AN ANALYSIS OF THE EMERGENCE AND CONCURRENT LEARNING OF A PROFESSIONAL TEACHING COMMUNITY

By

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TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER I

INTRODUCTION

The purpose of this dissertation was to document the development of a professional teaching community and the means of supporting its emergence and concurrent learning as situated in the institutional context of the school district.[1] In this process, I related the realized learning trajectory of this professional teaching community and of the participating teachers to both the means by which it was supported and organized, and to the institutional setting in which the teachers worked. The results of this analysis will generalize to other cases in that it will enable researchers and teacher educators to adapt the means by which the learning of the professional teaching community was supported to the organizational characteristics of the school systems in which they are working in a conjecture-driven manner.

The data for this study were collected during the first two years of collaboration with a group of nine middle school mathematics teachers who worked in five different schools in the Jackson Heights Public School District. This urban school district served a 60% minority student population and was located in a state with a high-stakes accountability program. The district had received an external grant to support its reform efforts prior to the research team's collaboration with the teachers. The research team began working in the district to provide teacher development in statistical data analysis at the invitation of the district's mathematics coordinator who selected the teachers with whom the research team collaborated. She was interested in professional development focused at the middle grades since reform efforts there were proving to be problematic. In particular, the district had adopted a new mathematics

[1] This study was part of a larger research project. Members of the research team included Paul Cobb, Kay McClain, Researcher McGatha, Teruni Lamberg, Jose Cortina, Jason Silverman, Lori Tyler, Qing Zhao, and myself.

curriculum, but significant proportions of the middle school teachers continued to use the traditional textbook series as the primary basis for their instruction. During the first two years in which the research team worked with the teachers, it conducted a three-day work session each summer as well as three one-day sessions during the first school year and six one-day sessions during the second school year. The long-term goal in doing so was to support the teachers' development of instructional practices in which they would place their students' reasoning at the center of their instructional decision making.

In the following chapter, I develop the construct of a professional teaching community by articulating its salient characteristics. This is important because I claim that at the beginning of the collaboration with the teachers in the research study, they did not constitute a professional teaching community. It was not until after approximately 19 months of working together that this group emerged into a professional teaching community. Following this, in Chapter III, I present an overview of the current literature on professional teaching communities from which critical issues emerged that build a case for the dissertation study. Next, in Chapter IV, I propose a conjectured learning trajectory for a professional teaching community and described the possible means of supporting learning along that trajectory. In Chapter V, I describe the institutional setting of the district in which the teachers worked and clarify the importance of situating the emergence and subsequent development of the professional teaching community within that context. Against this background, in Chapter VI, I describe the methodology that was used in conducting the analysis of the emergence and development of this professional teaching community. Then, Chapter VII documents the realized learning trajectory of the group as they engaged in the worksessions. Finally, Chapter VIII concludes the dissertation by delineating the significance of the study and the aspects of the process for supporting the emergence and concurrent learning of a professional teaching community that are generalizable.

CHAPTER II

PROFESSIONAL TEACHING COMMUNITIES

What differentiates a professional teaching community from a group of teachers that meet to discuss an issue of mutual concern? Grossman, Wineburg, and Woolworth (2001) argue for the importance of distinguishing between a professional teaching community and a group of teachers:

> Even a cursory review of the literature reveals the tendency to bring community into being by linguistic fiat. Groups of people become communities, or so it would seem, by the flourish of a researcher's pen. Researchers have yet to formulate criteria that would allow them to distinguish between a *community of teachers* and a *group of teachers* sitting in a room for a meeting. (p. 943)

Researchers who have collaborated with groups of teachers to establish professional teaching communities (cf. Franke & Kazemi, 2001; Grossman et al., 2001; Lehrer & Schauble, 1998; Rosebery & Warren, 1998; Stein, Silver, & Smith, 1998; Warren & Rosebery, 1995) make clear that a group of teachers who collaborate with each other in some way does not necessarily constitute a community. An important first step is therefore to clarify criteria for distinguishing a group from a community. Wenger (1998) proposes three interrelated dimensions that differentiate a community of practice from a group: joint enterprise, mutual engagement, and a shared repertoire. I will expand on these three dimensions and discuss how they relate to a professional teaching community.

A joint enterprise is a negotiated venture "produced by participants within the resources and constraints of their situations" (Wenger, 1998, p. 79). More than merely a stated goal, the joint enterprise creates a sense of mutual accountability that becomes an integral aspect of the practice of the community. Secada and Adajian (1997) introduce a similar notion when they describe community as a group of people who have organized themselves for a *shared purpose*. "They adopt or are assigned formal and informal roles, they organize additional structures (such as times for meeting and planning) as needed, and they take actions—all in order to achieve their purposes" (p. 194). For a professional teaching community, this shared enterprise might be teaching mathematics for understanding where there is a focus on students' learning of significant mathematical ideas.

Mutual engagement includes the social complexities and relationships that members develop as they collectively pursue a shared enterprise, as well as the norms of participation that are specific to the community. Bellah, Madsen, Sullivan, Swidler, and Tipton (1985) speak in similar terms when they define community as "a group of people who are socially interdependent, who participate together in discussion and decision making, and who share certain practices that both define the community and are nurtured by it" (p. 333). For a professional teaching community this would include both social norms of participation as well as norms that are specific to mathematics teaching such as the standards to which the members of the professional teaching community hold each other accountable when they justify pedagogical decisions and judgments (Cobb, McClain, Lamberg, & Dean, 2003).

A shared repertoire includes historical events, tools, styles, discourses, actions, stories, artifacts, and concepts. These have been produced or appropriated by the community in the course of its existence and have become an integral part of its practice. The elements of a repertoire "gain their coherence not in and of themselves as specific activities, symbols, or artifacts, but from the fact that they belong to the practice of a community pursuing an enterprise" (Wenger, 1998, p. 82). Because members develop this communal repertoire collectively during the process of the collaborative, coordinated effort to pursue a shared purpose, it is specific to the community and this shared purpose. For a professional teaching community, this shared repertoire includes normative ways of reasoning with resources when planning for instruction and making students reasoning visible.

Although the notion of a community of practice as developed by Lave and Wenger (1991), Rogoff (1995), and Wenger (1998), has been used relatively widely to characterize professional teaching communities (Franke & Kazemi, 2001; Lehrer & Schauble, 1998; Stein, et al., 1998; Warren & Rosebery, 1995), other researchers have proposed additional criteria for what constitutes a professional teaching community. Following Newmann and Associates (1996), Gamoran, Anderson, Quiroz, Secada, Williams, and Ashmann (2003) identify the elements of a professional teaching community as follows: (1) exhibiting a *shared sense of purpose* in their attention to student thinking, (2) *focusing collectively on student learning*, as opposed to teachers' more common conversations about administrative details and managing student behavior, (3) *collaborating* on ways to improve their students' understanding of mathematics, in contrast to teachers' usual practice of working in isolation, (4) engaging in *reflective dialog*, a conversation about the nature and practice of teaching, and (5) *making their own teaching practices public*, instead of keeping their practice private and confined within the classroom. The first, second, and fourth characteristics are variations on Wenger's notions of joint enterprise and shared repertoire previously discussed. However, the third and fifth characteristics bring to the fore an important aspect of Wenger's dimension of mutual engagement that has not been explicitly addressed—the deprivatization of teachers' instructional practices. Teachers working in isolation keep aspects of their instruction such as decisions made and tools used during planning, facilitation, assessment, and reflection private. Conversely, as teachers work together to pursue a shared purpose, they develop norms of participation which necessarily deprivatize their instructional practices. This in turn cultivates a regime of mutual accountability for justifying and critiquing pedagogical decisions within the professional teaching community.

Secada and Adajian (1997) operationalize the notion of a professional teaching community specific to mathematics teaching along four dimensions: (1) a shared sense of purpose, which they describe as the nature and extent of the school staff's shared values and goals; (2) a coordinated effort to improve students' mathematics learning including teachers working together and setting aside personal prerogatives in favor of shared goals; (3) collaborative professional learning, meaning how well and closely the teachers work together to learn about and to improve their practice as related to mathematics; and (4) collective control over important decisions affecting the school's mathematics program in which teachers have the power as a group to focus the direction of their program. Although similar to Wenger's (1998) and Gamoran, et al.'s (2003) descriptions, Secada and Adajian's dimensions raise two important issues when developing criteria for what constitutes a professional teaching community. First, in concert with much of the research literature on professional teaching communities (Franke & Kazemi, 2001; Grodsky & Gamoran, 1998; Grossman, et al., 2001; Newmann & Associates, 1996; Stein, et al., 1998), Secada and Adajian focus on the school as the location of the community. This obviously hints at the fact that the professional teaching community is institutionally situated. However, it is also important to note that a professional teaching community is not necessarily confined to the boundaries of a single school, but could include teachers who come from different schools within the same school district. Secondly, I believe Secada and Adajian's fourth dimension may need to be modified when taking the teachers' institutional context into consideration. Obviously, members of a professional teaching community should come to see themselves as the professionals most capable of making decisions that affect the mathematics teaching within their schools. However, whether or not they have the power as a group to do so does not make them any less a professional teaching community. This autonomy merely reflects the location of the community within the institutional context in which they work.

Wenger's three dimensions of community of practice form a base for my criteria for distinguishing a group from a community. However, taking into account the literature that I have reviewed on professional communities and professional teaching communities, my criteria are specific to mathematics teaching. Further, they take as a given the deprivatization of teachers' instructional practice as necessary for the initial emergence of a professional teaching community and also acknowledge the situatedness of the professional teaching community within the institutional setting of the school and district. Therefore, synthesizing the literature presented on communities of practice, professional communities, and professional teaching communities, I use the following criteria to determine when the group of teachers that were the subject of this dissertation evolved into a professional teaching community:

- *A shared purpose or enterprise* such as ensuring that students come to understand central mathematical ideas while simultaneously performing more than adequately on high stakes assessments of mathematics achievement
- *A shared repertoire of ways of reasoning with tools and artifacts* that is specific to the community and the shared purpose including normative ways of reasoning with instructional materials and other resources when planning for instruction or using tasks and other resources to make students' mathematical reasoning visible

- *Norms of mutual engagement* encompassing both general norms of participation as well as norms that are specific to mathematics teaching such as the standards to which the members of the community hold each other accountable when they justify pedagogical decisions and judgments

CHAPTER III

EMPIRICAL STUDIES OF PROFESSIONAL TEACHING COMMUNITIES

There is a significant body of research that marks considerable progress in recent years both in the conceptualizing and characterizing collegial relationships, and in specifying the attributes of professional communities (Achinstein, 2002; Grossman, et al., 2001; Gutierrez, 1996; King & Newmann, 1999; Little, 1990, 1999; Louis & Kruse, 1995; McLaughlin & Talbert, 2001; Stokes, 2001; Talbert, 1995; Westheimer, 1998; Witziers, Sleegers, & Imatns, 1999). However, there is relatively little research which examines the initial formation of professional teaching communities (Grossman, et al., 2001), the institutional settings in which the participating teachers develop and refine their instructional practices (Gamoran, et al., 2003), or the specific interactions and dynamics by which participation in a professional teaching community supports teacher learning and innovation in teaching practice (Wilson & Berne, 1999). In this section, I synthesize empirical studies that specifically address such issues and elucidate the critical themes that emerge.

Secada and Adajian (1997) argue that mathematics teachers' professional communities provide an important context in which to understand their practices, professional growth, and development. Secada and Adajian present the case of an elementary school to illustrate how teachers' participation in a professional community supported their efforts to teach according to the tenets of mathematical reform. Using classroom observations and teacher interviews, Secada and Adajian reported that the teachers changed their instructional practices to be more consistent with the recommendations of various Standards documents (NCTM, 1989, 1991) specifically in the areas of making mathematics more relevant to students' everyday lives through applications and realistic problem solving, and having students explain and justify their solutions. Self-reported data from interviews with teachers, the principal, and the district math consultant revealed that the teachers' efforts at changing their practices were supported in the context of their participation in the professional teaching community. Teachers claimed that their membership in the professional teaching community provided them with a shared sense of purpose that focused substantively on making mathematics more relevant to their students' everyday lives. It also created a place where they could risk sharing new ideas and work to solve problems created by their efforts to reform their instructional practices. Participation in the professional teaching community also made it possible for them to exert control over their program by making such decisions as who was hired into the program and by supporting them in inducting new teachers into the school's mathematics program. Although this self-reported data provides some indication that participation in a professional teaching community can support teachers' efforts to develop instructional practices consistent with reform mathematics standards, Secada and Adajian note that the relationship between the professional teaching community and teacher learning is a complex one that cannot be addressed using their interview methodology.

Franke and Kazemi (2001) also argue that we cannot separate teacher learning from the context in which it occurs, but they go beyond teachers' self reported data and observations of an established professional teaching community. Instead, they conducted a professional development project with the intention of creating a professional teaching community centered on elementary mathematics teaching and the ways in which tools support or constrain the practices and reasoning of the participating teachers. Using student work from the teachers' classes as the focus for the "workgroup" meetings, the researchers' goal was "to provide teachers the forum to develop relationships and create a community of practice not separate from their classrooms but one that could mirror the interactions and identities developed there" (p. 13). Franke and Kazemi were particularly interested in shifts in the teachers' participation within the professional teaching community. They provide examples of the teachers' contributions and how they used student thinking to support their arguments.

> The teachers used the detailed discussions of students' mathematical thinking to pursue discussions about broader issues about the teaching and learning of mathematics. For instance, throughout the workgroup meetings the teachers challenged each other's long-standing ideas about the standard algorithm. The teachers pushed each other, listened, but looked hard at the student work for evidence to support their positions. (p. 20)

Franke and Kazemi's study indicates the value of using student work as an artifact for generating learning opportunities within the professional teaching community. In a companion paper (Kazemi & Franke, in press), the researchers expand on the significance of using student work by analyzing shifts in participation in discussions centered around students' written or oral mathematical work to document students' learning. They claim that student work generated by "common problems that teachers posed to their students allowed teachers to focus on shared meaning, build common ground, and negotiate crossing the boundaries of the workgroup meetings and their classrooms" (p. 31). Although Kazemi and Franke emphasize the use of student work as a tool to develop a shared language among participating teachers, they do not document the process of the evolution of the professional teaching community.

Warren and Rosebery (1995) present a case study of one teacher's participation in a professional teaching community that focused on the learning and teaching of science. They document shifts in this teacher's participation by tracking the changes in her role in the discourse during seminars. They present three examples from different times during the year of the study to demonstrate how the nature of the teacher's talk changed from looking for approval from experts to speaking with authority and co-constructing theories about the teaching of science with other members of the professional teaching community. This is a compelling case of how an individual's development was supported by her involvement in a professional teaching community, but again, does not document the evolution of the community.

Stein, et al. (1998) endeavor to delineate the evolution of individual participation within an established professional teaching community by using Lave and Wenger's (1991) concept of legitimate peripheral participation to describe the nature of participating teachers' changing practices. Focusing on a middle-school mathematics community from one of their Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) research sites as an illustrative case, Stein, et al. describe the learning of the members of the professional teaching community as a movement from "newcomer" to "old-timer" status. In doing so, the researchers traced the trajectories of participation of the newcomers from peripheral to more substantial forms of participation in the professional teaching community. In order to analyze participation patterns as an index of teacher learning, Stein, et al. examine one particular activity of the community, implementation of the Visual Mathematics curriculum, and the ways in which old-timers' participation differed from that of newcomers. Similar to the studies conducted by Franke and Kazemi and Warren and Rosebery, Stein, et al.'s analysis of the changing participation of the members gives insight into the gradual evolution of individual participation within a professional teaching community. The authors acknowledge, however, that this particular case "has been presented as an established and somewhat static community" (p. 49) and call for studies that analyze the factors associated with the successful establishment of reform mathematics professional teaching communities.

Grossman, et al. (2001) agree that the actual formation of a professional teaching community needs further investigation.

> Studies of community typically examine already-formed groups. We have little sense of how teachers forge the bonds of community, struggle to maintain them, work through the inevitable conflicts of social relationships, and form the structures needed to sustain relationships over time. Without understanding such processes, we have little to guide us as we try to create community in settings where it doesn't already exist. (p. 943)

Drawing on their experience in a professional development project involving 22 English and social studies teachers from an urban high school, Grossman, et al. offer a model of markers of community formation as manifested in participants' speech and action. This trajectory of a professional teaching community formation is operationalized across four dimensions as summarized in a schematic (see Figure 1). The most practical aspect of this model is the attention to the changing norms of participation across time as the group evolves into a community.

Grossman, et al.'s model provides initial ideas of how a professional teaching community evolves, but does not address the issue of how this evolution was supported. The only scaffolding mentioned by the authors is the purposeful introduction (by the researchers) of two aspects of teacher development—one that focused teachers' attention on the improvement of student learning, and a second that emphasized teachers as students of subject matter. Grossman, et al. claim there is a contrast between these two foci in the promise of direct applicability and the more distant goal of intellectual renewal. In other words, they contend that a professional teaching community must be equally concerned with student learning and with teacher learning. This, for them, creates an "essential tension" pertinent to the breaking down of pseudocommunity and evolution of the group to community. Their findings emphasize the need

Beginning	Evolving	Mature
1. Formation of Group Identity and Norms of Interaction		
Identification with subgroups	Pseudocommunity (false sense of unity; suppression of conflict)	Identification with whole group
Individuals are interchangeable and expendable	Recognition of unique contributions of individual members	Recognition that group is enriched by multiple perspectives (sense of loss when member leaves)
Undercurrent of incivility	Open discussion of interactional norms	Developing new international norms
Sense of individualism overrides responsibility to group	Recognition of need to regulate group behavior	Communal responsibility for and regulation of group behavior
2. Navigation Fault Lines		
Denial of difference	Appropriation of divergent views by dominant position	Understanding and productive use of difference
Conflict goes backstage, hidden from view	Conflict erupts onto main stage and is feared	Conflict is expected feature of group life dealt with openly and honestly
3. Negotiation the Essential Tension		
Lack of agreement over purposes of professional community; different positions viewed as irreconcilable	Begrudging willingness to let different people pursue different activities	Recognition that teacher learning and student learning are fundamentally intertwined
4. Communal Responsibility for Individual Growth		
Belief that teachers' responsibility is to students not colleagues; intellectual growth is the responsibility of the individual	Recognition that colleagues can be resources for one's learning	Commitment to colleagues' growth
Contributions to group are acts of individual volition	Recognition that participation is expected from all members	Acceptance of rights and obligations of community membership (e.g., "intellectual midwifery," "press for clarification")

Figure 1. Model of Formation of Teacher Professional Community. (p. 988)

for an analysis that focuses on supports of this type that promoted the continuing evolution of the group into a professional teaching community.

The studies discussed to this point implicitly portray teachers' learning in a professional teaching community as occurring in an institutional vacuum. Talbert and McLaughlin raise the issue of how the development of a professional teaching community is supported (or constrained) by the institutional setting in which it is situated. Analyzing teacher professionalism in a survey study of sixteen high schools, Talbert and McLaughlin (1994) claim that strong teacher professional communities foster higher levels of shared standards for curriculum and instruction, evidence of a stronger service ethic in their relations with students, and show stronger commitment to the teaching profession. As a result, Talbert and McLaughlin not only call for investigations of "how" professional teaching communities develop, but analyses of the role of the institutional context in supporting or constraining that development. More specifically, they ask

> what division of functions and roles is played in building teacher communities by the different levels of the system…? How does department leadership work to promote collegial trust and collective problem solving, for example and what essential support is provided by district versus school administrators and staff? Can state policy and programs set the stage for, or facilitate, the development of local professional communities? What about outside organizations and networks? (p. 144)

Thus, Talbert and McLaughlin bring to the fore the importance of taking the institutional context into consideration when trying to understand the nature of teachers' development as they participate in professional teaching communities.

Gamoran et al. (2003) also note that there are currently no longitudinal analyses that report both the development of professional teaching communities from inception and the institutional settings in which those developments occur in either mathematics or science education. Gamoran, et al. conducted a four-year observational study of six "design collaboratives" to determine how districts and schools can support teaching for understanding in mathematics and science. They contend that the tensions and ambiguities that come to the fore as teachers attempt to teach for understanding can be managed by collaborations within professional teaching communities. Based on meeting observations, teacher interviews, interviews of school and district administrators, and survey data, Gamoran et al. report four types of resources that had a critical influence on the nature and success of professional teaching communities.

- Sufficient *time and other material resources*
- *Human resources*, including technical knowledge and expertise that supported the teachers' efforts to teach for understanding
- *Social resources*, including shared histories and purposes based on previous relationships among teachers, administrators, and researchers
- The development of *distributed leadership* in which administrators, researchers, and teacher-leaders all supported and sustained the professional community (p.129)

Gamoran, et al.'s study highlights the significant role of resources on the development of a professional teaching community. However, it fails to delineate a methodological approach for analyzing the institutional context of which these types of resources are integral aspects.

The theme that is evident across all the reviewed studies is the important role that participation in a professional teaching community plays in supporting teacher learning and innovations in teaching practice. However, the studies collectively indicate the need for additional research that focuses on the process of supporting the initial emergence and subsequent development of professional teaching communities. Several of the studies also emphasize that formation of professional teaching communities does not occur in a vacuum. The institutional setting in which these communities are situated must be taken into consideration. I address these issues in the next two chapters of this dissertation by outlining a conjectured process for supporting the emergence and development of a professional teaching community and for delineating the institutional context in which the community is situated.

CHAPTER IV

CONJECTURED LEARNING TRAJECTORY

Building from the work of Cobb and McClain (2001), Gravemeijer (1994), and Simon (1995), I use the construct of a conjectured learning trajectory as a way of thinking about the means of supporting the development of a professional teaching community. This notion of a conjectured learning trajectory is similar to Gravemeijer's (1994) description of developmental or design research in mathematics education. According to Gravemeijer, the instructional designer starts with an anticipatory thought experiment in which he or she tries to envision how the proposed teaching-learning process might be realized. During this thought experiment, the researcher develops conjectures about the course of students' development as well as the means of supporting it. Cobb and McClain (2001) add that

> conjectures about both a learning route and the means of supporting development along it are provisional and are tested and modified on a daily basis as we make local pedagogical judgments in the classroom. At the same time, the conjectured trajectory serves to guide the local decisions that lead to its revision. (p. 213)

Simon (1995) makes a related observation when he describes the reflexive relationship between a teacher's task selection and his or her consideration of students' thinking that might emerge as they participate in those tasks. "The consideration of the learning goal, the learning activities, and the thinking and learning in which students might engage make up the *hypothetical learning trajectory*" (p. 133). The teacher creates an initial learning goal and plan for instruction, which must be continually modified as students engage in the planned activities.

Gravemeijer (1994) and Simon (1995) refer to conjectured learning trajectories for individual students' mathematical learning in the classroom. However, as my focus is on the learning and development of a professional teaching community, it seems reasonable to appropriate Cobb and McClain's (2001) notion of developing a conjectured learning trajectory for classroom communities. Similar to Cobb and McClain's focus on the classroom community, I will focus on both the communal norms and practices that are developed collectively by the members of a professional teaching community and the diverse ways in which they participate in those practices.

The value of formulating a conjectured learning trajectory lies in the opportunity to test and revise the conjectures while working with the teachers. As attempts are made to support the emergence and subsequence development of the professional teaching community, the trajectory can be modified via a cyclic process of testing and revising conjectures based on ongoing analyses of collective norms and practices. Simultaneously, the evolving conjectured learning trajectory guides the local design decisions made as to how to support further learning. Therefore, at any point in time while working with the teachers, conjectures are formulated about both the potential learning of the emerging community and the means of supporting it. Thus, there is a reflexive relationship between the conjectured learning trajectory and the local decisions made to support the initial emergence and subsequent learning of the professional teaching community.

Consistent with this perspective on design, the goal when collaborating with teachers is not simply to assess whether an approach to professional development formulated at the outset works. Instead, the goal is to improve the approach by drawing on analyses of the instructional practices that the teachers develop, the activities in which they engage as members of a professional teaching community, and the institutional settings in which they work. In other words, the focus is on the actual *process* of the teachers' learning and the means by which it was supported and organized. This orientation offers the prospect that research on teacher professional development might become a design science characterized by repeated cycles in which designs are developed, tested, analyzed, and modified. A shift of this type would be compatible with the vision of educational reform as an ongoing, iterative process of improvement.

Preliminary thoughts for a conjectured learning trajectory for supporting the development of a professional teaching community were proposed by Cobb and McClain (2001) prior to beginning our work with the middle-school teachers in the Jackson Heights school district. Since we drew heavily on this proposed trajectory during our collaboration with the teachers, it is necessary to elucidate what those conjectures were. I will begin by delineating the overarching goal or endpoint of the trajectory. I will next outline the initial conjectures made by Cobb and McClain about a general approach to collaborating with teachers in order to support the development of a professional teaching community and the learning of the participating teachers. I will then discuss the process we used for delineating starting points for the proposed trajectory and outline the conjectured starting points of the trajectory.

In broad terms, the end point of the conjectured learning trajectory was to support the eventual development of instructional practices in which teaching is a generative, knowledge-building activity with students' reasoning at the center of instructional decision making. To the extent that instructional practices of this type become normative within a professional teaching community, implementation of instructional materials or innovations becomes a process of conjecture-driven adaptation in the course of which teachers elaborate and refine their understanding of both their students' thinking and the means of supporting its development as informed by ongoing assessments of their students' mathematical reasoning (Franke, Carpenter, Levi, & Fennema, 2001; Sebelli & Dede, in press; Stigler & Hiebert, 1999). This generative or on-going learning involves teachers evaluating new reform proposals, delineating the implications for their students' learning, and develop the skills to adapt the new ideas to their own classroom instructional practice (Franke, Kazemi, Carpenter, Battey, & Deneroff, 2002).

With this endpoint as a guide, Cobb and McClain (2001) build on findings from previous work with groups of second and third-grade teachers to outlined fragments of a conjectured learning trajectory for supporting the development of a professional teaching community. Three overarching goals Cobb and McClain formulated for working with teachers include supporting teachers in: 1) attempting to make sense of individual students' mathematical interpretations and solutions, 2) locating students' mathematical activity in social context by attending to the nature of the social events in which they participate in the classroom, and 3) appreciating the pedagogical intent of instructional sequences.

> This [appreciating the pedagogical intent of instructional sequences], we should stress, is not a separate "piece of knowledge" that informs pedagogical decision making. Instead, the pedagogical intent involves an envisioned developmental process and thus involves the teacher's understanding of students' mathematical thinking. Further, it involves a relatively deep understanding of the mathematical ideas that constitute the overall instructional goals *in relation to students who are attempting to learn them.* Finally, it involves specific conjectures about how the process of students' mathematical development might proceed in an instructional setting when proactive efforts are made to support their learning. (p. 216)

One of the primary conjectures underlying this envisioned approach is that instructional sequences of the type they had developed in previous classroom design experiments can serve as an important means of supporting the teachers' as well as students' learning (cf. Ball & Cohen, 1996; Gearhart, et al., 1999; Hiebert & Wearne, 1992). The key point they emphasize is that these sequences are justified in terms of 1) the trajectory of the classroom community's mathematical learning, and 2) the specific means of supporting and organizing that learning.

They argue that if the sequences were justified solely with traditional experimental data, teachers would know only that the sequences had proved effective elsewhere but would not have an understanding of the underlying rationale that would enable them to adapt the sequences to their own instructional settings. In contrast, a justification cast in terms of a mathematical learning trajectory offers the possibility that teachers will be able to adapt, test, and modify the sequences in their classrooms. To the extent that they do so, they would cease to be mere consumers of instructional innovations developed by others and would instead contribute to both the improvement of the sequences and the development of the local instructional theories that they embody.

In the case of the research team's collaboration with the group of middle school mathematics teachers in the Jackson Heights district, it drew on an instructional sequence for statistical data analysis (for a detailed analysis of the classroom design experiment see Cobb, 1999; McClain, Cobb, & Gravemeijer, 2000; Cobb, McClain, & Gravemeijer, 2003; McClain & Cobb, 2001). The intent of the instructional sequence was to support middle-school students' development of relatively sophisticated ways of reasoning statistically about univariate and bivariate data. The overarching mathematical idea that oriented the instructional design was that of distribution. The goal was that students would come to reason about data in terms of distributions. Thus, notions such as mean, mode, median, skewness, spread-outness, and relative frequency would then emerge as ways of describing how specific data sets are distributed and various statistical representations or inscriptions would emerge as different ways of structuring distributions. Based on a survey of the relevant research literature, student interviews, and classroom performance assessments completed in preparation for the design experiment, the goal for the learning of the classroom community was that reasoning about distribution of data in multiplicative terms would become an established mathematical practice that was beyond justification. It is important to note that an investigative orientation was taken when developing instructional activities to ensure a spirit of genuine data analysis, while simultaneously supporting the emergence of key statistical ideas (Cobb, 1999). Computer-based applets were developed as a primary means of supporting students' learning while simultaneously providing them with tools for data analysis. "Each [applet] offered students several ways of structuring data. Importantly, these options do not correspond to a variety of conventional inscriptions as is typically the case with commercially available data analysis tools. Instead, we drew on the research literature to identify the various ways in which students structure data when given the opportunity to conduct genuine analyses" (Cobb, 1999, p. 12). I describe these computer-based applets more specifically when I present an analysis of the normative practices that emerged during the work sessions with the teachers.

To support this conjecture about the role of instructional sequences in supporting the development of a professional teaching community, Cobb and McClain drew on implications from their prior work with teachers that remained viable when working with other groups of teachers. These included an initial emphasis on the teachers' role in guiding the renegotiation of classroom social norms so that their classrooms could become learning environments for them as well as their students. Another conjecture concerned the need for teachers to develop reasons and motivations to want to change their current instructional practices of mathematics. Cobb and McClain speculated this could be supported by framing

> selected teaching experiments as cases both of students' mathematical thinking and of how effective teachers build on that thinking to support and organize the emergence of significant mathematical ideas (cf. Barnett, 1991; Barron & Goldman, 1994; Bowers, Barron & Goldman, 1994; Franke, et al, 1998; Lampert & Ball, 1990; Schifter, 1990). In addition to the instructional activities and associated resources (e.g., computer-based tools used as part of a sequence), the case materials might involve CD-ROMs that are

> based on video-recordings of classroom sessions, student interviews, and copies of the students' written work. The specific types of activities in which teachers might engage as they investigate these cases could include: (a) using the instructional activities to conduct mathematical investigations oriented towards the mathematical ideas that constitute the overall intent of the sequences; (b) using the CD-ROMs to investigate the development of students' reasoning and the means that can be used to support it. (p. 217-218)

Consequently, the goal as teachers investigate these cases would be to enable them to reconstruct the rationale for the instructional sequences. The conjecture was that to the extent they did so, they would be in a position to begin to make informed decisions and judgments about how they might adapt the sequences.

Building from their experience of preparing for whole-class discussions during classroom design experiments by identifying mathematically significant issues that might emerge as topics of conversation, Cobb and McClain identified three pedagogically significant issues that could constitute productive topics of conversation both for the development of the professional teaching community and for the learning of the individual teachers: (1) developing norms for mathematical argumentation; (2) ensuring that students' activity remains grounded in the mathematical imagery of the situation they are investigating (Thompson, 1996); and (3) redescribing and notating students' solutions and explanations. The practical challenge is then to ensure that the pedagogical issues emerge in a way that is experienced as relatively natural by the participating teachers.

The complex and demanding nature of instructional practices of this type indicate the importance of social supports such as those afforded by membership in a professional teaching community (Gamoran et al., 2003). When situated in such a community, the process of instructional improvement becomes a collaborative, problem solving activity in which teachers generate knowledge about both students' mathematical reasoning and the process of supporting its development (Franke et al., 2001).

The process for identifying the starting points of a conjectured learning trajectory for supporting the initial emergence of a professional teaching community included documenting the participating teachers' current mathematical understandings, current instructional practices, and the institutional context in which they work. This analysis of mathematical understandings, instructional practices (including teachers' planning, facilitation of lessons, student assessment, reflection, adaptation, and the tools used during these aspects of practice), and the institutional context was essential in understanding how best to support the individual teachers' learning and the emergence of a professional teaching community. It is important to note, however, that this assessment was not cast in deficit terms of what teachers did not know or were unable to do, but instead emphasized the resources available on which to build on to support and organize the development of the professional teaching community and the learning of the participating teachers. In other words, the focus was not on assessing the teachers by using the envisioned endpoints as standards of evaluation, but to understand the teachers' current practices in order to access resources on which to capitalize in order to work towards the envisioned endpoints.

The research team assessed the teachers' initial mathematical understandings by presenting them with mathematical assessment tasks designed to bring to the fore teacher's reasoning about specific mathematical ideas involving statistical data analysis. As the teachers were working on the tasks together, this not only gave insight into the diversity of the mathematical understandings of the group, but also clarified initial norms of interaction between the teachers. Based on the teachers' work with these assessment tasks, the research team conjectured that teachers had limited experience with analyzing data, specifically in dealing with variability and distribution. Since a goal in working with the teachers was to support the pedagogical norm of making instructional decisions based on student reasoning, it was crucial that the teachers develop a relatively sophisticated understanding of the significant mathematical ideas their students were to learn (Bransford, Brown, & Cocking, 2000; Ma, 1999; Shulman, 1986; Thompson & Thompson, 1996). Therefore, the conjecture of using the statistics instructional sequence as a means of supporting the teachers' development of sophisticated ways of reasoning about data analysis seemed viable as a starting point.

In order to determine starting points related to teachers' classroom instructional practices, members of the research team conducted classroom observations of eight of the nine teachers. All attempts made to observe the ninth teacher failed due to conflicts with scheduling (school field trips, pep-rallies, parent meetings, and conferences). Early in the work with the teachers, it became apparent that the teachers were very private about their classroom instructional practices. Therefore, the first observations were casual "drop-in" visits to each of their classrooms that were neither audio nor video recorded. These initial classroom observations afforded opportunities for insights into teachers' classroom instructional practices as well as helping to establish an initial relationship of trust in which the researchers viewed their role of assisting rather than assessing the teachers (Tharp & Gallimore, 1988).

After working with the teachers for five months, the research team generated modified teaching sets (Simon & Tzur, 1999) for each teacher to better document the teachers' instructional practices. A teaching set consists of a series of classroom observations followed by a semi-structured interview with the teacher that focuses on instructional planning and on reflections of lessons. The interviews were audio-recorded, but as the teachers only allowed the research team to video-record their teaching after it had worked with them for 18 months, the team initially had to record classroom observations as field notes. It is important to reiterate that the research team's perspective when analyzing these teaching sets was to assume that teachers' perceptions of teaching and learning and specific classroom instructional practices they developed "are reasonable and useful from their perspectives" (Simon, Tzur, Heinz, Kinzel, & Smith, 2000, p. 588). Adopting this perspective enabled the research team to avoid characterizing the teachers' instructional practices in deficit terms and instead made evident the starting points on which to build towards our envisioned endpoints.

Analysis of the modified teaching sets revealed that there were identifiable regularities in the teachers' instructional practices when the research team first began collaborating with them even though they worked in almost complete isolation. In general, their instruction focused on students' acquisition and application of procedures for operating on mathematical symbols, and on the learning of definitions for mathematical terms. The organization of activities in each classroom typically involved a "problem of the day" or warm-up activity, a review of homework problems that had been assigned during the previous class session, a demonstration of the procedure for solving the types of tasks in a step-by-step manner that would be assigned as homework in the current session, and an assignment of individual work from a textbook or activity sheet in which the students began to work on homework tasks that were similar to the demonstrated problem. Teachers' assessments of students' reasoning were limited to the correctness of answers. The adjustments they made when students did not produce correct answers typically involved either explaining the procedure for a second time or asking students to check whether they had performed the steps correctly. For the most part, students were obliged to take notes on the solution procedures demonstrated by the teacher and answer questions posed by the teacher to solicit understanding of the demonstrated method.

There were two obvious outliers to this generalization of the teachers' initial classroom instructional practices. The first teacher was never observed actually teaching a math lesson.

Her students were either working on a competitive extra credit project or discussing a current event for the entire class period. The second teacher's classroom instructional practices were distinguished from the other teachers by the normative purpose of understanding and learning mathematics that appeared to be constituted in her classroom. For example, although this teacher also had a problem of the day, she gave no direct assistance to the students. Instead, she allowed students to struggle to make sense of the exercise and asked known answer questions to guide them. This teacher monitored students for understanding rather than merely their ability to produce correct answers. The students, rather than simply cooperating with the teacher, seemed to view the classroom mathematical activity as worthy of engagement and seemed to have a sense of responsibility for contributing to its development. Thus, taking the teachers' classroom instruction practices into account, a productive starting point seemed to be concentrating on the conjecture concerning the need for teachers to develop reasons and motivations to want to change their current instructional practices of mathematics.

Knowledge of teachers' current mathematical understandings and instructional practices was crucial in establishing viable starting points for a conjectured learning trajectory. The importance of situating these starting points within the institutional context cannot be overstated. Just as students' current orientations towards mathematics reflect their prior instructional history, teachers' current practices are partially constituted by the institutional context in which they developed those practices. Thus, an analysis of the institutional setting becomes crucial when both envisioning and understanding the development of the teachers' instructional practices.

CHAPTER V

ANALYSIS OF INSTITUTIONAL CONTEXT

A number of investigations document that teachers' instructional practices are profoundly influenced by the institutional constraints that they attempt to satisfy, the formal and informal sources of assistance on which they draw, and the materials and resources that they use in their classroom practice (Ball, 1996; Brown, Stein, & Forman, 1996; Feiman-Nemser & Remillard, 1996; Nelson, 1999; Senger, 1999; Stein & Brown, 1997). The findings of these studies indicate the need to take account of the institutional setting in which teachers develop and refine their instructional practices. These findings are consistent with Talbert and McLaughlin's (1994) and Gamoran, et al.'s (2003) claims elucidated earlier in this dissertation of the importance of investigating of the role of the institutional context in supporting or constraining this development. Therefore, to adequately explain the process of supporting the initial emergence and subsequent development of the professional teaching community, an analysis of the institutional setting in which the collaborating teachers worked was essential.

A concurrent analysis of the institutional setting was conducted during the first two years of the research team's collaboration with the group of teachers in Jackson Heights. A detailed elaboration of the analytical framework used and results of this analysis can be found in Cobb, et al. (2003). I will limit my discussion to a brief overview of the methods and findings of the analysis. My purpose for introducing the results of this ancillary analysis at this point is to locate the professional teaching community in the institutional setting of the school district. This will allow me to relate the realized learning trajectory of the professional teaching community to the organizational setting. To ground this discussion, I will first give a general overview of the Jackson Heights school district and the composition of the group of teachers with which we collaborated.

The district was established in 1993 following the court-ordered integration of a city and a county school district. It served a 60% minority population and is located in a state with a high-stakes accountability testing program with standardized tests administered at each grade level in late spring of each school year. The State Department of Education produces a Prescribed Instructional Program that specifies the mathematical objectives that teachers should address at each grade level. In an endeavor to support its reform efforts and consequently the raising of standardized test scores, the district received an externally funded grant. As part of this grant, the district had adopted an NSF-funded middle-school curriculum. A pacing guide was produced by the district's mathematics coordinator and three mathematics specialists to assist the teachers in using this curriculum, supplemented by the traditional textbook series, to address the objectives as outlined in the Prescribed Instructional Program. As stated previously, our introduction to the district was at invitation of the district's mathematics coordinator to provide teacher development. She was concerned that a significant proportion of the teachers at the middle grades level were continuing to use the traditional textbook series as the primary basis for their instruction.

The teachers that collaborated with the research team during the first two years were selected by the district mathematics coordinator. The group was composed of nine middle school mathematics teachers from five schools in the district. A mathematics specialist who supported teachers in their classrooms attended during our work sessions and the district mathematics coordinator attended lunches with the group and the summer work session at the end of year two.

As explicated in Cobb, et al. (2003), the research team shifted its focus from the competencies and actions of individual teachers working alone in their classrooms to the *functions of teaching* as they are accomplished in schools and school districts when analyzing the institutional setting. In the case of mathematics, these functions are not restricted to interacting with students in the classroom to support their mathematical learning but also include:

- *Organizing for mathematics teaching and learning* by, for example, delineating instructional goals and by selecting and adapting instructional activities and other resources.
- *Making mathematics learning and teaching visible* by, for example, interpreting test scores or posing tasks designed to generate a record of students' mathematical reasoning.

When analyzing how these latter two functions are actually accomplished in specific cases, it almost invariably proves to be the case that a number of persons in various designated positions within the school and district are involved in accomplishing them.

In addition to disclosing that a number of persons in a school or district are involved in accomplishing the functions of teaching, analyses of these functions of teaching also reveal that the contributors typically use a range of tools (e.g., documents listing State-mandated curriculum objectives, pacing guides, textbooks, classroom observation forms, reports of tests scores, copies of students' written work). It is in this sense that teaching can be viewed as a distributed activity that is accomplished collectively by a number of persons using a variety of tools. It is important to stress that this distributed perspective on teaching does not imply that people within a school or district necessarily coordinate their activities seamlessly or smoothly. As will become apparent when I describe the institutional setting in which the collaborating teachers worked, teaching was frequently a site of tension and conflict in that people within a school or district were pursuing conflicting agendas. It was these efforts of the members of different communities of practice to pursue sometimes-conflicting instructional visions and to gauge the extent to which their visions had been realized in classrooms that constituted the immediate institutional setting within which teachers developed and refined their instructional practices.

The approach that the research team took when analyzing the institutional settings in which the collaborating teachers worked involved identifying the communities of practice (Wenger, 1998) within a school or district whose missions or enterprises were concerned with the teaching and learning of mathematics. The potential value of this construct to the issue of locating teachers' instructional practices in institutional context stems from the manner in which it brings together 1) theories of social structure that give primacy to institutions, norms and rules, and 2) theories of situated experience that give primacy to the dynamics of everyday existence and the local construction of interpersonal events (cf. Wenger, 1998, pp. 12 – 13).

Methodologically, the research team used what Spillane (2000) refers to as a snowballing strategy and Talbert and McLaughlin (1999) term a bottom-up strategy to delineate the communities of practice within the school district whose missions or enterprises were concerned with the teaching and learning of mathematics. The first step in this process involved conducting audio-recorded semi-structured interviews with the collaborating teachers to identify people within the district who influenced their classroom instructional practices in some significant way (see Appendix A for complete list of guiding issues for these conversations with teachers). The issues addressed in these interviews included the professional development activities in which the teachers had participated, their understanding of the district's policies for mathematics instruction, the people to whom they were accountable, their informal professional networks, and the official sources of assistance on which they could draw. In order to corroborate these interview data, a survey was administered that addressed these same issues to all the mathematics teachers in the five schools in which the collaborating teachers taught (see Appendix B for survey). The second step in this bottom-up or snowballing process involved interviewing the people identified in the teacher interviews and surveys in order to understand both their agendas as they relate to mathematics instruction and the means by which they attempt to achieve those agendas. Those identified by the teacher interviews and surveys were school leaders, which included principals and assistant principals at each school, and the district mathematics leaders, which included the district mathematics coordinator and three mathematics specialists (see Appendix C and D for guiding issues used in conversations with school leaders and district mathematics leaders). This process was continued as additional people were identified in this second round of interviews who actively attempted to influence how mathematics was taught in the district.

The communities of practice that we identified by analyzing these data were the district-wide *mathematics leadership community* and the *school leadership communities* in the schools in which the teachers work. I describe each of these communities after first summarizing the results of the teacher interviews.

The Collaborating Teachers

The initial interviews that the research team conducted with the collaborating teachers indicated that their informal professional networks were extremely limited and that their mathematics instruction was highly privatized. These findings were corroborated by the survey responses of the other mathematics teachers in their schools. As will be explicated in my analysis, it was not until the research team had worked with the teachers for 19 months that the group could be said to constitute a community of practice with a joint enterprise. Prior to this, interactions between the teachers during the work sessions that were conducted frequently involved what Grossman, et al. (2001) term pseudo-agreements that serve to mask differences in viewpoints.

The tools with which the teachers reasoned as they organized for mathematics instruction included the two textbook series, the pacing guide, and the Prescribed Instructional Program. Student solutions from classwork tasks, homework, quizzes, and teacher or textbook produced tests were the primary means of making students' reasoning visible. The state standardized test was viewed more by the teachers as a means for others to assess their teaching instead of a means of making students' reasoning visible.

The Mathematics Leadership Community

The members of the mathematics leadership community included the mathematics coordinator and three mathematics specialists as core members and a number of teachers (none of whom were in our group) as more peripheral members. The joint enterprise of this community was to improve mathematics performance of all students but particularly minority students by assisting teachers to use the reform textbook series as the basis for their mathematics instruction. The core members of the Mathematics Leadership Community viewed themselves as participants in the broader social community of mathematics education reformers and had a relatively deep understanding of and a commitment to the general intent of reform proposals for mathematics teaching.

The tools with which the members of this community reasoned as they organized for mathematics instruction included the reform and traditional textbook series, the Prescribed Instructional Program, and the pacing guide that they produced and revised each year. However, the mathematics leaders all indicated that they relied almost exclusively on scores on the state-mandated test to make students' learning visible. A division of labor was evident in this community in that the mathematics coordinator spent the bulk of her time completing administrative tasks and coordinating with other groups both within and outside the district. For their part, the three mathematics specialists visited teachers' classrooms to assist them and also organized professional development sessions on the use of the reform textbook series that they conducted in collaboration with the teachers who were peripheral members of this leadership community.

The School Leadership Communities

The leadership communities in each of the five schools in which the collaborating teachers worked consisted of the principal and two or more assistant principals. In addition, one or more mathematics teachers in each school were peripheral members and, for example, occasionally conducted observations of other teachers at the request of a school administrator. The enterprise of each of the school leadership communities was to raise students' scores on the state-mandated achievement test. For example, one principal defined a good teacher as someone who has "good test scores" and another stated that "in mathematics an exceptional teacher has kids rank well in academics and behavior." When asked to whom she was accountable, this latter principal then went on to clarify that "the district office focuses on test scores."

The primary tool that the school leaders use to organize for mathematics teaching and learning is the Prescribed Instructional Program produced by the State Department of Education. In addition, members of the School Leadership Community in each school regularly conducted drop-in classroom visits during which they focus on the match between lesson objectives that teachers are required to write on their whiteboard and the objectives specified by the Prescribed Instructional Program, as well as on students' behavior and level of engagement.

The school leaders were aware of the mathematics leaders' efforts to reform mathematics instruction in the district. However, they indicated that they were open to a range of instructional techniques and to the use of the traditional textbook series as the primary basis for instruction provided that teachers met their expectations with respect to instructional objectives and student engagement. Consistent with the limited attention that they gave to the substance of teachers' instructional practices, each of the school leaders described current reform recommendations in mathematics education in terms of the de-mathematized generalities reported by Spillane (2000) (e.g., using small group work, manipulatives, and real world problems to achieve traditional instructional goals).

The school leaders, like the mathematics leaders, relied on scores on the state-mandated test to make students' learning visible. It was also apparent from both their interview responses and the collaborating teachers' comments that they also used these scores to make mathematics teaching visible by making inferences about the extent to which teachers had addressed the objectives specified by the Prescribed Instructional Program. Teachers whose scores were judged to be unsatisfactory were usually required to have their lesson plans reviewed by a school leader. The primary function of instructional leadership in mathematics appeared to be that of monitoring and assessing teachers' instructional practices with respect to content coverage and student engagement.

It is apparent from these cursory accounts of the mathematics leadership and school leadership communities that the visions for mathematics teaching and learning that they attempted to realize in classrooms differed significantly and were in partial conflict. For the school leaders, mathematics teaching is a relatively routine activity whereas for the mathematics leaders it is a complex and demanding activity that required a deep understanding of both students' mathematical reasoning and the mathematical ideas that are the focus of instruction. Whereas the school leaders viewed different instructional techniques as alternative ways to address traditional goals, the mathematics leaders conceptualized instructional goals in terms of central mathematical ideas and viewed mathematical communication and argumentation not as a possible instructional strategy but as an important goal in its own right. More generally, the school leaders appeared to participate in the Discourse of high stakes testing and accountability (cf. Confrey, Bell, & Carrejo, 2001) whereas the mathematics leaders participated in the Discourse of reform in mathematics education.

To this point, I have illustrated the potential value of analyzing schools and school districts as configurations of communities of practice and have described the relevant communities. The issue to which I now turn is that of how the relations between communities might be analyzed. In doing so, I distinguish between three types of interconnections: Boundary encounters, brokers, and boundary objects.

Boundary Encounters

The first type of interconnection arises when participation involves *boundary encounters* in which members of one community engage in activities with members of another community. The mathematics leaders in the district participated in the activities of their community by conducting both summer work sessions and study group meetings with teachers during the school year that focused on the use of the reform textbook series. As the collaborating teachers all gave positive accounts of these sessions and indicated that, as a result, they were better prepared to discern the mathematical intent of the non-standard problems in these textbooks, it would appear that these boundary encounters contributed to the human resources within the district (i.e., the knowledge, skills, and commitments of individuals).

I have also noted that the school leaders participated in the activities of their communities by conducting drop-in classroom visits to monitor and assess teachers' instructional practices. As a consequence of these boundary encounters, the teachers viewed classroom observations as situations for assessment rather than assistance and initially attempted to delimit the access that the researchers and other teachers had to their classrooms. When the researchers asked during a meeting of the teacher group why visitors to their classrooms made them uncomfortable, one simply stated to the others' agreement, "We don't want to be wrong."

This privatization of instructional practices within the district was evident in the teachers' initial view of themselves as independent professionals who worked within perceived institutional constraints by relying almost exclusively on their own resources. Gamoran et al.'s (2003) analysis of reform efforts in five school districts indicates that the institutionalization of teaching as a private activity in the district reduced the extent to which the teachers' classrooms could be sites for their learning, thereby curbing the generation of both human and social resources within the district (i.e., the relationships and methods of communication of groups of people engaged in joint activities).

Brokers

The second type of interconnection that I document when analyzing the institutional settings in which teachers work in a particular school or district concerns the activities of *brokers* who are at least peripheral members of two or more communities of practice. Brokers can bridge between the activities of different communities by facilitating the translation, coordination, and alignment of perspectives and meanings. Their role can therefore be important in developing alignment between the enterprises of different communities of practice. I therefore regarded it as highly significant that the research team was unable to identify a single broker between the teacher group, the mathematics leadership community, and school leadership communities when the team first began working in the district. This absence of brokers accounts to some extent for the lack of alignment of the agendas of the mathematics leadership and school leadership communities, and thus for the tensions that the teachers reported experiencing as they organized for mathematics teaching and learning.

Boundary Objects

The first two types of interconnections that I have discussed are based on participation and are usually visible both to observers and to participants. The third type of interconnection between the communities of practice, *boundary objects*, is typically less visible and is based on what Wenger (1998) terms reification rather than participation.

Star and Griesemer (1989) argue that reifying objects can play a significant role in enabling the members of different communities to coordinate their activities even when they are used differently and have different meanings. As they demonstrate, successful coordination does not require that members of different communities achieve consensus. Instead, the use of the objects in different communities makes it possible for them to function as common *boundary* *objects* around which the members of the different communities can organize their activity. Consequently, as Star and Griesemer emphasize, boundary objects do not carry meanings across boundaries but instead constitute focal points around which interconnections between communities emerge. In this respect, boundary objects can serve as tools for communication between the members of different communities even though they do not provide a ready-made bridge between perspectives and meanings.

The mathematics leaders in the district reified their vision of teachers building on students' reasoning to support their understanding of central mathematical ideas as they developed and revised the pacing guide. This guide mapped the two textbook series onto the Prescribed Instructional Plan that lists the objectives assessed by the state-mandated test. As I have indicated, when the research team first began working with the teachers, they used the pacing guide by developing lessons that tended to focus on performing and applying mathematical procedures. Thus, the pacing guide functioned as a boundary object even though it did not carry the mathematics leaders' instructional vision to the teachers.

The significance that I attribute to the teachers' participation in the teacher group becomes more apparent when I note that the pacing guide affords the formulation of teaching trajectories that are concerned with the coverage of content objectives. Although it does not preclude the formulation of learning trajectories that are concerned with students' reasoning and the means of supporting its development, such trajectories in effect have to be read into it. In this regard, the pacing guide can be contrasted with a planning tool used by Japanese teachers that specifies both the most frequent student solutions to particular types of problems and the ways in which teachers can capitalize on those solutions to achieve their instructional agendas (Stigler & Hiebert, 1999).

As stated before, the research team noted that the teachers' instructional practices were relatively homogeneous when the team first began working with them even though their informal professional contacts were limited. Most of the teachers in fact indicated that they were so familiar with the Prescribed Instructional Program that they rarely had to refer to it directly when they organized for mathematics teaching and learning. Importantly, the Prescribed Instructional Program affords the formulation of teaching trajectories that focus on a large number of relatively narrow goals rather than learning trajectories that focus on central mathematical ideas. The teachers had few social resources on which they could draw and apparently did not have the personal resources to read either more encompassing goals or a learning trajectory into the Prescribed Instructional Program as they attempted to meet school leaders' expectations. In my view, these aspects of the institutional setting in which the teachers worked account for the homogeneity that was observed in their instructional practices when the researcher team first began working with them even though they developed these practices in relative isolation and the school leaders were open to a range of instructional techniques.

As I have noted previously, this cursory overview of the institutional setting in which the collaborating teaches work is useful for explaining the teachers' initial practices and in formulating plans for how to work with them. Initial implications from this analysis of the institutional context include: 1) supporting teachers' learning can involve collaborating with them and members of other communities to bring about changes in the institutional settings in which they work and 2) teachers' understandings of the ways in which the institutional setting enables and constrains their instructional practices become centrally important.

CHAPTER VI

METHODOLOGY

As stated earlier, this analysis documents the development of a professional teaching community as situated in the institutional context of the school district. The analysis focuses on the process for supporting the emergence and subsequent learning of the professional teaching community during the first two years of our collaboration with the teachers. As my study focused on the initial formation of the professional teaching community and not the assimilation of new members into an established professional teaching community, I have limited my analysis to the first two years. The composition of the professional teaching community changed after the summer session following the second year. At the beginning of year three, three teachers from the original nine left the collaboration and six new teachers joined. In this section, I provide an overview of the methodology that was used to conduct the analysis.

Data Collection

I will recapitulate the data collected to document the institutional context and teachers' instructional practices, and then describe the data collected to document the learning of the professional teaching community and the process for supporting that learning. As I have already indicated, the institutional context was documented using a snowballing (Spillane, 2000) or bottom-up (Talbert and McLaughlin, 1999) process. The first step in this data collection process included audio-recorded semi-structured interviews with the collaborating teachers that each lasted approximately one hour (appendix A). The issues addressed in these interviews included the professional development activities in which the teachers had participated, their understanding of the district's policies for mathematics instruction, the people to whom they were accountable, their informal professional networks, and the official sources of assistance on which they drew. The research team drew from existing surveys (Bryk, Camburn, and Louis, 1997; Bryk and Schneider, 2002) to create a survey (appendix B) which addressed these same issues. This survey was administered to all mathematics teachers in grades six through eight in the five schools in which the nine collaborating teachers worked to corroborate the findings of the interviews. The second step in this bottom-up or snowballing strategy involved interviewing people identified in the teacher interviews and surveys as those people within the district who influenced the teachers' classroom instructional practices in some significant way. These included interviews that lasted an hour to an hour and a half with four school leaders (appendix C) and three members of the mathematics leadership community including the district mathematics coordinator and two mathematics specialists (appendix D). Issues addressed in the interviews with the school leaders included their subject matter background and grade level teaching experience, the background of their school, the major challenges they face, their views on mathematics teaching and learning, what they do to support this vision, to whom and for what are they held accountable, their perception of district policies with regards to mathematics instruction, and their views on cultural diversity and equity within the district. For the interviews with the mathematics leadership community, the issues addressed included background to the district, their perception of cultural diversity and equity within the district, how they make mathematics teaching and learning visible, their agenda for mathematics teaching and learning within the district, how their agenda is influenced by state and national policy, and how they attempt to achieve their agenda.

Teachers' instructional practices were documented by generating a modified teaching set (Simon & Tzur, 1999) each year for eight of the nine teachers. Although several attempts were made to generate a modified teaching set for the ninth teacher, the research team was unable to do so for several reasons (field trips, student/teacher basketball game, parent/teacher meetings, and conferences). Each teaching set consisted of a classroom observation followed by an audio-recorded semi-structured interview with the teacher that focused on instructional planning and on reflections on the observed lesson. As the teachers only allowed us to video-record their teaching after we had worked with them for 18 months, the research team initially had to record our classroom observations as field notes.

Data collected to document the learning of the professional teaching community included video-recordings of all work sessions using one video camera. The initial summer session, the three sessions during the first year, the summer session at the end of the first year, and the first two sessions of the second year were held at the district professional development center. The next four sessions were conducted at one of the middle schools in the district where two of the participating teachers worked. The summer session following the second year was held at the research team's university. In addition to video-recording all work sessions, the research team also audio-recorded these sessions in the second year using two audio-recorders. Field notes were generated for the initial summer session after the fact by one of the researchers. All subsequent field notes were generated during the work sessions by one member of the three person research team (a researcher, a graduate student, or a post-doc). After each session, a summary of the field notes was produced by the person who originally created the field notes for the session. All field notes and summaries were shared with all members of the research team. Data also included copies of all material artifacts produced by the teachers during work sessions such as copies of students' work that the teaches analyzed, their written analysis of statistics activities, and chart paper recording ideas of or issues raised by teachers during discussions.

Since my goal was to not only document the emergence and learning of the professional teaching community, but also to understand the means of supporting that emergence and learning, another data source was a log of the research teams' ongoing conjectures during the first two years of our collaboration. This included initial conjectures, critical issues that emerged, and revised conjectures.

As a member of the research team, I was involved in all aspects of data generation. This included contributing to the development of the semi-structured interview protocols, conducting interviews, creating and distributing of the survey, and conducting classroom observations. During the first year of collaboration with the teachers, I was part of a three person team that conducted the sessions and was responsible for developing field notes and making the video and audio recordings. During the second year, my role was more active in that I co-lead most activities and discussions during the sessions.

Data Analysis

My goal in analyzing the video-recordings, artifacts, and field notes generated to document the activities of the professional teaching community was to describe the collective development of the community. I did so by identifying the successive norms that became established in the community for 1) general participation, 2) pedagogical reasoning, 3) mathematical reasoning, and 4) strategic reasoning. The analysis of norms for general participation documented the evolving participation structure within the group (Lampert, 1990; Shulman, 1986). As an illustration, this analysis documented whether it became normative for the teachers to question and critique each others' reasoning or whether norms involved what Grossman, et al. (2001) term pseudo-agreement in which the teachers refrained from confronting issues that relate to their instructional practices. The analysis of norms for pedagogical reasoning documented the norms that became established as the teachers both reflected on their instruction and planned for instruction. In focusing on the key norm of what counts as an acceptable pedagogical argument, for example, I documented the extent to which the teachers became obliged to justify their pedagogical judgments in terms of analyses of students' mathematical reasoning. The analysis of norms for mathematical reasoning documented the forms of mathematical reasoning and argumentation that became normative as the teachers explored particular mathematical domains. When the teachers engaged in activities that involved analyzing data, for example, I documented whether the norms that became established for statistical reasoning involved additive or multiplicative reasoning. The analysis of strategic norms documented the evolution of the teachers' understanding of the institutional setting and its influence on their teaching of mathematics. As part of this analysis, I focused both on the extent to which the teachers viewed their practices as shaped by the institutional settings in which they worked and on the aspects of these settings that they believed they could change.

I identified three of these four types of norms a priori based on my review of the literature. Grossman, et al.'s (2001) claim that there is necessarily a change of norms of interaction as a group evolves from a pseudocommunity into a professional teaching community constitutes the rationale for my decision to document norms of general participation. Another reason for documenting norms of general participation comes from a focus on mutual engagement, the third criterion I use for determining when a group has evolved into a professional teaching community. As stated previously, mutual engagement includes the social relationships that members develop as they collectively pursue a shared enterprise, as well as the norms of participation that are specific to the community. This focus on mutual engagement also constitutes a rationale for documenting norms of pedagogical reasoning in that it includes not only general norms of participation but also norms that are specific to mathematics teaching such as the standards to which the members of the community hold each other accountable when they justify pedagogical decisions and judgments. It is also important to analyze pedagogical reasoning given the overarching goal of supporting the eventual development of teachers' instructional practices such that teaching becomes a generative, knowledge-building activity with students' reasoning at the center of instructional decision making.

The proposal to document norms of mathematical reasoning is derived from the literature that documents the importance of teachers developing a deep understanding of central mathematical ideas if they are to pursue an instructional agenda that focuses on these ideas by building on their students' reasoning (Bransford, Brown, & Cocking, 2000; Ma, 1999; Shulman, 1986). The focus on norms of strategic reasoning emerged as we worked with the teachers. Based on our log of on-going conjectures, it became evident that the teachers' view of the institutional context and how it supported or constrained their instructional practices was also an important aspect of their learning. Although Talbert and McLaughlin (1994) and Gamoran, et al. (2003) point to the importance of understanding the institutional context, previous studies have not attended to the norms of strategic reasoning.

Methodologically, it is important to clarify that norms are identified by discerning patterns or regularities in the ongoing interactions of the members of the professional teaching community. A norm is therefore not an individualistic notion but is instead a joint or collective accomplishment of the members of a community (Voigt, 1995). A primary consideration when conducting analyses of this type is to be explicit about the types of evidence used when determining that a norm has been established so that other researchers can monitor the analysis. A first, relatively robust type of evidence occurs when a particular way of reasoning or acting that initially has to be justified is itself later used to justify other ways of reasoning or acting (Stephan & Rasmussen, 2002). In such cases, the shift in the role of the way of reasoning or acting within an argument structure from a claim that requires a warrant, to a warrant for a subsequent claim provides direct evidence that it has become normative and beyond justification. A second, robust type of evidence is indicated by Sfard's (2000) observation that normative ways of acting are not mere arbitrary conventions for members of a community that can be modified at will. Instead, these ways of acting are value-laden in that they are constituted within the community as legitimate or acceptable ways of acting. This observation indicates the importance of searching for instances where a teacher appears to violate a proposed communal norm in order to check whether his or her activity is constituted as legitimate or illegitimate. In the former case, it would be necessary to revise the conjecture whereas, in the latter case, the observation that the teachers' activity was constituted as a breach of a norm provides evidence in support of the conjecture (cf. Cobb, Stephan, McClain, & Gravemeijer, 2001). Finally, a third and even more direct type of evidence occurs when the members of a professional teaching community talk explicitly about their respective obligations and expectations. Such exchanges typically occur when one or more of the members perceive that a norm has been violated.

The specific approach that I took when analyzing the data collected to document the learning of the professional teaching community involved a method described by Cobb and Whitenack (1996) that was developed for analyzing longitudinal data sets of the type generated during design experiments. This method is a variant of Glaser and Strauss' (1967) constant comparative method and is specifically tailored to the *systematic* analysis of longitudinal data sets in mathematics education. The distinguishing feature of their method is that as new episodes are analyzed, they are compared with currently conjectured themes or categories. This process of constantly comparing episodes leads to the ongoing refinement of the theoretical categories that remain grounded in the data. As Glaser and Strauss note, negative cases that appear to contradict a current category are of particular interest and are used to further refine the emerging categories.

The first phase of this analysis entailed reading through the field notes of the work sessions in chronological order and creating a log of the work sessions organized by theme of conversation or activity. In other words, I documented the major issues that emerged episode-by-episode, where the determining characteristic of an episode was that a particular mathematical or pedagogical theme was the focus of the teachers' and students' public activity and discourse. In this phase, my purpose was to index what happened in the sessions without differentiating between episodes in terms of how well they might reveal normative practices of the group. My intent was to develop a broad overview of what transpired in the work sessions and to create an organizational structure and brief summaries that would facilitate subsequent analysis of the data.

The second phase of the analysis involved working through the entire data corpus generated during the professional development sessions in chronological order identifying episodes that contained direct evidence of normative practices established within the group. I then supplemented the log created in the first phase by providing thick descriptions of these identified episodes. Also in this phase, I formulated, tested, and refined conjectures about the four types of norms that were established within the teaching group in each of these episodes, and documented the evidence for those claims. Each claim was substantiated or modified while analyzing subsequent episodes (see Figure 2). In other words, conjectures were tested and, as necessary, revised while analyzing subsequent episodes. The results of this first phase were a chain of conjectures, refutations, and revisions that were grounded in the details of the specific episodes.

In the third phase of the analysis, the resulting chain of conjectures and refutations from the second phase were analyzed and reorganized based on claims about how the shifts in normative practices were supported and organized by (1) the inter-relational nature of the four types of norms, (2) overviews of the teachers' classroom instructional practices, and (3) the analysis of the institutional setting. My focus on the inter-relational nature of the four types of norms was grounded in the assumption that they did not evolve independently. Instead, I conjectured that shifts in one type of norm could create certain affordances that precipitated shifts in another type of norm. For example, shifts in the teachers' norms of mathematical reasoning might precipitate certain shifts in the teachers' normative ways of participating within the group. As a second example, the fact that we could not video-record teachers in their classrooms until after 18 months of working with them revealed the private nature of the teachers' instructional practice. This privatization of instructional practices clarified the constraints of the development of norms of general participation when teachers discussed pedagogy. I relied on the prior analysis of the institutional setting to situate the chain of conjectures and refutations from the second phase. For example, the prior analysis of the institutional context clarified why there was such a privatization of classroom practice by providing insights into the constraints and affordances of the district in which they developed their instructional practices. The analysis of the institutional context was relevant when

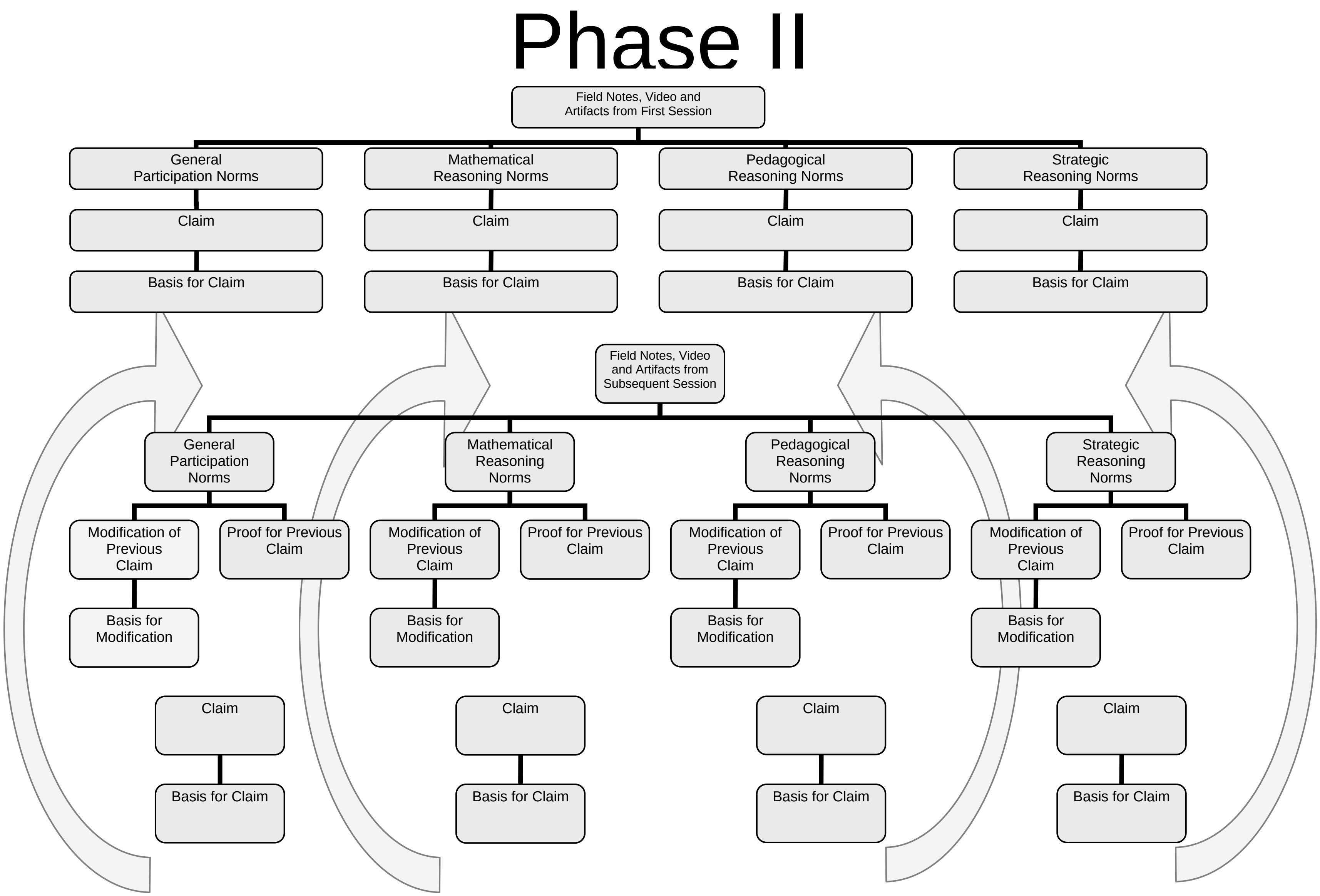


Figure 2. Schematic for Phase II of Analysis

explaining the initial formation and subsequent development of the professional teaching community. This was even more evident when delineating the supports and organization of the norms of strategic reasoning.

The final product of the third phase was a succinct, empirically-grounded account of the learning of the professional teaching community that consisted of a network of mutually reinforcing assertions that spanned the entire data set. Scrutinizing the conjectures developed during the second phase about the possible emergence of normative practices from a relatively global perspective resulted in an account of the realized learning trajectory of the professional teaching community.

CHAPTER VII

REALIZED LEARNING TRAJECTORY

Initial Summer Session—July 11-12, 2000

The two-day summer session in July of 2000 was the first meeting with the teachers. Thus, this session was particularly important for documenting the initial emergence of normative practices of the group, which included norms of general participation, mathematical reasoning, and strategic reasoning. The activities used in this session did not focus on issues of pedagogy. As a consequence, the initial formation of norms of pedagogical reasoning will be discussed in the section on the September 2000 session (year one, session one).

The research team's agenda for the summer initial session focused on identifying the starting points of the learning trajectory for supporting the initial emergence and subsequent learning of a professional teaching community. (For a list of activities for all sessions see Appendix E.) As an introduction, the research team planned to initiate a discussion with the teachers about the institutional context of the school and school district. The intent of this discussion was to begin to document the teachers' perceptions of the affordances and constraints of the setting in which they worked. As an example, the team was interested in how the reform curriculum was adopted and the role the teachers played in that process. The team was also interested in the teachers' perceptions of school organization, testing, and school meetings.

Next, the research team wanted to document the participating teachers' current understandings of central statistical ideas. This was to be accomplished by presenting the teachers with three mathematical assessment tasks specifically designed to determine the teachers' reasoning about statistical data analysis. The first task, How Much TV?, created a data analysis setting in which calculating say, the mean, did not address the question posed to the teachers because of the variability of the data set. The research team wanted to know if the teachers could find ways of accounting for the variability. The task was introduced by telling the teachers that *USA Today* conducted a survey of thirty seventh-grade students, asking them how much TV they watch in a week. The teachers were then handed a sheet of paper with the following values on it:

1.5	21	12.5	0	2.5	15
4	14	8	16	13.5	16.5
9	18	5	10.5	8.5	6
11.5	10	19.5	13	3.5	9
23	6	19	4.5	3	9

The teachers were instructed to come up with a quick and easy way to represent this data so that parents would have a sense of how much TV seventh-graders watch on average in a week.

The Fuel Mileage activity was the second in the series of mathematical tasks that were used. It was designed to assess how teachers would compare two data sets with unequal numbers of data values. One data set contained 12 values and the other 15. The research team wanted to know if the teachers would take account of this difference in their analysis. The task was introduced to the teachers using the following scenario: *Consumer Reports* had data on fifteen Ford Explorers and twelve Nissan Pathfinders that were tested to determine which cars get the better gas mileage. Based on the data presented with gas mileage as the sole criteria for making a purchase, which car would you recommend buying and why?

The third task in the series was designed to assess the teachers understanding of the mean in relation to the total. The researcher leading the session presented the task by discussing a survey conducted by *USA Today*. According to the survey the average cost of an 8-ounce bag of potato chips was $1.39. They apparently obtained this figure by averaging the prices of seven 8-ounce bags of different brands. The teachers were asked to determine what the prices of the seven individual bags might have been.

Finally, the research team planned to introduce the first two computer-based tools developed for the statistical instructional sequence. The first computer applet was designed to facilitate students' initial explorations of univariate data sets and provide means of ordering data values, partitioning, and otherwise organizing small sets of data in a relatively immediate way. Each individual data point is inscribed as a horizontal bar, the length of which signifies the numeral value of the data point. The color of each bar could be either pink or green, thus enabling two data sets to be entered and compared. In the case of the Batteries activity, which was the first computer applet scenario used in the initial session, data was generated to compare how long two different brands of batteries last (see figure 3).

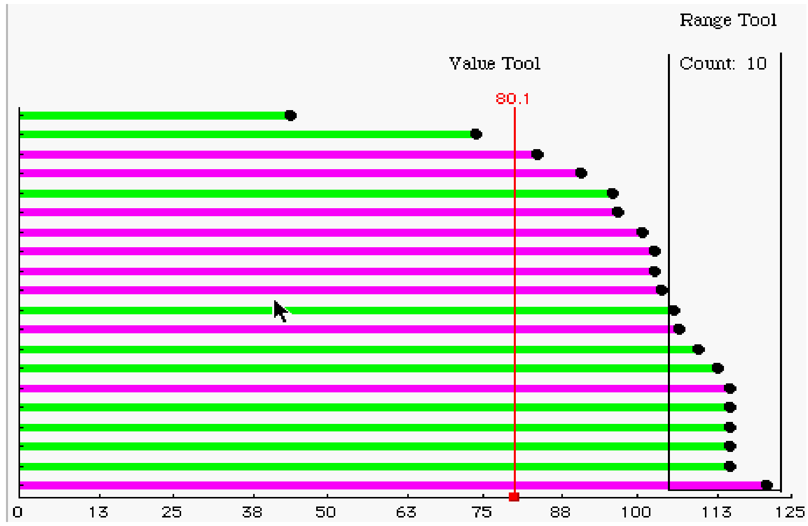


Figure 3. First Computer Applet—Batteries Dataset.

Each bar in Figure 3 shows a single data value. The teachers could sort the data by size and by color. In addition, they could hide either data set, and could also use the value bar to partition the data sets and to find the value of any data point by dragging a vertical red bar along the horizontal axis. Further, they could find the number of data points in any interval by using the range feature (Cobb, 1999; McClain & Cobb, 2001). The first scenario presented involved comparing the life span of two brands of batteries, Always Ready and Tough Cell. Ten batteries from each brand were used in the same type of appliance (e.g. CD player) and their life spans were measured in hours. The ten pink bars in Figure 3 represent the life span of batteries from the Tough Cell brand and the ten green from the Always Ready brand. The teachers were asked to decide, based on these data, which was the better brand of battery. The intent of the task was to support teachers' ability to view the data as measures of a relevant attribute of batteries rather then merely as numbers. Situations such as the life spans of batteries that involved linearity were chosen so that they would support the teachers' interpretation of the length of the bars as representing data values. A second goal was to support teachers' ability to organize the data in a manner that would take account of the variability in the data sets. The question posed necessitated the teachers taking into consideration the variability in one set versus the consistency in the other. Since the means were relatively similar (e.g. 100.29 for Always Ready and 102.55 for Tough Cell) the teachers would need to find ways to talk about how the data were actually distributed.

The Braking Distance task was presented following the Batteries activity. The Braking Distance data set (see Figure 4) is taken from braking tests conducted on two different types of automobiles, Oldsmobile Achieva and Toyota Corolla. Ten of each car were tested. The cars were driven on a test track at 30 mph. When the brakes were applied, the distance it took to come to a complete stop was measured. Each bar represents the distance in feet it took for one of the cars to stop. The ten pink bars represent the braking distances of the ten Oldsmobile Achieva cars and the green the ten Toyota Corolla cars. The teachers were asked to decide which would be the best type of car to buy based on the data, if it is a priority to have a car that stops fast. The intent of this task was similar to that of the Batteries task. Again, the linearity of the bars corresponded to the distance it took the bars to stop. Also, the data sets were designed to create a situation where the data set with the lower mean had the greater variability. In this instance, in would be important to make a decision based on what was important about braking.

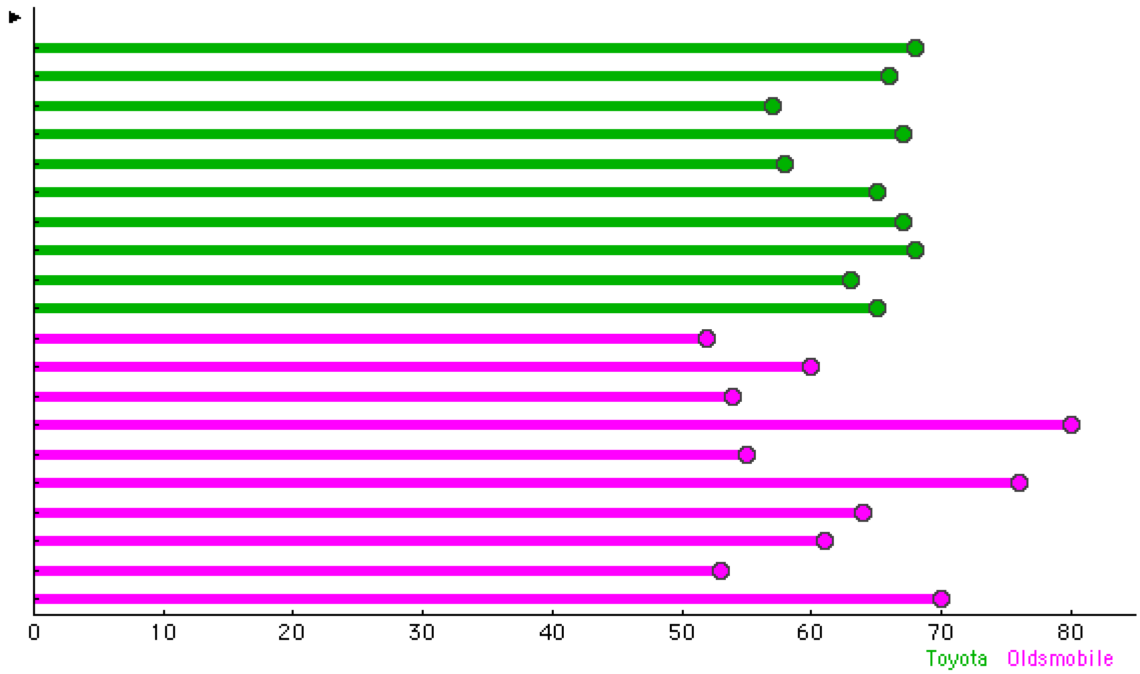


Figure 4. Braking Distance—Applet One.

The second applet can be viewed as an immediate successor of the first in that the endpoints of the bars that each signified a single data point that has, in effect, been collapsed down onto the axis so that a data set was now inscribed as an axis plot as pictured in Figure 5. The instructional intent when designing the second applet was to support the emergence of more sophisticated ways of comparing and analyzing datasets with larger numbers of data values. The tool offered a range of ways to structure data. Three available options did not correspond to graphs typically taught in school. These involved structuring the data by (1) making your own groups, (2) partitioning the data into groups of a fixed size, and (3) partitioning the data into two equal groups. The first and least sophisticated of these options simply involved dragging one or more bars to chosen locations on the axis in order to partition the data set into groups of points. The number of points in each partition was shown on the screen and adjusted automatically as the bars were dragged along the axis. The remaining two options are precursors to standard ways of structuring and inscribing data and corresponded to graphs typically taught in school. These involved organizing the data into four equal groups so that each group contained one-fourth of the data (precursor to the box-and-whiskers plot) and organizing data into groups of a fixed interval width along the axis (precursor to the histogram) (Cobb, 1999; McClain & Cobb, 2001).

The first task on the second applet involved analyzing speeds of cars before and after a speed trap was put in place (see Figure 5). The scenario used by the researcher was: There is a road in Nashville, called Briley Parkway that is notorious for speeding. The police recorded the speed of the first 60 drivers to pass their check point beginning at 3:00 on a Friday afternoon. The police then set up a speed trap at that location for two weeks. At the end of the two weeks, they collected the speeds on the first 60 drivers to pass the check point on a second Friday afternoon at 3:00. The teachers were asked to find way to structure or organize the data to make an argument to Chief of Police about whether or not the speed trap was effective in slowing traffic. The teachers asked what the speed limit was and were told it was 50mph.

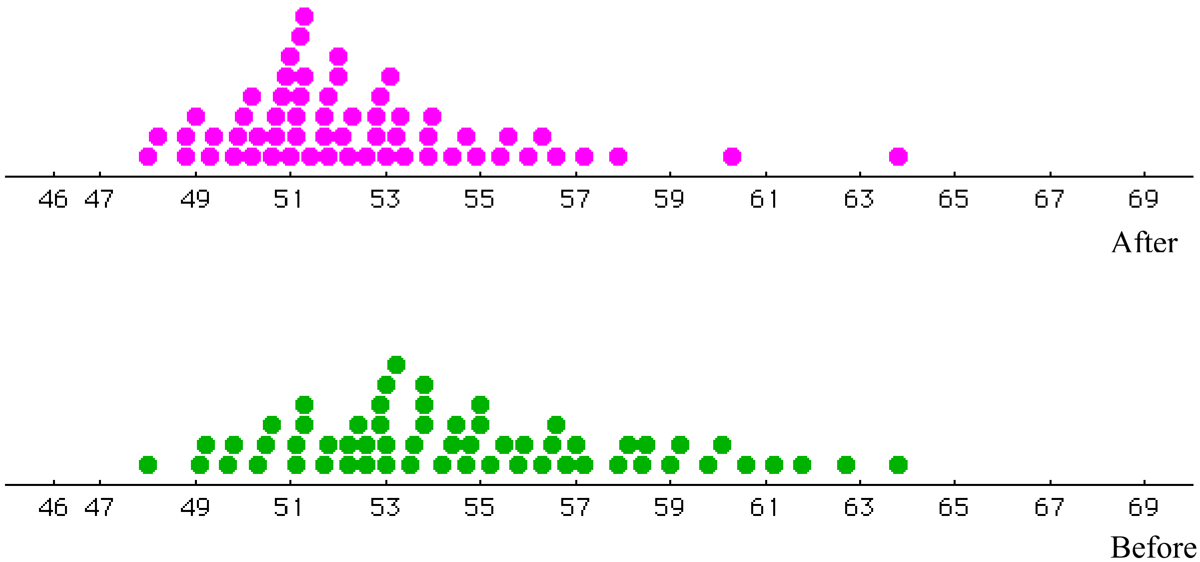


Figure 5. Speed Trap—Applet Two.

Each dot in Figure 5 represents the speed of a car in miles per hour. The 60 green dots represent the speeds of 60 cars measured before the speed trap was implemented, and the 60 pink dots represent the speeds of 60 cars in the same highway after the speed trap had been implemented. The intent of this task was to create a context in which teachers could focus on the data as aggregate by noticing patterns in the data. As an example, the cluster of data in the "before" data set has a wider range than that of the "after" data. Patterns such as these can be used to form initial conjectures which can be corroborated by organizing the data using the options available on the applet.

General Participation Norms.

The reader will recall that the general participation norms document the evolving participation structure within the group. Analysis of both the discussion of the institutional context and the discussions of the statistics activities revealed that the nature of the participation structure could be characterized as “turn taking.” Generally, one teacher would speak and when he/she was finished, another would take a turn to speak without necessarily referring to or building on what others said. The two-day initial summer session contained four hours and eight minutes of discussion in a group setting with approximately one thousand six hundred and seven turns at talk. During this time there were only twenty instances where a teacher either interrupted another teacher or one or more teachers talked simultaneously. Additionally, there were only five instances where a teacher acknowledged what someone else had contributed or ask clarification. Furthermore, only thirty teacher contributions were directed to other teachers; all other teacher offerings during the four hours and eight minutes were directed to the researcher.

Although the teachers would ask clarifying questions of others during statistics activities, it appeared to be unacceptable to challenge someone’s solution. During the initial session, only three attempts were made to challenge another teacher’s contribution. Each of these attempts at a challenge was halted by another teacher, in most cases by using humor. An example of this can be seen when Amy attempted to challenge Jeremy’s solution to the Fuel Mileage activity. After Jeremy had explained his reasoning, the researcher leading the session summarized Jeremy’s argument and asked if there were any questions.

Amy:	[addressing the researcher] Does it make a difference that you have 15 samples in one range and only 12 samples in the other one. Is it possible to put 15 Nissan samples and those three can change…
Jeremy:	Can I answer the question?
Researcher:	I would love for you to answer it.
Jeremy:	Based on what we just had, period, just looking at the numbers, looking at median and the mean and the smaller range, you could almost predict that Nissan did not have a large difference and Ford

did. If you look at Ford, the lowest you are going to get is 17. What's to say that someone isn't going to have a 15 miles per gallon Ford or a 32 miles per gallon Ford? So based on the consistency, as small as the difference is, you can almost pin point that Nissan would have a more…

Amy: But that argument is not going to hold water with me. Anyway, because you, [turns to face Jeremy] if you increase the difference of the sample, what I am hearing you say is it doesn't matter how many samples you use.

Jeremy: Right.

Dot: It shouldn't.

Marci: I am looking at the most…

Amy: But it does. If you are only going to use two and I am going to use fifteen, I am going to use…

Jeremy: That's a big difference. Three vehicles is really, really, really insignificant.

Amy: Take three vehicles that are all lower than the lowest Nissan you got.

[Percy raises his hand.]

Dot: Then take out a low and take out a high.

Amy: Or take three out of the low.

Muriel: It's still about eight and six.

Jeremy: Right, you're still getting a more consistent vehicle with a Nissan.

Percy: [in joking tone] I think Wesley [Percy's partner] has the better argument, so you guys need to just let him say what it is and then we can all stop arguing.

[Laughter from teachers and researcher.]

There are three important things to notice in this episode. One, Jeremy asked permission to respond to Amy's question. He was unsure if it was acceptable to talk directly to another teacher. Second, Amy actually turned and talked directly to Jeremy when she was responding to his justification. Third, their exchange, punctuated by others' comments, was a violation of the current participation structure of the group from Percy's perspective as he deemed it necessary to make a joke to end the discussion when a conflict in viewpoints became explicit. Furthermore, Percy's intervention was viewed as legitimate within the group which reinforces the claim that a challenge was a breach of the current participation structure.

At this point the group can be defined as a *pseudocommunity*:

> As community starts to form, individuals have a natural tendency to *play community*—to act as if they are already a community that shares values and common beliefs. Playing community, or pseudocommunity, draws on cultural notions of interaction often found in middle-class, typically White, settings. The imperative of pseudocommunity is to "behave as if we all agree." An interactional congeniality is maintained by a surface friendliness, hypervigilant never to intrude on issues of personal space. The maintenance of a pseudocommunity pivots on suppression of conflict. Groups regulate face-to-face interactions with the tacit understanding that it is "against the rules" to challenge others or press too hard for clarification. (Grossman, et al., 2001, p. 955)

Mathematical Reasoning Norms.

The analysis of the mathematical reasoning and argumentation that emerged as the teachers explored the first two assessment tasks, How Much TV? and Fuel Mileage, revealed that the teachers focused on 1) creating graphical representations with correct conventions and 2) producing calculations of mean, median, and mode. The importance teachers placed on rules for classifying or creating specific graphical representations was apparent in the issues discussed during the How Much TV? task. The discussions following *each* of the four representations presented did not focus on whether a certain type of graph accurately portrayed the data, but rather on whether the correct conventions for creating the graphical representation had been used. For example, the first representation presented was a histogram. The teachers' discussion of the histogram focused on the distinctions between a bar graph and a histogram. One teacher offered that the difference was whether or not the bars were adjacent or touching. Another said it was a bar graph if categorical data was used such as types of fruit. They were not aware of the difference in how the data are positioned on the horizontal axis in discriminating between the two (a bar graph organizes discrete data with no obligatory order and a histogram displays continuous data with an implied order). It is important to note that the teachers' focus on graphical conventions is understandable given that there are several objectives on their state Prescribed Instructional Program that require knowledge of how to create different types of graphs. From the teachers' perspective, the students must be taught the conventions for creating graphical representations in order to perform well on the standardized test.

The teachers initially approached the Fuel Mileage activity by calculating measures of center. In fact, since *every* group calculated the mean, median, and mode, the researcher leading the session first asked the teachers what they had calculated for these three measures of center so that everyone would be working with the same numerical values. The researcher then attempted to support the teachers in creating an argument that gave insight into the phenomenon under discussion (e.g. choosing a vehicle to buy based on fuel mileage) by removing the option of simply calculating measures of center. This appeared to be successful as only one group subsequently used these calculations as a way to support their arguments when presenting their solutions to the group.

Based on the teachers' work with the three assessment tasks (How Much TV?, Fuel Mileage, and Potato Chip), the research team conjectured that the teachers had limited experience with analyzing data, especially in dealing with variability and distribution. Since a goal in working with the teachers was to support the emergence of the pedagogical norm of making instructional decisions based on student reasoning, it was crucial that the teachers develop a relatively sophisticated understanding of the significant mathematical ideas their students were to learn (Bransford, Brown, & Cocking, 2000; Ma, 1999; Shulman, 1986; Thompson & Thompson, 1996). The immediate goal then became supporting the teachers in making data based arguments. In other words, the research team wanted the teachers to move beyond simply calculating measures of center or concentrating on graphical conventions. Instead, it wanted to support the teachers' engagement in exploratory data analysis, where they produced data-based arguments to justify decisions and judgments.

During the exploration of the Batteries data, all teachers partitioned the data to make their arguments. Teachers routinely gave justifications for the location of their cut points. On two occasions the legitimacy of choosing a specific cut point was questioned by another teacher. For example, one teacher, Muriel, combined and ordered both data sets and divided them in half, calling the cut point the "median of all" (see Figure 6).

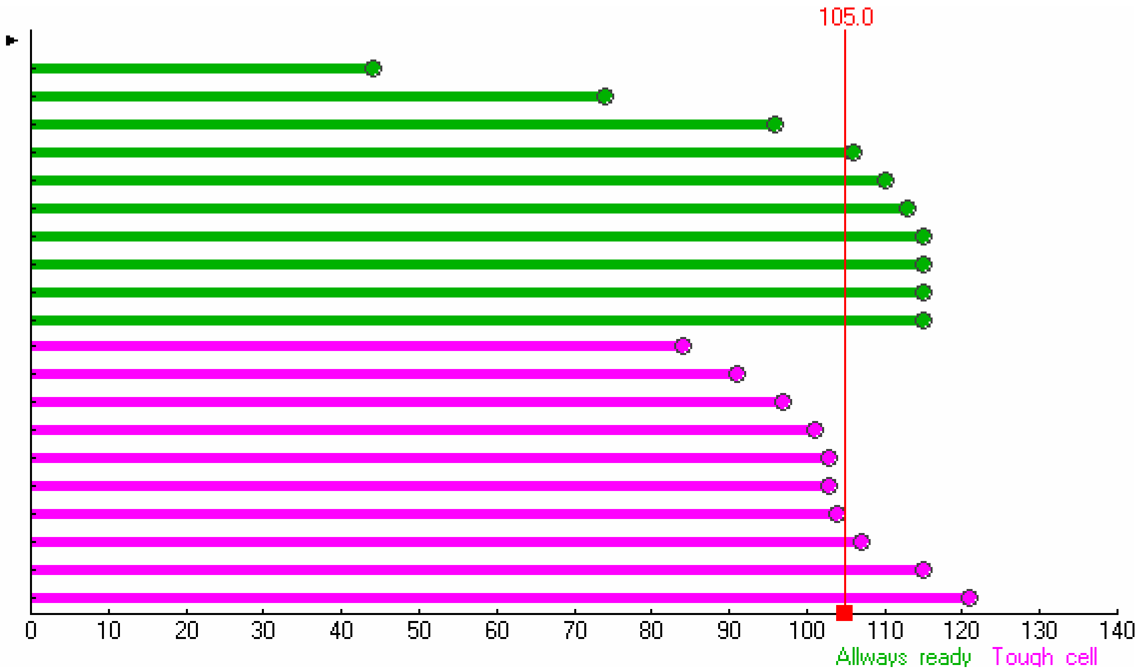


Figure 6. Muriel's Solution with Batteries Activity.

Muriel:	I sorted them by size and color and found the median of all, the median of each didn't help me. The median of all was 105, so I put the value bar at 105. And there are 7 out of 10 green ones that are above 105. And only 3 out of 10 pink ones above 105.
Researcher:	Does anyone have a question for Muriel?
Wesley:	"Yeah, is it, I liked very much what you did [talking to Muriel], but is it a significant thing to find the median of the total group?"
Muriel:	[playing like she is offended] "You're questioning me?"
[Group laughs.]	
Wesley:	"No, I think it's a great idea, I just…"

Note that the legitimacy of Muriel using this cut point as the basis of partitioning was challenged by Wesley. In making this challenge, Wesley contributed to the emergence of providing a justification for the location of a cut point as normative. However, it is important to note that Wesley prefaced his question with a positive statement, "I liked very much what you did." He seemed to be buffering what he thought might be perceived as a violation of general participation norms. Muriel's reaction, even though it was humorous, confirmed that he had indeed breached a norm of participation in her view. Thus, the claim that it was emerging as normative for the use of a cut point to require justification must be regarded as tentative. More generally, as this episode demonstrates, any claim about emerging norms based on one teacher challenging another's assertion must necessarily be tentative until challenging itself becomes a general participation norm of the group.

Throughout the activities using both computer-based tools (e.g. Batteries, Braking Distance, and Speed Trap), it became increasingly routine for the teachers to reason about the data in terms of percentages and part-whole relationships. In the Batteries activity, four solutions were not percentages or part-whole. Instead, teachers used a total or consistency argument. In the Braking Distance activity, only two arguments were not in terms of percentages and part-whole relationships. One argument focused on consistency and the other on how the data were "clustered." In the Speed Trap activities, all the teachers organized the data in terms of percentages and part-whole relationships, except one who "looked at the clumps."

Strategic Reasoning Norms.

In order to gain insight into the institutional context of the school and school district, the researcher leading the session began by asking the teachers to introduce themselves, tell what topics they taught, and describe their school and district. Topics discussed by the group included school organization, grade-level and department meetings, study groups, curriculum decisions, testing, tracking, and what statistics "looked like" in their classrooms.

Analysis of the institutional context discussion raised three issues: 1) the managerial nature of formal teacher meetings, 2) limited informal professional contacts, and 3) pressures associated with the state-mandated end-of-year standardized test. At least one teacher from each of the five schools represented described faculty, department, and team meetings as focusing on managerial rather than instructional issues. Textbook adoption, which entailed teachers voting on curricula from a list pre-selected by the district mathematics coordinator, was the only issue associated with curriculum discussed during these meetings. The only human resources that the teachers mentioned when discussing the forms of assistance on which they could draw when addressing instructional issues in mathematics were two district mathematics specialists. There was no mention of talking to colleagues about classroom instruction. Every teacher in the group discussed the pressures of the state-mandated standardized test and all stated that at least two weeks before the standardized test, they started reviewing and teaching for the test. One teacher stated that "there is more pressure to do well on the tests than to teach well" as monetary bonuses for teachers were based on results of the standardized tests. Every comment from each teacher was premised on the unarticulated assumption that aspects of the district were a source of frustration that they had no influence over. In other words, what became constituted within the group was that the institutional context was an obstacle that was beyond the teachers' control.

Based on this initial discussion, the research team conjectured that the institutional context significantly influenced mathematics teaching and learning in the district at least at the middle school level. As a result, it became important to develop a way to further document this context. Follow-up interviews were therefore conducted with each teacher in the group. The interviews, which focused on the institutional context, generated useful data for gaining a better understanding of the schools, the school district, and the teachers' understanding of the contexts in which they worked.

It is important to note the interrelation between the norms for mathematical reasoning and the teachers' perception of the institutional context. The mathematical reasoning that initially emerged within the group when working on the assessment tasks was reasonable given their understanding of the contexts in which they worked at the time. It was understandable that the teachers focused on procedures for creating graphical representations and calculating measures of center when one takes into consideration the pressure the teachers felt to make sure their students performed well on the standardized test. Thus, when working on the assessment tasks, the teachers' solutions reflected what they were held accountable for in terms of student learning.

Year One, Session One—September 21, 2000

During the two months following the initial summer session and preceding the first session of year one, the research team revisited the initial conjectures for supporting the learning of the teachers. One important goal was to support them in using records of student work as a resource for understanding student reasoning. At the end of the initial summer session, the research team had asked the teachers to do the How Much TV? assessment task with their students and to bring their work to the next session. The rationale was that, since this was an assessment task and not part of the statistics instructional sequence, the teachers would not need to understand the instructional intent of the sequence in order to use student work as a tool for focusing on student reasoning. However, the way in which the teachers conducted this activity with their students became an important avenue for documenting the initial formation of pedagogical reasoning norms and how those norms were influenced by the institutional context.

The agenda of the research team was to review the summer session, give an overview of the instructional intent of the statistics sequence in order to situate the computer applet tasks, have teachers share their student work from the How Much TV? task, and continue to support the teachers' statistical learning by completing an activity from the second applet, Cholesterol. The intent was to try to build from what had been accomplished during the summer session by revisiting the instructional sequence. The team judged that this could be accomplished by first talking through the statistical instructional sequence and then having the teachers share their students' work. Against this background, the research team would engage the teachers in the next series of task from the sequence. This iterative process of first engaging in the task as learners and then stepping back and thinking about the mathematical intent from the perspective of a teacher was intended to ground the teachers' subsequent activity with their students.

The Cholesterol (see Figure 7) activity involved investigating the effectiveness of a special diet program designed to reduce cholesterol levels. Cholesterol levels were measured on each of 60 patients both before and after the completion of the three-month study. Each dot represents the cholesterol level in the blood of a patient. The green dots represent cholesterol levels of the patients before enrolling in the new diet program. The pink dots represent the cholesterol levels of the same group of patients after being in the diet program for three months. The teachers were asked to help the medical personal decide whether the special diet program lowered the cholesterol levels of patients. The research team anticipated that the teachers might be able to find qualitative ways to describe the overall shift in the patients' cholesterol levels. This might be accomplished by focusing on the range of the cluster in each axis plot. The teachers could then use the computer applet to find ways to organize the data to support their initial claims.

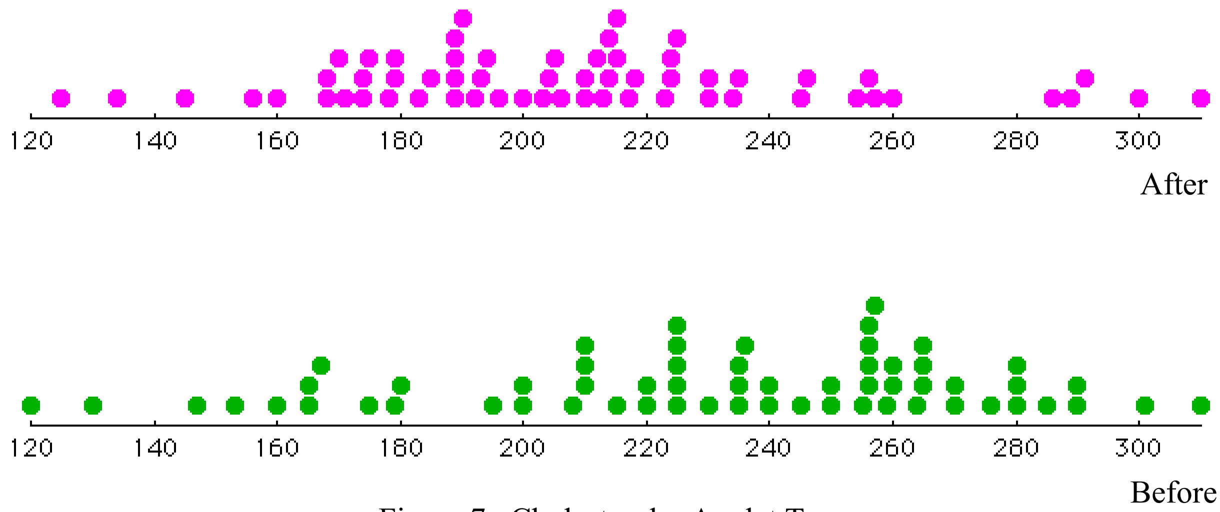


Figure 7. Cholesterol—Applet Two.

General Participation Norms.

The ongoing re-negotiation of the participation structure of the group was influenced by the type of activity in which the teachers were engaged. In each of the four different types of activities (review of summer session, discussion of instructional intent, discussion of student work, and analysis of the Cholesterol task) the teachers' participation was remarkably different.

Throughout the review of the summer session and the overview of the instructional intent of the sequence, which lasted 53 minutes, the teachers were unresponsive. This portion of the session resembled a lecture during which the researcher did the majority of the talking. Most of the teachers were attentive to what the researcher had to say, but there was very little interaction between the researcher and the teachers and no interaction between the teachers. There were only six instances in which the teachers asked questions or made comments during the entire 53 minute episode.

When the teachers shared their student work from the How Much TV? task, they returned to the normative practice of turn taking that emerged during the initial summer session. This part of the day-long session lasted one hour and 21 minutes with 312 turns at talk. There were only 16 occasions where a teacher either interrupted another teacher or one or more teachers talked simultaneously. Teachers would ask for clarification if they did not understand what a student had written (14 turns at talk), but there were no instances in which they offered alternative interpretations of students work. As before, the teachers would build on issues raised by others by drawing comparisons with what their students had done. For the most part, however, teacher contributions were directed to the researcher with only 27 of the 312 contributions directed to other teachers.

The last activity of the session involved the teachers analyzing the Cholesterol data. Because of time constraints, only two teachers had the chance to share their solutions. The norms of participation differed from those established during the prior discussion of students' work on the How Much TV? task. During the 13 minute session, 11 of the 33 teacher contributions were directed to other teachers and not the researcher. Also, although there were only two clarifying questions asked, both were asked directly to another teacher. This short data analysis discussion revealed efforts at the renegotiation of general participation norms in comparison to the initial summer session. When teachers were engaged in data analysis during the initial summer session, only 25 of the 1355 teacher contributions were directed to other teachers and not the researcher. In addition, there were five clarifying questions asked and these were all directed to the researcher.

Pedagogical Reasoning Norms.

Pedagogical reasoning norms were being negotiated and established during the discussion of the student work on the How Much TV? task. The researchers asked the five teachers that brought student work to the session to share student solutions, successes or problems that arose, and any issues that came up for the teachers and the students while participating in the How Much TV? task in their classrooms.

In order to understand the pedagogical reasoning norms that emerged, aspects of the teachers' instructional practices must first be described. The reader will recall that when modified teaching sets were generated to document the teachers' instructional practices, there were identifiable regularities across the teachers even though they worked in isolation. I use the term *regularity of instructional practice* to refer to these common aspects of their instructional practices such as a focus on student acquisition and application of procedures for operating on mathematical symbols and on students memorizing definitions for mathematics terms.

A derivative of existing regularities of instructional practice that became relevant during the discussion of student work was the teachers' emphasis on students' use of correct conventions and procedures students used. This was evidenced by *each* of the five teachers giving a report that described students' solution methods rather than students' reasoning. For example, Lisa discussed her student work in the following manner:

> Lisa: They took the first column and averaged those six and graphed it. Took the second column, averaged those six and graphed it. I have no idea why they did that. But that is what this group did. That one was very different…This was the, they just plotted each, which is the same as here. But on this one the group put at the top, they averaged. They did average the hours but then they plotted every single one of them. So this statement is totally different, they just decided to put that information in there. These put them in order. They said to show there was a wide range. The word range came up."

Another derivative of existing regularities of instructional practice that became apparent was that the teachers reviewed what they took to be relevant statistical topics before assigning the How Much TV? task. Of the five teachers that did the task with their students, four reported that they had presented lessons or had explicit conversations about measures of center or about the conventions of certain types of graphs with their students the day of or the day before the students analyzed the How Much TV? data.

These derivatives of regularities of instructional practice can be explained by taking the institutional setting of the school district into account. Interviews that documented the teachers' primary means of making student reasoning visible showed that the teachers focused almost exclusively on the correctness of student solutions to classwork tasks, homework, quizzes, and teacher or textbook produced tasks.

When considering the lack of informal professional networks and the privatized nature of mathematics instruction, the concept of reviewing prior to the task was rational. Sharing student work was a high-risk activity for these teachers because they perceived it as an assessment of their instruction. Since the teachers' classrooms were visited only in the context of evaluation, they judged that sharing their students' work was an evaluation of their instructional practices. Thus, reviewing a concept so that the students produced specific types of answers made sense.

During the discussion of student work, the teachers began to conjecture that students use of certain types of representations was based on what had been taught immediately prior to doing the How Much TV? activity. The reader will recall that every teacher who presented, except one, reported reviewing. The issue of reviewing became an explicit topic of conversation in which each of the teachers who presented student work conjectured that their student solutions were directly influenced by the topics addressed prior to engaging in the task. For example, Dot introduced her student work by saying, "We had just done mean, median, and mode, so the majority of mine came out with those things." Amy then used Dot's claim as a justification for the type of work her students created.

Amy: I think the way I presented it made the difference. I wrote mean, median, and mode and range stuff on the board for the first class and they all did that.

The issue came up again after Jeremy had talked about his students' work and Dot asked if he reviewed the concept of intervals.

Jeremy: Yes, briefly. And that stuck.

Dot: It's similar to Batteries; the different way you present the scenario can affect what students do.

Although the teachers focused on students' solution methods rather than on their underlying reasoning, they did begin to make claims about the relation between their teaching and student work.

Year One, Session Two—November 11, 2000

As stated previously, in order to support the goal of the teachers making pedagogical decision based on students' reasoning, the research team needed to support their development of relatively sophisticated ways of reasoning about statistical data analysis. In the previous session, the teachers reasoned about the data in terms of percentages and part-whole relationships. Although the teachers' thinking could be classified as multiplicative, the research team conjectured that the concept of the shape of the data needed to become an explicit topic of conversation. This was critical given the goal of reasoning about data in terms of distributions. Therefore, the research team dedicated the session to working on computer applet two activities: Cholesterol (see Figure 7), Airbags, and Aids. The Cholesterol data set had been introduced at the end of the previous session (September, 2000). However, because time had limited the discussion of the teachers' analyses, the research team decided to revisit it at the beginning of this session.

The second task involved investigating the effectiveness of automobile airbags in reducing the severity of head injuries. The crash test data consisted of measures of the severity of head injuries in 64 vehicles with airbags and 36 vehicles without airbags. The data sets are shown in Figure 8.

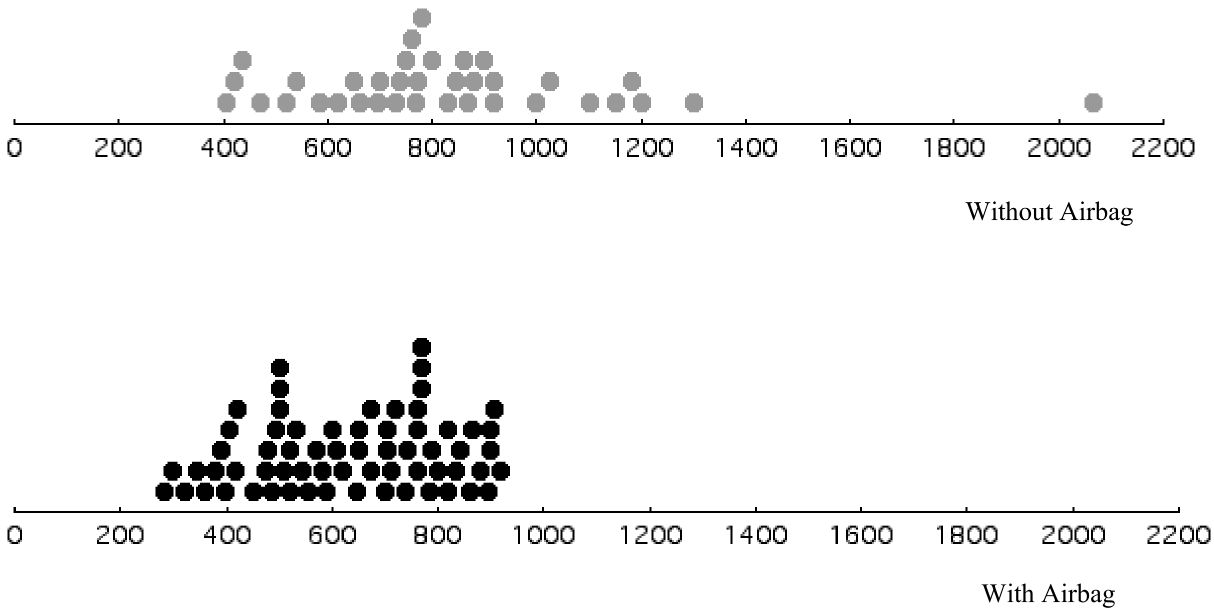


Figure 8. Airbags—Applet Two.

The intent of the task was to present an analysis situation in which the data sets contained unequal numbers of data points and one set contained an outlier. Given the unequal number of data points, the teachers would need to find reasonable ways to compare the data sets. In particular, the research team anticipated that the teachers would use multiplicative forms of reasoning such as proportions, relative frequencies, or percentages. The four-equal-groups feature on the computer applet would also be a useful option on this task. As the goal of the sequence was to promote a focus on distributions, the team also wanted to investigate how the teachers might reason in the context of outliers. For instance, would that become the significant data point, or would the teachers find a way to reason about the distributions while accounting for the outlier.

In the third task, data was presented on two groups of patients with AIDS as shown in Figure 9. Each dot represents the T-cell count of a patient. The pink dots represented the T-cell counts of AIDS patients treated with a traditional drug protocol and the green dots represent the T-cell counts of AIDS patients treated with an experimental drug protocol. The T-cell counts were collected from 186 patients using the traditional drug and 46 using the experimental drug. The teachers were asked to write a report in which they judge the relative success of each treatment program in raising patients' T-cell counts.

This was the second data analysis task on the computer involving data sets with unequal numbers of data points. The task was designed with the difference being relatively large so that the approach of eliminating data values from the larger data set would not be reasonable. (The reader will recall that the strategy of eliminating data values was discussed during the orientation session on the Fuel Mileage task). Again, the teachers would have to find reasonable ways to compare the data sets that took account of this difference.

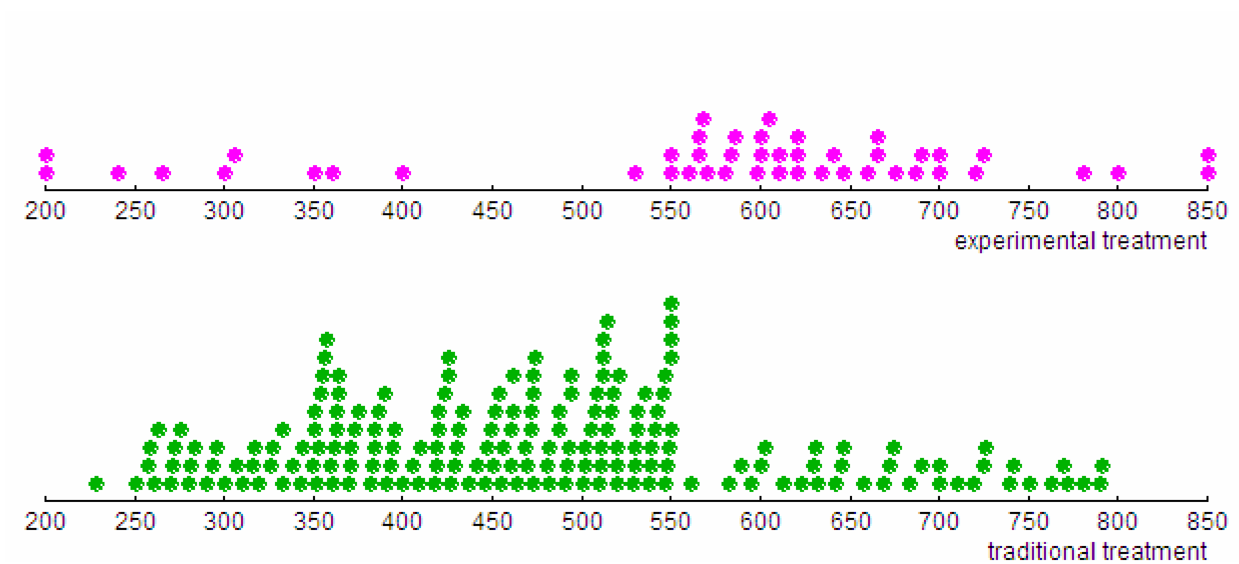


Figure 9. AIDS—Applet Two.

General Participation Norms.

During the discussion of the teachers' solutions to the three tasks in which they used the computer applet, the teachers continued to ask clarifying questions of others about their solutions. More importantly, they were directing these questions and comments to each other. This practice of addressing each other directly when discussing their results of analysis had begun to emerge at the end of the last session (September 2000) and was not perceived as a violation of norms by any member of the group. Furthermore, during the discussion of teachers' solutions to the Cholesterol activity, two teachers challenged each other's assertions. However, it remained a violation of norms to engage in confrontational talk within the group as evidenced by a joke and uncomfortable giggles that interrupted the two teachers who were disagreeing.

Muriel: If 77% are there then they are closer together than the ones that are 57%.

Researcher: What is the significance of that?

Amy: Space, if you are talking about area, what is the significance of that? You can stack one on top of one another, but it is still the same number.

Jeremy: Yeah, but that would still say at 77% more people fall in that range, more packed in, where as at 57% are fewer. They are more spread out.

Amy: [in defensive tone] But if you create a division 200 to 255 there are 24 people in both, the distribution or spreading out is not happening until after 255.

[Nervous giggling by several teachers.]

Amy: Are we having a little problem?

Wesley: I am having flashbacks to the Partridge Family and it is really starting to freak me out.

[Teachers laugh.]

Math Reasoning Norms.

When the teachers began working on the Cholesterol activity, they attempted to organize the data based on information that was specific to cholesterol levels. This was reflected in two ways: 1) by questions the teachers asked:

> Jeremy: What is the magic number to determine good cholesterol, again?
>
> Marci: What is good cholesterol?

And 2) by the unsolicited justifications teachers gave for their cut points in the data:

> Lisa: I used the 210 mark also. Before 75% were above and 25% were below and after the program, 48% were above and 52% below. That's using 210. I was using acceptable cholesterol levels.

This focus on a pre-determined cut point continued with the Airbags activity. As a result, at the end of the whole-group discussion about the Airbags data set, one of the researchers suggested that the teachers "eyeball the data" and make some initial conjectures before working with the applets on the next activity, AIDS. This suggestion appeared to influence how the teachers approached the AIDS task as *each* teacher based cut points on an initial inspection of the data. An example of this appeared in the discussion that followed Wesley's presentation of his argument (see Figure 10 for Wesley's representation).

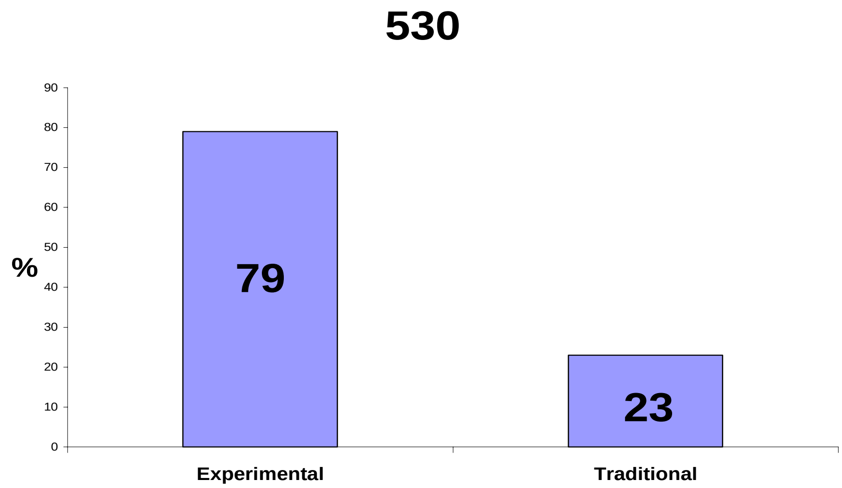


Figure 10. Wesley's Diagram.

Researcher (1): Why choose 530?

Wesley: There is such a big difference between 400 and 530. Such a blank spot, just visual.

Researcher (1): Oh, so because visually that is a good place to make a cut.

Researcher (2): And Lisa, kind of, do you want to explain?

Lisa: I did the same thing except I put it at 552, because that really shows the line.

Researcher (2): When you originally picked that you said it was…

Lisa: By majority, it was just totally visual. There's a break there.

The researchers' intent for suggesting that the teachers "eyeball the data" was to support solution methods that were based on the distribution of the data as opposed to information about the scenario (e.g. 200 to 210 being an acceptable cholesterol level).

Even thought the teachers created cut points based on an initial visual inspection of the data, they continued to develop arguments that focused on the percentage of the data in particular intervals rather than the shape. However, when prompted by the researcher, the teachers discussed the data in terms of shape in all three applet two activities (Cholesterol, Airbags, and AIDS). For example, after Dot drew her inscription for her solution to the AIDS task on the white board (see Figure 11), one of the researchers asked about the distribution of the data.

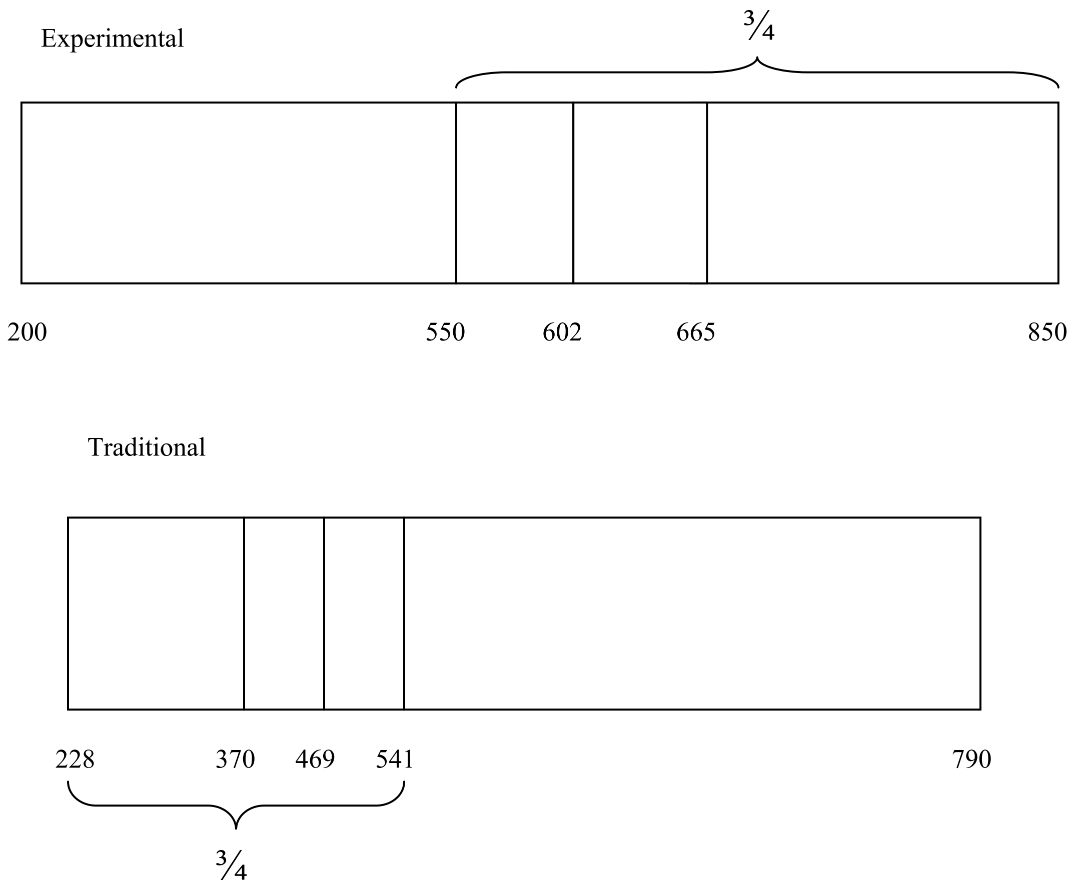


Figure 11. Dot's Diagram.

Researcher (1): What as you look at that, what, sort of, can you infer about data from the four equal groups sketch?

Dot: That the majority shifted. I mean if you look at it as a whole and how much is moving, that the majority, and that's about 550, where ya'll did 530 which is about the same thing, that the majority is up there. It's just looking at it in four parts as opposed to two parts. [Referring to Wesley and Lisa's solution method of partitioning the data and referring to percentages (see transcript above)] Which if you look at four parts and say 3 out of 4 parts, that might be more convincing. Then if you look at your percentages, that is about 75%. So it is basically the same thing, just divided differently.

Researcher (1): How would you infer the data are distributed, what is the shape of the data.

Dot: Over. [Makes a sweeping move with hand in shape of a curve] Do you mean like a bell curve?

Researcher (1): Yes.

Dot: Like that [draws shape of distribution with her finger in the air. Starting low and going higher in the shape of a hill to the right side] and the other like that [starts with hill shape on the left and then lowers—opposite of previous one].

Researcher (1): Does that make sense to other people?

Researcher (2): Does that make sense? What Dot said is that it would kind of go like this and then there would be a big hill there, and here there would be a hill, and then it would go gradually go off. Can everyone see that from this inscription? Can you see how the shape of the data would be in there? Does that make sense to everyone?

[Teachers nod their heads.]

Jeremy: Yes.

Amy: You can conclude that even though outliers [sic] are about the same, the body of the data is more on one end or the other.

This exchange was an important advance because prior use of four equal groups in the both the Cholesterol and Airbags tasks had involved comparing data sets in terms of the percentage in particular intervals. This last exchange was more sophisticated and indicated that some of the teachers were able to read how the data were distributed in the four equal groups without the actual data points on the graph.

Although the teachers could readily use the four equal groups option of applet two and when prompted could infer shape, the idea of describing the data in terms of shape originated with the researchers. The research team therefore concluded that it would be important to revisit this idea with additional applet two activities in the next session. The teachers' ability to reason about the data in terms of shape would be critical in supporting their understanding of the pedagogical intent of the statistics sequence.

Year One, Session Three—February 8, 2001

There were three objectives for this work session: (1) discuss the student work from the Batteries task that researchers has asked the teachers to do with their students between sessions, (2) support the emergence of inferring shape from the data as normative, and (3) introduce applet three. The overall intent was therefore threefold. The first goal was to support the teachers' ability to use student work as a resource for understanding students' ways of reasoning. The second goal was to continue to support the teachers' ability to see data as a distribution by continuing to engage them in tasks using the second computer applet. The third goal was to introduce the last segment of the instructional sequence which involved using the third of the three computer applets to analyze bivariate data.

The applet two activity the researchers used for this session was the Migraine task. The Migraine task involved analyzing data on the amount of time it took patients to get relief from a migraine headache (see Figure 12). Each dot therefore represented a patient's relief time measured in minutes. Two hundred and twenty people were treated with a traditional drug which was represented by the green dots. Sixty-eight people were treated with an experimental drug which was represented by the pink dots. Teachers were asked to make a recommendation to the hospital director about which drug should be used to treat migraines. The intent of this task was to build on the prior discussion of the AIDS task by posing a third activity that involved data sets with unequal N's. The issue of how to compare data sets with unequal N's was important as was being able to infer shape from the resulting representations.

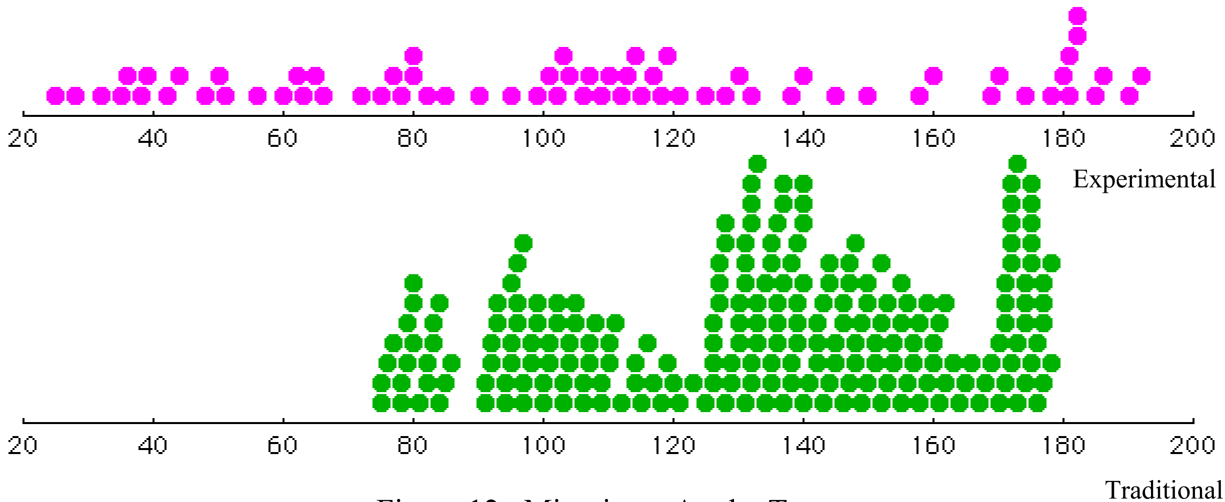


Figure 12. Migraine—Applet Two.

The first activity that involved analyzing bivariate data focused on the amount of carbon dioxide that the same care produced when it was driven at different speeds for one mile. Data were presented on 18 different speeds and the resulting carbon dioxide emissions measured in milligrams as shown in Figure 13. The teachers were asked to decide if the interstate highway speed limit was reasonable given these data on automobile pollution.

Speed of Car	Carbon Dioxide Emissions in Milligrams
15	430
20	450
24	425
32	350
38	320
42	340
52	290
82	320
55	262
60	260
75	270
88	310
99	325
103	380
93	350
26	400
9	375
66	245

Figure 13. Carbon Dioxide and Speed Data.

Teachers were asked to create a way of organizing and representing the data that would allow them to make a reasoned argument about the relation of the two measures. The intent was to see if the teachers could find ways to coordinate and investigate the relationship between the two sets of measures. The team anticipated that teachers might create double bar graphs and possibly scatter plots. The goal was for scatter plots to emerge as a tool for inscribing bivariate data. (Each teacher in fact created a scatter plot of the data.)

Following the Carbon Dioxide and Speed task, the third computer applet was introduced. With the third applet bivariate data are inscribed as a scatter plot such that each of the two measures of each case are represented on the horizontal and vertical axis as shown in Figure 14.

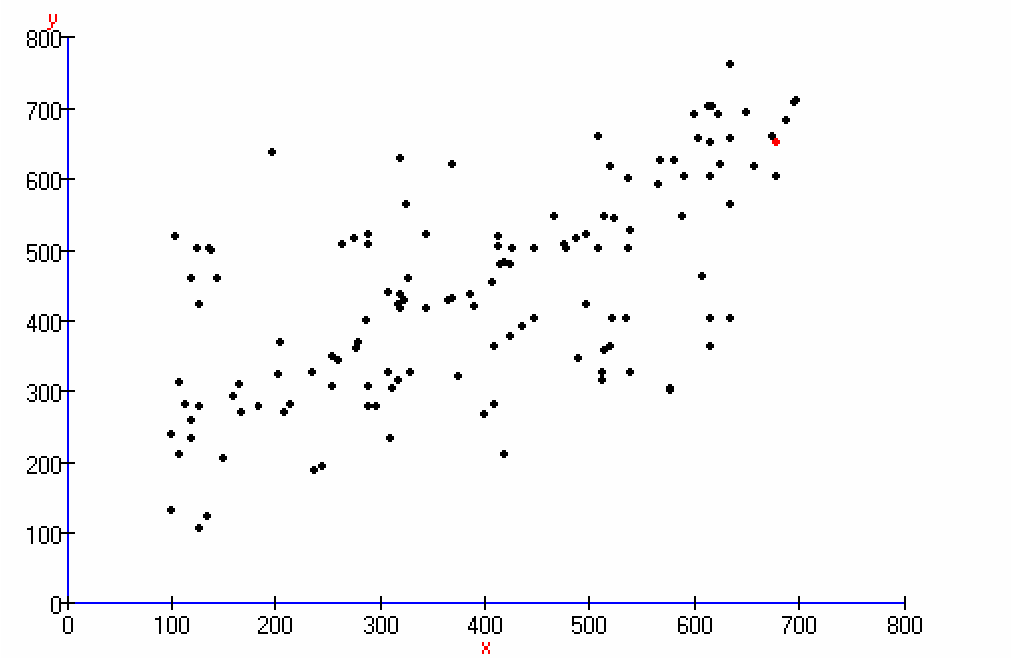


Figure 14. Computer Applet Three.

The features on the third computer applet included the ability to adjust the scales of the axes by changing the maximum and minimum values. Also included is a feature called Dots wherein if the teachers clicked on any data point, perpendiculars from the axes to the dot would be shown. This feature was used to aid the teacher in ensuring that discourse was about relationships between the two measures of each of a number of cases rather than about a mere configuration of dots scattered between two axes. Another feature to note was that the individual data points could be hidden. This option was designed to support conversations in which trends and patterns in the distribution of data are inferred from graphs.

Beyond these simple features, the applet offered four differing ways of organizing bivariate data: the cross, grids, two equal groups, and four equal groups. The cross option divides the data display into four cells and shows the number of data points in each cell. The teachers could drag the center of the cross to any location on the display, thereby changing the size of the cells. As they did so, the record of the number of data points in each cell adjusted automatically. The cross can be viewed as the two-dimensional correlate of the making your own groups option included in the second applet.

For the grids option, the teachers could select from a pull-down menu of grids that ranged in size from 4-by-4 to 10-by-10. The selected grid was shown superimposed on the data display and the number of data points in each cell was shown. The grids option can be viewed as the two-dimensional correlate of the fixed interval width option included in the second applet.

The two equal groups option partitions the data display into columns or vertical slices, the widths of which divide the horizontal axis into equal intervals. The minimum number of slices that the teachers could choose was four and the maximum was ten. Within each slice, the data points are partitioned into two equal groups (i.e., the display shows the median and the low and high values within each slice). This option can be viewed as the two-dimensional correlate of the two equal groups option included in the second applet.

The four equal groups option is similar to the two equal groups option except that the data points within each slice are partitioned into four equal groups. It can be viewed as the two-dimensional correlate of the four equal groups option included in the second applet (Cobb, McClain, & Gravemeijer, 2003).

The first activity introduced using the third applet involved investigating the effect of heat on the life of automobile tires. Data was presented on the life of tires in two cities: San Francisco and Phoenix (see Figures 14 and 15).

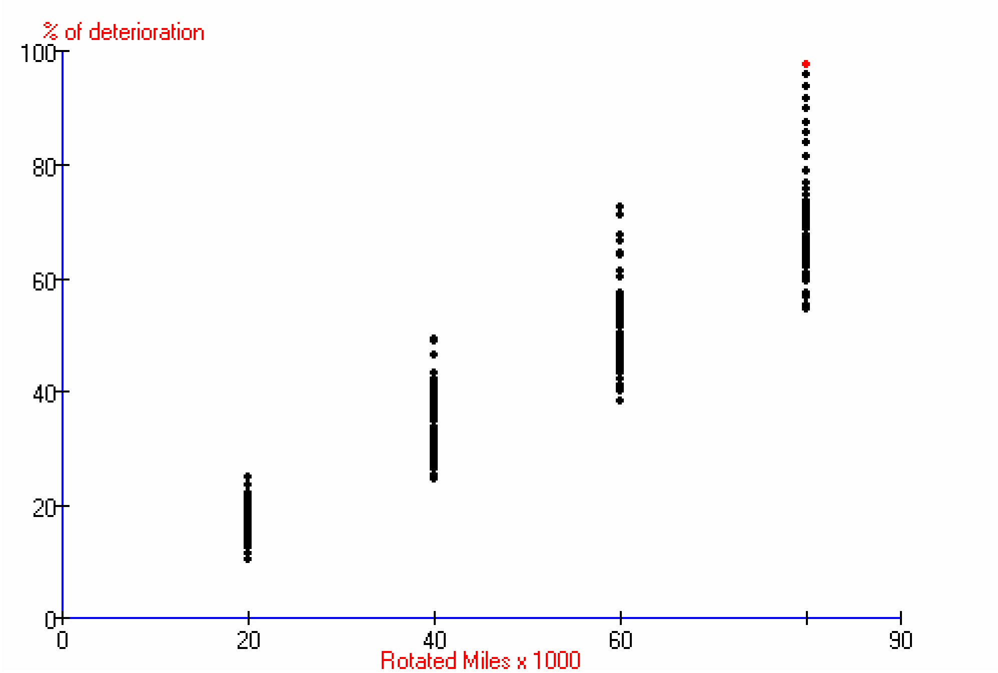


Figure 15. Tires San Francisco—Applet Three.

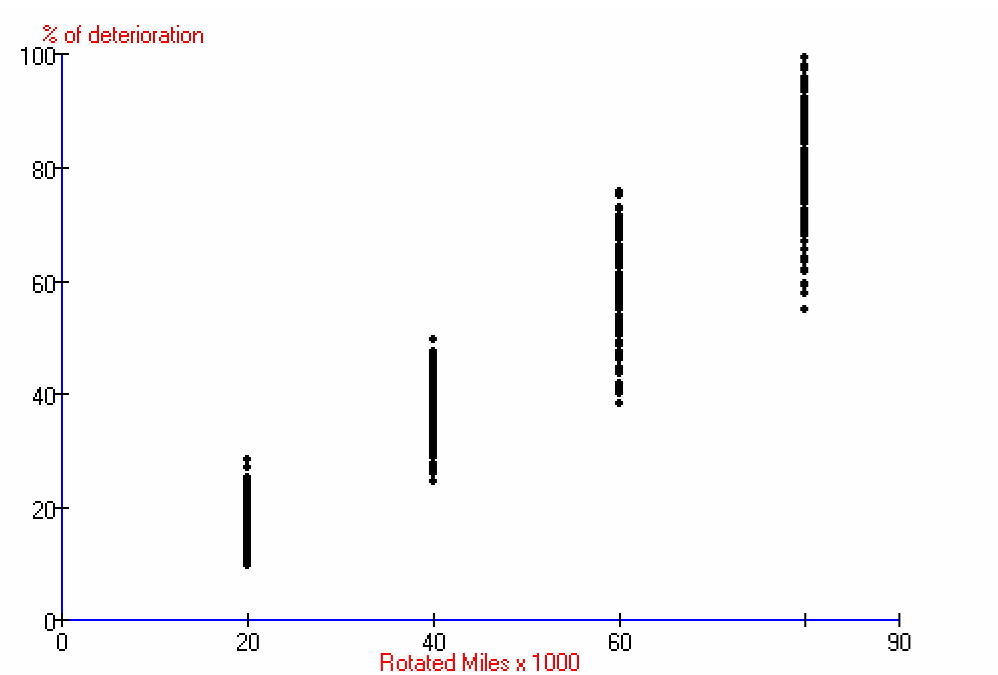


Figure 16. Tires Phoenix—Applet Three.

For each city the percentage of wear for one hundred tires was given after 20,000 miles, 40,000 miles, 60,000 miles, and 80,000 miles. Thus, there were 400 data points for each city. The teachers were asked to write an article for Motor Trend magazine analyzing the difference in tire wear in the two cities. The intent of the task was to build on the teachers' ability to compare univariate data sets by presenting the bivariate data in what the research team called "stacks." The immediate goal was to support teachers' ability to interpret each stack as a univariate data distribution and to analyze how these distributions changed as the second variable (number of miles driven) increased. If the teachers interpreted the stacked data in this way, they would come to view a bivariate distribution as a distribution of univariate distributions.

General Participation Norms.

In the previous session (November 2000), the general participation norms that emerged, directing clarifying questions and comments to each other, were apparent only when teachers were working on applet activities. As an example, when the teachers investigated the student work on the Batteries task, they routinely asked permission of the researcher to interrupt or build on what others had contributed: "Can I go on hers?"; "Can I interrupt?" Although the teachers referred to what others had conjectured, "I just want to piggy back on what Amy was saying…," the contributions took the form of turn taking.

However, this was not the case during data analysis discussions; turn taking was no longer expected. Interruptions were not seen as a violation of norms and the teachers often finished each other's thoughts. An example of this can be seen during the discussion of the Tires activity using the San Francisco data with the four equal groups option selected (see Figure 17).

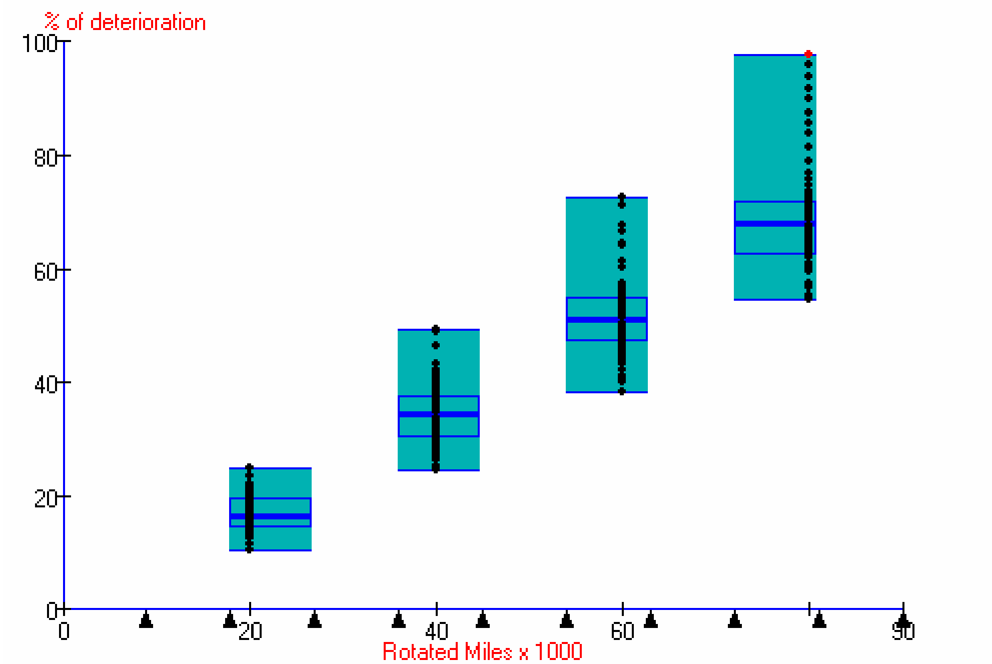


Figure 17. Tires San Francisco—Applet Three—Four Equal Groups Option

Researcher (1):	Draw some shapes.
Researcher (2):	Start down here with this one. Muriel, what did you say this one looked like?
Muriel:	Just a bell curve.
Researcher (1):	A bell curve. Like this up here and then down. Does that make sense, can you visualize that? How is this one [pointing to second group] going to be different from first one?
Jeremy:	The hump is going to be lower.
Researcher (2):	In this one the hill is kind of in the middle. Here it moved down a little. Does that make sense? Now what about this one [pointing to third group]?
Jeremy:	Almost going back to…
Amy:	Going back to the middle, but going to have to go in sharper and have to…
Jeremy:	And going to have a longer end.

Researcher (2):	And this one [pointing to the fourth group]?
Jeremy:	Again back down and have a long [draws line with finger].
Researcher (1):	Top 25% is stretching.
Researcher (2):	Does that make sense? Now let's talk about what 50,000 might look like.
Jeremy:	The bottom part not as low but then again it wouldn't be as high as 60 and your line after the curve comes down would not be as…
Amy:	Wouldn't be as long.
Jeremy:	Yeah.

Again, the general norms of participation that were emerging were dependant upon the activity in which the teachers were participating. When the teachers engaged in discussions about pedagogy, turn taking continued to be normative. However, when the teachers engaged in statistical activities, the participation norms began to evolve into a structure where interruptions and talking directly to each other were legitimate. I would conjecture that this contrast in norms of participation between statistical and pedagogical discussions can be explained by taking account of the private nature of the teachers' instructional practices. When working on the statistics activities, the teachers perceived themselves as learners, students of mathematics. However, discussing issues of pedagogy, particularly when examining their students' work, was a high risk activity that they perceived as involving the evaluation of their classroom practices. As a consequence of this privatization of instructional practices, the group continued to be a pseudocommunity when engaging in activities that were related to pedagogy.

Mathematical Reasoning Norms.

In an attempt to support the teachers in coming to reason about the data in terms of shape, the research team returned to the second applet and posed the Migraine task (see Figure 12). As the teachers shared the results of their analysis, it was readily apparent that they continued to reason in terms of percentages. However, one teacher did bring up the issue of shape in his solution.

Amy:	I thought about it in terms of, since the numbers being tested by each group was different, I should do it by percentages and not by numerical, the weight of the number of green dots and the number of, it had to be done by percentages. Thus, I did it the same way she did.
Researcher:	So you did it in terms of 50 % versus 25%.
Amy:	Yeah, instead of counting dots.
Jeremy:	I looked at it in terms of, uh, two parts. The time frame, the pink seems to more or less straddle the entire time continuum whereas the green is more compact and is in a specific time frame of relief.

Although this teacher focused on shape was without prompting, there was no evidence that reasoning in terms of shape was normative among the group. The solution was not discussed within the group because an unrelated question was raised by another teacher, and the issue was not revisited.

When working with the Tires activity by using applet three, four teachers could talk about the data in terms of shape when the data were displayed in four equal groups. They demonstrated this in four ways: 1) by tracing the shape of the data with a finger, 2) describing how the data were skewed, 3) describing what the "bell curve" would look like, or 4) describing where the "bubble," "hill," or "hump" was in the data. However, two teachers explicitly expressed their confusion with the concept of shape with bivariate data, one example of which is below.

> Researcher (1): We are running out of time. Does it make sense to go back to just one of those graphs. And go back to the data. And now 4 equal groups by 10. If you look at that, have you got a sense, could you estimate what data would look like at say 50,000 miles in SF? What would you expect the data to look like?
>
> Researcher (2): Could you anticipate what would look like at 50000?
>
> Wesley: Yes. [pause] You have a very linear progression in the maximums, minimums, and medians. It's very linear. You have linear divergence, but seems to me very linear.
>
> Muriel: In the upper 25% it is more scattered, it's getting wider every time.
>
> Researcher (1): What would you expect the shape of the data to look like at 50000 miles?
>
> Muriel: Slight lower bubble. Is that what you mean?
>
> Researcher (1): That is exactly what I am talking about. Hill or hump.
>
> Researcher (2): Yes a hill or hump in the middle.
>
> Rachel: I don't get it.
>
> Researcher (1): Ask a question.
>
> Researcher (2): Yes, Rachel, ask a question so we can talk about it a little bit more.
>
> Rachel: I understand when you were saying that upper 25% gets more spread out, you can estimate that at 50, but everything else…

As with the applet two activities in the previous session (Cholesterol, Airbags, and AIDS), the issue of shape had to be introduced by the researchers and there was no evidence that reasoning about data in terms of shape was normative. The teachers' ability to reason about the data inscribed in the second applet in terms of shape was important for supporting their investigations with applet three. This could explain the struggle some teachers had when analyzing bivariate data.

Pedagogical Reasoning Norms.

The reader will recall that two derivatives of regularities of practice were prevalent when the teachers previously discussed their student work on the How Much TV? task in the September, 2000 session. The first of these derivatives, focusing on solution methods rather than underlying reasoning, was also evident during the discussion of the Batteries student work. However, the second regularity of practice, reviewing specific statistical concepts prior to engaging their students in a task, was not as prevalent. Only half of the teachers reviewed specific procedures or methods this time.

Dot:	My Algebra kids did this in 10 minutes. Most did mode, median, mean, range. Used Average to find which batteries. Some did draw lines to show what they were looking at and others put numbers at end of each one. Most found average. Some made charts and gave a paragraph. Some wrote one sentence. My best kid focused on measures of center. All my kids did average. Algebra students are very concrete.
Researcher:	Were you surprised by this?
Dot:	No, I did this during the extra period we have. I have been doing mostly stats with them in AE. We talked about graphs and measures of center right before doing this. The kids I have are very "there has to be an answer to this." Not good at abstracting. Students were happy with finding average. Not a problem for them. The regular kids with the TV thing, took two class periods; 45 minutes class periods. I think regular kids would have done something different than Algebra kids.
Jeremy:	Can I go on hers? I went with certain students and gave specific parameters. Wanted 4 specific reasons one better than the other. Two students chose Brand A because of consistency, relied heavily on graph. Others look at Brand B because it had the longest bar. A few look at average. And they pulled the consistency thing on

their own. “Again, the visual, like Dot said, just really drove their answers.” Because it looks like it lasts longer.

Dot: Had you pre-taught any statistics yet? Taught any?

Jeremy: No, not yet.

The above exchange demonstrates that the teachers continued to conjecture about how reviewing specific statistical topics affected what their students produced. At a broader level, this is an instance of the relation the teachers perceived between teaching and learning (i.e. if we do this, we will get this student solution as a result). It indicates that the teachers analyzed their instructional practices in terms of its consequence on student outcomes. There were no challenges or alternative interpretations offered for these assertions. Thus, it appears to be normative for the teachers to justify their instructional decisions based on the resulting student outcome.

It is also important to note that comparing the data sets in terms of “consistency” was constituted as another student product or solution method as opposed to a qualitatively different approach in which data points were treated as measures rather than merely as numerical values. This was reasonable given the teachers’ instructional practice of evaluating student work based on correctness of solutions. Students’ production of “consistency” became a way to measure whether or not the students “got it.” Assessing student work based on whether they “got” a certain concept is indicative of the student work functioning more as a record of what had happened rather than a tool for supporting the development of student reasoning.

Strategic Reasoning Norms.

While teachers were discussing their student work on the Batteries task, issues pertaining to the institutional setting surfaced. It remained normative for aspects of the institutional context to be perceived by the teachers as frustrating and beyond the teachers' control.

Naomi:	Amy, one good example, we were doing this yesterday, my principal came in she saw me at the overhead and the room was kind of dark and the kids were talking about batteries. And she is looking at me like [standardized tests!] and you are talking about batteries? And she left the room and didn't say nothing.
Amy:	My principal took flack because the superintendent came in to my room and I was teaching Roman Numerals and they are not on the [standardized test]. I don't care.
Researcher:	Seriously, we want to understand about what it is like here.
Jeremy:	The pressure is on us in the next 50 days to make gains.
Amy:	The pressure is on every day. Our name is on that math test results.

However, there were instances where even as teachers were expressing frustrations about pressures they felt, they were starting to ask others for advice. This was the first instance of the deprivatization of their classroom instructional practices.

Rachel:	I just want to piggy back on what Amy was saying. I was telling [researcher (1)] that when you were saying that you would give a kid a half an hour to get a kid to discuss something that you asked them. I agree with that totally, but when I was talking to [researcher (1)], I said well my principal would say, you are not covering all your topics. I agree, I want kids to explain things, but administrators would say, when they come in to observe your class, and I have had several to observe my class, they say you are taking too long on this. You should ask them, maybe wait two or three minutes and then move on. So sometimes you can't get into that deep discussion because of time limits, because of behavior…
Amy:	Part of it is the fact that I have been at this a lot longer than you and I know they ain't gonna bother me.

Teachers laugh.

Amy: No, I am serious as a heart attack. Also, I can justify what I do by any standardized test you want to give them.

Jeremy: Right.

Amy: I know bright kids, but even they can get locked into blah math. The best conversations I have had have been with the low level kids. They have a lot of street smarts. They have to be able to reason about what they are doing.

Rachel: Well how do you like, if you are talking to one particular student, for example, you are talking to me and I am hesitant about talking to you, how do you keep the rest of the class engaged. Because sometimes if I am focusing on one particular student, the rest of the students are like ok…

Amy: Simply the force of my personality to a certain extent. They know, that in this class everyone has a right to speak and everyone has a right to make a mistake. And everyone has a right to an opinion. And by God, if I am going to listen to yours, you are going to listen to his. It is just a matter of directing them, if you don't want to participate, how about you stay an hour after school.

The research team considered this discussion to be productive because it focused on issues the teachers experienced with respect to the institutional setting. For the first time, the teachers openly discussed events from their classrooms as a way of describing problems and sharing solutions related to their perception of the institutional setting and how it affected their teaching.

The research team's intention for having the teachers share student work was to support a focus on their students' reasoning. However, the activity proved to be productive in supporting the initial deprivatization of instructional practice. A conversation occurred between one of the teachers and a researcher that showed the potential that student work had to support both the deprivatization and the development of the teachers' pedagogical reasoning.

Muriel: That's what I say with my kids. I talked to Dot and she said that all her kids did the average. So when my kids did the average, I put it on the board.

Researcher: Put what on the board?

Muriel: Like we did that time here. And then I said, "Well, does it tell me anything?" And they said, "No, it is too close." And then they started looking at other stuff.

Researcher: Interesting.

Muriel: And then they were asking, I could hear them, "So this one has the most, what is that word for that?" And then they would look it up. And so they started to figure out that they needed to find the mode or the range and telling each other, like when we were talking about it, they would say they found the highest high and the lowest low and they would say, "Oh you found the range." So it developed a need to find those things. So when you do study it, it will make more sense.

There were two very promising aspects of Muriel's comments to the researcher. First, it indicated that Muriel had talked with Dot about the Batteries activity outside the work session. This was important given the fact that our analysis of the institutional setting showed no informal networks in the schools or school district. Second, Muriel reflected on what Dot had told her about her students working on the Batteries task *and* what had happened during the work session to make decisions about how to lead the task with her students. Muriel started making connections between issues she had explored during the work session as a learner of mathematics and aspects of her classroom instructional practices.

Year One, Summer Session—June 4, 5, & 6, 2001

During the session on February 8, 2001, some of the teachers struggled with applet three activities. Thus, the research team conjectured that it would be important to support the teachers' reasoning about shape by revisiting univariate data using the second computer applet. The first task of the summer session therefore involved the teachers analyzing two univariate data sets. The data consisted of the response times of two different ambulance companies measured in minutes as shown in Figure 18. The times were measures of how long it took each ambulance company to respond to emergency calls from schools during a nine-month period. The task was to determine, based on the response time data, which company they would recommend to the School District. During the nine-month period, 162 emergency calls were made to the Acme Ambulance Company (pink dots) and 205 to the Lifeline Ambulance Company (green dots). The data sets were designed to allow the teachers to use the perceptual patterns in the data as a basis for preliminary analysis. It was intended that the teachers would then use features on the computer applet to quantify their conjectures based on shape.

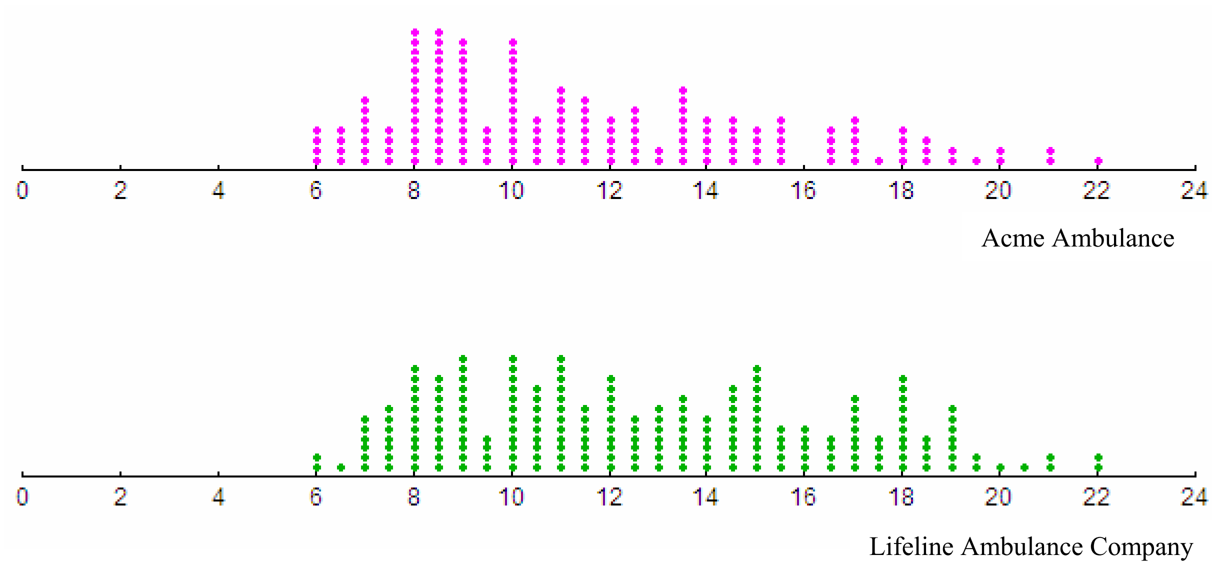


Figure 18. Ambulance—Applet Two.

After the Ambulance task, the researchers posed a task that was designed to support the transition to analyzing bivariate data. The task involved analyzing the reaction time of 50 people before they consumed any alcohol (e.g. 0 ounces) and then again after they had consumed two ounces of alcohol. In the Reaction Times dataset (see Figure 19), each dot is a case of a person and the two measured attributes are ounces of alcohol consumed and reaction time. Reaction time was measured by recording the number of centimeters a meter stick fell before the person closed their hand to catch it after it was released at the “0” centimeters mark. Thus, the greater the distance the meter stick fell before the subject caught it, the slower the reaction time. For most people drinking two ounces of liquor would result in a blood alcohol level higher than that permitted for driving a motor vehicle (.08%). The goal of the Reaction Times activity was to determine the reasonableness of the current standard under which someone is considered to be driving under the influence of alcohol (DUI). The research team anticipated that the teachers would interpret the two stacks as univariate data distributions. To do so, the teachers would have to use options on the computer applet to determine the density of the data since some of the dots were on top of each other. This task was similar to the Tires activity (see Figures 14 and 15) conducted during the session on February 8, 2001, except that there were only two stacks. For this reason, the research team planned to re-visit the Tires activity after the Reaction Time activity.

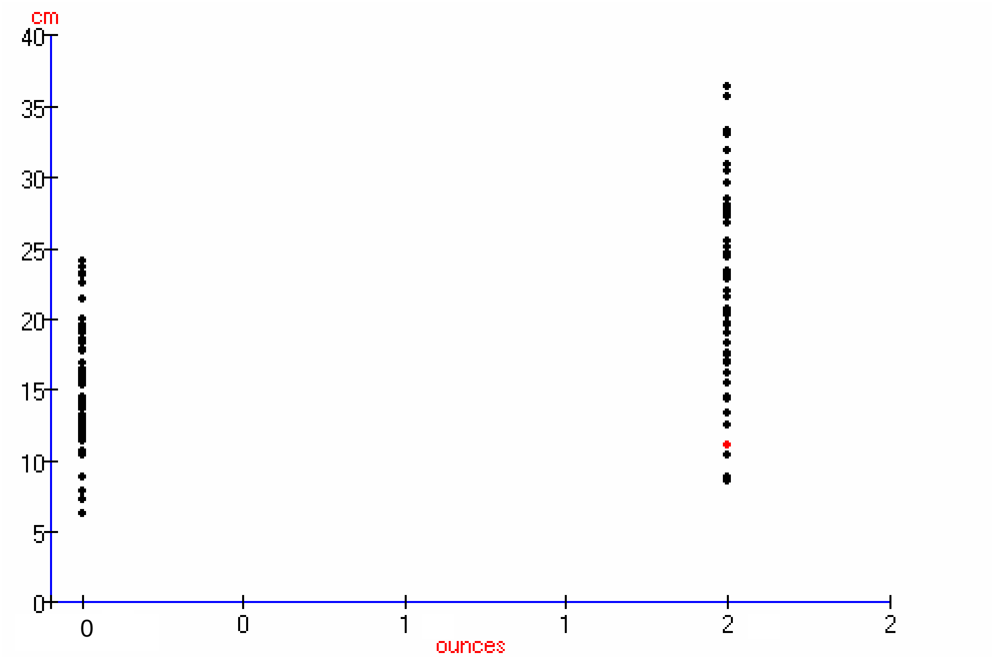


Figure 19. Reaction Times—Applet Three.

The reader will recall that the intent of the conjectured learning trajectory was, in broad terms, to support the teachers' eventual development of instructional practices in which teaching is a generative, knowledge-building activity with students' reasoning at the forefront of instructional decision making. Taking this potential end point as a guide, the research team therefore wanted to support a shift away from student solution methods and toward analyzing student reasoning. To achieve this goal, the research team developed a series of activities intended to highlight student reasoning. The first of these entailed a collection of student work from a series of tasks using each of the computer applets. The second was the use of classroom video-recordings from the seventh-grade design experiment. The last set of activities was the student work from the Carbon Dioxide and Speed task which the teachers brought with them to the session.

For the first of these three activities, the research team asked the teachers to analyze student work from a task involving the first computer applet. The task involved comparing the amount of juice produced from ten individual watermelons of each of two different brands to determine which brand to recommend to the owner of a juice bar (see Figure 20). The teachers were given a screen capture of the data as shown in Figure 20 along with four student reports (see Appendix F). The intent was to focus the discussion on the variety of ways that the students approached the analysis and how the solutions could be used as a resource for a productive whole-class discussion.

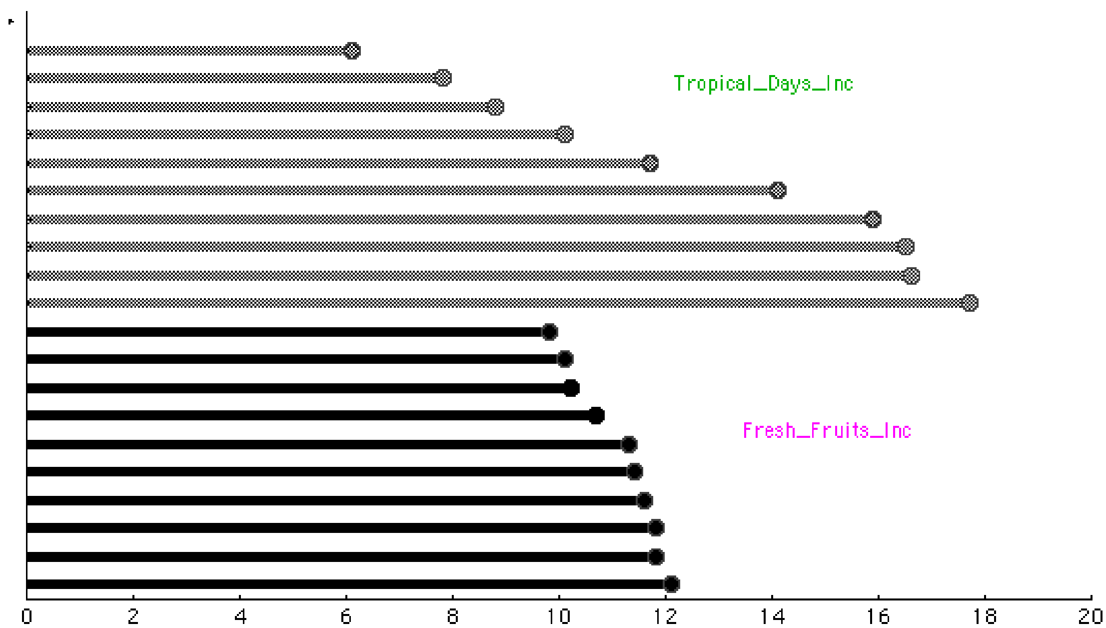


Figure 20. Watermelon—Applet One.

Similar resources were created for activities on the second computer applet. In particular, the teachers received four student reports the Speed Trap task (see Figure 5 for the data and Appendix G for the student work samples) and AIDS task (see Figure 9 for the data and Appendix H for the student work samples).

The video clips used by the researchers came from the seventh-grade design experiment classroom. The first clip was of the whole-class discussion of students' solutions to the Batteries task. The second clip was of students debating the adequacy of reports they had created to support their analysis of the AIDS data. These two clips were chosen because of the noticeable difference in the norms for argumentation. In the Batteries clip, students partitioned the data, but did not provide a warrant for their choice of cut point. However, in the course of discussion, questions arose that required such justifications. In the AIDS clip, the use of partitions was normative as was the requirement to provide a warrant that explained why making a particular partition gave insight into the questions under investigation, the relative effectiveness of the two AIDS treatments.

General Participation Norms.

The general participation norms evident during the statistical data analysis in this session involved teachers speaking directly to each other, interrupting each other, talking simultaneously, and finishing each others thoughts. An example of this can be seen in the discussion of one group's solution to the applet three activity, Tires. This group had used the grids option of the applet to organize the data with the data hidden (see Figures 20 and 21).

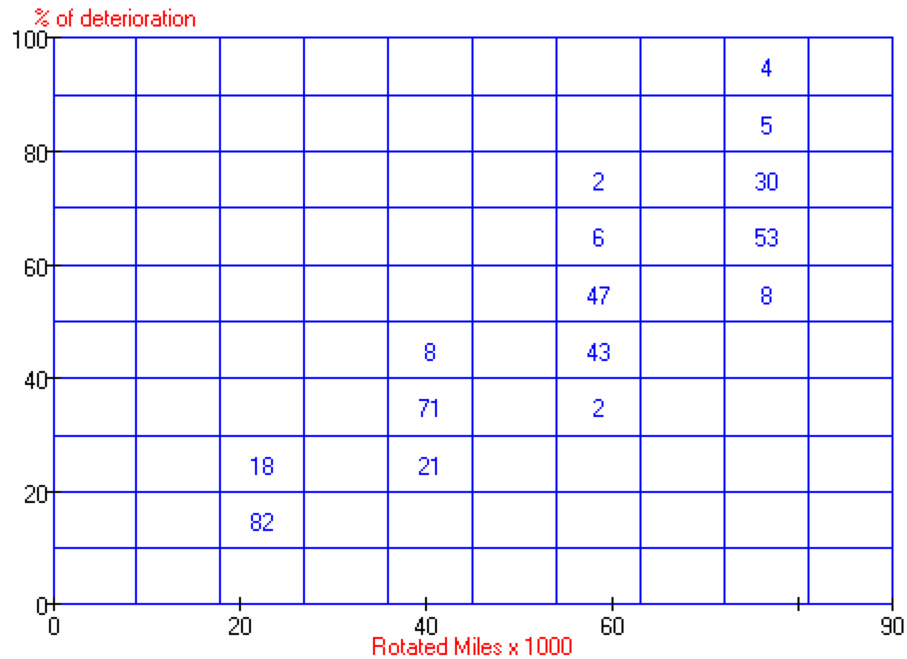


Figure 21. Tires San Francisco—Grids Option—Data Hidden.

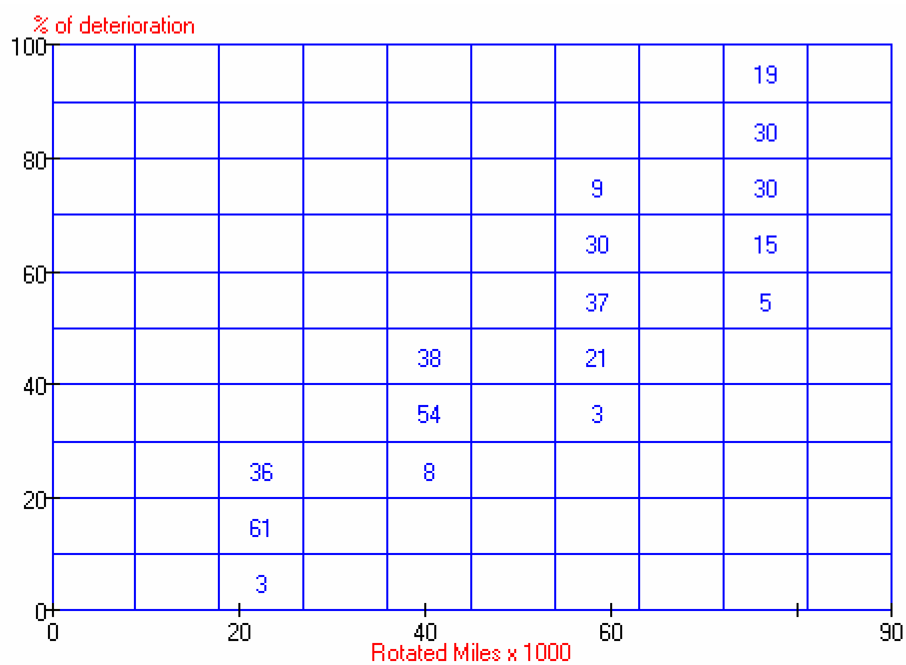


Figure 22. Tires Phoenix—Grids Option—Data Hidden.

Marci: My theory was that if heat played an important part in deterioration, then they should wear out a lot early [in Phoenix]. Look at the lower end of data around 22 or 25 thousand miles. I said, look at the numbers there. We have three on the lower end, 62, and then 35. At San Francisco all the sudden we have 83 and then 17. Heat played a factor and the tires should wear out a lot earlier than a lot later.

Researcher (1): So, could you make a conclusion from that?

Naomi: Since the tires seem to be deteriorating for the first data set almost twice as much in Phoenix as San Francisco. If you go to the next 40,000 it's almost four times as much, at 60,000 it's almost four times plus as much, at…

Researcher (2): What do you mean four times as much?

Marci: If you look at the number…

[Several teachers attempt to answer researcher's question]

Researcher (3): Remember what one data point is? One car, so each data point, each slice is a different car, same 100 cars tested at 40,000…

Naomi: Yes…

Marci: But still heat.

Researcher (3): So you say that at beginning, yeah?

Marci: Most cars deteriorated in San Francisco more than any of the, where the 83 and 17 is, than they did for Phoenix. So a lot more cars had more deterioration than the end.

[Teachers all talking at same time]

Jeremy: It should be the other way.

Rachel: Yeah, you are looking at the, this is San Francisco.

Marci: Sorry, sorry…

Rachel: You should be looking at Phoenix.

Marci: I told you I hadn't woken up yet.

Dot: Ok, but that is not what she was saying. She was looking at the last box and she was saying 17 goes into 35 two times…

Jeremy: Yeah, twice.

[Other teachers talking, agreeing]

Dot: And she looked at the eight and the 38 and said eight goes in about four times so that's four times as much and then look at the two and the nine…

Jeremy: Two and the nine [says with Dot]

Dot: Is about four times as much and the four and the 19…

Jeremy: Four and the 19 [again with Dot]

Dot: Is about four or five times as much.

Researcher (2): So can I ask a question? Interesting. So I am asking what does it mean, qualitatively what does it mean, in terms of shape, in terms of what is happening to these tires? When we say eight goes into 38 four times, not just for Naomi, but for everybody.

Jeremy: Two times more vehicles tires wear out than they do in the coolness of San Francisco.

Dot: And you are looking at the skew of the line as they go up. The curve moves to the right as you go up.

Researcher (2): The right being?

Lisa: The upper quartile.

Jeremy: Right, the top 75, or actually the 25.

Dot: Right. Like on her humps [referring to Rachel's drawing] the humps for…

Researcher (2): San Francisco.

Dot: Um, no that's for…

Jeremy: That's Phoenix.

Dot: Well, San Francisco would be more to the left and the bottom part of the data, and Phoenix would be to the right, upper part of the data [Lisa and Jeremy are saying stuff at the same time as Dot]

Researcher (2): So what they are proposing is a measure or indicator of skewness. Is that fair?

[Lots of teachers talking at the same time.]

In this example, the teachers are not only interrupting, talking directly to each other, and building on what others have said; they are also having side conversations about solutions, trying to answer questions at the same time, and giving justifications for other's solution methods. This participation structure also held for discussions around student work and classroom video. An example of this can be found during the discussion of student reports on the AIDS activity (see Appendix H).

Researcher: Let's talk about what student one was thinking.

Dot: They did a box and whiskers and looking at four equal.

Jeremy: They actually saw a visual difference between them. They were leaning towards the shape, but didn't say anything about it.

Muriel: Plus they are looking at a good argument, looking at what would be visually important if they were presenting it.

Researcher: Ok.

Wesley: The word I used, they picked the majority, but they qualified it by saying ¾. They picked up [applet] one with a justification and [applet] two by using that to present a majority.

Jeremy: Say that again.

Wesley: The difference between two and one is here we just have the most, but here we have ¾. They have a qualified most. So instead of saying ½, when get the kids to kind of estimate what part they are thinking about, is higher level.

Naomi: They are partitioning…

Jeremy: They divide into 4 equal groups, right?

Lisa: That is just like two, with four equal groups,

Dot: Yeah, but they are trying to use the data. They made a box and whiskers. We have looked at this before.

Researcher: They went a step…

Dot: further.

Over the course of the summer session, there were ten direct challenges made during statistical data analysis and during discussions of student work and classroom video. None of the challenges were interrupted or moderated by a member of the group and there was no discomfort displayed through humor or laughter. Challenges were generally prefaced with a statement such as, “I disagree with” or “I would argue that.” Only the very first challenge, which occurred during the applet two activity, Ambulance, was funneled through the researcher. Muriel had organized the data using fixed interval widths of six as shown in Figure 23.

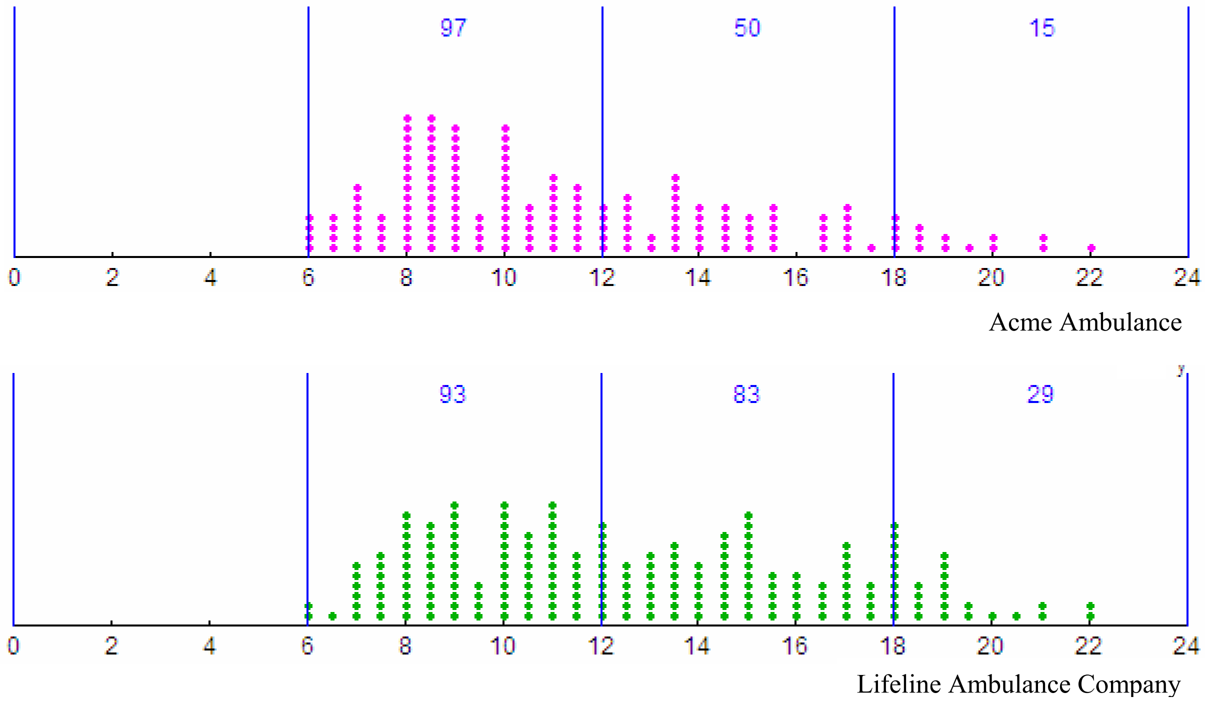


Figure 23. Ambulance—Fixed Interval Width (Six).

Muriel: I did fixed interval width at six, and then made a histogram.

Researcher (1): Ok, read your report. I need others to listen, pretend you are a board member who has not seen the data. Is this report adequate to help you make a decision?

Muriel: I found percents for each bar. The first one was 60% for pink, and 31%, and 9%. Then 45%, 40%, and 14% on green. So my thoughts were, over half of the pink calls were at 12 minutes or less at 60% and over half of the green calls were over 12 minutes at 54%. So 54% of the green were over 12 and 60% of the pink were less than 12 minutes.

[Jeremy raises hand.]

Researcher (1): Questions from you all? Jeremy.

Jeremy: With this type of representation, wouldn't she need to know the standard error of measurement, because there aren't the same number of data points?

Muriel: Well, I did the percents, so I don't know.

Researcher (1): I think her answer to that was she did the percentages. Does that work for you?

Jeremy: But considering, as the hard nose board member, there are 200 and something green and 167 pink. Those percentages aren't very indicative of the truth, because with the 167, there is an additional 32 or 33 that could be at the high end of the percentages, making one of the companies look bad.

Researcher (2): There's an additional…

Jeremy: So if you, because the numbers are not evenly split in half, 206 and 167, when you do percentages of each, it would help if both data sets had the same amount of numbers within.

Wesley: [talking directly to Jeremy] I would argue back that rarely would you have the same number of data. By taking percents you would norm out, or whatever, you reduce the data to an equivalent set.

Jeremy: Yeah, right…

Jeremy raised his hand to get permission and funneled his challenge through the researcher ("wouldn't *she* need to know"). However, Muriel answered Jeremy directly and when Wesley later disagreed with Jeremy, he directed his challenge to Jeremy. All other challenges made during the summer session were directed to the person being challenged rather than to the researcher. The pseudocommunity was breaking down as challenges and disagreements were no longer viewed as violations.

Previously, any claims about emerging norms in mathematical reasoning, pedagogical reasoning, or strategic reasoning that were based on a member of the group challenging another member's assertion had to be taken as tentative because challenging had yet to be established as an acceptable participation norm. However, given that disagreeing and challenging had emerged as normative during this session, claims based on perceived violations of normative practices evidenced by challenges no longer must be taken as tentative.

Mathematical Reasoning Norms.

The researchers intended to support the teachers' reasoning about shape by starting with the applet two activity, Ambulance. While the teachers were analyzing the data, one of the researchers asked the teachers to "eyeball the data" as they had in previous sessions. The researcher was attempting to support the issue of the shape of the data becoming a topic of conversation during the data analysis discussion. However, during the whole-group discussion of the data, all three solutions that were presented were based on percentages and contained no reference to shape. This is not to say teachers could not reason about data in terms of shape, as they had done so in previous sessions. Yet, there was no evidence to support the conjectures that reasoning about the data in terms of shape was normative.

Similarly, although more teachers were able to reason about shape with applet three activities by the end of the summer session, this had to be prompted by the researchers. There was no evidence that reasoning about shape with bivariate data was normative. This is reasonable given the conjecture that teachers ability to reason about the data inscribed in the second applet in terms of shape was important for supporting their investigations of bivariate data with applet three.

Pedagogical Reasoning Norms.

During *every* activity in which the teachers engaged, a topic of conversation that *always* emerged was the issue of prescriptions for implementation of the statistics activities. Whether the teachers were working on applet activities, discussing student solutions, watching video-recordings, or analyzing their students' work, they wanted specific instructions on how to *perform* as the research team had during the seventh- and eighth-grade classroom design experiments. Although the researchers attempted to address this concern by telling the teachers that it would depend on how their students' were reasoning and what their mathematical agenda was, teachers still wanted recommendations for the following issues:

- Timing
    - length of the data creation process
    - length of whole class data analysis discussion
    - moving onto next statistics activity
- Assessment
    - type of homework problems used
    - example of mid-term used
    - rubrics to assess students' work
- Questioning techniques
    - to solicit certain types of answers
    - for establishing classroom social norms
    - to solicit certain types of written reports

The teachers' requests for prescriptions further illustrated the normative practice of teachers justifying their instructional practices based on the resulting student outcomes. With this perception of the relation between their instructional practices and student outcomes, it was rational for teachers to reason that soliciting specific student outcomes required performing instructional strategies in a particular manner.

The research team used student work activities in an attempt to support the teachers in shifting their focus away from student products and toward analyzing student reasoning. The researchers asked the teachers to analyze the student work in order to make sense of how each student was reasoning. The next step was to anticipate how they would build on that reasoning to orchestrate a whole class discussion. With each of these tasks, the teachers began by first describing each student's method, much like they described their students' products in previous sessions. However, teachers evaluated the student solutions based on whether or not they involved a focus on shape. This became a focus of the analysis for every teacher in all activities with student work. An example was demonstrated in the following discussion of the third and fourth student reports of the AIDS data (see Appendix H).

Lisa:	They saw it.
All teachers:	They saw it.
Researcher:	They saw what?
Lisa:	They saw the shape.
Dot:	But I think they went four equal groups first, and instead of putting the partitions.
Researcher:	Say that again, you think…
Dot:	I think they went probably with four equal groups to also, begin with, to help them see that, and then backed up and saw the hump, went one step further than one.
Wesley:	Also have characteristics of the shape, one being wide and low and the other being narrow and tall. It's impressive.
Jeremy:	Very good.
Researcher:	Ok, student four. What were they thinking about?
Jeremy:	Again, if turn this box and whiskers, four equal groups, sideways, [hold paper up and turns] where the big group is more or less here, whereas the first three are in the chart, and the big groups had slid down to the bottom of the other chart.
Researcher:	Is that making sense? Not to Dot.
Jeremy:	Ok, Dot watch.

Dot:	You've got your numbers sideways.
Jeremy:	Right, if turn sideways, like the kid did 4 equal groups, but instead of…
Muriel:	Why turn sideways?
Researcher:	Because he is seeing the shape on the numbers.
Muriel and Dot:	Oh!
Dot:	Ok, I was looking at the numbers. I understand the visual.
Jeremy:	They were probably looking at create your own groups and traced with the partitions of the 4 equal groups and got the numbers.
Dot:	So you were saying they saw it in the numbers, as opposed to the picture.
Jeremy:	Yes, exactly. Thank you for finishing my thoughts.

Notice that the teachers described student solutions in terms of "seeing it." For the teachers, shape was not a type of student reasoning; shape became the "it" in "getting it." Again, student work was a record of what students had done and was evaluated in terms of correctness of solutions.

Strategic Reasoning Norms.

In an attempt to support a shift in focus from student products to student reasoning, the research team showed video clips from the seventh-grade statistics classroom design experiment. The research team anticipated this could initiate discussion of the different ways the students in the video were reasoning about the statistics. During the clip from the whole-class discussion of the AIDS task, the teachers laughed at students that were acting inappropriately in the video. (A student in the video laid his head down on the desk and covered it with his hood. The teacher asked the student to sit up and pay attention. Another student told the teacher he was asleep.) One of the researchers asked for the clip to be stopped.

> Researcher: I tell you why I am getting really annoyed, because you are not listening to the math. You are like the children, I am serious, I mean, yeah, you have got kids turning around and whatever. But my focus at the time in the classroom and Kay's by and large is on the math. And that took us a long, long way and took those kids a long, long way. And so at certain point in time that leads off, but that is where the focus has got to be. And when you have kids that are disrupting others we would intervene and do something, absolutely, but it is at that level, and is there was anything mathematical worth having?
>
> Muriel: What they are saying is…
>
> Researcher: Not saying there is or isn't; we can criticize it at that level. But I am saying if we want to focus on this other stuff, it is not worth our while coming or watching the tape. We could just watch any old classroom and come to those conclusions. Does that make some sort of sense? That is where I am coming from. It's not that you have to say this is great or not great. Let's just look at it for what it is.
>
> [Silence during long pause.]
>
> Researcher: So is this a good time to rewind or what? What do you think Researcher (2)?
>
> Naomi: Could you just rewind it a little bit, I couldn't hear the last part.
>
> [Teachers watch tape in silence.]

The purpose for the interruption was to redirect the teachers' focus to the student reasoning as they were attending solely to student behavior. Shortly after this incident, the group broke for lunch. At lunch the teachers attempted to explain to the researchers their focus on student behavior during the video clip. The teachers indicated that they focused on student behavior and classroom management issues because this was the primary criterion on which they were evaluated by school administrators. In other words, teacher effectiveness was equated with teachers' ability to manage their classrooms. Jeremy added that for the principals, "learning happens when everyone is paying attention." Other teachers confirmed Jeremy's statement explaining that they were evaluated based on number of students on task and whether the State Department of Education's Prescribed Instruction Program objective was written on the board. Lisa added that their focus on student behavior could also be related to the fact that they are isolated and have never had an opportunity to observe other teachers, even on video. Other teachers confirmed Lisa's statement and voice frustrations of having no chance to observe others' teaching practices.

The teachers had previously discussed their limited professional contacts and pressures they felt due to state-mandated end-of-year tests. However, the teachers now went beyond describing the institutional setting by drawing on it to explain how they engaged in activities during the session. The teachers did not attempt to give an excuse for their reaction to the video clip, but rather explained to the researchers what criteria administrators used to evaluate their teaching and how this affected their perspective on teaching.

As a result of this interaction, the nature of the research team's relationship with the teachers changed in two ways: 1) the researchers took the teachers' explanations seriously and began to attend to the institutional context explicitly during sessions and 2) although the researchers were still viewed as experts in statistics, the teachers began viewing themselves as authorities on what teaching entailed in their specific district.

Year Two, Session One—September 20, 2001

During the June 2001 summer session, teachers persistently requested prescriptions for implementing the statistics sequence. Thus, the research team intended to focus on ways to support the teachers in viewing the sequence as a resource that they could used in a process of conjecture-driven adaptation. It was important for the teachers to understand the instructional intent of the sequence. Therefore, the researcher team's agenda was to have a discussion about the statistics instructional sequence in an attempt to co-create the important "big ideas" of the sequence, or what the teachers termed the "benchmarks" of the sequence. The researchers decided to begin the discussion with the applet one activity, Cotton, with a focus on the data creation process to reinforce the importance of students actually analyzing data.

In the Cotton scenario (see Figure 24), the owners of a farm are trying to decide from which of two companies to buy seeds for growing cotton, Omega Land Company or PISCO Industries. They planted twenty-two acres of their farm with the cottonseeds, eleven acres with Omega cottonseeds and eleven acres with PISCO cottonseeds.

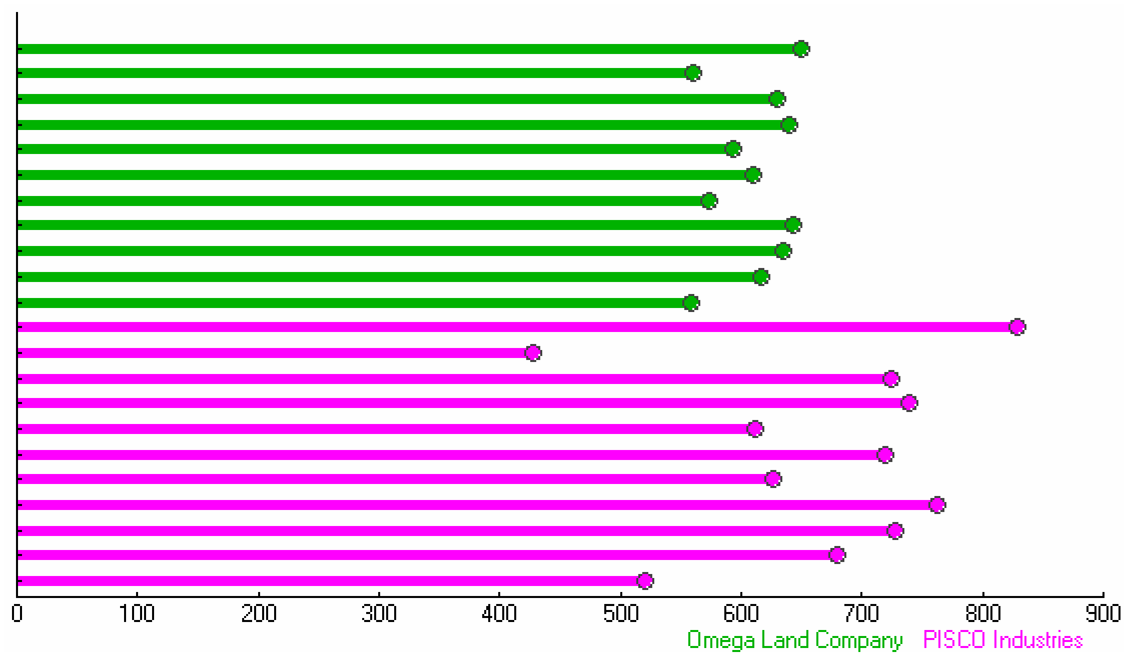


Figure 24. Cotton—Applet One.

Each bar in the data set represents the amount of cotton in pounds that was harvested from an acre of land. The green bars represent the cotton harvested using cottonseeds from the Omega Land Company and the pink bars represent cotton harvested using cottonseeds from PISCO Industries. The teachers were asked to analyze the data and help the owners decide from which of the two companies they should buy cottonseeds. The research team led an elaborate data creation process with the teachers. Cobb and Tzou (2000) describe this process and its importance when working with middle school students.

> From the research team's perspective, it was essential that the students come to view data as sets of measures that could be analyzed to address particular questions and issues rather than as sets of numbers to be manipulated because they were asked to do so by the teacher. The emphasis that the research team placed on the data generation process reflects the view that data do not simply come ready-made. Instead, data are typically generated to address specific problems or issues and reflect the choices that investigators make when they construe a situation. In addition, as Gravemeijer (1999) notes, data are usually generated with a certain audience in mind. The issues addressed during discussions of the data generation process therefore included the purpose for generating the data, the overarching problem or issue being addressed, and who would benefit from addressing this problem. The teacher typically initiated these discussions by introducing a general problem or issue. In the ensuing conversation, the teacher and students clarified why this problem or issue would be significant to them or to a particular audience. Next, the students and teacher discussed which attributes of the phenomenon under consideration were relevant to the issue they sought to address. Frequently, they also discussed how they would actually measure these attributes in order to generate the required data. It was not until these issues had been resolved that the teacher introduced the data the students were to analyze. In doing so, she often discussed the significance of possible data values with respect to the situation from which the data were generated (e.g., the significance of a resting heart rate of 90 beats per minute in an investigation of the effectiveness of exercise programs for patients' heart problems). The intent of these discussions was that the students would become familiar with how, why, and for whom the data were generated so that they might be able to interpret the variability of data values in terms of the situation from which the data were generated. (p. 8-9)

The conjecture was that the initial focus on the data creation discussion would then lead to a discussion about the other big ideas of the statistics instructional sequence. Following the discussion of the instructional intent, the research team planned to have the teachers discuss their student work from the Watermelon and Braking Distance activities.

Pedagogical Reasoning Norms.

During the Cotton Activity, the researchers initiated a discussion of the data creation process with the teachers in an attempt initiate a discussion of the importance of ensuring that the students' mathematical activity remained grounded in situation-specific imagery (McClain & Cobb, 1998). However, it became clear that the purpose of the data creation process from the teachers' perspective was primarily to solicit student engagement.

Researcher:	Let's take a step back and look at what we just did as a teacher. Why do you think I talked through about cotton? What did that do for you?
Rachel:	It gives us some background information; a purpose for growing cotton.
Researcher:	So thinking about why growing cotton. Why was that important?
Jeremy:	It gave us a lesson focus and got us interested in the subject.
Researcher:	So we talked about the data creation and we were trying to generate student interest…
Wesley:	It made me feel wonderful, as a know it all.
Researcher:	Why is it so hard to generate interest for students? What does it do?
Dot:	Makes them actually do the activity.
Researcher:	Ok, it gets them to start thinking about it. What trying to do with the statistics?
Jeremy:	Make an argument to try to decide which cotton seed company to choose?
Researcher:	So you are creating an argument to address a particular issue. Why is it important to do the data creation process?
Marci:	It makes it more personal. If just come in and give it to them there is no relevance to the child. It hooks them in, gets them interested and gets buy in to the idea.

Jeremy: It goes back to that authentic piece. This is their argument. No one can take it from them.

Only the researchers endeavored to question this line of thinking, none of the teachers perceived it as problematic. Thus, there was no evidence of shifts in the pedagogical reasoning of the group. Furthermore, asking for prescriptions by asking how to *perform* the data creation process with their students was evident.

Wesley: Is there a list of things we need to consider when presenting data sets to students? I am sure as move through more scenarios, it will take less effort on our part as we go along, kids will start asking more questions with experience.

In order to support the teachers in coming to understand the instructional intent of the statistics sequence, the research team decided to use the Cotton example as an introduction to a discussion of the big ideas of the instructional sequence.

Muriel: Is there anywhere examples of the sequence?

Researcher (1): Meaning?

Muriel: Like, I picture something like: if they can do this and having an example of student work or example of student argument then that is an example of them being able to justify their answer. You know?

Researcher (1): No, sorry.

Wesley: Like on a flow chart.

Muriel: Yes, like he showed us the example with the AIDS thing and then they could justify by shape of that.

Researcher (1): So you are wanting something that gives examples of big ideas?

Muriel: And when we are ready to move on to this.

Researcher (2): There are such examples.

Researcher (1): Actually we would call that a learning trajectory for the students.

Researcher (2): Yeah.

Researcher (1): Things we are wanting them to learn along the way.

Muriel: Yes, with examples

The researchers and teachers subsequently co-created a list of the "benchmarks" for the sequence:

Applet One:
Actually analyze data
Developing data based arguments
Justification with respect to question
Using alternative methods to analyze data
Partitioning data sets as a way to organize data in order to address question

Applet Two:
Focus on shape of data (trends and patterns in the data)
Comparing data sets (unequal numbers)
Majority (relative frequency)

In retrospect, the researchers and the teachers were participating in the discussion of the benchmarks with two different epistemologies. While the researchers were attempting to support the teachers in understanding the instructional intent of the sequence by co-creating the benchmarks, the teachers were creating a list of things that they should ensure students "got" as they went though the sequence. This claim is based on the teachers asking for the benchmarks to be worded as objectives similar to the ones found on the Prescribed Instructional Program created by the state. The teachers also asked if the particular objectives should be listed on the board prior to lessons or told to the students after the lesson. Again, the teachers' epistemology is reasonable given the institutional context in which they worked. As the teachers made explicit during the June 2001 summer session, one of the criteria used by school leaders to evaluate their instructional practices was having the State Department of Education's Prescribed Instructional Program objective written on the board. The reader will recall that the Prescribed Instructional

Program specified the mathematical objectives that teachers were expected to address at each grade level. Therefore, since the teachers were trying to fit the benchmarks of the sequence into the mold of state objectives, it was rationale that they saw these as a list of things that must be covered and that students must "get."

Year Two, Session Two—October 25, 2001

The research team, in a continued effort to support the teachers in understanding the instructional intent of the sequence, decided to start the session by briefly revisiting the benchmarks created during the last session. This was followed by the applet two activity, Recycling. The Recycling activity was to serve two purposes: 1) verify teachers' ability to use four equal groups to see shape and 2) support the teachers in listing more benchmarks for the instructional sequence.

The Recycling activity (see Figure 25) was presented to the teachers as follows: the Metro City Council wants to investigate the more effective of two methods for collecting plastic recycling material. During a trial period two methods were used. With the Curb Side method, people were asked to put the plastic recycling materials on the curb at some specified time. These containers were then collected separately from other garbage. With the Blue Bag method, people were asked to put their plastic recyclable material and garbage out at the same time, but to put the recyclable material in blue bags. Using the Blue Bag method, both the garbage and the recyclable material were picked up on the same day. Each dot represents the amount of recyclable material collected in one week, measured in thousands of pounds. The pink dots represent material collected using the Curb Side recycling method and the green dots represent material collected using the Blue Bag recycling method. The teachers were asked to summarize their findings from the data gathered and to make a recommendation to the City Council about which system to use in Metro.

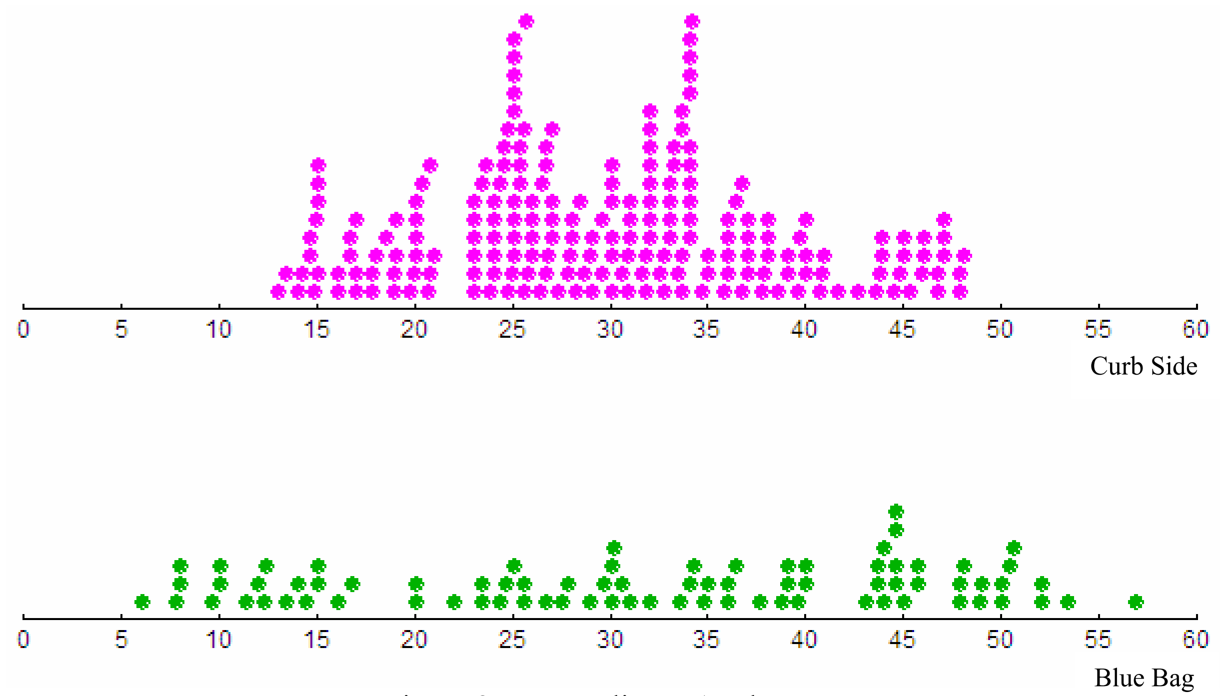


Figure 25. Recycling—Applet Two.

Once the teachers had completed this activity, the research team planned to revisit the benchmarks created during the last session (September 2001) before looking at student work re-created by a member of the research team on the applet two activity, Migraine (see Appendix I) and the student work on the Batteries task the teachers had brought with them to the session. The research team's conjectured that working on an applet activity with the teachers and revisiting the benchmarks would support the teachers in focusing on student reasoning when analyzing student work.

Mathematical Reasoning Norms.

The research team chose the Recycling data set because it did not lend itself to making cut points based on either an initial visual inspection of the data or a particular characteristic of the scenario (e.g. cut point at 55 in the speed trap data). Unlike previous discussions of statistics activities, the issue of shape emerged as a topic of conversation without having to be initiated by a researcher. Also, the solutions developed by all three groups involved inferring shape from either fixed interval widths or four equal groups. An example of this can be seen in Muriel and Dot's solution method (see Figure 26).

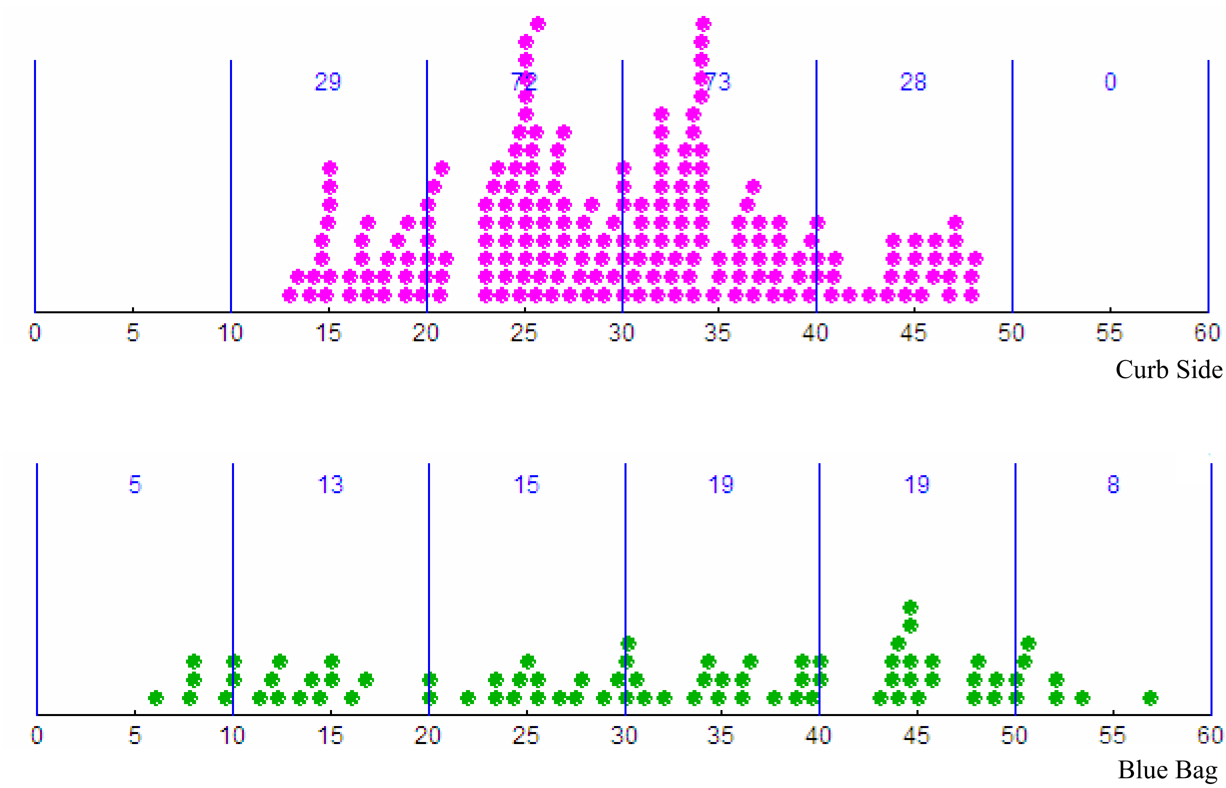


Figure 26. Recycling—Fixed Interval Width (Ten).

Muriel: Then we went to fixed interval width of 10. Then we can see the shape of it.

Dot: And we could see the shape, after we looked at where the median was. The hill, or the shape, or the central tendency shifted more farther down on the bags than it did on picking up curb side, so we decided to pick the bags because in the long run we thought it would shift even more later because it was a new thing.

Muriel: The plateau of the shape is right in the middle on top, but shifted to the right on the bottom and that group of eight down there; there is still a little bit left over, where as the one on top goes up and down and that's all.

Dot: So the majority was in the higher pounds than with the curbside, of what we had.

Even though it was normative at this point to question or challenge another teacher's solutions, there were no challenges to any of the three solutions. Thus, I would claim that it had become normative for the teachers to infer shape from either fixed interval widths or four equal groups.

Pedagogical Reasoning Norms.

After the Recycling activity, the group revisited the benchmarks and added to the applet two list (addition in italics):

Applet One:
Actually analyze data
Developing data based arguments
Justification with respect to question
Using alternative methods to analyze data
Partitioning data sets as a way to organize data in order to address question

Applet Two:
Focus on shape of data (trends and patterns in the data)
Comparing data sets (unequal numbers)
Majority (relative frequency)
Using plots and graphs to visualize the distribution
(hide the data/seeing the shape without seeing the data)

Following the revision of the benchmarks, the researchers distributed student work from the Migraine scenario (see Appendix I) and asked the teachers to think about how they would order the student reports for a whole-class discussion.

Researcher: So what did you decide?

Jeremy: We ordered them three, four, two, one. And here is the rationale…

Researcher: That is what Lisa did, too.

Jeremy: Reason being number three gave no visual evidence other than words, where does the interpretation come from? Evidence is lacking. The term consistency is stressed but not stressed. Try to change that jargon. Number four has evidence but it's not clear with the data points missing, if Average Joe picked it up off the street. The chart showing the 50/50 split, not showing data points, swaying one way or the other. With the bottom chart you can imagine where data points have shifted.

Researcher: Can you say that again?

Jeremy: On the top half the new drug, aside from the numbers, if look at the visual relationship of the bars to each other, you can't draw conclusions about how the data points spread. On bottom, the bars tell a story about, a little better, about the old drug about how the data moved one way or the other. Tell me if I am wrong [speaking to partner Marci]. Report number two is a little better. We are seeing more…

Marci: That the students are more or less thinking about the intervals and quartiles. And able to make relationships that ¾ of the people got relief in 140 minutes versus those who did not, compared to the new drug, between the new and old drug.

Jeremy: And the last one…[referring to report number one]

Marci: And on the last one, we had to get past the words "more packed up." When they say "packed up" I think about going home [giggles]. It's more packed up between 120 and 160, we know they have a good idea of where the numbers are being clumped and they have actually looked at it in smaller intervals than in report two. What if not look at in this area? Let's see how a visual model would be. With report 1 if they had to graph it or create a graph of it, they could see the rising and falling plateau more easily there than they could in report two.

Researcher: Lisa, you said you put them in the same order?

Lisa: Yeah, but I was just looking at it, but you could go from one, where it shows more of the data and this really hides all the data. You really have to see it in this one, have to visualize in this one.

Although the teachers' analysis of student work was more sophisticated, they continued to evaluate the student work based on whether the students were "getting it." The "it" had become the benchmarks of the instructional sequence. For example, Marci and Jeremy chose report three first because there was no visual (benchmark: using plots and graphs to visualize the distribution). In addition, even though they claimed that "two is a little better," report one was the last report because the students stated that the data were "more packed up" (benchmark: focus on shape of data) and created a visual that showed "the rising and falling plateau more easily than they could in report two" (benchmark: using plots and graphs to visualize the distribution). From the teachers perspective, ranking of student work was not about what mathematical issue this order could bring up in whole-class discussion, but rather were the students "getting" the benchmarks listed.

It was notable there were no questions from the teachers about prescriptions for soliciting these types of answers from their students. Instead, two conflicting viewpoints emerged during the discussion of the student work: covering the content and focusing on student "thinking."

Jeremy: Right. And I understand the need to teach thinking and reasoning. And this has definitely an avenue for thinking and reasoning. This is an avenue for them to become genius. But this is some seriously powerful stuff, but again, I guess I am argumentative too, but the harsh reality is, if our schools can structure our course offerings that would expose kids to having to think, to take the pressure off having to cover for a test, to create a course that allows risk taking and room for error and room for debate on why they think their answer is valid. Then our thinking would definitely improve, but unfortunately the people who need to hear this are not here. The reality of a course may never ever be created.

Naomi: But why can't we do that with anything we teach now?

Muriel: Yes, I think…

Jeremy: It would in essence mean that state department of public instruction would have to sit down with people and rethink and reword how they suggest, equilibrium for not instruction but for curriculum, there is a difference.

Naomi: I can think of so many things that I am supposed to teach now, or have taught in the last few of weeks, and I could use the strategy of actually sitting down and talking with my students getting to understand what they feel about the topic, what they really understood about it and gone on from there and they probably would have learned a whole lot more than standing up and saying, today we are going to do this, da, da, da, da, da.

Jeremy: But on the converse side of what we all would want, the reality is if we had that opportunity, let's be realistic about the students we serve, it would take two nine week marking periods before they get to where we wanted them to get as thinkers.

Naomi: But if I can do it with at least one class, just to start, it would make me happy.

Jeremy: Oh, yeah.

Naomi: And I am about making changes within the system. I am not happy with the way the system is now. So I can see doing this with any lesson, I would have to teach to make a difference.

Muriel: I think that is the hardest part with mine right now Jeremy. And with mine I was very, very frustrated, except with one class, I was very frustrated, because they just did not want to think. They wanted me to tell them what they were supposed to be coming up with and do it.

Naomi: Because they are so used to it.

Muriel: Yes…

Naomi: They expect us to do it.

Muriel: And if I can get them to think…

Researcher: But they are not born that way, right?

Naomi: No!

Muriel: No!

Researcher: They have learned it.

Muriel: Yeah, they have learned it.

Naomi: They have learned it.

Muriel: Yeah, my third grader is not like that.

This conflict between viewpoints was reasonable given the teachers institutional context. Although the teachers saw the importance of teaching with a focus on supporting the development of student thinking versus solely on student performance, they perceived of this as very time consuming. Focusing on student thinking was incompatible with the pressure the teachers felt to use their class time to cover the content because of the standardized test. However, it was encouraging that some of the teachers were now openly questioning the value of covering the content (and one the perceived institutional constraints). The end of this conversation also reinforced the conjecture about the emerging norm of teachers relating student responses to prior instruction. Teachers were frustrated by student expectations, but were not blaming those on the students. Rather, the teachers focused on the influence of prior instruction and the long-term consequences of traditional instruction.

Strategic Reasoning Norms.

During the June 2001 summer session, the teachers accounted for their focus on classroom behavior in terms of how they were evaluated by the school leaders. This issue became an explicit topic of conversation during this session.

Dot:	The principal is trying to get us to observe someone else, but we have to find someone to cover our class.
Researcher:	Would you feel comfortable with other teachers coming to see you teach?
Jeremy:	Yeah, come on in.

[Other teachers shake heads in agreement.]

Researcher:	Because having that opportunity would be very valuable.
Jeremy:	They are trying to require us to observe others during our planning period. We got results of what we are looking at. And how we looked at the video [referring to video that was interrupted during summer session], we looked at the lack of discipline of the kids, well that is what we, I guess by nature, have focused on instead of content. We looked at how someone handles the classroom first, and then we starting to break down the walls.
Dot:	Well that is how we have been evaluated for so long.
Muriel:	Exactly!
Jeremy:	Yeah.
Other teachers:	Yeah.

Jeremy and Dot's statements are direct evidence of the strategic reasoning norm of teachers' drawing on their perception of the institutional context to explain how they engaged in activities during the sessions.

It is also important to note the teachers' lack of hesitancy about others coming into their classroom was a dramatic change from previous discussions about people observing their instruction. And yet, now the teachers openly discussed their comfort level with having other teachers come into their classrooms and the desire to have the opportunity to watch other teachers. This was a consequence of the continuing deprivatization of teachers instructional practices.

The conflict between covering the content and focusing on student thinking also supported a conversation about the importance of collaboration and how this was hindered by the institutional context.

Jeremy:	The other problem is, but see, are there people in 6^{th} grade at your school teaching thinking?
Rachel:	Yeah…
Jeremy:	Well…
Muriel:	I don't know! I think so…
Naomi:	The one person in the building who was teaching my students how to think, I am hoping that person is me.
Jeremy:	Sure.
Naomi:	Ok, so there is at least one.
Jeremy:	And the bad part is, that because we are so separated, you know, seventh and eighth at Darlington, sixth at James Wood, both eighth at Sinclair, and myself at Grindley being eighth, no one, we are not receiving …
Researcher:	Ah, yes, a big point. One of the things we have noticed about this district, the schools you are in, there is very limited what we call professional networks, informal professional networks.

As teachers came to understand that teaching that focused on student thinking was difficult, they were beginning to think about the importance of collaboration. Thus, it appeared that changes in norms of pedagogical reasoning were beginning to influence the strategic norms established in the group.

Year Two, Session Three—November 27, 2001

The research team conjectured that it would be beneficial to build on the conflict that arose within the group in the October 2001 session between covering the content and focusing on student reasoning. To this end, the team decided to mail the teachers a video-recording from the Third International Mathematics and Science Study (TIMSS) that showed typical eighth-grade geometry lessons in three countries. The research team conjectured that the contrast between the two lessons might challenge the argument made in the summer session that content coverage and focusing on student reasoning are incompatible. The TIMSS video was mailed to the teachers on November 6, 2001 with a letter to focus their viewing of the video. The letter stated the following: "Watch the enclosed video based on the TIMSS study. This video contains typical examples of eight grade teachers teaching a lesson in geometry and algebra in America, Germany and Japan. The purpose of this video is not intended to figure out the "right or correct" way to teach. Rather, it is intended as a tool to reflect upon how teaching and student thinking are conceptualized in each country. Therefore, we would like you to watch the tape and think about how the approaches of the American and German schools differ from the Japanese schools. Think about how the lessons are presented in each situation with regard to the type of thinking that students are expected to engage in, and also how that type of thinking is facilitated in each classroom. What do the students in each country have to know and learn in order to be effective in the classroom mathematically? We hope that you find this video thought provoking. We are looking forward to discussing it with you."

The research team also structured time during the session for the teachers to plan and lead the Batteries task with a group of students from one of the teachers' classrooms. The researchers requested that the teachers not instruct or steer the students, but rather, attempt to understand how the students were reasoning and report that back to the group. The research team conjectured that this activity would support teachers in focusing on how students were reasoning about data.

Pedagogical Reasoning Norms.

At first, contributions of all but one teacher during the discussion of the TIMSS video concentrated on teaching strategies such as the order of presentation, questioning techniques, and providing the students with a context for the problem. In an effort to change the nature of the discussion, one of the researchers indicated that although it was sensible for the teachers to focus on teaching methods, the challenge was to focus on the original question: what does a student have to know and do mathematically to be effective in the classroom? After this intervention the teachers rarely focused on teaching strategies. In fact, only two such instances occurred, both from the same teacher, throughout the remainder of the discussion of the TIMSS video. All other teacher contributions focused on one of three topics: issues of student engagement supported by problem selection, student retention of previously taught concepts, and covering the content.

Student engagement became an issue when a teacher argued that the problem presented in the Japanese classroom was inherently worthwhile to students.

Muriel:	Plus the problem. It was like; they're spending all this time on a problem. The problem is seen as being valuable in itself.
Researcher:	By the students?
Muriel:	Well, just by the whole class. You know? This problem is important in itself, this one problem. Because I think he [the teacher] only did two [problems in the entire lesson], and then we do all these problems and it is like the problems are not valuable, just the answer. You know?

This led to a discussion about students not remembering concepts previously covered.

Naomi:	And what allows them to retain it? Um, is it because it is valuable?
Researcher:	The students?
Naomi:	Yes the students. What allows them to retain the skills they have learning in a previous lesson? To carry it…
Muriel:	Without practice, practice, practice.
Naomi:	And constantly repeating yourself.
Researcher:	And they don't have to [reteach, referring to Japanese teachers].
Muriel:	No.
Naomi:	Yeah, but we do. Why?

This conversation naturally led to the issue of content coverage arising again.

Muriel:	But see they [US teachers] do the same things every year. Every year! And I have seen all three of my [own] kids going through. Why don't they know it when they get to seventh-grade? And they don't.
Researcher:	Can I just say, when we were talking about issue of content converge before, those seventh-grade teachers I am sure covered the content.
Muriel:	Yeah, so did the 3rd and the 4th and the 5th.
Researcher:	And you have covered the content. And yet three days later they haven't learned it. That was your whole point. Right? [speaking to Naomi]
Naomi:	Yes.

It is important to note that in discussing all three issues, the teachers started to question their own instructional practices: why do we use so many problems? why don't students retain information we have taught? This was an advance beyond merely relating student responses to prior instruction.

Strategic Reasoning Norms.

The research team's agenda in having the teachers plan and lead the Batteries activity with a group of students during the session was two fold. First, the researchers wanted to further investigate the teachers understanding of the data creation process. Their interpretation of the data creation process remained a way "to get students engaged" or "to give a context for the problem." Second, the researchers hoped to support the teachers in focusing on how student reasoning was affected by the data creation process. As requested by the researchers, the teachers did not attempt to teach the students directly, but instead listened and took notes on what the students did during the activity. This environment was more conducive to listening to students than the teachers' own classrooms because there were no institutional expectations about covering content or ensuring that students "got" certain ideas or skills.

Although the teachers' discussion immediately following their observation of the lesson focused on the data creation process and student reasoning, it quickly turned to the importance of collaborating with each other. Each of the four times the researchers attempted to re-direct the discussion to the issue of student reasoning, the teachers raised the issue of needing the help of colleagues. An example of this occurs in the following discussion.

Researcher: It is so alien to us in this country, how we teach, to think how are students putting this together? How are students, it is very difficult, as opposed to evaluating did they get what I wanted them to learn? To suspend that in order to say HOW are they understanding? Does that makes sense? It is tough to do that.

Muriel: Sometimes it takes someone else in the room to do that.

Lisa: Yes it is [answering researcher].

Muriel: Because Ruth was in there. When she came to talk, I could watch my kids much easier and until you have somebody, it almost takes having someone else in there either to give you feedback or…

Researcher: I think you are right, so maybe we want to figure out how to set that up.

Throughout this discussion, none of the teachers expressed discomfort with the idea of having other teachers in their classrooms. Teachers focused instead on the benefits of being able to work with other teachers. This suggests that being able to plan and lead a statistics activity together and being able to focus on how students were reasoning supported the further deprivatization of teachers' instructional practices.

Year Two, Session Four—January 31, 2002

In order to further perturb the teachers' views of their current instructional practices and the resulting student reasoning, the research team planned an activity in which the teachers conducted informal student interviews. Analyzing student work from their classrooms had been less productive than the research team had hoped because it functioned more as a record of what had happened rather than a tool for supporting the development of their pedagogical reasoning. Therefore, the research team conjectured that interviewing students would better support the teachers in focusing on how students were reasoning about tasks. Because the teachers reasoned very similarly to students on the statistics tasks, the research team decided to use fraction tasks (see Appendix J) for the interviews as this was a content domain the teachers would view as self-evident and thus discover unanticipated student reasoning.

The research team mailed two short video clips from the seventh- and eighth-grade design experiment to the teachers prior to the session. The intent was to orient the teachers to think about data creation from the students' perspective rather than simply how the students were supposed to perform during such discussions. The first clip was the data creation process from the Batteries activity. The second clip was the data creation process from the Reaction Time activity. As the teachers watched the video, they were asked to focus on the following: 1) the aspects of the data generation process that the researcher teaching the class wanted the students to understand and 2) the various students' interpretations of her questions.

Pedagogical Reasoning Norms.

Before having the teachers interview the students, the researchers asked the teachers to discuss how they taught fractions. Unexpectedly, there was only one comment about a procedure for teaching students. Instead, the two issues that became the major topics of conversation were: 1) the need to re-teach fractions even though the material had been covered in previous years and 2) the need to make the fractions "real life" in order for students to understand.

Naomi:	My experience is that kids have no idea what 1/3 is of something. They are not being lazy, something is missing in their understanding of what a fraction actually is.
Dot:	And LD kids have no idea what that is.
Rachel:	Students can talk about 1/3 of a candy bar and relate it to a real life example.
Muriel:	And this is benchmark kind of stuff. 50% is ½, 75% is ¾, that basic understanding is foreign to them. If they can get that, they can get everything else.
Wesley:	They could figure out what 5/8 is.
Muriel:	Yes. If they can understand, if they can picture in their head what 1/3 is.
Researcher:	What do you mean picture in their head?
Muriel:	That it is breaking it into three pieces.

Dot: Yeah, and they have one piece of the whole three, or that it is three and they have one of those three. They don't visualize, they don't see that fractions and percents and decimals are all the same.

Muriel: You think they do. But they don't.

Dot: Yeah, you have to throw in realistic stuff all the time. If you don't do that, they have no conception of what you are talking about at all.

Muriel: And it is not real in elementary school. They have fraction pieces but that is not real. They don't see that as real.

Rachel: Yeah, they had fraction color coded circles, students need more realistic examples.

Each pair of teachers interviewed one student using a packet of fraction tasks (see Appendix J) prepared in advance by a member of the research team. All fraction tasks were problems with context, except the last page of problems which was intended to be similar to a textbook page and was referred to by the group as "school math." During the interviews, one teacher in each pair was responsible for posing questions and the other for taking notes. The researchers requested that the teachers not instruct the student or attempt to assist the student in figuring out the correct way to do the tasks, but endeavor to understand how the students were reasoning. In other words, the teachers were asked to try to figure why students were working the problems as they were.

In contrast to the data analysis activities, where the students and teachers reasoned very similarly, the teachers had not anticipated the ways in which the students had attempted to solve the fraction tasks. From the teachers' perspective, therefore, the students' reasoning was something that needed to be explained. In the discussion following the interviews, all pairs of teachers made conjectures about why the student they had interviewed reasoned about the tasks in particular ways rather than merely reporting what the student had done. Thus, the research team's conjecture that this activity could support teachers in focusing on student reasoning was viable. In fact, as the teachers listened to their colleagues discussing their interviews, they started offering conjectures about the reasoning of those students as well. An example of this can be seen in the following dialogue.

Rachel: The student drew the circles correctly but said that 5/8 is less than a half even with a picture.

Researcher (1): So she drew the circles and divided them correctly and shaded them correctly, but said that 5/8 was less than ½.

Wesley: I think that she is looking at the un-shaded part of the picture.

Rachel: She was. She was looking at the un-shaded part. She said she was.

Researcher (2): Makes you wonder what it meant to her.

Teachers: Uh, huh.

Naomi: I asked her but she wouldn't tell me.

Researcher (2): Why so you think she drew that picture?

Rachel: At first not draw a picture, we had to ask her to.

Naomi: It seemed she wanted to represent the quantity, and to her it just meant drawing the circle, and to me dividing it into eighths, to her it was just subtracting a piece or adding a piece on. But she just knew that that was what she had to do, so let me just go ahead and do that.

Rachel: It was like she was trying to please us. Saying "here let me give you something to represent these fractions, so that you can see I know something about fractions," even though her reasoning would not support her answer.

Researcher (2): She was operating in the "what am I supposed to do" world as opposed to the "I am thinking about these as quantities" world.

Rachel: Yes.

The teachers were stunned by *how* the students were reasoning. The issue for the teachers was not that the students struggled to understand fractions conceptually. Instead, the teachers claimed it was obvious that the students' lack of understanding was a result of the instruction they received.

Researcher: So, how did the kids get that way?

Muriel: I student-taught second grade and we did the same kind of, like the pizza activity that Dot does with her students, and they understood it. My third grader understands it.

Researcher: So it is not that these kids are incapable.

Lisa: I think maybe when you start, they understand it at a very basic level, but when you start throwing all this other stuff like adding them together and they have to reduce it and they are looking at improper fractions. I think when things start building on, they start loosing some understanding.

Researcher: So, can I push to turn it around? Maybe what might be useful to think about, so what is happening instructionally? What would be going on in your classroom? What would you be thinking about?

Lisa: All these little procedures that they have to learn.

Researcher: And how are the kids interpreting what is going on? Do you know what I am trying to say?

Muriel: It's math work now, instead of...

Lisa: Yeah, it's math work.

Researcher: And I am making the assumption that all of those teachers did it clearly with good intentions.

Lisa: That's right.

Researcher: They all wanted kids to learn. They all wanted them to learn with understanding. They all wanted them to remember it. And be meaningful and whatever.

Muriel: And they all probably did some type of graphical representation.

…

Researcher: Suppose I am going to review adding and subtracting fractions with unlike denominators, I am going to show how to get like denominators. I am going to break down the rules into bite size pieces, and going to suppose that students are going to get it. What would they have to do in order to produce right answers? I know what I intend as the teachers. How can the kids understand it, given what you found out this morning [as a result of the interviews]?

Muriel: It seems to me that we are just giving them more rules that they would add to their other list of rules and if they are building on misconceptions to begin with, then all they will do is apply those rules to the misconceptions they already have.

Lisa: They are learning procedures, lists of procedures.

Researcher: Or learning lots of more procedures. If our interviews were representative, then they got to the basics of what kids have got to make sense out of what we are doing when we are teaching fractions.

Muriel: Even going back and doing like the natural sequence of it. You know, the things that you teach before, like the least common multiples and greatest common factors and the divisibility rules and stuff like that, that you kind of see as the natural sequence of how you, a normal unit would go. Even that is…

Researcher: Would be [makes motion of hand flying over his head]

Muriel: Yeah. If they don't understand the basic things that we have seen they don't understand.

Teachers stated that they knew that fractions had been covered in lower grades, but the way students were reasoning was undoubtedly not what the teachers covering the material had intended. Therefore, the issue of simply "covering the content" had become problematic.

Strategic Reasoning Norms.

Since merely covering content had become problematic, the teachers' instructional practices had become problematic. The issue that became the topic of conversation was *how* to make changes in their instructional practices.

Lisa:	Kids are used to being told, used to us feeding it to them since kindergarten. We don't have the resources to break the cycle.
Researcher:	What is a resource?
Lisa:	Like the Japanese, where they give them the problem. We don't have those type of problems.
Researcher:	Can I mention another resource, I mention it every time.
Lisa:	Us.
Researcher:	Yes. You are isolated.
Lisa:	Its time, and its frustrating. Muriel is constantly trying different things to get them to understand, and we want these kids to understand. I can see myself up in front of the classroom. How do we get them to jump in there?
Amy:	A different curriculum would help tremendously.
Muriel:	Some things we give, give them rich problem, all things we want it to have, but if they don't have some kind of basic tools in side to be able to engage in it. That is what seems is missing with my kids.

In this discussion teachers raised the issue of needing resources in order to teach for understanding rather than merely covering the content. For their part, the researchers attempted to support the teachers in viewing collaboration as a primary resource when making these changes. The researchers built on Lisa's comment about the resources Japanese teachers have to introduce the concept of the Japanese Lesson Study, thereby further supporting the idea of teachers thinking of each other as resources.

Year Two, Session Five—February 28, 2002

Based on the productiveness of the Fraction interviews during the January 2002 session, the research team decided to have the teachers conduct more student interviews during this session. The research team conjectured that conducting the interviews could further support teachers' focus on student reasoning. The tasks chosen for interviews were the Potato Chip activity and the Allowance activity. These activities were designed to demonstrate the inadequacy of conceptualizing the mean solely in calculational terms. Because these tasks may not have been as self evident as the Fraction tasks, the research team had the teachers work through the activities prior to conducting student interviews. The scenario for the Potato Chip activity was: A recent report published in USA Today stated that the average price for an eight ounce bag of potato chips was $1.39. The information was obtained by checking the prices on seven different brands. What might the individual prices of the seven brands of potato chips have been? The students were given a picture of seven bags of potato chips with blank lines below each bag. The scenario for the Allowance activity was: Mr. Hodge's homeroom collected data about each other's allowances and they found that the average allowance in their homeroom was $3.50. Students were given tiles and a large card on which was drawn a horizontal axis that was marked in fifty cent increments from zero to ten dollars. The students were asked to place the tiles on the axis to create a bar graph that showed what the allowances of the students in Mr. Hodge's class could be.

The teachers had brought student work from the How Much TV? task. The researchers had chosen this task for two reasons: 1) it coincided with the other assessment tasks the teachers were using to interview the students and 2) it created an opportunity for a comparison between the teachers' interpretations of this work versus the first time they brought student work from the How Much TV? task during the first session of year one, September, 2000.

Prior to this session the teachers had been sent two items to follow up on the discussion of the Fraction interviews. The first was the article *Benny's Conception of Rules and Answers in IPI Mathematics* by S. Erlwanger. The focus of this article was on Benny's misconceptions of fractions and decimals. What made the article potentially relevant to the teachers following the fraction interviews, was that Benny was considered by his teacher to be "making much better than average progress through the IPI program, and his teacher regarded him as one of her best pupils in mathematics" (Erlwanger, 1982, p. 7). The researchers saw this article as a logical follow up to the Fraction interviews because both involved successful mathematics students who had developed misconceptions about elementary fraction concepts. The intent was to build from this discussion to the issue of generating evidence to make the consequences of typical instruction that focused on content coverage evident to school leaders.

Generating evidence to make student learning problematic for school leaders was one of two proposals the research team had for future projects. The intent was to support the teachers in gaining control over the resources they needed in order to make the modifications they had envisioned in their instructional practices. The second proposal built on the issues teachers had raised about the current state-adopted math curriculum units "lacking flow." The research team proposed beginning with adaptations to the statistics units of the math curriculum.

The second item mailed to the teachers prior to the work session was a videotape of an example of a Japanese Lesson Study. The video displayed a research cycle for the lesson *Can you Lift 100 Kg?* It began with excerpts of a meeting in which six teachers planned a lesson, followed by one of the teachers conducting the lesson, and then a faculty discussion of the lesson. The research team had sent this video-recording to the teachers in response to their questions about resources for teaching for understanding. The intent was that teachers could gain insight into how they could use each other as a resource as presently there were limited informal networks in the school and school district.

Pedagogical Reasoning Norms.

The first time the teachers brought the How Much TV? student work to a work session was the first session of year one (September 21, 2000). The reader will recall that when teachers shared their student work two issues were discussed. First, the teachers shared their students' work by presenting a list of what the students did. In doing so, they focused on the solution methods students used rather than how students were reasoning. Second, all the teachers had reviewed certain statistical topics prior to posing the How Much TV? task. As stated previously, both of these derivatives of regularities of instructional practice were reasonable given the institutional context.

As teachers shared their student work during this session, they continued to list what solution methods students used. However, their analysis of student work had become more sophisticated. During the discussion of each set of student work, the teachers made conjectures about why students used certain methods or how they understood the task. Also different from last time, there was no indication that any of the teachers had reviewed statistical topics prior to working on the task with students. The change provides more evidence that the teachers' instructional practices were becoming increasingly deprivatized.

A representative example of how teachers conjectured about student work can be observed in the following transcript. This example is of particular interest because of the challenge raised by other teachers when the teacher presenting the work claimed that her students did not know statistics based on the absence of numerical calculations.

Researcher: Who else has ones that were useful for making inferences about student reasoning?

Amy: 6^{th} graders found it difficult to keep opinion out of it and they would get out the colored pencils.

Researcher: How did students organize data?

Amy: Stem and leaf plot.

Researcher: How did they use the stem and leaf plot?

Amy: I was very vague about instructions. Most of them did a graph of some sort. Frequency chart, pie chart.

Researcher: Ok, that is what they did, what did you learn from that.

Amy: I learned that 6^{th} graders do not have a handle on statistics yet. Because not use any kind of numerical calculations.

Muriel: I disagree with you Amy, this is my lowest class, and every one of them did a numerical calculation and I would say that your kids, those kids right there, have a better grasp of stats than these do.

Amy: Why?

Muriel: Because they understand. Now, my focus is the data creation, and I did a really good job this time, because all of them understood, 30 kids, all very different, no average kid that watches TV. But even though they could all do mean, median, mode, they didn't do that, very few thought about it, and we talked about what is too much for a long time. They got it. They understood. They said what a good amount was, under 5 was a good amount. But they did not take it one step further, one wrote a ratio that wasn't even a ratio.

Amy: You said you wanted to know how the kids were thinking, but does the thinking include opinions

Researcher: Give us an example of an opinion.

Amy: This one here, they have four categories: looser, nothing to do, normal, and a little weird.

Muriel: Yeah! [raises arms]

Lisa: Did they put the range on those?

Amy: Yes.

Researcher: So we have very different opinions on how people are evaluating that. You are disappointed because they didn't do calculations, and others are saying, "great!"

Lisa: I would be ecstatic!

Amy: So what do you really want here? So define statistician for me. Does a statistician interject opinion? Draw conclusions based on what he feels is a level that indicates something?

Naomi: But in their opinion, numbers mean something to them.

Amy: But numbers are numbers and you assign values to numbers…

Naomi: Not talking about quantity or anything, those numbers mean something to the students. When you just see students calculating numbers, it means nothing.

It had emerged as normative for the teachers to use the student work as a tool for reasoning about student solution methods versus solely as a record of what students did. The teachers' challenged Amy's assertion that student production of calculations was sufficient to demonstrate understanding. This challenge was evident that Amy had breached a norm of pedagogical reasoning. The pedagogical norm that had emerged was that the numbers students generated must be a measure of something and calculations must be produced for a reason.

The discussion of the Benny article occurred briefly at the end of the session. The research teams' conjecture that it would be a logical follow up to the Fraction task was viable as the teachers had a similar reaction.

Naomi: It gave me a headache again, like the interview. I want to know what previous teachers were teaching, how they presented material to the student. I want to sit in class and hear what teachers are saying. I do not want to blame teachers.

Researcher: No, I don't either.

Naomi: I do not think it is fair to do that. But something is happening and I am trying to understand what that is. Because this is really serious stuff. It's mind boggling…

Researcher: It is, isn't it?

Naomi: How children think this way.

Similar to the fraction interviews, the teachers were amazed by how Benny reasoned about fractions and decimals and viewed this as the consequences of his prior mathematics instruction.

Strategic Reasoning Norms.

During a discussion of the agenda for the next session, one of the researchers proposed the two possible future projects for the group (i.e. generating evidence for principals and adapting aspects of the district math curriculum). The teachers voiced interest in both projects. Later in the work session, Wesley made the following proposal:

Wesley: I just had an idea, think about it. The middle school principals are going to be here on the 19th. Maybe if they are here for food, maybe we could be in here with them to convince them we are doing something good.

Ruth: It is a small group of them. But they are going to be looking at the schools.

…

Naomi: So maybe we should be doing an activity while they are here and invite them to come see the activity.

Muriel: Or with the kids.

Researcher: Or what the kids are doing.

Muriel: Yeah, I'd like for them to see what the kids are really thinking.

Wesley: Do you want me to look at the time frame for when they are here and see what we can get set up?

Researcher and Muriel: Yes!

Ruth: You can talk to Randolph Potts [Wesley's principal].

Naomi: I bet they would be surprised.

Researcher: I bet they would.

Muriel: I would like to do it [the interviews] with a couple of the principals.

[laughter]

Researcher: That idea might have merit. This is what we are finding. Letting them know that the 6th grade teachers are doing what you are telling them, they are covering the material, they are reviewing, but…

Muriel: I would like for them to see it and then hear the discussion afterwards.

Previously the teachers had referred to the institutional context to explain aspects of their instructional practices such as focusing on student behavior and covering the content. In this discussion, the teachers expressed the desire to make school leaders aware of the conflict between students performing well on the state standardized test and misconceptions students had about mathematical concepts. There were no challenges to the assertions that it was important to generate evidence to perturb the principals' view of mathematics teaching and learning. There were no declarations that this would not make a difference. Thus, I claim that this discussion marks a shift in the norms of strategic reasoning. The teachers now perceived of the institutional context as something they could affect.

The shifts that I have documented in the teachers' pedagogical reasoning norms were critical in making the shifts in the teachers' strategic reasoning norms possible. The teachers emerging focus on student reasoning resulted in their current instructional practices becoming problematic. This, in turn, led to the issue of collaboration in order to support their desire to focus on student understanding during mathematical instruction. However, collaboration required that they gain control over resources such as time to collaborate. Thus, the teachers decided it was necessary to generate evidence to challenge school leaders' assumptions about what students were learning when the focus of mathematical instruction is covering the content for the state standardized test. The institutional context in which the teachers worked was no longer something they had to live with, but instead became something they believed they could influence.

At this point in the collaboration with the teachers, I claim the group of teachers meet the criteria for and have emerged into a professional teaching community. I will expound on the specifics of the criteria and supports in Chapter VIII.

Year Two, Session Six—March 19, 2002

In previous sessions, the teachers had inquired about assessing students' reasoning when attempting to teach with a focus on student understanding. As a result, the researchers distributed three assessment articles during the February 2002 session: *Activating assessment alternatives in mathematics* by D. Clarke (1992), *Modifying our questions to assess students' thinking* by Chappell and Thompson (1999), and *Walking around: getting more from informal assessment* by Cole (1999). The researchers informed the teachers that these were not assessment articles in the sense of testing. Instead, these articles focused on ways to attend to student thinking as a part of instruction because assessment is not a separate event that occurs after instruction. The researchers also noted that it was not feasible for the teachers to interview every student and said that these articles offered practical suggestions for documenting student reasoning during instruction. The intent was to support discussion of assessing student reasoning in order to improve instruction.

During the February 2002 session, one of the researchers suggested two possible future projects for the group: generating evidence for school leaders and adapting the statistics units of the district math curriculum. Wesley had suggested meeting with the school leaders while they were visiting his school in March. Although time constraints made it impossible for school leaders to attend the session, generating evidence for the school leaders remained the focus for this session.

Pedagogical Reasoning Norms.

As teachers discussed the importance of generating evidence for principals, *each* teacher made unsolicited claims about how focusing on student reasoning had changed aspects of their instructional practices. An example of one of these claims is apparent in the following conversation.

> Wesley: Yesterday a child came up with a five sided figure as a right answer and I was trying to figure out how. And he was doing it in his head. I feel more prepared to deal with this child because of these sessions. I would have previously thought that this child was not doing anything.
>
> Muriel: And if you had gone on to the next child, you would not have known.
>
> Wesley: I would not have known. And it was a pretty complex thing to do. The kid did not want to do on paper. He finally did. Now he gets extended time on tests without question.

Marci:	I have a child in Algebra who does not put anything on paper. He does it in his head. If I had time to sit with him, he could write notes to defend his answers, but trying to get him to put something down on paper. I know exactly what you mean. The child takes a lot of time, but he is really a bright kid.

These types of claims by the teachers were possible starting points for further conversations about student reasoning being at the center of instructional decision making. This shift was important as it would support the other proposed project of adapting the current mathematics curriculum.

Strategic Reasoning Norms.

In the February 2002 session, teachers were able to articulate how school leaders' vision of mathematics teaching and learning constrained their instructional practices. This was problematic for the teachers because they saw themselves as the professionals who were more knowledgeable than the school leaders and should be making decisions about mathematics instruction.

Researcher (1):	The principal's thinking is just like the student's thinking. The students are doing fractions that way for a reason.
Muriel:	Yeah, not just a stupid student.
Researcher (1):	Yeah, and not just a stupid administrator.
Researcher (2):	All administrators are doing it a certain way for a reason.
Muriel:	Like Naomi said, they want to see students in rows.
Amy:	It is always top down; why not start with us in the trenches, the worker ants?
Naomi:	Because you are not supposed to know any better.
Amy:	But we are supposed to be professionals.

This exchange led the teachers to discuss how they might attempt to bring about changes in the institutional context. During the discussion of generating evidence about students' learning for school leaders, teachers defined the role they felt their school leaders should take as instructional leaders.

Marci:	Well, for one, in order to be an instructional leader, they have to establish a learning community. And by doing that they need to make sure time is allowed for, and I agree with what you said, Amy, they need to know exactly what is taught at each grade level. But part of a teachers being able to assess what these students know, that means we need time to collaborate. It may mean forfeiting a staff meeting once a month and having all the math teachers to work together collaboratively to create or think of solutions to avoid those problems the next time. Or to people, with us working together someone else can give some input into on, "hey I tried this and this really worked, or maybe your students are thinking such and such and such and such is happening," and then you may think, "hey you might be right," and then may be able to go back to drawing board and work with that child and try to get them up. But now principals are to a point where they are focusing now on mastery. Work with some children, if re-teach child same way as before, not get help from colleagues to help that child to improve, then the child is never going to achieve in that area.
Researcher (1):	So to be instructional leaders they need to give you resources such as time.
Marci:	And time is the biggest resource.
Researcher (2):	So their major goal should be to support teachers to get better.
Naomi:	Exactly.

This same topic was raised during the discussion of the assessment articles.

Researcher (1):	Our rationale for sharing articles is to get conversations going, not tell you what you should do.
Naomi:	This is what an instructional leader should be doing to help me.
Researcher (2):	Providing these types of resources?

Naomi: Yes. I have been asking for 3 years for help with asking higher order questions. But in observations they just tell me to ask higher order questions.

Researcher (1): So, basically they are there to assess you not to assist you

The teachers had a clear goal of where they wanted changes to occur and why. Again, this was a notable shift from merely referring to the institutional context in order to explain aspects of their instructional practices.

Year Two—Summer Session—June 5, 6, & 7, 2002

For the summer session in June of 2002, the teachers from Jackson Heights as well as teachers from a second research site met together in Nashville for a joint summer session. In order to clarify the similarities and differences across the two sites, the researchers worked separated with the teachers from each district on an activity designed to make public the perceived supports and constraints in each district.

The activity began by giving the teachers stacks of yellow index cards and asking them to write their thoughts about the things or people that support mathematics learning and teaching within their school or school district. Each support was written on a separate card. A researcher then collected the cards and, together with the teachers, organized the suggested supports into broad categories. These categories were then listed on chart paper and taped to the wall. Next, the teachers were given five blue dots and two red dots. The teachers were instructed to prioritize the categories by placing red and blue dots on the chart paper by the categories they perceived to be the most supportive. The researcher leading the activity clarified that they could place as many of their dots as they desired on any category and also explained that they should use the red dots to indicate those items that they considered to be especially important. This same procedure beginning with index cards was then repeated for things or people that the teachers perceived to hinder the teaching and learning of mathematics within the school district. Upon completing this activity, the two groups of teaching met to compare and contrast their results.

General Participation Norms.

The District Math Coordinator, Esther, joined the group for the first time on the second day of the summer session. Her only prior interaction with the group had been at lunch during the work sessions. However, the teachers treated her as a full member of the community, and challenged Esther's comments in much the same way that they challenged each others' contributions. Esther became visibly upset by these challenges and had to leave the room several times. The teachers were confused by Ester's behavior and asked the researchers why she was upset. The researchers speculated that Esther was not comfortable being challenged. The teachers noted that challenging was a norm of their group and they did not understand why she was offended by it. "We have always challenged each other." This comment provides direct evidence that the teachers were unaware of the dramatic shift in norms for general participation.

Strategic Reasoning Norms.

The results of the activity for Jackson Heights were as follows:

Supports:
District Math Leadership (three blue)
Statistics Project (two red; one blue)
District Math Grant (six red; two blue)
Working with Colleagues (one red; six blue)
Curriculum (one blue)
District Math Specialists (two red; eight blue)
Parents and Parent Organizations

Teachers (four blue)
Administration (one red; five blue)

Hindrances:
State accountability/Testing (one red; four blue)
Lack of Time (two red; seven blue)
Lack of Instructional Time (four red; seven blue)
Lack of Money (three red; two blue)
District Focus on Literacy (one red; one blue)
Parent Attitude and Support
Lack of Qualified Math Teachers (one blue)
Curriculum (one blue)
Lack of Communication (one blue)
Students (four blue)
Teachers (one red; one blue)
Administration (one blue)

According to the placement of the dots, the greatest supports were the District Math Specialists (such as Ruth), the District Math Grant, and Working with Colleagues. Working with colleagues was the form of support that the teachers claimed they needed the most during discussion.

> Wesley: It is hard to have a conversation with your self, in isolation. Having another person makes all the difference in the world, talking about student work. The professional conversations of what is going on with the mathematic has helped me learn what to look for with the kids. When you are collaborating you get the initial goals of mathematics. You get to see what your kids are doing.

The teachers related this to the opportunity to collaborate with planning.

> Wesley: This planning time does not include the traditional idea of making photocopies to plan, rather it is sit down and have professional conversations. We need to get out of box that planning is making photocopies and instead is having these type of conversations.

> Muriel: If everyone could plan together at the same time, it would be less stressful.

> Marci: It is a lack of time and planning, even though Naomi and I have the same planning periods, other stuff pulls me away from this time. She might have to substitute for another teacher or meet with a mentee.

All the teachers listed things that took priority over planning during the planning times: meetings with parents, faculty meetings, Special Education meetings, covering others' classes. They wanted time to plan collaboratively and they wanted this planning time to be protected from other commitments.

The placement of the dots revealed that Lack of Time and Instructional Time were the greatest hindrances that the teachers perceived to the teaching and learning of mathematics in the school and school district. However, as a consequence of the shifts in the strategic reasoning, the teachers now perceived of time as something they could influence.

> Amy: What we did this morning meant something for me. You get a sense of perspective. When you look at what mattered, when you look at the dots, what mattered were the grant and the people who worked with the grant. When we look at what is wrong, it looks like the lack of time and instructional times, those things I have no control of. The rest of it I could swallow for now. So, work on the two things you want the most. If nothing else, I could force the principals to design the schedule for more than 42 minutes [of instructional time] and get some planning time. I don't think that these are unreachable goals.

As Amy's comment indicates, engaging in the activity helped the teachers both clarify important aspects of the institutional context in their district and develop conjectures about possible changes they could make.

After completing the activity, the two groups of teachers met to discuss their results. Although there were several similarities and differences, the most prominent contrast emerged during a discussion of the role of standardized testing at the two sites. For the Jackson Heights teachers, the standardized tests led to enormous pressure to cover the content. In contrast, the primary issue for the teachers from the other district was the loss of instructional time due to testing. This difference raised the issue of how school leaders in the two districts were responding to the accountability pressures of state standardized tests. In the course of this discussion, the teachers and researchers clarified that the administrators in the Jackson Heights district responded by attempting to monitor and assess teachers. In the other district, the administrators responded by giving teachers access to knowledge and resources in order to support their efforts to improve their instructional practices.

Following this joint discussion, the researchers engaged the Jackson Heights teachers in a conversation about their administration by sharing their findings from interviews conducted with the school leaders. The researchers noted that the school leaders viewed mathematics teaching as a routine activity rather than a highly complex and demanding activity that required specialized knowledge. It was for this reason that the school leaders focused on class objectives, content coverage, and management issues, and saw no need for joint planning. This summary of research findings led to a conversation about how to support the school leaders' development of new views about mathematics teaching and learning.

Muriel:	But they see it as give the kids some notes and a little practice, they should have learned it. Then go on to the next day. They don't focus on if kids really understand the concepts.
Researcher (1):	If you view mathematics as a routine activity, one way to approach the principals is to show that this way is not working. One avenue is showing that students not reasoning about quantity the way they are doing it now.
Researcher (2):	If you follow the principals directive it would be a disaster. How are you going to deal with students who don't understand fractions?
Muriel:	Remediate.
Researcher (2):	What does remediate mean?
Muriel:	Teach the same rule similarly.
Researcher (2):	You have to make them understand that doing what they want you to do is really going to screw up those kids' standardized test scores. You are doing them a favor.

Wesley: I am still unsure of what happened to us that got us here. And the same question is what do we need to do to cause that perturbation as we undergird… What was the perturbation that happened to us, some version of that is what needs to happen to the principals.

Muriel: But God, look how long it took!

[laugher]

Muriel: We can't wait that long!

The teachers began to discussing how they might attempt to change the school leaders' vision of mathematics teaching. For the teachers, changing the school leaders vision was important for gaining access to resources for teaching mathematics with a focus on student understanding (e.g. release time for collaboration and professional development, more instructional time). Wesley suggested that the group start by thinking about the school leaders' current views and how the teachers eventually wanted them to think about mathematics teaching. The subsequently discussion resulted in the co-creation of the following learning trajectory for the school leaders.

Where are the school leaders right now?
***Teaching is a routine, predictable process**
***Focus on classroom management and covering content**
***Describe current reform efforts in terms of generalities (e.g. using small group work, manipulatives, and real world problems)**

Where do we want school leaders to be?
***Appreciate teachers' expertise**
***Knowledge or importance of understanding students' mathematical reasoning**
***Importance of collaboration to support focusing on student reasoning**
***See value in doing math**

Intermediate steps with the Principals
***Sensitize to what math teaching and learning is or should be**
***Come to understand and value focus on issues of student reasoning**
***Principals come to see value in making students' reasoning evident**
***Communicate mathematical goals of the curriculum**

After several suggestions about how to achieve some of the intermediate steps, the teachers revisited the fraction interviews.

Amy: The test scores don't necessarily reflect what the kids know. The students are capable of manipulating common denominators with no understanding and get the answers correct on the test. They could not tell how or why.

Researcher: What we need to address are the principals' current assumptions with a task that has face validity so that a non-math specialists would know it should be fractions and know what kids should be able to do. I conjecture it would be a shock.

Muriel: When I look at students' written work it has a lot of holes, a lot of student thinking cannot be determined by simply seeing the papers. Their thoughts are way more off than what I had seen. It blew my mind. Just paper would not show that. Talking to them and seeing what they say makes the difference.

The teachers then made revisions to the fractions tasks and set the following goals for a session with the principals in which they would interview students:

1. Get school leaders to say what they expect to see
2. Have school leaders work through activities themselves
3. Have school leaders conduct the interviews (high Algebra type students)
4. Make sure to relate findings back to what they said in the beginning and if these expectations are sensible

In previous sessions, teachers had expressed their desire to influence the institutional context by making school leaders aware of the conflict between covering content that would be tested and focusing on student reasoning. However, during this session, the teachers were able to state what resources they needed to achieve this goal. In addition, they articulated how the school leaders' vision would have to change if they were to gain access and control over resources that they considered critical. Further, they contributed to the co-creation of a detailed plan for how they might change the school leaders' vision of mathematics teaching.

Summary

In this analysis I have documented the emergence and concurrent learning of a professional teaching community as situated in the institutional context of the school district. I have traced the evolution of norms of general participation, mathematical reasoning, pedagogical reasoning, and strategic reasoning during the first two years of the collaboration with the group of teachers. In this summary, I will review the major changes in each of these four areas.

General Participation Norms.

Initially, the participation structure within the group could be characterized as turn taking with comments directed to the researcher, not each other. The group was a pseudocommunity in that challenges and conflicts were considered a violation of the participation structure. In later sessions, the participation norms were dependent upon the activity. For example, when the teachers were engaged in activities dealing with pedagogy, such as discussion of student work, the participation structure remained in the nature of turn taking. However, when teachers were engaged in statistical data analysis, they would build on other's contributions and direct their comments to each other, not the researcher. At this point, challenges were still considered a breach of the participation norms. After one year of collaborating, interruptions, finishing others' thoughts, and challenges were considered legitimate by the group.

Mathematical Reasoning Norms.

During the initial summer session, the teachers focused on correct conventions of graphs and calculations of measures of center when working on assessment tasks. As the teachers engaged in data analysis with the computer applets, it was normative for the teachers to provide justification for the location of cut points and to reason about the data in terms of percentages and part-whole relationships. The teachers could reason about the data in terms of shape, but did so only when pressed by the researchers. By the second session of the second year, it became normative for the teachers to infer shape from the fixed intervals or four equal groups options on the second computer applet.

Pedagogical Reasoning Norms.

At first, there were evident derivatives of the teachers' regularities of instructional practice as they analyzed student work. The teachers would list the students' solution methods instead of focusing on how the students reasoned about the task. Teachers would also review statistical topics with their students prior to assigning tasks. However, it was normative for teachers to make claims about the relation between their teaching and the student work. Pedagogical justifications were made based on the resulting student outcomes. The teachers evaluated student work based on whether or not they "got" specific mathematical topics. Thus, student work was a record of what students had produced, not a tool for reasoning about student solution methods. During the second session of the second year, a conflict between viewpoints emerged between covering the content and focusing on student thinking. This led to teachers questioning their own instructional practices in later sessions. Merely covering the content became problematic and it became normative for the teachers to focus on student reasoning. Students' use of calculations alone in their solution methods became insufficient. The calculations had to measure something and be produced for a reason.

Strategic Reasoning Norms.

Initially, the teachers viewed the institutional setting as an object of frustration that was beyond their control. During the summer session at the end of year one, the teachers were able to explain aspects of their participation in the sessions based on their perception of the institutional context. In later sessions, teachers voiced the desire to make changes in their instructional practices and the need for specific resources to do so. It became normative for the teachers to perceive of the institutional setting as something they could influence. This led to the co-creation of a conjecture learning trajectory for school leaders. The teachers' goal was to challenge the school leaders' view of mathematics teaching and learning and thus obtain control over essential resources needed to teach mathematics with a focus on student reasoning.

CHAPTER VIII

CONCLUSION

This analysis of the process for supporting the emergence and concurrent learning of a professional teaching community is significant as it builds on prior literature on the importance of professional teaching communities in supporting teacher learning. Although the prior research examines the specific interactions and dynamics by which a professional teaching community constitutes a resource for teacher learning, there are currently no longitudinal empirical studies that analyze the process by which a professional teaching community emerges and subsequently develops in either mathematics or science. In addition, prior research on teacher professional communities has given little attention to the institutional settings in which the participating teachers develop and refine their instructional practices. This analysis articulates the means by which the progression of the professional teaching community was supported and the conjecture-driven adaptations made given the institutional context.

The reader will recall that the criteria I used for distinguishing a group from a community were: a shared purpose, a shared repertoire, and norms of mutual engagement. For this particular professional teaching community, the shared purpose that emerged had a two-fold purpose: 1) ensuring that students come to understand central mathematical ideas while simultaneously performing more than adequately on high stakes assessments of mathematics achievement and 2) identifying, acquiring, and controlling resources to make that possible. The shared repertoire, which was specific to this professional teaching community and the shared purpose, included normative ways of reasoning with computer applet activities, student work, and student interviews when planning for instruction or making students' mathematical reasoning visible. The norms of mutual engagement included the general norms of participation such as building on others contributions, asking clarifying questions, and challenging others assertions. In addition, the norms of mutual engagement encompassed the norms that were specific to mathematics teaching such as the standards to which the members of the community held each other accountable when they justify pedagogical decisions and judgments. For example, it was unacceptable to justified pedagogical decisions based on the need to cover the content for the standardized test. It is important to note that these criteria take as a given the deprivatization of teachers' instructional practice as necessary for the emergence of a professional teaching community and also acknowledge the situatedness of the professional teaching community within the institutional setting of the school and district. Taking all of these criteria into consideration, I claim that this group of teachers did not emerge as a professional teaching community until after 19 months of working together.

These results are specific to this professional teaching community and are not meant to be prescriptive for the emergence and concurrent learning of other professional teaching communities. However, there are aspects of the process that are generalizable for supporting the initial formation and concurrent learning of a professional teaching community. The construct of a conjectured learning trajectory as a way of thinking about the means of supporting the emergence and concurrent learning of a professional teaching community remains pertinent.

The overarching goal, or end point, of the conjectured learning trajectory (support the eventual development of instructional practices in which teaching is a generative, knowledge-building activity with students' reasoning at the center of instructional decision making) remained the guiding principal behind the research team's decision making. This endpoint entails a view of instructional practices that are both complex and demanding and indicate the importance of the formation of a professional teaching community.

The process for identifying starting points of a conjectured learning trajectory also remains essential in understanding how best to support the individual teachers' learning and the emergence of a professional teaching community. It is important to document the participating teachers' current mathematical understandings, current instructional practices, and the institutional context in which they work. As stated previously, this assessment is to emphasize the resources available on which to build on to support and organize the development of the professional teaching community and the learning of the participating teachers. This is in contrast to casting the assessment in deficit terms of what the teachers did not know or were unable to do.

The means of supporting the emergence of a professional teaching community and the concurrent learning that remain pertinent included: deprivatization of instructional practices, problematizing current instructional practices, and making explicit aspects of the institutional context. As I have maintained previously, the deprivatization of instructional practices is essential to the emergence of a group of teachers into a professional teaching community. The activities that aided deprivatization included discussion of student work and the teachers planning and leading an activity with one of the participating teachers' students. At first sharing student work was a high risk activity for teachers as it was perceived as a way to evaluate their instructional practices. However, teachers came to use it as a tool for analyzing student reasoning as opposed to a record of what happened during classroom instruction. When the teachers planned and lead the activity with students, it was not as high risk because it was free of institutional pressures such as meeting certain state objectives.

Teachers must come to view their current instructional practices as problematic in order to motivate changes. Activities that were useful in problematizing instruction included the use of video (TIMSS) and having teachers conduct student interviews. The TIMSS video depicted contrasting methods of teaching in a US and Japanese classroom. This supported discussions about important aspects of teaching and the resources needed, especially since the teachers conceptualized the Japanese classroom as depicting "good teaching." The student interviews were successful in problematizing the teachers' current instructional practices because they were conducted in a content domain (fractions) that the teachers viewed as self-evident and would thus discover unanticipated student conceptions. Although the research team's focus as the time was on documenting the influence of the institutional context on teachers' instruction, the team could have capitalized on the discussions of the fraction interviews to raise the issue of the pedagogical significance of students' reasoning.

From the beginning of the collaboration with the teachers, the research team intended to investigate the institutional context in which the teachers worked. This was based on the philosophy that when making an attempt to support changes in teachers' mathematics instruction, the research team must first attempt to understanding what mathematics instruction is for these teachers. Without understanding the teachers' conceptions, our collaboration is generally ineffective (Simon, et al., 2000). The research team must assume that all teachers' ideas are reasonable and useful from their perspectives (Simon, et al., 2000).

Although it was not the goal of the research team to make aspects of the institutional context an explicit topic of discussions, it became imperative in supporting the emergence of the shared purpose of the professional teaching community. These explicit conversations contributed to the changing perceptions that teachers had about their institutional context. The activities that contributed to this included the discussion of the institutional context at the initial summer session, interviews conducted with the teachers about the affordances and constraints of mathematics teaching and learning in the school and school district, and the creation of the conjectured learning trajectory for school leaders.

The intent of the discussion about the institutional context during the initial summer session was to gain insight about the affordances and constraints of the settings in which the teachers worked. It became a method of documenting the current perceptions the teachers had of the institutional context. The interviews conducted with the teachers that focused on issues pertaining to the institutional context also made explicit the research teams interest in the institutional context. The creation of the conjectured learning trajectory for school leaders during the summer session became a tool for organizing an aspect of the shared purpose (i.e. acquiring and controlling the resources they deemed necessary for teaching with a focus on student understanding.

Again, this analysis is not meant to be a prescription for supporting the emergence and concurrent learning of a professional teaching community. However, the process that I have delineated in this conclusion is generalizable to other cases in that it will enable researchers and teacher educators to adapt the means by which the learning of the professional teaching community was supported to the organizational characteristics of the school system in which they are working in a conjecture-driven manner.

Appendix A

Guiding Issues for Conversations with the Teachers

Note: When appropriate, the discussion should be grounded in the specifics of the observed teaching/lesson. Also, ask the teachers to bring the textbooks and other resources that are currently using to teach statistics.

1. Orientation to students, thinking and what counts as a pedagogical argument
(Use observations of the lesson to frame this part of the conversation).

- How does the teacher take account of students, reasoning when planning for instruction?
- What sort of adjustments does the teacher make during instruction based on observations of students, reasoning?

2. Making students' reasoning/learning visible or observable

- What student reasoning/learning is visible to teachers in their classrooms?
- How do they make this reasoning/learning visible or observable? What artifacts do they use to make it visible?
- When students do not reason/learn as expected, how do the teachers go about accounting for it (i.e., do they locate the source of difficulty in the instructional materials, their teaching, the students, etc.)
- How do their principals (or whomever in the school) reads lesson plans. What do they make visible for the principal? Is the focus on compliance with the Pacing Guide or on what?

3. Current orientation to teaching statistics/understanding of students' statistical thinking
(Use their textbooks etc. as the basis for this part of the conversation)

- Most important topics or goals for statistics instruction
- How are the textbooks and other resources used or adapted?
- What is the influence of district policy or guidelines on statistics instruction?
- Are materials viewed as a blueprint or resource?

4. Informal professional networks

- To whom do they turn for information about students, and what types of information do they seek?
- Who does the teacher go to discuss instructional issues?
- Do math teachers in the school share ideas? How? When?
- Do math teachers observe each other's teaching? (deprivatization of practice)

5. Alignment within/across grade levels

- How is mathematics instruction coordinated and aligned within and across grade levels?
- Is there a shared sense of purpose among teachers in the school? If so, how is that accomplished?

6. Who "calls the shots" with regard to math instruction/Teacher's position within the school/Is the teacher situated in a regime of assistance or a regime of assessment?

- What are the district policies with regard to mathematics instruction (in their view)?
- How do the teachers learn about District policy? (reified means)
    - Curriculum materials
    - State and local guidelines
    - Memos etc.
- Who are the school leaders with respect to math instruction/Who do the teachers view as influential or having clout?
- How are teachers held accountable (see also #6)?
    - Teacher supervision
    - Student assessment
- (What is the influence of the NCTM Standards on their and/or other teachers' mathematics instruction?)

7. Official means of assistance/assessment (particiDotory - reified is addressed in #5)

- What sort of professional development activities do teachers particiDote in?
- Are the teachers expected to share their professional development experiences with other teachers?
- How are teachers supervised/assessed?
- How are students assessed?

8. Diversity (See also 4 above – information about students. Obviously, classroom observations will be critical to fully address this issue as so much is implicit)

- Perceptions of parents, students' families, and neighborhoods.
- Categories they use and which students are in which categories
- Is their tracking/ability grouping in their school?
    - Which types of classes do they teach, and which students end up in which classes?
    - What accommodations/adjustments do they make for these different classes?
    - For which classes/types of students are the various textbooks or resources most appropriate?
- How does their school compare to other middle schools in the district?
    - Are these views widely shared?

(On the basis of these conversations, identify school and district leaders to talk to).

Appendix B

Teacher Survey

Please mark the extent to which you disagree or agree with each of the following.

1: Strongly disagree 2: Disagree 3: Agree 4: Strongly Agree

1. Statistics mainly consists of finding mean, mode, median, and graphing skills.
   1 2 3 4

2. When teaching the topic of statistics, one of my goals is to present the material in terms of real life situations.
   1 2 3 4

3. One of my goals when teaching statistics is to ensure that students are capable of reaching the correct answer.
   1 2 3 4

4. I have taught statistics to my middle school students within the past three years.
   1 2 3 4

5. Each year that I have taught statistics the content has remained constant.
   1 2 3 4

6. A goal that is really important for me is to get all the material/topic covered.
   1 2 3 4

7. I am sometimes unclear where students' answers come from.
   1 2 3 4

8. My goal is not only to help my students get the right answer. It is also very important to me that I understand how they get it or why they don't get it.
   1 2 3 4

9. I use the Scott Foresman textbook as the main guide when planning my lessons.
   1 2 3 4

10. I use Mathscapes as the main guide when planning my lessons.
    1 2 3 4

If you circled either 1 or 2 for question 10 proceed to question 16.
If you circled either 3 or 4 for question 10 proceed to question 11.

Please mark the extent to which you disagree or agree with each of the following.

1: Strongly disagree 2: Disagree 3: Agree 4: Strongly Agree

11. Mathscapes lessons tend to be most effective with advanced students.
1 2 3 4

12. Most of what I learn in Mathscapes professional development addresses the needs of the students in my classroom
1 2 3 4

13. Most topics in the Mathscapes professional development are offered in the school once and not augmented.
1 2 3 4

14. When I need assistance teaching Mathscapes units, I turn to other math teachers in my school.
1 2 3 4

15. When I need assistance teaching Mathscapes units, my greatest resource is the district mathematics coordinator and/or the mathematics specialist.
1 2 3 4

16. I find that the pacing guide is a useful resource when I plan my lessons.
1 2 3 4

17. I base adaptations of my lessons on past successes and failures.
1 2 3 4

18. It is important that I include what is prescribed in the district policy and guidelines in my teaching.
1 2 3 4

19. I have a lot of difficulty in incorporating the district policy and guidelines into my lessons.
1 2 3 4

20. When I discuss students with other teachers, I describe them as advanced, regular, or low, to help others get a sense of their capabilities.
1 2 3 4

21. Math department meetings are used to discuss issues related to student learning.
1 2 3 4

Please mark the extent to which you disagree or agree with each of the following.

1: Strongly disagree 2: Disagree 3: Agree 4: Strongly Agree

22. Faculty meetings are used to discuss issues related to student learning.
1 2 3 4

23. Teachers in this school regularly have informal discussions about teaching and learning.
1 2 3 4

24. Teachers in this school share and discuss student thinking with other teachers.
1 2 3 4

25. Teachers in this school feel responsible to help each other do their best.
1 2 3 4

26. Teachers in this school are really trying to improve their teaching.
1 2 3 4

27. Teachers in this school work together to ensure that their instruction coordinates both within and across grade levels.
1 2 3 4

28. There are frequent opportunities for math teachers to observe each other teaching and provide feedback.
1 2 3 4

29. It is common practice for math teachers to give each other input when planning their lessons.
1 2 3 4

30. I am more comfortable teaching unfamiliar material when I have received assistance from district mathematics coordinator and/or the mathematics specialist.
1 2 3 4

31. In my experience, when students have difficulty with math topics, it is usually because they are unable to remember material that was covered.
1 2 3 4

32. Student misconceptions indicate that I need to reteach.
1 2 3 4

33. Math topics I cover in class are often originated by my students.
1 2 3 4

Please mark the extent to which you disagree or agree with each of the following.

1: Strongly disagree 2: Disagree 3: Agree 4: Strongly Agree

34. Student reactions to the material/problems I present often change the direction of my lesson.

1 **2** **3** **4**

35. When my students can't get it, I will ask them to explain in detail their reasoning, even if it is erroneous.

1 **2** **3** **4**

36. I find that many of the concepts in statistics are just too difficult for most of my lower level students to learn.

1 **2** **3** **4**

37. Which of the following topics do you think are most important to statistics? Please choose three and rank their importance from greatest to least.

collecting data
developing graphs (representing data)
distribution
mean
median
mode
percents
proportions
range
rates
ratios
sampling

Using the following scale:

1: Less than Once a Month
2: Two or Three times a Month
3: Once or Twice a Week
4: Almost Daily

Please mark how often have you had conversations with colleagues about:

1. What helps students learn best?
   1 2 3 4

2. Implementing curriculum?
   1 2 3 4

3. Managing classroom behavior?
   1 2 3 4

Using the following scale:

1: None 2: A Little 3: Some 4: A Great Deal

Please mark how much influence teachers have over school policy in each of the areas below:

1. Hiring new professional personnel.
   1 2 3 4

2. Planning how discretionary school funds should be used.
   1 2 3 4

3. Selecting books and other instructional materials used in classrooms.
   1 2 3 4

4. Hiring a new principal.
   1 2 3 4

5. Establishing the curriculum and instructional program.
   1 2 3 4

6. Determining the content of in-service programs.
   1 2 3 4

7. Setting standards for student behavior.
   1 2 3 4

1. Which of the following are characteristics of the issues discussed at mathematics department meetings (circle all that apply)?

Management issues

Curricular issues

Problem students

Ways to better support students' understanding

District mandated tests

Implementing the curriculum

Memos from the administration

Other___________________

2. Rank the circled items starting with 1 as the item that receives the most attention at department meetings.

_____Management issues

_____Curricular issues

_____Problem students

_____Ways to better support students' understanding

_____District-mandated tests

_____Implementing the curriculum

_____Memos from the administration

3. How often do you meet by department (circle the one that best describes your situation)?

Once a month
Twice a month
Once a week
Once a semester
Once each grading period

Please mark the extent to which you disagree or agree with each of the following.

1: Strongly disagree **2: Disagree** **3: Agree** **4: Strongly Agree**

5. I look forward to department meetings because I find them beneficial to my teaching and my students' learning.
1 **2** **3** **4**

6. Many of the resources I use in my classroom have originated from department meetings
1 **2** **3** **4**

7. I do not think that department meetings benefit my teaching.
1 **2** **3** **4**

8. The teachers in my department are a great resource for my teaching.
1 **2** **3** **4**

9. I have improved my teaching practice as a result of department meetings.
1 **2** **3** **4**

10. One thing I wish we would discuss at department meetings is ____________________.

Appendix C

Guiding Issues for Conversations with Principals/School Leaders

[In the course of the conversation, attempt to discern:
1. The membership of the School Leadership Community and specific responsibilities as they relate to mathematics teaching and learning
2. Their views about mathematics, mathematics teaching (and learning, and teachers' learning]

1. What is their subject matter background/grade level teaching experience?

2. Background on Their School
- Perceptions of parents, students' families, and neighborhoods.
- How does their school compare to other middle schools in the district?
- Are these views widely shared?

3. Major Challenges/Issues
- Making their school safe etc.
- Instructional challenges
- Instructional challenges specifically in mathematics

4. Building Community/Their School as an Institution With a Shared Sense of Purpose/Teacher Autonomy
- What is the role of teachers in their school in:
  - Hiring professional personnel
  - Planning how discretionary school funds should be used.
  - Selecting textbooks and other instructional materials used in classrooms.
  - Establishing the curriculum and instructional program.
  - Setting standards for student behavior.
- To what extent do they expect teachers in their school to work together to ensure that their mathematics instruction coordinates both within and across grade levels.
  - Do they expect teachers to have regular grade-level meetings?
  - Do they expect teachers to use each other as a resource?
  - How do they support/facilitate this (e.g., shared planning time, opportunities to observe others teach)
- What do they see as the function of Departments/What issues do they expect to be addressed in Department meetings?
- How often do they have faculty meetings?
  - What types of issues are addressed (e.g., Management issues, curricular issues, problem students, ways to better support students' understanding, End of Grade test, implementing Mathscapes, Memos from the administration).

5. To Whom Are They Accountable? What Are They Accountable For?
- In general
- Specifically with regard to mathematics instruction

    - End of Grade Test/Reward system
- How do they attempt to meet these obligations with respect to mathematics instruction?
    - How do they hold teachers accountable?
    - How do they make mathematics teaching and leaning visible (e.g., observations, artifacts)?
    - How do they deal with teachers who they perceive are not meeting their obligations? Assistance or assessment?
        - Why do they think this will enable teachers to improve?
- For whom do they make teaching and learning visible beside themselves (e.g., central administration, parents, the State)?

6. How do They Think Teachers can Improve Their Instructional Practices (and What Counts as Improvement)?

- What types of professional development activities do they expect teacher to engage in? Who organizes these activities?
    - Do they expect teachers to share what they learned with colleagues?
- What specific professional development activities do they organize at the school level?

7. What Are the District Policies With Regards to Mathematics Instruction (in Their View)?

- Adoption/use of Mathscapes and the supplementary textbook
    - For which classes/types of students are the various textbooks or resources most appropriate?
- The Pacing Guide
    - Mathscapes summer sessions
    - Mathscapes study groups
- How does Mathscapes fit with their goals/obligations for mathematics?
- What is their relationship with members of MLC?

8. Cultural diversity and equity

- How are students constructed:
    - In their school (ability groups tracking or whatever, and which students end up in particular tracks)

Appendix D

Guiding Issues for Conversations with District Mathematics Leaders (MLC)

1) History/backgound to the District
- The history of the district/schools. In NC, this includes the combining of the two districts and the problems/issues involved and the administrative/policy procedures developed in response.
- What are their views of differences between various schools?
- What are their views of parents, students' families, and neighborhoods?

2) Cultural diversity and equity
- How are students constructed:
    - at the school level (ability groups tracking or whatever, and which students end up in particular tracks)
    - at the district level (types of schools and which students attend which schools).

3) Adoption of Mathscapes
- How was Mathscapes adopted?
- What is the MLC's rationale for adopting Mathscapes?
- What are the MLC's assumptions /conjectures about the consequences of adopting Mathscapes for:
    - students' reasoning/learning?
    - Cultural diversity and equity?

4) How does the MLC attempt to achieve its agenda using:
- Professional development (particiDotion).
    - What happens at Mathscape summer training sessions?
        - goals/intent in terms of teachers' instructional practices and students' learning.
        - specific activities and their rationales.
    - Study groups
        - goals/intent in terms of teachers' instructional practices and students' learning.
        - Specific activties and their rationales
- Curriculum materials and associated resources (reification)
    - How do they anticiDote that teachers will use these resources?
- Other means that they use to communicate their intentions to teachers.
- More generally, what is their implicit model of teachers' learning etc. (see Spillane paper).

5) How is the MLC's agendas linked to and influenced by:
- State policy
- National policy (i.e., NCTM Standards).
    - How do members of the MLC interpret the Standards (e.g., do they interpret the Standards in terms of demathematized forms -- see the Spillane paper).
- What comDotibilities and conflicts do members of the MLC see between state and/or national policy and their agenda.

6) What classroom learning and teaching is visible to the MLC and how is it made visible?
- What artifacts does MLC use to make classroom learning and teaching visible?
- What is observable for member of MLC in various records, documents, classroom visits, etc.
- What do EoG scores make visible for MLC?
- For who are MLC making classroom learning visible besides themselves (e.g., the district superintendent, the State, funders?)

[Note that this set of issues encompasses teacher supervision (reification) student assessment (reification) as instruments of policy impementation]

7) How and to what extent does the MLC adapt its agenda?
- With respect to students' learning, what constitutes trouble for MLC?
- When trouble arises, how do members of the MLC go about attempting to account for it (i.e., do they locate trouble in the instructional materials, the teachers, the students, professional development, etc.)

8) To what extent does the MLC:
    - Support teachers' roles in school/district decision making as it relates to mathematics instruction.
    - Foster the school/district as an organization with a shared sense of purpose.

Appendix E

List of Session Activities

Initial Summer Session—July 11-12, 2000
Introductions Discussion
How Much TV?
Fuel Mileage
Potato Chips
Batteries— Applet One
Braking Distance—Applet One
Speed Trap—Applet Two

Year One, Session One—September 21, 2000
Review of Initial Summer Session
Overview of Sequence
Student Work—How Much TV?
Cholesterol—Applet Two

Year One, Session Two—November 11, 2000
Cholesterol—Applet Two
Airbags—Applet Two
AIDS—Applet Two

Year One, Session Three—February 8, 2001
Student Work—Batteries
Migraine—Applet Two
CO2/Speed
Tires—Applet Three

Year One, Summer Session—June 4,5, & 6, 2001
Ambulance—Applet Two
Reaction Time—Applet Three
Student Work (Research Team)—Watermelon
Student Work (Research Team)—Speed Trap
Student Work (Research Team)—AIDS
Video Clip from 7th Grade design experiment classroom—Batteries
Video Clip from 7th Grade design experiment classroom —AIDS
Student Work—CO2/Speed
Tires—Applet Three

Year Two, Session One—September 20, 2001
Cotton—Applet One
Co-Creation of Benchmarks
Student Work—Watermelon & Braking Distance

Year Two, Session Two—October 25, 2001
Revisit Benchmarks
Recycling—Applet Two
Student Work (Research Team)—Migraine
Student Work—Batteries

Year Two, Session Three—November 27, 2001
TIMSS Video
Student Work—Teachers choose task
Plan and Lead Batteries Activity with Wesley's Students

Year Two, Session Four—January 31, 2002
Student Interviews—Fractions
Video Clip from 7th Grade design experiment classroom —Data Creation Batteries
Video Clip from 8th Grade design experiment classroom —Data Creation Reaction Time

Year Two, Session Five—February 28, 2002
Potato Chip
Allowance
Student Work—How Much TV?
Student Interviews—Potato Chip & Allowance
Benny Article

Year Two, Session Six—March 19, 2002
Generating Evidence Discussion
Assessment Articles

Year Two, Summer Session
Dots Activity
Joint Discussion
Student Work—Fractions
Creation of CLT
Revision of Fraction Interviews
Induction of New Members Discussion

Appendix F

Re-created Student Work from Watermelon Activity—Applet One

Watermelon Juice
Students' Reports

Student Report 1

We added them up and we saw that the Tropical Days were better because they were more, and you want the most pounds. Tropical Days - 125 Good!!! Fresh Fruits - 110 Bad ☹

Student Report 2

The Fresh Fruits are more consistent because they are all between 12.2 and 9.7, so you want to buy those because you know what to expect. The Tropical Days can be anything from 6 to 18 pounds!!!!!!

Student Report 3

I think that the Fresh Fruits' Watermelons are better because they are all over nine pounds, and if you buy those you are going to get one that is at least 9 pounds, and with the Tropical Days, even though they have good ones they also have some very low it seems to me.

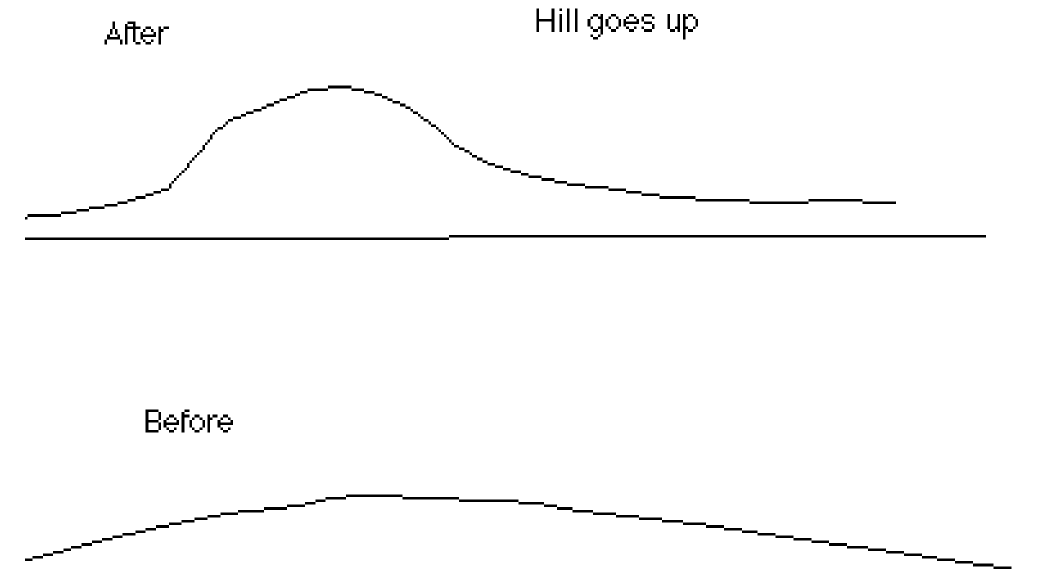
Student Report 4

The Tropical days have the higher mean so they are better because the average is higher: 12.7 vs. 11.3

Appendix G

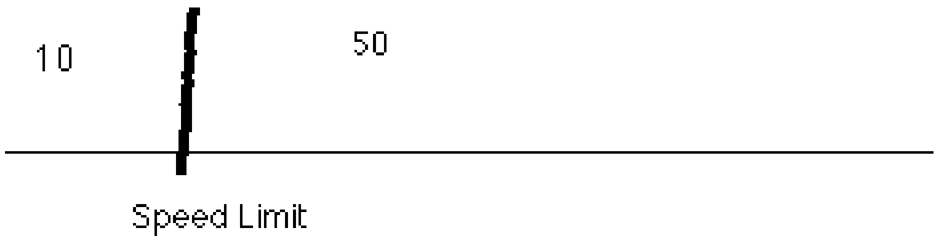
Re-created Student Work from Speed Trap Activity—Applet Two

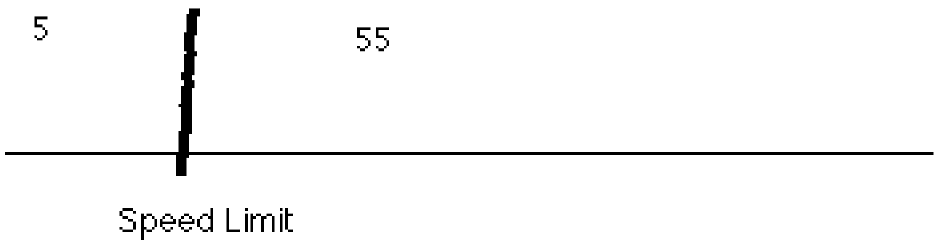
Speed Trap
Student Report 1



We saw that in the graph that is after they did it, the speeds bunched up more where it is lower, kind of like a hill goes up there, so the police did their job because now more people are not going very fast like before they did it.

Speed Trap
Student Report 2





We looked at how many were going at the speed limit before and after they did the speed trap and we saw that before 5 people were going below 50 and now 10 are going below. So it kind of worked but not very well because almost everybody is still going faster than the speed limit.

Speed Trap
Student Report 3

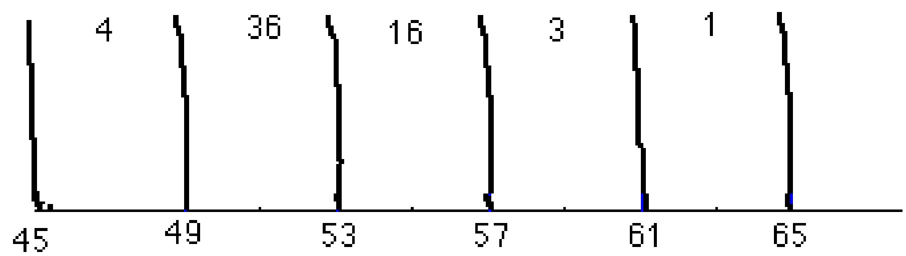
We used the interval thing and we looked how many people were going between 50 and 55 and we saw that after there is more people. We think that the police thing worked because now more people are not speeding.

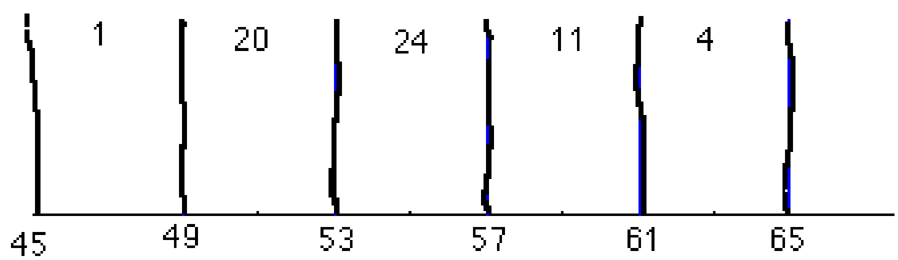
From 50 to 55

Before: 30

After: 42

Speed Trap
Student Report 4





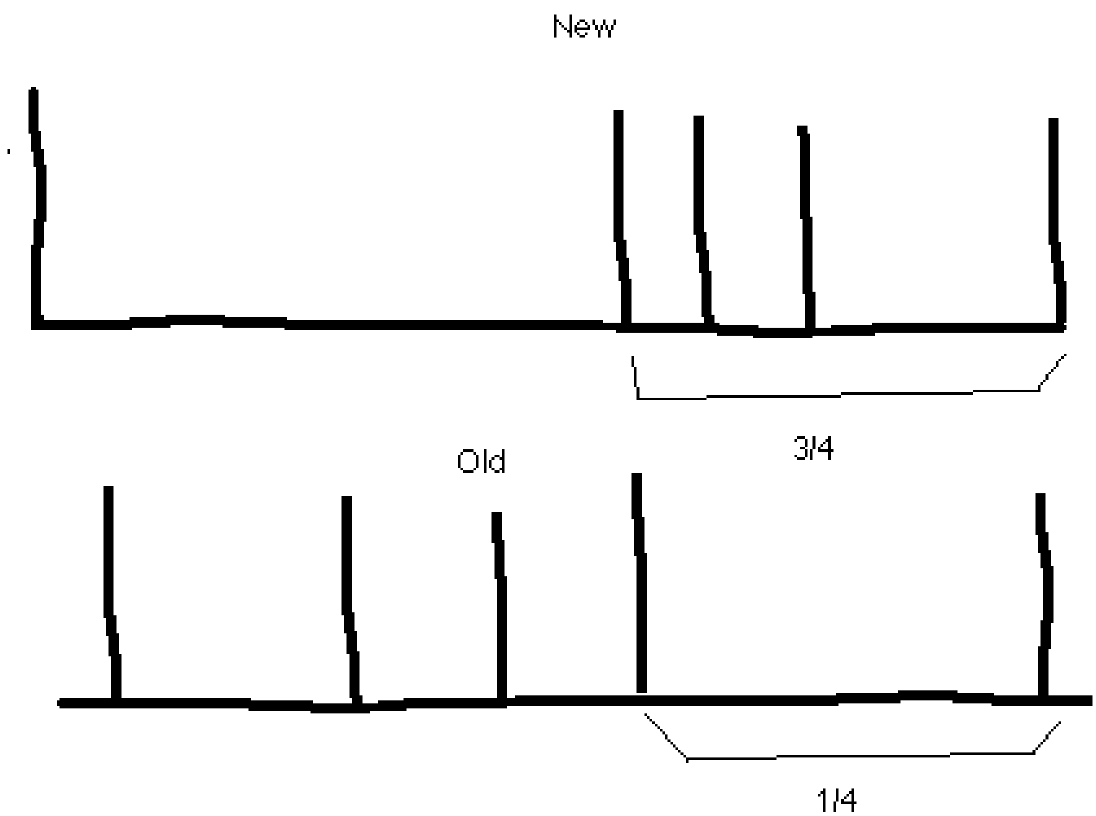
If you make intervals of 4 then you see that before the most people were between 53 and 57 and now they moved down to 49 and 36, so that shows that people are driving slower. The speed trap worked.

Appendix H

Re-created Student Work from AIDS Activity—Applet Two

AIDS
Student Report 1

I think that the new drug is better because 3/4 of the people that used it got more high scores vs. 1/4 of the old drug, so I think that the doctors should use the new drug because it looks to me that it works better for most of the people.



AIDS
Student Report 2

The most group of the old drug is lower than 550 (approx.) and the group of the new drug is above 55O, so that shows that the new drug is better because most people get more cells by using that drug.

Old Drug
200 - 550
151 THE MOST

500 – 850
35

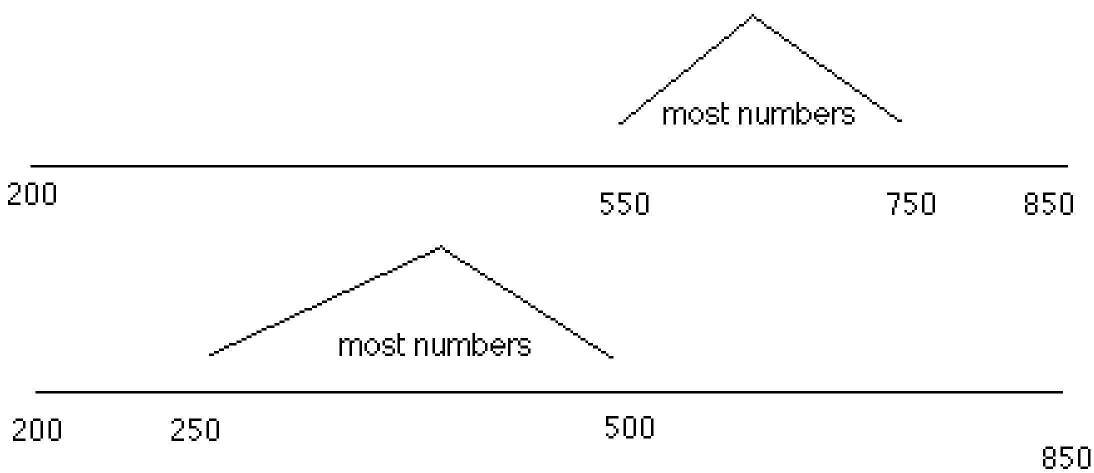
Experimental Drug
200 - 550
10

500 – 850
36 THE MOST

AIDS
Student Report 3

Dear Doctor Medical Director

In the analysis I did of the two drugs I saw that the new drug works better because if you look at the hills where the most numbers are, the hill of the new drug is higher and that means that it works better. I recommend that you give the new drug to everyone.



AIDS
Student Report 4

I did the intervals and it shows that the new drug is better because the groups with more people are higher in the new drug, and in the old drug are lower. I think that the hospital should recommend the new drug because more people get more.

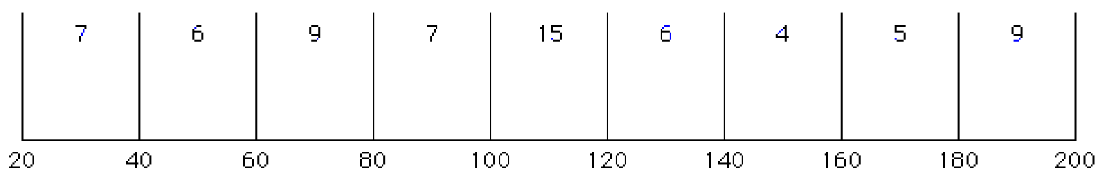
	OLD	NEW
200-300	16	4
300-400	44 big group	4
400-500	50 big group	1
500-600	45 big group	12 big group
600-700	16	17 big group
700-800	15	5

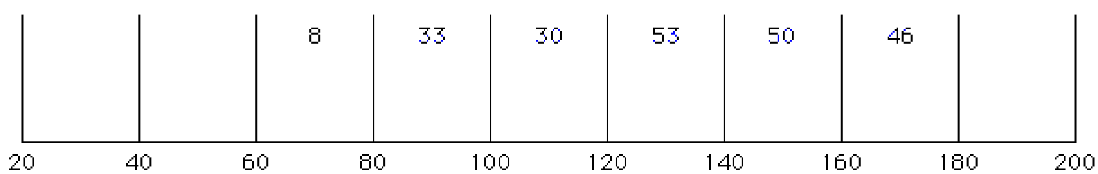
Appendix I

Re-created Student Work from Migraine Activity—Applet Two

Migraine
Student Report 1

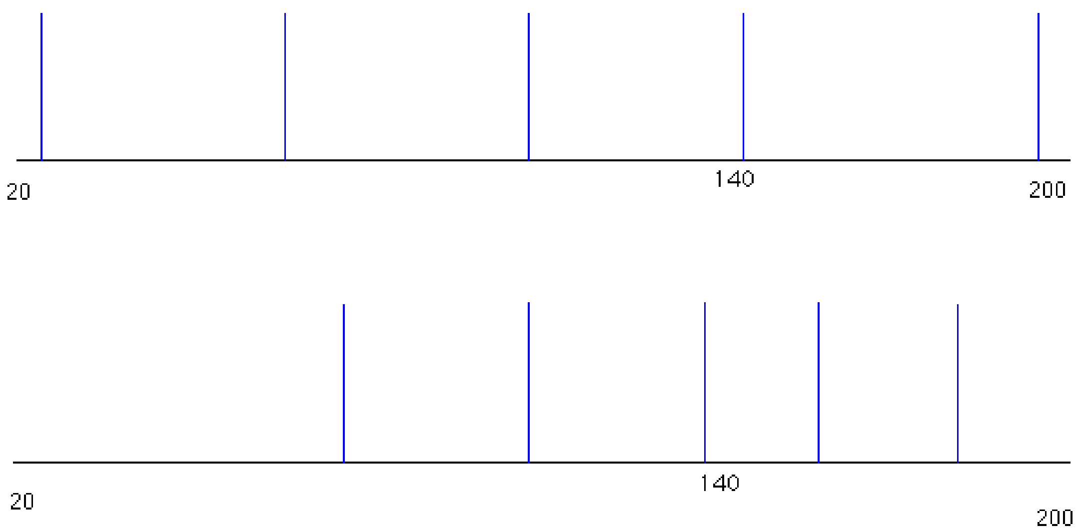
In the traditional drug the numbers are more packed up between 120 and 160, and in the new drug between 100 and 120 so the new drug is better because more people are getting rid of the headache more quickly. I would recommend that people use the new drug but it will not be good for everyone.





Migraine
Student Report 2

The new drug is better because 3/4 of the people got relief in about 140 minutes, and only half of the old drug got relief in 140 minutes. We think that the hospital should use the new drug.



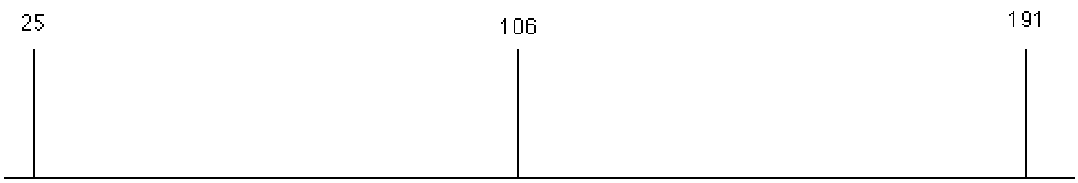
Migraine
Student Report 3

The old drug is more consistent than the new drug. The new drug worked well for some of the people but not for everyone. I think that the hospital should stay with the old drug because that way they can tell people what to expect. With the new drug people can get fast relieve but it can also take longer.

New drug: 24 to 193 NOT CONSISTENT
Old drug: 73 to 179. More consistent: 123 to 179

Migraine
Student Report 4

With the new drug it worked for half of the people in 106 minutes or less and the old drug in 136 minutes or less. It seems to me that the new drug works faster and the doctors can use that one.



Appendix J

Fraction Tasks used for Student Interviews during Year Two, Session Four—January 31, 2002

Lauren, Michael, and Lawrence are training for a Walk-a-Thon from Durham to Raleigh to raise money for cancer research. On the first day of training Lauren walked ⅓ of the distance, Michael walked ½ of the distance, and Lawrence walked ¼ of the distance. Volunteers are being placed on training teams according to their first day results. Who walked the farthest distance? Who walked the shortest distance?

Lauren $1/3$ of distance

Michael $1/2$ of distance

Lawrence $1/4$ of distance

Dana, Marcus, and Juanita have joined a beginning gymnastics class. Today they are learning how to walk on the balance beam. Dana walked across ½ of the beam before losing her balancing, Marcus walked ⅝ of the beam, and Juanita walked ¾ of the beam. The gymnastics teacher wants to give certificates for 1st, 2nd, and 3rd place according to who walked the longest across the beam. Which place goes to each student?

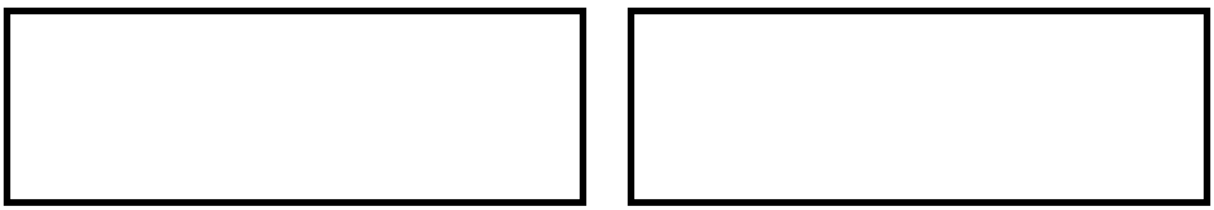
Dana $\frac{1}{2}$ of beam

Marcus $\frac{5}{8}$ of beam

Juanita $\frac{3}{4}$ of beam

Luis gets 1/2 of a candy bar and Jackie gets 2/3 of a bar. Who gets more? How much more?

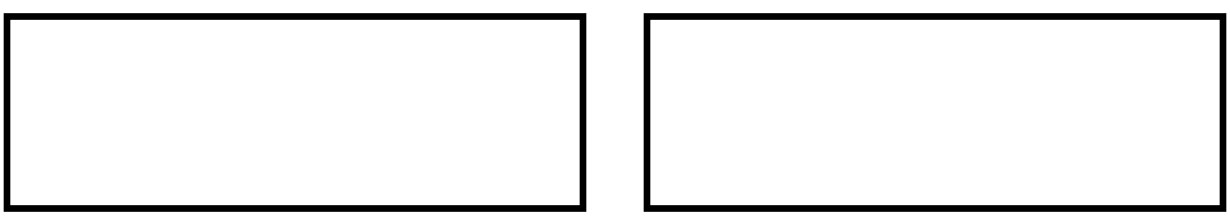
Luis Jackie



Maria gets 2/3 of a candy bar and Tony gets 5/6 of a candy bar. Who gets more?

How much do they get altogether?

Maria Tony



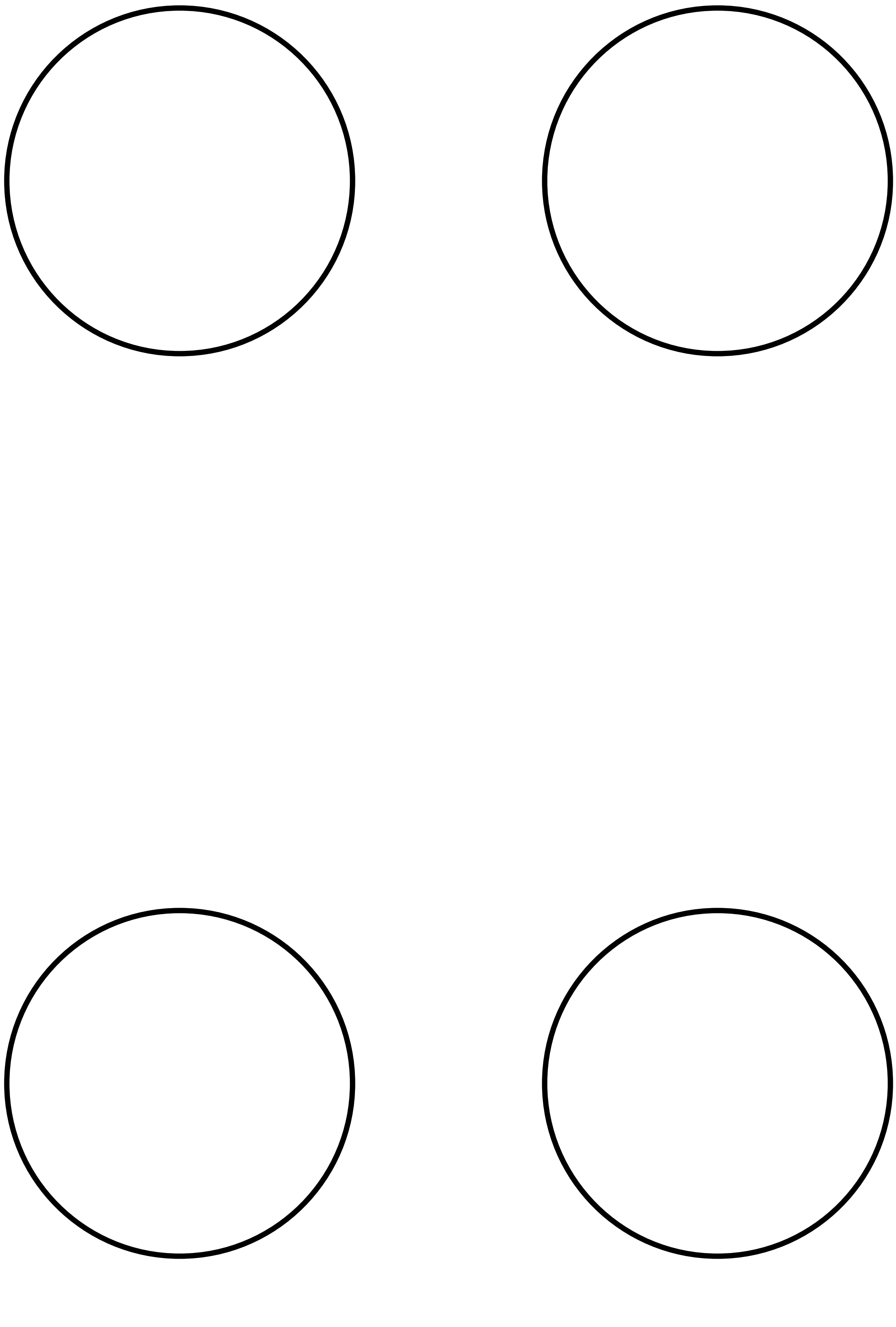
Anne gets 3/4 of a candy bar and Jeff gets 5/6 of a bar. Who gets more? How much more?

How much do they get altogether?

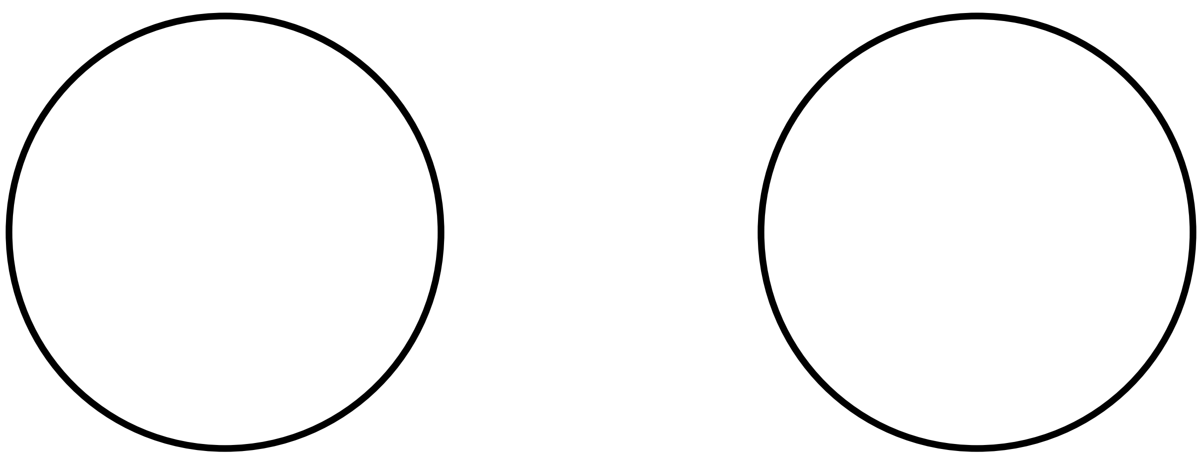
Anne Jeff

One pizza is divided into 1/4s and another is divided into 1/5s. If I take a piece from each pizza, how much pizza do I eat?

One pizza is divided into 1/3s and another is divided into 1/5s. If I take two pieces from each pizza, how much do I eat?

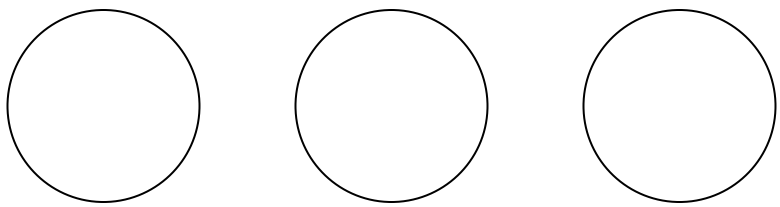


One pizza is divided into 1/5s and another is divided into 1/7s. Josh takes two pieces from the first pizza and Karen takes two pieces from the second pizza. Who eats more? How much more?

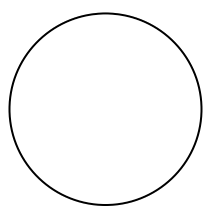


At the pizza house, several groups of people come for dinner. Show how the pizzas are distributed at each table.

1) 8 friends sit at the same table and order 3 pizzas to share equally. How much does each person get?

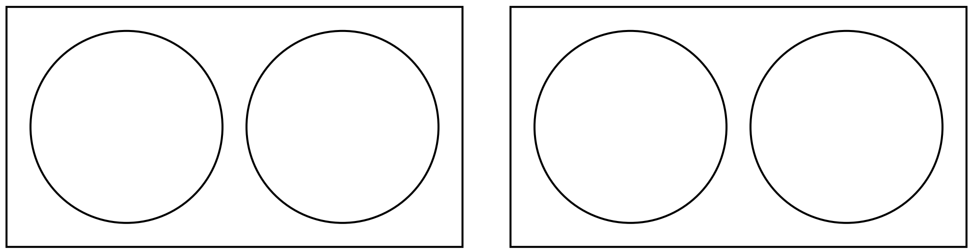


2) After they finish the first 3 pizzas, they order on more pizza to be shared equally. How much pizza does each person get to eat at this time?



3) How much did each person get to eat for dinner altogether?

4) Another group of 8 people sit at two separate tables. Four people sit at each table. Each group orders 2 pizzas. How much pizza does each person get to eat?



Place the following fractions in order from smallest to largest:

1/3 1/2 1/4

Place the following fractions in order from smallest to largest:

1/2 5/8 3/4

Which is larger? 2/3 or 3/5

3/4 or 5/6

Solve the following problems:

1/4 + 1/5=

2/3 + 2/5=

2/5 - 2/7=

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