SELF-EXPLANATION WORTH THE WHILE: A COMPARISON AGAINST PRACTICE AND TIME ON TASK

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CHAPTER I

INTRODUCTION

What types of instructional practices maximize student learning? In typical classrooms, students spend much of their classroom time practicing math skills (Hiebert, Givvin, Garnier, & Hollingsworth, 2005). Is having students work through practice problems an effective use of their time? Or rather, would scaffolding practice with a conceptually-oriented learning activity that pushes them to reflect on what they do and do not understand be better? Prior research suggests that a conceptually oriented learning activity, such as self-explanation, improves learning through focusing the learner's attention on attempting to understand the underlying concepts, but it has not been simultaneously contrasted with a group of students who received the same amount of practice and group who received the same amount of study time. In the introduction, the potential learning benefits of self-explanation and practice within mathematics will be discussed, and then compared. This study investigated the differential learning benefits of these two uses of instructional time with the aim of elucidating the roles of self-explanation, amount of practice, and study time in learning.

Conceptually Oriented Learning Activity: Self-Explanation

One conceptually oriented learning activity that has intuitive appeal and empirical backing is prompting students to generate explanations to themselves, referred to as *self-*

explanation. Self-explanation can be defined as *generating explanations to oneself that contain information that is not directly given in the learning materials in an attempt to make sense of new information* (Berthold, Eysink, & Renkl, 2009; Chi, 2000; Rittle-Johnson, 2006). The self-explanation effect was initially discovered while observing students learning physics by studying worked examples. The students who learned most spontaneously generated more and higher quality self-explanations, or statements that refined, expanded, or related the underlying principle to the problem at hand. The amount and quality of these statements were correlated with learning gains (Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Renkl, 1997). This finding inspired many studies that investigated self-explanation as a causal learning mechanism, in which people are prompted to self-explain correct materials. This effect has now been demonstrated across many domains, such as reading, electrical engineering, and biology, and in wide-ranging age groups, from 4-year-olds to adults (Ainsworth & Loizou, 2003; Calin-Jageman & Ratner, 2005; Mayer & Johnson, 2010; McNamara, 2004; Rittle-Johnson, Saylor, & Swygert, 2008). Prompting students to self-explain encourages them to make their knowledge explicit (Chi, Leeuw, Chiu, & Lavancher, 1994).

The act of explaining while making sense of new information is thought to increase understanding through a variety of mechanisms, and is generally thought to facilitate the integration of new and existing knowledge. Although prompting students to self-explain can be thought of as a conceptually-oriented activity, it can benefit both conceptual and procedural knowledge. Conceptual knowledge can be defined as an understanding of principles governing a domain and the interrelations between units of knowledge (Bisanz & LeFevre, 1992; Greeno, Riley, & Gelman, 1984; Rittle-Johnson, Siegler, & Alibali, 2001). Self-explanation is thought to benefit conceptual knowledge by focusing the learner's attention onto attempting to identify the relevant aspects of a domain and its underlying concepts and principles. Specifically, self-explanation helps students continuously update and correct their mental models of the domain and its principles (Chi et al., 1994), facilitates the construction of inference rules that are used in the formation of general principles and proceduralized into usable skills (Chi et al., 1989), and fosters generalization (Lombrozo, 2006). Self-explanation is thought to support the creation of novel goal structures that allow for generalization (Crowley & Siegler, 1999). Prompts to self-explain often encourage the learner to verbalize, and thus make explicit, their knowledge of concepts that govern the domain. As such, both implicit and explicit conceptual knowledge should be enhanced.

Self-explanation can also improve procedural knowledge, or action sequences for solving problems (Anderson, 1993; Rittle-Johnson & Alibali, 1999). The act of self-explaining is thought to broaden the range of problems to which children accurately apply correct procedures, and promote invention of new procedures (Lombrozo, 2006; Seigler, 2002; Rittle-Johnson, 2006). Conceptual and procedural knowledge help each other grow, and developing and linking conceptual and procedural knowledge is important for competence in mathematics (Hiebert, 1986; Rittle-Johnson & Alibali, 1999; Rittle-Johnson, Siegler, & Alibali, 2001).

Specifically within problem solving domains, self-explanation has been demonstrated to have positive effects. When self-explaining, students are often asked to study worked examples (e.g., the problem solving steps of a probability problem) (Große & Renkl, 2004), and then cite a principle or otherwise explain why each step in the worked example is correct. Specifically within mathematics, self-explanation has sometimes been shown to increase conceptual and procedural knowledge, as well as transfer of knowledge to novel tasks, compared to solving the same number of problems without explaining (e.g., Rittle-Johnson, 2006; Atkinson, Renkl, & Merrill, 2003; Hilbert, Renkl, Kessler, Reiss, 2008).

The conceptually oriented activity of prompting students to self-explain can benefit student learning. However, self-explanation requires a significant amount of time compared to working through the practice problems alone (Matthews & Rittle-Johnson, 2009). Additionally, self-explanation can be difficult for a teacher to administer on a whole-class basis. Therefore, it is important to consider if self-explanation is more effective than having students work through practice problems, a classroom activity that is easier to implement.

Practice

The practice and repetition of solving problems has long been upheld as a very practical route to learning, and might indeed be an efficient path to proficiency. There are two broad possibilities for why practice can increase learning (Jonides, 2004). First, practice could allow the learner to acquire greater skill at applying an initial problem solving strategy. Second, practice could allow the learner to acquire a new and more efficient strategy.

The first possibility has a wealth of supporting evidence. In regard to math learning in children, it has been said "practice is important because it can serve to make the use of rules, principles, and thinking strategies, as well as specific facts, automatic" (Baroody, 1987). Research in basic cognitive processes arrived at the same general conclusions. Practice results in the reorganization or restructuring of specific action sequences (Anderson, 1982; 1983; 1987; Rosenbloom & Newell, 1987), and increases the speed of these processes (Frensch & Geary, 1993). Practice on a narrow set of problems also automatizes the mental processes that underlie the problem solving procedure (Logan, 1990; Shiffrin & Schneider, 1977), thus reducing the load on working memory that serves as a single work-space for carrying out cognitive processes (Anderson, 1982; 1983; 1987; Rosenbloom & Newell, 1987). Durable, long-term memory traces of the acquired skill are formed with practice (Chi, Glaser, & Farr, 1988; Ericsson, Krampe, & Tesch-Romer, 1993). Practice also strengthens correct strategy application, and weakens incorrect strategies (Siegler, 2002).

Practice could also serve to help the learner discover a new problem-solving strategy. As a learner works through more practice problems, they have time to discover and test out new, possibly more efficient, problem solving strategies (Lemaire & Siegler, 1995; Siegler & Jenkins, 1989; Siegler & Stern, 1998). New strategies are often not immediately discovered but instead require an accumulation of experience to uncover. Indeed, high levels of practice are often necessary for the discovery of correct procedures (Brunstein, Betts, & Anderson, 2009). Even once a correct strategy is used for the first time, it is not used consistently until much later (Siegler & Jenkins, 1989). For example, when practicing complex arithmetic, learners modified their strategies and eventually settled on a stable problem solving routine (Cary & Carlson, 1999). These new strategies may facilitate generalization to new problem types (Rittle-Johnson, 2006). Thus, having the time to work through multiple practice problems is important for new strategy discovery and use.

Comparing Practice and Self-Explanation Prompts

Which learning activity is the best use of instructional time? Evidence from the self-explanation literature informs this question. A common approach for investigating the effects of self-explanation is to have two conditions work through the same number of practice problems. Both conditions solve the problems, but the self-explain condition spends additional time responding to explanation prompts. In studies with this design, there is evidence for a benefit of self-explanation (Calin-Jageman & Ratner, 2005; Pine & Messer, 2000; Rittle-Johnson, 2006; Siegler, 1995; 2002; Wong, Lawson, & Keeves, 2002). However, in these studies the self-explain condition had the double benefit of more time thinking about the material overall *and* of explaining, relative to the practice-only condition. Indeed, the amount of time on task has been shown to be a very reliable and powerful predictor of learning (Helmke & Renkl, 1992; Logan, 1990; Renkl, 1997), and thus one must attend to this when trying to determine the cause of learning.

Another way to investigate the comparative benefits of practicing versus prompting for self-explanations is to give both conditions the same amount of study time. There are still important differences between these conditions; namely that the control condition now has more practice problems to work through. There are only two studies on math learning with this design, and the results are mixed. When this design was used in a study on mathematical equivalence with elementary school students, there was no apparent benefit of self-explanation compared to students who practiced twice as many problems (Matthews & Rittle-Johnson, 2009). However, this study provided all students with conceptual instruction, which the researchers believed may have lessened the benefit of the explanation prompts. Another study did find a benefit of self-explanation with high school students learning geometry (Aleven & Koedinger, 2002). These students were asked to complete geometry proofs and provide a principle justifying each step of their proof. Importantly, the students were given feedback on the quality of their explanations and could not move along in the sequence until they gave a correct explanation. This feedback on explanation quality is uncommon in the self-explanation literature. In this case, the students who self-explained had improved transfer abilities compared to the control students, even though the self-explain students solved significantly fewer problems. Given the differences in these two studies, no definitive conclusions about the relative merits of practice and prompting for explanations can be drawn.

When solving the same number of problems, prompting for self-explanations was shown to be beneficial, however the self-explain condition had more study time. When both conditions have the same amount of study time, but the self-explain condition works through fewer problems, the results are mixed. Is it really the act of explaining that increases learning? Or, if given a comparable amount of time for practice, can the student come to the same level of proficiency on his or her own? In order to carefully evaluate these questions, the current study investigated the benefits of self-explanation while controlling for the amount of time and number of practice problems.

The Target Domain

We examined the benefits of self-explanation in the domain of mathematical equivalence. Mathematical equivalence is a foundational concept that links arithmetic to algebra (Baroody & Ginsburg, 1983; Carpenter, Franke, & Levi, 2003; Kieran, 1981; Knuth, Stephens, McNeil, & Alibali, 2006; MacGregor & Stacey, 1997). Mathematical equivalence problems (i.e. 3 + 5 + 6 = __ + 6) tap the idea that the amount on both sides of an equation are equal. Correctly solving problems such as these is challenging for elementary school children (Alibali, 1999; Carpenter et al., 2003; Perry, 1991; Perry, Church, & Goldin-Meadow, 1988; Rittle-Johnson, 2006), and many studies demonstrate that children have difficulties with the concept of mathematical equivalence (Alibali, 1999; Behr, 1980; Cobb, 1987; Falkner, Levi, & Carpenter, 1999; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Li, Ding, Capraro, & Capraro, 2008; McNeil, 2007; Perry, 1991; Powell & Fuchs, 2010; Rittle-Johnson, 2006; Rittle-Johnson & Alibali, 1999; Weaver, 1973). These difficulties stem in part from the fact that most students initially view the equal sign *operationally*, as a command to do the calculation on the left and place the answer to the right of the equal sign (Baroody & Ginsburg, 1983; McNeil & Alibali, 2005). They do not understand that the equal sign is really a *relational* symbol, denoting that the expressions or quantities on both sides of the equation are equivalent or of the same value (Baroody & Ginsburg, 1983; Behr, 1980; Carpenter et al., 2003; McNeil & Alibali, 2005). Indeed, the operational view of the equal sign can impede the development of the correct relational view (Kieran, 1981; McNeil & Alibali, 2005). This conceptual misunderstanding interferes with related procedural skills. When solving open equation problems (i.e. 3 + 5 + 6 = __ + 6), students with the operational view exhibit use of specific incorrect strategies. A common naïve incorrect strategy is to add the numbers before the equal sign while completely disregarding the addend on the right side of the equation (Matthews & Rittle-Johnson, 2009). Not only is having a relational understanding of equivalence necessary for correct problem solving, it is foundational for later algebraic reasoning (Jacobs et al., 2007; Kieran, 1992; Knuth et al. 2006; National Research Council, 1998; Steinberg, Sleeman, & Ktorza, 1991). Overcoming the operational view and adopting a relational view of equivalence was the goal of the current intervention. Also, this domain provides a proven learning domain for more rigorously testing the efficacy of self-explanation as a learning activity: self-explanation paired with procedural instruction has been shown to be beneficial for procedural knowledge within mathematical equivalence when the control condition solves the same number of problems (Rittle-Johnson, 2006; Siegler, 2002).

Current Study

In the current study we evaluated the effects of prompting students to self-explanation relative to students who received the same amount of practice and those who received twice as many practice prompts to approximate the same amount of time on task. When considered against comparable practice and comparable time on task, does self-explanation have a unique benefit for learning? This question was evaluated with three conditions. The Control condition received six practice problems, and no self-explanation prompts. The Self-Explain condition also received six practice problems, but was prompted to self-explain on each problem. The Additional-Practice condition received twelve practice problems, which has been determined to take the same amount of time as self-explaining with six problems (Matthews & Rittle-Johnson, 2009). The effect of condition was evaluated in terms of explicit and implicit conceptual knowledge, procedural learning and procedural transfer.

We hypothesized that (1) self-explanation prompts would focus the learner on verbalizing the principle of mathematic equality and broaden the range of problems initial strategies can be applied to. This will result in greater conceptual knowledge, as well as greater procedural knowledge than the control condition. (2) Additional practice problems would allow the learner to strengthen their initial problem solving strategy and allow for the discovery of new strategies, resulting in greater procedural knowledge than the control condition. (3) self-explanation would be more beneficial for learning than additional practice because conceptual knowledge would be greater and procedural transfer would be facilitated.

CHAPTER II

METHOD

Participants

The current study was conducted with students in second, third and fourth grade classes from two urban parochial schools serving similar middle-class, predominantly Caucasian populations. Consent to participate was obtained from 167 students and their parents. A pretest was administered on a whole-class basis, and students who did not already have a sufficient ability to solve math equivalence problems were selected to participate. Pretests were scored quickly, as to allow for intervention work to begin immediately. This initial pretest score of less than 85% correct overall identified the 108 children who participated in the instructional intervention. These students were randomly assigned to an intervention condition. After a more detailed pretest scoring, students who had a score of less than 80% correct on each of the conceptual and procedural knowledge measures were ultimately included, resulting in a sample of 80 students. Of these, 5 students were dropped. Three students' intervention sessions were interrupted by unexpected school activities, one student received extra math tutoring and we were asked by the school to exclude that student, and one student was accidentally run through the intervention twice. 75 students were included in the final sample, which included 35 second graders (16 girls and 19 boys), 28 third graders (17 girls and 11 boys), and 16 fourth graders (6 girls and 10 boys). The average age was 8.79 years (range 7.44 –

10.69). Teachers reported that the students had encountered math equivalence problems before, but not frequently. The intervention took place during the spring semester.

Table 1. Number of students in each condition by grade

	Grade			
	2nd	3rd	4th	**Total**
Control	11	7	2	**20**
Self-Explain	8	9	5	**22**
Additional Practice	16	12	5	**33**
Total	**35**	**28**	**12**	**75**

Design

Participating students completed a pretest, an intervention, an immediate posttest, and a two-week retention test. Students were randomly assigned to one of three conditions: the Control condition (n=20), the Self-Explain condition (n=22), and the Additional-Practice condition (n=33) (see Table 1). During the intervention, students were provided procedural instruction on two mathematical equivalence problems and then worked through either six or twelve practice problems on their own. Students in the Control condition worked through six practice problems, students in the Self-Explain condition also worked through six practice problems, but were prompted to self-explain, and students in the Additional-Practice condition worked through 12 practice problems. Answer feedback was given on all problems, and self-explain students were prompted to explain why example solutions of the problem they just solved were correct or incorrect.

Materials

Intervention

The intervention was conducted one-on-one using a computer interface designed using EPrime software (Psychology Software Tools, 2007). During the intervention, problems were presented on a computer screen. The program recorded the students' responses and amount of time they spent on each part of the intervention. Students were also video- and audio-recorded using the computer's built-in camera, and the current screen was captured concurrently. An additional audio recorder was used as well.

Assessments

The pre-, post- and retention tests were paper and pencil tests that were completed individually. The assessments used in this study were developed in previous measurement development projects (Matthews, Rittle-Johnson, McEldoon, & Taylor, under review; Rittle-Johnson, Matthews, Taylor, & McEldoon, 2011). These assessments have been developed to target an elementary student's understanding of mathematical equivalence and have been tested for validity and reliability (Rittle-Johnson et al., 2011). One version was used as the pretest, and an isomorphic version was used as the post and retention tests. The assessments were broken down into a *conceptual knowledge* section that focused on the meaning of the equal sign and allowable equation structures, and a *procedural knowledge* section that focused on solving equations with missing addends. The conceptual knowledge section had two components. One focused on explicit conceptual knowledge, namely on the meaning of the equal sign. This section contained 6 items, and included questions such as “What does the equal sign (=) mean? Can it mean anything else?” and asked students to define the equal sign in various contexts (See Table 2). The implicit conceptual knowledge section tested students on their knowledge of allowable equation structures. This section contained 11 items, such as judging equations (i.e. 8=8; 5 + 3 = 3 + 5) as true or false (5 items), explaining how they know equations of various structures are true or false (2 items), reconstructing equations from memory after a delay (3 items), and one filler item to ensure they were attending to the assessment (see Table 3). The memory items were administered by the researcher proctoring the assessment. Students were shown an equation in a non-traditional format (i.e. 5 + 2 = _ + 3) for five seconds, and were then asked to write it down exactly as they had seen it. Students are more able to encode equations into their memory if they already have schemas for those structures as allowable and familiar formats, and these items measured this capacity (Larkin, McDermott, Simon, & Simon, 1980; McNeil & Alibali, 2004; Rittle-Johnson et al., 2011).

Table 2. Explicit Conceptual Knowledge Assessment Items and Scoring Criteria

Explicit Conceptual Knowledge Items	
Pre, Post, & Retention	Scoring Criteria
What does the equal sign (=) mean? Can it mean anything else?	1 point if defined relationally at any time - keyword “same” in either spot (e.g. "same on both sides")
Which of these pairs of numbers is equal to 6 + 4? : 5+5, 4+10, 1+2, none of the above	1 point if selects '5+5'
Which answer choice below would you put in the empty box to show that five cents is the same amount of money as one nickel? : 5 cents, =, +, don't know	1 point if selects '='
Is "The equal sign means the same as" a good definition of the equal sign? Circle good or not good.	1 point if selects 'good'
Which of the definitions above is the best definition of the equal sign? The equal sign means the same as, The equal sign means add, or The equal sign means the answer to the problem.	1 point is selects 'The equal sign means the same as'
In this statement: 1 dollar = 100 pennies, What does this equal sign mean?	1point if defined relationally at any time - keyword “same”

Table 3. Implicit Conceptual Knowledge Assessment Items and Scoring Criteria

Implicit Conceptual Knowledge Items					
Judgment Items			Memory Items		
Pretest	Post & Retention	Scoring Criteria	Pretest	Post & Retention	Scoring Criteria
8 = 8	3 = 3	1 point if 'true'	5 + 2 = __ + 3	6 + 3 = __ + 2	If student puts numerals, operators, equal sign, and blank in correct respective positions; 1 point for each problem. Based on the coding of McNeil & Alibali (2004).
7 + 6 = 0	5 + 3 = 8	Filler	__ + 5 = 5 + 8 + 7	__ + 7 = 3 + 5 + 7	
5 + 3 = 3 + 5	31 + 16 = 16 + 31	1 point if 'true'	5 + 4 + 8 = 5+ __	4 + 3 + 9 = 4 + __	
8 = 5 + 10	5 + 5 = 5 + 6	1 point if 'false'	Judge & Explain		
3 + 1 = 1 + 1 + 2	7 + 6 = 6 + 6 + 1	1 point if 'true'	Pretest	Post & Retention	Scoring Criteria
4 = 4 + 0	6 = 6 + 0	1 point if 'true'	8 = 5 + 3 T/F? How do you know?	7 = 3 + 4 T/F? How do you know?	If student judges as 'true', *and* notes that both sides have the same sum or same value, or that inverse is true; 1 point for each problem
			4 + 1 = 2 +3 T/F? How do you know?	6 + 4 = 5 + 5 T/F? How do you know?	

The procedural knowledge section contained 12 items; 4 easy filler items that were included to motivate the student and ensure they were attending to the assessment, and 8 learning and transfer items (See Table 4). Learning items that had the same equation structure as those in the intervention (i.e. 3 + 4 + 5 = __ + 5) (4 at pretest and 3 at post and retention test), and transfer items had an equation structure unlike the intervention items that included subtraction or had the missing addend on the left (i.e. 8 + __ = 8 + 6 + 4; 6 - 4 + 3 = __ + 3) (4 at pretest and 5 at post and retention test). The learning items could be solved using the procedure learned during the intervention, but the transfer items required applying or adapting the procedures learned during the intervention- a standard measure of transfer (Atkinson et al., 2003; Chen & Klahr, 1999). Students were asked to show their work on these problems, so their problem solving strategy use could later be coded.

Table 4. Procedural Knowledge Assessment Items

Procedural Knowledge Items				
	Pretest		Post & Retention	
Item	Equation	Classification	Equation	Classification
1	__ + 5 = 9	Filler	4 + __ = 8	Filler
2	7 = __ + 3	Filler	8 = 6 + __	Filler
3	5 + __ = 6 + 2	Learning	3 + 4 = __ + 5	Learning
4	3 + 6 = 8 + __	Learning	7 + 6 + 4 = 7 + __	Learning
5	4 + 5 + 8 = __ + 8	Learning	__ + 2 = 6 + 4	Transfer
6	__ + 6 = 8 + 5 + 6	Transfer	8 + __ = 8 + 6 + 4	Transfer
7	8 + 5 - 3 = 8 + __	Transfer	6 - 4 + 3 = __ + 3	Transfer
8	5 - 2 + 4 = __ + 4	Transfer	5 + 6 - 3 = 5 + __	Transfer
9	13 = n + 5	Filler	10 = z + 6	Filler
10	y + 6 = 5 + 5	Filler	y + 4 = 8 + 2	Filler
11	6 + 3 + 2 = __ + 2	Learning	3 + 6 + 5 = __ + 5	Learning
12	7 + 9 - 4 = 7 + __	Transfer	2 + 8 - 4 = 2 + __	Transfer

Far transfer was assessed at retention test with eight items intended to tap a higher level of conceptual thinking (e.g., “17 + 12 = 29 is true. Is 17 + 12 + 8 = 29 + 8 true or false? How do you know?”). However, performance on the far transfer items was quite low, and no differences were found across conditions, so this subscale was not considered in further analyses.

Procedure

Pretests were administered on a whole class basis and took 30 minutes to complete. Eligible children participated in a one-on-one intervention session with an experimenter within a few weeks after the pretest. The intervention session lasted about 50 minutes, and consisted of procedural instruction, intervention problem solving, and an immediate post-test. The retention test was then administered on average 2 weeks after the intervention session on a whole class basis, and took about 30 minutes.

Instruction. At the beginning of the intervention, all students received procedural instruction on how to solve repeated addend problems. This was chosen because prior research has demonstrated a benefit of self-explaining in conjunction with procedural instruction (Rittle-Johnson, 2006). Students were shown two repeated addend problems, both with the blank in the final position (i.e. 3+4+6=6+__), and were taught an add-subtract procedure (Matthews & Rittle-Johnson, 2009; Perry, 1991; Rittle-Johnson, 2006). The experimenter described the procedure, and then asked the student to find the missing addend using the following procedure: "There are many ways to solve a problem like this, but one way you could solve it is to add up the 3, the 4, and the 6 on this side of the equal sign (gesture around the left side of the equation), and then subtract the 6 over here on the other side of the equal sign (gesture to the 6 on the right), and then that amount goes in this blank here. Now see if you can solve this problem using that strategy." Once the student provided an answer, they were asked to report how they solved the problem (a strategy report prompt) and were then provided accuracy feedback on the value of the missing addend. The experimenter led each student through two instruction problems.

Intervention Problems

All intervention problems were standard mathematical equivalence problems with a repeated addend on both sides of the equation, and the position of the blank alternated between the first or last addend on the right hand side of the equation (i.e. 3+4+6=6+__; or 3+4+6=__+6) (See Table 5). These were adapted from Matthews & Rittle-Johnson (2009), and have been used in several studies on this topic (i.e. Perry et al., 1988). The students were presented with either six or twelve of these repeated addend problems. Scratch paper was provided for the students to use if they wished. They were asked to quietly determine what number belonged in the blank, and to type their answer directly onto the computer. Their answer was displayed in the blank. Students were asked to verbally report how they determined their answer (*strategy report*). Once they reported their strategy, the correct answer was displayed on the screen below the original problem and the student's answer, and accuracy feedback was given ("You're right/Actually, 7 was the right answer"). In the Control and Additional-Practice conditions, the student then proceeded directly on to the next practice problem. Students in the self-explain condition were given an explanation prompt after each intervention problem.

Table 5. Intervention Items

Intervention Items		
Item	Equation	Condition
1	6 + 3 + 4 = 6 + __	All
2	3 + 4 + 8 = __ + 8	All
3	5 + 3 + 9 = 5 + __	All
4	9 + 7 + 6 = __ + 6	All
5	9 + 3 + 5 = 9+ __	All
6	7 + 8 + 5 = __ + 5	All
7	2 + 5 + 9 = 2 + __	Add'l Practice
8	3 + 7 + 8 = __ + 8	Add'l Practice
9	6 + 3 + 9 = 6+ __	Add'l Practice
10	8 + 3 + 7 = __ + 7	Add'l Practice
11	4 + 5 + 3 = 4 + __	Add'l Practice
12	6 + 7 + 3 = __ + 3	Add'l Practice

Self-Explanation Prompts

In the Self-Explain condition, after each practice problem, students were presented with two examples of the same problem they just solved, one with a correct answer, and another with an incorrect answer. They were told that these were answers from students at another school. “I showed this problem to students at another school. Allison came up with 7, which is the correct answer, and Jenny came up with 13, which is an incorrect answer.” Order of presentation of the correct and incorrect example alternated between problems. Both correct and incorrect examples were displayed simultaneously on the screen.

The students were then prompted to consider the strategies and reasoning behind each example. “*How* do you think Allison got 7, which is the correct answer? *Why* do you know that 7 is the right answer? *How* do you think Jenny came up with 13, which is a wrong answer? *Why* do you know that 13 is a wrong answer?” The *How* questions were designed to have students report the strategy by which they think the hypothetical students solved the problems. Students often gave replies such as, “He added the 3 and the 4 and the 6 and got 13.” The *Why* questions were self-explanation prompts. The goals of these prompts were for the students to consider the conceptual basis for why an answer was correct or not; and ideally, the equivalent nature of both sides of the equation. Students often gave answers such as “It’s the correct answer because he subtracted the 6.” A high quality, or conceptually-based explanation would appeal to the idea that both sides are equal, such as “because I know that 3+4+6 is 13, and then 7+6 is 13”. Once students replied to all four of these correct and incorrect *How* and *Why* prompts, they moved on to solve the next intervention problem.

Intervention Completion

The amount of time it took each student to solve a problem, report their strategy, and self-explain each problem was recorded. After the students completed the intervention problems, they completed a backward digit span task to measure their working memory capacity, as a metric for general processing ability. The students then completed an immediate posttest. A retention test was administered on a whole class basis on an average of two weeks after all students in a particular class completed the intervention session.

Coding

Assessments

Conceptual knowledge. The students' explicit and implicit conceptual knowledge was measured by their performance on items that concerned the meaning of the equal sign and allowable structures for equations. The six items in the explicit conceptual knowledge section were scored as correct or incorrect. See Table 2 for details. The implicit conceptual knowledge section contained 10 scored items that were scored as either correct or incorrect. See Table 3 for details. Internal consistency, as assessed by Cronbach's alpha, was good at both pretest ($\alpha = .701$) and posttest (α=.769).

Procedural knowledge. Procedural knowledge was measured in the assessment and during the intervention as the students' ability to use a correct problem solving strategy to solve open equation items. Based on their written work, students' problem solving strategies were inferred and classified as either a correct strategy, an incorrect strategy, or that the item was unattempted. See Table 9. Correct strategies include Equalizer (determine the values on both sides of the equation, and determine the value needed to make them equal), Add-Subtract (add all the numbers on the left side of the equation and then subtract the value on the right), Grouping (in problems with repeated addends, ignore the repeated value and sum the other two to determine the value of the missing addend), or they could have determined the correct answer but had an Incomplete Procedure (describe some operations but not enough to classify) or had Insufficient work. Incorrect strategies include Add All (add all numbers in the problem and place the sum in the blank), Add to Equal (add all the numbers up to the equal sign and place that sum in the blank), or used an Other Incorrect strategy.

Students could be given a correct problem solving strategy code even if they did not come up with the computationally correct answer. This coding is in line with prior research studies in this domain (Matthews & Rittle-Johnson, 2009; McNeil & Alibali, 2004). Accuracy measured by correct strategy use and by computational accuracy are highly correlated (R=0.95). See Table 9 for coding scheme details. Internal consistency, as assessed by Cronbach's alpha, was good at both pretest ($\alpha = .672$) and posttest (α=.821).

Intervention

Practice problems. Verbal strategy reports from the intervention problems were coded for problem solving strategy use, in the same way as the written procedural knowledge assessment items.

Explanations. In the intervention session in the Self-Explain condition, students' verbal explanations were coded for explanation quality. See Table 12 for coding scheme and student performance data.

Reliability

A primary coder coded all student work. An independent coder coded 20% of all student work. Kappa coefficients for interrater agreement were determined for assessments at all time points (see Table 6).

Table 6. Reliability Kappa Coefficients

Reliability Kappa Coefficients			
Assessment	Pretest	Posttest	Retention
Conceptual			
Structure	0.827	0.867	0.912
Equal Sign	1.000	1.000	1.000
Procedural	0.856	0.909	0.933
Far Transfer			0.970
Intervention			
Strategies	0.725		
Explanations			
How Right	0.936		
Why Right	0.687		
Why Wrong	0.877		

Missing Data

Our retention test was administered on a whole class basis, and five students were absent for the retention test. One of the absent students was in second grade, and the other 4 were in third grade. Two of the students were in the Additional-Practice condition, 2 were in the Control condition, and 1 of the students was in Self-Explain condition. The absent students had a range of assessment scores and did not differ from the other students on the pretest, so we could presume they were missing at random. Their missing data were replaced using a multiple imputation technique; because multiple imputation leads to more precise and unbiased conclusions than does case-wise deletion (Peugh & Enders, 2004; Schafer & Graham, 2002). We used the expectation-maximization (EM) algorithm for Maximum Likelihood Estimation via the missing value analysis module of SPSS, as recommended by Schafer and Graham (2002). The children's missing scores were estimated from all non-missing values on continuous variables that were included in the analyses presented below. Analyses using a case-wise deletion approach yielded the same pattern of findings.

Data Analysis

The hypotheses are tested in the following analyses by using and ANOVA model with three planned comparisons. The effect of additional practice is determined by comparing Additional-Practice to Control, the effect of explaining with the same amount of practice and more time is determined by comparing Self-Explain to Control, and the better use of additional time on task is evaluated by comparing Additional-Practice to Self-Explain.

CHAPTER III

RESULTS

We first discuss students' performance on the pretest. We follow this with a report of the effects of condition on students' posttest and retention test performance, as well as specific strategy use. Finally, we explore how condition affected performance on intervention activities, including the quality of students' explanations, and how this related to performance on the assessments.

Pretest Knowledge

There were no initial differences between conditions in grade (M=2.7, SD=.73), age at pretest (M= 8.8, SD=.80), or backwards digit span (M=4.6, SD=1.3). There were also no differences in their IOWA standardized national percentile rank scores in math (M=57.23, SD=24.44) or reading (M=61.40, SD=23.87).

The conditions were also similar in pretest scores. Means and standard deviations are listed in Table 7. There were no significant differences in conceptual knowledge overall or in explicit conceptual knowledge. However, the Additional-Practice condition had significantly higher implicit conceptual knowledge scores than the Self-Explain condition, $F(1,71)=4.21$, $p=0.044$, $\eta_p^2=0.056$. There were no differences on procedural learning items, but there were differences on procedural transfer items. The Additional-Practice condition performed better than the Self-Explain condition, $F(1,71)=7.854$,

$p=.007$, $\eta_p^2=0.10$. Differences in pretest scores were addressed in later analyses by having pretest score entered as a covariate.

Assessment Performance

To analyze differences between conditions on the post and retention test scores, a series of repeated measures ANCOVAs were performed, with the pretest measure of the respective outcome variable and backwards digit span entered as covariates. The covariates were used to control for prior knowledge and general ability differences. Planned contrasts were conducted to find specific differences between the Self-Explanation and Control conditions, Additional-Practice and Control conditions, and Self-Explain and Additional-Practice conditions. Tables present raw means and standard deviations unless otherwise noted, and the graphs in Figures 1-4 present post and retention test marginal means and standard error bars. Details of each analysis including the F value, significance level, and effect size are presented in Table 8.

Table 7. Assessment Component Scores by Condition. Raw mean percentage correct and standard deviations shown.

Assessment Component	Time	Control		Self-Explanation		Additional Practice	
		Mean	SD	Mean	SD	Mean	SD
Conceptual Knowledge	Pretest	38.3	21.7	36.6	17.7	43.4	20.4
	Posttest	45.0	23.6	55.1	22.5	47.8	20.4
	Retention	47.2	25.9	55.0	25.0	54.5	19.8
Explicit-Equal Sign	Pretest	35.8	29.8	39.4	27.0	35.9	24.0
	Posttest	50.0	32.4	47.7	30.1	44.9	29.3
	Retention	50.9	30.5	49.2	28.1	50.0	31.3
Implicit-Structure	Pretest	39.6	22.4	35.2	21.5	47.2	23.9
	Posttest	42.5	23.1	58.7	22.9	49.2	23.5
	Retention	45.4	26.5	57.9	27.9	56.7	21.9
Procedural Knowledge							
Learning Items							
Correct Strategy Use	Pretest	27.1	28.4	33	35.7	26.5	28.6
	Posttest	48.3	38.2	56.1	41.6	67.7	32.8
	Retention	38.5	38.4	57.6	42.7	58.5	42.7
Transfer Items							
Correct Strategy Use	Pretest	12.4	15.1	21.6	23.5	30.3	27.8
	Posttest	37	32	52.7	39.3	46.1	37.2
	Retention	43.5	37.2	60.8	32.5	49	37.8

Table 8. Assessment Performance by Condition using Planned Comparisons. Significant differences between conditions noted, as determined via ANCOVAs with backward digit span and the measure at pretest used as covariates.

Assessment Component	DFs	Additional Practice vs Control			Self-Explain vs Control			Self-Explain vs Additional Practice		
		F	p	η_p^2	F	p	η_p^2	F	p	η_p^2
Conceptual Knowledge	1,70	0.244	0.623	0.004	6.343	0.014*	0.089	5.272	0.025*	0.075
Explicit-Equal Sign	1,65	0.085	0.771	0.001	0.218	0.642	0.003	0.049	0.825	0.001
Implicit-Structure	1,65	1.081	0.302	0.016	14.06	0.000*	0.178	9.301	0.003*	0.125
Procedural Knowledge										
Learning Items										
Correct Strategy Use	1,70	4.607	0.035*	0.062	1.324	0.254	0.019	0.838	0.363	0.012
Incorrect Strategy Use	1,70	4.443	0.039*	0.060	2.285	0.135	0.032	0.241	0.625	0.003
Transfer Items										
Correct Strategy Use	1,70	0.080	0.777	0.001	2.207	0.142	0.031	1.858	0.177	0.026
Incorrect Strategy Use	1,70	0.016	0.900	0.000	2.931	0.091	0.040	3.974	0.050*	0.054

Effect of Condition on Conceptual Knowledge

Students in the Self-Explain condition generally had higher conceptual knowledge scores than students in the other two conditions. The conceptual knowledge part of the assessment included two sections. The explicit conceptual knowledge section focused on the meaning of the equal sign, and the implicit conceptual knowledge section focused on knowledge of equation structures.

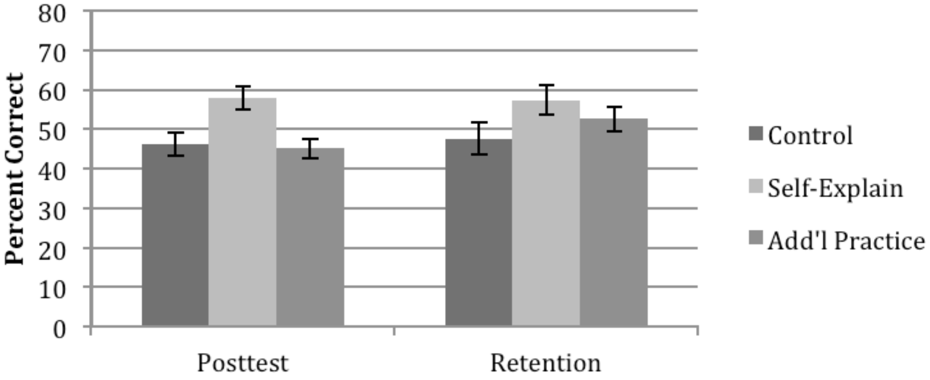

Figure 1. Conceptual Knowledge percent correct scores by condition. Posttest and retention test marginal means and standard errors shown, with covariates of pretest conceptual knowledge score and backward digit span included.

When considering conceptual knowledge overall, although the Self-Explain condition outperformed both the Control condition and the Additional-Practice condition a little (see Figure 1), differences between conditions were not significant.

The conceptual knowledge section had two sections, one on the explicit knowledge of the meaning of the equal sign, and one on implicit knowledge of equivalence as evidenced by student knowledge of different equation structures. There were no differences between conditions on the measure of explicit conceptual knowledge.

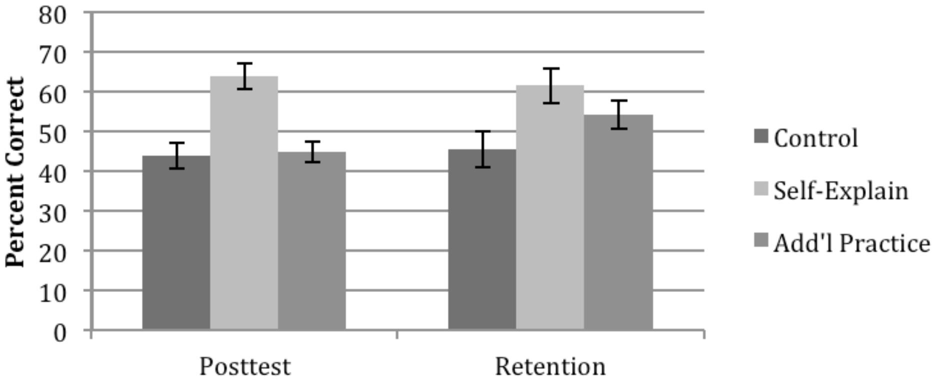

Figure 2. Implicit Conceptual Knowledge percent correct scores by condition. Posttest and retention test marginal means and standard errors shown, with covariates of pretest implicit conceptual knowledge score and backward digit span included.

The implicit conceptual knowledge section had significant differences between conditions (see Figure 2). The Self-Explain condition outperformed the Control and Additional-Practice conditions, and there was no difference between the Control and Addition-Practice conditions, see Table 8 for details. Follow-up analyses indicated the difference between Self-Explanation and the other conditions was due in part to the Self-Explanation condition marginally outperforming the Additional-Practice condition on the memory items, F(1,65)=3.45, p=.068, η_p^2 =.050, and outperforming the Control Condition on equation structure judgment items, F(1,65)=7.36, p=.009, η_p^2 =.102.

To summarize, conceptual knowledge was greatest overall in students who were prompted to self-explain their reasoning, relative to students who had the same problem solving experience but were not prompted to explain (Control), and students who had twice as much problem solving experience (Additional Practice). This advantage was found within implicit, but not explicit, conceptual knowledge.

Effect of Condition on Procedural Knowledge

The procedural knowledge items can be broken down into learning items (3) that were similar in equation structure to the intervention items, and transfer items (5) that had a novel equation structure. The following analyses will explore differences in procedural accuracy, as measured by correct strategy use between conditions on learning and transfer items. Follow-up analyses explored differences in incorrect strategy use, unattempted items, and use of specific strategies.

Procedural Learning. Correct strategy use on learning items was highest in the Additional-Practice condition, followed by the Self-Explain and then the Control conditions. There was a significant difference between the Additional-Practice and Control conditions, but Self-Explain didn't differ from either (see Table 8 and Figure 3). Follow-up analyses indicated that the Additional-Practice condition also used fewer incorrect strategies than the Control condition. There were no differences in unattempted procedural learning items between conditions.

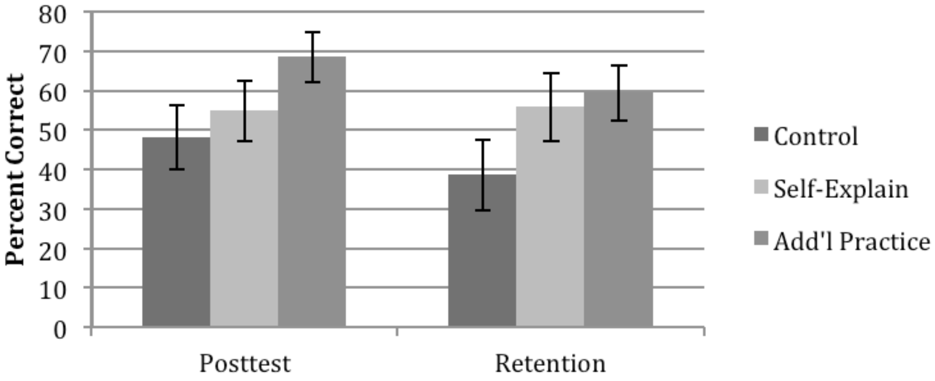

Figure 3. Procedural Learning percent correct strategy use by condition. Posttest and retention test marginal means and standard errors shown, with covariates of pretest procedural learning score and backward digit span included.

Procedural Transfer. Mean correct strategy use on procedural transfer items was highest in the Self-Explain condition, although there were no statistically significant differences between conditions. Follow-up analyses indicated that the Self-Explain condition used fewer incorrect strategies than the Additional-Practice condition and marginally fewer than the Control condition, $F(1,70)=3.974$, $p=.050$, $\eta_p^2=.054$ and $F(1,70)=2.931$, $p=.091$, $\eta_p^2=.04$, respectively. There were no significant differences in the amount of unattempted transfer items.

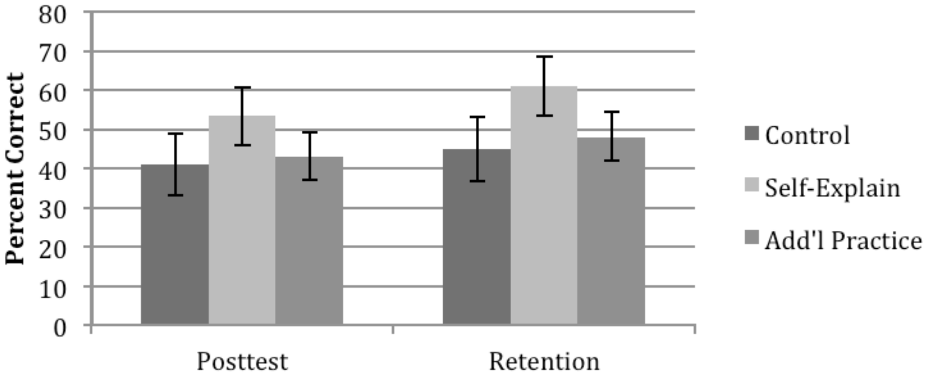

Figure 4. Procedural Transfer percent correct strategy use by condition. Posttest and retention test marginal means and standard errors shown, with covariates of pretest procedural transfer score and backward digit span included.

Specific Strategy Use

There were no differences between conditions in use of any specific strategy at pretest. The most common strategy was the Add-to-Equal strategy, being used on average on 47% of the items. This is a common mathematical equivalence misconception that elementary children have (McNeil & Alibali, 2005). Pretest use of the Add-Subtract strategy was very low (3%), but after they were taught to use this strategy, use increased to 65% during the intervention. Table 9 outlines all the different correct and incorrect strategies coded for, an example of student work using that strategy, and the raw means and standard deviations of percent strategy use at all time points. Significant differences in strategy use by condition are denoted by asterisks on the respective time point.

Table 9. Overall Strategy Use. Average percent strategy use and standard deviations. Significant and marginal differences between conditions noted, as determined via ANCOVAs with strategy use at pretest as a covariate.

Strategy	**Example for** 3 + 4 + 8 = _ + 8	**Time**	**Control**	**Self-Explain**	**Add'l Practice**
Correct Strategies					
Equalizer	3+4 is 7, 7+8 is 15, and 7+8 is also 15	Pretest	5.3 (9.6)	5.7 (13.2)	8.7 (15.5)
		Intervention	4.2 (7.4)	12.1 (25.8)	7.6 (13.1)
		Posttest	4.4 (14.8)	12.5 (29.1)	12.5 (27.1)
		Retention	14.6 (27.5)	17.9 (33.2)	23.4 (36.8)
Add-Subtract	I did 8+4+3 equals 15, and subtract 8	Pretest	1.3 (5.7)	4 (14.1)	3.4 (9.5)
		Intervention	68.3 (31.9)	64.4 (36.1)	64.1 (36.4)
		Posttest	16.3 (23.7)	22.2 (32.7)	24.2 (27.6)
		Retention	6.9 (24)	21.4 (32.1)	16.5 (30)
Grouping	I took out the 8s and I added 3+4	Pretest	0 (0)	0.6 (2.7)	0.4 (2.2)
		Intervention	5 (15.4)	0.8 (3.6)	4 (10.2)
		Posttest*	3.1 (14)	0.6 (2.7)	0.4 (2.2)
		Retention*	5.6 (15.6)	0 (0)	0.8 (3.1)
Incomplete Procedure	I added 7 plus 8 (gave correct answer)	Pretest	0.7 (2.9)	1.1 (5.3)	0.4 (2.2)
		Intervention	3.3 (11.6)	0.8 (3.6)	0.8 (2.4)
		Posttest	1.3 (5.6)	1.1 (5.3)	0 (0)
		Retention	0 (0)	0.6 (2.7)	0 (0)
Insufficient Work	I used my fingers	Pretest	11.2 (14.4)	15.9 (22.2)	15.5 (17.4)
		Intervention	10 (21.9)	7.6 (21.7)	3.8 (6.9)
		Posttest	16.3 (21.5)	17.6 (21)	17 (26.3)
		Retention	14.6 (18.8)	21.4 (27.1)	12.5 (18.5)
Incorrect Strategies					
Add All	I added 8 and 3 and 4 and 8 together	Pretest	5.9 (20.1)	10.2 (21.4)	9.1 (21.5)
		Intervention	1.7 (5.1)	4.5 (9.2)	3 (5.8)
		Posttest*	12.5 (26.9)	8 (16.6)	4.2 (11.1)
		Retention	8.3 (23.5)	12.5 (26.2)	4.8 (16)
Add to Equal	I just added 3+4+8	Pretest	46.7 (23.9)	45.5 (33.5)	47 (26.7)
		Intervention	5 (9.5)	3 (6.6)	6.1 (11.7)
		Posttest*	30.6 (20.9)	22.2 (25.6)	29.5 (27.7)
		Retention*	35.4 (31.6)	15.5 (18.1)	33.1 (32.4)
Other Incorrect	I just added 8 to 3	Pretest	10.5 (14)	9.7 (20.7)	4.9 (9.3)
		Intervention	0.8 (3.7)	6.1 (17.5)	5.1 (9.1)
		Posttest	3.1 (5.6)	5.7 (10)	8.3 (14.2)
		Retention	7.6 (15.5)	3 (5.5)	4 (9.3)
Blank		Pretest	18.4 (25.5)	7.4 (22)	10.6 (18.5)
		Posttest	12.5 (21.1)	10.2 (23)	3.8 (9.1)
		Retention	6.9 (14.4)	7.7 (17.4)	4.8 (12.8)

Specific problem solving strategy use at the three assessment times and during the intervention session was examined. ANCOVAs for strategy use at intervention, post-test, and retention test, with pretest strategy use as a covariate were performed. Backward digit span was not included as a covariate in these exploratory analyses so that differences in raw strategy use could be considered.

Follow-up analyses that examined differences in specific strategy use revealed some differences between conditions. The Self-Explain condition used the common and incorrect Add-to-Equal strategy significantly less overall than the Control and the Additional Practice conditions, F=(1,65)=4.30, p=.04, $\eta_p{}^2$ =.09 and F=(1,65)= 4.09, p=.05, $\eta_p{}^2$ =.059, respectively. As this is a prevalent naïve strategy, the dampening of this strategy was important. The Control condition used the correct Grouping strategy significantly more than the Self-Explain condition, F=(1,65)= 4.42, p=.04, $\eta_p{}^2$ =.064, although actual frequency of use was very low.

There were no differences between conditions in using the Add-Subtract strategy. This was not surprising given that this was the strategy demonstrated during the procedural instruction. Student use of this Add-Subtract strategy was related to students' implicit conceptual knowledge scores. Use of the Add-Subtract strategy during the intervention was related to implicit conceptual knowledge scores at posttest (R=.286, p=.02). Add-Subtract use during the retention test was also related to post and retention test implicit conceptual knowledge scores (R=.358, p=.003; R=.240, p=.053).

Assessment Results Summary

To summarize, there was an advantage of the Additional-Practice condition relative to Control condition on procedural learning items, in terms of more correct and less incorrect strategy use. There were no differences between the Additional-Practice and Self-Explain conditions on the learning items. On procedural transfer items, there was a small advantage for the Self-Explain condition relative to Additional-Practice condition, as they used incorrect strategies less often. The Self-Explain condition had the greatest mean procedural transfer accuracy, although this was not a significant difference. They did, however, use the fewest incorrect strategies. The Self-Explain condition also used the prevalent incorrect strategy Add-to-Equal strategy significantly less than the Control and Additional Practice conditions. At pretest, the Additional-Practice condition had significantly higher transfer performance than the Self-Explain condition, and this may potentially be contributing to the non-significant findings.

Effect of Condition on Intervention Activities

To explore reasons for differences in assessment performance by condition, intervention problem solving performance is examined. As expected, the Control condition took less time to complete the intervention practice problems (M=6.7 minutes, SD=2), and the Self-Explain (M=14.1 minutes, SD=4.75) and Additional-Practice (M=11.7minutes, SD=4.8) conditions indeed took additional time. Despite efforts to keep intervention time equal, students in the Self-Explain condition took significantly more time to complete the intervention than students in the Additional-Practice condition, $F(1,71)=4.825$, $p=0.031$, $\eta_p^2 = 0.064$.

There were no differences in accuracy on intervention problems between conditions, both when considering average intervention accuracy across all solved problems and when considering performance on only the first six items that students in all conditions completed. The average intervention accuracy was 73% correct (SD=

27%). The students in the Additional-Practice condition did improve slightly on the second six intervention items, with accuracy increasing from 67.7% to 78.3%, *t*(32)= -1.923, p(two-tailed)=.063. Their accuracy quickly stabilized on these additional items.

Intervention Strategy Repertoire

To explore the students' strategy use during intervention, we examined the number of different correct and incorrect strategies students' used (see Table 10). It is important to keep in mind that students in the Additional-Practice condition solved twelve intervention problems while the other conditions only solved six. The results were broken down into total counts, as well as counts for the first six intervention items and the second six intervention items.

Students used about 2 different strategies (M=2.3, SD= 1.3), regardless of condition. Thus, students in the Additional-Practice condition, who solved twice as many practice problems, did not use a wider range of strategies than the other two conditions. However, these students did use significantly fewer different strategies on the second six items than on the first six items. The number of total different strategies dropped from 2.3 to 1.7 (t(32)=2.89, p(two-tailed)=.007), and the number of different correct strategies dropped from 1.7 to 1.3 (t(32)=2.27, p(two-tailed)=.03). There was no difference in the number of different incorrect strategies between the two time points.

Table 10. Number of Different Intervention Strategies by condition. Means and standard deviations.

Number of Different Intervention Strategies				
	Condition	Out of All Items	First 6 Items	Second 6 Items
Total	**Total**	**2.3 (1.3)**	**2.1 (1.1)**	
	Control	2.1 (0.8)	2.1 (0.8)	
	Self Explain	2 (1)	2 (1)	
	Add'l Practice	2.7 (1.7)	2.3 (1.3)	1.7 (1.1)
Correct	**Total**	**1.7 (0.8)**	**1.6 (0.8)**	
	Control	1.7 (0.7)	1.7 (0.7)	
	Self Explain	1.5 (0.7)	1.5 (0.7)	
	Add'l Practice	1.9 (1)	1.7 (0.9)	1.3 (0.7)
Incorrect	**Total**	**0.6 (0.9)**	**0.5 (0.8)**	
	Control	0.4 (0.7)	0.4 (0.7)	
	Self Explain	0.6 (0.9)	0.6 (0.9)	
	Add'l Practice	0.9 (1)	0.6 (0.9)	0.4 (0.7)

Specific strategy use during the intervention was examined as well. There are no differences in overall percentage strategy use between conditions with the exception of Other Incorrect use, as discussed earlier and reported in Table 9. These results also remain true when considering strategy use on only the first six intervention items. To investigate the effects of additional practice, strategy use on the first and second set of intervention problems were compared. However, in the second six intervention problems, the amount of Insufficient Work significantly decreased from 36% to 9% (t(32)=2.058, p=.048). Additionally, the amount of the more sophisticated Grouping strategy increased from 12% to 36% (t(32)= -1.854, p=.073). Seven students in the Additional-Practice condition used the Grouping strategy for the first time during the intervention. Of these, 4 used it for the first time during the first six problems, and the other 3 used it for the first time during the second six problems.

High Quality Strategy Invention

In order to gain insight into individual students' learning and strategy acquisition, we examined when a student used a strategy for the first time and the percentage of students who had *ever* used a given strategy at least once. Percentages are presented in Table 11. Descriptively, the students who had additional practice problems during the intervention invented new correct strategies more frequently and sooner than students who only had six practice problems. There were no significant differences in the percentage of Equalizer strategy use by condition overall, although fewer students were Equalizer users in the Control and Self-Explanation conditions. The Control and Self-Explanation students who did use the Equalizer strategy must have been using it more frequently than the Additional Practice students who used the Equalizer strategy. Invention of the Grouping strategy during intervention was highest in the Control and Additional-Practice conditions (15% and 21% vs. 4.5%). The percentage of students who had ever used the Grouping strategy was the same between the Control (35%) and Additional-Practice conditions (33%), but there was a significant difference in percentage use *between all conditions*, with the Control students using it most. Therefore, the Additional Practice students who were Grouping-users must have been using this strategy less frequently.

Table 11. Strategy Invention and Use. Percentage of students who used the strategy for the first time at each time point, and running total of students who have used the strategy at least once, by condition.

Percentage of Students' Initial Use of Strategy and Running Total

Strategy	Condition	Pretest	Intervention		Posttest		Retention	
Correct								
Equalizer	Control	30	5	(35)	0	(35)	20	(55)
	Self-Explain	22.7	9.1	(31.8)	4.5	(36.4)	13.6	(50)
	Add'l Practice	36.4	24.2	(60.6)	12.1	(72.7)	3	(75.8)
Grouping	Control	5	15	(20)	0	(20)	15	(35)
	Self-Explain	4.5	4.5	(9.1)	4.5	(13.6)	0	(13.6)
	Add'l Practice	3	21.2	(24.2)	0	(24.2)	9.1	(33.3)

In summary, all students used an average of 2.3 different strategies during the intervention, with about 1.7 being correct and 0.6 being incorrect. Additional intervention practice facilitated the invention of the correct Equalizer and Grouping strategies, although their use was not retained at a high level. Overall, there was no immediate effect of condition on intervention problem solving performance. The additional practice did seem to enable students to stabilize their strategies repertoires and allowed for the invention of the more sophisticated Grouping strategy.

Self-Explanation Quality

After solving an intervention problem, students in the Self-Explanation condition were shown the same problem with a correct and an incorrect solution. They were prompted to give an explanation as to *why* the answer was correct or incorrect. See Table 12 for explanation types and percentage use. While responding to the prompt, students discussed the *concept* of the equal sign or equality (e.g., "Both sides have to equal the same thing”), mentioned a *procedure* (e.g., “You are supposed to add the first two numbers”), talked about the quality of the *answer* (e.g., “Because you get the right answer”), or made *other* observations (e.g., “He wasn’t paying close attention”). Students most frequently talked about procedures (57.5% of explanations), and they occasionally discussed the answer (15.1% of explanations). They only discussed the concept of equality on 6.4% of explanations, indicating that their explanations were often not focused on articulating the equivalence relationship. There were no differences in explanation quality when discussing correct and incorrect answers.

Table 12. Self-Explanation Quality Coding and Percent Use and Standard Deviations.

Explanation Type	Description	Example	Percentage Use		
			Why Right	Why Wrong	Why Total
Conceptual	Recognizes that both sides of the equation are equal, either verbally or in gesture OR displays some understanding of the equals sign. Often use the term "same" or "equals sign" *must be unambiguously referring to both sides of the equation	·"because this (points) makes 12 and so does this (points)" ·"they both have to equal the same thing" "both sides have to seem the same" ·"because there's an equal sign" ·"because it says equals after the 5"	3.8 (14.5)	9.1 (25)	6.4 (17.4)
Procedure	Talks about a specific procedure for solving the equation – mentions specific *operation or number*(s) (i.e., use terms "add", "subtract", "minus", etc.), even if the procedure is incorrect. Does not display any conceptual understanding of two equal sides or the equal sign.	*For the correct prompt* *"9+7+6=16+6"* ·"because you have to add 9+7" ·"he probably subtracted something" *For the incorrect prompt* *"9+7+6=22+6"* ·"because he left off the last 6" ·"because he added 9 + 7 + 6"	63.6 (36.2)	51.5 (38.8)	57.5 (33.3)
Answer	Refers to the quality of the answer, with no justification. Must talk *only* about the answer or focus mainly on the answer. Usually use the term "answer"	·"because you get the right/wrong answer" ·"because the answer/it makes sense/doesn't make sense" ·"because you'll get it right" ·"because the answer is 22"	13.6 (27)	16.6 (25.2)	15.1 (21.3)
Other	Does not fit into any other category, vague, *unintelligible*	·"that's how you're supposed to do it" ·"it's easier" · teachers taught you that way" ·"it makes sense" ·"…used all the numbers" ·"She learned it" ·"He wasn't paying close attention"	9.8 (18.3)	11.4 (18.8)	10.6 (16.7)
Don't Know	The student said they did not know or could not provide an explanation	·"I don't know"	9.1 (17.6)	11.4 (25.9)	10.6 (16.7)

The frequency of the explanation types given related to strategies the students used to solve the intervention practice problems. Students' explanation type was correlated with intervention strategy use, controlling for backward digit span and pretest conceptual and procedural knowledge. The use of an Equals explanation correlated with Equalizer strategy use (R=.812, p=.000), Procedure explanations correlated with Add-Subtract strategy use (R=.48, p=.044), Answer explanations negatively correlated with Equalizer strategy use (R= -.418, p=.085), Other explanations correlated with Insufficient work (R=.467, p=.051), and Don't Knows correlated with Add to Equal (R= .512, p=.03), Other Incorrect (R=.549, p=.018), Grouping (R=.775, p<.001), and negatively with Add-Subtract strategies (R= -.445, p=.064). The Grouping finding is not reliable, as only one Self-Explain student used Grouping during the intervention, and only two out of six times. There were no strong correlational patterns between explanation quality and strategy use on the assessments.

Overall, students' explanations of why example solutions were correct or not were mostly based on the strategies they had used to determine the solutions. Explanations that directly discussed an equivalence relationship were infrequent, occurring only about 6% of the time. The discussion of procedures was related to the students' own correct strategy use. Given the overall benefit of prompts to explain on implicit conceptual knowledge and procedural learning, this discussion of procedures may potentially play a role in improving their understanding of mathematical equivalence.

Summary

In regard to our hypotheses, we have found that (1) Self-Explanation increased conceptual but not procedural knowledge relative to the Control condition, (2) Additional-Practice increased procedural learning but not transfer relative to the Control condition, and (3) because conceptual knowledge was greater and incorrect strategy use on transfer items was lower, Self-Explanation was more beneficial for learning than Additional-Practice.

CHAPTER IV

DISCUSSION

Prompting students to self-explain benefitted conceptual and procedural knowledge when compared to students who received the same amount of practice problems or the same amount of instructional time. This benefit was maintained over a two-week delay. Specifically, prompts to self-explain increased implicit conceptual knowledge of allowable equation structures, decreased incorrect strategy use on procedural transfer items, and lessened the use of a prevalent naïve incorrect strategy. Additional practice problems benefitted learning relative to the control condition as well, but only on procedural learning items.

When prompted to explain why an example answer was correct or incorrect, students mostly appealed to the procedures used to determine that answer. These explanation prompts increased students’ understanding of equation structures, or implicit conceptual knowledge. The findings are in line with prior research that finds self-explanation prompts are beneficial for conceptual knowledge. Unlike prior studies, however, we demonstrated and refined this relationship in a sample in which self-explaining is not confounded with amount of practice or time on task. These findings expand upon past research by more carefully isolating and evaluating the effects of self-explanation prompts. The effect of self-explanation and additional practice on conceptual and procedural knowledge will be discussed.

Self-Explanation and Conceptual Knowledge

Self-Explanation supports increases in conceptual knowledge in mathematics. Our findings indicate that these gains are due to the process of explaining itself, and self-explanation is more beneficial for learning than working through additional practice problems for the same amount of time. This study complements a growing body of literature that self-explanation prompts can improve conceptual knowledge in problem-solving domains. Past research has demonstrated this when students in the control condition solve the same number of problems in less time (Calin-Jageman & Ratner, 2005), solve or study the same number of problems in the same amount of time in more complex domains (Atkinson et al., 2003; de Bruin, Rikers, & Schmidt, 2007; Hilbert et al., 2008) or solve more problems to equate time (Aleven & Koedinger, 2002). In this last study, however, students not only received prompts to self-explain; they also received feedback on their explanations and had to produce a correct explanation before continuing to the next problem, making it unclear if self-explanation prompts alone were sufficient to support conceptual knowledge more than additional practice.

Self-explanation could have increased conceptual knowledge through several mechanisms. First, the act of explaining is thought to update and correct the learner's mental model of the domain and its principles, primarily through drawing attention to gaps in the learner's understanding (Chi et al., 1994). In the current study, students were asked to explain why example answers were correct and incorrect. This may have encouraged students to notice that the incorrect answer added all the numbers up until the blank, which typifies a naïve conception of equality, and that the correct answer resulted in equivalent values on either side of the equal sign, which is the primary principle of mathematical equivalence. Second, explanations are also thought to benefit conceptual understanding because they facilitate construction of inference rules that are used in the formation of general principles, and are then proceduralized into usable skills (Chi et al., 1989). Explaining why an example answer was correct or incorrect may have also encouraged the learner to consider problem solving strategies more often, and this may have been driving students to infer the principle of equating both sides of the equation. In the current study, the majority of the explanations were based on procedures, and so this may have been enough to drive learning of concepts. However, there was little evidence in this study that suggested this greater conceptual knowledge was proceduralized into useable skills, as there was not a significant benefit of explaining for procedural learning.

Explanation is also thought to create novel goal structures within the problem-solving domain, and this allows for strategy generalization (Crowley & Siegler, 1999; Lombrozo, 2006). Children's naïve understanding of equivalence is that the goal is to compute operations and put an answer after the equal sign, and the more sophisticated goal is to make both sides equal. It has been found that students who correctly understand the principle of equivalence also have an increased understanding of allowable equation structures (Rittle-Johnson et al., 2011). Perhaps through explaining problem solving strategies, the goal of the problem is implicitly considered, which facilitates consideration of the equation structure itself. In the current study, students who self-explained had superior performance on implicit conceptual knowledge, which focused directly on allowable equation structures.

The most compelling mechanism that accounts for the benefit of self-explanation on conceptual knowledge in this case is that explanation supports an increase in awareness of the problem's goal-structure (Crowley & Siegler, 1999). Not only would a fuller understanding of a problem's goal structure allow for more appropriate strategy use, as evidenced by superior mean performance on procedural learning and transfer items, but it would allow for a firmer encoding of the problem structure itself. Indeed, McNeil & Alibali (2004) found that students who had more correct problem solving strategies were also more accurate at encoding equation structures in math equivalence problems exactly like the ones used in the current study. In the domain of mathematical equivalence, the goal is to find the missing value that will make both *sides* of the equation *equal*. This is the core principle of mathematical equivalence, and it is the key to correctly solving novel problems. This core principle focuses on understanding the structure of the equation. In the current study, prompts to explain did not result in increases in *explicit* conceptual knowledge of equivalence, as was the case in several prior studies (Große & Renkl, 2004; Mwangi & Sweller, 1998; Rittle-Johnson, 2006; Rittle-Johnson & Russo, 1999). However, there was a large benefit of explaining for *implicit* conceptual knowledge of allowable equation *structures*. Students are often unable to describe how they succeeded at a task despite being able to perform it (Siegler & Stern, 1998). Implicit knowledge often precedes explicit knowledge, and activating implicit knowledge prepares the learner to make their knowledge explicit (Broaders, Cook, Mitchell, & Goldin-Meadow, 2007). Overall, self-explanation prompts may benefit conceptual knowledge when prompts focus the learner on the underlying principle by making the problem solving goal structure more salient. Note that self-explanation is not the only route to improved conceptual knowledge. For example, in most studies, students do not receive direct instruction on correct concepts, and concept-based instruction may reduce or replace the benefits of self-explanation (Matthews & Rittle-Johnson, 2009).

Self-Explanation and Procedural Knowledge

In the current study, there were no strong benefits of self-explanation on procedural knowledge, although means were in the expected direction with slightly more correct strategy use and less incorrect strategy use. Specifically, self-explanation significantly lowered incorrect strategy use relative to Additional-Practice on procedural transfer items. Self-explanation has been proposed to benefit procedural knowledge by broadening the range of problems to which strategies can be applied (Lombrozo, 2006; Rittle-Johnson, 2006), and by promoting the invention of new strategies (Rittle-Johnson, 2006; Siegler, 2002). Neither of these mechanisms were supported in the current study. Broader strategy application, evidenced by superior performance on transfer items, was not found, and neither was an increase in strategy invention. Self-explanation did seem to dampen incorrect strategy use, with significantly less of the incorrect Add-to-Equal strategy being used relative to the other two conditions on the assessments.

The current study did not find a benefit of self-explaining on procedural knowledge, contrary to several prior studies (Aleven & Koedinger, 2002; Atkinson et al., 2003; Calin-Jageman & Ratner, 2005; de Bruin et al., 2007; Hilbert et al., 2008; Rittle-Johnson, 2006). This discrepant finding may be because we taught the students a strategy that worked on all learning items and required only minimal adjustment to be generalized to transfer items. For example, one of the most challenging transfer items was 6 – 4 + 3 = __ + 3. Students could still find the total value on left side of the equation, and subtract the quantity on the right to find the missing value. Most students used this one strategy the majority of the time, and use of alternative strategies was very low. This is consistent with the finding that children often do not use new strategies when a more familiar one is viable (Siegler & Jenkins, 1989). This may be why we did not see larger differences in correct strategy use on procedural learning and transfer items- the transfer items used did not require a great amount of adaptation of the taught strategy, and many students were able to do so. Generalization of specific strategies is a proposed mechanism of self-explanation (Crowley & Siegler, 1999; Lombrozo, 2006; Rittle-Johnson, 2006). Given prior work and our self-explain students' increased conceptual knowledge, it seems likely that their procedural skills would have been superior if they had been given transfer items that required more of an adaptation of the taught problem solving strategy. However, this study used the same design as Rittle-Johnson (2006) that found a benefit of self-explaining relative to our Control condition. They only major difference between this and the current study is that the students in Rittle-Johnson (2006) had considerably lower prior knowledge, with a pretest-in criteria of <50% at pretest, compared to the current criteria of <80% at pretest. This may be an important difference that drives the discrepancy in findings between these two studies.

Self-Explanation and Other Learning Factors

The efficacy of self-explanation for both conceptual and procedural learning may be dependent on other conditions in the learning environment. In designs in which both conditions have the same number of problems and time on task, self-explanation does have a benefit when the prompt focuses the learner on the underlying principles (Atkinson et al., 2003; de Bruin et al., 2007; Hilbert et al., 2008), but not when the focus is on the procedures used (Große & Renkl, 2004; Mwangi & Sweller, 1998). The type of instruction provided is important as well. In two studies nearly identical to the current study, conceptual instruction was given instead of procedural instruction, and no benefit of self-explanation was found relative to the Additional-Practice condition (DeCaro & Rittle-Johnson, under review; Matthews & Rittle-Johnson, 2009). Indeed, in DeCaro & Rittle-Johnson (under review), self-explanation was less beneficial for learning than practice for students with high (but not low) prior knowledge. This finding suggests that self-explanation is a better learning activity for students with low prior knowledge. This may, in part, shed light on why the current findings are not in line with Rittle-Johnson (2006) that compared self-explanation to our Control condition, and found a benefit in procedural, but not conceptual knowledge.

Additional Practice and Procedural Knowledge

Self-Explanation is not the only path to learning. Practice can benefit learning as well. Additional practice did increase student performance on procedural learning items.

This study supports the account that practice increases the learner's skill at applying an initial problem solving strategy (Jonides, 2004). This extra experience working through practice problems increased students' performance on procedural learning items that could be solved in the same exact way as the practice problems. Students with additional practice also had lower levels of incorrect strategy use on these items. Practice has been proposed to benefit learning precisely through the strengthening of correct strategies (Jonides, 2004) and dampening of incorrect strategies (Siegler, 2002). This may have been driven in part by the building of a strong memory trace of the correct problem solving strategy (Chi et al., 1988; Ericsson et al., 1993). The students' use of the taught correct strategy may have also become more automatic with practice (Logan, 1990; Shiffrin & Schneider, 1977). Descriptively, the Additional-Practice students trended towards attempting more procedural knowledge items than students in the other conditions. This suggests that they were more willing to attempt to apply the strategy they had learned, even if it was not successful.

Practice is also proposed to increase learning through allowing for the discovery of new, and presumably better, strategies (Jonides, 2004). This extra experience is thought to allow the learner to discover and test out new strategies (Lemaire & Siegler, 1995; Siegler & Jenkins, 1989; Siegler & Stern, 1998). New and more conceptually based strategies were indeed invented during the extra practice problems. However, these new strategies were not utilized more often overall on the subsequent assessments. This is likely due to the fact that all of the procedural knowledge items could be correctly solved using a modification to the taught strategy, makes the need to use the new strategies less salient. Indeed, children do not often use new strategies when a more familiar one is viable (Siegler & Jenkins, 1989).

Future Directions

While the current study gave us several important insights, when considering these findings with the prior literature, a few open issues remain. The learning benefits of self-explanation seem to be influenced by several other factors, and these factors should be studied systematically in order to have a more fine-tuned understanding of how exactly self-explaining enacts its benefit, and under what conditions this benefit is optimized. First off, how the self-explanation prompt focuses the learner's attention on the principle(s) of the domain is important. Prompts that in some way focus the learner on the underlying principles are more effective for learning than those that focus on the procedures used. Additionally, how these principle-based prompts interact with the problem-solving goal structure should be considered carefully. Second, the type of instruction given to the learner seems to greatly influence the efficacy of self-explanation prompts. Giving students conceptual instruction seems to wipe out the benefits of self-explaining (DeCaro & Rittle-johnson, under review; Matthews & Rittle-Johnson, 2009). However, this has only been found in the domain of mathematical equivalence, and the conceptual basis of this domain is fairly constrained. The main principle can be captured by one idea: that the equal sign means 'the same' and that both sides of an equation need to be the same amount. Students are told this directly when given conceptual instruction in these studies. If conceptual instruction can be given without 'giving it all away', perhaps in more complex domains, self-explanation may still be a beneficial learning activity. Finally, self-explanation may be a more effective learning activity for some learners relative to others. Evidence is starting to suggest that the learners' prior knowledge level interacts with the efficacy of self-explanation prompts. In DeCaro & Rittle-Johnson (under review), self-explanation was better for learners with low prior knowledge, and additional practice was better for learners with high prior knowledge. The current study did not find a benefit for explaining in procedural knowledge relative to Control, whereas Rittle-Johnson (2006) did. Perhaps it is important that the current study had students with higher prior knowledge than those in Rittle-Johnson (2006). Research from the cognitive load literature may be useful to inform this area of investigation (i.e. Paas, Renkl, & Sweller, 2004). Future research should consider how explanation prompt type, instruction, and prior knowledge interact with the benefits of self-explanation in order to more clearly understand the mechanisms of self-explanation, and to determine under what conditions learning with self-explanation is optimal.

Conclusions

Self-explanation as a pedagogical tool is a useful exercise, and its benefits go deeper than simply keeping the learner engaged for an extended period of time. Practice does indeed increase the learner's ability to carry out the practiced task. However, prompts to explain the underlying principle, even in subtle ways such as asking why an answer is right or wrong, benefit procedural learning as much as additional practice, and increase conceptual understanding more than additional practice. In this way, carefully designed explanation prompts are a worthwhile use of instructional time.

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