A Novel Measure of Effect Size for Mediation Analysis

By

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To my family, whose love and support made all this possible

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Chapter I

Introduction

Scholars in many academic fields, including psychology, have advocated that researchers move away from null hypothesis significance tests (NHSTs) and *p*-values as the primary source of support for their hypotheses (APA, 1999; Cohen, 1991). One of the primary criticisms of the NHST is that *p*-values do not provide information about the size or importance of effects, only whether effects are likely or unlikely to occur given that a null hypothesis is true. Indeed, the APA Task Force on Statistical Inference strongly recommended that researchers report effect sizes for the primary outcomes of their studies (APA, 1999). Whereas effect size measures have been developed and routinely employed for many hypotheses such as mean differences, proportions, and strengths of relationships, there are some methods for which consensus has not been reached regarding the most appropriate measures of effect size. One of the most notable among these methods is mediation analysis.

Mediation analysis is the study of the potential pathways through which a predictor variable has an effect on an outcome variable. The most basic example of mediation is a three variable model consisting of an independent variable X, an outcome variable Y, and an intervening variable M, or the *mediator*. In this model, changes in X are associated with changes in M, and those changes in M are associated with changes in Y. In other words, the effect of X on Y is partially or fully transmitted via M. The effect of X transmitted to Y through the mediator is the *indirect effect*. The indirect effect is often of primary interest to researchers in mediation analysis. Specifically, it allows for the examination of processes that give rise to their phenomena of interest. While much progress in mediation analysis has been made regarding statistical inference for indirect effects, less progress has been made in developing effect size measures for the indirect effect.

The goal of this research is to develop a general effect size measure for mediation analysis that can be applied to mediation models of any complexity. Currently available effect size measures for mediation analysis were developed for the special case of basic three-variable, single-level models, and generally have not been evaluated in the more complex models increasingly common in the social sciences (e.g., models involving multiple mediators, covariates, latent variables, etc.). Indeed, the currently available measures have properties that preclude or substantially limit their use for these complex models. A general effect size measure will be developed to address these limitations. First, the topic of effect size will be reviewed with a focus on effect size measures with properties relevant to mediation analysis. Next, mediation analysis will be reviewed, including effect size measures developed specifically for mediation analysis. Then, a general effect size measure for mediation analysis will be proposed, including a matrix method for obtaining the measure and demonstrations of its use in special cases. Finally, simulations will be conducted to examine the behavior of the statistic in various population models, as well as the finite sampling properties of the measure in a multiple mediator model. The study will conclude with a summary of findings, limitations, and future directions.

Effect size

Broadly speaking, an effect size is defined as the quantification of some phenomenon of interest to address a specific research question (Kelley & Preacher, 2012). In the social sciences, this is commonly translated as a measure of the extent to which a null hypothesis is false or a conveyance of the practical importance of results beyond their statistical significance (Cohen, 1988). Statistical significance often is determined by comparing a test statistic to a critical value from the null distribution of that test statistic. If the test statistic exceeds the critical value, it means the observation of the test statistic would have been unlikely in the null distribution, and the null hypothesis is rejected. The *p*-value is the translation of the test statistic into a probability of occurrence in the null distribution, where a *p*-value less than a specified critical value (e.g., $\alpha = 0.05$) provides evidence that the null hypothesis should be rejected. Whereas *p*-values play an important role in hypothesis testing, they do not provide information regarding the magnitude or practical importance of effects (APA 1999; Cohen, 1991). For example, *p*-values close to a .05 cutoff (.01 - .05) can correspond to large effect sizes and, alternatively, results with very small *p*-values (< .001) may correspond to small effect sizes. That is, smaller *p*-values do not necessarily correspond to larger effect sizes. This is because statistical significance is a function not only of the size of the effect, but also of sample size, the significance level, and sample variability.

Any sample statistic can function as a measure of effect size. However, there are several properties a statistic should have in order for it to be considered a useful measure of effect size (Kelley & Preacher, 2012). First, the statistic should have an interpretable scale. Whereas some measures of effect size are useful in their original metrics (e.g., mean differences on an established measure; Baguley, 2009), it is often necessary to standardize measures. This makes results comparable across studies by removing the variable metric. For example, Cohen's *d* is the standardized mean difference, or the difference in *z* scores, between two groups

$$d_{12} = \frac{\bar{X}_1 - \bar{X}_2}{s_p}, \tag{1}$$

where $\bar{X}_1$ is the mean for group 1, $\bar{X}_2$ is the mean for group 2, and s_p is the pooled standard deviation. Second, it is desirable to quantify the precision of the statistic. Because sample statistics vary about their corresponding population values, it should be possible to construct confidence intervals (CIs) based on the statistic's sampling distribution. Third, the statistic should be unbiased. This means that the expected value of the statistic is the population parameter across repeated sampling. If the expected value of a statistic is a value other than the population parameter across repeated sampling, the statistic is biased. Fourth, the statistic should be consistent, meaning that it approaches the population parameter as sample size approaches infinity. Fifth, the statistic should be efficient, meaning that its sample variance should be reasonably small. These properties ensure the effect size measure is a good estimator, and allow for the comparison of results over time and across studies, and facilitate meta-analyses and statistical power calculations (APA, 1999).

Effect size measures have been established for many primary outcomes. Likely the most often employed measure is the previously described standardized mean difference (Cohen's *d*). When the strength of relationship between two linearly related continuous variables is of interest, the Pearson bivariate correlation coefficient has been established as the optimal measure of effect size (Cohen, 1988). The correlation coefficient *r* between two variables is expressed as

$$r_{12} = \frac{\sum_{i=1}^{N}(X_{1i} - \bar{X}_1)(X_{2i} - \bar{X}_2)}{(N-1)s_1 s_2}, \tag{2}$$

where X_{1i} are the individual observations in group 1, X_{2i} are the individual observations in group 2, N is the sample size, s_1 is the standard deviation within group 1, and s_2 is the standard deviation within group 2. Like Cohen's *d*, the correlation coefficient is metric-free, and also can be expressed in terms of standardized mean differences

$$r_{12} = \frac{d_{12}}{\sqrt{d_{12}^2 + 4}} \ , \tag{3}$$

assuming that the number of subjects in each group is equal (Cohen, 1988).

When the strength of linear relationships between two variables is of interest controlling for the influence of a set of variables, or *conditional* on a set of variables, the effect size can be expressed using partial, semi-partial, and multiple correlation coefficients. The partial correlation coefficient ($r_{YX \bullet \mathbf{Z}}$) is the correlation between variables removing the influence of a set of variables from both. For example, the partial correlation between variables Y and X_2 with the variance shared with variable X_1 removed from both is expressed as

$$r_{YX_2 \bullet X_1} = \frac{r_{YX_2} - r_{YX_1} r_{X_1X_2}}{\sqrt{1 - r_{YX_1}^2}\sqrt{1 - r_{X_1X_2}^2}} \, , \tag{4}$$

where r_{YX_2} is the correlation between Y and X_2, r_{YX_1} is the correlation between Y and X_1, and $r_{X_1X_2}$ is the correlation between X_1 and X_2.

The semi-partial correlation coefficient ($r_{Y(X \bullet \mathbf{Z})}$) is the correlation between two variables removing the influence of a set of variables from only one variable. Using the previous example, the semi-partial correlation between Y and X_2 controlling for the association of X_1 and X_2 is expressed as

$$r_{Y(X_2 \bullet X_1)} = \frac{r_{YX_2} - r_{YX_1} r_{X_1X_2}}{\sqrt{1 - r_{X_1X_2}^2}} \, . \tag{5}$$

The semi-partial correlation coefficient is a scaling of the partial correlation coefficient. For example, the semi-partial correlation coefficient in (5) can be re-expressed in terms of a partial correlation

$$r_{Y(X_2 \bullet X_1)} = \left(\frac{r_{YX_2} - r_{YX_1} r_{X_1 X_2}}{\sqrt{1 - r_{X_1 X_2}^2}} \right) \left(\frac{\sqrt{1 - r_{YX_1}^2}}{\sqrt{1 - r_{YX_1}^2}} \right) = r_{YX_2 \bullet X_1} \sqrt{1 - r_{YX_1}^2} \,. \tag{6}$$

In other words, the semi-partial correlation is the partial correlation of Y and X_2 given X_1 scaled by the square root of the variance unaccounted for in Y by X_1. It can be seen from (6) that the semi-partial correlation is equivalent to the partial correlation when X_1 accounts for no variance in Y, and the semi-partial correlation is otherwise less than the partial correlation.

The multiple correlation coefficient ($R_{Y \bullet XZ}$) represents the strength of relationship between one variable and a set of other variables. From the previous example, the multiple correlation of Y with X_1 and X_2 is expressed as

$$R_{Y \bullet X_1 X_2} = \sqrt{\frac{r_{YX_1}^2 + r_{YX_2}^2 - 2 r_{YX_1} r_{YX_2} r_{X_1 X_2}}{1 - r_{X_1 X_2}^2}} \,. \tag{7}$$

When a variable is designated the outcome variable and another set of variables the predictors, the strength of relationship between variables can be expressed using regression coefficients. If the variables involved in the regression analysis are in their raw metrics, the coefficients are unstandardized, meaning they are scaled in terms of the predictor and outcome variables. Coefficients can be standardized by transforming variables to have means of 0 and unit variances. From previous examples, the standardized regression coefficient ($\beta_{YX \bullet Z}$) of X_2 predicting Y conditional on X_1 is

$$\beta_{YX_2 \bullet X_1} = \frac{r_{YX_2} - r_{YX_1} r_{X_1 X_2}}{1 - r_{X_1 X_2}^2} \,. \tag{8}$$

The standardized regression coefficient is a further scaling of the partial correlation coefficient, and can be re-expressed as

$$\beta_{YX_2 \cdot X_1} = \left(\frac{r_{YX_2} - r_{YX_1} r_{X_1X_2}}{1 - r^2_{X_1X_2}} \right) \left(\frac{\sqrt{1 - r^2_{YX_1}}}{\sqrt{1 - r^2_{YX_1}}} \right) \left(\frac{\sqrt{1 - r^2_{X_1X_2}}}{\sqrt{1 - r^2_{X_1X_2}}} \right) = r_{YX_2 \cdot X_1} \left(\frac{\sqrt{1 - r^2_{YX_1}}}{\sqrt{1 - r^2_{X_1X_2}}} \right). \tag{9}$$

In other words, the standardized regression coefficient represents the partial correlation scaled by the ratio of the square root of the variance in Y unaccounted for by X_1 to the square root of the variance in X_2 not accounted for by X_1. It should be noted that, unlike partial and semi-partial correlations, the standardized regression coefficient can be larger than 1.

Another common primary outcome is the proportion of variance explained (R^2). This statistic quantifies the amount of variability accounted for in an outcome by a set of predictors, or, equivalently, the proportion reduction in error variance accounted for in the outcome by the predictors. When a model consists of only two variables, R^2 is equivalent to the squared correlation coefficient (r^2_{YX}). Similarly, R^2 for two variables removing the effect of a set from both is the squared partial correlation coefficient ($r^2_{YX \cdot \mathbf{Z}}$), for two variables removing the effect of a set from one of the variables is the squared semi-partial correlation coefficient ($r^2_{Y(X \cdot \mathbf{Z})}$), and for the effect of a set of variables on another variable is the squared multiple correlation coefficient ($R^2_{Y \cdot X\mathbf{Z}}$). Because R^2 is a proportion, it is bounded by 0 and 1. In addition, because the standardized regression coefficient can be greater than 1, it should not be interpreted as a proportion.

R^2 can be decomposed into effect sizes for the unique and shared contributions of a set of variables. If the predictor variables are uncorrelated, the total R^2 is the sum of the unique R^2 of each predictor and the outcome. For example, if variables X_1 and X_2 are uncorrelated, the squared multiple correlation can be expressed as

$$R^2_{Y \cdot X_1X_2} = r^2_{YX_2} + r^2_{YX_1}. \tag{10}$$

When the predictors are correlated, the unique contributions of each variable can be obtained by subtracting the R^2 from a reduced model without the predictor of interest from the R^2 from the model with both predictors. For example, the unique proportion of variance accounted for by X_2 ($R^2_{Y \bullet X_2(X_1)}$) is

$$R^2_{Y \bullet X_2(X_1)} = R^2_{Y \bullet X_1 X_2} - r^2_{YX_1} . \tag{11}$$

$R^2_{Y \bullet X_2(X_1)}$ is equivalent to the squared semi-partial correlation coefficient. The proportion of variance in Y shared by X_1 and X_2 can then be calculated from the zero-order correlation between Y and X_2 and the squared semi-partial correlation coefficient

$$R^2_{shared} = r^2_{YX_2} - r^2_{Y(X_2 \bullet X_1)} . \tag{12}$$

Substituting the definition of the squared semi-partial correlation coefficient from (11) into (12) gives

$$R^2_{shared} = r^2_{YX_2} - (R^2_{Y \bullet X_1 X_2} - r^2_{YX_1}) . \tag{13}$$

That is, the proportion of variance accounted for in Y jointly by X_1 and X_2 is equivalent to the sum of the squared zero-order correlations $r^2_{YX_1}$ and $r^2_{YX_2}$ minus the squared multiple correlation of Y with X_1 and X_2.

Standardized mean differences, correlation coefficients, and proportion of variance measures are appropriate effect size measures for many quantities of interest in regression-based methods. However, there are some regression-based methods, notably mediation analysis, where these measures are not adequate to quantify the quantity of interest. In mediation analysis, the primary outcome of interest is often the indirect effect, which is defined as either the difference between regression coefficients or the product of regression coefficients. While the component regression coefficients have established measures of effect size, the effect size for the indirect effect is not completely captured by those measures. In other words, standardized regression coefficients can be estimated for the constituent paths of the indirect effect, but interpreting the effect sizes of these coefficients does not necessarily lead to a reasonable interpretation of the indirect effect. For example, if a mediation model has an indirect effect consisting of the product of two standardized regression coefficients, and the effect size of one coefficient is large and the effect size of the other is small, it is not clear the effect size of the indirect effect itself is large or small. A reason for this is that effect size measures of the indirect effect components were designed to quantify linear relationships defined by a single equation. The most basic representation of mediation, however, involves at least three variables and two equations, so it should not be unexpected that effect size measures for simpler models do not adequately capture a more complex effect.

Mediation Analysis

Mediation analysis is the study of the potential causal pathways through which a predictor variable has its effect on an outcome variable. Many natural processes can be described in terms of mediation. For example, observing an object in the environment may appear to be an instantaneous process, but there are many processes that must occur in order to perceive the object. Generally speaking, light must be absorbed by photoreceptors in the eye, and that light must be transduced into an electrical signal that travels through the brain to the visual cortex, which then must travel to the prefrontal cortex. Therefore, the observation of an object is mediated by many lower order processes, and the absence or malfunction of any one of these processes may preclude perception of the object. The lower order processes can be further broken down into the functioning of cells and molecules, and, theoretically, further still into an infinite causal chain that perfectly explains the phenomenon. In the social sciences, researchers often are concerned with phenomena that operate on a much larger scale. For example, is the relationship between maternal and child depression mediated by parental involvement? Or, is the relationship between leadership style and team success mediated by perceived charisma? These are the types of questions that mediation analysis was designed to address.

Figure 1 provides an example of a simple single-level, three-variable mediation model in which all variables are observed. The *total effect* of a predictor on an outcome variable is given by the ordinary least squares (OLS) regression equation

$$Y = d_{Y \bullet X} + \beta_{YX} X + e_{Y \bullet X} \,, \tag{14}$$

where *Y* is the outcome variable, *X* is the predictor variable, $d_{Y \bullet X}$ is the intercept term, or the mean of *Y* when *X* = 0, β_{YX} is the linear slope coefficient relating *X* to *Y*, and $e_{Y \bullet X}$ is the residual error term, where $e_{Y \bullet X} \sim N(0, \sigma^2_{e_{Y \bullet X}})$. The regression coefficient β_{YX} is the total effect, or the predicted change in *Y* for a unit change in *X*. For some research questions the total effect is of primary interest (e.g., the degree to which a drug reduces the risk of heart disease). However, it is also possible to examine how the effect of *X* on *Y* is transmitted via mediators. The relationships among *X*, *M*, and *Y* can be described in a system of two linear regressions. The relationship between *X* and *M* is expressed as

$$M = d_{M \bullet X} + \beta_{MX} X + e_{M \bullet X} \,, \tag{15}$$

where *M* is the mediator, $d_{M \bullet X}$ is the intercept term, β_{MX} is the linear slope coefficient relating *X* to *M*, and $e_{M \bullet X}$ is the residual, where $e_{M \bullet X} \sim N(0, \sigma^2_{e_{M \bullet X}})$. The relationships among *X*, *M*, and *Y* are expressed as

$$Y = d_{Y \bullet MX} + \beta_{YX \bullet M} X + \beta_{YM \bullet X} M + e_{Y \bullet MX} , \tag{16}$$

where Y again is the outcome variable, $d_{Y \bullet MX}$ is the intercept term, $\beta_{YM \bullet X}$ and $\beta_{YX \bullet M}$ are the slope coefficients of Y regressed on M controlling for X and on X controlling for M, respectively, and $e_{Y \bullet MX}$ is the residual, where $e_{Y \bullet MX} \sim N(0, \sigma^2_{e_{Y \bullet MX}})$. Because $\beta_{YX \bullet M}$ is the effect of X on Y independent of M, it is referred to as the *direct effect.* The effect of X on Y transmitted via M is referred to as the *indirect effect.*

For the basic three variable mediation in Figure 1, calculation of the indirect effect requires parameter estimates from two of the three regressions outlined in (14) - (16). There are two methods that can be used to compute the indirect effect. One calculates the indirect effect as the difference between the total effect (β_{YX}) from (14) and the direct effect ($\beta_{YX \bullet M}$) from (16) (Baron & Kenny, 1986). Another method calculates the indirect effect as the product of the regression coefficients β_{MX} from (15) and $\beta_{YM \bullet X}$ from (16). In this simple mediation model, the two methods yield equivalent indirect effects, but estimates will differ if the outcome is not continuous or data are nested (Bauer, Preacher, & Gil, 2006; Krull & MacKinnon, 1999; MacKinnon & Dwyer, 1993). Whereas the two approaches both yield unbiased and efficient estimates, the product of coefficients approach is preferred because when there is more than one indirect effect, $\beta_{YX} - \beta_{YX \bullet M}$ will return only the *total* indirect effect, or the sum of the indirect effects through each mediator (Krull & MacKinnon, 1999). The product of coefficients approach allows for computation of indirect effects through each mediator (i.e., *specific* indirect effects; Bollen, 1987) as well as the total indirect effect. The definitions of the indirect effect also show the relationships among the total, direct, and indirect effects:

$$\beta_{MX} \beta_{YM \bullet X} = \beta_{YX} - \beta_{YX \bullet M} \Rightarrow \beta_{YX} = \beta_{MX} \beta_{YM \bullet X} + \beta_{YX \bullet M} .$$

In other words, the total effect is the sum of the direct and indirect effects.

The incorporation of multiple mediators and/or covariates into mediation models is a straightforward extension of the three-variable model. A model with a single predictor (*X*) and outcome (*Y*) and two parallel mediators (M_1 and M_2) is represented in the four equations

$$Y = d_{Y\bullet X} + \beta_{YX} X + e_{Y\bullet X} \tag{17}$$

$$M_1 = d_{M_1\bullet X} + \beta_{M_1X} X + e_{M_1\bullet X} \tag{18}$$

$$M_2 = d_{M_2\bullet X} + \beta_{M_2X} X + e_{M_2\bullet X} \tag{19}$$

$$Y = d_{Y\bullet M_1M_2X} + \beta_{YX\bullet M_1M_2} X + \beta_{YM_1\bullet XM_2} M_1 + \beta_{YM_2\bullet XM_1} M_2 + e_{Y\bullet M_1M_2X}\ , \tag{20}$$

where β_{M_1X} and β_{M_2X} are the regression coefficients of M_1 and M_2 on *X*, respectively, and $\beta_{YM_1\bullet XM_2}$ and $\beta_{YM_2\bullet XM_1}$ are the regression coefficients of *Y* on M_1 and M_2, respectively, controlling for *X* and the other mediator. A path diagram for this model can be found in Figure 2. There are still a single total effect (β_{YX}) and a single direct effect ($\beta_{YX\bullet M_1M_2}$) of *X* on *Y* as in the three-variable mediation model, but now two indirect effects of *X* on *Y*, one through M_1 ($\beta_{M_1X}\beta_{YM_1\bullet XM_2}$) and one through M_2 ($\beta_{M_2X}\beta_{YM_2\bullet XM_1}$). It is still true that the indirect effect is equivalent to the difference between the total and direct effects (MacKinnon, 2008; Preacher & Hayes, 2008), therefore

$$\begin{aligned} &\beta_{M_1X}\beta_{YM_1\bullet XM_2} + \beta_{M_2X}\beta_{YM_2\bullet XM_1} = \beta_{YX} - \beta_{YX\bullet M_1M_2} \\ &\Rightarrow \beta_{YX} = \beta_{M_1X}\beta_{YM_1\bullet XM_2} + \beta_{M_2X}\beta_{YM_2\bullet XM_1} + \beta_{YX\bullet M_1M_2}. \end{aligned}$$

This means that the sum of the indirect effects through M_1 and M_2 is equivalent to the total indirect effect. This formulation also means that any correlation between the mediators is incorporated into the specific effects.

The inclusion of covariates is also a straightforward extension of the single or multiple mediator models in (14) - (20). For example, adding a covariate *Z* to a single mediator model is expressed as

$$Y = d_{Y \bullet XZ} + \beta_{YX \bullet Z} X + \beta_{YZ \bullet X} Z + e_{Y \bullet XZ} \tag{21}$$

$$M = d_{M \bullet XZ} + \beta_{MX \bullet Z} X + \beta_{MZ \bullet X} Z + e_{M \bullet XZ} \tag{22}$$

$$Y = d_{Y \bullet MXZ} + \beta_{YX \bullet MZ} X + \beta_{YM \bullet XZ} M + \beta_{YZ \bullet XM} Z + e_{Y \bullet MXZ} , \tag{23}$$

where $\beta_{YZ \bullet X}$, $\beta_{MZ \bullet X}$, and $\beta_{YZ \bullet XM}$ are the coefficients for the regression of *Y* on *Z* controlling for *X*, *M* on *Z* controlling for *X*, and *Y* on *Z* controlling for *M* and *X*, respectively. If *Z* is correlated with *X* and/or *M*, the estimates of $\beta_{YX \bullet Z}$, $\beta_{YX \bullet MZ}$, $\beta_{MX \bullet Z}$, and $\beta_{YM \bullet XZ}$ from (21) - (23) would be expected to differ from the estimates obtained in (14) - (16).

Much of the progress in mediation analysis has been concerned with inference regarding the indirect effect. In their seminal work on mediation analysis, Baron and Kenny (1986) first proposed a joint significance test, according to which an indirect effect was considered significantly different from zero if both β_{MX} from (15) and $\beta_{YM \bullet X}$ from (16) were significantly different from zero. This method, termed the causal steps approach, also required $\beta_{YX \bullet M}$ from (14) to be significant as well, which precluded the existence of indirect effects when the total effect was not significantly different from zero. It is possible, however, for non-zero indirect effects to exist in the absence of a significant total effect. These are cases of inconsistent mediation, or suppression. More generally, suppression is considered to occur when the magnitude of the direct or indirect effect is greater than the total effect, which also means the direct and indirect effects are of different signs (MacKinnon, Krull, & Lockwood, 2000). That is, suppression is said to occur when the effect of the predictor on the outcome is stronger, directly or indirectly, when incorporating the mediator than the effect of the predictor on the outcome alone. If one considers indirect effects only in circumstances where suppression is not evident, then the causal steps approach would be applicable, although other methods for determining the indirect effect have been shown to be superior in terms of power and Type I error rate (MacKinnon et al., 2004). However, if one considers indirect effects in circumstances where suppression is evident, then other methods must be employed. Specifically, this requires hypothesis testing for estimates of the indirect effect. The total and direct effects are OLS regression coefficients, so their sampling distributions are asymptotically normal, allowing hypothesis testing based on symmetric CIs. The indirect effect, however, is the product of the regression coefficients β_{MX} and $\beta_{YM \cdot X}$, the distribution of which is nonnormal (Aroian, 1947). Current methods for constructing CIs for the indirect effect include bootstrap CIs (Bollen & Stine, 1990; MacKinnon, Lockwood, & Williams, 2004; Shrout & Bolger, 2002), Monte Carlo CIs (MacKinnon et al., 2004), Bayesian credible intervals (Yuan & MacKinnon, 2009), and a method of constructing CIs based on the distribution of product terms (MacKinnon, Fritz, Williams, & Lockwood, 2007; Tofighi & MacKinnon, 2011). Although these developments in inference for the indirect effect have been crucial for the advancement of mediation analysis, less research has been devoted to establishing a general effect size measure for the indirect effect.

Chapter II

Effect size measures in mediation analysis

Success rate difference

A recently developed measure of effect size proposed by Kraemer (2014) interprets the indirect effect as a standardized mean difference, or the *success rate difference* (*SRD*). In general, *SRD* for two groups sampled from populations 1 and 2 is defined as

$$SRD = P(T_1 > T_2) - P(T_2 > T_1) , \tag{24}$$

where T_1 and T_2 are individuals sampled from populations 1 and 2, respectively, $P(T_1 > T_2)$ is the probability that the individual sampled from population 1 has a response that is clinically preferable to the individual sampled from population 2, and $P(T_2 > T_1)$ is the probability that the individual sampled from population 2 has a response that is clinically preferable to the individual sampled from population 1. If all the individuals from population 2 have a response preferable to those in population 1, then *SRD* = -1. If the converse is true, then *SRD* = 1. This can be re-expressed in terms of Cohen's *d*:

$$SRD = 2\Phi\left(\frac{d}{\sqrt{2}}\right) - 1, \tag{25}$$

where Φ( ) is the cumulative standard normal distribution function, and *d* is the standardized mean difference between groups (Cohen's *d*). Kraemer (2014) extended (25) as a measure of effect size for the indirect effect when the independent variable is dichotomous (e.g., group assignment). The effect size for the indirect effect is the difference between the overall *SRD* and the *SRD* under the null hypothesis. The overall *SRD* is

$$SRD_{Overall} = 2\Phi\left[\frac{b_1 + b_2\Delta M + b_3 M^*}{\sqrt{\left(b_2 + 0.5b_3\right)^2 P_1 + V + \left(b_2 - 0.5b_3\right)^2 P_2 + V}}\right] - 1, \tag{26}$$

where ΔM is the difference in the means of the mediator between groups, M^* is the mean of the mediator across groups, P_1 and P_2 are ratios of mediator variance in groups 1 and 2, respectively, to the average mediator variance across groups, V is the error variance in each group, B_1 is the difference in the outcome between the focal group and control group controlling for the other variables (i.e., the direct effect of group assignment), B_2 is the linear effect of the mediator on the outcome controlling for the other variables, and B_3 is the interaction effect of group assignment and the mediator. It should be noted that *SRD* was developed in an approach to mediation modeling known as the MacArthur Framework, which specifies an interaction between the independent variable and the mediator. The *SRD* under the null hypothesis is

$$SRD_{Null} = 2\Phi\left[\frac{b_1 + b_3 M^*}{\sqrt{2\left(\left(b_2 + 0.25b_3\right)^2 + V\right)}}\right] - 1, \tag{27}$$

The mediation effect size (*MedES*) is the difference between the overall *SRD* and *SRD* under the null:

$$MedES = 2\Phi\left[\frac{b_1 + b_2\Delta M + b_3 M^*}{\sqrt{\left(b_2 + 0.5b_3\right)^2 P_1 + V + \left(b_2 - 0.5b_3\right)^2 P_2 + V}}\right] - 2\Phi\left[\frac{b_1 + b_3 M^*}{\sqrt{2\left(\left(b_2 + 0.25b_3\right)^2 + V\right)}}\right], \tag{28}$$

CIs for *MedES* can be formed using a bootstrapping procedure. *MedES* has been demonstrated in cases of a binary *M* and continuous *Y*, binary *M* and binary *Y*, and binary *M* and any form of *Y* (Kraemer, 2014). It is theoretically possible for *MedES* to be applied in scenarios with multiple treatment groups, multiple outcomes, and multiple mediators, although its properties have not been examined in those cases.

MedES has several properties that limit its usefulness as a general measure of effect size in mediation analysis. If the goal of the researcher is to make strong causal inferences regarding the indirect effect, then an independent variable based on random assignment is desirable. However, the independent variable in many studies is continuous, and *MedES* would not be applicable. *MedES* could technically be applied if the continuous independent variable were dichotomized; however, in addition to the disadvantages inherent to dichotomizing continuous variables in general (MacCallum, Zhang, Preacher, & Rucker, 2002), the dichotomized variable would not be equivalent to a variable based on random assignment, and, therefore, strong causal arguments could not be made. In addition, the specification of the interaction term assumes non-additivity in the regression of the outcome on the predictor and mediator, an assumption that may not be appropriate. Although it can be argued that the mediation formulations in (14) - (23) impose an assumption of additivity, interaction terms can be added to any of equations to relax the assumption.

Ratio measures

The ratio measures of effect size measures in mediation analysis quantify the relative magnitudes of the indirect effect to either the total effect or direct effect. One often used measure is the proportion of the mediated effect (P_M), which is the ratio of the magnitude of the indirect to the total effect (Alwin & Hauser, 1975). In terms of the coefficients in Figure 1, the proportion of the mediated effect is

$$P_M = \frac{ab}{c}. \tag{29}$$

Since the total effect is the sum of the indirect and direct effects, this ratio can be expressed in terms of the direct effect as

$$1 - P_M = 1 - \frac{c'}{c}. \tag{30}$$

A similar measure proposed by Sobel (1982) is the ratio of the indirect effect to the direct effect, or R_M :

$$R_M = \frac{ab}{c'}. \tag{31}$$

There are several properties of P_M and R_M that limit their usefulness as general effect size measures for indirect effects. Mackinnon, Warsi, and Dwyer (1995) demonstrated via simulation studies that P_M is unstable in several parameter combinations, has excessive bias in small samples, has large variance over repeated samples, and stabilizes only in large samples ($N >$ 500). Preacher and Kelley (2011) identified several additional limitations. First, P_M can give misleading estimates of practical importance because it is possible to obtain relatively high values of P_M with relatively small total and indirect effects, and relatively small values of P_M with relatively large total and indirect effects. For example, consider two studies, one where $a\hat{b} = .01, \hat{c} = .02$, and $\hat{P}_M = .5$, and the other where $a\hat{b} = .2$, $\hat{c} = .6$, and $\hat{P}_M = .3$ (all estimates are statistically significant). Although P_M indicates that the effect size of the indirect effect from first study was larger than the second, it would be difficult to say the indirect effect from the first study was more meaningful. Second, although P_M is referred to as a proportion, it does not behave as a proportion; it can assume negative values or exceed 1 depending on the relationship of the total and direct effects. Third, focusing on the overall P_M could lead the researcher to neglect additional mediators. If the P_M is large, it may be that there are other correlated mediators that could explain a significant amount of variance but would likely not be included. Fourth, R_M, although not specifically referred to as a proportion, has the same limitations as P_M because it is based upon the same information.

Kappa squared

In their review of effect size measures in mediation analysis, Preacher and Kelley (2011) proposed two new measures. One was the ratio of the observed indirect effect to the maximum possible indirect effect that *could have been* observed given the study design (κ^2). The maximum possible indirect effect is computed as the product of the maximum possible values of β_{MX} from (15) and $\beta_{YM \cdot X}$ from (16) permissible to maintain a positive definite variance-covariance matrix. Preacher and Kelley (2011) showed that for a three-variable mediation model with variance/covariance matrix

$$\Sigma = \begin{bmatrix} \sigma_X^2 & \sigma_{MX} & \sigma_{YX} \\ \sigma_{MX} & \sigma_M^2 & \sigma_{YM} \\ \sigma_{YX} & \sigma_{YM} & \sigma_Y^2 \end{bmatrix} ,$$

where σ_X^2, σ_M^2, and σ_Y^2 are the variances of X, M, and Y, respectively, and σ_{MX}, σ_{YX}, and σ_{YM} are the covariances of X and M, X and Y, and M and Y, respectively, the bounds for β_{MX} holding the total effect and $\beta_{YM \cdot X}$ constant are

$$\beta_{MX} \in \left\{ \frac{\sigma_{YM}\sigma_{YX} \pm \sqrt{\sigma_M^2\sigma_Y^2 - \sigma_{YM}^2}\sqrt{\sigma_X^2\sigma_Y^2 - \sigma_{YX}^2}}{\sigma_X^2\sigma_Y^2} \right\} \quad (32)$$

and the bounds for $\beta_{YM\bullet X}$ holding the total effect and β_{MX} constant are

$$\beta_{YM\bullet X} \in \left\{ \pm \frac{\sqrt{\sigma_X^2 \sigma_Y^2 - \sigma_{YX}^2}}{\sqrt{\sigma_X^2 \sigma_M^2 - \sigma_{MX}^2}} \right\} \tag{33}$$

The maximum possible indirect effect ($Max(\beta_{MX}\beta_{YM\bullet X})$) is then

$$Max(\beta_{MX}\beta_{YM\bullet X}) = M(\beta_{MX}) \times M(\beta_{YM\bullet X}). \tag{34}$$

The sign of the observed indirect effect determines whether to use the upper or lower bound β_{MX} and $\beta_{YM\bullet X}$. κ^2 is then the ratio of the observed indirect effect to the maximum possible indirect effect,

$$\kappa^2 = \frac{\beta_{MX}\beta_{YM\bullet X}}{Max(\beta_{MX}\beta_{YM\bullet X})}. \tag{35}$$

There are limitations of κ^2 that restrict its utility as a general measure of effect size for indirect effects. For illustrative purposes and without loss of generality, variables will be assumed to be standardized with unit variance. Two of these limitations were identified by Wen and Fan (2015). The first is that κ^2 is not a monotonic function in raw or absolute value of the indirect effect. This means that as the indirect effect increases, the effect size corresponding to that indirect effect does not necessarily increase. This could make comparisons of κ^2 across studies problematic because equivalent effect sizes do not necessarily correspond to equivalent indirect effects. The second, and more problematic, limitation is that the maximum possible indirect effect is unbounded in situations where either r_{MX} or r_{YM} is not held constant such that the magnitude of the indirect effect becomes infinitely large as r_{MX} approaches 1. The correlation matrix will remain positive definite as r_{MX} approaches 1 if r_{YM} is allowed to approach the value of r_{YX}. A limitation not noted by Wen and Fan (2015) is that κ^2 as currently formulated does not hold $\beta_{YM \bullet X}$ constant when computing the bounds for β_{MX}. In fact, it is the correlations between *X* and *Y* (r_{YX}), and *M* and *Y* (r_{YM}) that are held constant. It is not possible to hold $\beta_{YM \bullet X}$ and β_{YX} constant when computing bounds for β_{MX}, because $\beta_{YM \bullet X}$ is determined in part by the value of β_{MX}, as evident by the formula for $\beta_{YM \bullet X}$ in (8). In addition, the current formulation underestimates the maximum value of $\beta_{YM \bullet X}$. The maximum value of $\beta_{YM \bullet X}$ was derived by first obtaining the bounds of the correlation coefficient r_{YM} with r_{MX} and r_{YX} fixed, then scaling by the ratio of the standard deviations of *Y* to *M*. However, $\beta_{YM \bullet X}$ achieves substantially larger values when r_{MX} is maximized as opposed to r_{YM}. This is also evident from the formula for $\beta_{YM \bullet X}$ in (8) because as *X* approaches its maximum correlation with *M* of 1, the denominator of (8) approaches 0, making the indirect much larger than could be possible by maximizing r_{YM}.

κ^2 may still be a useful measure of effect size for the indirect effect provided some additional constraints are applied. Specifically, it would be necessary to fix either r_{MX} or r_{YM} to a specific value to limit the size of the maximum possible indirect effect. This may be possible in situations where the relationship between the predictor and mediator or mediator and outcome is well established by prior research. For example, a treatment may be designed to influence a mediator that has a well-established relationship with an outcome of interest. It may be possible then to maximize β_{MX} (e.g., the maximum effect the intervention could have had on the mediator) with the mediator-outcome relationship fixed to its sample value. κ^2 would then be defined as

$$\kappa^2 = \frac{\beta_{MX}\beta_{YM \bullet X}}{Max(\beta_{MX}\beta_{YM \bullet X})} = \frac{\beta_{MX}\beta_{YM \bullet X}}{Max(\beta_{MX}) \times Max(\beta_{YM \bullet X} \mid \beta_{MX})}, \tag{36}$$

$Max(\beta_{YM \bullet X} \mid \beta_{MX})$ is the maximum value of $\beta_{YM \bullet X}$ when β_{MX} is maximized holding the total effect and r_{YM} constant. However, applying this logic to experimental or quasi-experimental studies where the relationships between the predictor and mediator or the mediator and outcome are not well established could be problematic.

Residual based index

The second effect size measure proposed by Preacher and Kelley (2011) is a residual-based index that compares the variance explained in *M* by *X* and the variance explained in *Y* jointly by *M* and *X* to the total variability of *M* and *Y*. This was based on a method proposed by Berry and Mielke (2002) for an effect size measure (*M*) in multivariate multiple regression. In this method, sums of Euclidean residuals from least absolute deviation (LAD) regression are calculated for models conforming to null and alternative hypotheses. The residual sum for the null, or reduced, regression model is

$$\sum_{i=1}^{N}\left(\sum_{k=1}^{r} e_{0ik}^{2}\right)^{1/2} = \sum_{i=1}^{N}\left(\sum_{k=1}^{r}\left(y_{ik} - \sum_{j=1}^{m_0} x_{ij}\beta_{0jk}\right)^{2}\right)^{1/2}, \qquad (37)$$

where subscript i refers to the i^{th} observation from a sample of size N, subscript k refers to the k^{th} of r dependent variables y_{ik}, subscript j refers to the j^{th} of m_0 covariates x_{ij} predicting the k^{th} dependent variable, and β_{0jk} are the regression coefficients. The residual sum for the alternative, or full, regression model is given by

$$\sum_{i=1}^{N}\left(\sum_{k=1}^{r} e_{1ik}^{2}\right)^{1/2} = \sum_{i=1}^{N}\left(\sum_{k=1}^{r}\left(y_{ik} - \sum_{j=1}^{m_1} x_{ij}\beta_{1jk}\right)^{2}\right)^{1/2}. \qquad (38)$$

The models in (37) and (38) differ in the number of covariates under the null and alternative hypotheses. The effect size (M_{BM}) is calculated as the ratio of sums of squared absolute deviations from the alternative to null model and subtracted from 1

$$M_{BM} = 1 - \frac{\sum_{i=1}^{N}\left(\sum_{k=1}^{r} e_{1ik}^{2}\right)^{1/2}}{\sum_{i=1}^{N}\left(\sum_{k=1}^{r} e_{0ik}^{2}\right)^{1/2}}\,. \tag{39}$$

In the special case of multiple regression with a single outcome variable, the effect size measure reduces to

$$M_{BM} = 1 - \frac{\sum_{i=1}^{N}\left|e_{1ik}\right|}{\sum_{i=1}^{N}\left|e_{0ik}\right|}\,. \tag{40}$$

Parameters in LAD regression are estimated by minimizing the sums of absolute deviations, which reduces the influence of extreme observations compared to ordinary least squares (OLS) regression (Berry & Mielke, 2002).

Preacher and Kelley (2011) proposed an extension to M_{BM} where the null and alternative models correspond to regression equations in mediation analysis. The residual sum under the null hypothesis in a simple three-variable mediation model corresponds to the sum of absolute residuals when X explains no variance in M,

$$\sum_{i=1}^{N}\left|e_{0M_i}\right| = \sum_{i=1}^{N}\left|M_i - \sum_{j=1}^{m_0} x_{ij}\beta_{0M_j}\right| = \sum_{i=1}^{N}\left|M_i - \bar{M}\right|, \tag{41}$$

and the sum of residuals when X and M explain no variance in Y,

$$\sum_{i=1}^{N}\left|e_{0Y_i}\right| = \sum_{i=1}^{N}\left|Y_i - \sum_{j=1}^{m_0} x_{ij}\beta_{0Y_j}\right| = \sum_{i=1}^{N}\left|Y_i - \bar{Y}\right|, \tag{42}$$

where $\bar{M}$ and $\bar{Y}$ are the means of *M* and *Y*, respectively. The residual sum under the limiting alternative hypothesis corresponds to the sum of residuals when that *X* explains all the variance in *M*,

$$\sum_{i=1}^{N} \left| e_{1M_i} \right| = \sum_{i=1}^{N} \left| M_i - \sum_{j=1}^{m_1} x_{ij} \beta_{1M_j} \right| = \sum_{i=1}^{N} \left| M_i - d_{M.X} - aX_i \right|, \tag{43}$$

and the sum of residuals when *X* and *M* jointly explain all the variance in *Y*,

$$\begin{aligned} \sum_{i=1}^{N} \left| e_{1Y_i} \right| &= \sum_{i=1}^{N} \left| Y_i - \sum_{j=1}^{m_1} x_{ij} \beta_{1Y_j} \right| \\ &= \sum_{i=1}^{N} \left| Y_i - d_{Y.X} - cX_i \right| + \sum_{i=1}^{N} \left| Y_i - d_{Y.M} - dM_i \right| - \sum_{i=1}^{N} \left| Y_i - d_{Y.MX} - c' X_i - bM_i \right|, \end{aligned} \tag{44}$$

The residual-based effect size measure Γ is then the ratio of the residual sum under the alternative hypothesis to the residual sum under the null subtracted from 1,

$$\Gamma = 1 - \frac{\sum_{i=1}^{N} \left(\left| e_{1M} \right| + \left| e_{1Y} \right| \right)}{\sum_{i=1}^{N} \left(\left| e_{0M} \right| + \left| e_{0Y} \right| \right)}. \tag{45}$$

Γ has several desirable properties for an effect size measure. Γ has a meaningful interpretation as a proportion bounded by 0 and 1 because the residual sum in the numerator of (45) will always be smaller than the denominator when suppression is not evident. Γ is also independent of sample size, and bootstrap CIs can be estimated. To further aid in the interpretability of Γ, it is often advisable to standardize variables to unit variance, as Γ can be influenced by the scales of *M* and *Y* .

There are several complicating factors that limit Γ as a general measure of effect size in mediation. One is that Γ can return a non-zero effect size when the indirect effect is zero. This can occur if *X* explains variance in *M*, but *M* and *X* explain no variance in *Y*, or *M* does not explain variance in *Y* controlling for *X* (i.e., $\beta_{YM \bullet X} = 0$). This suggests that the null hypothesis that X explains no variance in M and X, and M explain no variance in Y, is not the only null hypothesis under which the indirect effect is zero, and formulations for additional null hypotheses may need to be incorporated to cover all circumstances that correspond to a null indirect effect. In addition, it is possible for Γ to be greater than 1 in cases of suppression, meaning the interpretation of Γ as a proportion does not extend to these circumstances.

Proportion of variance explained

R^2 measures in mediation analysis aim to quantify the proportion of variance explained in the criterion that can be attributed to both the predictor and the mediator but to neither alone (De Heus, 2012; Fairchild, MacKinnon, Taborga, & Taylor, 2009; MacKinnon, 2008; Preacher & Kelley, 2011). One approach to quantifying this variance was developed by Fairchild and colleagues (2009) where the variance in Y jointly accounted for by M and X is

$$R^2_{med} = r^2_{MY} - (R^2_{Y \bullet MX} - r^2_{XY}), \tag{46}$$

where r^2_{MY} is the squared correlation between M and Y, r^2_{YX} is the squared correlation between X and Y, and $R^2_{Y \bullet MX}$ is the squared multiple correlation of Y with M and X. Equation (46) is equivalent to the result derived in (13). To isolate variance accounted for by M and X jointly, the proportion of variance in Y explained by X is added to the proportion of variance in Y explained by M. Since the addition of these probabilities counts their overlap twice, subtracting the total variance accounted for in Y by M and X leaves only the area of joint overlap.

The R^2_{med} method for obtaining shared variance is based on a multiple regression framework where a single equation represents the relationships among variables. However, this may not be an appropriate framework in which to determine shared variance in mediation analysis. Consider the mediation model presented in Figure 1. According to Fairchild and colleagues (2009), the proportion of variance accounted for in Y by X and M jointly is given in (46). However, as shown in (14) - (16), a mediation model consists of at least two equations specifying the causal ordering of variables (i.e., X causes M, X and M cause Y). That is, the causal ordering of variables imposes an assumption that M and Y are mutually dependent on X. This assumption implies that the observed correlation between M and Y is not the simple product-moment correlation of the two variables, but a composite of their correlation conditional on X and correlation due to their mutual dependence on X (Duncan, 1970; Wright, 1960). In path analysis literature, the conditional correlation is often referred to as "true" correlation, and correlation due to mutual dependence is referred to as "spurious" correlation (Blalock, 1962; Dillon & Goldstein, 1984; Simon, 1957). It should be noted that the designation of "true" and "spurious" correlation is due purely to the model's assumptions about the relationships among variables. That is, the observed correlation matrix does not make distinctions about which correlations are "true" and which are "spurious".

The assumptions of several models are illustrated in Figure 3. Panel A represents a model where all variables are simply correlated (i.e., no variable is a cause of another variable). In this case, while it is possible to obtain correlations among variables conditional on another variable, no assumptions are imposed regarding which relationships are conditional, so each observed correlation would be considered a "true" correlation. Panel B represents a multiple regression model where X and M are causes of Y. This model assumes the relationship between X and Y is conditional M and that the relationship between M and Y is conditional on X. These conditional relationships would be considered the "true" correlations. However, it is possible for these conditional relationships to differ from their respective observed correlations. If the "true" correlations differ from their respective observed correlations, the difference would be considered due to "spurious" correlation. In this case, "spurious" correlation arises from the correlation between X and M. It follows that for a multiple regression model, the observed correlations and "true" correlations would be equivalent when X and M are uncorrelated. Panels C and D represent simple three-variable mediation models where X causes M, and X and M cause Y. For both models, the model assumes the relationship between X and M is unconditional on any other variable, so the observed correlation would be considered "true" correlation. However, the model also assumes the relationship between X and Y is conditional on M, and the relationship between M and Y is conditional on X. Panel C represents a scenario where there is no relationship between M and Y conditional on X. In this case, although the observed correlation between M and Y would be non-zero, there would be no "true" correlation from the perspective of the model assumptions, meaning the relationship between M and Y was entirely "spurious". In terms of mediation analysis, the indirect effect for this model would be zero because none of the effect of X on Y is transmitted via M. Panel D represents a scenario where there is no relationship between Y and X conditional on M. In this case, the observed correlation between M and Y would be considered "true" correlation because M is the only cause of Y, precluding the presence of "spurious" correlation due to mutual dependence of M and Y on X.

To illustrate how "spurious" correlation is quantified in mediation analysis, consider the model in Figure 4 (adapted from Dillon & Goldstein, 1984) in which all variables are standardized with unit variance. X_2 and X_1 have direct effects on the outcome Y, as well as indirect effects on Y through the mediator M. In this example, the observed correlation between

X_2 and M is inflated due to the correlation of X_2 and X_1. Specifically, the observed correlation between X_2 and M is

$$r_{MX_2} = \beta_{MX_2 \bullet X_1} + r_{X_2 X_1}\beta_{MX_1 \bullet X_2}\,, \tag{47}$$

where $r_{X_1X_2}$ is the correlation between X_1 and X_2, $\beta_{MX_1 \bullet X_2}$ and $\beta_{MX_2 \bullet X_1}$ are the standardized regression coefficients for the regressions of X_1 predicting M controlling for X_2, and X_2 predicting M controlling for X_1, respectively. $\beta_{MX_2 \bullet X_1}$ would be considered "true" correlation between X_2 and M, and $r_{X_1X_2}\beta_{MX_1 \bullet X_2}$ would be considered "spurious" correlation. When considering the observed correlation between M and Y, additional "spurious" correlations among the variables need to be accounted for:

$$\begin{aligned} r_{YM} = {} & \beta_{YM \bullet X_1X_2} + \beta_{MX_1 \bullet X_2}\beta_{YX_1 \bullet MX_2} + \beta_{MX_2 \bullet X_1}\beta_{YX_2 \bullet MX_1} r_{X_1X_2} \\ & + \beta_{MX_2 \bullet X_1}\beta_{YX_2 \bullet MX_1} + \beta_{MX_1 \bullet X_2}\beta_{YX_1 \bullet MX_2} r_{X_1X_2}\,, \end{aligned} \tag{48}$$

where $\beta_{YM \bullet X_1X_2}$, $\beta_{YX_1 \bullet MX_2}$, and $\beta_{YX_2 \bullet MX_1}$ are the coefficients of the regression of Y on M, X_1, and X_2, and $\beta_{MX_1 \bullet X_2}$ and $\beta_{MX_2 \bullet X_1}$ are the coefficients of the regression of M on X_1 and X_2. $\beta_{YM \bullet X_1X_2}$ would be considered "true" correlation and all other terms "spurious" correlation.

In the simple three variable mediation model in Figure 1, the only "spurious" correlation to account for is that between M and Y,

$$r_{YM} = \beta_{YM \bullet X} + \beta_{MX}\beta_{YX \bullet M}\,, \tag{49}$$

where β_{MX} is the regression coefficient relating M and X, $\beta_{YM \bullet X}$ is the regression coefficient for the regression of Y on M controlling for X, and $\beta_{YX \bullet M}$ is the regression coefficient for the regression of Y on X controlling for M. Squaring r_{YM} to calculate the proportion of variance explained in Y by M in (48) thus yields an estimate that is a mixture of the "true" proportion of variance in *Y* accounted for by *M*, and a "spurious" proportion of variance due to their mutual dependence on *X*.

This can be seen more clearly when the relationship between *Y* and *M* is completely accounted for by *X* as in Panel C of Figure 3 (i.e., $\beta_{YM \bullet X} = 0$). The proportion of variance in *Y* accounted for by *M* is

$$\begin{aligned} r_{YM}^2 &= (\beta_{YM \bullet X} + \beta_{MX}\beta_{YX \bullet M})^2 \\ &= (0 + \beta_{MX}\beta_{YX \bullet M})^2 \\ &= \beta_{MX}^2 \beta_{YX \bullet M}^2 \, . \end{aligned} \tag{50}$$

This illustrates that even when the relationship between *Y* and *M* is 0 when controlling for *X, Y* and *M* appear to be correlated via an entirely "spurious" correlation. Therefore, the proportion of variance in *Y* explained by *M* in the case of simple three-variable mediation adjusted for the "spurious" correlation is

$$\beta_{YM \bullet X}^2 = (r_{YM} - \beta_{MX}\beta_{YX \bullet M})^2 \, . \tag{51}$$

Inserting this result into (51) gives an adjusted version of R_{med}^2 (R_{med}^{2*})

$$R_{med}^{2*} = (r_{YM} - \beta_{MX}\beta_{YX \bullet M})^2 - (R_{Y \bullet MX}^2 - r_{XY}^2) \, , \tag{52}$$

which can be interpreted as the joint variance accounted for in *Y* by *M* and *X* after correcting for "spurious" correlation induced by the ordering of variables.

This adjustment accounts for some contradictory results obtained using the Fairchild et al. (2009) R_{med}^2 formulation. Specifically, there are circumstances when R_{med}^2 returns a non-zero value when the indirect effect is in fact zero (Lindenberger & Pötter, 1998). In the three-variable mediation model in Figure 1, this occurs when $\beta_{MX} \neq 0$ and $\beta_{YM \bullet X} = 0$ (i.e., $r_{YM} = r_{MX} \times r_{YX}$). In this case, R_{med}^2 becomes

$$R_{med}^2 = r_{MY}^2 - (R_{Y.MX}^2 - r_{XY}^2) = r_{MY}^2 - (r_{XY}^2 - r_{XY}^2) = r_{MY}^2 \, .$$

Not only does this show that R^2_{med} returns non-zero effect sizes when the indirect effect is in fact 0, but also implies that R^2_{med} overestimates the variance in *Y* accounted for jointly by *X* and *M* when $\beta_{YM \bullet X} \neq 0$. To illustrate the effect of correcting for "spurious" correlation, consider the mediation model in Figure 1 with the correlation matrix

$$\begin{bmatrix} 1 & .7 & .5 \\ .7 & 1 & .6 \\ .5 & .6 & 1 \end{bmatrix},$$

where the first column contains the correlations with *X*, the second column correlations with *M*, and the third column correlations with *Y*. R^2_{med} unadjusted for spurious correlation returns the proportion of variance in *Y* accounted for jointly by *M* and *X* as

$$R^2_{med} = .6^2 + .5^2 - \frac{.6^2 + .5^2 - 2 \times .6 \times .5 \times .7}{1 - .7^2} = 0.2375.$$

However, because $r_{YM} \neq r_{MX} \times r_{YX}$ the estimate of r_{YM} is inflated by "spurious" correlation. The proportion of variance in *Y* accounted for by *M* accounting for spuriousness is

$$\beta^2_{YM \bullet X} = \left(.6 - .7 \times \frac{.5 - (.7)(.6)}{1 - .7^2} \right)^2 = .2403.$$

Substituting the adjusted value of r^2_{YM} into the R^2_{med} formula gives

$$R^{2*}_{med} = .5^2 + .2403 - \frac{.6^2 + .5^2 - 2 \times .6 \times .5 \times .7}{1 - .7^2} = 0.1177.$$

Contrasting this result using the value of r^2_{YM} without accounting for spuriousness (.2375) shows that the variance in *Y* jointly explained by *M* and *X* can be substantially inflated if the ordering of variables is not taken into account.

A circumstance in a three-variable mediation model where R^2_{med} would not need to be adjusted is when the "spurious" correlation is zero (i.e., $\beta_{MX}\beta_{YX\bullet M}=0$). This circumstance is only possible when either $\beta_{MX}=0$, $\beta_{YX\bullet M}=0$, or $\beta_{MX}=\beta_{YX\bullet M}=0$. Because mediation is not in evidence when $\beta_{MX}=0$, the only circumstance under which "spurious" correlation would not need to be accounted for is when $\beta_{YX\bullet M}=0$. In other words, the unadjusted R^2_{med} formula is appropriate only when the direct effect of *X* on *Y* is zero.

Adjusting the R^2_{med} formula for "spurious" correlation results in a noteworthy interpretation for the variance explained in *Y* jointly by *M* and *X*. Consider the previous three-variable mediation example. Calculating the indirect effect of *X* on *Y* through *M* gives

$$\beta_{MX}\beta_{YM\bullet X}=.7\times\frac{.6-.7\times.5}{1-.7^2}=.3431.$$

Squaring this result yields the value .1177, the same value obtained using R^{2*}_{med} in (52). In other words, for three-variable mediation, the standardized squared indirect effect is equivalent to the variance in *Y* jointly explained by *M* and *X*,

$$R^{2*}_{med}=(r_{YM}-\beta_{MX}\beta_{YX\bullet M})^2-(R^2_{Y\bullet MX}-r^2_{XY})=\beta^2_{MX}\beta^2_{YM\bullet X}\,. \qquad (53)$$

This result is commensurate with the interpretation of the indirect effect as a coefficient whereby a one-unit change in *X* results in change in *Y* through the mediator *M* (MacKinnon, 2008). For standardized variables, a standard deviation change in *X* results in a change in standard deviations of *Y* through *M*, which, when squared, gives the variance in *Y* explained by *X* indirectly though *M*. In this sense, the standardized squared indirect effect functions in the same manner as a standardized regression coefficient, except that the squared standardized indirect effect is confined to the variance explained jointly by *M* and *X*.

Equation (53) suggests that to obtain an R^2 effect size measure for the indirect effect, one standardizes *X*, *M*, and *Y*, squares the resulting regression coefficients, and computes the standardized squared indirect effect (υ). Whereas this is evident for the special case of three-variable mediation, it is unclear if the square of products of regression coefficients are *always* equivalent to the joint variance accounted for in an outcome by a set of predictors and mediators. It is therefore necessary to establish whether the squared product of these squared regression coefficients represents joint variance explained. It should be reiterated that standardized regression coefficients can be greater than 1, so the squared standardized regression coefficients should not be considered proportions. That is, squared path coefficients do not function as probabilities like squared correlation coefficients (or squared partial or semi-partial correlation coefficients). However, it should be also reiterated that joint variance explained can be negative, and therefore is also not a proportion (Cohen, Cohen, West, & Aiken, 2003), so the components of joint variance explained need not function strictly as probabilities.

Smith (1981) showed that the variance explained by squared path coefficients is related to that explained by the squared semi-partial correlation coefficients. For the mediation model in Figure 1, the equation for the squared semi-partial correlation of *Y* and *M* with *X* removed from *M* is

$$r^2_{Y(M\bullet X)} = \frac{(r_{YM\bullet X} - r_{YX\bullet M} r_{MX})^2}{1 - r^2_{MX}}. \tag{54}$$

The squared standardized regression coefficient represents a scaling of the squared semi-partial correlation by the proportion of variance in *M* unexplained by *X*,

$$\beta^2_{YM.X} = \frac{r^2_{Y(M\bullet X)}}{1 - r^2_{MX}}. \tag{55}$$

This shows that the squared path coefficient represents a scaling of the unique proportion of variance in *Y* accounted for by *X* controlling for *M*. To demonstrate that the product of squared regression coefficients represents joint variance explained, it is useful to first consider the following interpretation of the squared multiple correlation coefficient:

$$R^2_{Y \bullet X_1 X_2 \ldots X_{p-1} X_p} = r^2_{YX_1} + r^2_{Y(X_2 \bullet X_1)} + \ldots r^2_{Y(X_p \bullet X_1 X_2 \ldots X_{p-1})} . \tag{56}$$

This means that the squared multiple correlation between an outcome *Y* and *p* predictor variables is equivalent to the sum of the squared correlation of the outcome with the first predictor and the squared semi-partial correlations of the outcome with each subsequent predictor, partialing out the effects of the prior predictors. A general formula for the squared regression coefficient for the unique variance in *Y* accounted for by X_p partialing out the effects of the other predictors is

$$\beta^2_{YX_p \bullet X_1 X_2 \ldots X_{p-1}} = \frac{r^2_{Y(X_p \bullet X_1 X_2 \ldots X_{p-1})}}{1 - R^2_{X_p \bullet X_1 X_2 \ldots X_{p-1}}} . \tag{57}$$

Multiplying both sides of (57) by the right hand denominator shows that the squared semi-partial correlation is equivalent to

$$r^2_{Y(X_p \bullet X_1 X_2 \ldots X_{p-1})} = \beta^2_{YX_p \bullet X_1 X_2 \ldots X_{p-1}} (1 - R^2_{X_p \bullet X_1 X_2 \ldots X_{p-1}}) . \tag{58}$$

Substitution of this result into (56) yields

$$R^2_{Y \bullet X_1 X_2 \ldots X_{p-1} X_p} = r^2_{YX_1} + \beta^2_{YX_2 \bullet X_1} (1 - R^2_{X_2 \bullet X_1}) + \ldots + \beta^2_{YX_p \bullet X_1 X_2 \ldots X_{p-1}} (1 - R^2_{X_p \bullet X_1 X_2 \ldots X_{p-1}}) , \tag{59}$$

where each $p-1$ squared multiple correlation on the right hand side of the equation can be expressed in terms of squared standardized regression coefficients. Multiplying terms in (59) gives

$$\begin{aligned} R^2_{Y \bullet X_1 X_2 \ldots X_{p-1} X_p} = r^2_{YX_1} + \beta^2_{YX_2 \bullet X_1} + \ldots + \beta^2_{YX_p \bullet X_1 X_2 \ldots X_{p-1} X_p} \\ - \beta^2_{YX_2 \bullet X_1} r^2_{X_2 X_1} - \ldots - \beta^2_{YX_p \bullet X_1 X_2 \ldots X_{p-1} X_p} R^2_{X_p \bullet X_1 X_2 \ldots X_{p-1} X_p} . \end{aligned} \tag{60}$$

Considering the terms on the right hand side of (60), the squared coefficients on the left represent the scaled unique associations between the outcome and each of the *p* predictors partialing out the effects of prior predictors, and the products of squared coefficients on the right represent the shared variance components of the squared multiple correlation. Because each squared multiple correlation can be expressed in terms of squared semi-partial correlations, and the squared semi-partial correlations, in turn, can be expressed in terms of squared regression coefficients, it follows that the joint variance in an outcome accounted for by a set of predictors can be expressed as the product of squared standardized regression coefficients.

Chapter III

General formula for υ

A general formula for obtaining standardized squared indirect effects is derived using a matrix method developed by Bollen (1987). For recursive models, the **B** matrix can be expressed in general terms as

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{B}_{MX} & \mathbf{B}_{M} & \mathbf{0} \\ \mathbf{B}_{YX} & \mathbf{B}_{YM} & \mathbf{0} \end{bmatrix}, \tag{61}$$

where **B** is a $m \times m$ matrix of standardized regression coefficients, $\mathbf{B}_{MX}$ is a $q \times p$ partition of **B** of regression coefficients relating the *p* predictors to the *q* mediators, $\mathbf{B}_{YX}$ is a $r \times p$ partition of **B** of regression coefficients relating the predictors to the *r* outcomes controlling for the mediators, $\mathbf{B}_{YM}$ is a $r \times q$ partition of **B** of regression coefficients relating the mediators to the outcomes controlling for the predictors, and $\mathbf{B}_{M}$ is a $q \times q$ partition of **B** of regression coefficients relating the mediators. Because there are no regression coefficients relating predictors to other predictors or outcomes to other outcomes (some predictors or outcomes would then be considered mediators), the upper left $p \times p$ and lower right $r \times r$ submatrices are **0**. The $\mathbf{B}_{M}$ submatrix has zeros along its diagonal, and, whereas all elements of $\mathbf{B}_{MX}$, $\mathbf{B}_{YX}$, and $\mathbf{B}_{YM}$ may only appear in the lower or upper triangle, elements of $\mathbf{B}_{M}$ may be above or below the diagonal depending on the direction of relationships among mediators. In order for the model to remain recursive, however, it must be possible to arrange the rows and columns of $\mathbf{B}_M$ to yield a lower triangular matrix with **0** diagonal. For example,

$$\mathbf{B}_M = \begin{bmatrix} 0 & B_{M_2M_1} & 0 \\ B_{M_1M_2} & 0 & 0 \\ B_{M_1M_3} & B_{M_2M_3} & 0 \end{bmatrix}$$

is non-recursive. A matrix of total effects **T** can be derived from **B** as

$$\mathbf{T} = (\mathbf{I}-\mathbf{B})^{-1} - \mathbf{I} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{B}_{MX} + ((\mathbf{I}-\mathbf{B}_M)^{-1} - \mathbf{I})\mathbf{B}_{MX} & (\mathbf{I}-\mathbf{B}_M)^{-1} - \mathbf{I} & \mathbf{0} \\ \mathbf{B}_{YX} + \mathbf{B}_{MX}\mathbf{B}_{YM} + \mathbf{B}_{YM}((\mathbf{I}-\mathbf{B}_M)^{-1} - \mathbf{I})\mathbf{B}_{MX} & \mathbf{B}_{YM} + \mathbf{B}_{YM}((\mathbf{I}-\mathbf{B}_M)^{-1} - \mathbf{I}) & \mathbf{0} \end{bmatrix}, \quad (62)$$

where **I** is the identity matrix. This illustrates the potentially large number of direct and indirect effects possible if a model has multiple predictors, mediators, and outcomes. The matrix of indirect effects $\mathbf{I}_{IND}$ can be calculated by subtracting the coefficient matrix **B** from **T**:

$$\mathbf{I}_{IND} = (\mathbf{I}-\mathbf{B})^{-1} - \mathbf{I} - \mathbf{B} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ ((\mathbf{I}-\mathbf{B}_M)^{-1} - \mathbf{I})\mathbf{B}_{MX} & (\mathbf{I}-\mathbf{B}_M)^{-1} - \mathbf{I} - \mathbf{B}_M & \mathbf{0} \\ \mathbf{B}_{MX}\mathbf{B}_{YM} + \mathbf{B}_{YM}((\mathbf{I}-\mathbf{B}_M)^{-1} - \mathbf{I})\mathbf{B}_{MX} & \mathbf{B}_{YM}((\mathbf{I}-\mathbf{B}_M)^{-1} - \mathbf{I}) & \mathbf{0} \end{bmatrix}, \quad (63)$$

The four non-zero partitions in $\mathbf{I}_{IND}$ can consist of indirect effects. The partition in the first column, second row of $\mathbf{I}_{IND}$ consists of the indirect effects of a set of predictor variables on a set of mediators through another set of mediators. The partition in the second row, second column of $\mathbf{I}_{IND}$ consists of the indirect effects of a set of mediators on another set of mediators through another set of mediators. The partition in the third row, second column consists of the indirect effects of a set of mediators on the outcomes through another set of mediators. Finally, the partition in the third row, first column represents the indirect effects of a set of predictors on a set of outcomes through a set of mediators.

The indirect effects contained in $\mathbf{I}_{IND}$ in (63) represent the total indirect effects. That is, they represent the sums of specific indirect effects. Bollen (1987) proposed a method for obtaining these specific indirect effects from the coefficient matrix **B**. To omit paths through specific mediators, the rows and columns of **B** associated with those mediators are set equal to 0. This can be accomplished by premultiplying **B** by an elementary operator that has diagonal elements corresponding to the omitted mediators set to zero. Calculating $\mathbf{I}_{IND}$ from this modified **B** matrix gives specific indirect effects.

Bollen's method for obtaining total and specific indirect effects can be extended to obtain matrices of total standardized squared indirect effects ($\upsilon^{tot}_{\boldsymbol{YMX}}$) and specific standardized squared indirect effects ($\upsilon^{sp}_{\boldsymbol{YMX}}$), where superscripts of υ denote the type of standardized squared indirect effect (e.g., total, specific, etc.) and subscripts denote the variables utilized in estimating the standardized squared indirect effect. This can be accomplished by employing the procedure outlined in (62) and (63) and replacing the coefficient matrix **B** with a matrix of squared coefficients $\mathbf{B}^*$. This matrix $\mathbf{B}^*$ is computed by taking the Hadamard square of the **B** matrix

$$\mathbf{B}^* = \left[\mathbf{B} \circ \mathbf{B}\right] = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{B}^2_{MX} & \mathbf{B}^2_{M} & \mathbf{0} \\ \mathbf{B}^2_{YX} & \mathbf{B}^2_{YM} & \mathbf{0} \end{bmatrix}, \tag{64}$$

where each partition of $\mathbf{B}^*$ contains squared regression coefficients. The matrix of $\upsilon^{tot}_{\boldsymbol{YMX}}$ ($\boldsymbol{\Upsilon}^{tot}_{\boldsymbol{YMX}}$) can then be calculated

$$\boldsymbol{\Upsilon}^{tot}_{\boldsymbol{YMX}} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ ((\mathbf{I}-\mathbf{B}^2_M)^{-1}-\mathbf{I})\mathbf{B}^2_{MX} & (\mathbf{I}-\mathbf{B}^2_M)^{-1}-\mathbf{I}-\mathbf{B}^2_M & \mathbf{0} \\ \mathbf{B}^2_{MX}\mathbf{B}^2_{YM}+\mathbf{B}^2_{YM}((\mathbf{I}-\mathbf{B}^2_M)^{-1}-\mathbf{I})\mathbf{B}^2_{MX} & \mathbf{B}^2_{YM}((\mathbf{I}-\mathbf{B}^2_M)^{-1}-\mathbf{I}) & \mathbf{0} \end{bmatrix}. \tag{65}$$

Special case of two simultaneous mediators

Consider the mediation model shown in Figure 2 in which there is one predictor variable X, two simultaneous mediators M_1 and M_2, and one criterion Y. If all variables are correlated, and there is no directional relationship between the mediators (i.e., one does not predict the other), the joint variance accounted for in Y by M_1, M_2, and X can be decomposed into 1) the joint variance in Y accounted for by X and M_1, 2) the joint variance in Y accounted for by X and M_2, and 3) the joint variance in Y accounted for by X, M_1, and M_2. As with R^2 for multiple regression analysis, the joint variance accounted for in Y can be conceptualized as consisting of unique and shared joint variance. For example, the joint variance in Y accounted for by X and M_1 above and beyond M_2 could be considered a unique component of joint variance, and the joint variance in Y accounted for by all the variables could be considered a shared component of joint variance.

The addition of another simultaneous, non-directional mediator M_3 would expand the number of joint variance components to include three unique joint variance components, three shared joint variance components consisting of the predictor and two of the three mediators, and a shared joint variance component consisting of all variables. As more variables are added and the model complexity increases, this method of labeling joint variance components becomes unwieldy. To avoid confusion, joint variance components will hereafter be labeled using the convention for correlation and partial correlation coefficients. The most basic component of joint variance, the unique component (i.e., one predictor, mediator, and outcome), is zero-order joint variance, the shared joint variance component with four variables (e.g., two mediators, one predictor, and one outcome) is first-order joint variance, the shared joint variance component with five variables (e.g., three mediators, one predictor, and one mediator) is second-order joint variance, etc.

Returning to the special case of two simultaneous mediators, (46) can be expanded to account for additional spurious correlations. For example, the correlation r_{YM_1} can be decomposed into

$$r_{YM_1} = \beta_{YM_1 \bullet M_2 X} + \beta_{M_1 X}\beta_{YX \bullet M_2 M_1} + \beta_{YM_2 \bullet M_1 X}\beta_{M_2 X} r_{M_1 X} + \beta_{MX_2} r_{M_1 M_2}\,, \tag{66}$$

where $\beta_{YM_1 \bullet M_2 X}$ is the direct effect of M_1 on Y controlling for the other variables in the model, and the remaining terms are spurious common causes of Y and M_1 and residual correlation among the mediators controlling for X. Expanding (53) for the joint variance components results in two zero-order joint variance components, one for the joint variance in Y accounted for by M_1 and X,

$$R^{2*}_{med(Y \bullet M_1 X)} = (r_{YM_1} - (\beta_{M_1 X}\beta_{YX \bullet M_1} + \beta_{YM_2 \bullet \mathrm{X}}\beta_{M_1 X}\beta_{M_2 X}))^2 - (R^2_{Y \bullet M_1 X} - r^2_{XY})\,, \tag{67}$$

one for the joint variance in Y accounted for by M_2 and X

$$R^{2*}_{med(Y \bullet M_2 X)} = (r_{YM_2} - (\beta_{M_2 X}\beta_{YX \bullet M_2} + \beta_{YM_1 \bullet X}\beta_{M_1 X}\beta_{M_2 X}))^2 - (R^2_{Y \bullet M_2 X} - r^2_{XY})\,, \tag{68}$$

and one first-order joint variance component for the joint variance in Y accounted for by M_1, M_2, and X,

$$\begin{aligned} R^{2*}_{med(Y \bullet M_2 M_1 X)} = {} & (r_{YM_1} - (\beta_{M_1 X}\beta_{YX \bullet M_1} + \beta_{YM_2 \bullet X}\beta_{M_2 X}\beta_{M_1 X} + \beta_{YM_2 \bullet \mathrm{X}}(r_{M1M2} - \beta_{M_1 X}\beta_{M_2 X})))^2 \\ & + (r_{YM_2} - (\beta_{M_2 X}\beta_{YX \bullet M_2} + \beta_{YM_1 \bullet X}\beta_{M_1 X}\beta_{M_2 X} + \beta_{YM_1 \bullet \mathrm{X}}(r_{M1M2} - \beta_{M_1 X}\beta_{M_2 X})))^2 \\ & - R^2_{Y \bullet M_1 X} - R^2_{Y \bullet M_2 X} - R^2_{Y \bullet M_2 M_1} + R^2_{Y \bullet M_2 M_1 X}\,. \end{aligned} \tag{69}$$

The total joint variance accounted for in Y by X, M_1, and M_2 is

$$R^{2*}_{med} = R^{2*}_{med(Y \bullet M_1 X)} + R^{2*}_{med(Y \bullet M_2 X)} - R^{2*}_{med(Y \bullet M_2 M_1 X)}\,. \tag{70}$$

If the residual correlation of the mediators controlling for X is zero, the first-order joint overlap component $R^{2*}_{med(Y \bullet M_2 M_1 X)} = 0$. Substituting this result in (70) leaves R^{2*}_{med} as

$$\begin{aligned} R^{2*}_{med} &= R^{2*}_{med(Y \bullet M_1 X)} + R^{2*}_{med(Y \bullet M_2 X)} \\ &= \beta^2_{M_1 X} \beta^2_{YM_1 \bullet X} + \beta^2_{M_2 X} \beta^2_{YM_2 \bullet X}. \end{aligned}$$

In other words, when the residual correlation between mediators conditional on X is zero, the total joint variance accounted for is the sum of the zero-order joint variance components of the specific indirect effects.

This result also can be derived using the matrix expression outlined in the previous section. The standardized regression coefficients from multiple mediator model can be expressed in the matrix **B** from (61),

$$\mathbf{B} = \left[\begin{array}{c:cc:c} 0 & 0 & 0 & 0 \\ \hdashline \beta_{M_1 X} & 0 & 0 & 0 \\ \beta_{M_2 X} & 0 & 0 & 0 \\ \hdashline \beta_{YX \bullet M_1 M_2} & \beta_{YM_1 \bullet M_2 X} & \beta_{YM_2 \bullet M_1 X} & 0 \end{array}\right].$$

Because the relationships among the mediators are non-directional, the third row, second column entry is zero. Taking the Hadamard square of this matrix as in (64) gives

$$\mathbf{B}^* = \left[\begin{array}{c:cc:c} 0 & 0 & 0 & 0 \\ \hdashline \beta^2_{M_1 X} & 0 & 0 & 0 \\ \beta^2_{M_2 X} & 0 & 0 & 0 \\ \hdashline \beta^2_{YX \bullet M_1 M_2} & \beta^2_{YM_1 \bullet M_2 X} & \beta^2_{YM_2 \bullet M_1 X} & 0 \end{array}\right].$$

Solving for $\boldsymbol{\Upsilon}^{tot}_{YM_1 M_2 X}$ as in (65) gives

$$\boldsymbol{\Upsilon}^{tot}_{YM_1 M_2 X} = \left[\begin{array}{c:cc:c} 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hdashline \beta^2_{M_1 X} \beta^2_{YM_1 \bullet M_2 X} + \beta^2_{M_2 X} \beta^2_{YM_2 \bullet M_1 X} & 0 & 0 & 0 \end{array}\right].$$

To obtain a specific indirect effect of X on Y through, say, M_1, the paths through M_2 are set to zero. First, **B** is premultiplied by an elementary operator matrix that replaces the rows and columns associated with M_2 with zeros:

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{c:cc:c} 0 & 0 & 0 & 0 \\ \hdashline \beta_{M_1X} & 0 & 0 & 0 \\ \beta_{M_2X} & 0 & 0 & 0 \\ \hdashline \beta_{YX\bullet M_1M_2} & \beta_{YM_1\bullet M_2X} & \beta_{YM_2\bullet M_1X} & 0 \end{array}\right] = \left[\begin{array}{c:cc:c} 0 & 0 & 0 & 0 \\ \hdashline \beta_{M_1X} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hdashline \beta_{YX\bullet M_1M_2} & \beta_{YM_1\bullet M_2X} & 0 & 0 \end{array}\right].$$

The Hadamard square of $\mathbf{B}$ is taken:

$$\mathbf{B}^* = \left[\begin{array}{c:cc:c} 0 & 0 & 0 & 0 \\ \hdashline \beta^2_{M_1X} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hdashline \beta^2_{YX\bullet M_1M_2} & \beta^2_{YM_1\bullet M_2X} & 0 & 0 \end{array}\right].$$

Solving for the matrix of $\upsilon^{sp}_{YM_1X}$ ($\boldsymbol{\Upsilon}^{sp}_{YM_1X}$)

$$\boldsymbol{\Upsilon}^{sp}_{YM_1X} = \left[\begin{array}{c:cc:c} 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hdashline \beta^2_{M_1X}\beta^2_{YM_1\bullet M_2X} & 0 & 0 & 0 \end{array}\right].$$

As a practical example, consider the multiple mediator model described above with the correlation matrix

$$\begin{bmatrix} 1 & & & \\ .3 & 1 & & \\ .3 & .09 & 1 & \\ .4 & .2 & .2 & 1 \end{bmatrix},$$

in which the first row and column correspond to the predictor variable X, the second and third rows and columns correspond to the mediators M_1 and M_2, and the fourth row and column correspond to the criterion Y. Note that the correlation between M_1 and M_2 is equal to the product of the correlations r_{M_1X} and r_{M_2X}, meaning that their residual correlation controlling for *X* is zero. That is, the mediators are independent conditional on *X*. The standardized indirect effects are

$$\beta_{M_1X}\beta_{YM_1 \bullet XM_2} = .3 \times \frac{.2 - .4 \times .3}{1 - .3^2} = .0267$$
$$\beta_{M_2X}\beta_{YM_2 \bullet XM_1} = .3 \times \frac{.2 - .4 \times .3}{1 - .3^2} = .0267,$$

which, when squared and summed, is .00139. This means the total joint variance in *Y* accounted for by *X*, M_1, and M_2 is .00139, and the specific joint variance components of M_1 and M_2 are both .000695. Now consider the correlation matrix where the mediators are positively correlated:

$$\begin{bmatrix} 1 & & & \\ .3 & 1 & & \\ .3 & .2 & 1 & \\ .4 & .2 & .2 & 1 \end{bmatrix}.$$

The standardized indirect effects are now smaller ($\beta_{M_1X}\beta_{YM_1 \bullet XM_2} = \beta_{M_2X}\beta_{YM_2 \bullet XM_1} = .0235$), as is the sum of $\upsilon^{sp}_{YM_1X}$ and $\upsilon^{sp}_{YM_2X}$ (.0011). This result demonstrates that, all else being equal, conditionally independent mediators account for more joint variance than mediators with residual correlation.

Examination of the specific indirect effects in the above example with residually correlated mediators leads to a modification of the procedure for determining joint variance components to obtain unique joint variance attributable to certain variables controlling for other variables in the model. To examine the specific indirect effect through M_1, the rows and columns of the **B** matrix corresponding to M_2 are set to zero and $\Upsilon^{sp}_{YM_1X}$ is calculated as in (65), which results in a $\upsilon^{sp}_{YM_1X}$ of .0006. However, this value incorporates the residual correlation between the mediators, so it should not be considered a zero-order joint variance component. The zero-order joint variance in *Y* accounted for by M_1 and *X* can be determined using a procedure similar to that for finding unique variance components in multiple regression. In this case, the joint variance accounted for in Y by M_2 and X without controlling for M_1 is subtracted from the total joint variance. This results in the zero-order joint variance component of M_1 and X because the correlation between M_1 and M_2 is incorporated into the indirect effect through M_2, and subtracting the resulting joint variance from the total joint variance removes the shared component of M_1 and M_2. In this example, the joint variance accounted for by M_2 alone can be found by eliminating the row and column associated with M_1 from the correlation matrix

$$\begin{bmatrix} 1 & & & \\ .3 & 1 & & \\ .3 & .2 & 1 & \\ .4 & .2 & .2 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & & \\ .3 & 1 & \\ .4 & .2 & 1 \end{bmatrix}$$

and estimating regression coefficients unconditional on M_1:

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ .3 & 0 & 0 \\ .37 & .088 & 0 \end{bmatrix}.$$

The standardized squared indirect effect associated with M_2 alone, or the *unconditional* standardized squared indirect effect ($\upsilon_{\boldsymbol{YMX}}^{unc}$), is .00071. Subtracting this from the $\upsilon_{YM_1M_2X}^{tot}$ results in .00039, which is the zero-order joint variance component accounted for in Y by X through M_1. This can also be considered the unique joint variance in Y accounted for by X through M_1, or the unique standardized squared indirect effect ($\upsilon_{\boldsymbol{YMX}}^{uni}$).

A general procedure for removing the row and column associated with the variables to create a reduced correlation matrix of interest $\mathbf{R}_{-j}$ can be accomplished by pre- and post-multiplying the correlation matrix $\mathbf{R}$ by an elementary operator $\mathbf{O}$

$$\mathbf{R}_{-j} = \mathbf{O}'\mathbf{R}\mathbf{O} \quad , \tag{71}$$

where subscript $-\boldsymbol{j}$ refers to the variables removed from the correlation matrix. For the above multiple mediator example, to remove mediator M_1, **O** is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Standardized squared indirect effects are then calculated from $\mathbf{R}_{-\boldsymbol{j}}$ to obtain a matrix of $\upsilon^{unc}_{\boldsymbol{YMX}}$ ($\boldsymbol{\Upsilon}^{unc}_{\boldsymbol{YMX}}$). To obtain the matrix of $\upsilon^{uni}_{\boldsymbol{YMX}}$ ($\boldsymbol{\Upsilon}^{uni}_{\boldsymbol{YMX}}$) for $\boldsymbol{j}$, $\boldsymbol{\Upsilon}^{unc}_{\boldsymbol{YMX}}$ is subtracted from $\boldsymbol{\Upsilon}^{tot}_{\boldsymbol{YMX}}$. However, these matrices are not conformable for addition, so $\boldsymbol{\Upsilon}^{tot}_{\boldsymbol{YMX}}$ is reduced by the same elementary operator **O** employed in (71) to reduce **R.** The matrix $\boldsymbol{\Upsilon}^{uni}_{\boldsymbol{YMX}}$ is then

$$\boldsymbol{\Upsilon}^{uni}_{\boldsymbol{YMX}} = \mathbf{O}'\boldsymbol{\Upsilon}^{tot}_{\boldsymbol{YMX}}\mathbf{O} - \boldsymbol{\Upsilon}^{unc}_{\boldsymbol{YMX}} \tag{72}$$

R^2 measures of effect size in mediation analysis similar to υ

There are three proposed measures of effect size for mediation that resemble υ. Two measures proposed by MacKinnon (2008) for the simple three-variable mediation model in (14) - (16) are

$$R^2_{4.6} = r^2_{MX} \times r^2_{YM\bullet X} \tag{73}$$

$$R^2_{4.7} = \frac{r^2_{MX} \times r^2_{YM\bullet X}}{R^2_{Y\bullet MX}} \tag{74}$$

where (74) represents a scaling of (73) by the reciprocal of the proportion of variance in Y accounted for jointly by M and X. More recently, De Heus (2012) proposed a revision of (73)

$$R^2_{DH} = r^2_{MX} \times r^2_{Y(M\bullet X)} \tag{75}$$

which represents a scaling of the partial correlation coefficient by the proportion of variance in *Y* accounted for by *X* as in (6). These results are similar to that obtained for a three-variable mediation model in (53), except that the squared regression coefficient is replaced with squared partial and semi-partial correlations. These conceptions of the joint variance explained have an advantage and a disadvantage. The advantage is that because r_{MX}^2, $r_{YM\bullet X}^2$, and $r_{Y(M\bullet X)}^2$ are bounded by 0 and 1, R_{med}^2 in (73) - (75) is also bounded by 0 and 1. This is not true for υ_{YMX}^{tot} because the squared standardized $\beta_{YM\bullet X}$ can be greater than 1 when suppression is evident. The disadvantage of these conceptions is that (73) - (75) are not monotonically increasing functions of the indirect effect in raw or absolute value. This means that when the total effect, direct effect, and variances are held constant, an increase in the indirect effect does not necessarily result in an increase in R_{med}^2. This can be seen in panels A - C of Figure 5 where R_{med}^2 from (73) - (75) was calculated for 5,000 randomly generated positive definite correlation matrices. Most commonly used effect size measures (e.g., Cohen's *d*, Cohen's *f*, R^2, etc.) are monotonically increasing functions of the quantity of interest (Wen & Fan, 2015).

The conceptions of R_{med}^2 in (73) - (75) are not monotonically increasing functions of the indirect effect because the indirect effect is theoretically unbounded, and constraining R_{med}^2 to be bounded by 0 and 1 requires standardized indirect effects greater than 1 to be scaled ≤ 1. This is evident by rearranging (9) in terms of the partial correlation

$$r_{YM\bullet X} = \beta_{YM\bullet X}\left(\frac{\sqrt{1-r_{MX}^2}}{\sqrt{1-r_{YX}^2}}\right) \tag{76}$$

and substituting into (73):

$$R^2_{med} = r^2_{MX} \times \beta^2_{YM \bullet X} \left(\frac{1 - r^2_{MX}}{1 - r^2_{YX}} \right) . \tag{77}$$

The consequence of applying the scaling factor in (77) to $\beta^2_{YM \bullet X}$ is that the rank order of effect sizes is not necessarily equivalent to the rank order of indirect effects. In other words, if R^2_{med} from study 1 is larger than R^2_{med} from study 2 using (73) - (75), it does not necessarily mean that the indirect effect was larger in study 1 than in study 2.

υ^{tot}_{YMX} shares properties of the R^2 effect size measures in (73) - (75), but also has key advantages. When suppression is not evident (i.e., $ab < c$ and $c' < c$), υ^{tot}_{YMX} and (73) - (75) are bounded by 0 and 1, and are monotonic functions in absolute value of the indirect effect, differing by only a scaling constant. However, υ^{tot}_{YMX} is the only measure that remains a monotonically increasing function of the indirect effect in absolute value when suppression is evident. This can be seen in panel D of Figure 5 where υ^{tot}_{YMX} was calculated for the same 5,000 correlation matrices as in panels A - C. Monotonicity allows effect sizes from different studies to be compared directly regardless of suppression. In addition, whereas it appears that (73) - (75) have an advantage in interpretability as proportions, the total joint variance component should not be interpreted as a proportion of variance because it can be negative (Cohen, et al., 2003). Therefore, the advantage of interpretability afforded by having bounded R^2_{med} is illusory because it is technically not a proportion of variance explained.

Summary of υ

To this point it appears the υ is a promising measure of effect size for mediation analysis. υ has been shown to be the result from adjusting the R^2_{med} formula derived by Fairchild and colleagues (2009) for spurious and unanalyzed correlation induced by mutual dependence among variables. Whereas υ is not bounded by 0 and 1 like proportion of variance measures, it has a significant advantage in interpretability over related measures by maintaining a monotonic relationship with the indirect effect in absolute value regardless of whether or not suppression is evident. Thus, υ appropriately quantifies the variance in the outcome accounted for jointly by a set of predictors and mediators in circumstances common in applied research settings. A matrix method is also available for obtaining υ based on the work of Bollen (1987). It has also been demonstrated that the $\upsilon^{tot}_{\boldsymbol{YMX}}$ can be decomposed into joint variance components such that the unique and shared contributions of a set of predictors and mediators can be examined. Table 1 summarizes the decompositions of υ, including recommendations for use in applied research. υ components represent a novel set of statistics with largely unknown properties. The aims of the following simulations were to examine the behavior of these statistics across a large range of parameter values, and to investigate their finite sampling properties using a Monte Carlo method.

Chapter IV

Simulation studies

The first simulation study was conducted to examine the properties of the υ in two mediation models: 1) a four-variable model with a single predictor, single criterion, and two parallel mediators and 2) a four-variable model with a single predictor, single outcome, single mediator, and a covariate. All variables were considered continuous with means of 0 and variances of 1. A range of parameters was created by applying the models to various positive definite correlation matrices. The behavior of υ under some specific parameter combinations was of interest (e.g., correlation between mediators was 0, one indirect effect was 0, etc.), so correlation matrices were created with values systematically varied over 0, .1, .3, .5, .7, and .9. In total, the two mediation models were applied to 21,500 positive-definite correlation matrices.

It was expected that υ_{YMX}^{tot}, υ_{YMX}^{sp}, and υ_{YMX}^{unc} would be monotonic functions in absolute value of their respective indirect effects across the range of parameter combinations. Since υ_{YMX}^{uni} are differences in υ_{YMX}^{tot} and υ_{YMX}^{unc} and do not have directly corresponding standardized indirect effects, it was not expected that υ_{YMX}^{uni} would be a monotonic function of any particular standardized indirect effect. In addition, it was expected that when correlations among predictors in the covariate model and residual correlations in the multiple mediator model were zero, υ_{YMX}^{sp}, υ_{YMX}^{unc}, and υ_{YMX}^{uni} would be equivalent. Further, as correlations among predictors and residual correlations among mediators increased, values of υ_{YMX}^{sp}, υ_{YMX}^{unc}, and υ_{YMX}^{uni} would diverge, with the largest discrepancies occurring when correlations and residual correlations were strongest.

Finally, it was expected that $\upsilon^{tot}_{\boldsymbol{YMX}}$, $\upsilon^{sp}_{\boldsymbol{YMX}}$, $\upsilon^{unc}_{\boldsymbol{YMX}}$, and $\upsilon^{uni}_{\boldsymbol{YMX}}$ would be zero when their respective indirect effects were zero.

Two parallel mediators

The first four-variable mediation model considered was a model with a single predictor and two parallel mediators. A path diagram for this model can be found in Figure 2. There were seven standardized squared indirect effects examined in this simulation. Considering the regressions of M_1 on X, M_2 on X, and Y on X, M_1, and M_2, there are one total standardized squared indirect effect ($\upsilon^{tot}_{YM_1M_2X}$) and two specific standardized squared indirect effects through M_1 ($\upsilon^{sp}_{YM_1X}$) and M_2 ($\upsilon^{sp}_{YM_2X}$). $\upsilon^{tot}_{YM_1M_2X}$, $\upsilon^{sp}_{YM_1X}$, and $\upsilon^{sp}_{YM_2X}$ quantify zero- and first-order joint variance. Considering the regressions of M_1 on X, M_2 on X, Y on X and M_1, and Y on X and M_2, there are two standardized squared indirect effects, one for the indirect effect of X on Y through M_1 unconditional on M_2 ($\upsilon^{unc}_{YM_1X}$) and the indirect effect of X on Y through M_2 unconditional on M_1 ($\upsilon^{unc}_{YM_2X}$). In these models the residual correlation between the mediators is incorporated into the indirect effects. Zero-order joint variance components uniquely attributable M_1 ($\upsilon^{uni}_{YM_1X}$) and M_2 ($\upsilon^{uni}_{YM_2X}$) are determined by subtracting $\upsilon^{unc}_{YM_1X}$ and $\upsilon^{unc}_{YM_2X}$ from $\upsilon^{tot}_{YM_1M_2X}$. Although not studied in the present simulation, it is also possible to examine the joint variance component common to both M_1 and M_2 by subtracting the zero-order joint variance component from the unconditional component. In sum, a mediation model with a single predictor, single outcome, and two parallel mediators yields $\upsilon^{tot}_{YM_1M_2X}$, $\upsilon^{sp}_{YM_1X}$, $\upsilon^{sp}_{YM_2X}$, $\upsilon^{unc}_{YM_1X}$, $\upsilon^{unc}_{YM_2X}$, $\upsilon^{uni}_{YM_1X}$, and $\upsilon^{uni}_{YM_2X}$.

Plots of the joint variance components and standardized indirect effects can be found in Figures 6 - 9. As in the plot for the three-variable mediation in Figure 5 panel D, $\upsilon^{tot}_{YM_1M_2X}$ for the two mediator model is always greater than 0 and can be greater than 1. However, unlike Panel D of Figure 5, it appears in Figure 6 that $\upsilon^{tot}_{YM_1M_2X}$ is not a monotonically increasing functioning of the total standardized indirect effect. This is the case because even though all of the values for the correlations used to generate the parameters were within the positive manifold, some standardized indirect effects were negative. Squaring and summing these negative standardized indirect effects for $\upsilon^{tot}_{YM_1M_2X}$ thus creates this departure from monotonicity. For example, if two standardized specific indirect effects were .5 and -.5, the total standardized indirect effect would be 0, but $\upsilon^{tot}_{YM_1M_2X}$ would be .5. This apparent contradiction can be considered an advantage of the $\upsilon^{tot}_{YM_1M_2X}$ over the total standardized indirect effect. There are infinitely many ways two mediators could return a total standardized indirect effect of 0 (e.g., both indirect effects are 0, one indirect effect is -1,000 and the other +1,000, etc.), and they would be indistinguishable without examining specific indirect effects. $\upsilon^{tot}_{YM_1M_2X}$ returns different values for these various parameter combinations without requiring the examination of $\upsilon^{sp}_{YM_1X}$ and $\upsilon^{sp}_{YM_2X}$, showing that joint variance is accounted for even if the standardized indirect effects cancel. Plots of $\upsilon^{sp}_{YM_1X}$ and $\upsilon^{sp}_{YM_2X}$ in Figure 7 shows that these are monotonic increasing function of the standardized indirect effect in absolute value. It can be seen in Figure 7 that a significant number of the correlation matrices produced indirect effects greater than 1 (i.e., suppression was evident), and the $\upsilon^{sp}_{YM_1X}$ and $\upsilon^{sp}_{YM_2X}$ remained monotonically increasing functions of the indirect effect in absolute value.

Plots of $\upsilon^{unc}_{YM_1X}$ and $\upsilon^{unc}_{YM_2X}$ can be found in Figure 8. As expected, these were also monotonic functions of the respective standardized indirect effects in absolute value. To examine the differences in magnitude between $\upsilon^{sp}_{YM_1X}$ and $\upsilon^{unc}_{YM_1X}$ and between $\upsilon^{sp}_{YM_2X}$ and $\upsilon^{unc}_{YM_2X}$, values of each statistic for M_1 were plotted and can be found in Figure 9. It appears from this plot that $\upsilon^{unc}_{YM_1X}$ and $\upsilon^{unc}_{YM_2X}$ can vary substantially from $\upsilon^{sp}_{YM_1X}$ and $\upsilon^{sp}_{YM_2X}$. For example, the largest $\upsilon^{unc}_{YM_1X}$ was approximately 4, whereas the largest $\upsilon^{sp}_{YM_1X}$ was greater than 30. However, the Spearman rank correlation between $\upsilon^{sp}_{YM_1X}$ and $\upsilon^{unc}_{YM_1X}$ and between $\upsilon^{sp}_{YM_2X}$ and $\upsilon^{unc}_{YM_2X}$ was 0.88, suggesting that although the magnitudes of the estimates appeared to vary, the rank order is largely preserved.

Plots of $\upsilon^{uni}_{YM_1X}$ and $\upsilon^{uni}_{YM_2X}$ can be found in Figure 10. It can be seen that zero-order joint variance components can be negative, meaning that, for example, $\upsilon^{unc}_{YM_1X}$ was larger than $\upsilon^{tot}_{YM_1M_2X}$. In general, negative zero-order joint variance components occur when

$$\upsilon^{tot}_{\boldsymbol{YMX}} - \upsilon^{unc}_{\boldsymbol{YMX}} < 0.$$

In the two parallel multiple mediator example, the $\upsilon^{uni}_{YM_2X}$ of M_2 can be expressed as

$$\beta^2_{M_1X}\beta^2_{YM_1\bullet XM_2} + \beta^2_{M_2X}\beta^2_{YM_2\bullet XM_1} - \beta^2_{M_1X}\beta^2_{YM_1\bullet X} < 0,$$

Similarly, the $\upsilon^{uni}_{YM_1X}$ for M_1 is negative when

$$\beta^2_{M_1X}\beta^2_{YM_1\bullet XM_2} + \beta^2_{M_2X}\beta^2_{YM_2\bullet XM_1} - \beta^2_{M_2X}\beta^2_{YM_2\bullet X} < 0.$$

$\upsilon^{uni}_{YM_1X}$ and $\upsilon^{uni}_{YM_2X}$ are negative in situations similar to suppression multiple regression models. Consider the correlation matrix

$$\begin{bmatrix} 1 & .3 & .9 & .1 \\ .3 & 1 & .1 & .5 \\ .9 & .1 & 1 & 0 \\ .1 & .5 & 0 & 1 \end{bmatrix},$$

where the first column represents correlations with *X*, the second column correlations with M_1, the third column correlations with M_2, and the fourth column correlations with *Y*. In this example, regression coefficients are .3 for M_1 on *X* (β_{M_1X}), .9 for M_2 on *X* (β_{M_2X}), .51 for *Y* on M_1 controlling for *X* and M_2 ($\beta_{YM_1 \bullet XM_2}$), and -.014 for *Y* on M_2 controlling for *X* and M_1 ($\beta_{YM_2 \bullet XM_1}$). The total effect β_{YX} = .1 and the direct effect $\beta_{YX \bullet M_1M_2}$ = -.042. The specific indirect effect through M_1 is $\beta_{M_1X}\beta_{YM_1 \bullet XM_2}$ = .154 and the specific indirect effect through M_2 is $\beta_{M_2X}\beta_{YM_2 \bullet XM_1}$ = -.0125. Because the total indirect effect and direct effect were of different signs, this meant that suppression was evident (MacKinnon et al., 2000). The total indirect effect was .1415 ($\upsilon^{tot}_{YM_1M_2X} = .02$), the unconditional indirect effect through M_1 is .155 ($\upsilon^{unc}_{YM_1X} = .024$), and the unconditional indirect effect through M_2 is -.426 ($\upsilon^{unc}_{YM_2X} = .181$). Therefore, $\upsilon^{uni}_{YM_1X}$ through M_1 is .024 - .182 = -.1578, and $\upsilon^{uni}_{YM_2X}$ through M_2 is .024 - .024 = 0. In this case, the cause of the negative $\upsilon^{uni}_{YM_1X}$ was due to the difference between the specific indirect effect through M_2 ($\beta_{M_2X}\beta_{YM_2 \bullet XM_1}$ = -.0125) and the unconditional indirect effect through M_2 not controlling for M_1 ($\beta_{M_2X}\beta_{YM_2 \bullet X}$ = -.426). This illustrates how the magnitude of an indirect effect can be substantially affected by the residual correlation among mediators.

In general, correlation among predictors and residual correlation among mediators are the cause of differences between specific indirect effects and unconditional indirect effects. To illustrate this point, consider a scenario where mediators M_1 and M_2 are correlated, but the indirect effect through M_1 is zero (i.e., $\beta_{M_1X} = 0$, $\beta_{YM_1 \bullet XM_2} = 0$, or $\beta_{M_1X} = \beta_{YM_1 \bullet XM_2} = 0$). If M_1 and M_2 were independent conditional on X, then the unconditional indirect effects would be equivalent to the specific indirect effects. However, if there is any residual correlation between M_1 and M_2, the unconditional indirect effects through M_2 will be larger than its associated specific indirect effect, making $\upsilon^{tot}_{YM_1M_2X}$ smaller than $\upsilon^{unc}_{YM_2X}$, resulting in negative $\upsilon^{uni}_{YM_1X}$. If mediators are correlated, the unconditional indirect effects and specific indirect effects will certainly differ, and stronger correlations will be associated with larger differences.

Single mediator, covariate

The second four-variable model considered was a three-variable mediation model with a single predictor X, single mediator M, and a covariate Z. A path diagram of this model can be found in Figure 4 (X_1 or X_2 could be labeled Z). Z was considered as a baseline covariate (i.e., Z was measured concurrently with X). As with the four-variable model with two parallel mediators, there were seven υ s that could be examined. Considering the regressions of M on X, M on Z, and Y on X, Z, and M, there is a total standardized squared indirect effect (υ^{tot}_{YMXZ}), and two specific standardized squared indirect effects through M, one with predictor X (υ^{sp}_{YMX}) and the other with covariate Z (υ^{sp}_{YMZ}). υ^{tot}_{YMXZ}, υ^{sp}_{YMX}, and υ^{sp}_{YMZ} consist of zero- and first-order joint variance that can be decomposed to obtain estimates of the unique contributions of X and Z. Considering the regressions of M on X, M on Z, Y on X and M, and Y on Z and M, there are two unconditional standardized squared indirect effects, one the indirect effect of X on Y through M unconditional on Z (υ^{unc}_{YMX}) and the indirect effect of Z on Y through M unconditional on X (υ^{unc}_{YMZ}). In the unconditional models, the correlation between the X and Z is incorporated into the indirect effect. Finally, the zero-order joint variance attributable uniquely to X (υ_{YMX}^{uni}) and Z (υ_{YMZ}^{uni}) can be obtained by subtracting υ_{YMX}^{unc} and υ_{YMZ}^{unc} from υ_{YMXZ}^{tot}. In this example, the effect of interest could be υ_{YMXZ}^{tot} (i.e., the total joint variance accounted for by X, Z, and M), υ_{YMX}^{sp} (i.e., the joint variance accounted for by X and M without partialing out the effect of Z), υ_{YMX}^{unc} (i.e., joint variance accounted for by X and M including the variance shared with Z), or υ_{YMX}^{uni} (i.e., the joint variance in Y attributable uniquely to X and M). Z could also be considered another predictor of interest. In summary, the mediation model with a single predictor, mediator, outcome, and covariate can yield υ_{YMXZ}^{tot}, υ_{YMX}^{sp}, υ_{YMZ}^{sp}, υ_{YMX}^{unc}, υ_{YMZ}^{unc}, υ_{YMX}^{uni}, and υ_{YMX}^{uni}.

Plots of υ for the three variable mediation model with a covariate can be found in Figures 11 - 14. The overall behavior of υ in the three variable mediation model with a covariate appeared similar to the behavior of the mediation model with two parallel mediators in Figures 6 - 9. Specifically, υ_{YMXZ}^{tot} was not a monotonically increasing function in absolute value of the total standardized indirect effect, but υ_{YMX}^{sp}, υ_{YMZ}^{sp}, υ_{YMX}^{unc}, and υ_{YMZ}^{unc} were monotonically increasing functions of the specific and unconditional standardized indirect effects. However, there were some noteworthy differences between the models' behaviors. The plot of $\upsilon_{YM_1M_2X}^{tot}$ in Figure 6 is closer to a monotonic function of the indirect effect than υ_{YMXZ}^{tot} in Figure 11. This suggests that correlated predictors and residually correlated mediators differentially influence the magnitude of the indirect effects. In addition, the differences between the specific and unconditional υs were more disparate in the model with a covariate than in the multiple mediator model, with a Spearman rank correlation of only 0.33.

Chapter V

Monte Carlo simulation

Monte Carlo simulations are employed to examine the finite sample properties of statistical estimators. Employing this type of simulation allows for the behavior of estimators to be studied under various conditions (e.g., small sample sizes, nonnormality, misspecification, etc.) (Paxton, Curran, Bollen, Kirby, & Chen, 2001), and is particularly useful for estimators with asymptotic distributions that are overly complex or unknown. υ is the square of the standardized indirect effect, which is known to have a non-normal sampling distribution (MacKinnon, Lockwood, & Williams, 2004). Specifically, the asymptotic distribution of the indirect effect is a Bessel function of the second kind with a purely imaginary argument (Aroian, 1947). The square of this distribution is not easily derived, so a Monte Carlo simulation was employed to study the finite sample properties of υ.

The Monte Carlo simulation was designed to examine the sample behavior of υ from populations with varying magnitudes of indirect effects, total effects, and correlations among mediators. There were several υs that could be examined, and the simplest model that yielded $\upsilon_{\boldsymbol{YMX}}^{tot}$, $\upsilon_{\boldsymbol{YMX}}^{sp}$, $\upsilon_{\boldsymbol{YMX}}^{unc}$, and $\upsilon_{\boldsymbol{YMX}}^{uni}$ was a four-variable model with a single predictor, single outcome, and two parallel mediators. Of most interest for this set of statistics were bias, relative bias, mean square error (MSE), consistency, efficiency, coverage, and the proportions of estimates above and below the 95% confidence limits.

Bias was defined as the difference between the sample estimate and the population parameter,

$$bias(\hat{\theta}) = \hat{\theta} - \theta, \tag{78}$$

where $\hat{\theta}$ is the sample estimate and θ is the population parameter. Lower values of bias indicate that the mean sample estimate is closer to the population parameter, and an unbiased estimator has a bias of 0. It was hypothesized that because the sample estimate of the indirect effect is an unbiased estimate of the indirect effect (Bauer, Preacher, & Gil, 2006), then υ would be an unbiased estimate of υ. Relative bias was defined as the ratio of the bias to the population parameter:

$$bias_{rel}(\hat{\theta}) = \frac{\hat{\theta} - \theta}{\theta}. \tag{79}$$

Like bias, lower relative bias means the sample estimates are closer to the population parameter, and an unbiased estimator has a relative bias of 0. However, relative bias is on a different scale than bias, so whereas the measures convey essentially the same information, relative bias provides results that may be more interpretable. It was hypothesized that υ would be unbiased in terms of relative bias, and that relative bias would decrease as sample size increased.

MSE was defined as the sum of the variance of sample estimates and squared bias,

$$MSE(\hat{\theta}) = \mathrm{var}(\hat{\theta}) + (bias(\hat{\theta}))^2. \tag{80}$$

MSE is a measure of the bias and variability of a sample estimator, and lower values of MSE indicate a more precise estimator. When a sample estimator is unbiased, the MSE is equivalent to the variance of that estimator. It was hypothesized that the MSE would decrease as sample size increased.

Consistency of the statistics was evaluated by conducting the simulations across a range of sample sizes. As the sample size is increased, the statistics were expected to converge to their population values. That is, bias, relative bias, MSE, and CI width were expected to decrease as sample size increased. There are no explicit criteria for evaluating efficiency, but relative efficiency for MSE and CI widths of estimators (e.g., υ_{YMX}^{sp} vs. υ_{YMX}^{unc}) were compared to determine if υ s were equally efficient, and, if not, which estimators were the most efficient. Relative efficiency was evaluated by taking the ratio of pairs of estimators. Relative efficiency values greater than 1 indicated the estimator in the numerator was more efficient, and relative efficiency values less than 1 indicated the estimator in the denominator was more efficient. It was hypothesized that υ_{YMX}^{unc} would be the most efficient estimator because the inclusion of the additional mediator for υ_{YMX}^{sp} and υ_{YMX}^{uni} would result in less precise estimates.

Coverage was defined as the proportion of samples in which the population parameter was contained within the 95% CI. It was hypothesized that coverage would approach a nominal level of 95% as the sample size increased. If the population parameter was not contained within the 95% CI of a sample, it was recorded whether the parameter was above the upper CI limit or below the lower CI limit. A proper 95% CI requires 2.5% of samples outside of the CI to be above the upper confidence limit and 2.5% below the lower confidence limit. It was hypothesized that the proportion of these misses to the left and right of the CI would be equal.

Correlation matrices were used to generate the population data for the Monte Carlo simulations. Only positive definite matrices were included in the simulation. The values of the correlations varied among 0, .09, .1, .3, .5, and .9. Given the computational intensity of the simulation procedures, several restrictions were imposed to reduce the total number of population matrices. First, total effects were set to be only .3 and .5. Although cases in which the total effect is very small or zero may be of interest to investigate the properties of the statistic under those specific conditions of suppression, there still remained several parameter combinations in which suppression was evident with total effects of .3 and .5. Second, mediator correlations were set to be 0, .09, .3, and .9. These correlations represent conditions in which the mediators had no residual correlation ($r_{M_1M_2 \cdot X} = 0$), small residual correlation ($r_{M_1M_2} = \pm.15$), and strong residual correlation ($r_{M_1M_2} = \pm.9$). Third, the values for r_{M_1X} and r_{YM_1} were constrained to be equal and varied among .1, .3, .5, and .9. Fourth, values of r_{M_2X} and r_{YM_2} varied among 0, .3, and .9. These conditions created mediation models in which the magnitudes of the standardized indirect effects were in a range common in applied research settings, as well as models with more extreme parameter combinations. In addition, correlation matrices that resulted in similar parameters were removed. This resulted in 36 correlation matrices for use as population correlation matrices. Population data were generated from these matrices using the 'mvrnorm' function in R (version 3.1.2).

Monte Carlo simulations were performed on each of the 36 sets of population data. First, for each of the population correlation matrices, 500 samples were created from the population correlation matrix using the R 'mvrnorm' function with the setting 'empirical=FALSE'. Then for each of the 500 samples, 1000 bootstrap resamples were created by resampling with replacement. For each bootstrap resample, $\upsilon^{tot}_{YM_1M_2X}$, $\upsilon^{sp}_{YM_1X}$, $\upsilon^{sp}_{YM_2X}$, $\upsilon^{unc}_{YM_1X}$, $\upsilon^{unc}_{YM_2X}$, $\upsilon^{uni}_{YM_1X}$, and $\upsilon^{uni}_{YM_2X}$ were estimated. Values for the 2.5th and 97.5th percentiles, 95% CI width, and whether or not the population parameter was within the CI were recorded for each sample. If the population parameter was outside the 95% CI, it was recorded whether the parameter was above the upper confidence limit or below the lower confidence limit. For each of the 500 samples, $\upsilon^{tot}_{YM_1M_2X}$, $\upsilon^{sp}_{YM_1X}$, $\upsilon^{sp}_{YM_2X}$, $\upsilon^{unc}_{YM_1X}$, $\upsilon^{unc}_{YM_2X}$, $\upsilon^{uni}_{YM_1X}$, and $\upsilon^{uni}_{YM_2X}$ were also estimated. Means and variances of each statistic were computed across the 500 samples. This procedure was then repeated for four sample sizes of 50, 100, 250, and 500.

Results of Monte Carlo simulation

Complete results of the Monte Carlo simulation for all υs can be found in Tables 2-10. Included in the tables are estimates of the mean υ, bias, relative bias, MSE, mean lower and upper confidence limits, mean CI width, coverage, and proportions of misses to the left and right of the 95% CI. These values were estimated across sample sizes of 50, 100, 250, and 500, and for 36 different combinations of population correlations.

Graphical summaries across the generating populations of $\upsilon^{tot}_{YM_1M_2X}$, $\upsilon^{sp}_{YM_1X}$, $\upsilon^{sp}_{YM_2X}$, $\upsilon^{unc}_{YM_1X}$, $\upsilon^{unc}_{YM_2X}$, $\upsilon^{uni}_{YM_1X}$, and $\upsilon^{uni}_{YM_2X}$ can be found in Figures 15-21. For each sample size, plots include the mean $\hat{\upsilon}$ estimates for each sample, median $\hat{\upsilon}$ estimate across populations, medians for the lower and upper confidence limits, and the median population υ. Medians were used to summarize central tendency because the distributions of mean estimates across populations were substantially positively skewed. It can be seen in Figures 15-21 that the median estimate of $\hat{\upsilon}$ across populations approached the population median as sample size increased. In addition, CI widths about the median estimates decreased as sample size increased. Moreover, Tables 2, 5, and 8 show that for each set of population parameters, estimates approached the population parameters as sample size increased. This demonstrates that $\hat{\upsilon}$ is a consistent estimator of the population υ. It can also be seen from Figures 15 - 21 that the mean estimates of $\hat{\upsilon}^{tot}_{YM_1M_2X}$, $\hat{\upsilon}^{sp}_{YM_1X}$, and $\hat{\upsilon}^{sp}_{YM_2X}$ were consistently positively biased, and that the bias was more pronounced at smaller sample sizes. As with the estimates of the mean $\hat{\upsilon}$, the median bias across all parameter combinations approached zero as the sample size increased. This was also true for the relative bias and MSE.

Bias in the mean $\hat{\upsilon}$ estimates was more pronounced when the population υ was close to zero. For example, the maximum relative bias for $\hat{\upsilon}^{tot}_{YM_1M_2X}$ was 573.8742 for a sample size of 50. Even though the absolute bias for this parameter combination was 0.00284, the population $\upsilon^{tot}_{YM_1M_2X}$ was 4.9x10^{-6}, meaning the $\hat{\upsilon}^{tot}_{YM_1M_2X}$ estimate was of much larger magnitude relative to the population $\upsilon^{tot}_{YM_1M_2X}$. However, as the magnitude of υ increased in the population, the bias and relative bias of the estimated $\hat{\upsilon}$ decreased for all sample sizes.

Coverage and proportions of misses to the left and right of the 95% CI can be found in Tables 4, 7, and 10. There were several instances where the population parameters were not contained within the 95% CIs. This occurred when υ in the population was 0 (i.e., $\beta_{M_1X}\beta_{YM_1\bullet X}=0$). Because $\upsilon^{tot}_{YM_1M_2X}$, $\upsilon^{sp}_{YM_1X}$, $\upsilon^{sp}_{YM_2X}$, $\upsilon^{unc}_{YM_1X}$, and $\upsilon^{unc}_{YM_2X}$ can never be negative, all values close to zero were positive, making it impossible for a CI to include 0. This is why when examining the proportions of misses to the left and right of the 95% CI, all of the misses are to the left because the lower 95% confidence limit is always greater than the population parameter. Similarly, imbalances in the proportions of misses to the left and right appeared to occur more frequently across sample sizes when υ was small in the population (~.00005 or less). For υ larger than ~.00005, imbalances appeared to decrease as sample size increased. In addition, high coverage values (i.e., ≥ .98) were more common for smaller υ, suggesting that small υ may have overly wide CIs. It also appeared that, in general, coverage approached the nominal level of 95% as sample size and υ magnitude increased.

Examination of the CI widths in Tables 3, 6, and 9 and MSE in Tables 2, 5, and 8 showed that $\upsilon^{tot}_{YM_1M_2X}$ had the largest median CI width (*M* = .1013) and the largest median MSE (*M* = 1.3x10^{-5}) of all the statistics considered. Comparisons of the CI widths and MSE for $\upsilon^{sp}_{\boldsymbol{YMX}}$, $\upsilon^{unc}_{\boldsymbol{YMX}}$, and $\upsilon_{\boldsymbol{YMX}}^{uni}$ did not reveal clear optimal statistics in terms of efficiency. Plots of the relative CI widths for M_1 at sample size 500 for each set of parameters can be found in Figure 22, and plots of the relative MSE for M_1 at sample size 500 for each set of parameters can be found in Figure 23. For the majority of populations, the CI widths and MSEs for $\hat{\upsilon}_{YM_1X}^{unc}$ and $\hat{\upsilon}_{YM_2X}^{unc}$ were smaller than for $\hat{\upsilon}_{YM_1X}^{sp}$, $\hat{\upsilon}_{YM_2X}^{sp}$, $\hat{\upsilon}_{YM_1X}^{uni}$, and $\hat{\upsilon}_{YM_2X}^{uni}$. One pattern that emerged among the populations where $\hat{\upsilon}_{YM_1X}^{unc}$ and $\hat{\upsilon}_{YM_2X}^{unc}$ were the most efficient were that those populations tended to have high residual correlations among the mediators. Specifically, $\hat{\upsilon}_{YM_1X}^{unc}$ and $\hat{\upsilon}_{YM_2X}^{unc}$ were the most efficient in all populations that had the highest mediator residual correlation ($r_{M_1M_2} = .8684$). This suggests that high mediator residual correlations can result in decreased precision of parameter estimates in much the same way that multicollinearity among predictors in multiple regression results in less precise parameter estimates.

Chapter VI

Discussion

The goal of this research was to develop a general effect size measure for mediation analysis that could be applied to mediation models of any complexity. For the mediation models considered, υ was shown to be an appropriate measure of effect size. For the basic three-variable mediation model, it was demonstrated that υ represents the variance in the outcome that is accounted for jointly by the predictor and mediator. υ improves on a previous measure of joint variance (R^2_{med}) by using decomposition techniques from path analysis (Duncan, 1970; Simon, 1957; Wright, 1960) to account for spurious and unanalyzed effects that inflate the correlations between mediators and outcomes. Accounting for these effects corrected a contradictory result returned by R^2_{med} where an indirect effect of 0 could return a non-zero effect size. It was also shown that because υ is a squared product or sum of squared products of standardized regression coefficients, it always represents the variance in an outcome shared by a set of predictors and mediators. A general matrix technique was also developed that returns υ for various mediation models.

υ has many desirable properties of an effect size measure. First, υ has an interpretable scale. Because it is the squared product of standardized regression coefficients, υ is also standardized; i.e., it is not dependent on the scales of the predictors, mediators, or outcomes. This also means that it is invariant under linear transformations of the predictor, mediator, or outcome. υ is also a monotonically increasing function of the indirect effect in absolute value, meaning that larger magnitudes of υ directly correspond to larger magnitudes of the indirect effect. This is an advantage over similar measures of joint variance in (73) - (75) because those measures do not preserve the rank order of indirect effects, complicating the comparison of effect sizes across studies. Even though it is simply a scaling of (73) - (75), υ does not lose its property of monotonicity when suppression is evident. This means that results obtained under all conditions could be appropriately compared across studies. Second, although relative bias appeared excessively high in some cases, it was due to population effects being very small, and the absolute bias in those cases was relatively small. Third, υ is a consistent estimator. As demonstrated via Monte Carlo simulation, υ estimate approaches the population parameter as the sample size increases. Fourth, Monte Carlo simulation demonstrated that CIs can be created for υ using a bootstrap procedure. Fifth, although there are no criteria for absolute efficiency, Monte Carlo simulation also demonstrated that the width of the CIs decreases as sample size increases.

υ can also be used to decompose and compare magnitudes of indirect effects in models with multiple mediators and/or covariates. Current methods for determining indirect effects within a multiple mediator model estimate coefficients such that the sum of the specific indirect effects is the total indirect effect. However, it may be of interest to decompose the total joint variance accounted for into unique and shared components as in multiple regression analysis. This can be accomplished by estimating υ_{YMX}^{unc} for models without certain mediators, and subtracting υ_{YMX}^{unc} from those unconditional models from υ_{YMX}^{tot} from the full model. The resulting υ_{YMX}^{uni} represent joint variance components unique to certain mediators. υ also has the desirable property that when the total indirect effect is zero but the specific indirect effects are non-zero (i.e., when specific indirect effects cancel), it returns a non-zero total effect size, indicating that joint variance is indeed accounted for. This method of decomposition aids in determining the effects of mediators in the presence of covariates, such that the effects of the covariates can be partialed out from the effects of the mediators.

Limitations

υ has several limitations that should be addressed by future research. First, because υ is similar to R^2 in multiple regression, it likely has a bias towards overfitting in sample data. Simulation studies can address this potential limitation by 1) determining υ does in fact have an overfitting bias and the extent of bias using a resampling or simulated cross-validation procedure, and 2) adjust υ for inflation. Second, because υ can be larger than 1, it should not be interpreted as a proportion of variance. This somewhat limits the interpretability of υ, and highlights an advantage of the other joint variance measures (73) - (75) which are bounded by 0 and 1. In addition, benchmarks established for small, medium, and large effects sizes for R^2 measures would not be applicable. However, the forfeiture of the boundedness criterion comes with two significant gains. One is the aforementioned monotonicity property, but another is a simple detection of suppression or inconsistent mediation. Any $\upsilon > 1$ indicates that suppression or inconsistent mediation is evident, which could be valuable information for applied researchers. In addition, some commonly employed effect size measures such as Cohen's *d* do not have an upper bound and maintain interpretability. Benchmarks for small, medium, and large effect sizes can be established over time as the measure is employed in the reporting of results for a given field. Third, when using υ to decompose joint variance into unique and shared components, under some circumstances the unique joint variance components can be negative. There are some potential solutions to this limitation. One is to set a lower bound of 0 on the unique joint variance component. Because by definition υ is a squared value, it would be illogical to interpret negative values as meaningful. It may be more reasonable to interpret those variables as contributing no unique joint variance. Another potential solution could be to consider removing a variable with a negative unique joint variance component since it is unlikely to be contributing uniquely above and beyond the other variables in the model. However, this would be a more exploratory procedure, and appropriate steps should also be taken to control the Type I error rate.

Future research

υ is a promising general effect size measure for mediation analysis. The present study demonstrated the applicability of υ in single and multiple mediator models, as well as in mediator models with covariates. Future research should extend the results of this study to mediation models with latent variables, moderators, and multilevel mediation. In addition, variables considered in this study were continuous, and there may be approximations of υ in models with binary and count outcomes (e.g., pseudo-R^2). Future work should also develop an appropriate adjustment to correct for the consistent positive bias of υ. Given the expected behavior of a squared value, there may be a consistent adjustment to reduce the positive bias. Finally, future research could further examine methods of joint variance decomposition and the behavior of joint variance components. Given the similarities demonstrated between υ and standardized regression coefficients in multiple regression, there may be ways to increase effect sizes by adding independent predictors to reduce total error variance.

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Figure 1. *Path diagram for a three variable mediation model*

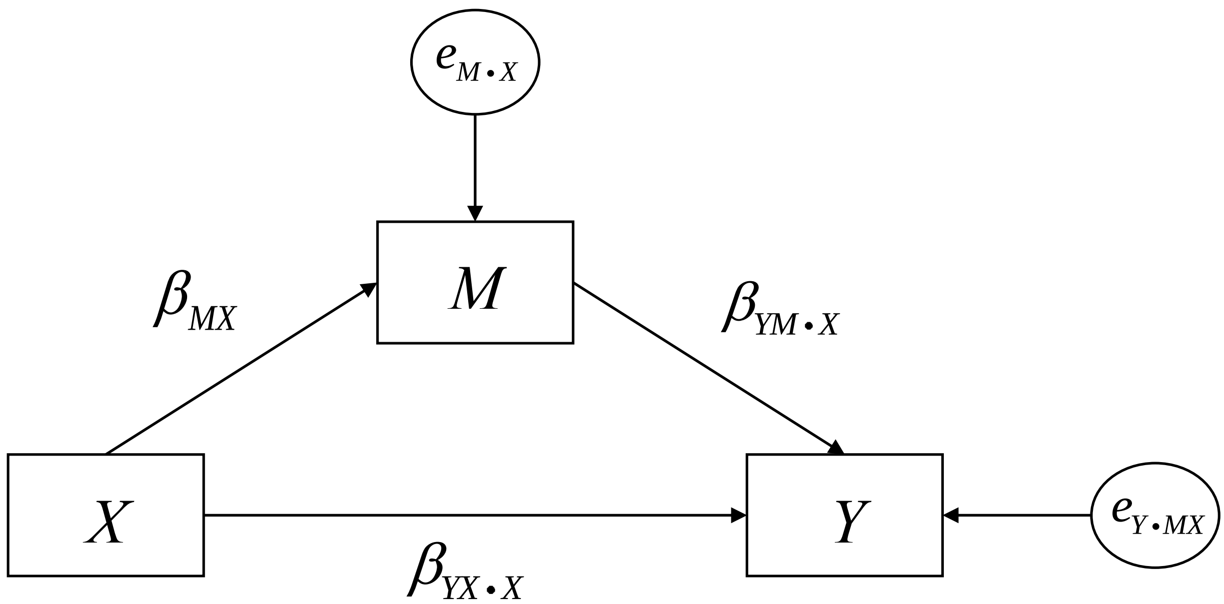


Figure 2. *Path diagram for a four variable mediation model with two parallel mediators*

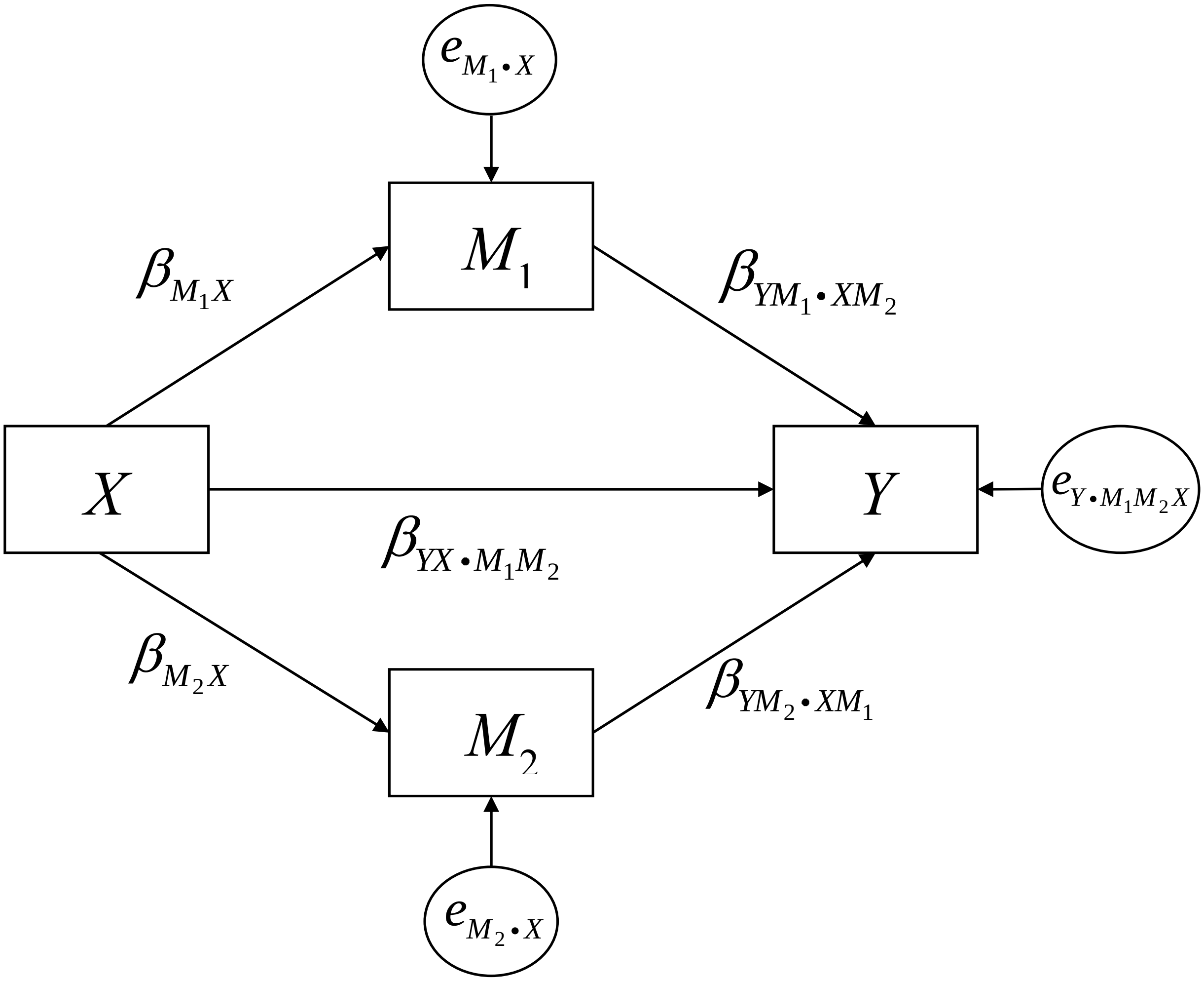


Figure 3. *Potential relationships among three variables*

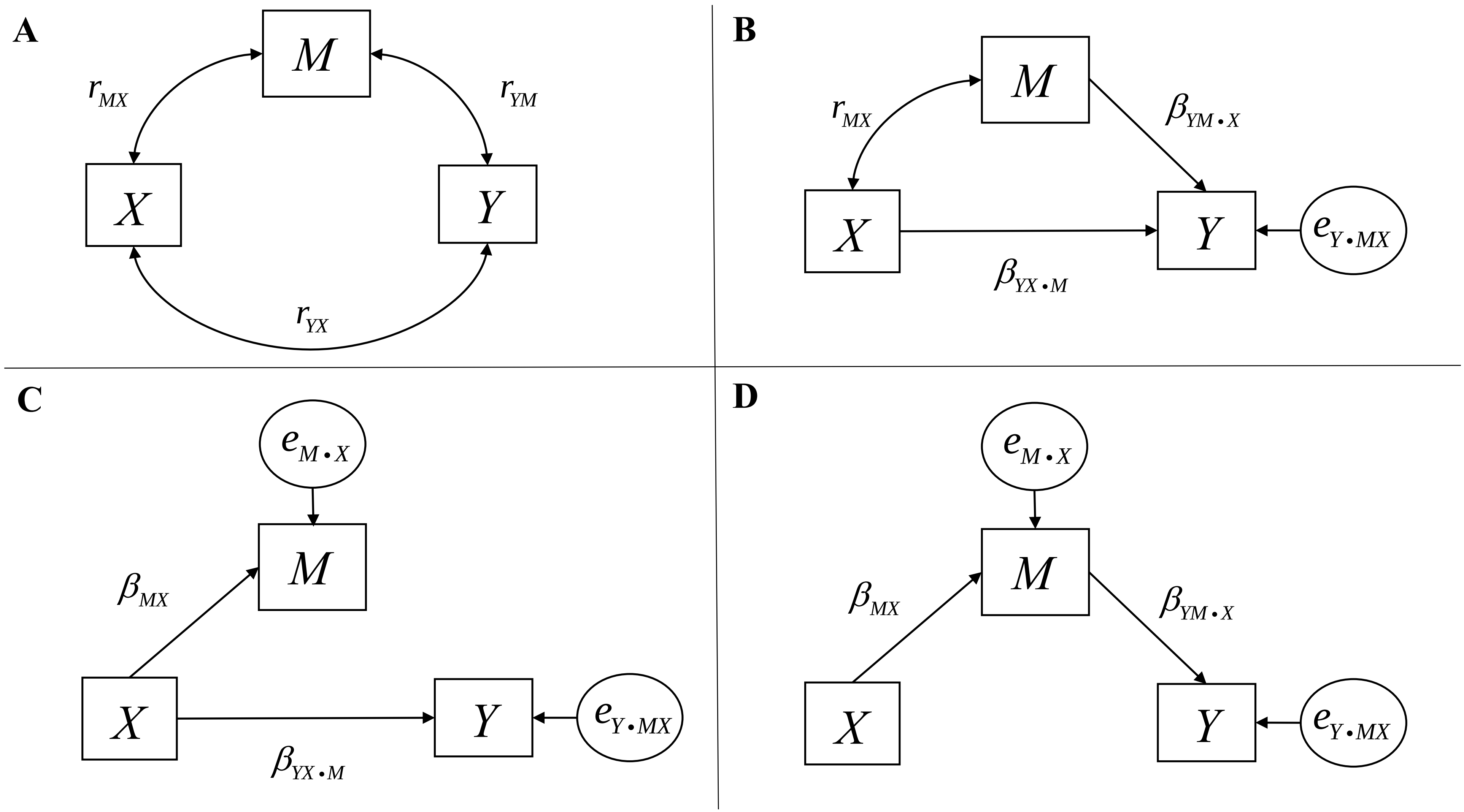


Figure 4. *Dillon & Goldstein 12.2-3 mediation model*

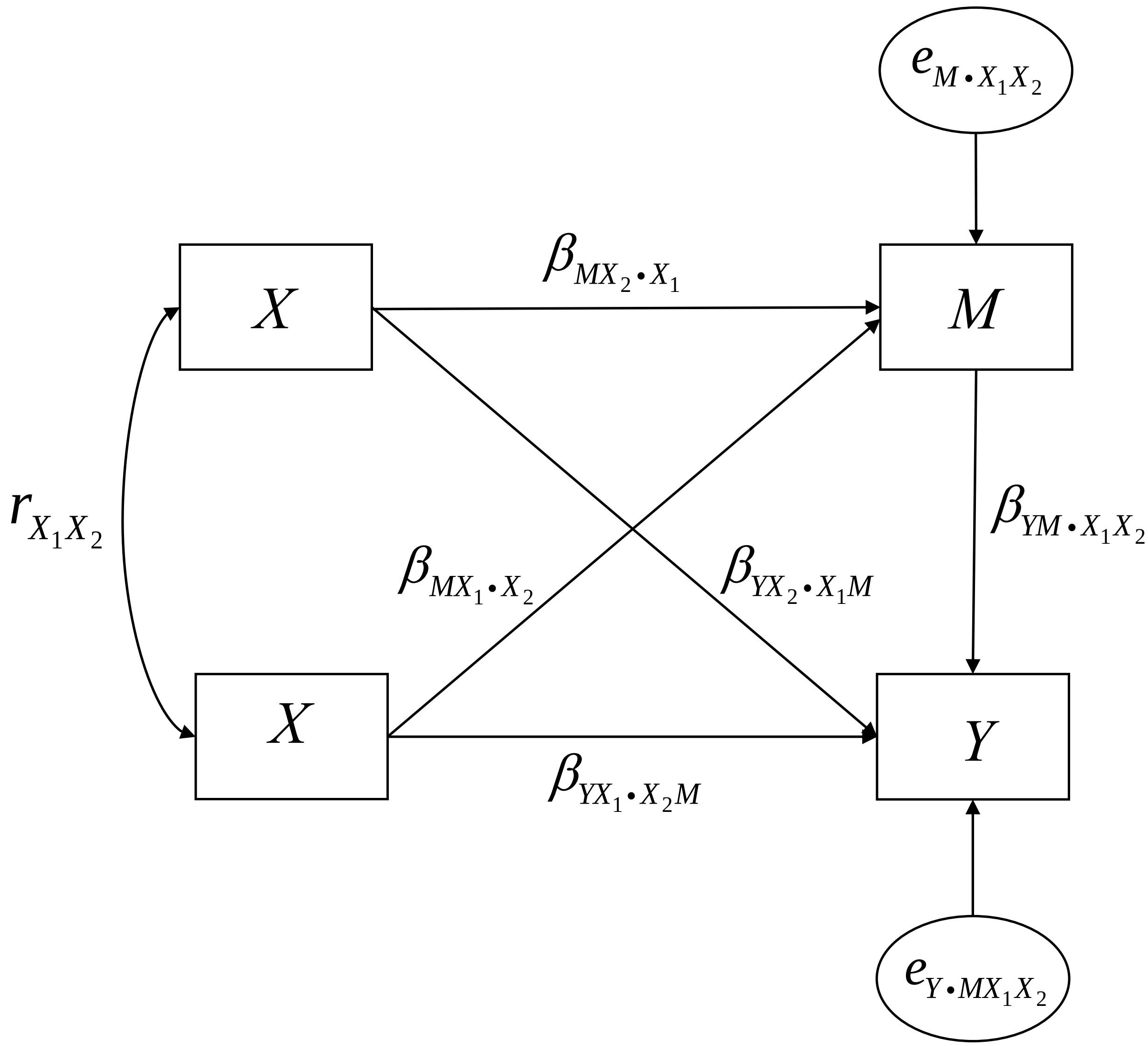
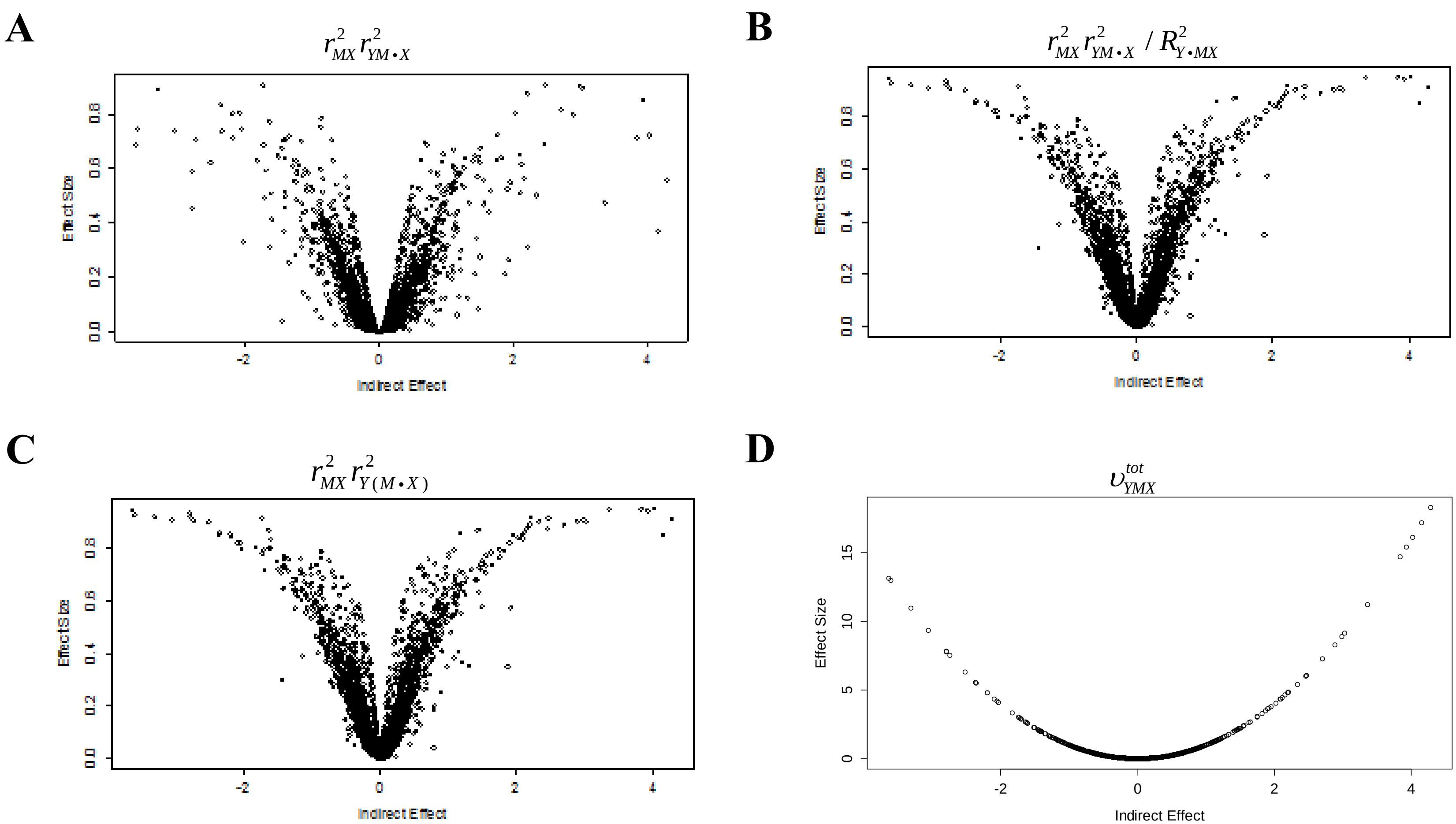


Figure 5. *Plots of R^2 effect sizes measures vs. the indirect effect for a three-variable mediation model for 5,000 indirect effects*



Note: Effect sizes in Panels A and B refer to equations (78) and (79) (MacKinnon, 2008); effect size in Panel C refers to equation (80) (De Heus, 2012); effect size in Panel D refers to equation (59) (υ^{tot}_{YMX}).

Figure 6. *Plot of* $\upsilon^{tot}_{YM_1M_2X}$ *vs. the total standardized indirect effect*

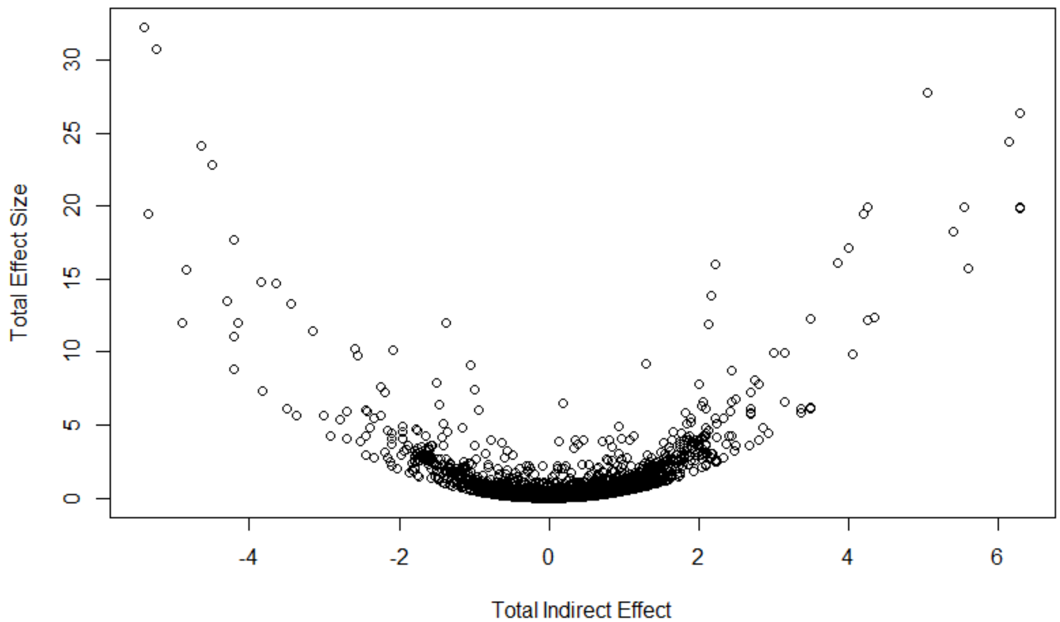
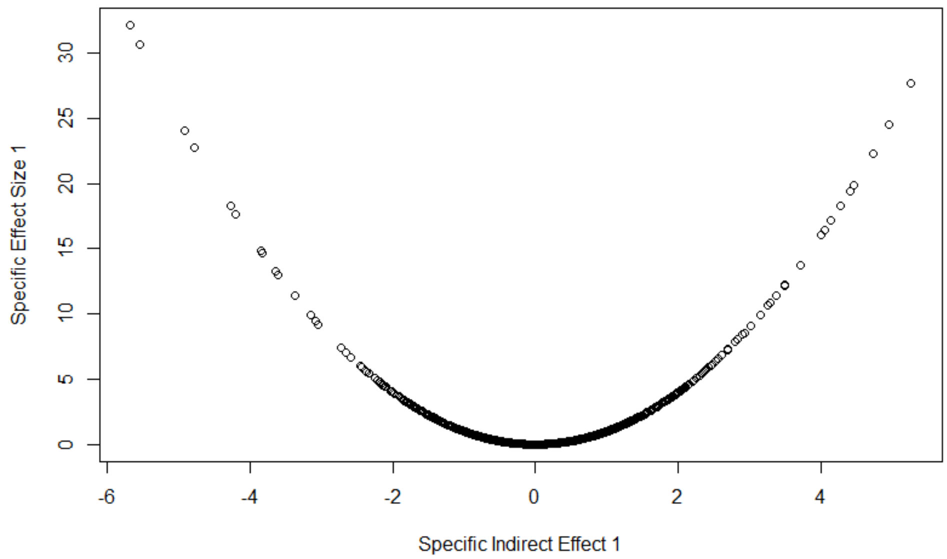


Figure 7. $\upsilon_{YM_1X}^{sp}$ and $\upsilon_{YM_2X}^{sp}$ *vs. the specific standardized indirect effects*



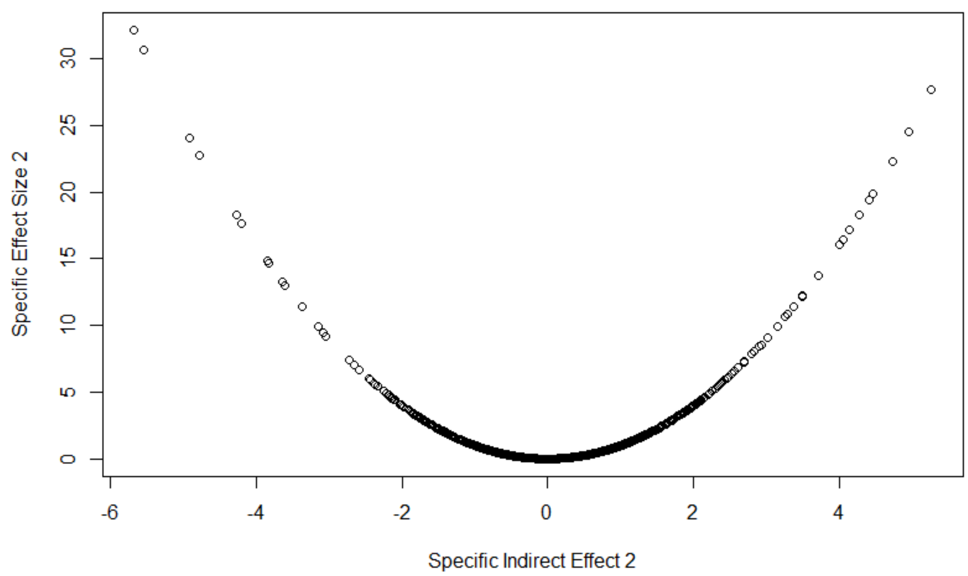


Figure 8. $\upsilon_{YM_1X}^{unc}$ *and* $\upsilon_{YM_2X}^{unc}$ *vs. unconditional standardized indirect effects*

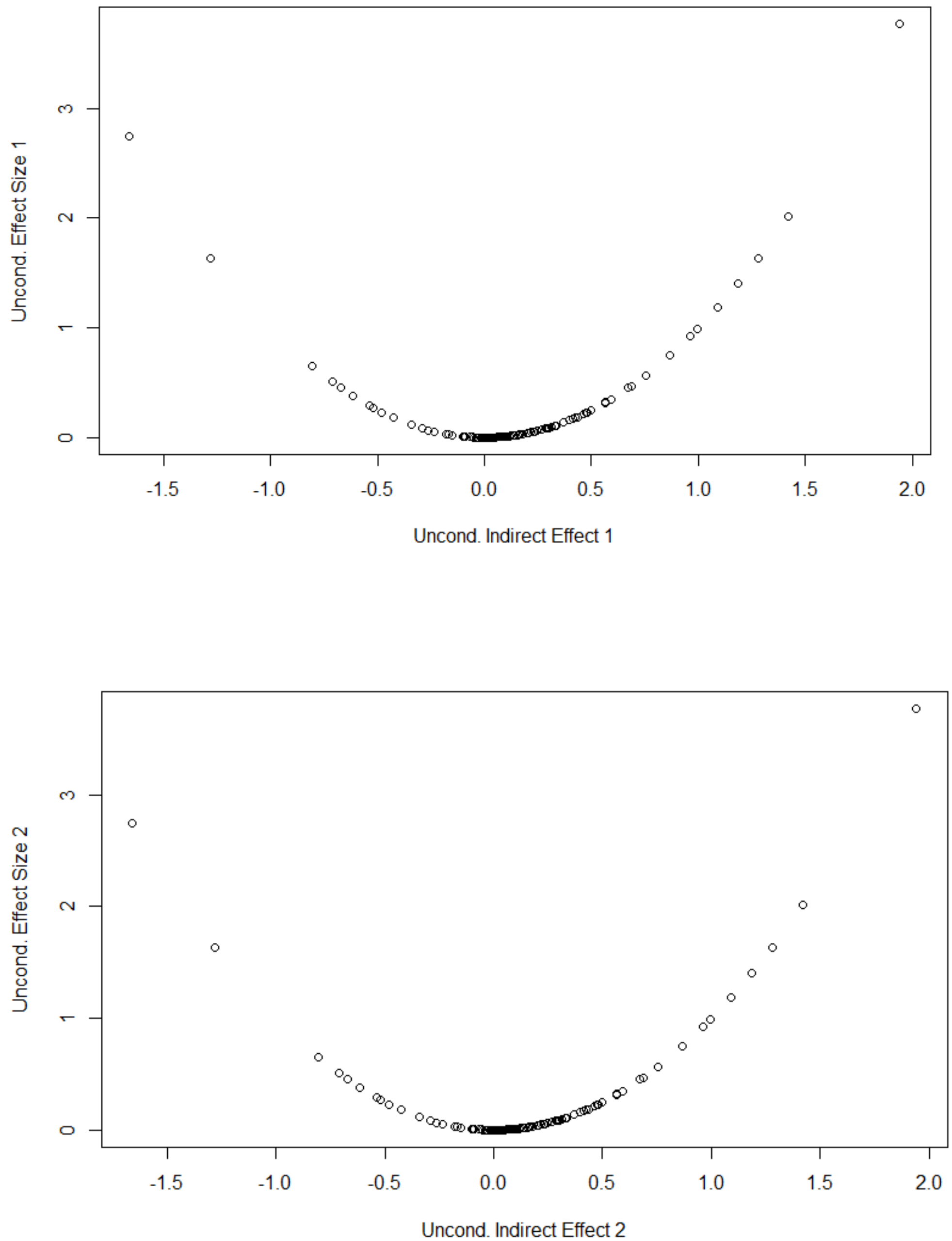


Figure 9. *Plot of* $\upsilon^{unc}_{YM_1X}$ *vs.* $\upsilon^{sp}_{YM_1X}$

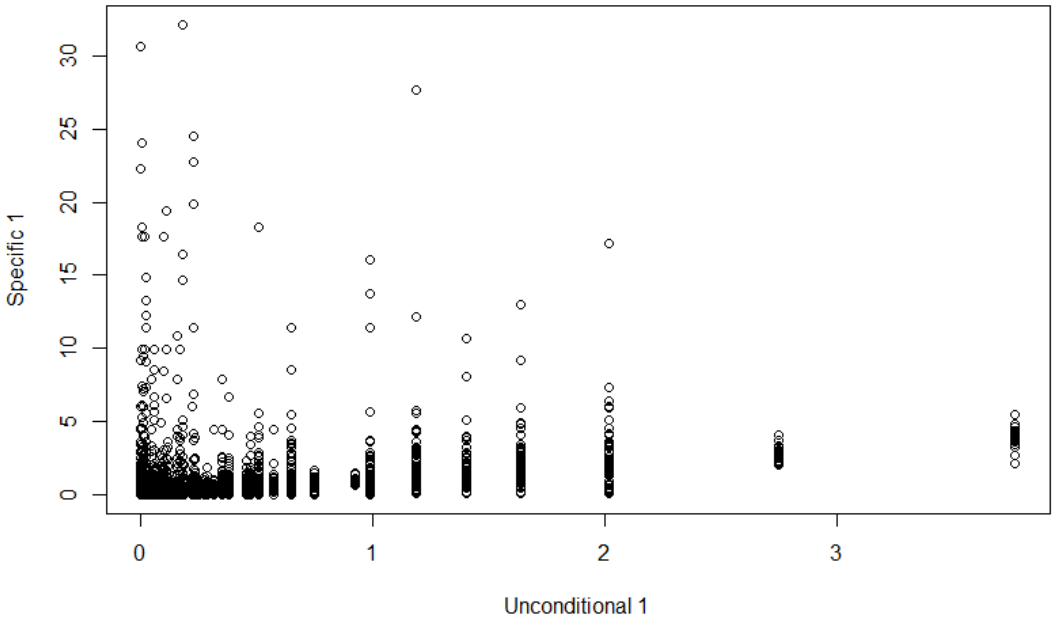
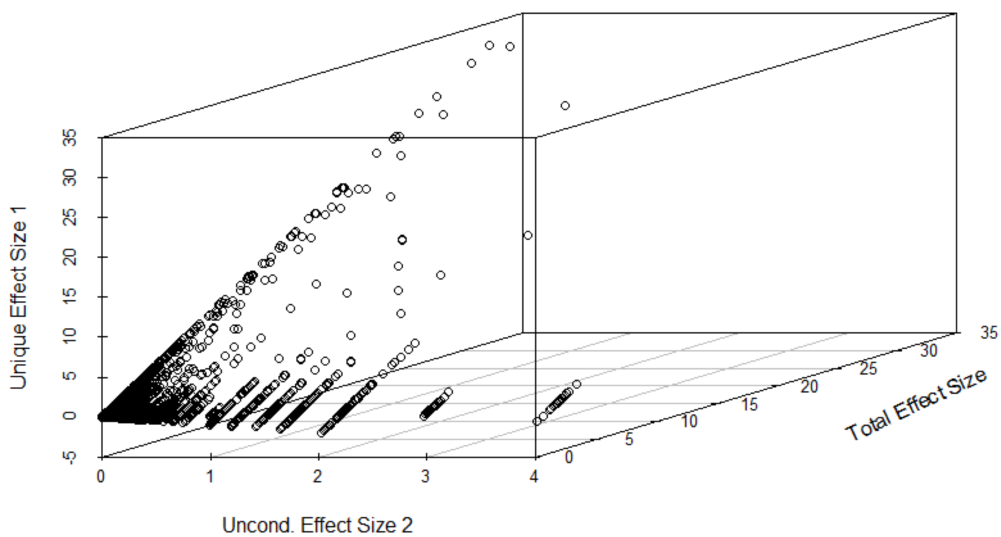


Figure 10. *Plot of* $\upsilon^{uni}_{YM_1X}$ *vs.* $\upsilon^{tot}_{YM_1M_2X}$ *and* $\upsilon^{unc}_{YM_1X}$



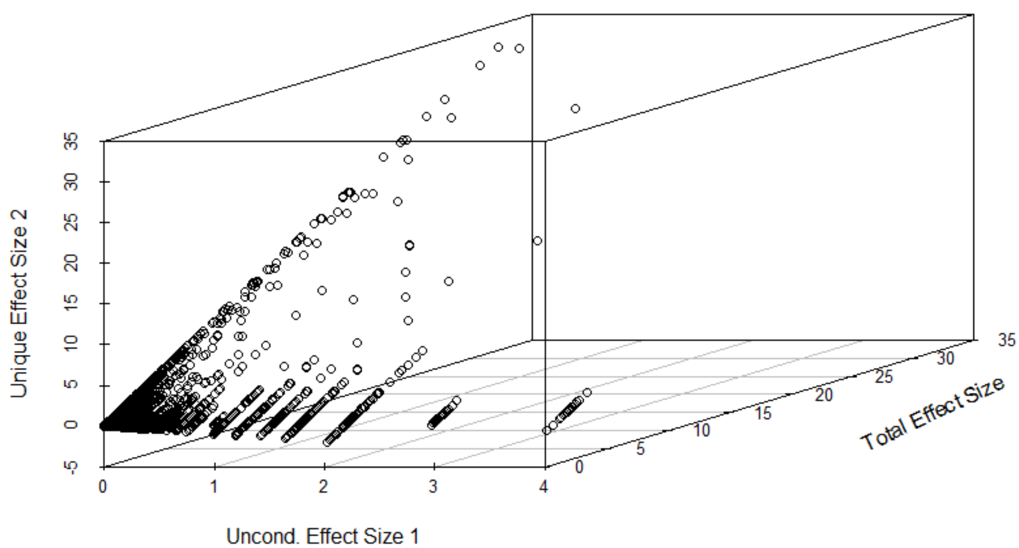


Figure 11. *Plot of the* υ_{YMXZ}^{tot} *vs. the total standardized indirect effect in a three variable mediation model with a baseline covariate (Z)*

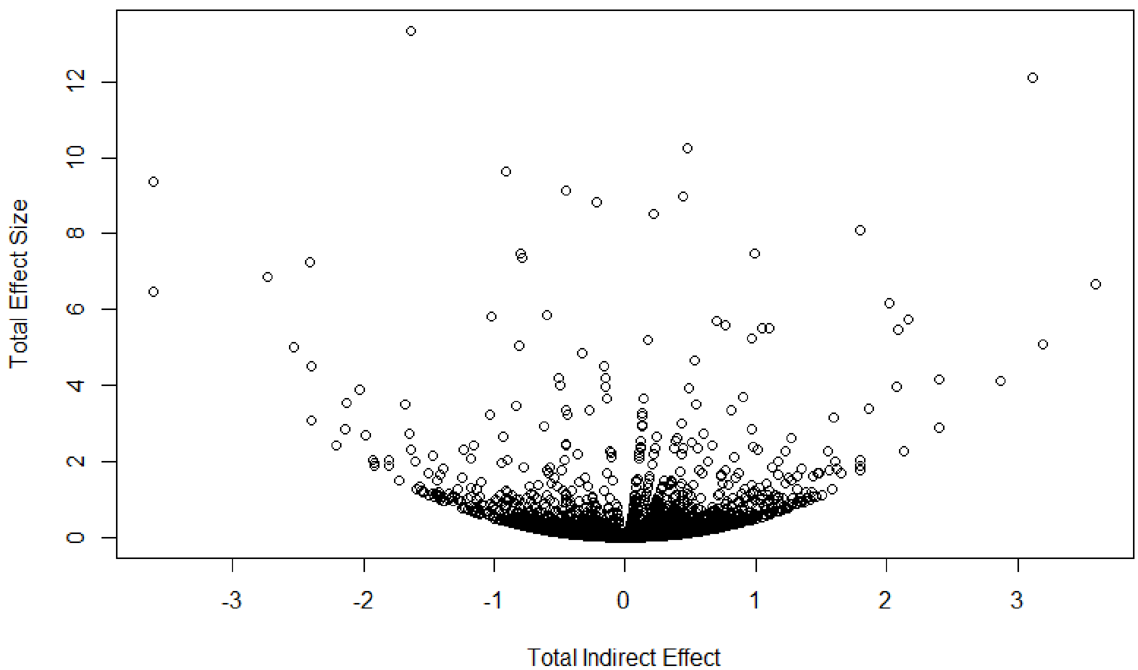


Figure 12. *Plot of* υ_{YMX}^{sp} *and* υ_{YMZ}^{sp} *vs. the specific standardized indirect effects in a three variable mediation model a baseline covariate (Z)*

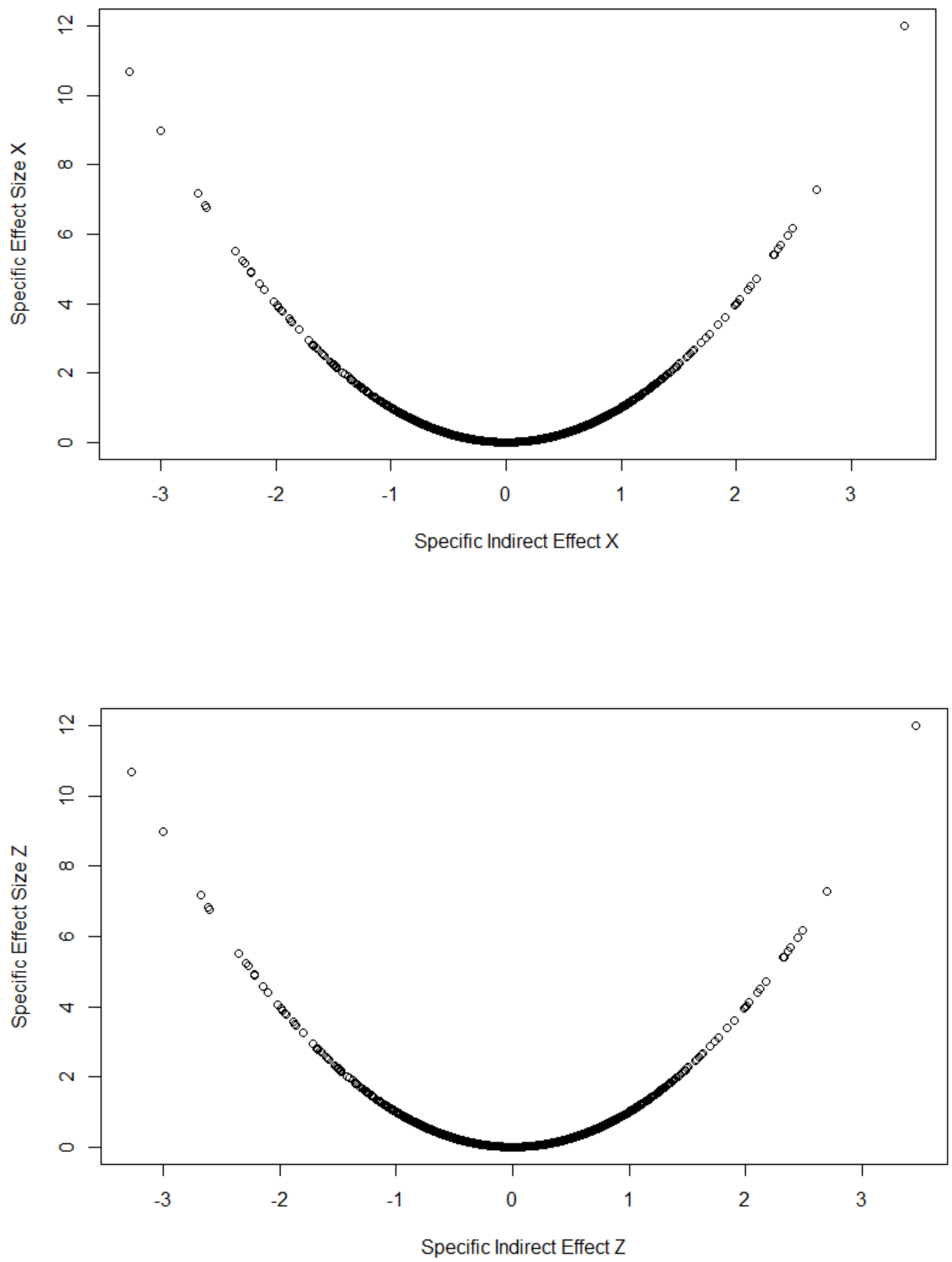
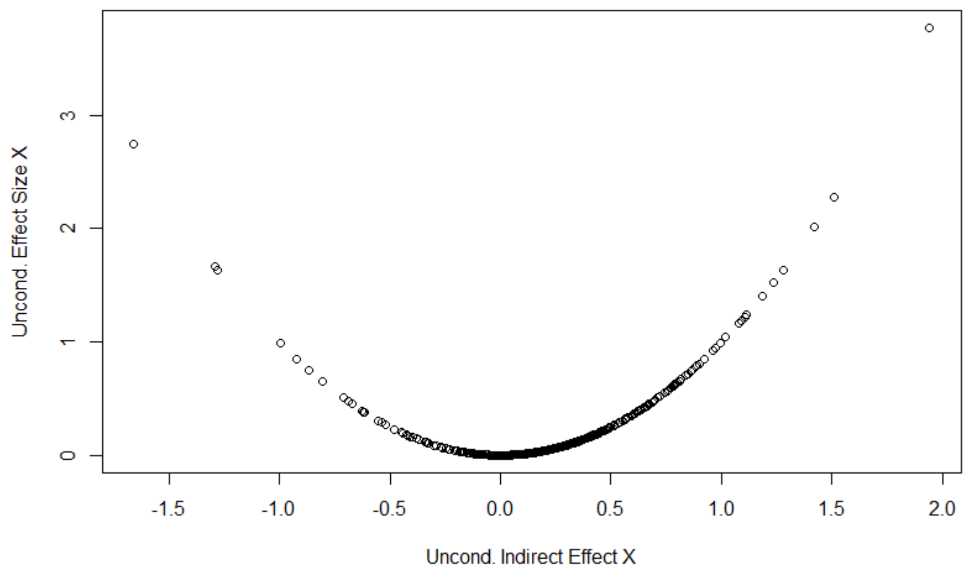


Figure 13. *Plot of* υ_{YMX}^{unc} *and* υ_{YMZ}^{unc} *vs. unconditional standardized indirect effects in a three variable mediation model a baseline covariate (Z)*



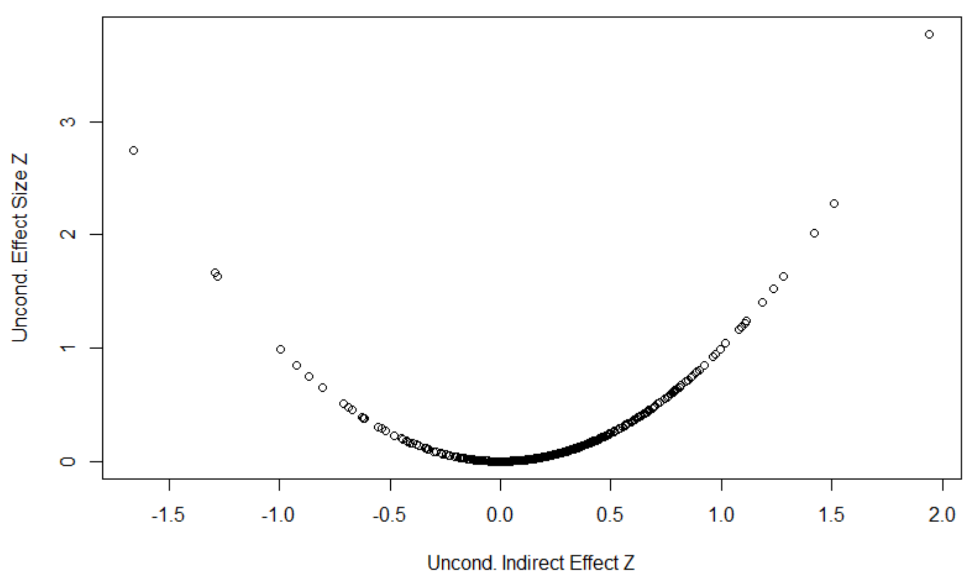
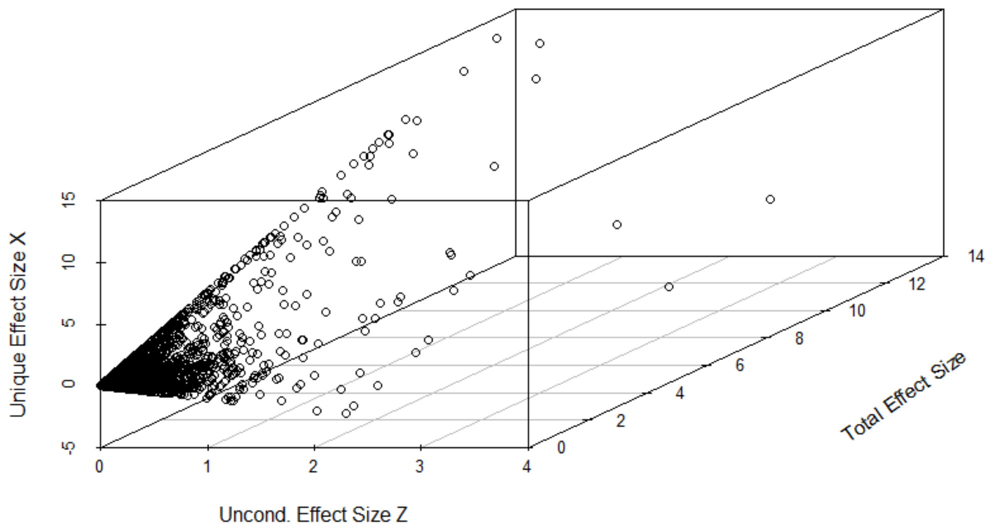


Figure 14. *Plots of* υ_{YMX}^{uni} *and* υ_{YMZ}^{uni} *vs.* υ_{YMX}^{unc} *and* υ_{YMZ}^{unc}, *and* υ_{YMXZ}^{tot} *in a three variable mediation model a baseline covariate (Z)*



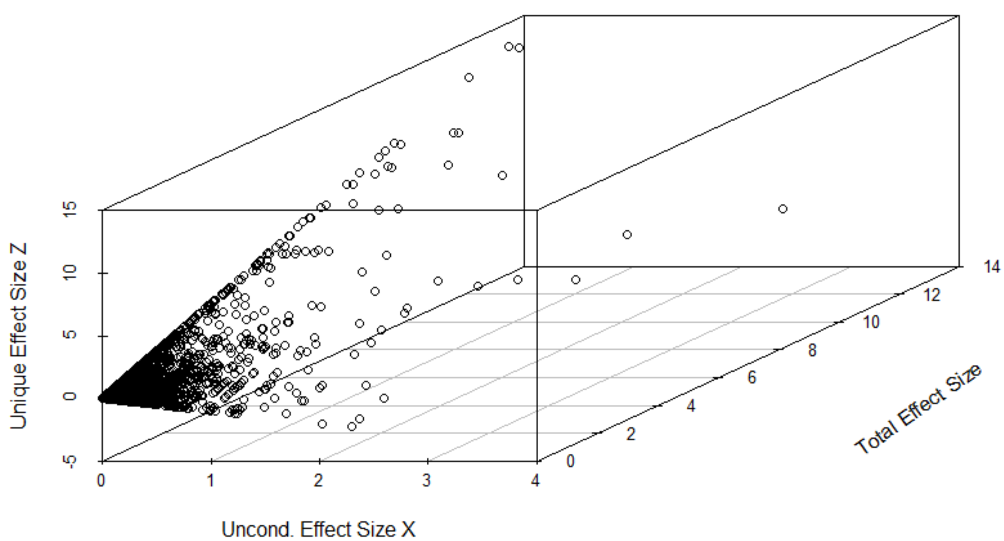
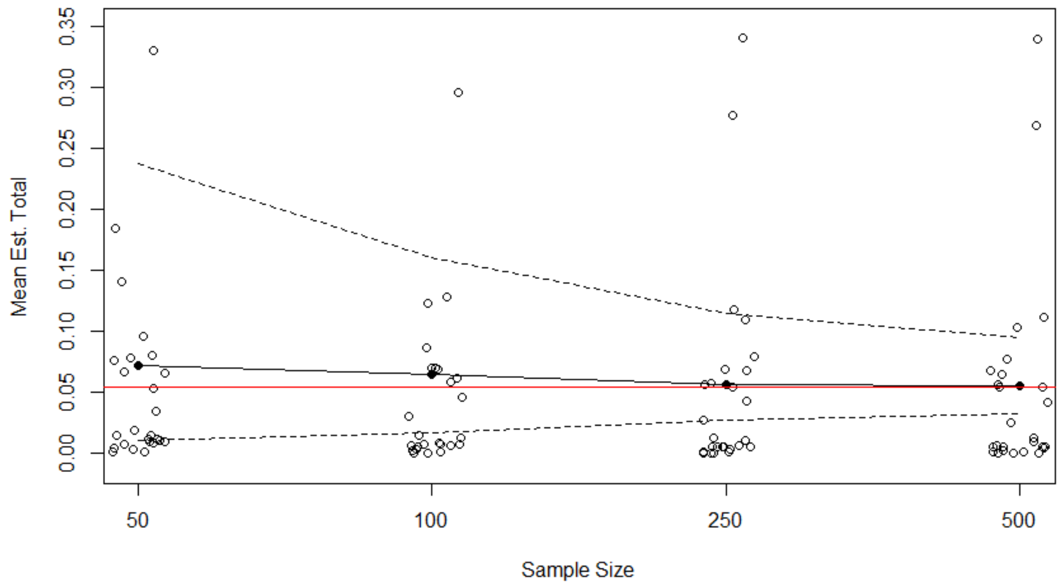
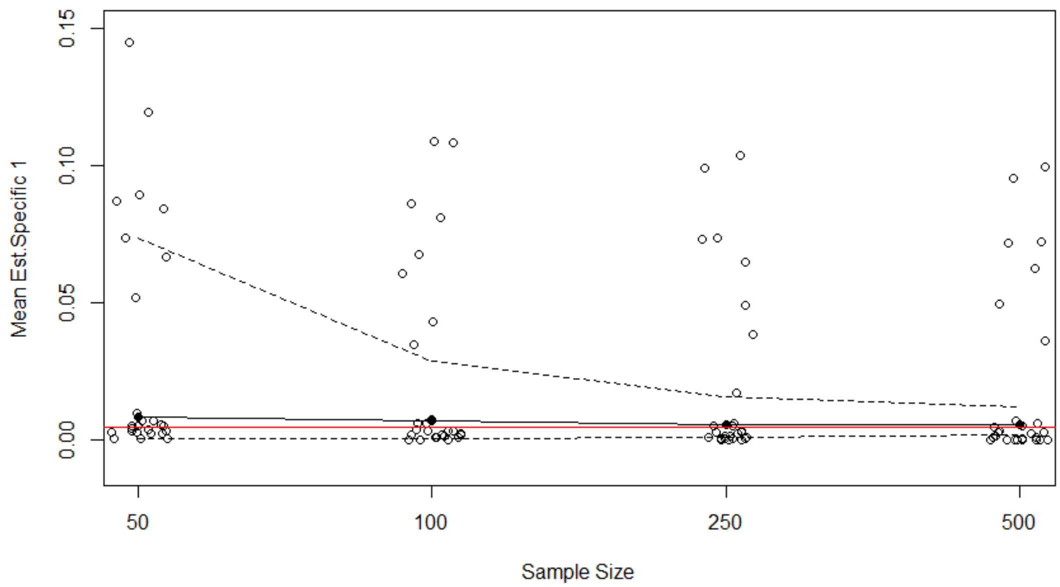


Figure 15. *Monte Carlo simulation results for* $\upsilon^{tot}_{YM_1M_2X}$



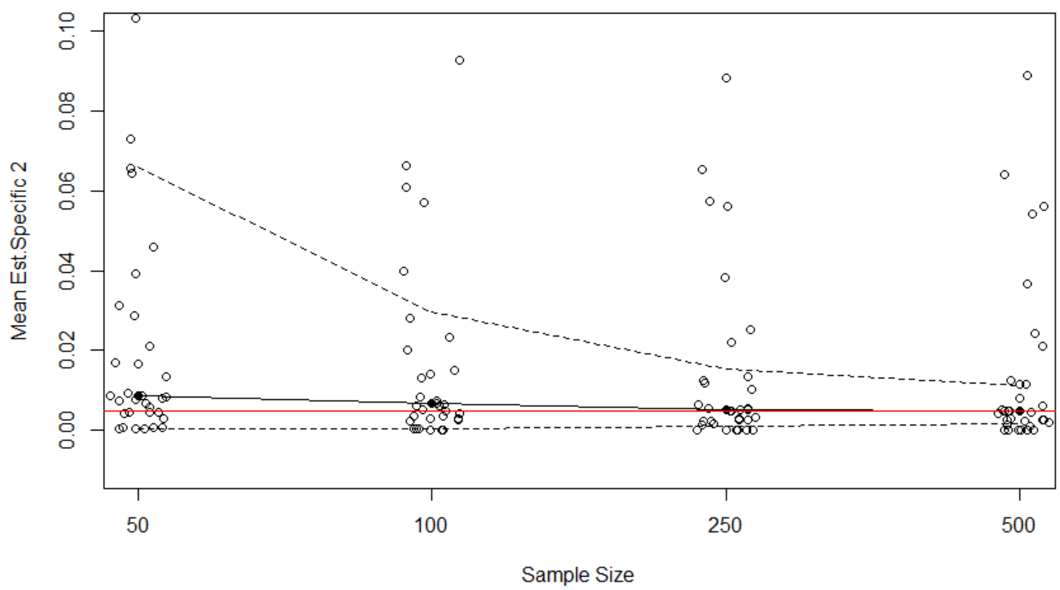
Note: Filled circles connect by a solid black line represent the median estimate of the mean $\hat{\upsilon}^{tot}_{YM_1M_2X}$ across combinations of population parameters for each sample size; unfilled circles represent mean $\hat{\upsilon}^{tot}_{YM_1M_2X}$ estimates for each combination of population parameters; dashed lines represent the median upper and lower 95% confidence intervals; the solid red line is the median population parameter

Figure 16. *Monte Carlo simulation results for* $\upsilon^{sp}_{YM_1X}$ *for* M_1



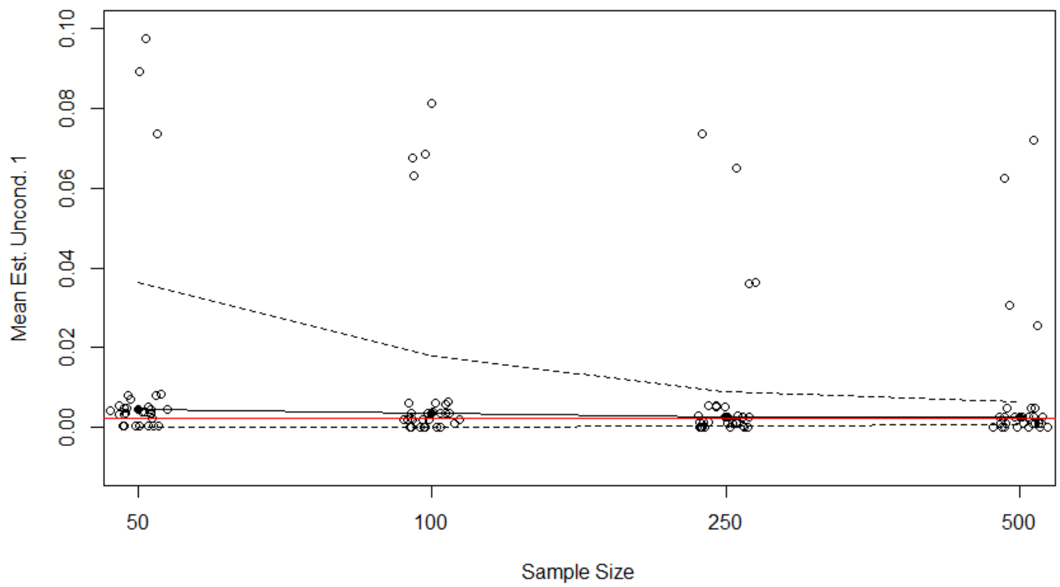
Note: Filled circles connect by a solid black line represent the median estimate of the mean $\hat{\upsilon}^{sp}_{YM_1X}$ across combinations of population parameters for each sample size; unfilled circles represent mean $\hat{\upsilon}^{sp}_{YM_1X}$ estimates for each combination of population parameters; dashed lines represent the median upper and lower 95% confidence intervals; the solid red line is the median population parameter

Figure 17. *Monte Carlo simulation results for* $\upsilon^{sp}_{YM_2X}$ *for* M_2



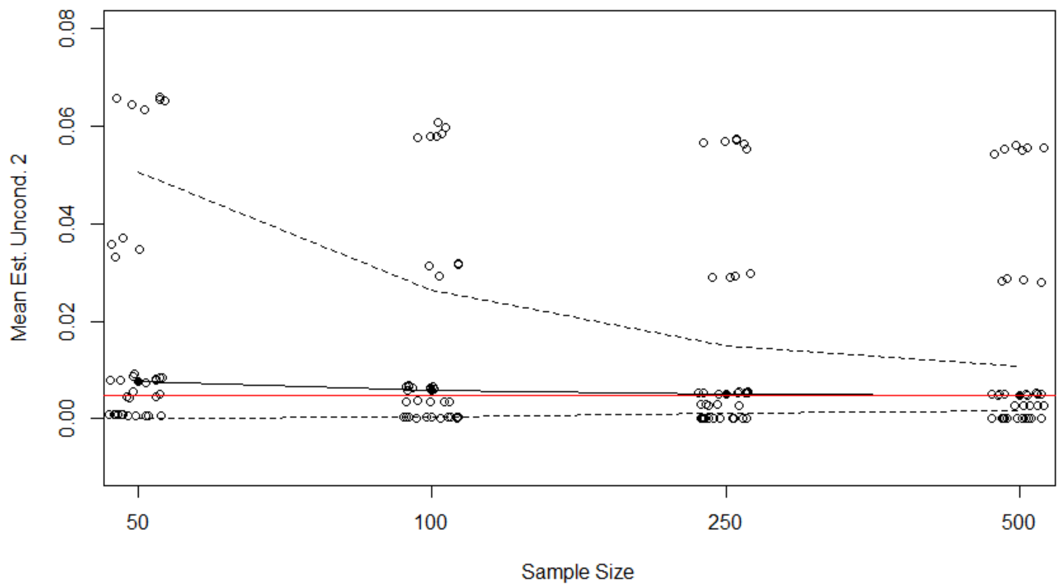
Note: Filled circles connect by a solid black line represent the median estimate of the mean $\hat{\upsilon}^{sp}_{YM_2X}$ across combinations of population parameters for each sample size; unfilled circles represent mean $\hat{\upsilon}^{sp}_{YM_2X}$ estimates for each combination of population parameters; dashed lines represent the median upper and lower 95% confidence intervals; the solid red line is the median population parameter

Figure 18. *Monte Carlo simulation results for* $\upsilon^{unc}_{YM_1X}$ *for M1*



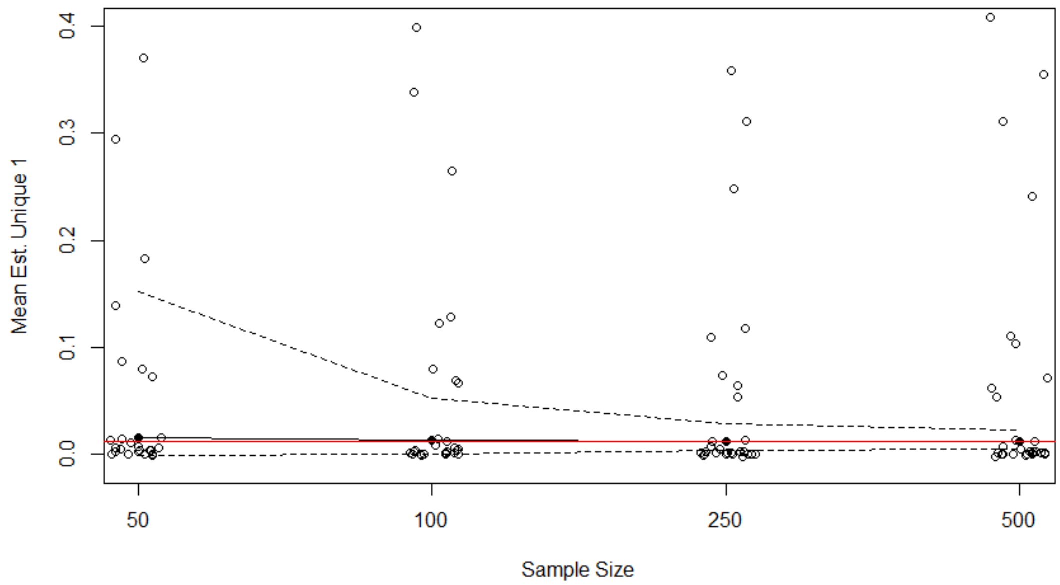
Note: Filled circles connect by a solid black line represent the median estimate of the mean $\hat{\upsilon}^{unc}_{YM_1X}$ across combinations of population parameters for each sample size; unfilled circles represent mean $\hat{\upsilon}^{unc}_{YM_1X}$ estimates for each combination of population parameters; dashed lines represent the median upper and lower 95% confidence intervals; the solid red line is the median population parameter

Figure 19. *Monte Carlo simulation results for* $\upsilon^{unc}_{YM_2X}$ *for* M_2



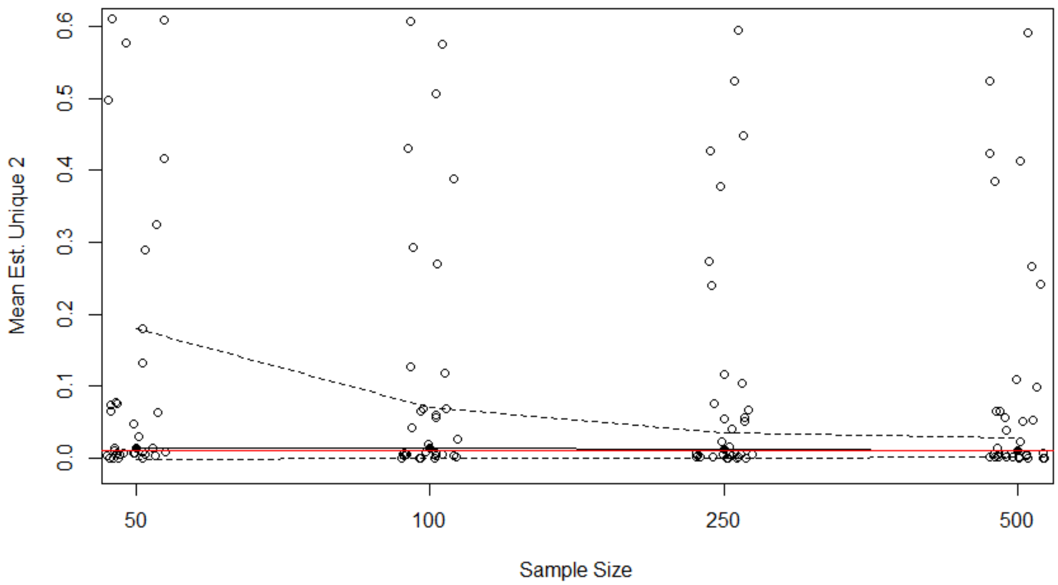
Note: Filled circles connect by a solid black line represent the median estimate of the mean $\hat{\upsilon}^{unc}_{YM_2X}$ across combinations of population parameters for each sample size; unfilled circles represent mean $\hat{\upsilon}^{unc}_{YM_2X}$ estimates for each combination of population parameters; dashed lines represent the median upper and lower 95% confidence intervals; the solid red line is the median population parameter

Figure 20. *Monte Carlo simulation results for* $\upsilon^{uni}_{YM_1X}$ *for* M_1



Note: Filled circles connect by a solid black line represent the median estimate of the mean $\hat{\upsilon}^{uni}_{YM_1X}$ across combinations of population parameters for each sample size; unfilled circles represent mean $\hat{\upsilon}^{uni}_{YM_1X}$ estimates for each combination of population parameters; dashed lines represent the median upper and lower 95% confidence intervals; the solid red line is the median population parameter

Figure 21. *Monte Carlo simulation results for* $\upsilon_{YM_2X}^{uni}$ *for M2*



Note: Filled circles connect by a solid black line represent the median estimate of the mean $\hat{\upsilon}_{YM_2X}^{uni}$ across combinations of population parameters for each sample size; unfilled circles represent mean $\hat{\upsilon}_{YM_2X}^{uni}$ estimates for each combination of population parameters; dashed lines represent the median upper and lower 95% confidence intervals; the solid red line is the median population parameter

Figure 22. *Monte Carlo simulation results for υ relative confidence interval width across 36 populations, n=500*

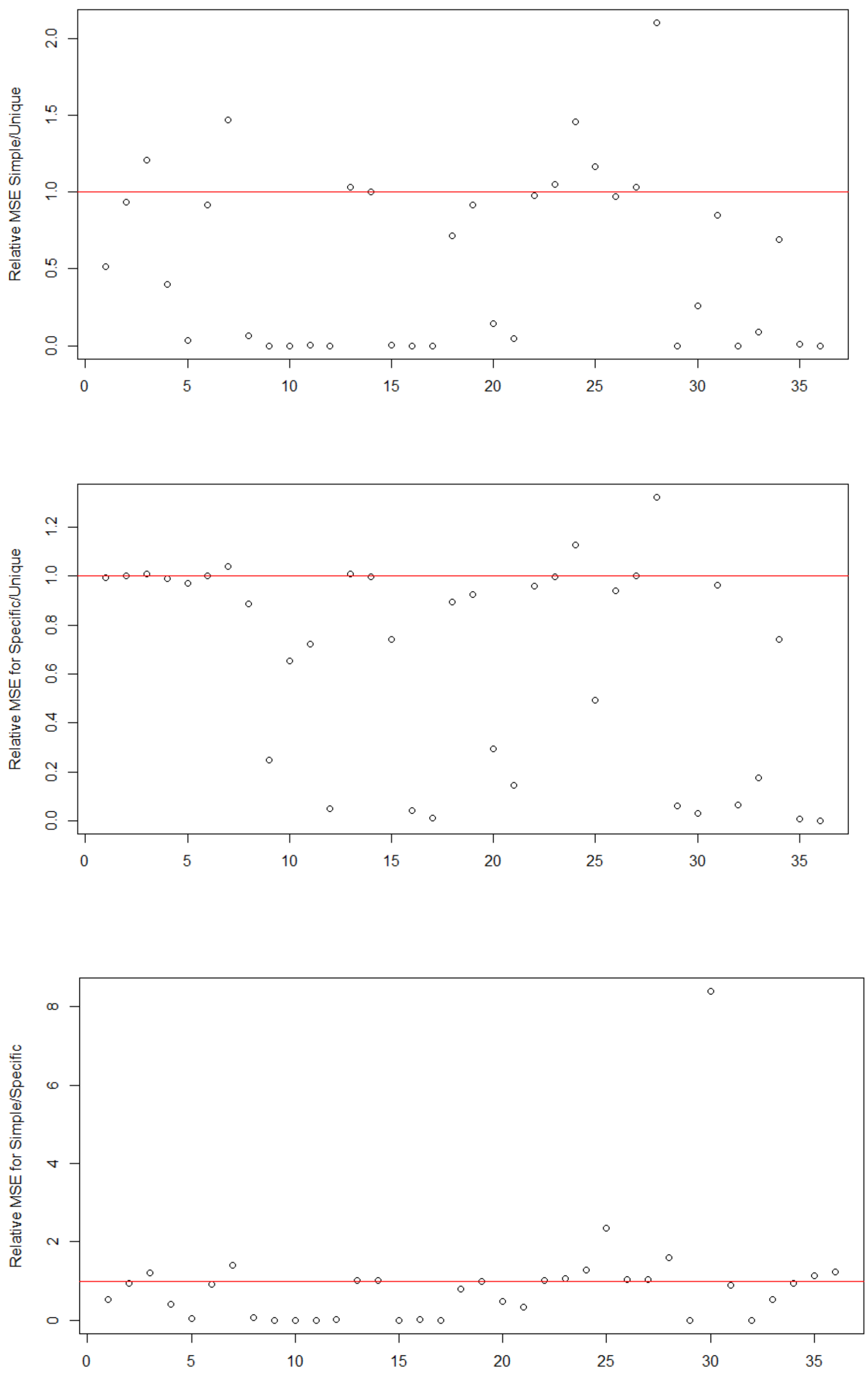


Figure 23. *Monte Carlo simulation results for υ relative MSE across 36 populations, n=500*

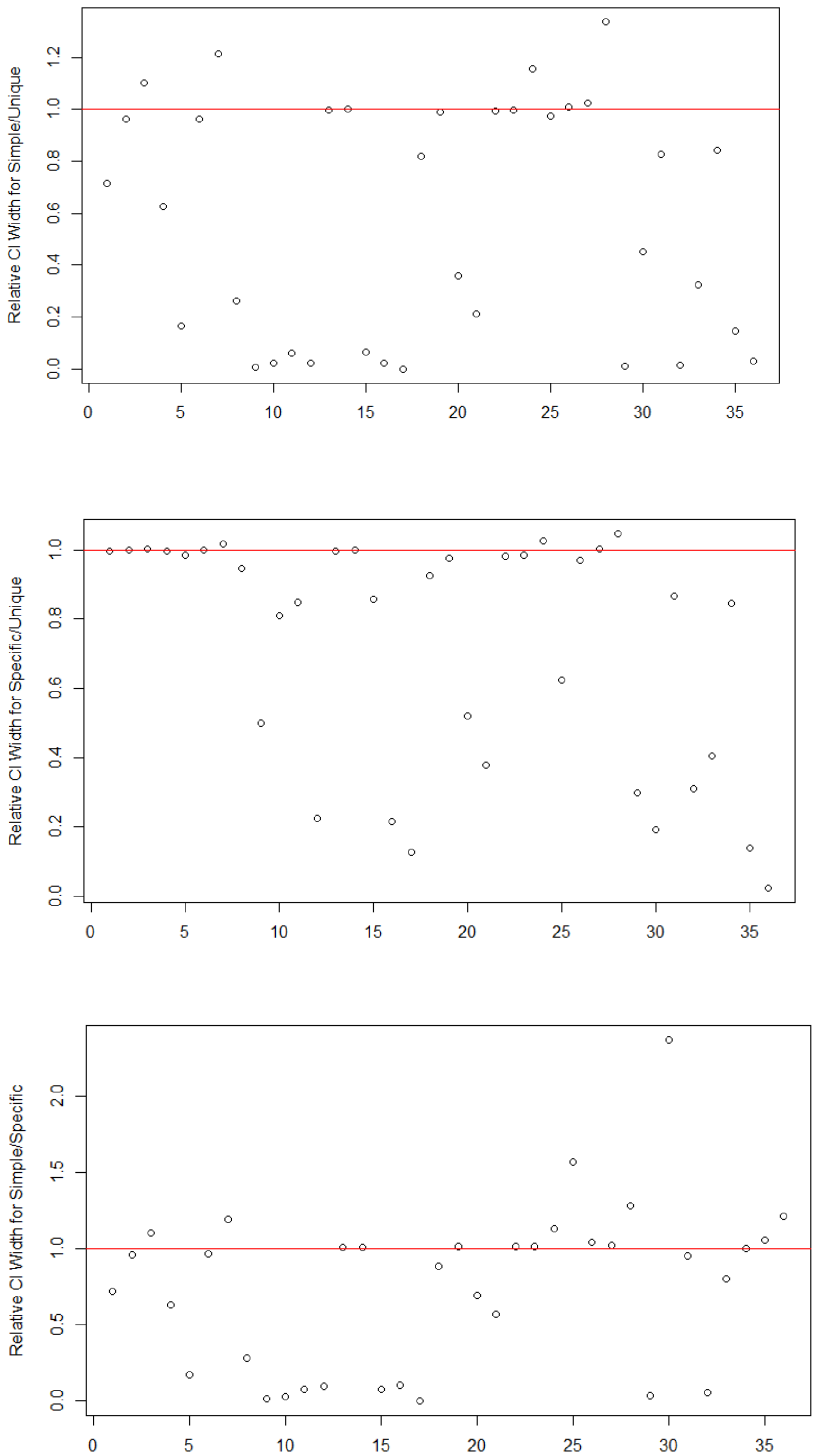


Table 1. *Summary of υ components and recommendations for use*

υ component (υ matrix)	Description	Recommendation
$\upsilon^{tot}_{\boldsymbol{YMX}}$ ($\Upsilon^{tot}_{\boldsymbol{YMX}}$)	Total standardized squared indirect effect	$\upsilon^{tot}_{\boldsymbol{YMX}}$ is appropriate when the effect of a predictor or set of predictors on an outcome through a set of mediators is the primary effect size of interest. Contributions of individual variables are of less interest.
$\upsilon^{sp}_{\boldsymbol{YMX}}$ ($\Upsilon^{sp}_{\boldsymbol{YMX}}$)	Specific standardized squared indirect effect	$\upsilon^{sp}_{\boldsymbol{YMX}}$ is appropriate when the effect of a specific predictor or set of predictors on an outcome through a specific mediator or set of mediators is the primary effect size of interest. Most appropriate when correlation among predictors or residual correlation among mediators are relatively small.
$\upsilon^{unc}_{\boldsymbol{YMX}}$ ($\Upsilon^{unc}_{\boldsymbol{YMX}}$)	Unconditional standardized squared indirect effect	$\upsilon^{unc}_{\boldsymbol{YMX}}$ is appropriate when the effect of a specific predictor or set of predictors on an outcome through a specific mediator or set of mediators unconditional on another variable or set of variables is the primary effect size of interest. Most appropriate when correlation among predictors or residual correlation among mediators are relatively large.
$\upsilon^{uni}_{\boldsymbol{YMX}}$ ($\Upsilon^{uni}_{\boldsymbol{YMX}}$)	Unique standardized squared indirect effect	$\upsilon^{uni}_{\boldsymbol{YMX}}$ is appropriate when the effect of a specific predictor or set of predictors on an outcome through a specific mediator or set of mediators controlling for another variable or set of variables is the primary effect size of interest. Appropriate when any correlation among predictors or residual correlation among mediators is present, specifically for controlling for the effects of certain variables (e.g., covariates).

Table 2. *Mean, bias, relative bias, and MSE of* $\hat{\upsilon}^{tot}_{YM_1M_2X}$, $\hat{\upsilon}^{sp}_{YM_1X}$, *and* $\hat{\upsilon}^{sp}_{YM_2X}$

	n	$\hat{\upsilon}^{tot}_{YM_1M_2X}$ Mean Est.	Bias	Rel. Bias	MSE	$\hat{\upsilon}^{sp}_{YM_1X}$ Mean Est.	Bias	Rel. Bias	MSE	$\hat{\upsilon}^{sp}_{YM_2X}$ Mean Est.	Bias	Rel. Bias	MSE
$\upsilon^{sp}_{YM_1X} = 0$	50	0.00122	0.00117	23.48416	0.00001	0.00050	0.00050	NA	0.00000	0.00072	0.00067	13.46716	0.00000
$\upsilon^{sp}_{YM_2X} = .00005$	100	0.00043	0.00038	7.61570	0.00000	0.00009	0.00009	NA	0.00000	0.00034	0.00029	5.89387	0.00000
$\upsilon^{tot}_{YM_1M_2X} = .00005$	250	0.00014	0.00009	1.77903	0.00000	0.00002	0.00002	NA	0.00000	0.00012	0.00007	1.46240	0.00000
$r_{M_1M_2 \bullet X} = 0$	500	0.00008	0.00003	0.69453	0.00000	0.00000	0.00000	NA	0.00000	0.00008	0.00003	0.62049	0.00000
$\upsilon^{sp}_{YM_1X} = 0$	50	0.00136	0.00130	21.48073	0.00001	0.00051	0.00051	NA	0.00000	0.00085	0.00079	13.03863	0.00001
$\upsilon^{sp}_{YM_2X} = .00006$	100	0.00044	0.00038	6.35318	0.00000	0.00011	0.00011	NA	0.00000	0.00033	0.00027	4.45902	0.00000
$\upsilon^{tot}_{YM_1M_2X} = .00006$	250	0.00015	0.00009	1.55980	0.00000	0.00002	0.00002	NA	0.00000	0.00014	0.00008	1.26890	0.00000
$r_{M_1M_2 \bullet X} = .3$	500	0.00011	0.00005	0.78100	0.00000	0.00001	0.00001	NA	0.00000	0.00010	0.00004	0.69405	0.00000
$\upsilon^{sp}_{YM_1X} = 0$	50	0.01477	0.01326	8.76496	0.00126	0.00532	0.00532	NA	0.00019	0.00945	0.00794	5.24977	0.00063
$\upsilon^{sp}_{YM_2X} = .00015$	100	0.00583	0.00432	2.85685	0.00013	0.00171	0.00171	NA	0.00002	0.00412	0.00261	1.72377	0.00006
$\upsilon^{tot}_{YM_1M_2X} = .00015$	250	0.00284	0.00133	0.88057	0.00001	0.00049	0.00049	NA	0.00000	0.00235	0.00084	0.55621	0.00001
$r_{M_1M_2 \bullet X} = .9$	500	0.00230	0.00079	0.52239	0.00001	0.00029	0.00029	NA	0.00000	0.00202	0.00050	0.33311	0.00000
$\upsilon^{sp}_{YM_1X} = 0$	50	0.00742	0.00664	8.61120	0.00032	0.00278	0.00278	NA	0.00006	0.00464	0.00387	5.01210	0.00015
$\upsilon^{sp}_{YM_2X} = .00077$	100	0.00333	0.00256	3.31918	0.00005	0.00101	0.00101	NA	0.00001	0.00232	0.00155	2.00671	0.00002
$\upsilon^{tot}_{YM_1M_2X} = .00077$	250	0.00169	0.00092	1.18833	0.00001	0.00032	0.00032	NA	0.00000	0.00137	0.00060	0.77639	0.00001
$r_{M_1M_2 \bullet X} = .9$	500	0.00107	0.00030	0.39113	0.00000	0.00014	0.00014	NA	0.00000	0.00093	0.00016	0.21144	0.00000

$\upsilon^{sp}_{YM_1X} = .00084$	50	0.00427	0.00338	3.80895	0.00006	0.00337	0.00253	3.01184	0.00005	0.00089	0.00085	18.40161	0.00001
$\upsilon^{sp}_{YM_2X} = .00005$	100	0.00241	0.00152	1.71568	0.00001	0.00214	0.00130	1.54346	0.00001	0.00027	0.00022	4.86849	0.00000
$\upsilon^{tot}_{YM_1M_2X} = .00089$	250	0.00151	0.00063	0.70487	0.00000	0.00139	0.00055	0.65402	0.00000	0.00012	0.00008	1.63583	0.00000
$r_{M_1M_2 \bullet X} = -.0316$	500	0.00115	0.00026	0.29182	0.00000	0.00107	0.00023	0.27326	0.00000	0.00007	0.00003	0.63176	0.00000
$\upsilon^{sp}_{YM_1X} = .09765$	50	0.18422	0.07684	0.71560	0.06153	0.14490	0.04724	0.48379	0.03157	0.03932	0.02960	3.04368	0.00630
$\upsilon^{sp}_{YM_2X} = .00972$	100	0.12874	0.02136	0.19890	0.01489	0.10872	0.01106	0.11327	0.00896	0.02002	0.01030	1.05890	0.00107
$\upsilon^{tot}_{YM_1M_2X} = .10738$	250	0.11740	0.01002	0.09331	0.00406	0.10395	0.00630	0.06447	0.00262	0.01345	0.00372	0.38298	0.00022
$r_{M_1M_2 \bullet X} = .8684$	500	0.11110	0.00372	0.03463	0.00179	0.09949	0.00184	0.01881	0.00116	0.01161	0.00188	0.19350	0.00009
$\upsilon^{sp}_{YM_1X} = 1.70303$	50	1.80044	0.09738	0.05718	0.42729	1.79986	0.09684	0.05686	0.42685	0.00058	0.00054	14.49267	0.00000
$\upsilon^{sp}_{YM_2X} = .00004$	100	1.74683	0.04377	0.02570	0.20381	1.74665	0.04362	0.02562	0.20380	0.00018	0.00015	3.94729	0.00000
$\upsilon^{tot}_{YM_1M_2X} = 1.70307$	250	1.69279	-0.01027	-0.00603	0.07966	1.69271	-0.01031	-0.00605	0.07966	0.00008	0.00004	1.12571	0.00000
$r_{M_1M_2 \bullet X} = -.2075$	500	1.71181	0.00875	0.00514	0.03665	1.71175	0.00873	0.00513	0.03665	0.00006	0.00002	0.48538	0.00000
$\upsilon^{sp}_{YM_1X} = 3.16064$	50	3.13782	0.02905	0.00935	0.75627	3.13031	0.02390	0.00769	0.74923	0.00751	0.00515	2.17940	0.00019
$\upsilon^{sp}_{YM_2X} = .00236$	100	3.18225	0.07348	0.02364	0.34669	3.17740	0.07099	0.02285	0.34453	0.00486	0.00249	1.05459	0.00004
$\upsilon^{tot}_{YM_1M_2X} = 3.10877$	250	3.11079	0.00202	0.00065	0.13699	3.10737	0.00097	0.00031	0.13634	0.00342	0.00106	0.44797	0.00001
$r_{M_1M_2 \bullet X} = .0925$	500	3.11837	0.00961	0.00309	0.06126	3.11544	0.00903	0.00291	0.06082	0.00294	0.00057	0.24313	0.00001
$\upsilon^{sp}_{YM_1X} = 0$	50	0.00924	0.00445	0.92847	0.00017	0.00043	0.00043	NA	0.00000	0.00882	0.00402	0.83933	0.00016
$\upsilon^{sp}_{YM_2X} = .00479$	100	0.00689	0.00210	0.43788	0.00005	0.00009	0.00009	NA	0.00000	0.00680	0.00201	0.41927	0.00005
$\upsilon^{tot}_{YM_1M_2X} = .00479$	250	0.00553	0.00074	0.15347	0.00002	0.00002	0.00002	NA	0.00000	0.00551	0.00072	0.15016	0.00002
$r_{M_1M_2 \bullet X} = 0$	500	0.00512	0.00033	0.06906	0.00001	0.00000	0.00000	NA	0.00000	0.00512	0.00033	0.06838	0.00001

$\upsilon^{sp}_{YM_1X} = 0$	50	0.60888	0.21198	0.53410	0.34775	0.08429	0.08429	NA	0.02484	0.52460	0.12770	0.32174	0.25391
$\upsilon^{sp}_{YM_2X} = .3969$	100	0.50651	0.10961	0.27617	0.10940	0.03493	0.03493	NA	0.00400	0.47158	0.07468	0.18816	0.09055
$\upsilon^{tot}_{YM_1M_2X} = .3969$	250	0.42787	0.03097	0.07804	0.03400	0.01698	0.01698	NA	0.00083	0.41089	0.01399	0.03525	0.03265
$r_{M_1M_2 \bullet X} = .9$	500	0.41289	0.01599	0.04028	0.01472	0.00720	0.00720	NA	0.00016	0.40569	0.00879	0.02214	0.01439
$\upsilon^{sp}_{YM_1X} = .00053$	50	0.01094	0.00593	1.18214	0.00022	0.00333	0.00280	5.26983	0.00005	0.00761	0.00313	0.69768	0.00015
$\upsilon^{sp}_{YM_2X} = .00448$	100	0.00759	0.00258	0.51392	0.00006	0.00153	0.00099	1.87072	0.00001	0.00607	0.00158	0.35312	0.00005
$\upsilon^{tot}_{YM_1M_2X} = .00501$	250	0.00568	0.00067	0.13325	0.00001	0.00079	0.00026	0.49488	0.00000	0.00489	0.00041	0.09039	0.00001
$r_{M_1M_2 \bullet X} = -.0989$	500	0.00526	0.00025	0.04933	0.00001	0.00069	0.00016	0.30793	0.00000	0.00457	0.00008	0.01869	0.00000
$\upsilon^{sp}_{YM_1X} = .19319$	50	0.50074	0.09552	0.23572	0.20734	0.23564	0.04245	0.21971	0.04871	0.26510	0.05307	0.25030	0.06210
$\upsilon^{sp}_{YM_2X} = .21203$	100	0.43311	0.02789	0.06882	0.08083	0.20535	0.01216	0.06295	0.01943	0.22775	0.01573	0.07417	0.02377
$\upsilon^{tot}_{YM_1M_2X} = .40522$	250	0.44979	0.04457	0.11000	0.03342	0.21399	0.02080	0.10764	0.00828	0.23581	0.02378	0.11214	0.00939
$r_{M_1M_2 \bullet X} = .8011$	500	0.42516	0.01995	0.04922	0.01639	0.20332	0.01013	0.05244	0.00398	0.22184	0.00981	0.04629	0.00464
$\upsilon^{sp}_{YM_1X} = .00203$	50	0.00862	0.00457	1.12736	0.00013	0.00406	0.00204	1.00551	0.00006	0.00455	0.00253	1.24922	0.00007
$\upsilon^{sp}_{YM_2X} = .00203$	100	0.00642	0.00237	0.58419	0.00004	0.00336	0.00134	0.65950	0.00002	0.00306	0.00103	0.50889	0.00002
$\upsilon^{tot}_{YM_1M_2X} = .00405$	250	0.00485	0.00080	0.19662	0.00001	0.00245	0.00043	0.21212	0.00000	0.00239	0.00037	0.18112	0.00000
$r_{M_1M_2 \bullet X} = -.0989$	500	0.00444	0.00039	0.09560	0.00000	0.00220	0.00017	0.08520	0.00000	0.00224	0.00021	0.10601	0.00000
$\upsilon^{sp}_{YM_1X} = 0$	50	0.06616	0.01172	0.21519	0.00295	0.00041	0.00041	NA	0.00000	0.06575	0.01131	0.20769	0.00294
$\upsilon^{sp}_{YM_2X} = .05444$	100	0.06110	0.00666	0.12232	0.00127	0.00008	0.00008	NA	0.00000	0.06102	0.00658	0.12085	0.00127
$\upsilon^{tot}_{YM_1M_2X} = .05444$	250	0.05740	0.00296	0.05433	0.00041	0.00001	0.00001	NA	0.00000	0.05739	0.00295	0.05414	0.00041
$r_{M_1M_2 \bullet X} = 0$	500	0.05610	0.00165	0.03038	0.00019	0.00000	0.00000	NA	0.00000	0.05610	0.00165	0.03032	0.00019

$\upsilon^{sp}_{YM_1X} = .00005$	50	0.06655	0.01311	0.24542	0.00276	0.00228	0.00223	48.06682	0.00003	0.06427	0.01088	0.20378	0.00271
$\upsilon^{sp}_{YM_2X} = .05339$	100	0.05808	0.00464	0.08686	0.00109	0.00095	0.00090	19.32938	0.00000	0.05713	0.00374	0.07011	0.00109
$\upsilon^{tot}_{YM_1M_2X} = .05344$	250	0.05637	0.00293	0.05486	0.00044	0.00038	0.00033	7.12958	0.00000	0.05599	0.00260	0.04870	0.00044
$r_{M_1M_2 \bullet X} = -.1816$	500	0.05438	0.00094	0.01759	0.00021	0.00022	0.00018	3.76555	0.00000	0.05415	0.00076	0.01433	0.00021
$\upsilon^{sp}_{YM_1X} = .00298$	50	0.07831	0.01208	0.18234	0.00331	0.00533	0.00235	0.79118	0.00007	0.07298	0.00972	0.15371	0.00305
$\upsilon^{sp}_{YM_2X} = .06325$	100	0.07032	0.00408	0.06165	0.00123	0.00398	0.00100	0.33614	0.00002	0.06634	0.00308	0.04874	0.00119
$\upsilon^{tot}_{YM_1M_2X} = .06624$	250	0.06857	0.00234	0.03526	0.00049	0.00325	0.00027	0.09164	0.00001	0.06532	0.00206	0.03261	0.00046
$r_{M_1M_2 \bullet X} = .1184$	500	0.06735	0.00111	0.01678	0.00023	0.00320	0.00022	0.07531	0.00000	0.06415	0.00089	0.01403	0.00022
$\upsilon^{sp}_{YM_1X} = 1.26563$	50	1.33497	0.05761	0.04510	0.29702	1.31798	0.05236	0.04137	0.30222	0.01699	0.00525	0.44744	0.00042
$\upsilon^{sp}_{YM_2X} = .01174$	100	1.28085	0.00349	0.00273	0.13441	1.26684	0.00122	0.00096	0.13759	0.01401	0.00227	0.19356	0.00014
$\upsilon^{tot}_{YM_1M_2X} = 1.27736$	250	1.30784	0.03048	0.02386	0.06814	1.29521	0.02959	0.02338	0.06958	0.01262	0.00089	0.07565	0.00005
$r_{M_1M_2 \bullet X} = -.8921$	500	1.28141	0.00405	0.00317	0.03196	1.26898	0.00336	0.00265	0.03273	0.01243	0.00069	0.05888	0.00003
$\upsilon^{sp}_{YM_1X} = 0$	50	0.00284	0.00283	573.8742	0.00003	0.00231	0.00231	NA	0.00002	0.00053	0.00053	106.7661	0.00000
$\upsilon^{sp}_{YM_2X} = 4.9 \times 10^{-6}$	100	0.00117	0.00116	235.0753	0.00001	0.00099	0.00099	NA	0.00000	0.00018	0.00017	34.47822	0.00000
$\upsilon^{tot}_{YM_1M_2X} = 4.9 \times 10^{-6}$	250	0.00042	0.00042	84.71810	0.00000	0.00036	0.00036	NA	0.00000	0.00007	0.00006	12.22098	0.00000
$r_{M_1M_2 \bullet X} = .3$	500	0.00023	0.00023	46.41417	0.00000	0.00021	0.00021	NA	0.00000	0.00003	0.00002	4.14990	0.00000
$\upsilon^{sp}_{YM_1X} = .09379$	50	0.14035	0.03973	0.39483	0.03026	0.11934	0.02556	0.27248	0.01900	0.02100	0.01417	2.07518	0.00180
$\upsilon^{sp}_{YM_2X} = .00683$	100	0.12335	0.02273	0.22589	0.01105	0.10821	0.01442	0.15377	0.00744	0.01514	0.00831	1.21637	0.00054
$\upsilon^{tot}_{YM_1M_2X} = .10062$	250	0.10929	0.00867	0.08617	0.00338	0.09916	0.00537	0.05722	0.00237	0.01013	0.00330	0.48374	0.00012
$r_{M_1M_2 \bullet X} = .8684$	500	0.10358	0.00296	0.02945	0.00156	0.09543	0.00164	0.01752	0.00112	0.00815	0.00132	0.19321	0.00005

$\upsilon^{sp}_{YM_1X} = .04785$	50	0.08016	0.02882	0.56142	0.01239	0.06656	0.01871	0.39099	0.00735	0.01360	0.01011	2.90199	0.00091
$\upsilon^{sp}_{YM_2X} = .00348$	100	0.06897	0.01764	0.34358	0.00482	0.06072	0.01286	0.26885	0.00320	0.00826	0.00477	1.36987	0.00022
$\upsilon^{tot}_{YM_1M_2X} = .05134$	250	0.05433	0.00299	0.05828	0.00119	0.04924	0.00138	0.02891	0.00085	0.00509	0.00161	0.46154	0.00004
$r_{M_1M_2 \bullet X} = .8684$	500	0.05404	0.00271	0.05273	0.00052	0.04981	0.00196	0.04088	0.00039	0.00424	0.00075	0.21554	0.00001
$\upsilon^{sp}_{YM_1X} = .5184$	50	0.60388	0.08547	0.16488	0.15958	0.60343	0.08503	0.16402	0.15950	0.00045	0.00044	89.55768	0.00000
$\upsilon^{sp}_{YM_2X} = 4.9 \times 10^{-6}$	100	0.57080	0.05239	0.10106	0.07399	0.57063	0.05223	0.10076	0.07396	0.00016	0.00016	32.05163	0.00000
$\upsilon^{tot}_{YM_1M_2X} = .5184$	250	0.52247	0.00406	0.00784	0.02600	0.52242	0.00402	0.00776	0.02599	0.00005	0.00004	8.48302	0.00000
$r_{M_1M_2 \bullet X} = .8684$	500	0.52614	0.00773	0.01492	0.01296	0.52612	0.00772	0.01489	0.01296	0.00002	0.00002	3.04503	0.00000
$\upsilon^{sp}_{YM_1X} = .0059$	50	0.01840	0.00659	0.55840	0.00033	0.00964	0.00374	0.63311	0.00018	0.00876	0.00286	0.48369	0.00013
$\upsilon^{sp}_{YM_2X} = .0059$	100	0.01463	0.00283	0.23947	0.00011	0.00735	0.00145	0.24598	0.00004	0.00728	0.00138	0.23296	0.00005
$\upsilon^{tot}_{YM_1M_2X} = .01181$	250	0.01262	0.00082	0.06918	0.00003	0.00631	0.00041	0.06968	0.00002	0.00631	0.00041	0.06868	0.00002
$r_{M_1M_2 \bullet X} = -.0989$	500	0.01230	0.00049	0.04154	0.00001	0.00611	0.00021	0.03490	0.00001	0.00619	0.00028	0.04819	0.00001
$\upsilon^{sp}_{YM_1X} = .57154$	50	0.70767	0.11540	0.19484	0.37014	0.67903	0.10749	0.18808	0.35757	0.02864	0.00790	0.38104	0.00122
$\upsilon^{sp}_{YM_2X} = .02074$	100	0.66984	0.07756	0.13096	0.17860	0.64636	0.07483	0.13092	0.17327	0.02347	0.00274	0.13194	0.00035
$\upsilon^{tot}_{YM_1M_2X} = .59227$	250	0.63014	0.03787	0.06394	0.05863	0.60820	0.03666	0.06414	0.05734	0.02194	0.00121	0.05818	0.00012
$r_{M_1M_2 \bullet X} = -.6493$	500	0.62123	0.02895	0.04889	0.03161	0.60012	0.02859	0.05002	0.03069	0.02110	0.00037	0.01771	0.00006
$\upsilon^{sp}_{YM_1X} = .7465$	50	0.86478	0.11699	0.15644	0.37863	0.86041	0.11392	0.15260	0.37503	0.00437	0.00307	2.36982	0.00007
$\upsilon^{sp}_{YM_2X} = .0013$	100	0.80687	0.05908	0.07901	0.15367	0.80412	0.05762	0.07719	0.15229	0.00275	0.00145	1.12265	0.00002
$\upsilon^{tot}_{YM_1M_2X} = .74779$	250	0.76067	0.01287	0.01722	0.05635	0.75903	0.01253	0.01679	0.05597	0.00164	0.00034	0.26373	0.00000
$r_{M_1M_2 \bullet X} = -.6493$	500	0.74374	-0.00406	-0.00542	0.02780	0.74223	-0.00427	-0.00571	0.02760	0.00150	0.00021	0.16118	0.00000

$\upsilon^{sp}_{YM_1X} = .06891$	50	0.58510	0.06617	0.12752	0.19085	0.08729	0.01838	0.26674	0.00937	0.49781	0.04779	0.10620	0.12161
$\upsilon^{sp}_{YM_2X} = .45002$	100	0.58192	0.06300	0.12140	0.09065	0.08615	0.01724	0.25019	0.00439	0.49577	0.04576	0.10168	0.05800
$\upsilon^{tot}_{YM_1M_2X} = .51892$	250	0.52967	0.01075	0.02072	0.02937	0.07313	0.00423	0.06135	0.00144	0.45654	0.00652	0.01450	0.01866
$r_{M_1M_2 \bullet X} = .7184$	500	0.52888	0.00995	0.01918	0.01381	0.07218	0.00327	0.04744	0.00060	0.45670	0.00668	0.01485	0.00901
$\upsilon^{sp}_{YM_1X} = .00465$	50	0.05282	0.01231	0.30370	0.00164	0.00691	0.00226	0.48703	0.00009	0.04591	0.01004	0.27994	0.00152
$\upsilon^{sp}_{YM_2X} = .03587$	100	0.04570	0.00518	0.12775	0.00063	0.00591	0.00126	0.27189	0.00004	0.03978	0.00391	0.10907	0.00059
$\upsilon^{tot}_{YM_1M_2X} = .04052$	250	0.04318	0.00266	0.06557	0.00026	0.00497	0.00032	0.06902	0.00001	0.03821	0.00234	0.06512	0.00025
$r_{M_1M_2 \bullet X} = -.1816$	500	0.04156	0.00104	0.02566	0.00010	0.00492	0.00027	0.05751	0.00001	0.03664	0.00077	0.02153	0.00009
$\upsilon^{sp}_{YM_1X} = .00116$	50	0.03422	0.00894	0.35376	0.00105	0.00283	0.00167	1.43819	0.00002	0.03139	0.00727	0.30151	0.00101
$\upsilon^{sp}_{YM_2X} = .02412$	100	0.03034	0.00505	0.19995	0.00042	0.00218	0.00101	0.87247	0.00001	0.02816	0.00404	0.16754	0.00041
$\upsilon^{tot}_{YM_1M_2X} = .02528$	250	0.02683	0.00155	0.06121	0.00014	0.00154	0.00038	0.32633	0.00000	0.02529	0.00117	0.04844	0.00013
$r_{M_1M_2 \bullet X} = .1184$	500	0.02563	0.00035	0.01368	0.00006	0.00135	0.00019	0.16298	0.00000	0.02428	0.00016	0.00649	0.00006
$\upsilon^{sp}_{YM_1X} = .03516$	50	0.33052	0.06577	0.24840	0.08218	0.05191	0.01675	0.47656	0.00413	0.27861	0.04901	0.21347	0.05205
$\upsilon^{sp}_{YM_2X} = .2296$	100	0.29585	0.03109	0.11744	0.03647	0.04291	0.00776	0.22067	0.00170	0.25294	0.02333	0.10163	0.02358
$\upsilon^{tot}_{YM_1M_2X} = .26476$	250	0.27680	0.01204	0.04548	0.00983	0.03859	0.00344	0.09777	0.00045	0.23821	0.00860	0.03748	0.00640
$r_{M_1M_2 \bullet X} = .7184$	500	0.26900	0.00424	0.01602	0.00588	0.03625	0.00109	0.03107	0.00023	0.23275	0.00315	0.01372	0.00394
$\upsilon^{sp}_{YM_1X} = .31641$	50	0.52815	0.12422	0.30754	0.16520	0.42503	0.10862	0.34330	0.14002	0.10312	0.01560	0.17826	0.00599
$\upsilon^{sp}_{YM_2X} = .08752$	100	0.45648	0.05256	0.13012	0.06826	0.36394	0.04753	0.15023	0.06175	0.09254	0.00502	0.05742	0.00221
$\upsilon^{tot}_{YM_1M_2X} = .40392$	250	0.41388	0.00995	0.02464	0.02332	0.32562	0.00922	0.02913	0.02072	0.08825	0.00073	0.00838	0.00080
$r_{M_1M_2 \bullet X} = -.8921$	500	0.41035	0.00643	0.01592	0.01017	0.32154	0.00514	0.01624	0.00877	0.08881	0.00129	0.01476	0.00035

$\upsilon^{sp}_{YM_1X} = .31641$	50	0.40313	0.07587	0.23185	0.10514	0.38643	0.07003	0.22132	0.10757	0.01670	0.00585	0.53875	0.00048
$\upsilon^{sp}_{YM_2X} = .01085$	100	0.36729	0.04003	0.12233	0.04577	0.35429	0.03789	0.11974	0.04703	0.01300	0.00214	0.19767	0.00016
$\upsilon^{tot}_{YM_1M_2X} = .32726$	250	0.34025	0.01299	0.03970	0.01602	0.32829	0.01188	0.03755	0.01661	0.01196	0.00111	0.10263	0.00005
$r_{M_1M_2\bullet X} = -.8921$	500	0.33955	0.01229	0.03756	0.00876	0.32809	0.01168	0.03693	0.00905	0.01146	0.00061	0.05607	0.00003
$\upsilon^{sp}_{YM_1X} = .00088$	50	0.01209	0.00642	1.13127	0.00025	0.00379	0.00291	3.30869	0.00008	0.00830	0.00351	0.73133	0.00015
$\upsilon^{sp}_{YM_2X} = .00479$	100	0.00859	0.00291	0.51339	0.00008	0.00208	0.00120	1.36021	0.00001	0.00651	0.00172	0.35785	0.00006
$\upsilon^{tot}_{YM_1M_2X} = .00567$	250	0.00639	0.00072	0.12617	0.00001	0.00120	0.00032	0.35865	0.00000	0.00519	0.00040	0.08347	0.00001
$r_{M_1M_2\bullet X} = -.0089$	500	0.00607	0.00040	0.07015	0.00001	0.00112	0.00024	0.27524	0.00000	0.00495	0.00016	0.03248	0.00001
$\upsilon^{sp}_{YM_1X} = .00245$	50	0.01141	0.00652	1.33306	0.00024	0.00567	0.00322	1.31695	0.00012	0.00574	0.00330	1.34917	0.00011
$\upsilon^{sp}_{YM_2X} = .00245$	100	0.00704	0.00215	0.43980	0.00004	0.00350	0.00105	0.42974	0.00002	0.00355	0.00110	0.44986	0.00002
$\upsilon^{tot}_{YM_1M_2X} = .0049$	250	0.00556	0.00067	0.13731	0.00001	0.00281	0.00037	0.15068	0.00001	0.00275	0.00030	0.12393	0.00000
$r_{M_1M_2\bullet X} = -.0089$	500	0.00525	0.00036	0.07400	0.00000	0.00265	0.00021	0.08469	0.00000	0.00260	0.00015	0.06331	0.00000
$\upsilon^{sp}_{YM_1X} = .00479$	50	0.01514	0.00555	0.57908	0.00030	0.00720	0.00241	0.50223	0.00012	0.00794	0.00314	0.65593	0.00014
$\upsilon^{sp}_{YM_2X} = .00479$	100	0.01252	0.00294	0.30634	0.00011	0.00622	0.00143	0.29799	0.00005	0.00630	0.00151	0.31468	0.00005
$\upsilon^{tot}_{YM_1M_2X} = .00959$	250	0.01053	0.00095	0.09900	0.00003	0.00517	0.00038	0.07956	0.00001	0.00536	0.00057	0.11845	0.00001
$r_{M_1M_2\bullet X} = -.0089$	500	0.01000	0.00042	0.04352	0.00001	0.00503	0.00024	0.04910	0.00001	0.00497	0.00018	0.03793	0.00001
$\upsilon^{sp}_{YM_1X} = .00245$	50	0.00966	0.00477	0.97471	0.00017	0.00496	0.00252	1.02968	0.00009	0.00469	0.00225	0.91973	0.00006
$\upsilon^{sp}_{YM_2X} = .00245$	100	0.00714	0.00225	0.46007	0.00005	0.00351	0.00107	0.43648	0.00002	0.00363	0.00118	0.48366	0.00002
$\upsilon^{tot}_{YM_1M_2X} = .0049$	250	0.00567	0.00078	0.15874	0.00001	0.00277	0.00033	0.13388	0.00000	0.00289	0.00045	0.18361	0.00001
$r_{M_1M_2\bullet X} = -.0089$	500	0.00530	0.00040	0.08271	0.00000	0.00265	0.00021	0.08464	0.00000	0.00264	0.00020	0.08078	0.00000

$\upsilon^{sp}_{YM_1X} = .07131$	50	0.09618	0.02008	0.26392	0.00663	0.08929	0.01798	0.25219	0.00649	0.00689	0.00210	0.43846	0.00005
$\upsilon^{sp}_{YM_2X} = .00479$	100	0.08627	0.01017	0.13367	0.00258	0.08101	0.00970	0.13601	0.00256	0.00527	0.00047	0.09887	0.00001
$\upsilon^{tot}_{YM_1M_2X} = .0761$	250	0.07883	0.00273	0.03585	0.00083	0.07382	0.00252	0.03532	0.00083	0.00500	0.00021	0.04371	0.00001
$r_{M_1M_2\bullet X} = -.0089$	500	0.07682	0.00072	0.00943	0.00041	0.07196	0.00065	0.00914	0.00041	0.00486	0.00007	0.01366	0.00000
$\upsilon^{sp}_{YM_1X} = .06113$	50	0.07653	0.01295	0.20365	0.00376	0.07350	0.01237	0.20231	0.00372	0.00303	0.00058	0.23710	0.00001
$\upsilon^{sp}_{YM_2X} = .00245$	100	0.07041	0.00683	0.10745	0.00160	0.06755	0.00641	0.10489	0.00159	0.00286	0.00042	0.17156	0.00000
$\upsilon^{tot}_{YM_1M_2X} = .06358$	250	0.06769	0.00411	0.06458	0.00068	0.06506	0.00393	0.06426	0.00067	0.00262	0.00018	0.07265	0.00000
$r_{M_1M_2\bullet X} = -.0089$	500	0.06490	0.00132	0.02081	0.00031	0.06235	0.00121	0.01986	0.00031	0.00255	0.00011	0.04466	0.00000

Table 3. *Confidence limits, mean confidence interval width for* $\hat{\upsilon}^{tot}_{YM_1M_2X}$, $\hat{\upsilon}^{sp}_{YM_1X}$, *and* $\hat{\upsilon}^{tot}_{YM_2X}$

		$\hat{\upsilon}^{tot}_{YM_1M_2X}$			$\hat{\upsilon}^{sp}_{YM_1X}$			$\hat{\upsilon}^{sp}_{YM_2X}$		
	n	Mean LCL	Mean UCL	Mean CI Width	Mean LCL	Mean UCL	Mean CI Width	Mean LCL	Mean UCL	Mean CI Width
$\upsilon^{sp}_{YM_1X} = 0$	50	0.00004	0.02194	0.02190	0.00000	0.01187	0.01187	0.00000	0.01440	0.01440
$\upsilon^{sp}_{YM_2X} = .00005$	100	0.00001	0.00572	0.00571	0.00000	0.00241	0.00241	0.00000	0.00437	0.00437
$\upsilon^{tot}_{YM_1M_2X} = .00005$	250	0.00001	0.00129	0.00128	0.00000	0.00040	0.00040	0.00000	0.00110	0.00110
$r_{M_1M_2 \bullet X} = 0$	500	0.00000	0.00055	0.00054	0.00000	0.00010	0.00010	0.00000	0.00052	0.00051
$\upsilon^{sp}_{YM_1X} = 0$	50	0.00004	0.02507	0.02502	0.00001	0.01319	0.01318	0.00000	0.01644	0.01644
$\upsilon^{sp}_{YM_2X} = .00006$	100	0.00002	0.00635	0.00634	0.00000	0.00293	0.00293	0.00000	0.00462	0.00462
$\upsilon^{tot}_{YM_1M_2X} = .00006$	250	0.00001	0.00149	0.00149	0.00000	0.00045	0.00045	0.00000	0.00128	0.00128
$r_{M_1M_2 \bullet X} = .3$	500	0.00001	0.00066	0.00066	0.00000	0.00011	0.00011	0.00000	0.00062	0.00062
$\upsilon^{sp}_{YM_1X} = 0$	50	0.00045	0.19333	0.19288	0.00002	0.08830	0.08828	0.00015	0.11532	0.11518
$\upsilon^{sp}_{YM_2X} = .00015$	100	0.00026	0.06033	0.06007	0.00001	0.02564	0.02563	0.00006	0.03891	0.03885
$\upsilon^{tot}_{YM_1M_2X} = .00015$	250	0.00024	0.01842	0.01818	0.00000	0.00616	0.00616	0.00008	0.01380	0.01372
$r_{M_1M_2 \bullet X} = .9$	500	0.00038	0.01004	0.00966	0.00000	0.00280	0.00279	0.00018	0.00815	0.00797
$\upsilon^{sp}_{YM_1X} = 0$	50	0.00018	0.12532	0.12514	0.00001	0.05880	0.05879	0.00003	0.07459	0.07456
$\upsilon^{sp}_{YM_2X} = .00077$	100	0.00011	0.04103	0.04093	0.00000	0.01775	0.01774	0.00002	0.02603	0.02601
$\upsilon^{tot}_{YM_1M_2X} = .00077$	250	0.00013	0.01193	0.01180	0.00000	0.00430	0.00430	0.00005	0.00884	0.00879
$r_{M_1M_2 \bullet X} = .9$	500	0.00015	0.00539	0.00525	0.00000	0.00157	0.00157	0.00006	0.00440	0.00434

$\upsilon^{sp}_{YM_1X} = .00084$	50	0.00018	0.04269	0.04251	0.00004	0.03474	0.03470	0.00000	0.01550	0.01549
$\upsilon^{sp}_{YM_2X} = .00005$	100	0.00015	0.01637	0.01622	0.00006	0.01481	0.01475	0.00000	0.00412	0.00412
$\upsilon^{tot}_{YM_1M_2X} = .00089$	250	0.00016	0.00674	0.00658	0.00010	0.00639	0.00629	0.00000	0.00111	0.00111
$r_{M_1M_2 \bullet X} = -.0316$	500	0.00018	0.00393	0.00375	0.00013	0.00379	0.00366	0.00000	0.00049	0.00049
$\upsilon^{sp}_{YM_1X} = .09765$	50	0.01902	0.89964	0.88062	0.01094	0.63854	0.62760	0.00102	0.29606	0.29503
$\upsilon^{sp}_{YM_2X} = .00972$	100	0.02431	0.46260	0.43829	0.01730	0.35359	0.33629	0.00087	0.12769	0.12682
$\upsilon^{tot}_{YM_1M_2X} = .10738$	250	0.03799	0.28750	0.24950	0.03395	0.23417	0.20021	0.00097	0.05946	0.05849
$r_{M_1M_2 \bullet X} = .8684$	500	0.04950	0.21793	0.16843	0.04654	0.18279	0.13625	0.00148	0.03843	0.03695
$\upsilon^{sp}_{YM_1X} = 1.70303$	50	0.77485	3.37822	2.60337	0.77320	3.37626	2.60306	0.00000	0.00952	0.00952
$\upsilon^{sp}_{YM_2X} = .00004$	100	1.01236	2.74188	1.72952	1.01193	2.74125	1.72932	0.00000	0.00255	0.00255
$\upsilon^{tot}_{YM_1M_2X} = 1.70307$	250	1.20609	2.27836	1.07227	1.20597	2.27821	1.07224	0.00000	0.00071	0.00071
$r_{M_1M_2 \bullet X} = -.2075$	500	1.35808	2.11396	0.75588	1.35802	2.11387	0.75586	0.00000	0.00033	0.00033
$\upsilon^{sp}_{YM_1X} = 3.16064$	50	1.75481	5.09787	3.34306	1.74552	5.07330	3.32778	0.00029	0.05668	0.05638
$\upsilon^{sp}_{YM_2X} = .00236$	100	2.15599	4.48603	2.33004	2.15164	4.47321	2.32157	0.00019	0.02835	0.02817
$\upsilon^{tot}_{YM_1M_2X} = 3.10877$	250	2.44743	3.88201	1.43458	2.44489	3.87512	1.43022	0.00028	0.01377	0.01349
$r_{M_1M_2 \bullet X} = .0925$	500	2.63762	3.65099	1.01337	2.63550	3.64590	1.01040	0.00038	0.00924	0.00886
$\upsilon^{sp}_{YM_1X} = 0$	50	0.00050	0.06105	0.06055	0.00000	0.01058	0.01058	0.00028	0.05706	0.05678
$\upsilon^{sp}_{YM_2X} = .00479$	100	0.00064	0.03011	0.02948	0.00000	0.00238	0.00238	0.00051	0.02956	0.02905
$\upsilon^{tot}_{YM_1M_2X} = .00479$	250	0.00116	0.01590	0.01474	0.00000	0.00038	0.00038	0.00112	0.01584	0.01472
$r_{M_1M_2 \bullet X} = 0$	500	0.00177	0.01132	0.00956	0.00000	0.00009	0.00009	0.00175	0.01131	0.00956

$\upsilon^{sp}_{YM_1X}=0$	50	0.16269	2.36346	2.20077	0.00256	0.73423	0.73166	0.06994	1.85019	1.78025
$\upsilon^{sp}_{YM_2X}=.3969$	100	0.17556	1.46359	1.28803	0.00079	0.31507	0.31428	0.09975	1.26075	1.16100
$\upsilon^{tot}_{YM_1M_2X}=.3969$	250	0.20744	0.88600	0.67855	0.00036	0.12758	0.12722	0.15535	0.81977	0.66442
$r_{M_1M_2\bullet X}=.9$	500	0.23781	0.71049	0.47267	0.00015	0.05891	0.05876	0.20945	0.67834	0.46889
$\upsilon^{sp}_{YM_1X}=.00053$	50	0.00094	0.07025	0.06931	0.00005	0.03352	0.03347	0.00024	0.05090	0.05066
$\upsilon^{sp}_{YM_2X}=.00448$	100	0.00106	0.03283	0.03177	0.00004	0.01258	0.01254	0.00044	0.02708	0.02664
$\upsilon^{tot}_{YM_1M_2X}=.00501$	250	0.00149	0.01616	0.01467	0.00003	0.00466	0.00462	0.00092	0.01447	0.01356
$r_{M_1M_2\bullet X}=-.0989$	500	0.00210	0.01139	0.00929	0.00006	0.00288	0.00282	0.00150	0.01030	0.00880
$\upsilon^{sp}_{YM_1X}=.19319$	50	0.07005	1.74996	1.67991	0.02770	0.85008	0.82237	0.03348	0.94250	0.90902
$\upsilon^{sp}_{YM_2X}=.21203$	100	0.09301	1.16959	1.07659	0.03976	0.56860	0.52884	0.04581	0.62410	0.57828
$\upsilon^{tot}_{YM_1M_2X}=.40522$	250	0.18014	0.87520	0.69505	0.08261	0.42413	0.34153	0.09138	0.46495	0.37357
$r_{M_1M_2\bullet X}=.8011$	500	0.22773	0.69881	0.47108	0.10630	0.33848	0.23218	0.11603	0.36890	0.25287
$\upsilon^{sp}_{YM_1X}=.00203$	50	0.00076	0.05505	0.05429	0.00010	0.03212	0.03202	0.00014	0.03431	0.03417
$\upsilon^{sp}_{YM_2X}=.00203$	100	0.00091	0.02743	0.02653	0.00016	0.01749	0.01733	0.00015	0.01631	0.01616
$\upsilon^{tot}_{YM_1M_2X}=.00405$	250	0.00138	0.01335	0.01196	0.00033	0.00843	0.00810	0.00031	0.00823	0.00792
$r_{M_1M_2\bullet X}=-.0989$	500	0.00187	0.00927	0.00740	0.00054	0.00568	0.00515	0.00055	0.00576	0.00521
$\upsilon^{sp}_{YM_1X}=0$	50	0.01029	0.22548	0.21519	0.00000	0.00970	0.00970	0.00931	0.22336	0.21405
$\upsilon^{sp}_{YM_2X}=.05444$	100	0.01721	0.15155	0.13433	0.00000	0.00200	0.00200	0.01691	0.15122	0.13430
$\upsilon^{tot}_{YM_1M_2X}=.05444$	250	0.02685	0.10578	0.07893	0.00000	0.00031	0.00031	0.02680	0.10573	0.07894
$r_{M_1M_2\bullet X}=0$	500	0.03350	0.08731	0.05382	0.00000	0.00008	0.00008	0.03349	0.08730	0.05382

$\upsilon^{sp}_{YM_1X} = .00005$	50	0.01123	0.22796	0.21674	0.00002	0.02677	0.02675	0.00862	0.22073	0.21212
$\upsilon^{sp}_{YM_2X} = .05339$	100	0.01692	0.14651	0.12959	0.00001	0.00968	0.00967	0.01516	0.14433	0.12917
$\upsilon^{tot}_{YM_1M_2X} = .05344$	250	0.02675	0.10418	0.07744	0.00001	0.00318	0.00317	0.02595	0.10352	0.07756
$r_{M_1M_2\bullet X} = -.1816$	500	0.03232	0.08535	0.05303	0.00001	0.00155	0.00155	0.03188	0.08502	0.05315
$\upsilon^{sp}_{YM_1X} = .00298$	50	0.01404	0.25576	0.24171	0.00013	0.03930	0.03917	0.01061	0.23945	0.22884
$\upsilon^{sp}_{YM_2X} = .06325$	100	0.02197	0.16877	0.14680	0.00022	0.01947	0.01924	0.01911	0.16051	0.14140
$\upsilon^{tot}_{YM_1M_2X} = .06624$	250	0.03428	0.12265	0.08837	0.00052	0.01028	0.00975	0.03171	0.11764	0.08593
$r_{M_1M_2\bullet X} = .1184$	500	0.04176	0.10260	0.06084	0.00094	0.00759	0.00664	0.03910	0.09833	0.05923
$\upsilon^{sp}_{YM_1X} = 1.26563$	50	0.52861	2.69499	2.16638	0.49113	2.68186	2.19073	0.00111	0.08611	0.08500
$\upsilon^{sp}_{YM_2X} = .01174$	100	0.67634	2.14192	1.46558	0.65069	2.13210	1.48141	0.00174	0.04931	0.04757
$\upsilon^{tot}_{YM_1M_2X} = 1.27736$	250	0.89096	1.82825	0.93729	0.87194	1.81967	0.94772	0.00331	0.03066	0.02735
$r_{M_1M_2\bullet X} = -.8921$	500	0.98023	1.63440	0.65418	0.96374	1.62481	0.66107	0.00511	0.02412	0.01901
$\upsilon^{sp}_{YM_1X} = 0$	50	0.00011	0.03601	0.03589	0.00002	0.02932	0.02930	0.00000	0.01301	0.01300
$\upsilon^{sp}_{YM_2X} = 4.9\times10^{-6}$	100	0.00005	0.01240	0.01234	0.00001	0.01100	0.01098	0.00000	0.00341	0.00340
$\upsilon^{tot}_{YM_1M_2X} = 4.9\times10^{-6}$	250	0.00002	0.00391	0.00388	0.00000	0.00361	0.00361	0.00000	0.00083	0.00083
$r_{M_1M_2\bullet X} = .3$	500	0.00001	0.00182	0.00180	0.00000	0.00173	0.00172	0.00000	0.00029	0.00029
$\upsilon^{sp}_{YM_1X} = .09379$	50	0.01462	0.73229	0.71767	0.00819	0.56436	0.55616	0.00054	0.20265	0.20211
$\upsilon^{sp}_{YM_2X} = .00683$	100	0.02063	0.44841	0.42779	0.01559	0.35967	0.34408	0.00045	0.10212	0.10167
$\upsilon^{tot}_{YM_1M_2X} = .10062$	250	0.03501	0.26568	0.23066	0.03220	0.22534	0.19314	0.00074	0.04457	0.04384
$r_{M_1M_2\bullet X} = .8684$	500	0.04632	0.20062	0.15430	0.04437	0.17525	0.13088	0.00099	0.02754	0.02655

$\upsilon^{sp}_{YM_1X} = .04785$	50	0.00561	0.48551	0.47990	0.00311	0.36352	0.36041	0.00018	0.14289	0.14271
$\upsilon^{sp}_{YM_2X} = .00348$	100	0.00960	0.27156	0.26197	0.00723	0.21973	0.21250	0.00026	0.06058	0.06033
$\upsilon^{tot}_{YM_1M_2X} = .05134$	250	0.01478	0.14429	0.12951	0.01343	0.12264	0.10921	0.00033	0.02430	0.02398
$r_{M_1M_2\bullet X} = .8684$	500	0.02212	0.11126	0.08914	0.02109	0.09749	0.07641	0.00047	0.01516	0.01469
$\upsilon^{sp}_{YM_1X} = .5184$	50	0.12976	1.60993	1.48017	0.12822	1.60805	1.47983	0.00000	0.01000	0.01000
$\upsilon^{sp}_{YM_2X} = 4.9\times10^{-6}$	100	0.18797	1.20827	1.02030	0.18757	1.20769	1.02012	0.00000	0.00266	0.00266
$\upsilon^{tot}_{YM_1M_2X} = .5184$	250	0.26521	0.87557	0.61036	0.26512	0.87542	0.61029	0.00000	0.00061	0.00061
$r_{M_1M_2\bullet X} = .8684$	500	0.33369	0.76759	0.43390	0.33365	0.76755	0.43390	0.00000	0.00021	0.00021
$\upsilon^{sp}_{YM_1X} = .0059$	50	0.00205	0.09123	0.08918	0.00039	0.05714	0.05674	0.00026	0.05473	0.05448
$\upsilon^{sp}_{YM_2X} = .0059$	100	0.00297	0.04932	0.04635	0.00061	0.03078	0.03017	0.00062	0.03009	0.02947
$\upsilon^{tot}_{YM_1M_2X} = .01181$	250	0.00476	0.02841	0.02365	0.00144	0.01723	0.01579	0.00142	0.01725	0.01583
$r_{M_1M_2\bullet X} = -.0989$	500	0.00638	0.02193	0.01554	0.00230	0.01287	0.01057	0.00234	0.01298	0.01064
$\upsilon^{sp}_{YM_1X} = .57154$	50	0.10957	2.35200	2.24243	0.09531	2.29055	2.19524	0.00165	0.14245	0.14080
$\upsilon^{sp}_{YM_2X} = .02074$	100	0.15726	1.68327	1.52600	0.14345	1.64411	1.50066	0.00301	0.08199	0.07898
$\upsilon^{tot}_{YM_1M_2X} = .59227$	250	0.25327	1.20268	0.94941	0.23767	1.17157	0.93391	0.00663	0.05192	0.04529
$r_{M_1M_2\bullet X} = -.6493$	500	0.33621	1.00245	0.66623	0.31953	0.97578	0.65625	0.00944	0.03988	0.03044
$\upsilon^{sp}_{YM_1X} = .7465$	50	0.17045	2.45299	2.28254	0.16525	2.43775	2.27250	0.00009	0.04177	0.04168
$\upsilon^{sp}_{YM_2X} = .0013$	100	0.24715	1.79324	1.54608	0.24456	1.78481	1.54025	0.00009	0.01867	0.01858
$\upsilon^{tot}_{YM_1M_2X} = .74779$	250	0.36966	1.30784	0.93818	0.36876	1.30364	0.93488	0.00011	0.00764	0.00753
$r_{M_1M_2\bullet X} = -.6493$	500	0.45363	1.11250	0.65887	0.45289	1.10926	0.65636	0.00019	0.00503	0.00484

$\upsilon^{sp}_{YM_1X} = .06891$	50	0.10312	1.90018	1.79706	0.00520	0.42598	0.42077	0.09009	1.50411	1.41402
$\upsilon^{sp}_{YM_2X} = .45002$	100	0.18652	1.36731	1.18079	0.01343	0.27787	0.26444	0.16828	1.10621	0.93793
$\upsilon^{tot}_{YM_1M_2X} = .51892$	250	0.26675	0.93647	0.66972	0.02368	0.16595	0.14227	0.23992	0.77861	0.53869
$r_{M_1M_2 \bullet X} = .7184$	500	0.33051	0.80053	0.47002	0.03382	0.13219	0.09837	0.29374	0.67303	0.37929
$\upsilon^{sp}_{YM_1X} = .00465$	50	0.00922	0.18377	0.17454	0.00023	0.04386	0.04363	0.00543	0.16548	0.16005
$\upsilon^{sp}_{YM_2X} = .03587$	100	0.01358	0.11603	0.10245	0.00053	0.02432	0.02380	0.00962	0.10567	0.09605
$\upsilon^{tot}_{YM_1M_2X} = .04052$	250	0.02070	0.07983	0.05913	0.00110	0.01364	0.01254	0.01659	0.07332	0.05674
$r_{M_1M_2 \bullet X} = -.1816$	500	0.02499	0.06500	0.04001	0.00185	0.01034	0.00849	0.02063	0.05921	0.03858
$\upsilon^{sp}_{YM_1X} = .00116$	50	0.00445	0.14137	0.13692	0.00003	0.02703	0.02699	0.00274	0.13166	0.12891
$\upsilon^{sp}_{YM_2X} = .02412$	100	0.00722	0.08678	0.07955	0.00011	0.01282	0.01271	0.00524	0.08293	0.07769
$\upsilon^{tot}_{YM_1M_2X} = .02528$	250	0.01084	0.05543	0.04458	0.00014	0.00610	0.00596	0.00911	0.05348	0.04437
$r_{M_1M_2 \bullet X} = .1184$	500	0.01367	0.04374	0.03008	0.00024	0.00397	0.00373	0.01217	0.04223	0.03006
$\upsilon^{sp}_{YM_1X} = .03516$	50	0.04561	1.22132	1.17571	0.00238	0.28624	0.28386	0.03826	0.95874	0.92048
$\upsilon^{sp}_{YM_2X} = .2296$	100	0.07375	0.79116	0.71741	0.00474	0.16143	0.15669	0.06654	0.64098	0.57444
$\upsilon^{tot}_{YM_1M_2X} = .26476$	250	0.12040	0.53845	0.41805	0.01032	0.09782	0.08751	0.10808	0.44646	0.33838
$r_{M_1M_2 \bullet X} = .7184$	500	0.15270	0.43587	0.28316	0.01500	0.07214	0.05714	0.13630	0.36704	0.23074
$\upsilon^{sp}_{YM_1X} = .31641$	50	0.10912	1.59245	1.48333	0.05298	1.42599	1.37301	0.01814	0.31625	0.29812
$\upsilon^{sp}_{YM_2X} = .08752$	100	0.13974	1.07702	0.93728	0.07804	0.94779	0.86974	0.02911	0.21403	0.18492
$\upsilon^{tot}_{YM_1M_2X} = .40392$	250	0.19141	0.75160	0.56019	0.12031	0.64264	0.52233	0.04475	0.15403	0.10928
$r_{M_1M_2 \bullet X} = -.8921$	500	0.24092	0.63659	0.39567	0.16458	0.53398	0.36940	0.05578	0.13319	0.07742

$\upsilon^{sp}_{YM_1X} = .31641$	50	0.07716	1.28390	1.20674	0.05511	1.26661	1.21150	0.00083	0.09234	0.09151
$\upsilon^{sp}_{YM_2X} = .01085$	100	0.10038	0.90209	0.80171	0.07930	0.89273	0.81343	0.00133	0.05007	0.04874
$\upsilon^{tot}_{YM_1M_2X} = .32726$	250	0.14913	0.63458	0.48546	0.13091	0.62608	0.49517	0.00264	0.03142	0.02879
$r_{M_1M_2\bullet X} = -.8921$	500	0.19214	0.53859	0.34646	0.17659	0.52977	0.35318	0.00409	0.02378	0.01969
$\upsilon^{sp}_{YM_1X} = .00088$	50	0.00104	0.07401	0.07297	0.00007	0.03469	0.03462	0.00028	0.05378	0.05350
$\upsilon^{sp}_{YM_2X} = .00479$	100	0.00129	0.03510	0.03381	0.00008	0.01417	0.01409	0.00054	0.02810	0.02756
$\upsilon^{tot}_{YM_1M_2X} = .00567$	250	0.00175	0.01743	0.01568	0.00008	0.00564	0.00557	0.00103	0.01497	0.01394
$r_{M_1M_2\bullet X} = -.0089$	500	0.00249	0.01274	0.01025	0.00015	0.00380	0.00366	0.00168	0.01097	0.00929
$\upsilon^{sp}_{YM_1X} = .00245$	50	0.00107	0.06516	0.06409	0.00017	0.03929	0.03912	0.00020	0.03958	0.03938
$\upsilon^{sp}_{YM_2X} = .00245$	100	0.00107	0.02835	0.02728	0.00021	0.01702	0.01681	0.00022	0.01767	0.01746
$\upsilon^{tot}_{YM_1M_2X} = .0049$	250	0.00163	0.01484	0.01321	0.00044	0.00909	0.00865	0.00040	0.00904	0.00864
$r_{M_1M_2\bullet X} = -.0089$	500	0.00226	0.01072	0.00847	0.00072	0.00654	0.00582	0.00070	0.00641	0.00570
$\upsilon^{sp}_{YM_1X} = .00479$	50	0.00150	0.08149	0.07999	0.00021	0.04792	0.04772	0.00027	0.05120	0.05094
$\upsilon^{sp}_{YM_2X} = .00479$	100	0.00240	0.04357	0.04118	0.00053	0.02633	0.02579	0.00054	0.02688	0.02634
$\upsilon^{tot}_{YM_1M_2X} = .00959$	250	0.00375	0.02463	0.02087	0.00107	0.01481	0.01374	0.00108	0.01521	0.01413
$r_{M_1M_2\bullet X} = -.0089$	500	0.00496	0.01846	0.01350	0.00176	0.01100	0.00925	0.00172	0.01093	0.00921
$\upsilon^{sp}_{YM_1X} = .00245$	50	0.00093	0.06071	0.05978	0.00017	0.03806	0.03789	0.00014	0.03563	0.03549
$\upsilon^{sp}_{YM_2X} = .00245$	100	0.00107	0.02887	0.02780	0.00020	0.01763	0.01742	0.00021	0.01788	0.01767
$\upsilon^{tot}_{YM_1M_2X} = .0049$	250	0.00165	0.01498	0.01333	0.00039	0.00908	0.00869	0.00044	0.00936	0.00892
$r_{M_1M_2\bullet X} = -.0089$	500	0.00227	0.01081	0.00854	0.00072	0.00652	0.00580	0.00071	0.00649	0.00577

$\upsilon^{sp}_{YM_1X} = .07131$	50	0.01832	0.29430	0.27598	0.01257	0.28481	0.27224	0.00064	0.02759	0.02695
$\upsilon^{sp}_{YM_2X} = .00479$	100	0.02361	0.20853	0.18492	0.01858	0.20232	0.18374	0.00091	0.01595	0.01504
$\upsilon^{tot}_{YM_1M_2X} = .0761$	250	0.03405	0.14667	0.11262	0.02910	0.14129	0.11220	0.00178	0.01066	0.00888
$r_{M_1M_2 \bullet X} = -.0089$	500	0.04280	0.12267	0.07987	0.03797	0.11756	0.07959	0.00239	0.00855	0.00616
$\upsilon^{sp}_{YM_1X} = .06113$	50	0.04280	0.12267	0.07987	0.03797	0.11756	0.07959	0.00239	0.00855	0.00616
$\upsilon^{sp}_{YM_2X} = .00245$	100	0.01750	0.17526	0.15776	0.01467	0.17187	0.15720	0.00045	0.00915	0.00870
$\upsilon^{tot}_{YM_1M_2X} = .06358$	250	0.02851	0.12706	0.09855	0.02580	0.12422	0.09841	0.00089	0.00581	0.00492
$r_{M_1M_2 \bullet X} = -.0089$	500	0.03570	0.10406	0.06836	0.03313	0.10136	0.06823	0.00121	0.00463	0.00342

Table 4. *Coverage, misses to the left and right for* $\hat{\upsilon}^{tot}_{YM_1M_2X}$, $\hat{\upsilon}^{sp}_{YM_1X}$, *and* $\hat{\upsilon}^{sp}_{YM_2X}$

	n	$\hat{\upsilon}^{tot}_{YM_1M_2X}$ Cov	<.025	>.975	$\hat{\upsilon}^{sp}_{YM_1X}$ Cov	<.025	>.975	$\hat{\upsilon}^{sp}_{YM_2X}$ Cov	<.025	>.975
$\upsilon^{sp}_{YM_1X} = 0$	50	84.2	15.8	0	0	100	0	99.2	0.8	0
$\upsilon^{sp}_{YM_2X} = .00005$	100	96	4	0	0	100	0	99.2	0.8	0
$\upsilon^{tot}_{YM_1M_2X} = .00005$	250	98.4	1.6	0	0	100	0	99.2	0.8	0
$r_{M_1M_2 \bullet X} = 0$	500	98.4	1.6	0	0	100	0	98.2	1.2	0.6
$\upsilon^{sp}_{YM_1X} = 0$	50	87.8	12.2	0	0	100	0	99	1	0
$\upsilon^{sp}_{YM_2X} = .00006$	100	96.8	3.2	0	0	100	0	99.2	0.8	0
$\upsilon^{tot}_{YM_1M_2X} = .00006$	250	99	1	0	0	100	0	99.4	0.6	0
$r_{M_1M_2 \bullet X} = .3$	500	98.4	1.2	0.4	0	100	0	98.4	0.4	1.2
$\upsilon^{sp}_{YM_1X} = 0$	50	94	6	0	0	100	0	98.6	1.4	0
$\upsilon^{sp}_{YM_2X} = .00015$	100	96.8	3.2	0	0	100	0	98.8	1.2	0
$\upsilon^{tot}_{YM_1M_2X} = .00015$	250	98.4	1.6	0	0	100	0	98.6	0.8	0.6
$r_{M_1M_2 \bullet X} = .9$	500	96.8	2.8	0.4	0	100	0	95.8	2.2	2
$\upsilon^{sp}_{YM_1X} = 0$	50	95.2	4.8	0	0	100	0	99.2	0.8	0
$\upsilon^{sp}_{YM_2X} = .00077$	100	98.2	1.8	0	0	100	0	99.4	0.6	0
$\upsilon^{tot}_{YM_1M_2X} = .00077$	250	97	3	0	0	100	0	96.4	1.8	1.8
$r_{M_1M_2 \bullet X} = .9$	500	98	1.4	0.6	0	100	0	95.2	1	3.8

$\upsilon^{sp}_{YM_1X} = .00084$	50	95.4	4.6	0	99	1	0	98.8	1.2	0
$\upsilon^{sp}_{YM_2X} = .00005$	100	96.6	3.4	0	98.4	1.6	0	100	0	0
$\upsilon^{tot}_{YM_1M_2X} = .00089$	250	95.6	4.4	0	96.6	3	0.4	99.8	0.2	0
$r_{M_1M_2\bullet X} = -.0316$	500	95.6	4.2	0.2	95.4	3.4	1.2	98	1.2	0.8
$\upsilon^{sp}_{YM_1X} = .09765$	50	95.6	2.2	2.2	93	2	5	98	2	0
$\upsilon^{sp}_{YM_2X} = .00972$	100	94.4	3	2.6	92.6	2.6	4.8	97.6	2.4	0
$\upsilon^{tot}_{YM_1M_2X} = .10738$	250	94.6	2.6	2.8	95	1.8	3.2	98	1.8	0.2
$r_{M_1M_2\bullet X} = .8684$	500	94.2	2.2	3.6	95.4	1.4	3.2	96.4	2	1.6
$\upsilon^{sp}_{YM_1X} = 1.70303$	50	95.6	1.6	2.8	95.6	1.6	2.8	99	1	0
$\upsilon^{sp}_{YM_2X} = .00004$	100	94.2	2.2	3.6	94.2	2.2	3.6	99.6	0.4	0
$\upsilon^{tot}_{YM_1M_2X} = 1.70307$	250	91.2	3.4	5.4	91.4	3.2	5.4	99.6	0.4	0
$r_{M_1M_2\bullet X} = -.2075$	500	96	2.6	1.4	96	2.6	1.4	97	1.2	1.8
$\upsilon^{sp}_{YM_1X} = 3.16064$	50	93.4	2.8	3.8	93.4	2.8	3.8	97.4	2.6	0
$\upsilon^{sp}_{YM_2X} = .00236$	100	94.6	2.6	2.8	94.8	2.4	2.8	97	3	0
$\upsilon^{tot}_{YM_1M_2X} = 3.10877$	250	94.8	2.6	2.6	95	2.4	2.6	98	2	0
$r_{M_1M_2\bullet X} = .0925$	500	95.2	1.4	3.4	95.2	1.4	3.4	96.4	3	0.6
$\upsilon^{sp}_{YM_1X} = 0$	50	98.6	1.4	0	0	100	0	96.8	1	2.2
$\upsilon^{sp}_{YM_2X} = .00479$	100	96.6	1.8	1.6	0	100	0	96	1.8	2.2
$\upsilon^{tot}_{YM_1M_2X} = .00479$	250	94.4	3	2.6	0	100	0	94.4	2.8	2.8
$r_{M_1M_2\bullet X} = 0$	500	94.6	2.2	3.2	0	100	0	94.8	2	3.2

$\upsilon^{sp}_{YM_1X} = 0$	50	94.6	5.4	0	0	100	0	93.2	4.2	2.6
$\upsilon^{sp}_{YM_2X} = .3969$	100	95.8	3.6	0.6	0	100	0	95.2	2.2	2.6
$\upsilon^{tot}_{YM_1M_2X} = .3969$	250	94.2	3.4	2.4	0	100	0	93.4	3.4	3.2
$r_{M_1M_2 \bullet X} = .9$	500	93.4	3	3.6	0	100	0	93.4	3	3.6
$\upsilon^{sp}_{YM_1X} = .00053$	50	96.2	3.8	0	98.6	1.4	0	97.4	1.2	1.4
$\upsilon^{sp}_{YM_2X} = .00448$	100	95.8	3.8	0.4	98.8	1.2	0	94.6	2	3.4
$\upsilon^{tot}_{YM_1M_2X} = .00501$	250	96.4	2.2	1.4	98.6	1.2	0.2	95	1.6	3.4
$r_{M_1M_2 \bullet X} = -.0989$	500	94.8	2.6	2.6	96	3.4	0.6	94.4	1.4	4.2
$\upsilon^{sp}_{YM_1X} = .19319$	50	94.4	2.4	3.2	93.8	2.2	4	94.4	2.6	3
$\upsilon^{sp}_{YM_2X} = .21203$	100	92.4	3.4	4.2	93.4	2.4	4.2	92.2	3.2	4.6
$\upsilon^{tot}_{YM_1M_2X} = .40522$	250	95	2.4	2.6	94.6	3.4	2	94.6	2.6	2.8
$r_{M_1M_2 \bullet X} = .8011$	500	92.2	4.4	3.4	92.6	4.2	3.2	93.2	3.2	3.6
$\upsilon^{sp}_{YM_1X} = .00203$	50	96.6	3.4	0	98.8	1	0.2	98.6	1.4	0
$\upsilon^{sp}_{YM_2X} = .00203$	100	96.2	3.6	0.2	97.4	2	0.6	97.2	1	1.8
$\upsilon^{tot}_{YM_1M_2X} = .00405$	250	98.2	1.4	0.4	95	1.6	3.4	95.4	1.6	3
$r_{M_1M_2 \bullet X} = -.0989$	500	95	3.2	1.8	93.8	2.4	3.8	95	2.4	2.6
$\upsilon^{sp}_{YM_1X} = 0$	50	94.6	2	3.4	0	100	0	94.4	1.6	4
$\upsilon^{sp}_{YM_2X} = .05444$	100	94.2	2.6	3.2	0	100	0	94.2	2.4	3.4
$\upsilon^{tot}_{YM_1M_2X} = .05444$	250	94.8	3	2.2	0	100	0	94.6	3	2.4
$r_{M_1M_2 \bullet X} = 0$	500	94.4	3.8	1.8	0	100	0	94.4	3.8	1.8

$\upsilon^{sp}_{YM_1X} = .00005$	50	96.4	1.8	1.8	95.8	4.2	0	94.2	1.6	4.2
$\upsilon^{sp}_{YM_2X} = .05339$	100	94.8	2.8	2.4	95.2	4.8	0	95.2	2.2	2.6
$\upsilon^{tot}_{YM_1M_2X} = .05344$	250	93.6	3	3.4	96.4	3.6	0	93.2	3	3.8
$r_{M_1M_2\bullet X} = -.1816$	500	94	2.6	3.4	97	3	0	94.2	2.4	3.4
$\upsilon^{sp}_{YM_1X} = .00298$	50	95.2	2	2.8	98.8	0.8	0.4	94.4	1.2	4.4
$\upsilon^{sp}_{YM_2X} = .06325$	100	94.4	1.8	3.8	97.6	0.6	1.8	94.6	1.4	4
$\upsilon^{tot}_{YM_1M_2X} = .06624$	250	95.4	2.4	2.2	93.2	1.8	5	95.8	2.2	2
$r_{M_1M_2\bullet X} = .1184$	500	94.6	2.2	3.2	94.8	2.6	2.6	94.4	2.4	3.2
$\upsilon^{sp}_{YM_1X} = 1.26563$	50	95	2.2	2.8	95	2.2	2.8	96.4	2.2	1.4
$\upsilon^{sp}_{YM_2X} = .01174$	100	95.4	2.6	2	95.4	2.6	2	95.2	1.2	3.6
$\upsilon^{tot}_{YM_1M_2X} = 1.27736$	250	90.8	4.4	4.8	91.2	4.2	4.6	95.4	1.4	3.2
$r_{M_1M_2\bullet X} = -.8921$	500	93.2	3.2	3.6	93.8	3.2	3	93.6	3.4	3
$\upsilon^{sp}_{YM_1X} = 0$	50	5.2	94.8	0	0	100	0	96.6	3.4	0
$\upsilon^{sp}_{YM_2X} = 4.9\times10^{-6}$	100	9.6	90.4	0	0	100	0	98.4	1.6	0
$\upsilon^{tot}_{YM_1M_2X} = 4.9\times10^{-6}$	250	24.4	75.6	0	0	100	0	98.4	1.6	0
$r_{M_1M_2\bullet X} = .3$	500	43.4	56.6	0	0	100	0	99.4	0.6	0
$\upsilon^{sp}_{YM_1X} = .09379$	50	97	1.8	1.2	95.4	1	3.6	98.8	1.2	0
$\upsilon^{sp}_{YM_2X} = .00683$	100	95.2	1.6	3.2	94.4	1.2	4.4	98.6	1.4	0
$\upsilon^{tot}_{YM_1M_2X} = .10062$	250	94.8	1.8	3.4	95	1.6	3.4	97.4	2.6	0
$r_{M_1M_2\bullet X} = .8684$	500	94	2	4	94.2	1.8	4	95.4	2	2.6

$\upsilon^{sp}_{YM_1X} = .04785$	50	97	1.4	1.6	94.8	1.2	4	99	1	0
$\upsilon^{sp}_{YM_2X} = .00348$	100	95.6	2	2.4	95	2	3	98.2	1.6	0.2
$\upsilon^{tot}_{YM_1M_2X} = .05134$	250	94.2	1.6	4.2	94.4	1.2	4.4	97.4	1.8	0.8
$r_{M_1M_2 \bullet X} = .8684$	500	92.8	2.8	4.4	93	2.4	4.6	95.2	2.2	2.6
$\upsilon^{sp}_{YM_1X} = .5184$	50	93.8	3.6	2.6	93.8	3.6	2.6	96.4	3.6	0
$\upsilon^{sp}_{YM_2X} = 4.9 \times 10^{-6}$	100	93.4	3.4	3.2	93.4	3.4	3.2	97.4	2.6	0
$\upsilon^{tot}_{YM_1M_2X} = .5184$	250	93.8	2.8	3.4	93.8	2.8	3.4	98.6	1.4	0
$r_{M_1M_2 \bullet X} = .8684$	500	95.8	3	1.2	95.8	3	1.2	99.4	0.6	0
$\upsilon^{sp}_{YM_1X} = .0059$	50	97.6	2.4	0	97.4	1	1.6	97.2	0.6	2.2
$\upsilon^{sp}_{YM_2X} = .0059$	100	96.4	2.2	1.4	95.4	0.8	3.8	94.6	1	4.4
$\upsilon^{tot}_{YM_1M_2X} = .01181$	250	96.2	2	1.8	94.4	1.8	3.8	94.6	1.2	4.2
$r_{M_1M_2 \bullet X} = -.0989$	500	94.6	3	2.4	93.2	2.2	4.6	94.4	1.6	4
$\upsilon^{sp}_{YM_1X} = .57154$	50	94.4	3.6	2	94.2	3.6	2.2	95.2	1.4	3.4
$\upsilon^{sp}_{YM_2X} = .02074$	100	94.2	3.8	2	93.8	3.8	2.4	94.8	1.4	3.8
$\upsilon^{tot}_{YM_1M_2X} = .59227$	250	95.4	2.2	2.4	95	2.6	2.4	96.8	1.4	1.8
$r_{M_1M_2 \bullet X} = -.6493$	500	93	4.6	2.4	92.6	4.6	2.8	96	1.8	2.2
$\upsilon^{sp}_{YM_1X} = .7465$	50	93.4	3.4	3.2	93.2	3.4	3.4	98.8	1.2	0
$\upsilon^{sp}_{YM_2X} = .0013$	100	95	2.2	2.8	95	2.2	2.8	98.6	1.2	0.2
$\upsilon^{tot}_{YM_1M_2X} = .74779$	250	94.4	2.6	3	94.4	2.6	3	97.8	1.4	0.8
$r_{M_1M_2 \bullet X} = -.6493$	500	94	3.6	2.4	94	3.6	2.4	96.6	1.4	2

$\upsilon^{sp}_{YM_1X} = .06891$	50	94.8	1.4	3.8	94.8	1	4.2	94.8	1.6	3.6
$\upsilon^{sp}_{YM_2X} = .45002$	100	93.8	3.4	2.8	94.4	3	2.6	93.8	2.8	3.4
$\upsilon^{tot}_{YM_1M_2X} = .51892$	250	95.2	1.2	3.6	95.2	1.4	3.4	95	1.2	3.8
$r_{M_1M_2 \bullet X} = .7184$	500	95.2	2.8	2	95	2.8	2.2	95.4	2.4	2.2
$\upsilon^{sp}_{YM_1X} = .00465$	50	97.2	2	0.8	98.4	0.6	1	95.4	1.8	2.8
$\upsilon^{sp}_{YM_2X} = .03587$	100	96.6	2	1.4	93.2	2	4.8	95.2	2.2	2.6
$\upsilon^{tot}_{YM_1M_2X} = .04052$	250	92.4	4.8	2.8	96.2	1	2.8	92.4	4.4	3.2
$r_{M_1M_2 \bullet X} = -.1816$	500	95.6	1.4	3	92.2	3.2	4.6	94.6	2	3.4
$\upsilon^{sp}_{YM_1X} = .00116$	50	97.4	2.2	0.4	99.4	0.4	0.2	96.6	1.6	1.8
$\upsilon^{sp}_{YM_2X} = .02412$	100	94.6	3	2.4	96.6	2.6	0.8	94.2	2.2	3.6
$\upsilon^{tot}_{YM_1M_2X} = .02528$	250	94.2	3.8	2	96.2	2	1.8	94.4	3.2	2.4
$r_{M_1M_2 \bullet X} = .1184$	500	95	1.6	3.4	95.2	2.2	2.6	94.8	1.6	3.6
$\upsilon^{sp}_{YM_1X} = .03516$	50	96.6	1.6	1.8	97	1	2	95.2	1.8	3
$\upsilon^{sp}_{YM_2X} = .2296$	100	93.4	3	3.6	93.6	2	4.4	93.8	2.6	3.6
$\upsilon^{tot}_{YM_1M_2X} = .26476$	250	95.6	1.6	2.8	96.2	1.4	2.4	96.2	1.2	2.6
$r_{M_1M_2 \bullet X} = .7184$	500	93.8	2.2	4	93	2.4	4.6	94	2.2	3.8
$\upsilon^{sp}_{YM_1X} = .31641$	50	94.4	3.8	1.8	94.4	3	2.6	93	2.4	4.6
$\upsilon^{sp}_{YM_2X} = .08752$	100	94.6	2.8	2.6	94	3.4	2.6	94.4	1.8	3.8
$\upsilon^{tot}_{YM_1M_2X} = .40392$	250	93.8	2.8	3.4	93	3	4	92	3	5
$r_{M_1M_2 \bullet X} = -.8921$	500	95.4	2.4	2.2	95.4	1.8	2.8	96.4	1.6	2

$\upsilon^{sp}_{YM_1X} = .31641$	50	95.2	3.8	1	94.4	3.8	1.8	97	1.2	1.8
$\upsilon^{sp}_{YM_2X} = .01085$	100	93.6	2.8	3.6	93.6	2.6	3.8	93.8	2.6	3.6
$\upsilon^{tot}_{YM_1M_2X} = .32726$	250	95.8	2.2	2	95.8	1.8	2.4	94.8	1.8	3.4
$r_{M_1M_2 \bullet X} = -.8921$	500	93.8	4	2.2	93.6	4.2	2.2	95	2.4	2.6
$\upsilon^{sp}_{YM_1X} = .00088$	50	96.2	3.6	0.2	98.4	1.6	0	97	1.8	1.2
$\upsilon^{sp}_{YM_2X} = .00479$	100	95.8	4	0.2	98.2	1.8	0	94	2	4
$\upsilon^{tot}_{YM_1M_2X} = .00567$	250	96	2.2	1.8	98.6	1.2	0.2	94.6	1.2	4.2
$r_{M_1M_2 \bullet X} = -.0089$	500	95.2	3.6	1.2	95.8	2.6	1.6	93.6	2.2	4.2
$\upsilon^{sp}_{YM_1X} = .00245$	50	94.2	5.8	0	97.8	2	0.2	96.2	3	0.8
$\upsilon^{sp}_{YM_2X} = .00245$	100	95.8	3.8	0.4	95.4	2	2.6	96.8	2.2	1
$\upsilon^{tot}_{YM_1M_2X} = .0049$	250	95.4	3	1.6	93.4	2.8	3.8	95.2	1.6	3.2
$r_{M_1M_2 \bullet X} = -.0089$	500	95.8	2.6	1.6	94.6	2.4	3	94.4	1.4	4.2
$\upsilon^{sp}_{YM_1X} = .00479$	50	97	2.6	0.4	97	1	2	97	1.2	1.8
$\upsilon^{sp}_{YM_2X} = .00479$	100	95.6	3.4	1	92.6	2.4	5	94.4	2.4	3.2
$\upsilon^{tot}_{YM_1M_2X} = .00959$	250	94.6	3.4	2	92.4	2.6	5	94.2	1.8	4
$r_{M_1M_2 \bullet X} = -.0089$	500	95.2	2.8	2	95.8	1.4	2.8	95	1.8	3.2
$\upsilon^{sp}_{YM_1X} = .00245$	50	96.8	3.2	0	97.8	1.6	0.6	98.4	1.2	0.4
$\upsilon^{sp}_{YM_2X} = .00245$	100	96	4	0	96.6	1.4	2	94.8	2	3.2
$\upsilon^{tot}_{YM_1M_2X} = .0049$	250	96	3.4	0.6	94.2	1.6	4.2	93.4	2	4.6
$r_{M_1M_2 \bullet X} = -.0089$	500	96.4	2.4	1.2	94.4	2.4	3.2	95.4	1	3.6

$\upsilon^{sp}_{YM_1X} = .07131$	50	94.6	4	1.4	94	4	2	95.6	1.8	2.6
$\upsilon^{sp}_{YM_2X} = .00479$	100	93.8	2.8	3.4	94	2.8	3.2	94.6	1.4	4
$\upsilon^{tot}_{YM_1M_2X} = .0761$	250	95	2.4	2.6	94.8	2.4	2.8	93	3.2	3.8
$r_{M_1M_2 \bullet X} = -.0089$	500	94.6	2.4	3	94.6	2.4	3	94.8	1.6	3.6
$\upsilon^{sp}_{YM_1X} = .06113$	50	95.6	3.4	1	94.8	3.4	1.8	96	0.8	3.2
$\upsilon^{sp}_{YM_2X} = .00245$	100	94.6	3	2.4	94.6	2.6	2.8	93	2.6	4.4
$\upsilon^{tot}_{YM_1M_2X} = .06358$	250	94.2	3	2.8	94.4	2.8	2.8	93.6	2.4	4
$r_{M_1M_2 \bullet X} = -.0089$	500	95.2	1.8	3	95.2	1.8	3	96	1.8	2.2

Table 5. *Mean, bias, relative bias, and mean square error of* $\hat{\upsilon}^{unc}_{YM_1X}$ *and* $\hat{\upsilon}^{unc}_{YM_2X}$

		$\hat{\upsilon}^{unc}_{YM_1X}$				$\hat{\upsilon}^{unc}_{YM_2X}$			
	n	Mean Est.	Bias	Rel. Bias	MSE	Mean Est.	Bias	Rel. Bias	MSE
$\upsilon^{unc}_{YM_1X}=0$ $\upsilon^{unc}_{YM_2X}=.00005$ $r_{M_1M_2\bullet X}=0$	50	0.00049	0.00049	NA	0.00000	0.00071	0.00066	13.11945	0.00000
	100	0.00009	0.00009	NA	0.00000	0.00034	0.00029	5.89384	0.00000
	250	0.00002	0.00002	NA	0.00000	0.00012	0.00007	1.46445	0.00000
	500	0.00000	0.00000	NA	0.00000	0.00008	0.00003	0.61593	0.00000
$\upsilon^{unc}_{YM_1X}=0$ $\upsilon^{unc}_{YM_2X}=.00005$ $r_{M_1M_2\bullet X}=.3$	50	0.00046	0.00046	NA	0.00000	0.00079	0.00074	14.71449	0.00000
	100	0.00009	0.00009	NA	0.00000	0.00029	0.00024	4.88295	0.00000
	250	0.00001	0.00001	NA	0.00000	0.00012	0.00007	1.39538	0.00000
	500	0.00000	0.00000	NA	0.00000	0.00009	0.00004	0.74116	0.00000
$\upsilon^{unc}_{YM_1X}=0$ $\upsilon^{unc}_{YM_2X}=.00005$ $r_{M_1M_2\bullet X}=.9$	50	0.00036	0.00036	NA	0.00000	0.00084	0.00079	15.87717	0.00001
	100	0.00010	0.00010	NA	0.00000	0.00028	0.00023	4.56369	0.00000
	250	0.00001	0.00001	NA	0.00000	0.00012	0.00007	1.34133	0.00000
	500	0.00000	0.00000	NA	0.00000	0.00008	0.00003	0.67602	0.00000
$\upsilon^{unc}_{YM_1X}=0$ $\upsilon^{unc}_{YM_2X}=.00003$ $r_{M_1M_2\bullet X}=.3$	50	0.00035	0.00035	NA	0.00000	0.00059	0.00056	21.96918	0.00000
	100	0.00008	0.00008	NA	0.00000	0.00020	0.00017	6.82512	0.00000
	250	0.00001	0.00001	NA	0.00000	0.00008	0.00005	2.11489	0.00000
	500	0.00000	0.00000	NA	0.00000	0.00004	0.00002	0.75773	0.00000
$\upsilon^{unc}_{YM_1X}=.00088$ $\upsilon^{unc}_{YM_2X}=.00005$ $r_{M_1M_2\bullet X}=-.0316$	50	0.00339	0.00251	2.85557	0.00005	0.00091	0.00086	17.20114	0.00001
	100	0.00219	0.00131	1.49091	0.00001	0.00029	0.00024	4.73949	0.00000
	250	0.00143	0.00055	0.62436	0.00000	0.00013	0.00008	1.53858	0.00000
	500	0.00111	0.00023	0.26085	0.00000	0.00008	0.00003	0.59550	0.00000
$\upsilon^{unc}_{YM_1X}=.00088$ $\upsilon^{unc}_{YM_2X}=.00005$ $r_{M_1M_2\bullet X}=.8684$	50	0.00397	0.00309	3.51436	0.00007	0.00081	0.00076	15.20565	0.00000
	100	0.00204	0.00116	1.32029	0.00001	0.00026	0.00021	4.13535	0.00000
	250	0.00129	0.00041	0.46122	0.00000	0.00012	0.00007	1.41732	0.00000
	500	0.00109	0.00021	0.24132	0.00000	0.00009	0.00004	0.73422	0.00000
	50	1.72213	0.08643	0.05284	0.37681	0.00080	0.00075	14.92462	0.00000
	100	1.67868	0.04298	0.02627	0.18677	0.00032	0.00027	5.36392	0.00000
	250	1.61725	-0.01846	-0.01128	0.07365	0.00011	0.00006	1.28428	0.00000

$\upsilon^{unc}_{YM_1X} = 1.63571$ $\upsilon^{unc}_{YM_2X} = .00005$ $r_{M_1M_2 \bullet X} = -.2075$	500	1.64555	0.00984	0.00602	0.03432	0.00008	0.00003	0.52046	0.00000
$\upsilon^{unc}_{YM_1X} = 1.63571$	50	1.67438	0.03868	0.02364	0.41555	0.00081	0.00076	15.26676	0.00001
$\upsilon^{unc}_{YM_2X} = .00005$	100	1.66801	0.03231	0.01975	0.18826	0.00027	0.00022	4.30325	0.00000
$r_{M_1M_2 \bullet X} = .0925$	250	1.63776	0.00205	0.00125	0.07417	0.00012	0.00007	1.43135	0.00000
	500	1.63789	0.00219	0.00134	0.03160	0.00008	0.00003	0.65382	0.00000
$\upsilon^{unc}_{YM_1X} = 0$	50	0.00044	0.00044	NA	0.00000	0.00874	0.00395	0.82313	0.00016
$\upsilon^{unc}_{YM_2X} = .00479$	100	0.00009	0.00009	NA	0.00000	0.00678	0.00198	0.41390	0.00005
$r_{M_1M_2 \bullet X} = 0$	250	0.00002	0.00002	NA	0.00000	0.00550	0.00071	0.14711	0.00002
	500	0.00000	0.00000	NA	0.00000	0.00512	0.00033	0.06816	0.00001
$\upsilon^{unc}_{YM_1X} = 0$	50	0.00049	0.00049	NA	0.00000	0.00785	0.00306	0.63823	0.00016
$\upsilon^{unc}_{YM_2X} = .00479$	100	0.00010	0.00010	NA	0.00000	0.00668	0.00189	0.39379	0.00004
$r_{M_1M_2 \bullet X} = .9$	250	0.00001	0.00001	NA	0.00000	0.00518	0.00038	0.08018	0.00001
	500	0.00000	0.00000	NA	0.00000	0.00510	0.00031	0.06426	0.00001
$\upsilon^{unc}_{YM_1X} = .00088$	50	0.00374	0.00286	3.25015	0.00006	0.00786	0.00306	0.63938	0.00015
$\upsilon^{unc}_{YM_2X} = .00479$	100	0.00194	0.00106	1.20765	0.00001	0.00641	0.00162	0.33767	0.00005
$r_{M_1M_2 \bullet X} = -.0989$	250	0.00116	0.00028	0.31589	0.00000	0.00522	0.00043	0.08911	0.00001
	500	0.00104	0.00016	0.18627	0.00000	0.00485	0.00006	0.01199	0.00001
$\upsilon^{unc}_{YM_1X} = .00088$	50	0.00341	0.00253	2.87388	0.00005	0.00844	0.00365	0.76140	0.00018
$\upsilon^{unc}_{YM_2X} = .00479$	100	0.00191	0.00103	1.16740	0.00001	0.00596	0.00117	0.24332	0.00004
$r_{M_1M_2 \bullet X} = .801$	250	0.00141	0.00053	0.60478	0.00000	0.00558	0.00079	0.16380	0.00001
	500	0.00106	0.00018	0.20769	0.00000	0.00535	0.00056	0.11712	0.00001
$\upsilon^{unc}_{YM_1X} = .00245$	50	0.00444	0.00199	0.81524	0.00007	0.00504	0.00259	1.06068	0.00007
$\upsilon^{unc}_{YM_2X} = .00245$	100	0.00381	0.00136	0.55707	0.00002	0.00350	0.00105	0.43014	0.00002
$r_{M_1M_2 \bullet X} = -.0989$	250	0.00289	0.00044	0.18050	0.00001	0.00281	0.00037	0.14938	0.00000
	500	0.00263	0.00019	0.07649	0.00000	0.00268	0.00024	0.09661	0.00000
	50	0.00050	0.00050	NA	0.00000	0.06539	0.01095	0.20109	0.00285
	100	0.00010	0.00010	NA	0.00000	0.06079	0.00634	0.11650	0.00124
	250	0.00001	0.00001	NA	0.00000	0.05741	0.00297	0.05453	0.00041

$\upsilon^{unc}_{YM_1X}=0$ $\upsilon^{unc}_{YM_2X}=.05444$ $r_{M_1M_2\bullet X}=0$	500	0.00000	0.00000	NA	0.00000	0.05609	0.00165	0.03027	0.00019
$\upsilon^{unc}_{YM_1X}=.00088$ $\upsilon^{unc}_{YM_2X}=.05444$ $r_{M_1M_2\bullet X}=-.1816$	50	0.00356	0.00268	3.04718	0.00004	0.06585	0.01140	0.20947	0.00291
	100	0.00203	0.00115	1.30978	0.00001	0.05792	0.00348	0.06390	0.00107
	250	0.00129	0.00041	0.46007	0.00000	0.05703	0.00258	0.04743	0.00044
	500	0.00110	0.00022	0.24581	0.00000	0.05517	0.00072	0.01328	0.00020
$\upsilon^{unc}_{YM_1X}=.00088$ $\upsilon^{unc}_{YM_2X}=.05444$ $r_{M_1M_2\bullet X}=.1184$	50	0.00325	0.00236	2.68628	0.00005	0.06346	0.00902	0.16560	0.00254
	100	0.00185	0.00097	1.10618	0.00001	0.05752	0.00308	0.05652	0.00101
	250	0.00120	0.00032	0.36443	0.00000	0.05654	0.00210	0.03852	0.00037
	500	0.00113	0.00025	0.28928	0.00000	0.05554	0.00110	0.02016	0.00018
$\upsilon^{unc}_{YM_1X}=1.63571$ $\upsilon^{unc}_{YM_2X}=.05444$ $r_{M_1M_2\bullet X}=.3$	50	1.68363	0.04793	0.02930	0.36434	0.06570	0.01125	0.20670	0.00273
	100	1.62788	-0.00782	-0.00478	0.16228	0.05834	0.00389	0.07150	0.00104
	250	1.66949	0.03378	0.02065	0.08242	0.05627	0.00183	0.03356	0.00037
	500	1.64077	0.00507	0.00310	0.03928	0.05566	0.00122	0.02232	0.00020
$\upsilon^{unc}_{YM_1X}=0$ $\upsilon^{unc}_{YM_2X}=.00005$ $r_{M_1M_2\bullet X}=-.8921$	50	0.00220	0.00220	NA	0.00002	0.00066	0.00061	12.29530	0.00000
	100	0.00095	0.00095	NA	0.00000	0.00029	0.00024	4.85074	0.00000
	250	0.00035	0.00035	NA	0.00000	0.00013	0.00008	1.52106	0.00000
	500	0.00020	0.00020	NA	0.00000	0.00009	0.00004	0.72596	0.00000
$\upsilon^{unc}_{YM_1X}=.00479$ $\upsilon^{unc}_{YM_2X}=.00005$ $r_{M_1M_2\bullet X}=.8684$	50	0.00804	0.00324	0.67681	0.00014	0.00077	0.00072	14.38349	0.00000
	100	0.00583	0.00104	0.21605	0.00004	0.00025	0.00020	4.03429	0.00000
	250	0.00536	0.00056	0.11743	0.00001	0.00013	0.00008	1.55388	0.00000
	500	0.00499	0.00019	0.04013	0.00001	0.00008	0.00003	0.53365	0.00000
$\upsilon^{unc}_{YM_1X}=.00245$ $\upsilon^{unc}_{YM_2X}=.00003$ $r_{M_1M_2\bullet X}=.8684$	50	0.00473	0.00228	0.93433	0.00007	0.00055	0.00052	20.39922	0.00000
	100	0.00352	0.00107	0.43837	0.00002	0.00021	0.00018	7.21530	0.00000
	250	0.00273	0.00029	0.11770	0.00001	0.00008	0.00005	1.96248	0.00000
	500	0.00261	0.00017	0.06896	0.00000	0.00004	0.00002	0.73152	0.00000
	50	0.58894	0.08410	0.16658	0.15123	0.00062	0.00059	23.17246	0.00000
	100	0.55087	0.04602	0.09115	0.06610	0.00019	0.00016	6.45737	0.00000
	250	0.50645	0.00160	0.00317	0.02451	0.00008	0.00005	2.09960	0.00000

$\upsilon^{unc}_{YM_1X} = .50485$ $\upsilon^{unc}_{YM_2X} = .00003$ $r_{M_1M_2\bullet X} = -.2075$	500	0.51244	0.00759	0.01504	0.01191	0.00005	0.00002	0.88739	0.00000
$\upsilon^{unc}_{YM_1X} = .00479$ $\upsilon^{unc}_{YM_2X} = .00479$ $r_{M_1M_2\bullet X} = -.0989$	50	0.00842	0.00363	0.75675	0.00017	0.00739	0.00260	0.54267	0.00011
	100	0.00608	0.00129	0.26930	0.00003	0.00604	0.00125	0.26080	0.00004
	250	0.00525	0.00046	0.09534	0.00001	0.00523	0.00044	0.09103	0.00001
	500	0.00501	0.00022	0.04627	0.00001	0.00508	0.00028	0.05890	0.00001
$\upsilon^{unc}_{YM_1X} = .02019$ $\upsilon^{unc}_{YM_2X} = .00479$ $r_{M_1M_2\bullet X} = -.6493$	50	0.09733	0.07714	3.81981	0.02665	0.00806	0.00327	0.68178	0.00013
	100	0.06303	0.04284	2.12128	0.00962	0.00586	0.00107	0.22222	0.00004
	250	0.03612	0.01593	0.78881	0.00195	0.00518	0.00039	0.08153	0.00001
	500	0.03061	0.01041	0.51567	0.00102	0.00490	0.00011	0.02321	0.00001
$\upsilon^{unc}_{YM_1X} = .50485$ $\upsilon^{unc}_{YM_2X} = .00245$ $r_{M_1M_2\bullet X} = -.6493$	50	0.57557	0.07072	0.14009	0.14659	0.00443	0.00199	0.81299	0.00006
	100	0.53660	0.03175	0.06289	0.06027	0.00352	0.00107	0.43901	0.00002
	250	0.52113	0.01628	0.03225	0.02388	0.00298	0.00053	0.21857	0.00001
	500	0.50188	-0.00297	-0.00587	0.01110	0.00268	0.00023	0.09472	0.00000
$\upsilon^{unc}_{YM_1X} = .00479$ $\upsilon^{unc}_{YM_2X} = .05444$ $r_{M_1M_2\bullet X} = .7184$	50	0.00820	0.00341	0.71164	0.00020	0.06429	0.00984	0.18078	0.00311
	100	0.00635	0.00156	0.32473	0.00005	0.05961	0.00517	0.09496	0.00115
	250	0.00555	0.00076	0.15774	0.00001	0.05674	0.00229	0.04211	0.00038
	500	0.00485	0.00006	0.01271	0.00000	0.05435	-0.00010	-0.00181	0.00018
$\upsilon^{unc}_{YM_1X} = .00245$ $\upsilon^{unc}_{YM_2X} = .02778$ $r_{M_1M_2\bullet X} = -.1816$	50	0.00440	0.00196	0.80051	0.00005	0.03711	0.00933	0.33581	0.00120
	100	0.00357	0.00112	0.45959	0.00002	0.03124	0.00346	0.12466	0.00045
	250	0.00269	0.00024	0.09961	0.00000	0.02971	0.00193	0.06947	0.00018
	500	0.00268	0.00024	0.09795	0.00000	0.02843	0.00066	0.02359	0.00007
$\upsilon^{unc}_{YM_1X} = .00245$ $\upsilon^{unc}_{YM_2X} = .02778$ $r_{M_1M_2\bullet X} = .1184$	50	0.00416	0.00171	0.69944	0.00004	0.03474	0.00696	0.25060	0.00113
	100	0.00354	0.00110	0.44837	0.00002	0.03173	0.00395	0.14211	0.00046
	250	0.00298	0.00053	0.21827	0.00001	0.02916	0.00139	0.04987	0.00016
	500	0.00266	0.00022	0.08860	0.00000	0.02794	0.00016	0.00573	0.00007
	50	0.00505	0.00260	1.06451	0.00007	0.03565	0.00787	0.28330	0.00120
	100	0.00357	0.00113	0.46030	0.00002	0.03154	0.00376	0.13554	0.00046
	250	0.00292	0.00047	0.19266	0.00001	0.02901	0.00123	0.04438	0.00014

$\upsilon^{unc}_{YM_1X} = .00245$ $\upsilon^{unc}_{YM_2X} = .02778$ $r_{M_1M_2 \bullet X} = .7184$	500	0.00256	0.00012	0.04734	0.00000	0.02817	0.00039	0.01420	0.00007
$\upsilon^{unc}_{YM_1X} = .02019$	50	0.11132	0.09113	4.51273	0.03267	0.06523	0.01078	0.19803	0.00299
$\upsilon^{unc}_{YM_2X} = .05444$	100	0.06843	0.04823	2.38852	0.01191	0.05789	0.00345	0.06334	0.00115
$r_{M_1M_2 \bullet X} = -.8921$	250	0.03623	0.01604	0.79420	0.00206	0.05530	0.00085	0.01566	0.00039
	500	0.02568	0.00548	0.27161	0.00062	0.05509	0.00065	0.01188	0.00017
$\upsilon^{unc}_{YM_1X} = .50485$	50	0.55697	0.05212	0.10324	0.12973	0.03328	0.00550	0.19798	0.00113
$\upsilon^{unc}_{YM_2X} = .05444$	100	0.53703	0.03218	0.06374	0.06716	0.02935	0.00158	0.05673	0.00045
$r_{M_1M_2 \bullet X} = -.8921$	250	0.51289	0.00804	0.01593	0.02410	0.02886	0.00109	0.03909	0.00015
	500	0.51909	0.01424	0.02821	0.01276	0.02879	0.00101	0.03637	0.00008
$\upsilon^{unc}_{YM_1X} = .00088$	50	0.00381	0.00293	3.33165	0.00007	0.00830	0.00350	0.73124	0.00015
$\upsilon^{unc}_{YM_2X} = .00479$	100	0.00214	0.00126	1.42933	0.00002	0.00649	0.00170	0.35485	0.00006
$r_{M_1M_2 \bullet X} = -.0089$	250	0.00121	0.00033	0.36937	0.00000	0.00519	0.00039	0.08218	0.00001
	500	0.00115	0.00027	0.30584	0.00000	0.00497	0.00017	0.03625	0.00001
$\upsilon^{unc}_{YM_1X} = .00245$	50	0.00559	0.00314	1.28516	0.00011	0.00567	0.00323	1.31941	0.00010
$\upsilon^{unc}_{YM_2X} = .00245$	100	0.00353	0.00109	0.44375	0.00002	0.00359	0.00115	0.46887	0.00002
$r_{M_1M_2 \bullet X} = -.0089$	250	0.00280	0.00035	0.14486	0.00001	0.00273	0.00029	0.11770	0.00000
	500	0.00266	0.00021	0.08694	0.00000	0.00260	0.00016	0.06430	0.00000
$\upsilon^{unc}_{YM_1X} = .00479$	50	0.00725	0.00245	0.51192	0.00013	0.00781	0.00301	0.62847	0.00013
$\upsilon^{unc}_{YM_2X} = .00479$	100	0.00623	0.00144	0.30053	0.00005	0.00640	0.00161	0.33570	0.00006
$r_{M_1M_2 \bullet X} = -.0089$	250	0.00518	0.00038	0.07987	0.00001	0.00538	0.00059	0.12239	0.00001
	500	0.00501	0.00021	0.04453	0.00001	0.00497	0.00018	0.03671	0.00001
$\upsilon^{unc}_{YM_1X} = .00245$	50	0.00498	0.00254	1.03806	0.00009	0.00462	0.00217	0.88874	0.00006
$\upsilon^{unc}_{YM_2X} = .00245$	100	0.00350	0.00106	0.43283	0.00002	0.00363	0.00118	0.48286	0.00002
$r_{M_1M_2 \bullet X} = -.0089$	250	0.00279	0.00035	0.14184	0.00000	0.00291	0.00046	0.18811	0.00001
	500	0.00266	0.00022	0.08866	0.00000	0.00265	0.00021	0.08442	0.00000
	50	0.08909	0.01779	0.24945	0.00647	0.00912	0.00432	0.90230	0.00020
	100	0.08135	0.01004	0.14079	0.00260	0.00655	0.00176	0.36757	0.00005
	250	0.07358	0.00228	0.03191	0.00083	0.00513	0.00034	0.06994	0.00001

$\upsilon^{unc}_{YM_1X} = .07131$ $\upsilon^{unc}_{YM_2X} = .00479$ $r_{M_1M_2 \bullet X} = -.0089$	500	0.07194	0.00064	0.00891	0.00041	0.00496	0.00017	0.03533	0.00001
$\upsilon^{unc}_{YM_1X} = .06133$ $\upsilon^{unc}_{YM_2X} = .00245$ $r_{M_1M_2 \bullet X} = -.0089$	50	0.07350	0.01237	0.20228	0.00380	0.00436	0.00192	0.78419	0.00005
	100	0.06756	0.00642	0.10507	0.00160	0.00358	0.00114	0.46592	0.00002
	250	0.06507	0.00393	0.06437	0.00067	0.00292	0.00048	0.19493	0.00001
	500	0.06237	0.00124	0.02023	0.00031	0.00271	0.00027	0.10955	0.00000

Table 6. *Confidence limits, confidence interval width for* $\hat{\upsilon}^{unc}_{YM_1X}$ *and* $\hat{\upsilon}^{unc}_{YM_2X}$

		$\hat{\upsilon}^{unc}_{YM_1X}$			$\hat{\upsilon}^{unc}_{YM_2X}$		
	n	Mean LCL	Mean UCL	Mean CI Width	Mean LCL	Mean UCL	Mean CI Width
$\upsilon^{unc}_{YM_1X} = 0$	50	0.00000	0.01159	0.01159	0.00000	0.01402	0.01402
	100	0.00000	0.00238	0.00238	0.00000	0.00433	0.00433
$\upsilon^{unc}_{YM_2X} = .00005$	250	0.00000	0.00041	0.00041	0.00000	0.00110	0.00110
$r_{M_1M_2\bullet X} = 0$	500	0.00000	0.00010	0.00010	0.00000	0.00051	0.00051
$\upsilon^{unc}_{YM_1X} = 0$	50	0.00000	0.01163	0.01163	0.00000	0.01464	0.01464
	100	0.00000	0.00252	0.00252	0.00000	0.00410	0.00410
$\upsilon^{unc}_{YM_2X} = .00005$	250	0.00000	0.00039	0.00039	0.00000	0.00113	0.00113
$r_{M_1M_2\bullet X} = .3$	500	0.00000	0.00009	0.00009	0.00000	0.00054	0.00054
$\upsilon^{unc}_{YM_1X} = 0$	50	0.00000	0.01097	0.01097	0.00000	0.01498	0.01498
	100	0.00000	0.00249	0.00249	0.00000	0.00409	0.00409
$\upsilon^{unc}_{YM_2X} = .00005$	250	0.00000	0.00037	0.00037	0.00000	0.00109	0.00109
$r_{M_1M_2\bullet X} = .9$	500	0.00000	0.00010	0.00010	0.00000	0.00052	0.00052
$\upsilon^{unc}_{YM_1X} = 0$	50	0.00000	0.00890	0.00890	0.00000	0.01169	0.01169
	100	0.00000	0.00204	0.00204	0.00000	0.00314	0.00314
$\upsilon^{unc}_{YM_2X} = .00003$	250	0.00000	0.00030	0.00030	0.00000	0.00079	0.00079
$r_{M_1M_2\bullet X} = .3$	500	0.00000	0.00008	0.00008	0.00000	0.00033	0.00033
$\upsilon^{unc}_{YM_1X} = .00088$	50	0.00005	0.03417	0.03412	0.00000	0.01529	0.01528
	100	0.00007	0.01491	0.01484	0.00000	0.00419	0.00419
$\upsilon^{unc}_{YM_2X} = .00005$	250	0.00011	0.00649	0.00639	0.00000	0.00114	0.00114
$r_{M_1M_2\bullet X} = -.0316$	500	0.00014	0.00388	0.00374	0.00000	0.00051	0.00051
$\upsilon^{unc}_{YM_1X} = .00088$	50	0.00007	0.03752	0.03745	0.00000	0.01426	0.01426
	100	0.00007	0.01427	0.01421	0.00000	0.00389	0.00389
$\upsilon^{unc}_{YM_2X} = .00005$	250	0.00008	0.00606	0.00598	0.00000	0.00109	0.00109
$r_{M_1M_2\bullet X} = .8684$	500	0.00014	0.00380	0.00366	0.00000	0.00052	0.00052
	50	0.74776	3.20794	2.46018	0.00000	0.01406	0.01405
	100	0.97733	2.62925	1.65193	0.00000	0.00425	0.00424
	250	1.15307	2.17626	1.02319	0.00000	0.00106	0.00105

$\upsilon^{unc}_{YM_1X} = 1.63571$ $\upsilon^{unc}_{YM_2X} = .00005$ $r_{M_1M_2\bullet X} = -.2075$	500	1.30539	2.03218	0.72679	0.00000	0.00049	0.00049
$\upsilon^{unc}_{YM_1X} = 1.63571$ $\upsilon^{unc}_{YM_2X} = .00005$ $r_{M_1M_2\bullet X} = .0925$	50	0.73240	3.12918	2.39677	0.00000	0.01519	0.01518
	100	0.95892	2.61786	1.65894	0.00000	0.00403	0.00403
	250	1.17283	2.19889	1.02606	0.00000	0.00110	0.00110
	500	1.29953	2.02435	0.72482	0.00000	0.00052	0.00052
$\upsilon^{unc}_{YM_1X} = 0$ $\upsilon^{unc}_{YM_2X} = .00479$ $r_{M_1M_2\bullet X} = 0$	50	0.00000	0.01087	0.01087	0.00027	0.05614	0.05587
	100	0.00000	0.00251	0.00251	0.00051	0.02938	0.02887
	250	0.00000	0.00040	0.00040	0.00111	0.01578	0.01467
	500	0.00000	0.00009	0.00009	0.00176	0.01131	0.00956
$\upsilon^{unc}_{YM_1X} = 0$ $\upsilon^{unc}_{YM_2X} = .00479$ $r_{M_1M_2\bullet X} = .9$	50	0.00000	0.01110	0.01110	0.00030	0.05139	0.05109
	100	0.00000	0.00246	0.00246	0.00046	0.02929	0.02883
	250	0.00000	0.00038	0.00038	0.00101	0.01502	0.01400
	500	0.00000	0.00009	0.00009	0.00173	0.01128	0.00955
$\upsilon^{unc}_{YM_1X} = .00088$ $\upsilon^{unc}_{YM_2X} = .00479$ $r_{M_1M_2\bullet X} = -.0989$	50	0.00006	0.03553	0.03547	0.00025	0.05166	0.05141
	100	0.00005	0.01433	0.01428	0.00049	0.02792	0.02743
	250	0.00006	0.00573	0.00567	0.00102	0.01514	0.01411
	500	0.00012	0.00373	0.00361	0.00164	0.01079	0.00915
$\upsilon^{unc}_{YM_1X} = .00088$ $\upsilon^{unc}_{YM_2X} = .00479$ $r_{M_1M_2\bullet X} = .801$	50	0.00005	0.03356	0.03351	0.00031	0.05332	0.05301
	100	0.00007	0.01394	0.01387	0.00047	0.02674	0.02628
	250	0.00009	0.00652	0.00643	0.00112	0.01597	0.01485
	500	0.00012	0.00380	0.00369	0.00186	0.01172	0.00986
$\upsilon^{unc}_{YM_1X} = .00245$ $\upsilon^{unc}_{YM_2X} = .00245$ $r_{M_1M_2\bullet X} = -.0989$	50	0.00012	0.03390	0.03378	0.00014	0.03613	0.03599
	100	0.00020	0.01893	0.01874	0.00020	0.01780	0.01760
	250	0.00043	0.00948	0.00905	0.00040	0.00925	0.00885
	500	0.00071	0.00651	0.00580	0.00072	0.00660	0.00588
	50	0.00001	0.01123	0.01122	0.00946	0.22060	0.21114
	100	0.00000	0.00241	0.00241	0.01695	0.15021	0.13326
	250	0.00000	0.00038	0.00038	0.02685	0.10568	0.07882

$\upsilon_{YM_1X}^{unc}=0$ $\upsilon_{YM_2X}^{unc}=.05444$ $r_{M_1M_2\bullet X}=0$	500	0.00000	0.00010	0.00010	0.03350	0.08730	0.05379
$\upsilon_{YM_1X}^{unc}=.00088$	50	0.00003	0.03507	0.03504	0.00947	0.22042	0.21095
$\upsilon_{YM_2X}^{unc}=.05444$	100	0.00006	0.01438	0.01432	0.01584	0.14436	0.12851
$r_{M_1M_2\bullet X}=-.1816$	250	0.00008	0.00612	0.00604	0.02681	0.10451	0.07771
	500	0.00014	0.00380	0.00367	0.03280	0.08611	0.05330
$\upsilon_{YM_1X}^{unc}=.00088$	50	0.00005	0.03343	0.03338	0.00865	0.21497	0.20632
$\upsilon_{YM_2X}^{unc}=.05444$	100	0.00005	0.01384	0.01379	0.01567	0.14275	0.12708
$r_{M_1M_2\bullet X}=.1184$	250	0.00007	0.00583	0.00576	0.02646	0.10426	0.07780
	500	0.00014	0.00390	0.00375	0.03302	0.08672	0.05370
$\upsilon_{YM_1X}^{unc}=1.63571$	50	0.74801	3.12402	2.37601	0.00926	0.22462	0.21535
$\upsilon_{YM_2X}^{unc}=.05444$	100	0.93959	2.55081	1.61122	0.01600	0.14579	0.12979
$r_{M_1M_2\bullet X}=.3$	250	1.19958	2.23711	1.03753	0.02624	0.10331	0.07707
	500	1.30200	2.02859	0.72659	0.03312	0.08709	0.05397
$\upsilon_{YM_1X}^{unc}=0$	50	0.00002	0.02751	0.02750	0.00000	0.01397	0.01397
$\upsilon_{YM_2X}^{unc}=.00005$	100	0.00001	0.01042	0.01041	0.00000	0.00410	0.00410
$r_{M_1M_2\bullet X}=-.8921$	250	0.00000	0.00345	0.00344	0.00000	0.00114	0.00113
	500	0.00000	0.00165	0.00164	0.00000	0.00053	0.00053
$\upsilon_{YM_1X}^{unc}=.00479$	50	0.00025	0.05185	0.05161	0.00000	0.01383	0.01383
$\upsilon_{YM_2X}^{unc}=.00005$	100	0.00036	0.02682	0.02646	0.00000	0.00400	0.00400
$r_{M_1M_2\bullet X}=.8684$	250	0.00106	0.01553	0.01447	0.00000	0.00114	0.00114
	500	0.00169	0.01109	0.00940	0.00000	0.00050	0.00050
$\upsilon_{YM_1X}^{unc}=.00245$	50	0.00013	0.03562	0.03548	0.00000	0.01131	0.01131
$\upsilon_{YM_2X}^{unc}=.00003$	100	0.00019	0.01784	0.01765	0.00000	0.00311	0.00311
$r_{M_1M_2\bullet X}=.8684$	250	0.00040	0.00897	0.00856	0.00000	0.00079	0.00079
	500	0.00071	0.00646	0.00576	0.00000	0.00033	0.00033
	50	0.12981	1.54579	1.41598	0.00000	0.01196	0.01196
	100	0.18246	1.16063	0.97817	0.00000	0.00306	0.00306
	250	0.25898	0.84512	0.58614	0.00000	0.00081	0.00081

$\upsilon^{unc}_{YM_1X} = .50485$ $\upsilon^{unc}_{YM_2X} = .00003$ $r_{M_1M_2\bullet X} = -.2075$	500	0.32663	0.74535	0.41872	0.00000	0.00034	0.00034
$\upsilon^{unc}_{YM_1X} = .00479$ $\upsilon^{unc}_{YM_2X} = .00479$ $r_{M_1M_2\bullet X} = -.0989$	50	0.00033	0.05252	0.05219	0.00019	0.05008	0.04989
	100	0.00042	0.02752	0.02710	0.00042	0.02707	0.02665
	250	0.00104	0.01519	0.01416	0.00102	0.01522	0.01420
	500	0.00172	0.01109	0.00937	0.00174	0.01122	0.00948
$\upsilon^{unc}_{YM_1X} = .02019$ $\upsilon^{unc}_{YM_2X} = .00479$ $r_{M_1M_2\bullet X} = -.6493$	50	0.00333	0.72769	0.72436	0.00024	0.05254	0.05230
	100	0.00289	0.38291	0.38002	0.00042	0.02627	0.02586
	250	0.00207	0.17651	0.17444	0.00102	0.01502	0.01400
	500	0.00286	0.11414	0.11128	0.00165	0.01090	0.00925
$\upsilon^{unc}_{YM_1X} = .50485$ $\upsilon^{unc}_{YM_2X} = .00245$ $r_{M_1M_2\bullet X} = -.6493$	50	0.12005	1.53416	1.41411	0.00012	0.03429	0.03417
	100	0.17722	1.13907	0.96185	0.00018	0.01790	0.01771
	250	0.26954	0.86336	0.59383	0.00045	0.00963	0.00918
	500	0.31808	0.73171	0.41363	0.00073	0.00656	0.00584
$\upsilon^{unc}_{YM_1X} = .00479$ $\upsilon^{unc}_{YM_2X} = .05444$ $r_{M_1M_2\bullet X} = .7184$	50	0.00028	0.05270	0.05242	0.00927	0.21634	0.20707
	100	0.00046	0.02850	0.02805	0.01613	0.14947	0.13334
	250	0.00114	0.01582	0.01468	0.02667	0.10398	0.07731
	500	0.00162	0.01085	0.00923	0.03227	0.08501	0.05274
$\upsilon^{unc}_{YM_1X} = .00245$ $\upsilon^{unc}_{YM_2X} = .02778$ $r_{M_1M_2\bullet X} = -.1816$	50	0.00009	0.03490	0.03481	0.00379	0.14266	0.13888
	100	0.00020	0.01803	0.01783	0.00642	0.08923	0.08281
	250	0.00036	0.00903	0.00867	0.01164	0.06033	0.04869
	500	0.00073	0.00660	0.00587	0.01491	0.04801	0.03310
$\upsilon^{unc}_{YM_1X} = .00245$ $\upsilon^{unc}_{YM_2X} = .02778$ $r_{M_1M_2\bullet X} = .1184$	50	0.00008	0.03359	0.03351	0.00344	0.13826	0.13482
	100	0.00023	0.01762	0.01739	0.00652	0.08950	0.08298
	250	0.00045	0.00962	0.00916	0.01131	0.05948	0.04817
	500	0.00071	0.00656	0.00585	0.01466	0.04732	0.03266
	50	0.00014	0.03698	0.03684	0.00364	0.13769	0.13404
	100	0.00019	0.01823	0.01804	0.00643	0.08964	0.08320
	250	0.00045	0.00941	0.00897	0.01122	0.05910	0.04788

$\upsilon^{unc}_{YM_1X} = .00245$ $\upsilon^{unc}_{YM_2X} = .02778$ $r_{M_1M_2\bullet X} = .7184$	500	0.00068	0.00635	0.00567	0.01485	0.04752	0.03268
$\upsilon^{unc}_{YM_1X} = .02019$ $\upsilon^{unc}_{YM_2X} = .05444$ $r_{M_1M_2\bullet X} = -.8921$	50	0.00455	0.77903	0.77448	0.00937	0.21790	0.20853
	100	0.00360	0.39684	0.39324	0.01564	0.14467	0.12904
	250	0.00212	0.17507	0.17295	0.02576	0.10196	0.07620
	500	0.00181	0.10425	0.10243	0.03264	0.08615	0.05351
$\upsilon^{unc}_{YM_1X} = .50485$ $\upsilon^{unc}_{YM_2X} = .05444$ $r_{M_1M_2\bullet X} = -.8921$	50	0.11741	1.50221	1.38480	0.00331	0.13348	0.13017
	100	0.18012	1.13515	0.95503	0.00591	0.08382	0.07791
	250	0.26428	0.85388	0.58960	0.01112	0.05894	0.04782
	500	0.33167	0.75308	0.42141	0.01523	0.04858	0.03335
$\upsilon^{unc}_{YM_1X} = .00088$ $\upsilon^{unc}_{YM_2X} = .00479$ $r_{M_1M_2\bullet X} = -.0089$	50	0.00007	0.03521	0.03514	0.00029	0.05326	0.05297
	100	0.00008	0.01472	0.01463	0.00052	0.02807	0.02755
	250	0.00008	0.00579	0.00572	0.00103	0.01498	0.01395
	500	0.00015	0.00395	0.00380	0.00168	0.01103	0.00935
$\upsilon^{unc}_{YM_1X} = .00245$ $\upsilon^{unc}_{YM_2X} = .00245$ $r_{M_1M_2\bullet X} = -.0089$	50	0.00017	0.03872	0.03855	0.00019	0.03956	0.03937
	100	0.00021	0.01735	0.01713	0.00022	0.01798	0.01776
	250	0.00043	0.00912	0.00869	0.00038	0.00909	0.00871
	500	0.00071	0.00660	0.00589	0.00069	0.00646	0.00577
$\upsilon^{unc}_{YM_1X} = .00479$ $\upsilon^{unc}_{YM_2X} = .00479$ $r_{M_1M_2\bullet X} = -.0089$	50	0.00022	0.04844	0.04822	0.00025	0.05135	0.05110
	100	0.00052	0.02671	0.02619	0.00053	0.02772	0.02719
	250	0.00104	0.01502	0.01398	0.00105	0.01545	0.01440
	500	0.00171	0.01109	0.00939	0.00168	0.01104	0.00936
$\upsilon^{unc}_{YM_1X} = .00245$ $\upsilon^{unc}_{YM_2X} = .00245$ $r_{M_1M_2\bullet X} = -.0089$	50	0.00018	0.03824	0.03806	0.00013	0.03539	0.03527
	100	0.00020	0.01778	0.01758	0.00020	0.01802	0.01782
	250	0.00039	0.00921	0.00882	0.00043	0.00947	0.00904
	500	0.00071	0.00659	0.00588	0.00071	0.00656	0.00585
	50	0.01243	0.28626	0.27383	0.00035	0.05563	0.05528
	100	0.01860	0.20381	0.18522	0.00052	0.02846	0.02794
	250	0.02891	0.14126	0.11235	0.00102	0.01483	0.01380

$\upsilon_{YM_1X}^{unc} = .07131$ $\upsilon_{YM_2X}^{unc} = .00479$ $r_{M_1M_2 \bullet X} = -.0089$	500	0.03789	0.11772	0.07983	0.00168	0.01101	0.00933
$\upsilon_{YM_1X}^{unc} = .06133$ $\upsilon_{YM_2X}^{unc} = .00245$ $r_{M_1M_2 \bullet X} = -.0089$	50	0.00969	0.24402	0.23433	0.00009	0.03458	0.03449
	100	0.01462	0.17231	0.15769	0.00019	0.01835	0.01815
	250	0.02576	0.12449	0.09873	0.00044	0.00951	0.00907
	500	0.03309	0.10155	0.06846	0.00073	0.00668	0.00594

Table 7. *Coverage, misses to the left and right for* $\hat{\upsilon}^{unc}_{YM_1X}$ *and* $\hat{\upsilon}^{unc}_{YM_2X}$

		$\hat{\upsilon}^{unc}_{YM_1X}$			$\hat{\upsilon}^{unc}_{YM_2X}$		
	n	Cov	<.025	>.975	Cov	<.025	>.975
$\upsilon^{unc}_{YM_1X}=0$ $\upsilon^{unc}_{YM_2X}=.00005$ $r_{M_1M_2\bullet X}=0$	50	0	100	0	99.4	0.6	0
	100	0	100	0	99.4	0.6	0
	250	0	100	0	99	1	0
	500	0	100	0	98	1.4	0.6
$\upsilon^{unc}_{YM_1X}=0$ $\upsilon^{unc}_{YM_2X}=.00005$ $r_{M_1M_2\bullet X}=.3$	50	0	100	0	98.6	1.4	0
	100	0	100	0	99	1	0
	250	0	100	0	99.6	0.4	0
	500	0	100	0	98.2	0.4	1.4
$\upsilon^{unc}_{YM_1X}=0$ $\upsilon^{unc}_{YM_2X}=.00005$ $r_{M_1M_2\bullet X}=.9$	50	0	100	0	98.6	1.4	0
	100	0	100	0	99.2	0.8	0
	250	0	100	0	99	1	0
	500	0	100	0	97	1	2
$\upsilon^{unc}_{YM_1X}=0$ $\upsilon^{unc}_{YM_2X}=.00003$ $r_{M_1M_2\bullet X}=.3$	50	0	100	0	98.8	1.2	0
	100	0	100	0	98.8	1.2	0
	250	0	100	0	99.4	0.6	0
	500	0	100	0	98.6	1.2	0.2
$\upsilon^{unc}_{YM_1X}=.00088$ $\upsilon^{unc}_{YM_2X}=.00005$ $r_{M_1M_2\bullet X}=-.0316$	50	98.6	1.4	0	98.8	1.2	0
	100	99	1	0	99.6	0.4	0
	250	96.4	3.2	0.4	99.6	0.4	0
	500	95.2	3.6	1.2	97.6	1.4	1
$\upsilon^{unc}_{YM_1X}=.00088$ $\upsilon^{unc}_{YM_2X}=.00005$ $r_{M_1M_2\bullet X}=.8684$	50	98.2	1.8	0	99.2	0.8	0
	100	98	1.8	0.2	99.8	0.2	0
	250	97.6	2	0.4	98.6	1.4	0
	500	96.2	2.6	1.2	97.4	1.4	1.2
	50	96	1	3	99	1	0
	100	93.8	2.6	3.6	99	1	0
	250	92	3	5	99	1	0

$\upsilon^{unc}_{YM_1X} = 1.63571$ $\upsilon^{unc}_{YM_2X} = .00005$ $r_{M_1M_2\bullet X} = -.2075$	500	95.6	2.8	1.6	98.8	0.6	0.6
$\upsilon^{unc}_{YM_1X} = 1.63571$ $\upsilon^{unc}_{YM_2X} = .00005$ $r_{M_1M_2\bullet X} = .0925$	50	94.2	2.8	3	98.6	1.4	0
	100	95.2	2.2	2.6	99.4	0.6	0
	250	93.6	3	3.4	99	1	0
	500	95	2.4	2.6	98.2	0.8	1
$\upsilon^{unc}_{YM_1X} = 0$ $\upsilon^{unc}_{YM_2X} = .00479$ $r_{M_1M_2\bullet X} = 0$	50	0	100	0	97	1	2
	100	0	100	0	96	1.4	2.6
	250	0	100	0	94.4	3	2.6
	500	0	100	0	95	2.2	2.8
$\upsilon^{unc}_{YM_1X} = 0$ $\upsilon^{unc}_{YM_2X} = .00479$ $r_{M_1M_2\bullet X} = .9$	50	0	100	0	97.2	1.2	1.6
	100	0	100	0	95.6	1	3.4
	250	0	100	0	94.4	1.4	4.2
	500	0	100	0	95.6	1.6	2.8
$\upsilon^{unc}_{YM_1X} = .00088$ $\upsilon^{unc}_{YM_2X} = .00479$ $r_{M_1M_2\bullet X} = -.0989$	50	98.6	1.4	0	96.8	1.4	1.8
	100	98.2	1.6	0.2	94.2	2	3.8
	250	98.4	1.2	0.4	95	1.6	3.4
	500	94.8	2.8	2.4	94.2	1.2	4.6
$\upsilon^{unc}_{YM_1X} = .00088$ $\upsilon^{unc}_{YM_2X} = .00479$ $r_{M_1M_2\bullet X} = .801$	50	98.4	1.6	0	97.2	1.4	1.4
	100	97.6	1.8	0.6	94.2	1.8	4
	250	97.2	2.4	0.4	94.8	2	3.2
	500	97.2	2.2	0.6	94.2	2.8	3
$\upsilon^{unc}_{YM_1X} = .00245$ $\upsilon^{unc}_{YM_2X} = .00245$ $r_{M_1M_2\bullet X} = -.0989$	50	98.4	1	0.6	98.4	1.2	0.4
	100	97.2	1.4	1.4	96	1.2	2.8
	250	94.4	1.4	4.2	94.6	1.6	3.8
	500	94.2	2.2	3.6	94.8	2	3.2
	50	0	100	0	94.6	1.4	4
	100	0	100	0	94.4	2.4	3.2
	250	0	100	0	95.2	2.8	2

$\upsilon^{unc}_{YM_1X} = 0$, $\upsilon^{unc}_{YM_2X} = .05444$, $r_{M_1M_2\bullet X} = 0$	500	0	100	0	94.6	3.8	1.6
$\upsilon^{unc}_{YM_1X} = .00088$	50	99.6	0.4	0	93.8	2	4.2
$\upsilon^{unc}_{YM_2X} = .05444$	100	97.2	2.4	0.4	95.4	2.2	2.4
$r_{M_1M_2\bullet X} = -.1816$	250	98.4	1	0.6	93.8	3	3.2
	500	95.4	3	1.6	94	2.8	3.2
$\upsilon^{unc}_{YM_1X} = .00088$	50	99	1	0	94.8	1	4.2
$\upsilon^{unc}_{YM_2X} = .05444$	100	98.8	1.2	0	94.2	1.8	4
$r_{M_1M_2\bullet X} = .1184$	250	97.8	1.4	0.8	96	2	2
	500	96	2.4	1.6	95	2	3
$\upsilon^{unc}_{YM_1X} = 1.63571$	50	94.6	2	3.4	93.8	1.8	4.4
$\upsilon^{unc}_{YM_2X} = .05444$	100	95.8	1.4	2.8	95.6	1.6	2.8
$r_{M_1M_2\bullet X} = .3$	250	92.4	4.4	3.2	95.4	1.4	3.2
	500	94.4	2.4	3.2	94.8	2.8	2.4
$\upsilon^{unc}_{YM_1X} = 0$	50	0	100	0	99	1	0
$\upsilon^{unc}_{YM_2X} = .00005$	100	0	100	0	99.8	0.2	0
$r_{M_1M_2\bullet X} = -.8921$	250	0	100	0	99.4	0.6	0
	500	0	100	0	98.6	0.4	1
$\upsilon^{unc}_{YM_1X} = .00479$	50	98.2	0.8	1	99	1	0
$\upsilon^{unc}_{YM_2X} = .00005$	100	95.6	0.4	4	99.6	0.4	0
$r_{M_1M_2\bullet X} = .8684$	250	94.4	2	3.6	99	1	0
	500	95	1	4	98.4	0.2	1.4
$\upsilon^{unc}_{YM_1X} = .00245$	50	97.2	1.8	1	98.2	1.8	0
$\upsilon^{unc}_{YM_2X} = .00003$	100	96.4	2	1.6	99	1	0
$r_{M_1M_2\bullet X} = .8684$	250	92.6	2.4	5	99.4	0.6	0
	500	94.6	2	3.4	99.2	0.4	0.4
	50	93.4	4	2.6	99.2	0.8	0
	100	93.2	3.4	3.4	99	1	0
	250	92.8	3.2	4	99	1	0

$\upsilon^{unc}_{YM_1X} = .50485$ $\upsilon^{unc}_{YM_2X} = .00003$ $r_{M_1M_2\bullet X} = -.2075$	500	95.4	3.6	1	99	0.6	0.4
$\upsilon^{unc}_{YM_1X} = .00479$	50	98.4	1.2	0.4	98	0.8	1.2
$\upsilon^{unc}_{YM_2X} = .00479$	100	96	0.6	3.4	95.6	0.6	3.8
$r_{M_1M_2\bullet X} = -.0989$	250	93.8	2.4	3.8	94.2	1.4	4.4
	500	93.4	2.8	3.8	93.8	2.4	3.8
$\upsilon^{unc}_{YM_1X} = .02019$	50	96.8	3.2	0	98	0.6	1.4
$\upsilon^{unc}_{YM_2X} = .00479$	100	95.8	4.2	0	95.2	1.2	3.6
$r_{M_1M_2\bullet X} = -.6493$	250	96.6	3.4	0	96.2	1.4	2.4
	500	97.2	2.8	0	95.4	1.2	3.4
$\upsilon^{unc}_{YM_1X} = .50485$	50	93.8	3.6	2.6	98.6	1.2	0.2
$\upsilon^{unc}_{YM_2X} = .00245$	100	94.8	2	3.2	97.2	1.2	1.6
$r_{M_1M_2\bullet X} = -.6493$	250	94.4	3.2	2.4	94.2	2.8	3
	500	94.8	2.4	2.8	94.6	2	3.4
$\upsilon^{unc}_{YM_1X} = .00479$	50	96.8	1.2	2	94.2	2	3.8
$\upsilon^{unc}_{YM_2X} = .05444$	100	95.8	1.6	2.6	94.8	2.2	3
$r_{M_1M_2\bullet X} = .7184$	250	94.4	2	3.6	95	1.6	3.4
	500	94.8	1	4.2	94	1.8	4.2
$\upsilon^{unc}_{YM_1X} = .00245$	50	99	0.6	0.4	95.2	2	2.8
$\upsilon^{unc}_{YM_2X} = .02778$	100	94.8	2.2	3	95.2	1.8	3
$r_{M_1M_2\bullet X} = -.1816$	250	96.6	1.2	2.2	93.4	3.4	3.2
	500	91.4	3.6	5	95.8	1.8	2.4
$\upsilon^{unc}_{YM_1X} = .00245$	50	99.4	0.4	0.2	96	1.6	2.4
$\upsilon^{unc}_{YM_2X} = .02778$	100	95	2.6	2.4	94.2	2.4	3.4
$r_{M_1M_2\bullet X} = .1184$	250	94.8	2	3.2	93	3.2	3.8
	500	95.6	1	3.4	94.6	1.4	4
	50	97.2	1.8	1	95.4	1.8	2.8
	100	97.2	1.4	1.4	92.8	2.4	4.8
	250	93.6	1.8	4.6	94.2	2.6	3.2

$\upsilon^{unc}_{YM_1X} = .00245$, $\upsilon^{unc}_{YM_2X} = .02778$, $r_{M_1M_2 \bullet X} = .7184$	500	96.4	1	2.6	94.8	2	3.2
$\upsilon^{unc}_{YM_1X} = .02019$, $\upsilon^{unc}_{YM_2X} = .05444$, $r_{M_1M_2 \bullet X} = -.8921$	50	95	5	0	92.8	3	4.2
	100	95.6	4.4	0	92.4	3.4	4.2
	250	97	3	0	93.6	2.8	3.6
	500	98	2	0	95.2	1.8	3
$\upsilon^{unc}_{YM_1X} = .50485$, $\upsilon^{unc}_{YM_2X} = .05444$, $r_{M_1M_2 \bullet X} = -.8921$	50	95.2	2.8	2	94.8	1.2	4
	100	92.2	3	4.8	93.8	2	4.2
	250	94.8	2.8	2.4	94.8	2	3.2
	500	93.6	4	2.4	94	3.6	2.4
$\upsilon^{unc}_{YM_1X} = .00088$, $\upsilon^{unc}_{YM_2X} = .00479$, $r_{M_1M_2 \bullet X} = -.0089$	50	98	2	0	97	1.6	1.4
	100	98.2	1.8	0	94.2	1.8	4
	250	98.4	1.4	0.2	94.4	1.2	4.4
	500	95.8	2.4	1.8	94	2.4	3.6
$\upsilon^{unc}_{YM_1X} = .00245$, $\upsilon^{unc}_{YM_2X} = .00245$, $r_{M_1M_2 \bullet X} = -.0089$	50	97.6	1.8	0.6	97	2.4	0.6
	100	95.8	2	2.2	96.2	2.2	1.6
	250	93.6	2.6	3.8	95.4	1.6	3
	500	94.2	2	3.8	94.2	1.6	4.2
$\upsilon^{unc}_{YM_1X} = .00479$, $\upsilon^{unc}_{YM_2X} = .00479$, $r_{M_1M_2 \bullet X} = -.0089$	50	97	1.2	1.8	98	0.6	1.4
	100	92.8	2.2	5	93.6	2.4	4
	250	92.4	2.2	5.4	94.8	1.2	4
	500	95.8	1.4	2.8	94.8	2.2	3
$\upsilon^{unc}_{YM_1X} = .00245$, $\upsilon^{unc}_{YM_2X} = .00245$, $r_{M_1M_2 \bullet X} = -.0089$	50	98	1.6	0.4	99	0.8	0.2
	100	96.4	1.6	2	95	2.2	2.8
	250	95	1.2	3.8	94.2	1.4	4.4
	500	93.8	2.6	3.6	95.2	1.8	3
	50	94.4	3.4	2.2	96.8	1.8	1.4
	100	93.2	3.4	3.4	94.6	2	3.4
	250	95	2.4	2.6	93	1.6	5.4

$\upsilon_{YM_1X}^{unc} = .07131$ $\upsilon_{YM_2X}^{unc} = .00479$ $r_{M_1M_2 \bullet X} = -.0089$	500	94.4	2.4	3.2	93.2	2.2	4.6
$\upsilon_{YM_1X}^{unc} = .06133$ $\upsilon_{YM_2X}^{unc} = .00245$ $r_{M_1M_2 \bullet X} = -.0089$	50	95.4	3.2	1.4	99	0.6	0.4
	100	94.4	2.8	2.8	97.2	1.2	1.6
	250	94.2	2.8	3	96	1.8	2.2
	500	95.2	2	2.8	93.8	1.8	4.4

Table 8. *Mean, bias, relative bias, and mean square error of* $\hat{\upsilon}^{uni}_{YM_1X}$ *and* $\hat{\upsilon}^{uni}_{YM_2X}$

		$\hat{\upsilon}^{uni}_{YM_1X}$				$\hat{\upsilon}^{uni}_{YM_2X}$			
	n	Mean Est.	Bias	Rel. Bias	MSE	Mean Est.	Bias	Rel. Bias	MSE
$\upsilon^{uni}_{YM_1X} = 0$	50	0.00052	0.00052	NA	0.00000	0.00073	0.00068	13.60807	0.00000
$\upsilon^{uni}_{YM_2X} = .00005$	100	0.00009	0.00009	NA	0.00000	0.00035	0.00030	5.90264	0.00000
$r_{M_1M_2 \cdot X} = 0$	250	0.00002	0.00002	NA	0.00000	0.00012	0.00007	1.46099	0.00000
	500	0.00000	0.00000	NA	0.00000	0.00008	0.00003	0.62223	0.00000
$\upsilon^{uni}_{YM_1X} = .00001$	50	0.00057	0.00056	53.70090	0.00000	0.00090	0.00084	13.82004	0.00001
$\upsilon^{uni}_{YM_2X} = .00006$	100	0.00015	0.00014	13.35426	0.00000	0.00036	0.00030	4.89039	0.00000
$r_{M_1M_2 \cdot X} = .3$	250	0.00004	0.00002	2.34272	0.00000	0.00014	0.00008	1.31498	0.00000
	500	0.00002	0.00001	0.97070	0.00000	0.00010	0.00004	0.72270	0.00000
$\upsilon^{uni}_{YM_1X} = .00146$	50	0.01392	0.01246	8.52180	0.00114	0.01441	0.01290	8.52871	0.00123
$\upsilon^{uni}_{YM_2X} = .00151$	100	0.00555	0.00409	2.79849	0.00012	0.00573	0.00422	2.78824	0.00012
$r_{M_1M_2 \cdot X} = .9$	250	0.00273	0.00126	0.86482	0.00001	0.00283	0.00132	0.87110	0.00001
	500	0.00222	0.00076	0.51713	0.00001	0.00230	0.00079	0.51974	0.00001
$\upsilon^{uni}_{YM_1X} = .00075$	50	0.00683	0.00608	8.15452	0.00030	0.00707	0.00630	8.16088	0.00031
$\upsilon^{uni}_{YM_2X} = .00077$	100	0.00313	0.00239	3.19932	0.00004	0.00326	0.00248	3.21883	0.00005
$r_{M_1M_2 \cdot X} = .9$	250	0.00161	0.00086	1.15666	0.00001	0.00168	0.00091	1.17607	0.00001
	500	0.00103	0.00028	0.37859	0.00000	0.00107	0.00030	0.38717	0.00000
$\upsilon^{uni}_{YM_1X} = .00084$	50	0.00336	0.00252	3.00896	0.00005	0.00087	0.00086	130.7847	0.00001
$\upsilon^{uni}_{YM_2X} = .00001$	100	0.00212	0.00128	1.53505	0.00001	0.00022	0.00021	31.65219	0.00000
$r_{M_1M_2 \cdot X} = -.0316$	250	0.00139	0.00055	0.65507	0.00000	0.00008	0.00008	11.42718	0.00000
	500	0.00107	0.00023	0.27368	0.00000	0.00004	0.00003	4.41702	0.00000
$\upsilon^{uni}_{YM_1X} = .10733$	50	0.18341	0.07608	0.70885	0.06104	0.18025	0.07375	0.69247	0.05951
$\upsilon^{uni}_{YM_2X} = .1065$	100	0.12848	0.02115	0.19707	0.01483	0.12670	0.02020	0.18963	0.01442
$r_{M_1M_2 \cdot X} = .8684$	250	0.11728	0.00995	0.09270	0.00404	0.11611	0.00961	0.09027	0.00398
	500	0.11101	0.00368	0.03430	0.00178	0.11001	0.00351	0.03292	0.00176
	50	1.79964	0.09663	0.05674	0.42711	0.07831	0.01095	0.16258	0.02837
	100	1.74651	0.04350	0.02554	0.20379	0.06815	0.00079	0.01179	0.00992
	250	1.69268	-0.01033	-0.00607	0.07967	0.07554	0.00819	0.12156	0.00393

$\upsilon^{uni}_{YM_1X} = 1.703$ $\upsilon^{uni}_{YM_2X} = .06736$ $r_{M_1M_2 \bullet X} = -.2075$	500	1.71173	0.00872	0.00512	0.03666	0.06626	-0.00109	-0.01622	0.00139
$\upsilon^{uni}_{YM_1X} = 3.10872$ $\upsilon^{uni}_{YM_2X} = 1.47306$ $r_{M_1M_2 \bullet X} = .0925$	50	3.13701	0.02829	0.00910	0.75568	1.46344	-0.00962	-0.00653	0.32424
	100	3.18199	0.07327	0.02357	0.34662	1.51424	0.04118	0.02795	0.15628
	250	3.11067	0.00195	0.00063	0.13698	1.47304	-0.00002	-0.00002	0.05126
	500	3.11829	0.00957	0.00308	0.06125	1.48048	0.00742	0.00504	0.02913
$\upsilon^{uni}_{YM_1X} = 0$ $\upsilon^{uni}_{YM_2X} = .00479$ $r_{M_1M_2 \bullet X} = 0$	50	0.00050	0.00050	NA	0.00000	0.00880	0.00401	0.83704	0.00016
	100	0.00011	0.00011	NA	0.00000	0.00680	0.00201	0.41859	0.00005
	250	0.00003	0.00003	NA	0.00000	0.00551	0.00072	0.15002	0.00002
	500	0.00000	0.00000	NA	0.00000	0.00512	0.00033	0.06834	0.00001
$\upsilon^{uni}_{YM_1X} = .39211$ $\upsilon^{uni}_{YM_2X} = .3969$ $r_{M_1M_2 \bullet X} = .9$	50	0.60103	0.20892	0.53282	0.33844	0.60839	0.21149	0.53286	0.34716
	100	0.49983	0.10773	0.27474	0.10728	0.50642	0.10952	0.27593	0.10933
	250	0.42270	0.03059	0.07801	0.03315	0.42786	0.03096	0.07800	0.03400
	500	0.40779	0.01568	0.03998	0.01439	0.41288	0.01598	0.04027	0.01472
$\upsilon^{uni}_{YM_1X} = .00022$ $\upsilon^{uni}_{YM_2X} = .00413$ $r_{M_1M_2 \bullet X} = -.0989$	50	0.00308	0.00286	12.96313	0.00006	0.00720	0.00307	0.74170	0.00016
	100	0.00118	0.00096	4.33967	0.00001	0.00565	0.00151	0.36618	0.00005
	250	0.00046	0.00024	1.09146	0.00000	0.00452	0.00039	0.09435	0.00001
	500	0.00041	0.00019	0.85985	0.00000	0.00422	0.00008	0.02017	0.00000
$\upsilon^{uni}_{YM_1X} = .40043$ $\upsilon^{uni}_{YM_2X} = .40434$ $r_{M_1M_2 \bullet X} = .8011$	50	0.49229	0.09187	0.22942	0.20056	0.49733	0.09299	0.22997	0.20455
	100	0.42715	0.02672	0.06673	0.07863	0.43120	0.02686	0.06643	0.08003
	250	0.44421	0.04379	0.10935	0.03271	0.44838	0.04404	0.10892	0.03313
	500	0.41981	0.01938	0.04841	0.01597	0.42410	0.01976	0.04888	0.01630
$\upsilon^{uni}_{YM_1X} = .0016$ $\upsilon^{uni}_{YM_2X} = .0016$ $r_{M_1M_2 \bullet X} = -.0989$	50	0.00358	0.00197	1.22897	0.00006	0.00418	0.00257	1.60301	0.00007
	100	0.00292	0.00131	0.81896	0.00002	0.00261	0.00100	0.62553	0.00002
	250	0.00204	0.00043	0.26861	0.00000	0.00196	0.00035	0.22119	0.00000
	500	0.00176	0.00015	0.09407	0.00000	0.00180	0.00020	0.12473	0.00000
	50	0.00077	0.00077	NA	0.00003	0.06566	0.01122	0.20607	0.00294
	100	0.00032	0.00032	NA	0.00001	0.06101	0.00656	0.12056	0.00127
	250	-0.00001	-0.00001	NA	0.00000	0.05739	0.00295	0.05409	0.00041

	N								
$\upsilon^{uni}_{YM_1X}=0$ $\upsilon^{uni}_{YM_2X}=.05444$ $r_{M_1M_2\bullet X}=0$	500	0.00001	0.00001	NA	0.00000	0.05609	0.00165	0.03032	0.00019
$\upsilon^{uni}_{YM_1X}=-.00101$ $\upsilon^{uni}_{YM_2X}=.05256$ $r_{M_1M_2\bullet X}=-.1816$	50	0.00070	0.00171	-1.69450	0.00014	0.06299	0.01043	0.19849	0.00272
	100	0.00015	0.00116	-1.15197	0.00003	0.05604	0.00349	0.06638	0.00110
	250	-0.00066	0.00035	-0.34612	0.00001	0.05508	0.00253	0.04807	0.00044
	500	-0.00079	0.00022	-0.21512	0.00000	0.05328	0.00072	0.01377	0.00021
$\upsilon^{uni}_{YM_1X}=.01179$ $\upsilon^{uni}_{YM_2X}=.06535$ $r_{M_1M_2\bullet X}=.1184$	50	0.01485	0.00306	0.25968	0.00031	0.07507	0.00971	0.14862	0.00316
	100	0.01280	0.00101	0.08538	0.00011	0.06846	0.00311	0.04758	0.00122
	250	0.01203	0.00024	0.02023	0.00004	0.06737	0.00201	0.03083	0.00048
	500	0.01180	0.00001	0.00119	0.00002	0.06621	0.00086	0.01311	0.00023
$\upsilon^{uni}_{YM_1X}=1.22292$ $\upsilon^{uni}_{YM_2X}=-.35835$ $r_{M_1M_2\bullet X}=-.8921$	50	1.26927	0.04636	0.03791	0.29777	-0.34866	0.00968	-0.02703	0.06959
	100	1.22251	-0.00040	-0.00033	0.13681	-0.34703	0.01131	-0.03157	0.03618
	250	1.25157	0.02865	0.02343	0.06852	-0.36165	-0.00330	0.00922	0.01320
	500	1.22575	0.00283	0.00232	0.03248	-0.35937	-0.00102	0.00285	0.00607
$\upsilon^{uni}_{YM_1X}=.00005$ $\upsilon^{uni}_{YM_2X}=4.9\times10^{-6}$ $r_{M_1M_2\bullet X}=.3$	50	0.00217	0.00222	-49.25457	0.00003	0.00064	0.00063	127.624	0.00000
	100	0.00087	0.00092	-20.38220	0.00000	0.00021	0.00021	41.77806	0.00000
	250	0.00030	0.00034	-7.59745	0.00000	0.00008	0.00007	14.62207	0.00000
	500	0.00015	0.00019	-4.28153	0.00000	0.00004	0.00003	6.13120	0.00000
$\upsilon^{uni}_{YM_1X}=.10057$ $\upsilon^{uni}_{YM_2X}=.09583$ $r_{M_1M_2\bullet X}=.8684$	50	0.13958	0.03901	0.38788	0.03004	0.13231	0.03648	0.38073	0.02802
	100	0.12310	0.02253	0.22400	0.01101	0.11752	0.02169	0.22638	0.01037
	250	0.10916	0.00859	0.08544	0.00337	0.10393	0.00811	0.08461	0.00314
	500	0.10350	0.00294	0.02920	0.00156	0.09860	0.00277	0.02891	0.00145
$\upsilon^{uni}_{YM_1X}=.05131$ $\upsilon^{uni}_{YM_2X}=.04889$ $r_{M_1M_2\bullet X}=.8684$	50	0.07961	0.02830	0.55156	0.01229	0.07543	0.02654	0.54277	0.01139
	100	0.06876	0.01745	0.34016	0.00479	0.06546	0.01657	0.33884	0.00447
	250	0.05425	0.00294	0.05733	0.00118	0.05159	0.00270	0.05530	0.00109
	500	0.05400	0.00269	0.05240	0.00052	0.05143	0.00254	0.05192	0.00048
	50	0.60326	0.08488	0.16374	0.15949	0.01493	0.00138	0.10143	0.00760
	100	0.57061	0.05223	0.10075	0.07399	0.01993	0.00637	0.46996	0.00395
	250	0.52239	0.00401	0.00774	0.02599	0.01602	0.00246	0.18160	0.00106

$\upsilon^{uni}_{YM_1X} = .5184$; $\upsilon^{uni}_{YM_2X} = .01356$; $r_{M_1M_2 \bullet X} = -.2075$	500	0.52609	0.00771	0.01487	0.01297	0.01370	0.00014	0.01040	0.00055
$\upsilon^{uni}_{YM_1X} = .00701$; $\upsilon^{uni}_{YM_2X} = .00701$; $r_{M_1M_2 \bullet X} = -.0989$	50	0.01100	0.00399	0.56915	0.00020	0.00998	0.00297	0.42283	0.00015
	100	0.00859	0.00158	0.22489	0.00005	0.00855	0.00154	0.21908	0.00007
	250	0.00739	0.00038	0.05424	0.00002	0.00737	0.00036	0.05130	0.00002
	500	0.00722	0.00021	0.02968	0.00001	0.00728	0.00027	0.03832	0.00001
$\upsilon^{uni}_{YM_1X} = .58748$; $\upsilon^{uni}_{YM_2X} = .57208$; $r_{M_1M_2 \bullet X} = -.6493$	50	0.69961	0.11213	0.19086	0.37076	0.61034	0.03826	0.06688	0.28488
	100	0.66398	0.07650	0.13021	0.17872	0.60680	0.03473	0.06070	0.13885
	250	0.62496	0.03748	0.06379	0.05879	0.59402	0.02194	0.03835	0.04663
	500	0.61632	0.02884	0.04910	0.03162	0.59062	0.01854	0.03241	0.02521
$\upsilon^{uni}_{YM_1X} = .74535$; $\upsilon^{uni}_{YM_2X} = .24294$; $r_{M_1M_2 \bullet X} = -.6493$	50	0.86035	0.11500	0.15429	0.37915	0.28921	0.04626	0.19043	0.16212
	100	0.80335	0.05801	0.07782	0.15366	0.27028	0.02733	0.11250	0.07005
	250	0.75769	0.01234	0.01656	0.05640	0.23954	-0.00341	-0.01402	0.02432
	500	0.74106	-0.00429	-0.00575	0.02784	0.24185	-0.00109	-0.00449	0.01188
$\upsilon^{uni}_{YM_1X} = .46448$; $\upsilon^{uni}_{YM_2X} = .51413$; $r_{M_1M_2 \bullet X} = .7184$	50	0.52081	0.05633	0.12127	0.16428	0.57689	0.06276	0.12207	0.18662
	100	0.52230	0.05783	0.12449	0.07911	0.57557	0.06144	0.11950	0.08926
	250	0.47294	0.00846	0.01821	0.02595	0.52412	0.00999	0.01944	0.02897
	500	0.47453	0.01005	0.02164	0.01204	0.52402	0.00989	0.01924	0.01363
$\upsilon^{uni}_{YM_1X} = .01274$; $\upsilon^{uni}_{YM_2X} = .03807$; $r_{M_1M_2 \bullet X} = -.1816$	50	0.01572	0.00298	0.23368	0.00028	0.04842	0.01035	0.27179	0.00159
	100	0.01445	0.00171	0.13450	0.00013	0.04213	0.00405	0.10644	0.00061
	250	0.01347	0.00073	0.05706	0.00004	0.04049	0.00241	0.06338	0.00026
	500	0.01313	0.00038	0.03015	0.00002	0.03887	0.00080	0.02101	0.00010
$\upsilon^{uni}_{YM_1X} = -.0025$; $\upsilon^{uni}_{YM_2X} = .03807$; $r_{M_1M_2 \bullet X} = -.1816$	50	-0.00051	0.00198	-0.79399	0.00007	0.03007	0.00723	0.31674	0.00099
	100	-0.00139	0.00111	-0.44359	0.00002	0.02679	0.00396	0.17335	0.00041
	250	-0.00233	0.00016	-0.06497	0.00000	0.02385	0.00101	0.04440	0.00013
	500	-0.00231	0.00019	-0.07474	0.00000	0.02297	0.00013	0.00566	0.00005
	50	0.29488	0.05790	0.24431	0.07229	0.32547	0.06316	0.24079	0.08068
	100	0.26431	0.02733	0.11532	0.03202	0.29228	0.02997	0.11424	0.03594
	250	0.24779	0.01081	0.04561	0.00884	0.27388	0.01157	0.04411	0.00973

$\upsilon^{uni}_{YM_1X} = .23698$ $\upsilon^{uni}_{YM_2X} = .26231$ $r_{M_1M_2\bullet X} = .7184$	500	0.24083	0.00385	0.01624	0.00527	0.26644	0.00413	0.01573	0.00584
$\upsilon^{uni}_{YM_1X} = .34948$ $\upsilon^{uni}_{YM_2X} = .38373$ $r_{M_1M_2\bullet X} = -.8921$	50	0.46292	0.11344	0.32460	0.15709	0.41682	0.03309	0.08624	0.10496
	100	0.39859	0.04911	0.14052	0.06757	0.38805	0.00432	0.01127	0.04197
	250	0.35858	0.00910	0.02604	0.02305	0.37764	-0.00609	-0.01586	0.01573
	500	0.35526	0.00578	0.01655	0.00992	0.38468	0.00095	0.00246	0.00741
$\upsilon^{uni}_{YM_1X} = .29948$ $\upsilon^{uni}_{YM_2X} = -.17759$ $r_{M_1M_2\bullet X} = -.8921$	50	0.36985	0.07037	0.23499	0.10494	-0.15384	0.02375	-0.13375	0.02288
	100	0.33794	0.03846	0.12841	0.04606	-0.16974	0.00785	-0.04422	0.01312
	250	0.31139	0.01191	0.03976	0.01601	-0.17264	0.00495	-0.02787	0.00405
	500	0.31076	0.01128	0.03768	0.00870	-0.17954	-0.00195	0.01098	0.00217
$\upsilon^{uni}_{YM_1X} = .00088$ $\upsilon^{uni}_{YM_2X} = .00479$ $r_{M_1M_2\bullet X} = -.0089$	50	0.00379	0.00291	3.30922	0.00008	0.00828	0.00349	0.72712	0.00015
	100	0.00209	0.00121	1.37655	0.00001	0.00645	0.00165	0.34516	0.00007
	250	0.00120	0.00032	0.36566	0.00000	0.00518	0.00039	0.08150	0.00001
	500	0.00110	0.00022	0.25471	0.00000	0.00492	0.00013	0.02686	0.00001
$\upsilon^{uni}_{YM_1X} = .00245$ $\upsilon^{uni}_{YM_2X} = .00245$ $r_{M_1M_2\bullet X} = -.0089$	50	0.00574	0.00329	1.34671	0.00012	0.00582	0.00338	1.38096	0.00012
	100	0.00345	0.00100	0.41073	0.00002	0.00351	0.00107	0.43585	0.00002
	250	0.00283	0.00038	0.15691	0.00001	0.00276	0.00032	0.12975	0.00000
	500	0.00265	0.00020	0.08370	0.00000	0.00259	0.00015	0.06106	0.00000
$\upsilon^{uni}_{YM_1X} = .00479$ $\upsilon^{uni}_{YM_2X} = .00479$ $r_{M_1M_2\bullet X} = -.0089$	50	0.00733	0.00254	0.52969	0.00013	0.00789	0.00310	0.64623	0.00016
	100	0.00612	0.00133	0.27698	0.00005	0.00629	0.00150	0.31215	0.00005
	250	0.00516	0.00036	0.07562	0.00001	0.00536	0.00057	0.11814	0.00001
	500	0.00503	0.00024	0.05032	0.00001	0.00500	0.00020	0.04251	0.00001
$\upsilon^{uni}_{YM_1X} = .00245$ $\upsilon^{uni}_{YM_2X} = .00245$ $r_{M_1M_2\bullet X} = -.0089$	50	0.00504	0.00259	1.06067	0.00010	0.00467	0.00223	0.91136	0.00007
	100	0.00351	0.00107	0.43728	0.00002	0.00364	0.00119	0.48730	0.00002
	250	0.00276	0.00032	0.12938	0.00000	0.00287	0.00043	0.17564	0.00001
	500	0.00264	0.00020	0.08100	0.00000	0.00263	0.00019	0.07676	0.00000
	50	0.08707	0.01576	0.22101	0.00647	0.00709	0.00230	0.47912	0.00013
	100	0.07972	0.00841	0.11795	0.00257	0.00493	0.00013	0.02770	0.00004
	250	0.07370	0.00239	0.03355	0.00084	0.00525	0.00045	0.09443	0.00001

$\upsilon^{uni}_{YM_1X} = .07131$ $\upsilon^{uni}_{YM_2X} = .00479$ $r_{M_1M_2 \bullet X} = -.0089$	500	0.07185	0.00055	0.00768	0.00040	0.00488	0.00008	0.01715	0.00001
$\upsilon^{uni}_{YM_1X} = .06113$ $\upsilon^{uni}_{YM_2X} = .00245$ $r_{M_1M_2 \bullet X} = -.0089$	50	0.07216	0.01103	0.18043	0.00370	0.00303	0.00058	0.23788	0.00006
	100	0.06683	0.00569	0.09311	0.00162	0.00285	0.00041	0.16707	0.00001
	250	0.06476	0.00363	0.05937	0.00068	0.00262	0.00017	0.07005	0.00000
	500	0.06219	0.00106	0.01726	0.00031	0.00253	0.00009	0.03540	0.00000

Table 9. *Confidence limits, misses to the left and right for* $\hat{\upsilon}^{uni}_{YM_1X}$ *and* $\hat{\upsilon}^{uni}_{YM_2X}$

		$\hat{\upsilon}^{uni}_{YM_1X}$			$\hat{\upsilon}^{uni}_{YM_2X}$		
	n	Mean LCL	Mean UCL	Mean CI Width	Mean LCL	Mean UCL	Mean CI Width
$\upsilon^{uni}_{YM_1X}=0$	50	-0.00099	0.01269	0.01368	-0.00077	0.01510	0.01587
$\upsilon^{uni}_{YM_2X}=.00005$	100	-0.00022	0.00255	0.00277	-0.00009	0.00442	0.00451
$r_{M_1M_2\bullet X}=0$	250	-0.00004	0.00042	0.00046	-0.00001	0.00110	0.00111
	500	-0.00001	0.00010	0.00012	0.00000	0.00052	0.00051
$\upsilon^{uni}_{YM_1X}=.00001$	50	-0.00184	0.01581	0.01765	-0.00139	0.01853	0.01991
$\upsilon^{uni}_{YM_2X}=.00006$	100	-0.00049	0.00361	0.00411	-0.00023	0.00502	0.00525
$r_{M_1M_2\bullet X}=.3$	250	-0.00013	0.00065	0.00078	-0.00004	0.00133	0.00137
	500	-0.00005	0.00023	0.00028	0.00000	0.00063	0.00064
$\upsilon^{uni}_{YM_1X}=.00146$	50	-0.00152	0.18542	0.18694	-0.00093	0.18836	0.18929
$\upsilon^{uni}_{YM_2X}=.00151$	100	-0.00027	0.05820	0.05846	-0.00007	0.05926	0.05933
$r_{M_1M_2\bullet X}=.9$	250	0.00016	0.01781	0.01765	0.00020	0.01828	0.01808
	500	0.00035	0.00972	0.00937	0.00037	0.01000	0.00963
$\upsilon^{uni}_{YM_1X}=.00075$	50	-0.00195	0.11966	0.12162	-0.00125	0.12159	0.12284
$\upsilon^{uni}_{YM_2X}=.00077$	100	-0.00040	0.03949	0.03989	-0.00020	0.04009	0.04029
$r_{M_1M_2\bullet X}=.9$	250	0.00004	0.01148	0.01145	0.00009	0.01181	0.01172
	500	0.00012	0.00520	0.00508	0.00014	0.00536	0.00523
$\upsilon^{uni}_{YM_1X}=.00084$	50	-0.00077	0.03541	0.03618	-0.00376	0.01848	0.02224
$\upsilon^{uni}_{YM_2X}=.00001$	100	-0.00002	0.01484	0.01486	-0.00128	0.00480	0.00608
$r_{M_1M_2\bullet X}=-.0316$	250	0.00010	0.00638	0.00629	-0.00038	0.00122	0.00160
	500	0.00013	0.00378	0.00366	-0.00019	0.00050	0.00069
$\upsilon^{uni}_{YM_1X}=.10733$	50	0.01817	0.89292	0.87476	0.01597	0.88048	0.86452
$\upsilon^{uni}_{YM_2X}=.1065$	100	0.02409	0.46065	0.43657	0.02329	0.45462	0.43133
$r_{M_1M_2\bullet X}=.8684$	250	0.03794	0.28693	0.24900	0.03738	0.28398	0.24659
	500	0.04947	0.21765	0.16818	0.04889	0.21563	0.16674
	50	0.77183	3.37633	2.60450	-0.27678	0.54342	0.82020
	100	1.01143	2.74119	1.72976	-0.12662	0.31838	0.44500
	250	1.20587	2.27815	1.07228	-0.03411	0.21631	0.25042

$\upsilon^{uni}_{YM_1X} = 1.703$ $\upsilon^{uni}_{YM_2X} = .06736$ $r_{M_1M_2 \bullet X} = -.2075$	500	1.35795	2.11387	0.75592	-0.00627	0.15514	0.16141
$\upsilon^{uni}_{YM_1X} = 3.10872$	50	1.75215	5.09384	3.34169	0.51769	2.81271	2.29502
$\upsilon^{uni}_{YM_2X} = 1.47306$	100	2.15541	4.48515	2.32974	0.83745	2.39612	1.55867
	250	2.44727	3.88167	1.43440	1.04241	1.98778	0.94536
$r_{M_1M_2 \bullet X} = .0925$	500	2.63753	3.65081	1.01328	1.16941	1.83185	0.66244
$\upsilon^{uni}_{YM_1X} = 0$	50	-0.00496	0.01517	0.02012	-0.00031	0.05734	0.05765
$\upsilon^{uni}_{YM_2X} = .00479$	100	-0.00180	0.00393	0.00573	0.00047	0.02955	0.02908
	250	-0.00058	0.00092	0.00151	0.00111	0.01583	0.01472
$r_{M_1M_2 \bullet X} = 0$	500	-0.00029	0.00035	0.00065	0.00175	0.01131	0.00956
$\upsilon^{uni}_{YM_1X} = .39211$	50	0.15960	2.33243	2.17284	0.16184	2.35785	2.19601
$\upsilon^{uni}_{YM_2X} = .3969$	100	0.17327	1.44415	1.27088	0.17525	1.46255	1.28730
	250	0.20507	0.87529	0.67023	0.20736	0.88587	0.67851
$r_{M_1M_2 \bullet X} = .9$	500	0.23504	0.70167	0.46663	0.23779	0.71046	0.47267
$\upsilon^{uni}_{YM_1X} = .00022$	50	-0.00435	0.03602	0.04037	-0.00453	0.05216	0.05669
$\upsilon^{uni}_{YM_2X} = .00413$	100	-0.00180	0.01284	0.01465	-0.00112	0.02687	0.02799
	250	-0.00062	0.00426	0.00488	0.00045	0.01406	0.01361
$r_{M_1M_2 \bullet X} = -.0989$	500	-0.00023	0.00246	0.00270	0.00114	0.00991	0.00877
$\upsilon^{uni}_{YM_1X} = .40043$	50	0.06811	1.71655	1.64844	0.06875	1.73304	1.66428
$\upsilon^{uni}_{YM_2X} = .40434$	100	0.09128	1.15235	1.06107	0.09234	1.16189	1.06955
	250	0.17774	0.86390	0.68616	0.17938	0.87177	0.69239
$r_{M_1M_2 \bullet X} = .8011$	500	0.22469	0.68995	0.46526	0.22707	0.69679	0.46972
$\upsilon^{uni}_{YM_1X} = .0016$	50	-0.00400	0.03311	0.03711	-0.00358	0.03534	0.03892
$\upsilon^{uni}_{YM_2X} = .0016$	100	-0.00129	0.01719	0.01848	-0.00139	0.01610	0.01749
	250	-0.00021	0.00792	0.00813	-0.00026	0.00770	0.00797
$r_{M_1M_2 \bullet X} = -.0989$	500	0.00013	0.00515	0.00502	0.00015	0.00523	0.00508
	50	-0.01932	0.02990	0.04922	0.00888	0.22339	0.21451
	100	-0.00850	0.01119	0.01970	0.01686	0.15114	0.13428
	250	-0.00338	0.00359	0.00697	0.02679	0.10573	0.07894

$\upsilon^{uni}_{YM_1X} = 0$ $\upsilon^{uni}_{YM_2X} = .05444$ $r_{M_1M_2 \bullet X} = 0$	500	-0.00163	0.00170	0.00332	0.03348	0.08730	0.05382
$\upsilon^{uni}_{YM_1X} = -.00101$ $\upsilon^{uni}_{YM_2X} = .05256$ $r_{M_1M_2 \bullet X} = -.1816$	50	-0.02582	0.04518	0.07100	0.00314	0.22050	0.21737
	100	-0.01188	0.01849	0.03037	0.01311	0.14349	0.13038
	250	-0.00659	0.00669	0.01328	0.02479	0.10264	0.07784
	500	-0.00455	0.00355	0.00810	0.03089	0.08417	0.05328
$\upsilon^{uni}_{YM_1X} = .01179$ $\upsilon^{uni}_{YM_2X} = .06535$ $r_{M_1M_2 \bullet X} = .1184$	50	-0.01115	0.08206	0.09322	0.00972	0.24567	0.23595
	100	-0.00201	0.04609	0.04810	0.01981	0.16452	0.14471
	250	0.00262	0.02868	0.02606	0.03303	0.12061	0.08758
	500	0.00479	0.02240	0.01761	0.04063	0.10100	0.06037
$\upsilon^{uni}_{YM_1X} = 1.22292$ $\upsilon^{uni}_{YM_2X} = -.35835$ $r_{M_1M_2 \bullet X} = -.8921$	50	0.44148	2.63193	2.19045	-0.96614	0.15430	1.12044
	100	0.60728	2.08633	1.47906	-0.73964	-0.02839	0.71125
	250	0.82901	1.77482	0.94582	-0.60122	-0.15952	0.44170
	500	0.92165	1.58065	0.65900	-0.52061	-0.21759	0.30302
$\upsilon^{uni}_{YM_1X} = .00005$ $\upsilon^{uni}_{YM_2X} = 4.9 \times 10^{-6}$ $r_{M_1M_2 \bullet X} = .3$	50	-0.00358	0.03062	0.03420	-0.00252	0.01598	0.01850
	100	-0.00143	0.01119	0.01262	-0.00077	0.00434	0.00510
	250	-0.00052	0.00363	0.00415	-0.00017	0.00106	0.00123
	500	-0.00029	0.00170	0.00199	-0.00005	0.00039	0.00043
$\upsilon^{uni}_{YM_1X} = .10057$ $\upsilon^{uni}_{YM_2X} = .09583$ $r_{M_1M_2 \bullet X} = .8684$	50	0.01323	0.72728	0.71405	0.01057	0.70093	0.69036
	100	0.02028	0.44658	0.42630	0.01862	0.43133	0.41271
	250	0.03494	0.26516	0.23022	0.03273	0.25454	0.22181
	500	0.04628	0.20036	0.15408	0.04368	0.19193	0.14825
$\upsilon^{uni}_{YM_1X} = .05131$ $\upsilon^{uni}_{YM_2X} = .04889$ $r_{M_1M_2 \bullet X} = .8684$	50	0.00425	0.48128	0.47703	0.00139	0.46538	0.46399
	100	0.00930	0.27034	0.26104	0.00785	0.26077	0.25292
	250	0.01471	0.14398	0.12926	0.01350	0.13845	0.12495
	500	0.02210	0.11112	0.08902	0.02069	0.10661	0.08592
	50	0.12788	1.60783	1.47996	-0.20112	0.28432	0.48544
	100	0.18741	1.20775	1.02034	-0.10200	0.17215	0.27415
	250	0.26506	0.87541	0.61035	-0.04931	0.09392	0.14323

$\upsilon^{uni}_{YM_1X} = .5184$ $\upsilon^{uni}_{YM_2X} = .01356$ $r_{M_1M_2 \bullet X} = -.2075$	500	0.33360	0.76752	0.43392	-0.03092	0.06365	0.09457
$\upsilon^{uni}_{YM_1X} = .00701$ $\upsilon^{uni}_{YM_2X} = .00701$ $r_{M_1M_2 \bullet X} = -.0989$	50	-0.00246	0.06265	0.06511	-0.00333	0.06047	0.06380
	100	0.00029	0.03364	0.03335	0.00024	0.03298	0.03274
	250	0.00185	0.01898	0.01713	0.00184	0.01895	0.01711
	500	0.00299	0.01441	0.01142	0.00302	0.01448	0.01146
$\upsilon^{uni}_{YM_1X} = .58748$ $\upsilon^{uni}_{YM_2X} = .57208$ $r_{M_1M_2 \bullet X} = -.6493$	50	0.10081	2.34011	2.23930	-0.11998	2.00102	2.12101
	100	0.15015	1.67637	1.52622	0.07047	1.47073	1.40027
	250	0.24740	1.19706	0.94967	0.22729	1.08952	0.86223
	500	0.33091	0.99738	0.66647	0.32255	0.92302	0.60047
$\upsilon^{uni}_{YM_1X} = .74535$ $\upsilon^{uni}_{YM_2X} = .24294$ $r_{M_1M_2 \bullet X} = -.6493$	50	0.16291	2.44720	2.28429	-0.30002	1.36388	1.66390
	100	0.24173	1.78934	1.54761	-0.14772	0.92446	1.07218
	250	0.36597	1.30473	0.93876	-0.03135	0.60049	0.63184
	500	0.45057	1.10975	0.65918	0.04523	0.48442	0.43919
$\upsilon^{uni}_{YM_1X} = .46448$ $\upsilon^{uni}_{YM_2X} = .51413$ $r_{M_1M_2 \bullet X} = .7184$	50	0.07157	1.75546	1.68389	0.09624	1.87684	1.78059
	100	0.15432	1.25956	1.10524	0.18235	1.35372	1.17138
	250	0.22888	0.85348	0.62460	0.26307	0.92734	0.66427
	500	0.28960	0.72876	0.43916	0.32697	0.79366	0.46669
$\upsilon^{uni}_{YM_1X} = .01274$ $\upsilon^{uni}_{YM_2X} = .03807$ $r_{M_1M_2 \bullet X} = -.1816$	50	-0.00792	0.07655	0.08447	0.00502	0.17158	0.16656
	100	0.00034	0.04500	0.04466	0.01067	0.10957	0.09890
	250	0.00438	0.02875	0.02437	0.01824	0.07628	0.05804
	500	0.00649	0.02281	0.01632	0.02245	0.06186	0.03942
$\upsilon^{uni}_{YM_1X} = -.0025$ $\upsilon^{uni}_{YM_2X} = .03807$ $r_{M_1M_2 \bullet X} = -.1816$	50	-0.02009	0.03319	0.05327	0.00040	0.12965	0.12925
	100	-0.01197	0.01254	0.02451	0.00405	0.08094	0.07690
	250	-0.00750	0.00277	0.01027	0.00797	0.05161	0.04364
	500	-0.00547	0.00053	0.00600	0.01109	0.04063	0.02954
	50	0.02210	1.13860	1.11651	0.03985	1.20726	1.16740
	100	0.05466	0.73228	0.67761	0.07044	0.78355	0.71310
	250	0.10095	0.49462	0.39368	0.11817	0.53376	0.41559

$\upsilon^{uni}_{YM_1X} = .23698$ $\upsilon^{uni}_{YM_2X} = .26231$ $r_{M_1M_2 \bullet X} = .7184$	500	0.13155	0.39839	0.26684	0.15066	0.43231	0.28164
$\upsilon^{uni}_{YM_1X} = .34948$ $\upsilon^{uni}_{YM_2X} = .38373$ $r_{M_1M_2 \bullet X} = -.8921$	50	0.06061	1.50833	1.44772	-0.13082	1.22065	1.35148
	100	0.08991	1.00997	0.92006	0.04061	0.86303	0.82241
	250	0.13887	0.69185	0.55298	0.16285	0.64191	0.47906
	500	0.18782	0.57857	0.39075	0.22918	0.56552	0.33634
$\upsilon^{uni}_{YM_1X} = .29948$ $\upsilon^{uni}_{YM_2X} = -.17759$ $r_{M_1M_2 \bullet X} = -.8921$	50	0.04685	1.24486	1.19801	-0.53998	0.19071	0.73069
	100	0.07021	0.87210	0.80189	-0.41477	0.02812	0.44289
	250	0.11876	0.60578	0.48702	-0.31183	-0.05910	0.25273
	500	0.16261	0.50977	0.34715	-0.27535	-0.09764	0.17771
$\upsilon^{uni}_{YM_1X} = .00088$ $\upsilon^{uni}_{YM_2X} = .00479$ $r_{M_1M_2 \bullet X} = -.0089$	50	-0.00356	0.03768	0.04124	-0.00342	0.05622	0.05964
	100	-0.00090	0.01498	0.01588	-0.00033	0.02835	0.02868
	250	-0.00014	0.00586	0.00599	0.00089	0.01502	0.01413
	500	0.00007	0.00384	0.00377	0.00161	0.01096	0.00935
$\upsilon^{uni}_{YM_1X} = .00245$ $\upsilon^{uni}_{YM_2X} = .00245$ $r_{M_1M_2 \bullet X} = -.0089$	50	-0.00304	0.04182	0.04485	-0.00290	0.04207	0.04496
	100	-0.00068	0.01745	0.01813	-0.00066	0.01809	0.01875
	250	0.00030	0.00925	0.00895	0.00026	0.00919	0.00893
	500	0.00066	0.00659	0.00593	0.00063	0.00645	0.00583
$\upsilon^{uni}_{YM_1X} = .00479$ $\upsilon^{uni}_{YM_2X} = .00479$ $r_{M_1M_2 \bullet X} = -.0089$	50	-0.00386	0.05075	0.05461	-0.00368	0.05318	0.05686
	100	-0.00104	0.02683	0.02787	-0.00068	0.02744	0.02812
	250	0.00073	0.01498	0.01426	0.00079	0.01538	0.01459
	500	0.00161	0.01111	0.00949	0.00160	0.01104	0.00944
$\upsilon^{uni}_{YM_1X} = .00245$ $\upsilon^{uni}_{YM_2X} = .00245$ $r_{M_1M_2 \bullet X} = -.0089$	50	-0.00224	0.03981	0.04204	-0.00304	0.03747	0.04051
	100	-0.00059	0.01804	0.01863	-0.00060	0.01833	0.01893
	250	0.00022	0.00918	0.00895	0.00028	0.00942	0.00914
	500	0.00065	0.00655	0.00589	0.00064	0.00652	0.00588
	50	-0.00385	0.28105	0.28490	-0.01664	0.04003	0.05667
	100	0.01399	0.20037	0.18638	-0.00787	0.02140	0.02928
	250	0.02812	0.14099	0.11287	-0.00135	0.01350	0.01485

$\upsilon^{uni}_{YM_1X} = .07131$ $\upsilon^{uni}_{YM_2X} = .00479$ $r_{M_1M_2 \bullet X} = -.0089$	500	0.03753	0.11745	0.07993	0.00048	0.01011	0.00962
$\upsilon^{uni}_{YM_1X} = .06113$ $\upsilon^{uni}_{YM_2X} = .00245$ $r_{M_1M_2 \bullet X} = -.0089$	50	0.00005	0.23894	0.23889	-0.01233	0.02301	0.03534
	100	0.01159	0.17045	0.15886	-0.00518	0.01335	0.01853
	250	0.02504	0.12370	0.09867	-0.00165	0.00777	0.00942
	500	0.03275	0.10109	0.06834	-0.00025	0.00578	0.00603

Table 10. *Coverage, misses to the left and right for* $\hat{\upsilon}^{uni}_{YM_1X}$ *and* $\hat{\upsilon}^{uni}_{YM_2X}$

		$\hat{\upsilon}^{uni}_{YM_1X}$			$\hat{\upsilon}^{uni}_{YM_2X}$		
	n	Cov	<.025	>.975	Cov	<.025	>.975
$\upsilon^{uni}_{YM_1X}=0$, $\upsilon^{uni}_{YM_2X}=.00005$, $r_{M_1M_2\bullet X}=0$	50	98.6	1.4	0	99.6	0.4	0
	100	98.8	1.2	0	99.4	0.6	0
	250	99.2	0.8	0	99.2	0.8	0
	500	98.4	1.6	0	98.2	1.2	0.6
$\upsilon^{uni}_{YM_1X}=.00001$, $\upsilon^{uni}_{YM_2X}=.00006$, $r_{M_1M_2\bullet X}=.3$	50	98.8	1.2	0	99	1	0
	100	99.2	0.8	0	99.4	0.6	0
	250	99.2	0.8	0	99.4	0.6	0
	500	99.4	0.6	0	98.8	0.4	0.8
$\upsilon^{uni}_{YM_1X}=.00146$, $\upsilon^{uni}_{YM_2X}=.00151$, $r_{M_1M_2\bullet X}=.9$	50	96.4	3.6	0	96.6	3.4	0
	100	97.8	2.2	0	97.2	2.8	0
	250	98.6	1.4	0	98.4	1.6	0
	500	97	2.6	0.4	96.8	2.8	0.4
$\upsilon^{uni}_{YM_1X}=.00075$, $\upsilon^{uni}_{YM_2X}=.00077$, $r_{M_1M_2\bullet X}=.9$	50	97.2	2.8	0	96.8	3.2	0
	100	98.6	1.4	0	98.2	1.8	0
	250	97.4	2.6	0	97	3	0
	500	98	1.2	0.8	97.6	1.4	1
$\upsilon^{uni}_{YM_1X}=.00084$, $\upsilon^{uni}_{YM_2X}=.00001$, $r_{M_1M_2\bullet X}=-.0316$	50	98.8	1.2	0	99.4	0.6	0
	100	98.6	1.4	0	99.6	0.4	0
	250	96.8	2.8	0.4	99.2	0.8	0
	500	95.6	3.2	1.2	99	1	0
$\upsilon^{uni}_{YM_1X}=.10733$, $\upsilon^{uni}_{YM_2X}=.1065$, $r_{M_1M_2\bullet X}=.8684$	50	95.6	2.2	2.2	95.6	2.2	2.2
	100	94	3	3	94.2	2.8	3
	250	94.6	2.6	2.8	95.2	2	2.8
	500	94.2	2.2	3.6	94.2	2.2	3.6
	50	95.6	1.6	2.8	97	0.8	2.2
	100	94.2	2.2	3.6	96.2	1.2	2.6
	250	91.4	3.2	5.4	94.2	2.4	3.4

$\upsilon^{uni}_{YM_1X} = 1.703$ $\upsilon^{uni}_{YM_2X} = .06736$ $r_{M_1M_2 \bullet X} = -.2075$	500	96	2.6	1.4	95.8	1.4	2.8
$\upsilon^{uni}_{YM_1X} = 3.10872$	50	93.2	2.8	4	95.2	1.4	3.4
$\upsilon^{uni}_{YM_2X} = 1.47306$	100	94.6	2.6	2.8	94.6	2.2	3.2
$r_{M_1M_2 \bullet X} = .0925$	250	94.8	2.6	2.6	96	1.6	2.4
	500	95.2	1.4	3.4	92.6	3.4	4
$\upsilon^{uni}_{YM_1X} = 0$	50	100	0	0	98.4	0.8	0.8
$\upsilon^{uni}_{YM_2X} = .00479$	100	100	0	0	96.6	1.4	2
$r_{M_1M_2 \bullet X} = 0$	250	100	0	0	94.4	2.8	2.8
	500	99.4	0.6	0	94.8	2	3.2
$\upsilon^{uni}_{YM_1X} = .39211$	50	94.6	5.4	0	94.6	5.4	0
$\upsilon^{uni}_{YM_2X} = .3969$	100	96.2	3.2	0.6	95.8	3.6	0.6
$r_{M_1M_2 \bullet X} = .9$	250	94.4	3.2	2.4	94.2	3.4	2.4
	500	93.4	3	3.6	93.4	3	3.6
$\upsilon^{uni}_{YM_1X} = .00022$	50	98.8	1.2	0	98.2	1.2	0.6
$\upsilon^{uni}_{YM_2X} = .00413$	100	99.2	0.8	0	95.2	1.8	3
$r_{M_1M_2 \bullet X} = -.0989$	250	99.2	0.8	0	95	1.6	3.4
	500	97.6	2.4	0	95.6	1.2	3.2
$\upsilon^{uni}_{YM_1X} = .40043$	50	94.6	2.2	3.2	94.2	2.4	3.4
$\upsilon^{uni}_{YM_2X} = .40434$	100	92.4	3.4	4.2	92	3.4	4.6
$r_{M_1M_2 \bullet X} = .8011$	250	95	2.4	2.6	95	2.4	2.6
	500	92.6	4.2	3.2	92.4	4.2	3.4
$\upsilon^{uni}_{YM_1X} = .0016$	50	99.2	0.8	0	98.2	1.8	0
$\upsilon^{uni}_{YM_2X} = .0016$	100	98	1.8	0.2	99.2	0.8	0
$r_{M_1M_2 \bullet X} = -.0989$	250	96	1.6	2.4	96	1.2	2.8
	500	94.4	2.6	3	95.8	2.2	2
	50	99.8	0.2	0	94	1.8	4.2
	100	100	0	0	94.2	2.4	3.4
	250	100	0	0	94.6	3	2.4

$\upsilon_{YM_1X}^{uni} = 0$ $\upsilon_{YM_2X}^{uni} = .05444$ $r_{M_1M_2 \bullet X} = 0$	500	100	0	0	94.4	3.8	1.8
$\upsilon_{YM_1X}^{uni} = -.00101$ $\upsilon_{YM_2X}^{uni} = .05256$ $r_{M_1M_2 \bullet X} = -.1816$	50	96.2	3.8	0	95	1	4
	100	96.2	3.8	0	94.8	2.2	3
	250	97.4	2.4	0.2	93.4	3	3.6
	500	94.2	4.6	1.2	94	2.6	3.4
$\upsilon_{YM_1X}^{uni} = .01179$ $\upsilon_{YM_2X}^{uni} = .06535$ $r_{M_1M_2 \bullet X} = .1184$	50	98	0.2	1.8	94.4	1.2	4.4
	100	95.8	0.2	4	94.6	1.4	4
	250	93	1.4	5.6	95.4	2.4	2.2
	500	93.4	2.2	4.4	94.6	2.4	3
$\upsilon_{YM_1X}^{uni} = 1.22292$ $\upsilon_{YM_2X}^{uni} = -.35835$ $r_{M_1M_2 \bullet X} = -.8921$	50	95.6	1.6	2.8	91.6	7.8	0.6
	100	95.4	2.8	1.8	92.2	6.6	1.2
	250	91.8	3.6	4.6	94.6	4	1.4
	500	93.6	3.2	3.2	93.8	4.2	2
$\upsilon_{YM_1X}^{uni} = .00005$ $\upsilon_{YM_2X}^{uni} = 4.9 \times 10^{-6}$ $r_{M_1M_2 \bullet X} = .3$	50	90	10	0	96.8	3.2	0
	100	83.2	16.8	0	97.8	2.2	0
	250	80.6	19.4	0	96	4	0
	500	80.8	19.2	0	97.2	2.8	0
$\upsilon_{YM_1X}^{uni} = .10057$ $\upsilon_{YM_2X}^{uni} = .09583$ $r_{M_1M_2 \bullet X} = .8684$	50	96.8	1.8	1.4	97	1.6	1.4
	100	95.2	1.6	3.2	95.4	1.4	3.2
	250	94.8	1.8	3.4	94.6	2	3.4
	500	94	2	4	93.6	1.8	4.6
$\upsilon_{YM_1X}^{uni} = .05131$ $\upsilon_{YM_2X}^{uni} = .04889$ $r_{M_1M_2 \bullet X} = .8684$	50	97	1.4	1.6	97.4	1	1.6
	100	95.4	2	2.6	95.6	2.2	2.2
	250	94.2	1.4	4.4	94	1.4	4.6
	500	92.8	2.8	4.4	93.4	2.2	4.4
	50	93.8	3.6	2.6	99.2	0.4	0.4
	100	93.4	3.4	3.2	97.4	1	1.6
	250	93.8	2.8	3.4	95.4	2	2.6

$\upsilon_{YM_1X}^{uni} = .5184$ $\upsilon_{YM_2X}^{uni} = .01356$ $r_{M_1M_2\bullet X} = -.2075$	500	95.8	3	1.2	95.2	2.6	2.2
$\upsilon_{YM_1X}^{uni} = .00701$	50	98.2	0.8	1	98	0.4	1.6
$\upsilon_{YM_2X}^{uni} = .00701$	100	96	0.8	3.2	94.8	1.8	3.4
$r_{M_1M_2\bullet X} = -.0989$	250	94.6	1.6	3.8	94	1.4	4.6
	500	93.4	2.4	4.2	95.8	1.2	3
$\upsilon_{YM_1X}^{uni} = .58748$	50	94.2	3.6	2.2	93	3.2	3.8
$\upsilon_{YM_2X}^{uni} = .57208$	100	93.8	3.8	2.4	93.6	3.2	3.2
$r_{M_1M_2\bullet X} = -.6493$	250	95.2	2.4	2.4	95.6	1.6	2.8
	500	92.8	4.6	2.6	93.4	3.6	3
$\upsilon_{YM_1X}^{uni} = .74535$	50	93.4	3.4	3.2	94.2	3	2.8
$\upsilon_{YM_2X}^{uni} = .24294$	100	95.2	2	2.8	94.2	2.6	3.2
$r_{M_1M_2\bullet X} = -.6493$	250	94.4	2.6	3	95.6	1.8	2.6
	500	94.2	3.6	2.2	95	2.2	2.8
$\upsilon_{YM_1X}^{uni} = .46448$	50	94.2	1.4	4.4	94.6	1.4	4
$\upsilon_{YM_2X}^{uni} = .51413$	100	93.2	3.4	3.4	93.8	3.4	2.8
$r_{M_1M_2\bullet X} = .7184$	250	95.4	1	3.6	95	1.2	3.8
	500	95.6	2.4	2	95.4	2.6	2
$\upsilon_{YM_1X}^{uni} = .01274$	50	98.4	1	0.6	96.2	1.4	2.4
$\upsilon_{YM_2X}^{uni} = .03807$	100	93.6	2.4	4	95.6	2	2.4
$r_{M_1M_2\bullet X} = -.1816$	250	94.8	1.4	3.8	91.8	4.8	3.4
	500	93.4	2.8	3.8	95.2	1.6	3.2
$\upsilon_{YM_1X}^{uni} = -.0025$	50	90.4	9.6	0	96.6	1.2	2.2
$\upsilon_{YM_2X}^{uni} = .03807$	100	90.2	9.8	0	94.4	2.2	3.4
$r_{M_1M_2\bullet X} = -.1816$	250	91.8	8.2	0	94.8	2.8	2.4
	500	92.6	7	0.4	95	1.8	3.2
	50	96.4	1.4	2.2	96.6	1.4	2
	100	93.8	2.2	4	93.4	2.8	3.8
	250	95.8	1.6	2.6	96	1.4	2.6

$\upsilon^{uni}_{YM_1X} = .23698$ $\upsilon^{uni}_{YM_2X} = .26231$ $r_{M_1M_2 \bullet X} = .7184$	500	93.4	2.2	4.4	94.2	2	3.8
$\upsilon^{uni}_{YM_1X} = .34948$ $\upsilon^{uni}_{YM_2X} = .38373$ $r_{M_1M_2 \bullet X} = -.8921$	50	94.6	3.2	2.2	92.6	1	6.4
	100	93.8	3.2	3	94.6	1.4	4
	250	93.6	2.8	3.6	93.8	2	4.2
	500	94.8	2.4	2.8	95	2	3
$\upsilon^{uni}_{YM_1X} = .29948$ $\upsilon^{uni}_{YM_2X} = -.17759$ $r_{M_1M_2 \bullet X} = -.8921$	50	95.4	3.6	1	90.8	9	0.2
	100	93.6	2.4	4	91.2	8	0.8
	250	95.4	2	2.6	94.8	4.2	1
	500	93.6	4.4	2	93.4	5	1.6
$\upsilon^{uni}_{YM_1X} = .00088$ $\upsilon^{uni}_{YM_2X} = .00479$ $r_{M_1M_2 \bullet X} = -.0089$	50	99.2	0.8	0	98.2	1.2	0.6
	100	98.6	1.4	0	94.6	2	3.4
	250	98	1.8	0.2	94.8	1	4.2
	500	95.8	2.4	1.8	93.8	2.2	4
$\upsilon^{uni}_{YM_1X} = .00245$ $\upsilon^{uni}_{YM_2X} = .00245$ $r_{M_1M_2 \bullet X} = -.0089$	50	99.2	0.8	0	98	1.8	0.2
	100	97.2	1.2	1.6	97.6	1.2	1.2
	250	94	2.6	3.4	95.6	1.4	3
	500	95.4	2.4	2.2	95.6	1.2	3.2
$\upsilon^{uni}_{YM_1X} = .00479$ $\upsilon^{uni}_{YM_2X} = .00479$ $r_{M_1M_2 \bullet X} = -.0089$	50	98.4	0.6	1	97.8	1.4	0.8
	100	93.8	2	4.2	95.6	1.8	2.6
	250	93	2.2	4.8	95	1.4	3.6
	500	95	2	3	94.2	1.8	4
$\upsilon^{uni}_{YM_1X} = .00245$ $\upsilon^{uni}_{YM_2X} = .00245$ $r_{M_1M_2 \bullet X} = -.0089$	50	99.2	0.8	0	99.4	0.6	0
	100	98	1.4	0.6	96.6	1.6	1.8
	250	94.6	1.8	3.6	93.8	2.2	4
	500	94	2.6	3.4	96.6	0.6	2.8
	50	94.4	3.2	2.4	95.8	3	1.2
	100	94	2.4	3.6	93.8	2.2	4
	250	94.4	2.4	3.2	94.4	3.4	2.2

$\upsilon^{uni}_{YM_1X} = .07131$ $\upsilon^{uni}_{YM_2X} = .00479$ $r_{M_1M_2 \bullet X} = -.0089$	500	94	2.6	3.4	95.6	2.4	2
$\upsilon^{uni}_{YM_1X} = .06113$ $\upsilon^{uni}_{YM_2X} = .00245$ $r_{M_1M_2 \bullet X} = -.0089$	50 100 250 500	95.4 94.6 94.4 95.6	2.4 2.6 2.8 1.4	2.2 2.8 2.8 3	95.4 96.4 95.4 97.4	2.4 1.4 2.2 1.2	2.2 2.2 2.4 1.4