The Role of Teacher Rehearsal in Classroom Mathematics Discourse

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CHAPTER I

INTRODUCTION

Classroom mathematics discussions can be difficult for teachers to orchestrate (e.g. Boerst, T., Sleep, Ball, D.L., & Bass, H., 2011; Ensor, 2001; Sherin, 2002). These discussions require attending to and responding to students' ideas about mathematics in ways that are responsive to their approaches, yet also guiding the group toward more sophisticated mathematical understanding (Jacobs, Lamb, & Philipp, 2010; Sherin, 2002). Teachers must identify and press on fruitful seeds of disciplinary concepts in ways that are accessible to students and generative of rich mathematical conversation. In spite of an element of unpredictability in these conversations, research suggests that the deliberate practice, with a goal of intentional revision and improvement, can strengthen these forms of activity (Borko & Livingston; Ericsson, Krampe, & Tesch-Römer, 1993; Yanow, 2001).

Consequently, teacher educators have turned to a form of deliberate practice in the field of teacher preparation that is referred to as rehearsal. Its components include teacher educator coaching interjections (Kazemi et al, 2009; Lampert et al, 2013), collaboration around problems of practice (Kazemi et al, 2009; Nelson, 2011; Fernandez, 2005; Ghousseini, 2008; Lampert & Graziani, 2009; Lampert et al., 2013), teachers role-playing students (Nelson, 2011; Kazemi et al., 2009), and alternation between rehearsal and classroom enactments of teacher-student interactions (Kazemi et al., 2009; Lampert & Graziani, 2009; Lampert et al., 2013).

Rehearsal studies are becoming more prominent in the field of math education, and this dissertation makes several novel contributions to the empirical literature surrounding rehearsals.

Most rehearsal studies to date are conducted with preservice teacher candidates who are considering the content of early number arithmetic for which decades of research have painted a rich portrait of student thinking and of ways to promote mathematically productive classroom dialogue. In contrast, I conduct this research with practicing teachers who were participating in extended professional development centered on statistical reasoning. Unlike early number, much less is known about typical forms of student thinking and how to leverage these forms of thinking to ensure productive mathematical conversations. Moreover, statistics is often less familiar to teachers than other areas of mathematics.

The existing corpus of literature illuminates the various learning opportunities and the role of teacher educators in the rehearsal space. However, because the focus of these studies is on the rehearsal space, relations between discourse in rehearsal and classroom spaces are relatively unknown. This dissertation closely examines discourse in both rehearsal and classroom settings to better characterize the nature and mechanisms of change across these settings and the role that rehearsal plays.

In Chapter II, I present the motivation for studying rehearsals through a synthesis of studies that have used various forms of deliberate practice in teacher education. I argue that rehearsals offer a number of promising learning opportunities—not just available to those role-playing the teacher, but also to those role-playing students. The authenticity of rehearsal experiences is often considered an indicator of how well learning will translate to classrooms. However, the rehearsal studies suggest that alternating both more and less authentic experiences actually strengthens deliberate practice because it provides the space for timely reflection and revision (Kazemi, et al., 2009). Through cycles of investigation and enactment, teachers strengthen their understanding and appropriation of classroom math discussions and learn *how* to learn from their own practice through critical reflection around successes and failures. These conclusions are considerations that designers of rehearsal experiences would find informative and that also formed the initial design of rehearsals that I studied empirically in this dissertation.

In Chapter III, I share results of an empirical analysis that traced changes in discourse from the suggestions instructors made during rehearsal. I examine consistencies and changes in the form and function of teacher turns of talk that I refer to as *discourse moves,* in the settings of subsequent rehearsal and classroom practice. An important piece of this analysis is a characterization of the patterns of change in ways consistent with the goals of instruction and individual moves. I described two such patterns, both of which originated as moments of conflict. The first is a case of internal conflict as teachers immediately self-corrected themselves from a pre-rehearsal form of discourse to one that aligned with a rehearsal suggestion. The second is change that resulted from classroom conflict, as students responded to teacher discourse with either confusion or vague answers. This prompted innovation, as teachers adapted the form or function of discourse to meet the needs of students.

In Chapter IV, I examine changes in the form and function of discourse routines over time between classroom and rehearsal settings. Discourse routines are socially shared ways of interacting that are easily recognizable by participants and allow for both stability and flexibility in classroom activity. The analysis identifies and traces one particular routine, the *transformation*, across rehearsal and classroom settings to identify the features that remained stable over time and those that provided flexibility in adapting to situated characteristics of different settings. I found patterned sequences of move types that constructed the transformation routine for individual teachers. These were somewhat resistant to change, but the individual phrasing of moves within the routines changed more readily. I also found evidence that the transformation routine was influenced iteratively through a series of successive rehearsal and classroom enactments.

Together, these papers provide explanatory power to the mechanisms of learning between rehearsal and classroom math discussions. They illustrate rehearsal's important role in changes of practice for inservice teachers, and the complexity of generating and sustaining classroom conversation. They set the stage for future research that can examine and quantify specific mechanisms of change in more detail.

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CHAPTER II

TEACHER LEARNING OPPORTUNITIES IN THE DELIBERATE PRACTICE OF CLASSROOM DISCUSSIONS

Abstract

The field of math education has turned to the foundational role of classroom talk for learning; however, researchers have been hard-pressed to find classrooms in which productive mathematics discussions are taking place. One way the field is trying to better prepare teachers for this work is through rehearsal experiences, where teachers (preservice candidates or practicing teachers) role-play the teacher and students while an instructor coaches the teacher through a live enactment and revision of an activity. In this paper, I synthesize literature on classroom dialogue with empirical findings about rehearsals to better illustrate how to support teachers in generating and sustaining productive classroom mathematics dialogue. My review is organized into three major findings. First, rehearsal helps teachers rehearse routines within complete instructional activities, nesting the technical aspects of eliciting and sustaining mathematics dialogue within a larger setting of instructional goals and purposes. Second, learning opportunities are not limited to the teacher role in rehearsal. Student role-playing provides an important way for teachers to think like their students might, which better supports their instruction. Further, teacher educators interject not only for the benefit of the teacher but also engage others for the benefit of establishing a professional practice of working through problems of practice collectively. My third finding suggests that rehearsal provides generative learning opportunities when used within cycles of investigation of enactment, and that these

cycles can be quite powerful when they move between more and less "authentic" types of learning activities.

Introduction

During the past two decades, research in mathematics education has been characterized by a commitment to the foundational role of classroom talk for learning (Stein & Engle, 2010; Ochs, 1986; O'Connor & Michaels, 1993). Scholars of mathematics education have identified whole-class discussion, which often serves as the culmination of an instructional activity, as a consequential site for the development of important mathematical ideas (Ball & Cohen, 1999; Spillane, 1999; Putnam & Borko, 1997; Wilson & Berne, 1999). In mathematically productive classroom talk, the teacher guides conversation to highlight important mathematical ideas, especially in ways that will support the future development of these ideas. Classroom conversations are ideally conducted in ways that give authority to students and structure a collective history of ideas (Cobb & Bauersfeld, 1995; Forman, McCormick, & Donato, 1998; O'Connor & Michaels, 1993; Sherin, 2001; Stein & Engle, 2010). However, teachers who manage this form of talk face a challenging and potentially unfamiliar task. They must notice how student ideas can connect to more sophisticated concepts and guide conversation in ways that, over time, lead to disciplined understandings.

Unfortunately, researchers have been hard-pressed to find classrooms in which this kind of teaching is taking place (Jacobs, et al., 2003; Hiebert, et al., 2005; Stein, Grover, and Henningson, 1996). In response, the field of teacher preparation has been investigating re-designs of teacher education experiences that might better prepare teachers for this kind of novel work.

Here I intend to review the nature of supports for these kinds of conversations that can be provided to teachers. Supports for instructional improvement come in many forms: access to better curricula, collaborative teacher communities, coaching, and reflection on practice, to name a few. Because this kind of instruction is unfamiliar to many teachers and requires fundamental *changes* in teachers' practice rather than improvement of what is already being done, I narrow my focus to opportunities for teachers to simulate, or approximate, this kind of teaching. Although there have been recent empirical studies of interventions of this kind (Kazemi & Franke, 2009; Lampert et al., 2013; Nelson, 2011), we do not yet understand what kinds of things are most productive to rehearse, the learning opportunities afforded by different roles and phases of rehearsal activity, and the role of rehearsals in the context of comprehensive teacher education programs that also include field experiences.

The first half of this paper will describe why approximations are a sensible investment for teacher education, to support both the more routine aspects of practice, such as eliciting student thinking, and the more improvisational components, such as responding to novel ideas in mathematically productive ways. In the second half of the paper I review empirical literature around teacher-student role-plays, or *rehearsals*, introduced as a form of professional development that cultivates the deliberate practice of discursive strategies. I conclude by synthesizing lessons learned from the empirical literature and identifying potentially fruitful next steps in the conduct of inquiry about the ways to support sustainable and mathematically productive dialogue.

Background

The Role of Language in Classroom Conversations

I begin by sorting out a number of terms concerning classroom talk that are germane to my discussion. *Discourse* is an umbrella term for all language whose meaning is defined through its function in specific contexts and aligns people through participation in particular activities or as members of particular communities. Discourse includes the role of talk, gestures, visual media, bodily orientation, pauses within and between utterances, among others (Gee, 2005). Here though, I focus specifically on *talk*, which is language that is produced through spoken utterances. Further, because I am interested in classroom talk, I focus on a specific form of talk, called *dialogue*, which engages at least two people in joint conversation. My discussion is concerned with how teachers and students build mathematical meaning during whole class dialogue.

One way teachers and students can build shared meaning together is through using types of dialogue that are familiar to them. Dialogue, like any form of discourse, is often used in routine ways as people engage in socially shared ways of interacting with each other, as well as with content and tools (Gee, 2005; Leinhardt & Steele, 2005). I will refer to these habitually shared interactions as *discourse routines*. Participants know what to expect when well-known routines are put into play. Teachers often use dialogue to carry out or accompany these routines, which can take many forms in classrooms, such as transitions between activities or taking a lunch count. Dialogue during these activities might consist of the teacher initiating a familiar phrase, followed by a sequence of predictable types of responses:

> *T: Liza?*
> *Liza: Here!*
> *T: Hot dog or hamburger?*
> *Liza: Hamburger*

T: Edwin?

The structure of the interaction, as well as the words themselves, can be routine. The individual turns of talk, such as the teacher's question or response, that compose these routines constitute *talk moves*, which are single turns of talk made by teachers and might not be routinely executed. Discourse routines are larger patterns of interaction that involve several turns of talk with beginnings and ends that are recognized by the participants. Routines develop over time through participants' shared understanding of their roles and expectations. Finally, *participation structures* include the roles and stances taken by participants during classroom dialogue (O'Connor & Michaels, 1993). The nature of group discourse often signals what participation structure is in play. For example, a teacher who stands off to the side while two students debate whether the number zero is odd or even communicates that the students share the central role in constructing meaning around the concept of zero, while the teacher's role is to listen and understand. The participation structure might shift if the teacher suddenly moved to the front of the classroom and interrupted the students' dialogue.

Dialogic routines help govern activity in classrooms because participants come to know what to expect. However, the timely incorporation of less routine teacher moves is also critical to the successful functioning of classrooms. For example, a teacher might pull two students aside to settle a disagreement or to introduce a visiting parent to the classroom. In this manuscript, I will restrict my focus to the kind of dialogue used to sustain complex, whole class conversations that are intended to build mathematics from student ideas. Both routines and spontaneous teacher talk play a role in classroom mathematics dialogue, and I will discuss the contributions of each of in greater detail. Because the direction of these conversations depends on student ideas, these discussions are difficult for teachers to sustain, but they are where the "rubber meets the road" in developing and advancing mathematical ideas. In the following section, I further characterize what I mean by productive dialogue in learning mathematics.

Discourse as Learning

Any kind of discourse is often viewed as a tool to aid thinking and learning. However, an alternative view is that discourse is "almost tantamount to the thinking itself" (Sfard, 2001, 2008) because participating in classroom dialogues *is* a target form of learning. In mathematics classrooms, for example, students' participation in mathematics is realized through claims that they communicate through mathematical language. Consistent with this view, productive teacher mathematical dialogue is language that 1) assigns meaning to student ideas by building on their mathematical significance, 2) makes connections between student ideas and disciplinary concepts, and 3) prioritizes particular forms of knowledge and ways of knowing mathematics (Gee, 2005). In the following sections, I will describe in greater detail how teachers generate and sustain productive mathematics dialogue in their classrooms.

Routines and Strategic Responses to Students: Two Essential Components of Productive Mathematics Dialogue

Next, I describe what teachers must know and be able to do to generate and sustain productive classroom mathematics dialogue, outlined in Figure 1. The literature around classroom mathematics dialogue suggests two categories of dialogue teachers must master to engage students in disciplinary practices and to collectively build mathematical ideas. I call these *routines* and *mathematical responses*. In a moment I will explain each category and its subcomponents. As part of this discussion, I will illustrate specific strategies that teachers use to incorporate routines and strategic, timely responses to student ideas through their dialogue. After I describe the unique role of each of these two components of classroom math dialogue, I will discuss their relation. Routines, student ideas, and mathematical responses inform each other in classroom dialogue. This discussion will set up my analysis of how teachers *learn* routine and mathematically strategic components of dialogue by demonstrating that learning how to relate both kinds of dialogue in the context of overarching instructional activity is a better investment for teacher educators than focusing exclusively on either one in isolation.

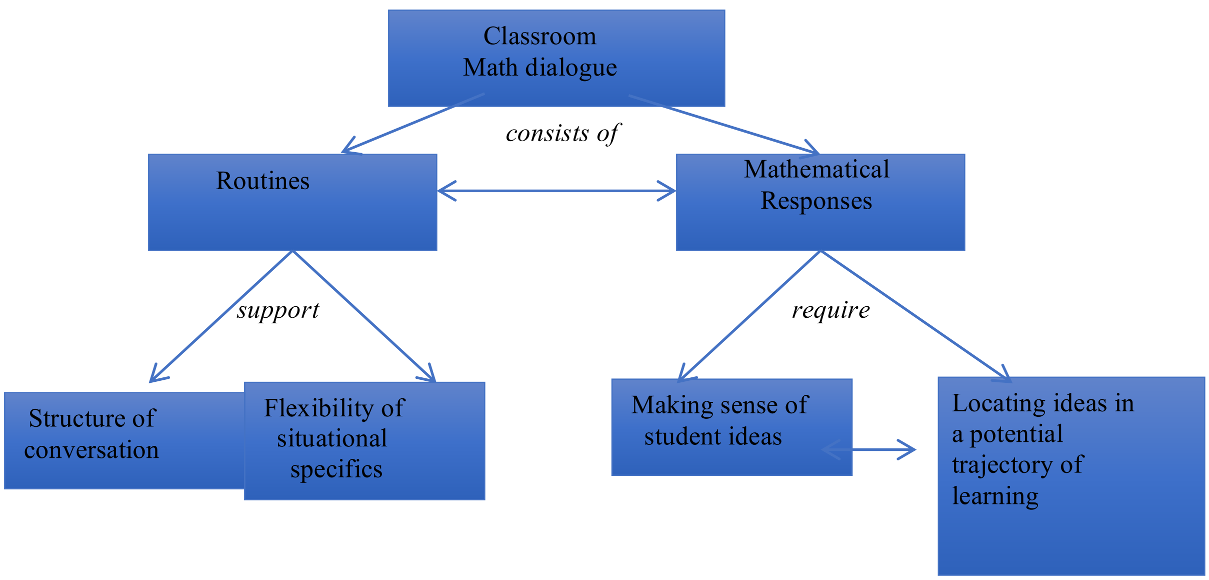

Figure 1. Components of classroom math dialogue

Structural support of routines. The first category of dialogue is *routines*, which organize, bound, and direct the flow of math discussions (Leinhardt & Steele, 2005). Routines, such as "Think (to yourself), pair (talk with a partner), share (to the class)," are characteristic of expert teachers' classrooms (Borko & Livingston, 1989). They are foundational to the work of teachers because they set students' expectations for participation and create the space for students to contribute ideas. In classrooms, dialogic routines are part of a larger composite of sociomathematical norms for participation (Yackel & Cobb, 1996). These norms communicate what counts as acceptable participation in classroom conversations (Yackel & Cobb, 1996).

Flexible support of routines. Although we generally think of routines as being stable building blocks that may be rather static, another body of literature shows that routines provide flexibility for teachers to adapt to situational specifics that come up in classrooms. Moreover, routines can, themselves, transform as teachers adapt them to situational variability, such as the routine's participants or purposes (Borko & Livingston, 1989; Feldmand & Pentland, 2003; Gee, 2005; Leinhardt & Steele, 2005). Consider, for example, the hiring routine in a business setting. The structural components of recruiting, interviewing, and selecting applicants are constants but do not specify details about how the routine is carried out. The situational aspects are variable and often require novel adaptations to previously specified routines. In the hiring routine of a university, for example, adjustments might need to be made to accommodate a joint appointment between two departments (Feldman & Pentland, 2003). Routines are flexible in other ways, as well. Teachers can adapt routines to novel situations or adjust them slightly to be useful in different ways. Some classroom dialogic routines intended for a single purpose can easily serve other purposes (Borko & Livingston, 1989; Leinhardt & Steele, 2005). For example, one teacher's management routines, usually used to guide the flow of conversation between participants, became useful to her instructional dialogue, as well (Leinhardt & Steele, 2005). The teacher used the management routine, "Get your notebooks and find spot x" not in its typical utilitarian way, but as a conversational starter for a discussion about notebook conventions. The teacher embedded this move into the routine structure of a problem solving discussion to elicit and develop ideas about why students might not all be on the same page. The teacher's adaptation transformed the routine from one intended to signal the start of an activity to one intended to support intellectual preparation for the morning's instruction (Leinhardt & Steele, 2005).

A repertoire of routines is necessary, but not sufficient to build mathematical substance. What I have written so far might appear to claim that teachers need only build a repertoire of discourse moves that can be called upon effortlessly during discussions to elicit mathematically productive student dialogue. Although a strong repertoire of routines is important, it is not sufficient to generate and sustain productive mathematics dialogue, even given the flexibility of routines. Even potentially fruitful routines can be enacted in ways more or less productive for the kind of learning I have described (Stein & Engle, 2010; Sfard & Kieran, 2001; Jurow, Hall, & Ma, 2008; Ghousseini, 2008). A classroom with established norms that support participation, explanation, and ownership of ideas can still lack mathematical substance. For example, if a teacher elicits and revoices ideas routinely from many students, but cannot use the ideas for a more advanced mathematical purpose, the resulting discourse will resemble a "show and tell," with many students talking but no mathematical insight growing (Stein et al., 2010). Imagine, for example, an expert literature teacher capable of using routines like these, but attempting to put them to work to teach mathematics. It is clear that regardless of their mastery of discourse routines, literature teachers would probably struggle with identifying productive ways to use student ideas as the basis for more mathematically productive dialogue.

Inherently "good talk moves," therefore, are not in themselves sufficient for sustaining good instruction. In fact, the emphasis on "talk moves" has led some to some draw unwarranted inferences, such as the principle that student talk should always be favored over teacher talk, or that teachers should never "tell" things to students because ideas should always be elicited from students via conversation. Like all hard and fast rules in education, this one is misleading. Many kinds of “telling,” for example, when teachers attach conventional mathematical language to distinctions that students make, remind students about a previously discussed idea, and can actually be a powerful form of discourse in math conversations when it is used at the right time (Chazan & Ball, 1999; Lobato et al., 2005).

In sum, routines are dependable and powerful ways for teachers to position student ideas as the foundation of mathematics dialogue, but they are not sufficient for teachers to advance mathematical ideas. Another category of dialogic moves is necessary for expertise in teaching mathematics. These, which are generally considered more difficult to use productively, rely more heavily on improvisation and response in the moment.

Advancing collective mathematical understanding through student ideas. I now move to the second theme related to the work of teachers who generate and sustain productive mathematics dialogue, namely the moves teachers use to respond to student ideas. Dialogic moves can alter the course of conversations. To move dialogue beyond “show and tell,” teachers must understand how student ideas, which are often highly variable, can build to more sophisticated concepts, and, must also try to guide the dialogue in those directions. For example, one teacher who initially encountered resistance from students eventually found success through a “what if” question that countered an argument the students were trying to make. The move provoked students to take a critical stance toward the hypothetical situation he established and pointed them in the direction he wanted them to take (Jurow, Hall, & Ma, 2008). Success in this case depended on more than a generally productive classroom culture, although that culture was undoubtedly useful for encouraging students to assume the critical stance that the teacher introduced. In this case, the teacher used his diagnosis of student thinking to craft a response that engaged students in disciplinary practices such as argumentation and justification while he guided from students' own scientific ideas.

Teachers can respond to student thinking through dialogic routines, but largely, exposing and engaging student thinking require considerable improvisation, because student ideas cannot always be anticipated. Moreover, some student ideas are more difficult to connect to disciplinary concepts than others. Because teachers do not know for sure what ideas students will bring to the table, they must act on their feet and make "rapid online diagnoses of students' understandings and compare them with disciplinary understandings, and then fashion a response" (Stein, Engle Smith, & Hughes, 2008). Dialogic routines create space for student ideas to emerge, which, in turn, inform teachers' responses to ideas.

Even though teachers might routinely use individual moves such as "What do others think about that?" or revoicing, their choices about *how* or *when* to use them are not in themselves routine. Should the teacher take up an idea immediately or elicit more thoughts? Introduce a new vocabulary word? Position two ideas against each other? Remind students of a previously discussed idea? Introduce a new idea for consideration? The timely incorporation of such words, ideas, or comparisons requires teachers to make connections between the idea on the table and the mathematical goal, guided by their map of student thinking.

The role of past successes and failures in developing expertise. Common to both routines and strategic mathematical responses is the importance of successes and failures in past practice that inform new enactment, even during improvisational moments (Feldman & Pentland, 2003). Classroom activity, like any activity, is continuously and actively rebuilt in the "here and now" (Gee, 2005). However, prior activity is still useful in making sense of the "here and now." Prior enactments help teachers innovate and flexibly use routines as well as respond to students. For example, discourse moves such as revoicing and critical questioning might initially fail to push student thinking in ways that support student understanding (Seymour & Lehrer, 2006). This was certainly the case for a math teacher who tried to enact mathematically responsive classroom dialogue with a new curriculum. After multiple enactments, she began to recognize particular kinds of student talk as the foundation of mathematically important ideas. In other words, these experiences helped build her map of student thinking. As a result, she adjusted her responses to particular patterns of student thinking and talk. For example, she noticed that when students identified and translated between different representations of linear relationships (such as tables and x-y coordinate grids), the students began to reason more easily about slope. She also began to revoice student contributions in ways that blended student and disciplined ways of talking as a way to connect to those more disciplined understandings because she knew that eventually students would have to understand their ideas in more conventional ways (Seymour & Lehrer, 2006). She began to couple her recognition of particular forms of student thinking with more specific conversational resources, such as this nuanced way of revoicing. As a result, her routines evolved as they became connected to specific kinds of student thinking. Her dialogic resources were routine, in the sense that her patterns of instructional support became predictable, but patterns were localized to particular situations. In this case, successes and failures informed the adaptation of her routines. As teachers adapt familiar routines to new situations like this one did, they begin to construct a framework for what are sometimes referred to as *situational discriminations* that inform future decisions and can be placed onto their maps. However, the flexibility of routines can only be realized when faced with novel situations. Over time, teachers can use different adaptations with more confidence in the nature of the outcome (Feldman & Pentland, 2003).

While both successes and failures in past interactions serve as resources when teachers respond to students, failures might be even more productive than successes. The failures of the teacher described above prompted situational discriminations that helped her link particular student ideas to particular forms of discourse she was able to use in the future. In her next year with the same curriculum, she made use of what she had learned from failures the previous year. She made changes to her interpretation of student thinking and her anticipation of productive discourse moves. The unpredictability of student ideas, while difficult for teachers to manage, actually served as a resource for this the growth of this teacher's knowledge about how students make sense of math content (Seymour & Lehrer, 2006). These past experiences help teachers construct both nuanced conversational resources and also their maps of student thinking.

Approximations of Practice in Teacher Education

The conundrum for teacher educators is how to provide experiences for teachers to learn through failure while minimizing the effects of that failure on students. The field of teacher education has identified *approximations of practice* as a way to simulate classroom activities, loosely defined as "core" to the profession, in a safe environment for the purpose of teacher education (Grossman et al., 2009). Theoretically, these approximations are built on the premise of *deliberate practice*, which improves performance expertise through prolonged engagement in activities specifically designed for the improvement of those skills (Ericsson, 1993). Many teaching activities can be approximated to prepare teachers for the work of teaching, including lesson plans, parent conferences, and adaptation of curriculum materials. However, Grossman and colleagues found that novice teachers were given fewer opportunities to approximate *interactive* practice than were novices within the other professions her team studied (Grossman et al., 2009). This is a notable dilemma for teacher educators, because interactive practice is considered the most difficult part of teaching. In response, several scholars have started to explore approximations of teaching in greater depth. The field of teacher education often refers to these kinds of approximations as "rehearsals" because they engage teachers in the *interactive* work of teaching, and they include live coaching from teacher educators to help them revise practice on the spot. Rehearsals in teacher education typically offer some form of simulated teaching, engage others who role-play "students," and include feedback from teacher educators, typically in the form of in-the-moment coaching and suggestions for revision. We typically think of rehearsal as belonging in the realm of artistic performances, such as music and theatre. Moreover, because it requires so much flexibility, we might consider mathematics dialogue to be an unlikely target for rehearsal. Yet, teaching aligns with these other kinds of performances in many ways. Even improvisational troupes rehearse to learn how to interact with each other and to anticipate each others' responses while they collectively construct a story (Yanow, 2001). Both teaching and improvisation contain routine and improvisational components, and both rely on interaction. All of these kinds of rehearsals are deliberate and occur in a training environment. However, even informal rehearsal helps teachers refine their practice, both as individuals and as participants in collaborative conversations (Borko & Livingston, 1989; Horn, 2010). For example, expert teachers think informally through imagined simulations of lesson details as they plan for instruction or collaborate with other teachers. These informal rehearsals help them plan tasks or think through how to introduce key mathematical representations (Borko & Livngston, 1989; Horn, 2010).

Teacher educators often provide valuable opportunities for teachers to investigate and analyze teaching, such as through video clubs that meet to critique videos of teachers' own classroom instruction (Sherin & Han, 2004; Kazemi & Franke, 2004). However, in addition, rehearsals offer teachers the chance to realize and refine knowledge during simulated teaching activity. For example, teachers might understand many different ways that students make sense of regrouping, but rehearsals offer them the chance to use that knowledge to carry out sustained dialogue to coordinate several student ways of thinking. Rehearsals provide opportunities to try things out, both to see how alternate strategies might play out and to learn how others might respond to dialogic moves. These experiences can add depth to the way teachers know routines and anticipate novel ways that students might deploy to make sense of mathematics. In sum, knowing how to do the work of teaching is constructed through opportunities to *do* teaching (Ball & Cohen, 1999; Cook & Brown, 1999; Beach, 1999).

Learning opportunities embedded in rehearsal. Rehearsals allow teachers to experiment and fail, and to revise in a safe environment (Grossman et al., 2009). The experiences teachers build through rehearsal are useful to both the routine and mathematically responsive components of teachers' classroom mathematics dialogue. First, and most simply, rehearsals can break less productive habits and help teachers master a better repertoire of dialogic routines. Second, teachers can practice using student ideas in the service of mathematical goals as instructors guide them through critical dialogic moments.

How rehearsals support the mastery of routines. Most basically, rehearsals help teachers routinize sets of discourse moves that make student ideas visible so they may be collectively understood (Lampert & Graziani, 2009). Once teachers master these structural routines, they are able to focus more on guiding the mathematical content (Kazemi et al., 2009). For example, the three-question routine mentioned earlier, "What do people think?" "Why?" and "What do other folks think about that?" could easily be mastered through repetitive use during rehearsal. Although structural routines are probably the simplest discursive component, their mastery cannot be taken for granted. The simple requirement to repeat a student's hypothesis proved difficult for novice teachers to incorporate in their rehearsal of a brief routine, even after they had watched other novice teachers stumble at trying to accomplish the same task (Lampert & Graziani, 2009).

How rehearsals support situational discriminations through successes and failures. Rehearsal sets up experiences in which teachers encounter successes and failures of their dialogic choices, as well as their developing maps of student thinking. In turn, their failures can build situational discriminations that inform future enactments. Discriminations are constructed when teachers try out variations in routines that are based on situational specifics, choose from the variations, and store them as part of a knowledge base about what doing the routine means and the conditions under which it might be useful (Feldman & Pentland, 2003).

For example, a teacher who rehearses how to build collective understanding around a student's idea might first use a routine to elicit an idea. Once an idea is on the table, candidates for the next move might include routines for clarification, extension of the idea, or comparison of the idea. A "failed" attempt might take the form of asking someone else to extend an idea that was not first clarified. The failure, indicated either by the coach's intervention or a confusion or unexpected type of response by students, signals to the teacher to ask for clarification the next time a similar idea is provided. In this case, students might misinterpret the idea or remain unresponsive after the teacher's question, signaling to the teacher that the idea could have benefited from further clarification. Similar failures can take place when teachers misinterpret student ideas or rush too quickly from a student's idea to conventional mathematical ideas. When failures like these occur, the teacher's map of student thinking undergoes refinement.

Certainly the kinds of past experience that are built through simulated experiences are not completely representative of those built through genuine classroom instruction. In a classroom, dialogic choices are informed by what a teacher knows about the students. Because of his or her history with a class, the teacher's knowledge of any student is likely to be deeper than mere knowledge of mathematical ideas of the kind that are elicited during rehearsal. This is a valid criticism. However, teachers can be coached not only through enactments of practice, but also through instances of considering *how to learn* from practice. Part of the coaching process includes helping teachers notice things and think through the process of how to respond, a process that is generalizable to any ideas, regardless of who contributes them (Lampert et al., 2013).

Scholars who study rehearsal generally agree about the learning opportunities afforded by simulated teaching. Current pressing questions relate to the best duration and complexity of "rehearsable" activities; the learning opportunities for teachers when they embedded receive instructor feedback, role-play students, and collaboratively debrief the choices made during rehearsals; and the challenge of how to fit rehearsals into comprehensive teacher education programs. These are the research questions I will explore through an analysis of the empirical literature on teacher rehearsal. I will aggregate these studies to identify characteristics of rehearsal designs that the field of teacher education has generally found to be useful. I will then identify remaining questions on which the field might direct further study.

Research Questions:

1. How do teacher educators break down the dialogic work of teaching for learning purposes, and which of these pieces are the best candidates for preservice teachers to rehearse?

2. What learning opportunities are embedded in the different roles that teachers take during rehearsal, and what is the teacher educator's role?
3. Where does rehearsal belong in longer sequences of teacher education experiences?

Theoretical Framework

Because my research questions investigate the utility of rehearsal and the activities teachers rehearse, I analyze the corpus of recent literature on rehearsals through two complementary characterizations of utility. The first relates to the second and third research questions: the nature of approximations themselves and how they develop through a series of learning experiences that increasingly approximate classroom practice. The second relates to the grain size of activity that is made a focus of rehearsals. Specifically, it characterizes the relation between large grain sizes of practice and smaller elements of practice that comprise them. The attention to both grain sizes in teacher learning experiences simultaneously brings novice teacher attention to both the "hows" and "whys" in dialogic classroom choices.

How to Rehearse: A Framework for Approximating Practice in Increasingly Authentic Ways

I begin by introducing one way that educators of preservice professionals structure rehearsal as part of a series of learning experiences. Grossman, Compton, Igra, Ronfeldt, Shahan, & Williamson (2009) identified rehearsal-like experiences in the education of novices in a variety of professions that prepare novices for relational work that relies on human interaction. These professions included the ministry, clinical psychology, and funeral directors. Their findings suggested that instructors in these fields typically guide novices through a variety of characteristic approximations in methods classes. The authors suggest that these different kinds of approximations fall along a continuum that begins with approximations that are “less authentic” and culminates with those considered “more authentic” (*Fig. 2*) (Grossman et al., 2009).

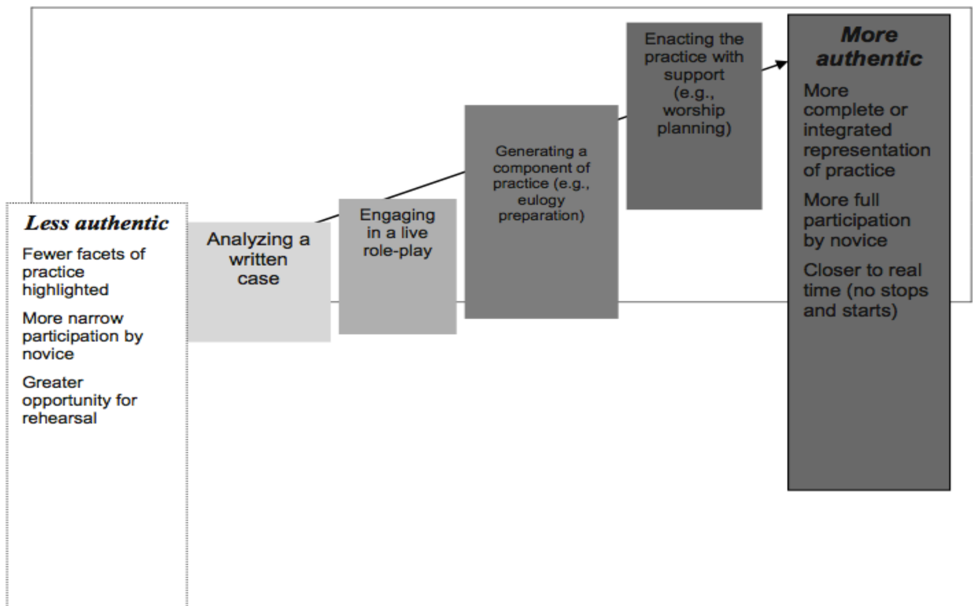

Figure 2. Increasingly authentic approximations of practice (Grossman et al, 2009)

The left end of the continuum characterizes what the authors mean by “less authentic approximations,” and the right end characterizes the “more authentic” approximations. The four boxes in the center are approximations that Grossman and colleagues (2009) found typical of methods courses in the professions that they observed. Less authentic approximations allow novices to practice isolated skills with the fewest authentic constraints, such as watching others demonstrate an activity. More authentic approximations require professionals to tackle more of the complexities of the profession. Grossman et al. found that teachers, as one type of the professional groups studied, benefit from this kind of educational approach, one that begins with the practice of less complex, isolated skills and progressively adds complexity and contingency. However, the team noted that teachers are provided with fewer opportunities to approximate interactive practice than the other professions, and might benefit from more. Instead, teachers tend to approximate things such as lesson planning.

Within this educational framework, the success of any rehearsal depends not just on its internal design elements, but also on the experiences that precede and succeed it. I adopt this framework for analyzing the components of rehearsals and for identifying the opportunities for preservice teachers to rehearse classroom mathematical dialogue.

A closer look at authenticity in the approximations of practice framework. One consideration central to the design of rehearsals is the construct of authenticity. Although the term is frequently used, its definition is rarely detailed, often making it too amorphous for characterizing activity. Broadly, the authenticity of professional practices refers to the degree to which an experience resembles the ways of doing the practices of a profession. One interpretation is that authenticity relies heavily on the setting in which teachers do their work. In this case, one would argue that conducting a lesson in a classroom is always more authentic than conducting one in a simulated environment. An alternative interpretation, and one I maintain, is that authenticity refers more centrally to the work someone is able to carry out, with less emphasis on the setting. In the words of Lampert and colleagues (2013), "Approximations of practice can be categorized as more or less authentic based on a number of characteristics, but those characteristics must be considered in relation to the teaching that one is trying to approximate." This interpretation places a larger priority on the authenticity of the desired teaching rather than on the complexity of the environment. Teacher educators, for good reason, do not always want novices to do things that resemble the actual work many math teachers do. Consider two scenarios. In the first, a novice teacher is placed in a classroom field placement and is asked to teach a math lesson by first demonstrating a procedure for dividing fractions and then giving students a worksheet to complete. In a second scenario, a novice teacher is allowed to teach a math lesson in the way she wants, but the guidance counselor frequently interrupts to pull students out of the classroom for testing. Therefore, the teacher finds herself doing more summarizing than usual to make sure everyone understands the concepts that they discuss. In both of these scenarios, the classroom setting, and its complexities, are authentic. However, the teaching is not authentic to that of mathematically responsive dialogue. Sometimes, practicing mathematics teachers teach in ways contradictory to those that teacher educators promote. In other cases, the complexities of schools prevent teachers from doing the work they are trying to learn to do or prove too difficult for a novice teacher to navigate. The most important consideration is not the authenticity of the setting but the authenticity of the target forms of practice that teachers are supported to enact. As novice teachers become ready, teacher educators can introduce them to more authentic settings, as long as they can still maintain authentic teaching in those more challenging settings.

With that said, consider the approximations of practice continuum described in Figure 3. The continuum suggests that there are three distinct dimensions of authenticity that co-develop in activities as they represent increasingly authentic engagement with practice. At the less authentic end, approximations are characterized by fewer facets of practice highlighted, narrower participation by novices, and greater opportunity for rehearsal. At the more authentic end of the continuum, approximations are described as more complete or integrated representations of practice, fuller participation by novices, and closer to real-time (without stops and starts). While not explicitly described this way in Grossman and colleagues' (2009) cross-professional study, these dimensions can be loosely defined as authenticity of *grain size* (larger activity v smaller components of activity), authenticity of *teaching enactment* (narrow v full participation as the teacher), and authenticity of *time* (real time v more unlimited/unrealistic time). Analyzing video or student work falls at the less authentic end of grain size, as this kind of analysis focuses on only a limited piece of practice. It represents a narrow teaching enactment, as the teacher is usually only looking back at a representation of teaching rather than doing it. Its time flow is less authentic, as teachers might pause a video at several points to engage in discussion. However, in this case, although video analysis may be less authentic in these senses, this does not mean it is not helpful. For a novice, learning is most easily enhanced when the learner is protected from an overwhelming amount of information or pressure to produce a large number of contingent responses (Goodwin, 1994).

What to Rehearse: A Framework for Breaking Down the Work of Teaching

The approximations framework addresses how to design a series of educative experiences for preservice teachers. Because teaching is complex, teacher educators break down, or *decompose,* the work of teaching into components that teachers can rehearse (Grossman et al., 2009). However, teacher educators often wonder how to identify the activities that are best to rehearse. Decomposition presumes that it is unproblematic to identify the single best place to slice practice, but in fact, this is a difficult problem, one that people sometimes solve in a hit or miss fashion. Ball (2000) points out that there are risks in slicing practice in the wrong places; if educators focus on activities that are not essential, they may harm the integrity of teaching, which is a complex, interactive system of practices. However, literature has not delivered a conclusive answer to the question of best parts of practice to rehearse. Whole class discussions, discourse routines, and discourse moves are all decompositions of practice. It might seem that the smaller the grain size, the more manageable the practice. Yet, asking teachers to rehearse only small, technical bits of dialogue would result in their "losing the forest for the trees." A sole focus on the structure of larger activities, such as whole class discussions, omits discussion of the techniques through which the conversation is accomplished. A well-documented challenge for teachers as they learn from experience is how to notice and interpret what they experience (Grossman et al., 2009; Goodwin, 1994). I argue that the best way to help novices notice and interpret the full anatomy of practice is for them to attend to both the smaller technical bits of dialogue and the structure of the larger activities in rehearsal, because relating them gives deeper meaning to the function of each (Grossman et al., 2009). Therefore, decompositions at both larger and smaller grain sizes have a complementary place in rehearsal design.

I suspect that the most productive rehearsal designs do not involve absolute choices between larger or smaller activities. Instead, they are those that make both the "hows" and "whys" of teaching visible and keep them both at the forefront of learning. Figure 3 is an illustration of the relationship between nested teaching practices. Specifically, it shows the relationship between the degree of specificity of a practice and the kinds of things each level makes visible about the meaning of activity. I will use this framework to discuss what different grain sizes of decomposition offer in rehearsal and how different rehearsal designs integrate them.

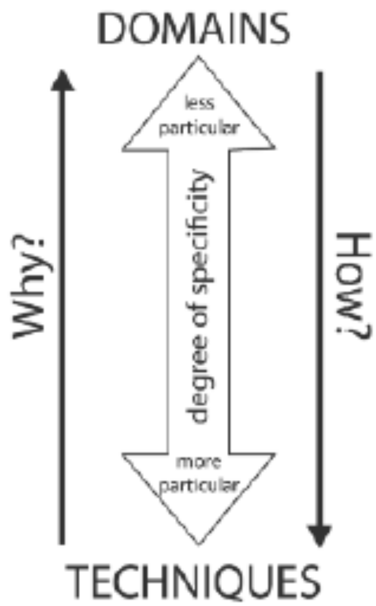

Figure 3. Continuum of grain sizes of practice. Adapted from "Preparing teachers to lead mathematics discussions," by Boerst, T., Sleep, Ball, D.L., & Bass, H., 2011. *Teachers College Record, 113(12),* p. 2855.

In this illustration, the work of teaching is first split into domains of practice, such as leading whole class discussions. Domains of practice are large core activities of teaching. They constitute the substantive work of teaching and are common to any teaching approach. Other domains, for example, are planning lessons and assessing students. Each domain can be increasingly specified, all the way down to the level of techniques, such as revoicing, that are nested within the domain itself (Boerst, Sleep, Ball, & Bass, 2011). Sets of moves such as eliciting or clarifying student thinking are positioned here as *intermediate-level practices* because they connect domains to techniques (*Fig 3).* For example, clarifying student thinking is a routine intermediate-level practice because it has an important purpose in relation to the domain of leading whole class discussions and can be further specified by techniques such as revoicing. Ideally, rehearsals provide opportunities to move back and forth between domains and techniques. The move from domain to technique answers successive questions about *how* a practice can be implemented. Similarly, the move from technique to domain answers successive questions about *why* a teacher would use a technique (Boerst et al., 2011). Because one might imagine that different whole class discussions have different goals, I have specified a particular kind of whole class math discussion in which a teacher generates and sustains mathematically productive dialogue. At this level of specificity, the intermediate-level practices and techniques are easier to specify.

Synthesis of Rehearsal Studies

Although consensus about robust designs for rehearsal are still emerging, some show promise for increasing the opportunities novices are given to practice mathematically responsive classroom dialogue. My primary goal is to inform teacher educators in the field of mathematics education, but because of the number of studies is limited, I will also incorporate studies from outside of mathematics education. In this review I am focusing on studies that report on instructor-facilitated rehearsal of dialogic teaching.

I identified the studies through searches on googlescholar.com, as well as Web of Knowledge and PsycInfo databases. The remaining studies were found from citations made in relevant literature or searches through googlescholar.com to identify literature that has cited the studies. Five studies are about the preparation of novice mathematics teachers, one is about the preparation of novice Italian language teachers, and one is about the preparation of novice science teachers. Two are dissertation studies, and the remaining five are published papers in scholarly journals. Of note, one author appears in three of the studies (M. Lampert), two of which represent successive iterations of one group's research (Kazemi et al., 2009; Lampert et al., 2013). Thus, there are few relevant studies and they do not represent very diverse perspectives on the issue of rehearsal. The rather limited current state of the literature suggests that teacher rehearsal is a fruitful area for new research. Table 1 summarizes key characteristics of each study, which will be discussed in more depth throughout the synthesis.

Table 1

Key characteristics of selected studies

	Lampert & Graziani, 2009	Ghousseini, 2008	Nelson, 2011	Kazemi et al., 2009	Fernandez, 2005	Lampert et al., 2013
Content Domain	Italian language	Math: early number	Science	Math: early number	Math: Unfamiliar content	Math: early number
Outlet	Peer-reviewed Journal	Dissertation	Dissertation	Peer-reviewed Journal	Peer-reviewed Journal	Peer-reviewed Journal
Teachers	Preservice	Preservice	Preservice	Preservice and 1st year inservice	Preservice	Preservice
Coaching/ Interjections	Live and retrospective	Live and retrospective	Live and retrospective	Live and retrospective	Retrospective only	Live and retrospective
Instructor modeling before rehearsal	x	x		x		
Field opportunities to practice	x	x		x		

The review is structured by examining the literature to identify what it has to say on issues that are key for designing rehearsal-based educational experiences for teachers. Accordingly, the first analysis examines the grain sizes of activities selected for rehearsal. Broadly, the rehearsal designs that I reviewed chose a few key foundational types of discussions and their component discourse structures as a focus of rehearsal. I will describe the nature of these supports in more detail in the first part of analysis.

Second, I will analyze the contribution of individual components of the educational activity that is based on rehearsal. Research tends to focus especially on teacher role-play. In addition, I will also analyze the potential learning opportunities in role-playing student perspectives and in developing ways and means of collaborating with other teachers, all-valuable in making the "whys" of practice more visible.

Finally, I will examine how rehearsals are situated in more comprehensive professional education experiences. The learning opportunities made available to teachers in prior experiences can strengthen or weaken what can be learned at a later time through rehearsal. The empirical studies generally focus on programs that use rehearsal as part of a cyclical design that engage novice teachers repeatedly in a sequence of investigations and approximations. This approach situates rehearsal as a generative way to tackle problems of practice and identify new ones for investigation in the next cycle. This back-and-forth model contrasts with the more typical approach that positions classroom teaching as a culminating experience, for example, as identified by Grossman and colleagues (2009). I will analyze those differences and motivations in the second piece of analysis.

In the following sections, I will explore how the teacher education designs support the mechanisms of rehearsal described earlier: mastery of structural dialogue and creation of space for constructing situational discriminations through successes and failures.

Finding 1: Integration of Core Activities and their Component Routines

A focus only on specific dialogic moves could quickly turn responsive mathematics conversations into prescribed mechanics, with little flexibility. On the other hand, a focus only on the "big picture" of a whole class discussion might leave teachers unprepared to execute specific routines. Most rehearsal designs that I examined balanced the hows and the whys of practice through a structure that nested discourse routines within activity structures identified as core to the domain. Table 2 summarizes these learning opportunities related to role-playing the teacher. In the center, I list the opportunities to learn the hows and whys of practice when teachers rehearse either core activities or routines alone. The far right lists the additional learning opportunities when rehearsal includes both. I will further describe evidence of these learning opportunities from the empirical findings.

Table 2

Learning opportunities embedded within different grain sizes of rehearsal

<table>
<tr><td rowspan="9">Playing Teacher</td><td>Grain size</td><td>Hows</td><td>Whys</td><td rowspan="9">Understand the contextual relation of routines within activities</td><td rowspan="9">Discriminate which parts of activity are routine and which depend on student thinking</td></tr>
<tr><td rowspan="5">Core activities</td><td colspan="2">How and why to strengthen professional judgment in relation to student ideas</td></tr>
<tr><td colspan="2">How and why to develop mathematical ideas over time</td></tr>
<tr><td colspan="2">How and why to integrate instructional resources</td></tr>
<tr><td colspan="2">How and why to adjust instruction to particular learners</td></tr>
<tr><td>How to master routine structural and dialogic components of activity</td><td></td></tr>
<tr><td rowspan="3">Routines</td><td>How to master dialogic routines</td><td></td></tr>
<tr><td></td><td>Why problems of practice arise</td></tr>
<tr><td></td><td>Why routines are useful</td></tr>
</table>

Learning opportunities in rehearsal of core instructional activities. Most rehearsal designs first identified a small set of *core instructional activities* as the target of rehearsal. These are routine classroom activities that structure student-teacher interactions in ways that maintain high expectations for student learning and adapt to particular interactions (Kazemi et al., 2009; Lampert & Graziani, 2009; Lampert et al., 2013). Their boundaries are signaled in reliable ways and typically take 10-40 minutes of instructional time. Generally, teacher educators selected activities that are ubiquitous and valuable to teachers of a content area, accommodate a variety of powerful mathematical content, and focus on interactions around student thinking (Ghousseini, 2008; Kazemi et al., 2009; Lampert & Graziani, 2009; Lampert et al., 2013). Further, instructional activities are typically described as generative, meaning they create more opportunities for novice teachers to question and learn about other practices and content in the process (Ghousseini, 2008; Kazemi et al., 2009; Lampert & Graziani, 2009; Lampert et al., 2013; Nelson, 2011). For example, the activity called *solving a sequence of related computational problems* can be done in the same way for simple addition and for division of fractions. Doing the activity provides opportunities for the teacher to learn how students think about the new content. Teacher educators generally considered rehearsal of intermediate-level activities, such as eliciting student thinking or clarifying student thinking, inadequate for supporting the development and justification of mathematical ideas (Fernandez, 2005), as well as for sufficient learning from practice to take place (Kazemi et al., 2009). The context of more complete instructional activities provided teachers more insight into why specific dialogic choices make sense (Kazemi et al., 2009; Lampert et al., 2013). For example, teachers who rehearse only how to elicit student thinking might not know how to identify mathematically promising elements of these ideas or bridge them to more sophisticated mathematical thinking (Kazemi et al., 2009; Lampert et al., 2013).

Rehearsals need not focus solely on the nature of dialogic interactions, and when intact, goal-directed activities are the unit of analysis, they often motivate attention to other important features, as well. For example, one design specified the function of materials, utilization of classroom space, and the teacher's physical positioning as part of the activity novices rehearsed (Lampert & Graziani, 2009). In this activity, called "Conversation Rebuilding," teachers of

Italian presented a four-part conversation to novice teachers, who played students, by miming, drawing, or describing ongoing action while the novice teachers made iterative hypotheses, in Italian, about what the actors might actually be saying in the situation. Novice teachers rehearsed using the notecards they would use with students, which specified the content of each of these conversations. They also used the board to draw clues to the conversation, just as they would with students. As a result, when novices rehearsed complete activities, they learned how to integrate resources with dialogue, as they would in practice (Lampert & Graziani, 2009).

Although rehearsals in these cases focused on the complete instructional activities, novice teachers also began to master some routine components of the activity, as well. This appears to be because they were coached through their execution of the activity, including relations between the smaller components embedded within the context of the larger activity (Kazemi et al., 2009; Lampert & Graziani, 2009; Lampert et al., 2013). Here, the opportunities for learning routines include both smaller routine exchanges, as well as the overarching activity structures that structure goals of the interaction.

The content of core activities in mathematics. Because of its foundational nature, early number is a target of most core instructional activities in elementary mathematics instruction. I found one important exception to this ubiquitous focus on early number. Fernandez and colleagues (2005) chose activities with content unfamiliar to prospective teachers, such as geometry investigations and inventing algorithms for finding permutations, because they were thought to make the student role during rehearsals more authentic for peer teachers to portray. The teachers took a test so the researchers could identify topics most problematic to them, although the topics were not provided in the article. The teacher educators also wanted novices to become accustomed to teaching topics unfamiliar to them, an experience that will almost certainly come up in their professional careers (Fernandez, 2005). The researchers concluded from videotapes of the microteaching that teachers had made considerable gains, beyond what was even measured in the pre-tests, in their content knowledge as a result of participating in the microteaching cycle. Participants indicated in a feedback survey that both the collaborative planning experience and the microteaching contributed to their deeper content knowledge (Fernandez, 2005).

All of these teacher educators prioritized the instructional activities that they considered to be of greatest benefit for novices to rehearse. However, I maintain that their differences can be traced back to commitments to different types of authenticity. Most studies prioritize structures of teaching (such as leading discussions, comparing solution strategies, or introducing warm-up problems) that recur in most classrooms, based on the belief that the ways teachers think through how to respond to students in these contexts should be consistent, regardless of the content of student ideas that come up in them. However, other teacher educators prioritize the authenticity of the *content* of student ideas so that teachers can rehearse how to respond to particular mathematical ideas, with less focus on orchestrating the particulars of the contexts in which they occur. I take the position that if teachers are able to consistently think through *how* to respond to student ideas by understanding and connecting typical student thinking to key mathematical ideas, the goal to expose them to the gamut of authentic student ideas is unnecessary.

Learning opportunities in the rehearsal of dialogic moves and routines. Teachers who begin their attempts at rehearsal by focusing on intact instructional activities could easily be overwhelmed, as activity structures in classrooms are complex and entail negotiating subgoals that often compete. Thus, many teacher educators first coached novices through rehearsal of smaller activity components. Some might argue that learning how to carry out the simple turn-taking structure characteristic of many routines can be accomplished easily without rehearsal. However, even the simplest of routines proved difficult to novice teachers during rehearsal, apparently because they clashed with teachers' intuitions, which were based on years of participating in IRE exchanges (Lampert & Graziani, 2009). Novice teachers who rehearsed smaller bits of dialogue mastered critical routines by breaking these conflicting habits (Lampert & Graziani, 2009). For example, as part of the *Conversation Rebuilding* activity mentioned earlier, teacher educators instructed novice language teachers to elicit student hypotheses and repeat the first hypothesis given. This simple move of repeating the student hypothesis initially was problematic for teachers because it clashed with their instinct to correct or ignore student ideas (Lampert & Graziani, 2009). Instructors interjected during rehearsals to correct novices who did not repeat students' hypotheses and to ask them to try it again. By the end of the rehearsal session, even other novices began to correct each other when one failed to repeat the student's hypothesis. However, even though the novice teachers showed that they knew *what* to do through their critique of other teachers, some of those teachers still needed prompting during their own rehearsal before executing the routine correctly (Lampert & Graziaini, 2009). This finding further suggests that teachers who watch others revise dialogic routines may not revise their own teaching unless they practice the instructional dialogue themselves by enacting it.

Teachers strengthen more than simple instructional habits when they rehearse routines; their responses to student ideas improve, as well. Teacher educators found that novice teachers were better prepared to anticipate problems of practice after rehearsal (Ghousseini, 2008). Further, the teachers began to use routines in service of instructional goals rather than just mechanically or because routines were considered "good practice." They were able to see how routines helped them uncover student thinking and to understand how to respond to student thinking in routine ways. In fact, three studies went so far as to script parts of these routines during isolated rehearsal because teacher educators believed that the purposes of routines would not be evident until teachers actually used them (Ghousseini, 2008; Kazemi et al., 2009; Lampert & Graziani, 2009). "Sticking to the script" was a powerful mechanism for showing teachers *how* to execute routines and also for provoking thinking about *why* routines or moves were useful to mathematics dialogue. Because routines were often rehearsed before students tried to enact the overarching activity, scripting provided a scaffold to the contextual cues that is typically provided by the unfolding goals within the larger activity. Scripting also supports both structure and flexibility in orchestrating discussions. It provides a structure that deliberately sets up opportunities for student ideas to become the focus of discussion. Teachers can then make informed instructional choices based on the student ideas that they hear. After understanding why routines are beneficial to the overarching activity through rehearsal, teachers can flexibly tweak the routine in classrooms as needed in response to instructional goals.

Learning opportunities in a nested structure of moves and routines within instructional activities. Typically, we might tend to think about isolated dialogic moves and routines as making the "hows" of practice more visible and the context of larger activities as making the "why'" more visible. However, most rehearsal designs nested routines within rehearsal of the larger activities, as shown in Figure 4, which illustrates the organization of this nested structure. Smaller components were often rehearsed first before rehearsing them in the context of the larger activities (Boerst et al., 2011; Ghousseini, 2008; Lampert & Graziani, 2009). Teacher educators recognized that rehearsing routines in isolation would strip away the contextual information that the larger activities provide. Thus, they took care to represent the overarching activity to novices in some way to provide some of the missing contextual information, either by modeling it themselves or by showing a video of the activity before zooming in on the components.

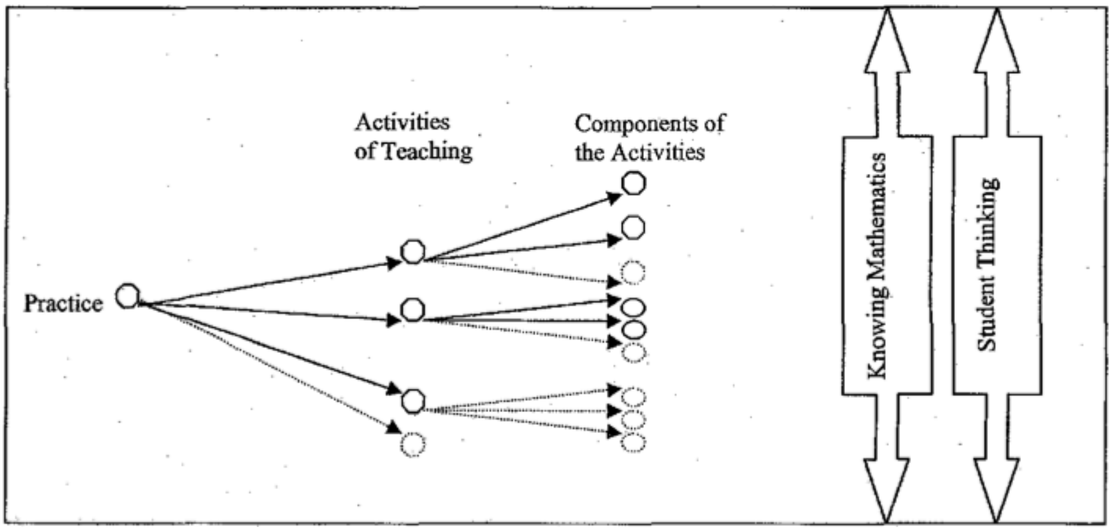

Figure 4. Components of teaching practice. Adapted from "Learning with routines: Preservice teachers learning to lead classroom mathematics discussions," by Ghousseini, N.H. (2008). *Dissertation Abstracts International Section A: Humanities and Social Sciences, 69(3-A),* p. 913.

Once novices had rehearsed components of activities within the full larger activity, they began to see benefits beyond those of rehearsing only de-contextualized routines or the goals of the larger activity. Rehearsals that focused on both together provided more space to work on their relation (Lampert et al., 2013). Novice teachers learned to distinguish between routine aspects of practice and those dependent on students and mathematical goals (Lampert, 2013). Integration helped teachers form new instructional "habits" and realize how those habits serve to generate and sustain mathematically productive dialogue. While "sticking to the script" exposed the role of individual moves within the context of the routine, the role of the routines in the larger activity was often revealed only after teachers rehearsed them within the context of the larger activity (Ghousseini, 2008). The nested structure also provided analytical opportunities for teachers to strengthen their responses to student thinking (Ghousseini, 2008; Kazemi et al., 2009; Lampert et al., 2013). Specifically, interjections from teacher educators provided opportunities for novices to relate the purposes of routine moves to the larger goal of using student ideas in more ambitious ways, goal that sometimes was realized only later in the activity (Kazemi et al., 2009; Lampert et al., 2013). Further, routines were tightly integrated into instructional activities in classroom practice. In that way, the nested structure more closely simulated the complexities of classroom teaching.

Therefore, rather than trying to identify "just right" grain sizes of teaching activity, teacher educators should design rehearsals in a nested way—sometimes focusing on overarching activity structures, sometimes zooming in through their interjections to emphasize the way a particular routine plays out within the activity structure. This hierarchical approach can promote a deeper understanding of the goals of the activity as well as the mechanisms through which productive mathematics dialogue is accomplished. While some might criticize what seems like the avoidance of tough choices, the nested structure of most classroom activities means that doubling the number of focal components emphasized in rehearsal does not always result in doubling the time spent on them in teacher education programs.

Finding 2: Learning Opportunities in Other Component Parts of Rehearsals

The most salient role in rehearsal is that of the teacher. Rehearsing the teacher's role strengthen both the hows and whys of practice through opportunities to tinker with and integrate routines and dialogic responses to student ideas. The utility of enacting the teacher's role cannot be underestimated. Recall, for example, that novice teachers in the "Conversation Rebuilding" corrected each other during rehearsals, showing that at some level they "understood" the importance of restating the student's hypothesis. However, they apparently incompletely understood the importance of this routine until they had the chance to role-play the teacher (Lampert & Graziani, 2009). However, the teacher's is not the only role that strengthen teacher learning during rehearsal. First, the teacher's role is supported by interjections from the teacher educators who guide the enactment. Research around what teacher educators need to know and be able to do is very new but suggests that they must be able to simultaneously consider how students make sense of mathematics and how teachers make sense of supporting students. They use more generalized knowledge about teaching, such as the different ways teachers might choose to represent a student's idea, and must decide when to interject and how. For example, a modeling strategy might be useful the first time a teacher educator suggests something, but a revise and redo is more appropriate in subsequent interjections (Lampert et al., 2013). Second, teachers who play the role of student strengthen their own understanding of teaching by imagining what students are likely to say in the contexts of instruction that the rehearsal presents. Finally, collaborative debriefings give the entire group opportunities to make sense of the teaching they experienced in either role. For teachers in the student/collaborative role, the whys of practice are primarily strengthened. Interjections attend to both hows and whys, because they support the teacher's role. Most notably, although the teacher is sometimes playing a student, the goal is not to strengthen his or her ability to be a student, but rather to be a teacher who can relate to students.

Interjections. In Table 3, the role of teacher educators as they interject is absent. This is because it strengthens every single learning opportunity that novices experience in the teacher role. Interjections set rehearsals apart from other forms of approximations. The coaching that teacher educators provide in response to teachers' instructional choices provokes critical revisions of practice on the spot, and coaching typically relates the whys of practice to the hows. For example, through interjections, novices reason about the purposes of dialogic moves in light of specific student ideas (Kazemi et al., 2009; Lampert et al., 2013). Based on their expert novice of students and pedagogy, teacher educators also insert representative or novel student thinking into the conversation for teachers to tackle at appropriate times.

Table 3

Learning opportunities embedded within different roles in rehearsal

<table>
<tr><td rowspan="9">Playing Teacher</td><td>Grain size</td><td>Hows</td><td>Whys</td><td rowspan="9">Understanding the contextual relation of routines within activities</td><td rowspan="9">Discriminating which parts of activity are routine and which depend on student thinking</td></tr>
<tr><td rowspan="5">Core activities</td><td colspan="2">Hows and whys of strengthening professional judgment in relation to student ideas</td></tr>
<tr><td colspan="2">Hows and whys of developing mathematical ideas over time</td></tr>
<tr><td colspan="2">Hows and whys of integrating instructional resources</td></tr>
<tr><td colspan="2">Hows and whys of adjusting instruction to particular learners</td></tr>
<tr><td>Hows of mastering routine structural components of activity</td><td></td></tr>
<tr><td rowspan="3">Routines</td><td>Hows and mastering discursive routines</td><td></td></tr>
<tr><td></td><td>Whys problems of practice arise</td></tr>
<tr><td></td><td>Why routines are useful</td></tr>
<tr><td rowspan="2">Playing Students</td><td rowspan="2"></td><td></td><td>Why knowing how students think is useful</td><td colspan="2" rowspan="5"></td></tr>
<tr><td></td><td>Why teacher moves evoke emotional responses</td></tr>
<tr><td rowspan="2">Collaboration</td><td rowspan="2"></td><td rowspan="2"></td><td>Why instructional choices are consequential</td></tr>
<tr><td>Why different instructional approaches solve problems of practice</td></tr>
<tr><td></td><td></td><td></td><td>Why mathematical knowledge informs instructional choices</td></tr>
</table>

Brief interjections to the teacher. Typically, interjections take one of two forms. The first is the brief interjections that address the enactment of specific questions or moves. These kinds of interjections do not disrupt the flow of the enactment and are directed at the teacher. They merely direct the role-player's attention to productive next moves or ask teachers to revise moves they have just made. In-the-moment coaching is built on the premise that rehearsal is more beneficial when teachers use the forms and functions of dialogue in the intended ways while they teach, rather than simply thinking about them retrospectively. In the Conversation Rebuilding activity, one might imagine that without these in-the-moment interjections, novice teachers' "bad habits" would have taken more time to break. Recall that novices who had not only been told to repeat student hypotheses but had also seen others prompted on the same move still fell prey to forgetting to do so when it was their turn to play teacher.

Four different types of interjections were noted in one study (Kazemi et al., 2009). However, this is the only study that described the nature of interjections at this level of detail:

1. *Questions*, such as, "What is your purpose for going in that direction?"
2. *Suggestions*, such as, "Try asking another student to restate that student's idea."
3. *Praise*, such as, "Nice representational choice."
4. *Typical student thinking*, such as, "All the numbers in the 50's are odd because they all start with 5."

These forms of interjection engage teachers in thinking both about hows and whys, and because they are contingent on the role-player's discursive choices, the balance of hows and whys depends on individual teachers' strengths and weaknesses.

The target of interjections in these studies is not just dialogue; it spans many aspects of practice, from time management to how representations are used. However, an analysis of the content of interjections found that teacher educators interjected most often around the dialogue of eliciting and responding to student thinking (Lampert et al., 2013). The authors attribute this finding to the salience of these routines in what they refer to as "ambitious teaching" and the critical importance of how these routines position students to interact with each other and with content. An alternative explanation for the prevalence of eliciting and responding moves is that each of these is quite complex and entails more opportunities for guidance than other routines.

Analytic interjections for the entire class. The second type of interjection consists of longer analytical conversations that engage the entire group of teachers and involve a "time-out" from the live role-play to engage everyone in analyzing the teacher role. These conversations guide teachers through anticipating how instructional dialogue might unfold through a series of question-response exchanges (Lampert et al., 2013). Together, novices build a deeper understanding of the characteristics of productive discourse by considering why specific moves are appropriate or effective (Lampert et al., 2013). However, these analytical conversations strengthen more than just the hows and whys of practice. The conversations are also critical to the development of a community of practice that is crucial to novices' identity formation as teachers of ambitious mathematics (Lampert et al., 2013).

While only two studies analyzed the nature and contribution of the types of interjections, all of the studies found that rehearsals were coupled with a collective analysis of practice, typically immediately following each rehearsal. Most of these analyses relied upon a combination of live and retrospective coaching. In the one exceptional study, teachers engaged in three cycles of design, enactment, and revision instruction based on retroactive analysis alone (Fernandez, 2005). The tradeoff of this approach was that teacher educators did not coach teachers through their teaching episodes; however, several iterations of teaching provided these teachers with chances to revise practice in a fairly quick turnaround cycle. Overall, interjections provided benefits in the refinement of both routine and non-routine components of practice to inform sound construction of repertoires of useful dialogic moves, sometimes tied to specific kinds of student thinking. In addition, they contributed to novices' identity formation.

Role playing students Typically, the teacher's role is conceptualized as the site of learning opportunities in a rehearsal. Indeed, the teacher role is the only place to practice and master routines. However, the student role strengthens some of the whys of practice that later inform novices' own teaching in rehearsal. When novices play students, they thoughtfully represent mathematical ideas in the ways students might and respond emotionally as students sometimes do to the choices that their teachers make (Lampert et al., 2013; Nelson, 2011). For example, one peer-student communicated a feeling of validation when the peer-teacher asked him to display his work on the board for discussion, even though the problem solution was not correct (Nelson, 2011). Some teachers felt better prepared to respond to unanticipated reasoning in their own rehearsals because they were required to think about how students might reason through a problem (Nelson 2011). However, others felt that because they already knew expectations for content and participation, they were swayed too easily to normative ideas, even when students were assigned a specific kind of thinking to portray. Teachers felt this was a shift that is unlikely in a real classroom (Nelson, 2011). To address this issue, other rehearsal designs restricted the voice of typical or novel student ideas to the teacher educators, which added some authentic complexity or novelty as needed when novice teachers were unable to fill those roles (Ghousseini, 2008; Kazemi et al., 2009; Lampert et al., 2013).

Collaboration Rehearsal designs also incorporated opportunities for teachers to collaboratively debrief after each teacher's rehearsal (Fernandez 2005; Ghousseini 2008; Kazemi et al., 2009; Lampert & Graziani 2009; Lampert et al., 2013; Nelson 2011). In fact, the strength of rehearsals was maximized when it was immediately followed by these opportunities to analyze what happened during the rehearsal (Kazemi et al., 2009; Lampert & Graziani, 2009; Lampert et al., 2013). Reflection is typically considered less authentic than other activities in the approximations of practice continuum, as Figure 2 shows. However, it is an important complement to enactments of dialogic teaching. Reflection engages everyone in thinking like the teacher but is based on the rehearsal that has taken place, in which one person did role-play the teacher. Novice teachers' critiques were typically guided by questions posed by instructors, such as "Julia said she started counting at 1 instead of 28. Do you get the difference?" (Lampert et al., 2013). Critiques were often directed at the reasons behind the choices teacher role-players made and the effects on the discussion with the class, as well as on individual students. Collaboration helped teachers anticipate responses to problems of practice that were brought by peers, plan revisions to their next enactment of the activity, build confidence, identify new instructional approaches, and strengthen mathematical knowledge (Fernandez, 2005; Ghousseini, 2008; Kazemi et al., 2009; Lampert & Graziani, 2009; Lampert et al., 2013; Nelson, 2011). Some rehearsal designs offered teachers chances to re-teach the activity immediately to experiment with new ideas or conjectures (Fernandez, 2005; Kazemi et al., 2009; Lampert & Graziani, 2009; Nelson, 2011).

On occasion, novice teachers found collaboration so helpful that they began to initiate collaborations on their own outside of class (Fernandez, 2005). Some teacher educators deliberately passed on some of the responsibility in choosing the topic of collaboration to novices in order to foster these relationships. For example, after novices taught in classrooms, they were asked to choose artifacts of practice, through reflection on their own teaching, for collaborative discussion in their methods class (Kazemi et al., 2009; Lampert & Graziani, 2009). Teacher educators found that this kind of "design research" generatively developed teachers' expertise even into their first year of professional teaching. It added depth to how teachers knew the instructional activities. The relationship between theory and practice was jointly constructed through the work of classroom enactments, reflective analysis, and hypothesized revisions carried out in subsequent rehearsal (Kazemi et al, 2009).

Finding 3: The Importance of the Opportunities Surrounding Rehearsal and the Cyclical Nature of Learning Opportunities

Thus far, I have examined the learning opportunities related to rehearsal itself. However, rehearsal is only one component of a larger set of learning experiences that are sometimes cyclical in nature. Here, I will first discuss how opportunities that precede rehearsal, such as discussing different purposes of the revoicing move, can strengthen what teachers learn from rehearsals. Second, some of my previous analysis alludes to the use of teacher education designs that cycle through a series of learning experiences several times. Here, I will examine the ways that these cyclical experiences strengthen the learning opportunities that rehearsals provide, compared to approaches that are more linear. As part of this analysis, I will more closely examine the structure of these cycles of rehearsal in relation to the approximations of practice framework illustrated in Figure 2. Although the three dimensions of authenticity embedded in the framework (grain size, time reality, and teaching enactment) are modeled as increasing in tandem, they do not typically follow that model in rehearsal designs. Instead, these cycles actually alternate between more and less authentic engagement with practice across the three dimensions of authenticity. Further, I describe a nested structure of learning designs, in which the three dimensions of authenticity increased on different timescales.

How investigation of practice strengthens rehearsal experiences. Although investigations of practice are considered less authentic than approximations of practice, they play an important role in the learning opportunities that rehearsals make available. Before rehearsing, teachers often first observe and analyze the component parts of instructional activities. These initial discussions often provide common language that teachers can use to notice and make sense of the hows and whys of practice during rehearsal (Kazemi et al., 2009). For example, in one study, after instructors modeled a complete instructional activity, they engaged teachers in discussion of the theoretical and empirical evidence suggesting that the activity would lead to student learning. Instructors then modeled key pieces of the activity protocol, specifying the hows of the participation structures to be used. Then the teacher educators led an analysis of their teaching with the class members, who discussed alternative solutions to problems of practice that were likely to arise (Kazemi et al., 2009). These complementary experiences introduce teachers to problems of practice that must be worked through before additional complexity is added. Further, they give teachers a chance to understand both the structure and utility of practices in preparation for immersion in field experience classrooms that might neither exhibit nor value practices like these (Grossman et al., 2009).

The structure of cycles of investigation and enactment. The approximations of practice framework positions classroom enactment as a culminating experience. However, rehearsal designs did not position them in that way. These designs often specified multiple cycles of enactment and reflection between methods classes and classrooms, referred to in Figure 5 as *cycles of enactment and investigation* (Kazemi et al., 2009; Lampert et al., 2013).

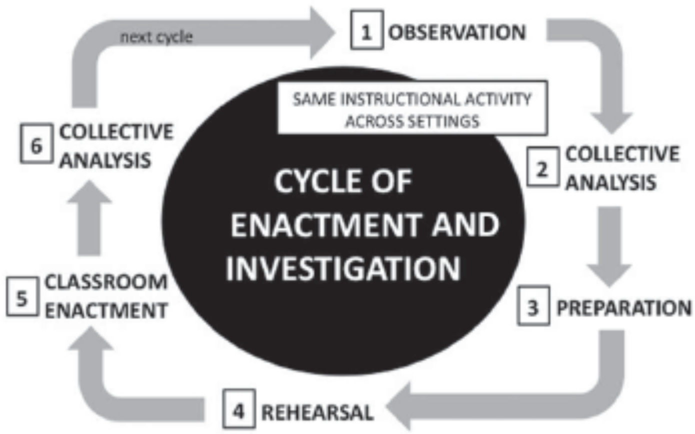

Figure 5. Description of where and how rehearsals are embedded in the larger cycle of enactment and investigation. Adapted from "Keeping it Complex: Using rehearsals to support novice teacher learning of ambitious teaching," by Lampert, M., Franke, M.L., Kazemi, E., Ghousseini, H., Turrou, A.C., Beasley, H., Cunard, A., and Crowe, K., 2013. *Journal of teacher Education, 64(3),* p. 229.

While the types of learning experiences vary slightly in different designs, they generally follow a structure similar to the one in Figure 5. Teacher educators first model the activity. Then they lead teachers through an in-depth content and pedagogical analysis of the activity. Rehearsal and sometimes classroom enactments follow. Collective analysis is typical after each phase.

I will analyze these cycles in relation to the approximations of practice framework in two ways. First, the framework models authenticity as increasing up to a culminating experience, such as classroom teaching. However, teacher educators guided novice teachers through approaches that alternated between more and less authentic learning opportunities. I will discuss what these back-and-forth learning opportunities afford. Second, I will show that rarely did the three dimensions of authenticity develop in tandem in these models. Instead, they followed different paths to authenticity. In this final section, I will illustrate how teacher educators sometimes spent more time gradually exposing novice teachers to some types of complexity while moving quickly into others.

Alternation between more and less authentic practice. In these cycles, teachers sometimes rehearsed an activity even after classroom enactment, which essentially backtracks in the continuum of authenticity. However, these opportunities required teachers to critically evaluate and revise their teaching as they realized solutions to problems of practice with the support of instructors and peers. As novices moved back and forth between classrooms and teacher education experiences, they added depth to the meaning behind the activities they are trying out. They built new experiences by resituating activities in classrooms, reflecting on those experiences in methods classes, and trying out new things again in classrooms (Cook & Brown, 1999; Kazemi et al., 2009). Teacher educators positioned classrooms as a place to learn from practice rather than just to apply what had been learned elsewhere (Bransford, Brown, & Cocking, 1999; Grossman et al., 2009; Franke et al., 2009). Classroom teaching experiences brought to light problems of practice that could then be further analyzed, rehearsed, and enacted in classrooms again to refine teaching. One design offered support to novice teachers during their first year in the classroom by using the ongoing university methods class as a place to further refine practice (Kazemi et al., 2009). Cycles between methods classes and classrooms let first-year teachers capitalize on successes and failures to iteratively revise their teaching over a longer timescale than single rehearsals.

Variable paths to authenticity Recall that the sequence of authenticity outlined by Grossman and colleagues (2009) specified three different types of complexities that were gradually increased for novice teachers as teacher educators led them through a sequence of learning experiences. These three dimensions were authenticity of grain size, authenticity of teacher enactment, and authenticity of enacting the activity in real time. Grossman and colleagues (2009) illustrated these dimensions of complexity, or authenticity, as developing in tandem, but other studies did not (Fernandez, 2005; Nelson, 2011; Ghousseini, 2008; Kazemi et al., 2009; Lampert & Graziani, 2009. Several studies gradually immersed teachers into some types of complexity while they held constant or even decreased other types of complexity in the process. These choices reflect the beliefs of teacher educators about the complexity of each dimension in relation to the activity.

Sometimes one dimension was the primary focus of a learning sequence. In one example, teachers studied a complete instructional activity, authentic in its grain size and time reality, throughout a sequence of learning experiences. The teachers first observed as the instructor modeled a lesson and analyzed a classroom video of the lesson. Their enactment in the teacher role was not authentic but the full complexity of grain size and time reality was authentic. Teachers then planned an enactment of the activity in small groups and took turns teaching the activity to peers, an activity quite high on all dimensions of authenticity (Fernandez, 2005). They repeated this step several times, reflecting together after each rehearsal.

Some deigns not only advance complexity at different rates but also reduce some complexity temporarily while focusing more on others. For example, novice teachers enacted science lessons through a series of four different experiences (Nelson, 2011). In the first activity, peer teaching, novice science teachers rehearsed a short (but complete) science lesson to peers, with live interjections and follow-up analysis. The second, "bite-size teaching," gave novices the chance to enact in turn the beginning, middle, and end of a longer science lesson on three different occasions in their field placements, followed by a brief written reflection. Once novices were comfortable with these shorter pieces of lessons, they gradually worked their way up to the third experience, reflective teaching, which asked them to teach two full-length science lessons in their field placements. Finally, the novices taught daily science lessons in their student teaching placements. In contrast to the previous study, this design suggests teacher educators envisioned authentic teacher enactment as less complex than grain size and time reality. Further, the initial decrease in grain size suggests they find authenticity of grain size the most complex.

In each of three additional studies (Ghousseini, 2008; Kazemi et al., 2009; Lampert & Graziani, 2009), teacher educators first modeled the activity while novice teachers role-played students. The modeling activity limited authentic teacher enactment but provided a more authentic grain size and time reality (Lampert & Graziani, 2009; Kazemi et al., 2009; Ghousseini, 2008). After teacher educators modeled the activity, they moved into an investigation and analysis of what they had modeled. Through these experiences, teachers considered the function of routines, typical student reasoning, and typical challenges as they anticipated how the activity would play out (Kazemi et al., 2009). Teacher educators sometimes gave novice teachers handouts that specified the phrasing and structure of the activity's dialogue (Kazemi, et al., 2009; Lampert & Graziani, 2009). This activity offered more authentic teacher enactment and grain size because novices took on the role of the teacher and worked with a complete classroom activity. Novices continued in the role of the teacher by rehearsing isolated discourse routines, gradually adding new routines until teachers had rehearsed the entire instructional activity (Kazemi et al., 2009; Lampert & Graziani, 2009). The move to rehearsal represents a shift to more authentic teacher enactment, time reality, and grain size. However, one design stipulated that only one pair of students rehearse the activity, which left the remaining teachers to observe (Ghousseini, 2008). Occasionally, novice teachers had the opportunity to teach the activity in their own field placements (Ghousseini, 2008; Kazemi et al., 2009; Lampert

& Graziani, 2009). Figure 6 illustrates the relation between the three dimensions of authenticity in this group of studies. It physically separates the three dimensions for the purpose of clarity, although they all begin and end around the same places.

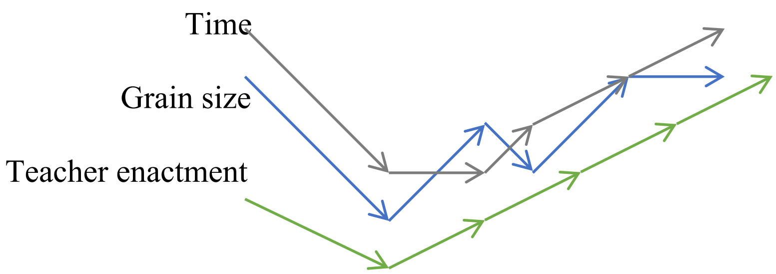

Figure 6. Reduction in all dimensions of complexity for analysis of practice

Similar to the Nelson (2011) design, complexity temporarily decreased when teachers moved to the investigation portion of the sequence. Teacher educators reasoned that modeling the activity first would provide context from which novice teachers could reason throughout the rest of the sequence of experiences (Kazemi et al., 2009; Lampert & Graziaini, 2009; Lampert et al., 2013). The previous studies suggested that teacher educators considered some types of authenticity to be more complex than others. Instead, Figure 6 suggests that teacher educators in these three studies gradually immersed novice teachers into all three dimensions. Teacher educators believed all three dimensions provided complexity that warranted a gradual introduction.

Discussion

Recent studies contribute some promising ideas for how rehearsal designs build novice teachers' expertise in productive classroom mathematics dialogue, although currently this work is still exploratory. Few studies in mathematics teacher education have reported the design and test of rehearsal designs; therefore, findings are suggestive, at best. However, literature from other fields has helped illustrate what the rehearsal design process might look like.

One critique of approximations of practice is that they do not fully represent classroom activity. Yet, the most important learning goal for teachers is to understand *how to learn* from practice, and more specifically, how to learn from student thinking. When teachers engage in the work of understanding the mathematical ideas of others and formulating responses, whether or not it comes from real students or accurate representations of their thinking, they engage in the authentic work of learning how to relate novel ideas to their goals and use successes and failures of dialogic moves and routines to make adaptations and connections to particular forms of student thinking. Rehearsal designers make purposeful choices about the mathematical content of rehearsals so that they can couple particular types of moves and routines to particular forms of student thinking (e.g. Lampert & Graziani, 2009).

When teacher educators consider the extent of content and activities required to teach novices, one challenge becomes fitting so many things into so little time. Rehearsal studies suggest another possible perspective. If the goal is for teachers to learn *how* to learn from practice, the investment in a few carefully chosen instructional activities will do more for novices than trying to prepare more superficially how to respond to everything students might think or say. Moreover, although it may seem that the time investment required to attend to both larger and smaller grain sizes of discussions would be too large, the amount of extra time required to work on both in rehearsal is less than the time it would take to rehearse each type of activity individually.

The most promising designs ask teachers to rehearse component dialogic moves and routines within the core activities they compose. This nested structure helps teachers better relate the hows of practice to the whys because the teachers:

- Learn routine activity structures, which save cognitive resources for use in the more difficult work of responding to students
- Use dialogic moves and routines to respond to typical and novel student thinking and build situational discriminations through successes and failures
- Begin to couple particular types of moves and routines to particular types of student thinking
- Develop conceptual maps that outline typical paths of student thinking and guide teachers' dialogic responses to students
- Resolve spontaneous problems of practice to help teachers further understand the activity and its components in relation to student thinking

The movement between rehearsal and classroom settings, or more and less authentic approximations of practice, generated new issues for teachers that could be explored further. Some designs accomplished this when they coupled rehearsals with enactments in field placements or teachers' own classrooms. However, teacher educators do not always have the luxury of classroom placements in which teachers can practice things.

In many ways, novel student thinking alone can constitute new situations for teachers to explore when classrooms are not available. Teachers add depth to how they understand routines when they draw from and adapt previous successes or failures to new situations. Some evidence suggests that discourse routines can even become coupled with particular forms of student thinking over time. Therefore, rehearsals can provide teachers with some of the same kinds of learning opportunities as classrooms.

Finally, the role of the teacher was the primary learning opportunity but not the only important one. When novices role-played the teacher, they attended to the hows of practice when they tinkered with dialogic moves and routines, while interjections, investigation, and collaboration strengthened the whys. However, role-playing students and participating in collaborative reflection also helped teachers strengthen the whys of practice.

Remaining Questions

The rehearsal studies often deliberately coupled particular types of discourse moves into the structure of the instructional activity in ways that were responsive to student thinking. For example, the requirement to restate a student's hypothesis incorporated teacher attention to student ideas as part of the design of the Conversation Rebuilding activity (Lampert & Graziani, 2009). However, most of the research on rehearsals in mathematics education has focused on instructional activities specific to early number, which leaves plenty of room for contributions in other domains. One might imagine a set of discourse moves that is specific to conversations around definitions in geometry or representational translations in algebra.

Further, rehearsal studies have been limited to preservice teacher education. In light of recent reforms that call for widescale reforms in mathematics teaching, rehearsals are a good candidate for inservice teacher education, as well. Responsive mathematics dialogue is difficult to master by those teachers new to the field, but arguably even more difficult for seasoned teachers, because it requires changes in teaching routines that have become automatized. Even teachers in the Conversation Rebuilding activity who had never taught before rehearsed to "undo" habits that had been built through their history of experiences as students. Changes in practice also require deep transformations of beliefs about students and teaching, instructional routines, and classroom participation structures.

Finally, and most important to the goal of my dissertation, teacher education literature leaves an open space to explore whether and how teachers enact practices differently or similarly across settings, because the rehearsal studies did not attend to this connection. The field would benefit from a better understanding of the mechanisms and common problems of practice embedded in the adaptations that take place across settings.

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CHAPTER III

THE ROLE OF REHEARSAL INTERJECTIONS IN TEACHER MATHEMATICS DISCOURSE

Abstract

I analyzed inservice teachers participating in rehearsals of classroom conversations intended to support the learning of sixth-grade students engaging in collective critique of student-invented ways of representing and measuring variability. During rehearsal, participants enacted whole class discussion by assuming roles of teacher and student, while instructors interjected at various points to provide immediate coaching and suggestions for revision. Employing discourse analysis, I examined how three cohorts, each consisting of four teachers, appropriated and adapted instructors' suggestions as they conducted whole-class conversation. First, I found that teachers made immediate self-corrections to reconcile internal conflict between the forms of discourse they used before and after a rehearsal suggestion. Second, I found that when teachers introduced new types of discourse moves during classroom conversations, they often adapted these moves productively in response to what they noticed about student confusion.

Rehearsals of classroom mathematics discussions

Recent research has turned to the role of deliberate teacher practice to improve the quality of instructional conversations (e.g. Kazemi, Franke, & Lampert, 2009; Lampert, Franke, Kazemi, Ghousseini, Turrou, Beasly, Cunard, & Crowe, 2013). In mathematics classrooms, improving the quality of conversation primarily entails attending to and responding to students' talk about mathematics in ways that create opportunities for students to develop and refine mathematical knowledge (Jacobs, Lamb, & Philipp, 2010). These dialogues are marked by attention to seeds of disciplinary concepts and values in student thinking.

Rehearsal is a form of deliberate practice that is now comparatively widespread (Lampert & Graziani, 2009; Kazemi, et al., 2009; Lampert et al., 2013). During rehearsal, teachers simulate episodes of classroom activity by animating the roles of teacher and students. As they do so, instructors provide immediate coaching and feedback. Coaching and feedback are usually accomplished by instructor interjections during the course of rehearsal. Ideally interjections help teachers understand better the purposes, forms, and functions of particular patterns of talk. For example, an instructor might ask a teacher to re-phrase a question so that students are given a chance to explain their thinking rather than simply provide an answer. In addition, interjections give teachers an opportunity to try new instructional methods through a simulated teaching episode. The resulting successes or failures can then inform their subsequent practice.

Although interjections are meant to help teachers learn how to teach in particular ways, the paradox is that doing things in exactly the same ways might lead to rituals that are insensitive to the perspectives and flow of a classroom conversation. Hence, teachers often must adapt activity in ways that maintain the integrity of the activity yet is responsive to novel or troublesome situations. From a sociocultural perspective (Lave & Wenger, 1991), these adaptations are changes in participation structures that signal learning. Although many forms of adaptation are possible and even likely as a teacher transitions between setting of rehearsal and classroom (e.g., body positioning, patterns of eye gaze, particular use of gestures), I chose to focus on transitions in teacher talk that appeared to be shaped by the interjections made by instructors during rehearsals. This commitment to teacher talk reflected the prominence of talk in rehearsal settings, although clearly other modalities of interaction also contribute to the joint construction of meaning in any conversation (Schegloff, 1987). In this analysis, I trace teacher learning by attending to changes in individual turns of teacher talk, or *discourse moves*, that appear to be in response to coaching, or *interjection,* by professional developers during rehearsals that followed earlier opportunities to learn about the types of discourse moves useful to the conversations around student work. The aim is to trace how the interjections, theoretically one of the most crucial pieces of rehearsal, are appropriated during the rehearsal and adapted and shaped into both subsequent classroom whole-class discussion and subsequent rehearsal.

Conceptual Framework

Discourse as Learning

Discourse, as a means of communication, is often viewed as a tool to aid thinking and learning. An alternative view, and one foundational to this study, is that changes in discourse are tantamount to learning itself (Sfard, 2001). From a sociocultural perspective, discourse varies between rehearsal and classroom contexts because interactions are actively constructed in the "here-and-now" (Gee, 2005). Interactions in each place, and the goals of students, teachers, and PD instructors, change as a result of new ideas that are contributed, problems that arise, and the affordances or constraints of available resources.

Consistent with this view of dialogic thinking, the spoken language of classroom mathematical discourse can function to: 1) make things (ideas, ways of knowing and learning) significant, 2) make connections between things (specifically mathematical representations and ideas), and 3) privilege ways of knowing and participating (Gee, 2005). My characterization of classroom and rehearsal contexts points to ways that students and teachers are positioned through their dialogue during the process of constructing mathematical ideas and ways of knowing. For example, one type of discursive strategy might be *eliciting student thinking*. A teacher asking, "Who can tell us what Star was thinking when she decided to group those values together?", is signaling that a strategy of representing similar values might be important. She is also nurturing a participation norm of attending to the thinking of others. In contrast, a teacher that elicits thinking by asking, "Who can tell us the answer to number 3?" might be signaling that answers and accuracy are most important, and she is privileging participation to students that can provide answers.

Discourse moves. *Eliciting student thinking* and other types of teacher discursive strategies constitute a unit of analysis called a *discourse move*. Discourse moves are individual turns of teacher talk that consist of purposeful statements or questions intended to promote student learning. Their form and function are often dependent on previous contributions by participants in the conversation. Discourse moves position students in relation to the teacher, other students, and content (Gee, 2005). Further, they can constrain student responses in ways that guide students toward mathematical norms and important ideas. For example, a teacher might ask a student to explain a previous student's idea or ask a student to show evidence that supports a claim he just made. The meaning of these moves relies not only on the language of the move itself but also on the context of the conversation, the teacher's goals, and the students' classroom and broader histories.

Discourse in Data Modeling Discussions

Discourse moves of interest in this study were embedded in classroom discussion about student-invented displays and statistics of variability. The aim of the discussion was to highlight mathematical concepts that were often tacit in student inventions, and to relate students' mathematical concepts to disciplinary conventions (e.g. Lehrer, Kim & Jones, 2011). Table 4 illustrates the theoretical intent of mathematical discourse in data modeling discussions as it relates to the functions of discourse suggested by Gee (2005), represented in the first column of this table.

Table 4

Framing discourse in data modeling discussions

Unit: Discourse functions	**1: Displaying Data**	**2: Measures of Center**	**3: Measures of variability**
What does discourse make significant?	• Student conjectures/ noticings • Interpretation of what displays show/hide about data • Tradeoffs of display design choices • Mathematical underpinnings of representation	• Student conjectures/ noticings • Statistics as measures of distribution • Algorithmic thinking (replicability, generalizability) • Tradeoffs of different measures • Features of data that a method values	
What connections does discourse make between ideas or representations?	• Display designs ⇔ features of data • Measurement process ⇔ qualities of displays	• This data set ⇔other data sets • Reasonableness of measures ⇔ qualities of data • Measurement process ⇔ measures of data	
What ways of [student] knowing and participating does discourse privilege?	• Sense-making • Explanation • Evidence identification • Hypothesizing • Revision of ideas • Student-student dialogue • Reflection • Students as authors of mathematical ideas and tools		

Many of these goals and formats of classroom conversation were new to teachers, so teachers were provided tools to support their conduct of productive mathematical dialogue. The primary tool consisted of an ensemble of five discourse routines that served as building blocks for accomplishing the functions outlined in Table 4. A routine refers to a well-structured sequence of discourse moves that participants collectively anticipate will contribute to directing and maintaining particular conversational goals. The routines were: (a) *eliciting student thinking*, (b) *building collective understanding, (c) responding to the hypothesis, (d) making connections, and (e) pulling it together.* Each of these five routines was composed of a set of discourse moves intended to support accomplishment of the goals suggested by the title of the phase. Inspection of Table 5 makes evident that although the structure of each discourse routine was preserved across different curricular units, the specific questions that teachers might pose to orchestrate discussion changed with the focal data practice in which students were learning to participate (i.e., visualizing variability, measuring variability).

Table 5

Data modeling template discourse routines

		Unit 1 (Displaying data)	Unit 2 (Measures of center)	Unit 3 (Measures of precision)
Eliciting Student Thinking	Description	Ask students to provide observations about what another group's display shows or hides about the data and the design choices that made that feature visible/hidden	Ask students to describe and provide observations about the relations between the procedure and the characteristics of the data it uses to find the best guess of the measure of center	Ask students to describe and provide observations about relations between the procedure and the characteristics of the data it uses to find the measure of precision
	Example	"What does this display show us about the measurements?" "How can we see that in this display?"	"What is the main idea behind this method?" "What part of the data does this method care about?"	"What is the main idea behind this method?" "What part of the data does this method care about?"

Building Collective Understanding ("Yes-anding"/making it public)	Description	Help the rest of the class understand the student's observation; clarify or extend thinking	Help the rest of the class understand the student's observation; clarify or extend thinking	Help the rest of the class understand the student's observation; clarify or extend thinking
	Example	"Can you restate that in your own words?" "Where do you see an example of that?"	"Can you restate that in your own words?" "What do you mean by ____?"	"Can you restate that in your own words?" "What do you mean by _____?"
Responding to Hypotheses	Description	Ask the authors to confirm/disconfirm claims about their display; Ask other students to form opinions about the claims	Ask the authors to confirm/disconfirm claims about their measure; Ask other students to form opinions about the claims	Ask the authors to confirm/disconfirm claims about their measure; Ask other students to form opinions about the claims
	Example	"Do you agree with his/her claim that the data shows ___?"	"Do you agree with his/her claim that this method uses ____ to show us the best guess?"	"Do you agree with his/her claim that this method uses ____ to show us the precision?"
Making Connections	Description	Ask questions about tradeoffs between different displays' features in understanding and interpreting data	Ask questions about tradeoffs (including replicability and generalizability) between different methods in relation to different qualities of data sets	Ask questions about tradeoffs (including replicability and generalizability) between different methods in relation to different qualities of data sets
	Example	"Which of these displays makes it easiest to see ____?"	"Would this method give me a good best guess if we had a value here?"	"Would this method give us a good measure of precision if we had a value here?"
Pulling It Together	Description	Make a summary statement that restates "big ideas" and tables ideas that remain unresolved	Make a summary statement that restates "big ideas" and tables ideas that remain unresolved	Make a summary statement that restates "big ideas" and tables ideas that remain unresolved
	Example	"In this display, we can see the extreme values more clearly than in this display because of the way they grouped the numbers. We call that "binning."	"What I'm hearing you say is that this method would give us a result but it might be a good estimate of the best guess when we have extreme values."	"What I'm hearing you say is that this method would give us a result but it might be a good estimate of precision when we have extreme values."

Although some of these routines, such as *eliciting student thinking*, resemble more generic routines that are used in a number of other kinds of instructional conversations, the discursive structure of these routines, and the suggested phrasing, is specific to the intentions of introducing students to practices of visualizing and measuring variability. (See Appendix A for supporting tools provided to teachers for Units 1, 2, and 3 designed to support visualizing and measuring variability).

Chronology of a discussion. To illustrate how the routines might be enacted within a classroom, consider a prototypical classroom conversation about students' invented displays. A typical display discussion uses two to four pieces of student work that take different approaches to the problem of displaying a set of data to make its features visible. The teacher typically begins with the most accessible approach, usually one that focuses on case-values, to ensure that all students can understand and participate in the discussion. The teacher begins with the first display and uses the *Eliciting Student Thinking* routine to find out what students notice. S/he uses these student contributions to highlight the affordances and constraints of the display in making specific aspects of the data set and distribution, such as the extreme values or the shape of the distribution, visible.

The *building collective understanding* routine is used to engage the rest of the class in making sense of what individual students notice and in coming to a consensus on what a display shows well and what it hides about the data. The teacher might return to ideas the students have contributed during the *eliciting student thinking* routine that are more fruitful for discussion than others. For example, an observation about the absence of a title is likely less productive than an observation about the use of tally marks to show frequency.

Once some conjectures have been established, the *responding to the hypothesis* routine directs the conversation back to the authors of the display, who can confirm or correct the conjectures made by the rest of the class about the purpose of the display and the reasoning behind the authors' design choices. However, it is likely that the teacher already knows the reasoning behind these choices after having conferenced with each group during the creation of their displays.

After this same sequence of routines has been run through again with other displays, the *making connections* routine uses these displays to compare and contrast design choices and what they make visible about the data. Looking across two different displays can show how the same piece of information, such as the highest value, is shown differently. The teacher can ask students to trace a cluster of values from one display to another (called tracing in the template for *making connections*). S/he can ask students to imagine a change in some of the values in a particular display, or the addition of one or more new values, and to then consider the effects of these transformations on the shape of the data using the mathematical approach of the designers of the display.

Finally, the teacher summarizes and anchors the ideas that have become consensus in the class in the *Pulling It Together* routine. This establishes a foundation from which subsequent conversation can continue and build while exploring new ideas. Because data modeling instruction typically requires a shift in mathematics talk in classrooms, many of these routines are only just developing, even for inservice teachers.

Each of these five routines employs questions and statements that might be commonly considered "best practice" discourse moves, such as asking whether students agree with an idea or asking what is similar and different about two methods. However, using these questions without considering their service to instructional goals can have detrimental effects on the interaction. For example, consider two classrooms in which students notice that the display is missing a title and that some bars in the display are taller than others. One teacher presses on the first noticing and the other teacher presses on the second noticing, but they use similar "best practice" questions during their conversation. Normally, the use of these questions could indicate good instructional decisions. However, in a data modeling discussion, the teacher who presses on the height of the bars is in a better place to move the conversation toward mathematically sophisticated ideas, such as a bar as a representation of a case-value or as a representation of the frequency of a particular case-value or class of case-values. As part of a larger study of this approach to statistics education, a tool was developed to helped the research team theorize and evaluate the quality of data modeling discussions by mapping categories and examples of approaches to using student inventions in discussion from least to most sophisticated approaches (Jones, 2015, see Appendix C). According to this tool, called a *Construct Map,* the teacher who presses on the height is more likely to reach instructional goals during the conversation and would fall higher in the level 4 category than a teacher who is not selective in which ideas are discussed in depth. Considering the construct map, a teacher who carefully selects two or three displays to illuminate an important mathematical idea is in a better place than a teacher who asks all groups in the class to share their displays, but does little to help students consider how the displays are related. The teacher who asks everyone to present would fall at a level 2 on the construct map, while the teacher who planned more carefully is likely to fall at a level 3 or higher.

The questions and related discourse moves embedded in the template routines, although representative of generically ambitious "best practice" questioning techniques, must be used more strategically in the data modeling conversations in order to reach mathematically productive discussion. Inevitably, they must be adapted or situated in ongoing activity. Hence, it is important to have a lens for characterizing continuity and change in discourse moves within and between settings. In the next section, I suggest a framework for such description with an eye toward employing it to characterize how teachers appropriated instructor suggestions/coaching as they moved from the context of rehearsal to that of classroom enactment.

Framing Consistencies and Changes in Discourse Moves by Characterizing Form-Function Relations

Figure 7 illustrates possible patterns of consistency and change in the form and function of discourse moves. Form refers to the phrasing of the move (the sequence of words in one or more utterances) while function refers to the goal served by the move in a particular context. Here, consistencies are marked by stability in both the form and function of a move. *Adaptations* are signified by stability in the function of a routine but the form, or the general sequence of discourse move types, changes. *Innovations* are signified by preservation of a sequence of discourse moves but their employment to serve a new function. When both form and function change but do not appear to be aligned with the goals of data modeling (see Table 4), the change is unrelated (perhaps a lethal mutation in design research terminology). This framework characterizes change and consistency without losing sight of the interplay between individual teachers and their surrounding contexts, as individual teachers and contexts shape each other (Gresalfi, 2009).

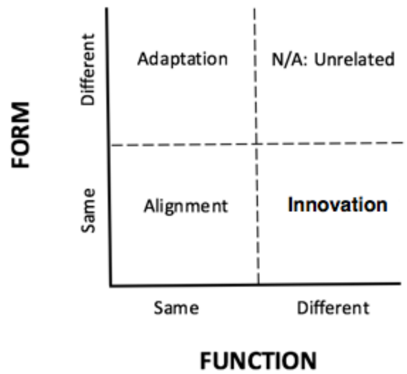

Figure 7. Discourse Framework: Consistencies and Changes in Discourse Moves. Adaptations to a move are indicated by a similar function but different form. Innovations of a move are indicated by a similar form but different function.

I turn now to consider each cell of Figure 7 in more detail.

Alignment. The phrasing of the move is the same when it is an exact or paraphrased copy of another move. For example, "Do you see the difference between these two displays?" would be considered the same as "Tell us whether you notice a difference in the two displays." The function of a move is its enacted role in the larger classroom discussion. For example, the teacher question about the difference between displays is intended to help students make connections between them.

Adaptation. A re-phrasing of the question about differences between displays, "Where do you see a difference between these two displays?" signals a change in form, because the question is now re-structured to demand that a learner indicate a specific region or aspect of difference. This too serves the function of supporting students to make mathematical connections between different types of displays, but it is likely that it will be more effective because a responding student must explain and not simply respond yes or no.

Innovation. The form of a discourse move can be re-purposed to fulfill a new function. For example, the question "What does the use of grouping make it easy to see?" is a move typically associated with the *eliciting student thinking* template routine. However, teachers also used these types of questions during the *making connections* routine. For example, while comparing two displays exhibiting different shapes for the same data, a teacher might ask, "And what did the use of grouping make it easy to see in this one?" Even though the phrasing of the question is nearly identical, the teacher's intention now is to highlight how grouping similar values influences the shape of the data. This is a shift in function.

Unrelated. Last, the form and function of a discourse move can change in ways that do not align with either the forms or functions of the discourse routines. For example, during the eliciting routine, a teacher could change a question to a statement, such as "this display groups values," with the intention of relating this feature of the display to the presence or absence of a title. These transformations to form and function would not be considered relevant.

Rehearsal as a Support for Adaptive Change

Successes and failures that teachers experience when solving instructional problems in new contexts provide depth to their understanding of content and pedagogy. Rehearsals provide simulated classroom situations from which instructional successes, failures, and revisions of discourse moves can take place. Rehearsals give teachers opportunities to construct specific instructional situations from which general discussions of teaching practice can take place and inform the remainder of the rehearsal as well as future practice.

One goal of rehearsal is to provide a place to change the form and function of discourse moves before teachers employ them with students. Rehearsal can reveal teachers' understanding (or misconceptions) of discourse moves in context before they are appropriated in the classroom. For example, a novice teacher in Lampert and colleagues' study (2013) explained the language of the number line during rehearsal, as she had interpreted it from the teacher educators. The rehearsal revealed that the teacher's explanation was so thorough that it did all the intellectual work for the students, contrary to another goal of keeping students involved in the intellectual work (Lampert et al, 2013). Rehearsal provided the space for this misappropriation to be uncovered and revised in training before entering the classroom. Had she not rehearsed, she might have done too much of the intellectual work for the students in her own classroom. Further, she would have reported back to her methods class that she had "explained the number line" as required in the materials, even though her enactment was not faithful to the intent of the activity's goals. The revision of such misappropriations during rehearsal is one example of an instance of learning. The teacher came into the rehearsal with an interpretation of the meaning of an activity she was expected to use. The refinement made following the interjection during rehearsal represents a change in the way the teacher understood the routine. This teacher might recognize similar situations to use the number line routine in her classroom and be able to draw from her rehearsal experience when interacting with her students.

Learning through a cycle of successive enactments. Rehearsals are based on a premise that learning through successive moments of activity does not only happen within one setting, but also across PD and classroom settings. Lampert and colleagues (2013) found that relating specific aspects and variations of practice to particular students or mathematical goals only became salient for novice teachers over the course of multiple instances of both rehearsal and classroom discussions. For example, novices began to learn which aspects of an instructional activity were fairly "routine" and which aspects were responsive to what students know and what they needed to learn. Like the design of Lampert and colleagues (2013), rehearsals in this study are embedded in a larger iterative cycle of observation, analysis, planning, and reflection (Figure 8).

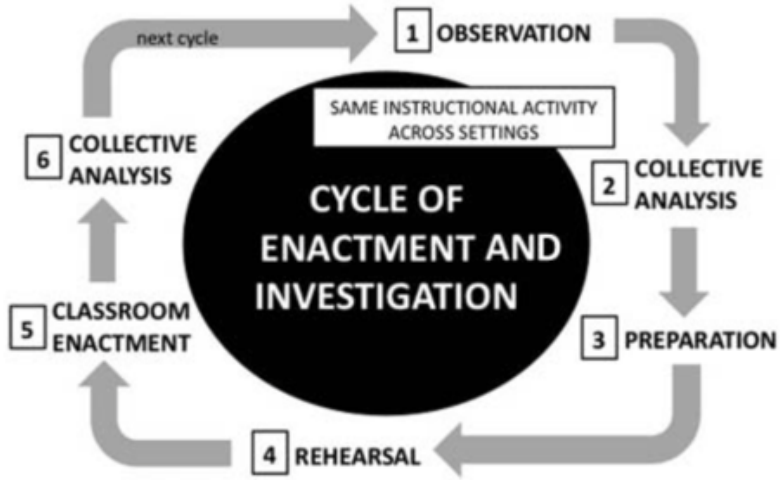

Figure 8. Description of where rehearsals are embedded in the larger cycle of enactment and investigation. Adapted from "Keeping it Complex: Using rehearsals to support novice teacher learning of ambitious teaching," by Lampert, M., Franke, M.L., Kazemi, E., Ghousseini, H., Turrou, A.C., Beasley, H., Cunard, A., and Crowe, K., 2013. *Journal of teacher Education, 64(3),* p. 229.

Cycles of enactment and investigation are based on a premise that teachers add depth and flexibility to their understanding of practice over the course of many instantiations of these activities across PD and classroom settings. In this theory of learning, a single instructional activity is taken through the cycle, and each phase of the cycle focuses on the same activity. The activity of focus in my study is a set of discourse moves used in the data modeling curriculum that teachers were learning.

Horn's (2010) study of a high school mathematics department provide further insight into teacher learning that results from many opportunities to situate aspects of practice between classrooms and collaborative workgroups. Horn identified two particularly important forms of discourse, replays and rehearsals, that created spaces for teachers to learn about teaching practice. Replays provided accounts of specific past, and often problematic, classroom episodes for further group analysis. Teaching rehearsals represented more generalized and often anticipated accounts of practice. Moving between replays and rehearsals in a single conversation helped teachers think about how to reframe and reorganize their teaching activity. For example, over the course of a single conversation, a teacher's description, or replay, of a past classroom episode, changed in response to a new question that a colleague asked of her, and the space provided for her to reflect in a new way on the same event. Her reconsideration then suggested pedagogical revisions, played out as she initiated an impromptu rehearsal of the conversation (Horn, 2010). She moved from a specific event to a general reflection, and then back again to a reframed specific event. Therefore, differences between contexts, such as novel student contributions or classroom management problems, offer learning opportunities for teachers to adapt familiar discourse structures in slightly different ways. In fact, novel student thinking can support changes in teachers' mathematical understanding (Seymour & Lehrer, 2006). As these understandings change, the ways teachers interact with students might change in turn in patterned ways, just as the teachers in Horn's workgroup did. In theory, a new diagnosis of student understanding might prompt a different, more productive response the next time the teacher experiences a similar episode in the classroom, but this particular question was outside the scope of Horn's study. The approach to studying rehearsal through many instantiations of both rehearsal and classroom activity reflects phenomena like these that are critically shaped by two different contexts and the relations between them. The cyclical nature of professional development as illustrated in Figure 8 ensures many opportunities to situate discourse moves in both PD and classroom settings.

For illustrative purposes, consider the following three trajectories of influence between rehearsal and classroom enactments below (Table 6), beginning with an instructor's suggestion to ask hypothetical questions to contradict a student's overgeneralization. These are just a few examples of the ways successive enactments might influence discourse.

Table 6

Sample trajectories of rehearsal-classroom activity

Trajectory	Explanation	Example
$R_1 \rightarrow C_1 \rightarrow R_2$	Rehearsal activity influences classroom practice, and the resulting change in classroom practice influences activity in the next rehearsal.	R1 C_1: The teacher asks the students a hypothetical question in the context of "partner talk," and many groups provide responses R_2: The teacher asks other types of questions in the context of "partner talk" during the next rehearsal
$R_1 \rightarrow C_1 \rightarrow C_{1\ or\ 2}$	Rehearsal activity influences classroom practice in an iterative fashion, and the resulting change of the first classroom instance further influences subsequent activity in the classroom (perhaps the same classroom enactment, perhaps the next one).	C_1: The teacher asks a hypothetical question $C_{1\ or\ 2}$: A student asks a hypothetical question to another student
$R_1 + C_1 \rightarrow C_{1\ or\ 2}$	Classroom activity reflects features of influence from two past but somewhat unrelated experiences together	C_1: Students have not been responding to any questions the teacher is asking $C_{1\ or\ 2}$: The teacher introduces a hypothetical question but changes the form to yes/no as a scaffold

For each of these trajectories of change, both settings are necessary, but not individually sufficient to influence change in practice. Each of these trajectories is dependent on the influence of both settings uniquely.

I am interested in how the form and function of discourse moves change over the course of trajectories like these in subsequent enactments. With this emphasis on teacher learning as signaled by continuities and changes in discourse moves, the conduct of my investigation was oriented by the question that follows.

How do teacher discourse moves change through rehearsal suggestions into subsequent enactments?

Method

Setting and Participants

Data were collected in two different contexts of professional development (Table 7). The first, Cycle 1, was a Masters class conducted during the fall semester of 2011 at a private university in the southeastern United States. The program of study placed middle school teachers at struggling urban schools while earning their degree, and it was designed to teach innovative ways to strengthen their knowledge and practice in the content areas they taught. Teachers were selected following a competitive application process and earned their degree free of charge, funded by the state's *Race to the Top* money. Each cohort took common classes around urban education topics, and each teacher took additional classes in their subject specialties (in this case, mathematics). Three of the teachers in the class were part of the urban Masters program, and one was completing a traditional Masters program straight out of her undergraduate program but was interested in this mathematics class. This teacher, Carina, was paired with one of the practicing teachers, Abby, in her classroom to co-teach the lessons. The four teachers in this class made up Cohort 1.

The second context was an experiment testing the efficacy of the Data Modeling curriculum and professional development model across four districts in an urban area in the southwestern United States. In the first year, 22 schools participated, and in the second year, 39 schools participated. Schools were randomly assigned to either the Data Modeling (treatment) condition or the practice-as-usual condition. 6th grade teachers in these schools were the targets of the experiment. Teachers in the Data Modeling condition received the professional development immediately, which consisted of three major components: The Data Modeling curriculum materials, professional development, and in-class coaching. Teachers in the practice-as-usual condition received curriculum materials and professional development after two years. While my study was not concerned with the experiment itself, I observed two cohorts (year 1, 2 of the experiment) of 4 teacher participants from the pool of treatment teachers in the experiment. Treatment teachers attend multiple workshops together over the course of the year, as listed in Table 7.

Table 7

Participants and PD schedule

	Cycle 1 (Cohort 1)	**Cycle 2 (Cohort 2a)**	**Cycle 3 (Cohort 2b)**
Total Teachers	4	19	38
Case Teachers	4	4	4
Experience (years)	0-4	4+ years	
Hours of PD	2.5 hours weekly (30 total)	32.5 hours (5 days) in summer + 8 hours 4-5 times during year (total)	39 hours (6 days) in summer + 8 hours 4-5 times during year (total)
Number of rehearsals	1-2 individual (Units 1-2)	2 group rehearsals (Units 1, 3)	3 group rehearsals (Units 1-3)
Number of classroom observations	3-5 Display/Measure Review discussions		
Classroom coaching	Co-reflection after each observation	Co-reflection after each observation	Pre-planning, co-teaching as needed, co-reflection 4 times a year

Participant selection. All four of the teachers in the Urban Masters class participated in the study as Cohort 1 (Table 7). I selected Cohort 2a teachers through observation during the week of summer training, which was my first interaction with them. I looked for teachers of varying experience levels, beliefs about mathematics instruction, and involvement during the training. In the following year, I selected a cohort of teachers (Cohort 2b) that all taught at the same school. While this was a convenience sample in part, I was also interested in informally observing the nature of collaboration between the teachers, if any, for purposes outside the scope of this analysis. Teachers who began the training in the first year of the project and continued into the second constitute Cohort 2a. They were also referred to as "veteran teachers" during the second year. Teachers who began the training in the second year of the project constituted Cohort 2b and were referred to as "new teachers." I selected the following teachers in each cohort.[1]

Cohort 1: Abby, Carina, Emma, Kristine

Cohort 2a (Cycle 2): Adam, Jill, Aspen, Lissa

Cohort 2b (Cycle 3): Amanda, Marissa, Rob, Chad.

The veteran teachers participated in a slightly different summer training than new teachers during Cycle 3, but all teachers attended the same follow-up workshops together during that school year. Instructional coaching took place in teachers' own classrooms about four times during the year. Coaches were teachers (or former teachers) who had taught the data modeling curriculum in their own classrooms before this project began, most of whom were from another locale. I took on the role of coach for the teachers I worked with in Cohort 2b. Coaches co-planned with teachers, co-taught with them as needed, and debriefed with them. The lessons that coaches worked on with teachers depended on the individual teacher's needs.

Professional Development

Professional development (PD) in this study was embedded in a larger instructional cycle around the data modeling conversations, including observation, investigation and analysis of

[1] Teacher names are pseudonyms.

practice, rehearsal, and classroom instruction (Figure 8). PD sessions were conducted generally by having teachers participate in the same measurement, invention, and discussion (as students) in which their own students would participate. After the teachers participated in the activity, we took time to make sense of the mathematical content together, in more depth than the teachers would require of their students. The additional depth was intended to serve as a resource from which teachers could make instructional decisions about when and how to make connections among ideas. In theory, learning the mathematical content strengthens content knowledge and pedagogical content knowledge that is drawn upon during the activity. For example, understanding the difference between precision and accuracy of measurements helped teachers understand and respond to a student's claim that the range of a set of measurements is a useless statistic because its value is nowhere near the value of the true measure. After doing math together, we discussed the discourse moves and routines that form the structure of the conversations the teachers would later orchestrate in their classrooms. We discussed categories of moves such as translations and transformations in the *making connections* routine, including their function in conversation as well as examples of what they might sound like. Figure 9 shows how teachers put all of this together using a planning template I created to map the student work, key points, and key questions that would form the basis of their conversations. Similar planning templates were created for each unit, and each one looked slightly different. The full set of planning templates can be found in Appendix E.

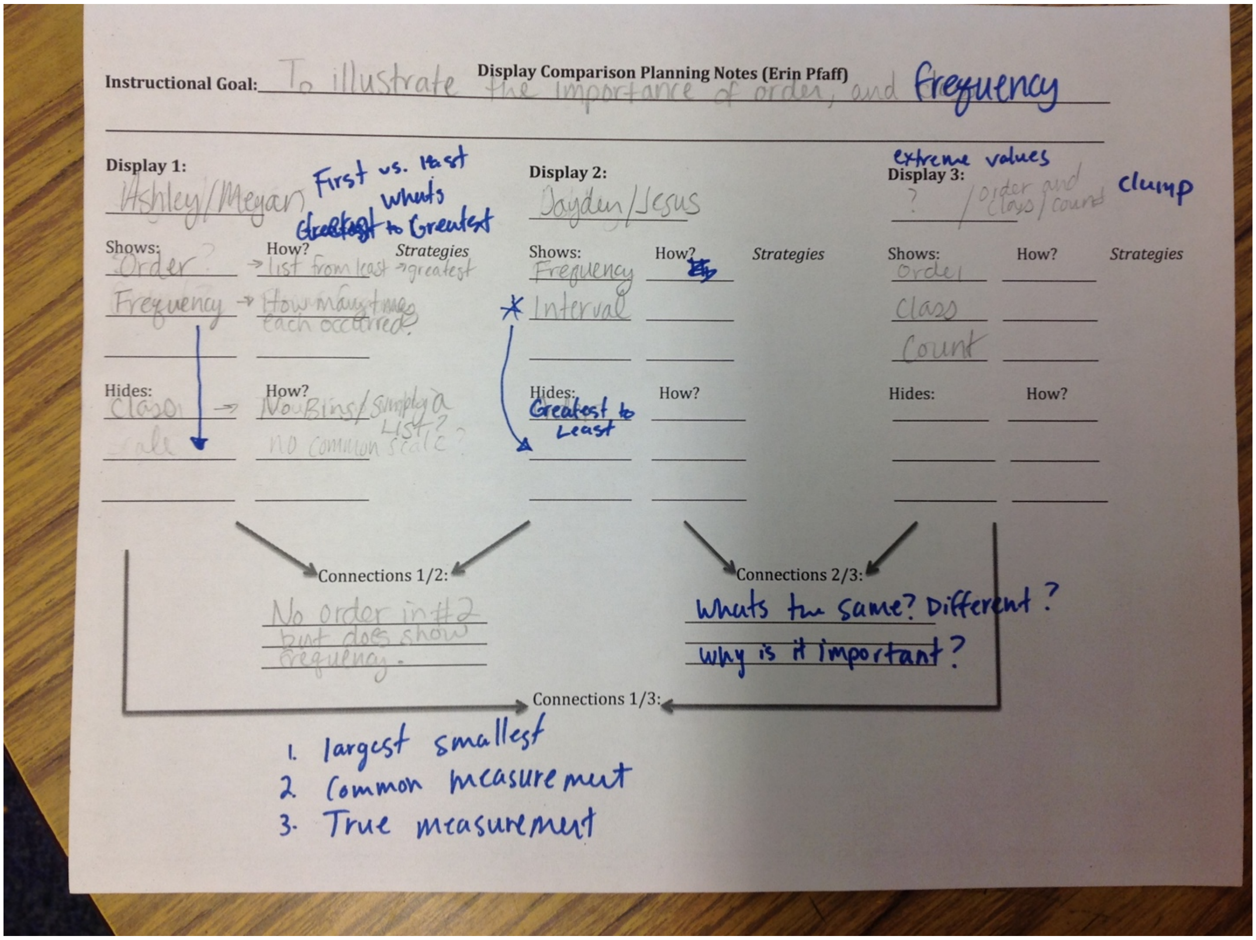

Figure 9. Display Comparison Planning Sheet. Teachers used these templates to plan both rehearsal and classroom discussions. Each sheet is intended to guide and cue teachers to key points and how to elicit them using the student work they have chosen.

Rehearsal. Finally, the teachers took turns rehearsing the discussions, either individually (Cohort 1) or in groups (Cohorts 2a and 2b), while the other teachers and instructors role-played students. During rehearsal, teachers were asked to choose student work to highlight from a corpus of student work provided by instructors and take on the role of the teacher individually as they used the student work as the basis of the conversation.[2] As part of the preparation for rehearsal, teachers were given discourse moves sheets that mapped categories and examples of discourse moves within each of the five template routines (Appendix A). Further, teachers were

[2] Cohort 1 teachers rehearsed with their own students' work during Unit 2 rehearsals.

provided with a planning template to prepare their discussions that mapped the key points about each piece of student work and how it could be connected to others through discourse moves to serve the teacher's instructional goal (Appendix E). Teachers worked in groups to plan, and instructors provided assistance and questions to guide their thinking .Peer-teachers portrayed students who authored the various displays. Peer-teachers engaged as they believed students might, guided by the ways their assigned pieces of student work appeared to reflect particular types of thinking, values, and commitments. Reasoning about the activity as both students and teachers was intended to support teachers' anticipation and reasoning of problems of practice by putting them into the position of the students (Nelson, 2011; Kazemi et al., 2009).

In the example of rehearsal below, a teacher, Kristine (*T* in the transcript) is rehearsing a Unit 1 discussion to compare student-invented data displays. She is coached on how to use gesture in coordination with eliciting evidence from students (Figures 10-13). This suggestion takes place right after a student (*S* in transcript) has made a claim about what the display shows. Another teacher is role-playing the student here. The instructor's interjections (*I* in transcript) are bolded.

1 S: Umm, it shows us what every person measured. It shows us the measurement that each person had.
2 T: OK, so this shows us that each person had a different measurement, is that what you're saying?
3 S: Yes
4 T: OK
5 I: Where do you see that?
6 S: Well some of them were the same, but each person had their own measurement.
7 T: OK, how do you -
8 I: Yeah, where do you see that? Where do you see those different measures?
9 S: Ummm - I'm interpreting the numbers on the bottom are different people. Person 1, person 2, person 3
10 I: So the teacher might actually point to that as you're saying that, so like you can actually say, "Down there, right,

Figure 10. Kristine points to values on the display in accordance with the instructor's language "so like you can actually say 'Down there, right'"

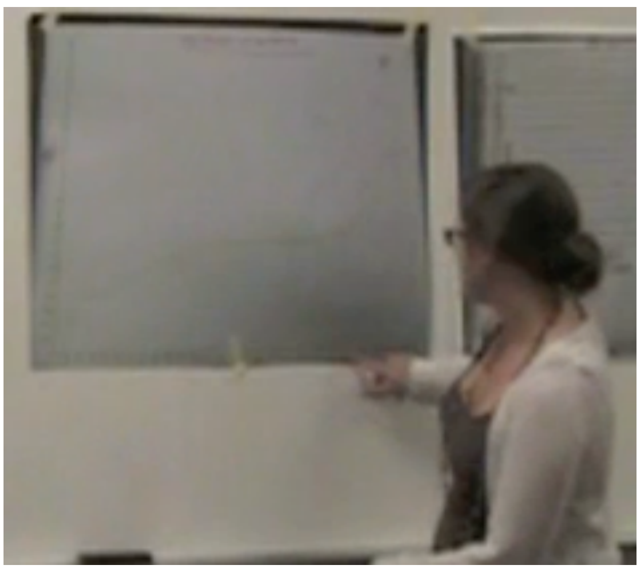

Figure 11. Kristine continues to point to values as the instructor models the form of the discourse move used to accompany her gestures.

11 ***I: and then up here, and then is that right?"***

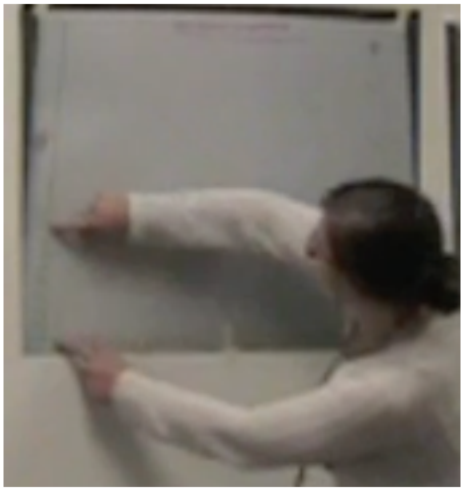

Figure 12. Kristine continues to point to values as the instructor models the form of the form of the discourse “and then up here, and then is that right?”

12 ***I: And so this is - and so you might point to a particular one and say –***

Figure 13. Kristine continues to point to values as the instructor says “and so you might point to a particular one and say -”

13 T: So person 9 had about -
14 ***I: Yeah***
15 T: 178 maybe.

Here, the teacher (Kristine) works through the task of building collective understanding about a student’s hypothesis that the display shows individual measurements clearly. The teacher uses a clarification move (turn 2) by repeating the student’s hypothesis, but the instructor interjects to suggest going a bit deeper for the benefit of others in the class (turn 5). Her suggestion promotes some interaction between the teacher or student and the display to show evidence of the claim on the display.[3] She suggests a norm for evidence-based reasoning: using the display as a text from which ideas can be supported or contested. The teacher's move is taken up first as a model of teacher talk, as the student responds directly to the instructor's turn of talk (turn 6). The instructor interjects again (turn 8), suggesting the teacher point to individual measurements on the display as the student mentions them. In this case, the interjection is taken up by the teacher through a revision in response to the student's contribution (turns 13-15).

Instructors made interjections during the rehearsal to coach teachers through their instructional decisions and use of discourse moves. Each rehearsal concluded with a short debrief, where the reasons for interjections, as well as questions posed by teachers, were discussed collectively. Teachers brought expertise of their own schools and classrooms to bear on these discussions. Instructors brought their knowledge of data modeling classrooms more generally, as well as their own teaching experiences. Through my position as a classroom coach, I was also able to initiate conversation around what I observed as common problems of practice among the cohorts of teachers, as a platform for discussing the different ways teachers responded or innovated. The instructors served a dual purpose in these conversations, with an eye to teacher learning but also an eye to the design of PD and the data modeling curriculum materials.

Researcher Role

I want to clearly describe my role in this project, as I was a member of different communities, each of which called for different roles. Each role provided unique perspective on my data collection, and each was reflected in the analysis. I tried to distinguish between the pieces of analysis that capitalized on each.

[3] This interjection concerns both teacher dialogue as well as gesture, but I focus only on dialogue in this analysis.

First, I was a member of the research team for the larger data modeling experiment as well as this smaller project. This means my history with the data modeling curriculum included portions of the design research, curriculum development, and assessment development. As a researcher, my primary goal was to account for the learning that took place as teachers participated in the PD and used the data modeling instructional system in their classrooms. More specifically to this analysis, I was concerned with the role of rehearsals in this learning. As an analyst, I tried to understand what was happening through the constructs I identified as central to my research questions. Because I took a sociocultural perspective on activity, I also wanted to understand interactions from the perspective of a member of both teacher communities and classroom communities (Schegloff, 1997). My analysis attended to both of these perspectives in making sense of rehearsal and classroom discussions.

Second, I was a member of the professional development team for the project. I interacted with other members of the PD team, who were current or former teachers of the curriculum in their own classrooms. They were not members of the research team. The PD team was responsible for designing and leading the PD, both during the summer as well as follow-up sessions during the school year. My primary goal as a member of this team was to provide teachers with opportunities to make sense of the mathematical content and the instructional demands surrounding the data modeling curriculum. We tried to be responsive to teachers' needs, meaning that sometimes the PD was revised "on the fly" as necessary to meet these needs. I personally designed and implemented the rehearsal piece of PD, with support from the PD team. These included templates that I designed to support particular discourse routines, which I describe more fully later.

Third, I was a classroom coach for some of the teachers I worked with. Specifically, I was the primary instructional coach for Cohorts 1 and 2b. In addition to my personal research goals, I was responsible for planning lessons with teachers, co-teaching (as needed) during some lessons, and reflecting and planning with teachers after each classroom coaching session. My primary goal as coach was to provide the right amount and right form of support to teachers on an individual basis, as needed. Another important, but secondary, goal in this role was to make sure the data modeling discussions were useful to students in the ways they made sense of the content at a collective level. This meant that sometimes I interjected with questions for the class or took on bits of the instruction myself. For instance, when a student made an important statement that the teacher did not attend to, I interjected to return to that student's idea and ask for responses from other students. In other cases, teachers explicitly asked me for direction, especially when they were having trouble communicating an idea with the class.

These roles and goals sometimes came into conflict. For instance, my role as researcher came into conflict with my role as a member of the PD team in the coordination of rehearsals during PD. Because I wanted to document my teachers rehearsing particular lessons, I often advocated to the PD team for rehearsals to take place more often than they had planned. In some cases when the PD was running short on time, the PD team wanted to shorten the allotted time for rehearsal and ask only one group of teachers to rehearse in front of the rest of the group. I successfully advocated for my case group of teachers to be the group selected, even though the team had intended different selection criteria to be used.

Other times, my role as classroom coach came into conflict with my role as researcher. For instance, because each teacher received individual coaching, this means that some teachers received much more support than others. I wanted to document each teacher's instruction following their rehearsal, but for the teachers I was coaching, this meant that I also had responsibility to make sure the discussion provided powerful learning opportunities for students. With the first cohort of teachers, I did not interject to co-teach. I was not assigned as coach for the second cohort, so each of those teachers received additional support from someone else from the PD team during some of their classroom instruction. For the third cohort of teachers, I was assigned as coach and did co-teach during some of their discussions as needed. This means that simply because of the varied nature of my involvement and the involvement of others from the PD team, a comparison of teacher discussions might show some as more "polished" than others. This is why my analysis looked at a series of enactments for individual teachers and examined what teachers did in relation to the surrounding context, to the extent I was able to document and characterize it.

In other ways, I see these roles as complementary. Each of these roles gave me a unique lens through which to make sense of what is happening in PD and classrooms. Therefore, each was visible in my analysis. My role as researcher was especially helpful in providing a theoretical account of what I saw happening. As a member of the PD team, I had unique insight into the thought behind the design decisions made both in advance and on the fly and how those decisions were intended to support teacher learning. My role as classroom coach was especially helpful in contributing a localized explanation of what was happening from the perspective of the individual classroom communities. I did my best to merge these different perspectives into my analysis while maintaining the individual perspectives of each.

Data Sources

The data I used to answer my research question, "How does teacher discourse change following rehearsal of data modeling discussions?" came from the video-recorded rehearsals and classroom enactments of the rehearsed discussion, corresponding field notes, and content logs. I performed line-by-line coding of each turn of talk in each of the rehearsal and classroom discussions to categorize discourse move types. Teacher interviews, while not a primary data source in this analysis, were queried as needed in search of supporting or disconfirming evidence for conjectures I made about teachers' goals during instruction. For example, I asked teachers about their decisions behind specific questions that they asked during the discussion (See Appendix B for interview protocol). Data includes all of the 12 teachers across the three cohorts. While I did not have strong illustrations of teachers' practice prior to the Data Modeling PD, I drew upon teacher self-reports and baseline rubric-based assessments of Cohort 1's classroom teaching that were required for their program.

Analysis

A focus on suggestions. I decided to parse the data into a smaller set so that I could explore select phenomena more deeply. To do this, I categorized each type of interjection into different categories based on what kind of work the instructor was doing. Of these, I narrowed my focus to instances of *suggestion*, which was the most frequent type in the data set. Table 8 shows each category of interjection that I found and the frequency of each.

Table 8

Categories and frequency of interjection types

	Cohort 1						**Cohort 2**			**Cohort 3**			**Total**
	K		**A**	**C**	**A, C**	**E**	**An**	**J, Au, L**	**All**	**All**			
	U1	U2	U1	U1	U2	U1	U1		U3	U1	U2	U3	
Model Teacher	1	1	1	1	0	1	0	0	3	1	1	3	13
Model Student	0	2	1	2	0	0	15		0	0	0	0	20
Praise	3	1	6	4	1	0	4		2	0	0	0	21
Suggestion: Taken up	3	2	7	13	0	5	11		5	13	10	7	76

Suggestion: Not taken up	0	0	4	1	2	4	8	3	9	3	3	37
Question	0	0	2	0	1	3	2	0	2	0	0	10
Total	13		46			13	53		52			**177**

Some of these interjection types were not useful to my analysis. Of the interjection types in the table, two types (*Model student* and *Questions*) proved untraceable and therefore unproductive for this analysis. For example, questions such as "What is your goal in asking that question?" could not be traced in any meaningful way into subsequent enactments. Further, *Model student* interjections did not appear to influence the classroom practice of teachers. In theory, representations of authentic student thinking can benefit rehearsal because teachers can face the same types of struggles that they would in the classroom; however, in practice these struggles were masked by a number of factors. In some cases, authentic student ideas were offered alongside other less authentic contributions, leaving the teacher to choose which idea to pursue instead of requiring them to address particular ideas. Authentic student ideas, typically submitted by instructors, tended to be more simplistic (e.g. A student notices a key or title on the display) than more sophisticated ideas submitted by teacher-students (e.g. A teacher-student notices the gaps in the data). One purpose of instructors representing these forms of thinking was to help teachers cope with these likely but more unproductive forms of thinking, but teacher motivations were not driven by the same purpose, nor were they informed by previous instruction of this nature as instructor contributions were. Therefore, teachers did not often press on the authentic responses because they were not as mathematically productive to the discussion. Second, even when authentic student contributions were offered, they often appeared during the first phase of discussion, which emphasizes an initial gathering of ideas rather than extended press and connection. Teacher responses to student thinking during this phase are often independent of the mathematical content of the idea. For example, a teacher might ask another student to restate, ask them to discuss the idea with their partners, or ask them to clarify their thinking. Therefore, teachers generally did not handle these ideas any differently than less authentic ones. Interjections that offered praise appeared to align well with subsequent practice, but the argument could be made that teachers would have used them anyway in subsequent practice, even without the praise or even the rehearsal itself. Further reflection and study from the instructors' point of view might illuminate the types of discourse they found worthy of praise, especially if the influence of praise spreads to other teachers as well.

Suggestions made sense as an analytical focus for a number of reasons beyond the elimination of the other interjection types. First, suggestions proved to be the most common type of interjection (Table 8). This finding was consistent with previous studies of rehearsal that emphasize suggestions for revision. Lampert and colleagues (2013) documented "directive feedback" as the most common type of interjection. Kazemi and colleagues described the work of the Teacher Educator (TE) as stopping the "action" to provide suggestions for revision (Kazemi, Franke, & Lampert, 2009). Second, suggestions were the best candidates for the type of evidence that could answer my research questions about the role of rehearsal in subsequent activity. While some suggestions simply directed teachers toward next directions for the conversation, most suggestions were provided in response to a discourse move that needed to be revised. Because I did not have a thorough account of what teachers' practice looked like before PD commenced, these "former" enactments of practice that preceded suggestions provided some valuable information about teachers at that point in time. Like a teaching observation, rehearsals provided insights to teachers' assumptions and beliefs about teaching, as well as their conceptions of best practice and what it looked like, in ways that analyzing or discussing practice could not (Ensor, 2001). Therefore, each episode of rehearsal interjections provided a kind of proxy for a teacher's "pre-rehearsal" teaching practice, which I could then situate in the larger developmental trajectories that I was interested in portraying. Many of these "former" enactments of practice suggested either teacher-directed or student-centered "show and tell" approaches to mathematics instruction. When teachers made choices that conflicted with Data Modeling instructional goals, instructors used suggestions to re-align teachers with the intended goals in the context of the teaching situation at hand. The hope during rehearsals was twofold: 1) that teachers might respond similarly in the future when faced with similar situations and 2) that building an understanding of the form and function of discourse in a simulated context might strengthen understanding about how the practice could be useful or adaptable to other situations.

After narrowing my focus to suggestions, I found that even some types of suggestions proved difficult to trace. For example, one interjection asked a teacher to increase the frequency of a move:

> *16 T: … What information can you see about the measurements with this display?*
> *17 S: That numbers here and numbers here.*
> *18 T: OK, so if I have this correct, you can see the numbers on both sides of the axis?*
> *19 S: Yes.*
> *20 T: OK, what else?*
> *21* ***I: "Press on that a little bit more, about those numbers. What else might you ask about that?"***

While instances of press could be identified through the coding scheme, determining whether a teacher's press constituted "more press" than they would have used otherwise was not an interjection my coding scheme could trace reliably. These few suggestions were not traced during analysis.

Focusing on suggestions around spoken language. The focus of my analysis is on the form or function of discourse moves, which take the form of spoken language. Sometimes instructors made suggestions around a teacher's use of gesture or material resources in addition to language. I consider these meaningful aspects of a teacher's practice in data modeling but eliminated them for the purposes of this analysis.

Coding types of suggestions. Next, I further categorized the remaining suggestions according to what type of work the teacher was asked to change about their discourse. I used the dimensions of form and function (Figure 7) to inform my categorization scheme. The types of categories I generated are listed and described in Table 9. The first type of suggestion simply asked teachers to use a move (or not) to at some point during the discussion. These are what I consider "best practice" moves that can be useful in any number of situations. Only instances of alignment of form and function were relevant to this category. They included the use of moves like restating and partner talk. Next, two types of suggestions concerned the function of moves in the discussion. I counted these as work on the function of a move because the instructor's coaching around the move concerned an aspect of its function rather than anything. For example, one of the instructor's suggestions in Table 9 concerns the timing of a move in the context of the larger discussion. The form of the move was not a concern for the instructor, but its timing was problematic and warranted a suggestion by the instructor. A final category concerned the phrasing of moves, irrespective of their context. I categorized these as suggestions around the form of a move.

Table 9

Categories and descriptions of instructor suggestions to teachers during rehearsal

Type of suggestion	Aspect	Example
Suggestion to use a move	N/A	"And then at some point you can go back to either the author or someone else: "Do you agree that that would be a good way to pick?" or you know "(inaudible) more possibilities?"
Suggestion to use a move at a different point in the discussion	Function: Context	"You wanna start with one. Just start with one. Pick your first one. Pick the one you think is easiest for those kids to understand."
Suggestion to use a move in response to a particular situation	Function: Context	"Now, make some connections back to the first ones, when they make a hypothesis about what this one can show.

		'Can we see that here in this other one? Where do you see it in the other one?'"
Suggestion to phrase a move in a particular way	Form	"Oh - again, you're being too specific. You wanna just point to the thing and say 'What does this one show about our measurements?'"

Coding types of discourse moves. Next, I coded the suggestions in all the categories above by categories of discourse moves they addressed. This phase of coding helped me later to match the types of moves in rehearsal suggestions to the types used in subsequent discussions so that I could then analyze the form and function of the moves further. I generated the codes through a combination of focused and open coding. Open coding reflected an emic approach, where meaning was derived subjectively from the perspective of the participants in interactions. Focused coding examined PD and classrooms using the a priori framework of the discourse routine templates, identifying types of discourse moves taught to teachers as part of the data modeling curriculum. This dual coding scheme provided better insight into the relation between discourse moves and contextual specifics for the purpose of characterizing instances of change in form and function of moves. Even though I had narrowed my data to the moves worked on during rehearsal suggestions, I had to code the entire corpus of data following the first rehearsal because I wanted to trace the moves into any and all subsequent instances, which potentially included any point in a future rehearsal or classroom discussion.

Open coding. I began by depicting an account of the classroom context using a grounded theory approach (Strauss & Corbin, 1990), where analytic categories came directly from my data through induction (Charmaz, 2001; Emerson, Fretz, & Shaw, 2011).

Chunking into episodes. I used field notes (classroom only), video, and transcripts of video during this process. As I watched the video and referred to my field notes, I made content logs of transcripts that bounded the conversations into episodes, or chunks of talk that were conceptually related in some way. In most cases, each episode contained a series of turns of talk related to a mathematical idea being discussed within a single method or pair of methods. In these cases, an episode included all the press related to a single idea before changing topics. For example, a teacher often elicited several noticings about a single display, and the follow-up exchange about each one constituted a different episode. Teachers typically initiated new episodes because they guided the topics of discussion. However, in a few cases, a teacher solicited several responses about what can be seen in a student's display without discussing any in depth. The motivation of the teacher is to gather responses and determine which ones might be fruitful to the discussion. I also counted this entire elicitation sequence as a single episode.

Characterizing turns of talk. Specifically, I defined turns of talk as the entirety of one participant's contribution before another participant began. In the case of overlapping talk, I identified the entirety of each participant's contribution individually to the extent possible. Chronologically, a single turn of talk that began before the overlapping talk still counted as occurring before any of the overlapping talk began. The overlapping talk (if audible) counted as a subsequent contribution. Cases in which many overlapping turns occurred simultaneously were chunked into one single turn of talk together and attributed to "students" more generally. For example, a choral response to a teacher's question was often coded as "yes/no/ummm" in the student column. When the students began answering before the teacher finished, the choral response was still coded as a student contribution that followed a teacher's contribution.
More than one code sometimes applied to a single turn as well. For example, a long teacher contribution often included at least a revoicing move and a new question, if not more. Open coding at this level allowed me to relate individual turns of talk to the move immediately following it and the episode more broadly.

Memoing to develop common themes. In the initial sweep through my data, I generated many codes. Using theoretical memoing (Glaser, 1998; Strauss & Corbin, 1990), I developed the themes around the core processes that characterized participant interactions in rehearsal and classroom settings. Memoing also helped me keep track of themes that might be related in different instances. In some cases, my memos assisted me with re-coding data after generating new codes.

Characterizing form and function. Next, I characterized the function of individual turns of talk within episodes, using the episode descriptions to help me subjectively determine the enacted function of each contribution from the perspective of the participants in those moments. To focus my coding a bit, I used three questions as a guide:

1. What are participants trying to accomplish?
2. What strategies do they use?
3. How is what I am seeing similar or different across instances?

Focused coding. After bounding and characterizing episodes through my open coding scheme, I blinded myself to the descriptions and applied an a priori framework, primarily generated by the move types of the discourse routine templates. As noted previously, these move types were previously identified and a focus of our work with teachers. For instance, the *Building Collective Understanding* routine (Figure 14) contains moves around restating and moves around extension/clarification questions.

Building Collective Understanding:

1. Restate the method and its basis to make the method shared and public.

- "_____________ said that if you add up all of the numbers and divide by how many there are, you will find the best guess."
- "______________, tell us what ___________ thinks the method for finding the best guess should be in your own words."
- "Can anyone explain what ___________ just said about their method for finding central measurement? "
- "So this method thinks repeated measures are really important."
- "Do you mean that this method depends on how many numbers there are rather than the value of the numbers themselves?"

2. Ask any extension/clarification questions if necessary to help others understand.

- "When you say, '*Divide by the number of guesses,*' what do you mean by *guesses*?"
- "You said to *put the numbers in order*. What kind of *order* do you mean?"
- "What do you mean when you say _____________?"
- "I'm not sure I understand what you mean by __________________?"
- "How could you give more specific directions so that anyone following your method would end up with the same best guess?"
- "Some of us knew to find the mean as the best guess. Some people call that a fair share. What might they have in mind?"

Figure 14: Excerpt from the Discourse Moves sheet provided to teachers for Unit 2 (measures of center). Two types of moves are described, restating and extension/clarification questions. Each of these types formed the basis for some of the categories of the focused coding scheme.

The complete discourse template routines for Units 1-3 are provided in Appendix A. Although the structure of the routines was consistent across units, individual discourse moves reflected variation in the instructional content. For instance, Units 2 and 3 did not contain the *translation* move as a part of the *making connections* routine because this type of move was germane only to the visibility of features of the data in relation to display design choices. Table 10 is a copy of Table 5 but also lists each of the discourse move types in each category (bolded), as well as the types of codes I generated using the discourse moves sheets to match each type (also bolded). The moves embed the person doing the talking as well as the move type. For instance, in Table 10, the move TEN in the *Eliciting Student Thinking* routine is composed of T (teacher) and EN (eliciting a noticing).

Table 10

Discourse move types within the five template routines of data modeling discussions

		Unit 1 (Displaying data)	Unit 2 (Measures of center)	Unit 3 (Measures of precision)
Eliciting Student Thinking	Description	Ask students to provide observations about what another group's display shows or hides about the data and the design choices that made that feature visible/hidden	Ask students to describe and provide observations about the relations between the procedure and the characteristics of the data it uses to find the best guess of the measure of center	Ask students to describe and provide observations about relations between the procedure and the characteristics of the data it uses to find the measure of precision
	Discourse move types	• Eliciting noticings about features of data and how they can be seen **(TEN)**	• Eliciting a hypothesis about the method (**TEHyp**)	• Eliciting a hypothesis about the method (**TEHyp**)
	Example	"What does this display show us about the measurements?" "How can we see that in this display?"	"What is the main idea behind this method?" "What part of the data does this method care about?"	"What is the main idea behind this method?" "What part of the data does this method care about?"
Building Collective Understanding ("Yes-anding"/making it public)	Description	Help the rest of the class understand the student's observation; clarify or extend thinking	Help the rest of the class understand the student's observation; clarify or extend thinking	Help the rest of the class understand the student's observation; clarify or extend thinking
	Discourse move types	• Restating (**TERest**) • Extension (**TEExt**) • Clarification (**TEClar**)	• Restating (**TEClar**) • Extension (**TEExt**) • Clarification (**TEClar**)	• Restating (**TERest**) • Extension (**TEExt)** • Clarification **(TEClar)**
	Example	"Can you restate that in your own words?" "Where do you see an example of that?"	"Can you restate that in your own words?" "What do you mean by ____?"	"Can you restate that in your own words?" "What do you mean by _____?"
Responding to Hypotheses	Description	Ask the authors to confirm/disconfirm claims about their display; Ask other students to form opinions about the claims	Ask the authors to confirm/disconfirm claims about their measure; Ask other students to form opinions about the claims	Ask the authors to confirm/disconfirm claims about their measure; Ask other students to form opinions about the claims
	Discourse move types	• Asking authors to confirm/disconfirm conjectures about their work (**TERA**) • Asking non-authors to agree/disagree with conjectures (**TERJ**)	• Asking authors to confirm/disconfirm conjectures about their work (**TERA**) • Asking non-authors to agree/disagree with conjectures (**TERJ**)	• Asking authors to confirm/disconfirm conjectures about their work (**TERA**) • Asking non-authors to agree/disagree with conjectures (**TERJ**)

	Example	"Do you agree with his/her claim that the data shows ___?"	"Do you agree with his/her claim that this method uses ____ to show us the best guess?"	"Do you agree with his/her claim that this method uses ____ to show us the precision?"
Making Connections	Description	Ask questions about tradeoffs between different displays' features in understanding and interpreting data	Ask questions about tradeoffs (including replicability and generalizability) between different methods in relation to different qualities of data sets	Ask questions about tradeoffs (including replicability and generalizability) between different methods in relation to different qualities of data sets
	Discourse move types	• Asking about tradeoffs of design choices (**TEC**) • Translation questions (**TET**) • Transformation questions (**TEH**)	• Asking about similarities/ differences between methods (**TEC**) • Transformation questions (**TEH**)	• Asking about similarities/ differences between methods (**TEC**) • Transformation questions (**TEH)**
	Example	"Which of these displays makes it easiest to see ____?"	"Would this method give me a good best guess if we had a value here?"	"Would this method give us a good measure of precision if we had a value here?"
Pulling It Together	Description	Make a summary statement that restates "big ideas" and tables ideas that remain unresolved	Make a summary statement that restates "big ideas" and tables ideas that remain unresolved	Make a summary statement that restates "big ideas" and tables ideas that remain unresolved
	Discourse move types	• Pulling it Together (**PiT**)	• Pulling it Together (**PiT**)	• Pulling it Together (**PiT**)
	Example	"In this display, we can see the extreme values more clearly than in this display because of the way they grouped the numbers. We call that "binning."	"What I'm hearing you say is that this method would give us a result but it might be a good estimate of the best guess when we have extreme values."	"What I'm hearing you say is that this method would give us a result but it might be a good estimate of precision when we have extreme values."

I also generated codes for student turns of talk that matched the teacher move types. For instance, a student turn of talk that provides a noticing about a display was coded as SN (Student noticing) instead of TEN (Teacher elicits a noticing). Once I applied the focused coding scheme to the turns of talk, I determined that the coding scheme I had formulated was not sufficient to adequately characterize every type of move. For patterns of moves that reappeared in the data, I generated additional codes. For instance, teachers often asked students to provide evidence of a claim. This was important to data modeling discussions and consistent with the goals, but it was not specifically called out in the coding scheme. In these cases, I generated additional codes to adequately describe these turns of talk. After generating all of these additional codes, I conducted another sweep of the data to apply these new codes to data I had already coded. A complete list of move types in each template routine can be found in Appendix D.

Recording the context of coded discourse moves. I used Excel spreadsheets to track my focused coding (Figure 15). I began with three columns: teacher turns of talk (blue), student turns of talk (green), instructor turns of talk (orange, top snippet only), and the code applied to each turn (white, middle column). Then I added more columns to characterize contextual elements of the conversation. The first column denotes the template routine of discussion. The last column indicates the central mathematical idea being discussed.

Rehearsal Coding Sheet

Responses to hypothesis	OK, good, so Clarissa, do you agree that you kind of put your graph in a descending order here?	TERA			Order
		SRA	C: Uh-huh.		
		TEClar		EP: You might want to clarify what's descending about it.	
	OK (pause) - I don't understand				
				EP: It's descending order of what? Like what is descending? What are those values representing?	
	OK, so what is the highest here? Cuz this is 158 and that number is lower than 169 so what did you do to go from your highest to your lowest?	TEClar			

Classroom Coding Sheet

Eliciting Noticings	OK, 5..4..3..2..1.. What did you guys talk about? What does this display show you? What does this display show you? Let's start with Natalie. Natlia, sorry.	TMan		Frequency
		TEN		
		SNot	N: It shows you how many times that person got the number.	
Building Collective Understanding	Can you come up and show us what you mean?	TEClar		
		SClar	N: How many times people chose the number, since there's different numbers.	
	OK, so give us an example.	TEEx		

Figure 15. Sample coding sheets for a rehearsal (top) and classroom (bottom) discussion. Each spreadsheet contains transcripts of teacher and student turns of talk, along with the assigned codes. The first and last columns contain contextual descriptors, aligned with the template routines of the discussion and the content being discussed.

Coding all turns of talk in rehearsal and classroom enactments made a few things clear at a glance: the types of moves teachers and students use and the primary ideas being discussed (such as frequency of case values in a display). In both the rehearsal and classroom episodes in Figure 15, the teacher used a clarification move (TEClar). These codes, in conjunction with the episode descriptions from the open coding scheme, helped me ultimately characterize whether situations constituted similar contexts as previous instances or different contexts, further described in the following section.

Matching discourse moves across rehearsal and classroom discussions. Recall that I began my analysis by identifying episodes of rehearsal suggestions. Once I had completely coded my data, I started by identifying the focused codes in these episodes, beginning with the first rehearsal for each teacher (when rehearsed individually) or cohort (when teachers rehearsed together). I copied each suggestion's transcript and code into a new Excel spreadsheet and then copied each instance of that code in a subsequent classroom and rehearsal enactment for that teacher or cohort. In Figure 15, for example, the rehearsal suggestion focused on a teacher clarification move (TEClar). Therefore, I searched the rest of my spreadsheets for TEClar moves that were used after the rehearsal took place and looked across all of the instances.

I recognized that this procedure might miss instances where teachers used a different type of move in a similar instructional situation. Therefore, I used my open coding scheme to identify any instances I may have overlooked from the focused coding scheme. The open coding scheme was particularly useful for identifying instances where the form of a move was different, and perhaps not coded in the same way in two different instances.

Characterizing consistencies and changes in the form and function of moves. The following sections describe how I characterize "different" contexts to determine whether a move is being generalized.

Context. Teacher practices are collectively constructed and rely on the contributions of others to maintain coherence and purpose. For the purposes of this analysis, I characterized context by the setting and participants, the goals of participants, mathematical content, and available tools or resources. Possible categories of each of these dimensions are represented in Table 11. Each dimension in a vertical column is independent of the next, unless the columns are merged. For example, the setting of "Classroom" includes the participants "Teacher(s) + Students" but is independent of "Mathematical Content," all of which are independent of "Tools." Any setting might intersect with any unit or tool. "Topics" may consist of a single student method or a single concept about that method. A single student method may include more than one topic of discussion, and more than one student method may be required to address a single topic. The participants labeled in bold print are those that were present in both settings.

Table 11

Context Framework: Contextual Dimensions and Categories

<table>
<tr><th></th><th></th><th colspan="4">FUNCTION</th></tr>
<tr><th>Setting</th><th>Participants</th><th>Goals of Participants</th><th colspan="2">Mathematical Content</th><th>Tools</th></tr>
<tr><td rowspan="3">Classroom</td><td rowspan="3">Students + Teacher(s) + Coach*</td><td rowspan="6">Instructor Goals

Teacher Instructional or Behavioral Goals</td><td rowspan="3">Unit 1</td><td>Topic 1</td><td rowspan="6">Student Displays

TinkerPlots

Other</td></tr>
<tr><td>Topic 2</td></tr>
<tr><td>Topic 3</td></tr>
<tr><td rowspan="3">Rehearsal</td><td rowspan="3">Teacher(s) + Instructor(s) +</td><td rowspan="3">Unit 2</td><td>Topic 1</td></tr>
<tr><td>Topic 2</td></tr>
<tr><td>Topic 3</td></tr>
</table>

Peer Teachers	Student Goals	Unit 3	Topic 1	
			Topic 2	
			Topic 3	

* The coach was always one of the instructors during rehearsals. For Cohort 1, the classroom coach did not intervene during instruction. For Cohorts 2 and 3, the classroom coach intervened on occasion during instruction.

The “form” of a discourse move, described earlier, only relates to the phrasing of that move in isolation of the context and is therefore not represented in this table. Even though the context of discourse includes the setting, the “function” of a discourse move is dependent only on its characterization in the dimensions of goals, mathematical content, and tools, but not setting or participants. This is an important distinction because I wanted to be able to identify adaptations to discourse moves in cases where the setting was different. Using the Figure 7 framework, an adaptation in a new setting would have otherwise counted as a change in both form and function, or “Unrelated.” Eliminating the setting and participants from the set of “Function” variables allowed me to count those instances as either adaptations or generalizations instead. It also did not limit me to looking at moves in two different settings. In cases when the same move was used two or more times in a single enactment, I could also look at changes in form and function within that single setting.

Goals of participants. To characterize the goal, I looked to the surrounding context to inform what participants were trying to accomplish. This often required consideration of the content being discussed (when the move is specific to content) and where the move fell chronologically in the discussion. For example, if an “eliciting noticings” move was used at the beginning of a discussion, its purpose was more likely related to generating ideas for discussion. If it was used at the end of a discussion, its purpose might have been to review what was discussed or prepare students for a homework assignment, among others. Thus, my analysis of the goals was not limited to activity that preceded the move but also activity that followed.

Consistent with my characterization of discourse described earlier (Table 4), I was particularly interested in ways the discourse moves functioned to: 1) make things (ideas, ways of knowing and learning) significant, 2) make connections between things (specifically mathematical representations and ideas), and 3) privilege ways of knowing and participating.

Mathematical content. The second consideration of a move's function was the mathematical content at hand. The mathematical content varied slightly throughout instruction, but even a single student's strategy provided a variety of fruitful mathematical concepts for discussion. Each different piece of student work contained some similarities and some differences between others. Each shift from one concept to another, even within a single student's strategy, constituted a new topic, regardless of the method(s) being discussed.

Tools. Finally, I considered any tools made available in the context of the move that influenced the interaction. Tools included the data displays that were used, sticky notes, Tinkerplots software, student worksheets, journals, the Smartboard, curriculum materials, or any other resource used by the teacher or students during the activity.

Combined framework. While the combined discourse (Figure 7) + context (Table 11) framework allowed for many types of categorical combinations in which to identify change, my first and primary concern was whether discourse moves were even present in classrooms after the work done during rehearsal interjections. My framework allowed me to identify consistencies and changes in discourse moves at broad grain sizes like this but also allowed me to look more closely at changes of a smaller grain size. Further, some discourse moves allowed for more general comparisons between settings, such as revoicing and partner talk. Each of these moves could be useful in many different classroom situations, regardless of the mathematical content or available tools. Looking at too many contextual variables in this case would be superfluous. On the other hand, some moves are germane to specific math content. The translation move is one example, used primarily in Unit 1 to compare design choices in two displays. Using a translation move with different mathematical content constituted a more noticeable and interesting change in function than a revoicing move, for instance.

Integrity of changes. Not all changes to discourse are consistent with the goals intended by instructors or data modeling discussions. To determine whether the enacted function of a move was consistent, I compared the surrounding discourse in each episode against the goals of each unit as described in Table 4. For example, a teacher who uses a translation as a way to hone in on a "best method" would not be using the move consistently with the data modeling goal to evaluate tradeoffs between methods. Because this study was also concerned with the PD activity setting and changes in practice, the intended goals of the instructors also played a role in the integrity of change.

Looking at the entire discourse + context framework together, one possible relation between discourse moves across rehearsal and classroom settings is that teachers might recognize a situation in the classroom as similar in some way to one in rehearsal and respond in a similar way. However, discourse moves also take shape from a cumulative history of contexts, building initially from what happened in rehearsal. They build meaning from successes and failures in these past experiences, which can change the ways teachers understand and use them. My analysis focused on instances of alignments and change to determine how teachers responded in the future when they noticed opportunities to use the same move again.

Table 12 shows the pieces of my coding scheme that pointed to matched instances. The table shows the same information as Table 9 but adds 2 more columns that describe the decision rules for how I identified cases in which a suggestion could possibly be found in subsequent enactments. Open coding was particularly useful when tracing suggestions about the function of a move.

Table 12

How open and focused coding schemes helped identify subsequent appropriation of a rehearsal suggestion

Type of suggestion	Aspect	Example	Coding scheme(s) used for identification	Helps to identify
Suggestion to use a move	N/A	"And then at some point you can go back to either the author or someone else: "Do you agree that that would be a good way to pick?" or you know "(inaudible) more possibilities?"	**Focused coding:** Matching move types by code	• Alignment
Suggestion to use a move at a different point in the discussion	Function: Context	"You wanna start with one. Just start with one. Pick your first one. Pick the one you think is easiest for those kids to understand."	**Focused coding**: Matching move types by code **Open coding:** Searching to identify matching points in discussion	• Alignment • Changes to function (innovation)
Suggestion to use a move in response to a particular situation	Function: Context	"Now, make some connections back to the first ones, when they make a hypothesis about what this one can show."	**Focused coding:** Matching move types by code **Open coding:** Searching to identify matching situations	• Alignment • Changes to function (innovation)
Suggestion to phrase a move in a particular way	Form	"Oh - again, you're being too specific. You wanna just point to the thing and say 'What does this one show about our measurements?'"	**Focused coding:** Matching move types by code	• Alignment • Changes to form (adaptation)

Comparing form and function of discourse moves. Next, I examined the form and function of a move's changes against the goals of data modeling discussions I described in Table 4. I determined whether the form, or phrasing, of an adaptation or generalization was consistent with ways of knowing and participating that are valued in data modeling. For instance, a question that limited student responses to a yes/no response were not as consistent with the goal of privileging student explanation than one that left the question open-ended. I also determined whether the function of a move was consistent with the same ways of knowing and participating. For instance, a teacher used a transformation question in service of identifying which display (in Unit 1) is the "best" is not using the move consistently with data modeling because the important idea in Unit 1 is understanding tradeoffs of different displays rather than trying to identify a "best" display.

I then further narrowed my data to only those subsequent enactments containing talk that aligned and/or changed in ways consistent with the characteristics of data modeling described in Table 4 because I was left with a large amount of data and was most interested in productive changes.

Characterizing patterns of change. For the final piece of my analysis, I was left with 752 subsequent enactments containing talk that matched a rehearsal suggestion and was consistent (either as an alignment or a change) with the goals of data modeling. With these cases, I characterized the timing of change (in relation to rehearsal and prior instances) and any influences to the change in discourse. I was particularly interested in examining patterns of change that showed compelling evidence that changes in teacher discourse could be traced to rehearsal suggestions.

Results

Changes in Teacher Discourse Consistent with Rehearsal Suggestions

In 752 of 1,072 cases, teachers made changes in subsequent enactments from pre-rehearsal discourse that were consistent with the rehearsal suggestion provided. One example of this consistency follows the illustration of rehearsal I described earlier, where Kristine was coached to use a discourse move that requested evidence for students' claims. Kristine's subsequent classroom discussion aligned with the suggestion because she appropriated the same type of move. In the example, Kristine's (T) request for evidence is in bold.

> *14 T: OK, and what does that show us about the numbers?*
> *15 S: Ummm …. that….948 is the highest.*
> *16 T: OK*
> *17 S: And then 27 - 27 is the lowest.*
> *18 T: Alright, excellent.* ***How could you tell which number was the lowest and which was the highest?***

In both instances, a student made a claim about what the display showed. Kristine recognized the classroom situation as similar to that of the occasion that prompted the suggestion during rehearsal. Like rehearsal, her question, "How could you tell? (turn 18)" pressed the student to submit evidence from the display. When the student offered evidence, Kristine asked specifically where the evidence could be found on the display (turn 23). Alignments of this nature were visible for all twelve teachers in the study. However, as Kristine's classroom discussion continues with the student's response to this question, it did not play out the same way as it had in rehearsal. There, Abby (in the student role) responded to the request for evidence with several examples from the display that supported her claim. In Kristine's class, the student's response did not reach the same level of detail and required further questioning. In this example, Kristine's adaptation is in bold.

> *19 S: Because I looked at all of 'em and then I looked to see if that was the lowest and which one was the highest.*
> *20 T:* ***Where was the lowest one?***
> *21 S: The first one.*
> *22 T: Yeah, the lowest one was first.* ***And where was the highest one?***
> *23 S: The last one.*
> *24 T: The last one was the highest. OK, good.*

This example represents an adaptation of the form of the question. Kristine's original question maintained the form of the question posed during rehearsal, but the student's response suggested that further questioning was needed to elicit the evidence Kristine was looking for. Therefore, she scaffolded the question to signal to the student what kind of evidence she was after. After she scaffolded the question, the student was able to provide the evidence. Taken out of context, each of the questions in bold would constitute questions in the category of *eliciting student thinking.* Looking at this entire episode together, it was clear that the function of the move, and specifically the teacher's goal, was to elicit evidence of a feature the student had already noticed. Therefore, this constituted a change in the form of the question, or an adaptation. However, this adaptation only happened because of the classroom situation that called for it. Many new discourse moves that teachers used in their classrooms integrated easily and aligned with rehearsal suggestions (Figure 7) without a need for adaptation. However, I found examples like the one above from Kristine's classroom intriguing because the need for change helped teachers build even more depth around their understanding of a move's form and function.

In other examples, I noticed that other productive changes in the form or function of a move happened as post-rehearsal discourse clashed with pre-existing discourse that had dominated teacher-student conversations in the past, requiring a reconciliation. As I looked further, I found two such types of conflict that led to change in discourse moves. In the first, teachers encountered internal conflict. In these cases, they self-corrected their own language immediately during a single turn of talk, to align with a suggestion provided during rehearsal. These types of changes tended to be coupled with suggestions about the phrasing of a move, as instructors guided teachers toward forms of the move more aligned with those intended in the template discourse routines. Second, instances of classroom conflict were important sites of change. In most of these cases, students showed confusion or provided vague responses following the teacher's use of a new discourse move. The teacher responded in one of two ways. Sometimes the teacher held steadfast to the move in an effort to change the ways of thinking and participating that are valued in the classroom. In other cases, the teacher responsively adapted the phrasing of a move to elicit the responses s(he) was seeking. The move served to resolve the conflict between the teacher's expectations and student behavior. In the sections that follow, I describe each of these patterns of change in more detail and provide examples of each.

Self-Corrections

Self-corrections are changes a teacher made to a discourse move within the span of one turn of talk. These types of changes tended to occur following suggestions around the phrasing of a move. The role of rehearsal was especially visible in these cases because elements of both a pre-rehearsal form of the move and a post-rehearsal form of the move were visible. Self-corrections are different than the changes teachers are asked to make following a rehearsal suggestion. However, sometimes self-corrections occurred later during the same rehearsal in which the suggestion was given.

Changes in cases of self-correction happened as a combination of two moments of activity, illustrated in Figure 16. The first change happened during the rehearsal interjection, when teachers were asked by the instructor to revise their language on the spot. (In most cases, teachers revised their language during the interjection, but not always.) The second change happened in the subsequent episode, where teachers made the self-correction on their own. Figure 16 maps the changes as a series of time events, where T stands for time and Δ stands for change.

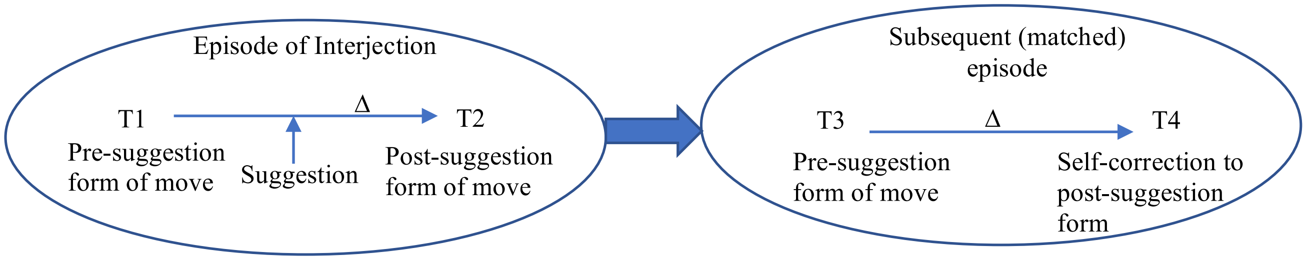

Figure 16. Map of change across time to instances of self-correction. The first two times take place within a rehearsal interjection. The second two take place during a subsequent rehearsal or classroom episode when the teacher self-initiates a correction from a prior form to a revised form of the move.

For example, Rob follows the sequence illustrated in Figure 16 as he self-corrects the phrasing of a move following a suggestion. The following vignette takes place during Cohort 3's Unit 2 rehearsal about student-invented measures of center, where Rob is playing the teacher (T) and an instructor (I) makes a suggestion around the phrasing of his first question.

T1 *22 T: So today I guess we're looking at measures of center, um -*
23 I: And don't start by saying that remember, because they're gonna come up with that word "center." You're talking about it as "best guess."
T2 *24 T: OK, well best guess. So it's gonna - so we're looking up here at this example of the data. How do you think that Alex tried to find best guess?*

In this example, Rob takes up the suggestion and re-phrases the question to incorporate the language of "best guess." The teachers have been instructed during a conversation prior to rehearsal to initially avoid using the word "center" because it naturally points students to the physical center of the data set, or the median. Instead, the use of "best guess" points students back to the context of the data collection and the purpose of measures of center. This is consistent with the Unit 2 goal of discourse that makes connections between the measurement context and the measures of data (Table 4). In spite of the prior discussion around this term, Rob still uses it during his rehearsal, suggesting the prior discussion about the term was insufficient to prompt a change in discourse.

Later, in the same rehearsal, Rob finds himself in a bit of a hybrid situation, where he uses both "best guess" and "center" to ask a question. He immediately stops himself, and the instructor confirms by modeling a re-phrasing of the question using the preferred term.

Same rehearsal (Rob):

T3 25 R: Right, with them being the same size, then with them being the same size, how would you then find the best guess of the center? Oh, we're not supposed to use that. Center.
26 I: Right, how would you find the best guess?

Rob has both been corrected and corrected himself in practice. Jumping to his subsequent Unit 2 classroom discussion, Rob never used the problematic term "center" when referring to measures of center. Notably, his discussion did not even take place until several months after this rehearsal.

Teachers did not always take up the instructor's suggestion immediately by revising the form of their move (i.e. T2 in Figure 16), but that did not always prevent subsequent change from taking place. In the following example, Chad tries to elicit the vulnerability of a student's method for precision (Unit 3) that couples the range with an interpretive rule, "If the number is small, your data is more precise." He wanted students to notice that the term "small" was relative, vague, and therefore problematic to the interpretation of the precision measure. In this Unit 3 rehearsal, Chad is playing the teacher (T), another teacher is playing the student (S), and the instructor (I) is providing a suggestion (bolded).

27 T: Now let's say that you think that 100 is considered small. How does that compare - how does that look on that data right there? Would you call it small - precise, or?
28 S: It's more precise than hers but it's still pretty far away.
T1 29 T: Yeah, so the number - what you consider small, changes, but the answer - what if I said that I consider 400 a small number? Would that all - would that change how we looked at that?
…

> *30* ***I: Chad, now that I know what you're getting at there, a better way might be just to ask to kids what they consider to be small? Like what's a small number to you? What's a large number? And then when they all say different things, then you bring out "How is that problematic?"***
> …
> *31* ***I: Right, like she was confused when you said 100's small, so I'm thinking like that might have been a better way. Like "What do YOU consider small?" Instead of telling her what she considers small or big.***

Here, the instructor suggests changing the form of a question in a way that requires students do the work of explaining and hypothesizing (Table 4) but simplifies the cognitive work required of them. This snippet is followed by further discussion about the suggestion; however, Chad does not immediately revise his question during the rehearsal. Upon return to his classroom, Chad made a self-correction that closely resembled this feedback. The students are discussing a measure of the distribution's range, just as in the rehearsal. A student suggests simplifying the strategy by eliminating any value with more than one occurrence from the data set before calculating range. In the following vignette, Chad (T) tries to help the student (S) see that this method is vulnerable when the highest or lowest values are repeated. The self-correction is in bold and represents T3 and T4.

> *32 S: Well, to make it easy, couldn't you have just said - couldn't he have - um, just said to put all of them - all of the numbers that have more than one in a group? And then the ones that are left that are - that don't have more than one number, look at those? And see if that - one of those are bigger than all the others?*
> *T3-T4 33 T: So - so you're talking about how the numbers look on the chart.* ***What if - what if - what's the largest number on that chart?***
> *34 S: 155*
> *35 T: What if there were three 155's? ... Would that solve the problem?*
> *36 S: It could mess up the problem.*
> *37 T: Yeah.*

In the first line, the teacher initiates a hypothetical question, just as in the rehearsal. While we do not know for sure, I suspect he was about to ask a question similar to the one in rehearsal, asking students to consider both an imagined "highest value" and also an imagined number of repetitions. He stops mid-sentence (turn 33) and changes his question, asking instead for students to use an existing value and only imagine its repetition, thus decreasing the cognitive work for students as they consider his hypothetical question. The last two turns suggest that the student indeed understands the teacher's revised phrasing of the question, although we do not know for sure what he means by "mess up."

In both of these illustrations, and in self-corrections more broadly, change in both rehearsal and subsequent enactments happened simply between a prior form of discourse and a revised one that an instructor suggested. They were coupled specifically with suggestions to phrase a move in a particular way (Table 12). Because the suggestion concerned phrasing of the move, its use in the larger context was irrelevant for the purposes of my analysis. Further, because a specific type of change to form was the focus of suggestion, any other kind of adaptation was not considered relevant or related to the suggestion.

Not all suggestions "stuck" after a single episode of self-correction. Rob struggled with a suggestion during his group's Unit 3 rehearsal to use the term "measurements" rather than "data" when referring to the set of measurements that students generated. This helps make a connection between the measurement process and the attribute of precision that students try to measure in Unit 3. Rob made a switch between "data" and "measurements" over the course of his discussion. Figure 17 shows each individual instance of these terms used in his discussion, in the form of a scaled timeline. Each instance is represented by a circle, and the y-axis is dummy-coded to indicate which form was used. A value of "1" corresponds to the term "data," or Marissa's pre-suggestion terminology. A value of "3" corresponds to the term "measurement," or the term suggested during rehearsal. A value of "2" indicates a hybrid between these two in the same utterance, either a self-correction or the hybrid term "data measurement." While the change did develop generally from “data” to “measurement” over the course of Rob’s classroom discussion, it was not in a continuous fashion.

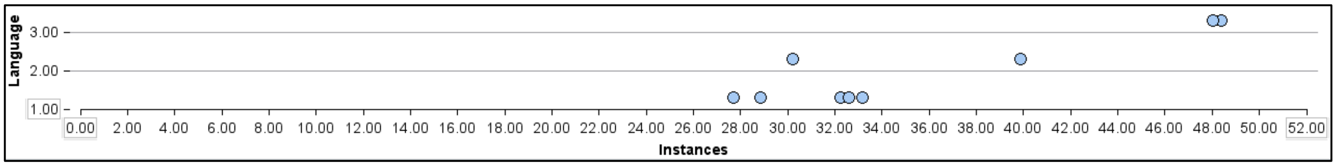

Figure 17. Rob’s Unit 3 classroom discussion: Instances of the term “data” and “measurement”

Time-stamped instances in detail:

> *27:43*
> *38 R: So her **data** was all spread out, right? And ours was closer together. Alright, what else about Mrs. Tomlinson's **data**? Can I pull it up?*
>
> *28:51*
> *39 R: Alright, let's take a look at our **data** now.*
>
> *30:15*
> *40 R: Talk about it at your table. Is that fair? [00:30:15.05] [00:30:44.04] Alright, let's wrap that up. Is that fair to keep that **data set**, or that **data measurement,** in this display?*
>
> *32:17*
> *41 R: The range would change, right? But if we get rid of it, is that an accurate representation of the range of Mrs. Tomlinson's **data**?*
>
> *32:36*
> *42 R: OK. But in this case, is range maybe a good measure of precision for Mrs. Tomlinson's **data**?*
>
> *33:10*
> *43 R: So is range a good method to use when measuring Mrs. Tomlinson's **data** of precision? What do you think? What did you talk about? Or what did you discuss at your table?...Is range a good measure of precision for her **data**?*
>
> *39:54*
> *44 R: What does that mean for precision of our two sets of **data**? Or **measurements**?*
>
> *48:03*
> *45 R: So our clusters, Jaden, you can go sit, thank you. Our clusters are our groups of **measurements**. And let's think back to when we actually measured our table. We agreed that our best guess was what?*

48.25
46 R: 450. And how precise were we or how alike were our ***measurements****?*

Rob self-corrected from "data" to "data set" to "data measurement" and later from "data" to "measurements." After the second self-correction, he used "measurements" consistently, but "data" was interspersed throughout his discourse here. This example represents the sometimes back-and-forth path of development. Not only did changes in practice take time, but they also sometimes reverted to prior forms of practice along the way, just as Rob's change moved between "data set," "data measurement," "data," and "measurement" following the suggestion during his group's rehearsal.

A preferred narrative might be that upon Rob's initial correction to the term "measurement," he continued using that term in every subsequent instance. However, this is not a realistic or practical expectation. In fact, Rob tended to use the word "data" when referring to Mrs. Tomlinson's display, while he used the term "measurements" when referring to his class's own display, possibly because the term "measurements" brought more contextual meaning to students for their own data than the data of a different class. When referring to both Mrs. Tomlinson's and his own displays, he self-corrected from "data" to "measurements." Perhaps this bifurcation resulted from contextual factors at play that my analysis could not identify.

Change resulting from classroom conflict

Alignments and changes to discourse also tended to happen within instances of classroom conflict. In these cases, discourse moves disrupted existing norms and had to be reconciled jointly by the teacher and students. As a result, the form of discourse moves was adapted to serve the same purpose to discussion that they had served during rehearsal. I also present the case of a teacher who responds to a conflict around expectations for participation the function of a *Pulling it Together* move in order to get the conversation back on track. Together, these examples illustrate change resulting from a combination of an alignment from rehearsal to classroom as well as a response to a specific classroom situation.

These instances of classroom conflict followed all types of suggestions. Whereas the map of change for self-corrections alternated between a pre-suggestion and post-suggestion form of a discourse move, this type of change included a third form of the move that was adapted from the post-suggestion form (Figure 18).

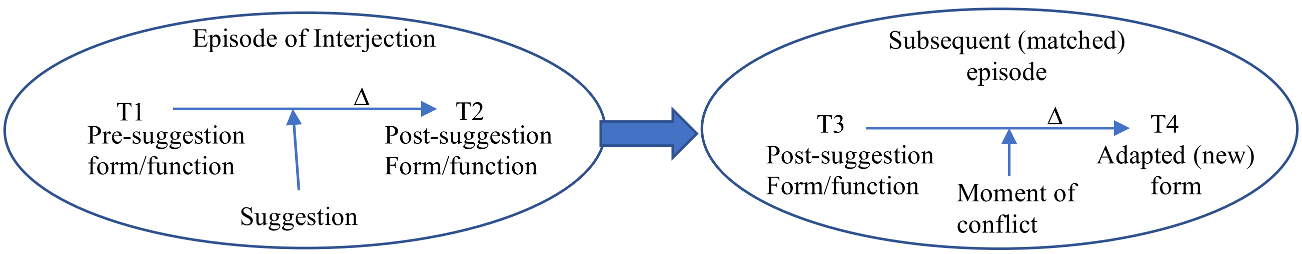

Figure 18. Map of change across time to instances of change resulting from classroom conflict. The first two times take place within a rehearsal interjection. The second two take place during a subsequent classroom episode when the teacher adapts a move in response to a moment of conflict around the use of the move. T4 results in a new form of the move.

As one might expect, changes to discourse often required a re-negotiation of student expectations for participation. I illustrate this through a series of examples that show follow the type of change in Figure 18 but in slightly different ways.

Adapting the form of a question through scaffolding. In an episode of rehearsal I described earlier, Kristine was coached around asking for evidence of a noticing. As part of the larger suggestion during this episode, the instructor modeled a discourse move that could be used to elicit evidence. Here, the instructor's (I) moves are bolded. Kristine is playing the teacher (T), and Abby is playing a student (S). Abby has just noticed that each measurement is represented on the display.

*47 I: **Where do you see that?***

48 S: Well some of them were the same, but each person had their own measurement.
49 T: OK, how do you -
50 I: ***Yeah, where do you see that? Where do you see those different measures?***
51 S: Ummm - I'm interpreting the numbers on the bottom are different people. Person 1, person 2, person 3

The instructor's request for evidence is answered easily by Abby, who explains her thinking. However, when Kristine uses the same type of question in her classroom discussion, she encounters a conflict (turn 53) when the student does not answer as easily as Abby had role-played during rehearsal.

T3 52 K: How could you tell which number was the lowest and which was the highest?"
53 S: Because I looked at all of 'em and then I looked to see if that was the lowest and (inaudible) was the highest.
T4 54 K: Where was the lowest one?
55 S: The first one.
T4 56 K: Yeah, the lowest one was first (points). And where was the highest one?
57 S: The last one.
58 K: The last one was the highest (points). OK, good.

In response, Kristine adapts the form of the move by scaffolding it into a series of two questions that guide the student to the identification of evidence. The adaptation is consistent with the goals of data modeling because the questions follow from the student's thinking about finding the lowest and the highest and still expects the student to provide the evidence. Kristine simply helps the student understand what pieces of information would be useful to satisfy her request for evidence.

Adapting the form of a question through directives about how to participate. In the following example, Kristine is given a suggestion during rehearsal (turn 62) to use a move at a different point during discussion. She asks the authors to respond to a hypothesis (Phase 3 of the 5 in discussion) before asking other students to interpret the display or provide hypotheses

(Phase 1 of the 5 in discussion). The instructor suggests asking follow-up questions about the display to students who did not create the display before asking the display's authors to explain the choices behind their methods. In this rehearsal example, Kristine plays the teacher (T), Carina plays the student (S), and the instructor's (I) suggestion is bolded. The group is discussing a display that uses the height of colored bars to represent the frequency of measurements in each bin, or interval.

59 S: Well, because of how tall they are represents how many people or represents each person. A measurement, so the orange category has the most amount of people on it.
T1 60 T: OK. When you say "orange category" how did you pick this category?
61 S: Well, see they all start with 5's, and the next one all start with 6's or -
62 I: So - so you wanna ask that question of someone who didn't do the display. You wanna say "What are they thinking - how did they do this?"
T2 63 T: What do you notice about their categories Emma?

In her following classroom discussion, Kristine indeed asked many questions, including both initial and follow-up questions about the displays, to non-authors before asking authors (T3). However, this example reflects an existing expectation by students that they share insights into their own work rather than waiting for others to provide insights. The following interaction reveals a conflict between the students' expectations for participation and the teacher's discourse move (turn 168). The function of the move (goal of the teacher, content, and tools - display) are the same in both instances. Similarly, Kristine (T) asks a student, Estaban (S), about the height of bars to represent frequency of measurements, and he provides a vague response.

T3 64 T: Alright, what do these bars show us Esteban?
65 S: Um, how or what the class data.
T4 66 T: Our class data, OK. This one's a little trickier because right here it's 500. I see 14,000. Do you know what that means, Esteban?
67 S: The mistake.
68 T: Shh, shhh, if you made this graph put your hand down, I'm gonna give you a chance to talk in a minute. Esteban do you know what that means?
69 S: They did it wrong?

70 T: Noooo, it doesn't mean they did it wrong. Alright, besides the people who made that graph, put your hand down if you made it, can anyone else figure out, because I'm serious. Why at 500 does their bar go all the way up to 14,000? You think you know, Mike?

In response to the student's vague response, Kristine points to the specific values 500 and 14,000 as a case example of the representational choice the student authors made. However, the student's first response assumes a mistake. The attribution to "mistakes" is a common way that students interpret either extreme values or design choices they do not understand. While gross measurement errors are quite common for a number of reasons, this is also possibly a signal that the students struggled with assuming interpretive positions or strategies other than their own. Here, Kristine demonstrated a commitment to the suggestion made during her rehearsal in spite of the conflict. She changed the move by not only further directing the student's attention but also by adding specific directives about how to participate to guide the students. She also passed up several opportunities for the authors to explain their thinking to the rest of the class. Many teachers trying to encourage class participation would have jumped at the enthusiasm of students to contribute responses. Instead, her choice to ask those student authors to refrain from contributing at this point remained consistent with her rehearsal activity. The student authors seemed anxious to explain the thinking behind their own work and continued to raise their hands, suggesting that "show and tell" was an established norm for the class. Like "show and tell" types of instruction, the Data Modeling curriculum embraces whole-class discussions centered around student work, but the mathematical onus of interpreting student strategies is shared by the whole class. The data modeling curriculum assumes that when students know that others will be interpreting their work, they become accountable for communicating their ideas in sensible ways that can be easily understood by others. Further, through the exercise of trying to make sense of the display, the class is in a better position to make sense of the authors' thinking once it is revealed. Consistent with the intent of the Data Modeling instruction, Kristine clearly pushed against the established “show and tell” norm. She explicitly instructed the student authors twice (turns 68 and 70) to withhold their explanations until later in the conversation, consistent with the instruction she had been given during her rehearsal. Kristine’s instructions to withhold student authors’ explanations were explicit attempts to reshape the norms for participation that had become routine in her classroom.

Further, Kristine remains consistent with her questioning in spite of Esteban’s first contribution (turn 65). While she might have chosen to count this as a conjecture to check with the authors, she is not satisfied with his response and presses him for further clarification using an example (notably, a strategy also suggested during her rehearsal). Kristine continues to press after Esteban’s “mistake” claim, asking Esteban what the value means. He again suggests the authors made a mistake. At this point, Kristine is not satisfied with his answer, yet she passes up another opportunity to elicit the authors’ thinking. Instead, she calls on another student to offer a conjecture, again reminding the authors to wait until later in the discussion to offer their reasoning. The commitment that Kristine shows to eliciting conjectures from non-authors and using press not only reveals alignment with her rehearsal activity but also a conceptual understanding of the purpose of the move. In contrast to Ensor’s (2001) findings that teachers often misappropriate moves from PD to classroom practice, Kristine’s example illustrates a maintenance of the move’s integrity and purpose across PD and classroom settings as she adapts the move through the phrasing and addition of more explicit directives about how to participate.

Choosing between different forms of a move in response to different goals. When teachers adapted discourse moves, they did not necessarily use the same adaptation in all subsequent appropriations of the move. Sometimes they reverted back to prior forms of the move, even though the prior form is what necessitated a change in the first place. One hypothesis is that this is a case of a teacher's internal conflict between prior and revised forms of a move. However, I argue there is a different reason for this alternation. Theoretically, as teachers adapt discourse, they build depth to how they understand the move's utility (Lampert et al., 2013). Teachers begin to couple specific forms of discourse with specific instructional situations (Seymour & Lehrer, 2005). I found examples of this same phenomenon in play as teachers adapted the form of moves.

For example, Abby was asked to use a translation move during her Unit 1 rehearsal to compare two displays once a student notices the density of values between 175 and 181 (a suggestion around a situational use of a move). Abby maintained the form of the translation move from a rehearsal suggestion to her classroom discussion but then altered it slightly because of the way students responded to the form of her question. The rehearsal served to give her an experience from which to build and then the classroom supported fine-tuning adaptation of the move to elicit the type of response the teacher wanted from students. In this rehearsal, Abby is playing the teacher (T), Kristine (S1) and Carina (S2) are playing students, and the instructor's (I) suggestion is bolded. Here, because the suggestion does not involve a prior and revised form of a move but rather the incorporation of the move, Time 1 (T1) is the Abby's first use of the translation move, following the suggestion to use it.

71 S1: Also cuz you have all those that are next to each other - the two and the three, so it kinda looks like most people were from 175 to 181. You can kinda see that cuz a lot of those only have one.
72 T: So from 175 to 181 there are multiple smiley faces in that -
73 I: Now, make some connections back to the first ones, when they make a hypothesis about what this one can show. "Can we see that here in this other one? Where do you see it in the other one?"
T1 74 T: Can you see that there are two measurements of 175 on this graph?
75 S2: You can tell when two numbers are the same on this graph cuz they're the same height.

The instructor made the suggestion immediately following Abby's revoicing move and modeled what a translation question would sound like. Specifically, the instructor recommended a set of two related questions. The first is "Can we see that here in this other one?" and the second is "Where do you see it in the other one?" Abby took up the modeled suggestion to translate but made some slight changes in the form of her question. First, she only asked the first yes/no question. Second, she embedded the specific feature Kristine had noticed into her question. In response, Carina moved beyond a simple yes/no response to offer a bit more explanation. Abby was satisfied with Carina's response and moved on.

Abby used the translation move several times in her subsequent Unit 1 classroom discussion, but it evolved slightly over the course of its use. Figure 19 shows each instance scaled on a timeline of the 54-minute classroom discussion. Each instance is further detailed below. The key phrasing is bolded. T2 (turn 76) is aligned with the form and function of the suggestion during rehearsal, and T3 represents the adaptation to the form of the move.

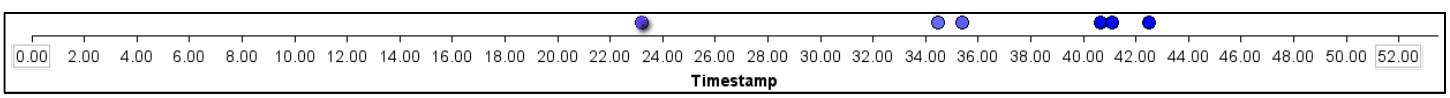

Figure 19. Instances of translation in Abby and Carina's Unit 1 discussion.

Examining the form of Abby's translation moves more closely, the similarity to the phrasing during her rehearsal is unmistakable. She began her first translation with the phrasing "Can you tell?" (turn 76) similar to the "Can you see" (turn 74) phrasing she used during rehearsal that drew out an explanation from Carina. Unfortunately, the same question posed to Mona in the classroom did not yield a similar result. Mona simply responds in the affirmative by nodding her head. This forces Abby to adapt the form of the move by asking "How do you tell?" which successfully elicits the response Abby was hoping for (turn 79).

Instance 1:

T2 76 A: ***Can you tell*** *on Chart number 1 how many people said each number? Can you tell?*
77 S: (nods head)
T3 78 A: ***How do you tell****? Mona, how do you tell?*
79 M: You see it more than once.

Now consider the following instances (2-5) around Abby's subsequent use of the form of the translation question (bolded in each). Of note, Instances 1-5 are interactions between Abby and the students. The only time Carina used a translation move during the discussion was to rephrase Abby's translation question in Instance 6 shortly after Abby asked it. Carina had not received coaching on the translation move during her rehearsal, so it is particularly interesting that the predominant use of translation move was by Abby, until Abby called upon Carina for help.

Instance 2:

80 A: Why is it so tall?
81 S: I think a couple more people got that number.
82 A: That's right, a couple more people got that number. Right here (Display 3) you can tell three people got it based on the labeling. ***How can you tell*** *- three people got it on this one (Display 2)? April?*
83 S: (IA)
84 A: Oh, OK. Javarius. I'll come back to you.
…
85 A: Come show us.
86 J: 1,2,3 (waits for teacher) She had one set of three so some people right here and some people right here so you said, you said there were one set of three so how many people have it.

Instance 3:

87 A: OK. And then what about this one? ***How can you tell*** *on that one how many people said it?*
88 S: On this, how many - how many numbers there are.
89 A: How many times it's listed?

Instance 4:

*90 A: **How can you tell** on this one that they have the same measurement?*
91 S: Cuz umm, like they're ..... Ummm ... they're not all spread out.

Instance 5:

92 S: They're the same - height.
*93 A: Yeah, the bars are the same height. Good. **Could you tell** on this one how different the number 60 and 103 are in measurement? **Can you tell**? That they're really far apart? What about on this one? **Can you tell** the difference between the measurement of 60 and the measurement of 109?*
94 S's: (no response)
95 A: Do you wanna help me with this Carina?

Instance 6 (Carina):

*96 C: ...So when we look at these - these two are right next to each other and these are next to each other even though they're way different. There's big differences between how far away they are. What about on this one? **Can we find that** on this - on this display? I see 62 is next to 80. **Can I tell** that those numbers aren't very close together?*
97 S's: Yeah (various responses)
98 C: Say it louder.
99 S: It almost looks the same.

Like Instance 1, translations in instances 2-4 began with the question "*How* can you tell?" instead of stopping short at the yes/no question. However, these were not permanent changes. Abby revisited the form of her original translation question in Instance 5 when she only desired a yes or no response. Looking at the larger context of that move, her goal shifted a bit from demanding evidence to making a comparison about the affordances and constraints of two display designs. Therefore, returning to the form "Can you tell" did not inhibit Abby's goal like it had earlier because she was not looking for an explanation. Each form became useful for a different purpose and might be called upon in future instances. In this case, the suggestion to use the translation move, followed by classroom conflict led to an adaptation of the move. Later, Abby generalized the move in service of a slightly different function.

In this case, the interjection and use of the move during rehearsal served as a starting point from which Abby and the students constructed an instance of translation together. Abby built personal meaning around the translation move through this series of interactions, both through the initial rehearsal instance and also through the first classroom instance and fifth classroom instance that called for changes to the form and function of the move. This is a vivid illustration of the way learning happens through successive rehearsal and classroom enactments. The initial appropriation of the move revealed opportunities for adaptations in response to the conflict of student confusion about how to respond.

Innovating the function of a move in response to classroom conflict. Innovations, or change to function, of discourse moves were not as common as adaptations. And sometimes they followed from an adaptation as in the previous example of Abby. However, they also happened in response to conflict. The example below follows an instructor's suggestion to Cohort 3 during their Unit 1 rehearsal to use a translation move for the purpose of highlighting which display makes the magnitude of a measurement more visible.

> *E: So they talked about - so Lester was just telling you that the height is clear because of the label - (IA) so one thing you might do, you might say "Can you see it here?" So yes you can, but (IA) - this one makes it -*
> *C: Well, without looking at the top numbers.*
> *E: Right. Yeah.*
> *C: Can you easily tell which one -*
> *E: And you could even like cover things with a post-it or something. Which one makes it easier to see which one had the most?*

This was a suggestion to use the move in a specific situation. Cohort 3 used this move toward that function in Unit 1, but the notable innovation took place during the Unit 3 discussion, where Ryan used the move in the face of a struggle to elicit what characteristic of the data a group's invented measure of precision (the range) attended to. He wanted the students to recognize that each method of precision measured a different feature of the data. The struggle took place after Rob asks the question in bold, where he asks about the relation between the "correct measurement" and the precision of the data.

T: I wanna ask you. This method - what does this method do a little bit differently from our last method? What does this one attempt to do? Our last one looked at range, right?
S's: Yes
T: This one's telling us to look at -
S's: The groups
T: Groups and clusters. It says "The group that has the most measurements a student chose is probably correct. What does that mean? What would that mean to our - graph, or our displays up there/ What do you think? Let's go - Joey and Jonathan, what do you think that means? So the group that has the most measurements a student chose is probably correct. ***How does that tell us how precise our display or our data was?...***
S: It shows us where more of our groups are.
T: Can you go show us where more of our groups are?........
S: Right here
T: OK. And can you show us in Mrs. Thompson's class where the groups were?
S: Right there.

The student's response attends to the grouping of the data, but Rob initiated a translation move to invite further clarification. By asking students to translate the location of the groups in two different data sets, he positioned students to attend to the difference in the resulting measure of precision.[4] In this example, Rob cleverly innovated the translation move in service of his goal to highlight how the grouping of data influences a measure of precision. The move served that goal, in that students noticed the relation between the density of the groups and the measure of precision. Rob learned that this move was useful in this situation, in addition to the original purpose during rehearsal. Therefore, he added depth to his understanding of the move by repurposing it to a new goal.

Discussion

[4] In this exchange, the group discusses the measure of center rather than the measure of precision because the student's method for precision first requires identifying what they consider to be the true measure of the data. The discussion continues from here into how the group used the true measure to find a measure of precision.

By examining discourse in settings of rehearsal and classroom, I found that teachers faithfully appropriated the suggestions made by instructors during rehearsals approximately 70 percent of the time. Employing a form-function framework for interpreting the nature of these appropriations, the most common forms of appropriation were either alignment with form or adaptation of form. Alignments of form often took the form of self-corrections, as teachers incorporated instructor suggestions made earlier during a rehearsal into ongoing dialogue in the rehearsal. Adaptations of form often followed classroom instances that called for slight modifications to accomplish the intended purpose. My research question sought further insight into the ways that interjections, and specifically suggestions, are appropriated into subsequent activity. Not only did I find that similar forms and functions of discourse moves were aligned with classroom discussions after their deliberate practice during rehearsal, but I also found evidence of their influence in subsequent rehearsals. The prevalence of adaptation and generalization of discourse in ways consistent with the goals of data modeling provided evidence of learning, both through a deeper understanding of mathematical content and of how forms and functions of discourse moves link to content and student thinking.

Generative Learning

Kazemi and Hubbard (2008) called for further study of the relations between PD and classroom practice. Specifically, they recommended relating collective learning in PD to individual learning in the classroom. Through these analyses, I have identified how each context deployed resources that contributed to the change. The PD environment contributed a simulation of a classroom situation, to which teachers could practice responding. Suggestions clarified or corrected the meaning behind moves that teachers used. In the classroom, the students provided further feedback, either indirectly or explicitly through their responses, that catalyzed further change to form or function Although each successive classroom discussion played a role in the refinement of discourse, they worked in relation to subsequent rehearsals and classroom coaching. Each setting served as a place of learning, and each served as a place to play out and refine something learned in the other.

Generative Nature of Rehearsals

Kazemi and Hubbard (2008) further call for more study of how enactments can support generation of new knowledge and ways of knowing. The development of the transformation showed how a single suggestion during a collective rehearsal had generative power to "snowball" over time. This was especially evidenced through the alternating use of moves, as in the case of Abby and the translation move. Second, suggestions evolved in both form and function in accordance with the contextual specifics of classrooms and rehearsals. Each time a move changed, teachers added depth to their understanding of the move. Each new form or function of a move could potentially evolve further, resulting in a network of forms and functions of a move that have been adapted to situational specifics of classroom instruction.

Implications for Rehearsal Design

The importance of moments of conflict to the innovation of moves suggests that perhaps such conflict can be a more purposeful design element of rehearsal. Perhaps instructors can plan specific problems of practice that teachers can cope with, such as vague responses that were particularly problematic in the examples here. Instructors in this study did aim to incite struggle for teachers, but usually around the content of the student thinking they represented. Peer teachers who role-played students were also quite cooperative in the way they responded to the role-playing teacher. They typically engaged in ways that were helpful and moved the conversation forward. Therefore, rehearsal in some ways served as a "best-case scenario" representation of a data modeling discussion. Representations of practice are certainly important to learning how to navigate professional interactions with students (Grossman et al., 2009).

Limitations

One of the major limitations of this study is that I focused primarily on characterizing the nature of change rather than quantifying their relative influence. My categories and characterizations served as a type of existence-proof of mechanisms of change that I hope can provide a framework for future research to begin to quantify instances of each. I also hope that further research designs attend to cross-teacher appropriation more purposefully to further the design work necessary to hone in on "best practice" forms of rehearsal for inservice teachers.

Finally, because each design of rehearsal was tailored to the opportunities and limitations of the surrounding context and research work, I did not maintain a consistent rehearsal design throughout the course of the study. While this was fruitful through the lens of design work and scalability, it limited my ability to generalize even to the extent of the population of teachers in the study because each cohort's rehearsal and classroom experiences were slightly different. I maintain that some of the design elements that varied, such as rehearsing with one's own student work, co-rehearsing with other teachers, and classroom coaching, had strong influences on how discourse was aligned or changed. I hope to isolate and study some of these variables in a more constant design in the future.

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CHAPTER IV

THE ROLE OF TEACHER REHEARSAL IN THE DEVELOPMENT OF MATHEMATICS DISCOURSE ROUTINES

Abstract

I analyzed teachers who participated in a form of professional development I refer to as "rehearsal," in which participants enacted whole class math discussions by taking on the roles of teacher and students, while instructors interjected at various points to provide live coaching and suggestions for revision. The rehearsals were guided by a set of five instructional routines introduced to teachers as templates to guide the conduct of classroom conversation about statistical approaches to variability. One of these template routines, "Making Connections," was particularly consequential for drawing student attention to mathematical foundations of statistical thinking. Accordingly, I focused my analysis on one component of the Making Connections template called "transformation." Transformation invited students to anticipate the effects of imagined changes to a distribution on displays of that distribution or on statistics that described it. My study followed teachers between rehearsal and subsequent classroom discussions to characterize the influence of rehearsals on their subsequent enactment of the transformation routine. I wanted to know how the enacted transformation routines that played out in classrooms resembled those that teachers rehearsed and particularly, which aspects of the transformation routine enacted in the classroom remained stable over time and which were flexible as teachers appropriated the routine during the course of several units of instruction. Finally, I analyzed the influence of each setting (rehearsal and classroom) on the other to characterize their *coevolution*,

or how they were mutually constituted, over time. I studied three cohorts of four inservice middle school math teachers. My results show that although the structure of teachers' existing classroom routines were generally stable, the content of individual turns of talk that composed the transformation routine was quite flexible and reflected elements of rehearsal.

Introduction

Recent visions of high-quality mathematics instruction hold teachers accountable to the complex work of orchestrating classroom discussions that guide students to understand important mathematical concepts in ways that are responsive to variations in how students think and talk about these concepts (National Council of Teachers of Mathematics [NCTM], 1991, 2000; Sherin, 2002). To manage this complexity, teachers usually develop conversational routines. Routines refer to patterns of talk that are recurrent, easily recognizable, and built jointly by participants (Leinhardt & Steele, 2005). They help teachers and students anticipate the goals and norms that govern conversation and to structure its enactment over time. For example, in the domain of early arithmetic, teachers exercising a "strategy sharing" routine first select several students' solutions to problems with an eye toward the potential of these solutions for making arithmetic concepts of number and relations among numbers visible to children (Kazemi, Franke, Lampert, 2009). Then, teachers ask students to present their solutions and to compare how a solution may be the same or different than another solution. But the strategy sharing routine encompasses more than simply juxtaposing children's strategies. Teachers must orchestrate sharing strategies in ways that help children extend their mathematical thinking. To do so, teachers employ "discourse moves" to structure the conversation in mathematically productive ways. A discourse move is an utterance that is consciously employed to serve an instructional goal. For example, to promote mathematical generalization, a teacher might ask children to consider: "Will this method work for any set of numbers?"

Unlike other classroom routines, such as taking attendance, the paradox in mathematical conversation routines like strategy sharing is that doing things in exactly the same ways might lead to rituals that are insensitive to children's perspectives. For example, instead of ritually prompting for generalization, a teacher might decide that what she has heard from students about a particular strategy suggests the need to ask a question about a particular element of the strategy that reveals a property of number. Hence, routines are ways for teachers and students to anticipate recurrent, stable elements of classroom activity, but they are ideally configured in the moment in ways that help teachers adapt to specific situations.

Because routines are complex forms of conversational exchange that usually emerge only gradually in teacher practice, the field of mathematics education has turned to deliberate practice as a way of supporting more robust development and enactment of routines. In this study, participating teachers participated in a form of deliberate practice called rehearsal, and as I soon elaborate, they did so to support classroom conversations about forms of mathematics, statistics, for which both concepts and students' ways of thinking about those concepts were novel. During rehearsals, teachers simulate episodes of classroom activity as others role-play students and instructors provide immediate coaching and feedback. Its components include instructor interjections (Kazemi et al, 2009; Lampert et al, 2013), collaboration around problems of practice (Kazemi et al, 2009; Nelson, 2011; Fernandez, 2005; Ghousseini, 2008; Lampert & Graziani, 2009; Lampert et al., 2013), teachers role-playing students (Nelson, 2011; Kazemi et al., 2009), and alternation between rehearsal and classroom enactments of teacher-student interactions (Kazemi et al., 2009; Lampert & Graziani, 2009; Lampert et al., 2013). Rehearsal helps teachers identify the meaningful aspects of instructional routines and find a balance between its conceptual and practical elements (Kazemi, Franke, & Lampert, 2009; Lampert et al., 2013). As teachers become familiar with the structure of a routine, cognitive capacity can be freed to attend to the more non-routine, demanding, and unpredictable elements of instructional interactions (Kazemi, Franke, Lampert, 2009).

Rehearsals create opportunities for teachers to learn how to learn from their own practice, making the learning of these routines generative in nature, and theoretically, a more efficient approach to learning how to teach (Grossman & McDonald, 2008; Kazemi & Hubbard, 2008; Lampert & Graziani, 2009; Lampert et al., 2013). For example, Horn (2010) studied informal teacher learning communities and found that one of the ways teachers learned from their own practice was to discuss problematic classroom episodes with each other. Their conversations not only "replayed" past accounts of classroom events as teachers analyzed the situations but also "rehearsed" anticipated future events in ways that changed their understanding of the situation. Therefore, these collaborations can take place in informal environments, outside the scope of formal professional learning activities led by instructors or coaches.

Although much of the research that examines rehearsal suggests a high potential for improving students' grasp of mathematical ideas and methods, there is comparatively little research that traces the form and function of conversational routines practiced (and adapted) during rehearsals into the classroom. Moreover, the few studies that seek to trace routines from rehearsal to classroom typically employs methods of teacher report (e.g., teacher accounts of what happened in classrooms), rather than examining sequences of discourse moves in each setting. Hence, it is difficult to describe (enacted) continuity and change in conversational routines across settings (Kazemi & Hubbard, 2008). A further gap in the research is that only influences from rehearsal to classroom settings are considered, rather than potential patterns of bilateral influence. Although instances of learning in both the rehearsal and classroom settings were germane to this analysis, I was not only interested in the individual contribution of each setting on subsequent activity. I was also curious about how the *interplay* between the settings influenced subsequent activity. This analytical approach has been referred to as a *coevolution of* *participation* (Kazemi & Hubbard, 2008). Attention to coevolution means that I was also concerned with the mechanisms in which activity in both settings mutually influenced subsequent classroom practice and forms of participation in rehearsals. As collective activity, the participants in both rehearsal and classroom settings shape the activity of subsequent individual activity, and individual activity shapes the way subsequent collective activity develops.

Two other limitations in the research on rehearsal are also problematic. First, much of the research in mathematics education focuses on conversations about early arithmetic, but the potential of rehearsal for other, generally more complex forms of mathematics, remains largely unexplored. Second, because preservice teachers are often the target of study, the field has limited accounts of how existing classroom routines influence the forms and functions of talk that teachers learn during rehearsals. This study explores how rehearsal can help shape the form and function of routines for inservice teachers with established histories of teacher-student interactions.

To address these gaps in studies of rehearsal, as I describe later in greater detail, I focused on teachers' rehearsal and subsequent enactment of a conversational routine designed to improve the quality of classroom discussion about important mathematical ideas and methods in the area of statistics. The particular focus chosen was on a routine called "transformation" which was embedded within a larger routine called "Making Connections." Much like the earlier discussion of strategy sharing, the Making Connections routine was designed to help students make sense of important statistical ideas as they compared student-invented displays (e.g., a visualization of a batch of variable data) and student-invented statistics (e.g. a method for measuring the variability of a distribution). Within this framing, "transformation" invited students to anticipate the effects of imagined changes to a distribution on the student-invented displays of that distribution or on the statistics that students invented. During professional development, middle-school teachers (grades 5-7) rehearsed Making Connections, including the transformation routine. After rehearsing this routine, they employed it to conduct classroom conversations.

The primary aim of this research was to characterize continuities and adaptations of the transformation routine as revealed by analysis of conversational exchanges in each setting over time. I also sought to describe the extent to which pre-existing conversational routines, such as Initiate-Respond-Evaluate influenced how teachers appropriated the transformation routine. Because there were multiple cycles of rehearsal and classroom enactment, in principle it was also possible to detect patterns of mutual influence, as teachers practiced during one rehearsal, conducted a classroom conversation and then participated in a second rehearsal (where, although the statistical topics changed, the same routine was employed). From a sociocultural characterization of learning, the phenomenon of adapting an activity such as Making Connections/Transformation to different social settings and contexts is referred to as *recontextualization* (Bernstein, 1977, 1990, 1996; van Oers, 1998; Ensor, 2001). As opposed to *decontextualization*, where thinking can be seen as independent from the context in which it was learned, recontextualization assumes that learning can never be separated from the context in which it was learned (van Oers, 1998; Gresalfi, 2009). Previous rehearsal literature is consistent with recontextualization, because the things teachers learn multiple aspects of practice in relation to each other and begin to couple particular teaching strategies with specific forms of student thinking (Lampert et al., 2013; Lehrer & Seymour, 2006; Horn, 2010). Hence, the research questions were:

(1) Considering multiple enactments of the routine across settings and over time, which elements of the routine were stable, and which tended to change over time, resulting in increased flexibility in use of the routine?

(2) What is the role of prior routines such as IRE on teachers' appropriation of the transformation routine?

(3) How, and to what extent, does participation in each setting lead to mutual influence?

Discourse and Learning

Discourse, as a means of communication, is often viewed as a tool to aid thinking and learning. An alternative view, and one foundational to my study, is that changes in discourse are tantamount to the learning itself (Sfard, 2001). Consistent with this view of dialogic thinking, I focus on classroom mathematical discourse as language that functions to: 1) make things (ideas, ways of knowing and learning) significant, 2) make connections between things (specifically mathematical representations and ideas), and 3) privilege ways of knowing and participating (Gee, 2005).

Discourse Routines

Discourse routines are patterns of interaction that involve several turns of talk, bounded in ways that are recognized by participants (Leinhardt & Steele, 2005). They are governed by a larger system of sociomathematical norms for participation, communicating what counts as acceptable participation in classroom conversations (Yackel & Cobb, 1996). The structure of the interaction, as well as the words themselves, can be routine to participants. For example, a routine common to many mathematics classrooms is the "show and tell" routine, where students present their work in a turn-taking format and are praised for their contributions and efforts. Sometimes other students ask questions of the authors to satiate their curiosity or confusion. In this routine, many students are contributing to the conversation but few collective mathematical insights are developed (Stein et al., 2010). Discourse routines like “show and tell” specify more than just the dialogue that is exchanged by participants. They embed expectations about the roles that participants play and the tools that help build meaning. In the “show and tell” example, the role of students is to explain their own thinking and listen to the thinking of others. However, the mathematical potential is limited by the ideas captured in the individual strategies rather than those that can be generated by doing the work of interpreting and connecting the thinking of others. To move dialogue beyond “show and tell” routines, teachers must do additional work to understand how student strategies connect and built to more sophisticated mathematical concepts. They also must guide the conversation in these directions. However, student ideas can be difficult to predict, so some of this work must be done during the moments of teaching. Teachers must work to understand a student’s response, compare it against disciplinary understandings, and then craft a response. (Stein, Engle, Smith, & Hughes, 2008; Jacobs, Lamb, & Philipp, 2010). This work positions the individual questions within the routine as particularly consequential to the routine’s function in discussion.

Structure and flexibility in routines. Although routines are sometimes assumed to be resistant to change, they also have been shown to provide flexibility in practice (Feldman & Pentland, 2003). Teachers can adapt routines to novel situations while still retaining the predictability and stability characteristic of the routine, and they can also be adapted to new functions (Borko & Livingston, 1989; Feldman & Pentland, 2003; Leinhardt & Steele, 2005). Even though activity is continuously built in the “here and now (Gee, 2005),” it can be informed by successes and failures accumulated from past practice (Feldman & Pentland, 2003). Teachers’ interactions with students might initially fail to elicit the types of mathematical responses they are hoping for, but successes and failures can point to more productive forms or functions of routines over time. For example, Seymour & Lehrer (2006) document how a teacher came to recognize particular kinds of student talk as foundational for mathematically important ideas during the course of several enactments of a new curriculum. The types of thinking she encountered in relation to specific forms of talk helped her construct maps of student thinking to guide her instructional choices. She even began to pair particular forms of thinking with nuanced forms of talk, resulting in an evolution of her routines that was flexible enough to be localized to particular instructional situations.

Discourse moves. As noted previously, discourse routines are structured patterns of conversational exchanges. Their structure originates in teachers' and students' generation of purposeful statements or questions with shared goals. These purposeful statements or questions are called discourse moves. For example, the discourse routine of strategy sharing might begin with a sequence of discourse moves similar to this:

(Purpose: Elicit student solution)	*T: Lexi, can you tell us about your strategy for solving 38 + 12?*
(Purpose: Respond, Justify)	*L: I knew that 38 and 10 more than that was 48, so I put the 2 more on afterward and then I had 48 + 2 and that's 50.*
(Purpose: Initiate collective consideration)	*T: Does anyone have a question for Lexi about her strategy?*

In this exchange, there are three discourse moves. The first and last moves are teacher moves and the middle one is a student move. The exchange in this example, built by the sequence of discourse moves, structures the relationship between the teacher and students around content in a way that communicates high expectations. This is accomplished by holding Lexi accountable to explain her thinking and also holds other students accountable to making sense of Lexi's strategy by taking the necessary steps (asking Lexi questions) to understand it.

The example above suggests that Lexi knows the expectations (classroom norms) for her explanation, and if this type of exchange is routine for the class, the students are likely already prepared with questions to ask Lexi. The sense-making conversation moves forward rather smoothly because the teacher and students recognize the forms of talk and know what to expect.

Unfortunately, the focus on explanation and student sense-making that is apparent in this exchange is not modal in most classrooms. Instead, the most common pattern of discourse moves found in mathematics classrooms is the IRE (Initiate, Respond, Evaluate) routine, in which the teacher initiates a question, a student responds with an answer, and then the teacher evaluates the student's response (Cazden, 1988). These three individual discourse moves are typically enunciated in tandem so that teachers and students can readily identify and participate in the conversation exchange. Like the Lexi example, there is stability in the structure of this routine that is easily recognized by its participants, but there is also flexibility in the content of each of these three moves. In fact, some researchers have renamed the routine IRF (Initiate, Respond, Follow-Up) because of the varying nature of the final teacher response (Lemke, 1990; Wood, 1992; Mehan, 1979; Sinclair & Coulthard, 1975). However, this use of this routine often does not lead to many learning opportunities for students, because the teacher holds tighter control over the direction of the conversation and evaluates student contributions as "right" or "wrong." There are circumstances in which this may be very informative for students, but all too often, use of IRE reduces mathematical learning to learning rituals.

Stability and Change in Discourse Routines

Routines are resistant to change for if they were not, students and teachers could not readily jointly identify the form of activity in which they were participating. But, as indicated earlier, to avoid mere ritual, teachers must adapt routines in light of situational variability, such as different participants or purposes (Borko & Livingston, 1989; Feldmand & Pentland, 2003; Gee, 2005; Leinhardt & Steele, 2005). For example, one teacher's management routines, usually used to guide the flow of conversation between participants, became useful to her instructional dialogue, as well (Leinhardt & Steele, 2005). The teacher used the management routine, "Get your notebooks and find spot x" not in its typical utilitarian way, but as a conversational starter for a discussion about notebook conventions. The teacher embedded this move into the routine structure of a problem- solving discussion to elicit and develop ideas about why students might not all be on the same page. The teacher's adaptation transformed the routine from one intended to signal the start of an activity to one intended to support intellectual preparation for the morning's instruction (Leinhardt & Steele, 2005). Studies of teacher learning, as well as studies conducted in fields other than education, have found that routines are typically constituted by relatively stable, highly repetitive elements as well as elements that appear to vary more in response to situational demands (Feldman & Pentland, 2003; Leinhardt & Steele, 2005). From a sociocultural perspective, teacher and student learning is visible through these stabilities and adaptations made during their enactment over time (Feldman & Pentland, 2003; Leinhardt & Steele, 2005). The ways teachers make sense of the goals, resources, and mathematics embedded in the routine are visible through its enacted forms and functions.

Learning as Changes in Participation

From a sociocultural perspective, the major focus in this study on tracing a particular routine from a setting of deliberate practice (the rehearsal) into a setting of classroom enactment is an opportunity to view learning by participating in each. Teaching activity varies and evolves between professional development (PD) and classroom contexts because interactions are actively constructed in the "here-and-now" (Gee, 2005). Van Oers (1998) illustrates this phenomenon through the vignette of children playing "shoe store" in a classroom. The children's play continually evolved into new activities as a result of the problems they encountered and therefore their goals of the activity. For example, one problem that arose was the need to know the size of the shoes the students were "selling." When the teacher introduced a measuring device to the children, the activity shifted to a focus on measuring feet and estimating different people's shoe sizes. Although measurement was a new activity with different goals, roles, and tools, it emerged from the shoe store activity and was situated within that context. The activity was transformed because the interaction between the students and teacher and the social organization of the shoe store took on new meaning (van Oers, 1998). Further, the change in activity was an opportunity to deepen the meaning and utility of measurement for the children. Learning the function of the measuring tool develops over many of these opportunities to experience successes and failures in new activity contexts.

Similarly, classroom interactions and the goals of both students and teachers change because new ideas are contributed, problems emerge, and sense-making is shaped by prior history and available classroom resources. Successes and failures that teachers experience when solving instructional problems in new contexts provide depth to their understanding of content and pedagogy. PD and classroom settings offer unique challenges and resources which influence the way that activity plays out in each over time.

Coevolution of participation. In the Oers (1998) vignette above, learning happened in successive moments of participation in the classroom. Rehearsals are based on a premise that this type of learning can also happen across PD and classroom settings. Rehearsals provide simulated classroom situations from which instructional successes, failures, and revisions can take place. However, research on professional development often looks at these shifts unilaterally, specifically attending to how teacher participation changes in their classrooms because of their participation in PD. Instead, I characterize both PD and classroom settings as sites of learning. Lampert and colleagues (2013) suggested that relating specific aspects and variations of practice to particular students or to mathematical goals only becomes salient for novice teachers over the course of multiple instances of both rehearsal and classroom discussion. They theorized that novices begin to learn which aspects of an instructional activity are fairly “routine” and which aspects are responsive to what students know and what they need to learn. For example, math teachers often facilitate discussions that compare a few select student strategies for solving problems. Their questions or comments to students as they work might be routine, such as restating a student’s explanation and adding “I think I understand your approach. Have you considered _____?” Although the form of this question is routine, the portion that completes the sentence will depend on how the teacher links the student thinking to the instructional goal. If the instructional goal is “Form groups of ten when adding,” a teacher might respond to a student who counts a large group of items by ones with “Have you considered a quicker way to count those up?” The routine portion is the sequence and type of questions the teacher asks, but the responsive portion is the select conceptual points the teacher asks the student to consider. Like the design of Lampert and colleagues (2013), rehearsals in this study are embedded in a larger iterative cycle of observation, analysis, planning, and reflection because a cyclical approach to learning routines helps teachers leverage what they have learned over the course of many rehearsal and classroom instantiations of the activity to build familiarity with routine components while also experiencing a number of situations that require flexible and responsive thinking. Further, the cycle helps teachers alternate between specific situations and general teaching principles, giving them multiple opportunities to experience the successes and failures around the moves and routines they employ (Horn, 2010; Seymour & Lehrer, 2006).

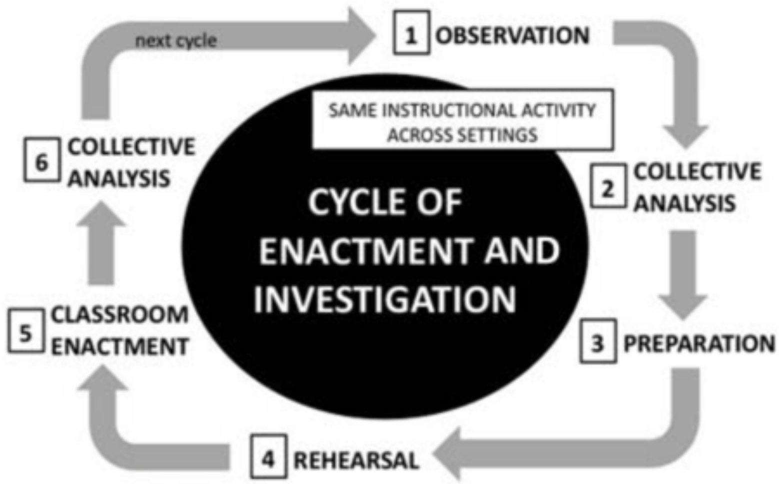


Figure 20. Description of where and how rehearsals are embedded in the larger cycle of enactment and investigation. Adapted from "Keeping it Complex: Using rehearsals to support novice teacher learning of ambitious teaching," by Lampert, M., Franke, M.L., Kazemi, E., Ghousseini, H., Turrou, A.C., Beasley, H., Cunard, A., and Crowe, K., 2013. *Journal of teacher Education, 64(3),* p. 229.

Horn's (2010) study of a high school mathematics department provide further insight into teacher learning that results from many opportunities to recontextualize aspects of their practice between classrooms and collaborative workgroups. Horn identified two particularly important forms of discourse, replays and rehearsals, that created spaces for teachers to learn about teaching practice. Replays provided accounts of specific past, and often problematic, classroom episodes for further group analysis. Teaching rehearsals represented more generalized and often anticipated accounts of practice. Moving between replays and rehearsals in a single conversation helped teachers think about how to reframe and reorganize their teaching activity. For example, over the course of a single conversation, a teacher's description, or replay, of a past classroom episode, changed in response to a new question that a colleague asked of her, and the space provided for her to reflect in a new way on the same event. Her reconsideration then suggested pedagogical revisions, played out as she initiated an impromptu rehearsal of the conversation (Horn, 2010). She moved from a specific event to a general reflection, and then back again to a reframed specific event. Therefore, differences between contexts, such as novel student contributions or classroom management problems, offer learning opportunities for teachers to recontextualize familiar discourse structures in slightly different ways. In fact, novel student thinking can support changes in teachers' mathematical understanding (Seymour & Lehrer, 2006). As these understandings change, the ways teachers interact with students might change in turn in patterned ways, just as the teachers in Horn's workgroup did. In theory, a new diagnosis of student understanding might prompt a different, more productive response the next time the teacher experiences a similar episode in the classroom, but this particular question was outside the scope of Horn's study. The approach to studying rehearsal through many instantiations of both rehearsal and classroom activity reflects phenomena like these that are critically shaped by two different contexts and the relations between them. Rehearsals give teachers opportunities to construct specific instructional situations from which general discussions of teaching practice can take place and inform the remainder of the rehearsal as well as future practice. The cyclical nature of professional development ensures many opportunities to recontextualize in each setting.

For illustrative purposes, consider the following three trajectories below (Table 13), beginning with an instructor's suggestion to ask hypothetical questions to contradict a student's overgeneralization. These are just a few examples of the types of forms coevolution might take between settings.

Table 13

Sample trajectories of rehearsal-classroom activity

Trajectory	Explanation	Example
$R_1 \rightarrow C_1 \rightarrow R_2$	Rehearsal activity influences classroom practice, and the resulting change in classroom practice influences activity in the next rehearsal.	C_1: The teacher asks the students a hypothetical question in the context of "partner talk," and many groups provide responses R_2: The teacher asks other types of questions in the context of "partner talk" during the next rehearsal
$R_1 \rightarrow C_1 \rightarrow C_{1 \text{ or } 2}$	Rehearsal activity influences classroom practice in an iterative fashion, and the resulting change of the first classroom instance further influences subsequent activity in the classroom (perhaps the same classroom enactment, perhaps the next one).	C_1: The teacher asks a hypothetical question $C_{1 \text{ or } 2}$: A student asks a hypothetical question to another student
$R_1 + C_1 \rightarrow C_{1 \text{ or } 2}$	Classroom activity reflects features of influence from two past but somewhat unrelated experiences together	C_1: Students have not been responding to any questions the teacher is asking $C_{1 \text{ or } 2}$: The teacher introduces a hypothetical question but changes the form to yes/no as a scaffold

For each of these trajectories of change, both settings are necessary, but not individually sufficient to influence change in practice. Each of these trajectories is dependent on the influence of both settings uniquely, and each is an example of what I consider coevolution of participation.

Framing Consistencies and Changes in Discourse Routines by Characterizing Form-Function Relations

An individual and their surrounding context shape each other (Gresalfi, 2009). As teachers interact with students, new goals, new ways of acting, and new strategies emerge that are slightly different than they were in the original context, just as in the Leinhardt & Steele (2005) and van Oers (1998) examples described earlier. In turn, the teacher and students learn together to interact in ways that, over time, shape how the routine is accomplished. Hence, even though rehearsals and classrooms are different environments, they can both shape a teacher's participation in a routine. To account for the relation between a routine and its context, I characterize the nature of consistencies and changes in the routine by considering the form, or structure of the discourse moves that compose the routine, and the function that these discourse moves serve in its context.

Figure 21 illustrates possible patterns of consistency and change in the form and function of discourse routines. Here, consistencies are marked by stability in both the form and function of a routine. *Adaptations* are signified by stability in the function of a routine but the form, or the general sequence of discourse move types, is different. *Generalization*s are signified by preservation of a sequence of discourse moves but their employment to serve a new function. When both are different, I conclude there is no relation between the two and hence no residue of the previous rehearsal or classroom activity that I documented. This framework allows me to characterize changes and consistencies without losing sight of the interplay between individual teachers and their surrounding contexts.

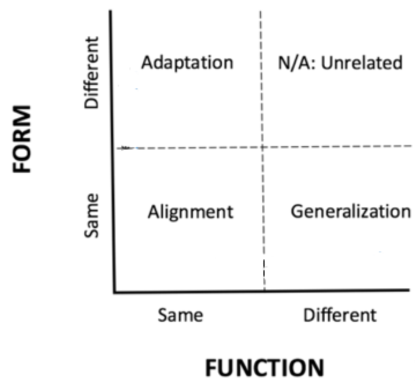


Figure 21. Discourse Framework: Consistencies and Changes in Discourse Moves

Form

I define the structure of a routine as a recurrent sequence of individual discourse moves that comprise the routine. For example, the IRE routine discussed earlier is a sequence of three distinct types of moves. The content of each of these moves might change, but the sequence of moves remains predictable to participants. The meaning of this routine relies not only on the individual discourse moves themselves but also the sequence of moves taken together in the context of the conversation, the teacher's goals, and the students' classroom and broader histories.

Function

The function of a routine is its enacted role in the larger context. The context of a routine shapes and informs its function. In the case of the IRE routine, the function might be to check students' understanding, engage students in the class discussion, or even discipline a student. For instance, consider the following exchange between a teacher and students.

T: What would have been a better choice in this situation?
S: Keeping hands to themselves.
T: Yes, that's right.

This exchange follows the IRE form (or sequence of moves) but the function is related to class discipline rather than to mathematics. While the function of this routine can likely be gleaned from looking at the routine in isolation, a look back or ahead in the larger context of conversation is often necessary to provide insight about a routine's function. Had this same exchange taken place in the context of a class discussion about the choices of a story book character, the function of the move might be related to comprehension.

Context. Routines are collectively constructed and rely on the contributions of others to maintain coherence and purpose. The context of routine enactment includes the setting and participants, the goals of participants, mathematical content, and available tools or resources. Possible categories of each of these dimensions are represented in Table 14. Each dimension in a vertical column is independent of the next, unless the columns are merged. For example, the setting of "Classroom" includes the participants "Teacher(s) + Students" but is independent of "Mathematical Content," all of which are independent of "Tools." Any setting might intersect with any unit or tool. "Topics" may consist of a single student method or a single concept about that method. A single student method may include more than one topic of discussion, and more than one student method may be required to address a single topic. The participants labeled in bold print are those that were present in both settings.

Table 14

Context Framework: Contextual Dimensions and Categories

FUNCTION (spanning Goals of Participants, Mathematical Content, Tools)

Setting	**Participants**	**Goals of Participants**	**Mathematical Content**		**Tools**
Classroom	Students + **Teacher(s)** + **Coach***	Instructor Goals	Unit 1	Topic 1	Student Displays
				Topic 2	
		Teacher Instructional or		Topic 3	TinkerPlots

Rehearsal	**Teacher(s)** + **Instructor(s) +** Peer Teachers	Behavioral Goals Student Goals	Unit 2	Topic 1	Other
				Topic 2	
				Topic 3	
			Unit 3	Topic 1	
				Topic 2	
				Topic 3	

* The coach was always one of the instructors during rehearsals. For Cohort 1, the classroom coach did not intervene during instruction. For Cohorts 2 and 3, the classroom coach intervened on occasion during instruction.

Considering these dimensions of context, the "function" of a discourse routine is dependent on goals, mathematical content, and tools, but not setting or participants. This is an important distinction because I want to be able to identify adaptations to discourse routines in cases where the settings and participants are different.

Goals of participants. To characterize the goal, it is necessary to consider the routine and context to make judgments about what participants were trying to accomplish. This requires consideration of the content being discussed, the ways participants are positioned, and where the routine falls chronologically in a discussion. For example, when the transformation routine is used at the beginning of a conversation, its purpose might be to review a previous discussion's main points. However, when it is used later in conversation, it may promote generalization. Thus, the characterization of the goals is not limited to activity that precedes the routine but also activity that follows. Consistent with my characterization of discourse, I was particularly interested in ways the discourse routines functioned to: 1) make things (ideas, ways of knowing and learning) significant, 2) make connections between things (specifically mathematical representations and ideas), and 3) privilege ways of knowing and participating. This third component concerns how discourse routines position participants in relation to mathematical thinking. Positioning includes attention to who is participating, how the teacher and students respond to each other, the authorship of the content being discussed, and the forms of teacher questions and statements. For example, if a teacher asks a student to interpret another student's strategy, the interpreter is positioned as an authority on his/her interpretation, but the student author is positioned as the authority of the strategy in question. Alternatively, the teacher might answer all questions relating to the student strategy, maintaining the authoritative role instead of building a collective understanding with students.

Mathematical Content. The second consideration of a routine's function in the dimensions of context framework is the mathematical content at hand. The mathematical content is from the perspective of participants and may not be formally recognized as such by the discipline.

Tools. Tools made available in the context of the routine can potentially influence the function of an interaction (Wertsch, 1998). In this study, these range from sticky notes, digital technologies (e.g. TinkerPlots, Smartboards), student worksheets, journals, curriculum materials, or any other resource used by the teacher or students during the activity.

Quality of Recontextualization

My combined discourse (Figure 21) + context (Table 14) framework identifies many potential types of changes in a routine as it is recontextualized within and between settings of rehearsal and classroom. However, it does not attend to the quality of the changes. For example, in Leinhardt & Steele's (2005) study, recontextualizing a problem-solving routine as a management routine to get students back on task was productive, as the teacher's goal of helping students re-focus and preparing them for the day's discussion was accomplished. However, one might also imagine a scenario in which the routine did not achieve this intended outcome. For example, students might have continued to be restless and unengaged. This would also be a case of an unproductive recontextualization, as it did not support the teacher's intended outcome.

Because this study looks at changes in practice influenced by PD rather than just changes within a classroom setting, the intended goals of the instructors play a role in the integrity of recontextualization. Ensor (2001) documents cases of recontextualization that are inconsistent with the goals of professional development. For example, a new teacher appropriated the notion of "visualization" to describe her own practice of displaying the "correct" summary graphs and triangles that she had asked students to produce in preparation for a discussion about the properties of sinusoidal trigonometric graphs. As she displayed these visual aids, she explained the solution herself with very little input from students. In contrast, the professional developers intended that teachers employing visualization would pose a problem with more than one solution, display student solutions, and have students to justify how the diagrams and inscriptions evident in their solutions helped them solve the problem. Instead, the teacher interpreted visualization in a manner that did not disrupt her instructional routines, despite her account of her practices as consistent with those of the teacher educators. In this case, the activity of visualization took on new meaning for the teacher when it was recontextualized into her classroom activity. Whether the teacher explanation component was an illustration of unproductive recontextualization or simply a misunderstanding, it did not carry the same purpose or process as the concept of visualization imagined by teacher educators.

Rehearsal as a Support for Productive Change

One goal of rehearsal is to learn and refine the form and function of discourse routines before teachers employ them with students. In the Ensor (2001) example above, the classroom discussion was the first opportunity the teacher had to try out the visualization routine on her own, as she understood it. Rehearsal can reveal teachers' understanding (or misconceptions) of discourse moves in context before they are appropriated in the classroom. For example, another novice teacher in Ensor's (2001) study explained the language of the number line during rehearsal, as she had interpreted it from the teacher educators. The rehearsal revealed that the teacher's explanation was so thorough that it did all the intellectual work for the students, contrary to another goal of keeping students involved in the intellectual work (Lampert et al, 2013). Rehearsal provided the space to simulate this instructional situation in the context of the relevant mathematical content so that the teacher's misappropriation could be addressed and revised in training before entering the classroom. Had she not rehearsed, she might have done too much of the intellectual work for the students in her own classroom. Further, she would have reported back to her methods class that she had "explained the number line" as required in the materials, even though her enactment was not faithful to the intent of the activity's goals. The revision of such misappropriations during rehearsal is one example of an instance of learning. The teacher came into the rehearsal with an interpretation of the meaning of a routine she was expected to use. The refinement made following the interjection during rehearsal represents a change in the way the teacher understood the routine. This teacher might recognize similar situations to use the number line routine in her classroom and be able to draw from her rehearsal experience when interacting with her students.

Recontextualization can happen either from one setting to the next or within a single setting. Looking at the entire discourse + context framework together (Table 14 and Figure 21), there are several possible relations between discourse routines from one enactment to the next. One is that of **alignment**, where teachers might enact a routine very similarly to a previous enactment. This is found in the lower left section of Figure 21. A second possibility is that teachers might change the sequence of discourse moves in the routine or the content of the discourse moves themselves. This would constitute an **adaptation** of form, pictured in the top left box of Figure 21. A third possibility is that teachers might maintain the form of a routine but change the purpose the routine serves. This would constitute a **generalization** of function, pictured in the bottom right box of Figure 21. Here, there is alignment in the structure of the routine across the two enactments, but it might fulfill a different function.

Looking at the quality of adaptations and generalizations, there are several possible relations in how well the new form or function is aligned with the goals of data modeling instruction. First, the new form or function might be **consistent** with the goals of data modeling. This means that the use of student inventions to guide instruction is consistent with the framework in Appendix C and students are positioned as mathematical sense-makers. Second, the new form or function might be **inconsistent** with the goals of data modeling. These cases might reflect surface-level features of data modeling routines but more deeply resemble prior forms of instruction that are inconsistent with the goals of data modeling. They have characteristics of teacher-directed instruction where students have very little input in sense-making. Further, they are characterized by procedural and accuracy orientations to mathematics, positioning students as seekers of correct answers. This type of instruction is also characterized by judgments about student thinking or methods as generally “good” or “bad,” without considering their potential or relation to mathematical goals. IRE routines would also typically fall in this category because of the limited ways students are positioned to participate and the evaluative nature of the teacher’s final turn.

However, while teachers have some agency over the types of moves they use, there is less predictability around what students will say, and routines must provide space for adapting moves in response to student contributions, whether they be novel ideas, misconceptions, or expressions of explicit confusion. These routines also take shape from a cumulative history of contexts, building initially from what happened in rehearsal as well as existing routines in the teacher's classroom. Routines build meaning from past enactments, which can change the ways teachers understand and use them. In the number line example described earlier, recognizing an opportunity to "explain the number line" in a slightly different situation or slightly different way would also signal an instance of learning, as the meaning of the routine continues to deepen. My analysis focuses on instances of recontextualization to determine teachers' subsequent and ongoing participation in the context of same or similar routines.

Research Context

Setting and Participants

Data were collected in two different contexts of professional development (Table 15). The first, Cycle 1, was a Masters class during the fall semester of 2011 in the context of an urban Masters program at a private university in the southeastern United States. The 2-year program worked with middle school teachers at struggling urban schools, and it was designed to teach innovative ways to strengthen their knowledge and practice in the content area they taught. Three of the participants were practicing teachers enrolled in an urban masters program, and one was completing a traditional Masters program straight out of her undergraduate program. This teacher, Carina, was paired with one of the practicing teachers, Abby, in her classroom to co-teach the lessons. The four teachers in this class made up Cohort 1.

The second context, seen in Cycles 2 and 3, was an experiment testing the efficacy of an instructional approach to teaching statistics in grade 6 by engaging students in participating in practices of visualizing, measuring and modeling variability. I refer to this approach as data modeling. Because participating teachers were typically not familiar with these forms of mathematics or with this approach to instruction, the efficacy study also included a professional development model. The efficacy study was conducted in four districts in an urban area in the southwestern region of the U.S. In the first year, 22 schools participated, and in the second year, 39 schools participated. Schools were randomly assigned to either the data modeling condition or to the practice-as-usual condition. The 6th grade teachers in these schools participated in the experiment. Teachers in the data modeling condition received the professional development immediately, which consisted of three major components: curriculum materials, professional development activities, and in-class coaching. Teachers in the practice-as-usual condition received curriculum materials and professional development after two years. While my study was not concerned with the experiment itself, I drew participants from the pool of treatment teachers in the experiment. As part of the professional development, teachers rehearsed how to conduct mathematically productive discussions central to the curriculum. Some of the rehearsals took place in the summer, and the rest took place during the school year. Treatment teachers attended multiple workshops together over the course of the year. Characteristics of each cycle are listed in Table 15.

Table 15

Participants and PD schedule

	Cycle 1 (Cohort 1)	**Cycle 2 (Cohort 2a)**	**Cycle 3 (Cohort 2b)**
Total Teachers	4	19	38
Case Teachers	4	4	4
Experience (years)	0-4	4+ years	
Hours of PD	2.5 hours weekly (30 total)	32.5 hours (5 days) in summer + 8 hours 4-5 times during year (total)	39 hours (6 days) in summer + 8 hours 4-5 times during year (total)
Number of rehearsals	1-2 individual (Units 1-2)	2 group rehearsals (Units 1, 3)	3 group rehearsals (Units 1-3)
Number of classroom observations	3-5 Display/Measure Review discussions		

Classroom coaching	Co-reflection after each observation	Co-reflection after each observation	Pre-planning, co-teaching as needed, co-reflection 4 times a year

Template Routines to Support Deliberate Practice

Discourse routines of interest in this study were embedded in classroom discussion about student-invented displays and statistics of variability. The aim of the discussion was to highlight mathematical concepts that were often tacit in student inventions, and to relate students' mathematical concepts to disciplinary conventions (e.g. Lehrer, Kim & Jones, 2011). For participating teachers, these goals and formats of classroom conversation were not routine, but we intended to support teachers to make them so. As noted previously, teachers can use the structure and function of routines to frame classroom conversations that are more responsive to student thinking and set expectations for students' roles in constructing mathematical understanding. Hence, we introduced and worked with teachers to rehearse a set of five core discourse routines designed to serve as building blocks of these forms of conversation[5]. The template routines were *eliciting student thinking*, *building collective understanding, responding to the hypothesis, making connections, and pulling it together.* Each of the five core discourse routines is summarized in Table 16. Inspection of this table makes evident that although the structure of each discourse routine is preserved across different student inventions, the specific questions that teachers might pose to orchestrate discussion changed with the data practice in which students were inducted into (i.e., visualizing variability, measuring variability).

Table 16

[5] I previously made analogies between the "Strategy sharing" routine in previous literature and the "Display Review" and "Measure Review" routine discussions in data modeling. For the remainder of this paper, I refer to the five core routines described here as "routines," even though they are sub-routines of the Display and Measure Review discussions.

Data modeling template discourse routines

		Unit 1 (Displaying data)	Unit 2 (Measures of center)	Unit 3 (Measures of precision)
Eliciting Student Thinking	Description	Ask students to provide observations about what another group's display shows or hides about the data and the design choices that made that feature visible/hidden	Ask students to describe and provide observations about the relations between the procedure and the characteristics of the data it uses to find the best guess of the measure of center	Ask students to describe and provide observations about relations between the procedure and the characteristics of the data it uses to find the measure of precision
	Example	"What does this display show us about the measurements?" "How can we see that in this display?"	"What is the main idea behind this method?" "What part of the data does this method care about?"	"What is the main idea behind this method?" "What part of the data does this method care about?"
Building Collective Understanding ("Yes-anding"/making it public)	Description	Help the rest of the class understand the student's observation; clarify or extend thinking	Help the rest of the class understand the student's observation; clarify or extend thinking	Help the rest of the class understand the student's observation; clarify or extend thinking
	Example	"Can you restate that in your own words?" "Where do you see an example of that?"	"Can you restate that in your own words?" "What do you mean by ____?"	"Can you restate that in your own words?" "What do you mean by _____?"
Responding to Hypotheses	Description	Ask the authors to confirm/disconfirm claims about their display; Ask other students to form opinions about the claims	Ask the authors to confirm/disconfirm claims about their measure; Ask other students to form opinions about the claims	Ask the authors to confirm/disconfirm claims about their measure; Ask other students to form opinions about the claims
	Example	"Do you agree with his/her claim that the data shows ___?"	"Do you agree with his/her claim that this method uses ____ to show us the best guess?"	"Do you agree with his/her claim that this method uses ____ to show us the precision?"
Making Connections	Description	Ask questions about tradeoffs between different displays' features in understanding and interpreting data	Ask questions about tradeoffs (including replicability and generalizability) between different methods in relation to different qualities of data sets	Ask questions about tradeoffs (including replicability and generalizability) between different methods in relation to different qualities of data sets
	Example	"Which of these displays makes it easiest to see ____?"	"Would this method give me a good best guess if we had a value here?"	"Would this method give us a good measure of precision if we had a value here?"

Pulling It Together	Description	Make a summary statement that restates "big ideas" and tables ideas that remain unresolved	Make a summary statement that restates "big ideas" and tables ideas that remain unresolved	Make a summary statement that restates "big ideas" and tables ideas that remain unresolved
	Example	"In this display, we can see the extreme values more clearly than in this display because of the way they grouped the numbers. We call that "binning."	"What I'm hearing you say is that this method would give us a result but it might be a good estimate of the best guess when we have extreme values."	"What I'm hearing you say is that this method would give us a result but it might be a good estimate of precision when we have extreme values."

Although some of these routines, such as *eliciting student thinking*, resemble more generic routines that are used in a number of other kinds of instructional conversations, the discursive structure of these routines, and the suggested phrasing, is specific to the intentions of introducing students to practices of visualizing and measuring variability. (See Appendix A for supporting tools provided to teachers for Units 1, 2, and 3 designed to support visualizing and measuring variability).

Chronology of a discussion. To illustrate how the routines might be enacted within a classroom, let's consider a prototypical classroom conversation about students' invented displays. A typical display discussion uses two to four pieces of student work that take different approaches to the problem of displaying a set of data to make its features visible. The teacher typically begins with the most accessible approach, usually one that focuses on case-values, to ensure that all students are able to understand and participate in the discussion. The teacher begins with the first display and uses the *Eliciting Student Thinking* routine to find out what students notice. S/he uses these student contributions to highlight the affordances and constraints of the display in making specific aspects of the data set and distribution, such as the extreme values or the shape of the distribution, visible.

The *building collective understanding* routine is used to engage the rest of the class in making sense of what individual students notice and in coming to a consensus on what a display shows well and what it hides about the data. The teacher might return to ideas the students have contributed during the *eliciting student thinking* routine that are more fruitful for discussion than others. For example, an observation about the absence of a title is likely less productive than an observation about the use of tally marks to show frequency.

Once some conjectures have been established, the *responding to the hypothesis* routine directs the conversation back to the authors of the display, who can confirm or correct the conjectures made by the rest of the class about the purpose of the display and the reasoning behind the authors' design choices. However, it is likely that the teacher already knows the reasoning behind these choices after having conferenced with each group during the creation of their displays.

After this same sequence of routines has been run through again with other displays, the *making connections* routine uses these displays to compare and contrast design choices and what they make visible about the data. Looking across two different displays can show how the same piece of information, such as the highest value, is shown differently. The teacher can ask students to trace a cluster of values from one display to another (called tracing in the template for Making Connections). S/he can ask students to imagine a change in some of the values in a particular display, or the addition of one or more new values, and to then consider the effects of these transformations on the shape of the data using the mathematical approach of the designers of the display.

Finally, the teacher summarizes and anchors the ideas that have become consensus in the class in the *Pulling It Together* routine. This establishes a foundation from which subsequent conversation can continue and build while exploring new ideas. Because data modeling instruction typically requires a shift in mathematics talk in classrooms, many of these routines are only just developing, even for inservice teachers.

Each of these five routines employs questions and statements that might be commonly considered “best practice” discourse moves, such as asking whether students agree with an idea or asking what is similar and different about two methods. However, using these questions without considering their service to instructional goals can have detrimental effects on the interaction. For example, consider two classrooms in which students notice that the display is missing a title and that some bars in the display are taller than others. One teacher presses on the first noticing and the other teacher presses on the second noticing, but they use similar “best practice” questions during their conversation. Normally, the use of these questions could indicate good instructional decisions. However, in a data modeling discussion, the teacher who presses on the height of the bars is in a better place to move the conversation toward mathematically sophisticated ideas, such as a bar as a representation of a case-value or as a representation of the frequency of a particular case-value or class of case-values. As part of a larger study of this approach to statistics education, a tool was developed to helped the research team evaluate the quality of data modeling discussions by mapping categories and examples of approaches to using student inventions in discussion from least to most sophisticated approaches (Jones, 2015, see Appendix C). According to this tool, called a *Construct Map,* the teacher who presses on the height is more likely to reach instructional goals during the conversation and would fall higher in the level 4 category than a teacher who is not selective in which ideas are discussed in depth. In another example, a teacher who carefully selects two or three displays that build on each other toward an instructional goal is in a better place than a teacher who asks all groups in the class to share their displays, with little thought as to how the displays are related. The teacher who asks everyone to present would fall at a level 2 on the construct map, while the teacher who planned more carefully is likely to fall at a level 3 or higher. Therefore, the questions embedded in the template routines, although representative of generically ambitious "best practice" questioning techniques, must be used more strategically in the data modeling conversations in order to reach mathematically productive discussion.

Professional Development

Professional development in this study was embedded in a larger instructional cycle around the data modeling conversations, including observation, investigation and analysis of practice, rehearsal, and classroom instruction (Figure 20). PD sessions were conducted generally by having teachers participate in the same measurement, invention, and discussion in which their own students would participate. After the teachers participated, we took time to make sense of the mathematical content together, in more depth than the teachers would require of their students. The additional depth was intended to serve as a resource from which teachers could make instructional decisions about when and how to make connections among ideas. Learning the mathematical content strengthens content knowledge and pedagogical content knowledge that is drawn upon during the activity. For example, understanding the difference between precision and accuracy of measurements can help teachers understand and respond to a student's claim that the range of a set of repeated measurements is a useless statistic because its value is nowhere near the value of the true measure. After doing math together, we moved next to consider how to support student invention and how to conduct productive conversations about their inventions. The latter were supported by consideration of discourse moves and routines that could structure the conversations the teachers would later orchestrate in their classrooms.

Rehearsal. Finally, the teachers took turns rehearsing the discussions, either individually or in groups, playing the role of teachers while the instructors and other teachers role-played students. During rehearsal, teachers were asked to choose student work to highlight from a corpus of student work provided by instructors and to take on the role of the teacher individually as they used the student work as the basis of the conversation.[6] Peer-teachers portrayed students who authored displays or invented statistics. Peer-teachers engaged as they believed students might, guided by the ways their assigned pieces of student work appeared to reflect particular types of thinking, values, and commitments. Reasoning about the activity as both students and teachers was intended to support teachers' anticipation and reasoning of problems of practice by putting them in the position of their students (Lave & Wenger, 1991).

Instructors made interjections during the rehearsal to coach teachers through their instructional decisions and implementation of the template discourse routines. Each rehearsal concluded with a short debrief, where the reasons for interjections, as well as questions posed by teachers, were discussed collectively. Teachers brought expertise of their own schools and classrooms to bear on these discussions. Instructors brought their knowledge of data modeling classrooms more generally. Through my position as a classroom coach, I was also able to initiate conversation around what I observed as common problems of practice among the cohorts of teachers, as a platform for discussing the different ways teachers responded or innovated. The instructors served a dual purpose in these conversations, with an eye to teacher learning but also an eye to the design of PD and the data modeling curriculum materials.

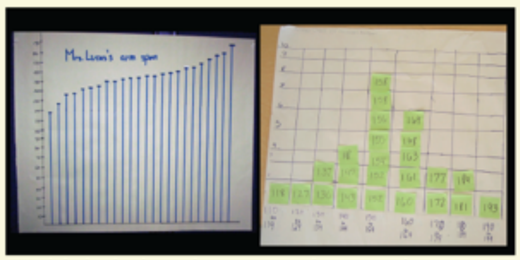
Material resources. Three tools were provided to teachers as resources to guide their discourse in their classrooms—the curriculum units, the discourse moves sheets (Appendix A),

[6] Cohort 1 teachers rehearsed with their own students' work during Unit 2 rehearsals.

and a planning tool (Appendix E). The curriculum units often situated questions with mathematical content, sometimes in response to particular forms of student thinking. For example, Figure 22 shows an excerpt from Unit 1 that illustrates a classroom example of a teacher's use of a transformation, or "what-if" question during a comparison.

Classroom Talk: Comparing different displays

In this example, a teacher asked students to compare the case-value display and the grouped-values display. She wanted students to see how their choices made the same data look different.



A student said, "I think theirs (the case-value graph) are started from the bottom and goes up but doesn't come back down, because they didn't do it like ours. They just did bars instead of doing different columns like what we were doing, 90s, 80s, 70s, 60s, and so on." Another student explained how design choices made differences in shapes of the two graphs. She mentioned that the case-value graph went up like stairs because its inventors ordered measurements from least to greatest and used lines to represent magnitude of each measurement. However, the grouped-values graph made a "mountain shape" because its creators used bins of 10s.

The teacher followed up with a 'what if' question: "What would the case-value graph look like if the whole class got 193?" A student answered, "It would just be the same line all across."

After some more conversation, the teacher posed a translation question: "If I want to find this group in there (grouped-values graph), what do I look for here (the case-value graph)? Can I find them easily?" A student said that it was very difficult because they had to go across to read each value from the y-axis. Another student said that she would look for plateaus to find similar values on the case-value graph.

Figure 22. Excerpt from Data Modeling Unit 1 that describes a case of the "transformation" move.

The discourse moves sheets listed examples of different questions embedded into each of the five template routines, and the routines were ordered chronologically as they are typically used in the discussion. Most routines further categorized questions into categories. For example, the *Building Collective Understanding* routine contained categories of restating, extending and clarifying; with suggested moves for each category provided. The questions here were isolated and did not pair moves with particular forms of student thinking like the curriculum units did. Both the curriculum units and discourse moves sheets referenced individual moves more than suggested sequences of moves, as these routines were expected to vary significantly.

The planning sheets provided to teachers served as templates for the key ideas that teachers wanted to discuss during their discussions, along with how and where they were situated in the larger discussion. The sheet provided space to list key features of each individual display or method, key points that the comparison of displays or methods would elicit, and key questions they planned to ask about each method or comparison in order to move the conversation forward. The primary support for developing routine sequences of discourse moves related to transformation (inviting students to anticipate the effects of a transformation) was instructor and peer suggestion during rehearsal.

Method

Participant Selection

All four of the teachers in the Urban Masters class participated in the study as Cohort 1. I made my selection of Cohort 2a teachers (Table 15) to invite into my project through observation during the week of summer training, which was my first interaction with them. I looked for teachers of varying experience levels, beliefs about mathematics instruction, and involvement during the training. I selected one teacher who had taught data modeling in her classroom the prior year and who appeared to place a high priority on developing a safe and collaborative classroom community, a teacher whose expertise was in writing instruction but had recently been moved to the math department and whose participation pointed to a procedural/accuracy orientation to math instruction, this teacher's wife, who was an established math teacher in the same school and grade level, and a teacher new to data modeling that appeared to embrace a student-centered instructional approach more generally.

In the following year, I selected a cohort of teachers (Cohort 2b, Cycle 3) that all taught at the same school. While this was a convenience sample, I was also interested in observing the nature of collaboration between the teachers, if any, for purposes outside the scope of this analysis. Teachers who began the training in the first year of the project and continued into the second constitute Cohort 2a. They were also referred to as "veteran teachers" during the second year. Teachers who began the training in the second year of the project constituted Cohort 2b and were referred to as "new teachers." The veteran teachers participated in a slightly different summer training than new teachers during Cycle 3, but all teachers attended the same follow-up workshops together during that school year. Instructional coaching took place in teachers' own classrooms four times during the year for both cohorts 2a and 2b. Coaches were teachers (or former teachers) who taught the data modeling in their own classrooms before this project began, most of whom were from another locale. I took on the role of coach for the teachers I worked with in Cohort 2b. Coaches co-planned with teachers, co-taught with them as needed, and debriefed with them. The lessons that coaches worked on with teachers depended on the individual teacher's needs.

Case Routine Selection

This analysis is performed through the lens of a single type of "Making Connections" routine, called transformation. Recall that during transformation, students anticipate the effects of imagined changes to a distribution on displays of that distribution or on statistics that describe it. The routine is named after the transformation question, which typically initiates the conversation around imagined changes to the distribution. I selected this routine for the following reasons:

- It is a constituent of the "Making Connections" routine: Much of the mathematical potential in student invention lies in the analysis of tradeoffs and perspective about variability guiding different inventions. This portion of the discussion also offers methodological benefits, as many of the discourse moves are unique to the data modeling curriculum and are thus more readily identified in a corpus of conversation.
- It is present across many rehearsal and classroom episodes: Focusing on routines that are present in both settings helps inform the nature of recontextualization.
- The basic structure of a transformation routine is visible as: TEH (teacher eliciting a hypothetical transformation question), SHyp (Student provides a hypothesis), and TRev (Teacher revoice) or some other follow-up move or follow-up hypothetical question.
- Although the basic structure is visible, transformations tended to be include many elaborations of this three-move sequence, perhaps because transformations often have high cognitive demand (Stein ref). Because of this high level of demand, the exchanges tended to be lengthier. I reasoned that in lengthier sequences I would be more likely to see adaptations to the routine.]

- Many teachers employed the transformation routine. This afforded possibilities for observing variability in teacher enactments and the extent to which teachers drew from *other* teachers' executions of the routine in their classroom discussions. Further, I was able to make broader generalizations about the function of the transformation in data modeling conversations.
- The transformation routine is native to data modeling and therefore less likely to have been used in the classroom prior to this professional development: Because I do not have solid accounts of teachers' former classroom practice, I am able to make stronger inferences about the role rehearsal played in the development of routines that were likely unknown to teachers at the beginning of this study.

Data Sources

My data for this analysis came from the videotaped and personally transcribed rehearsals and classroom enactments of the rehearsed discussion, corresponding field notes, and content logs. As I transcribed, I defined turns of talk as the entirety of one participant's contribution before another participant began. In the case of overlapping talk, I identified the entirety of each participant's contribution individually to the extent possible. Chronologically, a single turn of talk that began before the overlapping talk still counted as occurring before any of the overlapping talk began. The overlapping talk (if audible) counted as a subsequent contribution. Cases in which many overlapping turns occurred simultaneously were chunked into one single turn of talk together and attributed to "students" more generally. For example, a choral response to a teacher's question was often coded as "yes/no/ummm" in the student column. When the students began answering before the teacher finished, the choral response was still coded as a student contribution that followed a teacher's contribution. Teacher interviews, while not a primary data source in this analysis, were queried as needed in search of supporting or disconfirming evidence for conjectures I made about teachers' goals during instruction. Data included all of the 12 teachers across the three cohorts (all names are pseudonyms):

Cohort 1: Abby, Carina, Emma, Kristine

Cohort 2a: Adam, Jill, Aspen, Lissa

Cohort 2b: Amanda, Marissa, Rob, Chad.

While I did not have strong illustrations of teachers' practice prior to this PD, I drew upon teacher self-reports and baseline rubric-based assessments of Cohort 1's classroom teaching that were required for their program.

Analysis

Overview. I identified instances of the transformation routine through a comprehensive coding scheme that began with identification of individual discourse moves. I coded each turn of rehearsal and classroom talk using a combination of focused and open coding schemes, described in the sections below. In some cases, a single turn of talk contained several different types of discourse moves. Then I used the coding artifacts to find instances of the transformation routine.

Once I had identified instances of the routine, I first identified and analyzed the features that remained stable across instances of the routine within and between teachers and/or settings and the nature of changes in features that provided more flexibility, as characterized by Figure 21 and Table 14. I traced the instances of the routine forward and backward to try to identify the roots of the routine's development in earlier and subsequent enactments in case my coding scheme had not identified those as instances of transformation. For instance, a transformation in a teacher's second classroom discussion might have roots in the first classroom discussion where the teacher scaffolded the initial transformation move through a series of questions that were not coded as transformation questions themselves. I examined each of these subsequent instances to characterize the form and function of the discourse moves contained in different enactments of the transformation.

Because the perspective of participants in the study was central to the characterization of the “Goals of Participants” dimension, I used an analytic lens that attended to any inferences I could make about these situational specifics in each setting (Schegloff, 1997). For example, as an analyst I had access to what happened after a particular classroom interaction and sometimes to what other participants were doing at the same time. However, the participants did not have such information during the moments in which the interaction happened. I tried to account for only the relevant histories of each participant, with a closer focus on the teacher, in characterizing their motivations, including how other participants (i.e. students, teacher educators, and other teachers) positioned them in the interaction and the resources available to them in the moment. When possible and appropriate, I triangulated these inferences using teacher interviews.

Coding scheme. The quickest way to find instances of the transformation routine would have been simply finding individual discourse moves that met our definition of a transformation question according to its definition in the curriculum materials (Appendix A). However, this approach would have hidden some instances of the routine that were of interest to me. Because I am interested in how teachers adapted routines, my coding scheme had to be comprehensive enough to identify instances of the transformation routine not only with the same discursive makeup, but also instances in which the routine was accomplished through different discourse moves and used for different functions. Therefore, coding each type of move according to its definition but independently of the surrounding context might miss instances of the transformation routine in which different types of moves composed a transformation routine with the same function.

Therefore, I used a combination of both open and focused coding schemes. Open coding reflects an emic approach, where meaning is derived subjectively from the perspective of the participants in interactions. This helped me better characterize the surrounding context, and specifically the function, of moves and routines, which became useful for locating instances of the transformation routine where the function of the routine was similar to another instance but their composite structure of discourse moves was different. Theory-based focused coding examines PD and classrooms using a more a priori framework for conversation structure. In this case, the theory-based coding came from the way the transformation routine is characterized in the professional development (see Appendix A). Focused coding helped me locate instances of the transformation routine where the function of the routine was different.

Open coding. I began by depicting an account of the classroom context using a grounded theory approach (Strauss & Corbin, 1990) that ultimately helped me identify and describe similarities and differences in rehearsal and classroom enactments of the transformation routine. Consistent with this approach, analytic categories came directly from my data through induction (Charmaz, 2001; Emerson, Fretz, & Shaw, 2011). In the initial sweep through my data, I generated many codes. Using theoretical memoing (Glaser, 1998; Strauss & Corbin, 1990), I developed the themes around the ways participant interactions were accomplished, using teacher interviews as necessary to confirm or disconfirm my inferences. Memoing also helped me keep track of themes that might be related in different instances. In some cases, my memos assisted me with re-coding data after generating new codes.

I used field notes, video, and transcripts of video during this process. As I watched the video and referred to my field notes, I made content logs of transcripts that bounded the conversations into episodes, or chunks of talk that were conceptually related in some way. In most cases, these episodes contained a series of turns of talk related to a mathematical idea being discussed within a single method or pair of methods. An episode included all the press related to a single idea before changing topics. For example, a teacher often asked additional clarification questions as routines of any type routines played out. Teachers were typically the initiators of new episodes because they guided the topics of discussion. I briefly summarized each episode using two questions as a guide:

4. What are participants trying to accomplish?
5. What strategies do they use?

Next, I characterized instances of the transformation routine using the episode descriptions. I marked episodes for further examination whose description referenced discussion around a hypothetical question about a display or method or otherwise prompted students to generalize their thinking to other sets of data or methods. Open coding at this level allowed me to relate individual turns of talk to the move immediately following it and the episode more broadly.

Focused coding. Next, I blinded myself to the artifacts of the open coding process and applied an a priori framework taken from the professional development. I transferred the transcript to an Excel spreadsheet (Figure 23), using one column to denote teacher turns of talk and another to denote student turns. I used another column to chunk the talk according to the template routines described earlier: *Eliciting student thinking, Building collective understanding, Responding to hypotheses, Making connections*, and *Pulling it all together*. In rare cases, talk did not fall into any of the routines, such as when teachers disciplined students. These instances were coded as “Tman,” or Teacher Management, but some of these codes were embedded into the template routines. I also used another column to denote content codes that identified the ideas being discussed, such as "grouping" or "extreme values."

Ordered numbers; measurement as height of bars	Making Connections		SExt		S: You can see the numbers
		You can see the numbers easier? So if they had actually written these in black though, do you think it would be easier?	TRev		
			TEH		
			SHyp		S's: Yeah/yeah that would be

Figure 23. Sample coding sheet using Excel. First column denotes concepts, second column denotes template routine, third column denotes teacher turns of talk, fourth column denotes coded turns of talk, and last column denotes student turns of talk. Fifth column is where management or other codes unrelated to content fell.

Next, I searched the spreadsheets to find all of the instances of transformation questions, as described earlier and in Appendix A. I flagged any chunk of talk that contained a transformation question and coded every turn of talk in that chunk. These individual codes took a more "top-down" a priori approach to characterizing types of discourse moves because the codes were defined according to the data modeling curriculum materials. The codes identified each participant's role as either student or teacher and a more specific descriptor of the type of move. For example, in the code *TEH* that typically initiated a transformation routine, the first letter indicates that this move was used by a participant in the teacher role, either in the role of teacher in the classroom setting or the role of teacher in the rehearsal setting. The last two letters indicate "Eliciting a transformation,[7]" which describes the move's goal in more detail. Although transformations are generally characteristic of "Making Connections," they did not always fall neatly within this phase of discussion. In Figure 23, the teacher's turn of talk contains both a revoicing of the previous student response as well as the transformation question. The contextual

[7] Because another type of code called "Eliciting hypotheses" was a move used during the Eliciting Noticings routine, the code "TEHyp" was already taken.

codes helped me quickly determine at a glance whether two instances of the transformation routine differed in mathematical content or their role in the larger discussion.

Between 1-5 different types of moves were identified in each of the five template routine categories, and three additional codes indicated moves not specifically attributed to any particular routine, such as the "Teacher Management" code mentioned earlier. Student codes were fewer in number but generally corresponded with different types of teacher codes. For example, the teacher code "Teacher elicits an example" matches the student code "Student example." (See Appendix D for complete coding scheme.) This a priori coding scheme revealed different ways in which discourse moves shaped these conversational routines in slightly different ways. The dual emic/etic coding scheme helped me relate transformation routines to their context within episodes for the purpose of building meaning behind the routine and identifying instances of recontextualization in the next phase of analysis.

I characterized an "instance" of a transformation routine as the entirety of the episode that contained an initiation of the transformation routine, according to my open coding scheme. An episode contained talk that was conceptually related, but in some cases this meant that a single instance included many instances of discourse moves, especially teacher questions, and might be quite lengthy. For instance, a teacher might initiate the routine posing a question related to an imagined transformation of data, but then also ask students to agree or disagree with another student's response, talk with their partners, or actually work through a number of hypothetical situations in order to make the necessary generalization (e.g. "Would it work for 42?" "Would it work for 100?" "Would it work for 1000?"). Therefore, each instance was characterized by multiple codes. To communicate relative occurrences of phenomena, I counted occurrences across episodes in which the phenomena was observed, even if a conflicting phenomenon was observed during the same episode. For example, a teacher might constrain a student response much like an IRE sequence in one portion of an episode yet invite student explanation and conjecture in another portion of the episode.

Analysis of the recontextualization of the transformation routine. The presence of the transformation across two enactments, such as a Unit 1 rehearsal and a Unit 1 classroom discussion automatically signaled a recontextualization because the setting was different, but its recurring presence within an enactment, such as 2 instances in a Unit 1 rehearsal, also signaled recontextualization when its form was adapted or when the routine was used for a different function. I used my discourse + context framework in Figure 21 and Table 14 to further characterize the nature of stability and changes in the form and function of the transformation routine.

In this phase, I included the embedded codes that I previously excluded and examined the surrounding episode(s) to inform the contextual characteristics of the transformation routine. The embedded codes helped me identify adaptations that teachers made to the form of the routine in each setting. For example, the presence of interspersed management codes indicated that the teacher adapted the form of the transformation in response to a situation that was not present in the rehearsal. To identify generalizations, or adaptations to its function, I examined the context surrounding the transformation routine in relation to the goals of the data modeling discussion. One of the primary and overarching goals of instruction was supporting the development of sophisticated mathematical ideas in ways that built from the ways students made sense of them. I used the discourse as an insight into the way the mathematics was being communicated. Specifically, I looked for the ways the discourse communicated what ideas and ways of knowing and participation were valued, where mathematical authority lay (teacher, students) and who the routine gave access to (one student versus many students).

I then examined the form of the dialogue in the transformation routine, with a closer look at how the ways teachers construct responses to students help characterize alignment between rehearsal and classroom episodes. Together, these contextual variables helped me determine how, and to what extent, the transformation routine was recontextualized.

Characterizing significance of recontextualization. I anticipated that some adaptations would be more fruitful than others in supporting mathematical goals. To determine the utility of both adaptations and the new situations into which teachers bring routines, I examined the transformation routine in relation to the context of the classroom conversation. Was the content the same? Were participants positioned in the same way? How much of the mathematical work was being done by the teacher versus students? How was student work being used? I used the construct map in Appendix C (Jones, 2015), which mapped levels of the extent to which teachers used student inventions to guide discussion, to guide my analysis of how the goals, roles, and tools captured in my characterization of the routine built mathematical ideas from student ideas and inventions.

I looked for goals that provided mathematical access to more students or that bridged to more sophisticated mathematical concepts. For example, a transformation routine that communicated a goal of finding the “right” answer was less aligned with the curricular goals than one that focused on tradeoffs of different methods in relation to different data scenarios. Finally, I examined how the transformation routine positioned teachers in relation to students. For example, an exchange that concluded with a teacher’s evaluation of the student’s hypothesis communicated that the teacher’s role was an authoritative one, mathematically speaking, and the student's role was to find the right answer. In the category of tools, I looked at the ways teachers incorporated supporting resources, such as student displays, into the discourse. The mathematical utility of the ways student-invented strategies were incorporated into the discussion is also captured in Appendix C. For example, a turn-taking routine in which teachers asked students to tell the class about their displays was not as sophisticated as a routine in which teachers asked students to contrast the ways each display made repeated measures visible. Using these characterizations, I related the findings to the implications for teacher learning. Which aspects were stable and which were flexible across settings, teachers, and time? What was the role of rehearsal in recontextualization of the transformation routine? How were settings mutually constituted?

Results

I identified 109 episodes, referred to as instances for the remainder of the paper, of the transformation routine in the entire corpus. Table 17 parses the routine's instances based on individual teacher, cohort, unit, and setting. Across teachers and cohorts, inspection of the table suggests an increase of teacher use of the routine as instruction progressed (from unit 1 to unit 3). Comparison of incidence of use of the routine among cohorts or teachers may simply reflect differences in the conduct of professional development in each cohort. For instance, Cohort 1 rehearsed individually while the other cohorts rehearsed in groups. The design for Cohort 2 did not include a rehearsal for Unit 2, while that for Cohort 1 did not include rehearsal for unit 3. To examine my primary question of continuity and change in the enactment of the routine both within and between settings, I examined similarities and differences in the ways the turns of talk were composed around each instance of the transformation routine, with an eye to relations between form and function.

Table 17

Instances of the transformation routines across cohorts, teachers, settings, and units

		Cohort 1			**Cohort 2**				**Cohort 3**				**Total**
Unit		K	A/C	E	Ad	J	L	As	C	M	R	A	
1	**R**	0	3	0	4	4			4				15
	C	1	1	0	1	1	3	1	10	1	0	4	23
2	**R**	4	0		No Unit 2 rehearsal				2				6
	C	2	2	0	3	5	1	1	3	1	6		24
3	**R**	No Unit 3 rehearsals			4				4				8
	C				4	12	1	5	8	2	1		33
Total		13			46				50				**109**

Stable Sequences of Discourse Moves of the Routine

The most common sequence of discourse moves that comprised the transformation routine across settings and time was a teacher transformation question (TEH), followed by a student's hypothesis (SHyp), and another teacher transformation question (TEH). A transformation question was one directed toward asking students about some aspect of the effect of a hypothetical change. For example, during a conversation about the effect of repeated values on each display, this exchange typifies this triadic structure:

(TEH) *T: So what would possibly be the last number down here [if 575 had shown up three times]?*
(SHyp) *S:375*
(TEH) *T:Ooooh, does that change least to greatest?*

This pattern of discourse moves occurred 49 times across the 105 instances, and sometimes more than one of these sequences occurred in a single instance of transformation. For example, one such sequence might be followed with a teacher request for an explanation, followed by another TEH, SHyp, TEH sequence. Further, this did not necessarily indicate the entirety of a transformation routine, but the particular sequence of three moves was found more often than any other sequence within the instances of the transformation routine.

The second most common sequence of discourse moves consisted of the same first and second turns, followed by a teacher move to revoice the student hypothesis (TRev). For example, during a conversation about the choice to bin the data by hundreds, this exchange illustrates how the teacher uses a transformation, coupled with another class's data, to guide students toward the thinking that the method's choice to group into bins of 100 might not work for data

(TEH) *T: What if we took Mr. Simon's numbers that all the students in his class got for measuring that table? Could this still apply to his numbers?*
(SHyp) *T: Yes. Any numbers.*
(TRev) *T: So it would work with any numbers.*

This sequence occurred 37 times across the 105 instances. Sometimes a different transformation question was posed immediately following this sequence.

Analysis of the Content of Teacher Discourse Moves and of Productive Changes

The form of any discourse routine consists of first, its sequence of discourse moves and second, the content of individual moves that compose it. Of these, the element of form most susceptible to change across settings was teachers' construction of questions. These defined and constrained student participation: Not surprisingly, much of the mathematical productivity of the routine was determined by what teachers were asking, not merely the structure of the sequence of discourse moves.

Posing binary questions. In 58 of the 105 instances of the routine, teachers' questions imposed a binary choice on students (e.g., agree or disagree). The majority of these binary questions were framed a request for an explanation. In the example below, the group is discussing a choice to leave a bin (an interval) out of the display, because none of the values fell into it. A student reasons that the display will look more clumped because the space between bins will be hidden. The last move in the sequence below strays from the typical evaluative move in a typical IRE sequence and instead presses for explanation (why is leaving an interval out consequential for the shape of the data).

(TEH) *R: So do you agree or disagree that leaving it out changed the value of this one?*
(SHyp) *L: Well if we pull up the other one, hers would look like clumps, like all the data looks more of a clump than these two.*
(TEExt) *R: And what makes it look more clumped?*

Because the teacher presses, this sequence invites another contribution from the student, positioning students as sense-makers rather than simply seekers of correct answers.

However, in 23 of the 58 instances (40%), the teacher did not press for student explanation. In these cases, either the teacher provided the explanation after the students answered yes/no, or the teacher moved on to a different and unrelated question. It is important to keep in mind that many transformation routines contained more than one transformation question, so it is possible that both a yes/no question and a question that invited further press were present in what I counted as a single episode of the transformation routine.

Further investigation of adaptations of the binary choice format employed by many teachers are illustrated by considering similarities and differences in the enactments of three different teachers, all of whom employed binary (yes/no) questions but with different ends and goals in mind, as follows:

Teacher	Question
Kristine	*What if, in this column, there were no numbers that repeated? All of those numbers were different. But over here, three people had gotten 160. Do you still think that the number that happened the most is the best guess?*
Jill	*So if I had a display where we had all the exact same measurements, would it be really*

	really alike if I had a lot of 70's and a lot of 75's?
Chad	*So, you came up with a step that would work for this chart, right? But would it work equally for all charts?*

Note that each of these teachers posed a hypothetical scenario. Structurally, the questions were similar. Both Jill's and Chad's scenarios are based on an implied "any data set." Kristine's is posed more explicitly: a data set with no repeated numbers. Each transformation is embedded in a yes/no question for students to answer about the imagined scenario. The yes/no question invites a hypothesis, as students take a stance on what the transformation would produce. However, closer analysis revealed that the exchange that followed the initial question is where teachers diverged, as follows:

Teacher	Question	Follow-up
Kristine	*What if, in this column, there were no numbers that repeated? All of those numbers were different. But over here, three people had gotten 160. Do you still think that the number that happened the most is the best guess?*	*TEExt K: Yes, why yes?* *SExt S: It's because um you - usually if someone gets like one number the most, that means that's probably the closest to the exact number.* *TEClar K: So if the number that got the most was over here (motions to left of clump) ...* *SClar S: Then that would be the answer.* *THE K: OK. Who disagrees?* *SHhyp Ss: (Some raise hands)* *TEExt T: OK, Amar you disagree. Why do you disagree?*
Jill	*So if I had a display where we had all the exact same measurements, would it be really really alike if I had a lot of 70's and a lot of 75's?*	*TEExt/ J: Do you wanna add more?* *TEEvid Yes because?* *SExt S: Because the numbers are really close together IA* *TEC J: In which one are they more alike?*

Chad	*So, you came up with a step that would work for this chart, right? But would it work equally for all charts?*	*TRev C: Not necessarily, right? And our goal is to find something that works on all charts, so no matter what we're given, we can be successful.*

As in this example, Jill typically followed her initial scene setting and question by requesting evidence from students for the hypothesis they made. She often continued with a more specific question to either clarify or challenge the position that a student has taken. Like Jill, Kristine often followed her transformations with a request for evidence. And like Jill, she also followed up with a question to clarify and confirm the student's response. However, Kristine typically used the yes/no question to ultimately position students as representatives for a mathematical debate, usually by eliciting student agreement or disagreement with a position staked out by another student. In the example above, she asked Amar to explain his thinking after he adopted an opposing position. Chad, on the other hand, followed his yes/no question a little differently. After students provided their yes/no response, he added an explanation of his own, more consistent with an IRF triad (Chad exemplifies not pressing for student explanation).

Shifts in the content of a teacher's discourse moves to support elaboration of student thinking. As teachers enacted the routine, they often changed the content of their discourse moves to be more responsive to student thinking. For example, Abby and Carina (team teaching) first initiated the transformation routine during their first classroom enactment with yes/no questions that were followed up by an evaluation of the student's response, in traditional IRE form. For instance:

4 (Initiate) TEH *A: Now what if they had done a big 60 and a little 60 and a big 60 and the numbers hadn't been the same size? Would it have been as accurate?*
5 (Respond) SHyp *S: No*
6 (Evaluate)TE *A: No, you're right.*

Like most teachers enacting this routine, Abby posed a hypothetical, followed by a yes/no question about the hypothetical. When a student responded, she evaluated the response as “right” and continued to her next question. But during the last classroom enactment of this routine, Carina not only strayed from the yes/no question but also pressed for an explanation (“why”) from the student before the sequence concluded.

7 (Initiate)	*C: If I were to say that ten of us got that 96 inches was the measurement, and one of us got 80, which one would you think is the right measurement? 10 people got 96. One person got 80.*
8 (Respond) SHyp	*S: 96*
9 (Follow-up)TEExt	*C: Why?*
10 SExt	*S: Because more people got 96 (inaudible)*
11 TE	*C: OK, so that's a really important idea.*

Over time, teachers tended to include more “Why?” and “How?” questions, and their questioning sequences around a single idea were longer. The preceding exchange marks a shift from IRE/IRF triads because the teacher’s second question required a response from students. Similar shifts from IRE-style routines, even with the simple addition of a request for an explanation (line 9), are not trivial and marked important shifts that diverge from typical histories of instruction for these teachers, both as teachers and learners.

In addition to increasing frequency of why and how questions, teachers also adapted the routine to generate more mathematically fruitful forms of hypotheticals for students to consider. Some changed the hypothetical premise in anticipation of common student conclusions. For example, teachers resorted to “Let’s assume that they did not make a mistake” so many times during the routine that they began to tack it onto the initial transformation question. Other changes to the hypothetical scenario and teacher questions about it were more substantive. Recall that the routine usually began with a question in the form of “If X changed, what would happen to Y?" The X component (or hypothetical scenario) usually represented either a change to the data set or a change to a design element of a student invention. The Y component referred to reasoning about the effect of the change, as in "What would happen to the shape of the data?" Some teachers experimented with a more rigorous form of the question-scenario by switching the X and Y components. In these cases, they first posed a desired effect (Y) and then asked students to determine the scenario that would yield that outcome. In other words, they followed the structure: If we want Y, what must X have to be?

For example, in a discussion about student-invented statistics of variability, the teacher posed a question about generalization of a student-invented method, and asked a student (who was not the inventor) to take a position and imagine a scenario in which this student's doubts about the generalization of the method producing the statistic would be realized:

> *14 J: Do you think this method would work for any set of numbers Kayla?*
> *15 K: Me?*
> *16 J: Yes, you are Kayla.*
> *17 K: Well kinda.*
> *18 J: OK, what would be a set of numbers this wouldn't work for?*
> *19 K: Big numbers?*
> *20 J: But I mean like, does it give us a precise number, I'm saying does it work? Can you take any set of numbers out there and subtract the biggest and the smallest?*
> *21 S's: Yes*

This (Unit 3) classroom example about the range method marks the teacher's, Jill, first attempt at this type of adaptation to the transformation routine, although she has found past success with many instances of the if X, then effect on Y? format. Here, she links that prior form (line 14) to the more demanding form (line 20) in the same exchange. Jill's purpose in asking this question is to help students consider the generalizability of the range method to any set of data. Ultimately, she hopes they will see that while the range can be used on any data set, its vulnerability to extreme values does not always make it a good choice. Jill asks the question outright, using the hypothetical of "any set of numbers" and presses on a student's vague response ("Kinda") by asking her to pinpoint a set of numbers that would not produce a result for range. Jill interprets her response, "Big numbers," as a misunderstanding of the original question (line 18), so she rephrases the question, and the class agrees that the method is generalizable (without yet considering if it is always sensible). When she rephrases the question (line 20), she changes the form of the question to a yes/no question, reminiscent of an IRE exchange, in response to students' confusion with the form of her original question (line 18). Kayla's contributions to the discussion suggest that this sequence of questioning is not yet routine. Not only does Kayla appear unsure what the teacher is asking of her, but she even required some encouragement to participate in the first place. In spite of a shaky start to this routine, Jill incorporates the same form of question a bit later during the discussion, and with more success. Here, the class has moved past discussing the reliable application of the method and on to discussing whether the range is always a good *choice* for measuring precision (what Kayla was after in the earlier exchange) even though it will always produce a result.

22 J: Can someone tell me of an example of when range like this is not a good indication on if it is a good set of numbers or not? Samuel?
23 S: Cuz what if it's like 599 subtract 1?
24 J: So like what if we had a measurement where just about everybody had the same measure, but someone over here was crazy and someone over here was crazy, and those crazy people, they're the only measurements we're looking at? What's wrong with that?
25 S: Cuz they're crazy!
26 J: Does that tell us a lot about the data if all we look at is the two end numbers?
27 S: No
28 J: I don't think so.

Apparently, the routine has now gained some traction with the class, or at least with Samuel, who generates a contradiction to the claim that range is always a sound measure of precision (line 84). I position this new form as more demanding because where students originally reasoned from a given scenario to the resulting change, now they are expected to work backwards from a particular change to the scenario that will yield that result. Essentially, this is the same cognitive work that teachers do when crafting transformation questions in the first place, and as I show later in this analysis, this was not even a simple task for teachers, let alone students.

Chad also began to ask this more rigorous form of the question in his Unit 2 discussion and continued into his Unit 3 discussion. Marissa and Rob both used this form of questioning twice in their Unit 3 discussions. But only one teacher used this adapted form of questioning during a rehearsal. Teachers did explicit work on the transformation move during rehearsal, but the work focused more on signaling opportunities to initiate the routine or ways to construct a typical transformation question. For example, in the following exchange during the first rehearsal for Cohort 3, Chad is coached around an opportunity to initiate a transformation routine, but the focus is on the construction of the initial question:

29 C: OK, so what would be one thing that you saw was similar between the two charts?
30 S: The numbers are in sequence?
31 C: The numbers are in sequence. Good. We generally like them in sequence, it helps us understand the information.
32 I: Pose that to her instead. Why is it important – what does it tell – help us see about the data?
33 C: So you'd say – why do you think both authors put them in sequence and you see positive to putting them in sequence as opposed to just randomly placing the numbers on the chart?
34 S: You can see it quickly.
35 C: Good, OK.
36 I: And you might ask here what if they hadn't put them in order. How would that shape look different? Remember we want to try to get to shape eventually, so that would be an opportune moment to do that.

After the first suggestion made by the instructor (line 32), Chad revises his question not only by requiring a student to do the mathematical thinking but also using a hypothetical situation, placing values randomly, to support a visualization of the comparison (line 33). A student responds (line 34), and Chad praises the student, ready to move to another question (line 35). At this point, the instructor coaches him on the form of the transformation question and the ways that it functions in the larger context of the discussion's goals about display shape (line 36). The instructor suggests an adaptation, not to the hypothetical situation but to the content of the question that determines the resulting scenario we want students to consider. Rather than leaving the question open to any number of imagined results as Chad initially suggests (line 33), the instructor suggests a focus on the outcome of the display's shape (line 36). Similar work was done in other rehearsals to prompt teachers to use the move, modeling the phrasing of the move, and situating its purpose in the larger context of the discussion's goals.

Change in structure of the routine influenced by material resources. Teachers had different resources available to them as resources for conducting conversations around displays and statistics. Adaptations to the form of the transformation routine can partially attributed to the different resources teachers used in its service. Teachers used both tools native to their classrooms as well as a digital tool called Tinkerplots that was introduced as part of the professional development to help students with the conceptual work required of imagining hypothetical scenarios and the results of changes to the data or methods for displaying and measuring the data.

Use of an interactive whiteboard to combine elements of two displays. The material constraints to the transformation routine forced productive adaptations, and in some cases teachers used clever adaptations to launch a discussion about the data's shape, albeit an imagined one. While most of the transformation during Unit 1 rehearsals across all cohorts focused on changes to the shape of the display, rarely did a classroom display employ a design that revealed the shape of a normal distribution. Design elements of order and frequency were often used, but rarely were they used together in a single display. For example, Chad planned a transformation question that asked students to consider what a combination of two student displays would look like. One of the student-invented displays in his class suggested conceptual underpinnings necessary for a discussion about shape (Figure 24). Its design attended to order and frequency but separately. The group's display was split into two sections: a number line for non-repeated values, and a binning system for repeated values. Chad was able to use this group's display to launch a transformation about the effects of combining the two design elements into one display.

Figure 24. Student-invented display that contained the conceptual underpinnings necessary for a discussion about the data's shape.

Chad cleverly coupled the transformation with his Smartboard annotation tools that allowed for a quick sketch and manipulation of the hypothetical display (Figure 25).

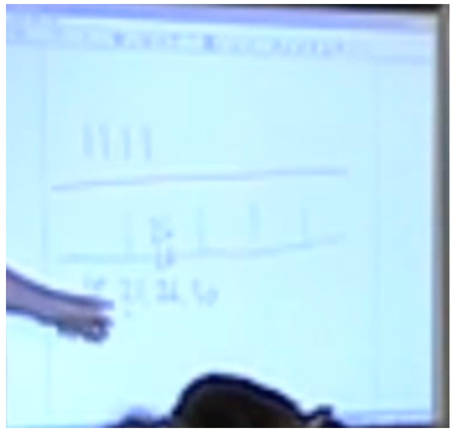


Figure 25. Chad's use of the Smartboard to combine features of order and grouping from two different displays into one single display.

Once this display had been created, students were able to begin thinking about the shape of a distribution and identify the middle of the data as the most likely location for the true measure of the table that they each measured. The earlier example of the display in Amanda's class that made a distinction between even and odd values (lines 37-41) serves as an example of the way the choice of displays is crucial to the mathematical opportunities afforded through the transformation.

Use of material resources to lighten the cognitive work of "imagining". As the material means shifted over the course of instruction, so did the nature of routines. With respect to the transformation routine, the use of Tinkerplots especially invited richer opportunities for transformation because of the easy data manipulation capabilities it afforded. Even aside from Tinkerplots, teachers planned their lessons in ways that anticipated the need to manipulate data while honoring students' work. Jill, in particular, embedded a variety of tools for this function.

In one case, Jill (Cohort 2) strategically built a display using sticky notes in anticipation of manipulating the data during her Unit 2 discussion. Her first transformation fell at the end of the class's discussion about the first method, which pointed to the highest bin as the most likely location of the true measure. She used the transformation to help students visualize the display that would result from removing three values from the highest bin:

> *138* ***J: What about this? Could that be my data set? (removes 3 sticky note measurements from display)***
> *139 S's: No*
> *140 J: OK, what if these three people were gone that day?*
> *141 S's: Uhh*
> *142 J: Bye bye to you three, you weren't here to measure my arm span.* ***Would that method still work?***
> *143 S's: No*
> *144 J: Why not? Someone raise your hand and tell me why that wouldn't work. Kay? And by the way, I'm hearing from some of the same people, just so you know, maybe you need to start being ready, because I'm hearing a lot from Kay and Samuel.*
> *145 S: It's smaller amount of the IA, smaller amount of that number.*
> *146 J: So there are only how many in this column now?*
> *147 S's: 3*
> *148 J: There's only 3. And Delia, how many are in the 160's?*
> *149 D: 3*
> *150 J: And is one taller than the other one now, Saul? Or are they the same?*
> *151 S: They're the same.*
> *152 J: So could I look for the tallest bar? But could I still get a best guess?*
> *153 S's: Yeah*
> *154 J: So based on this, Vanessa (display with 3 removed), what would you guess my arm span is?*
> *155 V: 150*

Removing the three values revealed at a glance that two bins were of equal height. Then the students were able to quickly notice and respond to the conundrum Jill had established. Later during the same discussion, Jill utilized an even simpler manipulation method, using only a pencil to eliminate one step of a student-invented method.

> *156 J: OK, so I have a really important question. What if this person who made this said - can I borrow your pencil for a minute, Gale? What if the person who made this said - get rid of this part for a second? (Crosses off part of Step 1) You don't have to, just watch, and they said order them from least to greatest, but don't put the repeated values. Meaning how many times would they put 150?*
> *157 S: 1*

158 J: How many times would they put 100?
159 S: 1
160 J: Do you think we would still get 150 as the median?
161 S's: No
162 J: As the center?
163 S's: No
164 J: Why not?
165 S: You have to -
166 J: Does the fact that 150 is there five times change something? You're nodding your head, Mandy. What do you think it changes?
167 M: Cuz when it says to cross off the numbers from left to right, it changes it.

Here, instead of transforming the data set, Jill transformed the method itself, showing flexibility in the form of the transformation routine. She removed a section from the method to help students envision its conclusion. Even before she asked her final question about the effect of the transformation, “Does the fact that 150 is there five times change something?” she scaffolded the transformation through a series of building questions. We can imagine that even without a pencil available, students would still have easily reached the conclusion here. However, the annotation did slightly ease the burden of considering the hypothetical that Jill had posed.

In Jill’s Unit 3 discussion about ways of measuring variability, some student methods relied on selected data. The first method, for instance, finds the range of the data after removing the highest and lowest values. Jen wanted to link this method to a more conventional method that finds the range of the middle 50% of the data. She employed her Smartboard as an additional resource for two different transformations. In both examples, Jill guided her students toward defining the clump as the middle 50% of the data values. She used a rational number line already posted in her room as a hint toward the type of thinking she wanted.

168 J: How about this? How about if I had a set of 100 numbers. And I want to take off a set from the side. Say I'm looking at the number line over there for a second.

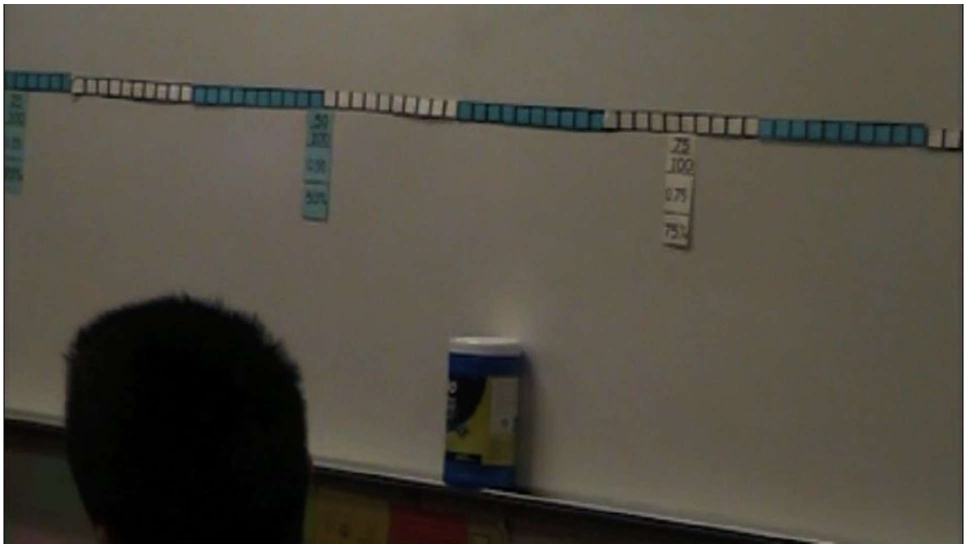


Figure 26. Rational number line posted in Jill's room, used to guide students toward thinking about how to narrow a set of 100 data values down to the middle 50% of values.

Some of you might say "Oh, I have an idea." Say I had 100 numbers and I wanted to take off like a certain part every time. the same amount each time. We're not gonna say take away 5 squares, I'm not gonna say take away 10 squares, but I'm gonna say something a little bit different.

169 J: Victoria?
170 V: Take away the ones on the side.
171 J: What are those things over there on the number line?
172 S: Oooh, the wholes! Take away the wholes!
173 J: The wholes would be 0 to 1.
174 S: They're cubes!
175 J: Could I take away part of it?
176 S: Yes
177 J: What part could I take?
178 S: 1/4
179 B: ½.
180 J: She's on the right track. Brandy said take off 1/2. OK, take away half the numbers. If I said take away half the numbers, how would I decide what's half?
181 D: Count them?
182 J: So Demi said count them. Let's say there's 30 numbers, cuz that's a good number to work with. So what's half of 30?
183 S: 15
184 J: So I can take away 15 numbers. What if I had a set of 100 numbers? How many would I take away? What if I had a set - and I wanna hear everybody so be listening. What if I had a set with 10 numbers?
185 S's: 5
186 J: What if I had a set with 30 numbers?
187 S: 15

188 J: What if I had a set with 50 numbers?
189 S: 25
190 J: OK, do I want to take away half the numbers?
191 S: Yes!/No/Too much
192 J: So try this. Cut it in half, get rid of the bottom. Done.
193 S: No/that's too much!
194 J: Well what about the top?
195 S: Well what about the top?
196 J: What about the top?
197 S: There's nothing at the top!
198 J: Well here's what I'm saying. Can I go like this and say "That's half," Get rid of them, done. Do you guys agree? Do you think that's OK?

Figure 27. Jill eliminating the lower half of data on her interactive whiteboard

199 S's: No!
200 J: I just found half the numbers and got rid of them.
201 S: Not OK. What about those people?
202 J: What half do I kind of care about?
203 S: The middle half!
204 J: The middle half, OK!

The student's comment that "There's nothing at the top!" (line 258) referred to the empty space at the top of the display, rather than the right side that the teacher referred to as the "top values" in the display. The teacher responded to this student's confusion by crossing out the values she refers to as the "bottom" so that the student can see the distinction she is making Line 359). Once she eliminated the "bottom half," she repeated the transformation question, "Can I go like this and say 'That's half,' get rid of them, done. Do you guys agree? Do you think that's

OK?” Consistent with Unit 3 discussions across teachers, Jill used a transformation in service of generalization. Her primary generalization question here concerned how to find the center clump of any data set using fractions. Then she used the hypothetical to hone the range of values that should constitute the most important half. Ultimately, the students reached consensus that the middle half was generally the best choice. Shortly after the last episode, Jill transitioned the class to another data set to further test the soundness of the middle 50% method.

205 J: So how about this? Say I'm looking at this set.

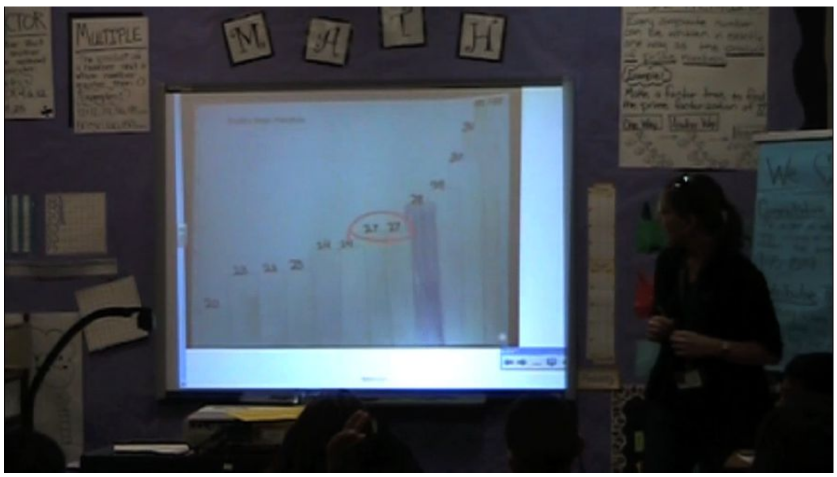


Figure 28. Jill using a display to illustrate the middle 50% of values

Sally's bean handfuls. And I said take off the bottom 25%, take off the top 25% and just focus on the middle 50%. Would that work?
206 S: Then the middle would be IA/why is it circled?
207 J: Do you think that would work?
208 S: No
209 J: Is that a good method?
210 S: Probably.
211 J: Well, let’s try it. I have 14 numbers and I want 25% of 14, so I get… OK, so I can take 3 and a half numbers off.

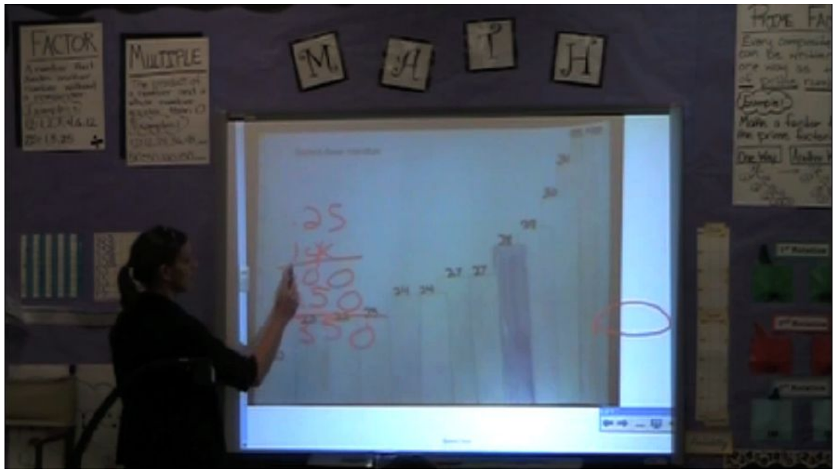


Figure 29. Jill calculating the middle 50% of 14 numbers

After hearing contradictory positions from students about whether the method would generalize to this new data set, she said, “Well, let’s try it.” Again, she used the Smartboard to annotate the display as they collectively eliminated 25% of the values at each end and reached the conclusion that the method “works.”

Jill’s Smartboard allowed her to manipulate a data display without marking on a student’s work. She began with a digital image of the student’s display, and she both added and removed elements of the display to help students envision a transformation. The tool served as a conceptual resource for students without doing too much of the mathematical work for them.

In all of these instances, Jill used digital resources to quickly generate and alternate between different distributions. She maintained the rigor of the transformation activity in spite of decreasing the conceptual work that is required for students to imagine a hypothetical scenario. Because the goal of transformation is using the outcome to generate larger conclusions about the tradeoffs or generalization of a method, students did not lose learning opportunities when technology assisted with the “imagining” work. Earlier I discussed some reasons that the transformation was better suited to mathematical questions of measure. In addition, material means such as these provided explanatory power to the increased use of the transformation following rehearsal suggestions that I found in Paper 2. Strategic incorporation of material resources within discussions likely contributed to why teachers' use of the transformation routine changed over the course of the three units.

Generalizing Routine to Accomplish New Functions

Adaptations to the form of routines, including both their content and sequencing, were much more common than generalization to new functions. Most implementations of the routine were aimed at fulfilling intended functions. There are two primary functions of the transformation routine in data modeling conversations, which are further described in the next section. However, in 11 instances, teachers employed the transformation routine toward different functions as they worked toward an understanding of its intended function in data modeling:

- *Function inconsistent with data modeling to identify the methods students "liked the best."* One teacher, Andy, used transformation in service of a goal to identify a single method that was the "best."
- *Using transformation around other math content.* One teacher, Amanda, employed the transformation when taking an aside from data modeling to discuss a pattern about even and odd numbers that a student posed.
- *Using transformation when the lesson plan says to → using transformation spontaneously to contradict problematic student thinking.* Teachers initially struggled to find useful places for the transformation and were coached during rehearsal about its timing. Over time, they began to use the transformation strategically in response to student thinking, even when it was unplanned.

- *[Teachers] Using transformation to pose a question → [Students] appropriating transformation questions to answer questions.* Students began to ask hypothetical questions themselves, both to initiate their own transformations and to answer teachers' questions. Consequently, the routine sometimes disrupted previously established roles, although these new roles were productive because teachers embraced these changes in student role and responsibility. Further, some students came to interpret the transformation question as an invitation to take opposing sides for the purpose of debate.

Before illustrating each of these phenomena, I first describe the intended functions of the transformation routine in more detail.

Intended functions of the transformation routine. The function of the transformation routine during discussion focuses largely on two different types of generalization. First, in Unit 1, teachers help students generalize that the shape of the data is dependent on representational choices. Moreover, the shape of many repeated measure data sets will generally take a similar shape when represented in the same way due to the composition of an observed value as signal and (random) noise. This type of generalization is employed to help students reason about similarities and differences across different displays. For instance, if a single class represents an identical set of measurements in three different ways, the shapes might look different even though the individual data points are identical. If three different classes of students measure the same teacher's arm span, and each class makes the same representational choices when creating displays of their own sample of data, the individual data points would be slightly different but the shape of the data would generally be the same across the three samples. However, to be able to access ideas about shape they first have to understand how the mathematics of order, interval, count and measurement scale influence the visible shape of the data. Transformation questions are also useful here, where students can imagine changes to the designs of the student-invented displays that would build to that shape. The second sense of generalization that is featured is the idea of a statistic as a measure of a characteristic of a distribution. As such, will the method of measure work for multiple data sets? And is it a sensible (robust) measure? For example, during Rob's Unit 2 discussion about measures of center, he asked, "So what if there were two modes, and what if 450 and 490 were exactly the same? Would Leo's (the student inventor) method still be a good method for finding the true measurement of the table?" Here, he was trying to contradict the notion that any of the student-invented statistics would be best in every case. The robustness of a method must be analyzed in relation to the features of the distribution. The transformation routine continued with Rob challenging the student's affinity toward mode as the best method by suggesting a transformation to the number of modes in the data set:

> *72 R: What if there were two modes in our data set?*
> *73 S's: Yeah*
> *74 R: Would Leif's method still be the best way?*
> *75 S's: Yeah*
> *76 R: OK. Tell your partner why.*

Even though the students initially respond affirmatively to his question, Rob continues to press on their reasoning to ensure that they understand and reason through the case he posed.

Function inconsistent with data modeling: identifying the "best" student method. Some Unit 1 instances contained surface features of the transformation question but functioned in ways inconsistent with the intended goal to make sense of important statistical ideas. Adam, a Cohort 2 teacher, routinely used the transformation in service of identifying the one display or method that students "liked" best. This use of the student inventions is located on the construct map (Appendix C) at level 3, where student inventions as "an instructional resource to promote a right/wrong orientation towards mathematical ideas" (Jones, 2015). In comparison, a teacher at the highest level (5) uses student inventions as "a resource to communicate different mathematical strategies in order to synthesize mathematical ideas into larger systems of meaning" (Jones, 2015). Adam posed a hypothetical scenario similar to the ways other teachers did, but he followed the hypothetical with a request for a conclusion about whether students would prefer a display or method over another as a result:

> *1 A: Anybody have any ideas if we just did that hypothetical situation what would happen to that mean? Cuz Karla likes number 4 better than anything. Karla, you still - you'd still like it if we had three numbers. One was 25, one was 52, and one was 549.*
> *2 K: Um -*
> *3 A: You'd still like - you'd still like to do it? Would you still like method 4?*

Adam used the transformation as a way to contradict problematic student thinking. Here, he was trying to highlight the vulnerability of the mean to critique that method. Ultimately, a major goal of discussions in the three units I observed was to highlight tradeoffs of methods for displaying or measuring data, and the transformation was well-suited to discussions about tradeoffs.

Generalizing the transformation's function to other math content. In other cases, even though the transformation routine was employed in ways consistent with sense-making about tradeoffs and generalizability, the content about which students were asked to reason did not prove fruitful to the larger discussion. For example, Amanda's first classroom attempt at the transformation question took place during the introduction of the second student display. Amanda's class data that was represented in the displays was generated by each student measuring the perimeter of a rectangular table in the classroom.

> *37 A: Let's move on to the next - to the next display. And I want you to think about what these authors were trying to show. Raise your hand if you were the author of this display, and I'm gonna ask that you don't give away any information; we'll come back to you. OK? Um, so look at evens and odds, so let's see, Table*

4, you wrote that evens were on the right, odds were on the left. Would it have been important if they had put odds on the right and evens on the left?
38 S's: Yes/no
39 A: I'm sorry, odds on the left and evens on the right?
40 S's: Yes/no
41 A: So it wouldn't matter? It's just evens and odds?

Typically the transformation question is used during a later portion of a display's discussion or during a comparison between two displays. Amanda tests the question during the introduction of this method, posing a hypothetical about reversing the position of the odd and even numbers. While teachers are asked to elicit insight from other students about each display that have selected, the insight Amanda requests through this question does not appear to serve the mathematical goals of the unit or her own self-identified goals for the discussion. It also does not reflect the rigor a transformation question offers when in used in service of the intended goals of generalizability. This example illustrates how the rigor of a particular question is not tied to the question itself but in the contextual appropriation of the move. Her second transformation uses unclear language, forcing a student to ask "What do you mean?" before attempting to answer her question.

Amanda's third attempt at the transformation represents a shift in function. This instance is an improvisational use in response to a student's claim that the sum of two odd numbers is always an even number, coupled with an example of 3 + 3 = 6.

42 M: No, because like he's saying that one - one - since it's one side's bigger it has to be odd but since it's odd and odd and odd plus odd equals an even, so it has to be an even, cuz it has to be - like um 3 plus 3 equals 6, it has to be even. Odd plus odd has to be even. Since like -
43 A: Would that work with any combination of numbers you use?
44 M: Yeah.
45 A: 6 and 6 and 3 and 3, it's always gonna be even?
46 M: Yes
47 A: What do you think? You guys agree with Matthew - it's gotta be on the even side, not the odd?

At face value, this exchange seems unrelated to statistical content. However, this student's claim counters a previous claim made by a student, that "the correct answer is the most likely to be the odds cuz we're measuring a rectangle. Rectangles aren't even. They're odd." In this example, the move is executed in service of generalization about the true measure of the rectangular table, and one that Amanda had certainly not anticipated. While many rich discussions about the measurement process could follow from such claims, Amanda uses a transformation to conduct an aside about sums of even and odd numbers and returns to the larger conversation. Between the first and second use of the transformation, Amanda refined its function in service of instructional goals. She moves from a somewhat arbitrary use of the move to a contextually responsive one that builds from a student's reasoning. The transformation also positions students against each other for the sake of mathematical argument, launching them into debate that begins to establish a routine for participation. Amanda's final Unit 1 transformation question served as a significant conceptual resource from which the students could begin to consider a shape to the data. She asked students to imagine a display that combines the key design elements represented in each of the two displays they discussed. Through this line of questioning, a student began to elaborate a conjecture that the table's true measure would become clearer because repeated values would be visible. Amanda continued to refine her use of the transformation question into the second rehearsal.

Adam also made a shift in his use of the transformation routine. In the example from the previous section, he used the transformation to ask a definitional question that limited students to the recall of the term "median." However, the very first instance of his Unit 3 classroom discussion shifted to a more typical use of the hypothetical. He asked "Which method do you think might work if I went ahead and showed you a new data set?" Which method wouldn't work if I showed you a new data set?" Most of the other instances in this same discussion followed the same form as this first instance, employing hypothetical situations to reason about the function or importance of a measure of precision (e.g. "That 41 tells me the difference. What happens if I had 82 and every other one of these numbers was 41. Is that still important?"). In only one case he employed a hypothetical to promote a right/wrong orientation of a student's method.

Using the transformation spontaneously in response to student thinking. As teachers became more familiar with the transformation routine in each unit, they began to employ it more strategically for a function of strategically responding to student thinking, whereas earlier attempts reflect an obligation to follow their plans. My data suggests that teachers did not initially understand the purpose of the transformation or how it could help them accomplish their instructional goals, whereas in Units 2 and 3 they considered it useful for a function of contradicting problematic student reasoning. To support teachers initially in using the transformation routine and anticipating hypothetical questions, they were asked to plan specific questions to ask about the displays or methods they chose (Appendix E). Planning ahead gave them a chance to carefully consider the questions they would use and, because they often referenced their plans several times during a discussion, the plans served as a cue as to when the questions might best fit into their conversations. In fact, some initial rehearsal suggestions even helped teachers think through the execution of the transformation relative to other landmarks of the discussion's structure. The earlier example of Abby provides a good example:

90 I: You did show and hide now, so now what you might want to do is transforming, mentally, and say, "What would this graph look like if - "
91 A: What would this graph look like if we put all the numbers in order?

However, teachers still struggled initially to use the transformation at opportune times during classroom discussions. In one instance, I asked a transformation question at the end of a discussion on behalf of the teacher, who had forgotten to ask it:

> *87 I: There's one more question - um what if this group 3 on the right over here? Instead of having like a display at the top and a display at the bottom, what if they had combined those?*
> *88 C: Oh, that's right.*
> *89 I: What would that look like?*

As classroom coach, I had pre-planned with Chad, so I knew that he had planned to ask this question himself. Unfortunately, the topics of conversation did not serve as a sufficient cue for Chad, even though the class had discussed what each display made visible and hid about the data. Like Chad, teachers initially planned to situate transformations into particular points in the structure of the conversation. Typically, they fell in the latter half of the discussion, during the "Making Connections" template routine.

The general shift in function occurred after Unit 1 into units 2 and 3, where teachers used the transformation more often on average (Table 17). Many teachers found the move useful to the discussion more generally for calling a student's reasoning into question immediately rather than at planned times, even though many of these moments required spontaneity in generating hypotheticals on the spot. For example, the following transformation took place during Aspen's Unit 3 discussion:

> *77 A: So Ike thinks this is only working because our numbers are less than 100. Is that what you're saying Ike?*
> *78 I: Yes*
> *79 A: All of your numbers are lower? So what if I had this Ike? (writes on board: 1002,1003,1002,1003). Do you think this method wouldn't work? Do you think I would still get a low measure of precision?*
> *80 I: You'd get a higher precision, but not higher than – it would be probably - so the number value off - but I would still think that how much numbers were in the data set would matter because -*
> *81 A: So the size of the number doesn't matter – right?*

82 I: Yeah
83 A: So this would still work, do you agree?
84 I: Yeah, it's just - it matters how close they are too.

Here, Aspen used a transformation question in response to Ike's claim that a precision method will only result in a small value for small numbers. She quickly generated a counterexample to contradict Ian's reasoning and recorded her example on the board for students to consider. In spite of the counterexample, Ike maintained his position that the magnitude of the numbers in the data set will directly correlate with the magnitude of the precision measure. In the same statement, Ike also suggested that the number of values in the data set would influence the precision measure. Aspen pressed on this piece of his statement to clarify that the magnitude of the numbers is irrelevant. Ike confirmed and added that the proximity of the values matters as well. Another student in the class, Ginny, began to calculate the precision of the data set Aspen had written on the board, using the invented measure they had been discussing. When Ginny finished her calculation, Aspen invited her to share her findings as a final piece of evidence to contradict Ike's original conjecture:

85 A: So [Ginny] did both methods, and she used the larger numbers for Ike, and she came up with the exact same measure of precision. Does anyone want to respond to that? Thank you, Ginny, you can have a seat. Does anyone want to respond? Ike let's start with you. Do you now - how do you feel about those larger numbers? Do you think they'll still work?
86 I: Yeah

These two exchanges in Aspen's classroom look markedly different from some earlier instantiations of the transformation routine in other classrooms, where students expressed some confusion in response to transformation questions asked by the teacher. Here, the transformation routine unfolds smoothly, as both student and teacher know what to expect and how to participate in this hypothetical space to build meaning around the method. However, because the transformation question did not prove problematic to the students in this class, even during the Unit 1 discussion, I cannot speak to its development in this particular classroom. I suspect that the norms and expectations were already in place for this class even before my study began.

The teacher thinking required to generate these kinds of responsive hypotheticals is not trivial. In fact, this type of responsive transformation was a focus of rehearsal in a Unit 2 rehearsal for Cohort 1, which I will discuss in more detail later. Kristine wanted to elicit that higher values in the data set would pull the mean up. In her first attempt, she asked students to imagine these larger numbers on their own. She asked, “What would happen, Emma, if we had more numbers up here further away from our median but the higher numbers? Where would our mean be on our chart?” At this point, an instructor stepped in to coach her through the thought process of generating a hypothetical data set that would serve as a conceptual resource to students without doing the key mathematical thinking for them. The instructor suggested, “So Kristine, give me a very very simple, say 5 numbers. Two sets that have a median of 3 but have different means.” This particular interjection continued for an extended period of time, as teachers worked to understand the utility and mechanisms for generating simplified, exaggerated examples to call out the vulnerability of different measures of distribution. Kristine did, in fact, use the transformation during her Unit 2 classroom discussion in response to a student overgeneralization that mode will always be the best method for measuring the center of the data. Her class measured the perimeter of a table in centimeters, and their data centered in the low 500’s. Kristine began the transformation with the question, “What if the most common was 27? What if 3 people had 27? Do you still think that’s gonna be the best measure of finding the center?”

Although teachers did not use transformations in a responsive or improvisational way during their Unit 1 rehearsals or classroom discussions, they began to improvise situational transformations as early as their Unit 2 rehearsals, which helped prepare them for the Unit 2 classroom discussions. The next few examples highlight the development of the spontaneous use of the transformation routine, and particularly, ways that teachers learned from failures in execution of the transformation that later supported more successful attempts. Chad, for example, asked a transformation question during his Unit 2 rehearsal in response to “a student’s” preference for her own measurement of the table as the best guess. Chad’s goal was to highlight that a student’s invented method was not replicable because the choice between the two most repeated numbers was too interpretive. The method required binning the data by tens, finding the 150’s bin, finding the repeated values in that bin, and then identifying the most reasonable of those repeated values. Chad used a transformation to contradict Helen’s reasoning in the following exchange:

96 C: So let's say that we give you a completely different set of numbers that you didn't measure yourself and you come up and you end up with two. Now you can't say "I picked, I actually came up with that so I pick that one" so you see how your process of using your own measurement as your determining factor, it doesn't work now, because it's a completely different set of numbers, so how could you adjust your decision on picking between the two doubles if you didn't personally come up with either one of them?

97 H: Maybe I'd go with the middle of it, because 152 is here, and 158 is here, and they're six apart, so I'm gonna go three in, so I'm gonna change my best guess to 155 cuz that's really the - that's really the middle.

In contrast to more polished student-centered classroom transformations that followed this episode, Chad fell into a bit of a trap here. Instead of pitching a transformation question to the group, his speech took the form of a monologue. He both posed the hypothetical and explained the conclusion himself in a single turn of talk. Rather than guiding the “students” to notice the weakness in Hillary's thinking, he pointed it out and explained it, almost as if he was forming that conclusion himself as he verbalized it.

Looking into his Unit 2 classroom discussion, Chad found ways to employ the transformation responsively. The first instance of the transformation in his Unit 2 discussion following this rehearsal showed a productive shift from answering his own questions to asking students to provide their reasoning about the result of a hypothetical situation. However, there is still residue of the rehearsal exchange in the way he still constrains some students' responses to single-word answers:

> *98 C: 138, right? So Matthew, you said find the highest number that has the most.* ***Can anybody see where that might be a problem if we used it on a second set of numbers?***
> *99 S's: Yeah/yes*
> *100 C: Who - who wants to give it a shot? Matthew said find the highest of the two numbers that have the most. If we wanted this to be general enough to work on all sets of numbers, how could that be a problem, OK? So he said find the highest number of the two that show up the most. If the numbers were different, how could they be different to make what he just said a problem? If our goal is 138? Ivan?*
> *101 I: One would be like 567, and 130 and (IA) those number but it's still telling me to pick the higher one.*
> *102 C: So if - what if there were three 575's and three 138's and one 26? Matthew* ***what number would you now be focusing on based off of what you just said?***
> *103 M: 575.*
> *104 C: 575.* ***Is that - is that his goal? Is that where he's thinking it is?***
> *105 S's: No*

In the rehearsal, the supposition of a "different set of numbers" was sufficient to guide Helen to a revision that was more replicable (line 97). Chad even asked a more challenging question in the classroom by following his initial question with one that required students to generate a hypothetical that would highlight the problem. When a student volunteered a hypothetical that did not highlight the particular point that Chad was trying to make, he generated his own hypothetical for students to consider. Although he moved beyond a monologue here by asking students for their thoughts, he alternated between two different forms of the transformation question (lines 98 and 102) and limited student participation to single-word responses after they did not submit the ideas he had hoped. Later during the same discussion, Chad used another transformation to elicit the vulnerability of a method that drew explicit focus to values between 100 and 200, again inviting students to reason about the result of the hypothetical but with residue of the rehearsal in which he provided much of the explanation:

106 C: OK, listen to this. What if we were looking at a group of um children's ages? Where are children ages gonna fall according to this?
107 S's: (various)
108 C: One through 12, and it says "Not over 200 and not under -
109 S's: 100
110 C: So if I had a chart of elementary school children's ages, how am I gonna find it if I can't use any of the ages, right? So you see why we gotta be careful when we make these rules, we gotta make sure, and this is the first time we've done it. Sit down Sami, sit down. So this is the first time we've done it, so I told you to expect - that's why we walked through 'em right?

Prior to this exchange, Chad had already tried unsuccessfully to elicit the limitation of the method for data sets outside of 100-200. There, he began by pointing to the step and asking why the method would "not work." He even asked students to discuss the question with a partner. However, the conversations did not yield anything fruitful. One student suggested, "There's a lot of numbers in between 100 and 200," which instead pointed to the method as a good match for the data set. Chad tried a bit more specificity by asking students to "think about using this on a different set of numbers." Again, students did not produce anything substantive. At this point, Chad used the transformation question in this example to point students in the right direction. "What if we were looking at a group of um children's ages? Where are children ages gonna fall according to this?" Because his original question sought a hypothetical scenario in which the method would fail, the hypothetical embedded in this transformation question actually provided the answer to his original question. Regardless, this episode illustrates a further shift from his original rehearsal attempt, where he posed and answered his own question. He shows progress in shifting the mathematical responsibility to the students. Not only does he provide more time for them to consider the questions, but he makes adaptations by embedding strategies like partner talk and re-phrasing. In Unit 3, he found success not only in students' conclusions to his hypothetical but also in generating hypotheticals of their own. The following example comes after another set of failed attempts to elicit the result of a difficult hypothetical scenario from students:

> *111 C: What would have to change on the numbers of the display in order for the steps that he's written to incorporate the biggest and the smallest number? How would the display have to change? Jamal, did you hear Jamal say - what did you say, Jamal? He seems to think you know.*
> *112 S: I was thinking that for that to work, the whole chart, that first number that was repeated like 18, would have to be repeated more than once.*
> *113 C: OK*
> *114 S: And then the last number, 575, would have to be repeated more than once too for his second step to work.*

Here, Chad's persistence has paid off. After difficult collective work with his students to construct this routine, he succeeded in both shifting the mathematical work to the students as well as shifting the work of generating hypotheticals to his students. This series of examples provides further evidence that the development of routines results from many opportunities to recontextualize in different contexts. Chad's trajectory includes both rehearsal and classroom episodes and shows shifts in the form and timing of his questions, even though he struggled with both of these pieces along the way.

Student appropriation of the transformation. One way that the transformation routine generalized to new functions was in the different goals of participants that were embedded in its use. Students began to initiate transformation questions on their own, and in some cases the initiation of a transformation question signaled the beginning of a debate for students. As students learned the transformation routine, some began to ask hypothetical questions themselves in service of a function toward answering a teacher's question, shifting the role of "hypothesizer" to the students. My initial conjecture assumed that this was an artifact of their familiarity with the routine over time. However, the data suggested instead that these instances took place sometimes very early into the development of the transformation routine. In other words, as the roles of the routine were negotiated, students experimented with initiating hypotheticals on their own. All of the student-initiated hypotheticals took place during a transformation routine initiated by the teacher, but as the students learned the "rules of engagement" for the transformation routine, they used hypotheticals for several different functions.

Students using the transformation question toward a function of becoming "hypothesizers". As early as Unit 1, students generated "What if" questions, both as authentic questions to which they sought an answer and as responses to questions the teacher posed. In the following Unit 1example, Chad's students discuss a student-invented design for a display of the data in which only one of any repeated value is represented. His goal was to elicit a conclusion that the number of measures taken would be difficult for a reader of the display to know.

> *212 C: What would be - what would be the possible mistake you could make if you saw a chart with just one 42 on it?*
> *213* ***S: What if you're - what if you're - like what if you answered the question "How many 42's are there?" And you're looking at the one that has one 42 chart you would like count them wrong.***
> *214 C: What would you say?*
> *215 S: One*

Adopting the display design that only represents repeated values once, Chad imagined a scenario in which all the measurements were 42. His question concerned the assumption that might be made about the number of data values. In response to his question, a student imagined misinterpreting how many 42's were measured. The student posed his own hypothetical in response to Chad's question: "…what if you answered the question 'How many 42's are there?'" The student's hypothetical identified the misinterpretation that the teacher envisioned. Chad helped him follow the scenario to the inevitable conclusion of the misinterpretation they collectively envisioned. Although this is only halfway through the class's first discussion (Unit 1), five instances of the transformation routine had already preceded this one, providing ample opportunities for students to become familiar with the "rules of the game."

Similar student hypotheticals were posed in two more teachers' classrooms during Unit 2 discussions. The first took place at the very end of Rob's discussion. His class had examined and compared mode and median strategies for finding the best guess of the table's perimeter. Rob closed the discussion with a transformation routine, although this was only the second transformation routine that he had initiated in his class. Scaffolding, leading, and limited response options were characteristic of the exchange, likely for that reason.

216 R: So we're gonna talk a little more about median on Monday, and we're gonna talk a little bit about a couple other methods that get us something slightly different, but I really wanna make sure that - you know that - you can use these two methods to get us to the best guess. But is mode always going to get us there?
217 S's: No
218 R: No, we were lucky and we got there. Is mode sometimes the easiest to do?
219 S's: Yeah
220 R: Cuz you're just looking for what?
221 S's: (various)/most common number
222 R: The number that appears the -
223 S's: Most!
224 R: The most! But it's not always gonna get us to our best guess of the table, right? So Aimee?
225 ***A: Um, it's never gonna get to an answer because what if you're measuring that table and this table?***
226 R: But did we do that?
227 A: No
228 R: But if we did, would those tables be the same?
229 S: No
230 R: Probably not, so we would have different - we would have a different best guess, right? So our numbers would look a little bit different. And we will

eventually look at our other data, cuz we measured using two things, right? We measured using a little ruler and then we used a big ruler so we're gonna have to compare in the future to see which one was better.

At the culmination of a series of questions leading to a conclusion that mode will not always be a reliable way to reach a best guess, Aimee interjected with a hypothetical of her own (line 225). She concluded that neither mode nor median would be reliable, although her hypothetical that the class would measure two different tables at once was not consistent with Rob's supposition (a repeated measure of the same table). Still, she tried to engage in the "rules of the game" and Rob adopted her supposition for further press (line 228). Up to this point, the routine had positioned Rob as the mathematical authority in relation to the students, who were the "answer-seekers." The roles changed when Aimee offered her hypothetical as an unsolicited explanation to a question that only required a yes/no answer from students, if one at all. Because the structure of the transformation had not yet been routinized in this classroom, this example illustrates a negotiation of structure and roles that will later develop into a more stable routine.

A similar sequence also took place in Marissa's classroom during her Unit 2 discussion. Marissa began the episode with a transformation question focused on generalizability of a student's method for producing a best guess with any data set.

231 M: Now can this be applied to any set of data? What if these weren't our numbers? What if we took Mr. Scott's numbers that all the students in his class got for measuring that table?... Could this still apply to his numbers?
232 T: Yes. Any numbers.
233 M: So it would work with any numbers. Even the first step right here? Evan?
234 E: I say it wouldn't apply to any numbers because it says group them into hundreds.
235 M: OK
236 ***E: And what if all the numbers were below 100?***
237 M: Aaahh, did you guys hear that? Ainsleigh, did you hear what Ethan said?
238 A: No
239 M: What did he - can you repeat for Ainsleigh?
240 E: I don't think it could work with any numbers because it says IA hundred and what if all the numbers are below 100?

241 M: OK so Mr. Scott's class measured that perimeter and say they got like 58, 24, 10, then you're saying that this wouldn't apply.

Initially, Tara responded that the method is completely generalizable (line 293). Marissa pushed back by pointing specifically to the first step of the method that asked students to bin the numbers by hundreds (294). At that point, Evan positioned himself against Tara by identifying a counterexample to her claim in the form of a hypothetical (line 297). He imagined a data set with values less than 100. Typically, the transformation routine requires a conclusion be provided to an imagined scenario. In this case, the scenario Evan has imagined *is* the answer to Marissa's question that points to a vulnerability of the method. Marissa's question only technically required a yes/no response. She asked "Could this still apply to his numbers?" and "Even the first step right here?" Evan went beyond what was necessary to fulfill Marissa's request and extended his response to a situation in which the method would not work, responding more to Tara's overgeneralized claim than the teacher's yes/no question.

Students also began to take more responsibility for transforming without being prompted. This serves as evidence that the transformation became more routine for the class over time. The following pair of examples represents two consecutive instances of the transformation routine in Jill's classroom. Both took place during the end of discussion around the first method. Here, the teacher had announced a transition ("Well we're gonna move on"), but the student responded to the teacher's cue by initiating a hypothetical question.

Instance 1:

242 J: So my last question is does that method work?
243 S's: Kinda/no/depends on the data.
244 J: OK depends on the data. There might be some examples where I don't wanna take off the top half. Last May, almost all my students were passing a division timed test. If I graphed that, almost all of my class would be on the far end. What would happen if I took off the top 25% and the bottom 25%, would I really get a number I'm looking for?

245 S's: No
246 J: Well we're gonna move on.
247 ***S: What if there were only ten?***
248 J: That's a really good point, what if there were only ten? And there were only ten numbers, 25 would be 2 and a half, 25 would be 2 and a half, and you'd be stuck with the middle 5. So you'd just look at the middle 5 numbers. What if I only had 4 numbers in a set? Take off a middle one or take off the far one, take off the far one, you're left with 2 in the middle. So if you don't take off a set amount, if you just take off a percent, that might be OK.
249 S: What if you only had 2 numbers?
250 J: What if you only had 2 numbers? I can't take anything away, cuz I'd have nothing left. So what if I had 100 and 1? I can't take off the 100, take off the 1 and be done, so maybe I'd just have to subtract what's there.

Here, the teacher and student essentially switched roles from question-asker to question-answerer (lines 247-248). The students realized that the method would only "work" if it results in a numerical value for any data set. One student posed a hypothetical about a single case, "What if there were only ten?" to further check the robustness of the method. After the student posed the hypothetical, the teacher modeled her thinking to determine the conclusion, assuming the student was asking for the number of values in the middle 50%. The student continued to alternate roles by the teacher, as they took turns posing and responding to hypothetical data sets. The student's questions mirror those of Jill prior to this exchange, where the students were asked the very same types of questions the student now posed to Jill:

251 J: So Demi said count them. Let's say there's 30 numbers, cuz that's a good number to work with. So what's half of 30?
252 S: 15
253 J: So I can take away 15 numbers. What if I had a set of 100 numbers? How many would I take away? What if I had a set - and I wanna hear everybody so be listening. What if I had a set with 10 numbers?
254 S's: 5
255 J: What if I had a set with 30 numbers?
256 S: 15
257 J: What if I had a set with 50 numbers?
258 S: 25

In fact, the very same example, a data set of 10, was already hypothesized by Jill during this earlier exchange. Later during this discussion, Jill initiated another transformation, but this time instead of asking the students to imagine a transformed data set or method, she provided the conclusion (Range is not a good measure of precision) and asked students to imagine a hypothetical that would lead to that conclusion.

Instance 2:

259 J: Can someone tell me of an example of when range like this is not a good indication on if it is a good set of numbers or not? Samuel?
260 ***S: Cuz what if it's like 599 subtract 1?***
261 J: So like what if we had a measurement where just about everybody had the same measure, but someone over here was crazy and someone over here was crazy, and those crazy people, they're the only measurements we're looking at? What's wrong with that?
262 S: Cuz they're crazy!
263 J: Does that tell us a lot about the data if all we look at is the two end numbers?
264 S: No
265 J: I don't think so.

Samuel suggested a data set where 599 was the highest value and 1 was the lowest, resulting in a range of 598. Jill extended his thinking a bit by suggesting that everyone except one person had the same measure. Together, Samuel and Jill built a new form of the hypothetical question for the class to consider. Jill added a question to their hypothetical scenario ("Does that tell us a lot about the data if all we look at is the two end numbers?") for the class to consider.

In these cases, which account for all of the instances of this phenomenon that I observed in my analysis of the transformation routine, students have adopted the "rules of the game" to answer the questions posed during the transformation routine, but they also began to experiment with the role traditionally held by the teacher. Not only were the roles for teacher and students fluid in these cases as students appropriated forms of questioning used by their teacher, but the students and teacher collectively built to the mathematical concepts through a joint effort.

The transformation routine functioning to initiate argument. As discussed earlier, the transformation became more meaningful for teachers spontaneously for the purpose of contradicting a student's overgeneralization. Teachers even provided "hints" through the form and intonation of their discourse (intentionally or unintentionally), such as "Do you **still** think…?" when asking questions for this purpose. However, some students began to ignore these cues in favor of adopting the positions they truly believed, even in opposition to each other, instead of only the ones suggested by the teacher's language.

Earlier, I discussed how the sequence of discourse moves in Kristine's transformation routine often positioned her students as debaters. In the following example, Kristine is reaching the end of her Unit 2 discussion about measures of center. The method of contention is the mode. Kristine initiates a transformation question to challenge a student's insistence on the mode as the best measure of center. Another method that has circulated during their discussion focuses on the most frequent value inside the highest bin. Many of the students prefer that method. This case shows how students have formed meaning around the transformation question as an invitation to debate.

> *266 K: 520. What if the most common was 27? What if 3 people had 27? Do you still think that's gonna be the best measure of finding the center?*
> *267 S1: Yeah.*
> *268 S's: No*
> *269 S1: I think yes because it's inside the biggest bin. The bin -*
> *270 K: 27, if there were three people who had 27 it still wouldn't be the biggest bin. The 500's would still be the biggest bin. Three people just all got the same answer -*
> *271 S2: I think it would be 520 because that's - that's the - first most one in the 500 bin.*
> *272 K: Nuh-uh, 515 shows up too. And 520 shows up three times.*
> *273 S3: 535 shows up two times.*
> *274 K: Yeah, and what if 27 showed up 6 times? Do we think that would be the right answer?*
> *275 S's: Yes/No*
> *276 K: Sounds like we have a bit of a debate going on.*

Kristine's first question (line 266) provided a subtle hint ("Do you *still* think that's gonna be the best method?") for the students to find fault with the method in a case where the mode was an extreme value. In this case, the value of 27 was the lowest on the display and far outside the central clump in the 500's area. While the student maintained her position (line 267), the rest of the students positioned themselves against her by answering "No." The same student responded to the class's dissent by explaining her reasoning that 27 would now fall in the highest bin (line 269), but the teacher clarified that the hypothetical scenario would not alter the highest bin of 500's. A second student took the teacher's lead and explained his reasoning that 520 was the best guess (line 271). While the teacher may have agreed with his conclusion, she pushed back on his reasoning with an observation that contradicted his claim (line 272). Another student added his own contradiction to the student's claim. The teacher exaggerated her hypothetical further to push students away from an attachment to mode as the best guess. However, even her exaggerated example of the extreme value represented six times, the class was still deadlocked. These types of instances were generally concentrated in three of the twelve teachers' classrooms, suggesting that the norms of these classrooms had a stronger influence on the frequency than professional development.

Relations Between Rehearsal and Classroom Enactments of Transformation

Typical moves that constituted the transformation routine in the classroom were not the same as typical moves in rehearsal. This was especially visible in the differences between how teachers positioned student ideas and methods in relation to content. In rehearsal, they positioned students as sense-makers, offering them opportunities to explain their thinking, but the classroom setting is where students were positioned as seekers of correct answers, characteristic of IRE sequences and an accuracy orientation. I also noticed the accuracy orientation in the classroom form of the transformation questions within the routine. As I show in the next section, the form of transformation routines, in both their sequencing and in the language of the transformation question itself, was more consistent with the data modeling goals in rehearsal instances, whereas classroom instances used language characteristic of an accuracy orientation.

However, in the final section, I document the mechanisms of a phenomenon of change to the transformation routine that required both the rehearsal and classroom settings. Although rehearsal and classroom instantiations of the transformation routine looked different, the two settings did rely on each other in the case of Kristine and Emma that I illustrate.

Accuracy orientations. Several teachers employed transformation questions during Unit 1 classroom discussions in service of accuracy orientations to the content. By accuracy orientations, I mean orientation to students' ideas or methods as right/wrong or better/worse than another. Unit 1 discussions focus on what different displays show and hide about the data rather than which displays are best or most accurate. Abby's rehearsal included coaching around the form (line 65) and timing (line 48) of the transformation question. Her rehearsal exchange did not employ an accuracy orientation, signaled by the ways she asks the students to provide explanations (lines 51, 56, 60) and uses the question to ask about resulting changes to the shape of the distribution (line 48) and what it would show (line 66).

48 ***I: You did show and hide now, so now what you might want to do is transforming, mentally, and say, "What would this graph look like if - "***
49 A: What would this graph look like if we put all the numbers in order?
50 C: It would get taller and taller.
51 A: What would get taller?
52 I: Good.
53 C: The size of the bars would get taller. Like that 526 would be huge and it would be all the way at the end.
54 A: OK
55 C: And 43 would be really little and would be all the way at the beginning.
56 A: At the beginning, you mean over here on the left?
57 C: Mmm-hmm.

58 A: OK.
59 K: It'd look kinda like stairs or something.
60 A: OK, so it may gradually go up. Would it go down at all?
61 I: Good question.
62 A: Would it kinda do this as it goes up?
63 K: It shouldn't cuz they should be in order. So it'd only get bigger.
64 A: It'd only get bigger because your numbers are increasing. Good. So this would be our smallest 43. And our tallest would be 526?
65 ***I: What would that help us see?***
66 A: What would that help you see?
67 C: Well, like before, I said that those two brown ones, you could tell they were the same cuz they were next to each other, but that there were probably more that were the same but we couldn't tell because it's really hard to tell like if they're really far away, if they're at the same height or not, but if they were right next to each other we could - we could tell.
68 A: Good, now would you be able to see gaps in between? Like how you spoke about 130 and 151 were not really close together? Would they be right next to each other or would you space them out?

Although this exchange does not employ an accuracy orientation, Abby's classroom discussion takes the form of an IRE exchange, coupled with an accuracy orientation, as she guides students through a transformation:

69 A: Now what if they had done a big 60 and a little 60 and a big 60 and the numbers hadn't been the same size? ***Would it have been as accurate?***
70 S: No
71 A: No, ***you're right.*** *Cuz if your numbers are the same size you can tell how many have three, they just the same height. The same ones that have one have the same height. They did a good job of planning that out. Even if it wasn't on purpose.* ***They knew it looked right.***

Here, the student is constrained to a yes/no response, while Abby assumes the mathematical authority as well as the responsibility for the mathematical thinking and the authors' reasoning (line 69). Instead of questions about what order shows or hides about the data, Abby asks, "Would it have been as accurate?" Granted, her question is likely intended to question how a representational choice of different-sized numbers might influence an interpretation about the relative frequency of the measurements. Even so, her use of the term "accurate" reflects residue of an accuracy orientation in past mathematics instruction. Perhaps she anticipates that students will understand this language more easily than questions about what a display "shows" or "hides."

However, markers of an accuracy orientation were not always inconsistent with the goals of instruction. Chad used a transformation question to ask, "*What mistake could you possibly make by reading a display with just one 42 on it?"* The use of the transformation question here is timely, as students are debating whether repeated values must be represented on a display. The language of mistakes would typically signal an accuracy orientation, but in this case, it does not position student authors as the ones who made a mistake, nor does it directly critique a display that students made. Like Abby, Chad is concerned with the interpretive potential of a display and the ability to communicate the message the author intended, easily upon first glance. Chad uses an extreme example to highlight a vulnerability of a representation of the data that ignores repeated values. Even though both Chad and Abby ask about interpretive potential, the form of Chad's question provides a stronger opportunity for student sense-making, as he tasks the students with the responsibility to anticipate the interpretive vulnerability. As the transformation routine evolved, residue of the accuracy orientation was still visible in Unit 2 and Unit 3 discussions, but less explicitly and across fewer instances.

Mutual constitution between rehearsal and classroom. The transformation routine is conceptually difficult to generate, as teachers must first think, for instance, about a vulnerability of a student's method and then generate examples (sometimes on the spot) that make the vulnerability even more visible. Because of its complexity, teachers initially struggled to understand its function and execution, as many of the previous examples communicate. Their learning was shaped through a series of learning opportunities between rehearsal and classroom settings, or a coevolution. Although the example below was mentioned earlier in the context of the shift from planned to responsive uses of the transformation, I use a more detailed illustration here to show how the relation between the two settings mutually influenced subsequent activity.

Transformation routines were often employed to examine the robustness of the mean to changes in a distribution, and particularly the presence of extreme values. If the class's own data set does not make this clear, the transformation routine can be useful for proposing a data set in which a tightly centered clump is paired with an [exaggerated] extreme value. Consequently, coaching on the transformation question often bootstrapped important discussions about content as well. For example, in Cohort 1's Unit 2 rehearsal, Kristine wanted to use a transformation to highlight the vulnerability of the mean.

277 K: What would happen, Emma, if we had more numbers up here further away from our median but the higher numbers? Where would our mean be on our chart?
278 E: It would move towards the end toward the highest bin.
279 K: It would move towards our higher bin. So is the mean always the best measure to use Carina?
280 C: I don't know.
281 I: So Kristine, give me a very very simple, say 5 numbers. Two sets that have a median of 3 but have different means.
282 K: OK, alright. 1, 1, 3, 7, oh, I didn't mean to do - I meant to do 5.
283 E: (whispers) I don't know if I know what the real mean is
284 K: 7
285 I: Now what's one that has a much bigger mean?
286 K: A much bigger mean?
287 I: Same median, bigger mean.
288 K: Uh, 2,
289 I: 1, 1
290 I: Yeah
291 K: What?
292 I: 1, 1, 3
293 K: OK
294 I: 5
295 K: 10
296 I: 100
297 K: 30. 100, OK. Alright, so
298 I: Do you see where I'm going with that?
299 K: So I see, OK, so I could stop here
300 K: And then -

301 I: Use the - use a simpler set of numbers to illustrate that point you're trying to illustrate about the mean pulling things -
302 K: OK alright.

Teachers struggled not only with the generation of hypotheticals, but also with the simple calculation of mean. While Kristine, who role-played the teacher in this example, has grasped the conceptual resource an extreme hypothetical like this serves, Emma continued to wrestle with the calculation of the hypothetical data set. The teachers continued to calculate for several minutes, after Kristine was ready to move on with her rehearsal. The exchange ended with the following resolution:

302 E: It's because of the 30 [extreme value].
303 K: What about the 30 Emma?
304 E: The 30's pulling it so far the other way, it's too big, yeah.

Ultimately, Emma articulated that the extreme value of 100 pulled the mean from the center clump of the data, but not in absence of a struggle with both the mathematics of mean as well as the pedagogical practice of generating a transformation question. Emma's struggle with mean in this case is particularly notable because earlier in this method's discussion, Emma used the rehearsal space to replay her own problematic classroom episodes around the concept of the mean.

305 K: OK, so Emma why did you think the mean might be the best guess?
306 E: Umm, I just knew that umm ... if you find the average of something that that's what's umm happens a lot that's the most likely one.
307 K: OK, so you just know when you find the average. What does it mean to find the average?
308 E: It means you add up all the numbers and you divide (laughs)
309 K: So let's pause here because I have no idea what my kids are gonna say when I try to prompt them in finding the average.
310 E: That's what they'll say. I promise, they even did it today (laughing)
311 K: So I'm gonna question them and I'm probably not gonna get anything real great.

Here, the rehearsal created space to talk about another problematic aspect of the mean, but this time it provided the space to represent perceptions of the students' conceptual weaknesses Emma replayed an instructional challenge that she had encountered recently and was unable to overcome in her classroom:

> *312 E: What does that number 336, that's what they came up with right here, and this is what y'all keep telling me, y'all keep saying it's the average.*
> *313 E: What does it mean for our measurements though?*
> *314 S: Uh, it means ... that you add all your measurements up, then divide by 17, then divide by how many numbers, how many measurements there are.*
> *315 E: OK*
> *316 E: I know how to do it, but what do I do with that number now? So I tell someone I have 336.*
> *317 E: They say "OK. Cool. What's next?" What does it mean though - I gave you all these measurements. What does 336 mean?*
> *318 S: That's ...*
> *319 E: Go ahead.*
> *320 S: It's there in answer to their measurements for after - they used all of the numbers and they - everyone's measurement and then they added them all up and they got that 712, so then they after they got the numbers by (inaudible) they got 336.*
> *321 E: Mm-hmm*
> *322 S: And so that's how the 336 has to do with these measurements of the numbers.*
> *323 E: OK I know how they got it, I know how.*

This exchange took place during Emma's Unit 1 classroom discussion, which focused on student-invented displays. However, one of the student displays showed the mean in very large print, and Emma chose this display as one of the focal displays. The student explanations were limited to procedural understandings of the mean, but Emma employed a number of discursive tools in an effort to uncover a conceptual explanation from her students. She continued to rephrase her questions and use examples, but her efforts were futile and eventually, she moved on. During Kristine's rehearsal, Emma recognized an opportunity to relate her troublesome experience by situating it in the context of a student she was role-playing. Her portrayal served several purposes. First, it created the space for Emma to seek solutions to the instructional problem she had encountered that she might be able to use in the future if and when such conversations arise again. Second, it helped Kristine anticipate similar problems of practice in her own classroom and use the rehearsal to practice how to respond in that situation. Third, it even created a learning opportunity for other teachers who might later recognize the utility of the transformation in a similar context. Because the exchange took place in the context of a simulated moment of instruction, it likely offered a stronger learning opportunity than simply discussing the possibility of this scenario together. In this case, Kristine makes the choice during rehearsal not to press on the meaning of the mean since Emma's replay suggests it would not be fruitful.

As a matter of fact, the same situation did arise in Kristine's classroom, and she was prepared for it. Rather than pressing on the meaning of the value, she focused her line of questioning on its position in relation to the tallest bin. This is where students believed the true measure was likely contained, according to the conversation leading up to this point. During the discussion of mean, Kristine leveraged their prior thinking by helping them see that the mean was outside of the tallest bin. As she expected, the students concluded that the mean was probably a bad estimate of the true measure in this case.

324 K: Alright, so there it is. Where is that up here? Come point to it for us, Diana.
325 K: Right here.

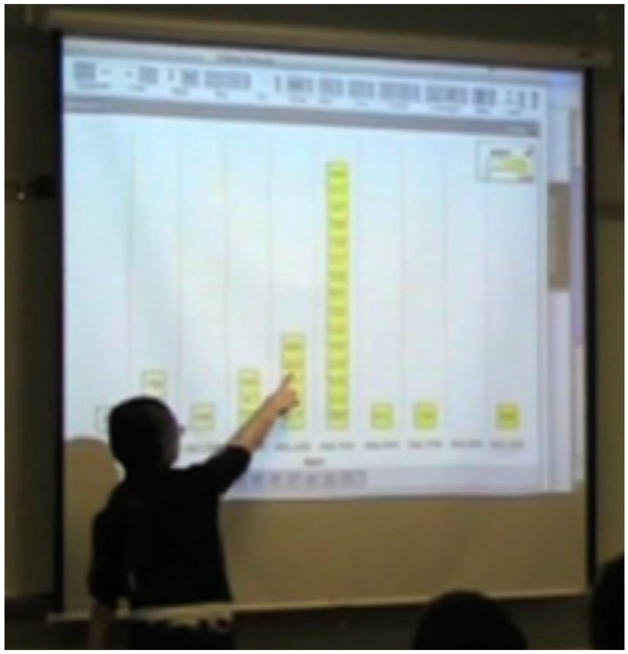


Figure 30. Student pointing to the mean of the data, which is outside the tallest bin, where the students believe the "best guess" of the true measure lies

326 K: Is that in our center clump, Juaquin?
327 S: (inaudible?
328 K: 463, that's what they got for the mean. Is that in the center clump?
329 K: It's not in the center clump. What do you think about that? Anybody.
330 K: Amin
331 S: Maybe it might be a lit - very close - 463, right?
332 K: Yep, 463 is right here.
333 S: It's above 450, so that's much closer than (inaudible)
334 K: So you think it's close to our center clump.
335 S: Yes
336 K: Kind of. Who's got a thought on that one? Is the mean going to give us the best guess this time?

Kristine's decision apparently was influenced by Emma's participation as a student during rehearsal. Kristine's press on the meaning of the mean during rehearsal ("*What does it mean to find the average?")* is evidence that she would have taken the same approach during her classroom discussion, had Emma not replayed the problematic student thinking that followed the same question in her own classroom. Kristine's classroom discussion was ultimately influenced through this trajectory:

Emma's classroom discussion → Kristine's rehearsal → Kristine's classroom discussion

Thus, not only were the settings of rehearsal and classroom mutually influenced, but they were even influenced across teachers, as Emma's practice influenced Kristine's practice.

These examples are somewhat rare in this study because the design feature that supported this phenomenon was unique to Cohort 1. However, this illustration serves as an existence proof of a mechanism that certainly warrants attention in future research. However, one of the design features that supported this kind of phenomenon was unique to Cohort 1. The teachers in this cohort rehearsed and taught subsequent discussions in a smaller amount of time. The rehearsals for Cohorts 2 and 3 took place during the summer before they had even begun instruction. Even with the subsequent professional development that took place during the school year, the amount of time between their rehearsal and classroom enactments was longer than for Cohort 1.

Discussion and Implications for Rehearsal Design

Role of Rehearsal in the Development of Routines

The data surrounding the transformation routine suggest that rehearsal did not play a large part in the establishment of neatly "packaged" routines, as in the case of Lampert & Graziani's (2009) study of Italian language teachers. In their study, teachers were provided with templates of specific sequences of moves that made up a routine. Teachers rehearsed these routines, line by line, following the recipe exactly. The routine in this study of language teachers still allowed for responsiveness to student thinking but in a more prescribed way. In my study, teachers of statistics were given examples of moves in each category of routine and some support during rehearsal about what type of moves might precede or follow others and how to strengthen the form or function of the sequences of moves they enacted, but largely the sequencing of moves was constructed by the teacher, and they were encouraged to use student thinking to guide their selection of "next moves." It makes intuitive sense that the routines that are the target of rehearsal are inherently different than those situated in classrooms, despite every effort to represent authentic forms of student thinking. And authentic student thinking is only one reason that rehearsal was markedly distinct from classroom contexts. However, this does not mean rehearsal was fruitless. It simply served a different role than I initially anticipated. First of all, it served as a "best-case scenario" that illustrated what teachers' classroom discussions might look like under the best circumstances: a small number of cooperative students who bring important ideas to the table. Although rehearsal provided some guidance around when and how to "try out" discourse moves and routines, the classroom was the place where their purposes began to take shape and where the bulk of innovation occurred.

Second, and in relation to the stabilities and changes in the form and function of routines, rehearsal initially served as a starting place to build the structure of what later became routine ways of interacting around content, although teachers' pre-existing routines may have strongly influenced the initial structure of the transformation routine. Rehearsals served as a place to try out new forms of questions that enhanced the rigor or shifted the roles of routines in the classroom, even though the patterned sequence of discourse moves within a routine tended to be resistant to change over time. Teachers often struggled with the execution of the routine in rehearsal. Over time, they began to strengthen the form of questions, especially from yes/no to more open-ended responses, and help shape new roles for students in math discussion. The function of moves in a routine also shifted in productive ways as students began to appropriate the transformation questions their teachers were asking. My analysis shows that adaptations were attached to particular functions, such that a single discourse move branched into a growing network of situation-specific adaptations that could be called upon in subsequent practice.

Rehearsal also offered authenticity, but not necessarily in the ways students and teachers would engage in routine classroom activity. The ways teachers engaged as students during rehearsal was inconsistent in many ways with the student thinking that emerged from classrooms. For instance, teachers role-playing students had already examined the collection of student displays and methods collaboratively and often knew the goals that rehearsing teachers had planned. In spite of some struggles with difficult content, they were cooperative participants who already knew the mathematical goals for the discussion. Accuracy (right/wrong/good/bad) orientations, revoicing moves, and triadic dialogue were all less likely to be seen in rehearsal than in classrooms. However, the type of thinking teachers did to respond to student thinking in rehearsal was authentic to classrooms because regardless of the authenticity of the student thinking portrayed, rehearsals provided the same opportunity to generate hypotheticals that would serve particular instructional functions as classrooms did. As shown in the discussion of the transformation both in rehearsal and classrooms, teachers did the same type of planning and improvising to generate hypotheticals in classrooms as they did (and were coached on) during rehearsal. Situating instructional moments in a problem context can support teachers in reasoning through the conceptual work that they must do in classrooms. In other words, the work of *professional noticing* (Jacobs, 2010) remained authentic even in simulated teaching environments for everyone sharing the teacher's role.

My findings also point to the role of rehearsal in reshaping teachers' beliefs and assumptions math instruction, although this was not a focus of the study. Teachers more broadly began to shift from discourse consistent with direct instruction to discourse characteristic of student-centered instruction, where students' ideas are used as a sense-making resource central to instruction.

Divergence-Convergence of Recontextualization

Ensor's 2001 recontextualization study found that a case teacher was able to reproduce tasks that were introduced in the teacher methods course but could not produce new tasks that were analogous to them in terms of the approach to teaching that they privileged. While I have not been able to show teachers producing new tasks from scratch, I have been able to illustrate a type of adaptive expertise that goes beyond simply reproducing tasks (such as "visualization" in the Ensor study). The convergence in the reproduction of discourse moves that different teachers made serves as some evidence that they understand the function of routines in relation to the instructional goals of this curriculum and can adapt elements of those routines in ways consistent with that the instructional goals. For example, both Rob and Marissa appropriated the transformation move in the same way into their Unit 3 discussions, but their recontextualization was a divergence from the form used during rehearsal, in which the teacher asked students to imagine a situation and communicate its effects on a display or statistic. While the four teachers in Cohort 3 used the transformation move many times, sometimes changing the data and sometimes changing the display or method in the scenario, only Rob and Marissa created hypotheticals that asked students to imagine the situation that would provide a given result. They had not done any collective work or planning together, so each of them individually adapted the transformation in this way because they understood how it could serve their [similar] instructional goals. While both of them diverged from the form of the rehearsal suggestion, and from their own previous appropriations, they converged upon a similar strategy in response to instructional demands and goals.

Implications for Rehearsal Design

One of the challenges our team faced as we designed rehearsals for each of the three cohorts was the practicality of allowing each teacher to rehearse individually. Because the rehearsal research studies enactments of individual teachers, coupled with research on deliberate practice, I preferred to maintain this design. While it was feasible for the first cohort of four teachers, it did not scale to larger groups with a more abbreviated schedule of PD. I initially considered the collective design a weaker form of deliberate practice, but I believe that this is quite possibly the most efficient model, as long as teachers are coping with instructional decisions and professional noticing in their collective teacher role-play. I suspect a more collective role is stronger than a "divide and conquer" model, where each teacher takes the lead for a predetermined phase of the discussion. In this model, teachers do not have to insert themselves into the teaching role until their turn, whereas the shared model holds teachers accountable for contributing teaching moves at any phase of the discussion.

Second, the longitudinal nature of analysis proved to be critical to identifying phenomena that would not have been seen in single rehearsal-classroom pairings. Learning to change practice in this way is sometimes gradual and does not always happen immediately. The instruction we asked of teachers was somewhat foreign to them. They were being asked to change many aspects of their instruction at once: participation structures, forms of questioning, rigor of mathematical content, and all while trying to make room for a lengthy curriculum sequence into an already-demanding curriculum map. Further, considering patterns of mutual constitution requires looking beyond single rehearsal-classroom pairings. Otherwise, tracing the coevolution between Emma and Kristine's efforts to build meaning around the mathematical mean would not have been possible. While the instructor could have accomplished the same work the Emma did in representing problematic student thinking during rehearsal for teachers to tackle, I suspect the validity of a peer-teacher replaying a classroom episode has a stronger influence than an instructor portraying the same kind of thinking. I would encourage longitudinal study in further studies of rehearsal-classroom relations to ensure that these kinds of changes can be documented.

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CHAPTER V

CONCLUSION

This dissertation has made several novel contributions to the empirical literature surrounding rehearsals. Most rehearsal studies to date are conducted with preservice teacher candidates who are considering the content of early number arithmetic. In contrast, I illustrated the unique challenges that practicing teachers who participated in professional development around data modeling. The domain of data modeling was conceptually challenging for teachers, as teachers did misappropriate the intent of discourse moves and routines on occasion. However, they faithfully aligned, adapted, and innovated the form and function of discourse more often than not, and these changes proved productive to navigating the challenges of classroom implementation that were not always seen during rehearsal.

The existing corpus of literature illuminates the various learning opportunities and the role of teacher educators in the rehearsal space. However, because the focus of these studies is on the rehearsal space, relations between discourse in rehearsal and classroom spaces has been relatively unknown. This dissertation closely examined discourse in both rehearsal and classroom settings to better characterize the nature and mechanisms of change across these settings and the role that rehearsal plays. I found that not only were both rehearsal and classroom spaces valuable in their own right to changes in teacher practice, but the relation of rehearsal and classroom enactments also initiated change. For example, teachers appropriated moves as they were suggested during rehearsal but then they had to adapt the move to yield the same type of response they elicited successfully during rehearsal without the adaptation. Both settings in these cases were necessary but not sufficient to the resulting form and function of discourse.

Another contribution of these analyses is methodological in nature. I developed a coding and analytical scheme that attempted to measure what I consider a type of adaptive expertise that developed in teachers over time across two distinct contexts of practice. This kind of analysis was only possible through a longitudinal approach, and my results suggest that because many phenomena were only visible across several enactments, a longitudinal approach is not only warranted but might be extended even further in time in future analyses.

Together, these papers provide explanatory power to the mechanisms of learning between rehearsal and classroom math discussions. They illustrate rehearsal's important role in changes of practice for inservice teachers, and the complexity of generating and sustaining classroom conversation. They set the stage for future research that can examine and quantify specific mechanisms of change in more detail.

APPENDIX A

DISCOURSE MOVES FOR UNITS 1, 2, & 3

Discourse Moves for Mathematics discussions:
Unit 1: Display Review
Erin Pfaff
Revised: 4/3/13

Eliciting a strategy/hypothesis:

Choose a strategy, display it, and ask someone else to state something they think the first display shows (1st) or hides (later). Alternate between questions in Box 1 and Box 2 to build relations between what can be seen and the design decisions the authors used to make those features visible:

Box 1: What can be seen about the data? (highest/lowest, repeated measures, gaps, outliers, etc.)		*Box 2: What did the authors do to make those features visible? (Ordered, binned, scaled, etc.)*
"What do you think this display helps us see about the measurements?" "What does this display tell us about the class's measurements?" "What is something that is hard to notice about the measurements in this display?" "What is something this display hides that maybe is easier to see in other displays?" "What would be hard to see in this display?"	⇔	"What did the authors do to make that visible?" "How can we see that in this display?" "What about this display makes that easy to see?" "Why is that hidden in this display?"

"Yes-anding"/making it public:

1. *Restate that student's hypothesis or have someone else restate the hypothesis to make the hypothesis public (yes-anding/making it public)*

"So you think it shows that _______________ ?"

"____________, can you restate that in your own words? What does _________ think this display shows about the measurements?"

" __________ claims that this display shows _______________."

2. Ask any extension/clarification questions if necessary to help others understand. Invite students to use the displays to make a point more clear.

"What do you mean when you say _____________ ?"

"Can you come point to where you see that on the display?"

"Where on the display do you see an example of that?"

"I'm not sure I understand what you mean by _____________."

Eliciting a response to the hypothesis:

Ask the authors if that is what they had intended to show, and open it up to other students for opinions

"Is that something you were trying to show in your display?"

"Do you agree with ___________ that this display makes it easy to see _______________?"

"What do you think about __________'s claim that this display helps us see ______________?"

"Can someone else help us understand how this display makes it easy to see _____________?"

Connective statements/questions:

Ask questions or make comments to promote thinking about tradeoffs of design choices and what is made visible. Avoid positioning one display as "better" than another. Each display shows something about the data.

"What can you see with this display that we haven't been able to see so far?"

"What do these displays both show?"

"What does this display show that that one hides?"

"What do we know about the measurements that we didn't know just by looking at this display?"

"Which of these displays hides ____________?"

Translation: Moves intended to elicit noticings about how a feature of the data is made visible differently across two displays

"So we see that this display shows ______________. Where do we see that in that display?"

"Which display makes it easiest to see ___________?"

"How did the authors of this display show the ________ we noticed in the first display?"

Transformation: Moves asking students to conjecture about how hypothetical data sets or data points would alter interpretation of the display.

> "How would the shape look different if the authors had binned by _________ instead of by __________?"
>
> "How would showing the gaps make the shape different?"
>
> "What do you think the shape would look like if we asked another class to take measurements?"

Pulling it together:
Make a brief summary statement with a "big idea" that students have come to through discussion. Think of it as a restatement, but you may want to add something extra to help make this idea salient.

> "In this display, we can see the outliers more clearly than in this display because of the way he grouped numbers together. We call that 'binning.'"
>
> "So this display makes it harder to notice the individual values, whereas this one makes each value clear."
>
> "I think the point we're agreeing on is that if we worked really hard, we could see that in each graph. But the question is: Do we want to have to work that hard? Or do we want the displays to make it easy for us to see?"

Discourse Moves for Mathematics Discussions
Unit 2: Invented Statistics of Center (Best Guess)
Erin Pfaff

Eliciting the rationale for the method:
Ask another student to try to describe the method that a student used to find the best guess. For each method, alternate between questions in Box 1 and Box 2 to build relations between the procedure itself and the characteristics of the data that each procedure uses to find the best guess.

Box 1: Summarizing the method

Box 2: Identifying which features of the data the method uses

"________, how do you think that ________________ tried to find the best guess?"

"Based on __________'s information, how did ____________ find the best guess?"

"Following this method, how do we get the 'best guess'?"

"Who can share with us what this group was thinking when they invented this method?"

"What do you think the authors of this method care about in the data?"

"What do these authors think is important about the data?"

"What part of the data does this method use to find best guess?"

"What about the collection of measurements is important for this method?"

Building Collective Understanding:
1. Restate the method and its basis to make the method shared and public.

- "_______________ said that if you add up all of the numbers and divide by how many there are, you will find the best guess."
- "________________, tell us what _____________ thinks the method for finding the best guess should be in your own words."
- "Can anyone explain what _____________ just said about their method for finding central measurement? "
- "So this method thinks repeated measures are really important."
- "Do you mean that this method depends on how many numbers there are rather than the value of the numbers themselves?"

2. Ask any extension/clarification questions if necessary to help others understand.

- "When you say, '*Divide by the number of guesses,'* what do you mean by *guesses*?"
- "You said to *put the numbers in order*. What kind of *order* do you mean?"
- "What do you mean when you say ________________?"
- "I'm not sure I understand what you mean by _____________________?"
- "How could you give more specific directions so that anyone following your method would end up with the same best guess?"
- "Some of us knew to find the mean as the best guess. Some people call that a fair share. What might they have in mind?"

Eliciting a response to the hypothesis:
Ask the author if that is what was intended, and/or open it up to other students.

- "___________________, did you intend to find the *mean, or average,* of the numbers when you *added them up and divided by 30*?"
- "___________________, do you agree that you thought *repeated values* were the most important part of the data?"
- "Can you explain your method to the class? Did your peers understand your method as you intended?"
- "Is that the "best guess" you were trying to show?"
- "Is that what you thought was important about the data?"
- "Do you agree with ____________ that this method shows us the best guess by ___________________?"

Connective statements/questions:
Ask questions or make comments to get students to think about similarities/differences between methods and which measures would be more informative with particular data qualities.

- "How does _____________'s method, which *doesn't included the highest and lowest numbers*, give a different best guess than __________'s method, in which *the mean was found*? Which method do you think gives us a more accurate estimate of the actual measurement?"
- "Let's compare Group 1 and Group 2's methods. What things did they do that were similar? "
- "How is this method like this other one?"
- "How were their methods different?"
- "Which method is the most helpful for________________________________? "
- "Where do you see _____________ in the other method?"
- "Which method might get us closer to the best guess when *there's an outlier*?"
- "Which of these methods focus on the *center clump*?"

Transformation:

Ask students to apply their reasoning to imagined data points or data sets. The purpose of these questions is to create situations that will ***very*** *clearly show why certain reasoning is problematic when it is generalized to other data sets. Therefore, this requires quick thinking about how the reasoning is problematic* ***and*** *what kind of situation would highlight that problem clearly. Both general suggestions as well as specific types of common problems are addressed below:*

General examples:

- "Will your method work with other data sets? Could anyone use this method by following your description?"
- "Which of these methods would work with other data too? Why do you think so?"
- "What would happen if we used this method on a bigger/smaller set of data?"
- "What if we use this method on a set of data that looks like this ("center" clump is way off to one side or the other)?"
- "What if this was our data (give 5 new numbers in a chosen order) and we used this method to find the best guess? Does this value really show what we think the best guess should be?"

Problem 1: Students favor the mean as the best method.

- "What would happen if one of the measurements is out here (outlier)?"
- "What if my data set was 2, 20, 22, 20? What would the median and mean be? Which one seems like the best guess?"

Problem 2: Students favor the mode as the best method.

- "Some of us found the best guess by finding the measurement that was repeated most often. This is called the mode."
- "Is the most common measurement always the best guess? What if the mode was (outside the center clump)?"
- "What if our data set had two modes?"
- "What if there were no repeated values? What would you think would be a good best guess?"

Pulling it together:
Make a brief summary statement with a "big idea" that students have come to through discussion. Think of it as a restatement, but you may want to add something extra to help make this idea salient. Include both points of consensus as well as issues to remain "on the table." Record on anchor chart.

- "_________'s group took out the highest few numbers and the lowest few numbers before finding the mean, **which helped to show** what *most* students measured close to but might be hard to repeat because they didn't give a specific rule for how many numbers to take out."

- "In this method, our best guess is affected by outliers in a very big way. This method might be best to use on data that ___________".
- "In this method, our best guess depends heavily on __________________. We might use it most often on data that ________________".
- "I think the point we're agreeing on here is that we want to use this method on data like this, because ______________________".

Discourse Moves for Mathematical Discussions
Unit 3: Invented Measures of Precision
Erin Pfaff

Eliciting a method:

Ask another student to try to describe the method that a student used to find the measure of precision. For each method, alternate between questions in Box 1 and Box 2 to build relations between the procedure itself and the characteristics of the data that each procedure uses to find the measure of precision.

Box 1: Summarizing the method

- "________, how do you think that _______________ tried to find the measure of precision?"
- "Based on __________'s information, what method did ____________ use to find the best guess?"
- "Following this method, how do we get the measure of precision?"
- "What is the main idea behind this method?"

Box 2: Identifying which features of the data the method uses

- "What part of the data do these authors think is important?"
- "What do these authors think is important about the data?"
- "What part of the data does this method care about? What part does it ignore?"
- "What about the collection of measurements is important for this method?"

Building collective understanding:

1. Restate that student's hypothesis or have someone else restate the hypothesis to make the hypothesis public.

- "_______________ found the measure of precision by adding up all of the differences between each value and the mean."
- "________________, tell us what _____________ thinks the method for finding the best guess should be in your own words."
- "Can anyone explain what _____________ just said about what these authors think is important about the data?"
- "So you think this method that subtracts the highest and lowest values only depends on the most extreme measurements and ignores the rest?"
- "_________ claims that this method of precision really values that middle clump of measurements."

2. Ask any extension/clarification questions if necessary to help others understand.

- "When you say, "*where the most people had their numbers,"* What do you mean by *most?"*
- "You said to count the "*number of same measurements*." How did you find that number?"

- "What do you mean when you say ________________?"
- "I'm not sure I understand what you mean by _____________________?"
- "How could you give more specific directions so that anyone following your method would end up with the same measure?"

Eliciting a response to the hypothesis:

Ask the authors if that is what they had intended to show, and open it up to other students for opinions.

- "___________________, did you intend to use only the repeated values when finding your measure? Why did you make that choice?"
- "___________________, do you agree with what ______________ said your method cares about?"
- "Can you explain your method to the class? Did your peers understand your method as you intended?"
- "Did this group find the same measure you did when they followed your procedure?"
- "Is that how you were trying to show your measure?"
- "Do you agree with ____________ that this method really only uses two data points – the highest and the lowest?"

Connective statements/questions:

Ask questions or make comments to promote thinking about tradeoffs of design choices and which measures would be more informative with particular data qualities. Avoid positioning one method as "better" than another. Different methods might more or less accurately characterize different types of data sets.

- "How does _____________'s method, which *only includes repeated values,* give a different best guess than __________'s method, in which *all the values were used?* Which method do you think gives us a more reliable measure of precision?"
- "Let's compare Group 1 and Group 2's methods. What things did they do that were similar?"
- "How is this method like this other one?"
- "How were their methods different?"
- "Which method is the most helpful for_______________________________?"
- "Where do you see _____________ in the other method?"
- "Which method is easiest/hardest to understand? Why?"
- "Which method would almost always be a good measure of precision, no matter what and how we measured? Why?"
- "Why do you think mathematicians use different methods for finding the measure of precision?"

Transformation:

*Ask students to apply their reasoning to imagined data points or data sets. The purpose of these questions is to create situations that will **very** clearly show why certain reasoning is problematic when it is generalized to other data sets. Therefore, this requires thinking about how the reasoning is problematic **and** what kind of situation would highlight that problem clearly. Both general suggestions as well as specific types of common problems are addressed below:*

General examples:

- "Will your method work with other data sets? Could anyone use this method by following your description?"
- "What would happen if one of the measurements is out here (outlier)?"
- "Which of these methods would work with other data too? Why do you think so?"
- "What would happen if we used this method on a bigger/smaller set of data?"
- "What if we use this method on a set of data that looks like this ("center" clump is way off to one side or the other)?"

- "What if this was our data (give 5 new numbers in a chosen order) and we used this method to find the measure of precision? Does this value really show what we think that measure should be?"

Problem 1: Students sum differences without finding an average of those differences.

- "What would happen if we used this method on a data set of 5 values that were really spread out?"
- "What would this method tell us about a set of 1,000 values that were tightly clumped like this?" (Illustrate with Tinkerplots or by drawing)
- "What could we do to make the method fair even if the number of measurements is not the same?"

Problem 2: Students propose the range.

- "What would happen if we used this method on a data set that was tightly clumped in the center but that had two poor measurements? Would the range be a good measure of precision/consistency of the measurements?"

Problem 3: Students propose an average deviation (perhaps to make comparisons between unequal sample sizes).

- "What might happen to the average deviation if the data had a few extreme scores, while most of the data was in the center clump?"
- Would the IQR be as vulnerable to the same extreme scores?

Pulling it together:

Teacher makes a brief summary highlighting a "big idea" that students have developed through discussion about the methods. The teacher may want to add something extra to help make this idea salient. Include both points of consensus as well as issues to remain "on the table." Record on an anchor chart.

- "_________'s group considered only repeated values in their measure of precision. Their method helps us get rid of outliers that probably don't represent the true measure."
- "In this method, our best guess is affected by outliers in a very big way. This method might be best to use on data that ___________."
- "What I think I hear people saying is that our best guess depends heavily on __________________. We might use it most often on data that ________________."
- "I think the point we're agreeing on here is that we want to use this method on data like this, because ______________________."

APPENDIX B

TEACHER INTERVIEW PROTOCOL

Teacher interview protocol (choose questions as appropriate)

General:

1. What were your main goals for this lesson?
2. What student strategies were you expecting to see in this lesson? Did you expect particular strategies from particular students or was it more trying to anticipate all possible strategies the class might use?
3. Can you tell me about what went into your decisions to choose these particular pieces of student work for your discussion? (only ones that had an answer)
    a. Probe: Is there anything else?
    b. Probe: Were you thinking about any classroom management factors? Like trying to stay within a time limit? Resources you have or don't have?
4. Can you identify a (AND/OR I identified this) point that was troublesome for you? Tell me about the decisions you made in the moment to work through that point toward your goal.
5. Can you identify a (AND/OR I identified this) point that you feel went well in working toward your goal? Tell me about what factors you think contributed to accomplishing what you had hoped.
6. Anything surprising?
7. Tell me about the ways you think your decisions (and tools you used) supported students in moving toward your goals for the lesson? Are there any decisions you made that you feel worked against your goals in ways you may not have anticipated?
8. Were there any turning points in the discussion? How did they come about?
9. If you were to teach this lesson again knowing what you know now, how would you do things differently (if anything)?
10. You used a few different talk moves than last time. How did you choose talk moves? What went in to your decision?
11. Is there anything you did this time that was a result of something you learned in teaching Unit 1 discussion?
12. What do you feel is becoming routine for you?

Effects of prep:

13. Some tools we gave you in class were lesson plan, construct map, and discourse moves sheet, and website. Did you find any of them particularly helpful for teaching the lesson? Is there anything about these that would have made them more helpful?
14. Question about constructs – did they help in how you ____________ ? Was the form with video examples any more helpful?
15. In what ways did rehearsing ahead of time help?
16. What did you do differently as a result of participating in rehearsal?
17. What did you do differently as a result of participating in model discussion?

Collaboration:

18. Are there any decisions you made that you weren't sure about that you'd like to discuss with the other teachers in class?
19. Which student work would you want to bring back to the class to discuss?

Enactment-specific:

1. I see that you paused for a while when this student asked this question. Can you tell me what you were thinking about during those moments?
2. You made the decision to take an aside from the lesson to (ex: review multiplication of fractions). Can you tell me about why you decided to make that aside? Did it accomplish what you had hoped in relation to your goals for the lesson?

3. Can you tell me about the reasons you chose to order the student strategies for discussion in the way that you did?
4. Can you tell me what you were hoping that question to accomplish and how you thought about the particular way you worded it in light of that goal?
5. You enacted this routine a little differently than you did when you practiced in class. Can you tell me about what you did differently and why you made the decision to change things up a little bit?
6. You enacted this routine very similarly to how you did in class. Tell me about how you think that helped you achieve your goals for the lesson or how it did not.
7. Did you find yourself having to think through this particular routine or do you feel it was automatic?
8. Did your students respond as you anticipated when you asked that question?

APPENDIX C

ROLE OF STUDENT INVENTION CONSTRUCT MAP

Level	Role	Classroom Interaction	Sub-Levels
5	Student invented methods are seen as a resource to communicate different mathematical strategies in order to synthesize specific mathematical ideas into systems of meaning.	Students invent a variety of methods. Teacher selects and juxtaposes contrasting examples and leads a whole class discussion that gives students the opportunity to think about the big mathematical ideas, but also works to establish the relationships among the different ideas in the invented methods.	
4	Student invented methods are seen as a resource to communicate different mathematical strategies in order to promote specific mathematical ideas.	Students invent a variety of methods. The teacher selects and juxtaposes contrasting examples with the intent of leading a whole class discussion to support student thinking, but does not establish the relationships among ideas. For example, big mathematical ideas might come out of the conversation, but they are not coordinated with each other.	4C: All intended mathematical ideas come out of the discussion.
			4B: More than one, but not all mathematical ideas come out of the discussions
			4A: One of the intended mathematical ideas comes out of the discussion
3	Student invented methods are seen as an instructional resource to promote a right/wrong orientation towards mathematical ideas.	Teacher gives students the opportunity to invent. The focus is on “right” or “wrong” methods with convention as the reference. The teacher might also guide students in an “invention” task so that the students produce primarily canonical methods. The class may discuss the invented	

		methods, or the teacher may tell the class the important ideas to attend to.	
2	Student invented method are seen as an instructional resource to primarily increase engagement.	Student-invented methods generated and shared using a "turn taking" strategy. Here the methods are invented and presented, but the mathematical components of them are not highlighted or discussed. The intent of inventing and sharing is to "keep students engaged" without reference to the learning opportunities found in the task.	
1	Student invented methods are not valued as an instructional resource.	Teacher does not give students the opportunity to invent. The teacher might lecture, lead a discussion, or give tasks to work on, but student-invented methods are not present during instruction.	
NL		Not Present	

APPENDIX D

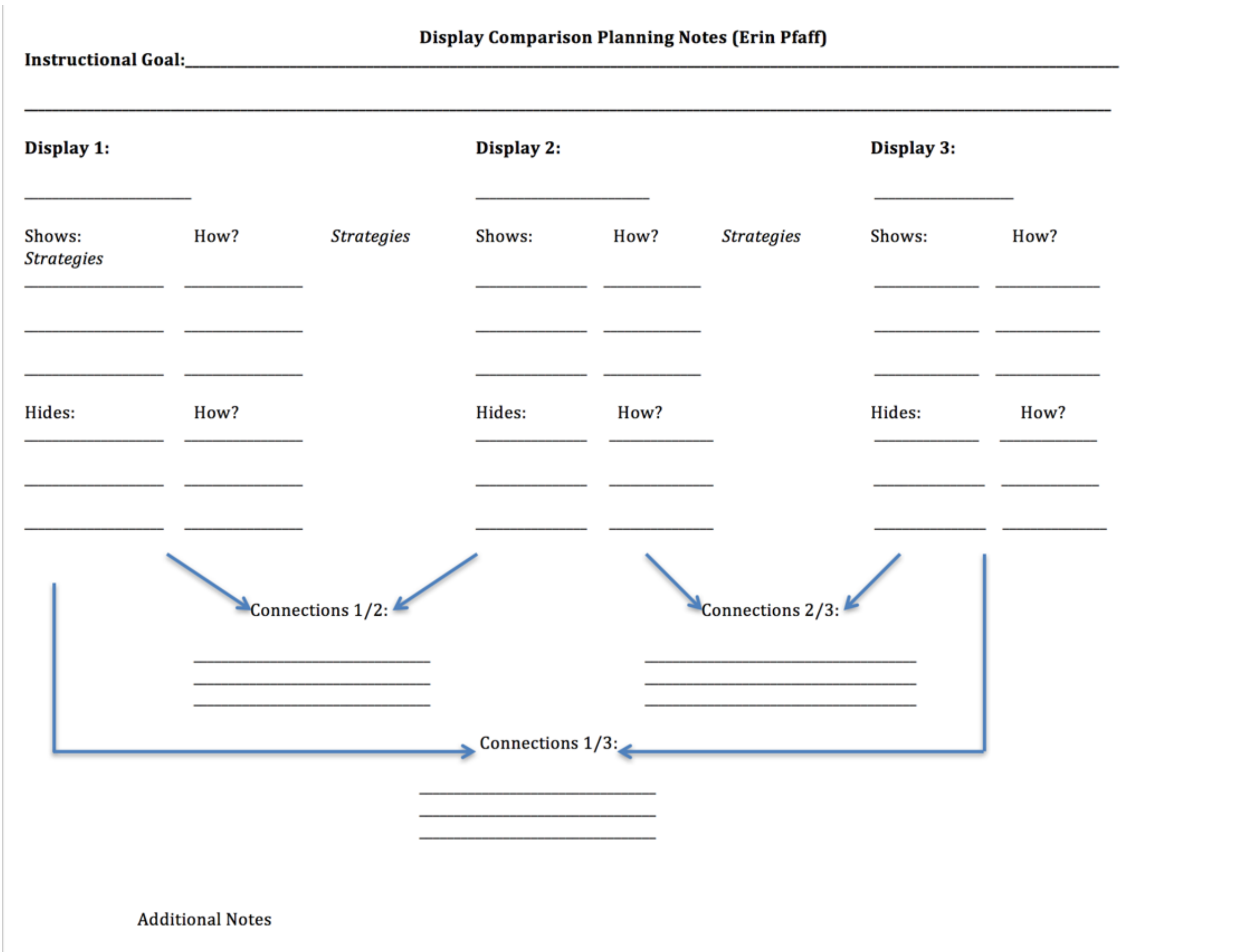
COMPLETE CODING SCHEME

Participant	Template routine	Name of move	Description
Teacher	Eliciting Noticings	Teacher noticing	Teacher provides a personal observation
		Soliciting open noticings	Asking students to provide an observation or summary
		Eliciting specific noticing/idea	Asking students to provide an observation with a specific response in mind
		Soliciting open hypotheses	Asking students to provide a hypothesis or conjecture for a claim
		Eliciting a specific hypothesis/conjecture	Asking students to hypothesize with a specific response in mind
	Building Collective Understanding	Eliciting a clarification	Asking a student to clarify their statement
		Clarification statement	Providing clarification in response to student request for clarification
		Eliciting a restatement	Asking for a student to repeat something in their own words
		Revoicing	Repeating or re-phrasing a student contribution
		Eliciting an example	Asking student to provide an example of a statement or claim
		Eliciting an extension	Pressing on a student contribution beyond what has already been stated
		Eliciting a definition	Asking definitional questions
		Eliciting evidence	Asking students to show evidence for a claim
	Response to hypothesis	Eliciting a response from author	Asking author to confirm/disconfirm other student hypotheses
		Eliciting a judgment about an idea or method	Asking non-author students to respond to another student’s claim
		Eliciting a revision	Asking students to suggest revisions to a method
	Making Connections	Eliciting a connection	Asking students to make a contribution that requires relating one method to another
		Eliciting a translation	Asking students to identify where a specific feature is visible in another method
		Posing a transformation	Asking a hypothetical question
	Pulling it all together	Pulling it all together	Providing a summary statement of what has been discussed.
	Other/non-specific	Partner Talk	Asking students to discuss a question with smaller groups
		Teacher Management	Any statement regarding organization of participants to activity

		Teacher Evaluation	Providing a judgment about accuracy of student contribution
		Teacher Other	Incomplete utterances/non-specified contributions
Student	Eliciting noticings	Student Noticing	Providing an observation or retelling
		Student Hypothesis	Providing a hypothesis about a method
	Building Collective Understanding	Clarification	Providing a clarification about one's idea
		Restatement	Repeating another student's contribution
		Example	Providing an example of a statement
		Extension	Answering further questions about a statement
		Evidence	Providing evidence for a claim
		Definition	Defining or identifying a term
		Request for clarification	Asking others to clarify a statement
	Response to Hypothesis	Author response	Responding (as author) to statements about method made by others
		Judgment	Giving an opinion about a method or statement
		Revision to method	Suggesting a way to revise a method
	Making Connections	Connection	Providing a statement that relates one method to another
		Translation	Shows how a feature of one method is visible in another
	Other/non-specific	Student Management	Any contribution regarding organization of participants of activity
		Student Other	Incomplete utterances/non-specified contribution

APPENDIX E

PLANNING SHEETS FOR UNITS 1-3



Display Comparison Planning Notes (Erin Pfaff)

Instructional Goal:____

____

Display 1: ____________ **Display 2:** ____________ **Display 3:** ____________

Shows: How? *Strategies* Shows: How? *Strategies* Shows: How?
Strategies

Hides: How? Hides: How? Hides: How?

Connections 1/2:

Connections 2/3:

Connections 1/3:

Additional Notes

Measures of Center Planning Notes:

Instructional Goal: ____

Strategy 1:		**Strategy 2:**		**Strategy 3:**	
____________		____________		____________	
Description: ____________		Description: ____________		Description: ____________	
Key points:	Strategies/questions for addressing each:	Key points:	Strategies/questions for addressing each:	Key points:	Strategies/questions for addressing each:
____________	____________	____________	____________	____________	____________
____________	____________	____________	____________	____________	____________
____________	____________	____________	____________	____________	____________
____________	____________	____________	____________	____________	____________

Connections 1/2:

____
____
____

Connections 2/3:

____
____
____

Connections 1/3:

____
____
____

Measures of precision Planning Notes:

Instructional Goal: ____________________

Strategy 1:		**Strategy 2:**		**Strategy 3:**	
Description: ____________		Description: ____________		Description: ____________	
Key points:	Strategies/questions for addressing each:	Key points:	Strategies/questions for addressing each:	Key points:	Strategies/questions for addressing each:

Connections 1/2:

Connections 2/3:

Connections 1/3:

Additional Notes: