Cognitive Predictors of Calculations and Number Line Estimation with Whole Numbers and Fractions

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CHAPTER I

INTRODUCTION

Fraction knowledge is one of the foundational forms of competence required to perform successfully in more complex and advanced mathematics, such as algebra (Booth & Newton, 2012; NMAP, 2008). In a longitudinal study examining the types of mathematical knowledge that predict later mathematical achievement in the United States and United Kingdom, students' fraction knowledge in fifth grade uniquely predicted their algebraic knowledge and overall mathematics achievement in high school, even after controlling for other types of mathematical knowledge, general intellectual ability, working memory, family income, and education (Siegler et al., 2012). Its predictive value compared favorably to whole-number addition, subtraction, and multiplication.

Yet, fractions is one of the most difficult mathematical topics to master (e.g., Bright, Behr, Post, & Waschsmuth, 1988; Lesh, Behr, & Post, 1987; Mack, 1990; Test & Ellis, 2005). Difficulty in understanding fractions is not new. In a national survey of algebra teachers, teachers reported that fractions is one of areas students have the poorest preparation (Hoffer, Venkataraman, Hedberg, & Shagle, 2007). Furthermore, more than 40 years of data from the National Assessment of Educational Progress (NAEP) have consistently indicated that students struggle with fractions. For example, results from 1996 NAEP indicated that only 49% of fourth -grade students correctly identified how many fourths are in one whole. In 2013 NAEP, only 60% of fourth-grade correctly identified the greatest fraction of three fractions with one in the numerator.

The difficulty with fractions has been often attributed to the fundamental differences between whole numbers and fractions. For example, there is no predecessor and successor of a fraction, and adding and subtracting fractions require a common denominator. In addition, quantities decrease with multiplication and increase with division in fractions (Stafylidou & Vosniadou, 2004). Thus, learning fractions has been considered different from and discontinuous with students' understanding of whole numbers, leading to the potential conflict between students' prior knowledge about whole numbers and new information about fractions (Cramer, Post, & delMas, 2002; Cramer & Wyberg, 2009; Sigler, Thompson, & Schneider, 2011).

Developmental Pathways

Despite these fundamental differences between whole numbers and fractions, evidence suggests that they may follow similar developmental paths. Using a nonverbal procedure of assessing calculation ability, Mix, Levine, and Huttenlocher (1999) suggested that whole-number knowledge and fraction calculation competence develop similarly in young children. They found that three to seven years old children's competence with whole-number and fraction calculations followed the same gradual rise in performance, not an abrupt shift of performance at a particular age. Furthermore, they found that understanding of important ideas about fractions is evident in children as young as four years old.

Additionally, according to a recently proposed integrated theory of numerical development, fraction understanding develops as students broaden their understanding of whole numbers to include magnitudes of fractions with specific locations on a number line. That is, Siegler et al. (2012) found that as with whole numbers, sixth and eighth graders' accuracy of fraction magnitude representation was strongly correlated with fraction calculation fluency and overall mathematics achievement. They argued that although whole numbers and fractions differ in many ways, their development requires an important commonality about understanding magnitudes. Siegler et al. further argued that fractions and whole numbers should, therefore, be considered within a single numerical developmental framework.

Thus, fraction and whole-number competence may develop in similar ways and therefore may rely on the same abilities. Prior studies examining cognitive abilities that underlie whole-number and fraction competence, namely calculation competence, provide some insights on the developmental pathways. That is, whole-number and fraction calculations draw upon shared cognitive abilities, such as working memory, attentive behavior, processing speed, and nonverbal reasoning. Yet, some evidence indicated that distinct cognitive characteristics also may underlie each form of competence, suggesting that differences between two calculation domains exist.

Prior Work on Potential Cognitive Predictors of Whole-Number and Fraction Calculations

Although only a few studies have investigated cognitive predictors of fraction calculations, prior research provides evidence for five cognitive characteristics that may affect both whole-number and fraction calculations: working memory, attentive behavior, processing speed, phonological processing, and nonverbal reasoning. In addition to these common predictors, language has been documented to uniquely affect fraction calculations.

Working memory. Studies consistently found that working memory, in a general sense, predicted whole-number calculation competence (e.g., Alloway, 2006; Fuchs et al., 2005, 2008, 2010b, 2013; Seethaler et al., 2011; Swanson, 2006; Swanson & Beebe-Frankenberger, 2004). Working memory provides temporal storage of information to support ongoing cognitive tasks (Baddeley, 1986). Whole-number calculation procedures require regulating and maintaining arithmetic combinations derived either through retrieval from long-term memory or by relying on counting while simultaneously attending to regrouping demands and place values. Therefore, students with low working memory would have difficulty holding sufficient information to complete a task (e.g., keeping track of where they are in a task; Alloway, Gathercole, Kirkwood, & Elliot, 2009).

As with whole-numbers, working memory has also been found to be a unique predictor of fraction calculations. Prior work suggests two potential mechanisms that may explain this connection. Working memory may influence whole-number arithmetic calculations, which in turn influences fraction calculations as in Hecht et al., (2003). This may be reflective of the hierarchical nature of whole-number and fraction calculations. That is, students' fluency with whole-number calculations is fundamental to executing more complex procedures for solving a fraction calculation problem, such as adding two fractions with different denominators. At the same time, working memory may also influence fraction calculations beyond its effects through whole-number calculations (Jordan et al., 2013; Seethaler et al., 2011). That is, besides supporting whole-number calculation tasks embedded within fraction calculations, working memory may help students regulate the interacting role of numerators and denominators as well as the planning and executing multiple steps to find common denominators and equivalent fractions when adding or subtracting fractions with different denominators.

Attentive behavior. Attentive behavior is an important cognitive predictor of whole-number calculations (e.g., Fuchs et al., 2005, 2006, 2008, 2010a, 2013; Swanson, 2006). In both Fuchs studies (2005, 2006, 2008, 2010a, 2013), in which teacher ratings of attention were used, and in Swanson (2006), in which direct measures of attention were used, attentive behavior was uniquely predictive of both arithmetic and procedural calculations. Given that considerable attention is necessary to execute calculation procedures and monitor errors simultaneously, it is not surprising that attentive behavior is a key determinant of whole-number calculations.

Attentive behavior has been also found to be a determining factor in fraction calculations (Hecht et al., 2003; Hecht & Vagi, 2010). As with working memory, attentive behavior appears to influence fraction calculations in two ways. In Hecht et al. (2003), attentive behavior influenced fraction calculations via fraction concepts and whole-number arithmetic knowledge. This suggests attentive students may perform better at whole-number arithmetic calculations, which in turn has a positive effect on fraction calculations because whole-number calculations tasks are embedded within fraction calculations. Furthermore, Hecht and Vagi (2010) provided evidence that attentive behavior also influences fraction calculations above and beyond its effects through whole-number calculations. Even greater attention may be required to carry out complex fraction calculation procedures, such as attending to the interacting role of numerators and denominators, and converting fractions so that fractions have the same denominators before carrying out addition or subtraction operations.

Processing speed. Processing speed, which refers to the efficiency with which cognitive tasks are executed (Bull & Johnston, 1997), is another leading candidate. In whole-number calculations, processing speed significantly predicted arithmetic and procedural calculations in Fuchs et al. (2006, 2008). Processing speed may facilitate the simple processes, such as counting or retrieving arithmetic facts from long-term memory (Bull & Johnston, 1997; Geary, Brown, & Samaranayake, 1991), which are required in whole-number calculations. Faster processing supports more automated mathematics performance, which permits more efficient processing of the mathematics, and this in turn improves performance (Bull & Johnston, 1997).

In fractions, only one study has investigated processing speed as a unique predictor of fraction calculations but found nonsignificant effects (Seethaler et al., 2011). However, there is evidence that processing speed may influence fraction calculations. In Fuchs et al. (2013), processing speed moderated students' responsiveness of fraction instruction for fraction calculations. That is, tutoring effects on fraction calculations decreased because control students with superior processing speed benefited more from classroom instruction. Although it is not clear under what mechanism processing speed affects fraction calculations due to the limited literature, this finding suggests that students with slower processing speed may experience challenges with fraction calculations in classroom instruction.

Nonverbal reasoning. Nonverbal reasoning refers to the ability to identify patterns and relations and to infer and implement rules (Nutley et al., 2011), and it allows students to organize and form stable representations of quantitative and qualitative relations among numbers in calculations (Primi, Ferrao, & Almeida, 2010).Whereas nonverbal reasoning has been consistently documented to affect word problems and general mathematics achievement, less consistent findings have been reported with whole-number calculations. Although researchers failed to find significant effects of nonverbal reasoning on whole-number calculations in four Fuchs et al. studies (2005, 2006, 2010a, 2010b), a recent study identified nonverbal reasoning as a unique contributor of whole-number calculations (Seethaler et al., 2011). Nonetheless, nonverbal reasoning was found to moderate responsiveness to first-grade calculations tutoring in a recent study by Fuchs et al. (2013).

Similarly, mixed findings exist in fractions. Although prior research failed to find significant effects of nonverbal reasoning on fraction calculations (Jordan et al., 2013; Fuchs et al., 2013), one study identified nonverbal reasoning as a unique predictor of rational number calculations, which include percents and decimals in addition to fractions (Seethaler et al., 2011). Despite the weak evidence, nonverbal reasoning is important to consider because it may play an important role in expanding and reorganizing students' initial knowledge of whole numbers to include fractions. More studies are needed confirm the role of nonverbal reasoning in fraction calculations.

Phonological processing. At the same time, prior studies (e.g., Fuchs et al., 2005, 2006) found phonological processing to be a unique predictor of whole-number calculations. Phonological processing abilities are required whenever phonological name codes of numbers are used (Geary, 1993). For example, students first convert numbers and operators of a calculation problem to a verbal code. Then, students must process the phonological information and either retrieve a phonologically based answer from long-term memory or use counting strategies to derive an answer, which are both dependent on phonological processing abilities (Hecht et al., 2001). Phonological processing skills may also play a role in the acquisition of arithmetic combinations as students orally practice repeating the problem until the problem stem/answer is committed to long-term memory and can be automatically retrieved (Robinson, Menchetti, & Torgesen, 2002). During this process, students form the associations between the arithmetic fact and phonological representations of the words, such as “four times three is 12.” Reliable connections facilitate both memorization and recall of the facts (Robinson et al., 2002). That is, if students have weekly connected and encoded representations of the numbers “four,” “six,” and “12,” it is harder to commit to memory the sequence of phonological representations, “four times six is 12.” This, of course, would make automatic recall of facts difficult.

Yet, the role of phonological processing has never been investigated for fraction calculations. However, phonological processing may be an important factor to consider because it is possible that strong phonological processing abilities facilitate fraction calculations by helping students to establish representations of fractions and fraction names. More studies are needed to investigate the potential role of phonological processing in fraction calculations.

Language. Lastly, in contrast to whole-number calculations, in which nonsignificant effects have been consistently documented for language (e.g., Fuchs et al., 2005, 2006, 2008, 2010a, 201b, 2013; Seethaler et al., 2011), language, in the form of vocabulary and listening comprehension, was found to support fraction calculations (Fuchs et al., 2013; Seethaler et al., 2011). Unlike whole-number calculations, fraction calculations require processing of the interacting role of numerators and denominators beyond adding and subtracting whole numbers (i.e., numerators). These processes, such as finding the same denominators and converting fractions with the same denominator, require conceptual understanding of fractions in addition to the ability to carry out rote calculation procedures. Prior research demonstrated that conceptual understanding of fractions is supported by language (Miura et al. 1999; Paik & Mix, 2003). For example, Miura et al. (1999) found that first and second grade Korean children, whose language transparently expresses part-whole concepts in fraction names, performed significantly better at associating fractions with their pictorial representations even prior to formal fraction instruction compared to those in the States, where their fraction-naming system do not directly support the part-whole concept.

As reflected by these relations between language and conceptual understanding of fractions, students with strong language ability may gain deeper conceptual understanding compared to those with weak language ability. Better understanding of fraction concepts may in turn facilitate fraction calculations. This is demonstrated in the literature where conceptual and procedural understandings were found to influence each other iteratively (e.g., Rittle-Johnson & Siegler, 1998; Rittle-Johnson, Siegler, & Alibali, 2001). More specifically, conceptual knowledge of fractions strongly influenced fraction calculations in Hecht and Vagi (2010).

Taken together, evidence may be converging on which cognitive characteristics are shared or distinct for whole-number and fraction calculations. That is, whole-number and fraction calculations seem to draw on working memory, attentive behavior, processing speed, and nonverbal reasoning, but language appears uniquely predictive of fraction calculations. Further investigation is, however, warranted for several reasons. First, only limited studies exist for cognitive predictors of fraction calculations. Second, conflicting findings exist with each cognitive factor because (a) some studies have not considered all cognitive abilities in their analysis (e.g., verbal working memory in Alloway, 2006; numerical working memory and attentive behavior in Hecht et al., 2003 and Hecht & Vagi, 2010), and (b) methodological differences (e.g., different outcome measures and study participants) exist across the literature. In fact, only one exploratory study (Seethaler et al., 2011) has considered cognitive predictors of whole-number and fraction calculations within the same study and thus with the same predictors and methodological features for both outcomes. However, even so, because two separate regression analyses were used for whole-number and fraction calculation outcomes, comparing the predictors across both outcomes was not possible.

Purpose of the Present Study

To address these limitations and extend the literature, the purpose of the present study was to examine the cognitive predictors associated with calculations and number line estimation with whole numbers and fractions. I chose calculations as an outcome because whole-number calculations are one main component of the primary-grade mathematics curriculum and represent a common deficit students experience. Also, difficulty with fraction calculations has been found to be persistent and stable. For example, in a recent study (Siegler & Pyke, 2012), low-achieving students' accuracy in solving fraction calculation problems remained similarly low across sixth through eighth grades compared to that of high-achieving students, despite that both groups had been in the same classrooms. Low-achieving students in both sixth and eighth grades primary relied on whole-number strategies to solve fraction calculation problems compared to high-achieving students.

I chose number line estimation as a contrasting outcome because students' ability to approximate numbers on a number line is another important form of mathematical development. Additionally, accuracy on number line representations has been found be a significant predictor of mathematics achievement and whole-number calculations (e.g., Booth & Siegler, 2006, 2008; Schneider, Grahber, & Paetsch, 2009; Siegler & Booth, 2004). As with whole numbers, accuracy of fraction magnitude representations is closely related to fraction calculation competence and overall mathematics achievement (Siegler et al., 2011; Sieger & Pyke, 2012). Despite the importance, limited studies exist regarding the underlying cognitive mechanisms of number line estimation. One study (Jordan et al., 2013) has examined cognitive predictors of the ability to approximate whole numbers on number lines and found that language, nonverbal reasoning, attention, working memory, reading fluency, and calculation fluency significantly predicted whole-number number line estimation skills. Because of the limited literature, I took an exploratory approach and examined whether the cognitive predictors that are found to be important to calculation skills also predicted whole-number and fraction number line estimation, and whether there were differences between the cognitive mechanisms that underlie whole-number versus fraction number line estimation.

The current study therefore extended the literature in three ways. First, the contribution of all of the potentially relevant, previously defined cognitive factors (i.e., numerical working memory, verbal working memory, language, attentive behavior, processing speed, nonverbal reasoning, and concept formation), were simultaneously assessed. Considering all important cognitive abilities that may affect calculation competence provides a more accurate and stringent test of each ability's contribution. This allows us to estimate unique contributions because effects of a cognitive ability in presence of other competing abilities may be different from when tested alone. However, we note that although we included a more complete set of previously identified cognitive resources, it is possible that there may be other important cognitive factors that have not been addressed in the literature yet. We also note that although we measured the cognitive factors in similar ways as previously assessed in the literature, there are other ways to measure these constructs. Second, the relation between cognitive predictors and both whole-number and fraction outcomes was analyzed within the same model using structural equation modeling and path analysis, allowing for direct comparisons across two outcomes. Third, although number line estimation has often been examined as a correlate and predictor of mathematics achievement, few studies have examined the cognitive mechanisms of whole-number estimation, and to my knowledge, this was first study to examine cognitive characteristics that underlie fraction number line estimation.

Examining cognitive predictors of both whole-number and fraction domains should produce insights on the cognitive mechanisms that underlie fraction calculation competence in relation to whole numbers. Such knowledge can help guide understanding development of fraction competence in comparison to whole-number competence. This in turn may provide insight into the nature of interventions for improving these mathematics outcomes.

CHAPTER II

METHOD

Participants

Data in the present study were collected as part of a larger study investigating the efficacy of a fraction intervention. As part of this larger study, 315 fourth-grade at-risk students were sampled from 53 classrooms in 13 schools in a southeastern metropolitan school district. We sampled two to eight at-risk students per classroom. When screening yielded more students than could be accommodated in the study, we randomly selected students for participation. We defined risk as performance on a broad-based calculations assessment (Wide Range Achievement Test–4 or WRAT-4; Wilkinson, 2004) below the 35th percentile. We excluded students (n = 18) with T-scores below the 9th percentile on both subtests of the Wechsler Abbreviated Scales of Intelligence (WASI; Psychological Corporation, 1999) because this study was not about intellectual disability.

Those 297 at-risk students were randomly assigned at individual level to fraction tutoring (n = 145) or a control condition (n = 152), stratifying by classroom. In the present study, we used data only from the control at-risk group because intervention was designed to disturb the predictive value of cognitive abilities. Of 152 at-risk control students, 12 moved before the end of the study, and one student had incomplete pretest data. These 13 students did not differ from remaining students. We therefore omitted these 13 cases, with 139 students comprising the final AR control sample. Their scores on the pretest WRAT averaged 9.01 (*SD* = 2.04). Their mean age was 9.49 (*SD* = .39). Of these 139 students, 58 (41.7%) were male, 12 (8.6%) were English learners, 114 (82.0%) received a subsidized lunch, and 12 (8.6%) had a school-identified disability. Race was distributed as 77 (55.4%) African American, 32 (23.0%) White, 25 (17.9%) Hispanic, and 5 (3.6%) "Other."

Screening Measure

With the *WRAT-4-Arithmetic* (Wilkinson, 2008), students have 10 min to complete calculation problems of increasing difficulty. In the beginning-of-fourth-grade range of performance, WRAT almost entirely samples whole-number items. Reliability at fourth grade on this measure is .85.

Cognitive Predictors

Nonverbal reasoning. *WASI Matrix Reasoning* (Wechsler, 1999) measures nonverbal fluid reasoning with pattern completion, classification, analogy, and serial reasoning tasks on 32 items. Students complete a matrix, from which a section is missing, by selecting from five response options. Reliability is .94.

Language. We used two tests of language, from which we created a unit-weighted composite variable using a principal components factor analysis. Because the principal components factor analysis yielded only one factor, no rotation was necessary. *WASI Vocabulary* (Wechsler, 1999) measures expressive vocabulary, verbal knowledge, and foundation of information with 42 items. The first four items present pictures; the student identified the object in the picture. For the remaining items, the tester says a word for the student to define. Responses are awarded a score of 0, 1, or 2 depending on quality. Split-half reliability is .86. *Woodcock Diagnostic Reading Battery* (*WDRB) - Listening Comprehension* (Woodcock, 1997) measures the ability to understand sentences or passages that the tester reads. With 38 items, students supply the word missing at the end of sentences or passages that progress from simple verbal analogies and associations to discerning implications. Reliability is .80.

Concept formation. With *Woodcock Johnson-III Tests of Cognitive Abilities* (WJ-III; Woodcock, Mather, & McGrew, 2001)-*Concept Formation*, students identify the rules for concepts when shown illustrations of instances and non-instances of the concept. Students earn credit by correctly identifying the rule that governs each concept. Cut-off points determine the ceiling. Reliability is .93.

Working memory. Mixed findings exist depending on what type of working memory was assessed, but prior work has found consistent evidence for the central executive component of working memory (Fuchs et al., 2005, 2008, 2010b). Therefore, we assessed the central executive component of working memory using The *Working Memory Test Battery for Children* (WMTB-C; Pickering & Gathercole, 2001)-*Listening Recall* and *Counting Recall*. Each subtest includes six dual-task items at span levels from 1-6 to 1-9. Passing four items at a level moves the child to the next level. At each span level, the number of items to be remembered increases by one. Failing three items terminates the subtest. Subtest order is designed to avoid overtaxing any component area and is generally arranged from the easiest to hardest. We used the trials correct score. Test-retest reliability ranges from .84-.93. For *Listening Recall*, the child determines if a sentence is true; then recalls the last word in a series of sentences. For *Counting Recall*, the child counts a set of 4, 5, 6, or 7 dots on a card and then recalls the number of counted dots at the end of a series. We opted to include both subtests, rather than creating a composite variable based on prior work (a) showing that listening recall may tap the verbal demands of word problems whereas calculations may derive strength from the specific ability to handle numbers within working memory (Fuchs et al., 2010) and (b) suggesting individual differences in working memory for numbers versus words (Siegel & Ryan, 1989; Dark & Benbow, 1991).

Processing speed. WJ-III (Woodcock et al., 2001) *Cross Out* measures processing speed by asking students to locate and circle five identical pictures that match a target picture in each row. Students have 3 min to complete 30 rows and earn credit by correctly circling the matching pictures in each row. Reliability is .91.

Attentive behavior. The Strength and Weaknesses of ADHD Symptoms and Normal-Behavior (SWAN; J. Swanson et al., 2004) samples items from the *Diagnostic and Statistical Manual of Mental Disorders* (4th ed.) criteria for attention deficit hyperactivity disorder (ADHD) for inattention (9 items) and hyperactivity impulsivity (9 items), but scores are normally distributed. Teachers rate items on a 1–7 scale. We report data for the inattentive subscale, as the average rating across the nine items. The SWAN correlates well with other dimensional assessments of behavior related to attention (www.adhd.net). Reliability for the inattentive subscale at fourth grade is .96.

Incoming Calculation Skill

We used the pretest scores from the *WRAT-4-Arithmetic* (Wilkinson, 2008) to index students' incoming calculation competence.

Whole-Number Outcome Measures

Whole-number calculations. We administered two subtests of *Double-Digit Calculation Tests* (Fuchs, Hamlett, & Powell, *2003*). The first subset, Double-Digit Addition, includes twenty 2-digit by 2-digit addition problems with and without regrouping. The second subtest, Double-Digit Subtraction, includes 20 2-digit by 2-digit subtraction problems with and without regrouping. Students have 3 min to complete each subtest. The score is the number of correct answers across both subtests. Alpha at fourth grade on this measure is .91. We also used the posttest scores from whole-number calculation items from the *WRAT-4-Arithmetic* (Wilkinson,

2008) to index whole-number calculation competence. Out of 40 items on WRAT 4-Arithemtic, 23 items are whole-number calculations. Cronbach's alpha on this sample was .77.

Whole-number number line estimation. *Number Line Estimation* (Siegler & Booth, 2004) assesses children's representations of numerical magnitudes. Following Siegler and Booth (2004), students estimate where numbers fall on a number line. Students are presented with a 25-cm number line displayed across the center of a standard computer screen, with a start point of 0 and an endpoint of 100. A target number is printed approximately 5 cm above each number line, and students place the target number on the number line. Target numbers are 3, 4, 6, 8, 12, 17, 21, 23, 25, 29, 33, 39, 43, 48, 52, 57, 61, 64, 72, 79, 81, 84, 90, and 96. Stimuli are presented in a different, random order for each child. The tester first explains a number line that includes the 0 and 100 endpoints and is marked in increments of 10. When the tester determines that the child recognizes the concept, a number line that includes the 0 and 100 endpoints only is presented, and the child points to where 50 should go. A model number line with the endpoints and the location of 50 marked is shown, and the child compares his/her response to the model. The tester explains how "the number 50 is half of 100, so we put it halfway in between 0 and 100 on the number line." Next, the tester teaches the child to use the arrow keys to place a red pointer on the line where 50 should fall on the computer screen. Then, the measure is administered, with only the end points of 0 and 100 marked. For each item, the tester asks, "If this is zero (pointing), and this is 100 (pointing), where should you put N?" There is no time constraint. The computer automatically calculates the absolute value of the difference between the correct placement and the child's placement of the target number (i.e., estimation of accuracy); this is averaged across trials to produce the score. This estimation accuracy score correlates with mathematics achievement (Geary et al., 2007; Siegler & Booth, 2004), and as Siegler and Booth showed, the source of improvement in estimation accuracy is increasing linearity of estimates. Cronbach's alpha as per Fuchs et al. (2010a) was .91.

Fraction Outcome Measures

Fraction calculations. We administered *Addition* (Hecht, 1998), in which students have 1 min to answer 12 fraction addition problems presented horizontally. Two items include adding a whole number and a fraction, six items with like denominators, of which two items involve adding a mixed number and a fraction, and four items with unlike denominators, of which two items involve adding a mixed number and a fraction. The score is the number of correct answers. Cronbach's alpha on this sample was .93.

From the 2010 Fraction Battery (Schumacher, Namkung, & Fuchs, 2010), *Fraction Subtraction* (Schumacher et al., 2010) includes five subtraction problems with like denominators and five with unlike denominators; half are presented vertically and half horizontally. Testers terminate administration when all but two students have completed the test. Scoring does not penalize students for not reducing answers. The score is the number of correct answers. Cronbach's alpha on this sample was .88.

Fraction number line estimation. *Fraction Number Line* (Siegler et al., 2011) assesses magnitude understanding by requiring students to place fractions on a number line with two endpoints, 0 and 1. For each trial, a number line with endpoints is presented, along with a target fraction shown in a large font above the line. Students practice with the target fraction 4/5 and then proceed to the 10 test items: 1/4, 3/8, 12/13, 2/3, 1/19, 7/0, 4/7, 5/6, 1/2, and 1/7. Items are presented in random order. Accuracy is defined as the absolute difference between the child's placement and the correct position of the number. When multiplied by 100, the scores are equivalent to the percentage of absolute error (PAE), as reported in the literature. Low scores indicate stronger performance. Test-retest reliability, on a sample of 57 students across 2 weeks, was .79.

Procedure

In August and September, testers administered the WRAT-4 in large groups and then administered the 2-subtest WASI individually to students who had met the WRAT-4 criterion for at-risk status. In September and October, testers administrated Double-Digit Addition, Double-Digit Subtraction, and Fraction Addition and Subtraction in three large-group sessions. Testers administered cognitive measures (WDRB Listening Comprehension, WMTB-C Listening Recall, WMTB-C Counting Recall, WJ-III Concept Formation, WJ-III Processing Speed), Number Line Estimation, and Fraction Number Line in two individual sessions. In early April, testers re-administered WRAT-4, Double-Digit Addition, Double-Digit Subtraction, and Fraction Addition and Subtraction in three large-group sessions and re-administered whole-number Number Line Estimation and Fraction Number Line estimation in one individual session. All test sessions were audiotaped; 20% of tapes were randomly selected, stratifying by tester, for accuracy checks by an independent scorer. Agreement on test administrating and scoring exceeded 98%.

CHAPTER III

DATA ANLYSIS AND RESULTS

Descriptive Data

Data analysis progressed in three stages. First, more than one measure was available for whole-number and fraction calculations allowing latent variables to be formed. A measurement model for theses outcome variables was estimated using confirmatory factor analysis to determine the factor structure among the calculation variables. Second, for whole-number and fraction calculations, the covariance structure of the data was modeled using structural equation modeling with seven cognitive predictors and one incoming calculation skill variable predicting latent variables representing whole-number and fraction calculations. Thirds, for whole-number and fraction number line estimation, in which only one outcome measure was available, path analysis was used to model the covariance structure between the eight predictors and number line estimation. In all analyses, because only one measure was available for all cognitive predictor variables, they were entered as manifest variables. All analyses were carried out using the Mplus statistical software (Muthen & Muthen, 1998).

Prior to conducting model estimation, we conducted preliminary analysis to identify outliers and univariate and multivariate normality. Univariate plots revealed no significant outliers (plus or minus three standard deviations from the mean for each variable used in the study). However, several variables were significantly skewed. These variables were normalized using transformations outlined by Howell (2007), and Tabachnick and Fidell (2007). This was the case for four variables: WASI Matrix, Fraction Addition, Whole-Number Number Line

Estimation, and Double-Digit Addition. Fraction Addition and Double-Digit Addition were substantially skewed and were log transformed. WASI Matrix was slightly skewed and was given square-root transformation. Whole-number Number Line Estimation was moderately skewed and was given reciprocal transformation. Scores on the fraction number line measure were reversed by multiplying by -1, so that higher scores mean higher performance. After normalizing the data, further analysis revealed that these variables were not multivariate normal. Therefore, models were constructed using a scaled chi-square estimated with robust standard errors using the robust maximum likelihood (MLR) estimator command in Mplus. Scaling correction factors ranged from 1.08 to 1.14 across models, suggesting little difference between the standard and scaled chi-square values.

Table 1 presents means and standard deviations on raw scores, as well as standard scores when available, on the cognitive predictors at the beginning of fourth grade (September and October, 2010) and on the math outcomes at the end of fourth grade (April, 2011). Table 2 presents correlations among all measures used in the study.

Whole-Number and Fraction Calculations

Outcome Measurement Model. The measurement model for whole-number and fraction calculations outcome included two correlated dimensions. The latent whole-number calculations variable comprised three manifest variables: WRAT-4 Arithmetic whole-number calculations, Double-Digit Addition, and Double-Digit Subtraction. The second latent variable, fraction calculations were represented by two manifest variables: Fraction Addition and Fraction Subtraction. A good model fit is indicated by (a) small values of chi-square relative to degrees of freedom, (b) large *p*-value associated with the chi-square, (c) root mean square error of approximation (RMSEA) approaching or equal to 0.0, (d) comparative fit index (CFI)

Table 1
Means and Standard Deviations

Variable	Raw score *M* (*SD*)	Standard score *M* (*SD*)
Language factor	.00 (1.00)	--
WASI Vocabulary	32.00 (6.33)	47.08 (8.93)
WDRB Listening Comprehension	21.05 (4.01)	91.12 (16.41)
Nonverbal reasoning	17.46 (6.16)	47.19 (10.11)
Concept formation	16.06 (5.18)	88.66 (9.16)
WMTB Listening Recall	10.37 (3.21)	91.35 (19.71)
WMTB Counting Recall	17.45 (4.76)	80.33 (16.23)
Processing speed	15.35 (2.74)	94.16 (11.29)
Attentive Behavior	34.94 (10.74)	--
Incoming calculation skill	24.34 (2.15)	--
WRAT whole-number calculation	12.07 (2.90)	--
Double Digit Subtraction	10.15 (4.82)	--
Double Digit Addition	16.93 (4.21)	--
Number Line Estimation	95.80(64.13)	--
Fraction Subtraction	4.06 (2.52)	--
Fraction addition	3.65 (2.41)	--
Fraction Number Line	0.32 (0.12)	--

approaching or equal to 1.0, (e) Tucker-Lewis index (TLI) approaching or equal to 1.0, and (f) standardized root-mean-square residual(SRMR) approaching or equal to 0.0 (Kenny, 2013). All manifest variables loaded significantly and reliably onto their respective factors (standardized coefficients: .65-.78, *ps* < .001). The overall fit of the two-factor model was excellent, $\chi^2(4, N = 139) = 3.23$, $p = .519$; RMSEA = 0.000, CFI = 1.000, TLI = 1.012, SRMR = 0.021. The correlation between two factors was significant, $r(137) = .49$, $p = .000$.

We contrasted this base measurement model with an alternative one-factor measurement model to confirm that both dimensions of calculations were necessary. Table 3 shows model fits and model comparisons for the measurement models. A adjusted chi-square difference tests (i.e., $\Delta\chi^2$) using the Satorra-Bentler scaling correction yielded a significantly worse fit of the one-factor measurement model, $\Delta\chi^2(1, N = 139) = 23.11$, $p = .000$. Therefore, both whole-number

Table 2

Correlations among All Measures

Measure	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. Language factor	--																
2. WASI Vocabulary	.88*	--															
3. WDRB Listening Comprehension	.88*	.56*	--														
4. Nonverbal reasoning	.30*	.28*	.25*	--													
5. Concept formation	.40*	.39*	.33*	.31*	--												
6. WMTB Listening Recall	.26*	.30*	.17*	.06	.20*	--											
7. WMTB Counting Recall	.03	.02	.03	.16	.00	.20*	--										
8. Processing speed	.09	.12	.03	.18*	.27*	.17*	.12	--									
9. Attentive Behavior	.11	.21*	-.00	.19*	.06	.03	.09	.10	--								
10. Incoming calculation skill	.17*	.24*	.06	.11	.22*	.17*	.24*	.14	.29*	--							
11. WRAT whole-number calculations	.16	.24*	.05	.22*	.25*	.12	.21*	.28*	.23*	.46*	--						
12. Double Digit Subtraction	.05	.09	.00	.11	.01	.05	.18*	.34*	.36*	.43*	.60*	--					
13. Double Digit Addition	.15	.23*	.03	.13	.05	.02	.23*	.30*	.32*	.38*	.49*	.53*	--				
14. Number Line Estimation	.17*	.20*	.11	.30*	.09	.15	.21*	.04	.08	.10	.34*	.15	.15	--			
15. Fraction Subtraction	.25*	.24*	.19*	.30*	.21*	.05	.16	.35*	.25*	.06	.32*	.22*	.23*	.16	--		
16. Fraction addition	.16	.15	.13	.18*	.12	.00	.04	.08	.15	-.00	.29*	.26*	.17*	.17*	.48*	--	
17. Fraction Number Line	.32*	.31*	.25*	.29*	.24*	.21*	.10	.01	.08	.15	.20*	.11	.14	.16	.14	.22*	--

Note. WASI = Wechsler Abbreviated Scale of Intelligence; WDRB = Woodcock Diagnostic Reading Battery; WMTB = Working Memory Test Battery; WRAT = Wide Range Achievement Test. **p* < .05

Table 3

Model Fits and Model Comparisons for the Measurement Models

Model	df	χ^2	*p*	RMSEA	CFI	TLI	SRMR	$\Delta\chi^2$Base Model
Two-factor model	4	3.23	.519	0.000	1.000	0.012	0.021	
One-factor model	5	26.17	.000	0.175	0.867	0.734	0.079	23.11*

*$p < .001$

and fraction calculations were incorporated into structural model.

Structural Model. Structural model, in which all cognitive predictors and incoming calculation skill had paths to both whole-number and fraction calculations, was tested. Figure 1 shows the results, with statistically significant paths in bold. Standardized path coefficient values are shown along the arrows. Table 4 shows the correlations among cognitive and incoming calculation manifest variables. The chi-square was statistically not significant, $\chi^2(28, N = 139) = 39.74$, $p = .070$, and the model fit was adequate, RMSEA = .055, CFI = .953, TLI = .916, SRMR = .034. The correlation between whole-number and fraction calculation factors was moderate, but not significant, $r(137) = .40$, $p = .107$. The model accounted for 51% and 32% of the variance in whole-number and fraction calculations, respectively. For whole-number calculations, significant predictors were processing speed, attentive behavior, and incoming calculation skill. For fraction calculations, significant predictors were language, processing speed, and attentive behavior.

Whole-Number and Fraction Number Line Estimation

Because only one measure was available for both whole-number and fraction number line estimation constructs, two-factor (whole-number and fraction number line factors) versus one-factor measurement (general number line factor) models could not be tested. However the correlation between whole-number and fraction number line estimation measure was low and not significant, $r(137) = .16$, $p = .070$, suggesting that the two represent different estimation skills.

Table 4

Calculations: Correlations among Cognitive and Incoming Calculation Manifest Variables

Variable	1	2	3	4	5	6	7	8
1. Language	--							
2. Nonverbal reasoning	.30	--						
3. Concept formation	.41	.32	--					
4. Working memory-Sentences	.25	.05	.21	--				
5. Numerical working memory	.04	.15	.01	.20	--			
6. Processing speed	.11	.20	.27	.19	.12	--		
7. Attentive behavior	.11	.19	.06	.03	.09	.10	--	
8. Incoming calculation skill	.18	.12	.22	.18	.24	.13	.29	--

Path analysis was used to estimate the relationship between each of the cognitive predictors and incoming calculation skill, in the presence of all other predictors, with whole-number and fraction number line estimation. All measures were entered as manifest variables, allowing whole-number and fraction number line outcomes to correlate. Because this was a saturated model, one non-significant path (Attention to Whole-Number Number Line Estimation) was set to 0. The chi-square was not statistically significant, and the model fit the data structure adequately, $\chi^2(2, N = 139) = 0.02$, $p = .992$; RMSEA = 0.000, CFI = 1.000, TLI = 1.494, SRMR = .001. As expected, the correlation between two variables was not significant, $r(137) = .03$, $p = .705$. The model accounted for 14% and 17% of the variance in whole-number and fraction number line estimation, respectively. Figure 2 shows the results, with statistically significant paths in bold. Standardized path coefficient values are shown along the arrows. Table 5 shows the correlations among cognitive and incoming calculation manifest variables. For whole-number number line estimation, the significant predictors were nonverbal reasoning and numerical working memory. For fraction number line estimation, the significant predictors were language and nonverbal reasoning.

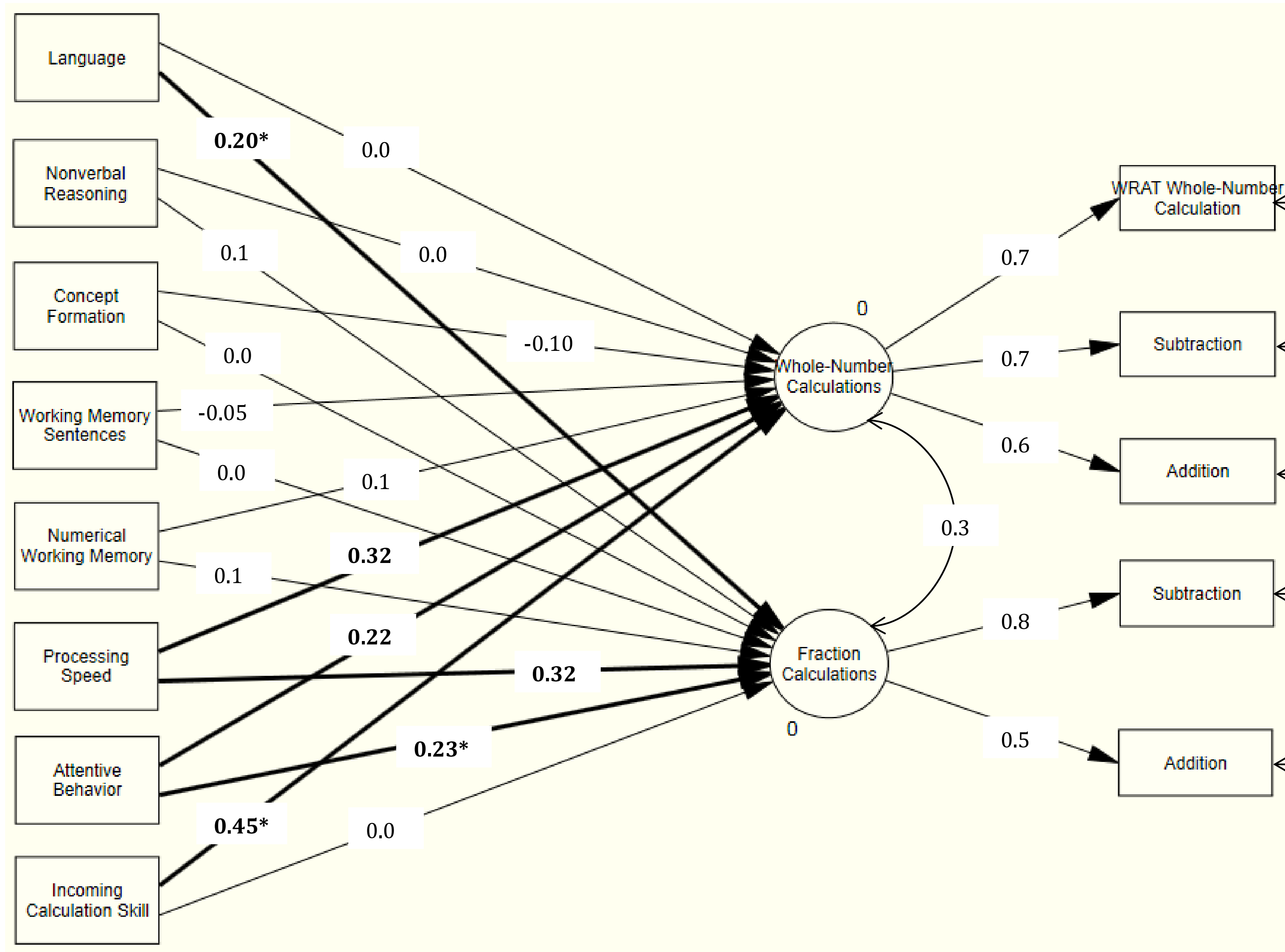

Figure 1. Whole-number and fraction calculation structural model. *p < .0

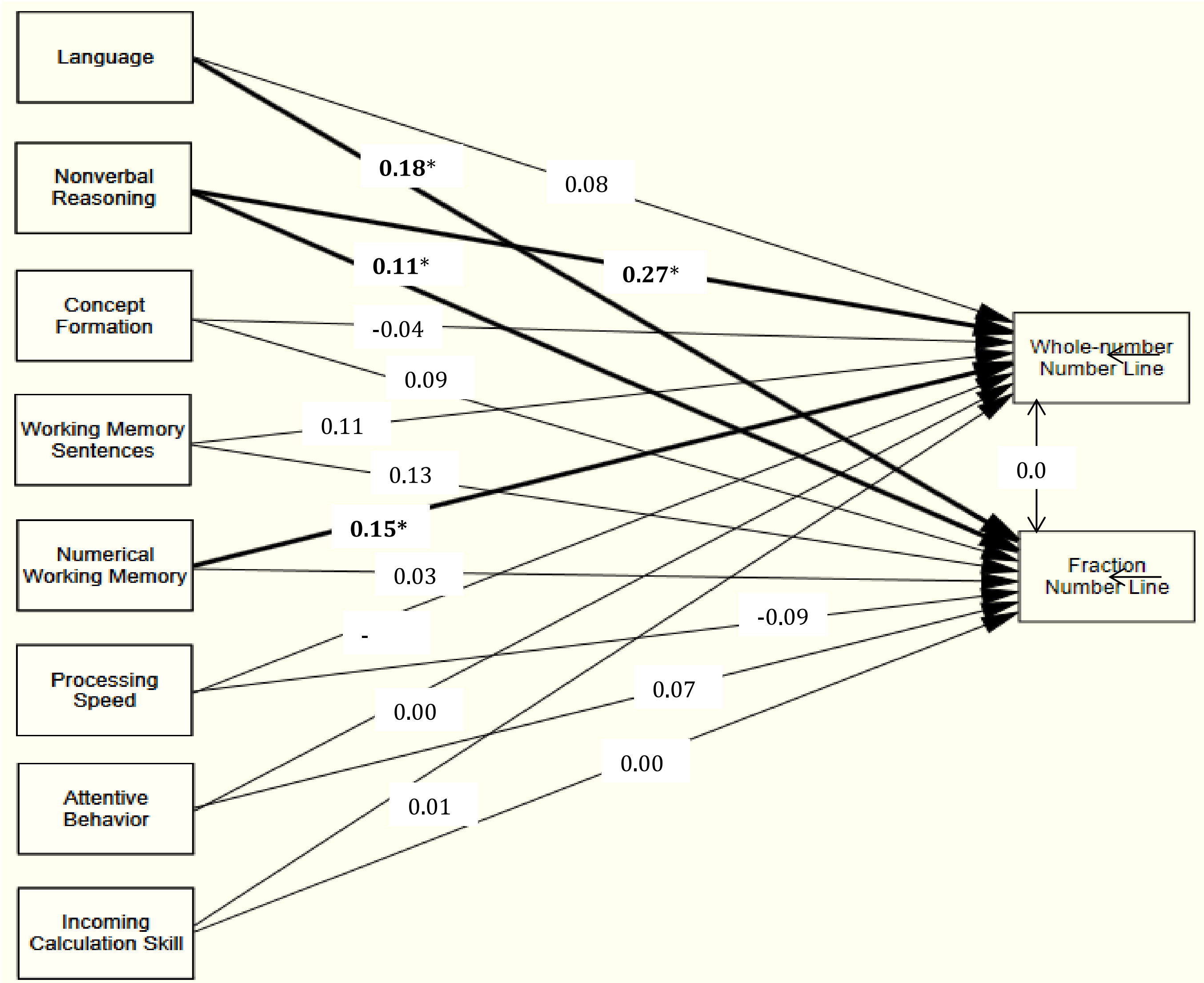

Figure 2. Whole-number and fraction number line path model. *$p < .05$

Table 5

Number Line Estimation: Correlations among Cognitive and Incoming Calculation Manifest Variables

Variable	1	2	3	4	5	6	7	8
1. Language	--							
2. Nonverbal reasoning	.29	--						
3. Concept formation	.41	.31	--					
4. Working memory-Sentences	.26	.06	.22	--				
5. Numerical working memory	.04	.16	.01	.20	--			
6. Processing speed	.10	.19	.26	.20	.12	--		
7. Attentive behavior	.11	.19	.06	.03	.10	.10	--	
8. Incoming calculation skill	.17	.11	.21	.19	.25	.12	.29	--

CHAPTER IV

DISCUSSION

The purpose of the present study was to examine cognitive predictors associated with calculations and number line estimation with whole numbers and fractions. At the beginning of fourth grade, students were assessed on seven cognitive abilities (i.e., language, nonverbal reasoning, concept formation, working memory-sentences, numerical working memory, processing speed, and attentive behavior) and one calculation skill measures. Then, at the end of fourth grade, they were assessed on whole-number and fraction calculation and number line estimation outcome measures. The relation between the predictors and calculation outcomes was analyzed using structural equation modeling, and the relation between the predictors and number line estimation was analyzed using path analysis. Results indicated that, in terms of calculations, processing speed, attentive behavior, and incoming calculation skills were significant predictors of whole-number skill whereas language, as well as processing speed and attentive behavior, significantly predicted fraction skill. In terms of number line estimation, nonverbal reasoning significantly predicted both whole-number and fraction number line competence; by contrast, for fraction competence, specific predictors were numerical working memory for whole-number competence and language for fraction competence.

Therefore, whole-number and fraction competence seem to draw upon some shared cognitive abilities: for calculation skill, processing speed and attentive behavior; for number line competence, nonverbal reasoning. Distinctive abilities also underlie whole-number and fraction competence. Language appears to be a key ability for both fraction calculations and number line estimation, but not for whole-number abilities. Similarly, incoming calculation was a distinctive predictor of whole-number calculations, and numerical working memory was a distinctive predictor of whole-number number line estimation. Before proceeding to discussing about how theses cognitive abilities are involved with whole-number and fraction calculations and number line estimation, it is important to distinguish the nature of calculation versus number line estimation tasks, and whole-numbers versus fractions within each task. In the sections below, we review how whole-number and fraction calculation and number line estimation competence develop. Then, we discuss the mechanisms for how the shared and distinct cognitive ability may affect whole-number and fraction competence. Finally, we provide limitations of the present study and instructional implications.

Calculation versus Number Line Estimation Development

Calculation competence. Calculation competence develops hierarchically. Children acquire quantitative abilities, such as discriminating between quantities, determining which of two sets represents the bigger amount, and understanding counting principles (e.g., one-to-one correspondence), that are fundamental to calculations during preschool years (Levine, Jordan, & Huttenlocher, 1992). With schooling, children develop procedural efficiency with addition and subtraction at first (Fuchs et al., 2006). Initially, children count the entire sets to derive the answer (e.g., one, two, three, four, five), then they count from the first number (e.g., three, four five), and eventually, they count from the larger number (e.g., four, five) to solve an addition problem (e.g., 2+3=5). As children become more fluent, they rely on automatic retrieval of addition and subtraction facts from long-term memory (Fuchs et al., 2006). Then, they learn to perform multi-digit addition and subtraction calculations with and without regrouping, in which students must keep track of place value and regrouping demands to derive the final answer.

With a continuum of skills, multi-digit calculation tasks are dependent upon successful execution of single-digit calculation tasks embedded. Along this continuum of calculation competence, fraction calculations are introduced at around fourth grade. Although whole-number calculation tasks are embedded within fraction calculations, such as adding and subtracting numerators, additional procedures are required with fraction calculations. Students must understand the interacting role of numerators and denominators, and find the common denominator and rewrite the fraction with the same denominators when adding or subtracting fractions with different denominators.

Number line estimation competence. On the other hand, making placements on a number line is one form of basic numerical representations that relies on a spatial representation of numerical magnitudes. Numerous psychophysical and neuropsychological studies found evidence to confirm the relation between number and spatial cognition that humans process number magnitudes as points on a continuous mental number line (e.g., Dehaene, 1997; Dehaene, Bossini, & Giraux, 1993; Hubbard, Piazza, Pinel, & Dehaene, 2005). Following the seminal work of Galton (1880), in which participants reported visualizing a left-to-right number line to process numbers, most evidence for the spatial representation of a number line comes from studies that found Spatial Numerical Association of Response Codes (SNARC) effects. The SNARC effects, which is thought to originate from the left-to-right orientation of the mental number line, refer to small numbers being associated with the left side of space whereas large numbers are being associated with the right side (e.g., Dehaene et al., 1993; Fias, Brysbaert, Geypens, & d'Ydewalle, 1996; Fischer, Castel, Dodd, & Pratt, 2003). Thus, number line estimation tasks should not require multiple procedures as in calculations, but require activating and processing numerical magnitudes as spatial representations on a number line.

It may be argued that students are engaged in multiple steps in estimating a location for a given number by using strategies, such as identifying a benchmark fraction (e.g., ½) and comparing it to the given fraction to figure out which side of the number line the given fraction belongs to (e.g., less than ½ or greater than ½). However, especially given that our measures of number line estimation were computer based with no paper and pencil support to rely on such strategies and given that students were instructed to guess where the number goes on the number line (as per the standard directions for that task), they were less likely to rely on complex strategies to locate the target number on a number line. Therefore, it appears that students rely on their internal, spatial representation of number magnitudes to locate a number on a number line, at least on a computer measure.

This spatial representation of magnitudes relies on logarithmic representations at first, which exaggerates the distance between the magnitudes of small numbers and minimizes the distance between magnitudes of large numbers (Feigenson, Dehaene, & Spelke, 2004; Geary, Hoard, Nugent, Byrd-Crave, 2006; Siegler & Booth, 2004). That is, children perceive that the distance between 5 and 8 is greater than that of 85 and 88. However, with schooling, children rely on linear representations of numerical magnitudes, with equal distances between two consecutive numbers at any point in the sequence. In fact, this shift from logarithmic to linear representations has been found to occur early on, between kindergarten and second grade for 0-100 number lines (Booth & Siegler, 2006, Geary et al., 2007; Laski & Siegler, 2007). Whereas kindergarteners rely on the logarithmic representations of magnitudes, most children generate a linear pattern of estimates by second grade (Booth & Siegler, 2008). Although how number line estimation develops in fractions has not been well investigated, Siegler et al. (2012) suggested that students expand their knowledge of whole-number magnitudes to include fractions with specific locations on a number line. So, it is possible that children may also form a spatial representation of fraction magnitudes on a number line as with whole numbers. However, one notable distinction exists between whole-number and fraction number line estimation; children cannot count fractions in sequence to locate a fraction on a number line whereas children can count by 10’s or 20’s to locate a whole number on a number line.

Shared Cognitive Mechanisms of Whole-Number and Fraction Competence

Attentive behavior. The relation between attention and academic tasks has been well documented in prior studies, where inattentive behavior is found to be correlated with poor academic achievement (e.g., Gross-Tsur, Manor, & Shalev, 1996; Shaywitz, Fletcher, & Shaywitz, 1994; Zentall, 1990). On this basis, we would expect attentive behavior to predict all four mathematics outcomes. Yet, attentive behavior uniquely predicted both whole-number and fraction calculations, but not number line estimation. Furthermore, the strength of predictive power for attentive behavior with whole-number ($\beta = 0.22$) and fraction calculations ($\beta = 0.23$) was similar. Executing both types of calculation tasks require keeping track of multiple numbers and steps, and therefore require considerable attention. For example, with whole numbers, students must attend to regrouping processes and keep track of each digit after regrouping to execute 35-19 while simultaneously monitoring for errors. With fractions, students must attend to the interacting role of numerators and denominators, and also to complex calculation procedures, such as finding the common denominator and rewriting fractions with the same denominator in order to add and subtract fractions with unlike denominators.

During these processes, inattentive students may commit more arithmetic and procedural errors than attentive peers (Raghubar et al., 2009) and may have less opportunity to persevere with given tasks (Fuchs et al., 2005, 2006). By contrast, as discussed earlier, students may access and rely on the spatial representations of number magnitudes to place a given number on a number line, rather than going through multiple steps to place the number. Thus, number line estimation may not involve as many procedures that are dependent upon each other as with calculations, and this would reduce demands on attention capacities. In this vein, it makes sense that attentive behavior helps predict both whole-number fraction calculations, but not whole-number or fraction number line estimation. Yet, as Fuchs et al. (2005, 2006) suggested, it is still possible that teacher rating forms of student attention used in the preset study may be a proxy for academic performance. That is, teachers may perceive students with low academic achievement as inattentive, which warrants further investigation.

Processing speed. Similarly, processing speed was found to be a key mechanism for both whole-number and fraction calculations, but not for either whole-number or fraction number line estimation. Furthermore, as with attentive behavior, the strength of predictive power for processing speed with whole-number ($\beta = 0.32$) and fraction calculations ($\beta = 0.32$) was similar. This finding corroborates previous research, in which processing speed has been shown to correlate with mathematics performance (e.g., Bull & Johnston, 1997; Kail, 1992; Kail & Hall, 1994). More specifically, Bull and Johnston found that students with calculation difficulties were slow in speed of executing operations, identifying numbers, and matching number and shapes. In comparison to the number line estimation tasks, in which students are not engaged in multiple tasks but rather rely on a mental representation of magnitudes to derive their answer, execution of multiple tasks embedded are required to derive answers to calculation problems. For example, with whole-number calculations, successful execution of operations in the larger task (e.g., regrouping in double-digit calculations) would depend on efficient processing of simple operations (e.g., single-digit calculations) embedded. Similarly, with fractions, successful executing of complex and multi-step procedures involved in adding and subtracting fractions would depend on efficient processing of each sub-step (e.g., finding common denominators, converting fractions, and adding numerators). Because processing speed facilitates simple processes necessary to carry out calculation procedures, such as counting or retrieving arithmetic facts from long-term memory, students with faster processing speed may be able to find answers more quickly and pair the problems with their answers in working memory before decay sets in (Bull & Johnston, 1997; Gery, Brown, & Samaranayake, 1991; Lemaire & Siegler, 1995).

Nonverbal reasoning. Nonverbal reasoning seems to play an important role in both whole-number and fraction number line estimation, but not in calculations. Nonverbal reasoning is important in drawing inferences and forming concepts when solving problems (Primi, Ferrao, & Almeida, 2010). Whereas calculations are taught as multiple procedures, in which processing speed and attentive behavior would play a significant role as discussed above, students must transfer and generalize their knowledge about number magnitudes when whey place numbers on whole-number and fraction number lines, which appear to draw upon their reasoning abilities. For example, students must think logically and systemically to infer connections between the target number, and 0 and 100 marked on the number line. Then, students must infer the location of the target number in comparison to their mental, spatial representations of number magnitudes. Therefore, it is not surprising that nonverbal reasoning helps to predict both whole-number and fraction number line estimation.

Distinct Cognitive Mechanisms of Whole-Number and Fraction Competence

Langauge. On the other hand, language appears to support the development of both fraction calculations and fraction number line estimation, but not whole-numbers. This is in line with previous findings, in which language was also found to play a significant role in fraction calculation development (Fuchs et al., 2013; Seethaler et al., 2011), but not in whole-numbers (e.g., Fuchs et al., 2005, 2006, 2008, 2010a, 201b, 2013; Seethaler et al., 2011). With respect to calculations, fraction calculations require conceptual understanding of fractions beyond being able to carry out rote procedures embedded. In fraction calculations, students must understand the interacting role of numerators and denominators and the concept of having the same denominators. This finding also corroborates the literature, which the significant role of language in acquiring conceptual understanding of fractions has been found (Miura et al. 1999; Paik & Mix, 2003). In particular with fractions, as discussed earlier, Miura et al. (1999) suggested that East Asian languages with transparent verbal labels of fractions that represent part-whole relations facilitate conceptual understanding of fractions. Better understanding of fraction concepts may in turn facilitate fraction calculations. Such relation between conceptual and procedural understandings is demonstrated in the literature, in which they were found to influence each other iteratively (e.g., Rittle-Johnson & Siegler, 1998; Rittle-Johnson, Siegler, & Alibali, 2001). More specifically, conceptual knowledge of fractions strongly influenced fraction calculations in Hecht and Vagi (2010).

The finding does, however, contradict Jordan et al. (2013), in which language was a significant predictor of fraction concepts, but not fraction calculations. However, in Jordan et al., fraction calculation measures composed of addition and subtraction items with like denominators whereas addition and subtraction items with unlike denominators were included in the present study and Seethaler et al. (2011). This suggests that students may rely on rote whole-number calculation procedures embedded within when they solve simple fraction calculations problems, such as adding and subtracting fractions with like denominators. By contrast, adding and subtracting fractions with unlike denominators require additional processes, such as finding the common denominator and rewriting equivalent fractions to have the same denominators, which draw upon conceptual understanding that is supported by language, as discussed above. Taken together, language ability may be essential at least for learning more advanced fraction concepts and calculations.

With respect to number line estimation, language abilities seem to be important in forming correct mental representations of fraction magnitudes. Whereas children's linear representation of whole-number magnitudes develop early on, prior literature suggest that students have difficulty with spatial representations of fraction magnitudes. Baturo and Copper (1999) found that sixth- and eighth-grade students had difficulty conceptualizing the number line representations of fractions. When these students were asked to place improper and mixed number fractions on number lines, they often associated the numerators with a whole-number marker on the number line and counted whole numbers instead of parts in fractions. Eighth-grade students performed even worse than sixth-grade students on placing improper fractions on number lines. As with fraction calculations, it is possible that conceptual understanding of fraction magnitudes, which is supported by language, is required to form correct mental representations of fraction magnitudes. Furthermore, students must learn novel words (e.g., "equivalent," "common denominator," and "improper fractions") and apply their vocabulary knowledge to solve problems in contrast to whole-numbers. Thus, their vocabulary knowledge, as assessed as a measure of language in the present study, may also play a significant role.

Incoming calculation. As expected, incoming calculation skill made the largest contribution to whole-number calculations, but it did not significantly predict both whole-number and fraction number line estimation. As we noted earlier, although our incoming calculation measure (i.e., WRAT4-Arithemtic) does include fraction, decimal, and percent calculations, it almost entirely samples whole-number items in the beginning-of-fourth-grade range of performance. Given the nature of incoming calculation tasks, in which students solely worked on deriving answers to whole-number calculation problems, it makes sense that incoming calculation skill is not predictive of number line estimation, which assesses number magnitude representations. However, it is surprising that incoming calculation skill did not significantly predict fraction calculations given the hierarchical nature of two calculation tasks that fraction calculations require competence with whole-number calculations. This finding suggests that fraction calculations may be distinct from whole-number calculations that fluency with whole-number calculations do not transfer to fraction calculations. This makes sense given the evidence that even those who are competent with whole numbers struggle with fractions (NMAP, 2008). The distinctive features of fraction versus whole-number calculations may be due to the fundamental differences between whole-number and fractions as noted earlier, such as infinite quantities existing between two fractions and requiring the same common denominator in calculation tasks.

This finding, however, contradicts, Jordan et al. (2013), in which calculation fluency was a significant predictor of fraction calculations. As discussed above, all fraction addition and subtraction items included in the outcome measure had the same denominators in Jordan et al. whereas items with unlike denominator were included in the present study. It is possible that simple fraction addition and subtraction tasks (e.g., like denominators) rely more on whole-number calculation competence. By contrast, cognitive resources, namely, processing speed, attentive behavior, and language, may play a more critical role for adding and subtracting fractions with unlike denominators above and beyond the ability to carry out whole-number calculation procedures embedded within fraction calculation tasks.

Working memory. Numerical working memory uniquely predicted whole-number number line estimation, but not fraction number line estimation. Nonsignificant effects of numerical working memory were found for whole-number and fraction calculations, and no significant effects of working memory-sentences were found in either forms of number line estimation and calculations. Whereas nonsigifincant effects of working memory-sentences found in the present study corroborate previous literature, in which working memory-sentences has been documented to uniquely predict word problem solving but not calculations (e.g., Fuchs et al., 2005, 2010b), it is interesting that numerical working memory also had nonsignificant effects on whole-number and fraction calculations. After all, both types of calculations require controlling, regulating, and maintaining numerical information while simultaneously carrying out calculation procedures and keeping track of where they are in the multi-step calculation procedures. However, mixed findings also exist in the literature regarding the contribution of numerical working memory. Fuchs et al. (2006, 2010a) did not find a significant influence for numerical working memory on arithmetic and procedural calculations whereas numerical working memory predicted whole-number calculations in Fuchs et al. (2008, 2010b). It is not clear why such conflicting results were found. Similar study participants and outcome measures, and the same working memory measures were used across Fuchs et al (2010a, 2010b) and Fuchs et al. (2006, 2008). Additional studies are needed to understand how and what components affect calculation competence.

With respect to number line estimation, in which numerical working memory significantly predicted whole-number, but not fraction number line estimation, it is possible that students may be using whole-number counting number sequence (e.g., counting by 10s or 20s), which is involved in the numerical working memory task, to place whole-numbers on a number line. Such counting is not applicable to fractions. This suggests a potential domain-specificity for numerical working memory. Previous literature provides evidence for domain-specificity that numerical working memory may be specific to tasks that involve numbers whereas working memory- sentences may be specific to verbal tasks (Fuchs et al., 2008; Hitch & McAuley, 1991; McLean & Hitch, 1999; Peng, Sun, Li, & Tao, 2012; Siegel & Ryan, 1989), but it appears that numerical working memory may be even more specific to whole-number tasks.

Limitations

As noted, we included a more complete set of predictors that are previously identified as relevant to whole-number and fraction competence. For calculations, those predictors accounted for 51% of variance on whole-number competence and 32% of variance on fractions; for number line, they accounted for 14% of variance on whole-number estimation and 17% of variance on fractions. This indicates there are other cognitive resources (e.g., phonological loop, inhibition) or environmental factors (e.g., socioeconomic status; quality of classroom instruction) that have not been identified in the literature yet. For example, we did not include measures of other components of working memory, such as phonological loop and visual-spatial sketchpad, because less consistent findings exit regarding the role of these other components of working memory on whole-number and fraction competence. However, it is possible that these tasks help predict whole-number and fraction competence.

Also, as noted, the percentages of variance accounted for whole-number and fraction number line estimation were significantly lower than those of calculations. This was expected in that we took an exploratory approach with number line estimation, and that the relevant predictors included in the present study were based on calculation competence. However, it does warrant further research with the goal of identifying the sources of individual differences in number line estimation. One potential cognitive predictor that future studies should include is a visuospatial component of working memory. Prior research found that brain regions associated with number and magnitude processing are located near areas that support visuospatial processing, and damage to these regions was found to disrupt forming spatial representations and imagining a mental number line (de Hevia, Vallar, & Girelli, 2008; Zorzi, Priftis, Meneghello, Marenzi, & Umiltà, 2006; Zorzi, Priftis, & Umiltà, 2002).

Another study limitation pertains to how we assessed each cognitive factor. We used measures that are similar to those used in previous studies, but there are other ways to measure these cognitive constructs. For example, the processing speed task involved finding five identical pictures that matched the target picture in a row of 19 pictures. Students need to maintain the representation of the target picture internally as they encode information for each picture. This may place demands on working memory. Therefore, the contribution of working memory may have been captured by the processing speed measure, leading to the nonsignificant effects of working memory in the present study. We note, however, that prior work has identified working memory as a significant predictor even when the same processing speed was controlled in the model (e.g., Fuchs et al., 2008; Fuchs et al., 2010b; Fuchs et al., 2013; Seethaler et al., 2011). Therefore, further research is warranted.

Instructional Implications

With these limitations in mind, the findings provide insight on the nature of interventions to remediate and compensate for weaknesses in cognitive resources in relation to whole-number and fraction calculations and number line estimation. With respect to whole-number and fraction calculations, interventions should incorporate effective strategies to improve students' attention and academic engagement, such as providing positive reinforcement for on-task behavior and implementing self-monitoring of attention (e.g., Edwards, Salant, Howard, Brougher, & McLaughlin, 1995; Harris, Friedlander, Saddler, Frizzelle, & Graham, S., 2005; Shimabukuro, Prater, Jenkins, & Edelen-Smith,1999). Providing instructional strategies that can compensate for slow processing may also be helpful in improving calculation skills. For example, students with mathematical difficulties often rely on counting the entire set of numbers when adding and subtracting. Teaching addition and subtraction strategies, such as counting up and counting down, may help them compensate for slow processing.

In terms of fraction calculations and number line estimation, instruction should be designed to reduce demands on language. For example, explicitly teaching fraction vocabulary, using simple language, and checking for students' understanding frequently may be helpful in reducing demands on language abilities. The present findings also suggest that practice on the whole-number calculation procedures that are embedded within fraction calculations may not lead to successful development of fraction calculations. Conceptual understanding of fractions that is supported by language appears to be a determinant of success with fractions. Therefore, fraction instruction should focus on improving students' conceptual understanding of fractions. Such instruction should address teaching fractions as numbers, providing multiple representations with number lines being the central representational tool, and helping students understand why procedures for fraction calculations make sense as outlined by the Institute of Education Science (Siegler et al., 2010) and as demonstrated as efficacious in randomized control trials (Fuchs et al., 2013; Fuchs et al., in press; Fuchs et al., in preparation).

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