more capable of navigating the tasks alone, they can take more responsibility for directing and managing the activity. In effective use of guided participation, teachers must be able to perform this transfer of responsibility. By doing so, children are practicing the skills that were once beyond their ability and internalizing the language and ways of reasoning (Rogoff, 1990). For example, the first time playing a math game, the teacher will have the sole responsibility for dictating the rules of play; however, with repetition, children will begin to direct one another's activity. Children who internalize the rules for play will notice if another child does not move the correct number of spaces or counts the cards incorrectly. In this way, children's learning is enhanced in two ways. First, children are attending to the activity even when it is not their turn in play. Another benefit is that interactions are enhanced because there are more participants than just the teacher and the child in play.

As children take more responsibility for the social activity, the teacher's role in the activity will shift. To begin, teachers act as an instructor of the task, reminding children what the goals are and instructing them in how to attain the goals. As children take more responsibility, teachers must shift among three roles. Lampert (1985) describes two roles: the role of participant and the role of commentator. As a participant, the teacher engages in the discourse, models mathematical language and ways of reasoning, revoices students' ideas in more exact language, and actively scaffolds children's levels of difficulty. As a commentator, the teacher states the rules of the activity and explains why a student's justification or explanation was representative of math talk. The changing role of the teacher aids children's mathematical development by providing explicit instruction in and models for using mathematical talk and ways of reasoning.

At the Intersections of Content, Instruction, and Social Activity

Mathematical content, instruction, and social activity are three independent constructs that, when linked together within a sociocognitive framework, provide an illustration of the types of interactions that occur at the intersections of any two constructs. Figure 1 displays this model that situates math talk at the center of the three constructs, demonstrating the importance of having all three constructs present to optimize learning opportunities for young students.

At the intersection of math content and instruction (a), there is an absence of social interaction; instruction is not reciprocal or shared interactions. Teachers are talking and/or they are presenting or modeling content. Examples include teachers lecturing, explaining, writing on the board, and pointing at materials. However, children may or may not attend to or engage in the content to be learned. At the intersection of content and social activity (b), there is an absence of instruction; children are *doing* mathematics collaboratively with (an) adult(s) or peer(s), but their thinking is not being extended beyond what they can conceive alone. Children are engaged in social activity that uses math content with which they are already comfortable; they are working at a level they already understand or are using skills they already have. At the intersection of instruction and social activity (c), teachers are using the principle of guided participation - activity that is guided by dialogic instruction and, equally important, activity in which children are as engaged as is the guide (Rogoff, 1990). While this interaction may be beneficial to children's development, in the absence of mathematical content, this activity is not fostering children's math development.

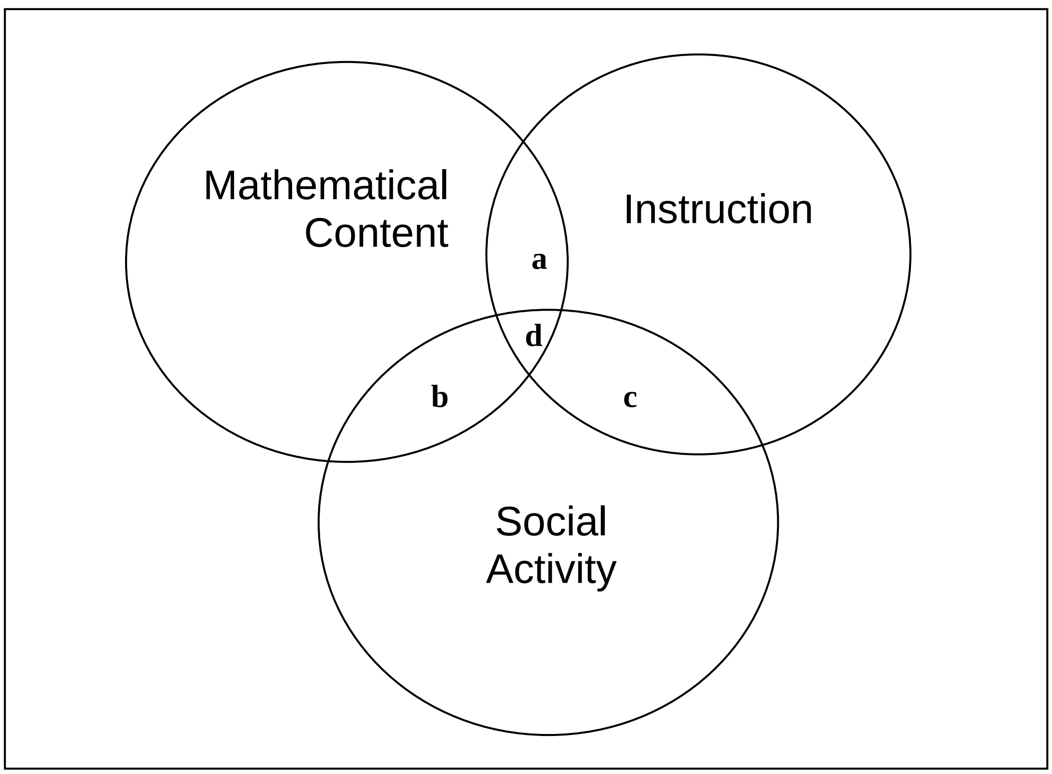


Figure 1. The Intersecting Circles of Instruction, Content, and Activity. This model serves as a representation of the independent constructs of instruction, content, and social activity as well as the intersections among them: (a) is the intersection of content and instruction, (b) is the intersection of content and social activity, (c) is the intersection of instruction and social activity, and (d) is the intersection of instruction, content, and social activity.

The math talk learning environment is situated at the intersection of instruction, content, and social interaction (d). For this study, a math talk learning environment is defined as guiding children's participation (instruction) in games (social activity) focused on number sense (content). Altogether, math talk is theorized to help math learning in three important ways. First, when children talk about mathematics it helps develop new knowledge. New structures are formed when children encounter new language and ways of thinking used by the community. As children use the language and reasoning skills, they internalize new knowledge.

Second, talking about mathematics helps make important connections between bits of previously disconnected information. When children first develop new knowledge structures, they make assumptions about the information that belongs within that conceptual circle. Through talk with a more knowledgeable other, assumptions are either reinforced or clarified.

Third, talking about mathematics provides information to a teacher or a more knowledgeable peer about the novice's thinking, what is understood or misunderstood, in order to provide feedback to the teacher. This feedback is critical if the teacher is to assist the novice's conceptual development.

Summary

Language and learning have long been theorized as interdependent processes; growth in one area facilitates growth in the other. The process by which teachers can initiate and sustain this iterative process is through the use of a math talk learning environment. This teaching strategy is situated at the intersection of math content, social activity, and instruction and, for this study, is defined as guiding children's participation (instruction) in games (social activity) focused on number sense (content). Math talk is theorized to help children develop new structures for knowledge; make important connections between previously distinct bits of information; organize their own ideas; and provide important clues for feedback from more knowledgeable teachers or peers.

Assumptions Underlying Math Talk in Preschool

The NCTM (2000) has asserted for more than a decade that math instruction needed to shift away from the instruction traditionally seen in American classrooms towards instruction focused more on the important relationship between language and learning. Traditional instruction is regarded as classrooms where teachers are the sole authority of right answers and students memorize procedures. Reformed instruction as advocated by the NCTM and many math educators is envisioned as classrooms where children use mathematical communication to reason together about mathematics. Even at preschool, they use logic and evidence as verification, and invented strategies to solve problems. The NCTM made recommendations for instructional programs from prekindergarten through grade 12 in their *Principles and Standards for School Mathematics* (2000). Among the standards, the Communication Standard for students in prekindergarten to grade two recommends that students are enabled to:

> Organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teachers, and others; analyze and evaluate the mathematical thinking and strategies of others; and use the language of mathematics to express mathematical ideas precisely. (p. 60)

The importance of communicating mathematically has also been voiced in the recently released draft of the *Common Core State Standards Initiative* (NGA/CCSSO, 2010) issued by the Council of Chief State School Officers and the National Governors Association Center for Best Practices. Among the standards for students in kindergarten, the CCSSI advocate that children say the number names, count, state whether the number of objects in a group is greater, less than, or equal to another group, represent addition and subtraction, describe objects in the environment using names of shapes and describe relative positions using specific terms. Among the standards for students in first grade, recommendations include children explaining and justifying properties of addition and subtraction, organizing and representing data, asking and answering questions about number. In order to execute any of the aforementioned tasks children must use and interpret formal mathematical language.

The National Research Council's Committee on Early Childhood Mathematics stated:

> Connecting and communicating are particularly important in the preschool years. Children must learn to describe their thinking (reasoning) and the patterns they see, and they must learn to use the language of mathematical objects, situations, and notation. Children's informal mathematical experiences, problem solving, explorations, and language provide bases for understanding and using this formal mathematical language and notation. The informal and formal representations and experiences need to be continually connected in a nurturing "math talk" learning community, which provides opportunities for all children to talk about their mathematical thinking and produce and improve their use of mathematical and ordinary language. (Cross, Woods & Schweingruber, 2009, p. 2.13)

Despite these recommendations for the use of math talk in preschool, there is no clear definition of what this means, no specific components identified, no guidance on how to teach this to children (particularly among those without former experience in using math language or in using language for reasoning), and no evidence that math talk matters to students' preparedness for elementary school mathematics.

What is Math Talk?

Teachers are instructed to use math talk in the preschool classroom in order to both model the language of mathematics and encourage children to develop the ability to communicate mathematically; however, few resources offer clear instructions or illustrations of how to do either. The majority of resources on using math talk (e.g., Chapin, O'Conner & Anderson, 2003; Lampert & Blunk, 1998; Sullivan & Lilburn, 2002) are written for teachers of grades one through grade six. It is becoming increasingly understood that to engage in math talk at older grades requires fostering the language and reasoning skills at much younger ages (Whitenack & Yackel, 2002), particularly among students from economically disadvantaged backgrounds without experiences in using math language in ways to think and reason mathematically.

How Should Practitioners Teach Math Talk?

In tandem with recommendations to help young learners communicate mathematically, several evidence-based preschool math curricula now include instructions for teachers to both use math talk and develop children's math talk. For example, in the preschool curriculum, *Building Blocks for Math PreK Curriculum* (herein *Building Blocks*) (2007a) teachers are encouraged to extend children's thinking, ask children to listen to and build upon other children's reasoning, and to use "every day" mathematics when interacting with children throughout the day. In the *Right Start* program, later called *Number Worlds* (Griffin et al., 1994), one of the five major instructional principles is to expose children to the major ways number is discussed in developed societies. In *Big Math for Little Kids* (BMLK) (Greenes, Ginsburg, & Balfanz, 2004), one of the seven major instructional goals is to develop children's familiarity with and use of the language of mathematics. Embedded in the BMLK lessons are mathematical symbolism and terminology, including vocabulary that promotes prediction (what might happen) and verification (evaluating mistakes, checking answers) in order to develop and enhance children's discussion skills. In addition, BMLK asks teachers to specifically develop a community of math learners by ensuring children learn to listen to peers, follow and comment on other children's line of reasoning, and formulate questions.

Despite encouraging teachers to use discussion of mathematics as it naturally occurs in preschool, Clements and Sarama (2007b) reported "no evidence of talk about mathematics" during their preliminary testing of *Building Blocks.* Although engaging children in math talk was an optional part of the curriculum and not enforced by the project support in the classroom, it is unclear why teachers failed to include this in their practice. It is possible that teachers did not know how to engage children in math talk or they did not believe such a practice was important to children's development.

Does Math Talk Matter to Student Learning?

Two studies demonstrate evidence that talk is important to children's math development; however, neither study was specific to the direct effects of children using math talk. Researchers tested whether preschool teachers' math-talk would be related to the growth in children's mathematical knowledge (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006). In a study of 26 preschool classrooms, data collection included 1-hour worth of audio recordings of teacher talk and the math achievement of 140 children measured at two time points. Results from this study showed that there was great variation in both the type and quantity of math language used by teachers. Teachers provided between one and nine types of math input between one and 104 instances of math-talk usage in the hour long recording of teacher talk. The type and quantity of teacher math-talk were highly correlated; teachers who used more diverse types of math language also used this language more frequently. Moreover, there was a statistically significant association of math input made by teachers with children's gain in math knowledge on a non-standard measure created by the investigators. What is less understood is whether the amount that children themselves are talking about mathematics plays a role in their mathematics development.

In a project funded by the US Department of Education's Institute of Education Sciences to assess the effectiveness of "scaling-up" a pre-K mathematics curriculum at sites different from that of the developers, Vanderbilt University was selected to test *Building Blocks* in Nashville, Tennessee. Twenty sites, 16 public schools and four Head Start centers, were randomly assigned to one of two conditions while blocking for system (public/Head Start). This process yielded 31 classrooms that participated in the new math curriculum training while 26 classrooms conducted business as usual. Individual children were assessed in the first and last six weeks of the implementation year using two subtests from the *Woodcock Johnson Achievement Battery* (Woodcock, McGrew, & Mather, 2001), a standardized measure of mathematical knowledge. Observations of children's behavior were collected over 12 hours of observations on three different days using the *Child Observation in Preschool* (COP) (Farran, Plummer, Kang, Bilbrey, & Shufelt, 2006). While children in both conditions gained in math achievement over the preschool year, children in the treatment condition gained significantly more.

In an analysis of data from this project, researchers (Cummings, Hofer, Farran, & Lipsey, 2009) tested mediators of the curriculum's effect on children's gain in mathematics. Because teachers of the new curriculum were encouraged to engage children in talk about mathematics by asking children to explain their answers or the answers given by their peers, one mediator tested was the proportion of observations children were talking while they were engaged in a math activity. Children in control classrooms were seen talking during math activities 2% of all observations while children in treatment classrooms were seen talking during math activities 4% of all observations. Despite the small proportion of observations in which children were talking during a math activity in either condition, doubling the percentage of observations children were talking during a math activity significantly explained the greater gain achieved by students in the treatment condition. This provides evidence that child-talk is important to children's learning of mathematics and should be investigated further as a developmentally appropriate intervention of early mathematics.

Summary

Math talk is discussed frequently in current literature on early childhood mathematics education and touted as both an important instructional tool for teachers and a key ingredient of student learning. Despite the number of years math talk has been advocated, it has just recently been included as a standard for preschool instruction and incorporated into a number of early childhood mathematics curricula. Despite this, there is relatively little known about how to define math talk or its principal components. While there are materials available to practitioners providing some examples of math talk, most are based on work in elementary school classrooms where students appear to be familiar with its use. However, preschool children require examples of and practice in how to use mathematical language for thinking and reasoning mathematically. Research shows that children from economically disadvantaged backgrounds do not demonstrate as much mastery as children from more advantaged homes and suggest early childhood experiences help foster this skill. Although there is empirical evidence to suggest that both teacher talk about mathematics and children talking during math activities are both associated with math learning outcomes at the end of the preschool year, it is still unknown how to develop child-talk about mathematics or whether doing so leads to improved early math skills among preschoolers.

Structuring a Math Talk Intervention

To address the assumptions underlying the usefulness of math talk in the preschool classroom, it was necessary to conduct a pilot study of activities and strategies that would compose an intervention. With support from the Administration of Children and Families' Office of Head Start, the author hired two former teachers to work as co-investigators on the *Talking About Mathematics in preSchool* (TAMS) project. The pilot work for the TAMS intervention was conducted by the three of us with 58 four to five year old children enrolled in three Head Start classrooms in 2009-2010.

Over six weeks, on two days each week, we spent between one and three hours conducting small group sessions with children. Sessions were recorded and notes were taken for later discussion. At the end of each day, we met to share results and recommend revisions to the tested activity before the next classroom visit. Our goal was to identify activities and strategies that would facilitate children's discussion of mathematical concepts and information. The group identified important challenges to engaging children in talk about mathematics and some successful strategies for overcoming those challenges. These next sections describe the challenges encountered during the pilot work and subsequent recommendations for developing a structured math talk learning environment for testing of its benefits to children's math readiness skills.

Challenges

Much published research on what preschool children know and are able to do upon entering kindergarten described abilities that exceeded those of the particular population we worked with in 2010. For example, when asked to count, more than one child responded with "A." When the investigator prompted the child saying, "One… What comes next?" the child responded "A, B." Children's skills were well below what was anticipated and therefore presented this project with the challenge of engaging children in both ideas and language that appeared to be completely novel to them. Conversely, children did not possess the language or prior experience required to talk about the mathematics activities they were engaged in. For example, the majority of children could not respond appropriately to questions asking if there were "too many or not enough" or "who has more blocks?" Therefore, the initial goal of engaging children in extended talk about mathematics was then modified to include explicit modeling of math language and requests that children revoice the language being modeled.

In addition to skill level and unfamiliarity with the language of math, another important challenge was identified. When asked open-ended math questions that required more than one-word responses, even children who were talkative in informal contexts would remain silent. Children were willing to talk to investigators during book readings, meal time, or free play and so their lack of responsiveness did not appear to be a result of unfamiliarity with the adults. There are three alternative explanations for children's reluctance to talk. Because these investigations were conducted in children's sixth and seventh months of preschool (for those who were attending preschool for the first time), the first possibility is that children had already learned the typical pattern of school discourse. Typical school discourse between teachers and students in formal settings follows a prescribed pattern of Initiation-Response-Evaluate (IRE)

(Buzzeli, 1996). Even children as young as three in school learn that the teacher *initiates* through closed questioning “What number is this?” Children *respond*, “One.” Teachers *evaluate* the accuracy of the response by saying, “Yes” or “No.” This pattern becomes engrained in children, and may have a detrimental effect; initiating conversation in any formal setting, including small group sessions, triggers the automated reaction to search for a one word response. Because investigators were using open-ended questions, children did not know how to respond.

Another possible explanation for children’s disinclination to provide longer responses to open-ended questions is their lack of familiarity with group discourse – the practice of listening to a speaker and responding appropriately to any given topic. A third possible explanation is that investigators and children have no experiences in common from which to draw for discussion purposes. During this exploratory phase of the study, children were generally more excited and talkative during the second or third trial of any given activity. Investigators speculated that having prior experience in playing each game gave children and investigators a shared context for discussing the mathematics embedded in the activity.

Recommendations

To overcome these challenges, the investigators arrived at the following recommendations for the structure of the math talk learning activity to be tested experimentally.

Math games. The math talk learning activity should not focus on developing new or including multiple activities, but rather focus on a few activities that could be played with little to no prior mathematical knowledge but then adapted to be increasingly more challenging and mathematical as children master each level. Focusing on fewer activities while making those activities increasingly more difficult solves several of the aforementioned problems by beginning at low levels of ability, providing familiarity of language and content and a shared context between the investigators and children on which to build conversations. In addition, adapting a game gradually places less cognitive demand on young children to learn new game rules thereby allowing them to focus more on learning the math talk.

To focus the math talk learning activity on a few activities that could be scaffolded according to children's early abilities, investigators recommended using games that targeted the very basics of mathematics known as number sense. Initially, eight activities were organized to be tested. After several iterations, three activities met all of the following conditions: the math activities (a) were successful in eliciting talk from children, (b) could be played with a comparison group of children without eliciting talk from children, and (c) could be played at varying skill levels so that, as children master a game at each level, the game could be made more challenging. The three games, their rules, and variations in difficulty are described in the next section.

Establishing an environment for talk. We decided that the math talk learning activity should begin early in the school year, before IRE patterns are engrained. Another important component of a math talk learning activity recommended by the investigators was to intentionally help children develop group discourse skills as described by Kantor, Green, Bradley, and Lin (1992). In Kantor's research, teachers were observed scaffolding preschool children's group discourse skills by modeling and explicitly guiding children to take turns to speak, listen to the speaker, and respond appropriately to what the last speaker just said. Because the development of group discourse as described by Kantor et al. (1992) required both a considerable amount of focus (children were completely immersed in this activity) and time (over the entire school year), investigators concluded that a study designed to test a math talk learning activity should administer a concentrated dosage of similar treatment. Our math talk learning activity, therefore, would be conducted with small groups of children in order to provide the instructional intensity necessary to scaffold their math talk as well as their conversations with one another. The small groups would be conducted outside the classroom to lessen distractions and avoid contamination of instruction to other children in the classroom.

Two specific strategies were spontaneously used by investigators during this first phase of the study and found to be successful in eliciting the use of math talk and engaging children in mathematical thinking. The first was to teach children hand-gestures for mathematical ideas – similar to sign language – for "greater/more than" (hands held facing each other shoulder distance apart), "smaller/less than" (same as greater/more than except the hands are held close together), and "same as/equal to" (both hands face down, fingers pointing towards each other and on the same plane). Using these hand gestures provided a useful scaffold for children's use of the language and developing the ideas the gestures represented. Also, by associating mathematical ideas with physical movement, children had an additional support for recalling the actual vocabulary.

Another strategy that proved useful towards scaffolding children's use of math talk was named the *doublecheck*. Each time a student completed his or her turn in any game, the investigator said, "Let's *doublecheck*" and would proceed to model a method for checking his or her accuracy. In the event a child had made an error in his or her calculation, the investigator would use the math hand gesture to represent if, for example, the number of apples removed from the tree was more than, less than or the same as the number on the spinner. After enough exposure to this modeling, children became very excited to *doublecheck* one another's moves during game-playing. Several children quickly began using the verbal language in place of the gestures and would *doublecheck* one another without the investigator having to ask them to.

Despite their spontaneous use by investigators, both of these supports are representative of strategies discussed in the literature on education and in cognition. Gesturing as a support for the growth and maturity of language and cognition has been examined among typically- and non-typically-developing children. Among typically-developing children, "gesturing provides children with a tool to expand their communicative repertoire, and children use this tool to convey increasingly complex ideas" (Özçalişkan & Goldin-Meadow, 2005, p. B110). Further, because children from disadvantaged homes show less competence than more advantaged children in performing verbal tasks of reasoning (Jordan et al., 1992) and metacognitive skills (Pappas, Ginsburg, & Jiang, 2003), some early childhood experts recommend a greater focus on developing these skills in early childhood. The *doublecheck* described above is a type of support for developing children's metacognition. By asking children to use language to express and justify mathematical relationships, we are asking them to think about their thinking.

Conclusion and Hypotheses

The mathematics performance gap between students from economically advantaged and disadvantaged backgrounds is large and has proven to be persistent over decades. This gap has long-term academic and economic consequences, disadvantaging individuals who perform on the lower end of this performance gap. Because of the relationship between having poor early math skills and later mathematics achievement, greater emphasis is being placed on early childhood education as a means of closing that gap early before formal schooling begins.

Research on early childhood mathematics instruction and environments finds critical elements of mathematics learning from the later grades absent from the typical preschool classroom. For example, the amount of time spent in instruction and the math content covered are important elements of math achievement among students in K-12. However, preschools generally allocate little time to math and few math topics are covered. Teacher's knowledge and attitudes about mathematics are also important to student math achievement; yet, early childhood practitioners often report feeling uncomfortable with and inadequately prepared to teach mathematics. Many national organizations and early childhood educators are concerned that the greater emphasis for academic outcomes in preschool could potentially lead to an inappropriate push down of the math curriculum. Instead, developmentally appropriate methods for engaging young children in meaningful experiences and interactions about mathematics should be researched.

To help guide ECME instruction it is important to consider the ways in which school is failing to connect children's informal ideas about mathematics with the formal curriculum introduced in school. Informal mathematics involve the ideas and strategies children invent in their daily activities. Formal mathematics relate to the symbols and representations taught in school and are not spontaneously acquired by children. There are several challenges specific to early math learning. There are several plausible theories used to explain why these challenges differentially affect children from economically advantaged and disadvantaged backgrounds. One theory is based in children's experience with using language to reason mathematically.

There is evidence that facilitating preschool children's talk in mathematics has a great number of benefits. Mathematical language and the use of language for reasoning and argumentation are not spontaneously acquired by children, rather they are fostered. These might be fostered in children's homes or schools – anywhere people practice mathematics. However, children from homes that do not use mathematical language or who have less experience in using language for collective reasoning are disadvantaged in school. Providing explicit instruction in the ways to use language for reasoning mathematically would help children bridge the divide between informal and formal mathematics.

Fostering environments that promote math talk requires that teachers consider the intellectual content, the social activity involved in developing new knowledge, and the teachers' role in constructing these interactions. Findings from research reported in this review suggest that mathematics instruction, particularly in programs serving children from economically disadvantaged backgrounds, focus the intellectual content on developing children's number skills. Additional research reviewed recommends using games to establish a context for the social interaction that optimized learning. To maximize learning opportunities for interactions that lead to language and conceptual development, there is much support for guiding children's participation in a math talk learning activity. Math talk is guiding children's participation in games focused on number sense with the goal of developing children's math language and reasoning skills. Research from the field informed the design of a math talk learning activity. The data collected from the field highlights specific challenges to engaging young children in discussion about mathematics and recommends strategies for successfully engaging children in social interactions about mathematical activity.

The present study was designed to examine the effect of implementing an early mathematics learning activity on preschool children's early math skills. More specifically, the purpose of this study was to compare the effects on children's early math skills of participating in math games to the effects of participating in math games enhanced by a math talk learning activity.

The two main hypotheses of this study were:

1. Children who participate in math games will gain more early math and reasoning skills than children who do not play these games.
2. Children who participate in math games <u>and</u> who are encouraged to engage in math-talk will gain more early math and reasoning skills than those who participate in the same games without that encouragement to engage in math talk.

CHAPTER III

RESEARCH DESIGN AND PROCEDURES

The Math Talk Intervention

The Math Talk Intervention had two components. The first component was playing games that involved number. The games allowed children to count, subitize, practice one-to-one correspondence, state the cardinal number, compare magnitude, learn numerals and the number line. The second component was establishing a math talk learning environment whereby children learned their roles as a part of that environment. This section describes how interventionists engaged children in the first component, facilitating the math games, and then how interventionists engaged children in the added component, facilitating the math talk learning environment.

Facilitating the Math Games

The first component of the intervention involved facilitating math games; this component did not include math talk. The three games – Hi-Hi-Cherry-O, Walk-the-Line, and Card Wars – all used materials found in a typical preschool classroom or could be easily acquired. Table 1 displays a summary of the math games – the number of sessions children would play each game, the learning objectives and rules for play at each level. For a more detailed account of the game materials and rules of play of each game, see Appendix A.

Table 1

Summary of Math Games to Be Played in Intervention.

SESSION	MATH GAME	LEARNING OBJECTIVES	HOW THE GAME IS PLAYED
1 - 3	Hi Ho Cherry O	Counting 1-3 1:1 Correspondence Cardinality Reasoning	Children spin the spinner that can land on the numbers 1, 2, 3, a bird, a dog, or a basket. If children land on a number, they remove that same number of fruit from their tree and place into the bucket that corresponds to that tree. If children land on the bird or dog, they must put one piece of fruit from their bucket back on the tree. If they land on the basket, they put all of the fruit that had been placed in their bucket back on the tree.
4 - 6	Walk-the-Line Level 1	Counting 1-10 1:1 Correspondence Subitizing 1-3 Numerals Number-line Composing numbers Reasoning	Children are placed into two teams. Each group has a number line on the floor with the numbers 1 -10 displayed. The teams share a large red foam die with dots ranging from 1-3 on two sides each. Each team rolls the die to determine who goes first. The teams then take turns: 1 player rolls the die and 1 player physically moves along number line. First team to move off the number line wins. Team members should switch roles between rolling die and moving along number line.
7 - 9	Walk-the-Line Level 2	Counting 1-20 1:1 Correspondence Subitizing 1-6 Numerals Number-line Composing numbers Reasoning	Level 2 is played identically to Level 1; however, the number line is extended to 20 and a large yellow foam die with dots ranging from 1-6 is used. If children become adept at this Level, the game can be made more difficult to include subtraction. Children roll one yellow die and one red die to "move forward" the number the yellow die is rolled and "move backward" the number the red die is rolled.
10 - 12	Card Wars Level 1	Counting 1-10 Subitizing Numerals Magnitude Comparison Reasoning	Children play in twos so that there are actually two games being played simultaneously. Each child has a shuffled deck of cards with a numeral from 0 -10 at the top, a grid with 10 spaces on the bottom, and the same number of dots in the grids as the numeral on the top. Children each turn over one card and determine who, between the two of them, has more. The child with the card of greater value wins the pair of cards and places them aside in his or her card holder. When children turn over cards of an equal value, a second set of cards is turned over and placed directly on top of the first set. When the children have used all the cards, they determine who has more cards.
13 - 16	Card Wars Level 2	Counting 1-10 Subitizing Numerals Composing Numbers Magnitude Comparison Reasoning	Level 2 is played *similarly* to Level 1, but now requires children to use addition. Children play in teams of two so that there is one game being played by all of them. Children each have a truncated set of cards (each deck includes cards 0 -5) that have been shuffled. Children each turn over one card and determine which *team* has more. The team with the higher sum then wins the four cards and places them aside in their team's card holder. When the sum of cards is of an equal value, a second set of cards is turned over and placed directly on top of the first set. When the children have used all the cards, they determine which team has more cards.

Hi-Ho-Cherry-O is a children's board game first published in 1960 where players race to fill their basket with cherries from the tree. Walk-the-Line is a simple game in which players roll a die and walk that many spaces along a number line mat on the floor. Card Wars is a simple version of the regular card game, War. Children turn over cards and the highest card wins the pair. Both Walk-the-Line and Card Wars were played at two levels of difficulty. Level one introduced children to the rules and expectations of each game and was less cognitively demanding. The second level of play used all of the same materials, but was played using greater numbers or by asking children to perform more cognitively demanding tasks.

Facilitating the Math Talk Learning Environment

The second component of the intervention involved facilitating math talk. Establishing the small group math games as a time to use math talk required that interventionists employ several different strategies. Teachers had to model and scaffold how to play group games, use math words, identify math concepts, engage one with another, respond appropriately to a peer's mistake or corrective actions, and attend to the game without losing focus. To do these required interventionists offering specific supports for math talk, asking open-ended questions, modeling appropriate types of responses, and maintaining certain expectations of children.

Scaffolding math talk. In the beginning of the intervention, as is true for many classroom activities, there were both verbal and physical cues established early on to help children know what to do and what to expect. In this intervention, the term "Let's talk math talk!" was the first such cue. This expression was meant to remind children that, during small group math games, we could talk and challenge each other in ways we might not otherwise in the regular classroom. Another such cue taught to children as the intervention progressed was the *doublecheck* as described earlier. This expression meant it was time to take a moment and consider if a peer played the game as he or she was supposed to, removed the right number of fruit, moved the correct number of spaces, etc. For example, the interventionist would model on Taylor's turn, "I will *doublecheck*. One, two. Yep. Taylor rolled a two and moved two spaces." Later on Ellis' turn, the interventionist reminded Taylor to *doublecheck* for Ellis.

In addition, interventionists needed to establish the ways in which members engaged in mathematical activity. Interventionists would model and ask children to *explain* what they were doing as they counted objects or moved a game piece (e.g., I am on the number two and rolled a two, so I move two spaces and now I am on four; two and two more is four.). Interventionists asked children to *revoice* what other children said (e.g., Can you tell me what Karen just said?) or whether they *agreed* with another child's calculation (e.g., Kurt, is that right? Do you agree that Cathy's piece should be on four and not three?). Interventionists asked children to *justify* their thinking (e.g., how do you think Kerry knew that her piece should be on the four after she rolled a two?) and *narrate* one's actions (e.g., Let's see I was here on the three and rolled a two and now I am on five.).

In addition to the aforementioned direct strategies of scaffolding children's math talk, there were instructional strategies used for facilitating talk in general. Interventionists provided ample wait-time to allow children to respond, listened carefully when children were speaking, and provided positive reinforcement (e.g., Oh, Carol! I like the way you described your thinking out loud for us!).

Asking open-ended questions. Through open-ended questioning, interventionists taught children to *extend* their own thinking as well as that of their peers (e.g., Jackie, do you have another way of explaining Ron's answer?). When children did not respond to these types of questions appropriately, interventionists provided examples of appropriate responses through explicit instruction and direct modeling (e.g., Kim, you can say: if we add one more we would have five because five is one more than four. Now you say it).

In playing preschool games it would be impossible not to ask children typical closed-ended questions like, “How many is that?” However, interventionists made great efforts to also ask open-ended questions like:

- “How do you know it is five?”
- “Is there another way you can make five?”
- “Can you tell me about what you are doing right now?”
- “What do you notice about that?”
- “What's the same (different)?”
- “What do you think might happen if...?”

Transferring responsibility. Children were expected to participate in the small group math games where they would perform the mathematical tasks as outlined in the activities. In addition, children were expected to learn the rules of play and come to monitor the other children’s play. In a sense then, children were being held accountable for their independent play, not just to do what the teacher or other children told them to do when it was their turn. This strategy could lead to contention between children at times and so it was important for interventionists to use such opportunities to help children understand the rules of the community as described in the *scaffolding math talk* section. For example, the teacher might say, “Good job, Mark, for noticing that Manya moved one too many spaces. Way to pay attention! Manya, do you agree that you moved one too many? You can *doublecheck* if you like. If you do not agree, you can say so. We’ll just have to explain our thinking.”

Children were also expected to interact with both the interventionist and their peers through verbal and physical communication when they recognized a mathematical idea. For example, two children turn over cards that are equal in value and they both use the hand gesture for "equals." This is different than what occurs in the typical classroom whereby children are expected to withhold random observations as to avoid verbal outbursts, or at the least, raise their hand when they want to share a thought or observation. Interventionists had to be cautious that children not overgeneralize the social nature of the small group activity time to include any interesting conversation or interaction. The social activity had to focus on mathematical observations the small group math games generated. Interventionists redirected wayward children's attention to the game by asking children to narrate what another player was doing.

To summarize, the TAMS intervention included facilitating math games and a math talk learning environment. Facilitators played three games with small groups of children. The games were Hi-Ho Cherry-O, Walk-the-Line, and Card Wars. Walk-the-Line and Card Wars were played at two levels of difficulty. A small group of children played these games outside the classroom with a trained facilitator. Facilitators scaffolded children's math talk by modeling math talk, asking open ended questions, expecting children to participate in play, modeling and asking children to talk about observations they made about mathematics.

Research Site and Participants

Research Sites

A Head Start Community Action Agency (CAA) partnered with the author in this study. This CAA serves several urban/suburban and suburban/rural counties in Middle Tennessee. To enroll a child in any Head Start, families must be at 100% of the poverty level and therefore all of the children were from economically disadvantaged backgrounds. From within this CAA, three sites located in three counties were selected to participate by the Director of the CAA. To be eligible for selection, the site had to contain classrooms with an English-speaking majority of children who were three-years and ten months of age or older.

1. One center is predominately suburban and is located in a county with a poverty rate of 9% and 12% minorities. This center serves approximately 100 children in five classrooms, only four classrooms met the criteria of this project.
2. A second center is predominately rural and is located in a county with a poverty rate of 11% and 14% minorities. This center serves approximately 60 children in three classrooms, only two met the criteria of this project.
3. A third center is both urban and rural; its county has a poverty rate of 10% and 17% minorities. This center serves approximately 80 children in four classrooms, only three met the criteria of this project.

Research Participants

To recruit children into the study, the author and seven researchers working for the project visited the nine eligible classrooms within these three sites during the first week of school to deliver consent forms to teachers and parents. As parent consents were returned, children who were younger than three years and ten months were excluded from the study. As children were assessed, those who were unable to complete an assessment in English were also excluded from the study. These criteria were established so that identifying intervention effects would not be confounded with age or children's inability to speak English, both of which might hinder a child's willingness to participate in the math talk intervention. Children had not yet been identified by the schools for Individual Education Plans or as having behavioral issues, but these were not characteristics expected to interfere with the intervention. For this reason, those who were expected to meet qualifications for these special services were not excluded. Up to the first 15 consented and eligible students per classroom were assessed, after which no more children in that classroom were considered for the study.

While 99 children were initially included in the study, 9 children were withdrawn from their schools after a week long fall break and not present at the time of post-testing. Five of those seven children were able to be located by the author and post-tested in a location outside of the initial setting of the intervention, either in their current school or in their home. The final analytic sample included 95 children: 46 children in four classrooms located in the first center, 21 children in two classrooms from the second center, and 28 children in three classrooms from the third center. Of the 95 children, 44 were female (46%) and 51 were male (54%). The children were a mean of four years, three months of age at the time of pretest (with a range of three years, ten months to four years, 11 months). The race/ethnicity of the participants were 32 white (34%), 30 black (32%), 11 Hispanic (12%), and 22 other or unknown (22%).

Research Design

As described above, the research sample was drawn from nine classrooms located in three centers serving children from economically disadvantaged backgrounds. The sample represented a range of urban, suburban, and rural locations serving demographically diverse children. Children were the unit of randomization and blocked by classroom. Within each block, at least nine children were randomly assigned to one of three conditions. The randomization was conducted independently from their pretest scores. Children were assigned identification numbers that were then entered into a computer program; the program randomized ID numbers per classroom. The first three numbers listed by the program were assigned to Treatment Group A, the next three numbers were assigned to Treatment Group B, and all remaining numbers were assigned to Control Group C. The number of children in this last group ranged from three to eight children depending on the number of consented and eligible children in each classroom. Randomization of children into three experimental conditions resulted in an equal representation of gender, age, and race/ethnicity. Figure 2 presents a visual representation of the randomized block design and the resulting numbers of students per condition by classroom and by center.

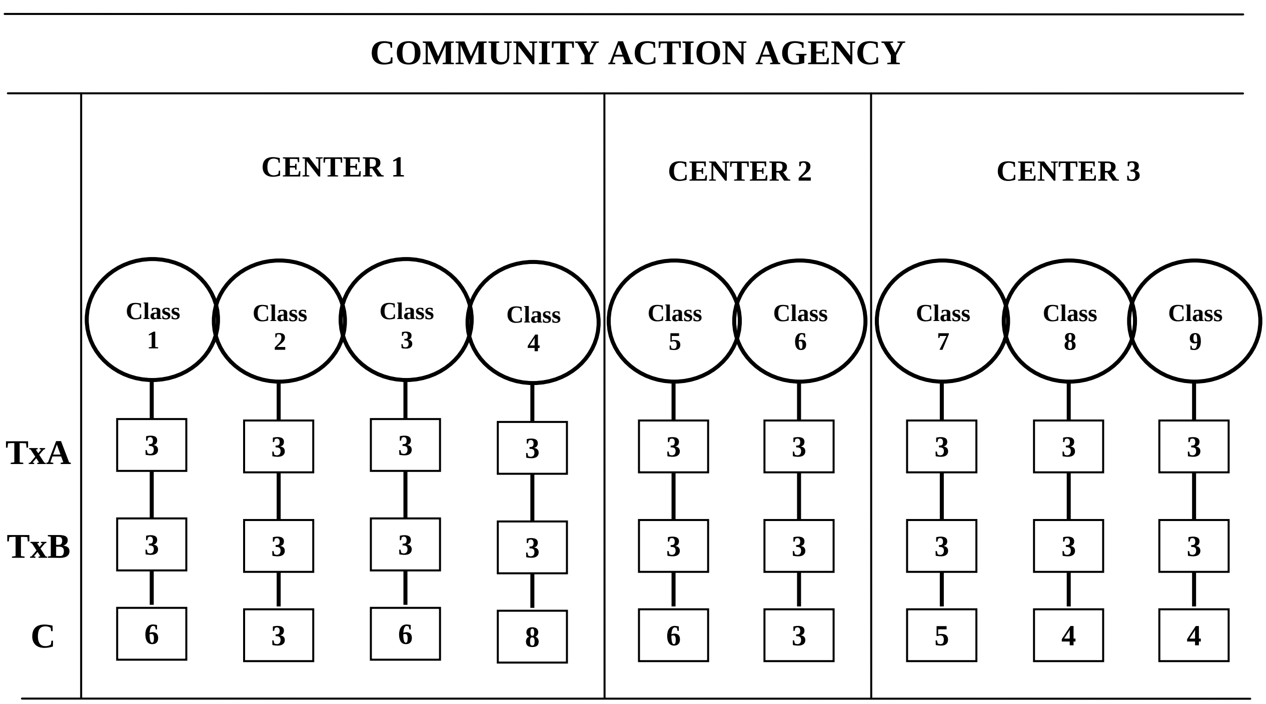


Figure 2. Randomized Block Design

The Experimental Groups

To test the effect of the math talk intervention, an 8-week intervention was implemented. The author trained the co-investigators from the pilot study to work as two additional interventionists so that there were three individuals facilitating small group math games with children. Each classroom had three experimental groups (two treatment groups and one control group). Each interventionist was responsible for three classrooms, conducting small group math games with the two treatment groups per classroom. The groups participated in small group math games, outside of the classroom, during free-play time so that no child missed his or her regular classroom instruction. Children in the treatment groups met in these small groups for 15- to 25-minutes of game time on two days each week. Therefore, children in Treatment Group A and children in Treatment Group B met with their interventionist for a total of 16 sessions and a total of 240 – 400 minutes of small group math games over the 8-week period.

Treatment group A: Math games with a focus on talk. Children in Treatment Group A participated in the full intervention as has been described. Interventionists facilitated math games and also facilitated a math talk learning environment. In order to test the benefit of focusing on math talk, and eliminate the benefits of simply *doing* more math, two treatment groups were necessary. Thus, a Math Games with a Talk Focus group (Math Games/Talk Focus) was compared to children who only played the math games (Math Games).

Treatment group B: Math games. Children in Treatment Group B participated in the small group math games as did the children in Treatment Group A; however, interventionists did not facilitate the math talk learning environment. In other words, interventionists did not scaffold the children's math talk nor provide similar supports in establishing a math talk learning environment. Children were not restricted from talking; interventionists guided the game being played through direct instruction. For example, interventionists might say, "Roll the die. That's two so move your game piece two spaces. One, two – now you are on four." Thus, the same language was heard by children in both Treatment Groups A and B; however, children in

Treatment Group B were not prompted to use the language themselves. Children in this group were not expected to add more talk than was required to play the games. They were not expected to support or correct their peers during play. They were not expected to share independent observations about mathematical ideas in either verbal or physical communication. It was anticipated that children in this group could spontaneously engage with one another concerning the mathematics at play, but this was not encouraged, or restricted by the interventionist. If children asked questions, interventionists answered them directly in order to continue with the game. Figure 3 shows the expected differences in interventionists' facilitation of the two treatment groups. For a more detailed account of expected differences between treatment groups by each game played, see Appendix B.

Math Games + *Focus on Talk*	Math Games
• Children hear facilitator model math-talk *and children are encouraged to use math talk*	• Children hear facilitator model math-talk
• Children are asked closed-ended questions *and open-ended questions*	• Children are asked closed-ended questions
• Interactions are between teacher and student *and interactions among peers are encouraged and supported*	• Interactions are between teacher and student
• Children are expected to participate *and take responsibility for game play (e.g., doublecheck for peers and explain why response was right or wrong)*	• Children are expected to participate in the game

Figure 3. Expected Differences in Facilitation of the Two Treatment Conditions

Control group C: Practice as usual. Children in this Control Group C were the "practice as usual" group. These children did not leave their regular classroom activities to participate in small group math games. They acted as the counterfactual condition in order to compare the math performance of playing math games or playing math games with math talk to the math performance of children who remained with their teachers during this time and engaged in all regular classroom activities.

Statistical Power

To determine the number of classrooms needed to detect statistically significant effects by the intervention, a power analysis was conducted using the Optimal Design software program (Liu, Spybrook, Congdon, Martinez, & Raudenbush, 2006). A justification for the estimates used in this analysis follows. With a randomized blocks design, each classroom is a block; within each block students are randomly assigned to conditions. To compare any two of those conditions, it was assumed that six children in a classroom would be randomly assigned to two different conditions (rather than nine children to three conditions). Therefore the number of children per classroom was estimated at n = 6. The blocks were treated as a fixed effect; that is to say, the classroom effect was not expected to vary significantly from classroom to classroom. The Intraclass Correlation Coefficient (ICC) values for similar outcome variables in other studies of pre-k curriculum effects generally range between .05 – .10. Because both classroom and school-level variance were assumed, an ICC value of .10 was used. In addition, it was assumed there would be an equal number of children per condition and no attrition. A school level pretest covariate that correlated at least .70 with the respective posttest was assumed; this is a figure consistent with prior pre-k studies with similar outcome variables. Therefore, with an alpha level of .10 and power of .80, the minimum detectable effect size was estimated to be .389. With a small sample, it was not realistic to suppose that inferences from the statistical results from this study could be generalized to the population of Head Start Centers in the U.S. or even to those in Tennessee. Thus, inferences about the efficacy of the intervention would be limited to this particular set of classrooms.

Measures

This study is interested in children's outcomes on five measures; four measures of children's early math skills and one measure of children's fluid reasoning. Three of the early math measures are standardized; the fourth is a non-standard measure created to closely align with the targeted skills of the intervention. The fifth is a standardized measure of children's fluid reasoning. These are each described in detail along with an account of the children's individual characteristics to be collected.

Math Outcomes

The *Test of Early Mathematics Ability* (TEMA) (Ginsburg, Baroody, & Pro 1983) measures the mathematics performance of children between the ages of three and nine. The test measures both informal and formal concepts and skills in the following domains: numbering skills, number-comparison facility, numeral literacy, mastery of number facts, calculation skills, and understanding of concepts. The TEMA has many items that do not require children obtain the correct answer to be counted correct if they apply the correct procedure or show evidence that they understand the concept. This is very useful in detecting small changes in children's informal mathematical knowledge and understandings. Descriptive results from the TEMA are presented using standardized scores while raw scores were used in all analyses.

The *Woodcock-Johnson III Tests of Achievement* (Woodcock, McGrew, & Mather 2001) is a more global measure of mathematical knowledge. There are two tests of math knowledge: *Applied Problems* (WJ-AP) and *Quantitative Concepts* (WJ-QC). *Applied Problems* requires children listen to the problem, recognize the procedure to be followed and then perform relatively simple calculations. Children's performance on these subtests is possibly related to their attention to mathematical language and so might detect differences between condition effects. *Quantitative Concepts* consists of two parts A and B; part A measures knowledge of mathematical concepts and symbols while part B measures understanding of the number line, which the TEMA does not measure. Another reason to include this assessment is because of the wide usage of WJIII in research on children's mathematical knowledge, thus allowing for comparison of results with other studies. Descriptive results from the WJIII subtests are presented using standardized scores; the IRT scaled *W* scores were used in all analyses.

Because the activities and games to be used in this study targeted specific number sense skills, it was important to have a more proximal measure of children's development than what the two global measures of math knowledge (TEMA and WJIII) might be able to capture. In addition, there were some targeted math skills that were not adequately represented by items on those two global measures. *The Number Sense Assessment* (NSA) was developed, therefore, to capture small changes in children's math skills and provide additional items to measure specific skills that were expected to improve as a result of the intervention.

Siegler (2009) defined number sense as the ability to approximate numerical magnitudes; for example, "estimating how many people can fit in a car, how much a car might weigh, and eventually, the product of 76 and 240." Siegler's (2009) description of the Number Knowledge Test used in his earlier studies included a magnitude comparison task and number line estimation task. However, Siegler reported that many preschoolers, even ones who could count perfectly from 1 to 10, were not able to estimate magnitudes accurately. For this reason, some of the tasks from his assessment were adapted to better suit the number sense abilities of preschool children.

The NSA asks children to complete four tasks. The first task measures children's ability to produce a set of objects and state the cardinal number of that set. This task was pilot tested in the exploratory phase of this project and found to be very effective in estimating children's ability to count and use one-to-one correspondence. The second and third tasks, taken directly from Siegler's test, measure children's ability to subitize (How many dots did you see?") and magnitude comparison ("Which side has more?" and "Which number is more?"). The fourth task is an adaptation of Siegler's number line task and was also piloted in the exploratory phase of the project. The zero and ten card are placed on the number line first. Then, children are prompted to place the 1-card where it belongs. If they cannot place it independently, the test administrator places the card where it belongs and says, "When we count we start with 1, so the 1 goes here. Can you now put the 1 where it belongs?" If the child is still unable to place the 1 on the number line in the correct location, the assessment ends. If, however, the child is able to place the 1 on the correct location on the number line then the child is given one card at a time with a numeral between 2 and 9 in a set random order.

A total raw score of 38 is possible on the NSA. Children's total raw scores are presented in descriptive results and were used in all analyses. The NSA instructions, scoring sheet, and testing booklet can be found in Appendix C. A detailed analysis of anticipated learning outcomes and corresponding test items can be found in Appendix D.

Reasoning Outcome

The *Leiter International Performance Scale-Revised* (Leiter-R) (Leiter, 1948) is a standardized, nonverbal measure of children's reasoning, visualization, attention, and memory. Neither the examiner nor the child is required to speak, and the child does not need to read or write and so this assessment is especially suitable for disadvantaged, nonverbal or non-English speaking preschoolers. Scores obtained by the Leiter-R have not been found to be significantly influenced by the child's educational, social, and family experiences (Leiter, 1948) and so was selected as an outcome to detect intervention effects on the reasoning skills of children that might not be identifiable by those measures that are language dependent. Two subtests of the Leiter-R were administered for this study, *Repeated Patterns* and *Sequential Ordering*. Together these subtests create a Fluid Reasoning Composite Score (LR-FR) (Leiter, 1948) that is strongly correlated with children's later school achievement. As instructed by the Leiter-R, this composite is calculated by doubling the scaled score on the *Repeated Patterns* subtest and adding the total to the scaled score received on the *Sequential Ordering* subtest. Descriptive results are presented using the standardized scores, although the Fluid Reasoning Composite Score (LR-FR) was used in primary analyses.

Covariates

The *Woodcock-Johnson III Tests of Achievement* also contains *The Picture Vocabulary* subtest. This subtest measures children's word knowledge; it is primarily an expressive language task. Because the intervention sought to use language to influence children's math and reasoning skills, their ability to express themselves might be correlated with growth in math knowledge. For this reason, it was important to collect baseline data on children's language and word knowledge. This assessment was also used at post-test in order to provide data for future exploratory analyses not to be included in this study. As with the other WJIII subtests, descriptive results are presented using standardized scores while IRT scaled *W* scores were used in all analyses.

The demographic characteristics of children are their gender, race/ethnicity, and age at time of testing. These variables provided descriptive information about the sample and for future exploratory analyses not to be included in this study.

Data Collection Procedure

Information from parental consent forms provided descriptive information for each child on gender, race/ethnicity, and date of birth. Assessors administrating pretests determined children's English Language Learning (ELL) status. Interventionists facilitating small group math games noted children who were emotionally or behaviorally challenged and likely to receive an Individualized Education Plan.

Assessors working for the project had to pass certification to administer assessments to consented children on the measures already described. Children who consented to go with the assessor outside of the classroom were then individually tested on two measures (The Leiter-R and all subtests of the WJIII) on one day and on the other two measures (TEMA and NSA) another day so that no single assessment period lasted for longer than 15-30 minutes. All assessments were conducted within 14 working days of the first day pretests began. After assignment to condition, the intervention began. During the eight-week intervention phase, video and audio recordings of sessions were collected for further exploratory analyses not to be included in this study. After the intervention ended, all participating children in the study were assessed within 14 working days of the first day post-testing began.

Statistical Analyses

The hypotheses of this study were:

1. Children who participate in Treatment Groups A and B will learn more math than children in Control Group C.
2. Children who participate in Treatment Group A will learn more math and reasoning skills than children in Treatment Group B.

Primary Analyses

Data analyses focused on the effects of the intervention on the child outcomes. Children were nested within classrooms, so all analyses were done using linear mixed modeling in SPSS. The first step was to examine the equivalence between the treatment and control groups on the pretests and other key descriptive variables.

To answer the two hypotheses, each analysis examined the intervention effects on each of the outcome variables. There were five outcome measures of interest (four measures of early math skills and one measure of fluid reasoning). The three standardized math measures were the *Test of Early Mathematics Ability* (TEMA), *WJIII Applied Problems* (WJ-AP), and *WJIII Quantitative Concepts* (WJ-QC). The fourth outcome of early math skills, *The Number Sense Assessment* (NSA), was a non-standard measure created as a proximal measure of the intervention effects. The standardized measure of children's fluid reasoning, the *Leiter-R Fluid*

Reasoning (LR-FR), was a composite score created from the *Repeated Patterns* and *Sequential Order* subtests (Leiter, 1948).

To test the first hypothesis, children in the Treatment Groups were compared to children in the Control Group; thus, children in Treatment Groups A and B are compared to children in the Control Group C. To test the second hypothesis, children in Treatment Group A were compared to children in Treatment Group B; thus, children in Control Group C were not included in these analyses. So there were a total of 10 independent models analyzed for the two hypotheses. An additional two models were conducted in order to test the effects of condition on *WJIII Picture Vocabulary* (WJ-PV); one tested the effects of being in the Treatment Groups as compared to the Control and the other compared the effects of being in Treatment Group A as compared to Treatment Group B. Each model was analyzed at the child-level using the respective pretest as a covariate in the model, making these residualized gain analyses. Besides pretest scores, children's personal characteristics were examined as child level covariates. The classroom blocking factor was included as a classroom level covariate for both hypotheses.

Effect sizes are often calculated to determine the magnitude of the difference in the mean scores of two groups on some measure. One advantage of examining effect sizes is they are expressed in standard deviation units and therefore allow for comparison of different measures within a study having different scales. A relatively large and educationally meaningful effect size can be achieved that does not reach statistical significance. Thus, in addition to the models described above, effect sizes were calculated to determine the magnitude of intervention effects on all outcome measures (TEMA, WJ-AP, WJ-QC, NSA, LR-FR, and WJ-PV).

Secondary Analyses

While standardized measures provide global and distal measures of children's growth that allow for comparisons to national norms and other similar studies, they often do not capture small but important changes targeted by an intervention. To capture changes in children's number sense not detected by the standardized measures, the individual items from all math measures (TEMA, WJ-AP, WJ-QC, and NSA) were conceptually grouped into 10 categories called early numeracy skills. Nine of the 10 early numeracy skills were skills targeted by the intervention; the 10th category was composed of test items measuring skills not expected to change as a result of the intervention. To test the practical significance of intervention effects, linear mixed models regressing early numeracy skills posttest scores on condition were conducted. Covariates included in the models were children's pretest score on the respective early numeracy skill, age at the time of pretest, and the blocking factor. Table 2 displays the 10 early numeracy skills, the total number of test items grouped into each skill, and examples of test items used to measure that skill. For a detailed analysis of the test items grouped into each skill, see Appendix D.

Table 2

Early Numeracy Skills.

Skill	No. of Test Items	Examples of Test Items
Counting	29	1, 2, 3, now you count by yourself and keep going until I tell you to stop; How many apples are in this picture?
One-to-One Correspondence	5	(Child has a mat and tokens in a cup. Assessor puts 3 tokens on her mat.) Make yours just like mine.
Producing Groups	12	Give me __ cubes.
Cardinality	9	How many stars did you count? How many cubes did you put in the cup?
Subitizing	14	(Child is shown a paper with __ number of dots on it for a count of 2 seconds and then it is covered). How many did you see?
Magnitude Comparison	15	(Child is shown a paper with two sets of dots on either side, divided by a vertical line) Which side has more?
Numeral Recognition	30	What number is this?
Number Line	25	What number comes after 2? What number comes between 5 and 7? What number comes before 3?
Adding/Composing Number	13	If you had two books and got two more books, how many books would you have?
Items not related to a TAMS learning objective	11	What is this called (circle)? Tell me the days of the week.
Overall	163	

CHAPTER IV

RESULTS

Initial Analyses

Ninety-nine children were originally consented and eligible to participate in this study. These 99 children had pretest scores and participated in one of three experimental conditions of the TAMS intervention. However, nine of the 99 children were withdrawn from Head Start following a weeklong break from school when posttesting began. One-way ANOVAs confirmed no identifiable differences in the personal characteristics (i.e., age, gender, race/ethnicity) or pretest scores of the nine children who were withdrawn and the rest of the original sample. Five of the nine withdrawn children were located and subsequently tested in their homes within four days following the testing of the ninety children still enrolled in Head Start. Four of the nine withdrawn children could not be located to collect posttest data and were excluded from further analyses. The original sample of 99 children was therefore reduced to 95 (one child was lost from the Math Games group and three were lost from the Control group). The following results are reported for the final analytic sample of 95 children who participated in the study and had valid pretest and posttest data.

Initial explorations of all variables of interest showed the assessment data to be normally distributed at both time points. Using linear mixed models, no statistically significant differences between condition groups were found on any of the pre-intervention measures. Therefore, the groups were assumed to be equivalent at time of pretest. One outlier was identified within the sample; this child (who was in the Math Games group) scored above the inter-quartile range on all math measures. Analyses using linear mixed models to detect treatment effects were run excluding this child's scores, yet the pattern of significance did not change from the pattern in analyses including this potential outlier. Therefore, data from this child were included in reporting results.

Using linear mixed models, a dummy-coded variable for interventionist was included in analyses to test for any significant association with outcome measures. This variable was not related to any of the outcomes, so there was no reason to believe that the effect of the intervention on children's outcomes would differ according to the effectiveness of one or two interventionists. As the inclusion of the interventionist variable did not change the overall pattern of significance, it was not included in any of the models.

The entire sample was from economically disadvantaged backgrounds; without variation in economic status, there was no variation in outcomes that might be explained by the inclusion of this variable in the final models. Neither gender nor race/ethnicity was significantly correlated with outcomes, so these were not included in subsequent models. The exclusion of gender from analyses was supported by prior research reviewed in this paper that has not found gender to be associated with math performance in early childhood (Bodovski & Farkas, 2007; Entwisle & Alexander, 1993; Jordan et al., 1992). Likewise, prior research has found that kindergarten children of different races do not differ on mathematics tasks when socioeconomic status is taken into account (Ginsburg & Russell, 1981).

Primary Analyses

Primary analyses sought to test the effects of condition on four main math outcomes and one fluid reasoning outcome. Before estimating the models, correlations were conducted in order to investigate covariation among the five outcomes hypothesized to be affected by the TAMS intervention. The scores on the outcomes to be correlated and used in statistical analyses were children's (a) raw scores on the TEMA, (b) *W* scores on the WJ-AP subtest, (c) *W* scores on the WJ-QC subtest, (d) raw scores on NSA, and (e) composite scores on the LR-FR. Results in Table 3 show medium strength intercorrelations among the math measures at the time of pretest ranging from r = .57 to .75, but LR-FR was related only to WJ-QC.

Table 3

Two-tailed Pearson Correlations Among Math and Reasoning Outcomes at Pretest

	WJ-AP	WJ-QC	NSA	LR-FR
TEMA	.67**	.69**	.75**	.15
WJ-AP	——	.64**	.57**	.06
WJ-QC		——	.65**	.21**
NSA			——	.08

* $p < .05$ ** $p < .01$

In addition, the covariation between age at pretesting, expressive language ability (as measured by the WJ-PV), and the math outcomes were investigated. Results in Table 4 indicated the need to control for age in subsequent analyses and warranted further exploration of language as a moderator of the effect of condition on math outcomes.

Table 4

Two-tailed Pearson Correlations Among Outcomes at Pretest and Selected Variables

	TEMA	WJ-AP	WJ-QC	NSA	LR-FR
Age	.32**	.31**	.23**	.30**	-.31**
WJ-PV	.26*	.46**	.34**	.23*	-.05

* $p < .05$ ** $p < .01$

Hypothesis I

To test the first hypothesis, c*hildren who participate in small group math games will gain more early math and reasoning skills than children who do not play these games*, descriptive and statistical results are presented for children who received math games intervention (treatment group) compared to children in the control group. The treatment group is comprised of children from both the Math Games/Talk Focus group and the Math Games group. The control group is comprised of children who did not participate in any TAMS activities.

Descriptive results. Table 5 presents the means, standard deviations, and ranges of children's scores on the four main math outcomes (TEMA, WJ-AP, WJ-QC, and the NSA). Standard scores are presented for the standardized measures (TEMA, WJ-AP, and WJ-QC) and raw scores are presented in the descriptive table for the non-standard NSA. The advantage of using standardized measures is the ability to compare the relative performance of the experimental groups with the population on which the tests were normed. A standard score indicates how a child performed relative to the population mean.

Table 5

Descriptive Statistics for the Main Math Measures: Treatment (N=53) and Control (N=42)

Source	Mean	Standard Deviation	Minimum	Maximum
TEMA				
Treatment Pretest	82.60	12.60	63	127
Treatment Posttest	86.68	13.67	64	133
Control Pretest	77.98	9.48	63	107
Control Posttest	78.19	9.94	62	104
WJ-AP				
Treatment Pretest	97.32	10.62	74	130
Treatment Posttest	101.81	9.93	80	124
Control Pretest	93.05	8.90	74	111
Control Posttest	94.36	10.67	74	117
WJ-QC				
Treatment Pretest	90.23	8.31	69	118
Treatment Posttest	96.85	12.34	72	141
Control Pretest	87.83	7.41	76	110
Control Posttest	90.43	10.97	74	126
NSA				
Treatment Pretest	10.30	5.86	0	26
Treatment Posttest	15.38	8.01	3	41
Control Pretest	9.52	6.08	0	28
Control Posttest	11.24	6.51	3	28

Note. Standard scores are reported for the TEMA, WJ-AP, and WJ-QC. Raw scores are reported for the NSA.

Table 6 presents the means, standard deviations, and ranges of children's scores on the fluid reasoning outcome (LR-FR) and children's expressive language (WJ-PV). The standard scores are presented for both the LR-FR and WJ-PV.

Table 6

Descriptive Statistics for the Leiter-R Fluid Reasoning and WJ-Picture Vocabulary: Treatment (N=53) and Control (N=42)

Source	Mean	Standard Deviation	Minimum	Maximum
LR-FR				
Treatment Pretest	92.30	14.77	56	127
Treatment Posttest	93.57	14.08	63	143
Control Pretest	89.76	15.21	56	114
Control Posttest	89.00	14.26	56	116
WJ-PV				
Treatment Pretest	97.02	18.34	35	131
Treatment Posttest	99.23	17.43	42	128
Control Pretest	94.93	16.77	45	128
Control Posttest	95.98	16.84	45	126

Estimates of main effects. To estimate the fixed effects of the TAMS intervention on children's outcomes, six linear mixed model analyses were conducted regressing posttest scores on condition. Two-level models, nesting children within classrooms, were conducted separately for each outcome. The raw or *W* scores were employed for each analysis, not the standard scores that were used for illustrative purposes in the descriptive tables. Covariates included in the models were children's pretest scores on each respective measure and age at the time of pretest. Estimated marginal means were generated for pairwise comparisons between children in the treatment group and children in the control group, then analyzed for statistical significance. In addition, Cohen's *d* effect sizes were calculated using the covariate adjusted mean difference between the two groups' scores on any outcome of interest divided by the unadjusted pooled standard deviation of those two groups. Table 7 displays the unstandardized beta coefficients, standard errors, and significance values for the treatment group compared to the control group.

Table 7

Effects of Condition on Main Outcomes: Treatment (N=53) Compared to Control (N=42)

	Treatment		
Source	*b*	*SE*	*p*
TEMA Total Score	1.74	0.60	.005
(Effect size)	0.33		
WJ Applied Problems	9.24	3.09	.004
(Effect size)	0.46		
WJ Quantitative Concepts	4.97	2.01	.015
(Effect size)	0.39		
Number Sense Assessment	3.54	1.14	.002
(Effect size)	0.48		
Leiter-R Fluid Reasoning	1.70	1.35	.210
(Effect size)	0.24		
WJ Picture Vocabulary	1.28	1.49	.394
(Effect size)	0.29		

Children in the treatment group made significantly greater gains than children in the control group on all of the main math measures. The effect sizes for all math measures were moderate, ranging from d = .33 for the TEMA to d = .48 for the NSA. There was no significant condition effect on the LR-FR. Therefore, Hypothesis I was partially supported; children who played the small group math games gained more early math skills; however, there were no effects on children's fluid reasoning skills.

Both the LR-FR and WJ-PV were first tested as outcomes and then as moderators of the effect of condition on math outcomes. No main condition effects were found for either outcome; the math intervention had effects only on children's math outcomes. To test whether the effect of condition on children's math outcomes differed by either their expressive language skills or their fluid reasoning skills, 10 additional linear mixed models were conducted. Each math outcome was regressed on condition with the respective math pretest, moderator pretest, and interaction term (i.e., Condition x LR-FR or Condition x WJ-PV) included in the model. No moderating effects were found; the effects of math games on math outcomes were not affected by children's initial fluid reasoning or expressive language skills.

Hypothesis II

To test the second hypothesis, *children who participate in small group math games and who are encouraged to engage in math-talk will gain more early math and reasoning skills than those who participate in the same games without that encouragement to talk about math*, descriptive and statistical results are presented for children in the Math Games/Talk Focus group as compared to the Math Games group. Children in the control group were not included in the following analyses.

Descriptive results. Table 8 presents the means, standard deviations, and ranges of children's scores on the four main math outcomes (TEMA, WJ-AP, WJ-QC, and the NSA). Standard scores are presented on the standardized measures (TEMA, WJ-AP, and WJ-QC) and raw scores are presented for the non-standard NSA.

Table 8

Descriptive Statistics for Main Math Measures: Math Games/Talk Focus (N=27) and Math Games (N=26)

Source	Mean	Standard Deviation	Minimum	Maximum
TEMA				
Math Games/Talk Focus Pretest	81.59	10.55	63	107
Math Games/Talk Focus Posttest	86.44	11.49	66	109
Math Games Pretest	83.65	14.57	63	127
Math Games Posttest	86.92	15.86	64	133
WJ-AP				
Math Games/Talk Focus Pretest	95.15	8.98	74	113
Math Games/Talk Focus Posttest	101.59	9.24	85	121
Math Games Pretest	97.50	12.27	74	130
Math Games Posttest	102.04	10.77	80	124
WJ-QC				
Math Games/Talk Focus Pretest	88.41	7.69	69	103
Math Games/Talk Focus Posttest	96.37	11.69	72	118
Math Games Pretest	92.12	8.66	80	118
Math Games Posttest	97.35	13.20	78	141
NSA				
Math Games/Talk Focus Pretest	10.41	4.73	0	19
Math Games/Talk Focus Posttest	16.00	7.06	5	41
Math Games Pretest	10.19	6.94	1	26
Math Games Posttest	14.73	8.97	3	37

Note. Standard scores are reported for the TEMA, WJ-AP, and WJ-QC. Raw scores are reported for the NSA.

Table 9 presents the means, standard deviations, and ranges of children's scores on the LR-FR and WJ-PV. Standard scores are presented for both measures.

Table 9

Descriptive Statistics for the Leiter-R Fluid Reasoning and WJ-Picture Vocabulary: Math Games/Talk Focus (N=27) and Math Games (N=26)

Source	Mean	Standard Deviation	Minimum	Maximum
LR-FR				
Math Games/Talk Focus Pretest	90.78	14.97	56	110
Math Games/Talk Focus Posttest	91.63	13.37	63	112
Math Games Pretest	93.88	14.67	65	127
Math Games Posttest	95.58	14.77	75	143
WJ-PV				
Math Games/Talk Focus Pretest	98.00	18.53	53	131
Math Games/Talk Focus Posttest	99.41	17.82	58	128
Math Games Pretest	96.00	18.45	35	121
Math Games Posttest	99.04	17.37	42	124

Estimates of main effects. To estimate the fixed effects of playing math games with a focus on math talk on children's outcomes, six linear mixed model analyses were conducted regressing the posttest scores on condition. Two-level models, nesting children within classrooms, were conducted separately for each outcome. The raw or *W* scores, not the standard scores, were used for each outcome. Covariates included in the models were children's pretest scores on each respective measure and age at the time of pretest. Estimated marginal means were generated for pairwise comparisons between children in the Math Games/Talk Focus group and children in the Math Games group, then analyzed for statistical significance. In addition, Cohen's *d* effect sizes were calculated using the adjusted mean difference between the two groups' scores on any outcome of interest divided by the unadjusted pooled standard deviation of those two groups. Table 10 displays the unstandardized beta coefficients, standard errors, and significance values for the Math Games/Talk Focus group compared to the Math Games group.

Table 10

Effects of Condition on Main Outcomes: Math Games/Talk Focus (N=27) Compared to Math Games (26)

	Math Games/Talk Focus		
Source	*b*	*SE*	*p*
TEMA Total Score	1.00	0.82	.228
(Effect size)	0.17		
WJ Applied Problems	1.49	3.50	.672
(Effect size)	0.09		
WJ Quantitative Concepts	3.52	2.49	.165
(Effect size)	0.28		
Number Sense Assessment	0.51	1.78	.775
(Effect size)	0.06		
Leiter-R Fluid Reasoning	-0.72	1.73	.679
(Effect size)	-0.10		
WJ Picture Vocabulary	-1.91	2.21	.392
(Effect size)	-0.18		

There were no statistically significant differences between the Math Games/Talk Focus group and the Math Games group. However, the effect sizes of being in the Math Games/Talk Focus group on children's gains on all four of the math measures were small to moderate, ranging from d = .06 for the NSA to d = .28 for the WJ-QC. Although not statistically significant, there was a greater effect on the LR-FR and WJ-PV by children in the Math Games group than children in the Math Games/Talk Focus group. Hypothesis II was not fully supported as it fails the test of statistical significance. Yet, the consistently larger effects on the Math Games/Talk Focus group over the Math Games group on all of the math measures warrants further investigation.

Secondary Analyses

Two analyses were conducted following the primary analyses. The first of these analyses sought to examine the effects of condition on children's specific early numeracy skills. The second of these analyses sought to examine treatment effects on children's learning gains by the number of days they participated in the small group math gains.

Early Numeracy Skills

The early numeracy skills were the 10 categories created by regrouping items from all the math measures (TEMA, WJ-AP, WJ-QC, and the NSA). Nine of the 10 early numeracy skills were skills targeted by the TAMS intervention; the 10th category was comprised of test items measuring knowledge not expected to be affected by the intervention (see Appendix D). The 10 categories (and number of test items included in each category) were: counting (29), one-to-one correspondence (5), producing groups (12), cardinality (9), subitizing (14), magnitude comparison (15), numeral recognition (30), number line (25), adding/composing number (13), and Not a TAMS Focus (11). This section presents descriptive and statistical results using the raw scores (the number of items answered correctly per skill) of children in all three experimental conditions (Math Games/Talk Focus group, Math Games group, and Control group).

Descriptive statistics. Table 11 presents the means, standard deviations, and ranges of scores on the 10 early numeracy skills for children in the three experimental conditions.

Estimate of fixed effects. To estimate the fixed effects of the TAMS intervention, 10 linear mixed model analyses were conducted regressing children's early numeracy skills posttest scores on condition. Two-level models, nesting children within classrooms, were conducted separately for each skill. Covariates included in the models were children's pretest scores on each respective skill and age at the time of pretest. Table 12 displays the unstandardized beta coefficients, standard errors, and significance values for the Math Games/Talk Focus group and Math Games group compared to the Control group.

Table 11

Descriptive Statistics for the Early Numeracy Skills

Source	Math Games/Talk Focus				Math Games				Control			
	M	*SD*	*Min*	*Max*	*M*	*SD*	*Min*	*Max*	*M*	*SD*	*Min*	*Max*
Counting at Pretest	18.67	6.88	6	28	17.54	7.33	6	36	15.10	7.23	4	30
Counting at Posttest	24.19	6.01	10	34	23.73	6.81	9	40	17.76	7.77	5	35
1:1 Correspondence at Pretest	1.00	1.21	0	4	1.15	1.66	0	6	0.67	1.14	0	5
1:1 Correspondence at Posttest	2.19	1.64	0	5	1.88	1.97	0	6	1.21	1.51	0	5
Producing Groups at Pretest	10.22	5.37	2	20	9.04	5.02	2	21	8.14	4.66	0	20
Producing Groups at Posttest	12.89	4.41	5	21	11.85	4.86	4	21	8.62	4.60	1	19
Cardinality at Pretest	5.07	2.27	1	9	5.62	2.47	1	10	3.88	2.59	0	9
Cardinality at Posttest	6.70	2.00	4	10	6.38	2.40	2	10	4.83	2.78	0	10
Subitizing at Pretest	4.70	2.69	0	9	4.42	2.82	0	10	3.64	2.53	0	10
Subitizing at Posttest	6.74	3.39	1	21	6.42	3.36	2	14	4.07	2.36	0	9
Magnitude Comparison at Pretest	8.44	4.84	0	17	8.77	6.23	0	23	7.90	5.49	0	23
Magnitude Comparison at Posttest	13.26	5.22	1	23	11.27	6.18	1	25	9.50	5.18	1	23
Numeral Recognition at Pretest	4.96	3.67	0	13	5.54	6.40	0	23	5.05	4.88	0	19
Numeral Recognition at Posttest	10.04	5.55	1	25	8.85	8.95	0	32	6.01	6.69	0	31
Number Line at Pretest	1.78	2.44	0	8	3.62	7.57	0	28	2.31	4.56	0	22
Number Line at Posttest	6.15	6.05	0	25	6.27	9.38	0	34	3.00	5.94	0	27
Addition/Composing at Pretest	2.44	2.56	0	8	2.35	2.48	0	11	1.64	2.07	0	8
Addition/Composing at Posttest	4.44	3.04	0	12	3.31	3.29	0	12	2.02	2.332	0	10
Not a TAMS Focus at Pretest	2.41	2.29	0	8	3.00	3.30	0	14	1.74	2.53	0	11
Not a TAMS Focus at Posttest	4.52	4.01	0	14	4.46	3.86	1	14	2.71	2.89	0	13

Table 12

Effects of Condition on Early Numeracy Skills: Math Games/Talk Focus (N=27) and Math Games (N=26) Compared to Control (N=42)

Source	Math Games/Talk Focus			Math Games		
	b	*SE*	*p*	*b*	*SE*	*p*
Counting	3.88	1.13	.001	4.41	1.14	.000
(Effect size)	0.54			0.58		
One to One Correspondence	0.67	0.32	.038	0.31	0.33	.347
(Effect size)	0.43			0.18		
Producing Groups	2.99	0.89	.001	2.80	0.90	.002
(Effect size)	0.66			0.60		
Cardinality	1.15	0.49	.021	0.63	0.51	.224
(Effect size)	0.46			0.24		
Subitizing	2.14	0.68	.002	2.02	0.68	.004
(Effect size)	0.76			0.72		
Magnitude Comparison	3.32	0.99	.001	1.42	1.01	.161
(Effect size)	0.64			0.26		
Numeral Recognition	4.19	1.07	.000	2.62	1.08	.017
(Effect size)	0.67			0.34		
Number Line	3.64	1.02	.001	2.20	1.04	.037
(Effect size)	0.61			0.30		
Addition/Composing	1.88	0.59	.002	1.01	0.60	.095
(Effect size)	0.72			0.37		
Not a TAMS Focus	1.04	0.56	.069	0.61	0.58	.296
(Effect size)	0.31			0.19		

Results show that children in the two treatment groups gained significantly more early numeracy skills than did children in the Control group. The Math Games/Talk Focus group gained significantly more than the Control group on all nine of the early numeracy skills targeted by the TAMS intervention. The Math Games group gained significantly more than the Control group on five of the nine early numeracy skills (counting, producing groups, subitizing, numeral recognition, and number line), but not significantly more on four of the skills (one to one correspondence, cardinality, magnitude comparison, and addition/composing). The magnitude of the TAMS intervention effect on children in both treatment groups (Math Games/Talk Focus group and Math Games group) was moderately large and positive for all 10 early numeracy skills. Although there were no statistically significant differences identified between the two treatment groups, the effect sizes for the Math Games/Talk Focus group were almost twice as large as those for the Math Games group on six of the nine early numeracy skills. The gains made by the two treatment groups were similar in magnitude on producing groups and subitizing. The gains made by the Math Games group were slightly larger than the Math Games/Talk Focus group on counting.

The Effect of Dosage

All prior analyses were conducted using the Intent-to-Treat principle in which participants' data are not excluded based on their compliance to the study. In other words, the previously presented results included all children assigned to condition and with complete pre- and posttest data, regardless of the number of days they were absent from school. The following analyses examine the effect of the treatment according to how much of the treatment was received.

The treatment was conducted two days per week over eight weeks; children in the two treatment groups had the opportunity to participate in 16 small group math game sessions. The mean (standard deviation, range) number of days children participated in the Math Games/Talk Focus group was 10.11 (*2.65, 2 – 13*). The mean (standard deviation, range) number of days children participated in the Math Games/Talk Focus group was 10.46 (*2.32, 4 – 13*). Attendance was not known for the children in the Control group; for this reason, their data were excluded from this analysis. A one-way ANOVA showed the mean number of days children participated in small group math games did not differ between the Math Games/Talk Focus group and the Math Games group ($F = .262, p = .61$).

Fifteen two-level models, nesting children within classrooms, were conducted regressing the number of days participated on outcomes. Covariates included in the models were children's pretest scores on each respective measure or skill and age at the time of pretest. Results are presented in Table 13. Results show the number of days participated is a significant predictor of the outcomes on the TEMA ($p < .05$) and NSA ($p < .10$), but not the WJ-AP or WJ-QC. Of the early numeracy skills, the number of days participated is a significant predictor ($p < .05$) of three early numeracy skills (counting, one to one correspondence, producing groups) and approaches significance ($p < .10$) on another five skills (subitizing, magnitude comparison, number line, addition/composing, and not a TAMS focus).

Table 13

The Effect of Dosage on Outcomes for Children in the Treatment Condition (N=53)

Source	*b*	*SE*	*p*
Main math outcomes			
TEMA Total Score	0.40	0.16	.014
WJ Applied Problems	0.09	0.74	.900
WJ Quantitative Concepts	0.61	0.52	.245
Number Sense Assessment	0.55	0.31	.100
Early numeracy skills			
Counting	0.48	0.23	.047
One to One Correspondence	0.06	0.24	.014
Producing Groups	0.38	0.18	.048
Cardinality	0.03	0.11	.803
Subitizing	0.27	0.17	.124
Magnitude Comparison	0.39	0.26	.132
Numeral Recognition	0.33	0.26	.199
Number Line	0.42	0.22	.066
Addition/Composing	0.24	0.16	.136
Not a TAMS Focus	0.25	0.15	.103

Summary

To test the two hypotheses of the TAMS intervention, primary analyses examined the effect of condition on four main math outcomes and one fluid reasoning outcome. As hypothesized, children who participated in the treatment group gained significantly more on all four main math measures than children who did not participate in the TAMS intervention;

however, no significant differences were made on fluid reasoning skills. The magnitudes of the intervention's effects on the treatment group as compared to the control group were small to moderate, ranging from *d* =.33 to .48. It was also hypothesized that, within the treatment group, children who participated in the Math Games/Talk Focus group would gain more math and reasoning skills than children in the Math Games group. However, no statistically significant differences between gains made by children in either treatment group were detected, thus this hypothesis was not supported. The magnitudes of the intervention's effects on all of the main math measures, however, were larger for the Math Games/Talk Focus group than for the Math Games group, ranging from *d* =.06 to .28.

Secondary analyses examined the effect of condition on children's early numeracy skills, 10 categories composed of test items from the four main math measures. Results showed children in the Math Games/Talk Focus group gained significantly more than children in the Control group on all of the early numeracy skills. Children in the Math Games/Talk Focus group gained significantly more early numeracy skills than children in the Control group on five of the early numeracy skills. While there were no statistically significant differences between gains made by the two treatment groups, the effects on the Math Games/Talk Focus group were larger than those on the Math Games group on two early numeracy skills and almost twice as large on six more skills.

Another secondary analysis was conducted to examine the effect of dosage on children's gains in math knowledge. This analysis considered how the number of days children participated in the TAMS intervention affected gains. Regardless of treatment group, the number of days children participated in small group math games had a significant effect on children's outcomes on the TEMA and NSA. When considering children's early numeracy skills, the number of days children participated in either treatment group significantly predicted three skills: counting, one-to-one correspondence, and producing groups.

In conclusion, results from statistical analyses show strong evidence in support of the TAMS intervention. Effect sizes for participating in the small group math games were positively and significantly related to children's gains in math knowledge. In addition, the magnitude of the effects on the Math Games/Talk Focus group consistently superseded those for participating in the Math Games group.

CHAPTER V

SUMMARY, DISCUSSION, AND CONCLUSIONS

This study examined the effect of a math talk intervention on the early math and reasoning skills of children from economically disadvantaged backgrounds. After being pretested on four math measures and one fluid reasoning measure, 99 children were independently and randomly assigned to one of three experimental conditions within their classrooms. Two of the three conditions participated in an 8-week intervention in which children played math games with a trained interventionist. In addition to playing math games, one of the two groups was engaged in a math talk learning environment. The third group carried on business as usual in their classrooms, serving as the counterfactual condition. When the intervention ended, 95 children were posttested on the same five measures used for the pretests (four children were withdrawn and unable to be located). The primary analyses used child-level residualized gain scores on the five measures as indicators of change in children's mathematical and reasoning competencies. Secondary analyses used child-level residualized gain scores on ten early numeracy skills derived from the testing measures to judge the intervention's practical significance. This chapter presents a summary of the analytical results, a discussion of the findings, and a review of the study's strengths and limitations.

Summary of Results

Primary Analyses

The first hypothesis of this study predicted that children who participated in the intervention by playing math games focused on number would gain more math and reasoning skills than would children who did not participate in these activities. To test this hypothesis, five linear mixed models were conducted to regress four math outcomes and one fluid reasoning outcome on experimental condition. Children were nested in classrooms and covariates included in the model were children's age at the time of pretest, their pretest score on the respective measure, and the blocking factor. Pairwise comparisons between children in the treatment group and children in the control group indicated this hypothesis was partially supported: children who participated in the intervention gained more math skills than children who did not participate; however, there were no changes in the fluid reasoning scores among children in any condition. The effect sizes for being in the treatment group as compared to the control group ranged from d = .33 - .48 on the math measures.

The second hypothesis of this study predicted that children who played math games with a focus on math talk would gain more math and reasoning skills than children who only played the math games. To test this hypothesis, five linear mixed models were conducted to regress four math outcomes and one fluid reasoning outcome on experimental condition, excluding those who were in the Control group. Children were nested in classrooms and covariates included in the model were children's age at the time of pretest, their pretest score on the respective measure, and the blocking factor. Pairwise comparisons between the two treatment groups indicated this hypothesis was not supported; there were no statistically significant differences found between children in the two treatment groups on gains made on math or reasoning skills. Despite the lack of statistical significance, the effects of being in the intervention group that played math games with a focus on talk were consistently larger on all math measures, ranging from d = .06 - .28, than they were for being in the group that only played math games.

Secondary Analyses

When items from the four math measures were regrouped into ten categories, it was possible to compare the three experimental groups' gains on specific early numeracy skills targeted by the intervention. The early numeracy skills on which students were being compared were count, cardinality, numeral recognition, one-to-one correspondence, producing a group, magnitude comparison, number line, addition, subtraction, and those items that were not targeted by the intervention (e.g., shape names, days of the week). Linear mixed models were conducted to regress outcomes from ten early numeracy skills on experimental condition, each in an independent analysis. Children were nested in classrooms and covariates included in the model were children's age at the time of pretest, their pretest score on the respective early numeracy skill, and the blocking factor.

Pairwise comparisons among the three conditions showed children in the group that played math games with a focus on talk gained significantly more than children in the Control group on all nine early numeracy skills targeted by the intervention. Children in the group that played math games gained significantly more than children in the Control group on five of the nine early numeracy skills (counting, producing groups, subitizing, numeral recognition, and number line). The effect sizes on the nine early numeracy skills targeted by the intervention ranged from d = .43 - .76 for being in the group that played math games with a focus on talk as compared to the control group and ranged from *d* = .18 - .72 for being in the group that played math games as compared to the control group. There was no statistically significant condition effect on the 10^{th} early numeracy category, items that were not a focus of the intervention, although the direction of effects favored children in either treatment group (*d* = .31 for those in the math games with a talk focus group and *d* = .19 for those in the math games group).

To examine the effect of dosage, another 15 linear mixed models were conducted regressing outcomes on the number of days children participated in small group math games. Dosage had a significant effect on children's outcomes on the TEMA (at $p < .05$) and NSA (at $p < .10$), but not the WJ-AP or WJ-QC. When considering children's early numeracy skills, the number of days children participated in either treatment group significantly predicted three skills: counting, one-to-one correspondence, and producing groups.

In summary, results from this study indicate that playing math games with preschool children from economically disadvantaged backgrounds leads to significant improvements in early math skills over not playing such games. There was not sufficient evidence to conclude that employing the strategies to engage children in the math talk learning environment as defined by this study leads to significantly greater results than just playing the math games. However, the greater gains made by the children who engaged in the math talk learning environment and larger effect sizes of being in this condition over children who only played the math games was a consistent trend across the math measures. Possible reasons for and implications of these findings are discussed in the next section.

Discussion

This study offers several important findings that might be used to inform early childhood educators and researchers on improving math outcomes among programs serving children from economically disadvantaged homes. To be discussed in this section are the degree to which playing math games improves children's early math skills, the consistency with which children who were included in the math talk learning environment made the greatest gains, and the implications these findings have for future research.

Playing Math Games Improves Children's Early Math Skills

Results from statistical analyses showed that the two groups who played math games learned significantly more than children who did not play these games, regardless of employing the math talk learning environment. Possible reasons for this finding include the intentional selection of games played, modifications made to those games, modifications made to group processes when playing games, and/or children's increased interaction with mathematical content.

Intentional selection of games. The games used in this intervention were selected because children had to use beginning counting and numeracy skills in order to play. To play Hi-Ho-Cherry-O, children had to count the fruit, use one-to-one correspondence when removing the fruit from the board into their hand, state the cardinal number of fruit in their own or their peer's hand, and count how many of their fruit remained to be removed from the board. To play Walk-the-Line, children had to subitize or count how many dots were on the top side of the die, count the number of spaces to move along the number line, and recognize the numeral they were standing on after moving the correct number of spaces. To play Card Wars, children had to recognize the numeral on the card or count the number of dots on the card then determine which of two numbers was greater. These games played over the 8-week intervention provided a context for focusing on count, number, and quantity as well as encouraged practice using many math skills across different contexts.

As has already been demonstrated by several early childhood mathematics interventions, children construct mathematical understandings through multiple modes of practice with mathematical concepts and materials (Clements & Sarama, 2007b; Griffin et al., 1994; Howell & Kemp, 2010). According to Howell and Kemp (2010), reinforcing the principles of count and number (i.e., how numbers represent quantity and relate to one another) leads to a more flexible number sense than just reinforcing counting skills (i.e., rote counting). The present study provides additional evidence in support of that claim.

Besides using skills in multiple contexts, the games used in the intervention also provided children with multimodal cues to experience number. The Walk-the-Line game was a physical embodiment of the game played by Siegler and colleagues that led to greater number sense among study participants (Booth & Siegler, 2006; Ramani & Siegler, 2008; Siegler & Opfer, 2003). When playing this game, children associated the number name with the numerical symbol; recognized the longer the game continued, the further distance traveled on along the line, and the more time passed since the game began (Siegler, 2009). The present study also supports Siegler's claim that through kinesthetic, auditory, visuospatial, and temporal cues, children develop a more robust understanding and flexible use of quantity and magnitude (Siegler, 2009).

Modifications to games. In addition to selecting games that provided practice in using count and number sense, games were also modified so that they started at the most basic level and were made more challenging over time as mathematical understandings developed. When children first learned to play Hi-Ho-Cherry-O, the mathematical learning objectives were to say the count words in the correct order, assign one number name to each fruit removed, and cease removing fruit from the board once the correct number of fruit was in the child's hand. Later, the learning objectives included recognizing when they had more than, less than, or the same number as one or more of their peers. Another level of difficulty included stating how many fruit were on their tree or bush at the start of the turn, how many fruit were removed, and how many were left on the tree or bush.

Walk-the-Line was modified so that children used small numbers and simple processes when learning the game, then larger numbers as children mastered the easier objectives. When children first learned to play Walk-the-Line, they used a die that had been painted so that there were two sides with one dot, two sides with two dots, and two sides with three dots, so that a child never rolled higher than a three. Learning objectives began with associating number names with numerical symbols and moving the correct number of spaces along the number line from 1 – 10. After playing the game in this fashion on two occasions, the painted die was replaced with a standard die with dots ranging from one to six. On the last day playing this game, children were to state the number on which they began, the number of spaces they moved, and on what number they landed. For example, a child might say, "I am on three now, I move one (moves a space), two (moves another space), and now I am on five."

The Card War game was also modified in similar ways so that learning objectives began simple and were made more difficult as children mastered the more simple processes. When learning to play the game, children were given a truncated set of cards ranging from one to five before being given the full set of cards ranging from one to ten. The learning objectives were first to place the cards side by side with the numeral situated at the top of the card, count the dots on the cards, and determine which card had more dots. The objectives were made more challenging by again removing cards with values greater than five and having children work in pairs (e.g., two children versus one child and interventionist) to determine which team's cards had the greater sum.

The large effects observed from scaffolding math games in this fashion, even without encouraging math talk, support the Cognitive Load Theory (CLT) (Paas et al., 2003). CLT is a major theory that considers both the cognitive processes involved in learning and the implications these processes have on instructional designs and procedures. Mathematical competencies are closely associated with working memory, which is "the ability to hold mental representation of information in mind while simultaneously engaged in other mental processes" (Geary, Hoard, Nugent, & Byrd-Craven, 2007, p. 88). Working memory can handle only a very limited number of novel elements; when too much information is presented at one time, there are not sufficient resources available to link the new information with existing schemas (Paas et al., 2003). To avoid cognitive overload, Van Merriënboer, Kirschner, and Kester (2003) suggest learners be presented with general, overarching principles when introduced to novel elements and then to more specific procedural information at the point when it is required.

The manner in which information is presented to learners and the activities in which they are expected to engage affect cognitive load and the learners' ability to construct more complex mathematical representations. The modifications made to selected games permitted interventionists to focus children's attention on those concepts of number requisite for playing the game. Once children were able to actively participate in the game, interventionists and children were in a better position to make mathematical observations and arguments. As game play progressed over sessions, children were able to perform those mathematical tasks they could do alone and the interventionists were able to suggest mathematical ideas just beyond what children could perceive alone. By starting the games with general principles for play and introducing more difficult processes as necessary, mathematical relationships were made accessible to children without overwhelming them.

Modifications to group processes. The use of group processes to engage children in mathematical activity was yet another possible explanation for the large gains made by children in the treatment groups. The use of (a) games, (b) small group instruction, and (c) peer interactions all likely contributed to increased interest and motivation among students. Research theorizes such increases in interest and motivation lead to greater effort towards and cognitive resources devoted to a task (Paas et al., 2003).

The mode of instruction used in the intervention was the use of games; instruction is rarely delivered in such a format. Children in the study were asked if they were familiar with Hi-Ho-Cherry-O as it was the first game played and the only commercial game they might have recognized. One child out of 99 children reported having played the game before, and all appeared to be unfamiliar with game playing in general, although they did appear to be excited when told that they would be leaving the classroom to go play games with their friends. Children recognized games are *fun* and so the primary goal of the children was to *play*; learning math skills was not the primary goal but instead a necessary part of learning how to play the game.

Another modification to typical classroom processes was the use of small group instruction to deliver the math content. According to the NCEDL's Multi-State Pre-K Study, on average, small groups are utilized only 6% of the total time spent in learning activities (Early et al., 2010), although they have been found to be positively related with math achievement in both elementary and secondary school (Webb, 1991) and language development in preschool (Smith, 2001). Children may have been excited to go play the math games because they were allowed to talk and share an adult's attention with only two peers as compared to sharing the teacher's attention with the entire class.

The game that Siegler and colleagues used in their studies was played between one child and one researcher (Booth & Siegler, 2006; Ramani & Siegler, 2008; Siegler & Opfer, 2003); the games used in the present study were played among three children and an interventionist. This study extends Siegler's work to show that strong intervention effects can be achieved for small groups of children as well as those realized by one-on-one activity. In addition, the present work extends research on the positive effects of playing games on math learning in elementary and secondary school (Randel et al., 1992) to include preschool-aged children.

Increased interaction with mathematical content. One more consideration concerning the greater math gains made by children in both treatment groups over the children in the control group is the increased interaction with mathematical content. Children in the treatment groups were doing more than just playing games; they were engaged in mathematical content. As has been demonstrated by Bodovski and Farkas (2007), time spent in learning activities does have a positive effect on primary school-aged children's math achievement. While this study did not collect any data pertaining to the presence or absence of classroom math instruction, classrooms did differ in their effect on the children in the control group. A few of the teachers of children in the study commented that they had not begun any math instruction to date. The present study ran concurrently with the first semester of the preschool year and some teachers stated that they would not introduce mathematical content until the second semester. It is likely that children in the treatment groups were exposed to more mathematical content than were children in the control groups. However, it is important to note that, even among classrooms that did engage in math instruction in the first semester, the pattern for math gains still held true: children in the Math Games/Talk Focus group had the greatest gains, children in the Math Games group had the second largest gains, and children in the controls had far fewer gains.

In summary, selecting games in which children had to use mathematics in order to play, modifying those games to children's ability levels, playing games in small groups with an adult, and the repeated exposure to mathematical content were all characteristics likely to have contributed to the very large effects on children's math gains on all measures, whether or not they were encouraged to use math talk.

Greater Gains Made by Children in Group with Focus on Talk

Children who were encouraged to use math talk consistently made greater gains on all math measures than did children who only played the math games, although these gains were not statistically significant. In fact, on all of the standardized measures, children in the Math Games group had the highest mean performance at pretest (no statistically significant differences from the other two groups); yet, children in the Math Games/Talk Focus group exhibited greater residualized gains than children in the Math Games group. This pattern suggests that the strategies employed to encourage the use of math talk during game play warrant further investigation.

Gestures to scaffold language. To encourage the use of math talk in the present study, interventionists used gestures to scaffold targeted mathematical language (*same as* or *equal to*, *more than*, *less than*, and *zero)*. Goldin-Meadow and colleagues have written extensively about the role that gesture plays in language development. One purpose gestures fulfill is allowing children to communicate information they cannot yet express verbally (Özçalişkan & Goldin-Meadow, 2005). In addition, research demonstrates that the mismatch between children's gestures and verbal speech is a marker of conceptual understanding (Perry, Church, & Goldin-Meadow, 1988).

Once children learned how to play the first game, Hi-Ho-Cherry-O, interventionists began using signs to represent mathematical ideas when they arose. For example, when a child in the Math Games/Talk Focus group spun the arrow and landed on the same number as did the child whose turn was before, the interventionist would have said, "Oh! You are going to take the *same* number of fruit from your tree as (the child before) did." While saying this, the interventionist would have made the hand gesture that meant *same* or *equal to*. The interventionists used hand gestures consistently and asked children to do the same when they made similar observations.

Children in the Math Games/Talk Focus group appeared to sincerely enjoy learning and using the hand gestures. They employed these hand symbols whenever appropriate and rarely when they were not. Take for example one occasion when one child stated that he had two fruit left on his tree and the next child spun a two on the spinner followed by making the sign for "the same as/equal to." The interventionist had to think why the child was making this sign before determining the child made the sign erroneously then realized, "Oh, I see. (Child 1) had two fruit left on his tree and you spun a two on the spinner. Two and two – those are the same number, yes." Despite this rare occurrence when odd connections had to be made, children came to understand and use the gestures regularly and accurately during the course of the intervention.

It is also noteworthy that, like learning any kind of new vocabulary, it was important that teachers probed children's conceptual understanding of the words the gestures represented.

There was an occasion when playing Card Wars that two children each turned over a six card and simultaneously made *the same as/equal to* gesture. It was obvious to the interventionist that the children knew the circumstances of this symbol's use, but chose to explore their understanding by asking, "So, who has more?" At this point, both children raised their hands and said, "Me." The interventionist addressed this dilemma by asking, "If you each had six cookies, who would have more? Or is it fair and you got the same number of cookies?" After some discussion between the children, they both seemed to come to a better understanding of the concept *having the same* or *an equal amount*. Upon future instances where these two children used the *equal to* gesture, they were able to accurately reason that neither had more because if they each had the same number of cookies then it was fair. Thus, it is possible that gesturing helped children remember the vocabulary and when to use that word, but fully linking the concept the word represented with children's existing knowledge still required verbal discussion between learner and teacher.

Although none of the children used the targeted mathematical language in the beginning of the intervention, after time several children dropped the gestures entirely and used only the appropriate verbal language while others continued to use the gestures with and without verbalizing the words the gestures represented. It is possible that children knew the words for these concepts and/or used these expressions under normal circumstances and prior to being taught the gestures; however, this was not apparent to the interventionists. Further, the concepts of equality, greater/less than, and zero were assessed on the math measures; children in the Math Games/Talk Focus group had greater gains on items measuring these concepts. Gesturing did appear to support children's use of mathematical language as well as a very basic understanding of the conditions under which that vocabulary should be used, although it was also apparent that children accurately used gestures to represent language even when their conceptual understanding was still developing.

Press to engage in talk. Children in the Math Games/Talk Focus group were *pressed* to engage in the math talk learning environment. In the present study, to *press* children meant that the interventionists exerted an intense level of energy towards employing strategies theorized to foster children's talk. The strategies included asking open-ended questions, modeling appropriate responses, facilitating peer interactions, and expecting children to assume increasing responsibility in the game activity. All of the aforementioned strategies provided important supports for children that likely explain their greater gains over the other treatment group, despite the finding that they proved challenging for both instructors and children.

At the start of the intervention, children appeared reluctant to answer open-ended questions, so teachers modeled responses and asked children to repeat them. For example, when asked "How did you figure out that these two cards are more than these two?" the interventionist would wait for the child to respond. It was necessary for the interventionist to pay close attention to children in order to determine the point at which they were no longer attempting to organize their ideas, but were beginning to mentally wander from the question. Only then would the interventionist offer assistance by repeating the question or providing a possible response such as, "Show me how you counted the dots to know that there were 9 altogether. Did you know this was 5 and then start counting here (pointing to the first dot on the second card)?" Usually children would agree with the adult by a nod of the head. The interventionist might ask children, "So, you say what you did. Say, 'I knew this was 5 (because the group had already discussed when one side of the grid was filled with dots, that was five) and so I counted from here (pointing to the first dot on the next card) 6, 7, 8, 9." Then the interventionist would wait in anticipation of children's imitation of this response, assisting as necessary.

There is much research that demonstrates the positive effects of generating explanations to learning outcomes. In line with Carey's Bootstrapping Theory (2004), generating an explanation encourages a reorganization or clarification of the ideas to be communicated by challenging the learner to put words with their intuitive number sense. Further, the benefits to preschool children generating explanations are maximized when an adult caregiver is actively listening, perhaps because generating explanations for a listener may press more upon children's cognitive abilities than generating explanations to self or just being told the correct answer (Rittle-Johnson et al., 2007). Despite the difficulty in eliciting clear verbal explanations from young children, Rittle-Johnson et al. found that children's attempts to explain to a listener led to greater learning outcomes than children who generated explanations to self.

While children in the present study were ineffective in generating explanations at the start of the intervention, modeling explanations for the children appeared to have, at the least, established an expectation by the children to do so. The press employed by the interventionists for children to generate explanations appeared to help them stay on track and make sincere attempts at generating explanations. By the end of the intervention, children understood the expectation of providing explanations although they still did not demonstrate skill at doing so.

For children who are not used to generating explanations and/or who are unfamiliar with the content to be explained, asking them to "explain how you know" or "tell how you got that answer" is a difficult expectation. In fact, there was a general consensus among the interventionists that this press irritated some children, possibly because children felt that this type of talk was awkward, artificial, and/or contrived. In addition, interventionists agreed that cultivating this math talk learning environment was extremely challenging and required far more effort than did directing game activities in the Math Games group. At first interventionists thought there might have been a glitch in the randomization process; children in the Math Games group appeared to be better self-regulated. However, analyses indicated these groups were the same at baseline. It became evident that the challenge of the Math Games/Talk Focus group was a product of the intervention; the press to engage children in talk slowed the speed of the games, required children develop a new way of talking and thinking, and changed the overarching goal of the activity (e.g., win the game).

In addition to asking open-ended questions, the press to engage in talk also included collaboration in structuring children's roles during game play. In the beginning of the intervention, the responsibility of directing game activities was carried by the adult; however, as children gained understanding and skill, children were able to assume increasing responsibility. In order to promote this transfer of responsibility, interventionists used the *doublecheck*. After the first child completed his or her turn, the interventionist checked to make sure that he or she had removed the correct number of fruit, moved the correct number of spaces on the number line, or accurately determined which card (or set of cards) was greater. This action was taken by interventionists for all children in the Math Games group. In the Math Games/Talk Focus group, however, interventionists facilitated this action among peers. After the second player completed his or her turn, the first player was expected to *doublecheck* for the second player, the second player *doublechecked* for the third, and so on. Thus, children were expected to follow the game even when it was not their turn to play, catch other children's mistakes or make interesting mathematical observations, and explain their reasoning when they challenged another student.

The press to engage children in talk about math was challenging to and demanding on interventionists and possibly depressed children's excitement in playing games to a degree. Nonetheless, the large effect sizes on children's mathematical development by the Math Games/Talk Focus group over the Math Games group indicate these strategies were effective and benefited children's development even over a relatively short period of time. Less was demanded from children in the Math Games group and less was gained. The present work to improve children's math skills included fostering their use of language for reasoning by generating explanations, promoting the use of metacognitive skills through peer interactions, and increasing their attention span by asking children to assume responsibility for the *doublecheck*. Prior to the current study, it was unknown whether teachers did not know how to engage children in math talk, did not judge math talk to be relevant, or possibly did not consider it to be a skill that preschool children could perform. The present study demonstrates that children who are not used to using language in the ways demanded by this intervention can be resistant, making game activities challenging for practitioners.

Future Directions

This study investigated the quantitative effects of an early math intervention on children's math development. Future research extending the current work might replicate this study with a larger sample size, extend the duration of the intervention, modify the games to include more levels of play, disentangle individual components of the intervention, and/or ascertain the level of support required for practitioners to implement the current strategies in their classrooms.

Other things being equal, effects are harder to detect in smaller samples. Increasing sample size is often the easiest way to boost the statistical power of a test. The effective sample size calculated was 150 students in 11 classrooms to obtain the power to detect effects. Although this study used a sample size large enough to detect the effects of being in either of the treatment groups as compared to the control group, there was likely not enough statistical power to detect any potential effects of being in the Math Games/Talk Focus group as compared to the Math Games group. An important line of research necessary to extend the present work would be to replicate the current study with a larger sample.

Another important extension to this work would be to test the effects of prolonged exposure to the math talk learning environment. Despite encouraging results from the present study, eight weeks might not have been a long enough period of time for children to become accustomed to the expectations of the math talk learning environment. It is possible that a longer duration of the Math Games/Talk Focus group would show significant benefits above those achieved in the Math Games group. It would important to learn if the effects of engaging children in a math talk learning environment for the entire preschool year, or across multiple years, has continuously increasing learning benefits or if there is a point when benefits are maximized before there are diminishing returns.

Another direction for future research might test the effects of a greater number of incremental levels of difficulty per game than was used during the TAMS intervention and how this might influence interactions during free-play in the classroom. The present study employed three math games over an eight week intervention period whereby the simplest rules were used for three sessions and then slightly more difficult rules were used for three additional sessions. However, any one of the games could have been played for all eight weeks at ever increasing levels of difficulty. For example, additional sessions of the Walk-the-Line game could have been played by extending the line to 20, using a pair of dice where one die indicated how many spaces to move forward and the other die indicated how many to move backwards, playing the same game moving from 20 to 1, and so on. This is also true for the Card Wars game; children could learn to put the cards out and put them in order along a number line, use regular playing cards whereby black cards are added together and red cards are subtracted from the total. As children learn to play each new level of a game, it is possible they might select to play together during free-play time, which would increase the time children are engaging in mathematical activity and math talk.

The findings in this study should not be interpreted to mean that the TAMS intervention successfully identified all of the critical elements of developing children's math talk. There may be components to developing children's talk and reasoning skills that were not included in this intervention. Likewise, there may be components that were included that are not critical to the goals of the intervention. Another direction for future research might consider explicating the effects of individual elements, such as the effects of gesturing separate from the effects of the *doublecheck*. Also, it would be helpful to early childhood practitioners to have concrete examples of the individual elements along with typical and atypical responses from children. Providing video of archetypal interactions would be a valuable resource for those interested in replicating the intervention strategies.

Finally, future research is needed to identify the type of training and subsequent support preservice and in-service teachers need to implement the intervention in their classrooms. As has already been discussed, gestures proved to be enjoyable and successful in aiding children's language development, although it remained necessary for the interventionists to explore children's conceptual understanding verbally. It was also necessary for interventionists to consider possible explanations for children's use of gestures at seemingly inappropriate times.

Interventionists had to be patient to wait for children's responses and also perceptive of when to interject additional support. Altogether, this press to engage children in talk required three equally important attributes: knowledge of children's math development, willingness to take the perspective of children, and a sincere and intense commitment to the goals of the activity.

Lacking one or two of the aforementioned attributes would likely fail to produce similar results to those obtained in this study; the presence of all three attributes was necessary to be sufficient. For example, a teacher with knowledge of children's math development, but without a willingness to take the perspective of the child, might make the common mistake of simply providing children with correct answers rather than helping the child link his or her intuitive understandings with the present mathematical activity. Conversely, a teacher who is willing to take the perspective of the child and committed to attaining the goals of the activity but who lacks understanding of children's math development will not likely be able to help the learner build bridges between their intuitive understandings and the mathematical activity. Finally, teachers who possess one or two of the aforementioned attributes but do not have the intensity and commitment to the goals of the activity will not likely maintain the motivation to persist in the game when children demonstrate irritation or discontentment with the mathematical activity.

Issues

Measuring Children's Math Skills

One issue that arose from this study is the large difference between children's standardized rankings on the TEMA and their standardized rankings on the Woodcock Johnson III Applied Problems and Quantitative Concepts. While the large majority of children ranked below the 25th percentile on the TEMA at pretest, the same children ranked between the 20th and 60th percentile on the WJ-AP and between the 5th and 45th percentile on the WJ-QC. This result not only demonstrates the wide variation in children's math knowledge upon entering preschool, it also demonstrates inherent limitations in measuring children's early skills associated with later school success. These divergent findings are likely due to one or both of two possibilities: the TEMA may underestimate children's performance and/or the WJIII may overestimate their performance.

There are differences between the TEMA and the WJIII subtests that can explain the differences in relative performance among children in a single sample. On both measures, a raw score of 90 places a child in the 25th percentile and a raw score of 100 places a child in the 50th percentile; however, it is harder to obtain a raw score of 90 on the TEMA and relatively easier to score a 90 on the WJIII. The TEMA is a measure of children's informal and formal mathematical knowledge. There are items on this assessment in which an incorrect response by children is counted as a correct response because it exemplifies a general understanding of a concept. For example, three chips are placed under a mat, then one more chip is added to the three hidden chips, and a child is asked, "How many chips did I place under the mat?" The child's response can be the number four or a number greater than four to be counted as correct because it demonstrates an understanding that addition leads to higher numbers. Although such responses are counted as being correct, it is often the case that a group of questions (such as three out of four) must be counted correctly for the group to receive a raw score of one. For this reason, children with poor informal skills will earn low raw scores and thereby fall into a low percentile ranking.

The WJIII subtests are very different from the TEMA; these assessments contain only a few items to measure early math skills before the test items become too difficult for preschool aged children. On the WJ-AP, the 10th test item involves subtraction and above the 20th test item involves manipulating money and mental representations (e.g., if you drew three more circles, how many circles would there be?). The 11th item on WJ-QC Part A asks children, "When you count, what number comes right *before* eight?" and Part B asks children to identify the number that goes in the blank space along a number line. Thus obtaining one more correct answer increases children's raw scores on the WJIII more so than on the TEMA. Although performance on WJIII is highly associated with later school success, its sensitivity to detect varying levels of math competencies is limited and children's raw scores yield higher rankings on a normed scale.

Measuring Children's Fluid Reasoning

Another issue that arose from this study was the inability to detect any growth in children's fluid reasoning skills as measured in this study. Fluid reasoning is the capacity to think logically and solve problems in novel situations, independent of acquired knowledge. It is necessary for all logical problem solving and includes inductive and deductive reasoning. There were a number of challenges to children's success on the Leiter-R, including the non-verbal delivery of instructions and the expectation that children would reason how to complete the tasks. The Leiter-R was designed to require little to no verbal communication to administer and the tasks were designed to be self-evident to children; the tasks are game-like in nature, yet children likely had little to no experiences with playing games or deciphering patterns. Children seemed to be at a loss for what to do, continuously looked for guidance from assessors, and often appeared to make response selections at random. Just as generating verbal explanations for their reasoning was challenging for children, so too was generating mental explanations for the tasks to be completed.

Another possible explanation for the lack of effects on the Leiter-R is that math knowledge and fluid reasoning develop independently and are differentially affected by schooling. Blair, Gamson, Thorne, and Baker (2005) conducted research to investigate the increasing population mean scores observed on measures of fluid reasoning over the last century. They observed the early elementary school math curriculum has moved increasingly farther from the rote memorization of mathematical facts towards a greater emphasis on skills associated with fluid reasoning (e.g., categorization, pattern recognition, approximating values, etc.). They theorize that these skills taught in school have influenced student performance on measures of fluid reasoning, although there has been no corresponding increase on measures of math achievement. Likewise, the TAMS intervention significantly influenced preschool children's math achievement without affecting their fluid reasoning. What is left unanswered are (a) whether these domains become more interdependent over the K-12 years, and (b) if affecting one domain is sufficient to improve children's later academic outcomes or if affecting both domains is necessary.

Fidelity of Implementation.

Fidelity of implementation is characterized as the degree to which an intervention is implemented in comparison with the original program design (Lipsey, 1999). There were three interventionists, each facilitating two experimental groups per classroom. While introductions and closings to activities were semi-scripted, general intervention activities were not, posing a potential threat to internal validity. To address this, the team of interventionists met on a weekly basis to discuss goals and strategies, review audiotapes for consistency, and attempt to anticipate children's questions or actions along with corresponding responses from interventionists. On the rare occasion an unanticipated incident occurred, the other two interventionists were notified the same day. For example, on the day that the two children made the hand gesture for *the same as/equal to*, but then responded that they both had *more than* the other, the other two interventionists not present were notified. It was decided that the other two interventionists would ask their children the same question (i.e., when two children had the same number in Card Wars, they were to ask the children, "Who has more?"), then respond similarly to way the interventionist who initially experienced this did (i.e., "If you both had that number of cookies, who has more? Or is it fair?"). As was humanly possible, the interventionists attempted to ensure the teaching strategies of one condition did not spill into the other condition, and that strategies for each condition were similar across classrooms and interventionists.

Student Absenteeism

Many of the children in this Head Start sample were routinely absent from school resulting in children who were assigned to participate in the intervention not receiving the treatment as it was intended to be received. Failure to include these students' data in the analyses could lead to biased estimates of the intervention effects. Intent-to-Treat (ITT) analyses do not exclude participants based on their compliance to the study. Rather, groups are compared based on the initial randomization scheme and thus provide estimates of the intervention's effectiveness for children who receive the treatment as they would in real world circumstances.

To consider the effect of the intervention based on how much of the intervention was received, analyses testing the effect of dosage on outcomes were conducted. Out of 16 possible sessions children could have participated in the intervention activities, children in both treatment groups were present an average of 10 days, or 62.5% of the time. The number of days children participated in the intervention significantly predicted children's outcomes on the TEMA ($p < .05$), the NSA ($p < .10$), and three early numeracy skills (counting, one to one correspondence, and producing groups) ($p < .05$). Thus, regardless of condition, the more intervention activities children participated in, the more math they learned. Moreover, the skills they most improved on were skills being practiced in the games: counting, one to one correspondence, and producing groups. It is likely that the number of days children participated in the intervention activities did not affect results on the WJIII subtests is because, as was discussed in the *Measuring Children's Math Skills* section above, they are not as sensitive to small changes in children's informal math skills as the TEMA and NSA.

Strengths and Limitations

This study's major strength lay in the research design. First, the sample was randomized within a block (classrooms) making the three groups of students as homogeneous as possible. The intervention was administered by the same interventionist to the two treatment groups within blocks over the ten weeks. Thus, the variability within each block is less than the variability of the entire sample and therefore each estimate of the treatment effect within a block is more efficient than estimates across the entire sample. When these more efficient estimates are pooled across blocks, the overall estimate is more efficient than it would be without blocking.

Another strength of this study was in its use of both standardized measures of achievement and the more proximal Number Sense Assessment (NSA), thus providing a more sensitive measure of children's early numeracy skills. The NSA was designed by identifying the math skills targeted by the intervention, categorizing items from the standardized measures according to the targeted skills, and selecting those skills that had an insufficient number of items. Going back to the literature, test items were adapted from Siegler and colleagues in order to increase the number of items measuring one-to-one correspondence, subitizing, and magnitude comparison. With a sufficient number of items measuring specific skills, this study was able to compare children's performance on nine early numeracy skills targeted by the intervention and an additional 10th category of items measuring skills not targeted by the intervention. By using multiple modes to evaluate children's math growth, small changes in their knowledge and/or understandings are more likely to be detected, and these changes can be compared to changes on nationally normed standardized tests as well as permit comparisons to other early childhood math interventions that may have also used these national measures.

A limitation of this study was its limited focus on number sense. While there is general consensus that early childhood mathematics should include topics beyond numeracy, such as geometry and measurement (Clements, 2004; Cross et al., 2009; NCTM, 2006), this study focused only on developing children's early number sense. The TAMS intervention was designed to test strategies for engaging children in meaningful mathematical activity and not as a curriculum for early childhood mathematics. Early childhood practitioners could implement these strategies to engage children in games focused on number; however, it remains unknown whether these strategies would have the same effect on children's development of skills related to other mathematical topics.

Another limitation of this study is the exclusivity of the population. All of the children in the study were from Head Start centers located in the surrounding counties of a major metropolitan city in Middle Tennessee. The results can not be generalized beyond this population of students. It remains unknown whether similar results would be obtained among children located in preschools administered by school systems or other childcare settings, among children located in different regions of the state or country, or even among children from other economic backgrounds.

Conclusion

The present dissertation study was designed to test the effects of an early mathematics intervention on the early math and reasoning skills of preschool children from economically disadvantaged backgrounds. In particular, this dissertation study sought to compare the effects of playing games intentionally selected to develop children's number sense to not participating in these games. An additional aim of this dissertation was to compare the effects of playing math games to the effects of participating in the same math games enhanced by a math talk learning environment.

The final sample was composed of 95 children from nine Head Start classrooms located in the surrounding counties of a major metropolitan area in Middle Tennessee. Children were pretested and posttested on three standardized math measures, one non-standard math measure, and one standardized non-verbal measure of fluid reasoning. They were also independently and randomly assigned to one of three experimental conditions within their classrooms. Two of the conditions participated in the intervention with one of those two conditions only playing math games and the other of the two conditions playing the same games enhanced by the math talk learning environment. The third experimental condition was used as a counterfactual to participating in any intervention activities. The math talk learning environment included strategies from the literature theorized to engage children in high quality interactions about mathematics.

Linear mixed modeling regressing outcomes on condition and controlling for children's pretest scores, age, and blocking factor were used to estimate effects. In addition, effect sizes were calculated and practical significance examined. Results indicated statistically significant gains made by children who participated in the math games over children who did not participate in the games. There were not statistically significant differences identified in the gains made by the two treatment groups. No changes in children's fluid reasoning were found in any condition. The effect sizes of the intervention on the two treatment groups were of moderate size, with consistently larger effects on the condition that participated in the math talk learning environment. The investigation of the intervention's practical significance showed children who played math games with a focus on talk learned more math than children who only played games, and both groups who played games learned more math than children who did not play the games.

There were two major findings of this study, each with its own set of potential interpretations. The first major finding was the impressive results playing math games had on children's early math development. It is likely this finding is a result of the intentional selection of games, the modifications made to those games, the modifications made to the group processes, and/or children's increased interaction with mathematical content. The second major finding was the consistently larger effect sizes and greater gains made by children who played math games with a focus on talk over children who only played math games. This finding could have been a result of the enhanced teaching strategies used to support children's mathematical language development, such as the use of gestures and the press by interventionists to engage children in talk about mathematics.

This study identified measuring children's math knowledge, measuring children's fluid reasoning, fidelity of implementation, and student absenteeism as major issues. The strengths of the study included its research design and the measures used to evaluate children's achievement. The limitations of the study were the limited focus on number sense and exclusivity of the population sample.

There are several directions future research might take to extend the present work. The study could be replicated with a larger sample size or for an extended period of time. Individual games could be adapted to include more levels of play. Individual elements of the intervention or potentially new elements could be tested to learn which are critical to developing children's math talk. Also, future research should determine the professional development activities and classroom support necessary for practitioners to implement these strategies in their classrooms.

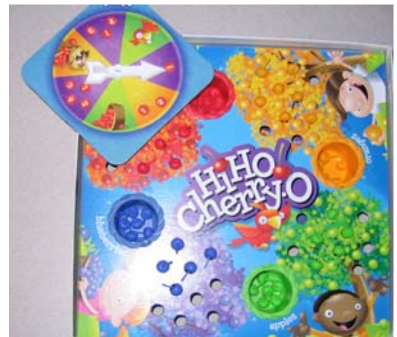
APPENDIX A

INTERVENTION ACTIVITIES

Activity 1: Hi Ho Cherry-O

Materials

- Gameboard with picture of two fruit trees and two bushes
  - Each tree or bush has 10 holes in which the fruit can be placed
  - Beside each tree or bush is a fruit bucket of the same color
- 10 pieces of fruit per tree/bush (10 oranges, 10 apples, 10 cherries, 10 blueberries)
- Spinner with arrow and base
  - The base is made of a cardboard square with a circle drawn in the center of the square.
  - The circle is divided into 7 equal sections by 7 radial lines. Each section has a picture or pictures drawn on it. Four sections have a picture of one, two, three, or four fruit and the fruit are numbered. One section has a drawing of a bird, another a drawing of a squirrel with fruit, and the last has a drawing of a spilled basket of fruit.
  - From the center of the base, there is a plastic piece that extends from the base, through a white plastic arrow. A plastic cap snaps on top of this piece securing the arrow to the base, but allowing the arrow to spin around.



Gameboard

Spinner

Get Ready to Play

- Place the 10 pieces of fruit on the matching tree/bush
  - 10 Orange oranges on the orange tree
  - 10 Green apples on the green tree
  - 10 Red cherries on the red bush
  - 10 Blueberries on the blue bush
- Spinner placed beside board, beside the facilitator
- Children sit in front of "their" tree or bush
- Facilitator should clarify to children whose fruit and bucket belong to whom.

Playing the Game

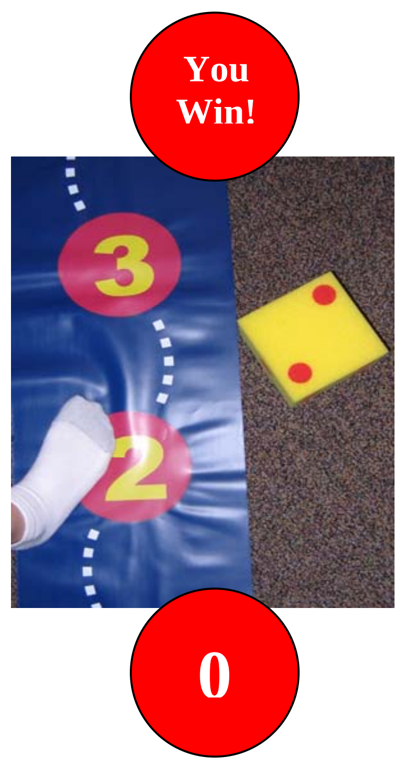
- The child to the facilitator's immediate left plays first. Play then passes to the left. This has been changed from the original game rules that call for the child with the next birthday to go first so that none of the 20-minute activity is wasted on explaining this concept to children.
- Each child spins the arrow on the spinner and completes one of the following tasks:
  - Land on one fruit: Pick one fruit from your tree/bush
  - Land on two fruits: Pick two fruits from your tree/bush

- Land on three fruits: Pick three fruits from your tree/bush
- Land on four fruits: Pick four fruits from your tree/bush
- Land on the bird or the squirrel – either is nibbling on the fruit in your bucket! Remove one fruit from your bucket. This has been changed from the original game rules that call for children to remove two fruit from their bucket. This change has been made for two reasons. The first reason is that the removal of two fruits is arbitrary while moving only one fruit from the bucket corresponds with the number of birds on the spinner. The second reason is that removing one fruit provides an opportunity for early learners to experience the relationship between consecutive numbers, for example, that three is one less than four and four is one more than three.
- Land on the spilled basket of fruit, and oh no! Take all of the fruit out of your bucket.

▪ *Doublecheck*: In condition A, the child *in play* (who pulled the fruit from his or her tree/bush) will be checked by another *not in play* (whose turn it is not currently) that the number of fruit in his or her hand is the same as the number on which the arrow landed and the cardinal number of that set will be stated. This is not a part of the original rules of this game, but has been added as a strategy for scaffolding mathematical development.

▪ To win the game, the first child with all of his or her fruit in his or her basket says "Hi-Ho-Cherry-*O*, I have zer-*O* fruit on my tree!" The latter part of that statement was not instructed in the original game rules, but was added to reinforce the concept of zero.
  - If time runs out before the game ends, then the child with the most fruit in his or her basket wins.

Activity 2: Walk The Line



Materials for Level I

▪ Two number line floor mats with the numerals 1 – 10 visible

▪ Four foam circles, 10 inches in diameter, ¼ inch in depth
  - Two circles, one red and one green, each have a zero on them
  - Two of the circles, one red and one green, each have "You Win!" on them

▪ Two 6-inch cubes (dice) with dots ranging in value from one to three.
  - Each cube has two sides with the same number of dots on them.
  - There are two sides with 1 dot, two sides with 2 dots, and two sides with 3 dots.

Materials for Level II

▪ Extend the number line floor mats so that the numerals 1 – 20 are visible

▪ Four foam circles, 10 inches in diameter, ¼ inch in depth
  - Two circles, one red and one green, have a zero on them

  - Two of the circles, one red and one green, have “You Win!” on them
- Two 6-inch cubes (dice). Each side of a cube has a different number of dots ranging in value from one to six similar to a typical die.

Get Ready to Play at Level I or II

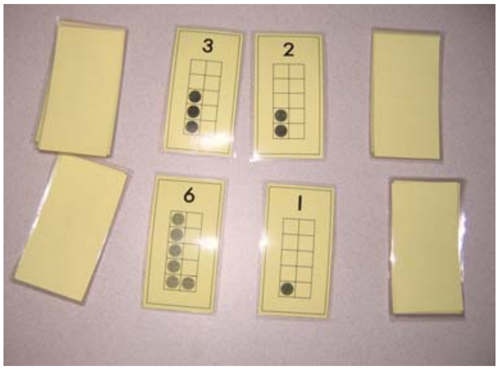
- Lay the number lines parallel to one another approximately three feet apart. For level I play with only the numerals 1 – 10 showing (the number line is folded so that 11 – 20 are not visible). For level II play with the full number line extended.
- Place a foam circle with the numeral 0 off the number line below the numeral one. Place the “You Win!” foam circle of matching color off the number line above the numeral 10 (or 20 for level II).
- Place the dice on the inside of the parallel number lines.
- Children are paired into teams (e.g., Red team and Green team).
- One red team member stands on the red zero, the other red team member is on the floor between the parallel lines with one die.
- One green team member stands on the green zero, the other green team member is on the floor next to the red team member and between the parallel lines with one die.

Playing the Game at Level I or II

- The children with the dice roll to see which team goes first
- The team with the higher number of dots goes first. If children get the same number, roll again.
- The team member due to go first then moves the number of spaces along the number line as was rolled by his or her partner. The line-walker counts each step while moving from one number to the next. The child must say the numbers he or she is walking on and not count the number of spaces that he or she was instructed to move. For example, a child is standing on the number line on the numeral three. Her partner rolls a two on the die and instructs her to move two spaces.
- *Doublecheck*: In condition A, children who are not *in-play*, meaning they are waiting for their turn, or the facilitator will check that those who are *in-play* (a) identified the correct number of dots and (b) moved the correct number of spaces.
- The other team is now *in-play*. The child with the die rolls his or her die, then their partner moves that number of spaces along the other number line saying the numerals, not counting the number rolled.
- The first team to move beyond the number line wins. That is to say, the first team to move past 10 and onto the “You Win!” wins the round. Landing *on* the 10 space does not end the round.
- Continue playing rounds until time runs out. On each round, team members change roles between rolling the die and moving along the number line.

Activity 3: Card Wars

Materials for Level I



- Four sets of cards
  - Cards are 4 inches by 2 inches in height and length
  - Each set has 11 cards
  - The 11 cards are numbered from 0 – 10 with the corresponding number of dots in a grid below the numeral.
  - The cards are blank on the opposite side of the numerals and dots

Get Ready to Play at Level I

- The first time the cards are introduced to children, let the children play with a complete set of cards to get familiar with them. Point out the salient features of the cards (numerals should be on top; all cards have a grid; the number on the card represents how many dots are in the grid, etc.).
- Pair children so that two children are playing against each other and the other two children are playing against each other. Children should be seated so that they are on the same side of the table. This way, children do not have to read cards upside-down.
- Each child receives 11 cards (0-10 in random order). These cards are to be stacked together and placed in front of the child with the numerals and dots not visible. Remind them not to turn them over or look at them until you say "go!"

Playing the Game at Level I

- When you say "Ready, set, go!" all the children take the top card from his or her stack and place them on the table in front of them with the numeral at the top of the card.
- The child with the highest card states the number that he or she has and then determines who has more. It is likely that some children will always say they have more. This provides an opportunity for discussion among the children and the facilitator.
- In condition A, the other child must *doublecheck*.
- When they agree, the one with the highest card wins the set of cards. These cards are set aside for the next round rather than added to the stack in play.
- When two children have "the same" number on their cards, then there is a "war." Those children turn over an additional card from their stack of cards and compare those two cards to determine who wins the war and all four cards in play.
- Rounds continue until one child has won all of the cards or until time runs out. If time runs out, the child with the most cards wins.
- At the end of the game, children should put the cards in order from least to greatest.

Materials for Level II

- Remove the cards 6 -10 from the 4 sets of cards. Use only the 0 – 5 cards.

- Pencil, paper, and/or tokens for children to use if they need to use representations to help with addition

Get Ready to Play at Level II

- Pair children into teams so that two children are playing against the other two children. Children should be seated so that they are on the same side of the table. This way, children do not have to read cards upside-down.
- Each child receives 6 cards (0-5 in random order). These cards are to be stacked together and placed in front of the child with the numerals and dots not visible. Remind them not to turn them over or look at them until you say "go!"

Playing the Game at Level II

- When you say "Ready, set, go!" all the children take the top card from his or her stack and place them on the table in front of them with the numeral at the top of the card.
- The facilitator will assist children in making and justifying claims about which team has more.
- The team with the highest sum of cards wins the set of cards. These cards are set aside for the next round rather than added to the stack in play.
- When teams have "the same" summation, there is a "war." Teams turn over one additional card to determine which team wins the war and all six cards in play.
- Rounds continue until one team has won all of the cards or until time runs out. If time runs out, the team with the most cards wins.
- At the end of the game, children should put the cards in order from least to greatest.

APPENDIX B

DIFFERENCES BETWEEN TREATMENT GROUPS' PARTICIPATION IN SMALL GROUP MATH GAMES

Activity 1: Hi Ho Cherry-O

SPECIFIC LEARNING OBJECTIVES & EXAMPLES OF INSTRUCTIONAL APPROACHES FOR ACTIVITY 1		
	Condition A	**Condition B**
Numeral Recognition (1 – 4)	Children count and say the number on the spinner; children will be assisted by a peer when they are incorrect. If the peer is unable to help, the facilitator will assist.	Children count the number on the spinner as evidenced by their producing the set; children will be corrected by the facilitator when they are incorrect
Count (1 – 10)	Children will count out loud how many fruit they are supposed to remove on the spinner and the number of fruit left on the tree	Children will count how many fruit they are supposed to remove on the spinner; the facilitator will ensure that the child is playing accurately and correct him or her as needed.
One-to-One Correspondence (1 – 4)	Children will count out loud and produce the set of fruit removed from their tree/bush	Children will count and produce the set of fruit removed from their tree/bush
Cardinality (1 – 10)	Children will state the cardinal number of the set produced by their peer	Children will hear the facilitator state the cardinal number of the set produced
Doublecheck, Sign Language, and Questions	Children will be asked to attend to what the child *in-play* is doing during the game. For example, Child 2 who is *not in play* will check that Child 1 who is *in play* has the same number of fruit as was spun on the spinner. Children will learn sign language for more than, less than, equals, all, some, none and be encouraged to use and say them whenever those apply. Children will be asked closed-ended questions (e.g, how many is that?) and open-ended questions (e.g., what would happen if that bird ate more than one?). Children will be asked to narrate their actions during the game (e.g., I landed on two, so I am taking two apples off my tree. One, two.) These differences are meant to facilitate math-talk, peer-to-peer interactions, and engagement. These three actions should work in tandem to accelerate math development.	Children will hear the teacher use math language for more than, less than, equals, all, some, none. Children will be asked closed-ended questions (e.g, how many is that?) only. Overall, the facilitator is to direct activities so that the interactions are teacher-student centered.

Activity 2: Walk-The-Line

SPECIFIC LEARNING OBJECTIVES & EXAMPLES OF INSTRUCTIONAL APPROACHES FOR ACTIVITY 2		
	Condition A	**Condition B**
Counting (1 – 20)	Children will count out loud the number of dots on the die, the spaces on the number line	Children will count the number of dots on the die, the spaces on the number line
Subitizing (1 – 6)	Children have the opportunity to subitize the dots on the die and say the number it is.	Children have the opportunity to subitize the dots on the die and say the number it is.
One-to-One Correspondence (1 – 6)	Children will move the same number of spaces as was rolled on the die	Children will move the same number of spaces as was rolled on the die
Numeral Recognition (1 – 20)	Children will say the numerals on the number line	Children will hear the facilitator say the numerals on the number line
Number Line (1-20)	The greater the number the child is standing on, the greater the distance the child has moved away from zero, the greater the number of discrete moves the child has made, the greater number of number names the child has spoken, and the longer the child has played.	The greater the number the child is standing on, the greater the distance the child has moved away from zero, the greater the number of discrete moves the child has made, the greater number of number names the child has spoken, and the longer the child has played.
Cardinality (1 – 6)	Children make associations between numerals and the quantity they represent. Children state the cardinal number of spaces the child moves along the line.	Children make associations between numerals and the quantity they represent. Facilitators state the cardinal number of spaces the child moves along the line.
Addition	Children must add the number rolled to their current position (e.g., a child is on 2 and rolls a 3 so 2 + 3 = 5). Children will be asked to narrate this.	Children must add the number rolled to their current position (e.g., a child is on 2 and rolls a 3 so 2 + 3 = 5). Facilitators will narrate this.
Doublecheck, Sign Language, and Questions	Children check that the other team correctly identified the correct number on the die and moved the correct number of spaces. Children will use sign language for more than, less than, equals, all, some, none and be encouraged to use and say them whenever those apply. Children will be asked closed-ended questions (e.g, how many is that?) and open-ended questions (e.g., What would happen if you had to move back two spaces?). Children will be asked to narrate their actions during the game (e.g., I was standing on two, we rolled one, so I moved to the three). These differences are meant to facilitate math-talk, peer-to-peer interactions, and engagement. These three actions should work in tandem to accelerate math development.	Children will hear the teacher use math language for more than, less than, equals, all, some, none Children will be asked closed-ended questions (e.g, how many is that?) only. Overall, the facilitator is to direct activities so that the interactions are teacher-student centered. Children will not be restricted from talking.

Activity 3: Card Wars

SPECIFIC LEARNING OBJECTIVES & EXAMPLES OF INSTRUCTIONAL APPROACHES FOR ACTIVITY 3		
	Condition A	**Condition B**
Counting (0 – 10) Subitizing (0 – 10)	Children count or subitize the number on the card.	Children count or subitize the number on the card.
Cardinality (0-10)	Association of numerals with the quantity they represent; Children will state the cardinal number of the card.	Association of numerals with the quantity they represent; Facilitators will state the cardinal number of the card.
Comparing (0-10)	Children reason together how they know that one is more than the other (e.g., you know this is 6 because this side of the grid is filled up and then there is one more on the other side)	Children will determine which card is "more." If they are unable to determine this independently, the facilitator will tell them.
Addition (with numbers 0-5 to a possible sum of 10)	Children reason together the sum of the cards with the assistance of the facilitator as necessary	Children determine the sum of the cards. If they are unable to determine this independently, the facilitator will tell them.
Doublecheck, Sign Language, and Questions	In the *doublecheck*, a child must give a reason how he or she knows that one card is more than the other (e.g., this one has more dots than that one). Only when the pair agrees can the cards *in-play* be taken off the table. Children will use sign language for more than, less than, equals, all, some, none and be encouraged to use and say them whenever those apply. Children will be asked closed-ended questions (e.g, how many is that?) and open-ended questions (e.g., How do you know which card has more?). These differences are meant to facilitate math-talk, peer-to-peer interactions, and engagement. These three actions should work in tandem to accelerate math development.	 Children will hear the teacher use math language for more than, less than, equals, all, some, none. Children will be asked closed-ended questions (e.g, how many is that?) only. Overall, the facilitator is to direct activities so that the interactions are teacher-student centered.

APPENDIX C

NUMBER SENSE ASSESSMENT

INSTRUCTIONS FOR SCORING SHEET

TASK 1: One to One Correspondence, Producing a Set and Cardinality

Materials

- 14 cubes and cup
- Test Booklet

Stop Rule

If child is unable to produce 4 and in addition, can not state the cardinality of the set he or she did produce, then continue to Task 2.

Procedure

Place 14 cubes in a pile in front of the child
Say: *Here are some cubes. Can you give me 4?*

Place the set that the child gives you in a cup. Holding the cup so that the child can not count the cubes in the cup
Say: *How many cubes are in this cup?*

Take the cubes out and let the child see that you return those to his or her pile and that the cup is empty.
Say: *I'm going to put them all back. Now, can you give me 7?*

Place the set that the child gives you in a cup. Holding the cup so that the child can not count the cubes in the cup
Say: *How many cubes are in this cup?*

Take the cubes out and let the child see that you return those to his or her pile and that the cup is empty.
Say: *Can you give me 10?*

Place the set that the child gives you in a cup. Holding the cup so that the child can not count the cubes in the cup
Say: *How many cubes are in this cup?*

Scoring

Children receive 1 point for each set that is produced correctly; Children receive 1 point for each cardinal number stated correctly; Total possible number of points: 6

TASK 2: Subitizing

Materials

- Booklet
    - Child and assessor sit side-by-side at a table with the booklet in front of them both.
    - The pages each show a certain number of dots between 0 and 10
    - The order of the dot quantities are: 3, 1, 2, 5, 0, 4, 8, 6, 7, 10, 9
    - The dots are arranged like those on a die.
    - Between the pages with the dots on them are blank pages of color so that the child can not see the dots on the next page.
    - In a spiral bound booklet1 set of blue dot cards 0 – 10

Stop Rule

Cease this task after 3 incorrect responses or failures to respond (in a row).

Procedure

Before opening the booklet, say: *When I turn this page, tell me how many dots there are as fast as you can. Ready?*

Turn the page so that the child can see the page with the dots on them for a count of 1 and then turn that page over to the next blank page. Do the same for all the dot pages, in the order given above.

If the child at any time asks to see the page again or says, "where are the dots?" say: *How many did you see?* If the child still does not respond with an answer, say: *Let's do the next one.*

If the child shows fingers after the first try, stop at the blank page following the first trial and say: *"How many was that?"* If no response, you can say: *Use your words to tell me how many dots there are.* However, if children accurately show you the same number of fingers as there were dots on the page, count these correct.

If the child counts the dots, even mentally counting after the page has been turned, it is not counted correct.

Scoring

Children receive 1 point for each correct response; Total possible number of points: 11

TASK 3: Numerical Magnitude Comparison (adapted from Ramani & Siegler)

Materials

- Booklet.
    - Each page has two sets of dots between 1 and 9 on it separated by a vertical line down the page.
    - The pairs are: (1, 9), (2, 6), (4, 1), (5, 3), (7, 8), (9, 9)

- Each page has two numbers between 1 and 9 on it separated by a vertical line down the page.
- The pairs are: (1, 9), (2, 6), (4, 1), (5, 3), (7, 8), (3, 3)

Stop Rule

Cease this task if the child is unable to make a determination after having been asked the prompt. Cease this task after 3 incorrect responses or failures to respond (in a row).

Procedure

Part A: With the booklet on a blank page, say: *I am going to show you some dots. Tell me which side has more.*

Practice item: Open the booklet to the first pair (1, 9)
If child responds correctly (i.e., 9), continue to the other items.
If child does not respond, say: *which side has more?*
If after being prompted, child does not respond or answer correctly, say: *John(Jane) has one cookie, Andy(Sarah) has nine cookies. Which is more: one cookie or nine cookies?*
If the child still does not understand, move on to the next task.
If the child is able to answer correctly, continue with the remaining items (On the remaining items, only ask the question, "*Which side has more*?" Do not prompt with the question, "*John(Jane) has X cookie, Andy(Sarah) has Y cookies. Which is more?"*).

Part B: With the booklet on a blank page, say: *I am going to show you two numbers. Tell me which side is more.*

When you get to the numeral section of this task (1, 9), say: *Which number is more?*
If child responds correctly (i.e., 9), continue to the other items.
If child does not respond, say: *which number is more?*
If after being prompted, child does not respond or answer correctly, say: *John(Jane) has one cookie, Andy(Sarah) has nine cookies. Which is more: one cookie or nine cookies?*
If the child still does not understand, move on to the next task.
If the child is able to answer correctly, continue with the remaining items (On the remaining items, only ask the question, "*Which number is more*?" Do not prompt with the question, "*John(Jane) has X cookie, Andy(Sarah) has Y cookies. Which is more?"*).

On the pairs (9, 9) and (3, 3), if the child gives the equal sign, say: *Do you remember the word for that? What does this* (showing the sign that the child made) *mean?* If the child says, "same" or "equal" or gives the sign for "equal" than the response is correct.

Scoring

1 point for each correct response; Total possible number of points: 12

TASK 4: Number Line (adapted from Ramani & Siegler)

Materials

- Number Line
  - White strip of paper approximately two feet in length and three inches wide with a black line that extends the length of the paper.
  - Along the black line there are 11 Velcro circles spaced two inches apart.
  - The number line strip is folded so that only the first six Velcro circles are accessible. When unfolded, all 11 Velcro circles are accessible.
- Numeral cards 0 – 10
  - Each card has a Velcro dot on the back so that it can be attached to the number line.

Stop Rule

If the child was unable to complete Tasks 1 -3 of this assessment, do not administer Task 4. If the child is administered Task 4, but is unable to place the 1-card on the number-line after having to be prompted where to place it, cease the assessment. If the child places the 1-card correctly on the number line before or after being prompted, cease the assessment after the child misplaces three numbers in a row.

Procedure

Child should be seated with the number line in front of them so that 11 Velcro circles are accessible. Lay the following numbers on the table, above the number line, in this order: 0, 1, 10. Place the 0 on the first Velcro dot to the child's left. Place the 10 on the last Velcro dot, furthest from the 0.
Say: *Look at this number line* (run your finger along the number line). *This is zero and this is 10* (pointing to each). *Can you put the 1 where it belongs on this line?*
If the child responds by placing the one on the number line in the correct or incorrect place, continue with the assessment. Remove the last number placed on the number line by the child before asking him or her to place the next number. Ask the child to place the following numbers in this order:
Say: *Can you put the 5 where it belongs?*
Say: *Can you put the 3 where it belongs?*
Say: *Can you put the 2 where it belongs?*
Say: *Can you put the 4 where it belongs?*
Say: *Can you put the 6 where it belongs?*
Say: *Can you put the 9 where it belongs?*
Say: *Can you put the 7 where it belongs?*
Say: *Can you put the 8 where it belongs?*
If the child is unable to place the one on the line the first time being asked, say: *When we count, we start with one, so the one goes here.* Remove the one and ask the child to place the one where it belongs. If the child is unable to complete this task, do not continue with the assessment. Do not give the child the point if the child requires a prompt to place the one.

Scoring

1 point for each correct response; Total possible number of points: 9

THE NUMBER SENSE ASSESSMENT SCORE SHEET

Child's Name ______________________________
Assessor's ID ______________________________
Date ______________________________

	PROMPT	RESPONSE	SCORE
TASK 1: One to One Correspondence, Producing a Set and Cardinality	*Here are some cubes. Can you give me __?How many cubes are in this cup?*		
	Produce 4		
	State cardinal #4		
	Produce 7		
	State cardinal #7		
	Produce 10		
	State cardinal #10		
			______
TASK 2: Subitizing	*When I turn this page, tell me how many dots there are as fast as you can. Ready*		
	3		
	1		
	2		
	5		
	0		
	4		
	8		
	6		
	7		
	10		
	9		
			______

TASK 3: Numerical Magnitude Comparison	*I am going to show you two numbers. Tell me which is the number is more.*		
	(1, 9)		
	(2, 6)		
	(4, 1)		
	(5, 3)		
	(7, 8)		
	(9, 9)		
	(1, 9)		
	(2, 6)		
	(4, 1)		
	(5, 3)		
	(7, 8)		
	(3, 3)		
			______
TASK 4: Number Line	*Look at this number line* (run your finger along the number line). *This is zero and this is 10* (pointing to each). *Can you put the 1 where it belongs on this line?*		
	1		
	5		
	3		
	2		
	4		
	6		
	9		
	7		
	8		
			______

TOTAL POSSIBLE POINTS ON THIS ASSESSMENT: 38

Total for tasks 1 - 4	

ASSESSOR'S GUIDE FOR USING THE TEST BOOKLET

To prepare the booklet, a blank page should be between each of the subitizing tasks. The entire booklet can be bounded in a spiral notebook. When ready to assess children follow these guidelines:

SUBITIZING TASK: Before opening the booklet, say:

When I turn this page, tell me how many dots there are as fast as you can. Ready?

- Open the booklet to the first dot, then turn to the blank page following.
- If the child uses sign to express how many, prompt child to use words
- The correct number of fingers held up to represent the number of dots are still counted as correct
- Stop Rule: 3 consecutive incorrect answers.

MAGNITUDE COMPARISON PART A: With the booklet on a blank page, say:

I am going to show you some dots. Tell me which side has more.

- Practice item: Open the booklet to the first pair (1, 9)
    - If child responds correctly (i.e., 9), continue to the next page.
    - If child does not respond, say: *which side has more?*
    - If after being prompted, child does not respond or answer correctly, say: *John(Jane) has one cookie, Andy(Sarah) has nine cookies. Which is more: one cookie or nine cookies?*
- If the child still does not understand, move on to PART B.
- If the child is able to answer correctly, continue with the remaining items (On the remaining items, only ask the question, "*Which side has more*?"
- Stop Rule: 3 consecutive incorrect answers.

MAGNITUDE COMPARISON PART B: With the booklet on a blank page, say:

I am going to show you two numbers. Tell me which number is more.

- Use same procedure as PART A.

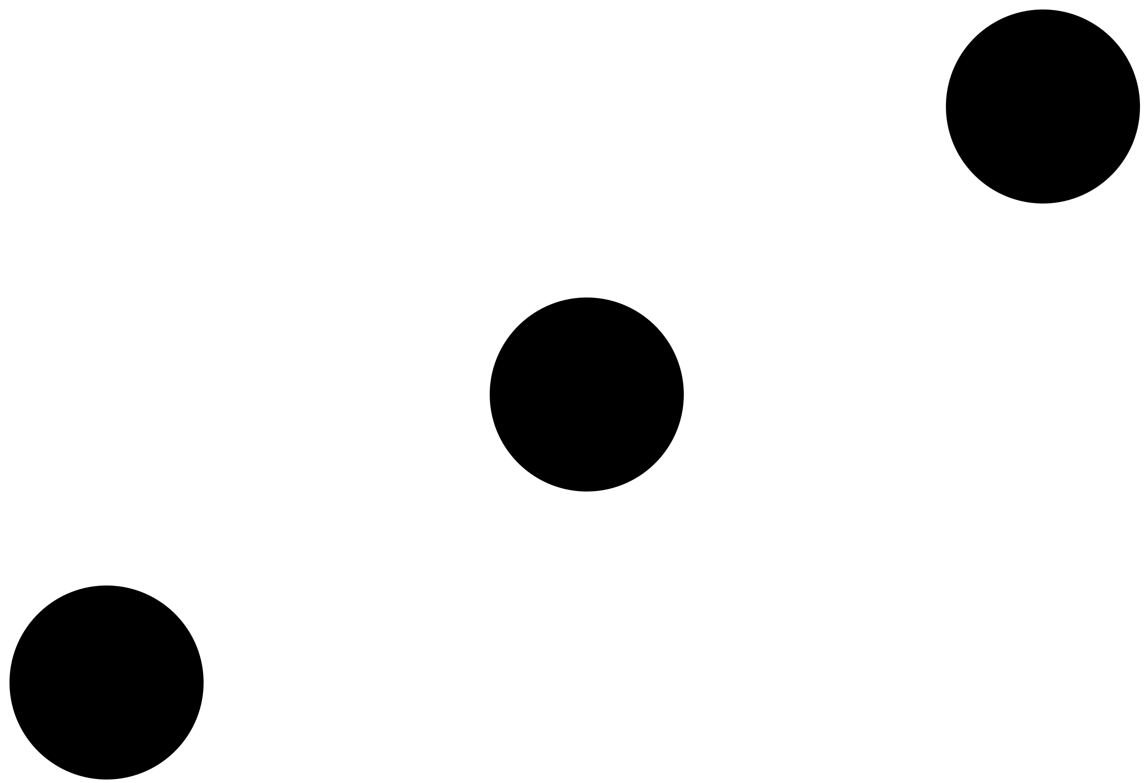
NUMBER LINE TASK: Say: *Look at this number line* (run your finger along the number line). *This is zero and this is 10* (pointing to each). *Can you put the 1 where it belongs on this line?*

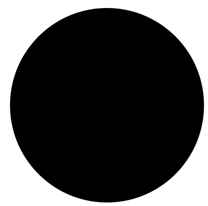
- If they do it correctly, say: "*That's right. When we count one comes first, so we place it first on the number line.*"
- If incorrect,
    - Say "*When we count one comes first, so we place it first on the number line.*"
    - Remove the 1 card from its incorrect place and ask the child to place the 1 where it belongs

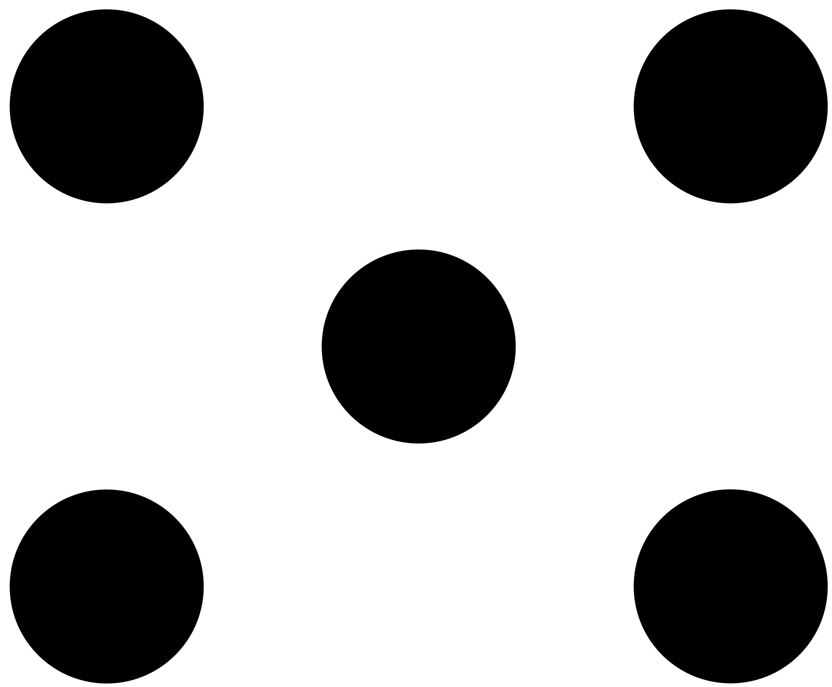
  - If the child is unable to complete the task, do not continue
- If the child places the 1 correctly, continue.
- Stop Rule: 3 consecutive incorrect answers.

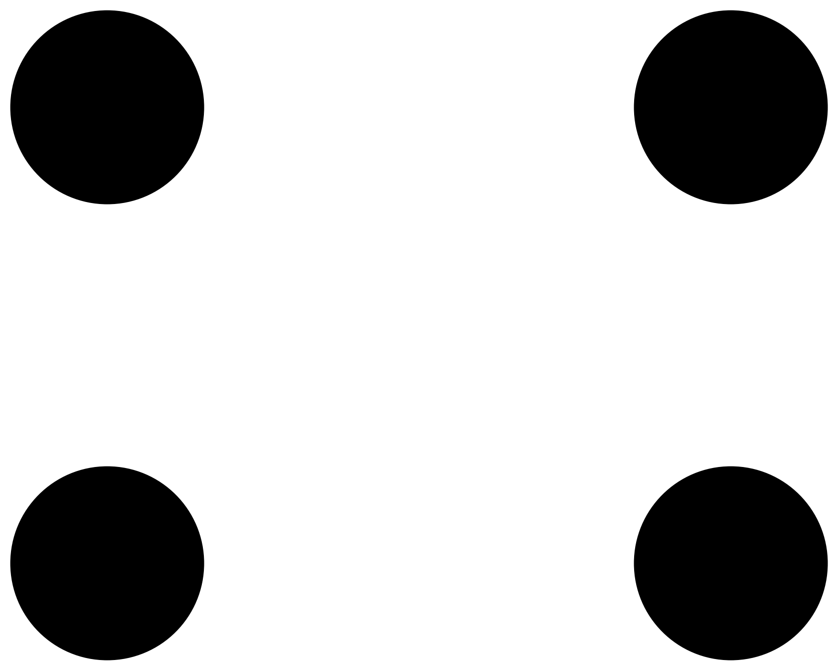
NUMBER SENSE ASSESSMENT

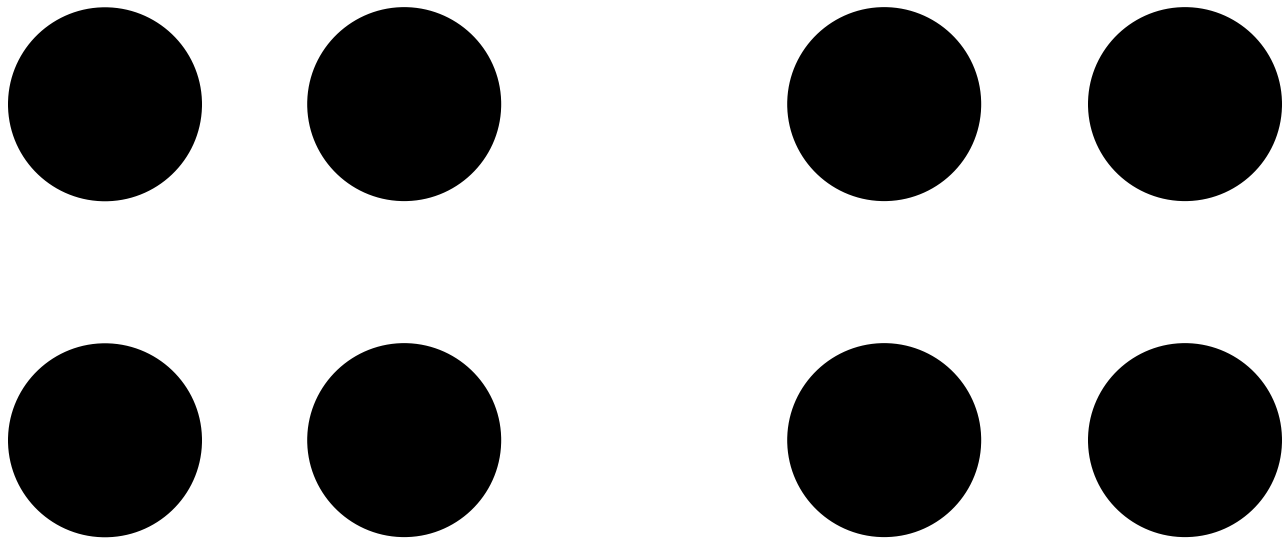
TEST BOOKLET

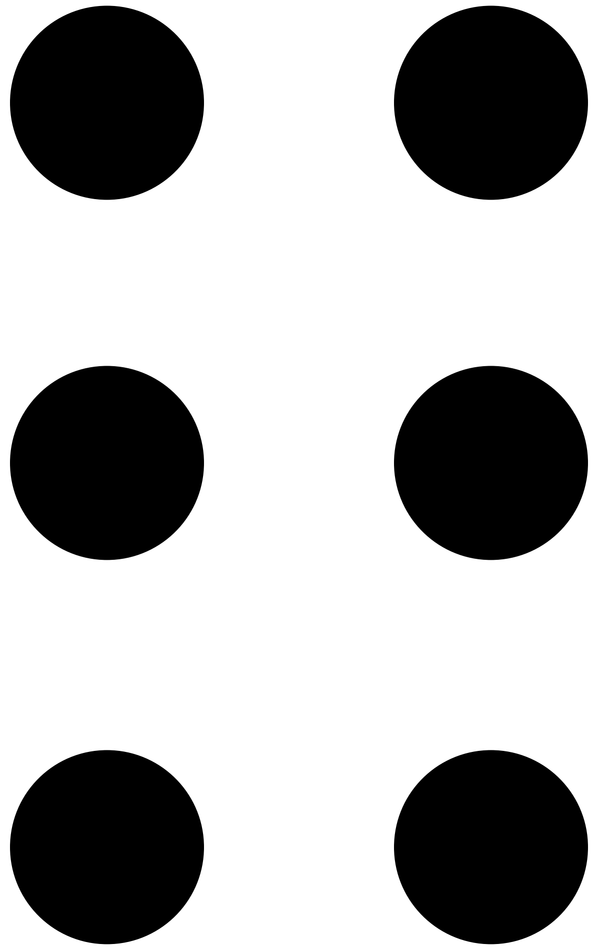


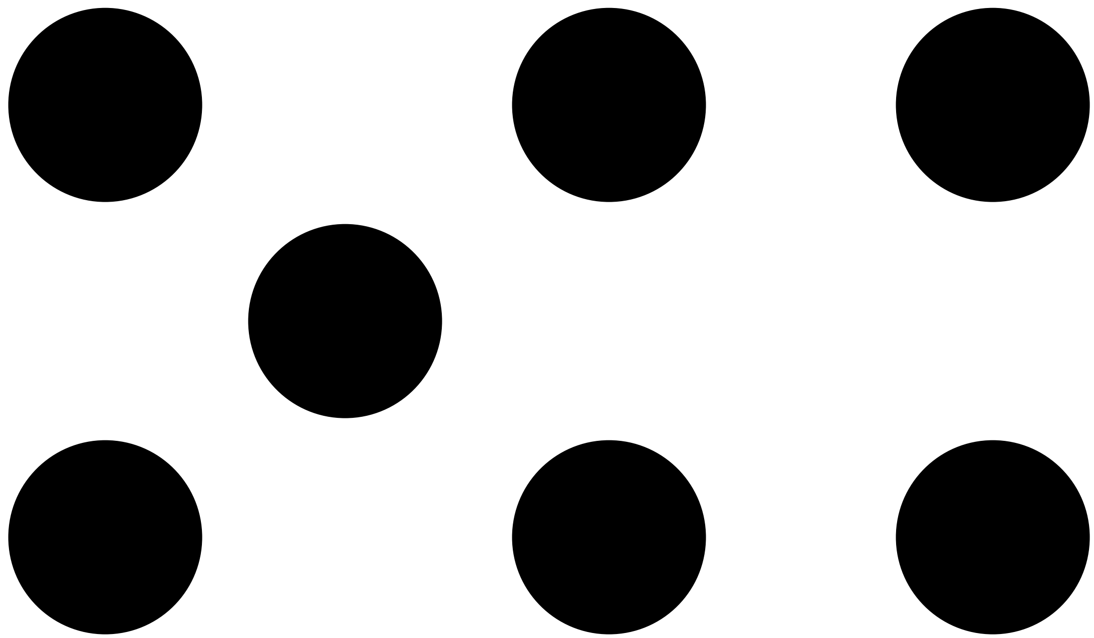


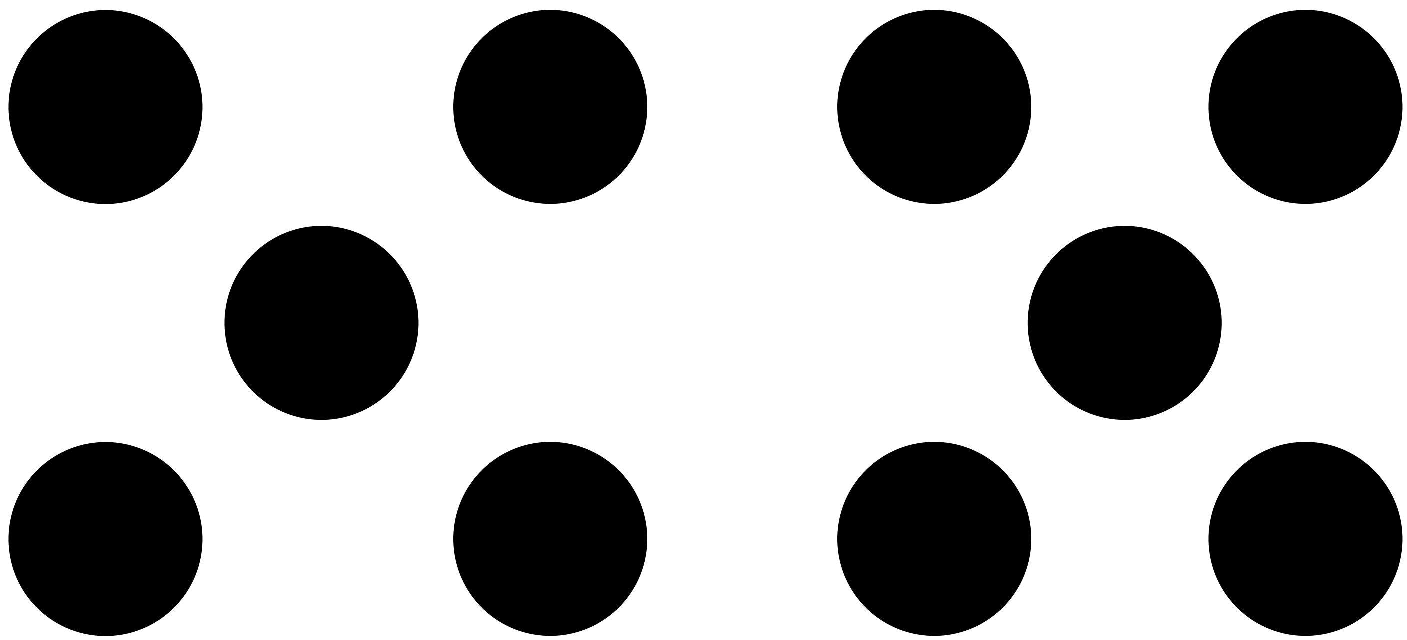


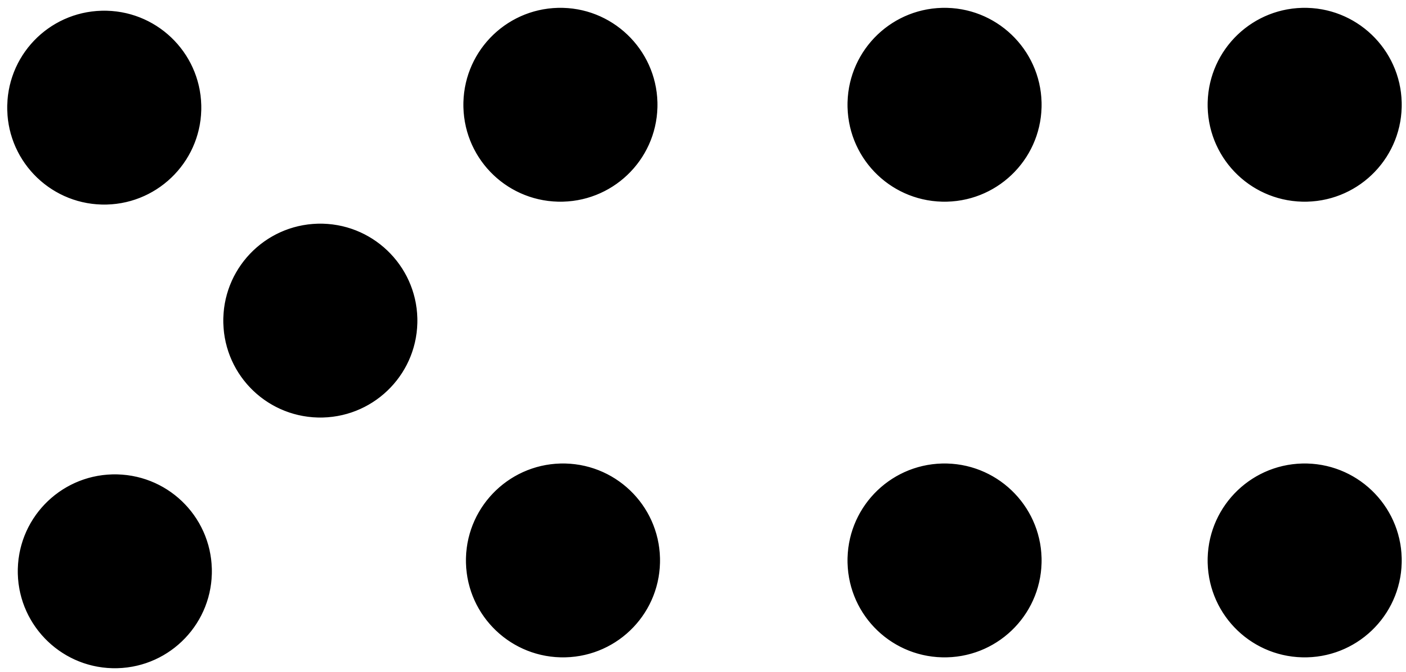


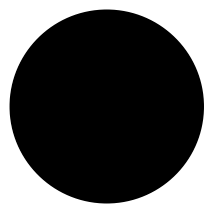


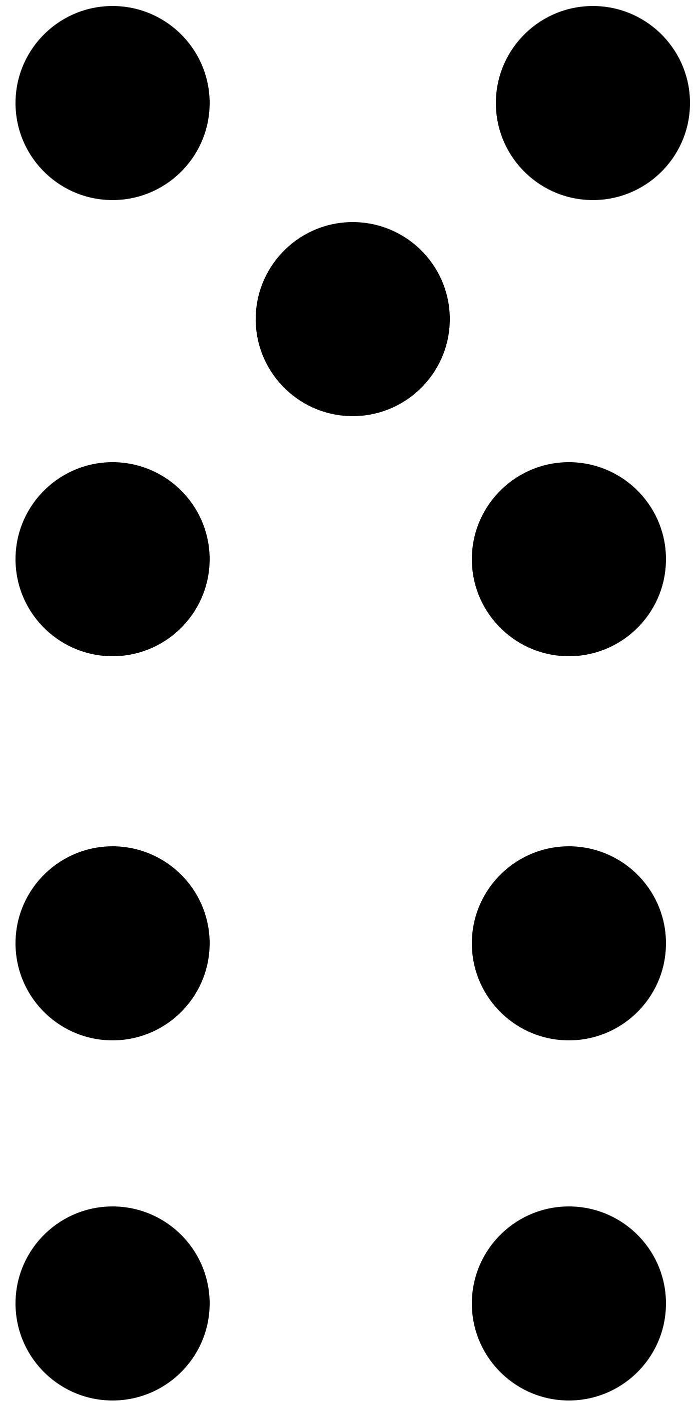


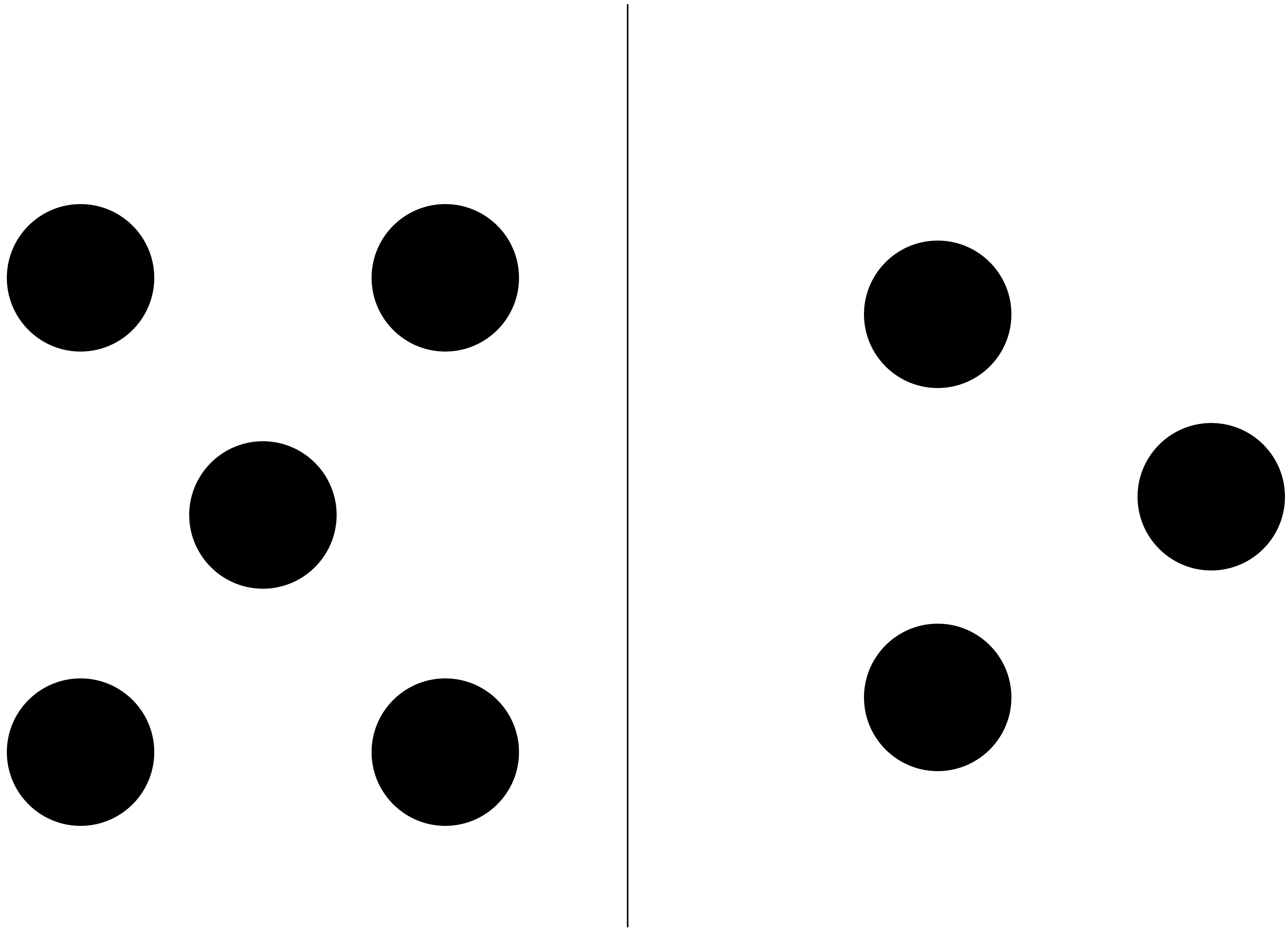


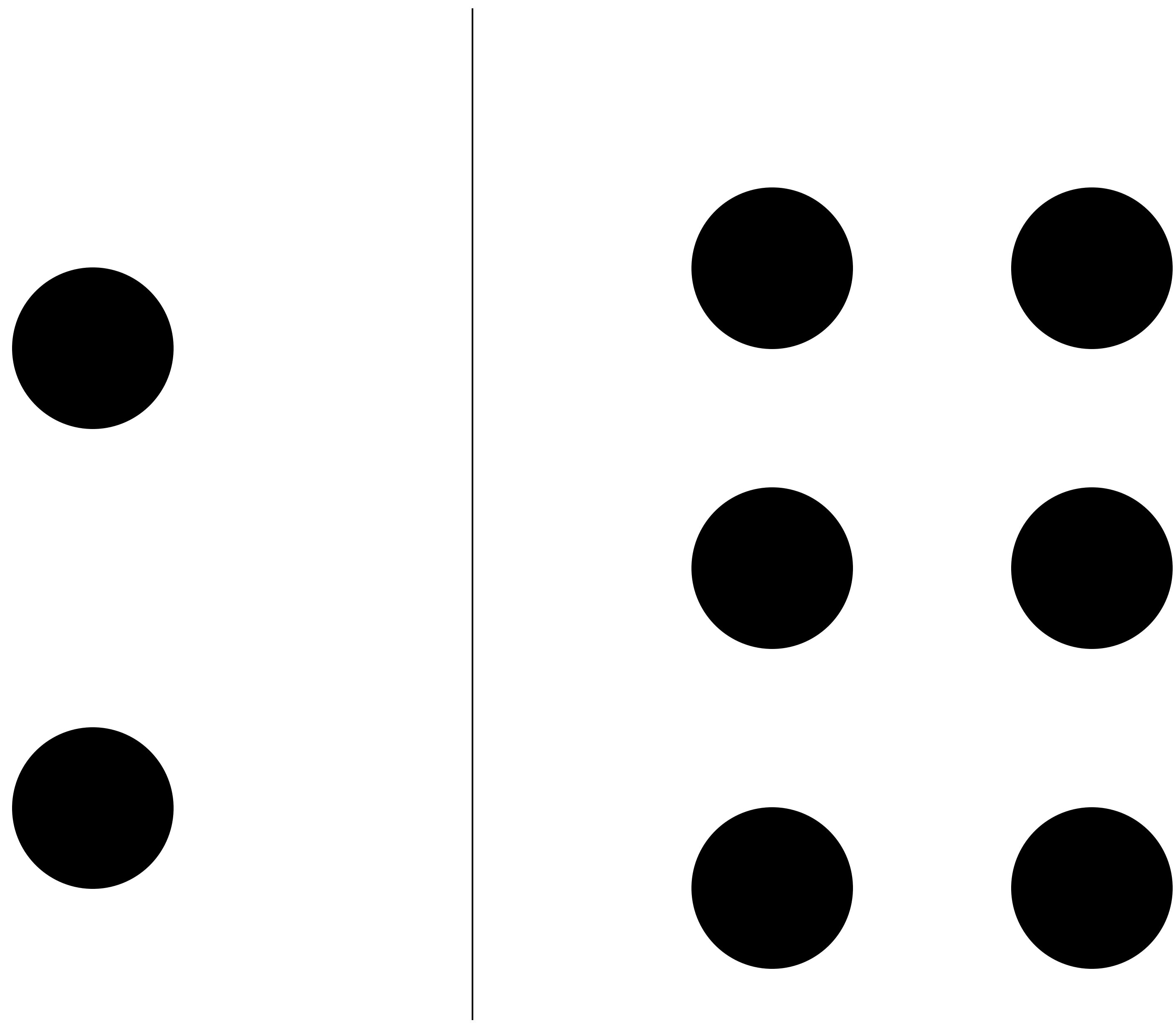


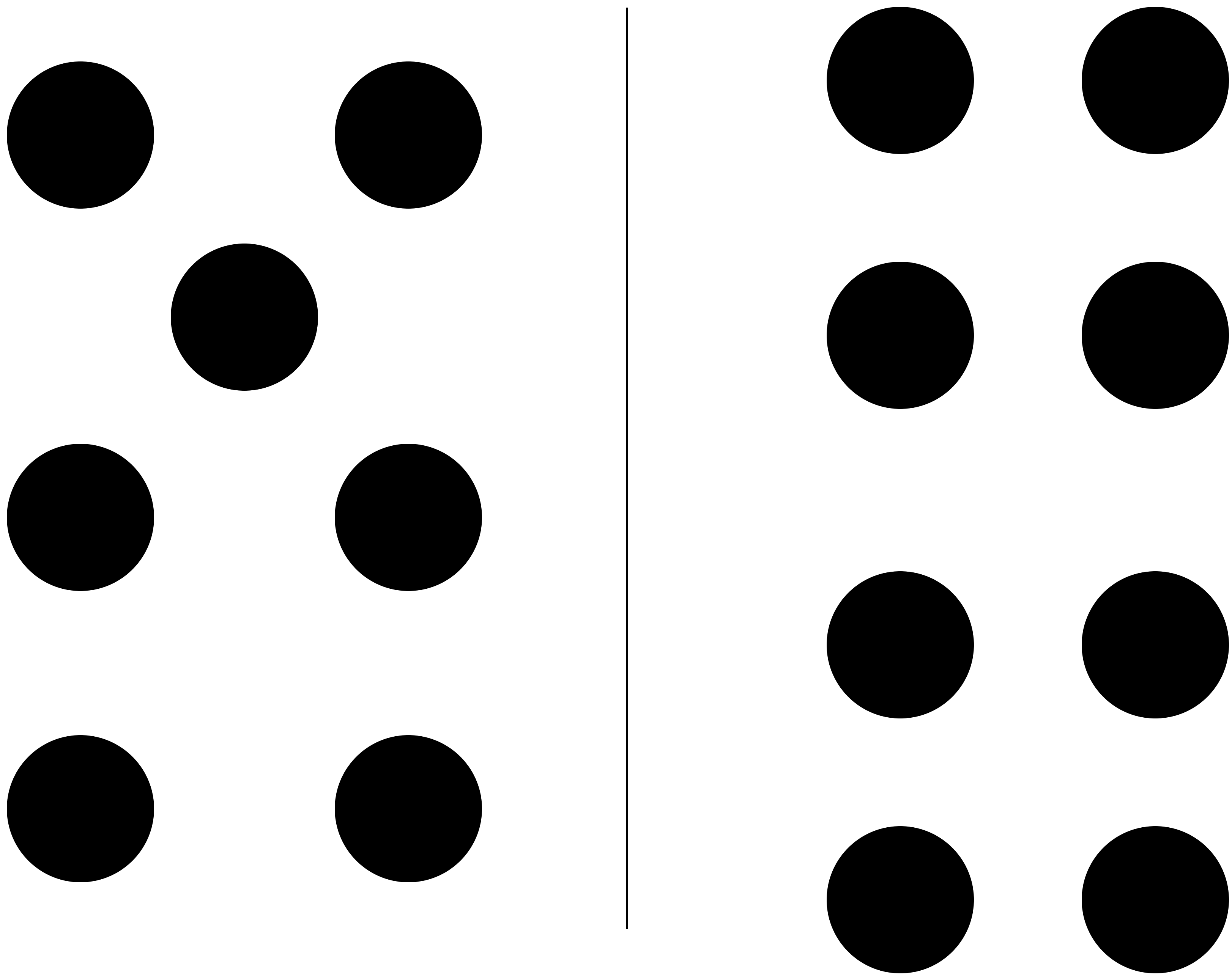


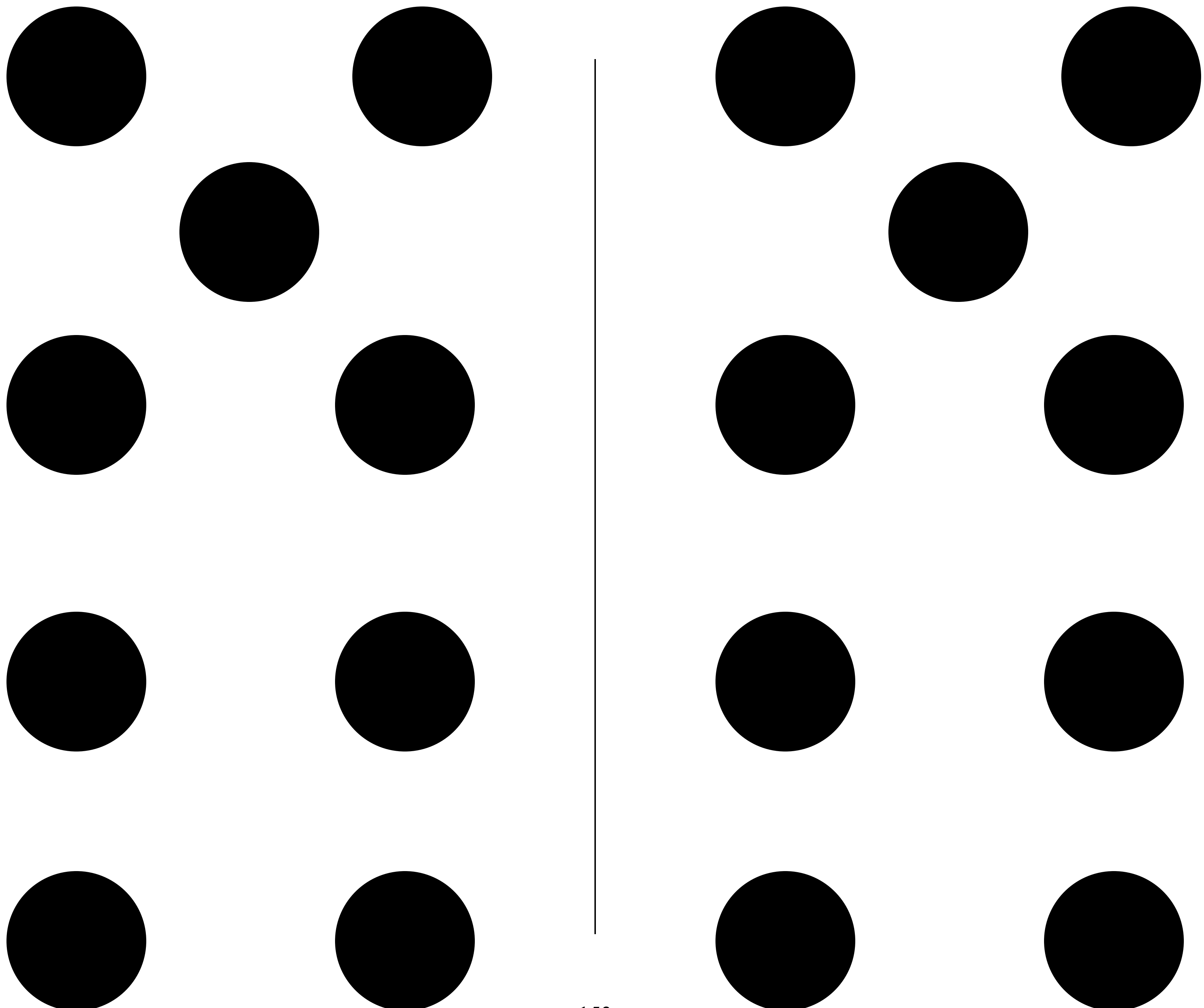


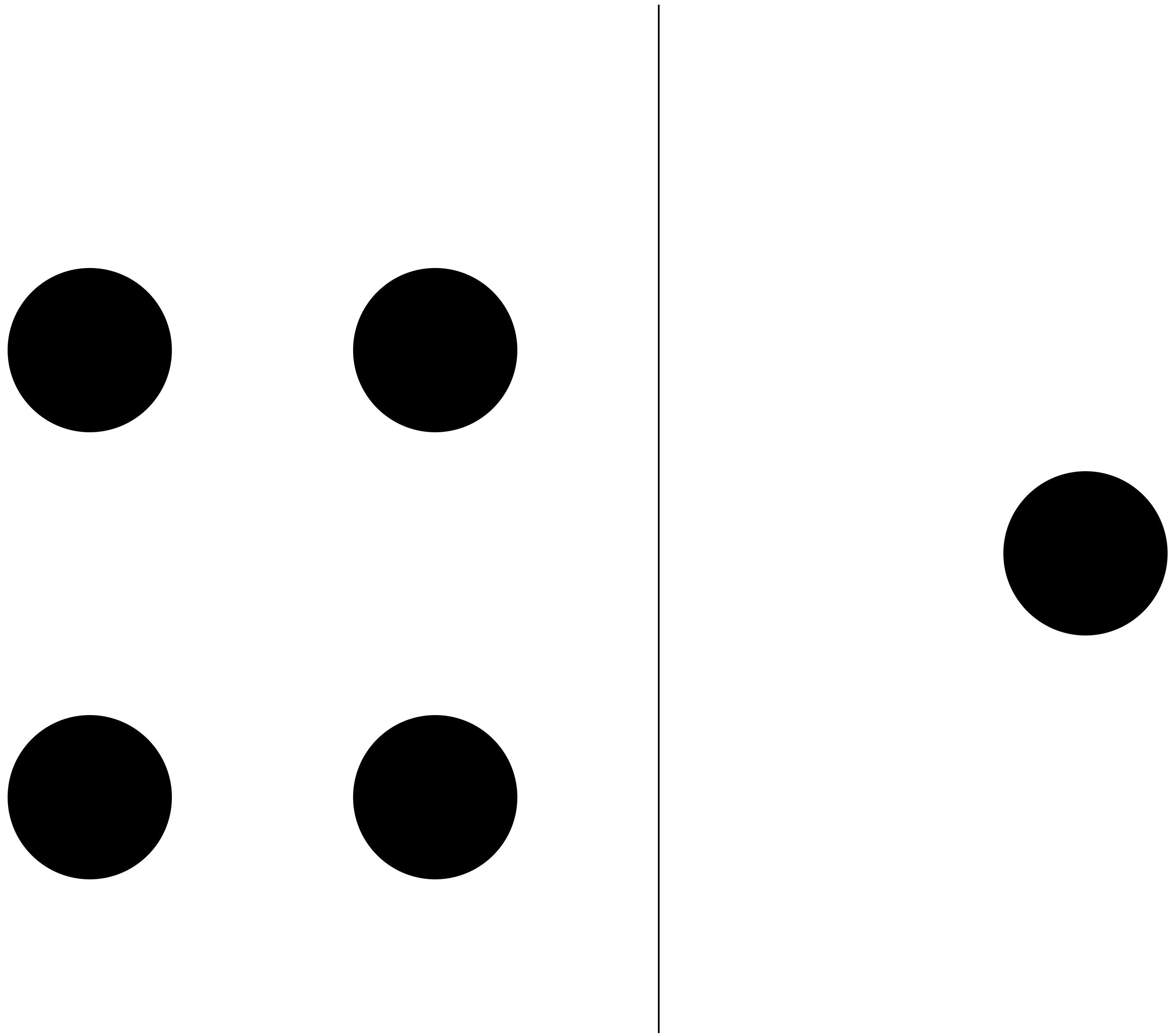












1

9

2

6

5

3

7

8

APPENDIX D

MATH OUTCOMES DEFINED & MEASURED

	Definition	Demonstrated By
Rote Counting	Knowing the number words; Reciting numbers in the correct sequence (up to how many?)	Being able to say the numbers in the correct order. Being able to count backwards in the correct order.
One-to-one Correspondence	Understanding that each object in a set should be counted only once (small sets?).	Being able to count objects, spaces on a number line, or dots on a die by touching each one only once and assigning one number name with each touch
Cardinality	Understanding that numbers tell how many are in a set; Understanding that the last number counted can represent the set, which includes all the objects counted before; Associating numerals with the quantity they represent.	Being able to answer the question "how many" without recounting the set of objects.
Subitizing	Knowing how many objects are in a group without counting. Perceptual subitizing is recognizing small groups of up to 5; Conceptual subitizing begins when children can determine how many were in a group by subitizing smaller groups and then combining the groups.	Being able to instantly know "how many" when a small number of objects or dots on a die are seen
Numeral recognition	Recognizing the written symbols for the spoken number	Being able to look at the symbol "1" or "4" and say the word "one" or "four"
Number line	Knowing the order of numbers (Arabic symbols or dots on a card) on the number line.	Being able to place numbers in order of magnitude. Being able to name the number that is before, after, or between two other numbers. For example, knowing that three comes before four, five comes after four, and that six is between five and seven. Being able to say that 5 is closer to 7 than 3 is.
Addition of Small Sets (Composing Number)	Combining two ore more collections of objects to form a larger group;	Being able to find the sum of two small groups. At first, children learn to combine small collections nonverbally. For example, a child is shown two objects that are then covered by a napkin. Afterwards, a third object is placed under the napkin and the child can produce a set of 3 objects "to match." At a higher level of ability, children can count on. For example, to add 2 and 3, a child might say, "3… 4, 5."
Magnitude comparison	Determining which set is larger, greater, or more than another, or that two sets are the same (equal).	At first, children are able to perceptually determine which group has more when there is a noticeable difference. Later this skill develops and children learn to put objects into one-to-one correspondence and say that the set with additional objects in it has more. When children have a developed number sense, they know that 8 is one more than 7.
Reasoning (Verbal and Non-verbal)	Generating conclusions from assumptions or premises. Verbal reasoning includes argumentation and negotiating. Non-verbal reasoning includes understanding the meaning of visual information and recognizing relationships between visual concepts.	In verbal reasoning, children are able to explain *why* and *how* or determine which is the *tallest* tree. In non-verbal reasoning, children can look at perceptual cues, such as a pattern, and decide how to complete the pattern or look at a sequence of pictures and determine what comes next.

TEMA **Item Number (3 = the child must answer 3 out of 3 trials correctly for item to be counted as correct; 4/5 means a child must answer 4 out of 5 trials correctly to be counted as correct)**	Count	One to one Crspnd.	Cardinality	Subitizing	Numeral Recognition	Number Line	Adding	Magnitude Comparison	Producing a	Reasoning	Not a TAMS focus
A1 (3) How many cats do you see? Why is this not cardinality? If a child counts, 1,2,3, doesn't the examiner ask how many is that?	X										
A2 (3) Show me __ fingers	X								X		
A3 (1) Count (your fingers) for me.	X								X		
A4 (4) Which side has more?								X			
A5 (3) Make yours just like mine (adult puts 3 tokens on table, covers them)	X								X		
A6 (2) Count the stars	X										
A7 (2) How many stars did you count?			X								
A8 (4/5) Make yours just like mine. (adult places two tokens, then adds 1 more)							X		X		
A9 (3) How many tokens are there? (conservation of number)											X
A10 (2) Give me __ tokens		X							X		
A11 (3) Hold up __ fingers									X		
A12 (1) 1, 2, 3, now you count by yourself…	X										
A13 (3) What number comes next? ___ and then…?	X					X					
A14 (3) What number is this?					X						
A15 (3) Write the number. (symbolic representation)					X						X
A16 (2/3) Word problems: How many altogether?							X				
A17 (4/4) Word problems: __ + 3 = 5, etc.							X				
A18 (3/4) Show me how many there are. (symbolic repres)											X
A19 (5) Which is more? ___ or ___ ? (1 to 5)						X		X			
A20 (5) Which is more? ___ or ___ ? (5 to 10)						X		X			
A21 (1) Count as high as you can (to 21)	X										
A22 (2) What number comes next; __ and then…?	X					X					
A23 (2) Count these dots with your fingers.	X										
A24 (1) Count backwards starting from 10											X
A25 (2) Share 12 between 2											X
A26 (2/3) How much are __ and __							X				
A27 (4/6) Which is closer to __, __ or __?						X					
A28 (1) Give me exactly 19 tokens.		X							X		
A29 (3) What number is this?					X						
Total	**10**	**2**	**1**	**0**	**3**	**5**	**4**	**3**	**7**	**0**	**5**

WJ-III Applied Problems	Count	One to one Crspnd.	Cardinality	Subitizing	Numeral Recognition	Number Line	Adding	Magnitude Comparison	Producing a group	Reasoning	Not a TAMS focus
AP1 Show me one finger	X								X		
AP2 Show me two fingers	X								X		
AP3 How many apples are there in this picture (if child counts, we ask for cardinal)	X		X								
AP4 How many boats are there (if child counts, we ask for cardinal)	X		X								
AP5 How many birds are there (if child counts, we ask for cardinal)	X		X								
AP6 Put your finger on the box with two kittens/ the box with three kittens (If they have to count to get the answer, do not count as subitizing.)	X			X							
AP7 How many children do not have balloons	X										
AP8 How many flowers are there (if child counts, we ask for cardinal)	X		X								
AP9 Put your finger on the flower with one bee/ three bees (If they have to count to get the answer, do not count as subitizing.)	X			X							
AP10 If you take away two cans, how many would be left							X				X
AP11 Show me the number that tells how many dogs there are	X				X						
AP12 Point to the group with 5 dots	X			X							
AP13 If the top man jumped off, how many men would be left							X				
AP14 If you took away two buttons, how many would be left							X				
AP15 If Jessica ate three, how many would be left							X				
AP16 If you take away three crayons, how many would be left							X				
AP17 If you had two books and got two more, how many would you have							X				
AP18 If the top two boxes were pushed off, how many would be left in the stack							X				X
AP19 If you had these balloons and someone gave you two more, how many would you have							X				
AP20 What time does this clock say											X
AP21 Point to 2 things you could buy with 50 cents											X
AP22 If you drew 5 more circles, how many would there be							X				
Total	11	0	4	3	1	0	9	0	2	0	4

WJ-III Quantitative Concepts	Count	One to one Crspnd.	Cardinality	Subitizing	Numeral Recognition	Number Line	Adding	Magnitude Comparison	Producing a group	Reasoning	Not a TAMS focus
QC-A1 How many dogs are there (if child counts, we ask for cardinal)	X		X								
QC-A2 What number is this					X						
QC-A3 Count for me; start with one…	X										
QC-A4 How many chairs are there	X										
QC-A5 What is this called (circle)											X
QC-A6 What number is this					X						
QC-A7 Point to the highest building/ lowest building											
QC-A8 Point to the tallest tree/ shortest tree											
QC-A9 This is the last tree. Point to the first tree/ middle tree											
QC-A10 What number comes between three and five	X					X					
QC-A11 What comes right before eight	X					X					
QC-A12 Tell me the days of the week											X
QC-B1 1 2 3 __					X	X					
QC-B2 1 __ 3 4					X	X					
QC-B3 5 6 7 __					X	X					
QC-B4 7 8 __ 10					X	X					
QC-B5 15 16 17 __					X	X					
QC-B6 18 19 20 __					X	X					
QC-B7 5 __ 3 2					X	X					
QC-B8 2 4 6 __					X	X					
QC-B9 6 5 4 __					X	X					
Total	5	0	1	0	11	11	0	0	0	0	2

Number Sense Assessment	Count	One to one Crspnd.	Cardinality	Subitizing	Numeral Recognition	Number Line	Adding	Magnitude Comparison	Producing a group	Reasoning	Not a TAMS focus
Task 1: Can you hand me 4 cubes?	X	X							X		
Task 1: Now tell me how many cubes are in the cup?			X								
Task 1: Can you hand me 7 cubes?	X	X							X		
Task 1: Now tell me how many cubes are in the cup?			X								
Task 1: Can you hand me 10 cubes?	X	X							X		
Task 1: Now tell me how many cubes are in the cup?			X								
Task 2: How many dots did you see? (3)				X							
Task 2: How many dots did you see? (1)				X							
Task 2: How many dots did you see? (2)				X							
Task 2: How many dots did you see? (5)				X							
Task 2: How many dots did you see? (0)				X							
Task 2: How many dots did you see? (4)				X							
Task 2: How many dots did you see? (8)				X							
Task 2: How many dots did you see? (6)				X							
Task 2: How many dots did you see? (7)				X							
Task 2: How many dots did you see? (10)				X							
Task 2: How many dots did you see? (9)				X							
Task 3: Tell me which side has more dots? (1, 9)								X			
Task 3: Tell me which side has more dots? (2, 6)								X			
Task 3: Tell me which side has more dots? (4, 1)								X			
Task 3: Tell me which side has more dots? (5, 3)								X			
Task 3: Tell me which side has more dots? (7, 8)								X			
Task 3: Tell me which side has more dots? (9, 9)								X			
Task 3: Tell me which number is more? (1, 9)					X			X			
Task 3: Tell me which number is more? (2, 6)					X			X			
Task 3: Tell me which number is more? (4, 1)					X			X			
Task 3: Tell me which number is more? (5, 3)					X			X			
Task 3: Tell me which number is more? (7, 8)					X			X			
Task 3: Tell me which number is more? (3, 3)					X			X			
Task 4: Can you put the one where it belongs?					X	X					

Task 4: Can you put the five where it belongs?					X	X					
Task 4: Can you put the three where it belongs?					X	X					
Task 4: Can you put the two where it belongs?					X	X					
Task 4: Can you put the four where it belongs?					X	X					
Task 4: Can you put the six where it belongs?					X	X					
Task 4: Can you put the nine where it belongs?					X	X					
Task 4: Can you put the seven where it belongs?					X	X					
Task 4: Can you put the eight where it belongs?					X	X					
	3	3	3	11	15	9	0	12	3	0	0

Leiter-R	Count	One to one Crspnd.	Cardinality	Subitizing	Numeral Recognition	Number Line	Adding	Magnitude Comparison	Producing a group	Reasoning	Not a TAMS focus
Fluid Reasoning Tasks										21	

Total number of MATH items grouped from each assessment to measure each skill	**Count**	**One to one Crspnd.**	**Cardinality**	**Subitizing**	**Numeral Recognition**	**Number Line**	**Adding**	**Magnitude Comparison**	**Producing a group**	Reasoning	Not a TAMS focus
TEMA	10	2	1	0	3	5	4	3	7		
WJ III- Applied Problems	11	0	4	3	1	0	9	0	2		
WJ III– Quantitative Concepts	5	0	1	0	11	11	0	0	0		
Number Sense Assessment	3	3	3	11	15	9	0	12	3		
	29	**5**	**9**	**14**	**30**	**25**	**13**	**15**	**12**		

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